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Kurzfassung

Ziel der vorliegenden Diplomarbeit ist es, den aktuellen Markt für Langlebigkeitsrisikotransferierung und verschiedene Risikotransferlösungen in Bezug auf Mortalität und Langlebigkeit zu erörtern.
Die in dieser Arbeit erörterten Risikotransferlösungen (Buy-in, Buy-out, an die Langlebigkeit oder Sterblichkeit gebundene Derivate, wie zum Beispiel Anleihen, Termingeschäfte oder Swaps, und Rückversicherungs-Sidecars) hängen von der Entwicklung der angegebenen Teilbevölkerung in Bezug auf Sterblichkeit oder Überleben ab. Daher werden einige extrapolative Sterblichkeits- und Langlebigkeitsmodelle (MIM-2021, Lee-Carter-Modell, CBD-Modell und das heat wave Modell) vorgestellt, die zur Messung und Vorhersage der Entwicklung der Sterblichkeit und Lebenserwartung einer bestimmten Teilbevölkerung verwendet werden können. Der Markt für Langlebigkeitsrisikotransferierung wird anhand der auf *artemis* veröffentlichten Abschlüsse analysiert: `https://www.artemis.bm/longevity-swaps-and-longevity-risk-transfers/` und die beschriebenen Risikotransferlösungen werden mit Beispielen bereits durchgeführter Trades ergänzt.
Diese Arbeit stützt sich weitgehend auf die Publikation *Still living with mortality: the longevity risk transfer market after one decade* von Blake, Cairns, Dowd und Kessler. Sie erweitert die in dieser Publikation behandelten Themen um aktuelle Literatur und Forschungsergebnisse.

Abstract

The aim of this thesis is to examine and discuss the current state of the longevity risk transfer market and to describe different solutions for transferring mortality or longevity risk.
The risk transfer solutions discussed in this thesis (buy-in, buy-out, longevity or mortality linked derivatives such as bonds, forwards and swaps, and reinsurance sidecar) depend on the development of the specified subpopulation in terms of mortality or survival. Therefore, some extrapolative mortality and longevity models (MIM-2021, Lee-Carter model, CBD-model and the heat wave model) are introduced, which can be used to measure and predict the development of the mortality and life expectancy of a specified subpopulation. The longevity risk transfer market is analyzed using financial statements published on *artemis*: `https://www.artemis.bm/longevity-swaps-and-longevity-risk-transfers/`, and the risk transfer solutions discussed are supplemented with examples of trades already carried out.
This thesis is largely based on the paper *Still living with mortality: the longevity risk transfer market after one decade* by Blake, Cairns, Dowd and Kessler. It expands on the topics covered in that paper to include recent literature and research.

Danksagung

Zuerst möchte ich Prof. Thorsten Rheinländer für die Unterstützung und Betreuung beim Verfassen meiner Diplomarbeit danken.

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Großer Dank gebührt meiner Familie Karin, Christoph und Georg sowie meinen Großeltern, welche mir mein Studium erst ermöglicht haben.

Eidesstattliche Erklärung

Ich erkläre an Eides statt, dass ich die vorliegende Diplomarbeit selbstständig und ohne fremde Hilfe verfasst, andere als die angegebenen Quellen und Hilfsmittel nicht benutzt bzw. die wörtlich oder sinngemäß entnommenen Stellen als solche kenntlich gemacht habe.

Wien, am August 9, 2022

Anna Schröckenfuchs

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1 Introduction

It is commonly known that risk management is necessary for companies exposed to mortality or longevity risk. The transfer of mortality and longevity risk is only one way to manage these risks. In Section 2 these risks are defined, and a brief overview of risk management is given in Section 1.1.

Before transferring any risk, companies must measure the extent of the risk and decide which scenarios to cover. Companies can use *mortality and longevity models* to measure their mortality and longevity risk, respectively. Some examples of mortality and longevity models are described in Section 4.

Since pensions and annuities must be paid until the end of life, the life expectancy of the cohort influences the upcoming size of liabilities the insurance company or pension plan has to cover. Therefore, these products are subject to longevity risk.[1] In order to estimate life expectancy, mortality improvement assumptions have to be made, which are subject to the subjectivity of the model user. Hence, the life expectancy is exposed to the risk of getting those assumptions wrong. In the past, those assumptions of mortality improvement have often been too low. Hence, there exists a substantial demand for managing longevity risk. For life insurances, especially term assurances, the current mortality rates determine that part of the cohort that dies within the next year, which influences the upcoming size of liabilities the insurance company has to cover. These products are subject to mortality risk. The estimation of mortality rates is more reliable and objective, but there is the risk of a rare or extreme event occurring, like the COVID-19 pandemic, and causing an unexpected jump in mortality rates.[2]

In Section 5 the *longevity transfer market* is described. Currently, most risk transfers are still traded over-the-counter (OTC) and therefore the market is dominated by major reinsurance firms. In [20, Section 13] it is stated that although the reinsurance capacity for the global longevity and annuity sector is currently sufficient, when the demand will exceed the supply, capital market solutions will be needed.

To open up the longevity market to new entrants, mortality and longevity indices based transactions can be relevant, as such provide lower legal and administrative costs and a quicker execution. Some historic and existing indices are described in Section 3.

Finally, the Sections 6 and 7 present some possibilities for transferring mortality and longevity risk, including some examples of realized trades.

[1] `https://www.actuaries.org.uk/news-and-insights/news/what-difference-between-longevity-and-mortality`

[2] Discussion by Matthew Edwards: `https://www.actuaries.org.uk/news-and-insights/news/what-difference-between-longevity-and-mortality`

1.1 Risk management

In their third longevity webinar, Club Vita gave a good overview of how to manage longevity risk for pension plans.[3] The fundamental concept of risk management applies to all companies exposed to risk. The goal of risk management is to bring the companies *risk profile* and *risk appetite* in an equilibrium.

Definition 1.1 (*risk profile*, *risk appetite*, *risk tolerance* and *risk capacity*)**.**

- The *risk profile* corresponds to the total risk to which the company is exposed.
- The *risk appetite* is determined by the *risk tolerance* and the *risk capacity*.
- The *risk tolerance* is the risk a company is willing to accept, and the *risk capacity* is the risk a company is able to accept.

Therefore, the goal is to change the risk profile using risk management tools to fit the companies risk appetite.

Risk management control cycle

A popular tool for managing risk is the *risk management control cycle* consisting of three (often more) repetitive steps: *risk measurement*, *risk management* and *monitoring*.

1. The first step is *risk measurement*, where companies can use mortality and longevity models (described in Section 4) to measure their mortality and longevity risk. This step was covered in the first two longevity webinars.[3]
2. An example of a *risk management* process is the 4 *T's of hazard response*: tolerate, treat, transfer and terminate (see [16, Chapter 15]), which can illustrate how each individual risk should be managed.
    - A risk is *tolerated*, if no action is taken. This response is usually used for risks that have low impact and are unlikely to occur. In this case risk monitoring is still necessary as the impact and the likelihood of this risk might change some day.

[3] The subject of the webinar longevity 101 was baseline longevity:
`https://www.clubvita.us/events/longevity-101-baseline`.
The subject of longevity 102 was longevity improvement and trends:
`https://www.clubvita.us/events/longevity-102-improvements-trends`.
The subject of longevity 103 was longevity risk management:
`https://www.clubvita.us/events/longevity-103-longevity-risk-management`

- Risks that likely occur, but have low impact, are mainly addressed by *treating* the risk through taking action to constrain the risk to an acceptable level.

  Example 1.2. Some options for a pension plan on the liability side are changing the structure of the plan by closing it to new entrants or increasing the retirement age. Further, the pension plan can adjust the size of future pension payments by PIE (Pension Increase Exchange), ETV (Enhanced Transfer Value), automatic indexation linked to life expectancy or by offering a lump-sum payment.[3] To treat the longevity risk on the asset side a pension plan may consider investing in longevity linked assets.

- Risks with high impact and low probability of occurrence are mainly *transferred*. This can also reduce financial and asset risks simultaneously. This response to manage risk is the main focus of this work, and the Sections 6 and 7 present some possibilities for transferring mortality and longevity risk.
- Risks that likely occur and have a too high impact might be *terminated*. This could be done by a buy-out, where the whole risk is transferred to another institution.

3. Finally, the last step in the risk management control cycle is *monitoring*, which includes the re-measurement of risks and a review of the risk management decisions. This step is important because by this the company is able to feed back into the control cycle and react to consistent changes through adjustments to ensure that the current risk profile and the risk appetite are in an equilibrium.

   Example 1.3. For pension plans, some important factors to monitor include the ration of actual to expected deaths, the longevity characteristics and demographic trends. Further, pension plans should reflect the mortality experience to adjust given assumptions and regularly update the calibration period.

Notation

Symbol	Meaning
x	The age of an individual or the age of the group considered.
t	The considered point in time, mostly given in years.

Life Table

Symbol	Meaning
q_x	The probability that a person aged exactly x dies before aging $x+1$. In other words, it is the probability that a person aged x dies within one year. (See also Item 1 in Section 2.2.)
p_x	The probability that a person aged exactly x will survive to age $x+1$. In other words, it is the probability that a person aged x survives one year.
${}_tq_x$	The probability that a person aged exactly x dies before aging $x+t$.
${}_tp_x$	The probability that a person aged exactly x will survive to age $x+t$.
D_x	The number of people that died over the last year, aged x last birthday at the beginning of the year.
L_x	The number of people alive at the beginning of the year, aged x last birthday at the beginning of the year.
E_x	The number of people exposed to the risk of dying over the year, aged x last birthday at the beginning of the year.
m_x	The central rate of mortality. (Defined in Section 2.2, Item 2.)
μ_x	The mortality intensity. (Defined in Section 2.2, Item 3.)
e_x	The life expectancy. (Defined in Section 2.2, Item 4.)

Models

Symbol	Meaning
$\mathrm{I}_{\mathrm{age}}$	contains all ages x used for the calculation.
$\mathrm{r}_{x,t}$	The mortality improvement rate for age x in year t.
s_x	The initial slope; precisely $\mathrm{s}_{x;d}$ is the diagonal initial slope for age x and $\mathrm{s}_{x;h}$ is the horizontal initial slope for age x.
$\hat{\mathrm{r}}_{x,t}$	The estimated mortality improvement rates for age x in year t.
$\hat{q}_{x,t}$	The estimated mortality rates for age x in year t.
$\hat{\mathrm{e}}_{x,t}$	The estimated period or cohort life expectancy for age x in year t.
a_x, b_x, c_x	are age specific parameters.
$\overline{x}$	is the average age of the used age range $\mathrm{I}_{\mathrm{age}}$.

σ^2	is the variance of the used age range $\mathrm{I}_{\mathrm{age}}$.
κ_t	is a stochastic process that is assumed to be measurable at time t.
γ_c	is a parameter modeling the cohort effect with $c = t - x$.

Longevity Linked instruments

Symbol	**Meaning**
$S(t)$	is the survivor index at time t.
N	is the notational amount, which is agreed at the interception of the contract.
m_{fixed}	fixed (or forward) mortality rate.
$m_{\text{realized}}(t)$	realized mortality rate at time t.

2 Mortality and longevity risk

Before presenting mortality and longevity risk transfer solutions, this section first defines mortality and longevity risk. In addition, some metrics for mortality and longevity are presented, and current developments in mortality and longevity are stated in this section.

2.1 Definition of Mortality and Longevity Risk

Definition 2.1 (The *one-year mortality* and *survival probability*)**.** In the actuary field q_x denotes *the one-year mortality probability*, which is the probability that a person aged exactly x dies before aging $x+1$ and p_x denotes *the one-year survival probability*, which is the probability that a person aged exactly x will survive to age $x+1$.[4]

Remark 2.2. The one-year mortality and survival probability are linked via $p_x = 1 - q_x$.

Remark 2.3. The one-year mortality probability for a person aged x varies with time due to factors such as advances in medicine, hygiene and food supply, or, as in recent times, due to a pandemic or war. For example, the one-year mortality probability for an $x = 65$ year old person q_{65} in 1950 differs from q_{65} in 2010. Due to COVID-19, the one-year mortality probability for a person aged $x = 70$ years in 2018 is of course also different from that in 2020. Therefore, it makes sense to define the one-year mortality probability q_x at time t.

Definition 2.4. $q_{x,t}$ denotes the probability that a person aged exactly x at the beginning of t dies within one year.

Remark 2.5. Unless otherwise stated, the age x is given in years and is implicitly understood as the age interval $[x, x+1)$. In general, t is a specific point in time, but unless otherwise stated, it is consider that also the time t is given in years, so that $q_{65,2020}$ denotes the probability that a person aged $x = 65$ years at the beginning of $t = 2020$ (on 01.01.2020) dies within one year (before 01.01.2021).

Definition 2.6 (*mortality rate* and *life expectancy*)**.**

- The natural estimator of $q_{x,t}$ is the *mortality rate*, which is calculated by dividing the number of deaths aged x in year t by the number of people at risk, who are those people of the considered (sub-)population, who are exposed to the risk of death during the considered period. (See also Section 2.2, Item 1 for closed populations and in standard life tables.)

[4] q_x is also known as the probability of an x-year-old person dying within the next year and p_x is also known as the probability of an x-year-old person surviving one year.

- The *life expectancy* of an x-year-old person is its average future lifetime. (See also Section 2.2, Item 4.) It can be expressed in total years or in years left.

Example 2.7 (total years vs years left)**.**
If a person aged $x = 65$ is expected to die at age 85, their life expectancy in total years is 85 and their years left is 20.

Definition 2.8 (*mortality improvement* and *deterioration*)**.** *Mortality improvement* is given if, over time, the probability of dying within the next year q_x declines, which implies that the survival probability and life expectancy increases. If, on the other hand, q_x increases over time, *mortality deterioration* prevails.

Definition 2.9 (*mortality risk* and *longevity risk* [4])**.**

- In the following, the term *mortality risk* describes the uncertainty that future mortality rates differ from the expected mortality rates.

- The term *longevity risk* describes the uncertainty in the long-term probability of survival[5] in both directions.

Example 2.10. Let $\hat{q}_{x,t}$ be the expected mortality rate for an x-year-old person in the year t. If $\hat{q}_{x,t} < q_{x,t}$, then the experienced mortality $q_{x,t}$ is higher than expected, which results in people living shorter than assumed. In the literature this scenario is often defined as mortality risk and analogously the scenario of people living longer than expected is defined as longevity risk. While an increase in mortality is generally correlated with higher financial obligations for life insurers, this results in a financial relief for pension plans. Otherwise, if $\hat{q}_{x,t} > q_{x,t}$, the roles are switched. Consequently, if future mortality rates differ from the expected ones, then this results in adverse financial consequences for either the life insurer or the pension plan. Similarly, financial obligations fall for life insurers and rise for pension plans when longevity is higher than expected.

Remark 2.11. As in most developed countries life expectancy is improving, mortality and therefore also longevity risk are asymmetrical risks, meaning that there is a higher probability of being exposed to unexpected mortality improvement than mortality deterioration.

Remark 2.12. Longevity risk can be divided into the three components individual or idiosyncratic risk, basis risk and trend risk.[6]

[5]Uncertainty in ${}_tp_x$ — the probability that a person aged exactly x dies before aging $x + t$ — where t equals 10, 20 or more years.
[6]Club Vita's Lexicon of Longevity: `https://www.clubvita.net/glossary`

- *Individual or Idiosyncratic risk* is the risk that certain members of a population live significantly longer or shorter than predicted.
- *Basis risk* arises when, for the calculation of the baseline mortality,[7] the underlying population differs from the cohort in question.
- *Trend risk* is the risk that experienced mortality rates decrease or increase at a different rate to that assumed.

The individual or idiosyncratic risk decreases the larger the pool of policyholders. Assuming that, in addition, the basis risk is minimized by referencing an appropriate population, the trend risk will make up most of the longevity risk for large companies.

Remark 2.13. In order to estimate life expectancy, mortality improvement assumptions must be specified, which are subject to the subjectivity of the model user. Therefore, life expectancy is exposed to the risk that these assumptions are wrong. In the past, these mortality improvement assumptions have often been set too low.
The estimation of mortality rates is more reliable and objective, but there is the risk of a rare or extreme event occurring, like the COVID-19 pandemic, and causing an unexpected jump in mortality rates.[8]

The organization Club Vita divides the calculation of future mortality rates and life expectancy in two steps:[9]

1. First, based on past experience the current state of mortality is estimated and in this way a baseline mortality is calculated. Currently, Life Tables are mainly used to calculate the baseline mortality and historical mortality improvement rates. Since there are cohort and period Life Tables, the resulting mortality rates and life expectancy are called cohort or period life expectancy and cohort or period mortality rates, respectively.

   Remark 2.14. At the Longevity 16 Conference in 2021 Razvan Ionescu, the Head of Biometric Risk Modelling at SCOR stated that Life Tables will probably remain around for the next decades, but more accurate

[7]The baseline mortality represents the background level of the mortality rate. This background level can be estimated by using data of the last 5+ years for the calculation of the chosen mortality model.

[8]Discussion by Matthew Edwards: `https://www.actuaries.org.uk/news-and-insights/news/what-difference-between-longevity-and-mortality`

[9]The subject of longevity 101 was baseline longevity:
`https://www.clubvita.us/events/longevity-101-baseline`.
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predictions are needed. This will eventually push actuaries to adopt more complex models. For example, one new approach is to include the use of Machine Learning techniques to also capture the impact of health (e.g. BMI), status (e.g. socio-economic group, marital status) and behavior (e.g. smoking) on the mortality rate.[10]

2. Afterwards, assumptions for future trends in longevity — more precisely assumptions for the future development of the mortality improvement rates — are determined by stochastic modelling (e.g. Section 4.2 and Section 4.3) and expert judgement[11] and applied to the baseline model. Naturally, the second step is more subjective.[9]

In Section 4 some examples for mortality and longevity models are discussed.

2.2 Mortality and Longevity Metrics

The following metrics are called *crude* if raw data are used for calculating, and *graduated* if the used mortality rates are averaged over time or ages.[12]

1. In Definition 2.1 two typical measures of mortality (q_x, p_x) have already been introduced. In [10, Section 3] q_x and p_x are called the *initial rate of mortality* and *initial rate of survival*, respectively. For a closed population and in standard Life Tables the initial rate of mortality q_x for a given age x is usually calculated by dividing the number of deaths over the last year, aged x last birthday at the beginning of the year D_x by the number of persons alive at the beginning of the year, aged x last birthday at the beginning of the year L_x, i.e. $q_x = \frac{D_x}{L_x}$.

2. The *central rate of mortality* m_x denotes the deaths per unit of exposure-to-risk over a year and is calculated as follows:

$$\begin{aligned} m_x &= \frac{D_x}{E_x} \\ &= \frac{\text{number of deaths over the year aged } x \text{ last birthday}}{\text{exposure-to-risk over the year aged } x \text{ last birthday}} \end{aligned}$$

The exposure-to-risk E_x over the year equals the sum of person-years of persons alive at the beginning of the year, aged x last birthday at the beginning of the year.[13] A *person-year* is the portion of the year in

[10] Longevity 16: `https://www.bayes.city.ac.uk/__data/assets/pdf_file/0007/632644/Ionescu-Razvan.pdf`

[11] This means that the model requires minimal input from a person with expertise in this area.

[12] The most common parametric model for smoothing mortality rates is the Gompertz-Makeham-Model, which assumes that mortality rates increase exponentially with age. (See [10, Appendix 2.1.1].)

[13] For a closed population, E_x, in contrast to L_x, takes into account that the number of people exposed to the risk may change during the year, e.g. due to death.

which the person was alive or more generally exposed to the risk, and is therefore expressed as the corresponding number between 0 and 1.[14] Assuming a uniform distribution of deaths over the year, the exposure of death is approximated by taking the number of lives at the middle of the year.[15]

3. The *force of mortality* or *mortality intensity* μ_x describes the instantaneous death rate for an x-year-old individual. Over a one-year period, the force of mortality and the initial rate of mortality q_x are linked as follows:

$$l_x q_x = \int_0^1 l_{x+u} \mu_{x+u} \, \mathrm{d}u,$$

   where l_{x+u} denotes the number of exposed lives aged exactly $x+u$ years, u being a fractional year. Under the assumption of a uniform distribution of deaths over the year, $\mu_x = -\log(1 - q_{x+\frac{1}{2}})$ applies.

4. The *life expectancy* measures the average future lifetime of an x year old individual. Particularly, the *period life expectancy* e_x is calculated as follows:

$$\mathrm{e}_x = \sum_{t=1}^{\infty} {}_t p_x,$$

   where ${}_t p_x$ is the probability of a person aged x to survive the next t years, which can be calculated by ${}_t p_x = \prod_{i=0}^{t-1}(1 - q_{x+i})$, given the mortality rates q_{x+i} for $i \in \{0, \dots, t-1\}$. Since each individual dies at some point, an ultimate age is assumed, which is often set to 120. Hence, ${}_t p_x = 0$ for $t \in \{u : x + u \geq 120\}$.

These metrics are used in Section 4 to calculate future mortality rates and life expectancy and in [10, Section 3] to create the LifeMetrics index.

Remark 2.15. In most use cases, a uniform distribution of deaths and a constant force of mortality over the year is assumed.

2.3 Current developments

In the 20th and 21st centuries, improving life expectancy in developed, high income countries was the predominant trend. In recent years, a decline in life expectancy has been observed in some of them. Ho and Hendi [15]

[14] Note that according to this definition especially for older ages x the central rate of mortality can be quite high.

[15] Note that in this case, if for example only one person age x is alive at the beginning of the year and dies within the first half of the year, the central rate of mortality is not defined.

highlighted that since 1990 life expectancy in the US was relatively low compared to other high income countries,[16] but the rate of increase did not differ substantially until 2016. Between 2010 and 2016 life expectancy plateaued in the US, compared to a steady increase in most high income countries. Between the years 2014 and 2015 Ho and Hendi concluded a decline in life expectancy for most of the countries, which may have been caused by a particular severe influenza season, as the gains in life expectancy in the following years 2015 - 2016 more than compensated for the declines. An exception to this rebound were the UK and the US, which experienced stagnation or continued declines in life expectancy during the following years. Hypothesis for these declines are the opium-crisis in the US and decreases in funding to healthcare and social welfare programs in the UK.
According to Blake [2], there are alternative expert views on how life expectancy will develop in the future. On the one hand, life expectancy might level off or decline, as has been observed in the US and the UK. On the other hand, due to future scientific and medical advances, such as regenerative medicine life expectancy might continue to improve in the overall trend.

COVID-19

In the study from Islam et al. [19] a reduction in life expectancy was detected in almost all the observed high-income countries in 2020. Exceptions are Denmark, Iceland, and South Korea, where no change in life expectancy was found and New Zealand, Taiwan, and Norway, where life expectancy increased in 2020.[17] Accordingly, the years of life lost[18] were higher than expected in all countries except Taiwan, New Zealand, Norway, Iceland, Denmark, and South Korea. Further, Islam et al. [19] concluded that due to the COVID-19 pandemic in 2020 the excess years of life lost were more than five times higher than those associated with the severe influenza season in 2015. However, this study did not differ whether these excess deaths were directly caused by SARS-CoV-2 or other causes of deaths and therefore other factors could have contributed to these results. Subsequently, the question remains how best to evaluate the years 2020, 2021 and 2022. For example, the MIM-2021 model in Section 4.1.4 adjusts the calculated improvement rates for the years $2020 - 2025$ using mortality shocks, given by expert judgement.

[16] Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom.

[17] The life expectancy was estimated as the difference between observed and expected life expectancy in 2020 using the Lee-Carter model, which is described in 4.2.

[18] The WHO expresses the years of life lost (YLL) per 100 000 population. It is a measure of premature mortality that takes into account both the mortality and the age structure of the population. The YLL is calculated from the number of deaths multiplied by a global standard life expectancy at the age at which death occurs.
Excess years of life lost were estimated as the difference between the observed and expected years of life lost in 2020 using the World Health Organization standard Life Table.

Furthermore, the effect COVID-19 might have on the longevity is currently unclear.[19] In 2021 Club Vita published 4 different COVID-19 longevity scenarios including the indicative key drivers (immediate increase in deaths due to COVID-19, disruption to non-COVID-19 medical care, changes to health and care systems and global recession) for those scenarios:[20]
In the following the first two scenarios considered are likely to occur, and the last two scenarios are tail events.

- Bump in the road
  This scenario predicts an increase in mortality in the years 2020 and 2021 due to COVID-19 following a return to the previous trend in longevity improvement from the year 2022 onwards without a catch-up for the years 2020 and 2021.

- Long road to recovery
  This scenario predicts an increase in mortality in the years 2020 and 2021 due to COVID-19 following a reduced excess mortality until 2025 driven by new strains of the virus and the relaxation of social distancing measures. Furthermore, the disruption to non-COVID-19 medical care will impact the mortality rates throughout the 2020s. Hence, also low levels of longevity improvements during this decade are anticipated.

- Innovation in adversity
  This scenario predicts an effectiveness of vaccines and reduced mortality rates due to COVID-19 from 2022 onwards. Furthermore, the impact of the disruption to non-COVID-19 medical care will be limited, and intensive efforts will be made to reduce the level of health inequalities that the pandemic has exposed. Simultaneously, the economy will recover to the pre-pandemic levels. Hence, there will be long term longevity and health improvements during the period $2025 - 2035$.

- Healthcare decline
  This scenario predicts further waves of excess mortality rates due to COVID-19 throughout this decade, driven by new mutations of the virus for which the vaccine is less effective and lockdown fatigue amongst the population. During each wave, the healthcare system is overwhelmed and disruptions to non-COVID-19 medical care continue, resulting in increased mortality for other causes of death over the coming decades. The growing strain on the healthcare system will

[19] https://www.actuaries.org.uk/news-and-insights/news/what-difference-between-longevity-and-mortality
[20] For more detailed information: https://www.clubvita.us/assets/images/general/clubvita_US_scenariospaper_covid19_f2_01.pdf
https://www.clubvita.co.uk/assets/images/general/Club_Vita_UK_COVID-19_scenarios_technical_appendix_March_2021-1.pdf

drive the growth in health inequality. Hence, life expectancy will stall or even decline for some sections of the population.

3 Mortality and longevity indices

Mortality and longevity indices track the mortality and the survivorship of a specified (sub-)population in a standardized way.[21] Using published indices for transactions can reduce administrative and legal costs and eliminate the information asymmetry regarding the development of the underlying population. On the other hand, since the development of the underlying index population does not exactly replicate the development of the hedger's cohort, there is a population basis risk for the hedger when entering into an index-based transaction.

3.1 Historical indices

QxX & QxX.LS.2 index

In 2008 Goldman Sachs launched the QxX and the QxX.LS.2 index for the life settlement market.[22] The QxX index referenced a pool of 46,000 US individuals aged 65+ with primary impairments, excluding AIDS and HIV, and the QxX.LS.2 index referenced a pool of 65,655 US individuals aged 65+ with impairments, including cancer, cardiovascular conditions and diabetes. The number of survivors of these underlying reference pools were calculated and published monthly. (See also: [29, Section 4].)
Both indices were designed for investments and not for hedging longevity risk.[23] Two years later those indices were shut down, partly because they did not sell. [24]

Xpect® indices

In 2008 Deutsche Börse launched Xpect® indices, which included the Xpect® cohort indices, the Xpect® Club Vita indices and the Xpect® customized indices.
Each Cohort index, classified in terms of cohort (1935 – 1939, 1940 – 1944, 1945 – 1949 and 1950 – 1954), population (England & Wales, Germany and the Netherlands) and gender (male, female), referenced a pool of 100,000 individuals. The current index value equaled the current number of survivors. Those indices were updated monthly and published on Bloomberg and ThomsonReuters. In later years, the Xpect® indices were expanded to include the US in terms of population. (See also: [6, 11.2].)

The Xpect® Club Vita indices were further classified into three different sociodemographic groups based on the annual pension volume ($\leq$ £5,000, £5,000 – 10,000, $\geq$ £10,000). On a monthly basis survivor rates were

[21] Definition by ClubVita: https://www.clubvita.net/glossary/longevity-index
[22] The life settlement market is defined in Definition 5.1.
[23] https://www.actuaries.org.uk/system/files/documents/pdf/steyn0.pdf
[24] https://www.investmentnews.com/goldmans-longevity-index-has-short-life-26030

calculated from monthly Xpect England & Wales data and Club Vita mortality rates, which were based on effective mortality rates of more than 100 UK pension schemes.

Future values of the Xpect® indices were calculated using the Xpect® forward curves, which project the future development of the underlying reference pool. For this purpose, future mortality rates q_x were estimated using inter alia the Lee-Carter- or CBD-model (described in Section 4.2 and Section 4.3) and then shocked by 30 % in both directions. The resulting differences in the Xpect® forward curve correspond to the bid-ask prices for maturities of 1, 2, 5, 10, 20 or 30 years.

In 2010 Tullett Prebon, a financial services firm in the UK, provided Xpect®-based longevity swaps.[25]

LifeMetrics index

In 2007 J.P. Morgan developed the LifeMetrics index. In 2011 the ownership of this index was transferred to the Life and Longevity Market Association (LLMA), which was dissolved on the 03.03.2020.[26]

The LifeMetrics index was based on data obtained from public sources, which were classified in terms of population (England & Wales, US, Germany and the Netherlands), gender (male, female), age ($x \in \{20, \dots, 89\} =: \mathrm{I}_{\mathrm{age}}$) and time period ($t$ as calendar years). This index provided the crude central rate of mortality m_x, the graduated initial rate of mortality q_x and the period life expectancy e_x for historical and then current periods, which were published annually on their website. Sub-indices could also be found on Bloomberg. (See also: [10, Section 5].)

The concept of the calculation of the index is summarized briefly below[27]:

First, the crude central rate of mortality $m(\text{population}, \text{gender}, x, t)$ is calculated as defined in Section 2.2 using the actual number of deaths $D_{x,t}$ and the mid-year population estimate $E_{x,t}$ in the year t, for each age x obtained from public data. Afterwards, the graduated central rate of mortality s_x was

[25] https://www.bayes.city.ac.uk/__data/assets/pdf_file/0017/113237/Pres_Rogge-and-Sachsenweger.pdf

[26] Transfer of ownership: https://www.artemis.bm/news/life-and-longevity-market-association-takes-ownership-of-j-p-morgans-lifemetrics-index/
LLMA: https://find-and-update.company-information.service.gov.uk/company/07081717

[27] Furthermore, the LifeMetrics toolkit provided also a framework for measuring and managing mortality and longevity risk and an accompanying software for forecasting future mortality rates.

obtained by minimizing[28]

$$p \sum_{x \in \mathrm{I_{age}}} (\log(m_x) - \log(s_x))^2 + (1-p) \int_{x \in \mathrm{I_{age}}} \left(\frac{\mathrm{d}^2 \log(s_x)}{\mathrm{d}x^2} \right)^2 \mathrm{d}x,$$

where $\log(s_x)$ is a set of cubic splines with a different spline between every two ages, continuity of the slope at each age x and $p = 0.375$. Using these graduated central rates of mortality, the graduated initial rate of mortality $q(\text{population}, \text{gender}, x, t)$ is estimated via: $q_x = \frac{s_x}{1+0.5 s_x}$. Since the assumed ultimate age is 120, but graduated initial mortality rates are only available up to age 90, for all $x > 90$ the graduated initial mortality rates q_x are estimated by fitting a single cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$ analogous to Section 4.1.3 and applying $q_x = \frac{f(x)}{1+0.5 f(x)}$.
At last, the period life expectancy $\mathrm{e}(\text{population}, \text{gender}, x, t)$ is calculated analogously to Section 2.2.

Remark 3.1. Example 7.7 describes a q-forward contract where the LifeMetrics index was used.

Credit Suisse longevity index
In 2006, Credit Suisse launched a longevity index in collaboration with the independent calculation company Milliman, which tracked the expected average life expectancy of the national US population on an annual basis. In addition, sub-indices quoted the expected average lifetime for different gender and ages. (See also: [27, Section 6].) The used methods were similar to those described for the LifeMetrics index. (See also: [29, Section 4].)
According to Blake in [2, Section 4.2.1] this index lacked transparency and there were no transactions executed using the Credit Suisse longevity index.

3.2 Existing indices

LIFE index
The longevity index for England (LIFE) results from research commissioned by the Actuarial Research Centre (ARC) in 2016. The Longevity and Morbidity Risk LMR program was led by Principal Investigator Professor Andrew Cairns of Heriot-Watt University and co-sponsored by the Society of Actuaries (SoA) and the Canadian Institute of Actuaries (CIA).

LIFE provides a group of indices classified in gender $g \in \{m, f\}$ (male, female), LSOA[29] index $i \in \{1, \ldots, L\}$ and age $x \in \mathrm{I_{age}} := \{40, \ldots, 89\}$.

[28] This method is called spline smoothing, which is presented in [10, Apendix 2.2.2].
[29] A Lower layer Super Output Area (LSOA) is a small, socially-homogeneous geographic area with an average population of 1,600 people. There are $L := 32{,}844$ LSOAs across England.

The used data, obtained from UK's Office for National Statistics (ONS)[30], include the deaths by Lower layer Super Output Area (LSOA), England & Wales, mid-year 2001 to 2018, from where the number of deaths $D(g,i,t,x)$ and the central exposure to risk $E(g,i,t,x)$ for
$(g,i,t,x) \in (\{\mathrm{m},\mathrm{f}\},\{1,\dots,L\},\{2001,\dots,2018\},\{35,\dots,94\})$ are derived. In addition, 12 observed predictive variables $V(i) = (V_1(i),\dots,V_{12}(i))$ available for each LSOA index i were used for the construction of the LIFE index. The first 9 are socio-economic variables,[31] of which the income and the employment deprivation are the principal socio-economic drivers, following 2 home-care variables,[32] whose influence is later removed by taking the national average of those variables, and finally, also an important driver is the urban-rural classification, consisting of 5 classes.[33] (See also: [1].)

The LIFE index LIFE(g,i,x) estimates the relative mortality rate for a given gender g aged x years in each LSOA i with the observed predictive variables $v(i)$ compared to the national mortality rate for the given gender g aged x years. Furthermore, the LIFE app provides LIFE deciles and percentiles, which estimate the position of the LSOA relative to all other LSOAs in England. Low deciles or percentiles represent higher deprivation and therefore higher mortality. At last, the estimated remaining life expectancy for the chosen gender g, LSOA i and age x based on mortality rates in 2019 is given.

As part of the "Modelling, Measurement and Management of Longevity and Morbidity Risk" research program, an app called the LIFE index app is being developed to allow non-expert users to explore the LIFE index and discover inequalities in mortality between different areas of England. In the webinar Introducing the New Longevity index for England (LIFE) App[34] the construction of the LIFE index was summarized as follows:
First, for each gender $g \in \{\mathrm{m},\mathrm{f}\}$ and age of interest $x \in \{40,\dots,89\} =: \mathrm{I}_{\mathrm{age}}$, the age range $\mathrm{I}_x := (x-5, x+5)$ and the time period $\mathrm{TP} := \{2001,\dots,2018\}$ are used to calculate the empirical relative risk $R^0(g,i,x)$ by dividing the

[30] website: `www.ons.gov.uk`, more specific (10772): `https://www.ons.gov.uk/peoplepopulationandcommunity/birthsdeathsandmarriages/deaths/adhocs/10772deathsbylowerlayersuperoutputarealsoaenglandandwalesmidyear2001to2018`

[31] old age income deprivation, employment deprivation (i.e. unemployment), education deprivation, housing standard (number of bedrooms), proportion of the population born outside the UK, deprivation in housing/living environment, employment/occupation: proportion in a management position, crime rate and proportion working more than 49h per week

[32] proportion of population aged 60+ in a care home with nursing and proportion of population aged 60+ in a care home without nursing

[33] urban conurbation (except London), urban city and town, rural town and village, rural hamlet and isolated dwellings, urban conurbation (London only)

[34] `https://www.actuaries.org.uk/learn-and-develop/research-and-knowledge/actuarial-research-centre-arc/arc-webinar-series-2021`

actual deaths by the expected deaths for each LSOA:

$$R^0(g,i,x) = \frac{\sum_{x\in \mathrm{I}_x, t\in \mathrm{TP}} D(g,i,t,x)}{\sum_{x\in \mathrm{I}_x, t\in \mathrm{TP}} m(g,t,x)E(g,i,t,x)},$$

where $m(g,t,x) = \frac{\sum_{i=1}^{L} D(g,i,t,x)}{\sum_{i=1}^{L} E(g,i,t,x)}$ is the national central rate of mortality.
To solve

$$f(v;g,x) = \mathbb{E}[R^0(g,x)|v],$$

where $R^0(g,x) = (R^0(g,1,x),\dots,R^0(g,L,x))$ and v denotes the 12 predictive variables, the random forest method, a supervised machine learning algorithm, is applied. The resulting estimator $\hat{f}^{RF}(v;g,x)$ is a piecewise constant function.
It follows that the LIFE index is given by

$$\mathrm{LIFE}(g,i,x) = \hat{f}^{RF}(\tilde{V}(i);g,x),$$

with $\tilde{V}(i) = (V_1(i),\dots,\bar{V}_{10},\bar{V}_{11},V_{12}(i))$, where for $i = 10, 11$ the national average $\bar{V}(i) = \frac{1}{L}\sum_{i=1}^{L} v_i$ is used.

A beta version of the LIFE index app is already available on `http://www.macs.hw.ac.uk/~andrewc/ARCresources/LIFEapp/`:[35]

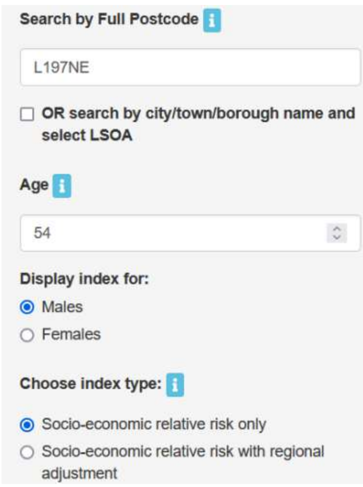


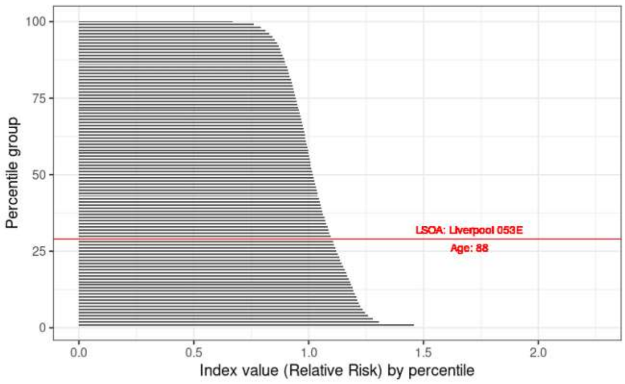
Figure 1: In the LIFE app the LIFE index is calculated after choosing the LSOA i by entering the corresponding Postcode, the age $x \in \{40,\dots,89\}$ the gender and the index type.
In the LIFE app, in addition to the LIFE index based on data obtained from the ONS, there is a second version with regional adjustments of the LIFE index resulting from the additional use of data from 106 Clinical Commissioning Groups (CCGs).

[35] This App, and the underpinning Longevity Index For England, is under the "Attribution 4.0 International (CC BY 4.0)" licence. See `https://creativecommons.org/licenses/by/4.0/`

Index and Life Expectancy of Selected LSOA

LSOA name	LSOA code	Region	Age	Index value	Index decile (*)	Index percentile (**)	Remaining life expectancy from age selected (in 2019) (***)
Liverpool 053E	E01006686	North West	88	1.1	3 /10	29 /100	4 years and 9 months

Relative Position of Selected LSOA to Others by Index Value

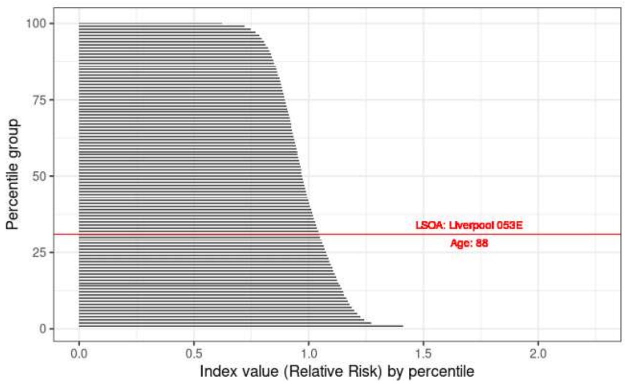


(a) Socio-economic relative risk only

Index and Life Expectancy of Selected LSOA

LSOA name	LSOA code	Region	Age	Index value	Index decile (*)	Index percentile (**)	Remaining life expectancy from age selected (in 2019) (***)
Liverpool 053E	E01006686	North West	88	1.05	4 /10	31 /100	4 years and 10 months

Relative Position of Selected LSOA to Others by Index Value



(b) Socio-economic relative risk with regional adjustment (CCG)

Figure 2: These screenshots show an example of the LIFE index for a 88 year old male person living in Liverpool 053E.
Overall both index types produce similar results. In some cases, as in this example, the resulting indices are in different index deciles.

The value-based longevity index
A theoretical approach developed by Chang and Sherris in 2018 is a value-based longevity index. (See [9].)
The calculation of the index is based on the following conditions:

- Consider nominal cashflows and set the ultimate age to 120.
- To estimate the expected number of survivors s_t at each year t a stochastic mortality model is used.
- To estimate the discount factor DF_t for each year t an Affine Term Structure Model (ATSM) is used.

The index value estimates the present value required to fund a lifetime annuity, taking into account longevity and interest rate risk, that pays a unit nominal income stream to the annuitant. For annuitants aged x:

$$PV_0 = \sum_{t=1}^{\tau} s_t DF_t,$$

where $\tau = 120 - x$,[36] s_i denotes the estimated expected number of survivors at the end of the year t and DF_t denotes the estimated discount factor for the year t.

3.3 Indemnity based vs index based transactions

- *Indemnity based transactions* track the experienced mortality rates of the hedger's cohort and usually have a run-off maturity, meaning that the term of the contract is defined by the time of death of the last member of the covered cohort. (See also the maturity of swaps: Remark 7.10.) Therefore, these transactions perfectly hedge the idiosyncratic risk, the basis risk and the trend risk.
- *Index based transactions* track an agreed index that reflects the experience of a standard population.

To date, most transactions have been indemnity based, and it appears that sophisticated investors do not have a strong preference between index-based and indemnity-based transactions.[37] One explanation is provided by Steven

[36] In [9, Section 6] $\tau = x - 65$ applies, but in the literature building on this, for example in [28, Section 4.1], τ was corrected to $120 - x$.
[37] Longevity 16 Conference speech by Luca Tres – Guy Carpenter: Capital Markets & Longevity: `https://www.bayes.city.ac.uk/__data/assets/pdf_file/0008/632645/Tres-Luca.pdf`

Baxte in a Mortality and Longevity Seminar from 2018,[38] in which he argues that there might have been a lack of confidence in the published indices. In addition, cedents demanded indemnity solutions for which plenty of capital was available, and systems to manage them. Nevertheless, index-based transactions can be relevant to open up the longevity market to new entrants. For example, when the LifeMetrics index — see Section 3.1 — was launched, the hope was that the development of such indices with accompanying standardized products would help to catalyze the development of a liquid market for traded mortality and longevity. (See [10, Section 1.2].)

Remark 3.2. If indices are mentioned in Section 6 and Section 7, both types of transactions can be used. Note that when using index-based transactions, basis risk arises.

[38] https://www.actuaries.org.uk/learn-develop/attend-event/mortality-and-longevity-seminar-2018

4 Models for mortality and longevity

For any company (especially life insurers and pension plan sponsors), which is exposed to longevity or mortality risk, it is of great importance to be able to measure and forecast its risk. Substantial tools for this are mortality and longevity models. Furthermore, mortality models play an essential part of constructing (design and pricing) capital market solutions. (See also: [6, Section 9].) Mortality models can be distinguished in *extrapolative* and *explanatory* models.

Extrapolative mortality models project historical data experiences into the future and therefore assume that historical trends will continue. In the following, all described models are extrapolative and use the parameters age, period and cohort. Furthermore, those models can be distinguished in deterministic models such as the MIM-2021 model (see 4.1) and stochastic models such as the Lee-Carter (see 4.2) model, the CBD model (see 4.3) or the heat wave model (see 4.4).

The aim of using *explanatory models* is to identify predictor variables for mortality changes and their impact on them. Some examples for predictor variables for mortality rates are the cohort (i.e. year of birth), the gender, the socio-economic status, the lifestyle, the geographical location, the education or medical advances and infectious diseases. Mortality projections are therefore a result of changes in these predictor variables. Cause of death models, causes of causes of death models or scenario based models are explanatory models.

First, an overview of the *MIM-2021 model*, a deterministic model for estimating mortality improvement rates, is described.

4.1 Mortality Improvement Model: MIM-2021

The following conceptual framework is modeled after the approach developed by the SOA's Retirement Plans Experience Committee (RPEC). The MIM-2021 is written in VBA and has a Data Analysis Tool and an Application Tool, which can be downloaded from the SOA's website. An annual updating process for the MIM-2021, (and associated tools) is anticipated.[39]
The key concept of the MIM-2021 model is to use recent experience for short-term mortality improvement rates, which should blend smoothly over an appropriate transition period to the assumed long-term mortality rates based on expert judgement. The MIM-2021 tool offers 5 datasets of US historical mortality rates, of which some are stratified into socio-economic

[39] Society of Actuaries. 2021. "Mortality Improvement Scale MP-2021". https://www.soa.org/resources/research-reports/2021/mortality-improvement-model/

categories. In the Application Tool, there are 5 steps to individually fit the parameters to your needs.

1. In the first step, the user can choose one of the available datasets.
2. In the second step the structure for the mortality improvement projection is fixed by setting the future dates (A, B, C and D) for the transition periods. Further, the projection point and the weight, which is placed on the cohort projection, is fixed.
3. In the third step, the historical periods, which are used to calculate the short-term, mid-term and long-term mortality improvement rates and the initial slopes are fixed.
4. In the fourth step, the calculated mortality improvement rates and the initial slopes can be validated. In the second version, a table was added for subjoining a COVID-19 adjustment for the years $2020 - 2025$.
5. Finally, the desired output, including annual mortality improvement rates, historical and projected mortality rates, period life expectancy by year and cohort life expectancy by year can be chosen.

Notation:

$\mathrm{I}_{\mathrm{age}}$	contains all ages x for which the mortality improvement rates are calculated. In the MIM-2021 Application Tool $\mathrm{I}_{\mathrm{age}} = [20, 120]$.
$\mathrm{r}_{x,t}$	the mortality improvement rate for age $x \in \mathrm{I}_{\mathrm{age}}$ in year t.
$q_{x,t}$	the smoothed historical mortality rate for an x-year-old individual in year t.
s_x	the initial slope; precisely $\mathrm{s}_{x;d}$ is the diagonal initial slope for age x and $\mathrm{s}_{x;h}$ is the horizontal slope age x.

4.1.1 Calculation of Mortality Improvement Rates

Let $q_{x,t}$ be the smoothed historical mortality rates, from which the short-term, mid-term and long-term mortality improvement rates, representing the mortality improvement rates for the time points A, B and D, are calculated by:

$$\mathrm{r}_x = 1 - \left(\frac{q_{x,T}}{q_{x,t_0}}\right)^{\frac{1}{(T-t_0)}}$$

for $x \in [20, 95]$ and for each period $[t_0, T]$, which were fixed in the third step. For $x \in [96, 115]$ the mortality improvement rates decline linearly to zero and stay zero for an age above 115. For the jumping-off point A, RPEC's practice so far is stepping back two years from the available historical data.

4.1.2 Calculation of the Initial Slopes

For the calculation of the initial diagonal slope set $x_{t_0} = \max\big(x-(T-t_0), 20\big)$:

$$\mathrm{s}_{x;d} = \frac{\frac{q_{x_{t_0},t_0}}{q_{x_{t_0},t_0-1}} - \frac{q_{x,T}}{q_{x,T-1}}}{T - t_0}$$

for $x \in [20, 95]$ and for each period $[t_0, T]$, which were fixed in the third step. Setting $x_{t_0} = x$ gives the horizontal slope $\mathrm{s}_{x;h}$. For $x \in [96, 115]$ the slope declines linearly to zero and stay zero for an age above 115.

4.1.3 Interpolation

The following interpolation is calculated horizontally (time-related) and diagonally (year-of-birth related). Afterwards, the results are combined linearly using the weight fixed in the second step to calculate the estimated mortality improvement rates $\hat{\mathrm{r}}_{x,t}$.

Between the first points $(A, \mathrm{r}_{x,A})_{x\in \mathrm{I}_{\mathrm{age}}}$ and $(B, \mathrm{r}_{x,B})_{x\in \mathrm{I}_{\mathrm{age}}}$ the cubic interpolation, which is presented below, is used by setting $t_0 = A$ and $T = B$. For the diagonal interpolation set $x_{t_0} = \max\big(x - (T - t_0), 20\big)$ for $x \in \mathrm{I}_{\mathrm{age}}$ and $\mathrm{s}_x = \mathrm{s}_{x;d}$. For the horizontal interpolation set $x_{t_0} = x$ and $\mathrm{s}_x = \mathrm{s}_{x;h}$. Between the years B and C the mid-term mortality improvement rates remain constant. From there on the mortality improvement rates converge linearly to the long-term mortality improvement rates, which are reached in the year D and remain constant for the years thereafter.

Cubic Interpolation

For $x \in \mathrm{I}_{\mathrm{age}}$ let t_0 be the initial year with mortality improvement rate $\mathrm{r}_{x_{t_0},t_0}$, T the end of the period with mortality improvement rate $\mathrm{r}_{x,T}$ and s_x the initial slope at t_0. As at the end of the period the mortality improvement rates remain constant, the slope at time T must be zero.
For any age $x \in \mathrm{I}_{\mathrm{age}}$ the cubic interpolation Cubic(t) for the year $t \in [t_0, T]$ is given by:

$$\mathrm{Cubic}(t) = \mathrm{r}_{x_{t_0},t_0} + \mathrm{s}_x(t - t_0) + b_x(t - t_0)^2 + a_x(t - t_0)^3,$$

where

$$a_x = \frac{\mathrm{s}_x(T - t_0) - 2(\mathrm{r}_{x,T} - \mathrm{r}_{x_{t_0},t_0})}{(T - t_0)^3}$$

$$b_x = -\frac{\mathrm{s}_x + 3a(T - t_0)^2}{2(T - t_0)} = -\frac{2\mathrm{s}_x(T - t_0) + 3(\mathrm{r}_{x,T} - \mathrm{r}_{x_{t_0},t_0})}{(T - t_0)^2}$$

This cubic interpolation meets all required conditions:

$$\mathrm{Cubic}(t_0) = \mathrm{r}_{x_{t_0},t_0}$$

$$\begin{aligned}\text{Cubic}(T) &= \mathrm{r}_{x_{t_0},t_0} + \mathrm{s}_x(T-t_0) - 2\mathrm{s}_x(T-t_0) + 3(\mathrm{r}_{x,T} - \mathrm{r}_{x_{t_0}t_0}) \\ &\qquad\qquad + \mathrm{s}_x(T-t_0) - 2(\mathrm{r}_{x,T} - \mathrm{r}_{x_{t_0},t_0}) \\ &= \mathrm{r}_{x_{t_0},t_0} - 3\mathrm{r}_{x_{t_0},t_0} + 2\mathrm{r}_{x_{t_0},t_0} + 3\mathrm{r}_{x,T} - 2\mathrm{r}_{x,T} = \mathrm{r}_{x,T}\end{aligned}$$

and for the derivatives

$$\begin{aligned}\text{Cubic}'(t_0) &= \big(\mathrm{s}_x + 2b(t-t_0) + 3a(t-t_0)^2\big)|_{t=t_0} = \mathrm{s}_x \\ \text{Cubic}'(T) &= \big(\mathrm{s}_x + 2b(t-t_0) + 3a(t-t_0)^2\big)|_{t=T} \\ &= \mathrm{s}_x - 2\frac{2\mathrm{s}_x(T-t_0) + 3(\mathrm{r}_T - \mathrm{r}_{t_0})}{(T-t_0)^2}(T-t_0) \\ &\qquad\qquad + 3\frac{\mathrm{s}_x(T-t_0) - 2(\mathrm{r}_T - \mathrm{r}_{t_0})}{(T-t_0)^3}(T-t_0)^2 \\ &= \mathrm{s}_x - 4\mathrm{s}_x + 6\frac{(\mathrm{r}_T - \mathrm{r}_{t_0})}{(T-t_0)} + 3\mathrm{s}_x - 6\frac{(\mathrm{r}_T - \mathrm{r}_{t_0})}{(T-t_0)} = 0\end{aligned}$$

4.1.4 Covid-19 adjustment

In the second version of the MIM-2021 model, a table for a Covid-19 adjustment was added. The mortality shocks ($\text{MShock}_{x,t}$) are given as the percentage by which mortality is expected to increase additionally in year t for age x due to COVID-19. They are applied to $(1 - \hat{\mathrm{r}}_{x,t})$, resulting in the following adjusted mortality improvement rates:

$$\hat{\mathrm{r}}^A_{x,t} = \begin{cases} 1 - (1 - \hat{\mathrm{r}}_{x,t})(1 + \text{MShock}_{x,t}), & t = 2020, \\ 1 - (1 - \hat{\mathrm{r}}_{x,t})\big(1 + \frac{1+\text{MShock}_{x,t}}{1+\text{MShock}_{x,t-1}}\big), & 2020 < t \leq 2025, \\ \hat{\mathrm{r}}_{x,t}, & \text{else.} \end{cases}$$

Part of the model output:	
mortality improvement rates:	$\hat{\mathrm{r}}^A_{x,t}$
projected mortality rates:	$\hat{q}_{x,t} = \min\big(q_{x,t-1}(1 - \hat{\mathrm{r}}^A_{x,t}), 1\big)$
x, t fixed:	
period life expectancy:	$\sum_{i=x+1}^{109} \prod_{j=x}^{i}(1-\hat{q}_{j,t}) + \frac{1}{2}\big(1+\prod_{j=x}^{110}(1-\hat{q}_{j,t})\big)$
cohort life expectancy:	$\sum_{i=x}^{109} \prod_{j=x}^{i}(1 - \hat{q}_{j,t+j-x}) + \frac{1}{2}\big(1 + \prod_{j=x}^{110}(1 - \hat{q}_{j,t+j-x})\big)$

The most common approaches in stochastic modelling are the *Lee-Carter model*, a single-factor model and the *CBD-model*, a two-factor model, which are presented below. Both are rather simple and robust mortality models, which can be expanded with further age parameters and a cohort effect.

First, an overview of some existing extrapolative stochastic mortality models is given below:

Model M1: The Lee-Cater Model	
$\ln(m_{x,t}) = a_x + b_x\kappa_t$	(2 const.)
Model M2: The Renshaw-Haberman Model	
$\ln(m_{x,t}) = a_x + b_x\kappa_t + c_x\gamma_{t-x}$	(4 const.)
Model M3: The Age-Period-Cohort Model	
$\ln(m_{x,t}) = a_x + b_x^{-1}\kappa_t + b_x^{-1}\gamma_{t-x}$	(3 const.)
Model M5: The Original CBD Model	
$\ln(\frac{q_{x,t}}{1-q_{x,t}}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x})$	(no const.)
Model M6: The CBD Model with a Cohort Effect Term	
$\ln(\frac{q_{x,t}}{1-q_{x,t}}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \gamma_{t-x}$	(2 const.)
Model M7: The CBD Model with a Cohort Effect and Quadratic Term	
$\ln(\frac{q_{x,t}}{1-q_{x,t}}) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\overline{x}) + \kappa_t^{(3)}((x-\overline{x})^2 - \hat{\sigma}_x^2) + \gamma_{t-x}$	(2 const.)

Table 1: In [8, Section 2.1] some existing extrapolative stochastic mortality models are listed. Here a_x, b_x and c_x are age specific parameters, $\overline{x}$ is the average age of the used age range, $\hat{\sigma}_x^2$ the variance of the used age range, $\kappa_t^{(i)}$ stochastic processes and γ_{t-x} a parameter modeling the cohort effect. The number of required parameter constraints for each model is shown on the right-hand side. In the following, the model M1 and M5 are described in more detail.

Assumption 4.1. As already mentioned in Section 2.1, let $q_{x,t}$ denote the probability that a person aged exactly x at the beginning of t dies within one year and $m_{x,t}$ be the central rate of mortality for age x in year t, as defined in Section 2.2, Item 2. Assuming a uniform distribution of deaths over each year implies the following link (see [8, Section 2.2]):

$$q_{x,t} = 1 - \exp(-m_{x,t}),$$
$$m_{x,t} = -\ln(1 - q_{x,t}).$$

4.2 Lee-Carter model

The Lee-Cater model models the age-specific death rates through (as in [22, Section 3.2.1]):

$$\ln(m_{x,t}) = a_x + b_x \kappa_t,$$

where a_x, b_x are age-specific parameters with $\sum_x b_x = 1$ and κ_t is a random walk with drift d and a sequence $\epsilon_t \sim \mathcal{N}(0, c)$ of independent and identically distributed (i.i.d.) normal random variables with a zero mean and a constant variance:

$$\kappa_t = d + \kappa_{t-1} + \epsilon_t,$$

and $\sum_t \kappa_t = 0$.
The parameters can be interpreted in the following way (see also: [21]):

- κ_t describes the general mortality changes over time,
- b_x describes how much of the mortality changes for each age x in the cohort depends on changes in κ_t and
- a_x gives the average mortality rate for age x.

Remark 4.2. In the original model, an error term $\mathrm{e}_{x,t}$ for age-period effects was added.

The change in log-central death rates for age x between years $t-1$ and t is

$$\begin{aligned}\ln(m_{x,t}) - \ln(m_{x,t-1}) &= a_x + b_x(d + \kappa_{t-1} + \epsilon_t) - a_x + b_x \kappa_{t-1} \\ &= b_x d + b_x \epsilon_t.\end{aligned}$$

Therefore, the mean forecast of the change in the log-central death rate between years t and $t-1$ equals $b_x d$ and only varies with age. Hence, the Lee-Cater model has a one dimensional mortality improvement scale. (See also [22, Section 3.2.1].)

4.3 The Cairns-Blake-Dowd (CBD) model

In the CBD model the logit transformation of $q_{x,t}$ follows:[40]

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \overline{x}),$$

[40]The logit transformation of a real number x is $\ln\left(\frac{x}{1-x}\right)$.

where $\overline{x}$ is the average age of the used age range, $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are stochastic processes that are assumed to be measurable at time t.[41] Set $\kappa_t = (\kappa_t^{(1)}, \kappa_t^{(2)})^\top$.

The parameters can be interpreted in the following way (see [8, Section 2.2]):

- $\kappa_t^{(1)}$ describes the general level of the mortality $q_{x,t}$ in year t after a logit transformation. A reduction in $\kappa_t^{(1)}$ implies an equivalent downward shift of $\ln(\frac{q_{x,t}}{1-q_{x,t}})$ and represents an overall mortality improvement.

- $\kappa_t^{(2)}$ does not influence the level of mortality for $x = \overline{x}$. The greater the difference $|x - \overline{x}|$, the greater the effect of changes in $\kappa_t^{(2)}$ on the level of mortality. An increase in $\kappa_t^{(2)}$ implies a mortality improvement for $x < \overline{x}$ and simultaneously a mortality deterioration for $x > \overline{x}$.

Remark 4.3. The two CBD mortality indices can be used to represent a logit-transformed mortality curve with slope $\kappa_t^{(2)}$ and level $\kappa_t^{(1)}$.

Remark 4.4. When comparing how the parameters $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ influence the longevity risk for a pension plan sponsor and a life insurer, one can see that changes in $\kappa_t^{(1)}$ affect the entities in the opposite way, but changes in $\kappa_t^{(2)}$ can affect both companies in the same way. This dynamic offers an explanation as to why natural hedging may not work perfectly in practice. Furthermore, it hints that the association between the two CBD mortality indices has a significant impact on the longevity risk exposure of a portfolio.

Remark 4.5. In [8, Section 1] three criteria for creating a mortality (or longevity) index from a stochastic model are defined:

1. The vector of indices should represent the varying age-pattern of mortality improvement, rather than the overall level of mortality.

2. The stochastic model should possess the new-data-invariant-property. This means that updating the model, by adding an additional year of mortality data, will not affect the historical values of the index.

3. The mortality index should be readily interpretable, so that they can be communicated easily to hedgers, investors and the general public.

The CBD model satisfies all of them, which is partly inferred from the fact that the model has neither age-specific nor cohort-specific parameters and furthermore no restrictions on κ are made. Hence, extending the sample

[41] By equivalent transformation of the equation above and the assumed link $m_{x,t} = -\ln(1-q_{x,t})$, the mortality probability $q_{x,t}$ and the central death rate $m_{x,t}$ are represented as follows:
$q_{x,t} = \frac{\exp\left(\kappa_t^{(1)}+\kappa_t^{(2)}(x-\overline{x})\right)}{1+\exp\left(\kappa_t^{(1)}+\kappa_t^{(2)}(x-\overline{x})\right)}$ and $m_{x,t} = \ln(1+\exp\left(\kappa_t^{(1)}+\kappa_t^{(2)}(x-\overline{x})\right))$

period will not affect historical optimal values for κ. (See [8, Section 2].) The missing age-specific parameter is also the reason that the CBD model is so well suited for projecting mortality rates for older ages.

Remark 4.6 (CBDX models). In 2020 Dowd, Cairns, and Blake [13, Section 2] published a workhorse mortality model from the Cairns-Blake-Dowd Family, which comes in three versions:

$$\ln(m_{x,t}) = \alpha_x + \sum_{i=1}^{K} \beta^i \kappa_t^{(i)} + \gamma_c,$$

where a_x is an age-specific parameter, γ_c describes the cohort-related effects,[42] where $c = t - x$ denotes the year of birth and $\sum_c \gamma_c = 0$, and

1. $K = 1$: $\kappa_t^{(1)}$ is a stochastic process that is assumed to be measurable at time t and $\beta^1 = 1$.

2. $K = 2$: $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ are stochastic processes that are assumed to be measurable at time t, $\beta^1 = 1$ and $\beta^2 = (x - \overline{x})$, where $\overline{x}$ is the average age of the used age range.

3. $K = 3$: $\kappa_t^{(1)}$, $\kappa_t^{(2)}$ and $\kappa_t^{(3)}$ are stochastic processes that are assumed to be measurable at time t, $\beta^1 = 1$, $\beta^2 = (x - \overline{x})$ and $\beta^3 = (x - \overline{x})^2 - \sigma_x^2$, where $\overline{x}$ is the average age and σ_x^2 is the variance of the used age range.

Depending on the version used, the following constraints apply:
$\sum_t \kappa_t^i = 0$ and $\sum_c c^i \gamma_c = 0$ for $i = 1, \dots, K$
In [13, Section 4], the CBDX models were estimated based on England & Wales male mortality data from $1971 - 2015$. The used method was Hunt and Blake's [17, Section 3] general procedure, where an adjusted age-period model is first obtained before adding in the cohort effect that then captures the residual year-of-birth effects. According to the results obtained, Dowd, Cairns, and Blake [13, Section 10] concluded that the preferred model for wider age ranges is the CBDX model with three period effects ($K = 3$).

Since the CBDX models include an age effect (through α_x), those models also face problems when projecting mortality at advanced ages. In order to provide a solution to this problem, Dowd and Blake [12] introduced an extension to the CBDX model to project cohort mortality rates to extreme old age in 2022. The approach taken in [12, Section 3] is to smooth and project α_x with a polynomial function of age x. Subsequently, this age-specific parameter is used to obtain the smoothed mortality rates from the CBDX model.

[42] The cohort-related effect γ_c is modeled as a residual that should be trend-less and mean-reverting if the model is well-specified.

4.3.1 κ modeled by a bivariate random walk with drift

In [4], κ is modeled as a two-dimensional random walk with drift $d = (d^{(1)}, d^{(2)})^\top$:

$$\kappa_t = d + \kappa_{t-1} + C\epsilon_{t-1},$$

where C is a constant upper triangle matrix and $\epsilon_t = (\epsilon_t^{(1)}, \epsilon_t^{(2)})^\top \sim \mathcal{N}(0, I)$. Here I is the identity matrix.
The change of the logit transformed mortality rate for age x between years $t-1$ and t is calculated using the matrix notation:

$$\begin{aligned}
\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) - \ln\left(\frac{q_{x,t-1}}{1-q_{x,t-1}}\right)& \\
&= (1, x-\overline{x})\kappa_t - (1, x-\overline{x})\kappa_{t-1} \\
&= (1, x-\overline{x})(d + \kappa_{t-1} + C\epsilon_{t-1}) - (1, x-\overline{x})\kappa_{t-1} \\
&= (1, x-\overline{x})(d + C\epsilon_{t-1})
\end{aligned}$$

Therefore, the mean forecast of the change in the logit transformed mortality rate between years $t-1$ and t equals $d^{(1)} + (x-\overline{x})d^{(2)}$ and only varies with age.

Remark 4.7. This mortality model performs well for mortality at higher ages, but is not suitable for short term longevity risks. Another disadvantage when κ is modeled by a bivariate random walk is that it cannot capture serial- and cross-correlations between the two CBD mortality indices. In [8] this problem is overcome by using a VARIMA(p, d, q) process instead of a bivariate random walk.

4.3.2 κ modeled by a VARIMA(p, d, q) process

Definition 4.8 (*Vector Autoregressive Moving-Average process* VARMA(p, q))**.**

First, let's introduce a *Vector Autoregressive Moving-Average* VARMA(p, q) process, which is defined as a combination of a *Vector Autoregressive process* VAR(p) and a *Moving Average process* MA(q) process:[43]

$$\kappa_t = c_0 + \sum_{i=1}^{p} \phi_i \kappa_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t, \tag{4.1}$$

[43]Let $\{\kappa_t\} = \{(\kappa_{1,t}, \dots, \kappa_{n,t})^\top\}$ be a multivariate time series.
$\{\kappa_t\}$ is a VAR(p) process if $\kappa_t = c_0 + \sum_{i=1}^{p} \phi_i \kappa_{t-i} + \epsilon_t$, where c_0 is a n-dimensional intercept vector, ϕ_i are $n \times n$-dimensional coefficient matrices and $\{\epsilon_t\}$ is a n-dimensional white noise process, a sequence of uncorrelated random variables with constant variance and constant mean. Often, the mean value is assumed to be zero.
$\{\kappa_t\}$ is a MA(q) process if $\kappa_t = \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}$, where $\{\epsilon_t\}$ is a n-dimensional white noise process.

where c_0 is a 2-dimensional intercept vector, ϕ_i and θ_j are 2×2-dimensional coefficient matrices and $\{\epsilon_t\}$ is an i.i.d process with zero mean and variance-covariance matrix Σ_ϵ.

Definition 4.9 (*order of integration*).
A VARMA(p, q) process *is integrated of order* d, if the d^{th} difference of the process is strongly stationary.

Definition 4.10 (*backward difference*).
The d^{th} *backward difference* is

$$\nabla^d \kappa_t = \nabla^{d-1}(\nabla \kappa_t) = \nabla^{d-1}\kappa_t + \nabla^{d-1}\kappa_{t-1}.$$

Using $\binom{d}{k} = \binom{d-1}{k} + \binom{d-1}{k-1}$ one can show by induction that

$$\nabla^d \kappa_t = \sum_{k=0}^{d} \binom{d}{k} (-1)^k \kappa_{t-k}.$$

Written with the backshift operator $\mathrm{B}^k \kappa_t = \kappa_{t-k}$:[44]

$$(1 - \mathrm{B})^d \kappa_t = \sum_{k=0}^{d} \binom{d}{k} (-B)^k \kappa_t.$$

Definition 4.11 (*Vector Autoregressive Integrated Moving-Average process* VARIMA(p, d, q)).
Assuming that the VARMA(p, q) process in Definition 4.8 is integrated of order d, then the d^{th} difference of the process:

$$\nabla^d \kappa_t = c_0 + \sum_{i=1}^{p} \phi_i \nabla^d \kappa_{t-i} + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t,$$

is a VARIMA(p, d, q) process.[45]

In [8, Section 3.2] a procedure for identifying an optimal VARIMA(p, d, q) model is shown using the multivariate generalization of the Box and Jenkins approach, an iterative process, which consist of three steps: model identification, model estimation and diagnostic checking.
For the model identification, a sample cross-correlation matrix and a sample partial autoregression matrix are estimated from a sample of historical values $\{\kappa_t\}_{t \in T_H} = \{(\kappa_t^{(1)}, \kappa_t^{(2)})^\top\}_{t \in T_H}$. If the sample cross-correlation matrix decays

[44] In time series theory usually used as lag operator $\mathrm{L}^k \kappa_t = \kappa_{t-k}$.
[45] Written with the lag operator: $(1 - \mathrm{L})^d \kappa_t = c_0 + \sum_{i=1}^{p} \phi_i (1 - \mathrm{L})^d \kappa_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} + \epsilon_t$

to zero with an increasing lag, the parameter p and q can be picked out of the sample matrices using the cut-off properties and $d = 0$. Otherwise, the underlying process $\{\kappa_t\}_{t \in T_H}$ is non-stationary. Therefore, the backshift operator is applied until the sample cross-correlation matrix estimated from the sample $\{(1 - \mathrm{B})^d \kappa_t\}_{t \in T_H}$ decays to zero and then p and q can be picked out of the corresponding sample matrices.

Subsequently, in the second step, the model parameters are estimated using the conditional maximum likelihood method. After eliminating statistically insignificant parameters, the remaining model parameters are re-estimated by the exact likelihood method.

Finally, to prevent model misspecification, diagnostic checking of the residuals was carried out.

4.4 The heat wave model

The goal of the first version of the heat wave model was to develop a stochastic mortality model, which produces a two-dimensional mortality improvement scale that fulfills the following properties (see also: [22, Section 1]):

- Higher short-term improvement rates converge gradually to lower long-term (ultimate) values.
- The model includes minimal expert judgment.[46]
- The model includes measures of uncertainty.
- The model is easy to implement.

In the improved version, two identified limitations of the original model were eliminated by allowing multiple heat waves and allowing the blend between period and cohort effects to interact with age using re-parameterization. (See also [23].)

Assumption 4.12 (See also: [23].)**.**

- mortality improvements consist of two components:
  - *the background* represents low levels of mortality improvements that always exist, and
  - *the heat waves* represent transient excesses over the background mortality improvements.

  The assumption above enables the heat wave model to distinguish between short- and long-term mortality improvements.
- The background component extends linearly into the future.
- The heat waves tape off over time.

In the heat wave model, the natural logarithm of the central rate of mortality at age x in year t follows:

$$\ln(m_{x,t}) = \underbrace{a_x + b_x \kappa_t}_{\substack{\text{background} \\ \text{Lee-Carter model}}} + \underbrace{\sum_{i=1}^{n} c_x^{(i)} g(x, t; \theta^{(i)})}_{\text{heat waves}}$$

where a_x, b_x and $c_x^{(i)}$ are age-specific parameters, κ_t is a stochastic processes measurable at time t (usually a random walk with drift), $n \geq 1$ gives the number of heat waves and the function g represents the cumulative effect of

[46] That means that the model requires minimal input from a person with expertise in this area.

the i^{th} heat wave and is a parametric function of age x, year t and parameter vector $\theta^{(i)} = (\mu_1^{(i)}, \ldots, \mu_{u^{(i)}}^{(i)}, h_1^{(i)}, \ldots, h_{u^{(i)}}^{(i)}, \sigma^{(i)})^\top$:

$$g(x, t; \theta^{(i)}) = \sum_{s=t_0}^{t} f(x, s; \theta^{(i)})$$

The i^{th} heat wave $f(x, s; \theta^{(i)})$ is assumed to be non-recurrent, symmetric and characterized by the probability density function of a normal distribution. To allow the blend between period and cohort effects to interact with age, the function f is defined as follows:
Let $[x_0, x_1]$ be the given age range, which is divided into $u^{(i)}$ segments: $[x_0, p_1^{(i)})$, ..., $[p_{u^{(i)}-1}^{(i)}, x_1]$. In any age segment $j \in \{1, \ldots, u^{(i)}\}$ the peak of the i^{th} heat wave aligns on a straight line with the gradient of $h_j^{(i)}$, a parameter that measures the extent to which the blend between period and cohort effect interact with the age over the age segment.
Then for x in the j^{th} age segment:

$$f(x, s; \theta^{(i)}) = \frac{1}{\sqrt{2\pi(\sigma^{(i)})^2}} \exp\left(- \frac{\left((s - t_0) - \mu_j^{(i)} - (x - x_0)h_j^{(i)}\right)^2}{2(\sigma^{(i)})^2} \right)$$

As $\lim_{s\to\infty} f(x, s; \theta^{(i)}) = 0$ for all $i \in \{1, \ldots, n\}$ the function f fulfills the assumption that heat waves taper off over time.

Interpretation of the Parameter $\theta^{(i)}$:

- $\sigma^{(i)}$ determines the speed at which the i^{th} heat wave tapers off over time. As the normal density becomes close to zero (0.054) at two standard deviations above mean, $2\sigma^{(i)}$ may be regarded as the approximated duration between the peak and the end of the i^{th} heat wave.

- $\mu_j^{(i)}$ determines the location of the peak of the i^{th} heat wave for age x in the j^{th} age segment, which occurs in the year $t_0 + \mu_j^{(i)} + (x - x_0)h_j^{(i)}$. To ensure that the line representing the peak of the i^{th} heat wave is continuous,

$$\mu_j^{(i)} + (p_j^{(i)} - x_0)h_j^{(i)} = \mu_{j+1}^{(i)} + (p_j^{(i)} - x_0)h_{j+1}^{(i)} \quad \forall j \in \{1, \ldots, u^{(i)} - 1\}$$

  must apply.

- $h_j^{(i)} \in [0, 1]$ determines how much of the transient excesses of the i^{th} heat wave in the j^{th} age segment can be regarded as a result of period (time-related) and cohort (year-of-birth related) effects. If $h_j^{(i)} = 1$, then the peak of the i^{th} heat wave in the j^{th} age segment would increase by one year as age increases by one year. Hence, the transient excess of the i^{th} heat wave over the background improvements are solely a consequence of cohort effects. If $h_j^{(i)} = 0$, the peak of the i^{th} heat wave in the j^{th} age segment occurs at the same time for all ages x. Hence, the transient excess of the i^{th} heat wave over the background improvements are solely a consequence of period effects.

The change in log-central death rates for age x between years $t-1$ and t:

$$\begin{aligned}
&\ln(m_{x,t}) - \ln(m_{x,t-1}) \\
&\quad = b_x d + b_x \epsilon_t + \sum_{i=0}^{n} c_x^{(i)} \Big(\sum_{s=t_0}^{t} f(x,s;\theta^{(i)}) - \sum_{s=t_0}^{t-1} f(x,s;\theta^{(i)}) \Big) \\
&\quad = b_x d + b_x \epsilon_t + \sum_{i=0}^{n} c_x^{(i)} f(x,t;\theta^{(i)})
\end{aligned}$$

Therefore, the mean forecast of the change in the log-central death rate between years t and $t-1$ equals $b_x d + \sum_{i=0}^{n} c_x^{(i)} f(x,t;\theta^{(i)})$ and varies with age and time. As the second part converges to zero over time, over the long run the log central death rate at age x is expected to change at a rate of $b_x d$ per year.

5 Longevity risk transfer market

In [2] Blake introduces the market for trading longevity-linked assets and liabilities as the Life Market, which is subdivided into two segments:

- In the Macro Life Market, participants deal assets and liabilities that are linked to a group of lives (e.g. members of a pension plan (see Definition 5.11)).
- In the Micro Life Market, assets and liabilities linked to individual lives such as individual life insurance policies or life settlements are traded.

Definition 5.1 (life settlement market)**.**
The primary market of the life settlement market is the life insurance market, where entities write life insurance policies for individuals.
The life settlement market includes the secondary and the tertiary market:

- In the secondary market, insured policyholders (usually seniors) sell their existing life insurance policies for a lump sum to a third party, who subsequently becomes the beneficiary in case the insured, who remains the seller, dies and takes over upcoming premium payments.
- In the tertiary market, these existing insurance policies are traded among third party investors.

Remark 5.2. The life settlement market is dominated by the US. (See in [2])

Remark 5.3. The global pension, insurance and reinsurance market is strongest in the following countries:

- Global pension market
  The largest pension market is in the US (about 65 % of the global pension market (OECD countries)), followed by the UK, which accounts for only 6.6 % of the global pension market, Canada and the Netherlands.[47]
- Global insurance market
  In 2020 Swiss Re published the market share of life and non-life insurers, calculated by the value of insurance premiums written.[48] The dominant country was the US (with about 40 % of the market share) followed by China, Japan and the UK. Taking into account only the 50 leading insurance companies, ranked by total assets as of the

[47] OECD-Report for 2020:
pension market: `https://www.oecd.org/finance/private-pensions/globalpensionstatistics.htm`
insurance market: `https://www.oecd.org/pensions/globalinsurancemarkettrends.htm`

[48] https://www.statista.com/statistics/217269/leading-countries-by-percent-of-total-world-life-and-nonlife-premiums-written/

31.12.2021, the four leading countries (USA, Japan, China and the UK) have a combined market share of 60 %.[49]

- Global reinsurance market
  The top four reinsurance companies ranked by unaffiliated gross reinsurance premiums written in 2020[50] (Munich Re, Swiss Re, Hannover Re and SCOR) are all located in Europe, and the 3 leaders are in German-speaking countries. Taking into account only the 50 leading reinsurance companies, ranked by gross written premiums in 2020, the four leading countries (Germany, Bermuda, USA and Switzerland) have a combined market share of 65 %.[51]

Remark 5.4. When speaking of the longevity risk transfer market or the Life Market in the following, we always refer to the Macro Life Market.

5.1 Longevity risk transfer market size

According to the digital trading platform Longitude Exchange, the total longevity-linked liabilities globally are estimated to be $100 trillions. The current global annual demand for longevity risk transfer amounts to $600 billions, of which at present $25 billions are transferred annually.[52] For instance, Legal & General, a multinational financial service and asset management company, estimated the market size for pension risk transfer for 2021 in the US at $38 – 40 billions and in the UK at $30 billions.[53]

On the following website `https://www.artemis.bm/longevity-swaps-and-longevity-risk-transfers/` artemis publishes a list of longevity swaps, longevity risk transfers and longevity reinsurance transactions that have taken place in the reinsurance and capital market.

[49] Data obtained from the following website: `https://www.advratings.com/insurance/worlds-top-insurance-companies`

[50] `https://www.reinsurancene.ws/top-50-reinsurance-groups/`

[51] `https://www.atlas-mag.net/en/article/top-50-global-reinsurers`

[52] `https://www.longitude.exchange/`

[53] `https://www.lgra.com/docs/librariesprovider3/lgra--knowledge-center/us-and-uk-joint-prt-monitor---feb-2022.pdf?sfvrsn=31bb07d9_14`

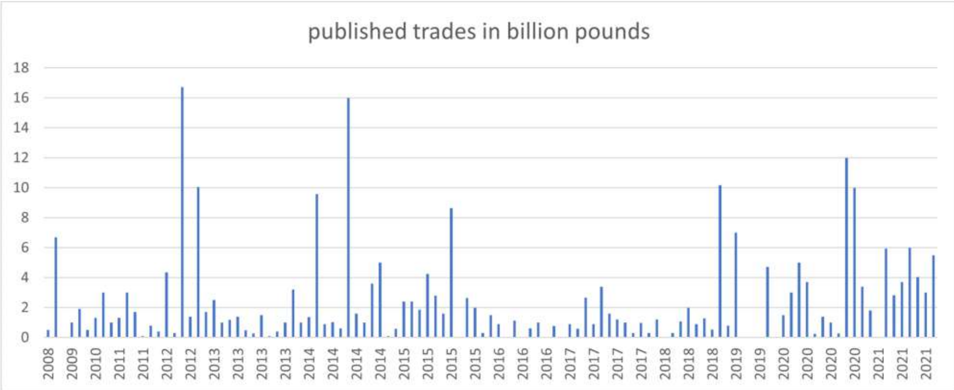


Figure 3: Note that this list is not necessarily a complete listing of all the deals that have taken place, and for some transactions the size is not known. In each of the years 2008, 2015, 2018 and 2021 one deal was affected. Two deals of unknown size were executed in the year 2016 and in 2019 there were even 4 deals, where the size is not known. Since most trades (about 60 %) were executed in pounds, the volume in the following graphs is also given in billions of pounds.

The largest transaction was a pension liability buy-out in 2012. In this way, General Motors transferred \$26 billions (or £16.7) worth of future pension obligations to Prudential. The largest pure longevity swap took place in 2014. BT plc, a multinational telecommunications holding company, established its own insurance company to which £16 billions of BT's longevity exposures were transferred. This insurance company in turn entered into a £16 billions longevity swap with the Prudential Insurance Company of America.

Since 2019 the average volume per trade has been at a consistently higher level compared to previous years.

Remark 5.5. So far in 2022 one transaction between the Lloyds Banking Group pension schemes, Scottish Widows (a life insurance and pensions company and a subsidiary of Lloyds Banking Group) and SCOR (a French reinsurance and financial services company) took place, which was a longevity swap and reinsurance deal over £5.5 billion.

As SCOR covers 100 % of the longevity exposure associated with the longevity swap by reinsurance, Scottish Widows is acting as an intermediary insurer in this transaction.

Based on this information, we can now take a look at the development of the longevity risk transfer market:

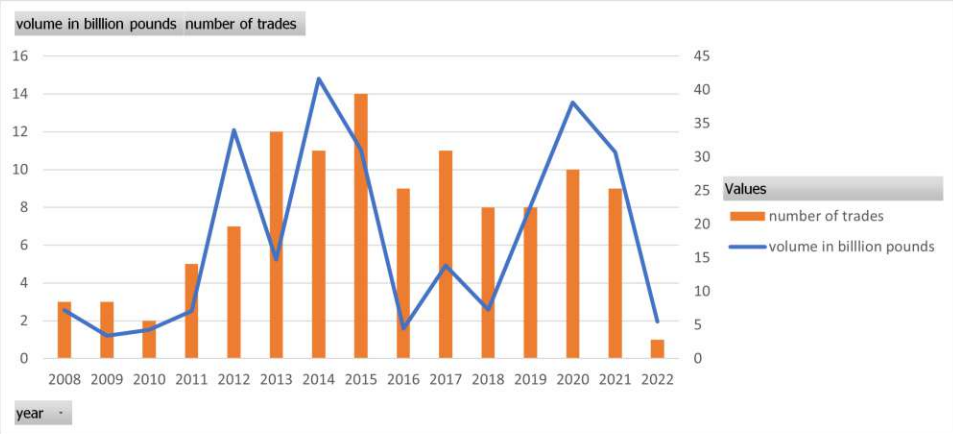


Figure 4: The primary y-axis shows the number of transactions in orange. In the early years, no more than 5 transactions were made each year. From 2010 onward, the number of transactions increased and peaked in 2013 and 2015. Since 2016, the number of transactions leveled off around the average of the last 9 years, which corresponds to 10 transactions per year.
The secondary y-axis shows the total annual size of transactions, given in billions of pounds (blue).
As the number of transactions increases, so does the annual volume traded. Low points can be observed in the years 2013 and 2016 – 2018, due to transactions of exclusively smaller size. (As shown in Figure 3.) On average the annual size of longevity risk transfer transactions amounts to £18.6 billions, which corresponds to about $23 – 25 billions, and is in line with the Longitude Exchange's estimate, bearing in mind that for some deals the size of the transaction is unknown.

The published longevity risk transfer transactions can be divided into the following four categories:

- SR denotes a longevity swap and reinsurance transaction, analogously to Remark 5.5 or Figure 16.
- S denotes a pure longevity swap as defined in Section 7.3.
- R denotes a reinsurance transaction as defined in Definition 5.19 and
- LLI denotes a Longevity-Linked Instrument other than a pure longevity swap. (See Section 7)

The following figure shows the share of the individual categories in the concluded transactions.

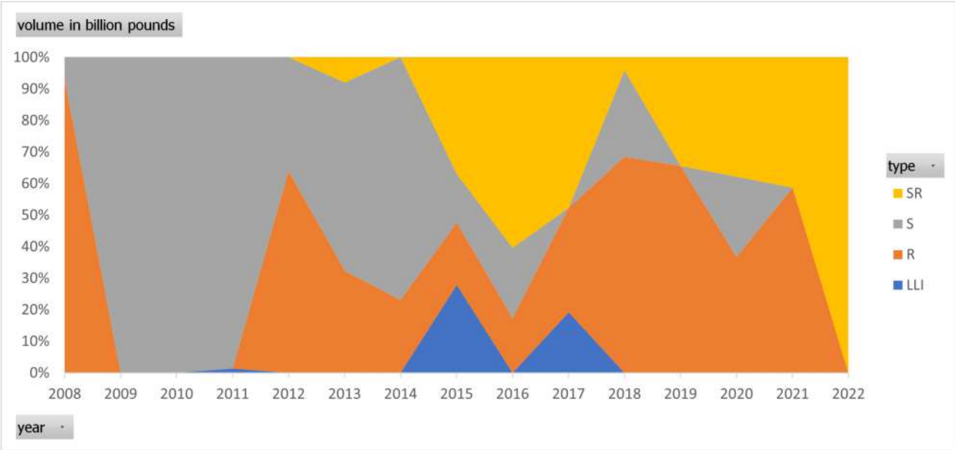
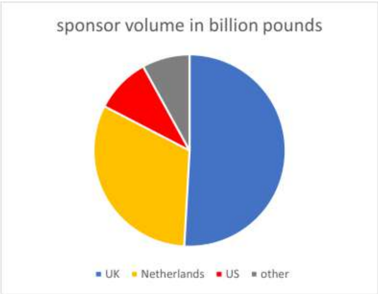


Figure 5: In the years 2009 – 2011 almost all transactions were pure longevity swaps with a total volume of approximately £15 billions. In the following three years 40 % of the transactions were pure longevity swaps with a total volume of approximately £53 billions. From 2015 onward, the number of pure longevity swaps transactions dropped.
At the same time, the longevity swap and reinsurance transactions (SR) got more attractive until 2018, when only one swap and reinsurance transaction was concluded. Since then, however, the number of trades has increased annually.
The number of reinsurance transactions has been fairly stable since 2012, with an average of 5 trades per year, in contrast to the annual volume traded. Most transactions so far were pure reinsurance deals. (In total, 50 transactions out of 113.) If the number of longevity swaps and reinsurance transactions (SR) are combined, these two types of deals cover almost all the transactions that have taken place for the last seven years. (Together never less than 70 %.)

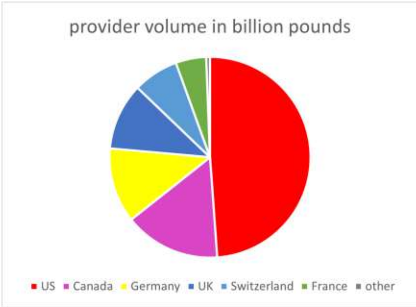
5.2 Longevity risk transfer marketplace

In [14, Section 1] it is stated that the longevity risk transfer market is still in an early state. It is currently an illiquid and incomplete market. To date, most longevity risk transfer deals are concluded over-the-counter (OTC) and the market is dominated by reinsurance companies. Moreover, since most transactions are concluded OTC, it is difficult to obtain reliable information on details such as investor returns.

Countries where companies have concluded longevity risk transfer deals during 2008 – 2021:
The following charts show that the UK and the US are the two countries dominating the Life Market based on the trades published on artemis.

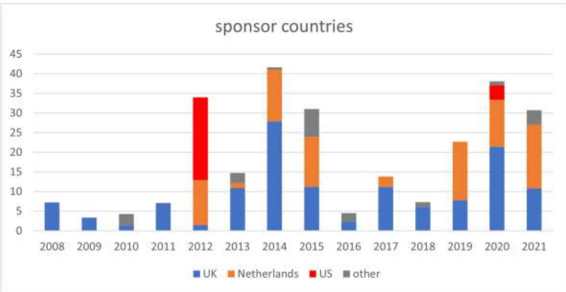


(a)

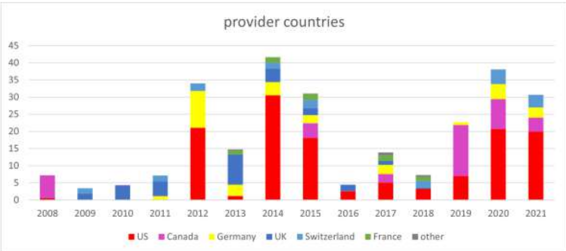


(b)

Figure 6: This graph shows that during 2008 – 2021 most longevity risks were transferred from companies in the UK, the Netherlands or the US to companies from the US, Canada, Germany, the UK, Switzerland, or France.



(a)



(b)

Figure 7: This chart shows the countries where companies have concluded longevity risk transfer deals in the period 2008 – 2021. In most years, the UK was the country with the highest demand for longevity risk transfer, followed by the Netherlands. The demand in the US resulted mainly from the largest published transaction, a buy-out in 2012, as mentioned in Figure 3.
On the provider side, currently the US and Canada dominate the market. In the years 2009 – 2011 and 2013 companies from the UK such as Rothesay Life or Abbey Life insured most of the longevity risk through bespoke longevity swaps, but since 2014 the influence of the UK has declined.

Industries in which companies have concluded longevity risk transfer deals during 2008 – 2021:

provider		sponsor	
industry	volume	industry	volume
Reinsurance	142.3	Financial services	81.93
Insurance	84.42	Pension plan	70.88
Financial services	39.38	Insurance	47.13
		Automotive	23.11
		Pension fund	13.18
		Aerospace	8.28
		others	1.58

Table 2: This table shows the transactions published on the artemis website during 2008 – 2021, broken down by provider and sponsor industries. The volume corresponds to the total volume of the transaction in billions of pounds.

Clearly, most longevity risk transfer transactions (85 %) were provided by insurance or reinsurance companies such as Prudential, Reinsurance Group of America (RGA) or Canada Life, and about 15 % of longevity risk transfer transactions were provided by financial services companies such as Deutsche Bank or Legal & General.

About almost 32 % of the demand for longevity risk transfer arises from pension plans or pension funds, followed by the demand from financial services companies such as Aegon or Delta Lloyd. Insurance companies such as NN Group meet about 18 % of the demand.

providers	industry	volume	sponsors	industry	volume
Prudential	Reinsurance	65.03	Aegon	Financial services	31.58
RGA	Reinsurance	43.63	Delta Lloyd	Financial services	18.21
Canada Life	Reinsurance	34.06	NN Group	Insurance	18.06
Pacific Life Re	Reinsurance	16.94	General Motors	Automotive	16.71
Hannover Re	Reinsurance	15.30	BT Pension Scheme	Pension plan	16
Swiss Re	Reinsurance	15.27	Lloyd's Banking Group	Pension plan	15.5
SCOR	Financial services	13.06	Athora Netherlands	Financial services	11.58
Abbey Life	Insurance	12.9	Pension Insurance Corporation	Insurance	9.28
Deutsche Bank	Financial services	10.05	HSBC UK Pension Scheme	Pension plan	7
Legal & General	Financial services	7.66	Standard Life	Financial services	6.7

Table 3: Top 10 companies (providers and sponsors) that transferred the most longevity risk by volume in billions of pounds.

In 2019 Kessler [20, Section 13] stated that while the reinsurance capacity for the global longevity and annuity sector is currently sufficient, when the demand will exceed the supply, capital market solutions will be needed.

For example, one step towards opening up the Life Market and attracting a wider audience of investors was the recent formation of a new digital trading platform, on which index based longevity-linked instruments are traded:[54]

Longitude Exchange
In March 2022 the digital trading platform called Longitude Exchange[55] — founded by CEO Avery Michaelson, CFO David Schrager of Longitude Exchange, who are also Founding Partner and Senior Partner of Longevity Solutions[56] and CTO Diederick Venekamp, who is the co-founder of VB Risk Advisory[57] — was launched in Bermuda. Shortly after the launch, Longitude Exchanged partnered with Club Vita to integrate Club Vita's longevity risk classification services into the platform.[58]
The purpose of this platform is to build a digital longevity marketplace by connecting hedgers and institutional investors interested in trading index-based longevity risk. Initially, the platform will focus on the North American and the European market, and aims to act on a global basis in the future.[59]

5.3 Pension plans, life insurance and reinsurance

Definition 5.6 (*annuity*)**.**
An *annuity* is a contract concluded between an insurance company and an individual (i.e. the policyholder), who makes a lump-sum payment or series of payments in the accumulation phase in exchange for a stream of periodic payments beginning at an agreed point in time after the accumulation period.

[54] artemis news: https://www.artemis.bm/news/digital-marketplace-for-index-based-longevity-risk-to-open-in-bermuda/
[55] https://www.longitude.exchange/
[56] Longevity Solutions is a U.S. transaction-oriented advisor with expertise in longevity risk, who advised the index-based longevity hedge between the NN Group and Hannover Re in 2017.
[57] VB Risk Advisory is a Dutch consultancy firm with expertise in actuarial science and technology development.
[58] Club Vita is a longevity analytic firm operating in the UK, Canada, and the U.S.
In 2014 Club Vita developed a longevity trend segmentation, called VitaSegments, in cooperation with UK's Pension and Lifetime Saving Association, for man (3 groups) and woman (2 groups) in the UK. See https://www.clubvita.net/glossary/vitasegments or the NAPF mortality model.
Furthermore, Club Vita provides the VitaCurves, a statistical model of the diverse range of survival patterns, which are updated annually with data that are collected and processed by Mercer. For more detailed information: https://www.clubvita.us/collaborative-research/2021-us-vitacurves-technical-papers.
[59] artemis news:
https://www.artemis.bm/news/digital-marketplace-for-index-based-longevity-risk-to-open-in-bermuda/
https://www.artemis.bm/news/club-vita-longitude-exchange-eye-more-transparent-efficient-longevity-market/

The pay-out phase, called the annuitization phase, can be agreed for a few years or last for the rest of the individual's life.

Remark 5.7. Annuities are designed to secure a steady cash flow for individuals during their retirement years and are a hedge against outliving their retirement income in case of an annuity with an annuitization phase for the rest of the individual's life.
There are 3 types of annuities:

- In case of *fixed annuities*, a specific rate of interest for the accumulation phase is agreed on, and the amount of the periodic payments is therefore fixed.
- In case of *indexed annuities*, the periodic payments correspond to the maximum of a guaranteed minimum value and a value linked to the performance of an agreed index.
- In case of *variable annuities*, the policyholder can choose from several investment options during the accumulation period. The resulting rate of return then determines the amount of the periodic payments.

Definition 5.8 (*defined benefit (DB) pension plan* [25])**.**
This type of pension plan (or pension scheme) is arranged by the pension plan sponsor, who for example can be a company or the government. Under the pension plan, the employees - member of the pension plan - receives a fixed annuity based on a predefined function that depends on the employee's salary, length of service and age at retirement. In addition to the longevity risk the sponsor also bears the investment risk.
Contributions can be made by either or both parties, which also applies to the DC pension plan.

Definition 5.9 (*defined contributions (DC) pension plan* [25])**.**
With this pension plan (or pension scheme), each member has an individual account into which the agreed contributions are paid. In addition to the contributions paid into the individual account, the annuity also depends on the returns on the investment and therefore fluctuates.

Remark 5.10. In this case of a DC pension plan, the entire risk (longevity and investment) lies with the member.

Definition 5.11 (type of members [25])**.**

- An *active member* is a member from whom contributions are currently being paid.
- A *deferred member* is a member from whom currently no contributions are paid and who does not receive his annuity yet.

- A *pensioner* or *retired member* is a member who receives his annuity.

Definition 5.12 (*pension plan trustee*)**.**
A *pension plan trustee* is someone who has legal control over the assets of the pension plan. They often have a fiduciary responsibility and are trusted to act in the beneficiary's best interest.

Definition 5.13 (*pension plan sponsor*)**.**
A *pension plan sponsor* is a company that offers a pension plan to their employees.

Remark 5.14. Often those pension plans are managed by a third-party (i.e. a pension plan trustee).

In different countries, there are preferential segmentation of pension plan members:[60]

Definition 5.15 (*top slicing*)**.**
To reduce longevity risk (in particular concentration of risk and idiosyncratic risk), *top slicing* selects those members with the highest pension amounts in the pension plan and transfers their liabilities. This is common in the UK.

Definition 5.16 (*bottom slicing* or *lift out*)**.**
In the US, pension plans that are qualified under the US tax code pay a flat-rate premium per member to the Pension Benefit Guaranty Corporation (PBGC), a federally chartered corporation. An option to reduce the number of members and therefor also the flat-rate premium at low cost is by *bottom slicing*, which selects those members with the lowest pension amounts to transfer their liabilities.

Definition 5.17 (*thin slicing*)**.**
In Canada, in case of an insolvency of a life insurer, Assuris — a nonprofit organization under Canadian Federal regulation — will seek to transfer the affected policies to a solvent life insurance company, guaranteeing to retain at least 85 % of the insurance benefits.[61] Thus *thin slicing*, which selects a specified percentage of the liabilities in the pension plan to transfer, is common in Canada.

Definition 5.18 (*life insurance* and *term assurance*)**.**

- A *life insurance* is a contract concluded between a life insurance company and an individual (i.e. the policyholder), who pays a specific amount of premiums. In return, the sum insured (or cover amount) is paid out to the designated beneficiaries upon the death of the policyholder.

[60] `https://www.clubvita.us/events/longevity-103-longevity-risk-management`
[61] For more detailed information: `http://assuris.ca/how-am-i-protected/`

- A *term life insurance or term assurance* is a contract concluded between a life insurance company and an individual (i.e. the policyholder), who pays a specific amount of premiums. In return, the sum insured (or cover amount) is paid out to the designated beneficiaries if the policyholder dies within an agreed period of time.

Definition 5.19 (*reinsurance*).
A *reinsurance* is a contract concluded between an insurance company and a reinsurer (a third party) to share risk associated with policies held by the insurance company.

Remark 5.20 (reinsurance terminology). In the reinsurance field, companies that hold longevity risk and want to hedge this risk by transferring it to another company are called cedents. By entering a (reinsurance) contract the cedent cedes its longevity risk to the counterparty (e.g. a reinsurer).

5.4 Tools for constructing longevity-linked solutions

Definition 5.21 (cashflow and value hedge).

- A *cashflow hedge* mitigates the risk that future cash flows deviate from the expected cash flows.
- A *value hedge* mitigates the risk of changes in the value of the hedged position.

Remark 5.22. In [6, Section 5.4.8.] it is stated that value hedges are more common in the capital markets, while cash flow hedges are more common in the insurance world.
An example of a value hedge is the q-forward for non-pensioners, which took place in 2011 and is described in Example 7.7. Examples of cashflow hedges include bespoke longevity swaps, such as the swap between Canada Re and J.P. Morgan in 2008 or the swap in 2013 between BAE Systems, a British multinational arms, security, and aerospace company, and Legal & General, covering $17,000$ pensioners from two of BA's pension funds.

Definition 5.23 (commutation function mechanism).
The commutation function mechanism attempts to hedge the remaining risk resulting from the fact that the period a company is exposed to longevity risk exceeds the maturity of the transaction entered into by estimating the expected net present value of the remaining exposure, taking into account the actual mortality rates over the transaction period. In [6, Section 5.5.5] the expected net present value of the remaining exposure is calculated using a re-parameterized version of the initial longevity model.

Remark 5.24. This mechanism is especially useful for index-based solutions, as according to Luca Tres, the Head of EMEA Strategic Risk and Capital Life Solutions,[62] most capital market investors are still not willing to enter a transaction with maturity of 20+ years.

Example 5.25. As mentioned in [6, Section 5.5] NN Life, part of NN Group N.V. — a financial services company, which primarily provides life insurance products in the Netherlands — entered into a Longevity Bull Call Spread, as described in Example 7.3, with Hannover Re in 2017. The maturity of the index based hedge transaction is 20 years, and via a subsequent commutation factor mechanism, NN Life is protected against the risk of pension and annuity policyholders living longer than expected over a longer time period.[63]
There was already a similar transaction between Aegon and Deutsche Bank in 2012. (See also: [6, Section 11.12].)

Definition 5.26 (*special purpose vehicle (SPV)*)**.**
A *special purpose vehicle (SPV)* is a legal entity created by a parent company. It is a separate company with its own assets and liabilities that can be continued in the event of the bankruptcy of the parent company.

Remark 5.27. Companies often set up an SPV to isolate financial risks, make assets easier to transfer or protect assets from risks associated with the parent company and secure these assets by selling shares to investors. For example, a reinsurance sidecar (as described in Section 7.4) is established as an SPV with limited maturity.

Example 5.28. In Example 7.19 SCOR sets up the sidecar Mangrove Insurance PCC Limited to transfer longevity risk arising from nine existing reinsurance contracts to the capital market.

Definition 5.29 (*protected cell company (PCC)*)**.**
A *protected cell company (PCC)* is a single legal entity, which consist of a core, to which at least one independent protected cell with separate assets and liabilities is attached. The PCC is managed by a single board of directors, which is authorized to create new cells. If the PCC holds a (re-)insurance licence, each cell may carry out (re-)insurance business.[64] Usually the core is owned by an insurance manager and the cells are owned by or rented to individual companies.[65]

[62] Longevity 16: `https://www.bayes.city.ac.uk/__data/assets/pdf_file/0008/632645/Tres-Luca.pdf`

[63] artemis news: `https://www.artemis.bm/news/nn-life-gets-index-based-longevity-hedge-from-hannover-re/`

[64] KPMG's Introduction to PCC: `https://home.kpmg/mt/en/home/insights/2017/01/protected-cell-companies-pcc.html`

[65] International Association of Insurance Supervisors (IAIS), Section 5: `https://www.fsb.org/2015/11/application-paper-on-the-regulation-and-supervision-of-captive-insurers/`

Definition 5.30 (*incorporated cell company (ICC)*)**.**
An *incorporated cell company (ICC)* — similar to a protected cell company — can create incorporated cells with separate assets and liabilities. Each incorporated cell is a separate legal entity and has its own board of directors.[66]

Remark 5.31. Since each incorporated cell is a separate legal entity, they can enter into transactions with each other.

Remark 5.32. In Example 7.13 an ICC was used in a longevity transaction where the incorporated cell entered into a longevity swap with a pension plan and subsequently reinsured that liability with a third party reinsurer.

[66]Definition: `https://www.careyolsen.com/briefings/guernsey-incorporated-cell-companies`

6 Buy-ins and buy-outs

Buy-ins and buy-outs are risk transfer solutions for (DB) plan sponsors interested in de-risiking or possibly eliminating pension liabilities. Often a buy-in is only an intermediate step towards a full buy out. It can also be an opportunity to lock in current interest rates or hedge the liability and risk without having to realize the fairly common losses on the balance sheet. The buy-out can then be completed at a later point in time.
A buy-in or buy-out should be well-considered, as these contracts are generally not renegotiable.

Definition 6.1 (*buy-in* and *buy-out* [24])**.**
A buy-in or buy-out contract is concluded between a pension plan sponsor and a (re-)insurer to transfer risks occurring from a DB pension plan. In order to reduce pension liabilities, the pension plan sponsor entering into a buy-in or buy-out sells bulk annuities to a (re-)insurer for an upfront premium. As the details of these contracts are determined individually, buy-in and buy-outs are viewed as customized indemnification solutions. (See also: [6, 4.1 Overview, P. 8].)

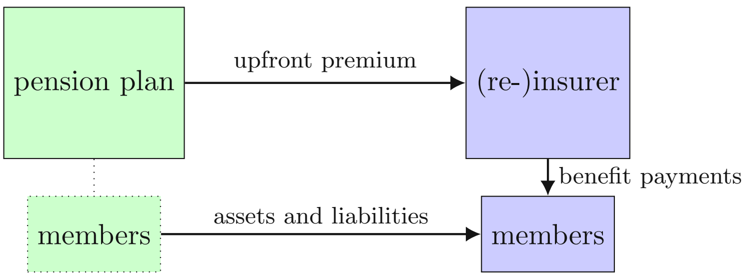


Figure 8: If the pension plan sponsor enters into a *buy-out* it transfers not only any investment risk, longevity risk and inflation risk — in case of indexed plans — but also the contractual obligations associated with the DB pension plan to the (re-)insurer. Consequently, all pension assets and liabilities are removed from the sponsor's balance sheets and the (re-)insurer takes over the responsibility for future payments to the members of the plan. If only part of the liabilities and assets are transferred, we speak of a partial buy-out. If all liabilities are transferred, the deal is called a full buy-out.

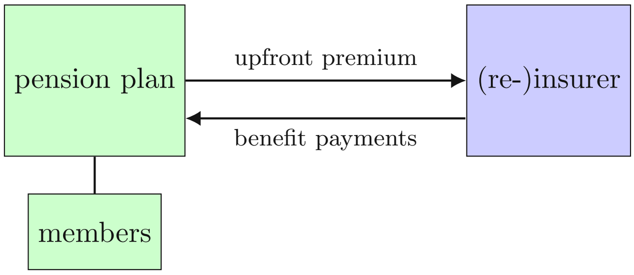


Figure 9: In case of a *buy-in* deal any investment risk, longevity risk and inflation risk, — in case of indexed plans — is transferred to the (re-)insurer, but the obligations stay with the pension plan sponsor, who must guarantee annuity payments to the members of the DB pension plan if the (re-)insurer defaults and is therefore liable to credit risk. Analogously to the buy-out, if only part of the liabilities are transferred, we speak of a partial buy-in, otherwise the deal is called a full buy-in.

Example 6.2. In 2019 and 2020 Aviva Staff Pension Scheme (ASPS) and Aviva Life and Pension UK Ltd (AVLAP) concluded a buy-in, where in total the investment and longevity risk of 8,668 members (deferred and retired) was transferred from ASPS to AVLAP for a premium of £2,528 millions.[67]

Example 6.3. Often a buy-in is just an intermediate step towards a full buy-out. One advantage of phased buy-ins is that this strategy can offer transfer pricing risk diversification over time compared to a direct full buy-out.
In anticipation of the conversion of the following buy-in to a full buy-out, which took place in 2021, the ASDA Group Pension Scheme concluded a £3.8 billion buy-in deal with Rothesay Life PLc involving 12,300 members of which 4,800 are pensioners and 7,500 are deferred members. With a final pension contribution of about £0.8 billions, ASDA could transfer all future liabilities and obligations in conjunction with the contractually defined members to Rothesay Life Plc.[68]

Example 6.4. If the upfront premium for a buy-in exceeds the budged of the pension plan sponsor, a synthetic buy-in is a way to transfer similar levels of longevity risk at lower costs. In the case of a *synthetic buy-in* or *DIY buy-in*, the pension plan sponsor enters into a longevity swap (as defined in Section 7.3) and an asset swap simultaneously. These swaps do not have to be concluded with the same counterparty.[69]

[67] Aviva annual report 2020: `file:///tmp/mozilla_anna0/aviva-plc-annual-report-and-accounts-2020.pdf`

[68] press release:`https://www.rothesay.com/media/o2mphou2/2019_10_17-agps-buy-out-final-external-announcement-with-logos.pdf`

[69] `https://www.actuaries.org.uk/system/files/documents/pdf/a4-nigel-bodie.pdf`

For example, in 2010 British Airways entered into a synthetic buy-in, where the longevity swap was concluded with Rothesay Life and the asset swap was concluded with Goldman Sachs.[70]

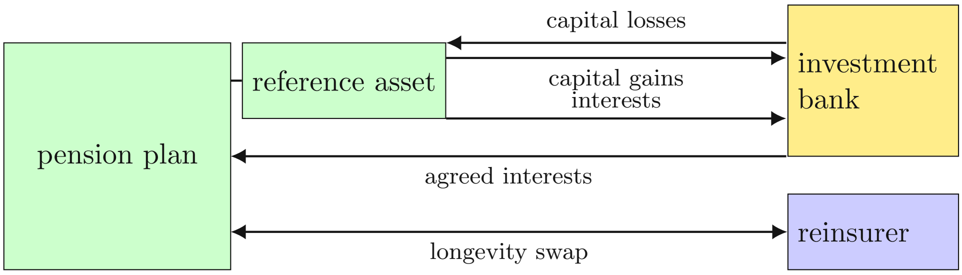


Figure 10: Let the asset swap be given in the form of a total return swap, which can be described as an interest swap plus a credit default swap.
First, the composition of the assets is chosen in such a way that the fixed leg of the total return swap and the fixed leg of the longevity swap match. In order to separate the agreed composition of assets, referred to as reference asset, from the rest of the pension plans assets, these assets are aggregated to a ring-fenced portfolio.
The reference asset generates income in the form of interests, which are regularly paid as the floating leg of the interest swap to the counterparty. In return, the pension plan receives the pre-agreed fixed leg payments.
In the event of a capital loss of the reference asset, the pension plan receives top up payments and collateral haircuts as compensation. In the case of capital gains of the reference asset, these are paid to the counterparty.
Therefore, the counterparty of the total return swap receives any income generated by the reference asset without owning it. On the other hand, the pension plan sponsor receives the demanded interest for the volatile reference asset and is protected against credit and market risk.

Example 6.5. Other alternatives to spread costs are (see also: [6, 11.52]):

- In a *deferred buy-in* or a *deferred buy-out*, a premium schedule (for example over 10 years) or a later date of premium payment is agreed on. In the second case, such a buy-in is also called *forward start buy-in*, in which additional options can be agreed on, such as bringing forward the start date against an additional fee.

[70] `https://www.rothesay.com/about-us/case-studies/british-airways/`
[70] Definition of total return swap: `https://www.gabler-banklexikon.de/definition/total-return-swap-61902/version-339483`
A ring-fenced portfolio is a portfolio of isolated assets, which may only be used for a certain purpose.
The collateral haircut is the difference between the market value of the reference asset and the value of the reference asset put as collateral.

- A case study of the Philips Pension Fund published by LCP[71] shows an example of *phased de-risking*, in which the Pension Fund completed a sequence of 4 buy-ins in the years 2013 – 2015 following a buy-out in late 2015.
  Phased de-risking solutions are usually accompanied by an umbrella contract, which ensures the same contractual terms and security arrangements for all buy-ins.

- In an *accelerated buy-in* in addition to a partial buy-in, a loan is provided by the (re-)insurer to the pension plan sponsor over the amount of the deficit to a full buy-in. After this loan has been paid off, the deal is automatically converted into a full buy-in.

- A partial buy-in is a buy-in, which transfers a particular category of members. For example, a top, bottom or thin slice buy-in or a buy-in, which covers members up to a certain age.[72]

Remark 6.6. According to Blake et al. [6, 11.3], Mercer launched a Pension Buy-out index for the UK in 2010 and extended this index to also cover the US, Canada, Ireland and Germany. The Pension buy-out index tracks estimated annuity prices, based on up-to-date pricing information provided by more than 20 insurers, expressed as a percentage of the accounting liability. Mercer publishes quarterly a report covering the development of the Pension Buyout index for all 5 countries.[73] For pension plan sponsors, this index can function as an indicator for potential buy-in or buy-out costs.

[71] case study: https://www.lcp.uk.com/pensions-benefits/case-studies/philips-35bn-staged-buy-ins-to-achieve-full-buy-out/

[72] For the definition of top, bottom and thin-slicing go to Definition 5.15 and the following.

[73] Mercer Global Pension Buyout Index Q4-2021: `https://www.mercer.com/content/dam/mercer/attachments/global/Retirement/monthly-report/gl-2021-mercer-global-pension-buyout-index-2021-q4.pdf`

7 Longevity-linked instruments

Longevity-linked instruments have cashflows that are linked to the mortality or the survival of a specific population. To express the development of the specific population in terms of mortality or survival, a mortality or survival index is used below.
On the hedger side, there are companies that are exposed to mortality or longevity risk and can use longevity-linked instruments to manage these risks. On the investor side, longevity-linked instruments offer an attractive opportunity to diversify a capital market participant's portfolio, as such instruments have a low correlation with other financial assets.
This section describes longevity-linked bonds, forwards, swaps and reinsurance sidecars, and gives examples of trades already executed in each case.

Remark 7.1. The longevity-linked instruments described in this section are based on chronological age, which measures the years past the individual's birth-year. A new approach is to base such instruments on the biological or physiological age, which is based on measuring biomarkers[74] of aging, such as the damage to various cells and tissues in the individual's body. Theoretically, instruments linked to biological age should better hedge mortality or longevity risk, as biological age correlates more strongly with age-related diseases than chronological age does with age-related diseases. So far, universal methods and biomarkers for the biological age have yet to be established. Until then metrics such as HALE (healthy life expectancy), DALE (disability-adjusted life year) and QALE (quality-adjusted life-year), which are assumed to have a high correlation with biological age, can be used instead to create new hedging instruments.[75]

7.1 Bonds

Definition 7.2 (longevity or mortality bond [5, Capter 6])**.**
A longevity (or mortality) bond is a bond, whose future coupon payments or principal repayment (face value) or both are linked to a survivor (or mortality) index.

[74]Some examples for biomarkers are omics biomarkers from genomics, epigenomics, transcriptomics, etc., functional testing (cognitive function, cardiovascular and respiratory system, etc.), blood based biomarkers, imaging (CT, MRI), microbiome and nutrition.
[75]Report — longevity derivatives and financial instruments: `https://analytics.dkv.global/deep-invest-solutions/longevity-derivatives-and-financial-instruments.pdf`

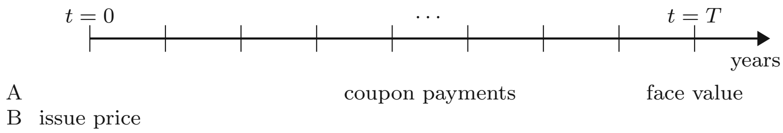


Figure 11: The price at which A originally sells the bond is called the issue price. Until maturity T A pays regularly coupon payments to B, which in case of a longevity bond are linked to a survivor index $S(t)$. At maturity in addition to the coupon payment, the buyer of the bond B also receives the principal repayment (face value), which in case of a principal-at-risk bond is linked to a survivor index $S(t)$.

There are different types of longevity bonds:

- A classical longevity bonds (survivor bond) (See also: [3, page 344].) is a longevity bond, whose coupon payments are linked to the survivorship of a reference cohort and matures with the death of the last surviving member of this cohort.

- A zero coupon longevity bonds
  is a longevity bond with a single coupon payment at maturity (face value), which is linked to a survivor index at the maturity. It can be used as building block to represent a tailor-fit portfolio to hedge the insurer's positions. As most such bonds are traded on a buy-and-hold basis, the market for these bonds is likely illiquid.

- A geared longevity bond or longevity spread bond
  is a longevity bond, whose payments depend on whether the index used lies between the two agreed barriers, namely the attachment and the exhaustion point.

- A deferred longevity bond
  is a longevity bond, whose payments are deferred to an agreed future point in time.

- A principal at risk longevity bond
  is a longevity bond, whose principal payment is linked to a survivor (or mortality) index. For instance, it may depend on a survivor index $S(t)$ crossing an agreed threshold $S_l(t)$.

Example 7.3 (geared longevity bond [5, Capter 6])**.** For the first example, let $S(t)$ be a survivor index, $S_l(t)$ the attachment point and $S_u(t)$ the exhaustion point. Furthermore, an SPV is set up and the issue price paid by the bondholders is deposited in a collateral trust account.

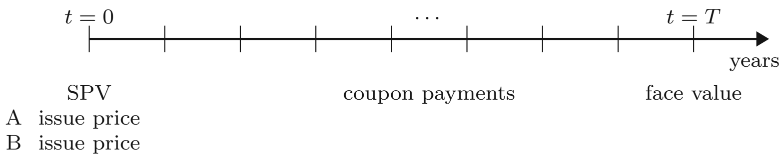


Figure 12: Let A and B be the bondholders. At any time $t \in (0, T)$ the SPV holds $S_u(t) - S_l(t)$ units of a fixed interest zero coupon bond, which matures at t. If $S(t) \in (S_l(t), S_u(t))$, then both bondholders receive payments from the SPV.

The coupon payments are spilt as follows:
A receives

$$\Big(\min\big(S(t) - S_l(t), S_u(t) - S_l(t)\big)\Big)^+$$

units and B receives

$$\Big(\min\big(S_u(t) - S(t), S_u(t) - S_l(t)\big)\Big)^+$$

units of the fixed interest zero coupon bond.

Hereinafter is demonstrated that the payoff to A is equivalent to a longevity bull call option on $S(t)$ with strikes $S_u(t)$ and $S_l(t)$. A benefit of a bull call spread is that the initial price is lower and the loss is limited, but potential gains are limited as well.
In this scenario, A could be thought of as a (re-)insurer, who would like to hedge its tail risks with a longevity bull call spread and sets up the SPV as described above.

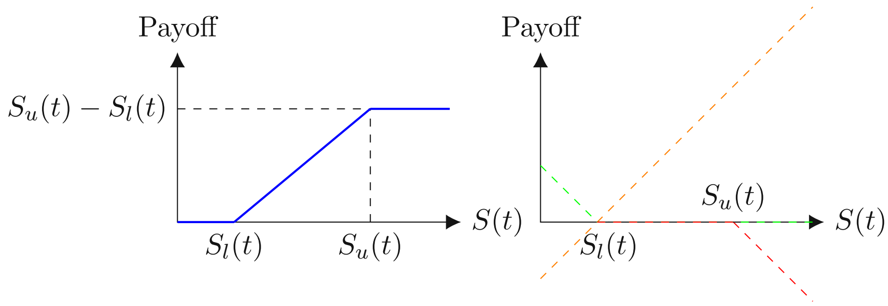


Figure 13: Payoff to A:
If $S(t) > S_l(t)$ A receives $S(t) - S_l(t)$ units, but never more than $S_u(t) - S_l(t)$ and otherwise nothing. The payoff can be equivalently written in a different way, thinking of A generally receiving $S(t) - S_l(t)$ units. In the case of $S(t) < S_l(t)$ A receives $S_l(t) - S(t)$ to get to zero and if $S(t) > S_u(t)$ A has to return the difference $S(t) - S_u(t)$. This therefore results in a combination of a long forward contract with strike $S_l(t)$, a long put option on $S(t)$ with strike $S_l(t)$ and a short call on $S(t)$ with strike $S_u(t)$:

$$\big(S(t) - S_l(t)\big) + \big(S_l(t) - S(t)\big)^+ - \big(S(t) - S_u(t)\big)^+ = \big(S(t) - S_l(t)\big)^+ - \big(S(t) - S_u(t)\big)^+$$

As the forward contract plus the put option is equivalent to a call option on $S(t)$ with strike $S_l(t)$ the payoff to A is the payoff of a bull call spread on $S(t)$ with strikes $S_l(t)$ and $S_u(t)$.

Example 7.4. In 2013 Deutsche Bank developed a longevity risk transfer instrument called Longevity Experience Option (LEO) as a cheaper and more liquid alternative to bespoke longevity swaps (see Section 7.3). LEO was an out-of-the-money bull call option spread traded over-the-counter under a standard ISDA contract.[76] The LEO had a duration of 10 years and was linked to 10-year forward survival rates based on indices published by the Life and Longevity Market Association (LLMA), more precisely males and females in five-year age cohorts (between 50 and 79) derived from England & Wales and the Netherlands. According to Blake et al.[6, 11.22] at least one LEO transaction was executed in 2014.[77]

Example 7.5 (longevity spread bond [6, Capter 5.2])**.** For the second example, let $S(t)$ be the LDIV (Longevity Divergence Index Value), $S_l(t)$ the attachment point and $S_u(t)$ the exhaustion point.

[76] The International Swaps and Derivatives Association (ISDA) published a standardized framework of documents, which is regularly used for OTC derivatives transactions.

[77] `https://www.artemis.bm/news/first-longevity-experience-option-to-be-traded-by-deutsche-bank-by-year-end/`

The LDIV measures the spread between the longevity trends of two different cohorts. It is constructed by calculating the averaged annualized mortality improvements over n years across all ages in the cohorts involved in the hedge, and then taking the difference of the two calculated values.
Again an SPV is set up by a (re-)insurer to hedge the tail risk of the spread described above, and the issue price paid by the bondholders is deposited in a collateral trust account to invest in high quality securities.
If the LDIV $\in \big(S_l(t), S_u(t)\big)$ part of the collateral is paid to the (re-)insurer to cover its hedged tail risk and as a consequence the principal repayment is reduced.
Up to now, this kind of bond was only issued once by Swiss Re, who could partially hedge two tail exposures: high-age English & Welsh males living longer than anticipated and middle-aged US males dying sooner than expected.

7.2 Forwards

Definition 7.6 (*q-forward* [6, Section 5.4])**.**
A *q-forward* is a contract concluded between two parties A and B, who exchange an amount proportional to the fixed mortality rate agreed at the inception of the contract (the fixed leg) against an amount proportional to the experienced mortality rate (the floating leg) at a future agreed point in time, the maturity of the contract.

Notation:	
N	Notational amount, which is agreed at the inception ($t = 0$) of the contract.
m_{fixed}	fixed (or forward) mortality rate agreed at the inception of the contract.
$m_{\text{realized}}(T)$	realized mortality rate at time T, the maturity of the contract.

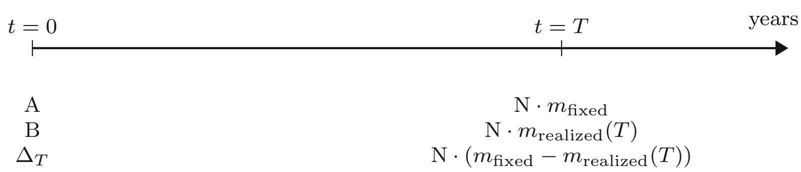


Figure 14: Looking at the situation from A as a life insurance company that wants to hedge its mortality risk by entering into a q-forward: The fixed (or forward) mortality represents the expected mortality proportional to the expected payments at maturity. If the realized mortality at T exceeds the fixed mortality, then the settlement $\Delta_T < 0$. In this case A receives the amount $|\Delta_T|$ from B and can use it to cover the costs occurring from the higher mortality rate. On the other hand, if $\Delta_T > 0$ A has to pay Δ_T to B, but at the same time A has lower costs than expected due to a lower mortality rate at maturity.
As receiving realized survival rate and paying fixed survival rate is equivalent to paying realized mortality rate and receiving fixed mortality rate, for example a pension plan sponsor or a pension fund could enter a q-forward as B to hedge its longevity risk.
Usually the counterparty to a q-forward is an investor like an investment bank.

Further similar products are:

- S-forward
  A survivor forward is linked to a survivor index, which is a function of multiple mortality indexes at multiple points in time. The natural example for a survivor index is the probability of survival of a cohort. In [18] Hunt and Blake introduced the e-forward as a forward linked to the period life expectancy, a natural index to describe the evolution of survival rates in a population.

- K-forward
  A K-forward is linked to a parametric mortality index.
  For example in [8, Section 5] Chan et al. suggest using the CBD mortality indices, resulting in a payoff in form of $N(\tilde{\kappa}_T^{(i)} - \kappa_T^{(i)})$ for $i \in \{1,2\}$, where $\tilde{\kappa}_T^{(i)}$ is the forward value of the i^{th} CBD mortality index.[78]

Example 7.7. Blake et al. [6] give two examples of q-forward transactions provided by J.P. Morgan.

1. In January 2008 Lucida, a buy-out company, entered into the first capital market q-forward with J.P. Morgan. The maturity of the contract was 10 years and the used mortality rates were linked to the LifeMetrics index, which is described in Section 3.1. Specifically, the mortality rates were based on England & Wales national male mortality for a range of different ages.[79]

2. In January 2011 the world's first q-forward for non-pensioners between the UK pension fund Pall and J.P. Morgan with a notational amount of £70 millions and 10 years maturity took place. The used index was J.P. Morgan's LifeMetrics longevity index.[80] Pall entered into the contract to mitigate the risk that non-retired members of the pension fund live longer than expected. In this way Pall received a pay-out if life expectancy improved at a greater rate than specified in the contract.

A fictional example for a q-forward transaction was constructed by Coughlan et al. in [11]:
This fictional contract is concluded between a pension plan sponsor ABC and J.P. Morgan over a period of 10 years (2006-2016) using the LifeMetrics index for 65 year old males in England & Wales. The notional amount is £50 millions and the fixed mortality rate equals 1.2 %. The settlement at

[78] Remark 4.4 describes how the parameters $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ influence the longevity risk for pension plans and life insurers.
[79] `https://www.actuaries.org.uk/system/files/documents/pdf/e6.pdf`
[80] The LifeMetrics index is described in Section 3.1

maturity then depends on the level of the LifeMetrics index for the reference year. Some potential outcomes are shown in the table below:

LifeMetrics index	$(m_{\text{fixed}} - m_{\text{realized}}(T))$	Settlement		
1	0.2	£	5	millions
1.1	0.1	£	10	millions
1.2	0	£	0	
1.3	−0.1	£	−5	millions
1.4	−0.2	£	−10	millions

Table 4: As mentioned before, the settlement is calculated by $\mathrm{N} \cdot (m_{\text{fixed}} - m_{\text{realized}}(T))$. In the first two cases, when the realized mortality rate was below the fixed mortality rate, which results in a positive settlement, the pension plan sponsor receives the settlement from J.P. Morgan to cover the increase in pension liabilities resulting from members living longer than expected. If at maturity, the level of the LifeMetrics index equals the fixed mortality rate, then there will be no exchange. In the last two cases, when the realized mortality rate exceeds the reference rate, which results in a negative settlement, the pension plan sponsor pays the settlement to J.P Morgan, but this is offset by the lower liabilities due to members living shorter than expected.

7.3 Swaps

Definition 7.8 (*longevity* or *mortality swaps* [6, Section 4.4])**.**
A swap is a contract concluded between two parties A and B who exchange one or more cash flows (of which at least one is random) at future agreed points in time. In case of a longevity (or mortality) those cashflows are linked to the survival (or mortality) of a specified population. For insurance-based longevity (or mortality) swaps (also called indemnity swaps), these cash flows are for example linked to the experienced survival (or mortality) in a specific cohort or population. In the case of a capital-markets-based longevity (or mortality) swaps (also called index-based swaps), these cash flows are linked to a mortality- or survivor-index.

Remark 7.9. Often swaps trade pre-agreed, fixed (fixed lag) against variable cash flows (floating lag), but there are also floating-for-floating swaps.

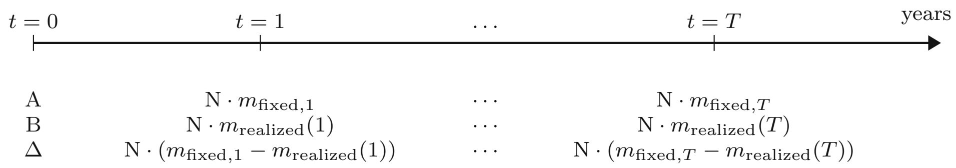


Figure 15: A morality swap is structured as follows:
At the inception of the contract $t = 0$, the contractual design is established. Specially, the maturity T, the settlement dates $t \in \{1, \cdots, T\}$, the fixed mortality rates $(m_{\text{fixed},1}, \cdots, m_{\text{fixed},T})$ for the fixed lag, the used realized mortality rate or mortality index $m_{\text{realized}}(t)$ for the floating lag and the notational amount N are agreed on.
For any $t \in \{1, \cdots, T\}$ the settlement $\Delta_t = \mathrm{N} \cdot (m_{\text{fixed},t} - m_{\text{realized}}(t))$ is calculated. Similar to the q-Forward, A can be interpreted as an life insurance company, that wants to hedge its mortality risk by entering into a mortality swap. If the floating lag at t exceeds the fixed mortality, then the settlement $\Delta_t < 0$. In this case A receives the amount $|\Delta_t|$ from B and can use it to cover the costs occurring from the higher mortality rate. On the other hand, if $\Delta_t > 0$ A has to pay Δ_t to B, but at the same time A has lower costs than expected due to a lower mortality rate. A mortality swap with only on swap payment at maturity is therefore simply a q-Forward.

Remark 7.10 (maturity)*.* The maturity of a longevity swap does not have to be given in years, it may also be defined as the time of death of the last member of the covered cohort. In this case, the term of the contract is run-off. A run-off maturity is often used for insurance based longevity swaps.

Remark 7.11. The pre-defined cashflows for the fixed leg are typically set to the best estimate projection of the affected liabilities plus a hedging fee. The fixed leg can also be contractually determined as the cashflows of an agreed asset portfolio. These cashflows should equal the best estimate projection of the affected liabilities. In this case, the asset risk lies with the sponsor of the hedge.[81]

Remark 7.12 (corridor solution). Similar to the structure of geared longevity bonds, which are defined in Section 7.1, attachment ($m_{\text{lower},t}$) and exhaustion ($m_{\text{upper},t}$) points can be used to cap the cashflows of a longevity swap. In this case, the settlement at time t can be written as

$$\Delta_t = \mathrm{N} \cdot \Big(m_{\text{fixed},t} - \max\big(m_{\text{lower},t}, \min(m_{\text{realized}}(t), m_{\text{upper},t})\big)\Big).$$

Example 7.13. The two most recent longevity swaps according to Artemis[82] are described below.

1. In May 2021 the ICL Group Pension Plan (Fujitsu) and Swiss Re concluded a £3.7 billion longevity swap and reinsurance agreement using a Guernsey incorporated cell company (GICC) as intermediary.

Figure 16: The longevity swap and reinsurance agreement is structured as follows: The ICL Group Pension Plan entered into a £3.7 billion longevity swap with the GICC Group. Simultaneously, the GICC entered into a reinsurance contract with Swiss Re that mirrors the terms of the longevity swap, so that no risk is retained in the cell. In this way, the ICL Group Pension Plan transferred the risk of members living longer than expected to the reinsurance market, covering 9000 members of the pension plan.

2. On February 27th 2021 the AXA UK Group Pension Scheme has entered into a £3 billion deferred longevity swap with the reinsurance firm Hannover Re covering predominantly non-pensioners. More specifically, the deal covers longevity risk associated with pensions that may come into payment after March 31st 2019. As in 2015 AXA UK already entered into a £2.8 billion longevity swap with Reinsurance Group of America (RGA) to transfer the risk of members living longer than expected, which covered around half the closed scheme's total liabilities, in total almost 93 % of the pension scheme's liabilities are protected against the chance of members living longer than anticipated once they come into payment.

[81] `https://www.actuaries.org.uk/system/files/documents/pdf/a4-nigel-bodie.pdf`
[82] `https://www.artemis.bm/longevity-swaps-and-longevity-risk-transfers/`

Example 7.14. In partnership with Zurich, Mercer launched longevity hedge solutions for different DB pension plans sizes under a project called SmartDB. Mercer SmartDB features streamlined longevity hedge solutions, including a streamlined longevity swap for small pension plans with liabilities above £50 millions. (See also: [6, Section 11.26].) In the years 2015 − 2017, six streamlined longevity swaps were executed.[83] All those deals were *named life longevity swaps*, covering the risk that the named pensioners and future dependants live longer than expected. Some differences to a traditional longevity swap are that the swap is uncollaterized, the deal can be converted to a bulk annuity later and a simplified contract is used. The fixed leg is linked to a standardized accessible mortality table and the settlement transactions are executed annually.[84]
Another longevity swap for pension plans with liabilities below £5 billions is the *fully intermediated longevity swap*, which is typically collaterized. In this case, the intermediated insurance company acts as counterparty for the pension plan and takes on all the operational tasks. Additionally, the pension plan is not exposed to the credit risk of the reinsurer.
Further longevity swap solutions Mercer offers are the *pass-through* and the *captive longevity hedge* for pension plans with liabilities above £0.5 billions. An example for a captive longevity hedge is shown in Figure 16. For a pass-through longevity swap the intermediary is again an insurance company, which takes on operational tasks. In both cases the pension plan is exposed to the credit risk of the reinsurer.

Example 7.15. In 2015 the second index-based longevity swap between the Dutch life insurance arm of the Delta Lloyd Group and Reinsurance Group of America (RGA) was completed. Delta Lloyd entered into this swap to cover the risk against pension and annuity policyholders living longer than expected. The maturity of this transaction is 8 years. The notational amount equals €350 millions and forms the maximum pay-out at maturity. The floating lag is based on Dutch population mortality data.[85]

[83] As of September 2020 8 streamlined longevity swaps have taken place. https://www.mercer.com/content/dam/mercer/attachments/global/webcasts/gl-2020-hedging-longevity-risk-during-a-global-pandemic.pdf

[84] https://www.mercer.ca/content/dam/mercer/attachments/north-america/canada/ca-2016-retirement-streamlined-longevity-agreements-webinar-en.pdf
https://www.mercer.com/content/dam/mercer/attachments/global/webcasts/longevity-risk-are-you-ready-to-hedge-mercer.pdf

[85] https://reinsurancegroupofamericainc.gcs-web.com/news-releases/news-release-details/rga-announces-second-dutch-longevity-transaction-delta-lloyd
https://www.artemis.bm/news/delta-lloyd-rga-in-second-e12-billion-longevity-swap-deal/

7.4 Reinsurance sidecars

The use of sidecars was developed in the P&C (property and casualty) reinsurance market for providing short-term additional capacity in the natural catastrophe market. (See also: [7].) For the purpose of transferring mortality or longevity risk, a transaction involving a reinsurance sidecar provides an opportunity to attract third party investors to cover a portion of the risk.

Definition 7.16 (*Reinsurance Sidecar*)**.** A *reinsurance sidecar* is a separate entity that is established as an SPV with limited maturity.
Usually, a reinsurance sidecar is set up for 2-3 years by an existing (re-)insurer (the sponsor of the sidecar) to raise capital for a pre-agreed book of business or categories of risk. The liability is limited to assets of the SPV and the vehicle is unrated. (See also: [6, 12.3.3].)

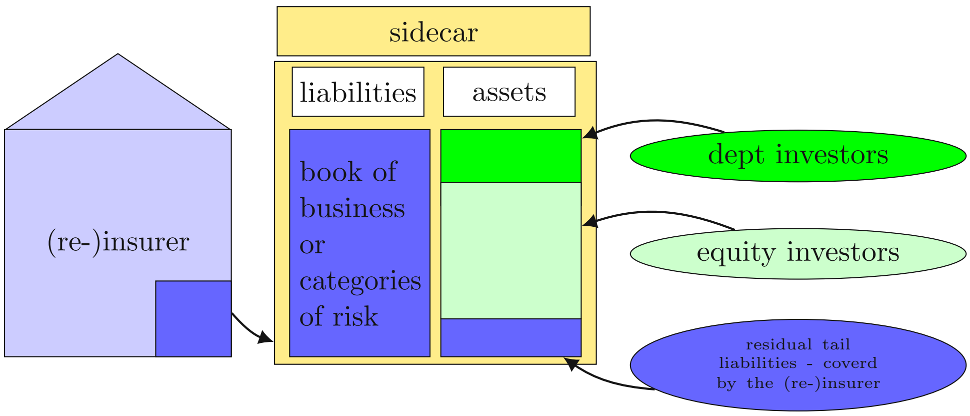


Figure 17: This chart shows how the assets and liabilities of a reinsurance sidecar are created.
The liabilities of the sidecar arise from the reinsurer ceding part of its liabilities to the sidecar.
Subsequently, third party investors contribute capital to the sidecar in the form of preference shares or dept instruments. Since in most jurisdictions SPVs must be fully funded, the tail residual liabilities are covered by the (re-)insurer.

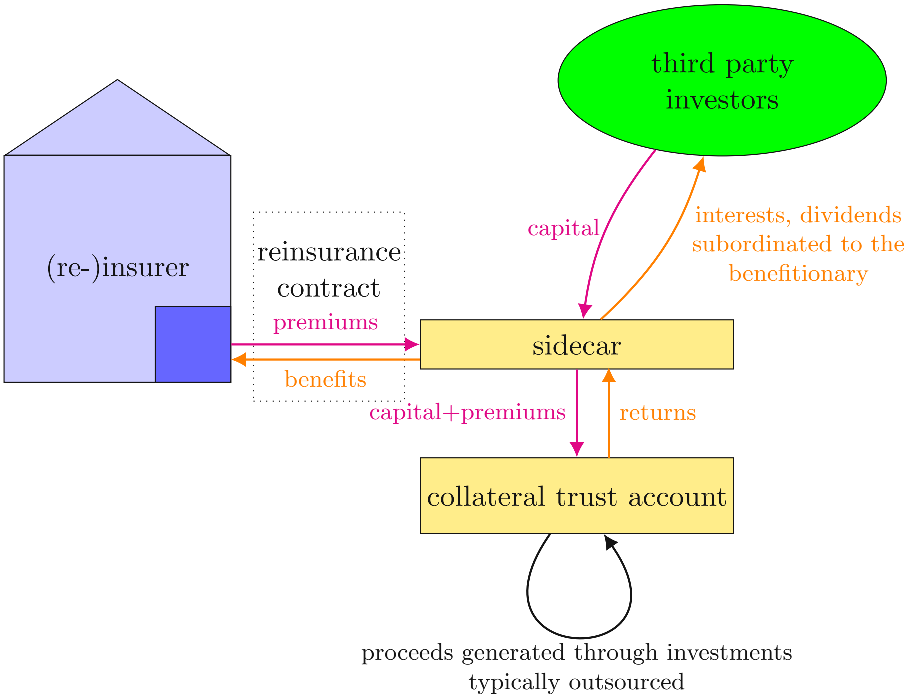


Figure 18: This chart shows the cashflows of a reinsurance sidecar transaction. After sufficient capital has been raised by third party investors, the sidecar enters into a reinsurance contract with the (re-)insurer — the sponsor of the sidecar. Note that in this context, the ceded risk equals the liabilities of the sidecar. Subsequently, the capital and the premiums are placed into a collateral trust account, whose management is usually outsourced. The money in the trust account is then used to make investments in high-quality securities that generate proceeds.
If the ceded risk occurs during the limited term of the reinsurance contract, the capital of the collateral trust account is used to settle the benefits under the reinsurance contract. At the end of the duration, the remaining capital of the collateral trust account will be dissolved and paid back to the investors in form of interests or dividends under an agreed arrangement.

Remark 7.17. A possibility to attract third party investors with different risk aversions and investment horizons according to Bugler et al. [7, Section 4.2] is to issue several securities in different tranches depending on their tenor. For example, the sidecar issues 3 tranches of securities with tenors of 5, $5-10$ and 10+ years. Each of those tranches is associated with a portion of the sidecar's liabilities for which a separate reinsurance contract is concluded. Therefore, the securities of the different tranches are issued with different coupon payments depending on the covered risk. For example, the tranche consisting of securities with tenors of 10+ years carries greater risk that the underlying liabilities will change over the years than the tranche consisting of the same securities, but with tenors of 5 years. Therefore, the long-tenor tranche would generate higher coupon payments, which are reduced over time if the associated liabilities decrease over time. This is particularly true if those liabilities arise from longevity risk. This structuring of the securities issued may be an attempt to reconcile the usually short-term investment horizon of the investors and the usually long-term structure of longevity risk. Based on the long-term relationship between investors and the sponsor of the sidecar mentioned in [7, Section 4], it follows that once third party investors invest in a sidecar, they are more likely to reinvest or continue their investment from year to year. It is therefore expected that there will be investors for long-term tranches. While investors who want to diversify their portfolio and expect an attractive return will more likely invest in short-tenor tranches, those who want to hedge their mortality risk will more likely invest in long-tenor tranches.

Remark 7.18. In the P&C reinsurance market, sidecars, as described above, whose sponsoring (re-)insurer is simultaneously the only counterparty who enters into a reinsurance contract with the sidecar to cover part of its liabilities are called *non market facing sidecars* or *true sidecars*. (See also: [26].)
Market facing sidecars are sidecars that enter into a reinsurance contract with at least one third-party (re-)insurer. Bugler et al. [7, Section 4.4] describes the construction of a market facing sidecar, which also deals with longevity risk. In this case, the sponsor of the sidecar is not the cedent and the sidecar is set up to enter into a pre-agreed pure longevity reinsurance contract with a third party reinsurer. The asset risk is transferred to the capital market through third-party investors and again the residual tail liabilities are covered by the cedent, the third-party reinsurer.

Example 7.19. Marsh, the subsidiary of Marsh McLennan, a global professional services firm, offers several cell captive facilities around the globe.[86] Mangrove PCCs were set up by Marsh & McLennan Companies — the sponsor of the PCCs — and are managed by Marsh Captive Solutions.[87]
In 2019 Société Commerciale de Réassurance (SCOR), a French reinsurance company, set up a sidecar named Mangrove Insurance PCC Limited. SCOR has entered into a quota share retrocession contract with the sidecar Mangrove Insurance PCC Limited to transfer longevity risk arising from nine existing reinsurance contracts with clients in the UK to the capital market. The contract period is 01.10.2019 – 01.10.2048, which corresponds to a long risk period for investors. According to artemis, the Mangrove Insurance PCC

[86] `https://www.marsh.com/us/services/captive-insurance/products/cell-captive-facility.html`

[87] On opencorporates i could find following PCCs, which are all limited by shares: Mangrove Insurance Europe PCC Limited (located in Isle of Man), Mangrove Insurance Solutions PCC Limited (located in Malta), Mangrove Insurance Guernsey PCC Limited (located in Guernsey)

Limited could be the first longevity risk focused quota share sidecar vehicle.[88]

Remark 7.20. At the Longevity 16 Conference in 2021 Luca Tres (Head of EMEA Strategic Risk and Capital Life Solutions) presented some options for possible longevity transfer deal structures.[89] Among others the following possible longevity transfer deal was shown, in which the sidecar passes on the longevity to the capital market through a longevity swap:

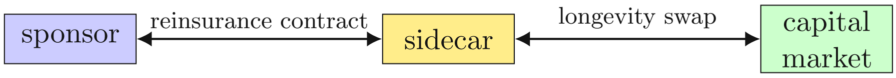


Figure 19: First, a sidecar is established as a reinsurance cell (e.g. a PCC with a reinsurance licence). While the sponsor enters into a reinsurance contract with the sidecar, which usually contains attachment and exhaustion points (similar to Remark 7.12), the sidecar passes on the longevity risk to capital market investors by entering into a longevity swap that mirrors the terms of the reinsurance contract so that no risk remains in the cell.

[88] artemis-article:
`https://www.artemis.bm/news/scor-gets-quota-share-longevity-retro-from-mangrove-sidecar/`

[89] Longevity 16 Conference Plenary Session 3, speech by Luca Tres – Guy Carpenter: Capital Markets & Longevity: `https://www.bayes.city.ac.uk/faculties-and-research/centres/pensions-institute/events/longevity-16/programme`

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