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Probabilistic based topology optimization in ANSYS

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Abstract

Optimization is a process or methodology of making something as fully perfect, functional or effective as possible. Structural optimization in the thesis focuses on optimal design of the loaded structures. The optimal topology design which is a type of structural optimization is defined as optimization problem of material distribution. The material densities of finite elements serve as variables and the maximum stiffness is defined as objective function. The interest for finding optimal design grows in recent years because it gives us a possibility to save the material that doesn't significantly contributes in the global stiffness of the structure. Probabilistic design is widespread discipline in engineering. It considers uncertainties in the structures. These uncertainties and random effects are related to quality and reliability of the structure. Probabilistic design is not used very often in the civil engineering because engineers rely on prescribed building codes and safety factors.

In recent years there has been a growing interest to combine probabilistic design with topology optimization. Techniques for combining these two analyses presented to this day are very time consuming and therefore expensive. They are also unreachable to every engineer. In this work a simplified method for engineers is presented. It combines the deterministic load cases that came from probabilistic analysis into an end topological solution. Monte Carlo simulation is applied for the probabilistic analysis.

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Optimierung ist ein Prozess oder eine Methode, um etwas so perfekt, funktional oder effektiv wie möglich zu machen. Struktur Optimierung konzentriert sich auf das optimale Design der belasteten Tragwerke. Das optimale topologische Design, die eine Art der strukturellen Optimierung darstellt, wird als Optimierungsproblem der Materialverteilung definiert. Die Materialdichten der finiten Elemente dienen als Variablen und die maximale Steifigkeit wird als Zielfunktion definiert. Das Interesse, ein optimales Design zu finden, wuchs in den letzten Jahren, da es uns ermöglicht Material einzusparen, welches nicht wesentlich zur globalen Steifigkeit des Tragwerks beiträgt.

Das probabilistische Design ist eine weitverbreitete Fach im Ingenieurwesen. Es berücksichtigt Unsicherheiten in Konstruktion. Diese Unsicherheiten und zufälligen Effekte hängen mit Qualität und Zuverlässigkeit der Konstruktion zusammen. Das probabilistische Design wird im Hoch- und Tiefbau nur selten angewandt, da sich Ingenieure auf Bauvorschriften und Sicherheitsfaktoren verlassen.

Das Interesse, probabilistische Design und topologische Design zu kombinieren, ist in den letzten Jahren gewachsen. Die bis heute verwendeten Methoden, um diese beiden Analysen zu kombinieren, sind sehr zeitaufwendig und daher teuer. Außerdem ist nicht jeder Techniker in der Lage sie anzuwenden.

In dieser Arbeit wird eine vereinfachte Methode für Ingenieure vorgestellt. Sie kombiniert die deterministischen Lastfälle, die aus der probabilistischen Analyse hervorgegangen sind, zu einer endtopologischen Lösung. Für die probabilistische Analyse wird die Monte-Carlo Simulation angewandt.

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Chapter 1 Topological Optimization

1.1 Introduction into Structural Optimization

Structural Optimization

Structural optimization is a field that studies optimal design of loaded structures. Topology optimization is a subfield of structural optimization. The typical problem of topology optimization is design of optimal structure that carries given loads in a design domain, supports, materials and external loads. To optimize a structure an optimal subdomain in the design domain should be found and filled with material. The objective may be to find minimal total weight of a constrained structure under given loads. In order to approach this problem numerically we should consider to discretize our design domain into finite element model. Now, when our structure is "discretized" it needs to be decided whether to place material in each finite element or not.

In order to formulate structural optimization problem we need to introduce following variables and functions:

- *Objective function (f):* Represents objective that should be reached. Usually our objective is weight of the structure but it can also be volume, stress, displacement and in some cases cost of production
- *Design variable (x):* this is the design variable, it mostly represents geometry or material of the structure
- *State variable (y):* variable of response of the structure. The response of the structures is characterized with displacement, stress, strain and force

Now that we have defined needed functions and variables, the general structural optimization problem takes following form:

$$(SO)\begin{cases}\min\limits_{x} & f(x,y(x)) \\ subject\ to & \begin{cases} design\ constraint\ on\ x \\ state\ constraint\ on\ y(x) \\ equilibrium\ constraint \end{cases}\end{cases}$$

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A problem with several objective functions a so called *multiple criteria*, or *vector* optimization problem would have following form:

$$\min_x f(f_1(x,y), f_2(x,y), \dots f_n(x,y))$$

Where n is the number of objective functions. The issue with this problem is that a global exact optimum can not be found since each objective is reached with different set of x,y. In this case we usually try to find a so-called *Pareto optimality*. We can say that our design is Pareto optimal if there does not exist any better design that would satisfy all of the objectives above.

A state function g(y) that represents the state variables can be introduced, for example a displacement in a certain direction. This state function can be incorporated as a constraint to the optimization task, where it is usually formulated such that g(y) ≤ 0. Consider the case where g(y) is represented by a displacement vector g(u(x)) in a finite element problem. To establish the state function in structural analysis, this requires that nodal displacement are solved for

$$\boldsymbol{u(x) = K(x)^{-1} f(x)}$$

where K is the global stiffness matrix and f is the global load vector. This means that the optimization task can be expressed in a so-called nested formulation where the equilibrium constraint is taken care of by the state function formulation

$$\begin{cases} \min\limits_x & f(x) \\ subject\ to & g(u(x)) \leq 0 \end{cases}$$

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Types of structural optimization

We will denote with x some sort of geometric features of the structure for a clearer description of the types of optimization bellow. Depending on the geometric features we can have following types of optimization:

- *Sizing optimization (*Figure 1-1 [1]*): Variable* X can represent structural thickness, cross section of a beam etc.

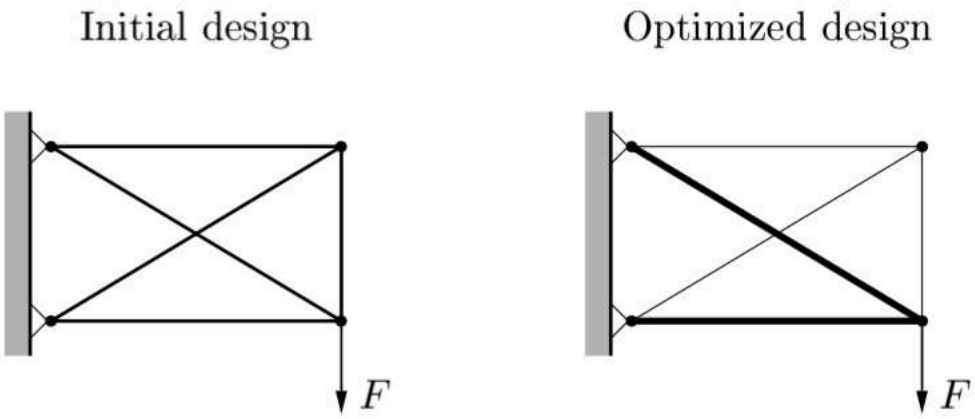

Figure 1-1 Sizing optimization problem

- *Topology optimization (*Figure 1-2 [1]*):* If we take a simple case, for example a truss, topology optimization is reached by taking its members as design variables and allow them to take 0 and 1 value. In this case connectivity of nodes varies and so topology changes. If it has value 1 it means that beam is present and if its 0 it means discontinuity.

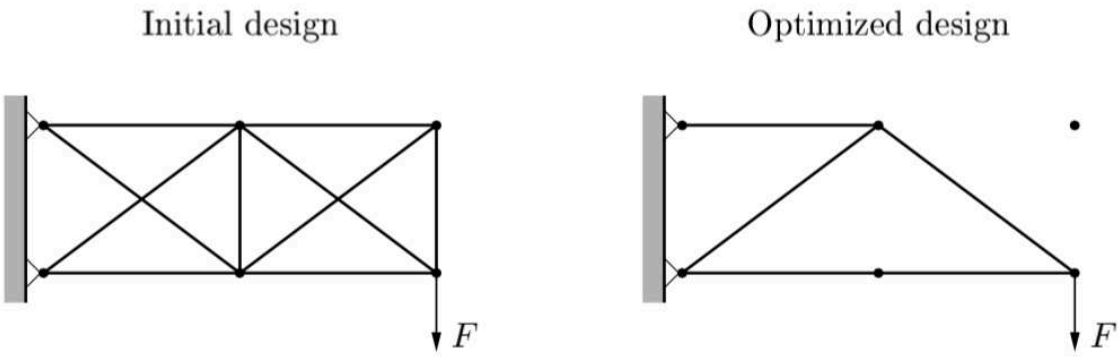

Figure 1-2 Topology optimization of a truss atructure

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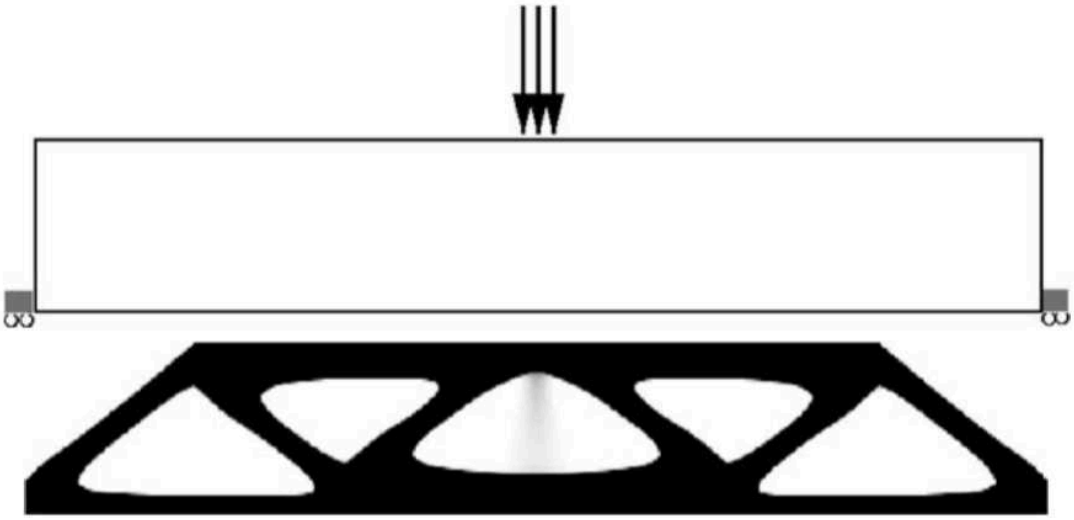

Figure 1-3 Topology optimization of a 2D continuum Beam [1]

- *Shape optimization (*Figure 1-4 [1]*):* X represents the contour of some part of the boundary of the structural domain. Our goal here is to find an optimal shape that minimizes certain cost functional. If we compare it to the topology optimization in shape optimization we have a constant volume but the shape of a structure is modified to reach and optimal design.

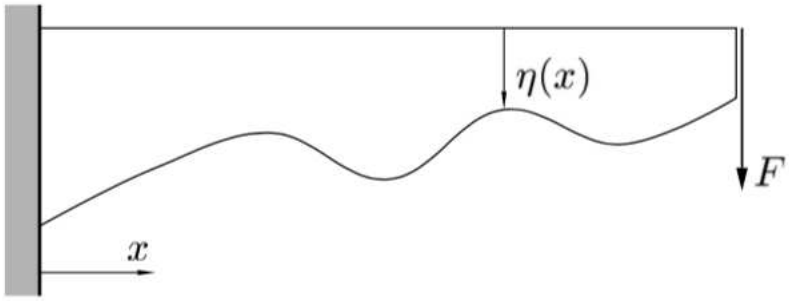

Figure 1-4 Shape optimization problem

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1.2 Introduction into Topology Optimization

Topological optimization is considered as type of “shape” optimization. The end result is a shape of a structure that has optimal material distribution for given boundary conditions. It is sometimes also called as “layout” optimization. The aim of topological optimization in structural analysis is to determine the most optimal use of material for a body that is loaded with one or multiple loads. This optimal or best use of the material in case of topological optimization described in this work represents “maximum-stiffness” design.

Compared to traditional structural optimization, topological optimization does not require to explicitly define optimization parameters. In topology optimization the material distribution serves as the optimization parameter. The objective function (function that should be minimized), state variables (variables that are constrains dependent), and design variables (independent variables that are being optimized) are all predefined. All we need to do is to define the structural problem. That includes defining the loads, geometry and material properties and off course the percentage of material that should be removed.

If we minimize the structural compliance energy, while satisfying a constraint on the volume (V) of the structure, then we achieved the objective of structural optimization. Minimal structural compliance is equivalent to maximum structural stiffness.

In this technique we assign internal pseudo-densities to our finite elements. For instance, if we say V=60, that means for the given load configuration, 60% of the material should be removed so that stiffness stays at a maximum.

On the Figure 1-5 [2] we have a rectangular area, constrained on the left edge and loaded at the bottom right corner. Figure 1-5(a) shows the geometry, boundary conditions and the loads and Figure 1-5(b) shows the “shape” results as density contours.

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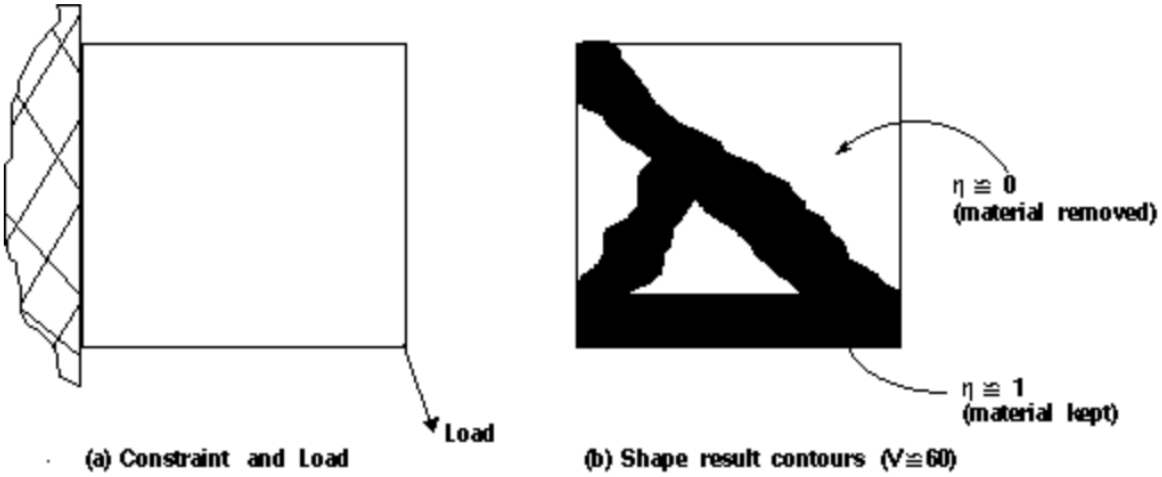

Figure 1-5 Topology optimization of a cantileaver wall

1.3 Topology optimization by distribution of isotropic material

In this chapter we will give an overview of the basics of the material distribution method for finding the optimal layout or shape of linear elastic structure. Al three problems of structural optimization (sizing, shape and topology) are being addressed simultaneously in the context “layout” of the structure and material distribution method.

Sizing, shape, and topology optimization deal with different features of the structural design problem. The main aim of the standard sizing problem is to find optimal thickness distribution of a linear elastic slab, or if we are dealing with trusses to get optimal cross section area in a truss. The optimal thickness distribution minimizes (or maximizes) a physical quantity such as the mean compliance (external work), peak stress, deflection, etc., while equilibrium and other constraints on the state and design variables are satisfied [3]. Slab thickness is the design variable and the deflection could be the state variable. The most notable characteristic of the sizing problem is that the model that we are designing and the state variables are known before the start of the optimization process and also that they are fixed through the optimization process. Shape optimization tries to find the optimal shape. Topology optimization of solid structures also includes determination of attributes such as the shape, shape of the holes in the structure and determination of the domain connectivity.

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1.4 Problem formulation

The following layout problem combines various characteristics of the traditional problems in the structural optimization. The objective of the topology optimization is to reach optimal layout or shape of a structure inside a defined region. The input variables in the problem are boundary conditions, applied loads, geometry, volume of the structure to be optimized and maybe location and size of the holes and other openings in the structure if there are any. Shape, size and connectivity of the structure in this problem can be unknown.

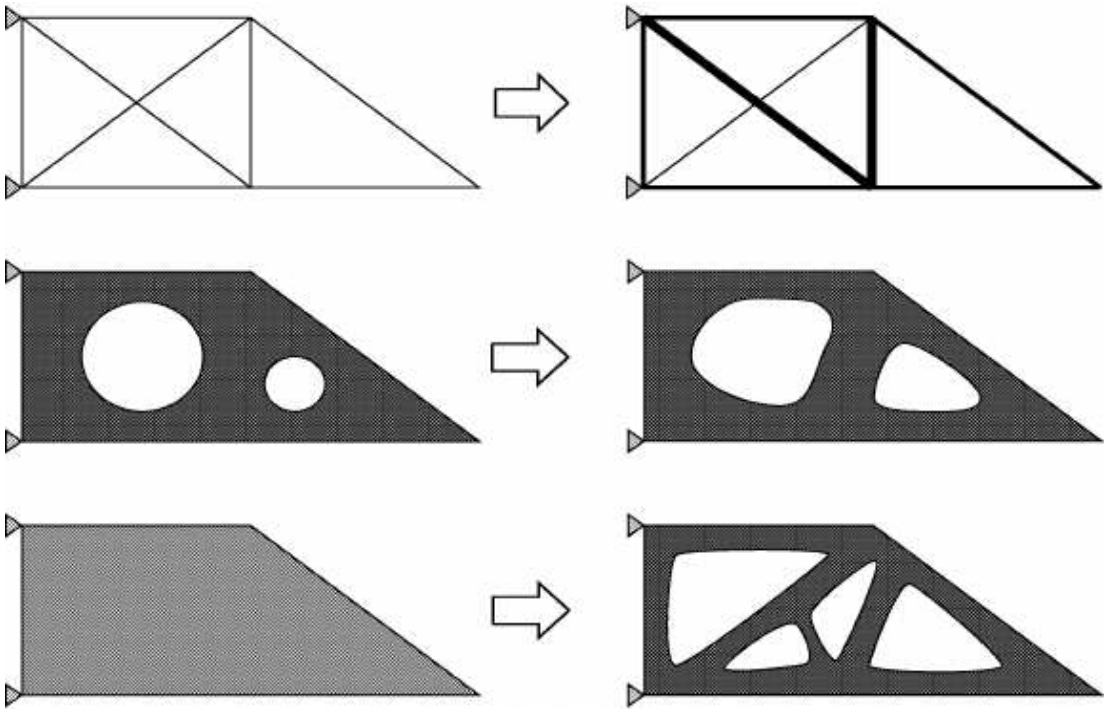

Figure 1-6 3 types of optimization; From top to bottom: sizing optimization, shape optimizatiom, topology optimization

The topology, shape, and the sizing Figure 1-6 [4] of the structure are not represented by standard parametric functions but by a set of distributed functions defined on a fixed design domain. These functions in turn represent a parametrization of the stiffness tensor of the continuum and it is the suitable choice of this parametrization which leads to the proper design formulation for topology optimization [3].

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1.4.1 Minimum compliance design

In the following text we will describe a general set-up for optimal shape design defined as a material distribution problem. The set-up is equivalent to truss topology formulations and to formulations for sizing problem for discrete and continuum structures.

It is important to note that the problem type we will consider is from computational point of view inherently large scale, both in state and in the design variables. For this reason the first problems treated in this area employed the simplest type of design problem formulation in term of objective and constraint, namely designing for minimum compliance (maximum global stiffness) under simple resource constraints. This is also conceptually a natural starting point for this exposition as its solution reflects many of the fundamental issues in the field.

Think of a element as a body Figure 1-7 in a region Ω^{mat} that is part of a larger reference region Ω in R^2 or R^3 . The reference region Ω is chosen so that loads and boundary conditions can be defined. If we were to use the same terminology as used in truss topology design then the reference region could be called the ground structure. Referring to reference region Ω, optimal design problem can be defined as the problem of finding the most optimal choice of stiffness tensor E_{ijki} (x) that varies across the region Ω.

Introducing the energy bilinear form (i.e., the internal virtual work of an elastic body at the equilibrium u and for an arbitrary virtual displacement v):

$$a(u,v) = \int_{\Omega} E_{ijkl}(x)\varepsilon_{ij}(u)\varepsilon_{kl}(v)d\Omega$$

With linearized strains $\varepsilon_{ij} = \frac{1}{2}\left(\frac{\delta u_i}{\delta x_j} + \frac{\delta u_i}{\delta x_j}\right)$ and the load linear form

$$l(u) = \int_{\Omega} fud\Omega + \int_{\Gamma_T} tuds\,,$$

The minimum compliance (maximum global stiffness) problem takes form

$$\min_{u\in U,E} l(u)$$

s.t.: $a_E(u,v) = l(v)$, for all $v \in U, E \in E_{ad}$. (1.1)

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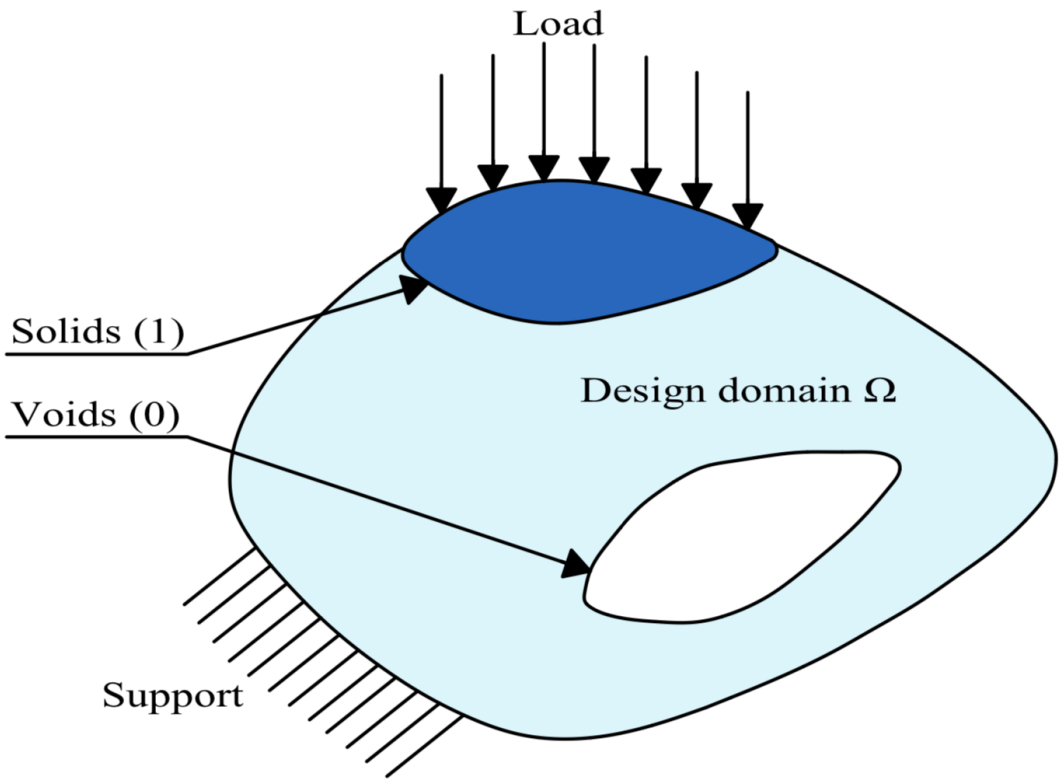

Figure 1-7 Generalized shape design problem. Goal is findig optimal material distribution in a 2D region

We wrote here the equilibrium equation on its weak, variational form, where *U* represent space of all applicable kinematical displacement fields, *f* represents body forces and *t* represents boundary tractions od the traction part $\Gamma_T \subset \Gamma \equiv \delta\Omega$ of the boundary. Here the index *E* is used to indicate that the bilinear form a_E is dependable on the design variables. In problem (1.1), E_{ad} represents the set of satisfactory stiffness tensors for our design problem. In the topology design, E_{ad} could be composed of all stiffness tensors that achieve material characteristics of a given isotropic material in the tensors that attain the material properties of a given isotropic material in the (unknown) set Ω^{mat} and none characteristics somewhere else, the limits of resource being expressed as $\int_{\Omega mat} 1 d\Omega \leq V$. We will give different possible definitions of E_{ad} in following section.

The standard procedure of solving the problems of the form given in (1.1) is to discretize it with finite elements. Note that in here we are interested in two fields in (1.1). The displacement field u and the stiffness field E. If we mesh both fields with the same finite element mesh and discretize *E* as constant in each element, the discrete form of (1.1) can be written as

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$$\min_{u,E_e} f^T u$$

s.t. : $K(E_e)u = f$ (1.2)

$E_e \in E_{ad}$

Here **u** and **f** are the displacement and load vectors, respectively. The stiffness matrix **K** depends on the stiffness E_e in the element e, numbered as e=1,…,N.

And we can write K in form

$K = \sum_{e=1}^{N} K_e(E_e)$,

Where K_e is the (global level) element stiffness matrix.

1.4.2 Design parametrization

When designing structure topology, we need to determine the best or in our case optimal position of isotropic material in the observed space, that means that the objective is the determining those points in space that should have material in them and determining of points that should be empty or simply void (material non existent). For better clarification, we represent the geometry of the structure similar as to black and white image render. This means black as where we have material and white being the empty space and the pixel represent our finite element discretization. If we restrict our space to the reference region Ω, we are looking to determine the optimal subset Ω^{mat} of material points. So, what does this approach implies then? It implies that for our above defined optimization problem the set E_{ad} is built from those tensors for which:

$$E_{ijkl} = 1_{\Omega^{mat}} E^0_{ijkl}, 1_{\Omega^{mat}} = \begin{cases} 1 \; if \; x \in \Omega^{mat}, \\ 0 \; if \; x \in \; \Omega/\Omega^{mat}, \end{cases} \quad (1.3)$$

$$\int_{\Omega} \; 1_{\Omega^{mat}} \, d\Omega = \mathrm{Vol}\left(\Omega^{mat}\right) \leq V.$$

The V in the last equation represents the limit on the material that is at our disposal, in the way that the design of minimum compliance is for fixed volume. The tensor E^0_{ijkl} represents the stiffness tensor of our isotropic material and it is usually written $E_{ijkl} \in L^{\infty}(\Omega)$ which indicates the relevant function -space for our problem. This kind of definition for E_{ad} means that we formulated a distributed, discrete valued design problem (a 0-1 problem).The usual way to solve this problem is by replacing the integer variables with continuous variables and then some form of penalty should be introduced that would direct the solution to discrete 0-

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1 values. Now our design problem for the fixed region is formulated as a sizing problem that through modification of the stiffness matrix is constantly dependable on a function which represents a density of a material, which is the design variable. Objective is to conclude our optimization results to areas that contain, or do not contain material (0-1 problem). This means that the values in between of this density function need to be penalized in the same way as other optimization approximations of 0-1 problems. One way which stands out in efficiency and popularity is the so-called penalized, proportional stiffness model or short, the SIMP model:

$$E_{ijkl}(x) = \rho(x)^p E^0_{ijkl}, p > 1,$$

$$\int_\Omega \rho(x) d\Omega \leq \mathrm{V}; 0 \leq \rho(x) \leq 1, x \in \Omega, \qquad (1.4)$$

In here we have the "density" $\rho(x)$ that represents our design function and E^0_{ijkl} which stands for material properties of our isotropic material. Under ρ we mean the density of the material in which the volume of the structure is evaluated as $\int_\Omega \rho(x) d\Omega$. The density of the material then interpolates between the values 0 and and E^0_{ijkl}:

$$E_{ijkl}(\rho = 0) = 0, \quad E_{ijkl}(\rho = 1) = E^0_{ijkl}, \qquad (1.5)$$

which means that our final result has density zero or one in each point. This design represents a black and white design for which we evaluate the performance with a correct physical model. In SIMP we choose that $p > 1$ so that the median values are unfavorable in the way that the stiffness that we obtain is small when we compare it to the volume of the material. So that means that by defining the value of p higher than one, it makes in unreasonable or in other words uneconomical to have values in between zero and one in the optimal design. Therefore we will achieve penalization without using any explicit penalization scheme. According to [1], for problems where the volume constraint is active, experience shows that optimization does result in such design if we choose p enough high (in order to obtain true "0-1" designs, p > 3 is usually required). According to [5], it has been proven for the minimum compliance problem in discrete form that for p large enough there is a optimal solution in 0-1 form. Off course the last statement is true only if the volume constraint is compatible with topological design. The SIMP method is the basis for every topological analysis that will be performed in this thesis.

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The original 0-1 problem is defined on a fixed reference region. This means that together with SIMP interpolation, the optimal topology problem becomes a form of a regular sizing problem on a fixed region.

1.4.3 Alternative problem definitions

It should be noted that the problem (1.1) can be expressed through several equivalent formulations. In these reformulations, special structure of the minimum compliance problem is used, and they can be very useful for the analysis if specialized computational procedures for these types of problems. We can express problem (1.1) in terms of th principal of minimum potential energy. That means that our displacement field u is a minimizer of the $F(v) = \frac{1}{2}a_E(v,v) - l(v)$ on *U* (the total potential energy). We notice that the value *F(u)* of the potential energy at equilibrium equals $-\frac{1}{2}l(u) < 0$. Then we can write our problem as:

$$\max_{E\in E_{ad}} \min_{\sigma\in S} \left\{\frac{1}{2}\int_\Omega C_{ijkl}\sigma_{ij}\sigma_{kl}d\Omega\right\} \qquad (1.6)$$

We can also formulate our Problem (1.6) in terms of stresses. If we express the inner equilibrium of problem (1.6) in terms of the principle of the minimum complementary energy we will get following formulation

$$\max_{E\in E_{ad}} \min_{\sigma\in S} \left\{\frac{1}{2}a_E(u,u) - l(u)\right\}, \qquad (1.7)$$

of the minimum compliance design problem. The $C_{ijkl} = (E^{-1})_{ijkl}$ represents compliance tensor, and the minimization with respect to stresses σ is taken over the set S of statically admissible stress fields, i.e [3].

$$S = \{\sigma |\ div\sigma + f = 0\ in\ \Omega, \sigma \cdot n = t\ on\ \Gamma_T\}$$

For the statement written in problem (1.6) it is seen that if we knew the displacement field in an optimal structure, then the optimal distribution of the stiffness is in that way that the strain energy is maximal. In the same manner, if we know stress distribution in the optimal structure then the complementary energy is minimized. This characterization helps us understand the theoretical and computational background of our design problem.

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1.5 Solution methods

The SIMP interpolation scheme allows conversion of the optimal topology problem into a sized problem, on a fixed region that is. If we compare it to the traditional sizing problem for example frames, stringers, etc., the difference is that in our problem we have a very large number of design variables. (the number of design parameters and the number of analysis variables is of the same order of magnitude)[1]. In our optimization design problem, the optimization procedure efficiency plays a very important role, and usually we have to utilize optimization settings that will exchange the number of constraints for number of design variables. An example of this is the compliance design problem. In the optimization problem we have only one constraint in added to the simple box constraints that give upper and lower limits to the density variable.

1.5.1 Optimality conditions

In the following text, the necessary condition of optimality for the density *p* will be derived. *p* is the density that refers to the minimum compliance design problem that utilizes the SIMP scheme. If we follow the standard optimality criteria methods that are being used in the structural optimization, the simple structure of the continuum, single load problem (1.1) can be used to generate very effective computational update scheme to solve problems that are being handled in this thesis. The goal is to develop iterative methods that for previously calculated design and its displacement field, update the design variables at every element of a finite element discretization. The design update of finite elements needs to be independent from the updates of other elements, based on the needed conditions of optimality

To this end, we first recapitulate the form of the minimum compliance problem (1.1) written for the case of the SIMP interpolation. In the continuum setting this is

$$\min_{u\in U,\rho} l(u)$$

$$\text{s.t. : } a_E(u,v) = l(v), \quad for\ all\ v \in U,$$

$$E_{ijkl}(x) = \rho(x)^p E^0_{ijkl}, \tag{1.8}$$

$$\int_\Omega \rho(x)d\Omega \leq \mathrm{V}; \quad 0 < \rho_{min} \leq \rho \leq 1.$$

A lower bound ρ_{min} is introduced to prevent a possible singularity of the equilibrium problem. ρ_{min} is usually chosen to be $\rho_{min} = 10^{-3}$. With Lagrange multipliers Λ, $\lambda^-(x)$, $\lambda^+(x)$ for the

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constraints of (1.8), the required optimality conditions for the sizing variable ρ are a subset of the stationarity conditions for the Lagrange function

$$\mathcal{L} = l(u) - \{a_E(u,\bar{u}) - l(\bar{u})\}$$
$$= \Lambda(\int_\Omega \rho(x)d\Omega - \mathrm{V}) + \int_\Omega \lambda^+(x)\Omega + \int_\Omega \lambda^-(x)\big(\rho_{min} - \rho(x)\big)\mathrm{d}\Omega,$$

In this equation $\bar{u}$ represents Lagrange multiplier for the equilibrium constraint, and $\bar{u}$ belongs to the set *U* of kinematically admissible displacement fields. If we assume that $\rho > \rho_{min} > 0$, the conditions of optimality respected to variations of the displacement field u give that $\bar{u}$ = u while the condition for ρ becomes:

$$\frac{\partial E_{ijkl}}{\partial \rho}\varepsilon_{ij}(u)\varepsilon_{kl}(u) = \Lambda + \lambda^+ - \lambda^-, \tag{1.9}$$

With the switching conditions

$$\lambda^- \geq 0, \lambda^+ \geq 0, \lambda^-\big(\rho_{min} - \rho(x)\big) = 0, \lambda^+(\rho(x) - 1) = 0. \tag{1.10}$$

For intermediate densities ($\rho_{min} < \rho < 1$) the conditions (1.9), can be written as

$$p\rho(x)^{-1}E^0_{ijkl}\varepsilon_{ij}(u)\varepsilon_{kl}(u) = \Lambda, \tag{1.11}$$

(1.11) expresses that the strain energy density-like left-hand side term is constant and equal to Λ for all intermediate densities. Therefore does this condition resemble very closely to the fully stressed condition in plastic design. Because we expect the high energy areas to be very low on stiffness, the following fix-point update scheme for the density is devised:

$$\rho_{K+1} = \begin{cases} max\{(1-\zeta)\rho_K, \rho_{min}\}\ if\ \rho_K B_K^\eta \leq max\{(1-\zeta)\rho_K, \rho_{min}\} \\ min\{(1+\zeta)\rho_K, 1\} \quad if \quad min\{(1+\zeta)\rho_K, 1\} \leq \rho_K B_K^\eta \\ \rho_K B_K^\eta \qquad\qquad otherwise \end{cases} \tag{1.12}$$

In here we have ρ_K that represents the value of the density at a iteration step K. B_K is given by following expression:

$$B_K = \Lambda_K^{-1} p\rho(x)^{p-1}E^0_{ijkl}\varepsilon_{ij}(u_K)\varepsilon_{kl}(u_K), \tag{1.13}$$

In the equation (1.13) u_K represents displacement field at the iteration step K. It is calculated from the equilibrium equation and depends on ρ_K. If the value of B_K results one for the densities ($\rho_{min} < \rho < 1$) then the optimum is reached (local optimum). If we look at the (1.12), we notice that the update scheme adds material to areas with a strain energy that has higher value than Λ (when $B_K > 1$) and deletes material if the energy is below Λ. This will only happen if the update does not violate bounds on ρ. Through integration of (1.11) it is notable that Λ

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is proportional (by a factor p) to the average strain energy density of the part of the structure that is given by intermediate values of the density [3].

In the equation (1.12) η represents a tuning parameter and ζ is a move limit. Their role is the control of the changes that occur at each iteration step. Both η and ζ can be adjusted in such manner that our method becomes more efficient. The update ρ_k *+1* is dependable on the present value of the Lagrange multiplier Λ. So, in order to satisfy the active volume constraint Λ should be adjusted in an inner iteration loop. We have already seen that the density values that are being updated is a continuous and decreasing function of the multiplier Λ [3] . Moreover, the volume is strictly decreasing in the interesting intervals, where the bounds on the densities are not active in all points (elements of a FEM discretization) [3]. Thus, using a Newton method we can calculate the value Λ. If we want a stable and rapid update scheme then the values η and ζ should be chosen by experiment (the usual value for η is 0.5 and for ζ is 0.2).

The optimality criteria method is related to the concept of fully stressed design but conditions 1.9 and 1.10 only imply that strain energy is constant in areas with density that is in middle. It has lower value in areas with density $\rho = \rho_{min}$ and higher value in areas where density equals 1. According to [3] the algorithm that we described above has had a great success in a wide spectrum of structural topology design studies. It is also very well accepted as an effective design method for solving large scale problems. The algorithm is so effective because each design variable is separately updated from other design variables, the only exception is rescaling that has to happen to satisfy the volume constraint.

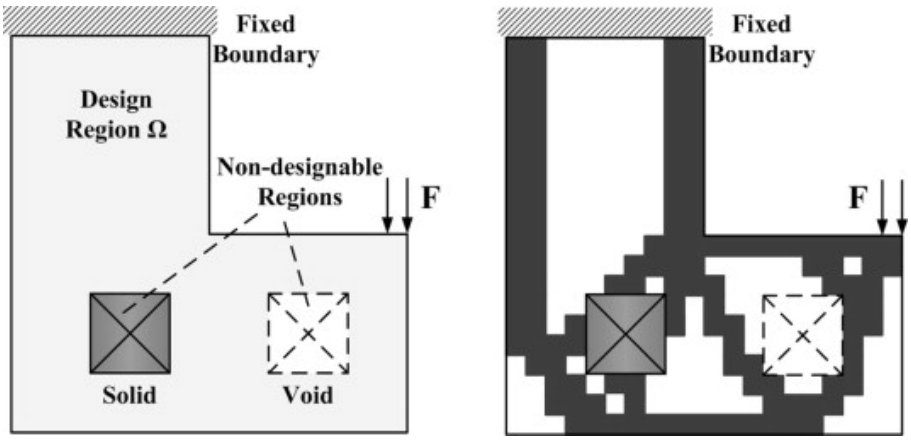

Figure 1-8 Possible sub areas in a refference region; Solid areas and Empty areas. [6]

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1.5.2 Implementation of optimality criteria method

In 1.1.2. and 1.2.1 basics of the optimality criteria method for the implementation of the material distribution procedure for topology designed have been described.

The basics of the optimality criteria method consist of the parametrization of design through the design-stiffness relation given by an appropriate interpolation scheme, and the update scheme for the density based on the optimality conditions. The update scheme rests on the ability to solve equations of equilibrium and therefore we expect this to be performed using finite element method.

Computational procedure

The direct method of topology design using the material distribution method is based on the numerical calculation of the globally optimal distribution of the density of material ρ which is the design variable. The main goal of the interpolations scheme is design that results 0-1(material-no material) design and that is best achieved through proper penalization of intermediate densities. To find the optimal topology of the structure using the optimality criteria method, following steps should be followed:

1. *Pre- processing of geometry and loading:*

- First we need to choose a reference domain where we can define surface tractions, fixed boundary conditions etc.
- Next, we need to define the parts of the reference domain that should actually be designed, and then we need to define parts of the reference domain that will either be solid domain or will be left as emptiness.
- After we defined the geometry of the structure we need to mesh our structure. The fines of the mesh should be so adjusted that the structure is described in a reasonable bit-map resolution representation. The mesh should also allow for defining the a priori areas of the structure that should stay fixed. The mesh always remains fixed through design process, that means that when design is being updated the mesh will not change. Construct finite element spaces for the independent fields of displacements and the design variables.

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2. *Optimization:*

In this step we compute optimal material distribution over the defined reference domain of the design variable p. The background of the optimization is finite element analysis and optimality update criteria for density. The algorithm has following structure:

- First, the initial design is made, which means homogenous material distribution. Then the iterative part of algorithm is as follows:
- For the current density distribution, use finite element method to compute resulting strains and displacements.
- Next, we compute compliance for the current design. If the resulting compliance has improved insignificantly over the last iteration, stop iterating, if not, iterations process should continue. When we make detailed studies, we should stop when the needed optimality conditions are satisfied.
- Based on the scheme in 1.2.1. update of the density variable is computed. This step has also an inner iteration loop that computes value of the Lagrange multiplies Λ. Repeat iteration loop.

In case that we have solid or void parts in the structure the design variable update should only be called for the areas of the structure that are being redesigned.

3. *Postprocessing:*

   In the last step we interpret the results i.e. the final topology. The final shape that we get is certainly not the shape of the end product design The shape is usually exported into drawing software and the edges are corrected for a smoother surface or if the geometry is too complicated to be built with disposable production line, it is corrected for easier production.

For the method that we described above, the choice of the basic interpolation scheme should be decided at the beginning of the design process, for instance SIMP. According to [1] topology optimization that uses SIMP with a high value of power p, gives rise to very well defined designs that consist almost completely of areas with material or no material. Very small part of the structure has areas with intermediate densities, in other words very little grey.

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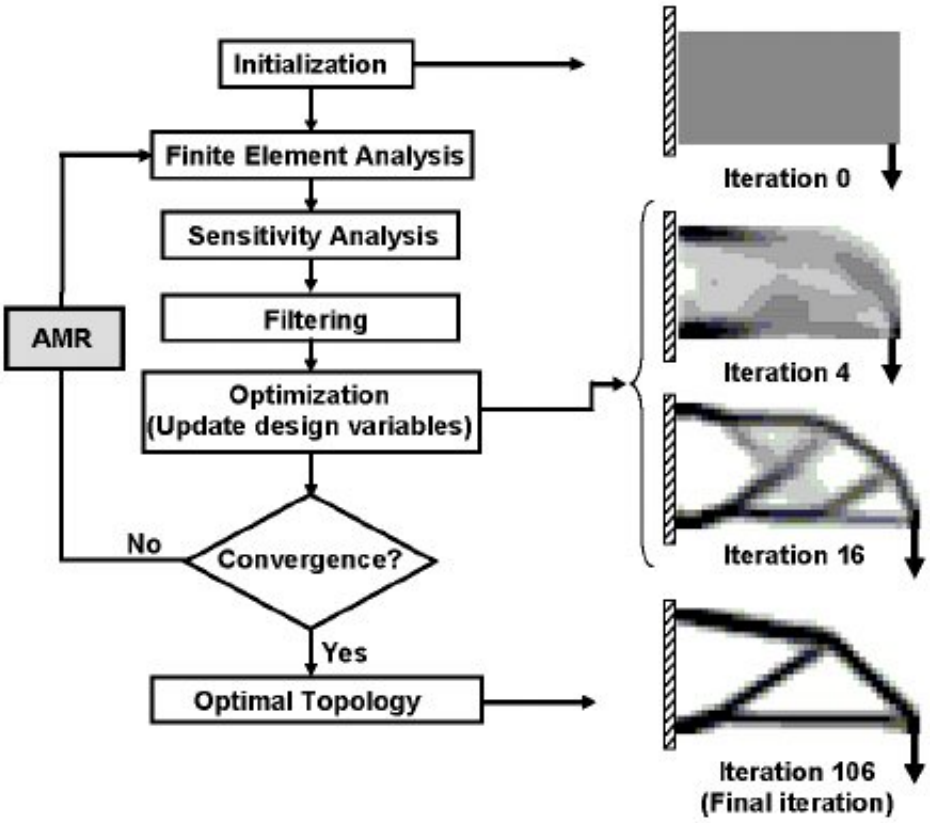

Figure 1-9 Flow chart for computing optimal topology

Finally, it is very important to state that the above described algorithm works for any type of finite element mesh, however the software that will we use (ANSYS Mechanical) for the topology optimization analysis supports only handful of finite element mesh types for the topology optimization analysis. Figure 1-9 [7] shows a flow chart for computing of optimal topology.

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1.5.2.1 Example for computational procedure

The steps noted in 1.4.2 will be shown on following example.

We will perform a topological optimization on the following example. Ansys Mechanical will be used, and the structure will be optimized for 50% of its volume. 50% of optimized volume means that the end result will have 50% less material compared to the body we started with. We will show the computing procedure that was explained in the previous chapter.

A 3.5x7 m cantileaver wall Figure 1-10 is loaded on the bottom edge with a point force and at the bottom edge and left edge is completely constrainted. (See Apendix A for APDL input code of the example). This means that we are solving a problem with 2 load cases.

Every iteration loop is presented on the Figure 1-11. Figure 1-13 to Figure 1-16 show the results for 60%, 70%, 80% and 90% of optimized topology.

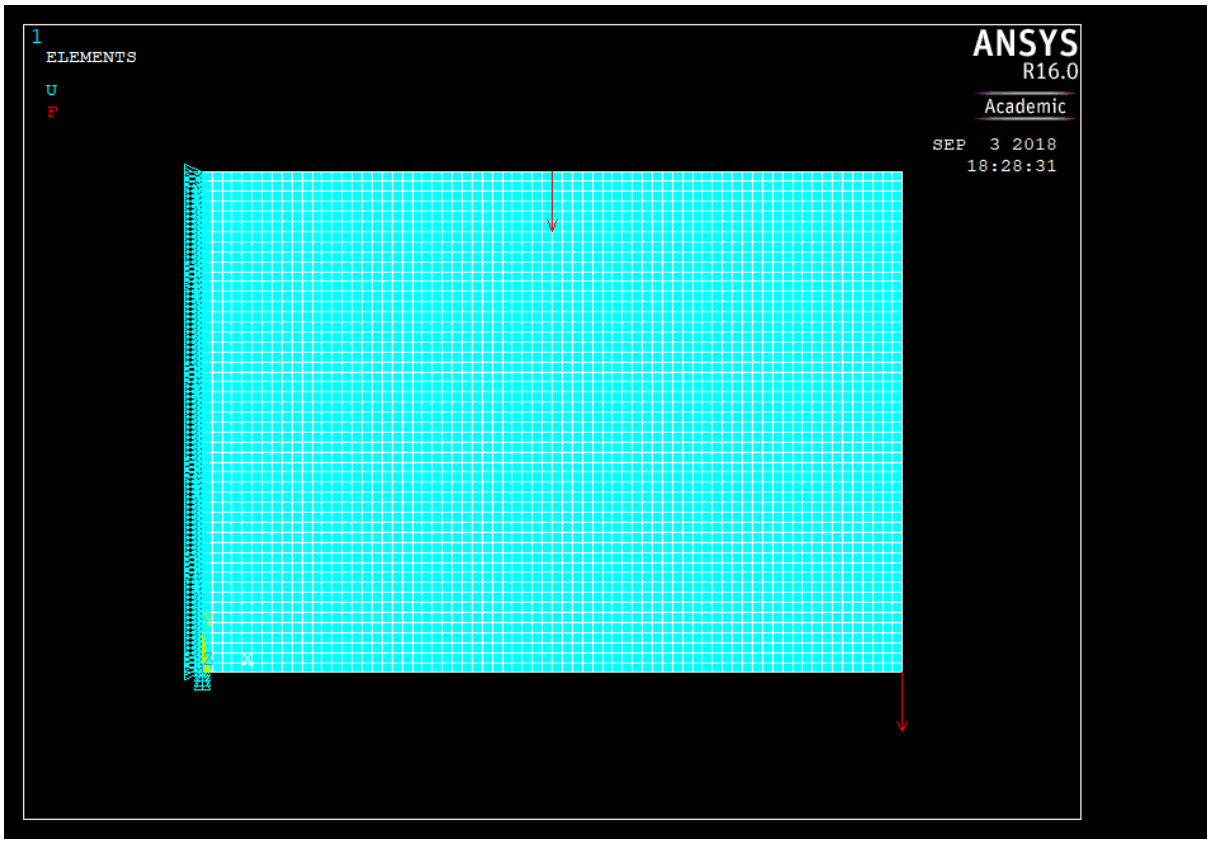

Figure 1-10 Wall constrainted at the left side, loaded with two forces.

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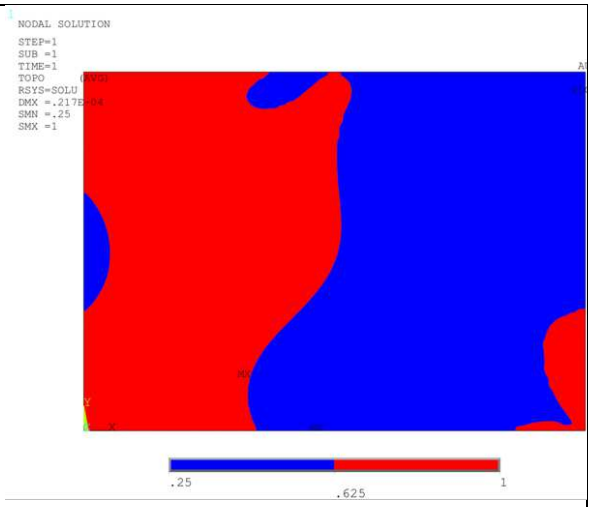
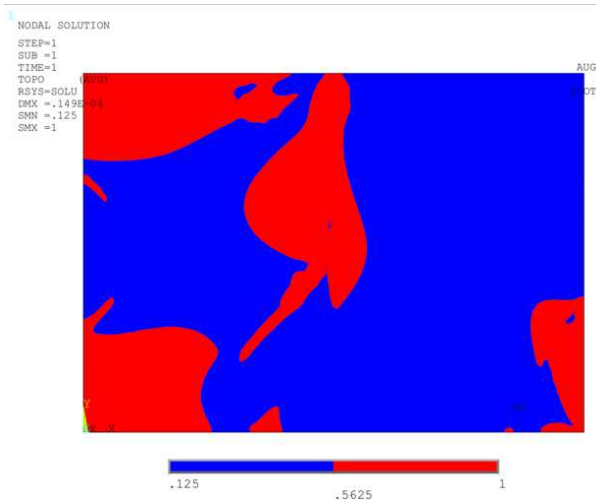
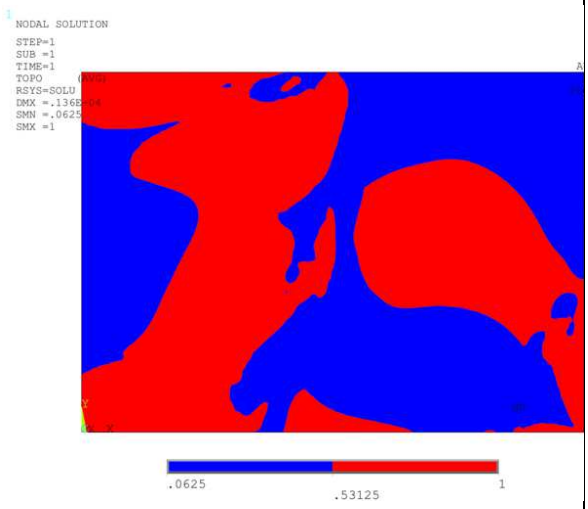
1. iteration loop



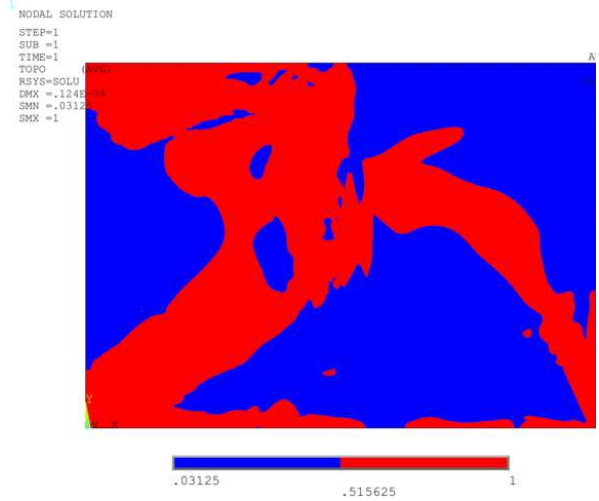
2. iteration loop



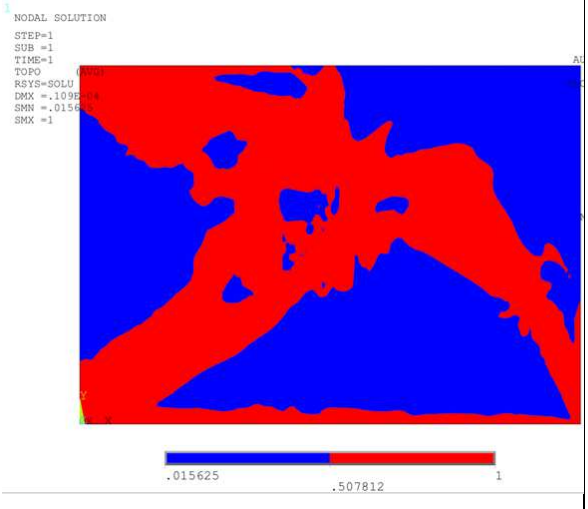
3. iteration loop



4. iteration loop



5. iteration loop



6. iteration loop

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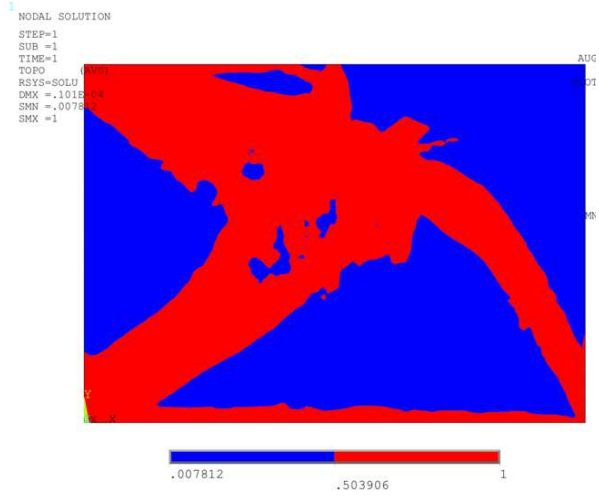
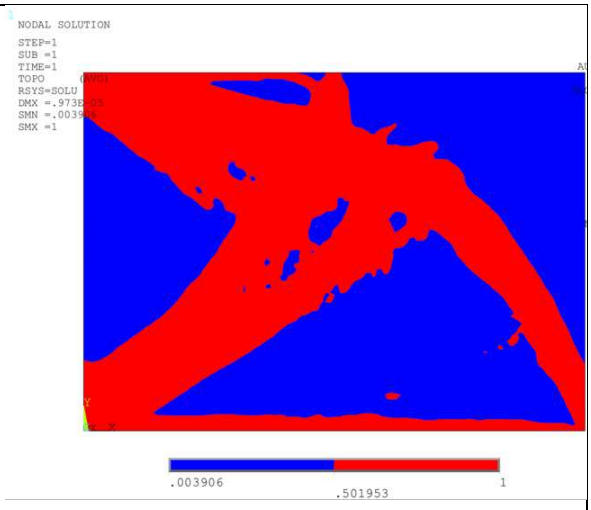
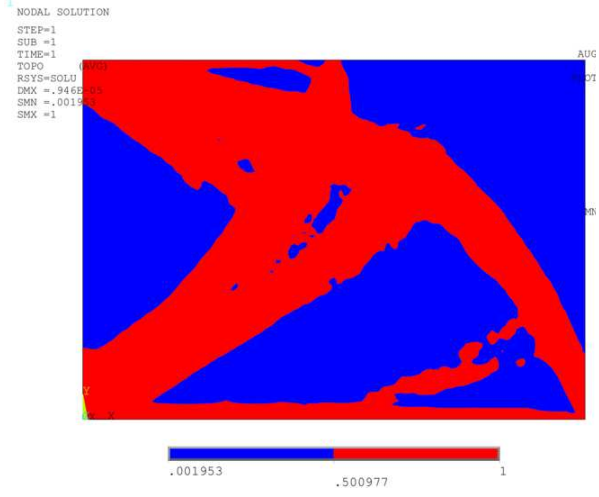


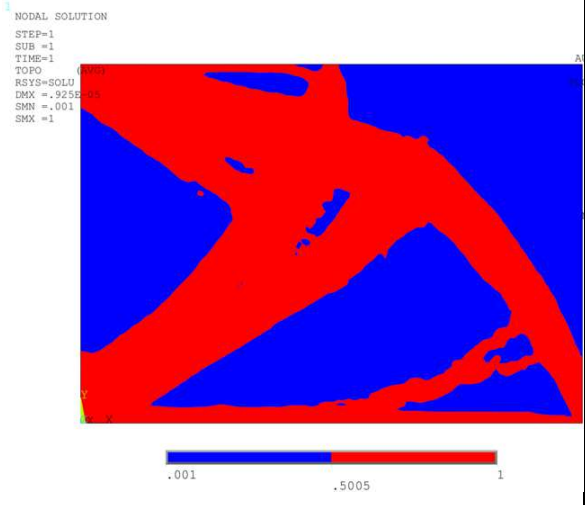
7. iteration loop



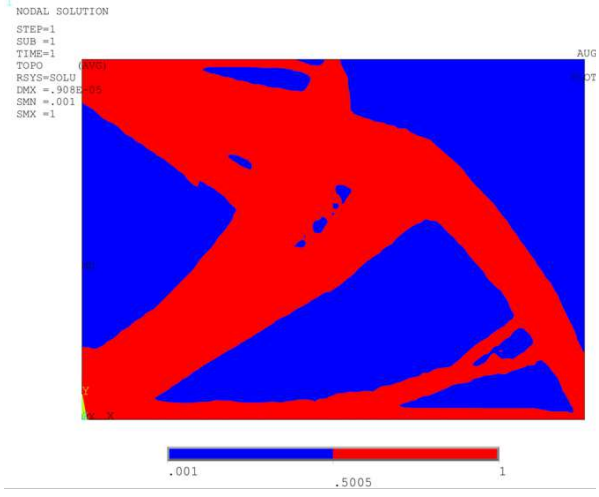
8. iteration loop



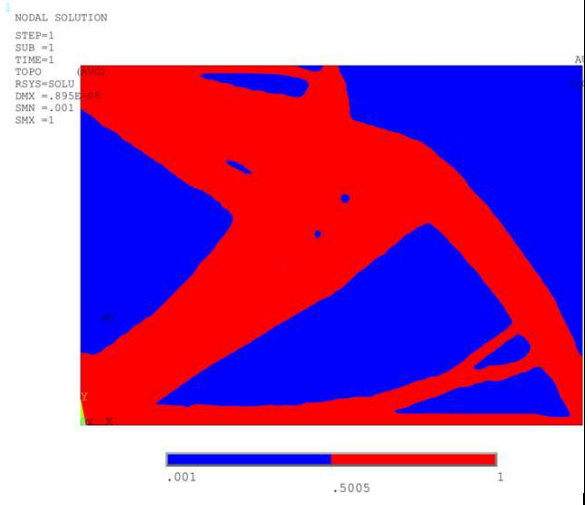
9. iteration loop



10. iteration loop



11. iteration loop



12. iteration loop

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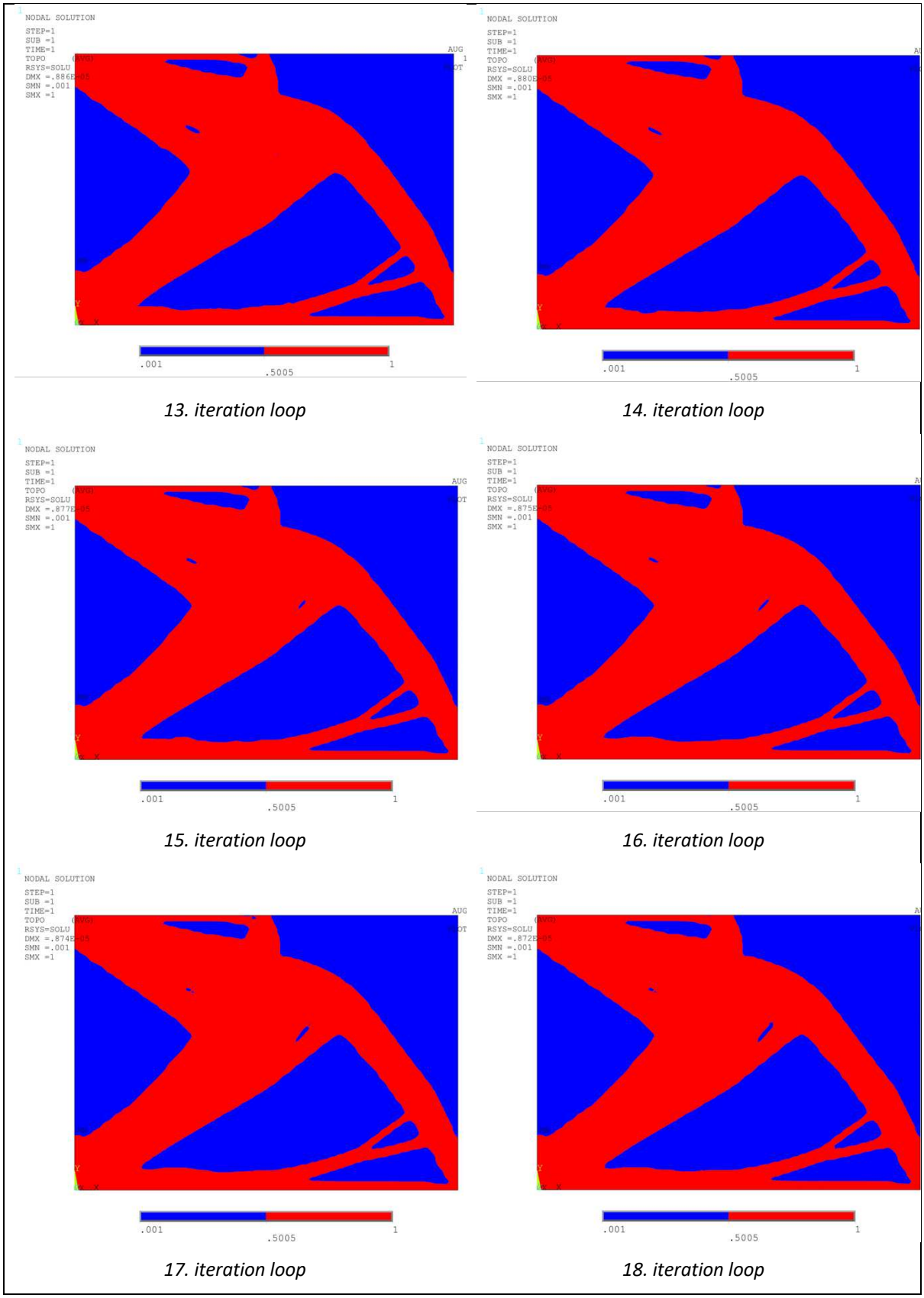

Figure 1-11 Topology for each loop of optimization

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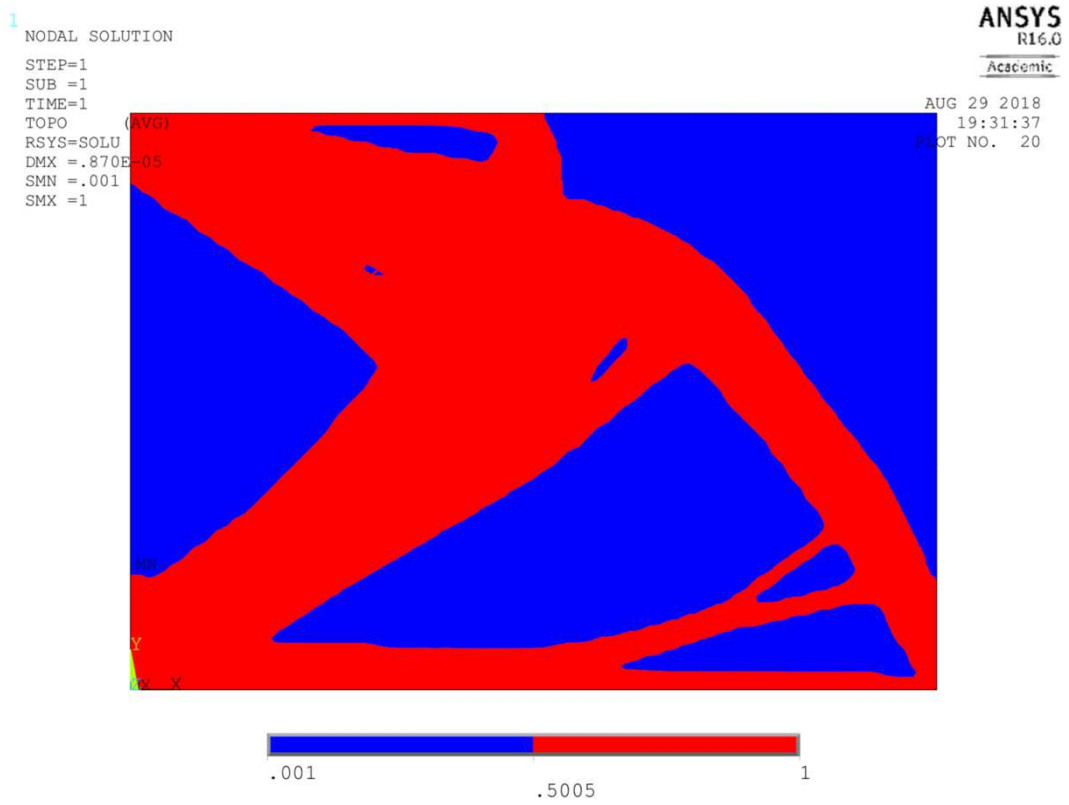

Figure 1-12 Topology for 50% volume optimization

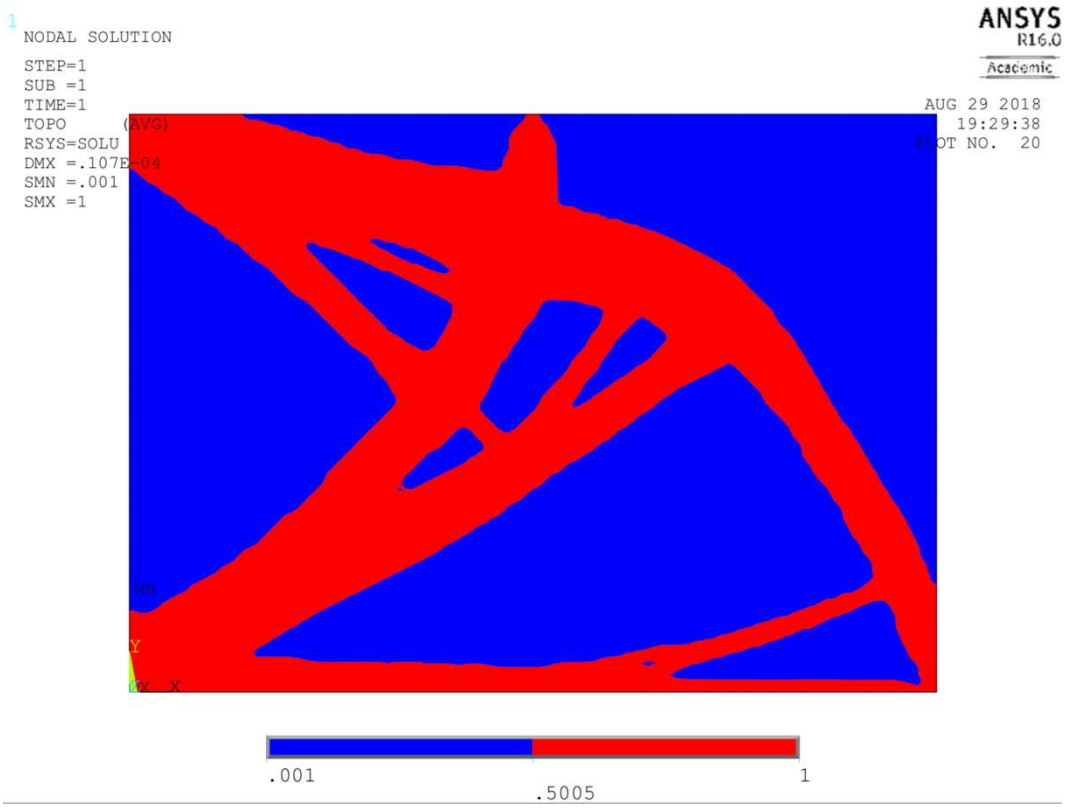

Figure 1-13 Topology for 60% volume optimization

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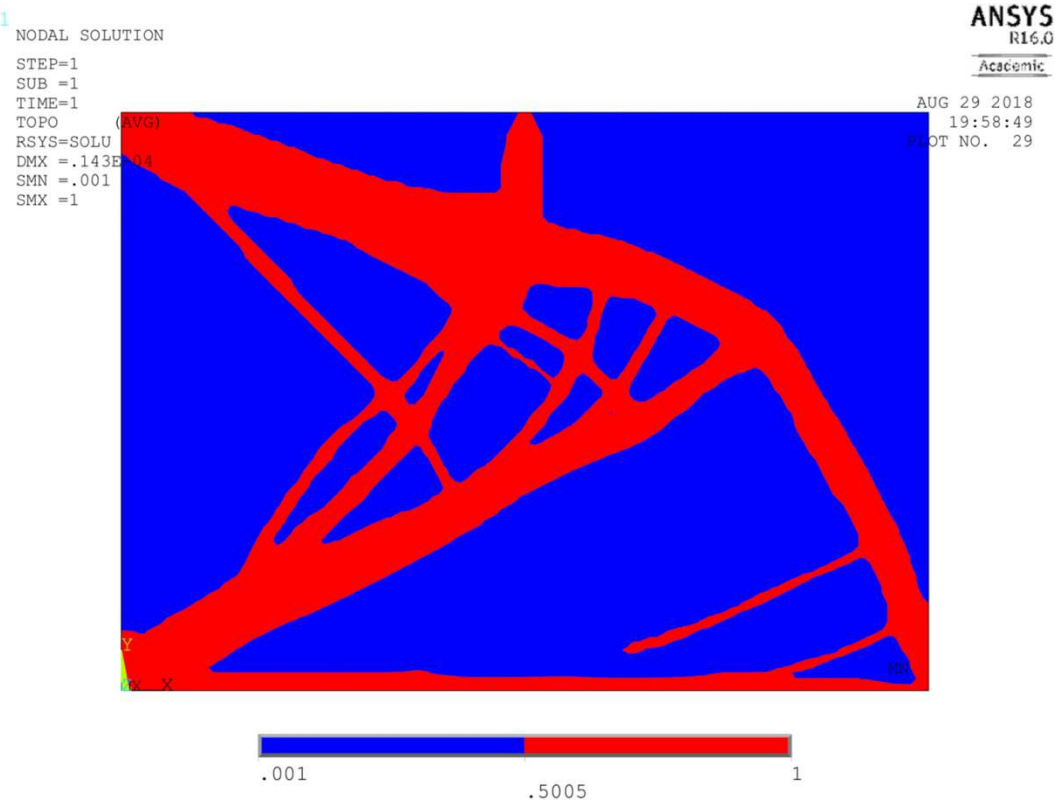

Figure 1-14 Topology for 70% volume optimization

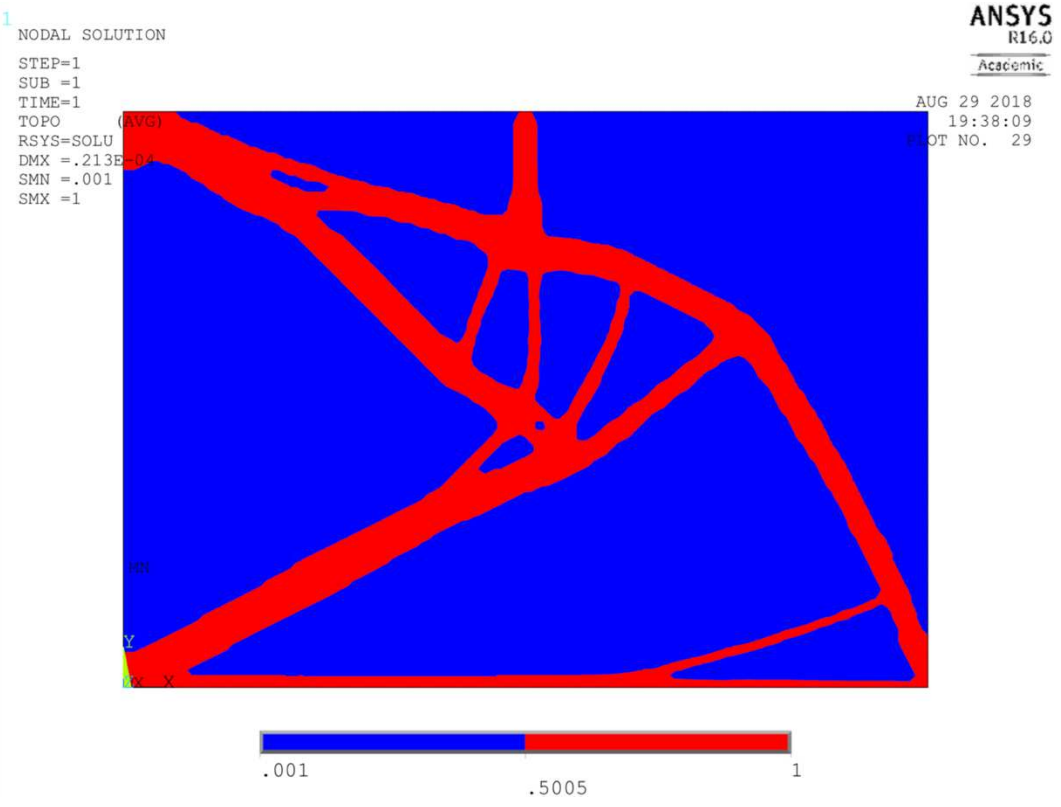

Figure 1-15 Topology for 80% volume optimization

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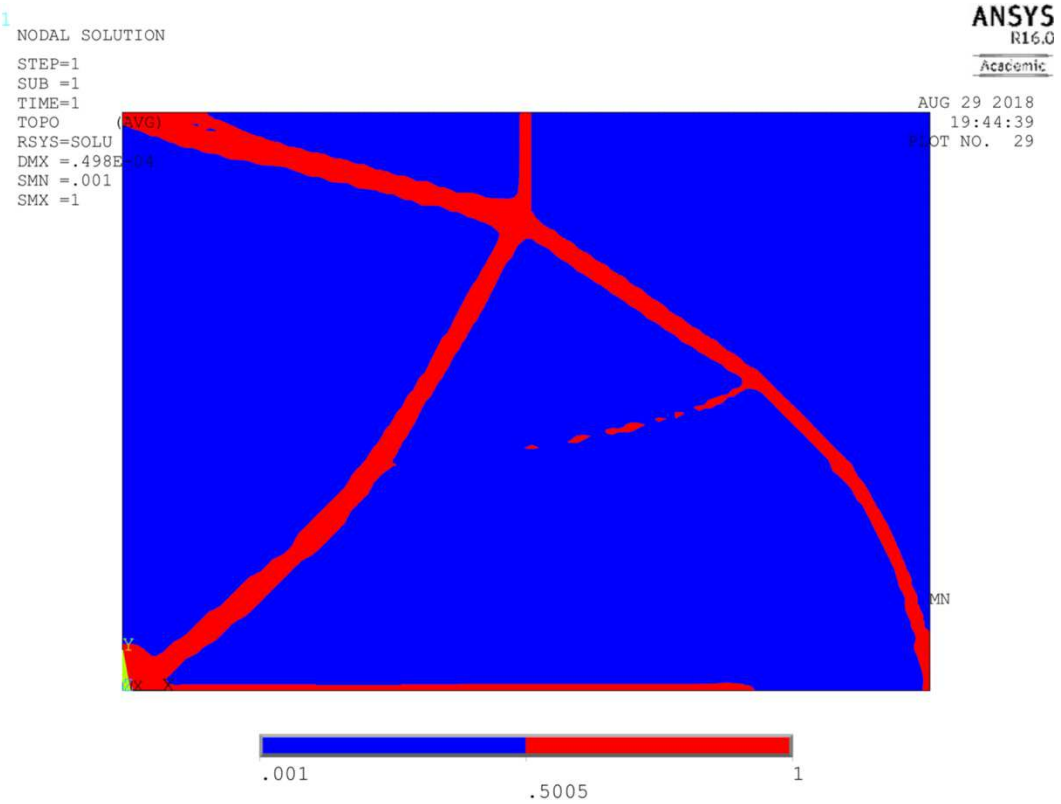

Figure 1-16 Topology for 90% volume optimization

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Chapter 2 Probability design analysis in Ansys

In the second part of this work basic elements of probability design and the tools that Ansys uses to solve different problems will be explained.

2.1 Probabilistic Design

Every structural model that we analyze with a computer software we describe with certain deterministic values. Material properties, geometry, contstraints, everything is described with a deterministic value. This kind of analysis based on a set of specific values is called deterministic analysis. So we ask ourselves, how good are our results? They are dependent on the input parameters. So the answer is, they are good and validated as the input parameters that we gave into analysis.

Every parameter and every aspect of structural model analysis has some degree of uncertainities, which means that it is subjected to a scatter. If we compare two concrete cubes put under a stress test we will never get the exact same pressure strength. The scatter of materials varies and depends on the type of the material and other material properties. In some cases, the scatter of the Youngs modulus is often described with Gaussian distribution that has standard deviation ±3 - 5%. In the same manner the geometric properties of structural components is only produced under specific production tolerances. The uncertainty also continues for the loads that load our structural model, and in some cases uncertainties are there because we lack engineering knowledge. For example, at a high temperatures, the thermal analysis is as reliable as the heat transfer coefficients that input. However, the heat transfer coefficients are almost impossible to measure. All of these reasons above imply that we almost don't have a single certain input parameter. Every input parameter has some sort of uncertainty.

If we wanted to reduce the scatter of material characteristics such as concrete, it would call for high production costs. Every batch of concrete would need to be produced from the exactly same stone granulation, same cement and same water and in same conditions. Therefore, if we accept scatter and rather dealing with it instead of trying to eliminate it, would give us materials that are cheaper and have the same quality, or almost same, as the materials with flawless manufacturing process.

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To embrace the uncertainties and scatter we need to answer following questions using the Probabilistic Design System (PDS):

- The first question we need to answer is how big is the scatter of our input variables and how robust are the output parameters? An output parameter can be any possible parameter that a software can calculate: stress, strain, deflection at a specific node, also maximal stress and strain, maximal temperature
- If we have scatter of output parameters because of the scatter of input parameters, then what is the probability that the design criteria that we gave as output is no longer achievable? What is the failure probability if an unwanted event happens.
- The last question that we need to answer is which input variables have the biggest effect on the scatter of an output parameter and what is the failure probability? How dos the sensitivity plot of the output parameter look with reference to the input variables?

We can use Probabilistic design evaluate the effect of input variables on the outcome of the whole structural system that we analyzed.

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2.1.1 Deterministic vs. Probabilistic Design

In deterministic analysis we always either ignore or apply conservative assumptions for all uncertainties.

We can ignore uncertainties if we know that our input parameters have small effect on the behavior of our analyzed structure. In this case we use only mean values or some nominal values in our analysis. However, sometimes uncertainties have significant impact on the analysis but is still neglected; for instance, the Youngs modulus or the thermal expansion coefficient, where we typically ignore the scatter.

For example, lets say we have a structure and would like to determine thermal stresses. Thermal stresses are directly proportional to thermal expansion coefficient and to Youngs modulus. So that means that our thermal stress looks like this:

$\sigma_{therm} = E\,\alpha\,\Delta T$

Let us assume now that our Youngs modulus has a Gaussian distribution with a 5% standard deviation, what we now get are the stresses that have 16% chance to be more than 5% higher then what we would assume if we were approaching deterministic way. This number increases if the thermal expansion coefficient performs under Gaussian distribution which typically is the case.

2.1.2 Reliability and Quality Issues

If reliability and quality are of great importance then Probabilistic design should be used. We concern us with reliability when product failure has great financial effects. This includes repair cost, product replacement, product warranty etc. The biggest disaster that can happen is human injury or even loss of life. Even though perfection is not physically achievable nor costly approvable, we can use probabilistic design to design reliable and safe products while simultaneously keeping the production cost at a reasonable level.

We can say that the Quality is the customers perception of a product. The product performs as we expected it to perform or even better. When we have a quality product we shouldn't have unexpected event that leads to a failure of the product. By nature, these rare "failure" that happen, happen due to uncertainties in the design. We can use probabilistic design methods in these cases to help us determine how often a "failure" event may occur.

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2.2 Probabilistic Design Terminology for ANSYS

To understand the terminology involved in probabilistic design, consider the following problem.

Find the scatter of the maximum deflection of a beam under a random load of snow Figure 2-1 [8]. The snow should have a linear distribution along the length of the beam with a height of H_1 at one end and a height of H_2 at the other. The beam material is described by various parameters including the Young's modulus, for which a mean value and standard deviation have been derived.

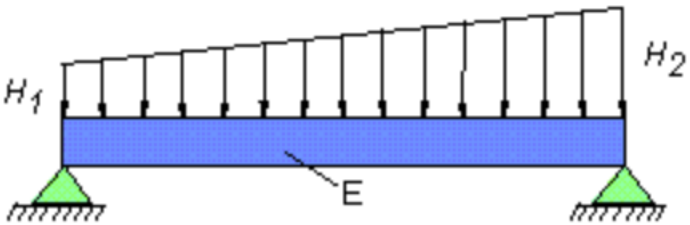

Figure 2-1

random input variables (RVs)

Quantities that influence the result of an analysis.In probabilistic design, RVs are often called "drivers" because they drive the result of an analysis. You must specify the type of statistical distribution the RVs follow and the parameter values of their distribution functions.For the beam example, the heights H_1 and H_2 and the Young's modulus E are clearly the random input variables. Naturally, the heights H_1 and H_2 cannot be negative and more often there will be little snow and only a few times there will be a lot of snow. Therefore, it might be appropriate to model the height of the snow as an exponential or a lognormal distribution, both of which have the bulk of the data at lower values. [8]

correlation

Two (or more) RVs which are statistically dependent upon each other.

In the beam example, it is unlikely that one side of the roof (beam) supports a great deal of snow while almost no snow exists on the other. It is not necessary that H_1 and H_2 are exactly identical, but with a lot of snow then H_1 and H_2 both likely have larger values and with little snowfall then both would be lower. Therefore, H_1 and H_2 are correlated, although the correlation must not be mistaken for a direct mathematical dependency.

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In the beam example, no numerical dependency exists but rather a certain trend between the two heights; with this particular H_1 and H_2 it is unlikely that their values will be drastically different at any given point in time. [8]

Note: Mathematical dependencies have some numerical dependence on each other. For example, a true correlation exists if, when one parameter value doubles, another parameter value also doubles. [8]

random output parameters (RPs)

The results of a finite element analysis.

The RPs are typically a function of the RVs; that is, changing the values of the random input variables should change the value of the random output parameters. In our beam example, the maximum beam deflection is a random output parameter. [8]

probabilistic design variables

The random input variables (RVs) and random output parameters (RPs) are collectively known as *probabilistic design variables*.

In the Probabilistic Design System (PDS), you must identify which parameters in the model are RVs and which are RPs. [8]

sample

A unique set of parameter values that represents a particular model configuration.

A sample is characterized by random input variable values. If you measure the Young's modulus of a given beam, and the snow height on a given day at each end of that beam, then the three values for E, H_1, and H_2 would constitute one sample.

Think of a sample as one virtual prototype. Every component manufactured represents one sample, because you can measure its particular properties (material, geometry, etc.) and obtain specific values for each. [8]

In statistics, however, *sample* also has a wider and more general use. For example, any single measured value of any physical property is considered to be one sample. Because a probabilistic analysis is based on a statistical evaluation of the random output parameters (RPs), the values of the RPs are also called samples. [8]

simulation

The collection of all samples that are required or that you request for a given probabilistic analysis.

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The simulation contains the information used to determine how the component would behave under real-life conditions (with all the existing uncertainties and scatter); therefore, all samples represent the simulation of the behavior. [8]

simulation loop

A single pass through the analysis file.

In each loop, the PDS uses the values of the RVs from one sample and executes the user-specified analysis. The PDS collects the values for the RPs following each loop. [8]

probabilistic model

The combination of definitions and specifications for the deterministic model (in the form of the analysis file). The model has these components:

- Random input variables
- Correlations
- Random output parameters
- The selected settings for probabilistic method and its parameters

If you change any part of the probabilistic model, then you will generate different results for the probabilistic analysis (that is, different results values and/or a different number of results). For example, modifying the analysis file may affect the results file. If you add or take away an RV or change its distribution function, you solve a different probabilistic problem (which again leads to different results). If you add an RP, you will still solve the same probabilistic problem, but more results are generated. [8]

mean value

A measure of location often used to describe the general location of the bulk of the scattering data of a random output parameter or of a statistical distribution function.

Mathematically, the mean value is the arithmetic average of the data. The mean value also represents the center of gravity of the data points. Another name for the mean value is the *expected value*. [8]

median value

The statistical point where 50% of the data is below the median value and the 50% is above. For symmetrical distribution functions (Gaussian, uniform, etc.) the median value and the mean value are identical, while for nonsymmetrical distributions they are different. [8]

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standard deviation

A measure of variability (dispersion or spread) about the arithmetic mean value, often used to describe the width of the scatter of a random output parameter or of a statistical distribution function. [8]

The larger the standard deviation, the wider the scatter and the more likely it is that there are data values further apart from the mean value. [8]

solution set

The collection of results derived from the simulation loops performed for a given probabilistic analysis.

The solution set includes the values of all random input variables and all random output parameters for all simulation loops of a probabilistic analysis. A unique label identifies each solution set. [8]

response surface set

The collection of response surfaces derived from a fitting procedure (regression analysis) *and* the results derived from using the response surfaces for a probabilistic analysis. A response surface set is identified by a unique response surface label. [8]

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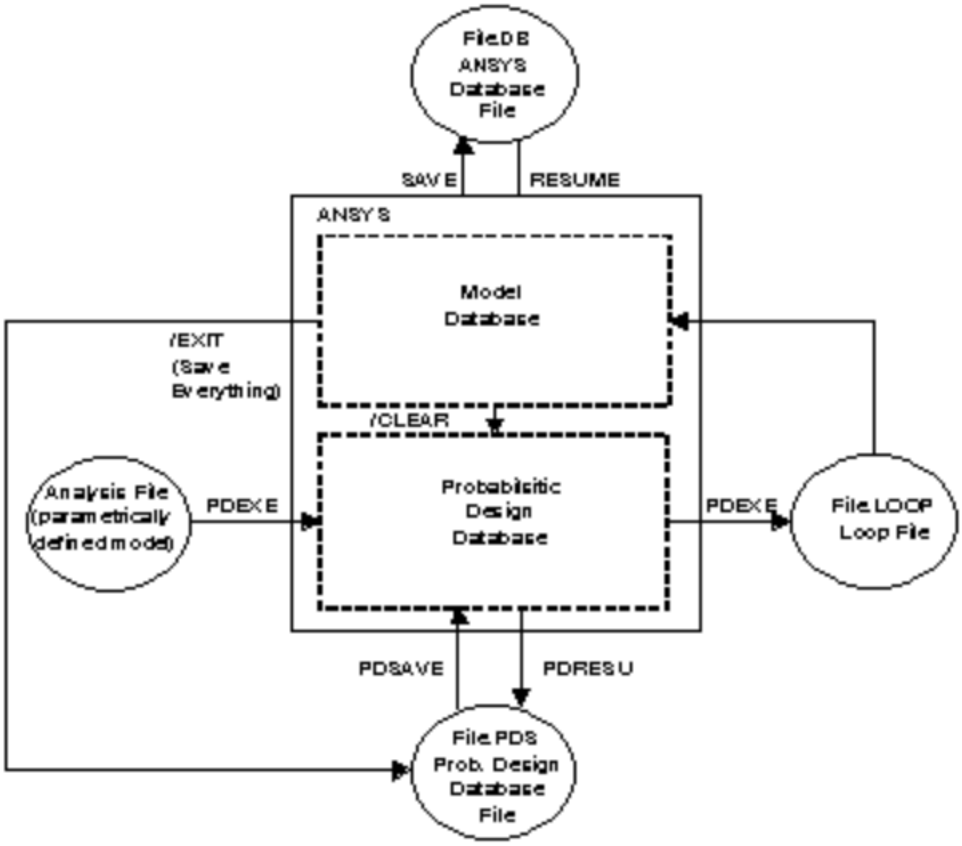

Figure 2-2 Flow chart for PDS analysis process [8]

2.3 Guidelines for Selecting Probabilistic Design Variables

Here are presented useful guidelines for defining our probabilistic design variables

2.3.1 Choosing and Defining Random Input Variables

In the next chapters we will give few tips that help us determine the right distribution function for each of our random input variables and what do the parameters in the distributions represent.

First, it is important to:

- Determine a range of values for each of our random input variables. These values should be kept in reasonable boundaries
- We should set a variability limit for each Random Variable.

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2.3.1.1 Random input Variables for Monte Carlo Simulations

The number of simulation loops that we need to perform for a Monte Carlo Simulation are not dependent on the number of random input variables. The number of simulation loops that we need depends only on the scatter amount of the output parameters and the results type we expect from our analysis. In a Monte Carlo simulation, it is advisable to include all every random input variable we can think of, even if we are unsure about their influence on the random output parameters. Only those random input variables, where we are 100% certain that they have not influence on random output parameters are to be excluded. Then, the probabilistic design system will tell us automatically which random input variables are significant and which are not. The simulation number that we need in a Monte Carlo analysis to provide for us this kind of information depends from case to case, but the more simulation loops we perform the more precise the results will be.

2.3.1.2 Choosing a Distribution for a Random Variable

The type of data that we have basically determines the distribution function that are best for our analysis. In this chapter we will only go through basics of choosing the right distribution and mention those that are available in ANSYS. It is needed to mention that there are numerous papers on this topic and will not be a subject of this thesis.

2.3.1.2.1 Measured Data

If we have measured data then the first thing we need to know if that data is reliable and how much the it is reliable. The scatter of the data is not an inherent physical effect, but it also includes inaccurate measurement. We have to take into account that the person that did the measurement maybe “tuned” the data. For instance, if the load has been measured, the person that did the measuring maybe rounded the values it measured, this means that the information we received are not the values that are truly measured. This could maybe provide a deterministic bias in the data, depending on the tuning amount, that we need to separately address. This bias that could have been built into the data should be discussed with the person that did the measurements, or with the person that provided the data for us.

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In case that we are certain about the data precision, then how we continue depends on how much data and information we own. If we have a single field of production, the quantity of data is mostly wide scattered. If we don't have much data then it would be reasonable to use the data only to evaluate a rough figure for standard deviation and mean value. What we do in these cases is that we assume that our physical effect follows Gaussian distribution. This is only possible if our effect has no maximum and minimum since Gausain distribution goes from -∞ to +∞. The other option that we have is to use the data at our disposal and determine the upper and lower boundaries for a uniform distribution. In a field with a mass production there is a lot information and a lot of data, in which case we could use a commercial statistical package which would let us fit a distribution function that describes the scatter of our data in best possible way.

2.3.1.2.2 Mean Values, Standard Deviation, Exceedence Values

The scatter of our data is typically described through the mean value and standard deviation. In many cases the value of a physical quantity is given as; for example, "100±5.5". This means that the value "100" represents mean value and "5.5" represents standard deviation. If we use this way to specify the data, that would mostly imply to a Gaussian distribution, but whether it truly is Gaussian distribution or not should be verified. The verification should be done because sometimes man value and standard deviation is given for any data collection regardless of its true distribution. If we are provided with more information for example we know that our input variable must follow lognormal distribution, then the PDS lets us use the standard deviation and the mean value for a definition of a lognormal distribution.

In some cases we are given the specification of the data through mean value and and exceedance confidence limit. Commonly is the yield strength of a material provided in this manner; for instance, based on a 95% confidence level a 99% exceedance is provided. What this means is that we can be 95% certain that 99% of all cases, its measured property will be greater than specified boundary , and will sink bellow the specified boundary in only 1% of all cases.

The person that passes this information to us is using the number of the measured data sample, mean value and standard deviation to extract this type of information. If the data scatter is passed this way it would be for good to ask for even more detail from the person

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that gave them to us. The exceedance limit is determined from the measured data and its statistical evaluation, and because of this the person that provided us with the information might be able to give us more details that were used in statistical assessment. For the case that we aren't provided with more further details we could consider to assume that measured sample number was great.

If we denote exceedance limit with $z_{1-\alpha/2}$ and mean value with z_{μ} the we can derive standard deviation from following equation:

$$\sigma = \frac{|z_{1-\alpha/2} - z_{\mu}|}{C}$$

The values for the coefficient *C* can be taken form the table bellow:

Exceedence Probability	C
99.5%	2.5758
99.0%	2.3263
97.5%	1.9600
95.0%	1.6449
90.0%	1.2816

2.3.1.2.3 No Data

If we found ourselves in situations when there is no information or data available, there is no just one solution or just one right answer. Which physical quantities are usually described of what functions of distribution are given in the hints below. This could help with certain physical quantity. There will be also given a list below of which distribution functions are typically used for what sort of situation.

Geometric Tolerances

- When designing a prototype we assume that the real dimensions of the produced parts are near manufacturing tolerances. If this is the case then using uniform distribution would be reasonable since we could describe those tolerances with upper and lower limit.

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- Sometimes the manufacturing process generates a skewed distribution; for example, one half of the tolerance band is more likely to be hit than the other half. This is often the case if missing half of the tolerance band means that rework is necessary, while falling outside the tolerance band on the other side would lead to the part being scrapped. In this case a Beta distribution is more appropriate.
- The most often used distribution is Gaussian, but the problem with it is that it has not bounds and we are making a hard theoretical error since geometrical extensions can only be positive numbers. On the other hand this is in practice irrelevant because the standard deviation is very small when compared with geometric extension value.

Material Data

- In many cases we have Gaussian distribution that describes out material data scatter
- Sometimes the strength of a certain part behaves according to "weakest-link-theory". This means that the structure fails when the weakest spot fails. In these cases where structure behaves according to "weakest-link", it is advisable to use Weibull distribution to describe material properties.
- Sometimes if we have data scatter of a material with similar material properties, it is acceptable to use those. For example, we have information about material that is similar to the one we will use. Its material property is described with Gaussian distribution and has standard deviation ±5% of mean value. If we knew only mean value of the material we are using, we could assume that its properties have also Gaussian distribution with ±5% standard deviation of its mean value.
- For temperature-dependent materials it is prudent to describe the randomness by separating the temperature dependency from the scatter effect. In this case you need the mean values of your material property as a function of temperature in the same way that you need this information to perform a deterministic analysis. If *M(T)* denotes an arbitrary temperature dependent material property then the following approaches are commonly used:
    - Multiplication equation:

$$M(T)_{rand} = C_{rand}\overline{M}\,(T)$$

    - Additive equation:

$$M(T)_{rand} = \overline{M}\,(T) + \Delta M_{rand}$$

    - Linear equation:

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$M(T)_{rand} = C_{rand}\bar{M}(T) + \Delta M_{rand}$

- Here, $\bar{M}(T)$ represents mean value of the property as a function of temperature. The mean value in the "multiplication equation" is scaled with a coefficient C_{rand} and this random number describes the scatter of the material property. In the "additive equation" a random variable ΔM_{rand} is added on top of the mean value function $\bar{M}(T)$. The "linear equation" combines both approaches and here both C_{rand} and ΔM_{rand} are random variables. However, you should take into account that in general for the "linear equation" approach C_{rand} and ΔM_{rand} are, correlated.

In order to make a decision on which approach is most usable to describe the scatter temperature dependent material property we need raw data of this material property. Only by reviewing the raw data and plotting it versus temperature you can tell which approach is the better one.

Load Data

- For loads, we typically have only average or maybe nominal value. We could ask the person that provided the nominal value these questions: If there are 10000 parts that work under real life conditions, what would be the lowest value of the load that puts only one of these 10000 parts under stress and if all other parts are excited to a higher load? What load value would most of these 10000 parts have? What would be the highest value of the load that puts only one of these 10000 parts under stress and if all other parts are excited to a lower load? If we want to be safe, we should ask these questions not just the person who gave us the nominal value but also the people that are knowledgeable with how our products and structures are operated. Combining all of the answers above we can then determine what is probably the lower and the upper limit of the value. This is not an easy task since a lot of knowledge about statistics and the operating conditions of the parts or components is needed. As a check or a confirmation we compare this representation with the nominal value we would use in a deterministic analysis. In case that we have a conservative assumption for the nominal value that could mean that its value is close to maximum, if we do not then it is probably close to most likely value. If we have maximum, minimum and the most likely value, we can use triangular distribution to describe our input variables.
- If we use a computer program to generate the load parameter then it is advisable to a probabilistic analysis of ANSYS as the solver mechanism. It is also advisable to use

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probabilistic design technique Use a probabilistic design technique on that computer program to assess what the scatter of the output parameters are, and apply that data as input to a subsequent analysis. In other words, first run a probabilistic analysis to generate an output range, and then use that output range as input for a subsequent probabilistic analysis.

2.3.1.3 Distribution Functions

In this chapter we will give basic explanation of the distribution functions that are available in ANSYS and when they are useful. There are many more in literature where they are also in detail explained. Some of them include the following:

Beta Distribution

We use Beta distribution when describing random variables that are limited at both ends. When performing linear operations on random variables that follow uniform distribution, then the solutions that come from these operations are typically described by Beta distribution. For instance; if we have assembly line, and parts are being assembled, then the individual tolerances of each assembly follows uniform distribution. Now if we take a look at overall tolerances of the assembly, they are the result when summing geometrical extensions of each individual part. As a consequence, we describe the overall tolerances of the assembly with a beta distribution. We can also use the Beta distribution to describe the scatter of each individual geometrical extensions of parts, it does not have to be uniform distribution, the uniform distribution is just a special case of the Beta distribution.

Exponential Distribution, (Figure 2-3 [9]***)***

If we have a physical reason that pdf is decreasing as the number of random input variable increases it is advisable to use exponential distribution. In most cases these are time related effects. It can describe time between events that happen at a constant rate. For this reason is it very popular for lifetime related systems reliability, and its used for life distribution of systems without redundancy. In most cases we use it if the rate of failure is constant with time, and if there is no wear-out in the lifetime. Mechanical parts and components are very life limited because of wear-out and this would prevent us from using exponential distribution for mechanical components. On the other hand, if there is a preventive maintenance before

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wear-out, then we could still use exponential distribution to describe time until maintenance occurs.

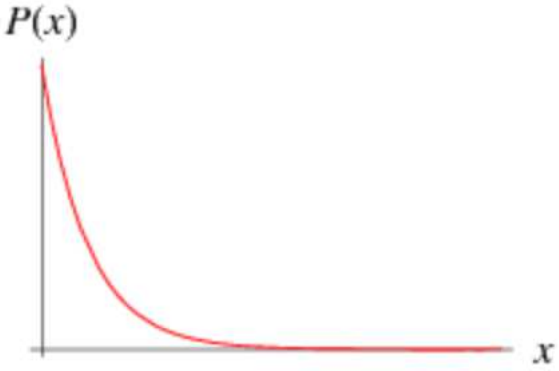

Figure 2-3 Exponential distribution

$$D(x) = P(X \leq x) = 1 - P(X > x) = 1 - e^{-\lambda x}$$

Gamma Distribution, (Figure 2-4 [9]***)***

The Gamma distribution is a distribution function that is time related. If we assume that some phenomena happen at the constant rate, we can use Gamma distribution to describe the time that is needed for these events to happen. It can be used if we want to describe the time to failure of a system that has standby components.

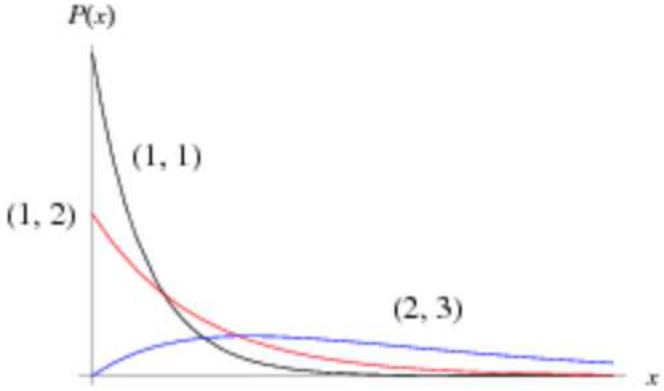

Figure 2-4 Gamma distribution

$$D(x) = P(X \leq x) = 1 - P(X > x)$$

$$= 1 - \sum_{k=0}^{h-1} \frac{(\lambda x)^k e^{-\lambda x}}{k!} = 1 - e^{-\lambda x} \sum_{k=0}^{h-1} \frac{(\lambda x)^k}{k!} = 1 - \frac{\Gamma(h, x\lambda)}{\Gamma(h)}$$

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Gaussian (Normal) Distribution (Figure 2-5 [9]***)***

The Gaussian distribution also called normal distribution is the most basic and most commonly used distribution. It can describe the scatter of many physical effects. If a random variable is generated from a linear combination of a large sample of other random effects, that random variable follows Gaussian distribution. This applies even if the other random effects follow some other distribution.

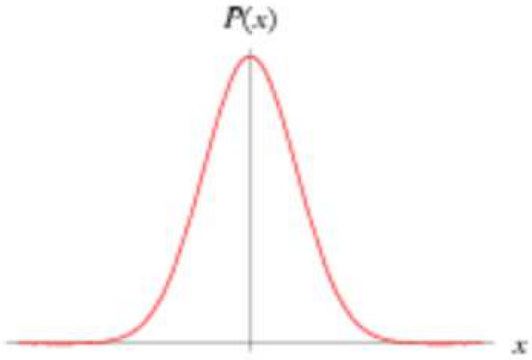

Figure 2-5 Gaussian (Normal) Distribution

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}$$

Lognormal Distribution (Figure 2-6 [9]***)***

The lognormal distribution together with Gaussian distribution belongs to most basic and most commonly used distribution. We use it when describing the scatter of the measured data and here the logarithm of the data that we measure follows normal distribution. When describing events that come from multiplication of a great number of error effects, the lognormal distribution is very suitable to use. This also applies for when describing random variable that comes from multiplication of two more random effects. These effects follow also lognormal distribution. In cases of lifetime distributions lognormal distribution is the commonly used. For instance, if we have a cyclically loaded material that survives until low-cycle-fatigue, the scatter of the strain amplitude is described with lognormal distribution.

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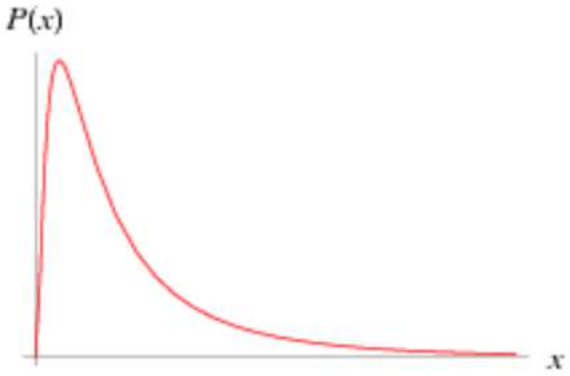

Figure 2-6 Lognormal distrinution

$$P(x) = \frac{1}{S\sqrt{2\pi}x} e^{-(lnx-M)^2/(2S^2)}$$

Uniform Distribution (Figure 2-7 [9]***)***

The most common use for the uniform distribution is when we are provided with only lower and upper boundary. This is mostly helpful when we are describing geometric tolerances. In the cases where there is no evidence that a certain value of the random variable has greater probability than any other then we can also use uniform distribution. It is also called rectangular distribution because it has a constant probability.

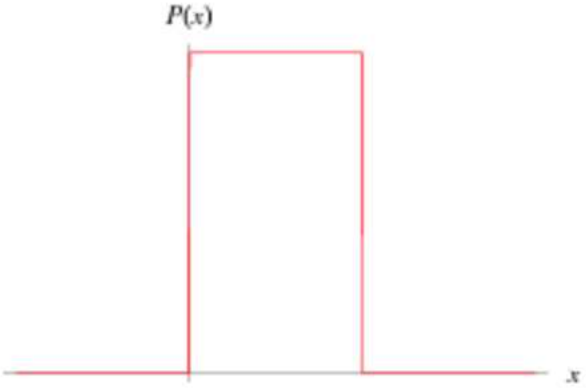

Figure 2-7 Uniform distribution

$$P(x) = \begin{cases} 0 \; for \; x < a \\ \dfrac{1}{b-a} \; for \; a \leq x \leq b \\ 0 \; for \; x > b \end{cases}$$

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Triangular Distribution (Figure 2-8 [9]***)***

When modeling random variable and the actual data is not at hand the most commonly used distribution is the Triangular distribution. We used to mathematically describe an expert opinion. It does not matter what kind of random variable we are modeling we can always ask experts in the observed field some questions. It is also advisable to involve the approximation calculated in a computer program.

Figure 2-8 Triangular distribution

$$P(x) = \begin{cases} \dfrac{2(x-a)}{(b-a)(c-a)} & for\ a \le x \le c \\ \dfrac{2(b-x)}{(b-a)(b-c)} & for\ c < x \le b \end{cases}$$

Weibull Distribution

Weibull distribution is mostly used in engineering for strength related parameters. Its typical use is for strength parameters of exceptionally brittle materials.

$$P(x) = \alpha\beta^{-\alpha}x^{\alpha-1}e^{-(x/\beta)^{\alpha}}$$

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2.4 Probabilistic Design Techniques

For running the probabilistic design in ANSYS it is advisable to understand the algorithm used by the computer program. Here we will present some details on different types of methods and the sampling that each of them uses.

2.4.1 Monte Carlo Simulations

The Monte Carlo Simulation is according to most literature the most used method for probabilistic analysis. Monte Carlo lets us simulate how pieces behave the way they are made. The way that one manufactured peace is put under certain set of load or its boundary conditions are all simulated with just one simulation loop.

For Monte Carlo simulation we can utilize Direct Sampling method or Latin Hypercube Sampling method, both of these will be explained in further text.

When a piece or component is manufactured, its geometry and other material properties can be measured. Same way, when the component is put under use, the loads loading its could be also measured. At the end of the line, once we hold the component in our hand and put it in use, all the input parameters have their own particular value that could be measured. When the second component is manufactured, the same could be done, and if these first two components were to be compared we would discover that there is small discrepancy between the two. This correlation between neighbouring components i.e. first to second, second to third and so on represents the scatter of the input parameters. So what does Monte Carlo Simulation technique do, it actually imitates this process. This way, we can “virtually” operate and manufacture parts one after the other.

The next main question that needs to be answered is what are the advantages of the Monte Carlo Simulation method?

- First, this method is independent from the physical effect that we model in a finite element analysis, that means that it is almost always appropriate for use. It is not bounded to random output parameters in the way that if some assumptions of the random output parameters were satisfied, probabilistic analysis would run faster. Under the condition that the deterministic model is free from errors and we perform a large number of loops, then Monte Carlo techniques will always give us a correct

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probabilistic results. However, it is not possible to run infinite number of loops, for that reason, the only assumption is that loop number is high enough for the probabilistic results to be representative. PDS analysis also provides verification of this assumption through confidence limits.

- Second, if we take into account all of the reasons above we can say that Monte Carlo Simulation methods are the best suted probability methods for validation purposes and benchmarking.

- And third, all of the simulation loops are inherently independent one from another; a single simulation loop does not depend on the outcome of other simulation loops. It is from this reason that Monte Carlo Simulation is suited for parallel processing.

However, negative side of the Monte Carlo Simulation technique is its inefficiency in simulation loop number.

2.4.1.1 Direct Sampling

The most used type of Monte Carlo analysis is Direct Monte Carlo Sampling Figure 2-9 [8]. It imitates natural processes that is suitable for everyone to observe and that makes it very easy to understand. It is from this reason that it is so popular. Using this method, we can simulate how our parts and components act based on the manner they are built. One loop depicts one part that is set under load and has its own boundary conditions. However, Direct Monte Carlo Sampling technique can not be considered the most efficient one. None the less it is still very popular for use, particularly for benchmarking and validation of the probabilistic results. Still, a great number of loops is needed for benchmarking and validating and some times that is not achievable. The Direct Sampling method is less efficient because the sampling process possesses no “memory”.

For instance, there are two random input variables X1 and X2, and they both have a uniform distribution from 0.0 to 1.0, and 15 samples are generated. Then it is possible to get a cluster

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of two samples that happen very near to each other. It can occur that one sample owns input value near to some other sample, and from this we do not get new information and understanding of the components behavior.

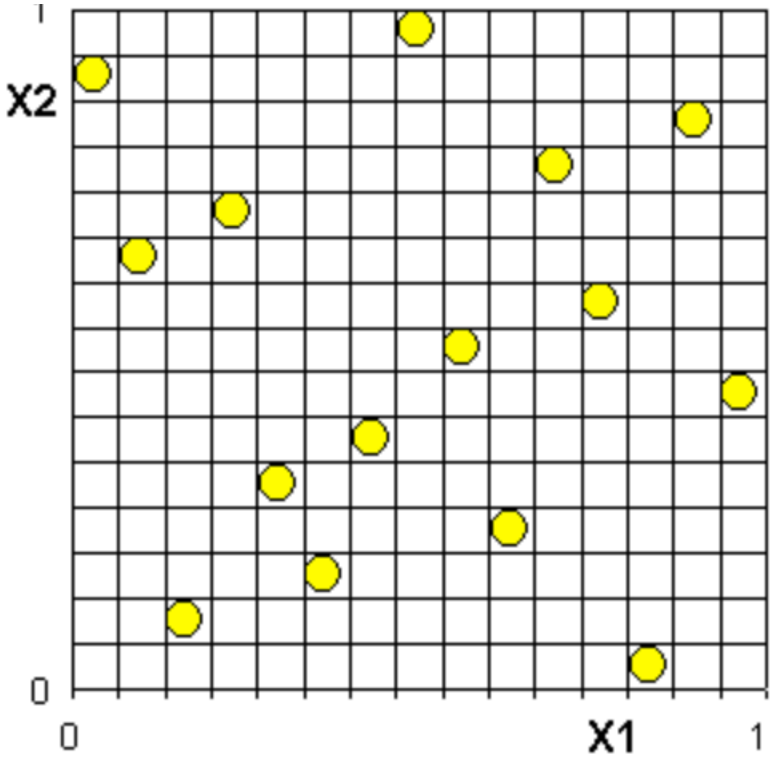

Figure 2-9 Direct sampling [8]

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2.4.1.2 Latin Hypercube Sampling

The Latin Hypercube Sampling (LHS) technique (Figure 2-10 [8]) is a form for Monte Carlo Simulation methods that is even more advanced and more effective. The difference between the Direct Monte Carlo Sampling and the LHS is that LHS possesses a sample "memory", which means that the samples that are already evaluated will not repeat themselves. Its basically goes around clustering samples. Another trait of LHS is that the tails of distribution also take part in the sampling process. Basically, LHS requires les simulation loops when compared to Direct Monte Carlo Simulation technique and the results will not lack accuracy.

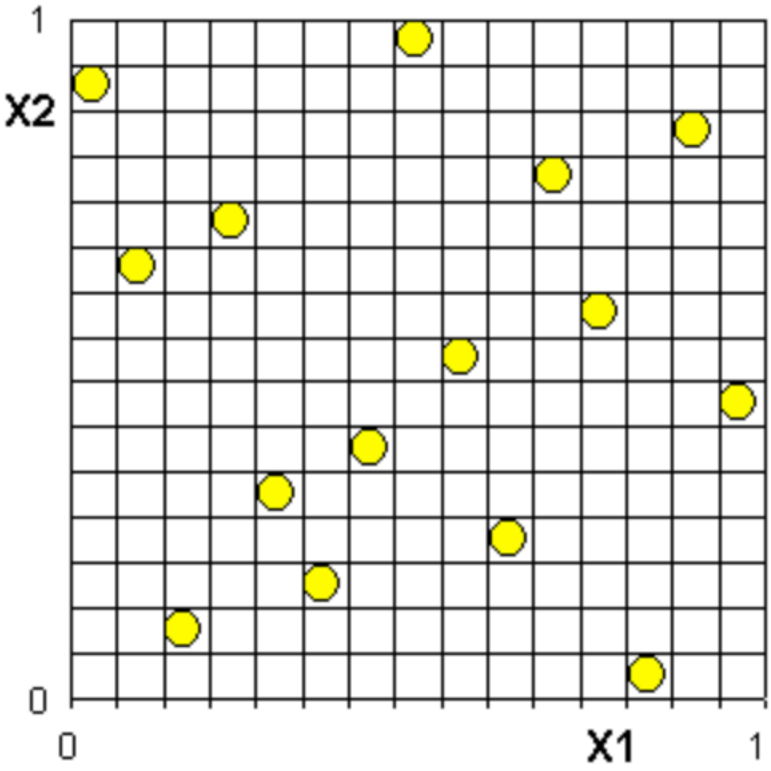

Figure 2-10 Latin Hypercube sampling [8]

2.4.1.3 User-Defined Sampling

The third option from Monte Carlo Simulation in ANSYS is that we provide the file that contains the samples. Here, we have the control over the sampling data. It is only needed that we input the file name and its path.

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Chapter 3 Probability based topology optimization

Engineering field of topology optimization is older than 100 years and it is still a major topic in development. In 1973 Rossow and Taylor [10] elaborated topology optimization using the finite element method (FEM), and later during the 80s the topic of the topology optimization started to expand. Most of the papers that were published during this time have been dealing with deterministic problems. Until the 2000s there were almost no published papers that dealt with any kind of uncertainties.

During the late 90s there were almost no released papers that concerned themselves with some kind of stochastic behavior in the topology optimization. In 2002 Marti and Stöckl [11] [12] gave some insight on this topic in their paper Stochastic Material and Topology Design. In 2000 Duan B Y [13] released one of the first papers that approached topology optimization from the stochastic point of view. In the 2005 ,Alvarez and Carrasco [14] dealt with truss topology optimization under stochastic loading. It is showed in their work that the problem of finding minimal compliance in truss under stochastical loading is eqivalent to the dual of a special convex minimax problem. In 2010 and 2011 Dunning P D [15] [16] approached to robust structural topology optimization. The aim was to minimize expected compliance with stochastic loading quantity and loading direction. The stochastic parameters were normally distributed and statistically independent. This perspective is same as problem with multiple load cases where load cases are analytically derived and from there expected compliance is accuretaly computed. Guest J K and Igusa T [17] in 2008 released a paper on topology optimization with uncertain magnitude and location of the load and with uncertain location of the structural nodes. They presented a method where the loading uncertainties are covered with “safety factors” of the deterministic load cases. They also estimated geometric uncertainties with second order stochastic perturbation, form there the uncertainties in structural stiffness were transformed into a equivalent system of auxiliary loads.

Later in 2010, this method was extended for uncertainties in material properties and in 2011 for nonlinear effects of global instability.

Around the same time Asadpoure A [18] presented a technique that combined an approximation method for assessment of uncertain structural stiffness, material properties and geometry, and deterministic topology optimization. This technique was significantly faster than Monte Carlo based algorithms.

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In the final chapter of this thesis we will try to combine topology optimization with the probabilistic design analysis using Ansys Mechanical APDL. There is very limited number of papers published on this topic, and the iterative procedures described in them are time consuming and therefore expensive. We will introduce a simplified way of obtaining probability based topological optimizations.

Some input parameters will be input probabilistic which means that some input parameters will follow some kind of distribution, rather than just be input deterministically. Then when we acquire number of topological optimizations based on number of loops for Monte Carlo simulation we will overlay those solutions and get the final result. This is more conservative approach a more engineer approach but its solution is valid, it will consider every probable input parameter combination and combine them into one.

This approach of topology optimization could play a great role when designing structures that have a probabilistic side to them, and even a slight aberration may lead to failure of the part of the structure and maybe the whole..

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Chapter 4 Examples

4.1 Example 2-Load cases and Topological Optimization

In this example we will see how are load cases treated in Ansys. There are 3 types of loading cases that can be used in Ansys:

- We can obtain for example 5 topological solutions, each for different load case
- We can obtain 1 topological solution for 5 independent load cases
- We can obtain 1 topological solution for 5 combined load cases

4.1.1 Example 2.1 -1 Topological solution for 2 independent load cases

A 5x7 shear wall is loaded with two independent forces and constrained at the left edge. One force is applied at the top edge at the coordinate (x,y)=(5,5) and has intensity F_1=1000 kN and the second force is applied at the bottom edge at the coordinate (x,y)=(7,0) and has the same intensity F_2=1000 kN. The optimized volume will be 65%.

Topological solution is shown on Figure 4-1:

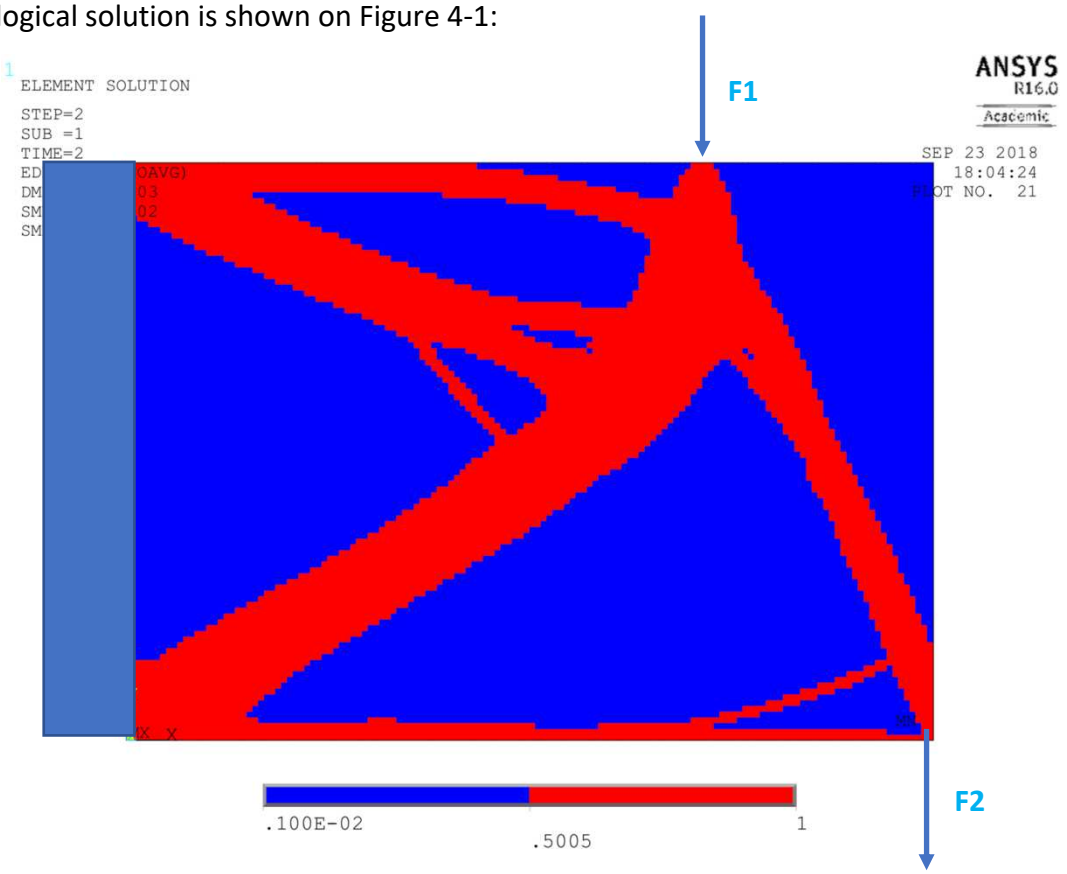

Figure 4-1 Topological solution for 2 independent load cases

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4.1.2 Example 2.2 -1 Topological solution for 2 independent load cases

We will now perform topological optimization on the same system as in the example 2.1. The geometry and the location of the forces is the same, F_2=1000 kN, the only difference is the intensity of the force F_1 which will now be F_1=333 kN.

Topological solution is shown on Figure 4-2.

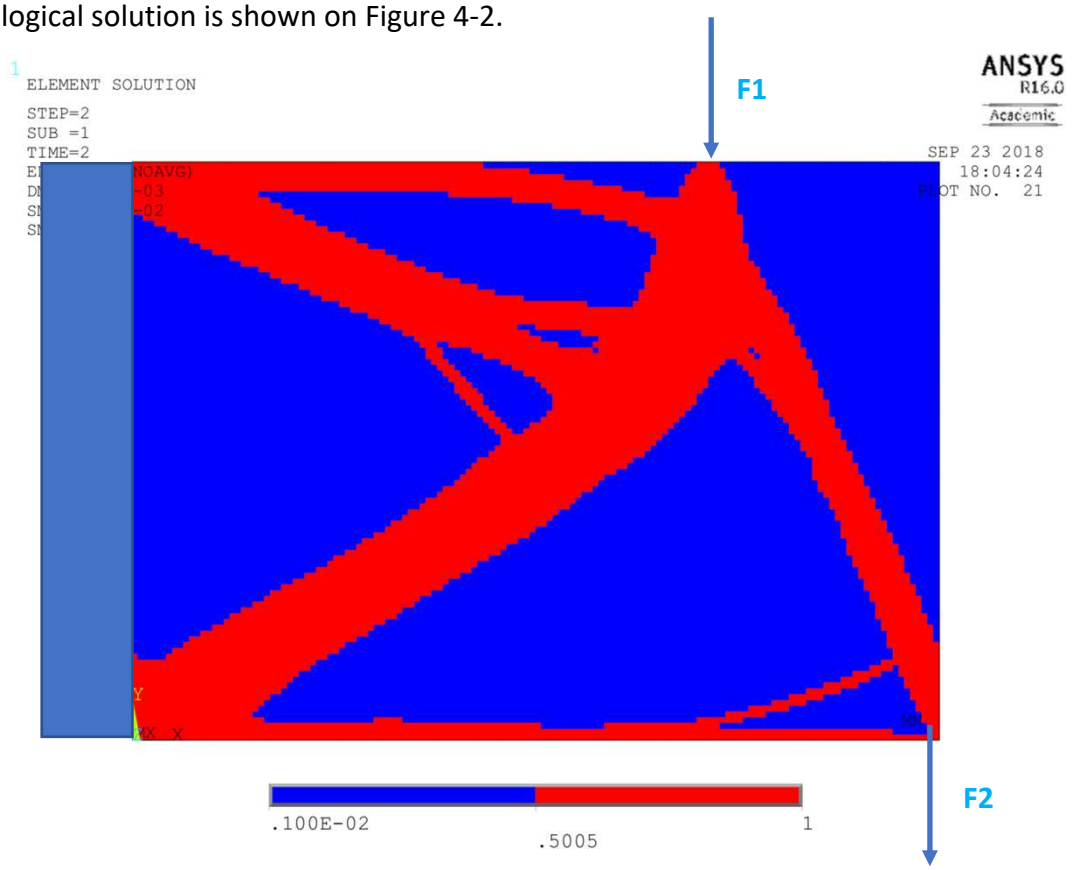

Figure 4-2 Topological solution for 2 independent load cases that have different force intensity

If we look at the solutions of the examples 2.1 and 2.2 we will notice that the solutions are exactly the same, which means that by this type of loading with independent load cases force intensity does not play a role. This is very important for the analysis we will perform later on.

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4.1.3 Example 2.3 -1 Topological solution for 2 combined load cases

A 5x7 shear wall is loaded with two combined forces and constrained at the left edge. One is applied at the top edge at the coordinate (x,y)=(5,5) and has intensity F_1=1000 kN and the second force is applied at the bottom edge at the coordinate (x,y)=(7,0) and has the same intensity F_2=1000 kN. The optimized volume will be 65%.

Topological solution is shown on Figure 4-3.

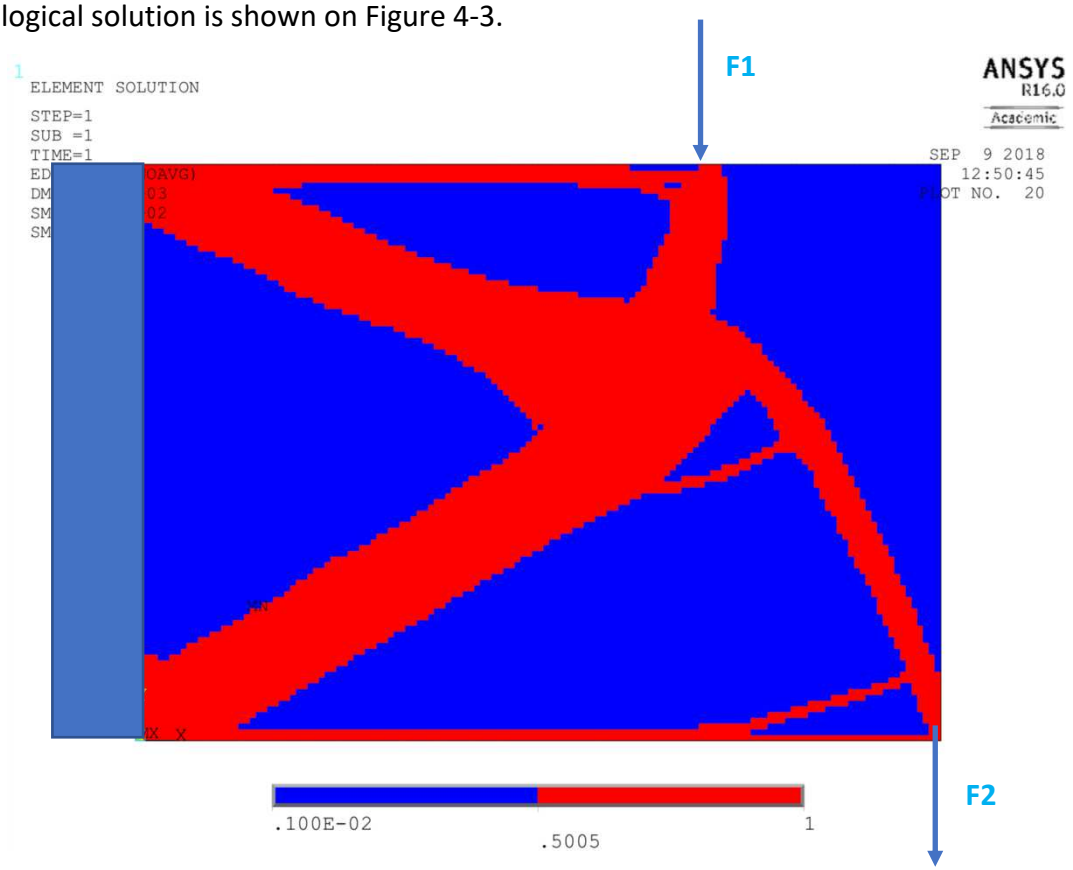

Figure 4-3 Topological solution for 2 combined load cases

If we compare the topological solution we acquired in this example with the solutions acquired in the examples 2.1 and 2.2 we can notice that the shapes are very similar but not exactly the same.

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4.1.4 Example 2.4-1 Topological solution for 2 combined load cases

We will now do the same as with examples 2.1 and 2.2 a topological optimization will be performed on the same system as in the example 2.3. The geometry and the location of the forces is the same, the only difference is the intensity of the force F_1 which will also in this case be F_1=333 kN .

Topological solution is shown on Figure 4-4.

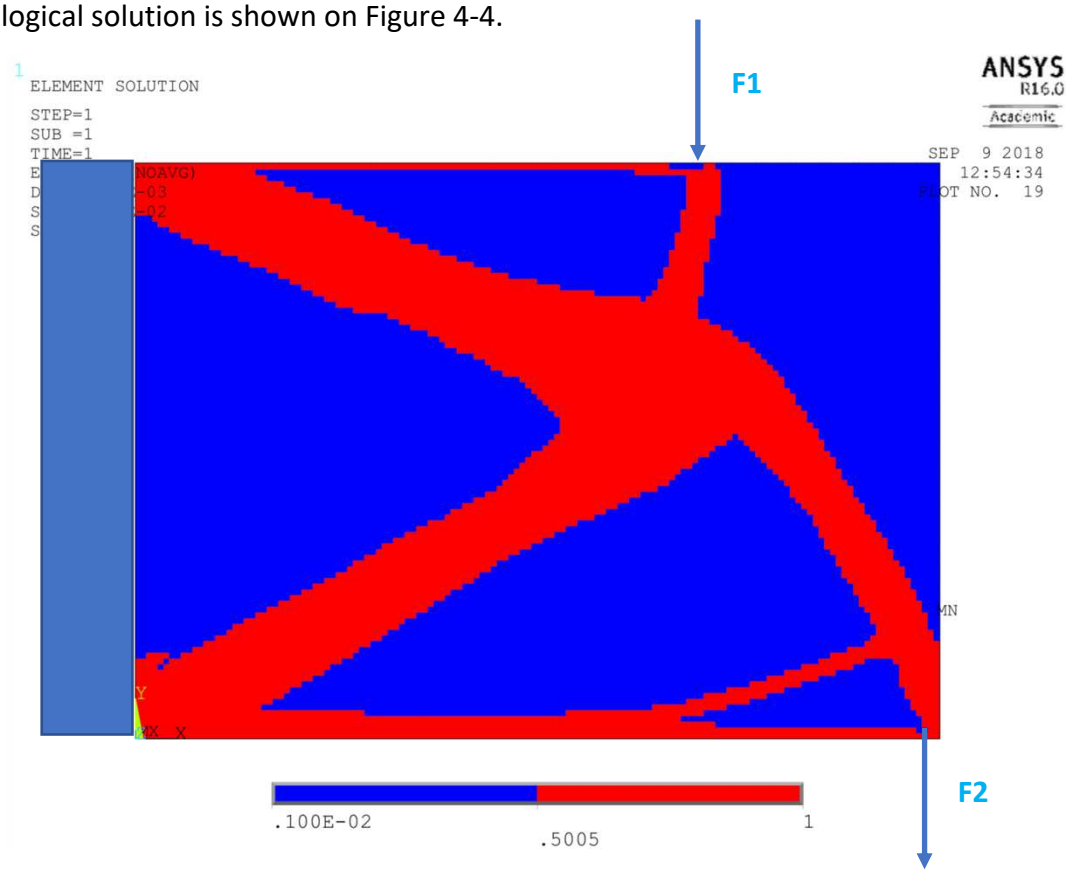

Figure 4-4 Topological solution for 2 combined load cases with different load intensity

If we now do the comparison between the topological solutions of examples 2.3 and 2.4 we will notice that the solutions are similar but not exactly the same. There is now less material in the area of the force F_2 which has smaller intensity of the two forces. This means that by this type of loading, the different force intensity plays a role on end result. If we perform a topological analysis on a system where the force intensity significantly influences our results, then this sort of loading may be better suited for the analysis.

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4.1.5 Example 2.5 – 2 Topological solutions for 2 different load cases and their superposition

In the final example related to topological optimization we will perform two topological optimizations on the same system which means same geometry and same boundary conditions. The only difference is the load location and its intensity. We will try to "adapt" the optimized volume according to the load intensity to try to obtain the end (solutions added together) optimized volume of 65%. If we input for both load cases 65% optimized volume and add them together, the end solution would actually be 30% plus the areas where two solutions overlay. If we have 65% optimized volume that means that the percentage of the material that is left is 35%. If we add together two solutions with 35 % of material each, we obtain 70% of material, which means that the end solution is only 30% optimized. The error on this 30% are those finite elements where both solutions have material.

Example A (Figure 4-5):

In the example A we will perform a topological optimization on the same system as on the examples 2.1, 2.2, 2.3, 2.4 which means same geometry, same boundary conditions. We will load it with only one force F_1=333 at the coordinates (x,y)=(5,5). The optimized volume will be 92 %. Why 92%? It will be explained on the following example, example B.

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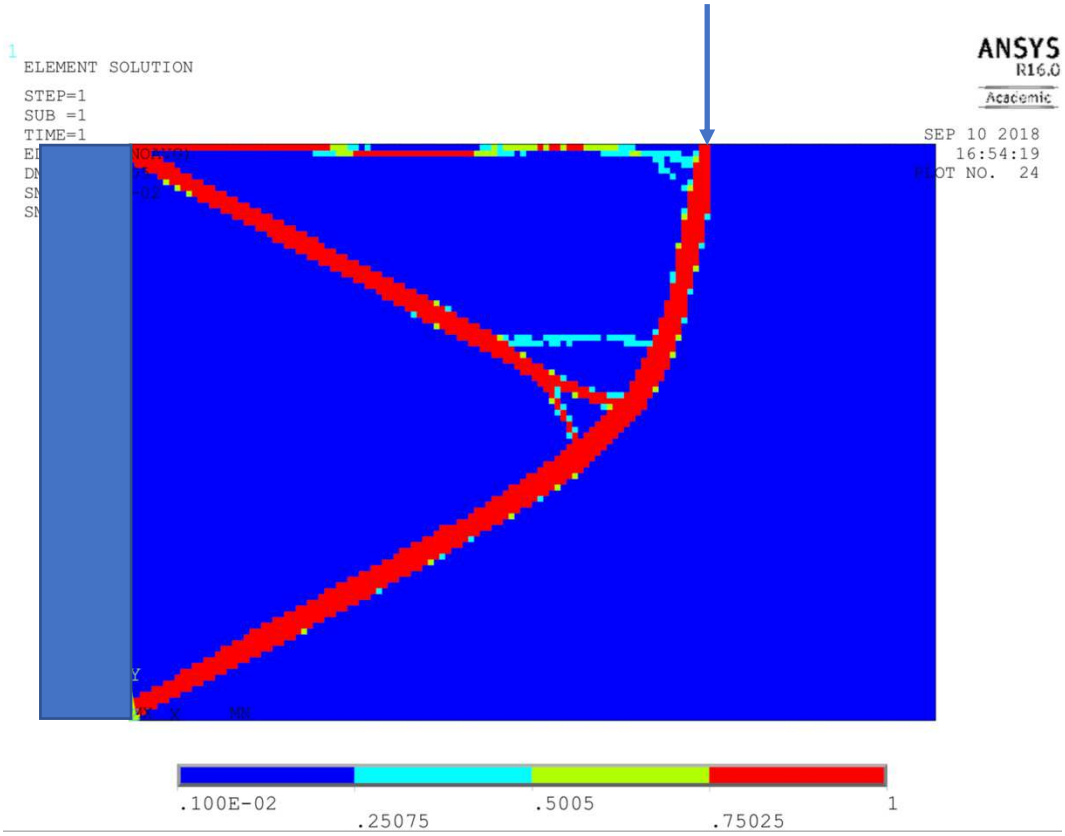

Figure 4-5 Topological solution with 1 load case

Example B (Figure 4-6):

In the example B we will perform a topological optimization on the same system as on the examples 2.1, 2.2, 2.3, 2.4 which means same geometry, same boundary conditions. We will load it with only one force F_2=1000 at the coordinates (x,y)=(7,0). The optimized volume will be 73 %. Now we will explain why are the examples A and B optimized for 92% and 73% respectively.

Well, we are trying to achieve somehow topological optimization solution that acknowledges load intensity, so if we are looking for 65% end optimized volume, that means that our body has 35% of material. This 35% has to be divided between the two. In the example A we have a force of F_1=333, so the corresponding optimized volume is:

$$V_A = 100 - 35 \cdot \frac{F_1}{F_1 + F_2} \approx 92\%$$

In the example B we have a force of F_1=1000, so the corresponding optimized volume is:

$$V_B = 100 - 35 \cdot \frac{F_2}{F_1 + F_2} \approx 73\%$$

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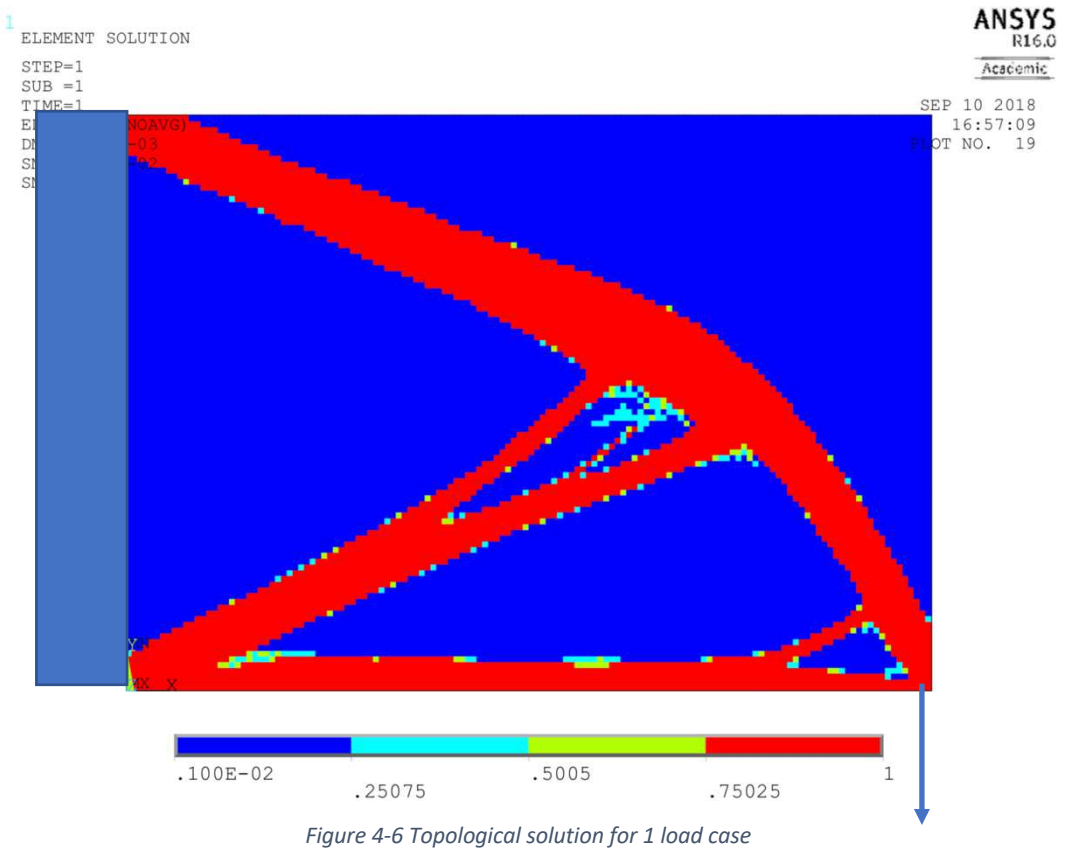

Figure 4-6 Topological solution for 1 load case

And now, if we were to overlay the topological solutions from example A and example B we would receive image shown on Figure 4-7.

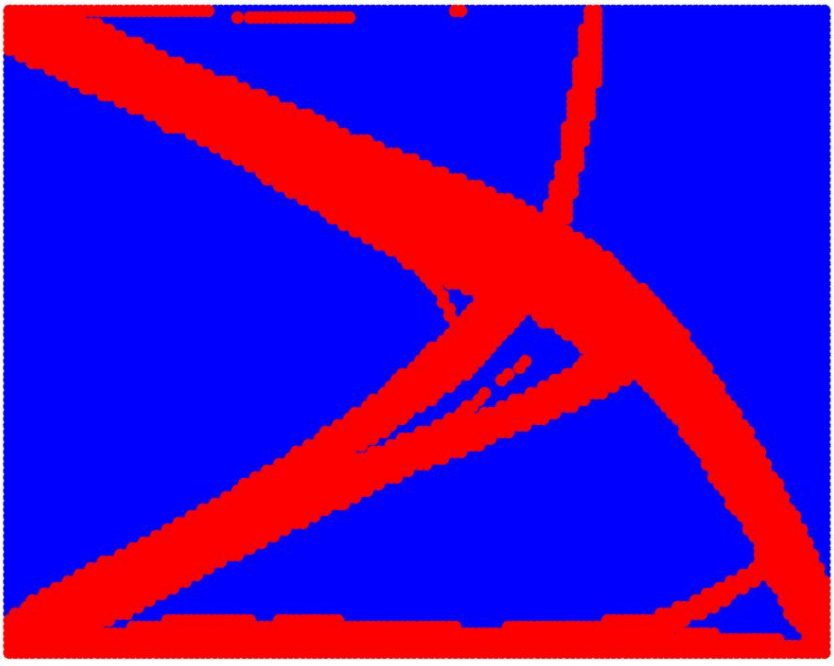

Figure 4-7 Topology received when overlaying figures 3-5 and 3-6

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4.1.6 Commentary on the examples 2.2

Lets us now look closely to figures 3-7 and 3-4. We can notice that they are very similar but not completely identical. This means that if we are designing a part of structure or the whole structure, both methods would give the same product since the topology that we acquired from the software solutions needs to be postprocessed to get the shape of the end product that would be produced. The conclusion from all of this above is that with overlaying several topologies from different topological solutions would lead to the same product as if we were to use one topological solution with independent load cases. It should be noted that when using overlaying method, there can be a part of volume that is contained in all overlayed solutions, and in this case only one is taken, since the volume can not be greater than 1. This conclusion is very important for the following examples.

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4.2 Example 3.1 “Bridge” - Optimization of topology with probabilistic input parameters

The first example that will be analyzed is following:

A 4x12 beamlike wall (Figure 4-8) with circular openings is supported on end bottom points and in the middle. It is loaded with a continuous load at the top edge. The load has a constant value of a 1000 kN but it is divided into three combined load cases. Load case 1 is acting along top edge of the wall between X=0 and X=4 and follows gaussian distribution with mean value of 1000 and standard deviation of 50. Load case 2 is acting between points X=4 and X=8 and is also normally distributed with mean value 1000 and standard deviation of 100. Load case 3 is acting between X=8 and X=12. It intensity is also normally distributed with mean value 1000 and standard deviation of 200. The distance of the middle support from the origin will also be a probabilistic parameter. It will be uniformly distributed with upper boundary of 6.5m and lower boundary of 5.5m. The last thing that will have a probabilistic dimension will be the young modulus od the material. We will assume that our wall is not casted at once, so we will divide its propperties into three sections as well. The first third of the wall has E=120E9 mean value and 6E9 standard deviation. The second third has also mean value of E=120E9 and standard deviation of 24E9 and the last third has the same mean value but 48E9 standard deviation. The young modulus in all cases follows lognormal distribution. The goal optimization will be set to 65%.

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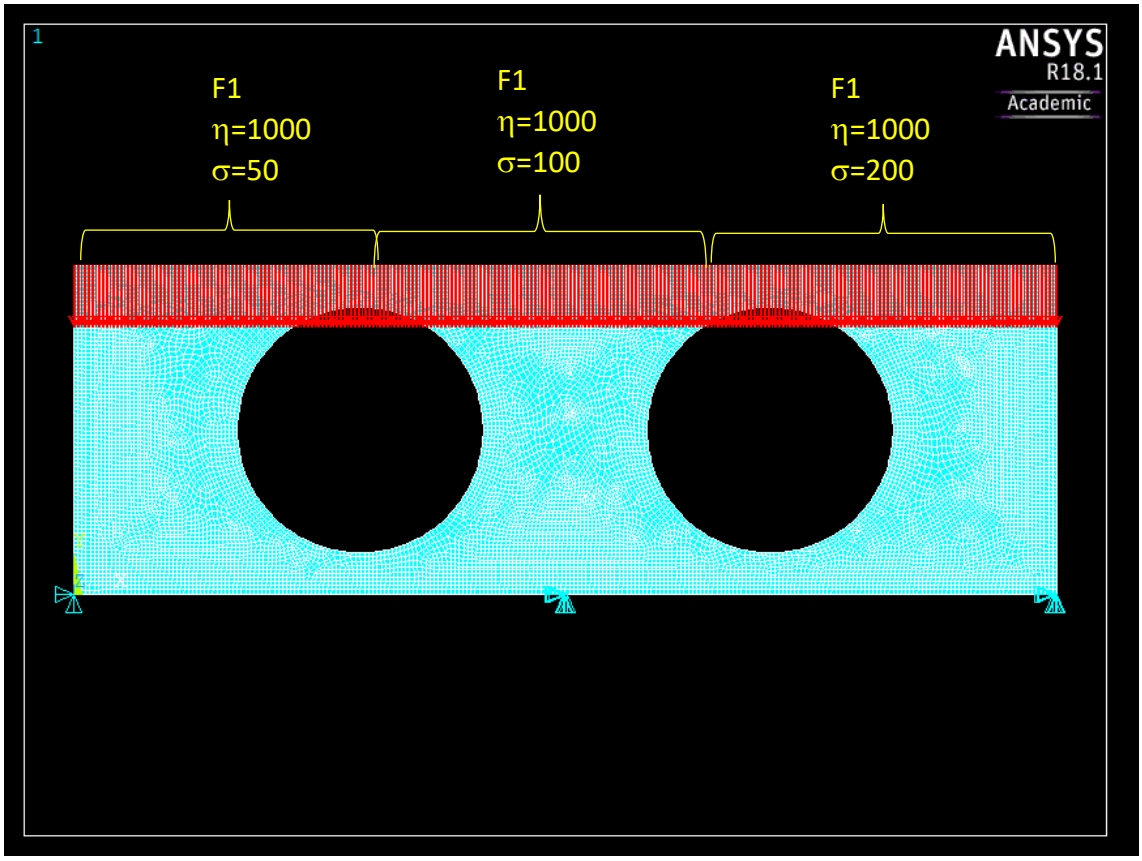

Figure 4-8 Example 3.1; "Bridge"

After performing 50 loops of Monte Carlo simulation we get 50 results. Combining those results gives us results as shown on the Figure 4-9

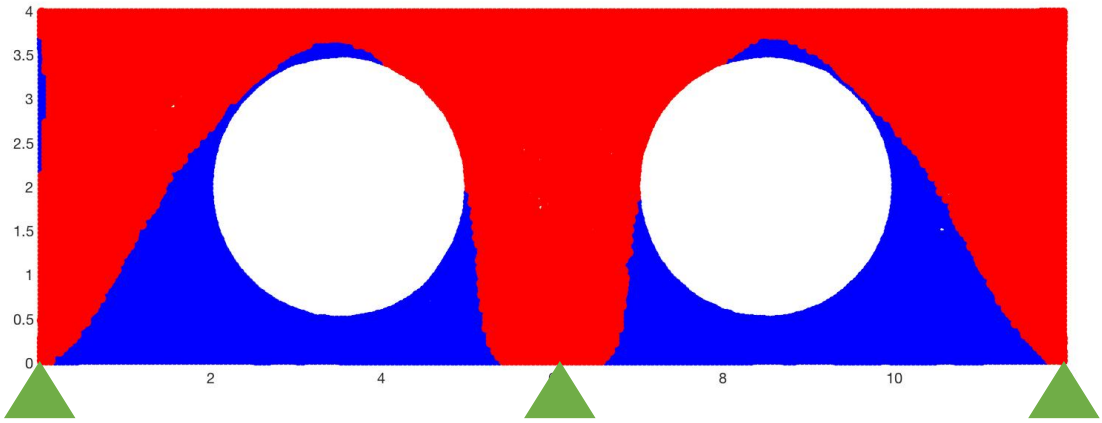

Figure 4-9 "Probabilistic" solution

To make any comparison we would also need to know the “deterministic” solution. The “deterministic” optimized topology is given on the Figure 4-10.

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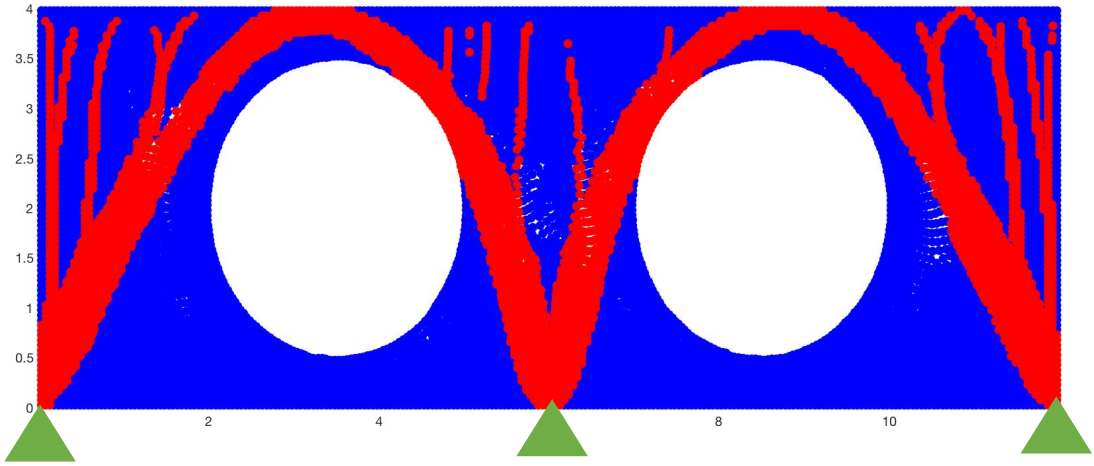

Figure 4-10 Deterministic solution

4.3 Example 3.2 "Tower"

A 15x5 wall Figure 4-11 with circular openings is supported at the bottom corner. It is loaded with 2 independent point forces at (x,y)=(0,10) and (x,y)=(5,10) respectively. As we learned so far with independent load cases the intensity does not play any role so in our example intensity of the forces will be given deterministically. However the location of the forces will be given probabilistically, which means the coordinate y of loading point for both forces will vary. Y coordinate for both forces will follow normal distribution with mean value of 10m. The standard deviation of the force acting on left side of the wall is 0.5m and the standard deviation of the force acting on right side of the wall is 1m. Since the loading point will be variable we need to ensure that there is a body to act on during each loop. The first idea was to analyze a 10x5 wall, but if the y coordinate of any forces gets a higher value of 10 during looping, ANSYS will show an error. That's why a 15x5 wall is being analyzed to make sure that we have a body for forces to act on during each loop. Theoretically, when using normal distribution, it would be possible to generate a very high value which would mean that we would need a very high structure, but that wasn't the case in our example. ANSYS generated lower number than 15 and higher than 0 each loop.

Like in the previous example we will assume that the material of the structure does not have the same distribution properties. As it was the case with previous example we will divide the wall in three equal parts: from 0=<y<5; 5=<y<10; 10=<y<=15.

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The mean value for Youngs modulus will be the same for each part with μ_E=120E9 but the standard deviation will be different:

σ_{E1}=6E9; σ_{E2}=24E9; σ_{E3}=48E9 respectively. The goal optimization will be 87%. This would be around 80% for a 10x5 wall, so we are trying to have same volume optimized as if there wasn't the upper third of the wall.

Side by side comparisons of probabilistic and deterministic solutions are shown on Figure 4-12 and Figure 4-13

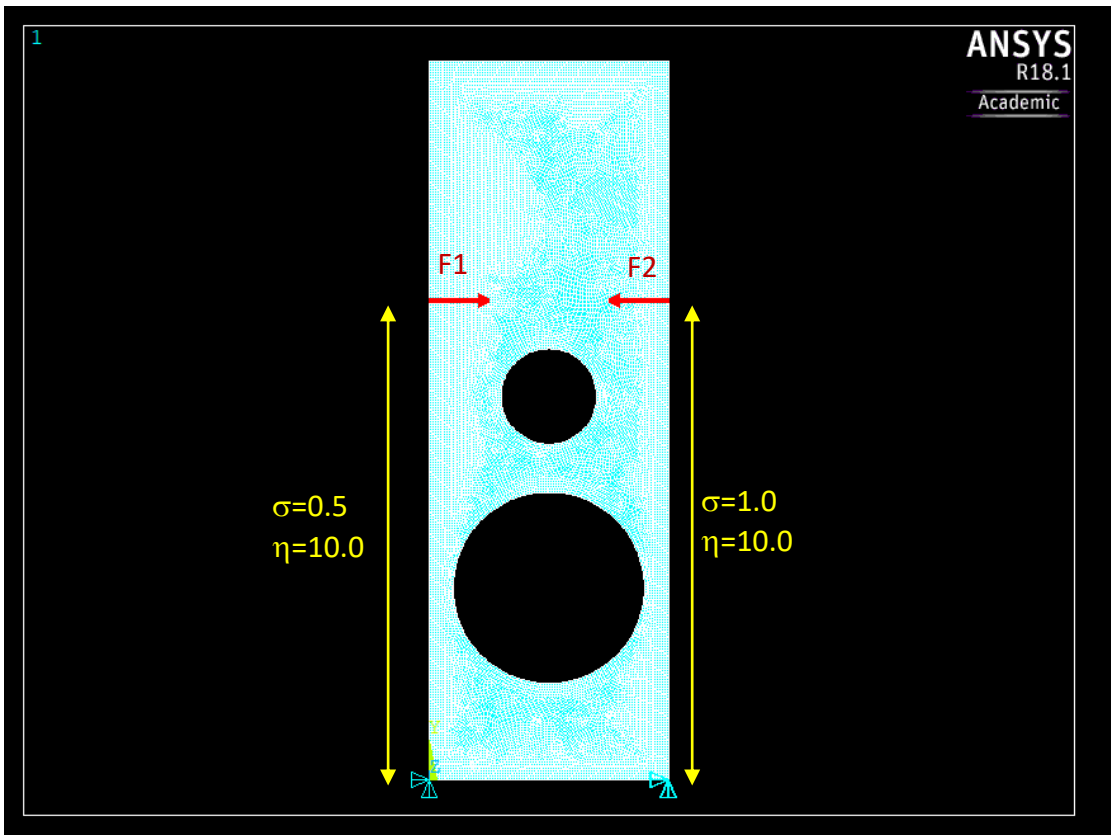

Figure 4-11 Example 3.2 "Eifel Tower"-Loading point coordinate given probabilistically

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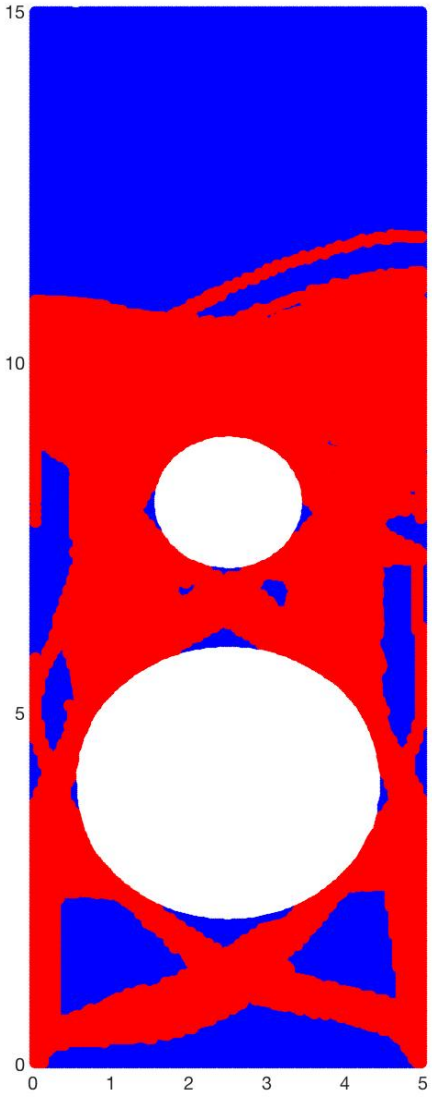

Figure 4-13 "Probabilistic" solution

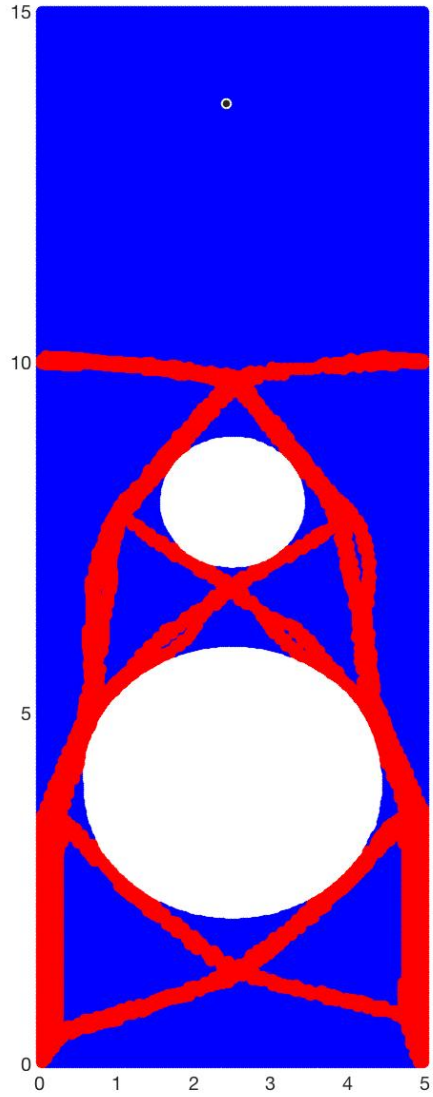

Figure 4-12 Deterministic solution

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4.4 Example 3.3 “Tower”

For the last example we will analyze similar input parameters as for example 3.2. This time we will analyze 10x5 wall (Figure 4-14) with same boundary conditions as the previous example but the points where the forces act will be fixed, which means that in this case coordinate y of the loading point will be given deterministically. In this example we are trying to see how great is the impact of different distribution properties for Youngs modulus on topological solution.

The goal volume to be optimized during each Monte Carlo loop is 80%.

Side by side comparisons of probabilistic and deterministic solutions are shown on Figure 4-15 and Figure 4-16.

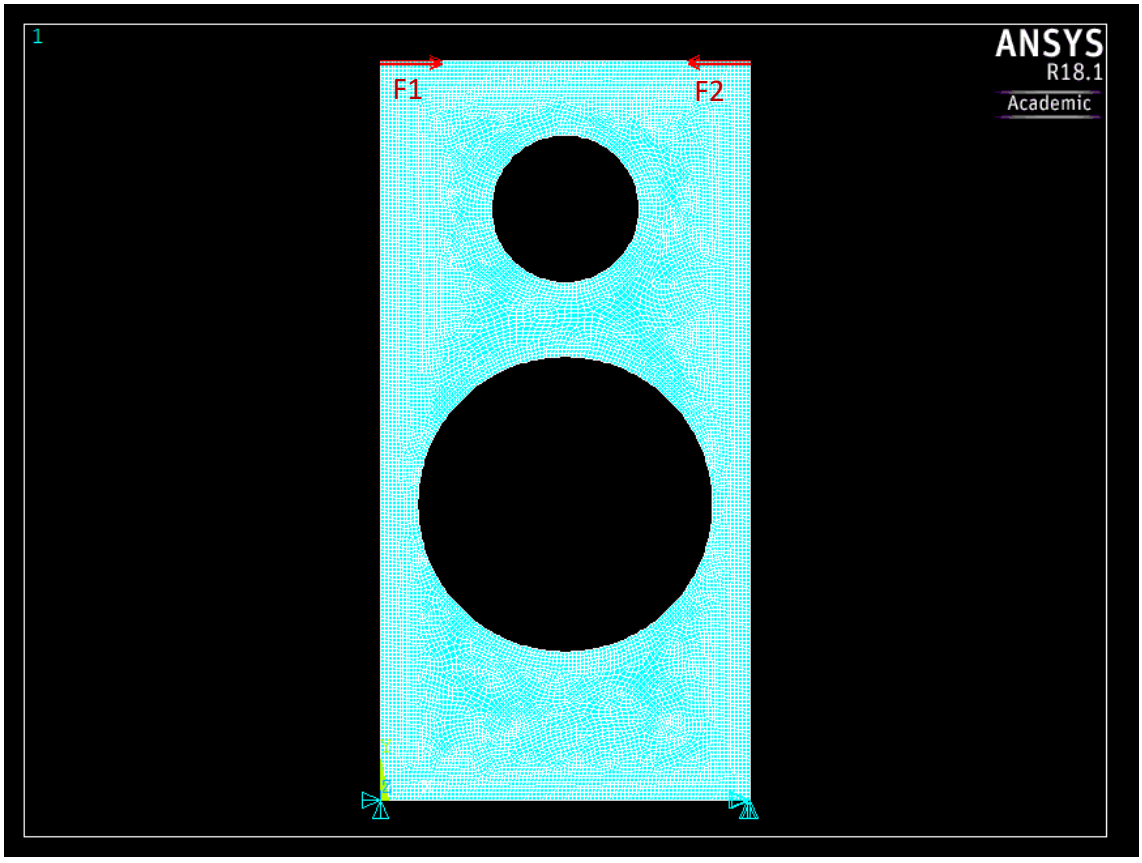

Figure 4-14 Example 3.2 “Eifel Tower” Loading point given as deterministic value

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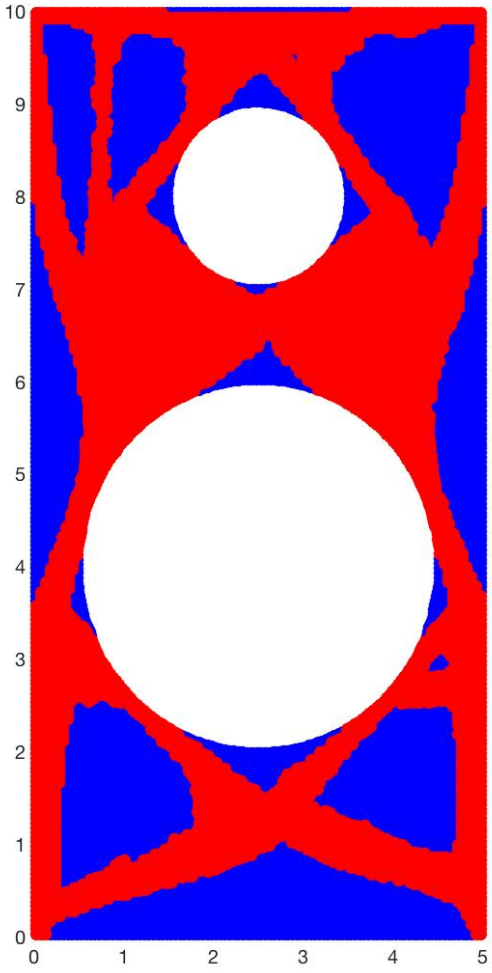

Figure 4-16 "Probabilistic" solution

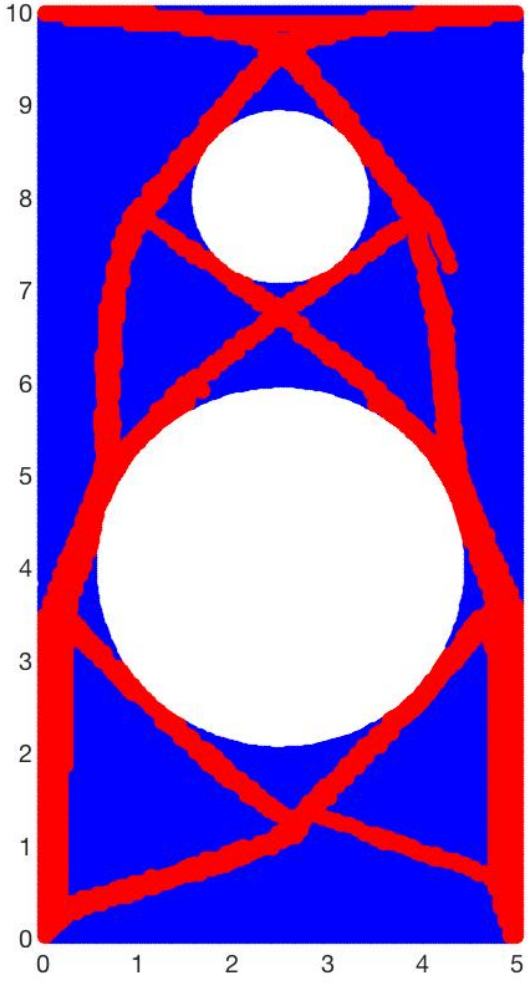

Figure 4-15 Deterministic solution

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4.5 Commentary on examples 3

In simple cases (examples 1 and examples 2) the shape of optimized topology can be predicted, it is always a truss looking shape, but how thick should be the truss elements, the best answer gives us the presented technique. In some cases, like in the example 3.1, we don't have a structure that as a result resembles a truss. Here we need material at the whole upper part of the structure. In the example 3.2, the result shape is not symmetrical body, and it does not have a "regular" shape. After the optimization and overlaying process, we have very small amount of material to remove from the original body. In these cases, we have to even ask the question whether to save material on structure or not. This will mostly be the case with problems where uncertainties have a very wide distributions where it doesn't pay of to save material at all and the answer to that question provides this technique. Looking at the result of the example 3.2 we can see that the input variables parameters are do wide spread that in the upper part of the structure there wouldn't be any logic to save material, but at the lower part where the supports are, there we have parts of the structure where material could be saved.

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Chapter 5 Conclusion

The interest for finding optimal design grows in recent years because it gives us a possibility to save the material that doesn't significantly contributes in the global stiffness of the structure. Structural optimization in the thesis focuses on optimal design of the loaded structures. The optimal topology design which is a type of structural optimization is defined as optimization problem of material distribution. The material densities of finite elements serve as variables and the maximum stiffness is defined as objective function. Probabilistic design is widespread discipline in engineering. It considers uncertainties in the structures. These uncertainties and random effects are related to quality and reliability of the structure. The thesis combines probabilistic design with topology optimization. Techniques for combining these two analyses presented to this day are very time consuming and therefore expensive. They are also unreachable to every engineer. In this work a simplified method for engineers is presented. It combines the deterministic load cases that came from probabilistic analysis into an end topological solution.

The restriction of this method (obtaining probabilistic solution through superposition of deterministic solutions) is that the location of the finite element nodes must be the same during each loop of Monte Carlo analysis. During every loop of Monte Carlo analysis finite elements are generated. The optimized volume of each finite element of the body during one loop is added to the optimized volume of the corresponding finite element of the next loop. Therefore we can only sum their volumes together if these finite elements have the exact same nodes location during every loop of Monte Carlo simulation. If a probabilistic parameter would cause the change of the geometry of a body and with it the finite element mesh during some other loop, we wouldn't be able to use this method.

The examples 3.1,3.2 and 3.3 illustrate this approach and its effectivity. It is a very powerful tool for assessment of probability based topology of a structure. Its end result is a shape of structure that could cover almost any possible uncertain parameter. Since this method does not give an exact solution of a stochastic problem, it is more of an engineering approach. This method is very useful when deciding what part of the material will be removed after optimization. It can simulate every possible combination of input parameters and form the end optimized shape.

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Appendix

Appendix A Example 2.2

```
/prep7
FORCEone=10
FORCEtwo=10000
FORCEPOINTUNTEN=7
FORCEPOINTOBEN=5
blc4,0,0,7,5
ET,1,82
MP,EX,1,3E9
MP,NUXY,1,0.3
ESIZE,0.05
type,1
mat,1
amesh,all
NSEL,S,LOC,x,0,,0
d,all,all,0
NSEL,S,LOC,X,FORCEPOINTUNTEN
NSEL,R,LOC,Y,0
F,ALL,FY,-FORCEone
ALLSEL
LSWRITE,1
FDEL,ALL
NSEL,S,LOC,X,FORCEPOINTOBEN
NSEL,R,LOC,Y,5
F,ALL,FY,-FORCEtwo
ALLSEL
LSWRITE,2
FDEL,ALL
TOPDEF,65,2
/SHOW,topo,grph
/DSCALE,,OFF
/CONTOUR,,2
TOPITER,20,1
FINISH
```

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Appendix B Example 2.3

```
/prep7
FORCEone=10000
ForceTwo=10000
FORCEPOINTUNTEN=7
FORCEPOINTOBEN=5
blc4,0,0,7,5
ET,1,82
MP,EX,1,3E9
MP,NUXY,1,0.3
ESIZE,0.05
type,1
mat,1
amesh,all
NSEL,S,LOC,x,0,,0
d,all,all,0
NSEL,S,LOC,X,FORCEPOINTUNTEN
NSEL,R,LOC,Y,0
F,ALL,FY,-FORCEone
NSEL,S,LOC,X,FORCEPOINTOBEN
NSEL,R,LOC,Y,5
F,ALL,FY,-FORCEtwo
ALLSEL
LSWRITE,1
FDEL,ALL
TOPDEF,65,1
/SHOW,topo,grph
/DSCALE,,OFF
/CONTOUR,,2
TOPITER,40,1
FINISH
```

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Appendix C Example 2.4

```
/prep7
FORCEone=10000
ForceTwo=3333
FORCEPOINTUNTEN=7
FORCEPOINTOBEN=5
blc4,0,0,7,5
ET,1,82
MP,EX,1,3E9
MP,NUXY,1,0.3
ESIZE,0.05
type,1
mat,1
amesh,all
NSEL,S,LOC,x,0,,0
d,all,all,0
NSEL,S,LOC,X,FORCEPOINTUNTEN
NSEL,R,LOC,Y,0
F,ALL,FY,-FORCEone
NSEL,S,LOC,X,FORCEPOINTOBEN
NSEL,R,LOC,Y,5
F,ALL,FY,-FORCEtwo
ALLSEL
LSWRITE,1
FDEL,ALL
TOPDEF,65,1
/SHOW,topo,grph
/DSCALE,,OFF
/CONTOUR,,2
TOPITER,40,1
FINISH
```

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Appendix D Example 2.5 A

```
/prep7
FORCE=10000
FORCEPOINT=5
blc4,0,0,7,5
ET,1,82
MP,EX,1,3E9
MP,NUXY,1,0.3
ESIZE,0.05
type,1
mat,1
amesh,all
NSEL,S,LOC,x,0,,0
d,all,all,0
NSEL,S,LOC,X,FORCEPOINT
NSEL,R,LOC,Y,5
F,ALL,FY,-FORCE
ALLSEL
LSWRITE,1
FDEL,ALL
TOPDEF,92,1
/SHOW,topo,grph
/DSCALE,,OFF
/CONTOUR,,4
TOPITER,60,1
FINISH
/post1
*get,ecomp,topo,,comp
*GET,TOPCV,TOPO,,CONV
etable,edens,topo
*get,density,etab,1,elem,29
pletab,edens
esel,all
*get,ncount,elem,,count
*dim,topodens,array,ncount,2
esel,all
*vget,list,elem,,elist
*vget,toci,elem,,etab,edens,,,2
*cfopen,'result',txt,,append
*vwrite,'elemno','edens'
(a10,a10)
*VWRITE,list(1),toci(1)
(f8.0,g16.8)
*CFCLOSE
finish
```

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Appendix E Example 2.5 B

```
/prep7
FORCE=10000
FORCEPOINT=7
blc4,0,0,7,5
ET,1,82
MP,EX,1,3E9
MP,NUXY,1,0.3
ESIZE,0.05
type,1
mat,1
amesh,all
NSEL,S,LOC,x,0,,0
d,all,all,0
NSEL,S,LOC,X,FORCEPOINT
NSEL,R,LOC,Y,0
F,ALL,FY,-FORCE
ALLSEL
LSWRITE,1
FDEL,ALL
TOPDEF,73,1,.001
/SHOW,topo,grph
/DSCALE,,OFF
/CONTOUR,,4
TOPITER,60,1
FINISH
/post1
*get,ecomp,topo,,comp
*GET,TOPCV,TOPO,,CONV
etable,edens,topo
*get,density,etab,1,elem,29
pletab,edens
esel,all
*get,ncount,elem,,count
*dim,topodens,array,ncount,2
esel,all
*vget,list,elem,,elist
*vget,toci,elem,,etab,edens,,,2
*cfopen,'result',txt,,append
*VWRITE,list(1),toci(1)
(f8.0,g16.8)
*CFCLOSE
Finish
```

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Appendix F Example 3.1

```
EMODUL1=120E9
EMODUL2=120E9
EMODUL3=120E9
KRAFT1=1000
KRAFT2=1000
KRAFT3=1000
ABSTAND=6
/PREP7
BLC4,0,0,12,4
CYL4,3.5,2,1.5
CYL4,8.5,2,1.5
asba,1,2,,delete,delete
asba,4,3,,delete,delete
ET,1,82
MP,EX,1,EMODUL1
MP,NUXY,1,0.3
MP,EX,2,EMODUL1
MP,NUXY,2,0.3
MP,EX,3,EMODUL1
MP,NUXY,3,0.3
esize,0.05
type,1
MAT,1
amesh,all
NSEL,S,LOC,X,4,8
ESLN
type,1
mat,2
emodi,all
allsel
NSEL,S,LOC,X,8,12
ESLN
type,1
mat,3
emodi,all
allsel
NSEL,S,LOC,X,0
NSEL,R,LOC,Y,0
D,ALL,ALL,0
NSEL,S,LOC,X,ABSTAND
NSEL,R,LOC,Y,0
D,ALL,ALL,0
NSEL,S,LOC,X,12
NSEL,R,LOC,Y,0
D,ALL,ALL,0
```

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```
NSEL,S,LOC,Y,4
NSEL,R,LOC,X,0,4
F,ALL,FY,-KRAFT1
NSEL,S,LOC,Y,4
NSEL,R,LOC,X,4,8
F,ALL,FY,-KRAFT2
NSEL,S,LOC,Y,4
NSEL,R,LOC,X,8,12
F,ALL,FY,-KRAFT3
ALLSEL
LSWRITE,1
FDEL,ALL
TOPDEF,65,1
/SHOW,topo,grph
/DSCALE,,OFF
/CONTOUR,,2
TOPITER,30,1
FINISH
/post1
*get,ecomp,topo,,comp
*GET,TOPCV,TOPO,,CONV
etable,edens,topo
*get,density,etab,1,elem,29
esel,all
*get,ncount,elem,,count
*dim,topodens,array,ncount,2
esel,all
*vget,list,elem,,elist
*vget,toci,elem,,etab,edens,,,2
*cfopen,'result',txt,,append
*VWRITE,list(1),toci(1)
(f8.0,g16.8)
*CFCLOSE
Fini
% Input for PDS analysis
/inp,input1,txt
/pds
pdanl,input1,txt
pdvar,KRAFT1,gaus,KRAFT1,KRAFT1*0.05
pdvar,KRAFT2,gaus,KRAFT2,KRAFT1*0.1
pdvar,KRAFT3,gaus,KRAFT3,KRAFT1*0.2
pdvar,EMODUL1,LOG1,EMODUL1,EMODUL1*0.05
pdvar,EMODUL2,LOG1,EMODUL2,EMODUL1*0.2
pdvar,EMODUL3,LOG1,EMODUL3,EMODUL1*0.4
pdvar,ABSTAND,UNIF,ABSTAND-0.5,ABSTAND+0.5
pdvar,ecomp,resp
pdvar,density,resp
```

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```
pdmeth,mcs,dir
pddmcs,50,none,all,,,,123457
pdexe,input
fini
```

Appendix G Example 3.2

```
EMODUL1=120E9
EMODUL2=120E9
EMODUL3=120E9
KRAFT1=1000
KRAFT2=1000
KRAFTABSTAND1=10
KRAFTABSTAND2=10
/PREP7
BLC4,0,0,5,15
CYL4,2.5,4,2
CYL4,2.5,8,1
asba,1,3,,delete,delete
asba,4,2,,delete,delete
ET,1,82
MP,EX,1,EMODUL1
MP,NUXY,1,0.3
MP,EX,2,EMODUL2
MP,NUXY,2,0.3
MP,EX,3,EMODUL3
MP,NUXY,3,0.3
esize,0.05
type,1
MAT,1
amesh,all
NSEL,S,LOC,Y,5,10
ESLN
type,1
mat,2
emodi,all
allsel
NSEL,S,LOC,Y,10,15
ESLN
type,1
mat,3
emodi,all
allsel
NSEL,S,LOC,X,0
NSEL,R,LOC,Y,0
D,ALL,ALL,0
NSEL,S,LOC,X,5
NSEL,R,LOC,Y,0
```

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```
D,ALL,ALL,0
NSEL,S,LOC,X,0
NSEL,R,LOC,Y,KRAFTABSTAND1
F,ALL,FX,KRAFT1
ALLSEL
LSWRITE,1
FDEL,ALL
NSEL,S,LOC,X,5
NSEL,R,LOC,Y,KRAFTABSTAND2
F,ALL,FX,-KRAFT2
ALLSEL
LSWRITE,2
FDEL,ALL
TOPDEF,87,2
TOPITER,30,1
FINISH

% Input for PDS analysis

/inp,input1,txt
/pds
pdanl,input1,txt
pdvar,EMODUL1,LOG1,EMODUL1,EMODUL1*0.05
pdvar,EMODUL2,LOG1,EMODUL2,EMODUL1*0.2
pdvar,EMODUL3,LOG1,EMODUL3,EMODUL1*0.4
pdvar,KRAFTABSTAND1,GAUS,KRAFTABSTAND1,KRAFTABSTAND1*0.05
pdvar,KRAFTABSTAND2,GAUS,KRAFTABSTAND2,KRAFTABSTAND2*0.1
pdvar,ecomp,resp
pdvar,density,resp
pdmeth,mcs,dir
pddmcs,50,none,all,,,,123457
pdexe,input
fini
```

Appendix H Example 3.3

```
EMODUL1=120E9
EMODUL2=120E9
EMODUL3=120E9
KRAFT1=1000
KRAFT2=1000
KRAFTABSTAND1=10
KRAFTABSTAND2=10
/PREP7
BLC4,0,0,5,10
CYL4,2.5,4,2
CYL4,2.5,8,1
asba,1,3,,delete,delete
```

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```
asba,4,2,,delete,delete
ET,1,82
MP,EX,1,EMODUL1
MP,NUXY,1,0.3
MP,EX,2,EMODUL2
MP,NUXY,2,0.3
MP,EX,3,EMODUL3
MP,NUXY,3,0.3
esize,0.05
type,1
MAT,1
amesh,all
NSEL,S,LOC,Y,3.3,6.6
ESLN
type,1
mat,2
emodi,all
allsel
NSEL,S,LOC,X,6.6,10
ESLN
type,1
mat,3
emodi,all
allsel
NSEL,S,LOC,X,0
NSEL,R,LOC,Y,0
D,ALL,ALL,0
NSEL,S,LOC,X,5
NSEL,R,LOC,Y,0
D,ALL,ALL,0
NSEL,S,LOC,X,0
NSEL,R,LOC,Y,KRAFTABSTAND1
F,ALL,FX,KRAFT1
ALLSEL
LSWRITE,1
FDEL,ALL
NSEL,S,LOC,X,5
NSEL,R,LOC,Y,KRAFTABSTAND2
F,ALL,FX,-KRAFT2
ALLSEL
LSWRITE,2
FDEL,ALL
TOPDEF,80,2
TOPITER,30,1
FINISH

% Input for PDS analysis
```

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```
/inp,input1,txt
/pds
pdanl,input1,txt
pdvar,EMODUL1,LOG1,EMODUL1,EMODUL1*0.05
pdvar,EMODUL2,LOG1,EMODUL2,EMODUL1*0.2
pdvar,EMODUL3,LOG1,EMODUL3,EMODUL1*0.4
pdvar,ecomp,resp
pdvar,density,resp
pdmeth,mcs,dir
pddmcs,50,none,all,,,,123457
pdexe,input
fini
```

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