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Robust Project Portfolio Management and Optimal Budget Allocation between Subportfolios

ausgeführt am
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unter der Anleitung von
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durch

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Hannes Demel

Abstract

Organizations continuously increase their efforts to improve their project portfolio management (PPM). As a consequence, software that supports the processes of PPM is a standard application in organizations. Common PPM software provides various methods to support the decision process, collecting informative data to do so. The lack of data was in the past the main reason for the failure of mathematical optimization methods in the area of PPM. Thus, the connection of mathematical optimization methods with frequently used PPM software seems to be a promising approach to enhance the applicability of mathematical optimization methods in the area of PPM.

Therefore, the first part of this work outlines how a mathematical optimization model must be designed so that it can be embedded into existing PPM software. Based on these requirements, a mathematical optimization model is formulated. Thereby, the main focus lies on incomplete information as well as on optimal budget allocation among strategic buckets. Incomplete information refers to vaguely formulated project parameters due to prediction difficulties. To process vaguely formulated parameters, robust optimization concepts are used. In the context of strategic buckets, we discuss the divisibility of the entire portfolio into subportfolios so that every strategic bucket is represented by a subportfolio. The main goal of strategic buckets is to enforce a certain budget allocation among projects to implement the desired strategy. To support the allocation of the budget among strategic buckets, we define their marginal values and use those values as decision criteria.

Contents

List of Abbreviations

AHP	Analytical Hierarchy Process
CNDP	Computation of Non-Dominated Portfolios
DEA	Data Envelopment Analysis
DMs	Decision Makers
ECV	Expected Commercial Value
ERP	Enterprise Resource Planning
ILP	Integer Linear Programming
IRR	Internal Rate of Return
KPIs	Key Performance Indicators
MDSFPPM	Mathematical Decision-Support Framework for Project Portfolio Management
MOZOLP	Multiobjective Zero–One Linear Programming
NPV	Net Present Value
OM	Operations Management
PI	Productivity Index
PM	Project Management
PPM	Project Portfolio Management
ROI	Return On Investment
RPM	Robust Portfolio Modelling

Chapter 1

Introduction

Project Portfolio Management (PPM) is a set of processes for the selection and maintenance of a set of projects. Usually there are more projects available for selection than can be undertaken within the organization's physical and financial constraints. Thus, choices must be made in making up a suitable project portfolio (Archer and Ghasemzadeh 1999a).

The aim of PPM is to select and maintain the set of projects which best supports the corporation's mission as described by a set of objectives. Typically, objectives are efficient use of resources, financial values—e.g., return on investment (ROI), net present value (NPV), and internal rate of return (IRR)—and ancillary nonfinancial benefits, including ensuring a balance with respect to various key parameters and market share among many more (see Levine 2005).

PPM is a highly dynamic process. On the one hand, this dynamic is caused by uncertainty, stemming from both the determination of the corporation's mission as well as the implementation of projects. The determination of the mission is made unpredictable by such external influences as market conditions and competitors' actions. Even if the implementation of projects were not directly subject to external influences, its result may be unpredictable. For instance, technical issues can occur or the consumption of budget and resources may increase unexpectedly. On the other hand, the dynamic is simply described by the life cycle of projects and new project proposals. If a project has reached the end of its life cycle, resources become available to be allocated to other projects. A new available project proposal, in contrast, needs an accept or reject decision.

Thus, the environment of the project portfolio and the project portfolio itself are constantly changing. This requires the selection of a new project portfolio at regular intervals. Therefore, the corporation's decision makers (DMs) meet periodically for so-called *portfolio reviews*. Usually, portfolio reviews take place biannually or quarterly, but emergency reviews may also occur. During a portfolio review, the corporation's mission, strategies, active projects, and new project proposals are evaluated so that a new project portfolio can be selected.

For this purpose a DM can choose from various techniques that support the evaluation and selection process. Some of the earliest techniques of the 1960s and 1970s were already based on mathematical optimization approaches to automatically select the optimal project portfolio. However, mathematical approaches for project portfolio selection largely failed to gain user acceptance. Reasons for their failure are reported by Mathieu and Gibson (1993). Other techniques, such as financial models, strategic approaches, scoring models, checklists, and the analytical hierarchy process (AHP), have been developed and experienced more user acceptance. However, most of those techniques address only some of the above issues. In contrast, a mathematical approach is designable to address multiple issues simultaneously.

This thesis will pick up the mathematical approach again. The major obstacle for the failure of mathematical approaches in PPM was the enormous amount of data it required. However, since that failure almost every big corporation has begun using enterprise resource planning (ERP) software from such vendors as SAP, Oracle, and others. Thus, the data required for a mathematical approach should be available, especially since vendors of ERP software also supply PPM products. In these products, data are stored and visualized to give the DM immediate access to the information required for the selection decision. PPM software also supports the decision techniques mentioned above. Furthermore, vendors of PPM software recently started to implement a mathematical approach as an additional feature in their products—e.g., *HP Project and Portfolio Management: Portfolio Management Module*. The implementation of the mathematical approach into PPM software has several advantages. First, available data about projects and resources are already connected with the mathematical model. Second, the mathematical approach can be combined with other features from the software like scoring models, presentation techniques, etc. Finally, it allows the integration of the mathematical model's control into the existing software, allowing DMs to feel comfortable with the mathematical approach from the beginning.

The aim of this thesis is to describe a mathematical decision-support framework for PPM (MDSFPPM) that satisfies the needs of DMs and is easily embedded into existing PPM software. Furthermore, we show how well current mathematical models actually meet the MDSFPPM. For the description of the MDSFPPM, Chapter 2 introduces the methods of PPM. This serves as basis for the formulation of numerous requirements which must be met by the MDSFPPM. Chapter 3 then outlines a mathematical approach for project portfolio selection. More precisely, the *robust portfolio modelling* (RPM) approach from Liesiö et al. (2008) is described and a new budget-dependent core index is introduced. Chapter 4 extends the RPM model so that strategic buckets are supported. Chapter 5 explains the algorithm from Liesiö et al. (2008) for the calculation of desirable portfolios, namely non-dominated portfolios. Finally, Chapter 6 discusses the applicability of the RPM model and the corresponding algorithm by comparing it with the MDSFPPM.

Chapter 2

Project Portfolio Management Background

The brief introduction of PPM in this chapter is intended to outline the background of the Requirements for the MDSFPPM which are formulated in this chapter. Detailed descriptions of PPM can be found in various textbooks, including Project Management Institute (2006), Levine (2005) and Cooper et al. (2001b). A particularly valuable work with respect to the description of the MDSFPPM is Archer and Ghasemzadeh (1999a) in combination with Archer and Ghasemzadeh (2000).

Section 2.1 describes PPM and motivates its use. The split of the entire PPM process into several phases is presented in Section 2.2 as well as a task summary of, for our purposes, less significant phases. The remaining sections focus on the significant phases for the description of the MDSFPPM. Section 2.3 outlines the impact of strategy on the selection process. Section 2.4 presents the critical factors of the project selection phase itself, while Section 2.5 covers the DM's need for interaction features. Section 2.6 focuses on the effect of active projects on the project selection phases. Finally, Section 2.7 summarizes the requirements for the MDSFPPM.

2.1 The Why and What of PPM

With growing competitive pressures in the global economy, effective PPM practices is becoming increasingly critical to business corporations. All corporations, large and small, must select and manage their investments wisely to reap the maximum benefit from their investment decisions. To select and manage investments wisely means, in business terms, investing in the most promising *projects* and implementing them prudently. A *project* is as a complex effort, usually less than three years in duration, made up of interrelated tasks, performed by various organizations, with a well-defined scope, time frame, and requirements on resources (Archibald 2003).

The difficulty is that there are usually much more projects available than can be undertaken within resource constraints. If resources are unlimited, the decision rule would be simple. *Do every project whose impact on the portfolio is positive.* In reality, resources are limited, so decisions about resource distribution must be made. This is already the first reason *why* we need PPM. In a business world where the efficient use of resources is highly valued, allocating resources to low-value projects is not advisable.

Another reason we need PPM is to identify the projects that are the tomorrow's new product winner. Only a project that will become tomorrow's new product winner can ensure high revenue, high market share, a desirable competitive position, cost reduction, technical advantage and so on. Other specific reasons for the importance of PPM, cited by managers in a survey of 205 firms, are set out in Table 2.1

Table 2.1: Key Reasons that PPM is vital

- To maximize return, maximize R&D productivity, and achieve financial goals.
- To maintain the competitive position of the business—to increase sales and market share.
- To properly and efficiently allocate scare resources.
- To forge the link between project selection and business strategy. The portfolio is the expression of strategy; it must support the strategy.
- To achieve focus—not doing too many projects for the limited resources available and providing resources for the great projects.
- To achieve balance—the right balance between long- and short-term projects, and high-risk and low-risk ones, consistent with the business's objectives.
- To better communicate priorities within the organization vertically and horizontally.
- To provide better objectives in project selection and weed out bad projects.

Source Cooper et al. (1999) and Cooper et al. (2001a).

Taken together, corporations practice PPM to better achieve their missions. No matter how the mission of a corporation looks, it is only achieved if the entire *project* *portfolio* as a whole, not only single projects, support it. A *project portfolio* is defined as any subset of active projects and project proposals that are available for a corporation. Thus, DMs who are responsible for the selection of the project portfolio are advised to identify the project portfolio that best suits the corporation's mission.

During the selection of the best project portfolio, a DM has also to take into account project-specific objectives that address the successful implementation of the project. In the literature, these objectives are summarized as doing the project on time, within resources, and on scope. Figure 2.1 illustrates these objectives as well as the conflict between them.

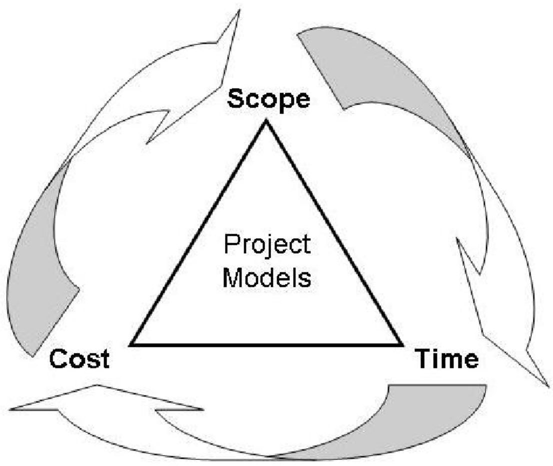


Figure 2.1: Magic triangle of project objectives

A project within the project portfolio will only drive the corporation to its mission if its own objectives about its implementation are met. For example, the scope of a project is the project's support for the corporation to meet its mission. Thus, any change in the scope of a project endangers the achievement of the mission. Another example of the interdependency between project objectives and the corporation's mission is given by doing projects within resources. If a project's resource consumption exceeds the planned level, the implementation of the project itself and perhaps of other projects is endangered. In the worst case, this leads to the termination of the project and perhaps to the termination of other projects as well, which may lead to the corporation not fulfilling its mission.

While the responsibility for project objectives belongs to the project management (PM) group, the responsibility for the corporation and its mission belongs to the operations management (OM) group. More generally, activities concerning the corporation level are the responsibility of OM, activities concerning the project level are the responsibility of PM. Table 2.2 outlines some activities for both groups.

Table 2.2: Specific roles of either side of the bridge

Operations Management	**Project Management**
Strategies	Schedule/time
Mission, objectives	Project cost
Business performance	Project performance
Stockholder satisfaction	Stakeholder satisfaction
Project selection mix	Scope/change control
Resource availability	Resource utilization
Cash flow, income	Cash usage

Source Levine (2005).

It is shown that in OM the mission is determined as well as strategies to achieve it. Further, the OM group is responsible for the selection of the project portfolio as well as for the available resources to implement projects within the project portfolio. In short, in OM many effort goes into doing the right projects. In contrast, in PM many effort goes into doing projects right. Since the only way for a successfully future of a corporation is to do the right projects **and** to do them right, a strong connectivity between PM and OM is fundamental. Now we are by the **What** of PPM. It is the bridge between OM and PM. Hence, the basics of PPM aren't new and consists of already broad used techniques.

The importance of the bridge (PPM) is shown in a study undertaken by the Standish Group. Their study shows that across industries, only 28 percent of IT projects are successful on average and 23 percent fail outright. The remaining 49 percent are considered challenged based on the triple project objectives of being on time, on resources, and on original scope (Figure 2.2).

Failed projects are those that are terminated before completion. Challenged projects may be completed and operational, but they are either over resources, over schedule, or delivered less than the originally specified functionality, or any combination of these triple project objectives (Levine 2005).

The violation of project objectives is characteristically for reasons of a poorly practiced PPM. If the PPM is poorly practiced or not part of the business, the collaboration between OM and PM is weak. Thus, DMs in OM select the project portfolio and determine the availability of resources without detailed knowledge about the de-

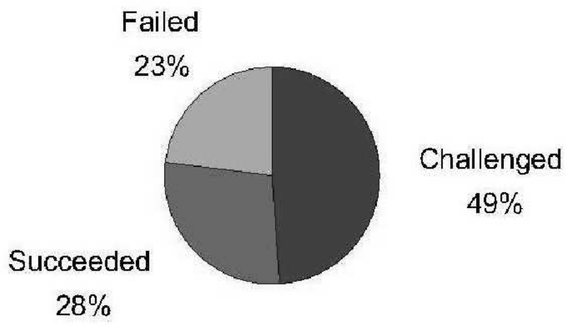


Figure 2.2: Project Success Rates
Source: Standish Group International.

sign of projects and their resource needs. In contrast, the DMs of PM design projects without detailed knowledge of the corporation's mission and strategy.

2.2 Project Portfolio Selection Phases

The selection of the best project portfolio is a highly complex process. First, it requires a panoply of preparation tasks. This includes the definition of project proposals and the estimation of their key parameters (resource requirements, benefits, schedule plan, etc.). Further preparation tasks are the evaluation of active projects as well as the determination of the mission and strategies. Second, project portfolio selection is a group decision process and requires perfect collaboration between different DM groups such as PM and OM. Finally, in the project portfolio selection process, DMs have to deal with multiple and often conflicting objectives, alignment of requirements on resources and their availability, project interactions through direct dependencies or resource competition, etc.

To simplify the project portfolio selection, it is suggested to decompose the selection process and also PPM into a series of discrete phases which progress from initial broad strategy considerations toward the final solution (Archer and Ghasemzadeh 1999a). Figure 2.3 depicts a decomposition of PPM into several phases. It starts with a set of project proposals, each of which must pass several screening, evaluation and selection phases before it enters the development phase. During the development phase, as each project reaches certain milestones it must pass the evaluation, screening, and selection phases again. Only projects that successfully pass these phases repeatedly are successfully implemented.

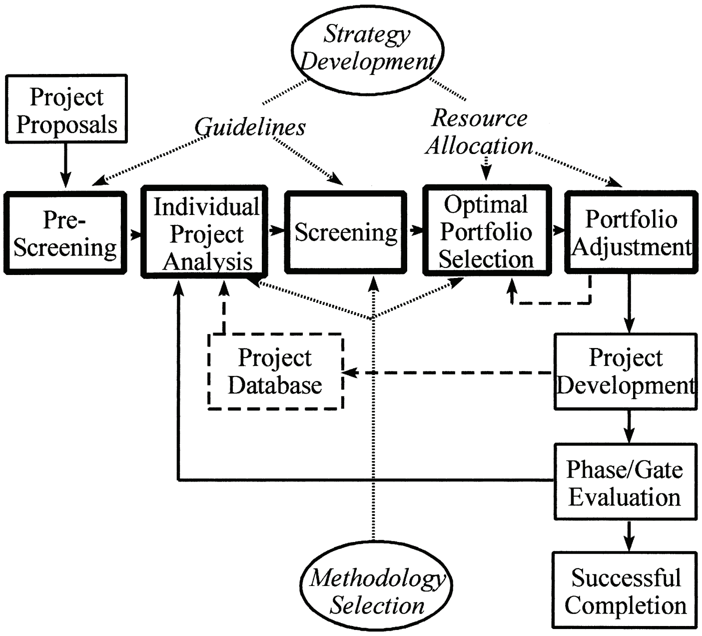


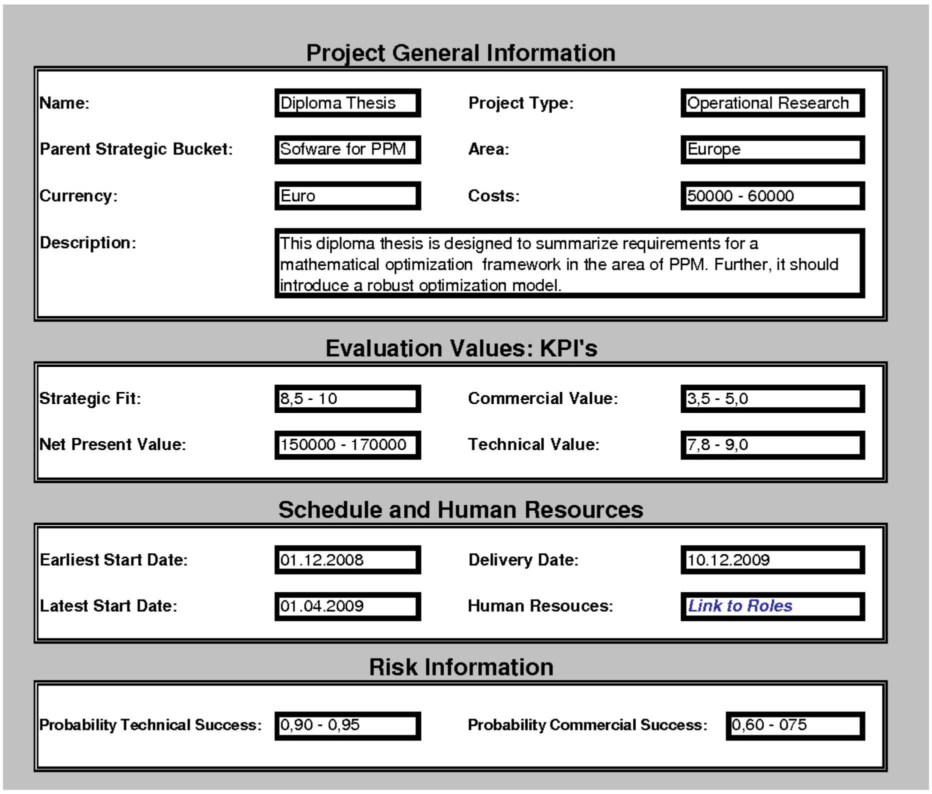
Figure 2.3: Phases for Project Portfolio Management
Source Archer and Ghasemzadeh (1999a).

Proposals are the trigger of this process and can derive from different sources, such as customers or employees. Prosperous corporations spend considerable effort and resources receiving many new proposals. Thus, the assumption that the number of available project proposals and active projects exceeds the number that can be effectively executed in a reasonable time is appropriate.

The **Prescreening** phase ensures that all passing projects fulfill at least minimum requirements, such as strategic fit and completeness. Essential requirements before the project passes this phase should also include a feasibility analysis and estimates of parameters needed to evaluate each project. Mandatory projects are also identified at this point, since they will be included in the remainder of the portfolio selection process. Mandatory projects are projects agreed upon for inclusion, including improvements to existing products no longer competitive, projects without which the corporation could not function adequately, etc. For the MDSFPPM, this means that mandatory projects must be scheduled subject to their time constraints. Further, resources required by mandatory projects must be reserved for them.

Requirement 2.1 (Mandatory Projects)**.** *There must be functionality to flag projects as mandatory.*

In the **Individual Project Analysis** phase, a common parameter set is calculated separately for every project. This parameter set serves as the decision base in the next phases. The calculation is based on estimates available from feasibility studies and from a database of previously completed projects. Common parameters are the project's benefits, budget and resource requirements over the project's timeline, and the project's development schedule. See Figure 2.4.



Project General Information

Name: Diploma Thesis
Project Type: Operational Research
Parent Strategic Bucket: Sofware for PPM
Area: Europe
Currency: Euro
Costs: 50000 - 60000
Description: This diploma thesis is designed to summarize requirements for a mathematical optimization framework in the area of PPM. Further, it should introduce a robust optimization model.

Evaluation Values: KPI's

Strategic Fit: 8,5 - 10
Commercial Value: 3,5 - 5,0
Net Present Value: 150000 - 170000
Technical Value: 7,8 - 9,0

Schedule and Human Resources

Earliest Start Date: 01.12.2008
Delivery Date: 10.12.2009
Latest Start Date: 01.04.2009
Human Resouces: Link to Roles

Risk Information

Probability Technical Success: 0,90 - 0,95
Probability Commercial Success: 0,60 - 075

Figure 2.4: Common Project Details

The estimation of these parameters is a difficult task even for highly experienced DMs. The estimation of project parameters is nothing less than operating in a variable environment where some conditions are beyond the direct control of DMs. A slight delay, a change in an exchange rate, a failure of an experiment, an act of nature: Each will change the values of project parameters so that the initial parameter prediction is wildly inaccurate. Thus, information about project parameters must be considered as *incomplete*, meaning that project parameters are estimated through lower and upper bounds rather than through point estimates; see Figure 2.4. For instance, the net present value (NPV) of a project is estimated to lie between 150 and 170 thousand euros. As a consequence, the MDSFPPM must support project portfolio selection on the basis of *imprecise* project parameters.

Requirement 2.2 (Imprecise project information)**.** *The estimation of project parameters shall be possible by using lower and upper bounds instead of point estimates.*

Note, the concept of incomplete information also supports group-decision environments. For example, the estimation of a project's NPV may differ among DMs since they have different backgrounds (some DMs correspond to the OM group and some DMs correspond to the PM group). The concept of incomplete information then allows the system to account for the estimates of all DMs through corresponding lower and upper bounds for the project's NPV.

Requirement 2.2 is one of the most crucial requirements for the MDSFPPM. The concept of incomplete information accounts for prediction errors in project parameters, which are based on uncertainty. By allowing for imprecision, the portfolio-selection procedure and also the finally selected project portfolio take prediction errors and uncertainty into account. Cooper et al. (2001a) reported experiences of selection techniques that do not take prediction errors into account (this includes conventional mathematical approaches). It is stated that such selection techniques deliver the worst performance and may be one of the main reasons for past failures of mathematical methods in PPM.

In the **Screening** phase, project or interrelated families of projects are terminated if they do not meet preset criteria such as estimated rate of return, except for mandatory projects or those required to support other selected projects (Archer and Ghasemzadeh 1999a). The phase is necessary to reduce the number of projects for the following phases because the complexity of these phases increases exponentially with the number of projects.

In the **Optimal Portfolio Selection** and **Portfolio Adjustment** phases and during **Project Portfolio Maintenance** (shown in Figure 2.3 through the arrow starting at the box *Phase/Gate Evaluation* and ending at the box *Individual Project Analysis*), project proposals and active projects that have passed the foregoing screening and evaluation phases are considered simultaneously. These are the phases where the MDSFPPM is used for project portfolio selection. Thus, these phases are the main focus of the adaptation of the MDSFPPM to the needs of DMs. Therefore, we describe these phases in Sections 2.4 to 2.6 explicitly.

Finally, **Strategy Development** is a preprocessing phase performed by the OM group. As shown in Figure 2.3, *Strategy development* impacts the *Optimal Portfolio Selection* phases as well as *Portfolio Adjustment* through high-level guidelines. These guidelines are described in the next section.

The expectation of this phase approach is that proposals and projects should be successively screened or culled out at each phase. Selecting only the best projects for the project portfolio with a funneling approach. Once into development, most of the poor projects have been weeded out, so the funnel begins to resemble a tunnel; see Figure 2.5. The aim is to eliminate the need to terminate projects that are already under development since a project's resource requirements increase enormously at the development phase.

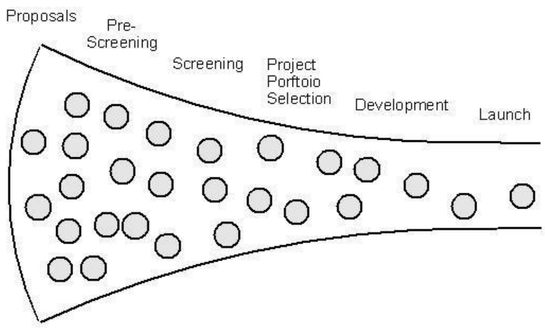


Figure 2.5: A Funnel Leading to a Tunnel to Weed Out Poor Projects Early
Source Cooper and Edgett (2005).

2.3 Strategic Guidelines for the Project Portfolio Selection

2.3.1 Strategy development

There are two main stages in the development of strategy. The first stage starts with the strategic analysis to collect information and is divided into *internal analysis* and *external analysis*. The *internal analysis* examines a corporation's internal environment to establish the corporation's strengths and weaknesses. The *external analysis* examines the market and competitive environment in which the corporation operates. The purpose of the external analysis is to establish opportunities and threats for the corporation. In the literature, this strategic analysis is often referred to as a SWOT (Strengths, Weaknesses, Opportunities, Threats) analysis (see Bohlander and Snell 2009).

The information collected in the strategic analysis serves as the basis for the definition of the corporation's mission. The mission just reflects the corporation's targeted future position, which can be characterized by such factors as size, market share, market types, technologies, wealth, and product lines, among others. Usually, a corporation's mission is expressed by a set of quantifiable objectives. Examples of an objective are *become market leader in technology Y in the next two years* and *increase market share to* 50 *percent in the next five years.*

Once the corporation knows where to go in the future, a plan is established to fulfill the mission in the second stage of the strategy development. Establishing such a plan just means to establishing a strategy to fulfill the mission. Therefore, DMs start by generating a list of strategies that are open to the corporation and that address the mission. After this, each strategy is evaluated by the DMs using a number of criteria so that finally the most appropriate strategic option is selected.

It is the projects that implement the strategy. Thus, strategic development must take place before projects can be considered for a project portfolio (shown in Figure 2.3 by the oval form of the corresponding box). In the *Individual Project Analysis* phase, DMs must then establish a project's strategic fit. Later, in the *Optimal Portfolio Selection* phase, the process selects the project portfolio that optimally supports the strategy to ensure the mission is achieved. In the literature and in practice, this is often referred to as the *strategic alignment* of the project portfolio and is a main goal of PPM.

Figure 2.6 outlines the strategic development for a fictional corporation. It starts with the collection of information and the definition of the corporation's mission. Next, for every objective, an appropriate strategy is established so that projects that support the strategy can be identified. Note that a project is able to support multiple strategies, as Figure 2.6 shows.

2.3.2 Strategic resources planning

Beside the strategy, DMs have to establish the resources that are available in pursuit of the mission. *Resources* are the essential inputs for the normal functioning of the organizational process: the inputs without which a corporation simply could not

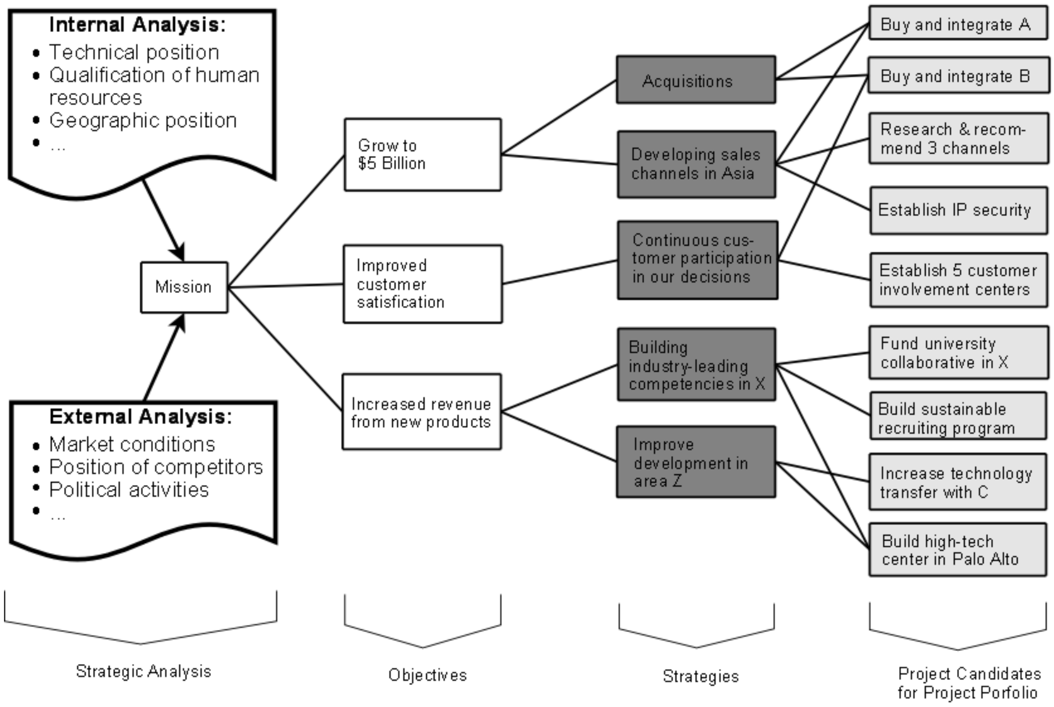


Figure 2.6: Strategic Linkage in Project Portfolio
Source Levine (2005); original Figure modified.

continue to exist or meet its mission since the implementation of any project is simply not feasible without resources. Resources fall into three basic categories (see Campbell et al. 2002):

1. *Budget resources* are money for capital investment and working capital. Sources include shareholders, banks, and bond holders, among others.

2. *Human resources* are appropriately skilled employees to add value in operations and to support those that add value (e.g., supporting employees in marketing, accounting, personnel, etc.). Sources include the labor markets for the appropriate skill levels required by the organization.

3. *Physical resources* can be land, buildings (offices, warehouses, etc.), plants, equipment, stock for production, and so on. Sources include real estate agents, builders, and trade suppliers.

In the strategic-development phase, DMs should identify the core competencies of human resources the corporation needs to achieve its mission. Thus, human resources are further classified into *resource categories*. A *resource category* is a collection of one or more individual human resources with skills that are needed to support the strategy. Most corporations have between 25 and 50 resource categories defined; see Table 2.3. For every resource category, as well as for the budget, the available amount

Table 2.3: Example of Resource Capacities

Capacity Group	**Availability Total**	**Availability 1st Month**	**Availability 2nd Month**	**Availability 3rd Month**
IT Design	264–301 hours	95–110 hours	86–97 hours	83–93 hours
IT Development	1278–1468 hours	450–530 hours	422–481 hours	406–447 hours
Fin. Analysis Eng.	280–315 hours	101 hours	86–97 hours	93–117 hours
Drawings	583–686 hours	203–240 hours	170–203 hours	210–343 hours
Industrial Eng.	870–1110 hours	311–401 hours	290–363 hours	269–336 hours
Production Process	2548–3050 hours	985–1187 hours	845–995 hours	718–868 hours
Production Mgmt.	120–153 hours	49–60 hours	33–44 hours	38–49 hours
Purchasing	245–282 hours	89–104 hours	85–97 hours	71–81 hours
Op. Analysis	223–253 hours	97–113 hours	81–88 hours	45–53 hours
Marketing	334–394 hours	120 hours	99–129 hours	115–145 hours
Sales	386–400 hours	148 hours	105 hours	133–147 hours

must be established. In many cases, the availability level is adjustable to some extent. For instance, the available budget amount can be increased through outside capital or a bank loan. In the opposite direction, a corporation has the opportunity to save some budget along the planning horizon with what equates to a decrease of the budget stock. The availability of human resources and their corresponding categories is flexible through external personal or through recruiting arrangements.

So, the aim is to identify for each resource category, as well as for the budget, the amount that will drive the corporation to its mission and that fits the available project proposals and active projects. Since the determination of the optimal availability amounts is associated with the selection of the optimal project portfolio, DMs here confine themselves to formulate "loose" availability constraints for resource categories and the budget; see Table 2.3. Based on these "loose" availability constraints, DMs of the *Optimal Portfolio Selection* phase are encouraged to identify the best availability amounts; see Section 2.4.1.

2.3.3 Implementation of strategy using strategic buckets

The strategy is implemented by projects which in turn are implemented through the consumption of resources. As a consequence, we can assume that the implementation of strategy equates to spending resources on specific projects. The *strategic buckets* approach operates exactly on this simple principle and is most popular among corporations. In Cooper (2003), it is reported that some 65 percent of corporations use it. Consequently, strategic buckets are a basic feature in PPM software. Thus, it is vital to design the MDSFPPM so that it supports the strategic-buckets approach.

The strategic-buckets approach starts with the classification of projects into different categories, the strategic buckets. The structure of strategic buckets is flexible even into multilevel hierarchies and is frequently adjusted in practice. Common categories for strategic buckets are mission objectives, product lines, types of projects, and technologies, among others. Figure 2.7 presents an example of a strategic bucket structure with a multilevel hierarchy. Every project proposal, as well as every active

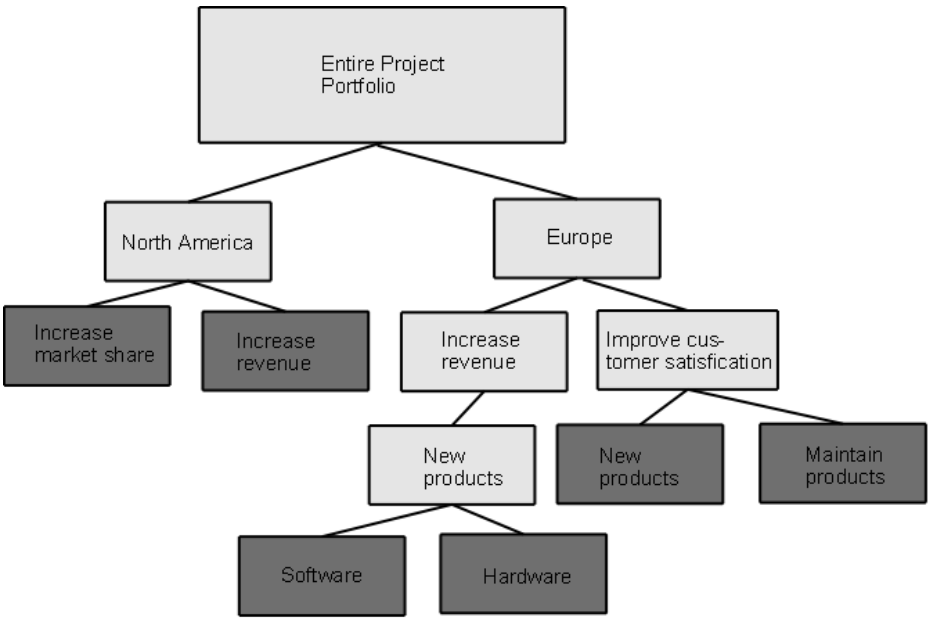


Figure 2.7: Strategic Buckets: Multilevel Structure of a Project Portfolio

project, is contained in exactly one strategic bucket, and, in the case of a multilevel hierarchy, only strategic buckets at the lowest level contain projects. For instance, in Figure 2.7, all project proposals and all active projects are contained in the shaded buckets.

The next step of the strategic-buckets approach is the allocation of resources among strategic buckets. It begins with the top-level decision of how much resource funding every top level bucket receives. For example, in Figure 2.7, a decision must be made about how to split the resources between two buckets, *North America* and *Europe*. Moving one level down, the same issue is faced again. Thus, the allocation of resources across strategic buckets is continued until the lowest level is received. Every single split decision must be aligned with the well-defined strategy so that the bucket structure together with the resource allocation fits the corporation's strategy. If each project is only allowed to consume resources from the strategic bucket it belongs too, the final selected project portfolio will implement the corporation's strategy.

Requirement 2.3 (Strategic buckets and resources). *There shall be a functionality to define strategic buckets and to allocate the available resources among them. Each project is stored in a unique strategic bucket and is only allowed to consume resources from this strategic bucket.*

In reality the resource allocation between buckets is often simplified since the allocation of human resources may be too complex due to their classification into resource categories. Since budget applies to every project, the simplification results in a simple budget breakdown. Thus, only the overall budget stock is allocated across strategic buckets while the availability of human resources for projects is independent of strategic buckets. For corporations using the simplified strategic buckets approach Requirement 2.3 is replaced by

Requirement 2.4 (Strategic buckets and budget). *There shall be a functionality to define strategic buckets and to allocate the available budget resources among them. Each project is stored in a unique strategic bucket and is only allowed to consume budget resources from this strategic bucket. The availability of human resources is independent of the strategic buckets.*

The allocation of (budget) resources among strategic buckets is a balancing act. On the one hand, the allocation must ensure the implementation of the corporation's strategy. On the other hand, the (budget) resource allocation must not terminate promising project proposals or active projects. As a consequence, the allocation of (budget) resources across strategic buckets must not be rigid; rather, it must be negotiable to a certain degree. Since the information about promising project proposals and active projects is missed in the strategic-development phase, the MDSFPPM should support the identification of the optimal (budget) resource allocation among strategic buckets.

Requirement 2.5 (Strategic resource allocation)**.** *There shall be a functionality that supports the allocation of (budget) resources across strategic buckets so that the implementation of strategy is ensured and promising project proposals and active projects are taken into account.*

In practice, the strategic bucket structure often equates to the corporation's hierarchy. Strategic buckets of the lowest level are then the smallest units for which a project portfolio or rather a *project subportfolio* must be selected. The project portfolio itself is obtained through the union of the project subportfolios. The selection of project subportfolios does not necessarily take place at a single point in time. In some cases DMs are just interested into the selection of project subportfolios of a single or a few strategic buckets. For example, assume that in Figure 2.7, the DMs responsible for Europe hold a portfolio review. They are interested in projects within the corresponding branch in Figure 2.7. Thus, they select the strategic bucket *Europe*, which includes the corresponding subbuckets, for the portfolio review. Strategic buckets and projects within *North America* are not affected in this portfolio review.

Requirement 2.6 (Subportfolios)**.** *The MDSFPPM must support the selection of project subportfolios based on strategic buckets.*

Taken together, strategic buckets are constraints designed by upper management for the project selection phase to guarantee that the final chosen project portfolio is aligned with strategy.

2.4 Project Portfolio Selection

In the *Project Portfolio Selection* phase results from the foregoing phases are consulted to identify the most valuable project portfolio for the corporation. The main difference from the foregoing evaluation and screening phases is that at the project-portfolio-selection phase all available project proposals and active projects are evaluated simultaneously (or all project proposals and active projects within strategic buckets currently subject to the portfolio review). The simultaneous evaluation of all project proposals and active projects is responsible for the complexity of this phase. It requires that the following factors be taken into account during the project portfolio selection: implementation of the corporation's strategy, the value of a project portfolio, the balance of a project portfolio, the risk associated with a project portfolio, the utilization of resources, scheduling of projects, and dependencies between projects. The factors are interrelated and often conflict with each other, so attention must be paid to each of them along the whole selection process. We examine the aims of every factor in the subsections below. Since strategy implementation from the foregoing section and utilization of resources are particularly closely interlinked, we will start with resource utilization.

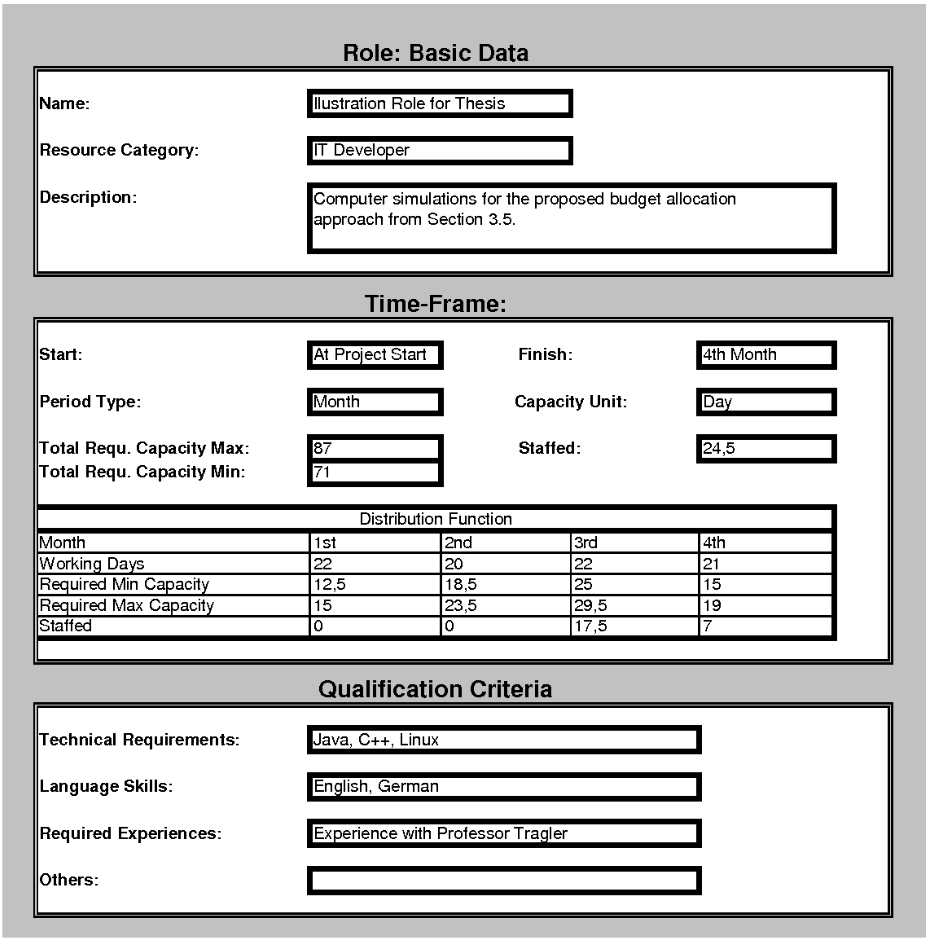
2.4.1 Resource planning

The realization of any project requires various resources. A project must be designed, developed, tested, introduced into the market, maintained, and so on. Every single stage requires special resources, and resource requirements usually reach their peaks during the development stage. However, resources are limited for any corporation so that the realization of all project proposals as well as active projects is not achievable and projects must be selected. Thus, limited resources are the main reason that a project portfolio must be selected and the PPM procedure is necessary.

Human Resources

For the project portfolio selection, DMs must be familiar with availability and skills of their human resources. Thus, software for PPM is integrated into an ERP System where information of every *individual human resource* is stored. An individual human resource is either an internal employee or an external individual who is available to the corporation. The information about an individual resource contains data about his or her qualification, availability, working experiences, and so on.

In the same way, DMs must be familiar with human resource requirements of available project proposals and active projects. Therefore, a project's parameter set contains an arbitrary number of parameters for the detailed description of its human resource requirements. In practice such parameters are called *roles*. Similar to an individual human resource, a role contains information about qualification criteria, required capacities, required working experience, and so on. Since the information about required capacities is affected by estimation errors, Requirement 2.2 is relevant for those parameters; see Figure 2.8.



Role: Basic Data

Name: Illustration Role for Thesis

Resource Category: IT Developer

Description: Computer simulations for the proposed budget allocation approach from Section 3.5.

Time-Frame:

Start: At Project Start — Finish: 4th Month

Period Type: Month — Capacity Unit: Day

Total Requ. Capacity Max: 87 — Staffed: 24,5

Total Requ. Capacity Min: 71

Distribution Function				
Month	1st	2nd	3rd	4th
Working Days	22	20	22	21
Required Min Capacity	12,5	18,5	25	15
Required Max Capacity	15	23,5	29,5	19
Staffed	0	0	17,5	7

Qualification Criteria

Technical Requirements: Java, C++, Linux

Language Skills: English, German

Required Experiences: Experience with Professor Tragler

Others:

Figure 2.8: Overview of a Common Role

In the context of project portfolio selection and human resource planning, the information about time plays a key role. On the one hand, the information about time is essential since the realization of a project corresponds to a time line and resource requirements within this time line change in terms of qualification criteria and capacity amounts. On the other hand, the information about time is essential since the planning horizon for a project portfolio consists of a time interval (the time between portfolio reviews) and availability of human resources fluctuates within the planning horizon or can be adjusted through recruitment measures.

As a consequence, the information of roles and individual human resources must contain appropriate time data. Therefore, a role is defined for a certain time frame.

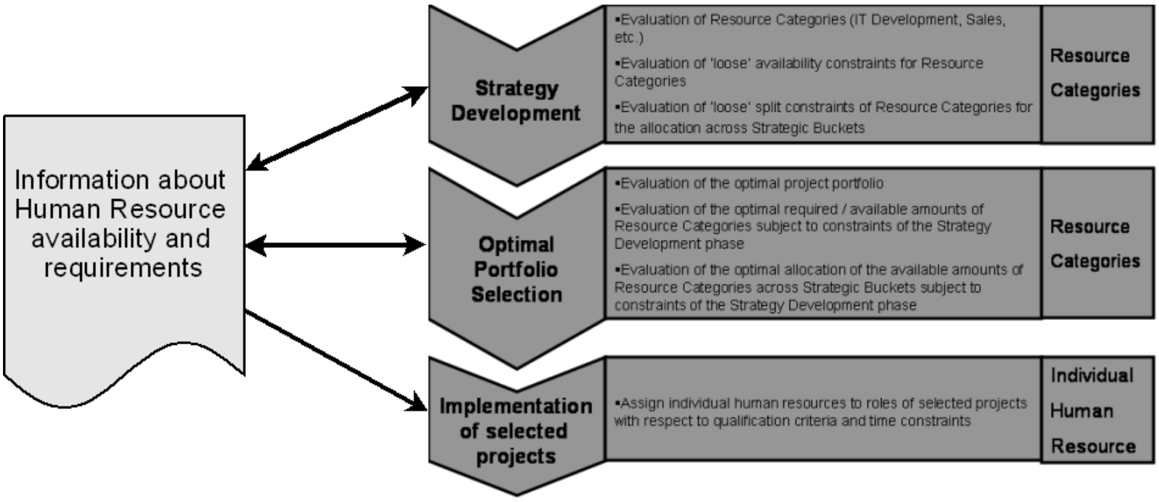


Figure 2.9: Project Portfolio Selection and Human Resource Planning. At the beginning, the focus of DMs is on the level of resource categories and later switch to the level of individual resources.

Either a total amount represents the required capacity for this time frame, or the required capacity amount is further distributed into time periods so that for every single time period a required amount can be defined; see Figure 2.8. The period type as well as the capacity unit of a role is driven by the parent strategic bucket. As a consequence, roles of projects with different parent strategic buckets may have different period types as well as capacity units. For example, in strategic buckets within *North America*, the period type week and the capacity unit hour are used while in strategic buckets within *Europe* the period type month and the capacity unit day are used.

In the description of an individual human resource, availability is denoted in hours per business day. This description allows the DM to configure the availability of an individual human resource into the time-period type and capacity unit of any role.

The information about human resources and about active projects' and project proposals' human resources requirements serves as a foundation for *human resource planning*. The process of human resource planning affects the strategic-development phase, the optimal-portfolio-selection phase, and the implementation of the selected project portfolio; see Figure 2.9. The process starts in the strategic-development phase where resource categories are identified and the information about individual resources is rolled up to the resource-category level so that "loose" availability constraints for resource categories can be determined; see Section 2.3.2.

The DM's goal in the optimal-portfolio-selection phase is to identify the most beneficial project portfolio with respect to resource guidelines of the strategy-development phase and qualification criteria. Therefore, DMs focus on a few resource categories which more than any others determine how many projects the corporation can implement (Kendall and Rollins 2003). It is assumed that those resource categories contain the most valuable human resources and if the project portfolio's human resource requirements with respect to those resource categories are aligned with their availabilities, the entire human resource requirements of the project portfolio are aligned with the entire availability of human resources. Thus, the challenge for DMs is to identify the most beneficial project portfolio for which the following conditions hold.

- total human resource requirements are aligned with "loose" total availability guidelines
- period human resource requirements are aligned with "loose" period availability guidelines
- human resource allocation across strategic buckets is satisfied

In other words, in the optimal-portfolio-selection phase, DMs are challenged to identify the availability level of resource categories with the best ratio between benefit and requirements from the project portfolio that is within availability constraints.

Requirement 2.7 (Human resource planning)**.** *There shall be a functionality to support the identification of the most beneficial and allowed human resource availability level, based on multiple resource categories. This functionality must support time-applied resource requirements and availability based on time periods and capacity units that may differ between strategic buckets.*

Note, the applicability of the condition about resource allocation among strategic buckets depends on the corporation.

The reason for the detour across resource categories for the alignment of resource availability and the project portfolio's resource requirements is the long planning horizon of the project portfolio; the planning horizon of the project portfolio is too long to make plans for individual human resources (Levine 2009). For instance, an individual human resource can resign, become ill, or worse. However, the project portfolio must be robust against unpredictable events affecting individual human resources. The approach via resource categories provides this robustness and simultaneously considers qualification criteria.

The last step of human resource planning refers to the implementation of the selected project portfolio. The aim here is to staff roles with the best-fitting individual human resource and therefore considers a short-term planning horizon, in contrast to the project-portfolio-selection planning horizon. For instance, the staffing of roles with individual human resources may be adjusted daily or weekly while the project portfolio itself may be selected biannually or annually. Although the first two stages of human resource planning ensure that sufficient human resources are available, usually projects compete for some individual human resources. This requires a compromise (postpone a project until the resource becomes available, find the best substitute resource, etc.). The complexity of this step motivates the use of mathematical features. However, since the implementation of the selected project portfolio occurs after the phase and the planning horizon of role staffing is different from the planning horizon of the portfolio, the MDSFPPM should not directly support DMs in this issue. Rather, it should allow the adoption of a mathematical optimizer that supports the staffing of roles with individual human resources (for such mathematical optimizers, see Gutjahr et al. 2008, Tereso et al. 2001). Thus, DMs can, in a first step, use the MDSFPPM to select the optimal project portfolio with a planning horizon of, for instance, one year. In the second step, they use the mathematical optimizer for role staffing, for instance, at the begin of every business week to adjust the role staffing to unexpected events of human resources (sick leave, increasing resource requirement, etc.) with the selected project portfolio as input.

Requirement 2.8 (Role staffing)**.** *The MDSFPPM must allow for the adoption of a mathematical optimizer for role staffing with individual human resources.*

Budget resources

As with human resources, DMs must be familiar with budget availability and budget requirements of project proposals and active projects—i.e., the costs of project proposals and active projects. In contrast to human resource planning, however, budget planning fully encapsulates the information about costs and availability of budget since budget resources are not distinguished in terms of skills. As a consequence, the parameter set of a project contains usual parameters, instead of complex roles, to describe its costs. Common examples for cost parameters are launch costs, development costs, personal costs, overhead, and so on. These parameters are easy used to calculate the project's total cost. Attention must be paid, however, because an exact estimation of cost parameters is not possible so that Requirement 2.2 must be taken into account; compare with Figure 2.4.

The available budget may be decomposed into multiple sources: credits, provisions, financial revenues of implemented projects, and so on. Some of those sources are time sensitive. As a consequence, cost parameters of a project are distributed into time periods so that, as with roles, they are adaptable to the availability of budget resources. The period type of cost parameters as well as the currency are driven by the project's parent strategic bucket. Summarizing, the information about budget and costs differs from the information about human resources in terms of qualifications.

The information about available budget and costs is used for the *budget planning* process. Similar to the human resource planning process, the budget planning process affects the optimal-portfolio-selection and strategy-development phases and the implementation of the selected project portfolio; see Figure 2.10.

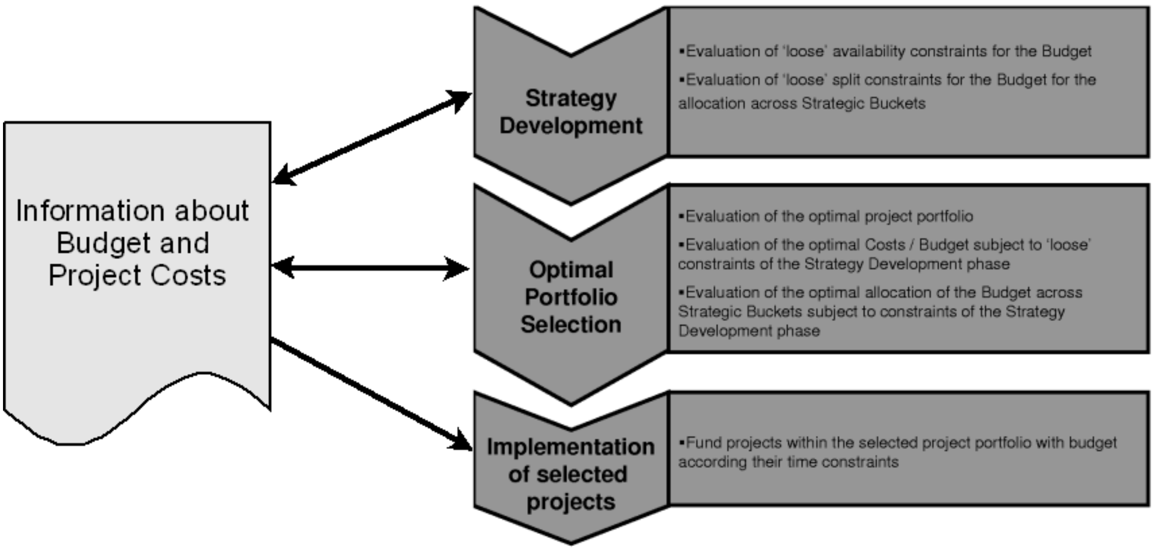


Figure 2.10: Project Portfolio Selection and Budget Planning

Budget planning starts at the strategic-development phase where "loose" availability constraints for the budget are determined; compare with Section 2.3.2.

In the optimal-portfolio-selection phase, one aim is to identify the most beneficial project portfolio such that the following conditions apply.

- total costs do not exceed "loose" budget constraints
- period costs do not exceed "loose" period budget constraints

- budget allocation across strategic buckets is satisfied

In other words, DMs of the optimal-portfolio-selection phase are challenged to identify the budget level with the best cost–benefit ratio that is within the guidelines from the strategic-development phase. Therefore, the MDSFPPM must support DMs in identifying the most beneficial budget level for which the conditions above hold.

Requirement 2.9 (Budget resource planning)**.** *There shall be a cost–benefit analysis functionality to support the identification of the most beneficial budget level. This cost–benefit analysis must support time applied costs and budget availability based on time periods and currencies which may differ between strategic buckets.*

For the implementation of the selected project portfolio, it is necessary to fund the selected projects with budgets according their costs. Since the first two steps of budget planning ensure that sufficient budget is available to fund all selected projects, the funding step should go smoothly as long as projects are within their planned costs.

2.4.2 The value of a project portfolio

The successful implementation of any project brings the corporation closer to its mission. For example, it may drive the corporation closer to high financial revenues, to a higher market share, or to more technical power. In short, it will support the corporation's successful implementation of its strategy. To estimate how well a project drives the corporation to its mission, it is necessary to establish a set of *evaluation criteria*. Evaluation criteria fall into three categories (Kleinmuntz and Kleinmuntz 1999):

1. **Financial criteria**
   The most common financial criteria include net present value, expected commercial value (ECV), productivity index (PI), and return on investment.

2. **Strategic criteria**
   Strategic buckets are only one technique to implement the corporation's strategy. Another technique is to estimate a project's strategic alignment so that a project portfolio can be identified that implements the corporation's strategy. The approach is combinable with the strategic-buckets approach and their combination is a common method in practice.

Evaluation Criterion 1: Strategic Fit

	Rating Scale					
Attributes	1	4	7	10	**Rating**	**Comments**
Congruence	Only peripheral fit with business strategies	Modest fit, but not with a key element of the strategy	Good fit with a key element of strategy	Strong fit with several key elements of strategy		
Impact	Minmial impact; no noticeable harm if program dropped	Moderate competitive, financial impact	Significant impact; difficult to recover if project unsuccessful or dropped	Business unit future depends on this project (mandatory)		

Figure 2.11: Scoring Model to Evaluate a Project's Strategic Fit
Source Cooper et al. (2001b).

3. **Qualitative criteria**
   Common qualitative criteria include customer satisfactory, technical advantages, and competitiveness.

In the *Individual Project Analysis* phase, every project must be rated with respect to every evaluation criterion. Thus, for every project a set of values is estimated that describes how well the project drives the corporation to its mission if successfully implemented. In praxis, these values are often referred to as key performance indicators (KPIs); see Figure 2.4.

In Cooper et al. (2001b), it is reported that it is common to use between two and five evaluation criteria with different priorities. It is suggested that using more than five evaluation criteria can cause the selection process to become confusing.

An important fact for the adoption of the MDSFPPM is that evaluation criteria may vary between strategic buckets. Projects within different strategic buckets are designed for different purposes so that it is not appropriate to compare them on the same evaluation criteria. Consequently, a strategic bucket has its own set of evaluation criteria and for every project a set of values corresponding to the evaluation criteria (KPIs) of its strategic bucket must be estimated.

A common tool to estimate a project's value for a certain evaluation criterion is the *scoring model*. In a scoring model a list of attributes is created and it is assumed that each attribute exhibits a correlation with at least one evaluation criterion. To estimate a project's value on this evaluation criterion, a DM just has to rate the project on every attribute. The rating is then used to calculate the project's value for the corresponding evaluation criterion. Figure 2.11 outlines a simple scoring model for the evaluation criterion *Strategic Fit*. The attributes are *Congruence* and *Impact* and the scoring scale allows scores from 1 through 10.

Although scoring models may help improve the quality of value estimates, they do not replace Requirement 2.2. Whether or not a scoring model is used, the environment of a corporation as well as the implementation of a project remains uncertain. To take the uncertainty into account in the estimation of project values, one could create two scoring models for every evaluation criterion. The first scoring model is then used to calculate a project's minimum performance. In contrast, the second scoring model is used to calculate a project's maximum performance.

The value of a project portfolio with respect to a certain evaluation criterion is described by the projects it contains and their values associated with the evaluation criterion. Thus, it is assumed that a project portfolio with the highest values at all evaluation criteria drives the corporation most rapidly and most safely to its mission. However, the identification of the project portfolio with the highest values at all evaluation criteria is a complex task. First, evaluation criteria conflict with each other: A high value at evaluation criterion X may include a low value at evaluation criterion Y. Second, the evaluation criteria may have different priorities. Finally, to achieve the corporation's mission it may be necessary to ensure a minimum performance of the project portfolio at every evaluation criterion (Edwards et al. 2007). As a consequence, the process seeks the project portfolio with the best trade-off between the evaluation criteria values with respect to their priorities.

Requirement 2.10 (Trade-off between portfolio values)**.** *There shall be a functionality to support the identification of the project portfolio with the best trade-off between evaluation criteria values based on different priorities. Additionally, the functionality must ensure minimum performance restrictions. The MDSFPPM must support different sets of evaluation criteria between strategic buckets and it must be adaptable to multiple scoring models.*

2.4.3 Seeking the right balance for the project portfolio.

As with an investment portfolio, a project portfolio must be balanced. The concept of balance in the project portfolio simply means that the overall portfolio should reflect some level of proportionality across certain dimensions. For instance, DMs may prefer a project portfolio that balances the overall risk. Typically, a high risk project will also have the greatest value and also the greatest potential to bring the corporation closer to its mission. A balanced portfolio might include a small investment in high-risk, high-value projects as well as investment in low-risk projects with more modest expected values. A mixture of projects with different risks will allow a corporation to achieve acceptable results while taking on some risk in large, unstructured, or relatively high-technology projects (Archer and Ghasemzadeh 1999b). Common dimensions where balance is sought are

- Risk balance
- Strategic balance
- Product balance
- Market balance
- Geographic balance
- Time balance
- Technology balance

The desired proportionality for any dimension is driven by the corporation's mission. For instance, if a corporation's mission includes becoming market leader in Technology Y, the proportion for high-risk projects is large since technology projects are usually subject to higher risk. In contrast, if the mission for a corporation is driven by cost reduction, the proportion of high-risk projects is low since cost saving projects are generally not affected by much risk. Since project portfolio balancing is driven by the corporation's mission, it is related with strategy alignment. As a matter of fact, the strategic buckets approach benefits from the implementation of balance. To realize this, note that dimensions for which a balance is sought and categories for strategic buckets (see Section 2.3.3) overlap. The allocation of (budget) resources across strategic buckets ensures that the selected project portfolio is balanced as desired.

However, there may be some dimensions for which a certain balance is sought, but which are not subject to the strategic-buckets approach. For those dimensions, Archer and Ghasemzadeh (1999b) suggested enforcing a budget allocation between projects, independent of strategic buckets, so that the desired balance is achieved. For instance, DMs could agree that the budget amount consumed by high-risk projects must not exceed 30 percent of the total budget. Another approach to implement a desired balance for a certain dimension is to specify a relationship between projects.

For instance, the number of low-risk projects should be at least twice the number of high-risk projects.

Requirement 2.11 (Portfolio balancing)**.** *There should be a functionality that supports the balancing of the project portfolio for both approaches: project numbers and budget allocation across projects.*

Balancing a project portfolio as outlined above is not popular in reality. The formulation of trade-offs in advance is too abstract. Instead, project-portfolio balancing is done by adjustments once a first proposal for a project portfolio is available. The project portfolio proposal allows DMs to use diagrams and charts for the visualization of the proposal's balance, allowing DMs to identify necessary adjustment steps. We revisit this topic again in Section 2.5.

2.4.4 The impact of risk

Probably every project is affected by risk. On the one hand, risk is driven by uncertainties in the implementation of a project. The successful implementation of a project depends on many tasks whose outcomes are not predictable since they are influenced by factors beyond the control of DMs. For instance, technical issues can occur, an experiment can fail, or some necessary core resources can become unavailable. Thus, in practice it is common to estimate the probability of the successful implementation of a project in the individual-project-analysis phase. This probability is a standard parameter of a project, so it is supported by PPM software and usually is referred to as the *probability of technical success*.

On the other hand, risk is driven by uncertainties in the environment in which the corporation operates. Projects are designed to drive the corporation to its mission. The mission of a corporation is, among other things, based on the corporation's opportunities (see Section 2.3). However, the opportunities may change while the project is under construction so that its promised value does not manifest. Thus, although a project may be successfully implemented, it still may not drive the corporation to its mission since the position of the mission has changed.

For instance, look at a corporation producing cars which designed a project to create a *new car for its prestige collection* since the corporation's mission addresses financiers as customers. However, an unexpected banking crises occurs so that the demand for such cars decreases. Additionally, the government launches a cash-for-clunkers initiative to support the demand on economic, environmentally friendly cars. As a consequence, the demand on cars from the prestige collection disappears entirely so that the corporation must shift its mission towards economical cars to survive. Even if the project to create a new car for its prestige collection was implemented successfully, it no longer has any significant value for the corporation.

In practice it is common to estimate the probability that a project supports the corporation's mission in the individual-project-analysis phase. This probability is also a standard project parameter supported by PPM software, where it is referred to as the *probability of commercial success*.

Since the most valuable projects are usually affected with considerable risk, a corporation may not achieve its mission with only low-risk projects in its project portfolio. Hence, risk cannot and should not be avoided in most project portfolios. Rather, it should be accepted and managed honestly so that the corporation drives to its mission **and** drives safely (Levine 2005). Accepting and managing risk honestly consist of three steps:

1. Define the risk of a project
2. Consider risk in the project portfolio selection
3. Monitor and maintain the project portfolio

The first step, define the risk of a project, just means that risk must influence the description of a project. Since a project is described by a set of parameters, this set must reflect the project's risk. Therefore, the parameter set usually contains some explicit risk parameters such as probability technical success and probability commercial success. Apart from explicit risk parameters, a project's risk is described by fluctuation ranges for its parameters. Recall, project parameters are estimated using upper and lower bounds based on a project's risk level; see Requirement 2.2. Thus, a high fluctuation range in a project's parameters is associated with high risk while a small fluctuation range in a project's parameters is associated with low risk.

In the second step, consider risk in the project portfolio selection, explicit risk parameters are used to select a risk-balanced project portfolio since they enable the discrimination between high-risk and low-risk projects. Thus, explicit risk parameters are the basis for the formulation of proportional ratios between project groups of different risk levels; see Requirement 2.11. In the same way they are used to visualize projects risk levels in diagrams which are used for project portfolio balancing; see Section 2.5. Fluctuation ranges of parameters are utilized to simulate different scenarios about project outcomes. For instance, if the evaluation values for a project are set to its lower bounds and the costs are set to its upper bounds, a DM can simulate the worst case for this project and its impact on the entire project portfolio. In general, DMs are interested in various simulations characterized by specific scenarios of parameters for some or all the projects. Through the extensive analysis of such scenarios, DMs learn where the risks lie and where the opportunities are so that the best project portfolio can be identified.

Requirement 2.12 (Risk analysis)**.** *The MDSFPPM shall provide a functionality which calculates simulations of a project portfolio based on a specific scenario of project parameters.*

The last step, accepting and managing risk, monitors active projects as well as the corporation's environment with respect to critical risk indicators. Thus, this step is not part of the optimal portfolio selection; rather, it is a part of the maintenance of the project portfolio; see Section 2.6.

Finally we will remark that the collection of such project risk parameters as probability of technical success into a project portfolio risk parameter is inappropriate since such a parameter does not have sufficient expressiveness (Levine 2006). Thus, a functionality to minimize a certain project portfolio risk parameter in the MDSFPPM is superfluous.

2.4.5 Project scheduling

Section 2.4.1 about resource planning is the foundation for project scheduling. There, we have established that the implementation of a project corresponds to a time line and the project's resource requirements along this time line vary in terms of qualifications and capacity amounts. Further, we have seen that the availability of resources may vary along the portfolio's planning horizon. Thus, how many projects can be selected into the project portfolio, and therefore are implemented, correlates with the schedules of those resources. Since the schedule requirements of projects may be loose with respect to the start date as well as task dates, DMs in the optimal-portfolio-selection phase are challenged to schedule projects so that resources are optimally utilized, optimizing the portfolio with respect to the corporation's mission.

We start our investigation with the *schedule* of a project. The schedule of a project is a collection of tasks and dependencies between them. Based on these dependencies and on the project's delivery date start and end dates for every task can be calculated; see Figure 2.12. From the structure of tasks and their resource requirements, the time-dependent resource requirements of a project are calculated. The structure of tasks is also the basis for the identification of the *critical path*. The critical path of a project is the sequence of tasks that takes the longest time to complete. In other words, the critical path determines the length of a project and is in practice often referred to as the *time to market*. Every task which corresponds to the critical path cannot be delayed without the length of the project increasing. Thus, if the start date of a project is chosen in a way which does not allow any delays, start and end dates of a critical path task are rigid to ensure that the project is delivered on time. On-time delivery of projects is a main component of the corporation's path to its mission since it is closely linked with competitive advantages and market share (see Pfähler and Wiese 2008). In contrast to critical path tasks, start and end dates of noncritical-path tasks may be flexible to a certain degree which is recorded through earliest start date, latest start date, earliest finish date, and latest finish date. The flexibility of noncritical-path tasks can be used to optimize resource utilization through implementing them when sufficient resources are available.

The project's start date is not necessarily rigid, rather it is constrained by its delivery date on one hand and by an earliest project start date on the other hand. The project's earliest start date results from preliminary work for the project which must be completed before the project can start (e.g., dependency on the result of another project). The delivery date of a project together with its critical path indicate the latest possible start date of a project so that it is delivered on time. Thus, the start date of a project can be chosen from a window in time. As a consequence, DMs in the optimal-portfolio-selection phase are challenged to schedule project starts so that resources are optimally utilized. Thereby, attention must be paid that the project start date sets the general conditions for task schedules, even for critical path tasks.

Requirement 2.13 (Project scheduling)**.** *The MDSFPPM must support project scheduling based on the critical path so that projects are delivered on time and resources are optimally utilized. For the utilization of resources, flexibilities of noncritical-path tasks must be taken into account.*

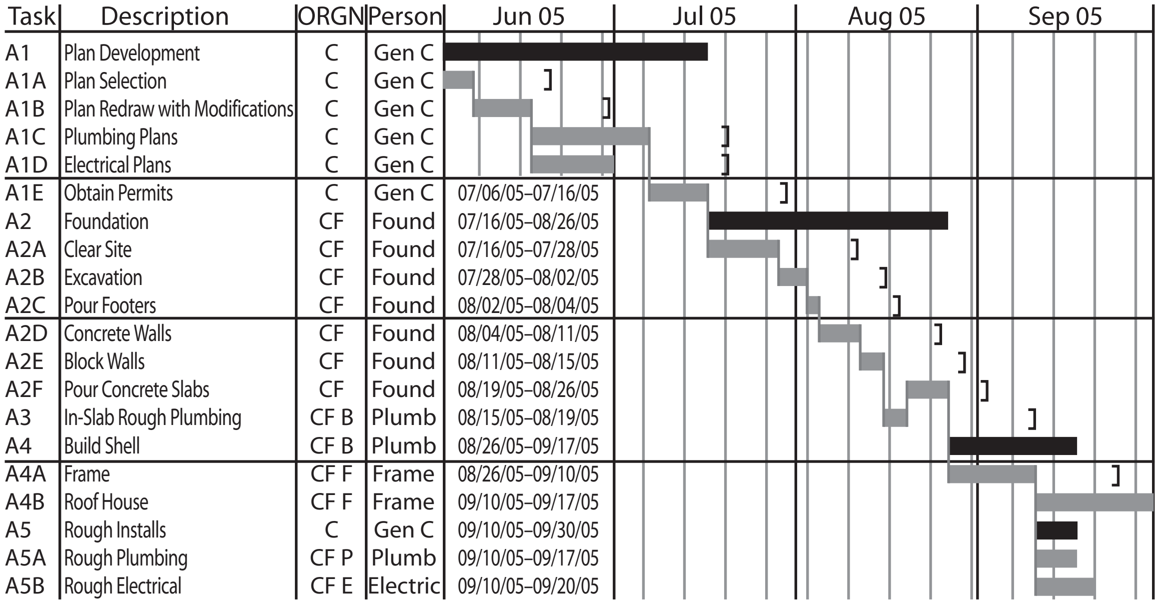


Figure 2.12: Gantt Chart for the Project *Built my House.* Tasks within the critical path are outlined in black. Tasks in grey can by delayed until the bracket without endangering the delivery date of my dream house.

The scheduling of project tasks is already covered in great detail and is the purview of the PM department. Therefore, in the optimal-portfolio-selection phase, DMs may content themselves with scheduling the project start dates. When the implementation of the selected project portfolio is in progress, the flexibility of noncritical-path tasks can be used for the optimal staffing of roles with individual resources and therefore corresponds to the competencies of the mathematical optimizer for role staffing; see Section 2.4.1.

2.4.6 Project dependencies

Projects cannot always be considered in isolation. There are many possible dependencies among projects that require the effects of one project on another to be taken into account in the project portfolio selection procedure. We distinguish between two categories of project dependencies. The first category is about *technical project dependencies*, while the second category is about *economic project interactions.*

Technical project dependencies indicate that the implementation of a project, the so called *successor project*, is dependent on the implementation of another project, the so called *predecessor project.* In general, a successor project can have multiple predecessor projects and, conversely, a single project may be predecessor for multiple successor projects. Technical project dependencies are supported by PPM software where the following relations are allowed.

- The *finish-to-start* dependency states that the predecessor project must be complete before the successor project can begin. For example, the predecessor project represents the collection of data and the successor project represents statistical analysis of this data.
- The *start-to-start* dependency states that the successor project may begin once the predecessor project has begun. For example, as soon as the collection of data has started, one could start entering data into the statistical analysis program.
- The *start-to-finish* dependency states that the predecessor project cannot be finished sooner than the successor project has started. For example, suppose a corporation has built a new information system. It does not want to eliminate the legacy system until the new system is operational. When the new system (the successor project) starts to work the old system (the predecessor project) can be disconnected.
- The *finish-to-finish* dependency states that the successor project cannot finish sooner than the predecessor project. For example, refer back to our data collection and analysis example. Data analysis (successor project) cannot finish until data collection (predecessor project) has finished.

Requirement 2.14 (Technical project dependencies)**.** *The MDSFPPM must support technical project dependencies based on various timing relations.*

An economic project interaction addresses the interaction of projects with respect to their support of the corporation's mission. The support of the corporation's mission from project A may be different depending on whether project B is implemented or not, and vise versa. An economic project dependency does not necessarily consist of just two projects, rather it consists of a project family. According to which projects out of the family are implemented, the effects of the economic project interaction occur. The effects of an economic project interaction on the projects' support of the corporation's mission can be synergistic, cannibalistic, or mutually exclusive. (Since synergy and cannibalization effects are hard to predict, Requirement 2.2 is also valid for parameters that describe those effects.)

An economic project interaction with a synergy effect contains additional benefit, or resource savings, or a combination of them. For an example, assume that the implementation of four projects is based on the development of a certain technology. As soon as one project out of this four is selected, the technology must be developed. If DMs chose a second project out of these four for the project portfolio, they have the choice to save development resources, take the resources to develop a better technology, or a combination of these choices. The same is true for the remaining two projects.

An economic project interaction with a cannibalization effect contains benefit reduction, additional resource requirements, or a combination of these effects. For example, assume two projects are designed for the same market using different technologies. If both projects are implemented, they may compete for the same customers so that the benefit of the projects is reduced. Further, the resource requirements may increase due to maintenance work for different technology types.

A special case of economic project interaction is given by mutually exclusive projects. In the situation of mutually exclusive projects, the choice must be made between two or more projects, each vying to meet same mission need. For instance, assume that several different versions of a project are available. The most attractive version should be selected, while the others should be rejected (Heerkens 2006).

Requirement 2.15 (Economical project interaction)**.** *The MDSFPPM must support economic project interactions with synergy, cannibalization, and mutually exclusive project effects. An economic project interaction can exists among an arbitrary number of projects.*

In reality some projects are so closely related that only their concurrent implementation is beneficial for a corporation. This may be due to technical dependencies or synergy effects of this projects. Such a project family must managed in a coordinated and logistically sound way called *program management*. Thus, a program is a collection of projects with a common vision to support the corporation on its way to the mission. Although projects within a program are strongly dependent, the final composition of a program must be established by DMs at the optimal-portfolio-selection phase. This is because more projects are usually submitted for a program than are necessary for its implementation, or at least different versions of projects within the program are available where choices must be made. Taken together, DMs are challenged to make a decision about whether the program is accepted or rejected, and, for the case that the program is accepted, they must further establish the final composition of the program. Program management is a standard feature in PPM software where a program is referred to as an *initiative*. Therefore, it is essential that the MDSFPPM support program management.

Requirement 2.16 (Program selection and composition)**.** *The MDSFPPM must support the selection decision of programs (initiatives) as well as the decision about program composition.*

2.5 Portfolio Adjustment

2.5.1 Interaction features for decision makers

The initial project portfolio calculated in the optimal-portfolio-selection phase is probably not the final portfolio. Rather it is the root for an interactive adjustment process that leads to the final portfolio. An adjustment process is necessary for several reasons. First, some information is difficult to anticipate and to include in a mathematical model, so its solution will require some modifications (see Archer and Ghasemzadeh 1999a). Second, for user acceptance, it is essential to include a controlling mechanism where DMs can include their knowledge and experiences in the final solution (Baker and Freeland 1975). Finally, with the initial project portfolio, new information that was not available in advance may come to light and require some adjustments.

The first step for adjustments is to analyze the initial project portfolio to identify its weaknesses. Therefore, PPM software includes a set of charts and tables in a *reporting cockpit* functionality. In the reporting cockpit, DMs can start their analysis at the portfolio level to drill down into the strategic bucket level or project level as well as into any collection of projects.

In the second step, DMs can set adjustment actions to eliminate weaknesses of the initial portfolio. The adjustment actions must be controlled via the corresponding charts and tables to promote the identification of desirable project portfolios. Common adjustment actions are addition and deletion of projects, shift of projects between strategic buckets, adjustment of the trade-off between evaluation criteria, changes in resource/budget availability levels, changes in the resource/budget allocation among strategic buckets, definition of project dependencies (e.g., identification of

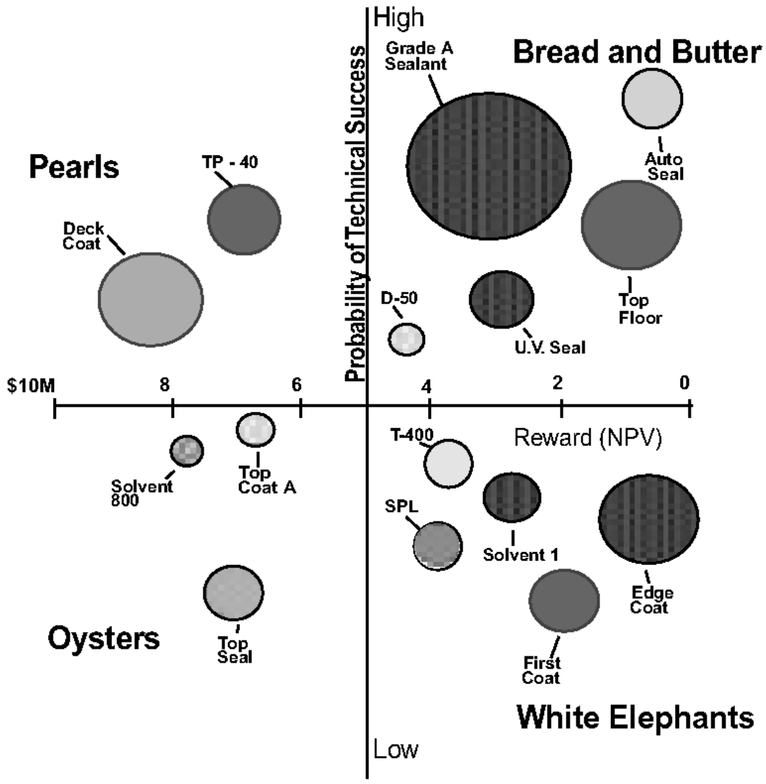


Figure 2.13: Risk–Reward Bubble Diagram with Circle Size as Annual Resource Needs
Source Cooper (2003).

extra mutually exclusive projects), rescheduling of projects, and portfolio balancing.

In the final step, the MDSFPPM is required to recalculate the new "optimal" project portfolio based on the adjustment actions. It is crucial that the recalculation be done in real time to facilitate an interactive adjustment process.

Requirement 2.17 (Portfolio adjustment features)**.** *The MDSFPPM must support an interactive adjustment process. The controlling of the adjustment process must be feasible via charts and tables from the reporting cockpit of the corresponding PPM software to give DMs immediate feedback on the consequences of their adjustments.*

For the sake of completeness, we must mention that achieving some form of balance is a significant part of the adjustment process because portfolio balancing is done by interactive charts, which are an information display rather than a decision model per se; compare with Section 2.4.3. Figure 2.13 shows a representative bubble diagram, one of the most frequently used charts. A bubble diagram shows projects on a two-dimensional X–Y plot, for instance NPV versus probability technical success as in

Figure 2.13. Due to the analysis of bubble diagrams, projects may be added or deleted from the project portfolio. The MDSFPPM must then recalculate the new "optimal" project portfolio to show DMs the impact of the adjustment actions with regard to any parameters of interest (e.g., impact on budget/resource requirements via tables and bar charts, or project schedules via Gantt charts).

2.5.2 What-if scenarios

What-if scenarios are very much a learning tool for DMs and therefore are a part of the third reason for portfolio adjustment. The what-if scenario continues the idea of new information in the initial project portfolio: Various project portfolio scenarios are created and recorded to give the DM as much information as possible and to compare alternative scenarios against each other. Scenarios are simply obtained through adjustments like those outlined above and the recalculation of the new "optimal" project portfolio. In addition to the adjustment steps from above, DMs are also interested in scenarios characterized by changes to project parameters.

The aim of all this analysis and comparing is to learn about opportunities and risks facing the corporation; see Section 2.4.4. For learning purposes, it is especially important to control the adjustments via corresponding interactive charts and tables to immediately receive feedback about the changes. This is the main drawback of current what-if scenarios in PPM software. Although PPM software vendors offer the *what-if-scenario* feature, DMs do not receive a recalculated project portfolio or anything else as feedback let alone interactive charts and tables. In present what-if-scenario features, it is only possible to copy the current project portfolio into a virtual offline scenario to make changes without modifying the real data. In other words, current what-if-scenario features are useless until they are connected to a reasonable mathematical model. Due to Requirements 2.12 and 2.17, the MDSFPPM provides an excellent combination to present what-if scenarios since it addresses exactly this drawback.

2.6 Maintaining the Project Portfolio

As mentioned in the foregoing chapter, PPM is a highly dynamic process due to the environment of the project portfolio as well as its implementation. To address this dynamic, DMs must *maintain* the project portfolio with respect to both of its sources. Maintaining the project portfolio with respect to the dynamic environment means that DMs must periodically update or confirm the corporation's mission and, if necessary, adjust the project portfolio. Maintaining the project portfolio with respect to its implementation means evaluating the status and performance of projects.

2.6.1 The stage–gate process: Monitor projects' performance

The dynamic with respect to the project portfolio implementation is given by uncertainties in the development of projects. Thus, maintaining the project portfolio with respect to its implementation means monitoring project development and adjusting the project portfolio if some projects have performance deficits; compare with Section 2.4.4. The most popular approach for performance monitoring of individual projects is the *stage–gate process*, which is supported by common PPM software. In the stage–gate process, the development of projects is decomposed into an arbitrary number of development phases; see Figure 2.14. During any development phase, project parameters are updated to, for example, indicate the remaining human resource requirements, schedule dates, and so on. (Attention should be paid that with each successive development phase the uncertainty in parameter estimates decreases so that their uncertainty intervals are smaller.) At the end of every development phase a gate is placed where the updated parameters are used to evaluate whether the project is progressing as assumed.

The stage–gate process is used to observe the developmental progress of individual projects. Based on the received information, a *confirm*, *on hold*, or *terminate* decision must be made. Although in some cases the decision about a project's future may be manageably isolated—i.e., just on the project's parameters—its contribution to the entire portfolio should be part of the decision as well. As a consequence, it is common to merge portfolio reviews and the gate decisions of projects together. Thus, the selection of new project proposals and the reevaluation of active projects must work in harmony: The selection of a project portfolio consists of the selection of new project proposals **and** the repeated reconfirmation of active projects (see the arrow leading from the *Phase/Gate Evaluation* box to the *Individual Project Analysis* box in Figure 2.3).

It is often assumed that projects at an advanced development stage have higher priority than new project proposals. However, the priority of a project should be

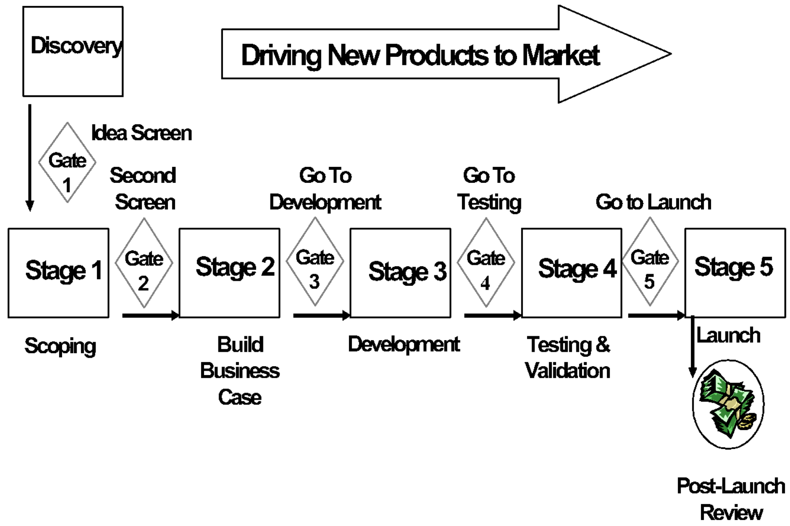


Figure 2.14: Stage–Gate Model with a Discovery Stage
Source (Cooper et al. 2002).

calculated independently of its development stage. An appropriate approach is given by the concept of *sunk costs*. Sunk costs are unrecoverable past expenditures and should not be taken into account in the decision of whether an active project is confirmed or terminated (see Levine 2005).

In common PPM software, the stage–gate process provides information about the development progress of individual projects. Nevertheless, it does not provide any information on how the development progress of individual projects impacts the entire project portfolio since the software lacks a mathematical approach. Further, the software does not inform DMs how their potential project decisions (confirm, on hold, or terminate) affect the portfolio. Thus, the MDSFPPM must be the counterpart to the stage–gate process, informing DMs how the development progress of individual projects impacts the entire portfolio. The MDSFPPM must additionally support simulations of how potential project decisions affect the entire portfolio.

Requirement 2.18 (Project development progress evaluation)**.** *The MDSFPPM must be compatible with the stage–gate process for project progress monitoring. It must inform DMs how project development progresses and how it impacts the entire portfolio, and it must simulate potential decisions. The MDSFPPM must also support the simultaneous selection of new project proposals and confirmation of active projects.*

2.6.2 Project-portfolio environment

Probably every corporation deals in an uncertain environment. The choices of a competitor or main customer, an actions of government, an economical crisis, all of them can require the reevaluation of the corporation's mission and may change its position. Changes to the corporation's mission may trigger adjustments in the strategy, the structure of the strategic buckets, resource availability, the allocation of resources among strategic buckets, or the evaluation criteria. No matter how the change of the mission looks, it is necessary to put the project portfolio back on track. Through the selection of new project proposals, termination of active projects, and rescheduling of active projects, a new project portfolio must be selected to successfully drive the corporation to its new mission.

Requirement 2.19 (Mission update and portfolio adjustment)**.** *The MDSFPPM must support the adjustment of the ongoing project portfolio in case of changes in the corporation's mission.*

Finally, one question continually comes up: If a hot new project proposal comes up, what will its impact be on resources and the portfolio (see Kendall and Rollins 2003). The aim is to implement the hot new project proposal while limiting its impact on resources and the remaining portfolio is as much as possible. Thus, the MDSFPPM must support the smooth integration of the hot new project proposal into the remaining portfolio through rescheduling or termination of active projects as well as adjustments on the availability of resources. (Here, it may be difficult to differentiate the competencies of the MDSFPPM and the competencies of the mathematical optimizer for role staffing.)

Requirement 2.20 (Hot new project proposal)**.** *The MDSFPPM must support the smooth integration of hot new project proposals into the ongoing portfolio.*

2.7 Requirements Summary

For the purposes of a better overview, this section provides a table summarizing the requirements of the MDSFPPM. The table also includes economic and mathematical classifications of the requirements. The economic classification just outlines which phase of PPM is affected by the requirement and therefore whether the competencies are on the project or on the portfolio level.

For a smooth crossover to the formulation of the mathematical model, the mathematical classification states how a requirement influences the formulation. In addition to the common classes, *constraint* and *objective*, we have the classes *interface* and *simulation*. While the class *interface* deals with the embedding of the MDSFPPM into PPM software, the class *simulation* deals with the creation of portfolio scenarios and sensitivity analysis. Note, some requirements are so comprehensive that they affect multiple classes.

Table 2.4: Summary of Requirements for the MDSFPPM

Requirement Number	**Requirement Name**	**Economic classification**	**Mathematical classification**
Requirement 2.1	Mandatory projects	Preselection	Constraint
Requirement 2.2	Imprecise project information	Preselection	Interface, Simulation
Requirement 2.3	Strategic buckets and resources	Strategy	Constraint
Requirement 2.4	Strategic buckets and budget	Strategy	Constraint
Requirement 2.5	Strategic resource allocation	Strategy	Constraint, Objective
Requirement 2.6	Subportfolios	Strategy	Constraint
Requirement 2.7	Human resource planning	Selection	Objective
Requirement 2.8	Role staffing	Maintain	Interface, Objective
Requirement 2.9	Budget resource planning	Selection	Objective
Requirement 2.10	Trade-off between portfolio values	Selection	Constraint, Objective
Requirement 2.11	Portfolio balancing	Selection	Constraint
Requirement 2.12	Risk analysis	Selection	Simulation
Requirement 2.13	Project scheduling	Selection	Constraint, Objective
Requirement 2.14	Technical project dependencies	Selection	Constraint
Requirement 2.15	Economic project interactions	Selection	Objective
Requirement 2.16	Program selection and composition	Selection	Constraint, Objective
Requirement 2.17	Portfolio adjustment features	Selection	Interface, Simulation
Requirement 2.18	Project development progress evaluation	Selection, Maintain	Interface, Simulation
Requirement 2.19	Mission update and portfolio adjusting	Selection, Maintain	Constraint, Objective
Requirement 2.20	Hot new project proposal	Maintain	Simulation

Chapter 3

A Robust Model to Support Portfolio Selection under Incomplete Information

The literature about project portfolio optimization offers a broad body of different mathematical models. Many of these models are formulated as multidimensional knapsack problems. Archer and Ghasemzadeh (1999b) presented a multidimensional knapsack model with a weighted multicriteria objective function. In this model, linear constraints can easily be added to satisfy the underlying business case. A literature review of the multidimensional knapsack problem is given in Freville (2004).

A different way is introduced in Eilat et al. (2006). They use a branch-and-bound algorithm to generate the set of all feasible portfolios. Subsequently, they use data-envelopment analysis (DEA) to identify the set of efficient portfolios. The decision maker units are the portfolios generated by the branch-and-bound algorithm. A further approach is given through the use of the analytic hierarchy process. Liberatore and Nydick (2008) presented a literature review of the application of the analytic hierarchy process to important decision problems in the context of medicine and health care.

Almost all models presented in the literature share the same disadvantage. They do not accept incomplete data as input, which we identified in the foregoing chapter as a main reason for the failure of mathematical approaches in the area of PPM. They necessitate precise data for every project about costs, resource requirements, benefits, schedules, and so on. It is also necessary to offer exact information about availabilities of budget and human resources.

Liesiö et al. (2007) and Liesiö et al. (2008) did present one model that allowed imprecise data as input. Called *robust portfolio modelling* (RPM), the model accepts interval data for project evaluation as well as for project costs. Furthermore, preferences between evaluation criteria are modelled through inequalities.

The remainder of this chapter describes the RPM approach from Liesiö et al. (2007) and Liesiö et al. (2008) in detail. Section 3.1 addresses the value of a portfolio while Section 3.2 provides an introduction to portfolio feasibility. Section 3.3 presents the dominance concept used as preference evaluation between portfolios. Incomplete cost and a cost–benefit analysis are introduced in Section 3.4.

3.1 Portfolio Overall Value with Project Interactions

3.1.1 Evaluation criteria and project scoring

Assume that a corporation considers the set $X = \{x^1, \ldots, x^m\}$ of m projects for evaluation. Project versions are treated as individual projects. X can contain ongoing projects as well as new project proposals. From a mathematical point of view, we do not have to distinguish between these two groups of projects. This follows from Requirement 2.18, which states that ongoing projects should be evaluated independently of sunk costs.

A project portfolio $p \subseteq X$ can be any subset of available projects. Hence, the theoretical set of all possible project portfolios is the power set $P = 2^X$. To discriminate between portfolios $p, p' \in P$, it is necessary to define the overall value for a project portfolio p. Therefore, let us suppose that n evaluation criteria are identified to be correlated with the corporation's mission. As stated in Requirement 2.10, DMs consider some of these n evaluation criteria especially important while others are not as crucial for the mission. Consequently, it is necessary to define a set of evaluation criteria weights, $\boldsymbol{w} = (w_1, \ldots, w_n)^T$, where each weight, w_i, reflects the relative importance of evaluation criterion i for $i = 1, \ldots, n$. Without loss of generality, we can scale the weights $\boldsymbol{w} = (w_1, \ldots, w_n)^T$ so that the following condition holds

$$\boldsymbol{w} \in S_w^0 := \left\{ \boldsymbol{w} \in \mathbb{R}^n \mid w_i \geq 0, \sum_{i=1}^{n} w_i = 1 \right\}. \tag{3.1}$$

Every project, x^j, must be evaluated with respect to every evaluation criterion i (e.g., through scoring models). The score of project x^j is stored in the vector $\boldsymbol{v}^j = (v_1^j, \ldots, v_n^j)$. The vectors $\boldsymbol{v}^j$ form the rows of the score matrix $\boldsymbol{v} \in \mathbb{R}^{m \times n}$ such that $[\boldsymbol{v}]_{ji} = v_i^j$. The overall value of project x^j is simply given by the weighted sum of its scores $V(x^j) = \sum_{i=1}^n w_i v_i^j$. For any portfolio $p \in P$, the corresponding overall value is modelled as the sum of the overall values of the projects in the portfolio. For a given score matrix $\boldsymbol{v}$ and criterion weights $\boldsymbol{w}$, the overall value of portfolio p is

$$V(p, \boldsymbol{w}, \boldsymbol{v}) = \sum_{j=1}^{m} \sum_{i=1}^{n} z_j(p) w_i v_i^j = \boldsymbol{z}(p)^T \boldsymbol{v} \boldsymbol{w}, \tag{3.2}$$

where $\boldsymbol{z}(\cdot)$ is a bijection $\boldsymbol{z} : P \to \{0,1\}^m$ such that $z_j(p) = 1$ if $x^j \in p$ and $z_j(p) = 0$ if $x^j \notin p \; \forall j = 1, \ldots, m$. Theoretical presuppositions about the additivity assumption in (3.2) are found in Golabi (1987) and Golabi et al. (1981).

As already mentioned several times before, the elicitation of exact project scores and evaluation criteria weights is hard and often impossible. Thus, it is essential to provide the opportunity to accept incomplete information about project scores and evaluation criteria weights. In this model, any project x^j can be evaluated by *score intervals* instead of *point estimates* of every evaluation criterion i. Lower and upper bounds of all score intervals are denoted by $\underline{v}_i^j$ and $\overline{v}_i^j$ and the corresponding score vectors are $\underline{\boldsymbol{v}}^j$ and $\overline{\boldsymbol{v}}^j$, respectively. It is assumed that all score intervals contain the "true" score, $\underline{v}_i^j \leq v_i^j \leq \overline{v}_i^j$ for all $j = 1, \ldots, m$ and $i = 1, \ldots, n$. The set of feasible scores is denoted by $S_v := \{\boldsymbol{v} \in \mathbb{R}^{m \times n} | v_i^j \in [\underline{v}_i^j, \overline{v}_i^j] \; \forall j = 1, \ldots, m \; \forall i = 1, \ldots, n\}$.

Incomplete information about evaluation criteria weights is captured through a set of linear inequalities corresponding to the DM's preference statements. For instance, the statement that evaluation criterion k is most important can be described through

$$w_i < w_k \text{ for } i = 1, \ldots, n \text{ and } i \neq k.$$

This results in a set of feasible weights, denoted by $S_w \subseteq S_w^0$ with S_w^0 given by (3.1), which is assumed to be a convex polyhedron. $S_w = S_w^0$ is the largest possible weight set, which corresponds to lack of any weight information, while point estimates in S_w correspond to complete information. The literature on preference programming provides several methods for the elicitation of both complete and incomplete weight information (see Salo and Punkka 2005, Wang and Chin 2008).

Corresponding to the imprecise information provided by a DM in S_v and S_w, the overall value (3.2) of project portfolio p is not unique. For a given portfolio p, the selection of different feasible scores and weights in S_v and S_w defines an interval of the overall portfolio value such that for any $\boldsymbol{w} \in S_w$ and $\boldsymbol{v} \in S_v$,

$$V(p, \boldsymbol{w}, \boldsymbol{v}) \in \left[\min_{\boldsymbol{w} \in S_w} \underline{V}(p, \boldsymbol{w}), \max_{\boldsymbol{w} \in S_w} \overline{V}(p, \boldsymbol{w}) \right], \tag{3.3}$$

where $V(p, \boldsymbol{w}, \boldsymbol{v})$ is given by (3.2) and

$$\overline{V}(p, \boldsymbol{w}) = \sum_{j=1}^{m} \sum_{i=1}^{n} z_j(p) w_i \overline{v}_i^j, \tag{3.4}$$

$$\underline{V}(p, \boldsymbol{w}) = \sum_{j=1}^{m} \sum_{i=1}^{n} z_j(p) w_i \underline{v}_i^j. \tag{3.5}$$

Upper and lower bounds of the portfolio overall value are linear in $\boldsymbol{w}$. Moreover, for a given weight vector $\boldsymbol{w}$, the overall value of portfolio p ranges over the entire interval $[\underline{V}(p, \boldsymbol{w}), \overline{V}(p, \boldsymbol{w})]$ when feasible scores $\boldsymbol{v}$ are allowed to vary in S_v.

3.1.2 Project interactions

The definition of the portfolio overall value by (3.2) has a substantial drawback. It considers values of included projects $x^j \in p$ separately and does not take into account project interactions. However, Requirements 2.15 and 2.16 state that it is essential to include project interactions in the evaluation of project portfolios. More precisely, it is required to take synergy and cannibalization effects between projects into account. Therefore, we assume that synergy or cannibalization effects between projects may arise if a portfolio p contains at least a number $\tilde{m}_k$ of projects that are elements of some interaction subset $\tilde{X}_k \subseteq X$ whereby $\tilde{m}_k \leq |\tilde{X}_k|$ and K such interaction subsets are defined—i.e., $k = 1, \ldots, K$. Additionally, we assume that synergy or cannibalization effects between projects may arise if a portfolio p contains at most a number $\hat{m}_l$ of projects that are elements of some interaction subset $\hat{X}_l \subseteq X$ whereby $\hat{m}_l \leq |\hat{X}_l|$ and

L such interaction subsets are defined—i.e., $l = 1, \dots, L$.

Even though synergy and cannibalization effects are exceptions to value additivity, the portfolio overall value remains linear if we follow the approach from Stummer and Heidenberger (2003). Therefore, we define dummy projects, $\tilde{x}^k$ and $\hat{x}^l$, for every interaction subset $\tilde{X}_k$ and $\hat{X}_l$. Dummy projects are added to X so that $|X| = m+K+L$ and $|P| = 2^{m+K+L}$. The score vectors for dummy projects $\tilde{\boldsymbol{v}}^k$ and $\hat{\boldsymbol{v}}^l$ represent the synergy and cannibalization effects that occur if conditions for interaction subset $\tilde{X}_k$ and $\hat{X}_l$, respectively, are met. These vectors form the rows for the score matrices $\tilde{\boldsymbol{v}} \in \mathbb{R}^{K \times n}$ and $\hat{\boldsymbol{v}} \in \mathbb{R}^{L \times n}$. As with the usual projects x^j, dummy projects can be evaluated using score intervals—i.e., $[\underline{\tilde{v}}_i^k, \overline{\tilde{v}}_i^k]$ and $[\underline{\hat{v}}_i^l, \overline{\hat{v}}_i^l]$. The sets of feasible scores are denoted by $S_{\tilde{v}} := \left\{ \tilde{\boldsymbol{v}} \in \mathbb{R}^{K \times n} \mid \tilde{v}_i^k \in [\underline{\tilde{v}}_i^k, \overline{\tilde{v}}_i^k] \ \forall k = 1, \dots, K \ \forall i = 1, \dots, n \right\}$ and $S_{\hat{v}} := \left\{ \hat{\boldsymbol{v}} \in R^{L \times n} \mid \hat{v}_i^l \in [\underline{\hat{v}}_i^l, \overline{\hat{v}}_i^l] \ \forall l = 1, \dots, L \ \forall i = 1, \dots, n \right\}$.

The composite set of feasible weight and score parameters for projects (interactions) is denoted by the Cartesian product $\boldsymbol{S} := S_w \times S_v \times S_{\tilde{v}} \times S_{\hat{v}}$ and $s = (\boldsymbol{w}, \boldsymbol{v}, \tilde{\boldsymbol{v}}, \hat{\boldsymbol{v}}) \in \boldsymbol{S}$ is equivalent to $\boldsymbol{w} \in S_w, \boldsymbol{v} \in S_v, \tilde{\boldsymbol{v}} \in S_{\tilde{v}}$, and $\hat{\boldsymbol{v}} \in S_{\hat{v}}$. Such a nonempty set $\boldsymbol{S}$ of feasible weights and scores will be referred to as the *information set*. To keep the notation simple, we will from now on use the notation s instead $(\boldsymbol{w}, \boldsymbol{v}, \tilde{\boldsymbol{v}}, \hat{\boldsymbol{v}})$. Further, let us set $\underline{s} := (\boldsymbol{w}, \underline{\boldsymbol{v}}, \underline{\tilde{\boldsymbol{v}}}, \underline{\hat{\boldsymbol{v}}})$ and $\overline{s} := (\boldsymbol{w}, \overline{\boldsymbol{v}}, \overline{\tilde{\boldsymbol{v}}}, \overline{\hat{\boldsymbol{v}}})$ with $\boldsymbol{w} \in S_w$ free.

The prediction of project-interaction scores is especially hard and the interval concept is appropriately important. The interval concept allows a DM to set the corresponding interval boundary to 0 if interaction effects *may* arise but are not certain.

The project portfolio overall value with project interactions appears by adding the dummy projects to (3.2)

$$\begin{aligned} V(p, s) &= V(p, \boldsymbol{w}, \boldsymbol{v}, \tilde{\boldsymbol{v}}, \hat{\boldsymbol{v}}) \\ &= \sum_{j=1}^{m} \sum_{i=1}^{n} z_j(p) w_i v_i^j + \sum_{k=1}^{K} \sum_{i=1}^{n} \tilde{z}_k(p) w_i \tilde{v}_i^k + \sum_{l=1}^{L} \sum_{i=1}^{n} \hat{z}_l(p) w_i \hat{v}_i^l \\ &= \boldsymbol{z}(p)^T \boldsymbol{v} \boldsymbol{w} + \tilde{\boldsymbol{z}}(p)^T \tilde{\boldsymbol{v}} \boldsymbol{w} + \hat{\boldsymbol{z}}(p)^T \hat{\boldsymbol{v}} \boldsymbol{w} \end{aligned} \tag{3.6}$$

with $\tilde{\boldsymbol{z}}(p)$ and $\hat{\boldsymbol{z}}(p)$ chosen appropriately as described below. Instead of (3.5) and

(3.4), we have

$$V(p,\underline{s}) := \underline{V}(p,\boldsymbol{w}) = \sum_{j=1}^{m}\sum_{i=1}^{n} z_j(p)w_i\underline{v}_i^j + \sum_{k=1}^{K}\sum_{i=1}^{n} \tilde{z}_k(p)w_i\underline{\tilde{v}}_i^k + \sum_{l=1}^{L}\sum_{i=1}^{n} \hat{z}_l(p)w_i\underline{\hat{v}}_i^l \quad (3.7)$$

and

$$V(p,\overline{s}) := \overline{V}(p,\boldsymbol{w}) = \sum_{j=1}^{m}\sum_{i=1}^{n} z_j(p)w_i\overline{v}_i^j + \sum_{k=1}^{K}\sum_{i=1}^{n} \tilde{z}_k(p)w_i\overline{\tilde{v}}_i^k + \sum_{l=1}^{L}\sum_{i=1}^{n} \hat{z}_l(p)w_i\overline{\hat{v}}_i^l. \quad (3.8)$$

From a DM's perspective, decisions are made only on real projects x^j. The decision on dummy projects $\tilde{x}^k$ and $\hat{x}^l$ is made implicitly through the decision on the real projects x^j. Thus, the definition of bijection $\boldsymbol{z}(p)$ is unchanged from that given after equation (3.2). $\tilde{\boldsymbol{z}}(p)$ is a K-dimensional vector with the functions $\tilde{z}_k(p)$ as elements. Analogously, $\hat{\boldsymbol{z}}(p)$ is an L-dimensional vector with the functions $\hat{z}_l(p)$ as elements. These functions control the selection of dummy projects $\tilde{x}^k$ and $\hat{x}^l$ into the overall portfolio value. Therefore, $\tilde{z}_k(p)$ and $\hat{z}_l(p)$, respectively, must take the value 1 iff portfolio p meets the condition about interaction subset $\tilde{X}_k$ and $\hat{X}_l$, respectively. Otherwise, they must be equal to 0. To achieve these properties for $\tilde{z}_k(p)$ and $\hat{z}_l(p)$, we define vectors $\tilde{\boldsymbol{a}}_k \in \{0,1\}^{1\times m}$ for $k = 1,\ldots,K$ and $\hat{\boldsymbol{a}}_l \in \{0,1\}^{1\times m}$ for $l = 1,\ldots,L$, as well as two sets of constraints. The vectors $\tilde{\boldsymbol{a}}_k$ are characterized through $\tilde{a}_k^j = 1$ if $x^j \in \tilde{X}_k$ and $\tilde{a}_k^j = 0$ if $x^j \notin \tilde{X}_k$, so that $\tilde{\boldsymbol{a}}_k\boldsymbol{z}(p)$ represents the number of projects in portfolio p that are affected by the interaction set $\tilde{X}_k$. Similarly, the vectors $\hat{\boldsymbol{a}}_l$ are characterized through $\hat{a}_l^j = 1$ if $x^j \in \hat{X}_l$ and $\hat{a}_l^j = 0$ if $x^j \notin \hat{X}_l$, so that $\hat{\boldsymbol{a}}_l\boldsymbol{z}(p)$ represents the number of projects in portfolio p that are affected by the interaction set $\hat{X}_l$. Given $\tilde{m}_k$ as the necessary minimum number of projects for interaction subset $\tilde{X}_k$, the constraints

$$\begin{aligned} \tilde{\boldsymbol{a}}_k\boldsymbol{z}(p) - m\tilde{z}_k(p) &\leq \tilde{m}_k - 1 \\ -\tilde{\boldsymbol{a}}_k\boldsymbol{z}(p) + m\tilde{z}_k(p) &\leq m - \tilde{m}_k \\ \tilde{z}_k(p) \in \{0,1\} \text{ for } k &= 1,\ldots,K \end{aligned} \quad (3.9)$$

ensure that $\tilde{z}_k(p) = 1$ and also dummy project $\tilde{x}^k$ is included into p iff the condition for project interaction subset $\tilde{X}_k$ is met by p.

Given $\hat{m}_l$ as the maximum number of projects for interaction $\hat{X}_l$, the constraints

$$\begin{aligned} -\hat{\boldsymbol{a}}_l \boldsymbol{z}(p) - m\hat{z}_l(p) &\leq -\,\hat{m}_l - 1 \\ \hat{\boldsymbol{a}}_l \boldsymbol{z}(p) + m\hat{z}_l(p) &\leq \hat{m}_l + m \\ \hat{z}_l(p) &\in \{0,1\},\ \text{for } l = 1,\dots,L \end{aligned} \tag{3.10}$$

ensure that $\hat{z}_l(p) = 1$ and also dummy project $\hat{x}^l$ is included into p iff the condition for project interaction subset $\hat{X}_l$ is met by p.

For given project interaction sets $\tilde{X}_k$ and $\hat{X}_l$, we can define the vectors $\tilde{\boldsymbol{a}}_k$ and $\hat{\boldsymbol{a}}_l$ for all $k = 1,\dots,K$ and for all $l = 1,\dots,L$. These vectors form the rows of the matrix $\boldsymbol{A} \in \{0,1\}^{2(K+L)\times m} = [\tilde{\boldsymbol{a}}_1, -\tilde{\boldsymbol{a}}_1, \dots, -\hat{\boldsymbol{a}}_L, \hat{\boldsymbol{a}}_L]^T$. Further, we define matrix $\tilde{\boldsymbol{A}} \in \mathbb{Z}^{2(K+L)\times K}$ through $\tilde{a}_k^j = -m$ iff $(k+1)/2 = j$, $\tilde{a}_k^j = m$ iff $(k)/2 = j$ for $k = 1,\dots,2K$. Otherwise, we have $\tilde{a}_k^j = 0$. Similarly, we define the matrix $\hat{\boldsymbol{A}} \in \mathbb{Z}^{2(K+L)\times L}$ through $\hat{a}_l^j = -m$ iff $(l+1)/2 = j$, $\hat{a}_l^j = m$ iff $(l)/2 = j$ for $l = 2K+1,\dots,2(K+L)$. Otherwise, we have $\hat{a}_l^j = 0$. Finally, let us record the right sides of (3.9) and (3.10) in vector $\boldsymbol{\alpha} \in \mathbb{Z}^{2(K+L)\times 1}$. Any portfolio $p \in P$ satisfying

$$\begin{gathered} \boldsymbol{A}\boldsymbol{z}(p) + \tilde{\boldsymbol{A}}\tilde{\boldsymbol{z}}(p) + \hat{\boldsymbol{A}}\hat{\boldsymbol{z}}(p) \leq \boldsymbol{\alpha} \\ \boldsymbol{z}(p) \in \{0,1\}^m,\ \tilde{\boldsymbol{z}}(p) \in \{0,1\}^K,\ \hat{\boldsymbol{z}}(p) \in \{0,1\}^L \end{gathered} \tag{3.11}$$

is feasible in terms of project interactions. This means that, for portfolio p, the overall value (3.6) reflects the true value since it is modified by dummy projects that represent the value of fulfilled project interaction subsets.

From now on, we always refer to (3.6) if the discussion is about portfolio overall value. Like portfolio overall value (3.2), the portfolio overall value (3.6) varies in an interval. This interval is given by (3.3) with (3.7) and (3.8) instead of (3.4) and (3.5). Section 3.3 shows how this interval can be used to discriminate between portfolios $p \in P$. However, before doing so, we discuss the *feasibility* of portfolios $p \in P$ since feasibility in terms of project interactions is not sufficient for a portfolio to be feasible.

3.2 Feasibility of Portfolios

Theoretically, all possible portfolios are given by P. However, as a result of the requirements formulated in Chapter 2, not every portfolio p is feasible. In RPM, the set of feasible portfolios $P_F \subseteq P$ is defined by linear constraints, which may be summarized into four categories:

- **Resource constraints** reflect fixed project (interaction) requirements $c^j, \tilde{c}^k$, and $\hat{c}^l$ and fixed availability R. Note, that c^j, can be negative if x^j corresponds to a resource saving project. Similar, $\tilde{c}^k$ and $\hat{c}^l$, respectively, can be negative if $\tilde{x}^k$ and $\hat{x}^l$, respectively, represent synergy effects.

- **Logical constraints** model mutually exclusive projects and technical project dependencies.

- **Positioning constraints** reflect such issues as starting a minimum number of projects in different project subsets such as strategic buckets.

- **Threshold constraints** help ensure that the performance of the portfolio and its constituent projects meet minimum requirements. For example, the aggregate NPV may have to exceed a minimum acceptable level.

The mathematical formulation of these constraints is straightforward. With respect to the requirements of Chapter 2, we outline some examples for every category.

For the budget and resource categories with the property that project (interaction) requirements $c^j, \tilde{c}^k$, and $\hat{c}^l$ are exact to predict and there is a rigid availability level R, a **resource constraint**,

$$\sum_{j=1}^{m} z_j(p)c^j + \sum_{k=1}^{K} \tilde{z}_k(p)\tilde{c}^k + \sum_{l=1}^{L} \hat{z}_l(p)\hat{c}^l \leq R, \tag{3.12}$$

is created. Such a constraint can be formulated for an arbitrary number of resource categories as well as for budgets. Nevertheless, these constraints, whether they account the time dimension of Requirement 2.7, do not allow for imprecise resource-requirement information. Furthermore, the availability amount is assumed to be rigid, which is inconsistent with Requirement 2.7 and Section 2.3.2. Section 3.4 upgrades the model so that it partly satisfies the requirements above through the introduction of a cost–benefit functionality that is useable for any resource category and for budget.

Next, we introduce two examples of **logical constraints**. The first example is about mutually exclusive projects. Assume a set of projects $\tilde{X} \subseteq X$ which contains mutually exclusive projects. The following constraint ensures that at most one project out of subset $\tilde{X}$ is selected:

$$\sum_{j=1}^{m} e_j z_j(p) = \boldsymbol{e}\boldsymbol{z}(p) \leq 1 \tag{3.13}$$

with $\boldsymbol{e} \in \{0,1\}^{1\times m}$ so that $e_j = 1$ if $x^j \in \tilde{X}$ and $e_j = 0$ if $x^j \notin \tilde{X}$. This type of constraint conforms to Requirement 2.15. It can be used, for example for project versions. At the beginning of Section 3.1, we mentioned that project versions are treated as individual projects. Such a constraint ensures that a feasible portfolio p contains at most one version of a project.

The second kind of logical constraint reflects technical project dependencies. If a successor project x^s has $\tilde{m}$ predecessor projects, $x^{p_1}, \ldots, x^{p_{\tilde{m}}}$, the corresponding constraint is given by

$$\sum_{j=1}^{m} e_j z_j(p) = \boldsymbol{e}\boldsymbol{z}(p) \geq \tilde{m} z_s(p), \tag{3.14}$$

where $\boldsymbol{e} \in \{0,1\}^{1\times m}$ so that $e_j = 1$ if $x^j \in \{x^{p_1}, \ldots, x^{p_{\tilde{m}}}\}$ and $e_j = 0$ otherwise. Such constraints partially fulfill with Requirement 2.14 because the time dimension is not considered.

To illustrate **positioning constraints**, assume again a set of projects $\tilde{X} \subseteq X$. $\tilde{X}$ can represent any subset of projects, such as for technological or geographical areas. The aim is to ensure that the selected portfolio p contains exactly, at least, or at most $\tilde{m}$ projects out of $\tilde{X}$ whereby $\tilde{m} \leq |\tilde{X}|$. This can be achieved by attaching

$$\sum_{j=1}^{m} e_j z_j(p) = \boldsymbol{e}\boldsymbol{z}(p) \begin{cases} \leq \tilde{m} & \text{if at most} \\ = \tilde{m} & \text{if exact} \\ \geq \tilde{m} & \text{if at least} \end{cases} \Bigg\} \begin{array}{l} \tilde{m} \text{ projects out of } \tilde{X} \text{ are} \\ \text{allowed/required} \end{array} \tag{3.15}$$

to the model as a constraint. We have again $\boldsymbol{e} \in \{0,1\}^{1\times m}$ so that $e_j = 1$ if $x^j \in \tilde{X}$ and $e_j = 0$ if $x^j \notin \tilde{X}$. The main application of this type of constraint is given by Requirement 2.11—i.e., for portfolio balancing. Here, it is also conceivable that $\tilde{X}$ represents a strategic bucket. However, this is not an adequate treatment for strategic buckets. For a better treatment of strategic buckets see Chapter 4.

Another positioning constraint is needed for mandatory projects, Requirement 2.1. This constraint is simply given by $z_j(p) = 1$ for every mandatory project x^j and is just a special case of (3.15).

A **threshold constraint** relates to an evaluation criterion i (e.g., NPV) for which a minimum performance is required; see Requirement 2.10. To guarantee that the selected portfolio p meets the defined minimum performance V_i, it is necessary to add

$$\sum_{j=1}^{m} z_j(p) w_i \underline{v}_i^j + \sum_{k=1}^{K} \tilde{z}_k(p) w_i \underline{\tilde{v}}_i^k + \sum_{l=1}^{L} \hat{z}_l(p) w_i \underline{\hat{v}}_i^l \geq V_i$$

to the set of constraints.

Depending on the situation at hand, many other constraints can be added. One could specify required relationships for different project groups. For example, the number of projects in a certain subset should be at least twice the number of projects in another subset. Outlining all the possibilities is beyond the scope of this thesis. Other common constraints can be found in Archer and Ghasemzadeh (1999b) and Stummer and Heidenberger (2003).

For a given set of constraints, the coefficients can be added to the matrices $\boldsymbol{A}, \tilde{\boldsymbol{A}}$ and $\hat{\boldsymbol{A}}$. Thus, $\boldsymbol{A} \in \mathbb{R}^{(2K+2L+T)\times m}, \tilde{\boldsymbol{A}} \in \mathbb{R}^{(2K+2L+T)\times K}$ and $\hat{\boldsymbol{A}} \in \mathbb{R}^{(2K+2L+T)\times L}$ if T represents the amount of nondummy project selection constraints. The matrix $\boldsymbol{A}$ contains the coefficients corresponding to real projects x^j, while the matrices $\tilde{\boldsymbol{A}}$ and $\hat{\boldsymbol{A}}$ contain coefficients corresponding to dummy projects $\tilde{x}^k$ and $\hat{x}^l$, respectively. The set of feasible portfolios is then denoted through

$$\begin{aligned} P_F := \ & \{p \in P \mid \boldsymbol{A}\boldsymbol{z}(p) + \tilde{\boldsymbol{A}}\tilde{\boldsymbol{z}}(p) + \hat{\boldsymbol{A}}\hat{\boldsymbol{z}}(p) \leq \boldsymbol{\alpha}, \\ & \boldsymbol{z}(p) \in \{0,1\}^m,\ \tilde{\boldsymbol{z}}(p) \in \{0,1\}^K,\ \hat{\boldsymbol{z}}(p) \in \{0,1\}^L\} \end{aligned} \tag{3.16}$$

where $\leq$ holds componentwise and the vector $\boldsymbol{\alpha} \in \mathbb{R}^{(2K+2L+T)\times 1}$ contains the feasibility limits. For the special case that the information set $\boldsymbol{S}$ contains complete information about project (interaction) scores and evaluation criteria weights, RPM becomes a general multiobjective zero–one linear programming (MOZOLP) problem:

$$\begin{aligned} \max_{p \in P_F} V(p,s) = \max_{\boldsymbol{z}(p)} \ & \{\boldsymbol{z}(p)^T \boldsymbol{v}\boldsymbol{w} + \tilde{\boldsymbol{z}}(p)^T \tilde{\boldsymbol{v}}\boldsymbol{w} + \hat{\boldsymbol{z}}(p)^T \hat{\boldsymbol{v}}\boldsymbol{w} \mid \\ & \boldsymbol{A}\boldsymbol{z}(p) + \tilde{\boldsymbol{A}}\tilde{\boldsymbol{z}}(p) + \hat{\boldsymbol{A}}\hat{\boldsymbol{z}}(p) \leq \boldsymbol{\alpha}, \\ & \boldsymbol{z}(p) \in \{0,1\}^m,\ \tilde{\boldsymbol{z}}(p) \in \{0,1\}^K,\ \hat{\boldsymbol{z}}(p) \in \{0,1\}^L\}\,. \end{aligned} \tag{3.17}$$

Nevertheless, complete information is an exception. Therefore, it is much more interesting to discuss how RPM can be used for decision support in situations where $\boldsymbol{S}$

contains incomplete information.

3.3 Portfolio Preference Evaluation Based on the Overall Value

3.3.1 Non-dominated portfolios

While a MOZOLP identifies the portfolio with the highest overall value, the situation with an imprecise information set $\boldsymbol{S}$ is different. In this case, the overall value (3.6) is a function in $s \in \boldsymbol{S}$ and can take any value in the interval mentioned at the end of Section 3.1. Thus, the value intervals of two portfolios $p, p' \in P_F$ can overlap; at the first glance, it may not be clear whether p or p' has a higher overall value. However, it may be possible to identify one of them as inferior through the dominance concept, even if their intervals do overlap.

Definition 3.1. *Let $p, p' \in P$. Portfolio p dominates p' with regard to the information set $\boldsymbol{S}$, denoted by $p \succ_{\boldsymbol{S}} p'$, if $V(p,s) \geq V(p',s)$ for all $s \in \boldsymbol{S}$ and $V(p,s) > V(p',s)$ for some $s \in \boldsymbol{S}$.*

We denote $p \succ p'$ when there is no risk of confusion about the information set $\boldsymbol{S}$. Following Requirement 2.10, which specifies that the overall value of the project portfolio should be maximized, we can discard dominated portfolios. Thus, the analysis can focus on the set of non-dominated portfolios.

Definition 3.2. *The set of non-dominated portfolios with regard to the information set $\boldsymbol{S}$, denoted by $P_N(\boldsymbol{S})$, is*

$$P_N(\boldsymbol{S}) := \{p \in P_F \mid \nexists p' \in P_F \ s.t. \ p' \succ_{\boldsymbol{S}} p\}.$$

We denote $P_N \equiv P_N(\boldsymbol{S})$ when there is no risk of confusion about the information set $\boldsymbol{S}$. The computation of P_N is a key step in supporting project portfolio selection under incomplete preference information. With regard to Requirement 2.10, the computation eliminates unacceptable (dominated) portfolios from further consideration while retaining the interesting (non-dominated) ones. An algorithm for the computation of P_N is given in Chapter 5.

Dominance between two portfolios can readily be checked using the bounds (3.7) and (3.8) and by noting that (i) projects that are included in both portfolios contribute equally both of them and (ii) projects' scores may vary across the full range of their respective intervals, regardless of what the other scores or weights are. Thereof, we can formulate a theorem.

Theorem 3.1. *For any $p, p' \in P$ and information set $\boldsymbol{S} = S_w \times S_v \times S_{\tilde{v}} \times S_{\hat{v}}$*

$$p \succ_{\boldsymbol{S}} p' \iff \begin{cases} \min\limits_{\boldsymbol{w} \in S_w} [V(p \setminus p', \underline{s}) - V(p' \setminus p, \overline{s})] \geq 0 \\ \max\limits_{\boldsymbol{w} \in S_w} [V(p \setminus p', \overline{s}) - V(p' \setminus p, \underline{s})] > 0, \end{cases}$$

with $V(\cdot, \underline{s})$ and $V(\cdot, \overline{s})$ given by (3.7) and (3.8), respectively.

For the proof, see Liesiö et al. (2007).

It can be shown that the dominance relation of Definition 3.1 is asymmetric ($p \not\succ p$), irreflexive ($p \succ p' \Rightarrow p' \not\succ p$) and transitive ($p \succ p' \wedge p' \succ p'' \Rightarrow p \succ p''$). Consequently, P_N cannot be empty unless the set of feasible portfolios P_F is empty. Furthermore, for each dominated portfolio $p' \in P_F \setminus P_N$, there exists at least one non-dominated portfolio $p \in P_N$ such that $p \succ p'$.

The set of non-dominated portfolios, $P_N(\boldsymbol{S})$, is not just nonempty, in general it consists of multiple portfolios so that the size of $P_N(\boldsymbol{S})$ is probably unmanageable. Nevertheless, the initially calculated set of non-dominated portfolios is just a first step in the decision-making process. It provides informative decision recommendations to a DM that are useful for modification or formulation of additional information, resulting in a smaller set of non-dominated portfolios. Thus, the RPM model is an iterative process that allows a DM interaction under informative decision recommendations until a solution is identified. This process fits well with Requirement 2.17.

3.3.2 Impact of additional information

Before we outline which informative decision recommendation are provided by non-dominated portfolios, we investigate how *additional information* modifies the set of non-dominated portfolios, $P_N(\boldsymbol{S})$. Additional information refers to narrower score intervals or additional constraints on the feasible weights and reduces the information set $\boldsymbol{S}$ to $\boldsymbol{S}^* \subset \boldsymbol{S}$. We already know that point estimates lead to an ordinary MOZOLP, usually resulting in a few optimal portfolios. In contrast, loose preference statements and wide score intervals typically result in a large number of non-dominated portfolios.

For the purpose of examining the impact of additional information, it is assumed that the "true" parameter values are contained in $\boldsymbol{S}^*$ as well as in the (relative) interior of $\boldsymbol{S}$, defined as

$$\text{int}(\boldsymbol{S}) := \{s \in \boldsymbol{S} \mid \forall s' \in \boldsymbol{S}\ \exists \delta > 0 \text{ s.t. } s + \epsilon(s - s') \in \boldsymbol{S}\ \forall \epsilon \in [0, \delta]\}.$$

If this condition holds, the impact of additional information on $P_N(\boldsymbol{S})$ is given by

Theorem 3.2. *Let $\boldsymbol{S}^*, \boldsymbol{S}$ be information sets such that $\boldsymbol{S}^* \subset \boldsymbol{S}$ and $\text{int}(\boldsymbol{S}) \cap \boldsymbol{S}^* \neq \emptyset$. Then, $P_N(\boldsymbol{S}^*) \subseteq P_N(\boldsymbol{S})$.*

For the proof, see Liesiö et al. (2007).

A major computational impact of Theorem 3.2 is that the set of non-dominated portfolios needs to be computed for the initial information set $\boldsymbol{S}$ only. Later on, additional information may eliminate some portfolios from the previous set of non-dominated portfolios, but cannot add any new portfolios to it. $P_N(\boldsymbol{S}^*)$ can be obtained from $P_N(\boldsymbol{S})$ by pairwise dominance checks (Theorem 3.1):

$$P_N(\boldsymbol{S}^*) = \{p \in P_N(\boldsymbol{S}) \mid p' \not\succ_{\boldsymbol{S}^*} p\ \forall p' \in P_N(\boldsymbol{S})\}. \tag{3.18}$$

3.3.3 Decision information obtained by non-dominated portfolios

Now that we know the value of additional information for the selection process, we investigate which information $P_N(\boldsymbol{S})$ provides for the formulation of additional information. The aim is to use $P_N(\boldsymbol{S})$ to receive information about how to modify the information set $\boldsymbol{S}$ into $\boldsymbol{S}^* \subset \boldsymbol{S}$ so that the resulting set of non-dominated portfolios, $P_N(\boldsymbol{S}^*)$, is considerably smaller. Another aim is to link $P_N(\boldsymbol{S})$ and the selection of individual projects. In doing so, the robustness of the selection of a project with respect to incomplete information is of interest. We start this investigation with the definition of the *core index of a project x^j*.

Definition 3.3. *The core index of project $x^j \in X$ with regard to the information set $\boldsymbol{S}$, denoted by $CI(x^j, \boldsymbol{S})$, is*

$$CI(x^j, \boldsymbol{S}) = \frac{|\{p \in P_N(\boldsymbol{S}) \mid x^j \in p\}|}{|P_N(\boldsymbol{S})|}.$$

The core index leads to a transparent project selection process, because for each project it transforms information about non-dominated portfolios into a single performance measure. Furthermore, the core index concept is transferable to project interactions $CI(\tilde{x}^k, \boldsymbol{S})$ and $CI(\hat{x}^l, \boldsymbol{S})$. The core index of a dummy project indicates how significant the interaction effect is at the portfolio level. Also, if the dummy project represents a program, the core index is a performance measure for the program and fits well with Requirement 2.16.

If the core index of a project (interaction) is 1, the project (interaction) is included (active) in *all* non-dominated portfolios and is consequently called a *core project* (*interaction*). In contrast, if the core index of a project (interaction) is 0, the project (interaction) is not included (active) in *any* non-dominated portfolio and is referred to as an *exterior project* (*interaction*). Finally, the remaining projects (interactions) whose core index is strictly greater than zero but less than one are called *borderline projects* (*interactions*).

Definition 3.4. *With regard to the information set $\boldsymbol{S}$, we define the sets of core projects $X_C(\boldsymbol{S})$, borderline projects $X_B(\boldsymbol{S})$, and exterior projects $X_E(\boldsymbol{S})$:*

$$\begin{aligned} X_C(\boldsymbol{S}) &:= \{x^j, \tilde{x}^k, \hat{x}^l \in X \mid CI(x^j, \boldsymbol{S}), CI(\tilde{x}^k, \boldsymbol{S}), CI(\hat{x}^l, \boldsymbol{S}) = 1\}, \\ X_B(\boldsymbol{S}) &:= \{x^j, \tilde{x}^k, \hat{x}^l \in X \mid 0 < CI(x^j, \boldsymbol{S}), CI(\tilde{x}^k, \boldsymbol{S}), CI(\hat{x}^l, \boldsymbol{S}) < 1\}, \\ X_E(\boldsymbol{S}) &:= \{x^j, \tilde{x}^k, \hat{x}^l \in X \mid CI(x^j, \boldsymbol{S}), CI(\tilde{x}^k, \boldsymbol{S}), CI(\hat{x}^l, \boldsymbol{S}) = 0\}. \end{aligned}$$

Core projects can be surely recommended and all exterior projects can be safely rejected independent of additional information. This is because core and exterior projects (interactions) remain core and exterior projects (interactions) even in light of additional information. This is easily shown using Theorem 3.2.

Corollary 3.1. *Let $\boldsymbol{S}^* \subseteq \boldsymbol{S}$ such that $\mathrm{int(S)} \cap \mathrm{S}^* \neq \emptyset$. Then, $X_C(\boldsymbol{S}) \subseteq X_C(\boldsymbol{S}^*)$ and $X_E(\boldsymbol{S}) \subseteq X_E(\boldsymbol{S}^*)$ holds.*

As a consequence, the decision to select or to reject a project can be made as soon as the core status or the exterior status is first established for a project x^j.

The core index of projects (interactions) has an additional useful property. It tells a DM on which projects (interactions) to spend effort for additional information to reduce the set of non-dominated portfolios, $P_N(\boldsymbol{S})$.

Corollary 3.2. *Let* $\boldsymbol{S}^* \subseteq \boldsymbol{S}$ *such that* $\mathrm{int(S)} \cap \mathrm{S}^* \neq \emptyset$*. If* $S_w^* = S_w$, $\underline{v}_i^{j*} = \underline{v}_i^j$, $\overline{v}_i^{j*} = \overline{v}_i^j$, $\underline{\tilde{v}}_i^{k*} = \underline{\tilde{v}}_i^k$, $\overline{\tilde{v}}_i^{k*} = \overline{\tilde{v}}_i^k$, $\underline{\hat{v}}_i^{l*} = \underline{\hat{v}}_i^l$, *and* $\overline{\hat{v}}_i^{l*} = \overline{\hat{v}}_i^l$, $\forall i = 1, \ldots, n$, $\forall x^j, \tilde{x}^k, \hat{x}^l \in X_B(S)$, *then* $P_N(\boldsymbol{S}) = P_N(\boldsymbol{S}^*)$.

Thus, the elicitation efforts can be focused on obtaining narrower score intervals for borderline projects (interactions) and more restrictive weight information. In this sense, core indices help to identify further information needs, which as such is one of the key purposes of sensitivity and robustness analysis.

Summarizing the decision process, a DM is advised to start with loose preference information. This implies large feasible sets of the parameter values and typically results in a large number of non-dominated portfolios. Core index analysis helps identifying core and exterior projects. This also helps focusing the effort of eliciting additional information on borderline projects (interactions). Due to narrower score intervals and stricter weight statements, the set of non-dominated portfolios becomes smaller and new core and exterior projects may be identified. This process can be continued until the final portfolio is identified.

3.4 Portfolio Costs and Optimal Budget Level

3.4.1 Portfolio costs with project interactions

By now, we are able to model budget and resource categories if the assumptions about exact predictable costs and resource requirements and a rigid availability level hold. Therefore, we can formulate Constraint (3.12). This section introduces a cost–benefit analysis which is applicable to any resource category as well as the budget. In keeping with the notation of Liesiö et al. (2008), we outline the cost–benefit analysis just for the budget.

The cost–benefit analysis supports the identification of the most valuable budget level based on incomplete cost information so that Requirements 2.7 and 2.9 are supported as well as Requirement 2.2. Let us define the costs of portfolio $p \in P$ with

$$\begin{aligned} C(p, s_c) = C(p, \boldsymbol{c}, \tilde{\boldsymbol{c}}, \hat{\boldsymbol{c}}) &= \sum_{j=1}^{m} z_j(p) c^j + \sum_{k=1}^{K} \tilde{z}_k(p) \tilde{c}^k + \sum_{l=1}^{L} \hat{z}_l(p) \hat{c}^l \\ &= \boldsymbol{z}(p)^T \boldsymbol{c} + \tilde{\boldsymbol{z}}(p)^T \tilde{\boldsymbol{c}} + \hat{\boldsymbol{z}}(p)^T \hat{\boldsymbol{c}}. \end{aligned} \tag{3.19}$$

The vector $\boldsymbol{c} = (c^1, \ldots, c^m)^T$ represents costs for projects $x^1, \ldots, x^m$. Similarly, the vectors $\tilde{\boldsymbol{c}} = (\tilde{c}^1, \ldots, \tilde{c}^K)^T$ and $\hat{\boldsymbol{c}} = (\hat{c}^1, \ldots, \hat{c}^L)^T$ represent the costs for dummy projects $\tilde{x}^1, \ldots, \tilde{x}^K$ and $\hat{x}^1, \ldots, \hat{x}^L$, which still reflect project interactions. Thus, portfolio costs (3.19) are adjusted to project interaction effects, similar to the overall value. Like for project scores it is allowed to estimate project costs via an interval $[\underline{c}^j, \overline{c}^j]$. Analogously intervals are used for dummy projects.

The set of feasible costs for projects x^j is denoted by $S_c := \{\boldsymbol{c} \in \mathbb{R}^{m \times 1} \mid c^j \in [\underline{c}^j, \overline{c}^j]\ \forall j = 1, \ldots, m\}$, from which the definition of $S_{\tilde{\boldsymbol{c}}}$ and $S_{\hat{\boldsymbol{c}}}$ should be obvious. The composite set of feasible costs for projects and interactions is denoted by the Cartesian product $\boldsymbol{S}_c := S_c \times S_{\tilde{c}} \times S_{\hat{c}}$ and $s_c = (\boldsymbol{c}, \tilde{\boldsymbol{c}}, \hat{\boldsymbol{c}}) \in \boldsymbol{S}_c$ is equivalent to $\boldsymbol{c} \in S_c, \tilde{\boldsymbol{c}} \in S_{\tilde{c}}$, and $\hat{\boldsymbol{c}} \in S_{\hat{c}}$. Such a nonempty set of feasible costs will be referred to as the *cost information*. The information set $\boldsymbol{S} = S_w \times S_v \times S_{\tilde{v}} \times S_{\hat{v}}$ still corresponds to the feasible regions of weights and scores associated with the n evaluation criteria.

In keeping with the notation of the portfolio overall value, we set $\underline{s}_c = (\underline{\boldsymbol{c}}, \underline{\tilde{\boldsymbol{c}}}, \underline{\hat{\boldsymbol{c}}})$ and $\overline{s}_c = (\overline{\boldsymbol{c}}, \overline{\tilde{\boldsymbol{c}}}, \overline{\hat{\boldsymbol{c}}})$. The feasible cost of a portfolio p is then captured in the interval $[C(p, \underline{s}), C(p, \overline{s})]$, similarly to the overall value.

3.4.2 Portfolio preference evaluation based on overall value and costs

To identify a set of "interesting portfolios," we have to consider the overall value as well as the cost. Thus, non-dominated portfolios in their current shape are not usable since they are based on the overall value only. Instead of non-dominated portfolios, we identify *efficient* portfolios as the "interesting" ones. We call a portfolio efficient if no other feasible portfolio gives a higher overall value at a *lower cost*.

Definition 3.5. *The set of efficient portfolios with regard to information set $\boldsymbol{S}$ and cost information $\boldsymbol{S}_c$ is*

$$P_E(\boldsymbol{S}, \boldsymbol{S}_c) := \left\{ p \in P_F \mid \nexists p' \in P_F \ s.t. \ \left\{ \begin{array}{c} V(p', s) \geq V(p, s), \forall s \in \boldsymbol{S} \\ C(p', s_c) \leq C(p, s_c), \forall s_c \in \boldsymbol{S}_c \end{array} \right\} \right\}$$

with at least one strict inequality for some $s \in \boldsymbol{S}$ or $s_c \in \boldsymbol{S}_c$.

Similar to the set of non-dominated portfolios, the further analysis about the portfolio selection procedure can be focused on the set of efficient portfolios. If one selected a portfolio outside the efficient set, there would exist an efficient portfolio that yields a higher overall value for all feasible weights and scores, and cost less no matter what the project costs are within their intervals.

A relationship between the sets of non-dominated portfolios and efficient portfolios is the aim. Such a relationship should enable the use of decision recommendations developed for non-dominated portfolios. To achieve this relationship, the total cost, $C(p, s_c)$, is modelled as a criterion to be *minimized.* Thus, the $(n+1)$th score of project x^j is the opposite of its cost—i.e., $-c^j$. The $(n+1)$th scores for dummy projects $\tilde{x}^k$ and $\hat{x}^l$ are given by the corresponding opposite costs $-\tilde{c}^k$ and $-\hat{c}^l$. The costs are associated with a weight w_{n+1} that varies so that $w_{n+1} \in [0, 1]$. This arrangement allows the formulation of a theorem that guarantees a relationship between the sets of non-dominated portfolios and the efficient portfolios we seek.

Theorem 3.3. *Consider information set $\boldsymbol{S} = S_w \times S_v \times S_{\tilde{v}} \times S_{\hat{v}}$ and cost information $\boldsymbol{S}_c = S_c \times S_{\tilde{c}} \times S_{\hat{c}}$. Let the extended information set $\check{\boldsymbol{S}} = \check{S}_w \times \check{S}_v \times \check{S}_{\tilde{v}} \times \check{S}_{\hat{v}}$ be defined by:*

$$\check{S}_v = \{[\boldsymbol{v}, -\boldsymbol{c}] \in \mathbb{R}^{m \times (n+1)} \mid \boldsymbol{v} \in S_v, \boldsymbol{c} \in S_c\},$$

$$\check{S}_{\tilde{v}} = \{[\tilde{\boldsymbol{v}}, -\tilde{\boldsymbol{c}}] \in \mathbb{R}^{K \times (n+1)} \mid \tilde{\boldsymbol{v}} \in S_{\tilde{v}}, \tilde{\boldsymbol{c}} \in S_{\tilde{c}}\},$$

$$\check{S}_{\hat{v}} = \{[\hat{\boldsymbol{v}}, -\hat{\boldsymbol{c}}] \in \mathbb{R}^{L \times (n+1)} \mid \hat{\boldsymbol{v}} \in S_{\hat{v}}, \hat{\boldsymbol{c}} \in S_{\hat{c}}\},$$

$$\check{S}_w = \{\boldsymbol{w} \in \check{S}_w^0 \mid \tfrac{1}{1-w_{n+1}}(w_1, \ldots, w_n)^T \in S_w, w_{n+1} < 1\}$$

$$\cup \{\boldsymbol{w} \in \check{S}_w^0 \mid w_1 = \cdots = w_n = 0, w_{n+1} = 1\},$$

where $\check{S}_w^0 = \{\boldsymbol{w} \in \mathbb{R}^{n+1} \mid w_i \geq 0, \sum_{i=1}^{n+1} w_i = 1\}$. *Then,* $P_E(\boldsymbol{S}, \boldsymbol{S}_c) = P_N(\check{\boldsymbol{S}})$.

For the proof, see Liesiö et al. (2008).

This theorem allows the use of decision recommendations for non-dominated portfolios on the set of efficient portfolios. First, additional information, $\boldsymbol{S}^* \subset \check{\boldsymbol{S}}$, can only reduce the set of efficient portfolios, because $P_N(\boldsymbol{S}^*) \subseteq P_N(\check{\boldsymbol{S}})$ by Theorem 3.2 and $P_N(\boldsymbol{S}^*)$ is obtained from $P_N(\check{\boldsymbol{S}})$ by pairwise checks between portfolios as in (3.18). Second, the share of efficient portfolios that contains project x^j can be interpreted as the core index of project x^j for the information set $\check{\boldsymbol{S}}$—i.e., $CI(x^j, \check{\boldsymbol{S}})$. This is also true for dummy projects $\tilde{x}^k$ and $\hat{x}^l$. Moreover, the results of Corollary 3.1 and Corollary 3.2 can be carried over to the set of efficient portfolios. Finally, the algorithm introduced in Chapter 5 for the computation of non-dominated portfolios can be used for the computation of efficient portfolios as well.

3.4.3 Cost–benefit analysis

Efficient portfolios help to analyze how the portfolio overall value changes as a function of the budget level. Since both overall values and costs are intervals, efficient portfolios do not result in a unique cost–benefit *curve*. Rather they form a *band* in the overall value–total-cost plane. For the analysis, we define the set of feasible portfolios, $P_F(s_c, R)$, that are attainable with fixed cost, $s_c \in \boldsymbol{S}_c$, and budget, $R \in \mathbb{R}$, and the corresponding set of non-dominated portfolios, $P_N(\boldsymbol{S}, s_c, R)$, with regard to information set $\boldsymbol{S}$:

$$P_F(s_c, R) := \{p \in P_F \mid C(p, s_c) \leq R\}, \tag{3.20}$$

$$P_N(\boldsymbol{S}, s_c, R) := \{p \in P_F(s_c, R) \mid p' \not\succ_{\boldsymbol{S}} p \ \forall p' \in P_F(s_c, R)\}. \tag{3.21}$$

The set $P_N(\boldsymbol{S}, s_c, R)$ contains all non-dominated portfolios in terms of overall value whose project (interaction) costs are s_c and budget is R. Dominance is determined by incomplete information $s \in \boldsymbol{S}$ and (3.20) is the budget (resource category) constraint, as is (3.12). Thus, $P_N(\boldsymbol{S}, s_c, R)$ is just an alternative notation to $P_N(\boldsymbol{S})$ in Definition 3.2 to highlight the dependence on s_c and R.

For the cost–benefit analysis, it is reasonable to investigate the relationship between the set of efficient portfolios $P_E(\boldsymbol{S}, \boldsymbol{S}_c)$ and the set of non-dominated portfolios with fixed costs s_c and budget R—i.e., $P_N(\boldsymbol{S}, s_c, R)$. The set of efficient portfolios corresponds to the union of all sets of non-dominated portfolios in the sense that: (i) every efficient portfolio is included in the set of non-dominated portfolios for some fixed costs $s_c \in \boldsymbol{S}_c$ and budget R and (ii) non-dominated portfolios for any fixed $s_c \in \boldsymbol{S}_c$ and R are efficient. However, there is an exception to the latter property. If two non-dominated portfolios have equal overall value for all feasible weights and scores, by definition only the less expensive one is efficient. The relationship between $P_E(\boldsymbol{S}, \boldsymbol{S}_c)$ and $P_N(\boldsymbol{S}, s_c, R)$ is formalized by Theorem 3.4.

Theorem 3.4. *Consider information set* $\boldsymbol{S} = S_w \times S_v \times S_{\tilde{v}} \times S_{\hat{v}}$ *and cost information* $\boldsymbol{S}_c = S_c \times S_{\tilde{c}} \times S_{\hat{c}}$. *Then,*

$$(i) \quad p \in P_E(\boldsymbol{S}, \boldsymbol{S}_c) \Rightarrow \exists R \in \mathbb{R}, s_c \in \boldsymbol{S_c} \text{ s.t. } p \in P_N(\boldsymbol{S}, s_c, R)$$

$$(ii) \quad p \in P_N(\boldsymbol{S}, s_c, R) \Rightarrow \exists p' \sim p, p' \in P_N(\boldsymbol{S}, s_c, R) \text{ s.t. } p' \in P_E(\boldsymbol{S}, \boldsymbol{S}_c),$$

where $\sim$ *is the equivalence relation* $p \sim p' \iff V(p, s) = V(p', s) \ \forall s \in \boldsymbol{S}$.

For the proof, see Liesiö et al. (2008).

This relationship implies that $P_N(\boldsymbol{S}, s_c, R)$ can be obtained from $P_E(\boldsymbol{S}, \boldsymbol{S}_c)$ for any given values of s_c and R: Discard portfolios that do not meet the budget constraint $C(p, s_c) \leq R$ and use pairwise dominance checks (3.21) in the resulting $P_F(s_c, R)$ to obtain $P_N(\boldsymbol{S}, s_c, R)$.

The cost–benefit band describes the ranges of overall values that non-dominated portfolios can assume at different budget levels R, subject to incomplete information $s \in \boldsymbol{S}$ and $s_c \in \boldsymbol{S}_c$. For each available level, R, this band is given by the interval

$$\left[\min_{\substack{p \in P_N(\boldsymbol{S}, \underline{s}_c, R) \\ s \in \boldsymbol{S}}} V(p, s), \max_{\substack{p \in P_N(\boldsymbol{S}, \underline{s}_c, R) \\ s \in \boldsymbol{S}}} V(p, s) \right]. \tag{3.22}$$

The bounds are attained by maximizing/minimizing the overall portfolio value while project costs vary within their intervals $s_c \in \boldsymbol{S}_c$. The optima are achieved when all costs are at their lower bounds $s_c = \underline{s}_c$, where the budget is least restrictive.

The lower bound does not necessarily increase with R because a higher budget may suffice to fund a new portfolio which is non-dominated but has a lower worst-case overall value. Since this may confuse a DM, we use the maximum worst-case overall value over $P_N(\boldsymbol{S}, \overline{s}_c, R)$, instead of $\min_{\substack{p \in P_N(\boldsymbol{S}, \underline{s}_c, R) \\ s \in \boldsymbol{S}}} V(p,s)$, as the lower bound of the cost–benefit band.

Definition 3.6. *Lower and upper bounds for the cost–benefit band of the project set X with regard to information set $\boldsymbol{S}$ and cost information $\boldsymbol{S}_c$ are*

$$\text{Maximal overall value: } MV(R) := \max_{p \in P_N(\boldsymbol{S}, \underline{s}_c, R)} \max_{s \in \boldsymbol{S}} V(p,s),$$

$$\text{Guaranteed overall value: } GV(R) := \max_{p \in P_N(\boldsymbol{S}, \overline{s}_c, R)} \min_{s \in \boldsymbol{S}} V(p,s).$$

Note, since the $GV(R)$ curve is based on $P_N(\boldsymbol{S}, \overline{s}_c, R)$, every portfolio associated with the $GV(R)$ curve is feasible no matter what the costs are ($C(p, s_c) \leq R \, \forall s_c \in \boldsymbol{S}_c$). Both $MV(R)$ and $GV(R)$ are non-decreasing in R and can be taken as the basis for the cost–benefit ratio $MV(R)/R$ and $GV(R)/R$. The selected availability level, R, may maximize one of these measures.

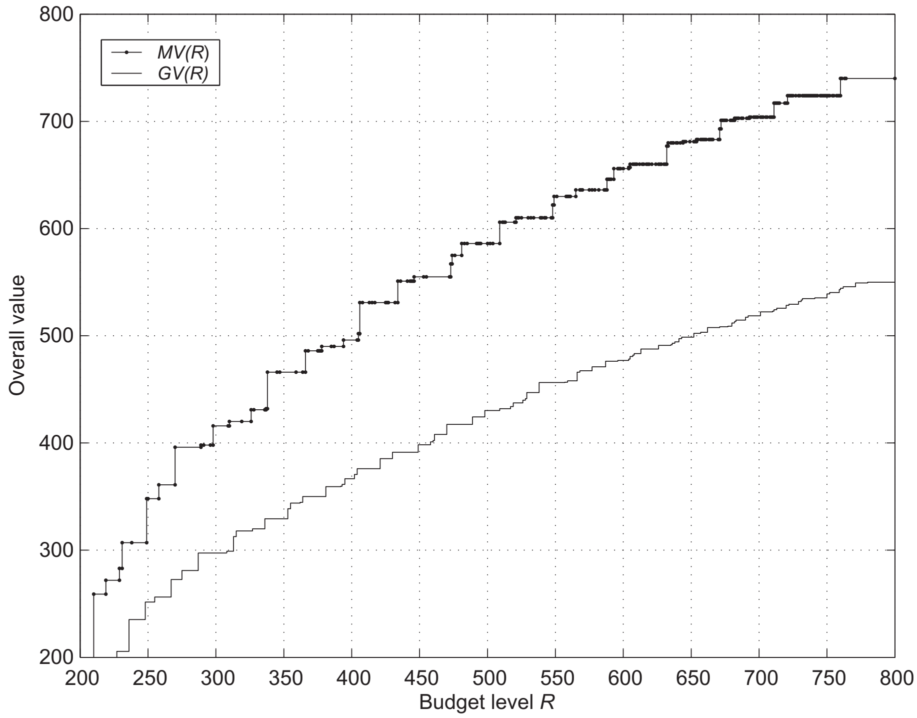


Figure 3.1: Guaranteed and Maximal Overall Values as Functions of Budget Level R
Source Liesiö et al. (2008).

Figure 3.1 illustrates an example of a cost–benefit band for a project set X containing 40 projects and 3 project interaction sets. It shows how the $GV(R)$ and $MV(R)$ curves develop as functions of the budget level R. The GV curve shows that $R = 227$ is the lowest level which *guarantees* that a feasible portfolio can be constructed. From MV, this first portfolio's cost may be as low as $R = 210$. The range $[GV(R), MV(R)]$ expands with increasing R, since the more expensive portfolios include more features (interactions) that add to the overall value and cost intervals. Further, Figure 3.1 shows that both $MV(R)$ and $GV(R)$ are piecewise constant because the composition of $P_N(\boldsymbol{S}, s_c, R)$ for some fixed s_c (including $\underline{s}_c$ and $\overline{s}_c$) can only change at levels $R \in \{C(p, s_c) \mid p \in P_E(\boldsymbol{S}, \boldsymbol{S}_c)\}$. Thus, the number of jumps in $GV(R)$ and $MV(R)$ is limited from above by the number of efficient portfolios.

3.4.4 Budget-dependent core index

The sets $P_N(\boldsymbol{S}, \overline{s}_c, R)$ also help analyze the robustness of individual projects (interactions) as a function of R. The budget-dependent core index of project (interaction) x^j ($\tilde{x}^k$ or $\hat{x}^l$) measures the share of non-dominated portfolios which contain x^j ($\tilde{x}^k$ or $\hat{x}^l$) and are certainly attainable—i.e., $s_c = \overline{s}_c$—at budget level R.

Definition 3.7. *The budget-dependent core index of project x^j at budget level R is*

$$CI(x^j, \boldsymbol{S}, R) = \frac{|\{p \in P_N(\boldsymbol{S}, \overline{s}_c, R) \mid x^j \in p\}|}{|P_N(\boldsymbol{S}, \overline{s}_c, R)|}.$$

The budget-dependent interaction core indices—$CI(\tilde{x}^k, \boldsymbol{S}, R)$ and $CI(\hat{x}^l, \boldsymbol{S}, R)$—are defined analogously. Interestingly, the core index is not necessarily increasing in R. For instance, a small increase δ in R may be enough for a new portfolio p to enter $P_N(\boldsymbol{S}, \overline{s}_c, R+\delta)$. In this new portfolio, project (interaction) x^j ($\tilde{x}^k$ or $\hat{x}^l$) may not be included. However, the new portfolio p must contain projects and interactions that are alternatives to x^j ($\tilde{x}^k$ or $\hat{x}^l$). These alternatives fit into the budget level $R + \delta$, even if $s_c = \overline{s}_c$, and yields a higher overall value than x^j ($\tilde{x}^k$ or $\hat{x}^l$) for some $s \in \boldsymbol{S}$. Thus, project (interaction) x^j ($\tilde{x}^k$ or $\hat{x}^l$) can lose its core status when these alternative projects or interactions become attainable at the higher budget $R + \delta$.

Figure 3.2 offers a graphical description of the budget-dependent core index for 40 projects and 3 project interactions. The budget-dependent core index of project x^{17}, for instance, seems to be very sensitive to changes in R, demonstrating how a core index can change with the budget level, R.

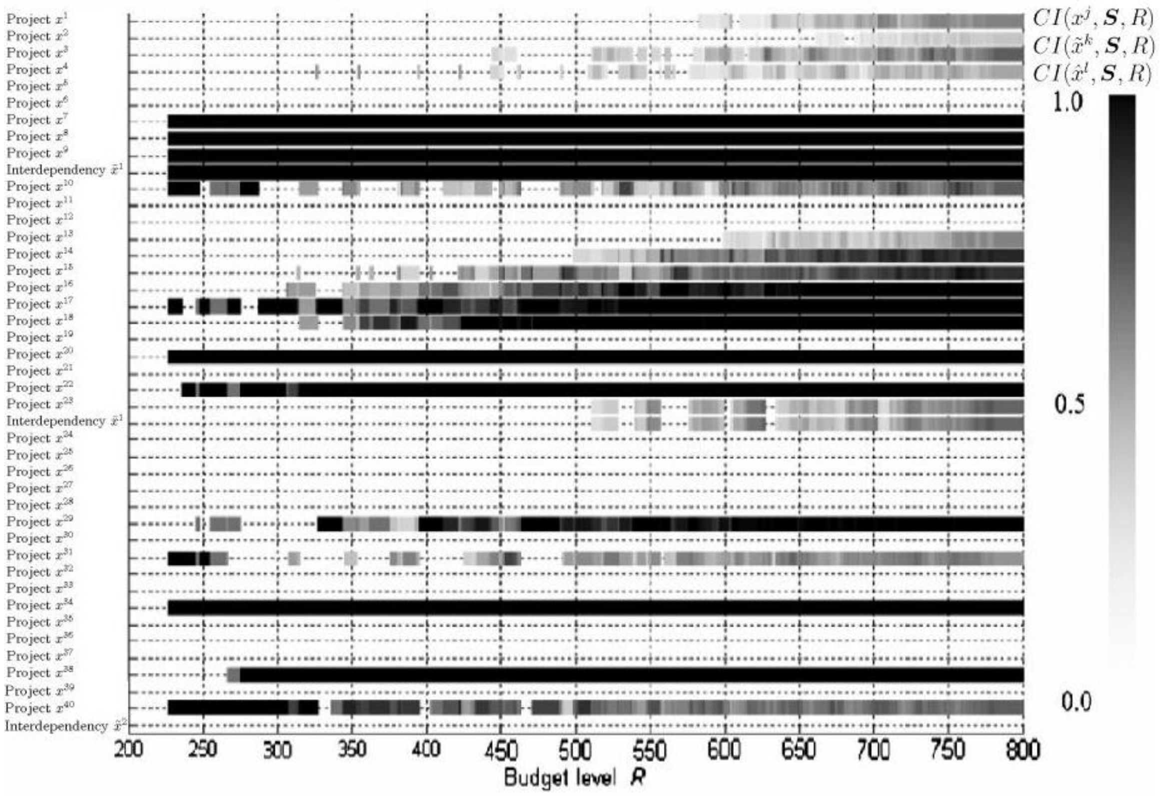


Figure 3.2: Budget-Dependent Core Indices of Projects and Project Interdependencies
Source Liesiö et al. (2008).

3.4.5 Alternative budget-dependent core index

The budget-dependent core index is based on the single cost scenario $\overline{s}_c$ although the interval cost information indicates that a DM is interested in different cost scenarios. Instead of restricting the analysis to a set of non-dominated portfolios, $P_N(\boldsymbol{S}, \overline{s}_c, R)$, we will focus the analysis on *all non-dominated portfolios* for a fixed budget level R:

$$P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) := \{p \in P_F(\underline{s}_c, R) \mid \exists s_c \in \boldsymbol{S}_c \text{ s.t. } p \in P_N(\boldsymbol{S}, s_c, R)\}\,. \tag{3.23}$$

Liesiö et al. (2008) stated that $P_N(\boldsymbol{S}, s_c, R) \subseteq P_N(\boldsymbol{S}, \underline{s}_c, R)$ holds for any $s_c \in \boldsymbol{S}_c$ and therefore $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) = P_N(\boldsymbol{S}, \underline{s}_c, R)$. However, we doubt the correctness of this statement. Note that $P_F(s_c, R) \subseteq P_F(\underline{s}_c, R)$ holds for any fixed $s_c \in \boldsymbol{S}_c$. While $P_N(\boldsymbol{S}, s_c, R)$ is received from pairwise dominance checks between portfolios $p \in P_F(s_c, R)$, is $P_N(\boldsymbol{S}, \underline{s}_c, R)$ obtained through pairwise dominance checks between portfolios $p \in P_F(\underline{s}_c, R)$. We can assume a portfolio p for which $p \in P_F(\underline{s}_c, R)$ and

$p \notin P_F(s_c, R)$. If we further assume a portfolio $p' \in P_F(s_c, R)$ so that $p \succ_{\boldsymbol{S}} p'$, then $p' \notin P_N(\boldsymbol{S}, \underline{s}_c, R)$. However, since $p \notin P_F(s_c, R)$, $p' \in P_N(\boldsymbol{S}, s_c, R)$ may hold.

As a consequence, we use $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ as a base for the alternative budget-dependent core index. The base for the calculation of $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ is given by $P_F(\underline{s}_c, R)$—i.e., efficient portfolios for which $C(p, \underline{s}_c) \leq R$ holds, see Theorem 3.4. To examine whether for portfolio $p \in P_F(\underline{s}_c, R)$ also $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ holds, we use pairwise dominance checks and collect portfolios $p' \in P_F(\underline{s}_c, R)$ with $p' \succ_{\boldsymbol{S}} p$ in the auxiliary set $P_C(p)$. Iff we can find a cost scenario $s_c \in \boldsymbol{S}_c$ for that portfolio p is feasible and all portfolios $p' \in P_C(p)$ are infeasible, $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ holds. The set of cost scenario $s_c \in \boldsymbol{S}_c$ for which the mentioned conditions hold is described by the following inequalities:

$$\begin{array}{ll} C(p', s_c) \geq R + \epsilon & \forall p' \in P_C(p) \\ C(p, s_c) \leq R & \\ \underline{c}^j \leq c^j \leq \overline{c}^j & \forall j = 1, \ldots, m \\ \underline{\tilde{c}}^k \leq \tilde{c}^k \leq \overline{\tilde{c}}^k & \forall k = 1, \ldots, K \\ \underline{\hat{c}}^l \leq \hat{c}^l \leq \overline{\hat{c}}^l & \forall l = 1, \ldots, L \end{array} \tag{3.24}$$

with $\epsilon > 0$ marginal. Thus, iff the set described by (3.24) is not empty then $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ holds. Whether (3.24) is an empty set or not can be verified using the first phase of the two phase simplex method—i.e., through the solution of an auxiliary linear optimization problem. Summarizing, the set of all non-dominated portfolios $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ can be established through pairwise dominance checks and solving an auxiliary linear optimization problem for portfolios $p \in P_F(\underline{s}_c, R)$ for which $\exists p' \in P_F(\underline{s}_c, R)$ s.t. $p' \succ_{\boldsymbol{S}} p$.

Since $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ is compound by sets of non-dominated portfolios, Theorem 3.2 indicates that additional information can only reduce $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$. Thus, if $\boldsymbol{S}^* \subseteq \boldsymbol{S} \neq \emptyset$ with $\text{int}(\boldsymbol{S}) \cap \boldsymbol{S}^*$, it follows $P_N(\boldsymbol{S}^*, \boldsymbol{S}_c, R) \subseteq P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$. $P_N(\boldsymbol{S}^*, \boldsymbol{S}_c, R)$ is therefore obtained from $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ as base instead of $P_F(\underline{s}_c, R)$. As with additional information, it is interesting how additional cost information $\boldsymbol{S}_c^* \subset \boldsymbol{S}_c$ impacts the set $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$. The following Theorem shows that additional cost information can eliminate some non-dominated portfolios, but cannot add new ones.

Theorem 3.5. *Let* $\boldsymbol{S}_c^* \subset \boldsymbol{S}_c$. *Then,*

$$P_N(\boldsymbol{S}, \boldsymbol{S}_c^*, R) \subseteq P_N(\boldsymbol{S}, \boldsymbol{S}_c, R).$$

Proof. Since $\boldsymbol{S}_c^* \subseteq \boldsymbol{S}_c$, we can find every set of non-dominated portfolios $P_N(\boldsymbol{S}, s_c^*, R)$ with $s_c^* \in \boldsymbol{S}_c^*$ defined on the basis of $\boldsymbol{S}_c$. □

As with additional information, we can use $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ as a basis for the calculation of $P_N(\boldsymbol{S}, \boldsymbol{S}_c^*, R)$.

Taken together, we have outlined that additional information, $\boldsymbol{S}^*$, and additional cost information, $\boldsymbol{S}_c^*$, cannot add new portfolios to $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$. Thus, $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ satisfies important properties in the context of the core index concept. Therefore, we can use the set of all non-dominated portfolios $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ to introduce a core index that fits to the interval-cost concept.

Definition 3.8. *The alternative budget-dependent core index of project x^j at budget level R is*

$$CI(x^j, \boldsymbol{S}, \boldsymbol{S}_c, R) = \frac{|\{p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) \mid x^j \in p\}|}{|P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)|}.$$

As with the budget-dependent core index, the alternative budget-dependent core index is applicable for project interactions as well—i.e., core indices $CI(\tilde{x}^k, \boldsymbol{S}, \boldsymbol{S}_c, R)$ and $CI(\hat{x}^l, \boldsymbol{S}, \boldsymbol{S}_c, R)$. Further, the alternative budget-dependent core index facilitates the establishment of the sets of core projects $X_C(\boldsymbol{S}, \boldsymbol{S}_c, R)$, borderline projects $X_B(\boldsymbol{S}, \boldsymbol{S}_c, R)$, and exterior projects $X_E(\boldsymbol{S}, \boldsymbol{S}_c, R)$. Due to the validity of Theorem 3.2 for $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$, the results of Corollaries 3.1 and 3.2 hold. Thus, for a fixed R, the decision information with respect to project values and evaluation criteria weights comes from $CI(x^j, \boldsymbol{S}, \boldsymbol{S}_c, R)$ as from $CI(x^j, \boldsymbol{S})$. Additionally, $C(x^j, \boldsymbol{S}, \boldsymbol{S}_c, R)$ provides decision information with respect to project costs. Specifically, core (exterior) projects (interactions) remain core (exterior) projects (interactions) even if additional cost information is given. The corresponding corollary can be derived from Theorem 3.5.

Corollary 3.3. *Let $\boldsymbol{S}_c^* \subseteq \boldsymbol{S}_c$. Then, $X_C(\boldsymbol{S}, \boldsymbol{S}_c, R) \subseteq X_C(\boldsymbol{S}, \boldsymbol{S}_c^*, R)$ and $X_E(\boldsymbol{S}, \boldsymbol{S}_c, R) \subseteq X_E(\boldsymbol{S}, \boldsymbol{S}_c^*, R)$ holds.*

As with score information, we can reduce the set of projects for which additional cost information is useful. The following theorem states that additional cost information can reduce the set of all non-dominated portfolios only if it relates to borderline or core (dummy) projects.

Theorem 3.6. *Let* $\boldsymbol{S}_c^* \subset \boldsymbol{S}_c$ *so that* $\underline{c}^{j*} = \underline{c}^j$, $\underline{\tilde{c}}^{k*} = \underline{\tilde{c}}^k$, $\underline{\hat{c}}^{l*} = \underline{\hat{c}}^l$ *and* $\overline{c}^{j*} = \overline{c}^j$, $\overline{\tilde{c}}^{k*} = \overline{\tilde{c}}^k$, $\overline{\hat{c}}^{l*} = \overline{\hat{c}}^l$ $\forall x^j, \tilde{x}^k, \hat{x}^l \in X_B(\boldsymbol{S}, \boldsymbol{S}_c, R) \cup X_C(\boldsymbol{S}, \boldsymbol{S}_c, R)$. *Then,* $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) = P_N(\boldsymbol{S}, \boldsymbol{S}_c^*, R)$ *holds.*

Proof. (i) $P_N(\boldsymbol{S}, \boldsymbol{S}_c^*, R) \subseteq P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$: Follows from Theorem 3.5.

(ii) $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) \subseteq P_N(\boldsymbol{S}, \boldsymbol{S}_c^*, R)$: Assume contrary to the claim $\exists p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ s.t $p \notin P_N(\boldsymbol{S}, \boldsymbol{S}_c^*, R)$. From $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) \Rightarrow \exists s_c \in \boldsymbol{S}_c$ for which $p \in P_N(\boldsymbol{S}, s_c, R)$. Due to the presumptions $\exists s_c^* \in \boldsymbol{S}_c^*$ for that $C(p', s_c^*) = C(p', s_c)\ \forall p' \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ including p. This implies that $p \in P_F(s_c^*, R)$ so there must be a $p' \in P_N(S, s_c^*, R)$ s.t. $p' \succ_{\boldsymbol{S}} p$. Since $p' \in P_N(S, s_c^*, R)$ we have $p' \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ and therefore $C(p', s_c^*) = C(p', s_c)$. This means that $p' \in P_F(s_c, R)$ wherefore $p \notin P_N(S, s_c, R)$ which is a contradiction.

□

Although core projects are robust choices in the sense that they are unaffected by additional information and additional cost information, Corollaries 3.1 and 3.3, a change in their cost bounds may eliminate non-dominated portfolios. This is since there may exist portfolios $p, p' \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ so that $p' \succ_{\boldsymbol{S}} p$ wherefore a certain cost scenario is necessary for which p is feasible but not p'. Additional cost information concerning a core or a borderline project may eliminate this cost scenario. In the opposite, additional cost information relating exterior projects cannot eliminate this cost scenario. Increasing lower cost bounds of core projects or borderline projects may also eliminated some non-dominated portfolios if they are not feasible anymore. Thus, an increase of the core or exterior projects' lower cost bounds tells a DM how sensitive a $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) \backslash P_N(\boldsymbol{S}, \overline{s}_c, R)$ is with respect to the costs of core and exterior projects.

The alternative budget-dependent core index approach allows one to define additional information and additional cost information simultaneously. However, for the purpose of promoting learning, we suggest defining either additional information and generating the new set of all non-dominated portfolios or defining new cost information and generating the new set of all non-dominated portfolios. This allows one to backtrack if a discarded portfolio is sensitive with respect to project scores and criteria weights or with respect to project costs. Therefore, the alternative budget-dependent core index approach allows DM's to investigate the robustness of projects and portfolios with respect to costs as well as score and weight information for a fixed budget level R.

This section has introduced costs of a portfolio. These costs are incorporated into the evaluation process through efficient portfolios and a relationship-evaluation concept for non-dominated portfolios was given (Theorem 3.3). Further, a cost–benefit band was defined to support the determination of the budget level R. The link to the selection of single projects and programs is given by the budget-dependent core index or by the alternative budget-dependent core index.

The concept is usable for resource categories with incomplete information on resource requirements and availability. Further, the concept may be upgraded to consider multiple resource categories and budget simultaneously. In this case, Definition 3.5 (efficiency) can be extended to form another cost inequality for each additional resource category, and Theorem 3.3 can be applied successively to incorporate resource categories one by one into the extended information set.

Chapter 4

Implementation of Strategic Buckets in the Robust Model

The main purpose of strategic buckets is to enforce a strategic resource/budget allocation between them; see Requirements 2.3 and 2.4. (To keep with the notation of Section 3.4, we focus the investigation on budget, but the concept is also valid for any resource category). The strategic budget allocation is *flexible* in the sense that a strategic bucket receives more budget than is proposed strategically if it contributes extra to the overall portfolio value; see Requirement 2.5. More budget than strategically proposed for some strategic buckets means less budget than strategically proposed for some others. Therefore, it is desirable to extend the cost–benefit analysis from Section 3.4 such that: (i) for any budget level $R \in \mathbb{R}_+$ the allocation of R among strategic buckets with the highest portfolio overall value is identified and (ii) *flexible* strategic allocation constraints for the budget allocation are satisfied. Therefore, Section 4.1 introduces strategic buckets for the RPM model. Sections 4.2 and 4.3 address the extension of the cost–benefit analysis with respect to (i). Section 4.4 covers the extension of the cost–benefit analysis with respect to (ii). Section 4.5 covers the allocation of a fixed budget level R among strategic buckets. Finally, Section 4.6 combines results from the foregoing sections and introduces the extended cost–benefit analysis which satisfies the requirements of a strategic-bucket structure.

4.1 Introduction to Strategic Buckets

Let us assume that the set of projects X is split into strategic buckets $X_1, \ldots, X_Q \subseteq X$ so that X_q represents a strategic bucket of the lowest level for $q = 1, \ldots, Q$. Every (dummy) project, $x^j, \tilde{x}^k, \hat{x}^l \in X$, is contained in exactly one X_q so that $X_q \cap X_{q^*} = \emptyset$ for $q \neq q^*$ and $\bigcup_{q=1}^{Q} X_q = X$.

A subportfolio, $p_q \subseteq X_q$, is a subset of (dummy) projects within one single strategic bucket. The set of all theoretically possible subportfolios for X_q is the power set $P^q := 2^{X_q}$. The relationship to the set of all theoretically possible portfolios, P, is given by

$$P = \bigoplus_{q=1}^{Q} P^q := \Big\{ p_1 \cup \cdots \cup p_Q \mid p_q \in P^q \text{ for } q = 1, \ldots, Q \Big\}. \tag{4.1}$$

Thus, any portfolio $p \in P$ has a unique description through a union of Q subportfolios $p = \bigcup_{q=1}^{Q} p_q$ with $p_q \in P^q$. We refer to the subportfolios of p as $p_1, \ldots, p_Q$.

To meet Requirement 2.10, we assume that every X_q has its own set of n_q evaluation criteria. As a consequence, every X_q is associated with an information set $\boldsymbol{S}^q$ and we assume that for every information set, $\boldsymbol{S}^1, \ldots, \boldsymbol{S}^Q$, the same assumptions hold as for the information set $\boldsymbol{S}$ in the foregoing chapter. A particular realization of criteria weights and (dummy) project scores for strategic bucket X_q is denoted by $s^q \in \boldsymbol{S}^q$. The notations $\underline{s}^q$ and $\overline{s}^q$ are used to indicate particular realizations where (dummy) project scores are set to their lower or upper bounds, respectively. The definitions of $\boldsymbol{S}^1, \ldots, \boldsymbol{S}^Q$ are allowed since $X_q \cap X_{q^*} = \emptyset$ for $q \neq q^*$ so that every project is evaluated with respect to a unique set of evaluation criteria.

The information set for X under a strategic bucket structure appears as the Cartesian product $\boldsymbol{S} := \boldsymbol{S}^1 \times \ldots \times \boldsymbol{S}^Q$, and $s := (s^1, \ldots, s^Q) \in \boldsymbol{S}$ is equivalent to $s^1 \in \boldsymbol{S}^1, \ldots, s^Q \in \boldsymbol{S}^Q$. This implies that s^q can be selected independently from s^{q^*} for $q^* \neq q$, which has important implications for the further investigations.

First, we investigate the overall value of subportfolios. The overall value of subportfolio p_q is the sum of the overall values of its (dummy) projects. Thus, the definition of the overall value of a subportfolio is analogous to the definition of the overall value of a portfolio (3.6) and is therefore denoted by $V(p_q, s^q)$.

For the overall value of project portfolio p, notice that p in general consists of (dummy) projects within different strategic buckets. Thus, (dummy) projects within portfolio p are evaluated by multiple sets of evaluation criteria so that we are interested in premises to keep up the additivity assumption of the overall value of a portfolio. In Golabi et al. (1981), premises for the additivity assumption are discussed, assuming a certain set of evaluation criteria for every single project. Therefore, we can further assume that the overall value of a portfolio is the sum of the overall values of its (dummy) projects. This implies that the overall value of portfolio p under a strategic bucket structure is the sum of the overall values of its subportfolios:

$$V(p,s) := \sum_{q=1}^{Q} V(p_q, s^q). \tag{4.2}$$

Nevertheless, subportfolios are evaluated by different sets of multiple evaluation criteria. For the evaluation of projects on multiple criteria, in the RPM approach the project scores for each evaluation criterion are scaled so that $v_i^j \in [0,1]$ (for scaling issues see Clemen and Smith 2009). Therefore, the overall value of subportfolio p_q increases with the number of evaluation criteria, n_q, so that the impact of subportfolio p_q on the overall value of portfolio p depends on the number of evaluation criteria, n_q. Since a strategic bucket, X_q, is represented in the overall value of a portfolio by the corresponding subportfolio, the priority of a strategic bucket would also depend on the number of its evaluation criteria, n_q. In our discussion about budget allocation among strategic buckets, however, we will assume that every strategic bucket, $X_1, \ldots, X_Q$, has the same priority (although other approaches are conceivable, see Section 7). Hence, we have to replace the overall value of a portfolio (4.2) with the normalized overall value:

$$V(p,s) := \sum_{q=1}^{Q} \frac{1}{n_q} V(p_q, s^q). \tag{4.3}$$

Despite the redefinitions of $V(p,s)$ and $\boldsymbol{S}$, the dominance concept and its decision procedure introduced in Section 3.3 holds. In the following, we show that under certain conditions the dominance concept is transferable to subportfolios and decisions about subportfolios allow conclusions for decisions about the entire portfolio. Therefore, notice that Section 2.3 about strategy motivates the separation of constraints between strategic buckets. The only exceptions would be resource constraints. However, Section 3.4 demonstrated that resources are not formulated as constraints in the RPM. Thus, we assume that constraints are divided among strategic buckets. Separated constraints between strategic buckets means that matrices $\boldsymbol{A}, \tilde{\boldsymbol{A}}$ and $\hat{\boldsymbol{A}}$ from (3.16) are defined as follows.

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{A}_1 & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{A}_Q \end{pmatrix}, \tilde{\boldsymbol{A}} = \begin{pmatrix} \tilde{\boldsymbol{A}}_1 & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{\boldsymbol{A}}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \dots & \boldsymbol{0} & \tilde{\boldsymbol{A}}_Q \end{pmatrix}, \hat{\boldsymbol{A}} = \begin{pmatrix} \hat{\boldsymbol{A}}_1 & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \boldsymbol{0} & \hat{\boldsymbol{A}}_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \dots & \boldsymbol{0} & \hat{\boldsymbol{A}}_Q \end{pmatrix}$$

with $\boldsymbol{A}_q$ being matrices containing coefficients for projects within X_q. Analogously, matrices $\tilde{\boldsymbol{A}}_q$ and $\hat{\boldsymbol{A}}_q$ store coefficients about dummy projects within X_q.

The bijection $\boldsymbol{z} : P \to \{0,1\}^{m\times 1}$ can be divided into $\boldsymbol{z}_1(\cdot), \dots, \boldsymbol{z}_Q(\cdot)$ so that $\boldsymbol{z}_q(\cdot)$ represents the share for projects within X_q. In the same way the vectors $\tilde{\boldsymbol{z}}$ and $\hat{\boldsymbol{z}}$ are dividable. Further, we split the vector with the limits $\boldsymbol{\alpha}$ into Q shares so that $\boldsymbol{\alpha}_q$ can be associated with $\boldsymbol{A}_q$. Thus, the following description for feasible portfolios is similar to that in (3.16):

$$P_F := \{p \in P \mid \boldsymbol{A}_q \boldsymbol{z}_q(p_q) + \tilde{\boldsymbol{A}}_q \tilde{\boldsymbol{z}}_q(p_q) + \hat{\boldsymbol{A}}_q \hat{\boldsymbol{z}}_q(p_q) \le \boldsymbol{\alpha}_q \text{ for } q = 1, \dots, Q\}. \quad (4.4)$$

This means a portfolio p is feasible if all its subportfolios, $p_1, \dots, p_Q$, comply with the corresponding subconstraints. The assumption about separated constraints has therefore important implications. First, it allows one to define the set of feasible subportfolios for any X_q, $P_F^q := \{p_q \in P^q \mid \boldsymbol{A}_q \boldsymbol{z}_q(p_q) + \tilde{\boldsymbol{A}}_q \tilde{\boldsymbol{z}}_q(p_q) + \hat{\boldsymbol{A}}_q \hat{\boldsymbol{z}}_q(p_q) \le \boldsymbol{\alpha}_q\}$. The set of feasible portfolios is given by $P_F = \bigoplus_{q=1}^{Q} P_F^q$. Second, it allows even the definition of the set of non-dominated subportfolios for any X_q with the information set $\boldsymbol{S}^q$, $P_N^q(\boldsymbol{S}^q) := \{p_q \in P_F^q \mid \nexists p'_q \in P_F^q \text{ s.t. } p'_q \succ_{\boldsymbol{S}^q} p_q\}$. Even here, the relationship $P_N(\boldsymbol{S}) = \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q)$ holds. These results are summarized in Theorem 4.1.

Theorem 4.1. *Consider a set of projects, X, divided into strategic buckets, $X_1, \dots, X_Q$ with information set $\boldsymbol{S} = \boldsymbol{S}^1 \times \dots \times \boldsymbol{S}^Q$. If feasibility constraints $\boldsymbol{A}\boldsymbol{z}(p) + \tilde{\boldsymbol{A}}\tilde{\boldsymbol{z}}(p) + \hat{\boldsymbol{A}}\hat{\boldsymbol{z}}(p) \le \boldsymbol{\alpha}$ are dividable into $\boldsymbol{A}_q \boldsymbol{z}_q(p_q) + \tilde{\boldsymbol{A}}_q \tilde{\boldsymbol{z}}_q(p_q) + \hat{\boldsymbol{A}}_q \hat{\boldsymbol{z}}_q(p_q) \le \boldsymbol{\alpha}_q$ for $q = 1, \dots, Q$ then the following holds.*

(i) $P_F = \bigoplus_{q=1}^{Q} P_F^q$

(ii) $P_N(\boldsymbol{S}) = \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q)$

Proof. (i) For any $p \in P_F$,

$$\begin{aligned}
&\boldsymbol{A}\boldsymbol{z}(p) + \tilde{\boldsymbol{A}}\tilde{\boldsymbol{z}}(p) + \hat{\boldsymbol{A}}\hat{\boldsymbol{z}}(p) \leq \boldsymbol{\alpha} \\
\iff& \boldsymbol{A}_q\boldsymbol{z}_q(p_q) + \tilde{\boldsymbol{A}}_q\tilde{\boldsymbol{z}}_q(p_q) + \hat{\boldsymbol{A}}_q\hat{\boldsymbol{z}}_q(p_q) \leq \boldsymbol{\alpha}_q \text{ for } q = 1, \dots, Q \\
\iff& p_q \in P_F^q \text{ for } q = 1, \dots, Q.
\end{aligned}$$

(ii) $P_N(\boldsymbol{S}) \subseteq \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q)$:

Assume, contrary to the claim, that $\exists p \in P_N(\boldsymbol{S})$ s.t. $p \notin \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q)$. Since $p \in P_N(\boldsymbol{S})$, we have $p \in P_F$ and, due to (i), we have $p_q \in P_F^q$ for $q = 1, \dots, Q$. From $p \notin \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q)$, $\exists q^*$ s.t. $p_{q^*} \notin P_N^{q^*}(\boldsymbol{S}^{q^*})$. Therefore, $\exists p'_{q^*} \in P_N^{q^*}(\boldsymbol{S}^{q^*})$ s.t. $p'_{q^*} \succ_{\boldsymbol{S}^{q^*}} p_{q^*}$ and the portfolio $p' = \bigcup_{\substack{q=1 \\ q \neq q^*}}^{Q} p_q \cup p'_{q^*} \in P_F$. From this, $p' \succ_{\boldsymbol{S}} p$, which is a contradiction.

$\bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q) \subseteq P_N(\boldsymbol{S})$:

Assume, contrary to the claim, that $\exists p_1 \in P_N^1(\boldsymbol{S}^1), \dots, p_Q \in P_N^Q(\boldsymbol{S}^Q)$ s.t. $p = \bigcup_{q=1}^{Q} p_q \notin P_N(\boldsymbol{S})$. From (i), we have $p \in P_F$. Therefore, $\exists p' \in P_N(\boldsymbol{S})$ s.t. $p' \succ_{\boldsymbol{S}} p$. If $p'_1, \dots, p'_Q$ are the subportfolios of p', we have $p'_q \in P_F^q$ for $q = 1, \dots, Q$ due to (i) again. Since $p' \succ_{\boldsymbol{S}} p$ we have:

$$\begin{aligned}
p' \succ_{\boldsymbol{S}} p \quad &\Longrightarrow \quad V(p', s) - V(p, s) \geq 0 \ \forall s \in \mathrm{S} \\
&\iff \quad \min_{s \in \boldsymbol{S}} \left[V(p', s) - V(p, s) \right] \geq 0 \\
&\iff \quad \min_{s^1 \in \boldsymbol{S}^1, \dots, s^Q \in \boldsymbol{S}^Q} \left[\sum_{q=1}^{Q} \tfrac{1}{n_q} V(p'_q, s^q) - \sum_{q=1}^{Q} \tfrac{1}{n_q} V(p_q, s^q) \right] \geq 0 \\
&\iff \quad \sum_{q=1}^{Q} \tfrac{1}{n_q} \left[\min_{s^q \in \boldsymbol{S}^q} \left[V(p'_q, s^q) - V(p_q, s^q) \right] \right] \geq 0.
\end{aligned}$$

If there $\exists q^*$ so that $\min_{s^{q^*} \in \boldsymbol{S}^{q^*}} [V(p'_{q^*}, s^{q^*}) - V(p_{q^*}, s^{q^*})] < 0$ it follows from the last equation $\exists q^{**}$ s.t. $\min_{s^{q^{**}} \in \boldsymbol{S}^{q^{**}}} [V(p'_{q^{**}}, s^{q^{**}}) - V(p_{q^{**}}, s^{q^{**}})] > 0$. However, this means that $V(p'_{q^{**}}, s^{q^{**}}) > V(p_{q^{**}}, s^{q^{**}}) \ \forall s^{q^{**}} \in \boldsymbol{S}^{q^{**}}$ and $p'_{q^{**}} \succ_{\boldsymbol{S}^{q^{**}}} p_{q^{**}}$ which is a contradiction. As a consequence we must assume that $\min_{s^q \in \boldsymbol{S}^q} [V(p'_q, s^q) - V(p_q, s^q)] \geq 0 \ \forall q = 1, \dots, Q$. Since $p' \succ_{\boldsymbol{S}} p$ there $\exists s_\circ \in \boldsymbol{S}$ for that $V(p', s_\circ) > V(p, s_\circ)$ holds implying that $\exists q^\diamond$ for that $V(p'_{q^\diamond}, s_\circ^{q^\diamond}) > V(p_{q^\diamond}, s_\circ^{q^\diamond})$ so that $p_{q^\diamond} \notin P_N^q(\boldsymbol{S}^{q^\diamond})$ what is a contradiction and finishes our proof.

□

Theorem 4.1 gives the set of non-dominated portfolios, $P_N(\boldsymbol{S})$, from the sets of non-dominated subportfolios, $P_N^q(\boldsymbol{S}^q)$. This has positive consequences for the calculation time of $P_N(\boldsymbol{S})$ since it depends heavily on the number of projects; see Chapter 5.

From Section 3.3, we know that additional information may eliminate some non-dominated portfolios but cannot add new ones. That is, $P_N(\boldsymbol{S}^*) \subseteq P_N(\boldsymbol{S})$ for $\boldsymbol{S}^* \subset \boldsymbol{S}$ with $\text{int}(\boldsymbol{S}) \cap \boldsymbol{S}^* \neq \emptyset$. Since the results of Theorem 4.1 also hold for $\boldsymbol{S}^*$ we obtain $P_N(\boldsymbol{S}^*) = \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^{*q})$. Thus, additional information can eliminate some non-dominated subportfolios, but cannot add new ones to any $P_N^q(\boldsymbol{S}^{*q})$, so $P_N^q(\boldsymbol{S}^{*q})$ can be obtained from $P_N^q(\boldsymbol{S}^q)$ by pairwise dominance checks. Further, it is obvious that additional information with respect to $\boldsymbol{S}^q$ —i.e. $\boldsymbol{S}^{*q} \subset \boldsymbol{S}^q$ with $\text{int}(\boldsymbol{S}^q) \cap \boldsymbol{S}^{*q} \neq \emptyset$— can just eliminate subportfolios within $P_N^q(\boldsymbol{S}^q)$ but not within any other $P_N^{q'}(\boldsymbol{S}^{q'})$ for $q' \neq q$. In short, the decision information introduced in Section 3.3 also holds for the sets of non-dominated subportfolios and the set of non-dominated portfolios can be calculated using Theorem 4.1.

Thus, the decision about the optimal portfolio can be divided into the independent decisions of optimal subportfolios. That is useful if strategic buckets $X_1, \ldots, X_Q$ have their own DM or one is going to drill down the analysis into a X_q, see Section 2.

4.2 The Marginal Value of a Strategic Bucket

One approach for the optimal allocation of a fixed $R \in \mathbb{R}_+$ among $X_1, \ldots, X_Q$, is given by define the marginal value of X_q (further conceivable approaches are outlined in Section 7). The marginal value of X_q describes how efficient X_q uses a marginal budget increase and therefore whether it is a candidate for additional funding. For the definition of the marginal value of X_q, we apply cost–benefit analysis for the project set X from Section 3.4 to X_q. Therefore, notice that the entire cost information $\boldsymbol{S}_c$ can be separated into $\boldsymbol{S}_c^1, \ldots, \boldsymbol{S}_c^Q$ since $X_q \cap X_{q^*} = \emptyset$ for $q \neq q^*$. The entire cost information can be regarded as their Cartesian product $\boldsymbol{S}_c = \boldsymbol{S}_c^1 \times \ldots \times \boldsymbol{S}_c^Q$. A particular realization of project costs for strategic bucket X_q is denoted by $s_c^q \in \boldsymbol{S}_c^q$. The notation $\underline{s}_c^q$ and $\overline{s}_c^q$, respectively, is used to indicate the realizations where project costs are set to their lower or upper bounds, respectively.

The cost for subportfolio p_q is given by $C(p_q, s_c^q)$ with $C(\cdot, \cdot)$ from (3.19). Consequently, the cost of a portfolio p is the sum of costs of its subportfolios $p_1, \ldots, p_Q$:

$$C(p, s_c) = \sum_{q=1}^{Q} C(p_q, s_c^q). \tag{4.5}$$

Taking the total cost of subportfolios, we can define the set of efficient subportfolios for any strategic bucket X_q:

$$P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q) := \left\{ p_q \in P_F^q \mid \nexists p_q' \in P_F^q \text{ s.t. } \left\{ \begin{array}{l} V(p_q', s^q) \geq V(p_q, s^q) \ \forall s^q \in \boldsymbol{S}^q \\ C(p_q', s_c^q) \leq C(p_q, s_c^q) \ \forall s_c^q \in \boldsymbol{S}_c^q \end{array} \right\} \right\},$$

with at least one strict inequality for some $s^q \in \boldsymbol{S}^q$ or $s_c^q \in \boldsymbol{S}_c^q$. Using Theorem 3.3, we find the relationship $P_N^q(\check{\boldsymbol{S}}^q) = P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q)$, and through Theorem 4.1, we obtain $P_E(\boldsymbol{S}, \boldsymbol{S}_c) = \bigoplus_{q=1}^{Q} P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q)$.

Let us denote the share of R allocated to X_q by R_q. Then we can define the set of feasible subportfolios, $P_F^q(s_c^q, R_q) := \{p_q \in P_F^q \mid C(p_q, s_c^q) \leq R_q\}$, that are *attainable with fixed* $s_c^q \in \boldsymbol{S}_c^q$ *and* R_q and the corresponding set of non-dominated subportfolios, $P_N^q(\boldsymbol{S}^q, s_c^q, R_q) := \{p_q \in P_F^q(s_c^q, R_q) \mid p_q' \not\succ_{\boldsymbol{S}^q} p_q \ \forall p_q' \in P_F^q(s_c^q, R_q)\}$, with regard to information set $\boldsymbol{S}^q$. Since the results of Theorem 3.4 also hold for $P_N^q(\boldsymbol{S}^q, s_c^q, R_q)$ and $P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q)$, the cost–benefit band for X_q is defined analogously to the cost–benefit band in Section 3.4. The bounds of the cost–benefit band for strategic bucket X_q are then given by

Maximal overall value for X_q:

$$MV_q(R_q) := \max_{p_q \in P_N^q(\boldsymbol{S}^q, \underline{s}_c^q, R_q)} \max_{s^q \in \boldsymbol{S}^q} V(p_q, s^q),$$

Guaranteed overall value for X_q:

$$GV_q(R_q) := \max_{p_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)} \min_{s^q \in \boldsymbol{S}^q} V(p_q, s^q).$$

The cost–benefit bands for $X_1, \ldots, X_Q$ are the basis for the calculation of marginal values $\Delta_1(R_1), \ldots, \Delta_Q(R_Q)$. The marginal value $\Delta_q(R_q)$ measures how efficiently subportfolios $p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q)$ use a marginal increase of R_q to $R_q + \delta$ and is therefore a measure of whether X_q is a candidate for additional funding. However, the cost–benefit band for X_q depends on information $s^q \in \boldsymbol{S}^q$ as well as on costs $s_c^q \in \boldsymbol{S}_c^q$, implying $\Delta_q(R_q) = \Delta_q(R_q, s^q, s_c^q)$. The dependency on information s^q and costs s_c^q

impacts the definition of $\Delta_q(R_q, s^q, s_c^q)$ as well as the preference evaluation among $\Delta_1(R_1, s^1, s_c^1), \ldots, \Delta_Q(R_Q, s^Q, s_c^Q)$.

For the definition of $\Delta_q(R_q, s^q, s_c^q)$, let us consider the cost–benefit band of X_q. It is described by the sets of non-dominated subportfolios, $P_N^q(\boldsymbol{S}^q, s_c^q, R_q)$, with fixed costs s_c^q. To ensure that all subportfolios in the set of non-dominated subportfolios are feasible, even for their worst case cost scenario, we use costs $\overline{s}_c^q$ so that we have $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$. Since the composition of $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ can change only at levels $R_q \in \{C(p_q, \overline{s}_c^q) \mid p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q)\}$, the cost–benefit band of X_q changes at these levels only; see Figure 3.1. Thus, if $\delta_q(R_q)$ represents the marginal increase of R_q, it should be the smallest increase so that a new subportfolio $p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q)$ enters into the set of non-dominated subportfolios with cost scenario $\overline{s}_c^q$:

$$\begin{aligned}\delta_q(R_q) := \inf_{p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q)} \; &\{C(p_q, \overline{s}_c^q) - R_q \mid C(p_q, \overline{s}_c^q) > R_q, \\ &p_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta_q(R_q))\}.\end{aligned} \tag{4.6}$$

Note, for given costs, $\overline{s}_c^q$, any increase of R_q less than $\delta_q(R_q)$ is a waste of money since no new opportunity becomes feasible. Conversely, if R_q is increased by $\delta_q(R_q)$, it is ensured that a new opportunity becomes feasible even for the worst-cost scenario, $\overline{s}_c^q$. The marginal value $\Delta_q(R_q, s^q, s_c^q)$ is then received through comparing subportfolios between $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ and $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta_q(R_q))$.

In what follows, let arg min and arg max denote the *sets* of variables that minimize and maximize, respectively, the given functions. For a given $s^q \in \boldsymbol{S}^q$, let us introduce the following notation:

$$P_M^q(s^q, R_q) := \operatorname*{arg\,max}_{p_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)} V(p_q, s^q)$$

and

$$P_M^q(s^q, R_q + \delta_q(R_q)) := \operatorname*{arg\,max}_{p_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta_q(R_q))} V(p_q, s^q).$$

If we use this notation, the definition of the marginal value for a strategic bucket reads as follows.

Definition 4.1. *Consider budget level R_q, scenario $s^q \in \boldsymbol{S}^q$, and cost scenario $s_c^q \in \boldsymbol{S}_c^q$. Further, let $p'_q \in P_M^q(s^q, R_q + \delta_q(R_q))$ and $p''_q \in P_M^q(s^q, R_q)$ such that $p'_q \in \arg\min_{p_q \in P_M^q(s^q, R_q + \delta_q(R_q))} C(p_q, s_c^q)$ and $p''_q \in \arg\min_{p_q \in P_M^q(s^q, R_q)} C(p_q, s_c^q)$. Then* *is the marginal value for* X_q *given by*

$$\Delta_q(R_q, s^q, s_c^q) := \frac{V(p'_q, s^q) - V(p''_q, s^q)}{C(p'_q, s_c^q) - C(p''_q, s_c^q) + \epsilon}$$

with $\delta_q(R_q)$ *given by* (4.6) *and a small* $\epsilon > 0$.

The ϵ in the denominator ensures that the denominator cannot assume a value less than or equal to zero. In the definition of $\Delta_q(R_q, s^q, s_c^q)$, we always use those subportfolios $p'_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta(R_q))$ and $p''_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ that maximize the overall value for given information, s^q. This approach is motivated by the assumption that a rational DM would for given information, s^q, choose the subportfolio with the highest overall value, since every considered subportfolio is feasible no matter what its costs. Further, the subportfolios that maximize the overall value remain in the set of non-dominated subportfolios, $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$, even if additional information is defined that confirms the subportfolios as optimal, see Theorem 3.2. For the case that $P_M^q(s^q, R_q + \delta_q(R_q))$ and/or $P_M^q(s^q, R_q)$ consist of multiple subportfolios, we choose the subportfolio(s) with the lowest cost for the given cost scenario, s_c^q. Summarizing, we will choose those subportfolios p'_q and p''_q which best represent the sets $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta_q(R_q))$ and $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ subject to a maximal overall value, with s^q and s_c^q given.

Consider the subportfolio that becomes feasible at budget level $R_q + \delta_q(R_q)$, denoted by p_q^*. The set of non-dominated subportfolios, $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta(R_q))$, is easily obtained through $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$. We just have to add p_q^* to $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ and remove all subportfolios dominated by p_q^* through dominance checks (3.18). Therefore we can derive the following properties for $\Delta_q(R_q, s^q, s_c^q)$.

(i) $\Delta_q(R_q, s^q, s_c^q) \geq 0 \; \forall s^q \in \boldsymbol{S}^q$ and $\forall s_c^q \in \boldsymbol{S}_c^q$,

(ii) $\Delta_q(R_q, s^q, s_c^q) > 0 \Longleftrightarrow V(p_q^*, s^q) > \max_{p_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)} V(p_q, s^q)$.

Condition (i) reflects the fact that the cost–benefit band of X_q is non-decreasing. Since the denominator is bigger than zero, condition (i) is straightforwardly obtained from the explanation of the composition of $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta(R_q))$ above. Just take p'_q and p''_q from Definition 4.1 and if $p''_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta_q(R_q))$ set $p'_q = p''_q$ otherwise set $p'_q = p_q^*$. Condition (ii) also arises from the explanation above which implies that $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta_q(R_q)) \backslash p_q^* \subseteq P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$.

Conditions (i) and (ii) are helpful for the calculation of $\Delta_q(R_q, s^q, s_c^q)$. Instead of the identification of $P_M^q(s^q, R_q+\delta_q(R_q))$, the set of subportfolios with maximal overall value for a given $s^q \in \boldsymbol{S}^q$, we can simply use p_q^*. For a given $s^q \in \boldsymbol{S}^q$, $\Delta_q(R_q, s^q, s_c^q) \neq 0$ iff $V(p_q^*, s^q) - V(p_q, s^q) > 0$ with $p_q \in P_M^q(s^q, R_q)$, otherwise $\Delta_q(R_q, s^q, s_c^q) = 0$ so that

$$\Delta_q(R_q, s^q, s_c^q) = \max\left\{\frac{V(p_q^*, s^q) \ - V(p_q'', s^q)}{C(p_q^*, s_c^q) \ - C(p_q'', s_c^q) + \epsilon},\ 0\right\}, \tag{4.7}$$

where $p_q'' \in \arg\min_{p_q \in P_M^q(s_c^q, R_q)} C(p_q, s_c^q)$. Thus, for given information $s^q \in \boldsymbol{S}^q$ and given cost information $s_c^q \in \boldsymbol{S}_c^q$, the marginal value $\Delta_q(R_q, s^q, s_c^q)$ is easily obtained. We just have to identify $P_M^q(s^q, R_q)$ and subsequently the subportfolio $p_q'' \in P_M^q(s^q, R_q)$ with the lowest cost for s_c^q. However, for the budget allocation between strategic buckets, DMs may be more interested in the range of values which a marginal value $\Delta_q(R_q, s^q, s_c^q)$ can assume—i.e., DMs are interested in $\underline{\Delta}_q(R_q, s^q, s_c^q)$ and $\overline{\Delta}_q(R_q, s^q, s_c^q)$ with

$$\begin{aligned}
\underline{\Delta}_q(R_q, s^q, s_c^q) &:= \min_{s^q \in \boldsymbol{S}^q, s_c^q \in \boldsymbol{S}_c^q} \Delta_q(R_q, s^q, s_c^q), \\
\overline{\Delta}_q(R_q, s^q, s_c^q) &:= \max_{s^q \in \boldsymbol{S}^q, s_c^q \in \boldsymbol{S}_c^q} \Delta_q(R_q, s^q, s_c^q).
\end{aligned}$$

For the calculation of $\overline{\Delta}_q(R_q, s^q, s_c^q)$ notice that the set $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta(R_q))$ is represented by the subportfolio p_q^* as specified in equation (4.7). Further, notice that the maximization with respect to s^q corresponds to the maximization of the numerator in (4.7) and the maximization with respect to s_c^q corresponds to the minimization of the denominator in (4.7).

The maximization of the numerator in (4.7) is achieved if (dummy) project scores are set to their upper bound if they relate to p_q^* and to their lower bound if not. We will denote those scenarios by $s^{p_q^*}$. Therefore, for a given $p_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ the maximization of the numerator in (4.7) depends only on the weights of evaluation criteria $\boldsymbol{w}^q \in S_{w^q}$. Thus, the maximum of the numerator in (4.7) for a given $p_q \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ is given by the solution of the following linear optimization problem:

$$\begin{aligned}
&\tilde{\Delta}_q(p_q) := \max_{\boldsymbol{w}^q \in S_w^q} \left(V(p_q, s^{p_q^*}) - V(p_q, s^{p_q^*})\right) \\
&\text{subject to} \\
&V(p_q''', s^{p_q^*}) \leq V(p_q, s^{p_q^*}) \quad \forall p_q''' \in P_N(\boldsymbol{S}^q, \overline{s}_c^q, R_q)\backslash p_q
\end{aligned} \tag{4.8}$$

The constraints in the optimization problem (4.8) ensure that $p_q \in P_M^q(s^q, R_q)$ holds.

Since $P_N(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ is not represented by a certain subportoflio p_q we have to calculate $\tilde{\Delta}_q(p_q)$ for all $p_q \in P_N(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$.

To receive the maximum from the fraction in (4.7) we must divide the obtained maximums, $\tilde{\Delta}_q(p_q)$, with the corresponding minimums of the denominator. The minimum of the denominator in (4.7) is obtained if (dummy) project costs are set to their lower bound if they relate to p_q^* and to their upper bound if not. We will denote this cost scenario by $s_c^{p_q^*}$. Thus, we simply divide the maximums of the numerators $\tilde{\Delta}_q(p_q)$ by the corresponding cost differences $C(p_q^*, s_c^{p_q^*}) - C(p_q, s_c^{p_q^*}) + \epsilon$. The received maximum is then $\overline{\Delta}_q(R_q, s_c^q, s^q)$ if it is greater than zero. Otherwise we have $\overline{\Delta}_q(R_q, s_c^q, s^q) = 0$ according to (4.7). (Note, if for a p_q the value $\tilde{\Delta}_q(p_q)$ is also achieved by another subportfolio p_q''', we have to remove p_q from the calculation of $\overline{\Delta}_q(R_q, s^q, s_c^q)$ if its costs at the cost scenario $s_c^{p_q^*}$ are higher than the costs from p_q'''. This is since in this case $p_q \notin \arg\min_{P_M(s^q, R_q)} C(p_q, s_c^{p_q^*})$).

The calculation of $\underline{\Delta}_q(R_q, s^q, s_c^q)$ works analogously. However, if $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta(R_q)) \neq p_q^*$ we receive $P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta(R_q)) \cap P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q) \neq \emptyset$. Thus, there exists a subportfolio p_q'' for which $p_q'' \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q + \delta(R_q))$ and $p_q'' \in P_N^q(\boldsymbol{S}^q, \overline{s}_c^q, R_q)$ holds. For p_q'', we can find a certain scenario, s^q, so that $V(p_q^*, s^q) \leq V(p_q'', s^q)$. This implies that $\underline{\Delta}_q(R_q, s^q, s_c^q) = 0$ according to (4.7).

4.3 Strategic-Bucket Preference Evaluation Based on the Marginal Value

When different weights and/or scores are selected from the information set $\boldsymbol{S}$ and/or different costs are selected from cost information $\boldsymbol{S}_c$, the marginal values of strategic buckets vary within the intervals $[\underline{\Delta}_q(R_q, s^q, s_c^q), \overline{\Delta}_q(R_q, s^q, s_c^q)]$ for $q = 1, \ldots, Q$. For the identification of the marginal value of the preferred strategic bucket, we could use the dominance concept again. However, the dominance concept outlines a unique solution only on rare occasions. For the case of multiple solutions, interactions with the DM are required. This is a serious disadvantage in the context of budget allocation between strategic buckets. The budget level, R, is determined through a cost–benefit analysis employing the cost–benefit band for X; see Section 3.4. The overall cost–benefit band, however, depends on the allocation of R among $X_1, \ldots, X_Q$. Thus, to calculate the cost–benefit band for X it would be necessary to set interaction effects for all $R \in \mathbb{R}_+$ where the cost–benefit band can change. Since this is too time consuming, we need another preference measure.

Hence, instead of the dominance concept, we will use the *minimax–regret* rule as preference criterion. The minimax–regret rule suggests the marginal values for which the maximum loss is smallest. Theory and algorithms for the calculation of the minimax–regret rule are to find in Kasperski (2008) as well as in Kouvelis and Yu (1996). If $\boldsymbol{\Delta}$ denotes the collection of marginal values $\Delta_1(R_1, s^1, s_c^1), \ldots, \Delta_Q(R_Q, s^Q, s_c^Q)$, the suggested marginal values are given by

$$\Delta_{mmr} := \min_{\Delta_q \in \boldsymbol{\Delta}} \max_{\substack{s \in \boldsymbol{S},\ s_c \in \boldsymbol{S}_c, \\ \Delta_{q'} \in \boldsymbol{\Delta}}} \left\{ \frac{1}{n_{q'}} \Delta_{q'}(R_{q'}, s^{q'}, s_c^{q'}) - \frac{1}{n_q} \Delta_q(R_q, s^q, s_c^q) \right\}. \tag{4.9}$$

Although the minimax–regret rule does not guarantee a unique solution neither, the solution set may be incomparably smaller than the set of non-dominated marginal values. Thus, the minmax–regret rule should be a suitable decision criterion for the funding of strategic buckets.

The calculation of Δ_{mmr} depends on the calculation of the extreme points of the marginal values. Therefore, notice that the maximization with respect to $s \in \boldsymbol{S}$ and $s_c \in \boldsymbol{S}_c$ can be separated so that the minimax–regret rule appears as

$$\Delta_{mmr} = \min_{\Delta_q \in \boldsymbol{\Delta}} \max_{\Delta_{q'} \in \boldsymbol{\Delta}} \left\{ \frac{1}{n_{q'}} \overline{\Delta}_{q'}(R_{q'}) - \frac{1}{n_q} \underline{\Delta}_q(R_q) \right\}. \tag{4.10}$$

4.4 Strategic Budget Allocation Constraints

To guarantee a certain strategic alignment of the selected portfolio, it is necessary to constrain the allocation of R among $X_1, \ldots, X_Q$; see Section 2.3. Thus, every R_q is allowed to vary between a minimum and maximum ratio to R, generating the following set of constraints.

$$\underline{r}_q R - \underline{\iota}_q(\overline{s}_c^q) \leq R_q \leq \overline{r}_q R + \overline{\iota}_q(\overline{s}_c^q) \tag{4.11}$$

$$R_1 + \cdots + R_Q \leq R \tag{4.12}$$

$$\underline{r}_1 + \cdots + \underline{r}_Q \leq 1 \tag{4.13}$$

$$\overline{r}_1 + \cdots + \overline{r}_Q \geq 1 \tag{4.14}$$

$$0 \leq \underline{r}_q \leq \overline{r}_q \leq 1 \tag{4.15}$$

$$q = 1, \ldots, Q$$

The constants $\underline{r}_q$ and $\overline{r}_q$ are the minimum and maximum proportional ratio for R_q to R. Constraint (4.11), therefore, ensures that every R_q is within its flexible range. The functions $\underline{\iota}_q(\overline{s}_c^q)$ and $\overline{\iota}_q(\overline{s}_c^q)$ in (4.11) only adjust the minimum and maximum ratios to costs of subportfolios:

$$\underline{\iota}_q(\overline{s}_c^q) := \underline{r}_q R - \max\{C(p_q, \overline{s}_c^q) \mid p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q),\ C(p_q, \overline{s}_c^q) \leq \underline{r}_q R\},$$
$$\overline{\iota}_q(\overline{s}_c^q) := \min\{C(p_q, \overline{s}_c^q) \mid p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q),\ C(p_q, \overline{s}_c^q) \geq \overline{r}_q R\} - \overline{r}_q R.$$

The adjustment of the lower bound $\underline{r}_q R$ by $\underline{\iota}_q(\overline{s}_c^q)$ is necessary because if a strategic bucket X_q does not receive additional funding, the budget amount $\underline{\iota}_q(\overline{s}_c^q)$ would be wasted in the sense that it does not enable a new opportunity. In the opposite scenario, if a strategic bucket X_q is funded by the lower bound, the adjustment by $\underline{\iota}_q(\overline{s}_c^q)$ does not destroy any opportunity, see the allocation procedure in the next section.

The adjustment of the upper bound by $\overline{\iota}_q(\overline{s}_c^q)$ is just necessary if the solution space, described by the constraint set above, is empty. Although Constraints (4.13) and (4.14) ensure that R can be continuously allocated among $X_1, \ldots, X_Q$, this is not true if the allocation is based on subportfolio costs, $\overline{s}_c^q$. That is, if strategic bucket X_q is considered to be valuable for additional funding at budget level R_q, the additional budget amount for X_q is given by $\delta_q(R_q)$. Thus, we can imagine a situation in which $R_q + \delta_q(R_q) > \overline{r}_q R$ for $q = 1, \ldots, Q$ but $R - \sum_{q=1}^{Q} R_q \geq \delta_q(R_q)$ holds for at least one $q \in \{1, \ldots, Q\}$. If we increase the upper bound for strategic bucket X_q, we can increase its funding level to $R_q + \delta_q(R_q)$ so that a new opportunity becomes available. Otherwise, the budget amount $R - \sum_{q=1}^{Q} R_q \geq \delta_q(R_q)$ is not utilized, see the allocation procedure in the next section. Constraint (4.12) just ensures that at most the available budget amount R is allocated among $X_1, \ldots, X_Q$.

4.5 Strategic, Preference-Based Budget Allocation

For the allocation of a fixed budget level R among strategic buckets $X_1, \ldots, X_Q$ we will assume that their cost-benefit bands have a concave form, like shown in Figure 3.1. This assumption is motivated by the fact that efficient portfolios at lower budget levels include mostly projects with the best individual cost-benefit ratios. As the budget level increases, projects with less attractive cost-benefit ratios are added to maximize the overall value of the portfolios, see Liesiö et al. (2008). This implies that the increase of the overall value of portfolios drops down with an increasing budget level R_q and the cost-benefit band is concave.

If the assumption about concave cost-benefit bands holds, the most valuable allocation of a fixed budget level R among strategic buckets can be identified via their marginal values. Just start the allocation by the lower bounds for every strategic bucket and allocate the remaining budget stock to strategic buckets with the "best" marginal value. Thus, for the allocation of a fix budget level, set $R_q = \underline{r}_q R - \underline{\iota}_q(\overline{s}^q_c)$ for $q = 1, \ldots, Q$ so that the allocation of $\sum_{q=1}^{Q} R_q$ is already fixed. The remaining budget amount $R - \sum_{q=1}^{Q} R_q$ is optimally allocated among $X_1, \ldots, X_Q$ using their marginal values. Therefore, identify the set of strategic buckets $\boldsymbol{X}_F$ which is not fully funded:

$$\boldsymbol{X}_F := \left\{ X_q \mid R_q < \max\{C(p_q, \overline{s}^q_c) \mid p_q \in P^q_E(\boldsymbol{S}^q, \boldsymbol{S}^q_c)\} \right\}. \tag{4.16}$$

As long as $R < \max\{C(p, \overline{s}_c) \mid p \in P_E(\boldsymbol{S}, \boldsymbol{S}_c)\}$, the set $\boldsymbol{X}_F \neq \emptyset$. From $\boldsymbol{X}_F$, we can determine the set $\boldsymbol{X}_A$ that is qualified for additional allocation:

$$\boldsymbol{X}_A := \left\{ X_q \in \boldsymbol{X}_F \mid \delta_q(R_q) \leq R - \sum_{q=1}^{Q} R_q,\ R_q + \delta_q(R_q) \leq \overline{r}_q R \right\}. \tag{4.17}$$

If $\boldsymbol{X}_A = \emptyset$ and $R - \sum_{q=1}^{Q} R_q \geq \delta_q(R_q)$ for at least one $X_q \in \boldsymbol{X}_F$, set $\overline{r}_q R = \overline{r}_q R + \overline{\iota}_q(\overline{s}^q_c)$ for all $X_q \in \boldsymbol{X}_F$. Once $\boldsymbol{X}_A$ is established, calculate marginal values $\Delta_q(R_q, s^q, s^q_c)$ for all $X_q \in \boldsymbol{X}_A$ and identify the set of Δ_{mmr}. If it is affordable, every strategic bucket within Δ_{mmr} receives additional fund—i.e., we set $R_{q^*} = R_{q^*} + \delta_{q^*}(R_{q^*})$ for all q^* for that $\Delta_{q^*}(R_{q^*}, s^{q^*}, s^{q^*}_c) \in \Delta_{mmr}$ holds. Otherwise, we will identify that combination of strategic buckets within Δ_{mmr} with the highest available number of additionally funded strategic buckets. If we set the corresponding $R_{q^*} = R_{q^*} + \delta_{q^*}(R_{q^*})$ the budget level R should be used optimally. For all increased R_{q^*} check whether $X_{q^*} \in \boldsymbol{X}_F$

holds and for the corresponding ones calculate the new $\delta_{q^*}(R_{q^*})$. The procedure can be started again with the determination of $\boldsymbol{X}_A$ and it can be repeated until the entire budget stock, R, is allocated among $X_1, \dots, X_Q$. That is, $\min_q \delta_q(R_q) > R - \sum_{q=1}^{Q} R_q$. If the remaining budget amount $R - \sum_{q=1}^{Q} R_q > 0$, it is wasted in that sense that it does not enable a new opportunity for costs $\overline{s}_c$.

4.6 Strategic-Bucket–Based Cost–Benefit Analysis and Additional Information

4.6.1 Cost–benefit analysis

The cost–benefit analysis from Section 3.4 is based on sets of non-dominated portfolios, $P_N(\boldsymbol{S}, s_c, R)$. These sets, $P_N(\boldsymbol{S}, s_c, R)$, on the other hand, are based on the sets of feasible portfolios, $P_F(s_c, R)$ from (3.20), which do not consider *flexible* budget allocation constraints (4.11) to (4.15). As a consequence, the sets of $P_N(\boldsymbol{S}, s_c, R)$ are not applicable for a cost–benefit analysis in the case of a strategic-bucket structure.

However, for a fixed R, we can find the optimal levels for $R_1, \dots, R_Q$ through the allocation procedure given in Section 4.5. The budget levels R_q are usable to determine the sets of non-dominated subportfolios $P_N^q(\boldsymbol{S}^q, s_c^q, R_q)$ for any fixed $s_c^q \in \boldsymbol{S}_c^q$. Furthermore, the budget levels R_q are useable to define the set of feasible portfolios with respect to *flexible* allocation constraints (4.11) to (4.15),

$$P_F(s_c, R, \boldsymbol{R}_Q) := \{p \in P_F(s_c, R) \mid C(p_q, s_c^q) \leq R_q \text{ for } q = 1, \dots, Q\}, \tag{4.18}$$

whereby the vector $\boldsymbol{R}_Q \in \mathbb{R}^Q$ contains the budget levels $R_1, \dots, R_Q$ to highlight the dependency on them. From $P_F(s_c, R, \boldsymbol{R}_Q)$, we can determine the set of non-dominated portfolios with respect to *flexible* budget allocation constraints:

$$P_N(\boldsymbol{S}, s_c, R, \boldsymbol{R}_Q) := \{p \in P_F(s_c, R, \boldsymbol{R}_Q) \mid p' \not\succ_{\boldsymbol{S}} p \ \forall p' \in P_F(s_c, R, \boldsymbol{R}_Q)\}. \tag{4.19}$$

Therefore, we can determine the guaranteed overall value and the maximal overall value of the cost–benefit band for X at any fixed R. Moreover, since $P_F(s_c, R, \boldsymbol{R}_Q)$

meets the assumptions for Theorem 4.1, $P_N(\boldsymbol{S}, s_c, R, \boldsymbol{R}_Q) = \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q, s_c^q, R_q)$, so

$$\begin{aligned} MV(R, \boldsymbol{R}_Q) &:= \max_{p \in P_N(\boldsymbol{S}, \underline{s}_c, R, \boldsymbol{R}_Q)} \max_{s \in \boldsymbol{S}} V(p, s) = \sum_{q=1}^{Q} \tfrac{1}{n_q} MV_q(R_q), \\ GV(R, \boldsymbol{R}_Q) &:= \max_{p \in P_N(\boldsymbol{S}, \overline{s}_c, R, \boldsymbol{R}_Q)} \min_{s \in \boldsymbol{S}} V(p, s) = \sum_{q=1}^{Q} \tfrac{1}{n_q} GV_q(R_q). \end{aligned} \tag{4.20}$$

In contrast to $P_N(\boldsymbol{S}, s_c, R)$, which can change its composition at levels $R \in \{C(p, s_c) \mid p \in P_E(\boldsymbol{S}, \boldsymbol{S}_c)\}$ for some fixed s_c only, the set $P_N(\boldsymbol{S}, s_c, R, \boldsymbol{R}_Q)$ may change its composition also for budget levels $R \notin \{C(p, s_c) \mid p \in P_E(\boldsymbol{S}, \boldsymbol{S}_c)\}$. Therefore, note that the set $P_N(\boldsymbol{S}, s_c, R, \boldsymbol{R}_Q)$ depends on the budget levels $R_1, \ldots, R_Q$. The determination of R_q, on the other hand, starts at $\underline{r}_q R - \underline{\iota}(\overline{s}_c^q)$ and is inserted into the calculation of the marginal value $\Delta_q(\underline{r}_q R - \underline{\iota}(\overline{s}_c^q), s^q, s_c^q)$, which serves as a decision base for the final R_q. The allocation procedure stops as soon as no new opportunity for costs $\overline{s}_c$ is available and adjusts upper limits by $\overline{\iota}_q(\overline{s}_c^q)$ if it is necessary. Thus, $P_N(\boldsymbol{S}, s_c, R, \boldsymbol{R}_Q)$ may change its composition at levels $R \in \{C(p_q, \overline{s}_c^q)/\underline{r}_q,\ C(p_q, \overline{s}_c^q)/\overline{r}_q,\ C(p_q, \overline{s}_c^q),\ C(p_q, s_c^q) \mid p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q),\ q = 1, \ldots, Q\}$. For the cost–benefit band this means that the value of the $GV(R, \boldsymbol{R}_Q)$ curve can increase at levels $R \in \{C(p_q, \overline{s}_c^q)/\underline{r}_q,\ C(p_q, \overline{s}_c^q)/\overline{r}_q,\ C(p_q, \overline{s}_c^q) \mid p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q),\ q = 1, \ldots, Q\}$. The levels where the $MV(R, \boldsymbol{R}_Q)$ curve may increase are given by $R \in \{C(p_q, \overline{s}_c^q)/\underline{r}_q,\ C(p_q, \overline{s}_c^q)/\overline{r}_q,\ C(p_q, \overline{s}_c^q),\ C(p_q, \underline{s}_c^q) \mid p_q \in P_E^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q),\ q = 1, \ldots, Q\}$.

4.6.2 Additional information

The distribution of R into $R_1, \ldots, R_Q$ depends on the marginal values, $\Delta_q(R_q, s^q, s_c^q)$, and therefore on the information set $\boldsymbol{S}$ as well as on the cost information $\boldsymbol{S}_c$. The dependency of R_q on $\boldsymbol{S}$ and $\boldsymbol{S}_c$ has adverse effects. If additional information is defined, $\boldsymbol{S}^* \subset \boldsymbol{S}$ with $\mathrm{int}(\boldsymbol{S}) \cap \boldsymbol{S}^* \neq \emptyset$ and/or $\boldsymbol{S}_c^* \subset \boldsymbol{S}_c$, the values of R_q may change even if the overall level R is fixed.

If we highlight the dependency of R_q on $\boldsymbol{S}$ and $\boldsymbol{S}_c$ by $R_q = R_q(\boldsymbol{S}, \boldsymbol{S}_c)$, we may find $R_q(\boldsymbol{S}, \boldsymbol{S}_c) \neq R_q(\boldsymbol{S}^*, \boldsymbol{S}_c^*)$ for fixed R and in further sequence

$$P_N^q(\boldsymbol{S}^{*q}, s_c^{*q}, R_q(\boldsymbol{S}^*, \boldsymbol{S}_c^*)) \nsubseteq P_N^q(\boldsymbol{S}^q, s_c^{*q}, R_q(\boldsymbol{S}, \boldsymbol{S}_c))$$

with fixed $s_c^{*q} \in \boldsymbol{S}^{*q}$. This includes

$$P_N(\boldsymbol{S}^*, s_c^*, R, \boldsymbol{R}_Q) \nsubseteq P_N(\boldsymbol{S}, s_c^*, R, \boldsymbol{R}_Q)$$

since

$$\bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q, s_c^{*q}, R_q(\boldsymbol{S}, \boldsymbol{S}_c)) = P_N(\boldsymbol{S}, s_c^*, R, \boldsymbol{R}_Q)$$

holds for any information set $\boldsymbol{S} = \boldsymbol{S}^1 \times \ldots \times \boldsymbol{S}^Q$ and any fixed cost $s_c^* \in \boldsymbol{S}_c^*$.

However, this means that the results of Theorem 3.2 no longer hold and subsequently Corollaries 3.1 and 3.2 no longer hold either. Since their results are essential for the decision process, the allocation of R among strategic buckets can be calculated just once at the base of $\boldsymbol{S}$ and $\boldsymbol{S}_c$. The minimax–regret rule, which serves as a decision criterion for the distribution of R into $R_1, \ldots, R_Q$, ensures that the greatest possible loss of value relative to other distribution scenarios is smallest for any $\boldsymbol{S}^*$ and $\boldsymbol{S}_c^*$.

Summarizing this chapter, we have introduced a strategic-bucket structure on the set of projects X. The analysis is based on the assumption of separated constraints as well as separated sets of decision criteria and shows the relation of feasible, non-dominated, and efficient subportfolios to feasible, non-dominated, and efficient portfolios (Theorem 4.1). It is also shown how additional information impacts non-dominated subportfolios. The main focus, however, was on the support of strategic budget allocation between strategic buckets. We defined the marginal value for a strategic bucket which is used to identify the optimal allowed budget allocation with respect to the minimax–regret criterion. Finally, the optimal allocation of budget levels, R, was used to define a cost–benefit band for X that satisfies budget-allocation constraints.

Chapter 5

Generation of Non-Dominated, Efficient Portfolios

The RPM model corresponds to a general MOZOLP problem with interval-valued objective-function coefficients. This problem has not been widely studied, so this small chapter outlines the algorithm from Liesiö et al. (2008) for the computation of non-dominated portfolios.

The computation of non-dominated and efficient portfolios is identical in the sense that the set of efficient portfolios is equal to the set of non-dominated portfolios with regard to the extended information set (Theorem 3.3). Thus, we only consider the computation of the set P_N without taking a stance on whether or not the information set includes the cost criterion.

The calculation time of any algorithm depends heavily on the number of projects (see Stummer and Heidenberger 2003). This leads to the disadvantage that problems with a large number of projects may not be solvable in a reasonable time. However, if the assumptions about separated constraints between strategic buckets and own sets of decision criteria hold, Theorem 4.1 may provide a solution. In this case, any algorithm for the calculation of $P_N(\boldsymbol{S})$ can be also used to calculate $P_N^q(\boldsymbol{S}^q)$ $\forall q \in \{1, \ldots, Q\}$. The set of non-dominated portfolios, $P_N(\boldsymbol{S})$, is then obtained by their composition, $P_N(\boldsymbol{S}) = \bigoplus_{q=1}^{Q} P_N^q(\boldsymbol{S}^q)$. Since every strategic bucket, X_q, contains only a subset of the projects in X, the calculation time of $P_N(\boldsymbol{S})$ via the sets $P_N^q(\boldsymbol{S}^q)$ should be considerably shortened. For simplicity of notation, we consider the computation of the set $P_N(\boldsymbol{S})$ only. The computation of any $P_N^q(\boldsymbol{S}^q)$ is then identical.

For the formulation of the algorithm, we often focus the discussion on the non-dummy project-selection constraints without taking care about the dummy project-selection constraints. Therefore, we define matrices $\boldsymbol{B} \in \mathbb{R}^{T\times m}, \tilde{\boldsymbol{B}} \in \mathbb{R}^{T\times K}$, and $\hat{\boldsymbol{B}} \in \mathbb{R}^{T\times L}$ which contain the coefficients of the non-dummy project-selection constraint. Thus, matrix $\boldsymbol{B}$ corresponds to the last T rows from matrix $\boldsymbol{A}$. Analogously, matrices $\tilde{\boldsymbol{B}}$ and $\hat{\boldsymbol{B}}$, respectively, correspond to the last T rows from matrices $\tilde{\boldsymbol{A}}$ and $\hat{\boldsymbol{A}}$, respectively. Additionally, we record the corresponding feasibility limits in the vector $\boldsymbol{\beta} \in \mathbb{R}^T$ which therefore corresponds to the last T rows from the vector $\boldsymbol{\alpha}$. If the discussion is just about the non-dummy project-selection constraints we use matrices $\boldsymbol{B}, \tilde{\boldsymbol{B}}$, and $\hat{\boldsymbol{B}}$ as well as the vector $\boldsymbol{\beta}$ to highlight that we do not consider non-dummy project-selection constraints.

The algorithm builds upon dynamic programming which, in this case, is equivalent to the breadth-first search strategy. Let us define $P^0 := \{\emptyset\}$ and a recursive iteration scheme,

$$P^h = P^{h-1} \cup \{(p \cup \{x^h\}) \mid p \in P^{h-1}\}, \tag{5.1}$$

for $1 \leq h \leq m$. Further, we define $P^{-h} := \{x^{h+1}, \ldots, x^m, \tilde{x}^1, \ldots, \tilde{x}^K, \hat{x}^1, \ldots, \hat{x}^L\}$ so that each auxiliary set, P^h, has the property that if $p \in P^h$, then $p \cap P^{-h} = \emptyset$. Since $P_N \subseteq P_F \subseteq P$, a complete enumeration approach would start with P^0 and go through the iteration (5.1) for all $h \in \{1, \ldots, m\}$ and would further use the same iteration scheme to include dummy projects $\tilde{x}^k$ and $\hat{x}^l$ to obtain P. Infeasible portfolios could be discarded using the feasibility check (3.16), and P_N could be computed from P_F using pairwise dominance checks; see Theorem 3.1. However, since the size of P is 2^{m+K+L}, this approach becomes infeasible in terms of memory requirements and computation time when $m + K + L$ grows even if the assumptions for Theorem 4.1 hold. " For instance, if the generation of P with 20 (dummy) projects takes one second then it would take 1×2^{20} seconds (about 12 days) to generate P with 40 (dummy) projects," Liesiö et al. (2007). As a consequence, more efficient optimization algorithms are needed.

Therefore, at each hth stage of the iteration, we identify and discard portfolios $p \in P^h$ that cannot become non-dominated even if (dummy) projects from the set P^{-h} are added to them. The computational benefits of discarding such portfolios are amplified by the iteration scheme, since if $p \in P^h$ is discarded, any portfolios $p' = p \cup p'', p'' \subseteq \{x^{h+1}, \ldots, x^m\}$ are not included in any of the auxiliary sets $P^{h+1}, \ldots, P^m$. Furthermore, for every portfolio $p' = p \cup p'', p'' \subseteq \{x^{h+1}, \ldots, x^m\}$ it is not necessary to check which interaction effects are active to add the corresponding dummy projects.

However, the infeasibility of portfolio $p \in P^h$ is not a sufficient condition for discarding it. Lemma 5.1 presents a sufficient condition for discarding p on the basis that it cannot become feasible, and thus non-dominated, by including (dummy) projects from the set P^{-h}.

Lemma 5.1. *Let $p \in P^h$. If*

$$\sum_{j=1}^{h} z_j(p) b_t^j + \sum_{j=h+1}^{m} \min\{0, b_t^j\} + \sum_{k=1}^{K} \min\{0, \tilde{b}_t^k\} + \sum_{l=1}^{L} \min\{0, \hat{b}_t^l\} > \beta_t$$

for some $t \in \{1, \ldots, T\}$, then $(p \cup p'') \notin P_N$ for any $p'' \subseteq P^{-h}$.

For the proof, see Liesiö et al. (2008).

Lemma 5.2 compares the overall values and constraint values of two portfolios $p, p' \in P^h$ to determine if p cannot become non-dominated.

Lemma 5.2. *Let $p, p' \in P^h$. If $p' \succ p$ and*

(i) $\boldsymbol{B}\boldsymbol{z}(p') \leq \boldsymbol{B}\boldsymbol{z}(p)$ *(where $\leq$ holds componentwise)*

and

(ii) $\tilde{\boldsymbol{a}}_k \boldsymbol{z}(p') = \tilde{\boldsymbol{a}}_k \boldsymbol{z}(p)$ *for $k = 1, \ldots, K$*
$\hat{\boldsymbol{a}}_l \boldsymbol{z}(p') = \hat{\boldsymbol{a}}_l \boldsymbol{z}(p)$ *for $l = 1, \ldots, L$,*

then $p \cup p'' \notin P_N$ for any $p'' \subseteq P^{-h}$.

For the proof, see Liesiö et al. (2008).

Lemma 5.3 states that a portfolio, $p \in P^h$, for which $\boldsymbol{B}\boldsymbol{z}(p) \leq \boldsymbol{\beta}$ holds cannot become non-dominated if it is *full*. That is, no (dummy) projects can be added to it without losing feasibility, and there exists a feasible portfolio $p' \in P_F$ that dominates p.

Lemma 5.3. *Let $p \in P^h$ for which $\boldsymbol{B}\boldsymbol{z}(p) \leq \boldsymbol{\beta}$ holds. If*

(i) $\sum_{j=1}^{h} z_j(p) b_t^j + \min_{j \in \{h+1, \ldots, m\}} b_t^j > \beta_t$

(ii) $\sum_{j=1}^{h} z_j(p) b_t^j + \min_{k \in \{1, \ldots, K\}} \tilde{b}_t^k > \beta_t$

(iii) $\sum_{j=1}^{h} z_j(p)b_t^j + \min_{l\in\{1,\dots,L\}} \hat{b}_t^l > \beta_t$

for some $t \in \{1,\dots,T\}$*, then* p *is full (i.e.,* $p \cup p'' \notin P_F$ *for any nonempty* $p'' \subseteq P^{-h}$*). Furthermore, if there exists* $p' \in P_F$ *such that* $p' \succ p$*, then* $p \cup p'' \notin P_N$ *for any* $p'' \in P^{-h}$*.*

For the proof, see Liesiö et al. (2008).

For Lemma 5.4, recall that S_w is assumed to be a convex polyhedron so that for fixed score matrices, $\boldsymbol{v}$, $\tilde{\boldsymbol{v}}$, and $\hat{\boldsymbol{v}}$, the overall value, $V(p,s) = \boldsymbol{z}(p)^T\boldsymbol{v}\boldsymbol{w} + \tilde{\boldsymbol{z}}(p)^T\tilde{\boldsymbol{v}}\boldsymbol{w} + \hat{\boldsymbol{z}}(p)^T\hat{\boldsymbol{v}}\boldsymbol{w}$, achieves its maximum at extreme points of S_w (for linear optimization, see Hillier and Lieberman 2004). We denote the extreme points of the convex polyhedron S_w by $\{\boldsymbol{w}^1,\dots,\boldsymbol{w}^E\} := \text{ext}(S_w)$ and the extreme point matrix by $\boldsymbol{W}_{ext} := [\boldsymbol{w}^1,\dots,\boldsymbol{w}^E]$. In Lemma 5.4, the extreme points of S_w are used to calculate an upper bound of how much a portfolio, $p \in P^h$, can still be increased. Thus, we are interested in the maximal overall value of portfolio $p'' \in P^{-h}$, measured at the extreme point $\boldsymbol{w}^e$ of S_w, achievable with the slack in constraints $\boldsymbol{\alpha}_* = \boldsymbol{\alpha} - \boldsymbol{A}\boldsymbol{z}(p)$. This corresponds to an integer linear programming (ILP) problem,

$$\max_{\boldsymbol{z}_* \in \{0,1\}^{m-h}} \Big\{ \boldsymbol{z}_*^T \overline{\boldsymbol{v}}_* \boldsymbol{w}^e + \tilde{\boldsymbol{z}}^T \tilde{\overline{\boldsymbol{v}}} \boldsymbol{w}^e + \hat{\boldsymbol{z}}^T \hat{\overline{\boldsymbol{v}}} \boldsymbol{w}^e \mid \boldsymbol{A}_* \boldsymbol{z}_* + \tilde{\boldsymbol{A}} \tilde{\boldsymbol{z}} + \hat{\boldsymbol{A}} \hat{\boldsymbol{z}} \leq \boldsymbol{\alpha}_*,\ \tilde{\boldsymbol{z}} \in \{0,1\}^K,\ \hat{\boldsymbol{z}} \in \{0,1\}^L \Big\}, \tag{5.2}$$

where $\overline{\boldsymbol{v}}_* = [\overline{\boldsymbol{v}}^{h+1^T},\dots,\overline{\boldsymbol{v}}^{m^T}]^T \in \mathbb{R}^{(m-h)\times n}$ and $\boldsymbol{A}_* = [\boldsymbol{a}^{h+1},\dots,\boldsymbol{a}^m] \in \mathbb{R}^{2(K+L)+T\times(m-h)}$. Finding an exact solution at each extreme point may be too time consuming. Instead, the Lagrangian dual of (5.2) is solved with subgradient optimization, which gives an upper bound, $U_e^{h+1}(\boldsymbol{\alpha}_*)$, for the exact solution with less computational effort.

In Lemma 5.4, the overall value of portfolio $p \in P^h$ plus the upper bound of the increase, denoted by vector $\boldsymbol{U}^{h+1}(\boldsymbol{\alpha}_*) = [U_1^{h+1}(\boldsymbol{\alpha}_*),\dots,U_E^{h+1}(\boldsymbol{\alpha}_*)]^T$, is compared to the overall value of a reference portfolio, $p' \in P_F$, to determine whether p cannot become non-dominated.

Lemma 5.4. *Let* $p' \in P_F$ *and* $p \in P^h$*. If*

$$\boldsymbol{z}(p'\backslash p)^T\underline{\boldsymbol{v}}\boldsymbol{W}_{ext} + \tilde{\boldsymbol{z}}(p')^T\underline{\tilde{\boldsymbol{v}}}\boldsymbol{W}_{ext} + \hat{\boldsymbol{z}}(p')^T\underline{\hat{\boldsymbol{v}}}\boldsymbol{W}_{ext} \gneqq \boldsymbol{z}(p\backslash p')^T\overline{\boldsymbol{v}}\boldsymbol{W}_{ext} + \boldsymbol{U}^{h+1}(\boldsymbol{\alpha} - \boldsymbol{A}\boldsymbol{z}(p))$$

(denoted by $p' \succ^U p$*), then* $(p \cup p'') \notin P_N$ *for any* $p'' \in P^{-h}$*.*

For the proof, see Liesiö et al. (2008).

For Lemma 5.4 to discard portfolios effectively, the reference portfolio p' should have a high overall value over the whole information set $\boldsymbol{S}$. The first step is to generate a set of reference portfolios $P_D \subset P_F$ by solving the ILP-problem (3.17) with random feasible weights and scores. An algorithm for the computation of non-dominated portfolios (CNDP) is formulated as follows.

$P_N = \textbf{CNDP}(\boldsymbol{W}_{ext}, \overline{\boldsymbol{v}}, \underline{\boldsymbol{v}}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\alpha}, \boldsymbol{\beta})\{$

1. Generate P_D
2. $P^0 \leftarrow \{\emptyset\}$
3. **For** $h = 1, \ldots, m$ **do**
   (a) $P^h \leftarrow \{(p \cup \{x^h\}) \mid p \in P^{h-1}\} \cup P^{h-1}$
   (b) $P^h \leftarrow \{p \in P^h \mid \sum_{j=1}^{h} z_j(p) b_t^j + \sum_{j=h+1}^{m} \min\{0, b_t^j\}$
   $+ \sum_{k=1}^{K} \min\{0, \tilde{b}_t^k\} + \sum_{l=1}^{L} \min\{0, \hat{b}_t^l\} \leq \beta_t \ \forall t \in \{1, \ldots, T\}\}$
   (c) $P^h \leftarrow \{p \in P^h \mid p' \not\succ^U p \ \forall p' \in P_D\}$
   (d) $P^h \leftarrow \{p \in P^h \mid p' \not\succ^U p \ \forall p' \in \{p' \in P^h \cap P_F, p'' \not\succ p' \ \forall p'' \in P_D\}\}$
   (e) $P^h \leftarrow \{p \in P^h \mid \nexists p' \in P^h \text{ s.t. } p' \succ p, \ \boldsymbol{B}\boldsymbol{z}(p') \leq \boldsymbol{B}\boldsymbol{z}(p),$
   $\tilde{\boldsymbol{a}}_k \boldsymbol{z}(p') = \tilde{\boldsymbol{a}}_k \boldsymbol{z}(p) \ \forall k \in \{1, \ldots, K\}, \ \hat{\boldsymbol{a}}_l \boldsymbol{z}(p') = \hat{\boldsymbol{a}}_l \boldsymbol{z}(p) \ \forall l \in \{1, \ldots, L\}\}$
4. **For** $k = 1, \ldots, K$ **do**
   $P^m \leftarrow \{p = (p \cup \{\tilde{x}^k\}) \mid p \in P^m, \ \tilde{\boldsymbol{a}}_k \boldsymbol{z}(p) \geq \tilde{m}_k\} \cup \{p \mid p \in P^m, \ \tilde{\boldsymbol{a}}_k \boldsymbol{z}(p) < \tilde{m}_k\}$
5. **For** $l = 1, \ldots, L$ **do**
   $P^m \leftarrow \{p = (p \cup \{\hat{x}^l\}) \mid p \in P^m, \ \hat{\boldsymbol{a}}_l \boldsymbol{z}(p) \leq \hat{m}_l\} \cup \{p \mid p \in P^m, \hat{\boldsymbol{a}}_l \boldsymbol{z}(p) > \hat{m}_l\}$
6. $P^m \leftarrow \{p \in P^m \mid \sum_{j=1}^{m} z_j(p) b_t^j + \sum_{k=1}^{K} \tilde{z}_k(p) \tilde{b}_t^k + \sum_{l=1}^{L} \hat{z}_l(p) \hat{b}_t^l \leq \beta_t \ \forall t \in \{1, \ldots, T\}\}$
7. $P_N \leftarrow \{p \in P^m \mid p' \not\succ p \ \forall p' \in P^m\}$

$\}$

In Step 3, the algorithm runs through the iteration scheme (5.1) for all $h = 1, \ldots, m$. In Step 3(a), P^h is structured by taking each portfolio in P^{h-1} with and without project x^h. Step 3(b) discards portfolios that cannot become feasible by Lemma 5.1. Steps 3(c) and 3(d) discard portfolios by Lemma 5.4, first by using portfolios in P_D as reference portfolios (c) and then using feasible portfolios $p \in P^h$ that are not dominated by any portfolio in P_D as reference portfolios (d). By Lemma 5.3, zero upper bounds are used for full portfolios p when determining whether $p' \succ^U p$ in Steps 3(c) and 3(d). Step 3(e), discards portfolios within P^h by Lemma 5.2. In Steps 4 and 5, dummy projects are added to portfolios $p \in P^m$ so that every portfolio $p \in P^m$ is feasible in terms of project interactions. Step (6) discards portfolios for which $\boldsymbol{B}\boldsymbol{z}(p) + \tilde{\boldsymbol{B}}\tilde{\boldsymbol{z}}(p) + \hat{\boldsymbol{B}}\hat{\boldsymbol{z}}(p) \leq \boldsymbol{\beta}$ does not hold so that $P^m \subseteq P_F$. Finally, in Step 7, P_N is obtained from P^m by discarding dominated portfolios through pairwise dominance checks.

We will remark that instead of Steps 4 and 5 we could add an additional step between 3(a) and 3(b) where the conditions for project interactions are checked and dummy projects are added. This would increase the set of reference portfolios in Step 3(d) by comparison to the outlined approach since every portfolio in P^h is made feasible in terms of project interactions. However, therefore we must check conditions for project interactions in every iteration loop.

Chapter 6

Selection Procedure of the Robust Model and Its Alignment with PPM

This chapter outlines the RPM decision process for comparison with the MDSFPPM requirements. This will facilitate the illustration of the strengths and weaknesses of the RPM model. The construction of the chapter follows the RPM's decision process which is coincident with the decision process of the MDSFPPM. Thus, Section 6.1 presents strategy guidelines and individual project analysis. Section 6.2 focuses on the calculation of non-dominated portfolios while Section 6.3 covers the interactive adjustment and portfolio analysis procedure. Finally, Section 6.4 offers a table to outline which of the MDSFPPM requirements are supported by the RPM model.

6.1 Preselection: Strategy and Project Analysis

6.1.1 Strategic guidelines

For the demonstration of the decision process of the RPM model, we assume again that the entire project portfolio is decomposed into Q lowest-level strategic buckets. Since projects are stored only in the lowest-level strategic buckets, we do not have to confuse ourselves with multilevel hierarchies. If the assumptions for Theorem 4.1 hold, we can establish any set of strategic buckets that are subject to the (sub)portfolio selection. Without loss of generality, we can assume that the first $Q^* \leq Q$ strategic buckets are considered for the portfolio selection. The remaining $Q - Q^*$ buckets are unaffected by the selection process. Thus, as long as the assumptions for Theorem 4.1 hold, Requirement 2.6 is satisfied. For situations where assumptions for Theorem 4.1 are violated, one could try to merge some strategic buckets for the selection process so that the assumptions are met again. Otherwise, the isolated selection of subportfolios is not supported in general.

For the Q^* strategic buckets that are subject to the selection process, it is necessary to determine available amounts of resource categories and budget. In the RPM model, budget and resource categories are treated in the same way, so we can use the budget concept from Section 3.4 and Chapter 4 for each resource category as well. As a consequence, we can define an arbitrary number, $F \in \mathbb{N}$, of resource categories for the decision process. Through resource constraints (3.12), DMs are able to set upper bounds for the budget availability, $\overline{R}$, as well as for the availability of resource categories, $\overline{R}_1, \ldots, \overline{R}_F$. In the same way, we can set lower bounds for budget availability, $\underline{R}$, and availability of resource categories, $\underline{R}_1, \ldots, \underline{R}_F$. To set lower and upper limits for the availability of budget and resource categories means discarding unacceptable availability levels in advance of the cost–benefit analysis.

The opportunity to set lower and upper bounds for the overall availability of budget and resource categories only partly meets the requirements of Table 2.4. Time sensitivities in the availability of budget and resource categories are disregarded since the RPM model does not, in general, consider the time dimension. This is the main weakness of the RPM model since budget and resource utilization are strongly correlated with project scheduling (see Archer and Ghasemzadeh 1999b).

For the case of the budget-based strategic-bucket approach, Requirement 2.4, the budget level R must be allocated among $X_1, \ldots, X_{Q^*}$. To guarantee a strategic allocation of R among the Q^* strategic buckets, DMs can define minimum proportional ratios $\underline{r}_q$ as well as a maximum proportional ratios $\overline{r}_q$ for the strategic buckets. The RPM model supports the identification of the most beneficial allocation of R among strategic buckets with respect to strategic allocation constraints. Summarizing, the combination of Requirements 2.4 and 2.5 is satisfied.

The situation is different for the case of the resource-based strategic-bucket approach, Requirement 2.3. The issue is that allocation decisions among strategic buckets for resource categories and budget cannot be taken independently from each other; they must be taken collectively. The RPM model, however, only paves way for the necessary collective treatment and its implementation needs further research.

The isolated allocation of resource categories and budget among strategic buckets is just usable for a very small number of resource categories—e.g., $F = 1$ or $F = 2$. In this case, DMs may learn how the allocation of resource categories and budget interact each other. However, if the number of resource categories increases, it is not possible to keep an overview about the interactions. Thus, the combination of Requirements 2.3 and 2.5 is not satisfied if the number of resource categories increases.

Although the resource allocation concept from Chapter 4 paves way to implement interaction effects, it may also be valuable to use interval-based data envelopment analysis (DEA). Therefore, strategic buckets represent decision-maker units so DEA can identify the most efficient ones. For interval-based DEA, see (Despotis and Smirlis 2002) or (Kao 2006).

For the evaluation of non-dominated portfolios, DMs define a set of n_q evaluation criteria and priority statements between them for every strategic bucket, i.e. $q = 1, \ldots, Q^*$. The definition of different sets of evaluation criteria between strategic buckets corresponds to Requirement 2.10.

Finally, DMs can analyze projects to identify mandatory projects and balancing criteria. If a project, $x^j \in X_q$ for any $q \in \{1, \ldots, Q^*\}$, is considered mandatory, a simple constraint ensures that every non-dominated subportfolio, $p_q \in P_N^q$, contains x^j, satisfying Requirement 2.1. The DM can also define balancing requirements that can be identified at this early decision stage. Thereby, it does not matter which balancing dimension from Section 2.4.3 is considered with the exception of time. Further, it does not matter whether the budget-based or the project-based approach from Requirement 2.11 is chosen. Summarizing, aside from the time dimension, the preselection requirements for portfolio balancing are met.

6.1.2 Project analysis and project dependencies

Project proposals as well as active projects within the strategic buckets, $X_1, \ldots, X_{Q^*}$, must be evaluated with respect to the corresponding n_q evaluation criteria. For active projects, it may be rather a confirmation or adjustment process. If we assume that $X_1, \ldots, X_{Q^*}$ contain m project proposals and active projects, x^j, DMs must estimate or confirm intervals $[\underline{v}^j_{i_q}, \overline{v}^j_{i_q}]$ for $j = 1, \ldots, m$ and $i_q = 1, \ldots, n_q$. Thereby it does not matter whether a scoring model is used or not.

As with the estimation of evaluation criteria scores, costs, $[\underline{c}^j, \overline{c}^j]$, and resource requirements, $[\underline{c}_f^j, \overline{c}_f^j]$ for $f = 1, \ldots, F$, must be estimated for all projects, $x^1, \ldots, x^m$. For active projects, that means estimating the remaining costs and resource requirements to conform to the concept of sunk costs, Requirement 2.18. The estimation of project parameters by the interval concept is the main strength of the RPM model and conforms to Requirement 2.2. The interval approach takes estimation errors into account and is necessary for a high-quality solution (see Cooper et al. 2001b).

Besides the evaluation of single projects, it is necessary to identify project dependencies as well as programs. With regard to technical project dependencies, DMs can for a successor project, x_s, define a set of predecessor projects, $x_p^1, \ldots, x_p^G$; see Constraint (3.14). However, certain relationships between x_s and any x_p^g for $g \in \{1, \ldots, G\}$, as stated in Requirement 2.14, are not possible since the RPM model ignores the time dimension.

For economical project dependencies, assume a project set, $\tilde{X}^k \in X_q$ or $\hat{X}^l \in X_q$, for which a synergy or cannibalization effect is expected. Synergy or cannibalization effects with respect to evaluation criteria and requirements on resource categories are modelled by dummy projects $\tilde{x}^k$ or $\hat{x}^l$ so that the interval concept for parameter evaluation holds. Since mutually exclusive projects are simply modelled by constraint (3.13), Requirement 2.15 is satisfied.

The approach for economical project interactions is usable to define programs, as stated in Requirement 2.16. Therefore, assume a project set, $\tilde{X}^p$, which is considered as a program. The difference from a common economic project interaction is that just some compositions of the projects within $\tilde{X}^p$ are allowed (mutually exclusive projects, technical dependencies, mandatory projects, etc.). As long as the composition of the program does not take time constraints into account, it can be modelled via constraints as outlined in Section 3.2. With respect to the evaluation process, benefits and resource requirements of a program should primarily be described by the corresponding dummy project, $\tilde{x}^p$. That is, by the intervals $[\underline{\tilde{v}}_{i_q}^p, \overline{\tilde{v}}_{i_q}^p]$ $\underline{\tilde{c}}^p, \overline{\tilde{c}}^p$, and $[\underline{\tilde{c}}_f^p, \overline{\tilde{c}}_f^p]$. For any project, $x_p^j \in \tilde{X}^p$, the interval parameters should just be used to describe project-specific contributions to the program; see Section 6.3.

6.2 Generation of Efficient Portfolios

If the assumptions for Theorem 4.1 hold, one can calculate the sets of efficient subportfolios, $P_N^1, \ldots, P_N^{Q^*}$. Since the computational effort increases exponentially with every additional project, the partition of the entire calculation problem into subproblems offers a considerable advantage. Instead of a single calculation problem with m projects, we have to solve Q^* subproblems with $m_1, \ldots, m_{Q^*}$ projects and $m = m_1 + \ldots + m_{Q^*}$ holds.

To further speed up the calculation of P_N^q, notice that X_q contains a set of active projects, $X_q^{act} \subseteq X_q$. In the previous subportfolio selection procedure, X_q^{act} was a part of the selected subportfolio p_q^{sel}. Projects within the set $p_q^{sel} \backslash X_q^{act}$ are either successfully finished or terminated. Since we can assume that p_q^{sel} has a high overall value and $X_q^{act} \subseteq p_q^{sel}$, feasible combinations of X_q^{act} with new project proposals $x^j \in X_q \backslash X_q^{act}$ are useful as reference subportfolios in P_D^q, which are required for Lemma 5.4 to accelerate the calculation of P_N^q.

Within the scope of this thesis, we do not perform calculation experiments with the dedicated dynamic programming algorithm from Chapter 5. However, Liesiö et al. (2008) reported that the algorithm handles problems with some 60 projects on a personal computer in a reasonable time. It is further reported that the calculation time is highly dependent on the types of constraints and correlates between project scores and constraint coefficient so that further simulation studies are necessary. However, it should be taken into account that the calculation has to be performed just once because in the subsequent interactive decision process, additional information can only reduce the set of non-dominated subportfolios (Theorem 3.2 in combination with Theorem 4.1). As a consequence, the calculation of the initial set of non-dominated portfolios by the dedicated algorithm may be started days before DMs meet for the interactive decision process so that the calculation time may not be critical for many corporations (Stummer and Heidenberger 2003).

According to experiences collected during an internship, corporations store up to 100 projects within a single strategic bucket. Since, from a technical point of view, project versions are treated as an ordinary project x^j, the number of *technical projects*—i.e., the number of projects m which is considered in the RPM model—may by considerably higher than 100. If in the future the time dimension can be incorporated, the number of technical projects will further increase (see Stummer and Heidenberger 2003). Thus, further research into algorithms for the calculation of non-dominated portfolios based on incomplete information is necessary. To our knowledge, the RPM research group has developed in the interim an algorithm which handles up to 100 projects. Additionally, there is the discussion of a heuristic algorithm that can work with hundreds of projects.

The allocation of the overall budget level R among strategic buckets is not adjusted to additional information; see Section 4.6. Hence, the allocation procedure from Section 4.5 to distribute R into $R_1, \ldots, R_{Q^*}$ can be started in advance to the interactive decision process. Arguments about the calculation time of non-dominated portfolios therefore also hold for the calculation time of the allocation procedure of R (and $R^1, \ldots, R^F$ for the case of the resource-based strategic-bucket approach).

6.3 Interactive Decision Process

6.3.1 Cost-benefit analysis

The interactive decision process starts with the efficient subportfolios, $P_E^1(\boldsymbol{S}^1, \boldsymbol{S}_c^1), \ldots,$ $P_E^{Q^*}(\boldsymbol{S}^{Q^*}, \boldsymbol{S}_c^{Q^*})$, and the set of efficient portfolios, $P_E(\boldsymbol{S}, \boldsymbol{S}_c)$. Further, overall cost–benefit bands for budget and resource categories are available since the allocation of overall availability levels, $R, R^1, \ldots, R^F$, among strategic buckets has already been calculated in advance. To be consistent with foregoing chapters, we outline the cost–benefit decision by reference to budget.

The bounds of the budget overall cost–benefit band $GV(R, \boldsymbol{R}_{Q^*})$ and $MV(R, \boldsymbol{R}_{Q^*})$ are helpful for the identification of the most beneficial budget level R. For instance, R may be set at a level where $GV(R, \boldsymbol{R}_{Q^*})$ first exceeds a given threshold. Further, one could employ the cost–benefit ratios $GV(R, \boldsymbol{R}_{Q^*})/R$ and $MV(R, \boldsymbol{R}_{Q^*})/R$ to determine the final budget level. Additionally, one can employ the budget-dependent core-index plot, see Figure 3.2, to identify a budget level R^* at which a reference project first enters a non-dominated portfolio (Liesiö et al. 2008). The determination of the overall availability level R implicitly determines the availability levels R_q for strategic buckets; see Section 4.6. Thus, DMs may also drill down their analysis into the cost–benefit bands for $X_1, \ldots, X_{Q^*}$ to determine R.

For the case that multiple cost–benefit bands and resource categories are analyzed, DMs need to take into account interaction effects between them. For instance, an isolated increase of the availability level R^f is useless if the portfolio also lacks a different resource category, f^*. In an overview, Kendall and Rollins (2003) suggested focusing the cost–benefit analysis just on the most critical resource categories. Summarizing, the cost–benefit analysis concept supports Requirements 2.9 and 2.7, apart from the time dimension.

6.3.2 Core-index–based portfolio selection

If DMs have identified an interesting combination of availability levels, $R, R^1, \ldots, R^F$, they can drill down their analysis into the corresponding set of non-dominated (sub)-portfolios. The core-index approach is especially valuable for this purpose. However, in this thesis, we only have introduced core indices that depend on budget, not on multiple resource categories. Although especially the alternative budget-dependent core index may be expandable for situations with multiple resource categories, the restriction to budget is a main weakness at the moment. Since the further decision process is based on the core index, we must restrict the discussion to the situation without resource categories, knowing that this is a considerable weakness.

Thus, we assume that DMs have identified an interesting budget level R and therefore will identify the most preferred project portfolio for this budget level. The most preferred portfolio for budget level R, information set $\boldsymbol{S}$, and cost information $\boldsymbol{S}_c$ is contained in

$$P_N(\boldsymbol{S}, \boldsymbol{S}_c, R) = \bigoplus_{q=1}^{Q^*} P_N^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q, R_q). \tag{6.1}$$

To ensure that every portfolio, $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$, is subject to the analysis, we use the alternative budget-dependent core index $CI(x^j, \boldsymbol{S}, \boldsymbol{S}_c, R)$ from Definition 3.8. Due to Corollary 3.2, core projects, $x^j \in X_C(\boldsymbol{S}, \boldsymbol{S}_c, R)$, can surely be selected, even under their worst scenarios. Similarly, exterior projects, $x^j \in X_E(\boldsymbol{S}, \boldsymbol{S}_c, R)$, can surely be discarded even under their best scenarios. Thus, the choice about core and exterior projects takes all predictable situations into account and therefore fits Requirement 2.12; this is surely a main strength of the RPM model.

The choice about borderline projects, $x^j \in X_B(\boldsymbol{S}, \boldsymbol{S}_c, R)$, is undetermined. However, the interactive decision process of the RPM model supports the decision about borderline projects. According to Corollary 3.2 DMs are advised to reduce their efforts to define additional information $\boldsymbol{S}^*$ to borderline projects, and, due to Theorem 3.6, efforts to define additional cost information $\boldsymbol{S}_c^*$ can be reduced to borderline projects and core projects. Considering fewer projects considerably reduces the complexity of the decision process. The impact of additional information $\boldsymbol{S}^*$ or cost information $\boldsymbol{S}_c^*$ on $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ is a straightforward calculation from Theorems 3.2 and 3.5. Thus,

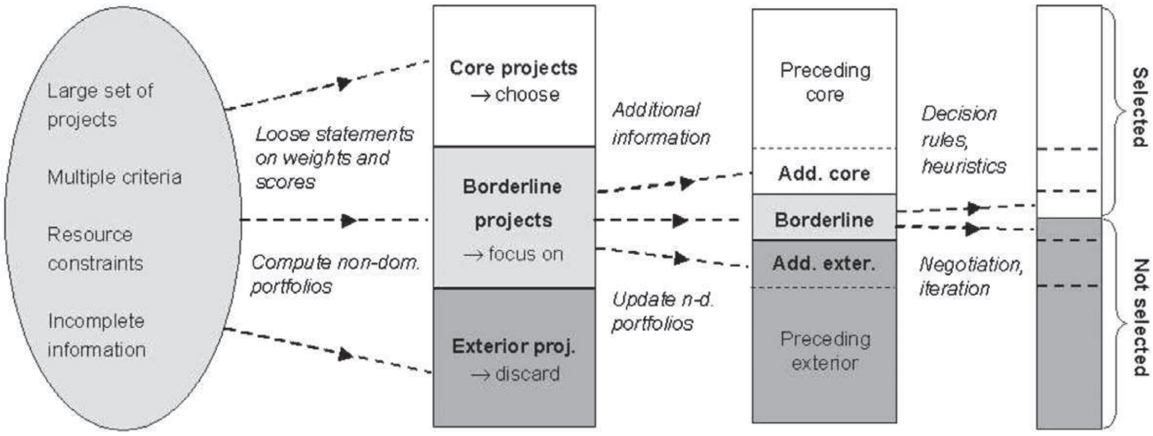


Figure 6.1: RPM—Decision Support Process
Source (Liesiö et al. 2007).

DMs immediately receive feedback about their actions as called for in Requirement 2.17.

The definition of additional information and cost information and the calculation of their impact on $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ can be understood through a simulation. This facilitates the generation of What-If scenarios of Section 2.5.2 (simulations of portfolios with different input parameters). The RPM model even guides DMs through the choice of input parameters. It tells DMs that only adjustments to parameters of borderline projects or adjustments to costs of core projects have effects. To promote learning, it may be advisable to proceed from a relatively incomplete information set towards a more complete one. In the course of such an iterative process, the DM can learn, for instance, when a particular project is identified as one of the core or exterior projects. Moreover, the selection of the final project portfolio can be defended by showing which projects were among the core and borderline projects, respectively. It is even possible to backtrack and show at what stage core projects acquired their status (Liesiö et al. 2007). Figure 6.1 outlines this phase of the decision process.

The concept of the alternative budget-dependent core index is also applicable to project interactions $CI(\tilde{x}^k, \boldsymbol{S}, \boldsymbol{S}_c, R)$ and $CI(\hat{x}^l, \boldsymbol{S}, \boldsymbol{S}_c, R)$. The core index of a dummy project illustrates how significant the corresponding interaction effects are at the portfolio level. Furthermore, we can assume that dummy project $\tilde{x}^p$ represents a program whose benefits and costs are estimated as stated in Section 6.1. Then, the decision support obtained by $CI(\tilde{x}^p, \boldsymbol{S}, \boldsymbol{S}_c, R)$ is similar to the decision support

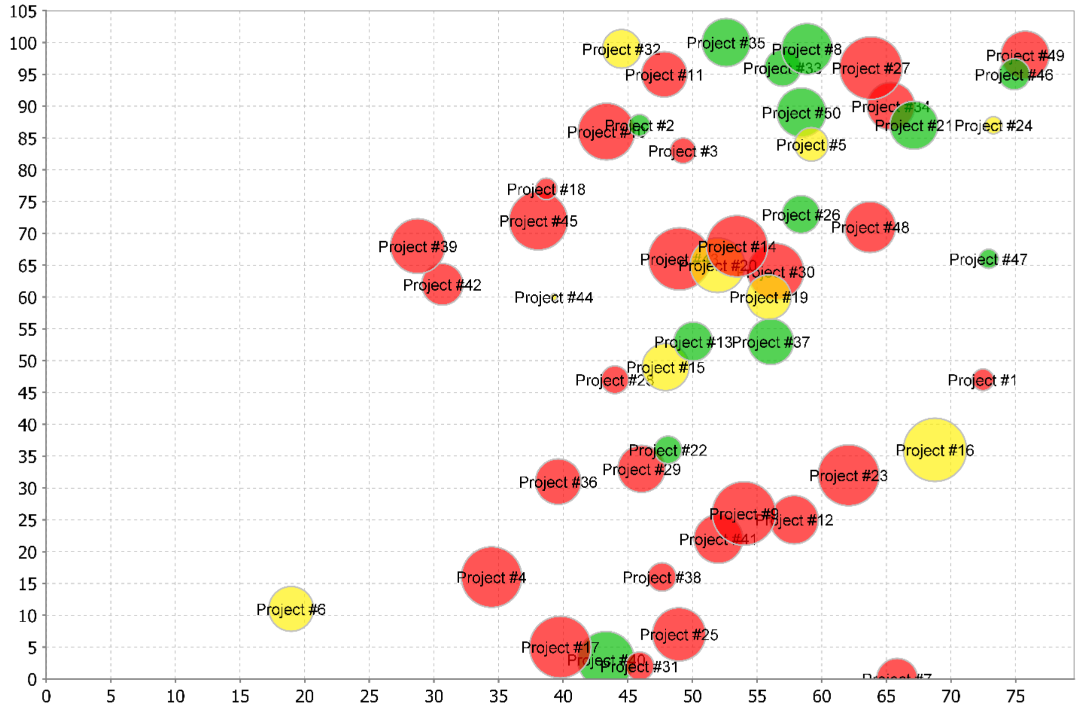


Figure 6.2: Bubble Diagram with Core, Exterior, and Borderline Projects
Source RPM Group.

obtained by the core index of a common project $CI(x^j, \boldsymbol{S}, \boldsymbol{S}_c, R)$. If program $\tilde{x}^p$ is selected, DMs can further drill down their analysis to core indices of projects that belong to that program, $x^j \in \tilde{X}^p$. This may help make a decision about the final composition of the program and fulfills Requirement 2.16.

Another way the core index supports the project portfolio decision is the charts frequently used to achieve a desired balance. For instance, in a bubble diagram like the one outlined in Figure 2.13, we can insert all m projects and highlight core projects as green, exterior projects as red, and borderline projects as yellow. Figure 6.2 represents an example with $m = 50$ projects and programs.

Borderline projects can be used to balance the project portfolio. Therefore, DMs can mark a certain borderline project, $x^B \in X_B(\boldsymbol{S}, \boldsymbol{S}_c, R)$, as mandatory so that all portfolios, $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$, for which $x^B \notin p$ holds are immediately discarded. The rejection of portfolios may indicate further borderline projects, $x^j \in X_B(\boldsymbol{S}, \boldsymbol{S}_c, R)$, with $x^j \neq x^B$ as core projects or as exterior projects. For instance, the selection of borderline project x^5 in Figure 6.2 indicates that borderline project x^{19} is a core and projects x^{20}, x^{24}, and x^{32} are exterior; see Figure 6.3

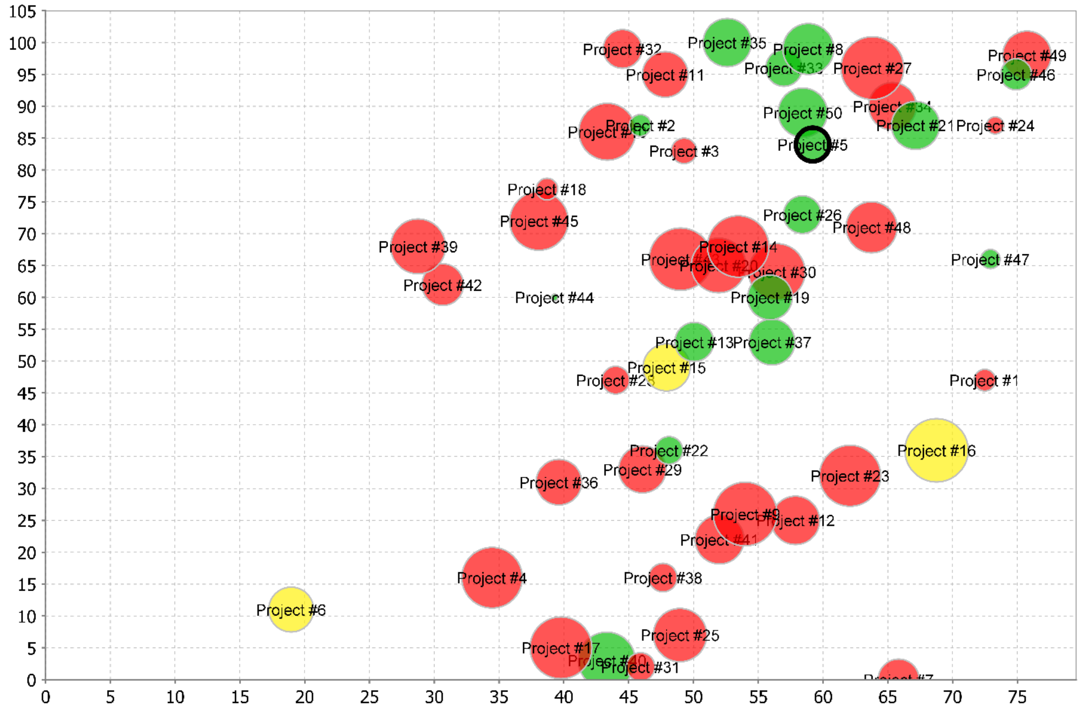


Figure 6.3: Bubble Diagram After Identifying Borderline Project x^5 as Mandatory
Source RPM Group.

The set of non-dominated portfolios, $P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$, provides further opportunities for the interactive decision process, even aside from the core index. For instance, it could be used to analyze scores of individual evaluation criteria to identify thresholds. Portfolios $p \in P_N(\boldsymbol{S}, \boldsymbol{S}_c, R)$ not meeting those thresholds are rejected. However, for our purpose, it is sufficient to know that the RPM model provides excellent opportunities for an interactive decision process so that we can consider Requirement 2.17 as satisfied. Details about an interactive decision process based on evaluation criteria scores can be found in Stummer and Heidenberger (2003).

Regarding portfolio maintenance, we will mention that the core index of a project also supports gate decisions of individual projects; see Requirement 2.18. The termination of a borderline project has a limited impact on the entire portfolio since there are non-dominated portfolios available that do not contain the borderline project. In contrast, the termination of a core project endangers the entire project portfolio so that an emergency portfolio review may take place.

Finally, how the RPM model supports decision with respect to Requirements 2.19 and 2.20 depends on the situation at hand. If the impacts on the ongoing portfolio are expected to be considerable, it may be necessary to start the decision process from the beginning. Otherwise, it may be sufficient to find the optimal portfolio starting with the interactive decision process. For instance, if the mission update corresponds to additional information about evaluation criteria weights, $S_w^* \subseteq S_w$, it is sufficient to use the interactive decision process.

6.4 Strengths and Weaknesses of the Robust Model

For the purposes of a better overview of how the RPM model supports the requirements of the MDSFPPM, we repeat the table of requirements for the MDSFPPM. The last column in Table 6.1, therefore, describes how the corresponding requirement is handled in the RPM model. *Supported* means that the requirement is totally covered by the RPM model. In contrast, *Unsupported* means that the requirement is not covered in general by the RPM model. *Partly supported* means that just parts of the requirement are covered by the RPM model, but the entire requirement is not.

We identified the missing time dimension as a main weakness of the RPM model. Therefore, we highlight those requirements whose treatment in the RPM model is unsatisfactory due to the missing time dimension by the word *Time*. Further, we use the word *Resources* to highlight an insufficient treatment of a requirement if it is based on the isolated treatment of resource categories and budget, the second major weakness of the RPM model. Note, the isolated treatment of resource categories and budget implies a weaknesses of the core index; see Section 6.3.

Table 6.1: Requirements for the MDSFPPM supported by the RPM model

Requirement Number	Requirement Name	Satisfaction in RPM
Requirement 2.1	Mandatory projects	Supported
Requirement 2.2	Imprecise project information	Supported
Requirement 2.3	Strategic buckets and resources	Partly supported, Time, Resources
Requirement 2.4	Strategic buckets and budget	Partly supported, Time
Requirement 2.5	Strategic resource allocation	Partly supported, Time, Resources
Requirement 2.6	Subportfolios	Supported
Requirement 2.7	Human resource planning	Partly supported, Time, Resources
Requirement 2.8	Role staffing	Not discussed
Requirement 2.9	Budget resource planning	Partly supported, Time, Resources
Requirement 2.10	Trade-off between portfolio values	Supported
Requirement 2.11	Portfolio balancing	Partly supported, Time
Requirement 2.12	Risk analysis	Supported
Requirement 2.13	Project scheduling	Unsupported, Time
Requirement 2.14	Technical project dependencies	Partly supported, Time
Requirement 2.15	Economical project interactions	Supported
Requirement 2.16	Program selection and composition	Supported
Requirement 2.17	Portfolio adjustment features	Partly supported, Time, Resources
Requirement 2.18	Project development progress evaluation	Partly supported, Time, Resources
Requirement 2.19	Mission update and portfolio adjusting	Partly supported
Requirement 2.20	New hot project proposal	Partly supported, Time

Chapter 7

Conclusions

7.1 Summary

The primary focus of this thesis is the design of a mathematical optimization model that can be embedded into existing PPM software. Therefore, we formulated numerous requirements which must be satisfied by the mathematical model. However, since we were not addressing a certain software product, we tried to formulate the requirements very generally. If a certain PPM software is considered, it is certainly necessary to specify the set of requirements more precisely. Chapter 2, which describes the requirements, should also help to avoid misunderstandings between developers of mathematical optimization models and their users.

As the mathematical optimization model, we introduced the RPM approach from Liesiö et al. (2007) and Liesiö et al. (2008) since it seems to be a promising approach to be embedded into PPM software. Thereby, we explicitly considered project interactions. However, the main focus here is the extension of the RPM model to permit a strategic-bucket structure, optimizing budget allocation among buckets. Therefore, we divided the entire project set into subsets which represent strategic buckets. We introduced restrictions so that decisions within strategic buckets can be taken independently of each other. Furthermore, we defined the marginal value for strategic buckets which serves as the decision criterion for the optimal budget allocation between them. We outlined a possible budget allocation procedure subject to strategic-budget allocation constraints. Nevertheless, in this thesis, we only outlined the theory, leaving simulations of the behavior of the extension for the future.

Besides the strategic buckets, we enhanced the RPM model with the alternative budget-dependent core index. Even here, simulation experiments have not been performed, so some adjustments may be necessary during implementation.

Finally, we presented the algorithm from Liesiö et al. (2008) for the computation of non-dominated/efficient portfolios as well as the decision process from the RPM model. The demonstration of the decision process served as a basis to outline the strengths and weaknesses of the enhanced RPM model.

7.2 Further Research Areas

Several areas of research remain unsolved in the extension of the RPM model. First, the definition of the overall value of a portfolio under a strategic-bucket structure needs a more detailed investigation. We introduced a normalization so that there are no priorities between strategic buckets (priorities of strategic buckets are rather formulated as strategic-budget allocation constraints). However, the normalization of project scores, which is necessary for the case of multiple attributes, is an open issue, see Clemen and Smith (2009). As a consequence, the normalization of the overall values of subportfolios by $\frac{1}{n_q}$ may be inappropriate. Besides the normalization issues, it may be interesting to introduce weighting factors for the overall values of subportfolios to incorporate the priority of strategic buckets into the overall value of a portfolio.

Second, the outlined budget allocation among strategic buckets is, so far, just a theoretical approach; simulation studies may reveal adjustments to be made to the extension. For instance, in the definition of the marginal value, it may be valuable to replace $P_N^q(\boldsymbol{S}^q, \overline{s}^q, R_q)$ with $P_N^q(\boldsymbol{S}^q, \boldsymbol{S}_c^q, R_q)$ to account for all non-dominated subportfolios. Also, besides the definition of the marginal value, it may be beneficial to use a decision criterion different from the suggested minimax–regret rule.

Third, the outlined approach for the optimal budget allocation among strategic buckets is just one of several conceivable approaches. For instance, one could take a representative score and weight scenario as well as a representative cost scenario for every strategic bucket. For the budget allocation, weights are assigned to strategic buckets; to comply with possible uncertainties, imprecise information about the weights must be respected. For a certain budget stock, one could identify a robust allocation among strategic buckets by using an interactive decision process. Another approach may be given by using an interval-based DEA concept. Strategic buckets would represent the decision-maker units so that the DEA concept identifies the efficient strategic buckets.

Finally, the main weakness of the RPM model in its current form is the lack of the time dimension. Project scheduling, however, is important for various reasons, so research efforts must be placed on selecting and scheduling projects. When doing so, one must know how much schedule work is required from the RPM model and how much schedule work should be done by the optimizer for role staffing, see Requirement 2.8.

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