TECHNISCHE UNIVERSITÄT WIEN

VIENNA UNIVERSITY OF TECHNOLOGY

D I S S E R T A T I O N

FROM MAP AND COMPASS TO UBIQUITOUS NAVIGATION

How navigation tools, strategies and errors work in a natural environment

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A thesis submitted in partial fulfilment of the requirements for the degree of
Doctor of Technical Sciences

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by

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ABSTRACT

The thesis demonstrates the hypothesis, that the *error* characteristics of the navigation *tool* determine the *strategy* of navigation.

We first define the basic terms related to the navigation, wayfinding, cognition and locomotion. Among them, the terms dead reckoning, path integration and updating are exposed. We introduce the ideas of the optimum path and the different least-cost paths, including the fastest path.

We introduce the navigation with map and compass, and the technique of orienteering in a natural environment. We decompose the strategy and the execution of such a navigation into primitive actions in the framework of the sense-plan-act architecture. The optimum path is hierarchically divided into legs, runs, and segments, where legs as a part of the orienteering course lie between two control points, runs lie between two waypoints, and segments represent the chunks of optimum path with homogeneous friction and risk properties. The planned and the executed paths are in principle different. A separate treatment is dedicated to the errors in navigation, where we focus on the cognitive and physical background of errors, and not on the positional accuracy of navigation.

To compare the different tools and strategies, we describe the technically augmented navigation, as an opposition to the classical orienteering with map and compass. Two devices are presented: a GPS receiver with a screen map, and a GNSS receiver with a screen map which is hypothetically functioning everywhere. The strategy, the execution, and the errors are analysed in a comparable way to the orienteering navigation case.

The most important part of the thesis is the simulation, where we numerically demonstrate the hypothesis with the vector-type cognitive approach. We take an orienteering map, we choose the origin and the destination, and we draw several optimum paths between both. Then we cognitively define the waypoints and segments along each of the different optimum paths. The optimum path condition requires, that the (fictive) navigator has to travel the distance in the shortest time.

We empirically and experientally construct the spreadsheets of frictions and risks for each of the three tools. The following are computed: the resistance to locomotion, the navigation risk, the anisotropic slope friction, and the dead reckoning and waypoint discernibility risk. From the distances, frictions, risks, and the average running pace of the navigator, we then compute the cost and time for each run, each optimum path, and each tool. From the results we infer the general characteristics of risks and strategies regarding the three tools used on open areas and in a forest. The general strategy of navigation with map and compass is dead reckoning, aided by feature matching, while for the GPS and GNSS receiver cases the strategy is positioning aided by the display of straight direction to the next waypoint on a screen map.

We observe, that the time of travel functionally depends on cost and pace, where the pace depends on the physical condition of the navigator, and the cost depends on distances, frictions and risks. The distance is influenced by the position of origin, destination and waypoints. The frictions depend on the environment, however the risks depend on the strategy, where the strategy depends on the tool. Finally, we conclude that the tool provides affordances for the emerging errors. The series of formal statments within an IF clause positively demonstrates the hypothesis.

Keywords
Navigation, orienteering, cognition. Tool, strategy, error.
Map, compass, GPS, GNSS. Least-cost path, fastest path, waypoint. Friction, risk.

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I have first met professor Frank ten years ago when we were jointly working on a training centre project. I was immediatelly impressed by his skill of lecturing and profound scientific vision. One late afternoon, just before the delivery of the project, he concluded our discussion with a firm statement that I must make a PhD. I still have that image in my head.

In the mean time I learned, that for the choice of a suitable dissertation theme one has to know much more than myself. With my geodetic spatial reasoning in the head I figured out that within a reasonable driving distance from my hometown professor Frank would be the best choice for a supervisor. I was definitely not wrong. He gave me inspiration and ideas, but above all, he taught me to think so different than I was used to.

Thank you, professor.

CONTENTS

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1. INTRODUCTION

In the introduction, we present the research topic of navigation and the general practical applications, which motivated the investigation. We then focus on a single navigation problem in a natural environment to develop the goals and the hypothesis, which has to be formalized and demonstrated. We present the methodology and organization of the work, which will finally lead to the new insights into the topic.

1.1. Introduction of the research topic

This thesis deals with the topics of navigation, wayfinding, and orientation. Every human being and mobile animal must use a certain degree of navigation skills while moving through space. People navigate every day: they move through the rooms at home, walk to the neighbouring buildings and back home, stroll in the park or in a forest, or travel with a vehicle to job. Sometimes they make a voyage on a longer distance, travel to other cities, trek accross the mountains, they fly by plane, sail at sea, or even circle the globe in space. Every such dynamic action needs some kind of navigation.

When people travel in unknown or less known places, they use navigation tools to successfully reach the destination. The use of tools requires planning and execution of the movement, which among other processes, activates perception, cognition, and knowledge. The tools are interrelated with the method of navigation. In a complex world, the navigation process is error prone. We try to investigate the relations between the navigation strategy, the tools, and their associated errors, for a special case of human navigation in a natural environment.

1.2. Motivation

The first primitive maps appear as far away back as in the Stone age. The compass was invented in the 11th century by the Chinese and revolutionized navigation at sea and on land. Many other navigation techniques and tools were developed in the history of a mankind, however never was the quest for ubiquitous navigation services and tools more important than after the invention of satellite navigation with GPS (Global Positioning System) and other GNSSs (Global Navigation Satellite Systems), and after the introduction of mobile telecommunications into everyday life.

While extensive research has been done on the technical development of tools, computational methods, and navigation services, few research exists on the explanation how navigation tools and the errors produced with their usage are interrelated with the respective cognitive navigation strategies. Many new research questions arise if we try to conceptually connect the tools, the errors, and the strategies of navigation, eg.:

- How does the strategy of navigation change if we change, amend, or alter the tools?
- How does the optimum path with regard to a chosen condition change if we change the tools?
- Which are the potential errors of navigation with specific tools, and what are their consequences on the optimum path and the travel as a whole?
- Which errors are caused in turn by the tool, by the planning of navigation, and by the execution of navigation?
- How do in turn, the potential errors of the tools affect the planning, how do the errors of planning affect the execution, and how do the errors of execution alter the realization of the planned optimum path?
- How do people cognitivelly cope with generalization, fuzziness, and complexity of the real world in all phases of planning and execution with specific tools?

- Can cognitive navigation skills be completely replaced by some technically augmented navigation device, and what would be the corresponding computational model?
- What is the remaining role of human cognition in the planning of navigation, if we use technically augmented tools?
- How does the decreasing number of potential errors, when using technically augmented tools, increase the feeling of false safety by the user, and what are the consequences and strategies when the tool fails to operate in the middle of navigation? How can we measure this risk?

Some of these question will be partly answered in this thesis. Many strategies used by the specific tools have arisen directly out of the praxis in a real environment, without any cognitive and computational scientific explanation of the corresponding risks of errors. The formalization, evaluation and numerical computation of risks is also an important topic, which is addressed in the thesis. For example, a switch from the classic method of navigation with map and compass to the navigation with technically augmented navigation device can reduce the risks of travel, but would lead to modifications of the strategy of navigation, and to different habits of future navigators.

Most research focuses upon navigation in a urban environment, which has a totally different spatial and visual structure than a natural environment. These differences affect the choice of tools, the strategy, and the potential errors. We argue that the urban environment is just a special, relatively predictable, repetitive, and geometrically well organized case of structures found in a natural environment.

1.3. Definition of the problem

In this chapter, we describe the general assumptions of the research, which help the reader to understand the hypothesis. The specific navigation case can be described as follows.

A single person on foot with the selected tools for navigation is going to navigate over a physical topographic surface in a natural environment from a given standpoint to the destination object.

In principle, the *person* is not familiar with the area of navigation, and the person's navigation skills are adequate for proper use of the tools. The person can locomote, percept, sense, and deliberate normally and rationally.

The *physical topographic surface* exhibits different micro- and macromorphological features, surface ruggedness, vegetation cover, or unpassable topographic objects (ie. natural or built-up obstacles). We presume, that the area of navigation is a normally differentiated natural environment or countryside, where we exclude urbanized areas, and navigable water surfaces.

For the investigation, we concentrate on two cases. The person's *tools for navigation* could be either (a) compass and paper map, or (b) technically augmented navigation device (ie. a portable, electronic, integrated navigation device). Regardless of the tool, the goal of the person is to navigate along an *optimum path* regarding a chosen condition referring to some least-cost criterium such as eg. the fastest path. Both techniques of navigation are prone to *errors* of various origin. The chosen optimum path has different risks of errors for each set of tools. It may appear optimal for one tool but not for the other. We argue that the differences in the tools and the types of errors influence the choice of optimal strategy.

1.4. Research goals

We research how the tools, the potential errors, and the strategies depend on each other, and how the dependence can be demonstrated and formalized. Since the navigator wants to minimize the errors, we propose that he chooses the optimal strategy, which will allow him to use efficiently the tools for navigation. Consequently, the goals of the thesis are to:

a) describe in detail the process of navigation with compass and map,
b) describe the technically augmented navigation procedure,
c) describe the errors of navigation,
d) assess and compute the different frictions and risks of navigation along the optimum path regarding the chosen optimum path condition,
e) simulate the navigation and calculate the cost and time of navigation using the numerical values of frictions and risks,
f) draw inferences about the impact of imperfect tools, the associated errors, and the navigation strategy on the cost of travel along the optimum path,
g) demonstrate and formalize the dependence between the navigation parameters.

1.5. Hypothesis

By summarizing the ideas from the chapters above, we can write the following *hypothesis*.

The *error* characteristics of the navigation *tool* determine the *strategy* of navigation.

In a more causal and formal way, we can write the following explanation of the hypothesis.

Bearing in mind that:

- in the evolution of a mankind, the navigation strategy was always adapted to the navigator's physical and cognitive sources of information about the natural environment, which have been provided by the tools for navigation, maps and topographic data, observations, perceptions, beliefs, experiences, and knowledge,

the thesis argues that:

- the errors of navigation are a consequence of the disparity between the navigator's cognitive model of the environment, formed from his imperfect and generalized sources of information, and infinitely complex natural environment. The navigator adapts to the error characteristics of the tools, and selects the appropriate strategy in order to follow the optimum path. The navigator compensates the imperfection of the tools and other information sources with the appropriate strategy to sustain the quality of navigation, reduce risks, and raise probability to reach the destination.

1.6. Research methodology

The thesis combines several research fields spanning from cognitive to geographic information science. We first show how the cognitive part of the navigation process and the type of environment affect the success of navigation. We suppose that the individual navigator chooses the optimum path criterium (eg. the fastest path) to reach some destination by travelling along some optimum path which is person-specific. We describe the environmental frictions (eg. vegetation, slopes) and risks (eg. to get lost) which produce fictive costs (eg. in terms of time) spent for the travel.

We suppose that the navigator masters the orientation with compass and the map reading technique. The orienteering sport is taken to study the frictions and risks. Before we model them, we distinguish the planning and the execution phases of orienteering as much as possible. Each of the two phases is divided into the basic procedures which are all prone to errors.

From the basic actions we learn what kind of errors are possible, and how as a consequence will the navigator react to them in the cognitive delineation of the optimum path. To cope with the errors, we hierarchically divide the optimum path into legs, runs, and finally into segments of homogene friction and risk properties. In order to cognitively and physically command the segmentation, the navigator chooses the waypoints and executes the navigation with rough and precise techniques. He has to follow topographic features, distances, and directions to provide himself an updated position and orientation.

If we know the detailed procedures, we can simulate which optimum path would the navigator choose to avoid the errors. In that way we can also make numerical estimates of the different kinds of frictions and risks. If we simulate them on the same optimum path for different tools, we can inference about the functional interconnections between the errors, the tools and the strategies. For this reason, we additionally choose two technically augmented navigation tools to study the costs of the optimum paths: the GPS receiver, and the GNSS receiver which is hypothetically functioning everywhere. For each, we first have to describe the navigation strategy by separate actions, and the possible errors for the tools used.

In the simulation part of the thesis, we analyse the raster and the vector types of simulation. The vector simulation with cognitive definition of optimum paths is chosen for the demonstration of the hypothesis. We assess several frictions and risks to get the costs of the selected paths, ie. the resistance to locomotion, the navigation risk, the slope friction, the dead reckoning risk, and the waypoint discernibility risk. For each segment, run and optimum path, the costs and the travel times are calculated by the equations which connect the frictions and risks with the distance. The comparison between the costs gives the general functional dependencies between all parameters. They enable the construction of the formal demonstration of the hypothesis.

1.7. Organization of the thesis

The thesis has ten chapters, as follows.

Chapter 1 is the introduction, where we present the research topic, the goals, and the hypothesis, which is formalized and demonstrated at the end of the thesis.

Chapter 2 introduces the terminology regarding the real world and its representations, the processes of spatial travel, the orientation cues, and the navigation components. We explain the paramount importance of cognition, including perceptions, senses, attention, experiences, knowledge, spatial abilities, and spatial reasoning. We discuss the role of beliefs and misbeliefs within the process of navigation, and introduce the term affordance. We show how humans organize cognition about the environment, and how they cognitively structure space and knowledge.

Chapter 3 introduces the idea of the least-cost path and considers the path selection from three general aspects: cognitive, physical, and theoretical. We also define the terms friction, risk, and cost, which are crucial so for the theoretical considerations, as for the simulation part of the thesis.

Chapter 4 describes the navigation with map and compass. Both tools are presented in detail: a simple orienteering compass, and a map as a complex symbolic representation of environment. We compare the topographic and the orienteering maps. We introduce the technique of orienteering, which serves as a showcase of navigation strategies in the thesis. We compare the orienteering in a natural environment and in a city. We add also the explanation of the principles of navigation with incomplete tools.

Chapter 5 decomposes the strategy of navigation into primitive actions in the framework of the sense-plan-act architecture. The optimum path is hierarchically divided into legs, runs, and segments. We show the fundamental method of orientation with map and compass. The planning of navigation is explained as a sequence of rough optimum path selection, definition of waypoints, and detailed navigation techniques.

Chapter 6 decomposes the execution part of navigation into primitive actions, considering separately locomotion, updating of position and orientation, estimation of distances, and orientation procedures. The result is the executed optimum path, which is in principle different than the planned one.

Chapter 7 classifies, studies, and quotes the detailed errors of the tools, the planning, and the execution. We shortly analyse also the impact of physical ability, mental concentration, and emotions on the navigation. The discussion about the consequences of errors, and about the relocation process is added.

Chapter 8 describes the technically augmented navigation, as an opposition to the classical orienteering with map and compass. Two devices are presented: the GPS receiver with a screen map, and the GNSS receiver with a screen map, which is hypothetically functioning everywhere. The strategy, the planning, the execution, and the errors of the technically augmented navigation are analysed in a comparable way to the orienteering navigation case.

Chapter 9 simulates the navigation with the three different tools in a natural environment. We describe the raster and the vector approaches to simulation. The vector cognitive approach is then used with the aid of numerically simulated risks and frictions of navigation, to compute the cost and time for several optimum paths in a real-world example. Finally, we combine the functional dependencies between the parameters to construct the formal demonstration of the hypothesis.

Chapter 10 gives the conclusion. The results of the thesis are summarized and the possible future work is suggested.

1.8. Expected results

The thesis will connect the findings of different scientific disciplines to conceptually and methodologically cover human navigation in a natural environment. Several studies quoted below will lead to the demonstration of the hypothesis:

- the study of cognitive and environmental aspects of navigation,
- the study of optimum path conditions,
- the description of the tools, strategy, planning, and execution of navigation process,
- the description of errors of tools, and errors of respective phases of navigation,
- the description of the technically augmented navigation and its errors.

The optimum paths will be mapped on an orienteering map, a special version of the topographic map. The risks of navigation and the costs for optimum paths will be shown in the spreadsheets. The tools and the relevant strategies will be compared regarding the risks. From the spreadsheets we will draw formalized conclusion about the dependence between the tools, the errors, and the strategies of navigation. This will demonstrate the hypothesis.

2. NAVIGATION, ENVIRONMENT AND COGNITION

We introduce the terminology which is used throughout the thesis to show the differences between:

- the real world and its representations (ie. topography vs. natural environment vs. map vs. cognitive map),
- the processes of spatial travel (ie. locomotion vs. mobility vs. navigation vs. wayfinding vs. orientation vs. orienteering),
- the orientation cues (ie. landmark vs. waypoint),
- the navigation components (ie. position, distance, direction), and their basic use.

To stress the role and the importance of cognition, we describe how the navigation process is affected by perceptions, senses, attention, experiences, knowledge, spatial abilities, and spatial reasoning. They all provide information and mental imagery, which serve for decision making. With the aid of inferencing, we build true or wrong beliefs within the process of navigation. We make use of affordances in specific situations.

The last chapters explain how humans organize cognition about the environment while navigating. We show how they use different types of knowledge, and how they cognitively structure space and knowledge. To represent the spatial orientation of the navigator, we present personal reference frames that humans use to manage spatial travel.

2.1. Definition of basic terms

Navigation is a research topic in many technical, natural and social sciences. To avoid confusion and ambiguity, we define the basic terms which are widely used throughout the thesis.

2.1.1. Topography

We use the term *topography* for the entirety of the Earth's surface features, or *topographic features*. Topography is represented by the shape of the surface of local detail, including the three-dimensional landsurface and landforms of *relief* (ie. ground *terrain*), vegetation, hydrography, and the man-made features (Glossary of the mapping sciences, 1994).

2.1.2. Natural environment

The thesis deals with a *natural environment*, where the topography consists of few or no man-made features. It can have only rudimentary network of footpaths. The topography in a natural environment has undergone minor antropogene interventions over time, by forestry, hay harvesting, agriculture, or land reclamation. We presume that the areas of a natural environment are devoid of any artificial semiotic cues for navigation. They are not easy for navigation with any navigation tool, and structurally different from the urban areas. We use also the terms 'countryside' and 'natural landscape' as synonyms for a natural environment.

2.1.3. Map

To represent topography, we use maps. A *map* is an abstract, reduced and generalized representation of reality. It is a symbolic representation of spatial information about the environment (Pick et al. 1995). The older definitions of a map are usually describing a two-dimensional representation of reality on a paper sheet. Namely, maps have the capacity to portray a large-scale space as a small-scale space on a piece of paper (Hutchins 1995).

One possible concise contemporary definition presented in (Kraak, Ormeling 2003) says that a map is a graphic model of the geospatial aspects of reality. A similar definition posits that a map is a representation or an abstraction of geographic reality, and a tool for presenting geographic information in a way that is visual, digital, or tactile (Board 1992). Since the topographic type of a map is usually drawn in orthogonal projection, it reperesents the world in a perspective that can never be achieved from any actual viewing point (Hutchins 1995).

Maps can be categorized relative to their medium into a single real and three virtual types. The real maps are permanently tangible and directly viewable as a cartographic image. The virtual ones can fulfil only one of these two characteristics at a time, or none of them (Moellering 1980, 1991). Thus the paper map is of a real type, but a map visualized on a computer screen, a map stored on a digital memory media, or a cognitive map recalled in a human mind, all belong to different virtual types. Every form can be transformed into the others, so every virtual map can also be materialized as a real map. In the thesis we use all four listed types of maps.

2.1.4. Mental imagery

Humans organize knowledge and information about geographic space into *mental imagery*. Places, paths, and their relationships are inferred from the sequence of perceived images (Kuipers 1983a). A person's spatial behaviour, which is a research topic in behavioural geography, is tightly correlated with such cognitive representations (Raper 2000). In the following subchapters, we present how we mentaly remember and perceive objects, actions, or scenes, when they are not actually present in our visual space.

2.1.4.1. Cognitive map

Cognitive map is a widely accepted metaphor for human maplike mental construct which mentally represents the environment (Tversky 1993). The term was introduced by behavioural psychologist Tolman (1948), while publishing his experiments on wayfinding behaviour of rats in mazes. Cognitive map is a long term information stored or imagined in human mind, depicting relative locations and properties of objects and spatial phenomena encountered in a physical environment (Tversky 1993). It is not a cognitive copy of an ordinary map, since it is often schematized, distorted, false, incomplete, and can contain fictional, or past-time information (Tversky 1993, Golledge, Stimson 1990). However, metrically correct paper map can provide most information for the development of a metrically correct cognitive map (Thorndyke, Hayes-Roth 1982).

Cognitive map is built over time through associative learning about the environment (Kuipers 1983a). All navigation processes have crucial impact on the construction of cognitive maps, and vice versa: cognitive maps enable humans to navigate. A cognitive map is a person's model of objective reality, which is used to structure, visualize, and store spatial knowledge. It enables recall and learning of spatial information, necessary for navigation. Neurophysiologically, a part of brain, named hippocampus, is responsible for cognitive map construction and storage (Montello 2005).

2.1.4.2. Cognitive collage

When the area of navigation is not known in detail, the knowledge about the environment appears in a variety of forms, like memory snippets of maps we have seen, routes we have taken, facts about distances and directions we have learned before, and other environmental attributes (Tversky 1993). Regardless of the scale of space, the mental imagery is incomplete, distorted, and contains errors. Humans recall such information according to what is relevant, and try to integrate it with various degree of success into a *cognitive collage*, rather than to a fully coherent cognitive map (Tversky 1993).

2.1.4.3. Cognitive atlas

The term cognitive map was also extended to *cognitive atlas*, which is a complex formation of spatial, visual, and declarative knowledge that is typical for atlases (Hirtle 1998). Instead of a map, geographical information system is used as a metaphor for spatial memory, since it can allow simultaneous consideration of object related vector and field related raster data, combination of data into overlays, provision of data integrity, and distinction between scale and resolution. All these concepts can not be found in a single map structure, but are worthy of consideration with respect to mental imagery. Such mental representations of space are more or less integrated complex mixture of (Golledge 1992a):

- location of objects, actions, or occurences,
- spatial patterns and configurations of geographic objects,
- areal phenomena,
- hierachical phenomena and geographic objects,
- networked links and objects,
- homogene spatial associations of objects,
- natural surfaces and textures.

2.1.4.4. Spatial mental model

Humans gradually learn about pointlike elements (eg. landmarks) and linelike elements (eg. routes) of environment. They finally combine them into a metric survey information about the area of navigation (Tversky 1993). When the navigation area is simple and well-known, people capture coherent categorical spatial relations. Such spatial knowledge usually consists of coarse spatial relations among landmarks in a form of *spatial mental model*, which acts as inferencing platform instead of serving as precise cognitive map with metric information (Tversky 1993).

2.1.4.5. Mental route directions

A part of the mental imagery are also *mental route directions*. Route directions are instructions that explain how to get from one place to another. Usually they are communicated verbally, however here they are constructed internally as intuitive self-instructions how the path will be followed and executed. For a short part of the path the navigator tries to establish and remember route self-directions by memorizing landmarks and waypoints, read from the map, including their types, positions, distances between them, and turns to reach them (Lovelace et al. 1999). Route self-directions differ for paths in familiar and unfamiliar environments. In unfamiliar environment, mental route self-directions can be supplemented by the risks and recovery from potential errors along the path section, eg. in the form 'if I go too far, I will reach a swamp, and will have to turn left to find the footpath again '.

2.1.5. Locomotion and mobility

The ability of *locomotion* is a neccessary prerequisite for navigation as will be dealt in this research. Locomotion is body movement coordinated to the local or proximal surrounds, ie. to the environment that is directly accessible to the navigator's sensory and motor system (Montello 2005). During locomotion, we solve behavioural tasks, such as reacting to to the type of ground surface, avoiding obstacles, and moving toward landmarks.

The *mobility* skill enables humans to travel in a natural environment. Mobility involves the ability to travel safely, comfortably, and independently (Cheesman, Perkins 2002). It is a complex, determined activity combining spatial cognition, navigation, and motoric.

2.1.6. Navigation

Navigation is ubiquitous task performed by humans and mobile animals. It is one of the primary functions of vision in all biological systems (Golledge 1995b). Every human being has at least basic ability to orientate and navigate in space. Most people use navigation mainly on the ground, while specialist navigate also in the air and at sea (see eg. Hutchins 1995). Navigation is a fundamental skill required for independent mobility of humans, where it relies upon wayfinding and orientation skills (Cheesman, Perkins, 2002).

Navigation is coordinated and goal-directed movement through the environment by organisms or intelligent machines. It is successful when we reach the destination in an efficient and safe manner. Navigation includes two components, locomotion and wayfinding (Montello 2005). It is determination of a position, planning and determination of a route, and guidance of the movement of a person or a vehicle in a certain direction along a planned route (Pick et al. 1995). It implies the steering or course-setting of oneself, or of one's vehicle (McGranaghan et al. 1987).

2.1.7. Wayfinding

Wayfinding is a cognitive and behavioural ability of a person to find his way from a specified starting point (origin) to a specified target (destination) (Golledge et al. 1996). The term *wayfinding* was introduced by Lynch (1960) as a process based on a consistent use and organization of definitive sensory cues from the external environment.

Human wayfinding describes the mental process humans use to orient and navigate on foot or by vehicle (Gluck 1991). Wayfinding is a goal-directed planning and decision-making process coordinated to the distal as well as local surrounds (Montello 2005). It consists of planning and determination of a route with respect to instant position and orientation.

Navigation and wayfinding are quite similar terms, however navigation is more associated with movement, and wayfinding more with cognition. While wayfinding is supported by cognition, pure locomotion is fed only by perceptual stimuli. Locomotion does not need a specific place goal, but wayfinding is always targeted to some usually distal destination defined in advance.

Wayfinding requires internal and external sources of memory to find the destination (Montello 2005). Internal sources could be knowledge and cognitive maps. External artifacts are usually maps, verbal path descriptions, or other tools for navigation. Wayfinding is also the most common means of acquiring place knowledge (Golledge 1992b).

2.1.8. Orientation

Geographic *orientation* is the ability of a person to know her location and heading in the environment (Montello 2005). Thus, orientation is determination of position and rotation of the body with respect to the surrounding objects, or to the compass cardinal directions (eg. north, south, east, west). Humans use symbolic representations to maintain orientation, including language, artificial signs, and maps (Montello 2005). Orientation of a person is usually compared and aligned with the orientation of a map and with the real objects represented on a map.

2.1.9. Orienteering

Orienteering is an outdoor running and navigation sport discipline, which combines map reading and compass skills (Bagness 1995). It is a unique blend of mental and physical abilities (Bratt 2002). A competitor must select the fastest path between the starting point, the marked intermediate control points which represent the course, and the final destination point. The winner is the one who finds and visits all control points in a specified sequence in the shortest time.

Orienteering has been recognized Olympic sport since 1977, however the discipline has not yet to make it onto the regular program of the games (Bratt 2002). The only allowed navigation tools for orienteering are orienteering map and magnetic compass. The rules of the sport and the contents of orienteering maps are standardized world-wide by the International Orienteering Federation (IOF 2000).

2.1.10. Landmark

We navigate, wayfind, and orientate with the aid of topographic features around us, called landmarks. *Landmarks* are distinctive discrete objects or scenes that are stored in memory and recognized when perceived (Montello 1998). They can be local or distal, small or large, simple or complex. They act as passive beacons. The salience or 'landmarkness' of a feature is the degree of differentiation from the background, and can be measured regarding the structure, visual appearance, and semantic (Winter et al. 2005, Klippel, Winter 2005, Raubal, Winter 2002, Sorrows, Hirtle 1999).

Lynch (1960) explained the role of landmarks in the context of imageability of a city and differentiation of urban landscape. Landmark is any element which can serve as a *point of reference*, or as a *spatial reference point*. Spatial reference can refer to multiple things. Landmark is a spatial cue associated with a location (Presson, Montello 1988). It is cognitively distinct from other elements in spatial memory.

Landmarks are unique configurations of perceptual events or patterns (Siegel, White 1975). They help organize and structure spatial knowledge during planning and execution of navigation (Presson, Montello 1988). Large landform features act as global landmarks as the navigator can align his external (ie. cartographic) or internal (ie. cognitive) map with them (Montello 2005).

Landmarks appear in two types (Presson, Montello 1988):

- *route decision landmarks* represent cues to turn at decision points,
- *route maintenance landmarks* represent intermediate cues to prove that the person is still on the right way, thus they serve as course-maintaining devices (Siegel, White 1975).

They are usually objects with the following outstanding characteristics (Sorrows, Hirtle 1999, Lynch 1960):

- singularity (ie. with a sharp contrast with its surroundings),
- prominence (ie. of positional and visual exceptionality within the environment),
- accessibility (ie. with good physical or visual access),
- content (ie. with distinct meaning, use, or significance),
- prototypicality (ie. with the capacity to represent an object category).

In sufficiently differentiated environment, humans navigate with the aid of cognitive segmentation or chunking of the path (Golledge 1992b). Landmarks often divide such segments. The landmark nodes can provoke the change of direction of travel, or not (Klippel, Winter 2005). Landmarks provide key information about the relationships of locations, objects, and paths. Therefore, landmarks serve also as identification of choice points for navigation, such as origin, destination, and intermediate turning or verification points (Sorrows, Hirtle 1999).

2.1.11. Waypoint

If a landmark is represented with a location point along, or in the vicinity of the route, and if it is defined with coordinates, or with a physical location, it is called *waypoint*. Waypoints are checkpoints which can serve as a course-alteration, or course-maintaining points.

The definition and selection of waypoints depend on the task and the navigation tools. A waypoint may not be strictly tied to a physical landmark. In robotics, waypoints are treated as routemarks which decompose the route and trigger sequences of basic actions or behaviours (Röfer 1999). In the navigation with a GPS receiver, eg. in search and rescue, or in military operations, a waypoint can simply be a coordinate pair or triplet where the route changes or turns. However, the navigator usually needs sensory cues from the environment. The number of decision points (ie. waypoints) directly influences the difficulty of navigation, and vice versa (Raubal 2002c).

2.1.12. Navigation components: position, distance, directions

For geometric representation of orientation, wayfinding, and navigation, we have to apply certain knowledge of positions, distances, and directions (Montello 2005). We provide here the definitions of elementary components used in navigation.

Positioning is determination of absolute or relative *position* (ie. location). In the navigation process, position is usually directly or indirectly represented on a map. While wayfinding and navigation are linear processes referring to a curved route between origin and destination, positioning and orientation are related to the current standpoint.

Two or more points with a known position are necessary to define the distance. A *distance* is a metric relation between points corresponding to their separation in space (Klatzky 1998).

In navigation, positions and distances are combined with directions of different kind. The term *direction* has several diverse meanings as follows (Loomis et al. 1998a, Klatzky 1998).

A *reference direction* is a fixed direction to which all direction measurements refer. Physically, it can be realized by, eg. distinctive, distal, and high landmark. When we use a compass or a map, such reference is usually magnetic or geographic north, respectively (Figure 2.1.).

A *course* is the intended direction of motion or travel over the ground. It is defined with respect to some reference direction.

A *heading* is a facing direction of the body of the navigator. It is defined with respect to some reference direction, too. In ideal conditions, such as on a smooth and flat ground without obstacles, the course and the heading can be practically the same, however if the ground is uneven, the heading can deviate from the course locally by skidding, sidestepping, and backstepping.

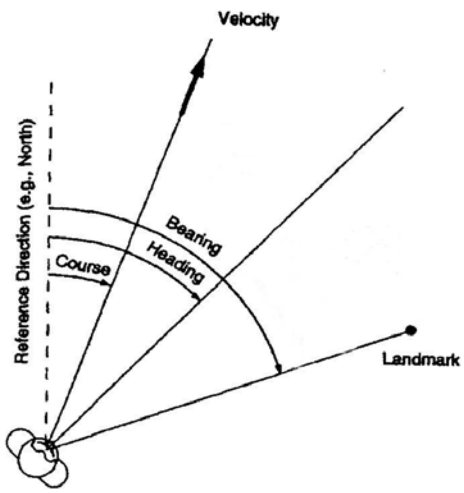


Figure 2.1. Reference direction, course, heading and bearing
(in adaptation to (Loomis et al. 1998a))

A *bearing* is the direction from the standpoint to another point, eg. to a landmark, with respect to a reference direction. If this landmark is the destination, the bearing is equal to the course. A bearing can also be heading-relative, or course-relative. The angle between the bearings to two landmarks is a *bearing difference*.

An *azimuth* is a horizontal angular measurement of any kind of direction used to locate an object (Neufeldt 1988). It is usually measured with a compass to express compas directions in angular units (eg. in degrees).

2.1.13. Dead reckoning, path integration, and updating

When humans navigate from a certain known position to the destination only with keeping track of the navigation vector components such as, position, distance, direction, speed, acceleration, and time, we call such a navigation *dead reckoning*. When the process of dead reckoning is not aided with landmark recognition to update the position and orientation, it leads to increasing errors of navigation. *Updating* is an important activity which maintains a sense of the current position with respect to map as that position changes with locomotion (Pick et al. 1995).

Dead reckoning can be a reliable way of navigation if the path is mentally integrated over time. *Path integration* is a perceptually directed action referring to updating of position, where an animal or a human senses translation (ie. velocity, acceleration) and turns with respect to time. It would not be effective without a sequence of cognitive snapshots imaging the environment passed by the navigator (Loomis et al. 1998a).

Unfortunately, humans are poor path integrators. Most human navigation in unfamiliar environment, which is not assisted by technical aids, is error-prone (Golledge 1992b). While animals are able to integrate the extent of movement over time, humans additionally need tools, language, and maps, for efficient updating (Montello 2005).

Path integration, updating, and dead reckoning are closely related processes, which are essential for the understanding of navigation with map and compass, and human behaviour in the wild in general (Hutchins 1995). Sometimes, dead reckoning is also termed *velocity-based navigation*, and path integration is called *acceleration-based navigation*, or, inertial navigation (Loomis at al. 1993, 1998a).

2.2. The goals of navigation

Navigation is a fundamental behavioural problem to get from one place to another, regardless of the distance (Montello 2005). In the past, humans moved about their environment, eg., to find food, shelter, mates, to chase animals, or to communicate with other groups. It still is important for humans, and crucial for survival of animals, that the navigation act is safe, economic and efficient, considering that the available time, water, calories, knowledge about environment, and other resources, are limited (Montello 2005).

In the evolution of a mankind, the navigation has been developed firstly to spatially master the home territory of a group in which an individual cohabitated with other subjects, and secondly, to explore the environment behind the borders of home territory. In both cases an individual had to travel and return home safely in a reasonable time. The navigation process which aims at finding way back home is termed *homing*. It is essential part of human and animal behaviour. Homing is a high-level task, however in order to avoid cumulative errors, it is frequently accomplished by a collective interaction of various low-level operations (Wehner 1999), such as:

- *path integration*, where travelled distances and angles are cognitively and iteratively integrated into a global vector as in dead reckoning,
- acquiring and using *cognitive map*, where relative positions of landmarks are represented symbolically in the mind,
- *landmark guidance*, where homing is supported by a recognition of certain features from different vanatage points, distances, and angles of view,
- *systematic search*, which helps to pin-point the destination, when wandering in the vicinity of it.

In the contemporary world, the functions of navigation are not any more linked only to survival in the wild but rather to movement in a natural or urban environment for a certain reason and with respect to some activity. The basic goal of navigation essentially remains always the same: to move safely and economically from the origin to the destination.

2.3. The role of navigation tools

Navigation is essentially innate in every human and can be further learned and trained. Nevertheless, humans usually need technical aids for wayfinding. Many cultures have developed navigation aids. Tools for navigation are inventions with cultural background. The Western culture's primary navigation aid is a map, usually used in a combination with some other navigation tool. In a familiar environment, a map at a suitable scale could be sufficient, while for more precise navigation to distal destinations additional tools are recommended, eg. a compass, a GPS receiver, or some other integrated electronic device. The types of tools and maps influence the way of navigation.

The tools and methods of navigation relieve the navigator from algebraic and arithmetic reasoning (Hutchins 1995). They have emerged in the history as a substitute for human inaccuracies and errors in recognizing places and coding geometrical components of landscapes (Golledge 1995b). To sustain the quality of navigation, the navigator substitutes his lack of geographic knowledge and navigation experience with the tools for navigation. Tools do not amplify cognitive abilities of the navigator, but just represent the solution as an apparent and simple cognitive process (Hutchins 1995).

2.4. Cognition and navigation

The main human factors for navigation are *cognitive* and *motor abilities* (Montello 2005). In this chapter, we deal with factors involved into the cognitive part of wayfinding. With the term *cognition*, we refer to mental process, or mental content. The *mental process* includes perception, memory, thinking, or expression of desire, while *mental content* refers to a system of knowledge such as language, mathematics, music, wayfinding, and maps (Olson 1984). Cognitive aspects of navigation are multidisciplinary covered by psychologists, geographers, linguists, anthropologists, neuroscientists, computer scientists, specialists for artificial intelligence, and others (Montello 2005).

2.4.1. The importance of cognition

Locomotion as pure coordinated movement without cognition works directly from motor ability, however wayfinding without cognition can not exist. If the distance between the origin and destination is very short, or if the way is familiar to a person to the last relevant detail, one can hardly declare such a movement as navigation, since most movements rely only upon sensoric signals, and not to a conscious spatial cognition. We suppose in the thesis, that the terrain is unfamiliar to the navigator, and that every navigation tool and method in a physical world is error-prone. Therefore the navigator has to use his cognitive and behavioural abilities to avoid getting lost.

We assume that the tools for navigation are near impeccable, and that we know how to use them. Understanding the risks of wayfinding then stands for the understanding of the processes of cognition in wayfinding, the functioning of the mind during wayfinding, and the intelligence of the navigator. The wayfinding process merges all main topics of cognitive science (Raper 2000):

- *perception* (eg. of the environment),
- formation of *mental representations* (eg. cognitive maps),
- thinking as *inferencing* (eg. about the optimum path and the morphology of the terrain),
- thinking as *learning* (eg. when adapting to a certaint type of terrain),
- cognition related to the use of *language* (eg. if a map symbology is considered as a graphic replacement for a written language).

Thus, the declaration of "knowing where you are" represents a mixed psychological state, which merges perceptual experiences about environment, knowledge about navigation and optimum path routing, and feelings of safety and security, leading to false or correct beliefs about your own position (Hill 1998).

The principal metaphor of cognitive science posits that cognition can be represented as *computation* (Hutchins 1995). Many mental representations are location specific and have explicit geographical properties, therefore navigation practice can also be treated as a computational and information-processing activity (Hutchins 1995). So mental states provoked by eg. thoughts, observations, sensations, knowledge, experiences, affordances, and beliefs, are computable in a human inferencing machine, if we use such a metaphor for brain. Before making any attempt to simulate navigation in this thesis, it is evidently relevant to deal with these cognitive factors (in the following chapters) to show the tight relation between cognition and navigation.

2.4.2. Perceptions and senses

Cognition is initiated by *percepting* and *sensing*. Several sensory modalities provide information for navigation and variety of cognitive systems are involved in processing information from senses and from memory (Montello 2005). Humans observe and sense the environment with receptors and percept it with neural system. Perception exists only in relation to conscious sensation, which means that the stimulus that caused it, has reached the brain (Smith 1990). So, perception is about human senses: what we sense and how we sense it (Raper 2000).

If *perceptions* refer to stimuli from external environment of the body, we term them as *allothetic*, else if they come from internal feelings, we call them *idiothetic* (Montello 2005). Perceptions and observations integrated over longer time can construct *mental imagery* (Kuipers, Levitt 1988). They infer spatial structures, the effect of actions, and build beliefs about the environment. Since the navigator's spatial knowledge comes from perception of only fragmented parts of the world, perceptions, observations and sensations are incomplete and imprecise (Raubal 2002c).

Humans have five *senses* that receive information from outside the body, termed *exteroceptors* (Golledge, Stimson 1990, Smith 1990):

- sight - visual sense,
- hearing - auditory sense,
- touch - haptic sense,
- smell - olfactory sense,
- taste - gustatory sense.

More than 80% of the exterior information perceived by humans come from *sight* (Gebhardt 1990). The navigation process is no exception in this case. Vision is the most precise channel for spatial and pattern information, particularly when the landmarks for orientation are distal (Montello 2005). Beside landmark recognition, visual sensing of patterns of texture movement in the visual space is also important. *Visual space*, or *visually perceived space*, is independent of person's spatial behaviour, and offers immediate and accurate assessment of (egocentric) navigation elements, like distances and directions (Fukusima et al. 1997). Most incoming spatial information depend upon vision, though the navigator can use also other perception modalities.

Acoustic flow properties like acoustic paralax, azimuth of the sound source, and acoustic tau, which specifies the time to contact with the sound source, enable *spatial hearing*. Spatial hearing localizes sources of sound, and percepts reflection of sound from surfaces. Audition therefore reinforces visual information, and enables proprioception (Loomis et al. 1998b).

Touch can help navigation only in the dark, or if the person is blind, when the distance to the destination is very short, and if the destination can be differentiated by the surface texture. Visually impaired persons frequently use tactile maps to navigate and to acquire spatial knowledge. Tactile maps have to be touched to read them, so the sense of touch indirectly enables the navigation of blind. The navigation of blind has been extensivelly covered in cartographic and psychologic literature (eg. Tatham 1991, Coulson et al. 1991, Golledge et al. 1998, Loomis et al. 1993), and is not within the scope of this thesis.

Additionally, *smell* can be a weak and fuzzy cue in navigation of humans, but *taste* practically has no relevant meaning.

Another three *internal senses* enable the perception of pain, temperature, and pressure, however they have only indirect impact on navigation. Additionally, two senses related to movements and position of the body in space exist. They are termed with a common expression *proprioceptors*, or *self-sensors* (Smith 1990). Proprioceptors are the *vestibular sense*, or *equilibrioception,* which enables the perception of balance and acceleration of the body, and the *kinesthetic sense*, which enables the perception of body awareness, consisting of own bodily movements and muscular tensions. The kinesthetic sense is also explained as a low level reflex for maintaining equilibrium, while wayfinding uses several higher level cognitive processes for navigating (Gluck 1991).

In navigation, the body motion triggers receiving and translating sensory feedback provided by self perception (proprioception) of motion over time (Loomis et al. 1992). Normally, the proprioceptors are used to support and react to visual information. Self-motion in path integration process is mostly perceived by proprioception, optical flow, and acoustical flow (Loomis at al. 1993). By means of proprioception we sense bodily translations and rotations in space, and update position from information about velocity and acceleration (Loomis ate al. 1998a). Individual perception of distances, angles, and locations is integral psychometric part of navigation (Gluck 1991). Distance and direction to object during navigation are captured by visual observations on a ratio scale and in the relative orientation of the object with respect to the navigator (Frank 2005).

Many research has been dedicated also to the possibility of existence of *magnetic sense of direction*, however there is no evidence, that humans possess such an ability, even in non-Western cultures, where such a sense would be a matter of survival in the wild (Becker, Marino 1982, Hill 1998). Humans also lack the innate ability to use the sun and stars for navigation, however they have a great capacity for learning (Ross 1974). People who have good sense of direction have the ability to take the advantage of environmental cues, rather than having some fictional "sixth sense" (Hill 1998). Instead of developing senses and instincts, humans have invented sophisticated navigation tools which made the use of primitive cues for navigation obscure or even unnecessary (Ross 1974).

2.4.3. Attention and experiences

The efficiency of perception depends on *attention*, or *perceptual focusing*, that can spur a subconscious process to a conscious state (Golledge, Stimson 1990). Attention is also very important factor in navigation as it raises self-consciousness. Simply said, it concentrates the navigator on the wayfinding process and prevents the distraction caused by disturbing environmental (external) or cognitive (internal) stimuli. Maintaining orientation on longer distances in unfamiliar environment demands high attention, while dead reckoning on short distances can be executed without awareness of the navigation act (Montello 2005). Conscious and intentional navigation procedures, where attention is necessary, are termed *explicit navigation strategies*.

From a conscious perception and sensing grow experiences. Humans need *experience* and *awarenes about space* to navigate efficiently. Different cultures develop different degrees of spatial skill in navigation, and of awareness of space in relation to their environment and social structure (Hutchins, 1995). The part of environment of which the person is aware is called the *perceptual environment*; its image is a cognitive map (Golledge, Stimson 1990).

2.4.4. Spatial knowledge and intelligence

Philosophical considerations about knowledge are dating back to Plato, who postulated that *knowledge* is justified true belief. Knowledge can be specific and problem-related (eg. spatial knowledge), or a common one. In land navigation, the *spatial knowledge* is initially acquired from the map, then from physical exploration, ie. from a combination of two major means of learning about environment (Tversky 1993, Tversky, Taylor 1998). The navigator has first to plan the route, and then to 'get used' to the terrain and the map. Afterwards, he reconsiders his knowledge sequentially along the route. In such manner, the spatial knowledge acquisition continues over long time periods, even after the navigation process is concluded.

Some spatial knowledge is arguably neurologically innate. Neuroscientists have identified certain brain structures responsible for maintenance of orientation, and have even uncovered an evidence for neurons that enable updating of animal's location and heading (Montello 2005).

Intelligence is a capability to adapt to the environment, and to solve problems. Psychologists discern amongst many different kinds of intelligence: numeric, verbal, graphic, spatial, motoric, memorizing, perceptive, inductive, deductive, social, and emotional (Goleman 2001). In the wayfinding process nearly all types must be present in various circumstances, so a well trained navigator is the one who is able to switch smoothly between them at the right moment.

2.4.5. Spatial abilities and inferencing

Spatial abilities necessary for navigation are a consequence of the described chain of mental preparation states, including consciousness, attention, awareness of space, experiences, knowledge, and intelligence. Specifically, the wayfinding abilities include the ability to (Golledge 1992a):

- think geometrically,
- image and imagine spatial relations in various scales,
- recognize spatial patterns, distributions, and functions,
- interprete spatial relations,
- comprehend directions and distances,
- execute path integration, short-cutting, and other wayfinding procedures,
- integrate partial information into configurational ones,
- orientate and re-orientate after translation and rotation of body in space.

Humans differ regarding their spatial inference abilities, the way to acquire spatial knowledge, and how they represent it (Montello 1998). Some people are learning about the environment from maps, the others learn from direct experiences. Those learning from maps seem to form mental representations in north-up orientation from the facts that are direcly observable from maps, like distances and topological relations, whereas those learning from experiences form more flexible and detailed representations of features along the route (Tversky, Taylor 1998). Therefore, the ability to build spatial knowledge, to cognitively represent that knowledge, and to inference about own position, direction and locomotion in environment is also a wayfinding ability (Kuipers, Levitt 1988, Raper 2000).

2.4.6. Navigation skills and spatial reasoning

Specific spatial abilities are *navigation skills*. They can be studied regarding two overlapping functions: *competence* and *performance*. The competence view discusses the cognitive models of wayfinding, and the performance view discusses the ability of humans to find the way (Gluck 1991). They both involve cognition. Competence can be enhanced by learning, while performance can not be improved without additional information. The peak performance is associated with moderate arousal, while extreme arousal levels, ie. drowsy and panic, lead to poor performance (Hill 1998).

The human system of navigation skills interacts with, and adapts to the environment in which it is navigating (Golledge 1995a). It includes the following skills (Golledge 1995b):

- cue or landmark recognition,
- turn angle estimation and reproduction,
- optimum path sequencing,
- network comprehension,
- frame of reference identification,
- optimum path strategic planning, and,
- execution (eg. dead reckoning and path integration; measurements of position, direction, and distance; environmental simplification; en-route rough and precise path selection; shortcutting).

To combine the skills above, the navigator uses spatial reasoning to navigate. *Spatial reasoning* is concerned with cognitive, computational, and formal aspects of making logical inferences about the environment (Worboys, Duckham 2004). A major part of spatial reasoning is mereological thinking in terms of the part relation (Casati et al. 1998). Spatial reasoning is infinitely complex, as spatial world (ie. natural environment) is infinitely complex (Timpf, Frank 1997).

Spatial reasoning in navigation is a mixture of quantitative and qualitative approaches. While the quantitative reasoning uses numerical data (eg. distances, height differences, number of steps, time passed), the qualitative one is recognizing, classifying, and spatially ordering real-world topographic objects by comparison with their representation on the map.

The values for *quantitative reasoning* can be simply acquired by measurements, while in *qualitative reasoning* which is often termed as *naive reasoning,* values are limited to a finite number of coarse estimations about spatial phenomena and actions (Montello, Frank 1996). They are called *qualitative metrics*. Spatial knowledge of natural cognitive systems tends to be qualitative rather than quantitative (Freksa 1992). A sample of quantitative metrics is the egocentric categorization of direction by the values 'front' (ie. 'same'), 'back' (ie. 'opposite'), 'left', and 'right', or a proximity description with vague surrogates for distance measures, such as linguistic variables 'here', 'near', 'far', 'very far' (Frank 1998, Freksa 1992, Gahegan 1995). The perceptions and the resulting actions can not be split into qualitative and quantitative ones. Although the theory treats both types of spatial reasoning as antonyms, the navigator has to combine both at the same time.

2.4.7. Affordances

The term *affordance* explains what the environment enables a person to do, that is, which potential actions can a person afford in a certain environment (Raubal 2002b, 2002c). Affordances create potential activities for a navigator. They may be (a) physical, when they are related to spatial actions, (b) social, when they involve social interaction, and (c) mental, when they stimulate decisions (Raubal 2002b, 2001).

The term affordance comes from ecological psychology, which studies human and other biological systems' response to the environment and vice versa (Gibson 1979). The environment is a passive element but acts as an information source. The navigator observes, percepts and reacts to the information flow with a specific action. If the navigator is motivated and self-conscious, the flow is permanent but not constant. He is balancing between his information need and the data input.

Affordances are determined by a person's cognitive frame of reference, ie. by his commonsense knowledge, past knowledge, social setting, culture, intentions, experiences, and their mental interpretations (Gibson 1979, Raubal, Worboys 1999, 2002, Raubal 2002b, 2002c). She decides about the affordances according to her preferences (Raubal 2002c). Therefore, the affordances are person-specific.

In the process of navigation, the amount of information increases on the decision points, and decreases after they are passed. The decision points serve as the checkpoints for assessment of distances and directions. The decision points are mostly objects on, or near the route, where the network of possible routes can change. We can observe that the topological properties of a route are much more important for the evaluation of affordances than the metric ones.

The object of the environment is set into some medium, consists of some substance, and can be discriminated from other objects which offer less affordances, by the morphology of its surface. High morphological differentiation near the route may offer many affordances to an experienced navigator, while for the other that same objects may seem confusing. Some affordances are interrelated: the affordance at the specific object acts as a pointer to the other object along the route, where new meaningful affordances can be perceived and interpreted. Such relations are also person-specific.

2.4.8. Beliefs vs. knowledge

Belief is a person's own knowledge or thought about something which is considered true by herself. While knowledge represents the objective truth about reality, a belief depends upon the mental state of the person (Krek 2002). Relative to objective truth, the beliefs can be correct or erroneous. If they are erroneous, they can lead to a false interpretation and action, eg. to errors in navigation. Therefore in formal representations, where the distinction between true and false beliefs is computationally important, the *two-tiered reality and beliefs model* can be used to separate the real world and the belief model (Frank 2000).

An important component of knowledge in this context is *meta-knowledge*, ie. knowing what you know. In wayfinding this means, that you are able to track the quality of information necessary to reach the destination (Hill 1998). The meta-knowledge must also be justified true belief, ie. the meta-knowledge is correct, only if the beliefs are correct, too. A rational navigator should never cross this mental limit of illusion about his knowledge.

In the navigation process, the beliefs are accumulated through the information flow and affordances given by stimuli from the environment (Raubal 2002c). If new information emerge, which disprove current beliefs, we change beliefs. The process of updating, enhancing, or changing the information upon which the beliefs are formed is called *belief revision*. Belief revision (and likewise, a theory revision) is caused by frequent mismatches between observations and predictions (Twaroch 2007). Belief revision alters our knowledge base.

The preferences of a person treat the mismatches as a violation of expectations, therefore the false theories and beliefs tend to persist strong in the memory (Twaroch 2007). A belief about something is built upon the meaning of it within a person's mind. The meaning supersedes what has been remembered and stored in a person's memory (Olson, 1984). Often, only very strong sensoric stimulus followed by a cognitive process is needed to override the erroneous beliefs and to revise these beliefs in order to achieve the correspondence with reality.

A belief is the navigator's conviction in the truth of the statements regarding his travel along the optimum path. Once the optimum path is selected, it firmly persists in the navigator's positive belief, since it was cognitively build and confirmed upon the knowledge about the map and landmarks, and about the routes, their distances, orientation and topology (Gluck 1991).

Spatial orientation is difficult in an unfamiliar environment, where the navigator does not know his exact position but only believes he knows the way. Such an illusion of being oriented is a kind of erroneous belief (Hill 1998). When the navigator admits to himself that his beliefs are in a conflict with the true situation, he has to evaluate the strength of beliefs, range them into preference relation and review, revise, or retract some of them. The navigator might be mentally and physically tired. He will tend that the amount of change will be minimal, which is a general principle also in normal cases of belief revision. The navigator will favor to retain beliefs which have been learned from the phenomena which are spatially or temporally near his position. This effect is generally called the *nearness principle* (Worboys, Duckham 2004).

Frequently, the initiation of belief revision is seriously hindered by the removal of subconscious *cascading beliefs*, ie. subordinated beliefs that are founded on a principal belief (Gärdenfors, Rott 1995). When we recognize an error of belief we should cascade through the subdued beliefs and regain consistency of all beliefs in the chain. Intentional disregarding or overlooking the occurence of essentially new information can lead to serious errors which can cause the navigator to become lost.

2.5. Cognition and environment

Humans execute navigation in a physical environment. They cognitively build mental representations about it, resulting in a spatial knowledge and in an enhancement of navigation skills. Their actions depend on the scale of space perceived with senses and derived from symbolic representations. They relate their behaviour to various frames of reference and reason about space quantitatively and qualitatively.

2.5.1. Representation of environment

The real environment is represented by humans in two different ways (Casati et al. 1998):

- *observed* and *percepted representation* - it realizes a link between the external world and human senses; it directly affects human behaviour and response to the environment,
- *mental representation* - it enables spatial reasoning tasks and affects human behaviour indirectly.

To study both types of representations referring to geographical objects, regions, and events, one has to apply various types of theoretical tools like: topology, positioning, kinematics, mereology (theory of part-whole relations), and morphology.

Geographic phenomena are infinitely complex, so any attempt to store, process, interprete, or measure them must involve *approximation*, *generalization*, and *scaling* (Goodchild, Proctor 1997). A generalized, symbolic, or schematic representation of environment is provided by a map, being real or virtual (eg. cognitive) one. Map symbolization allows map reader to understand complex geographic concepts of large scale space, which could be otherwise expressed in more complicated and less consistent form only by natural language.

There is a profound difference between what is represented on a map, and what a person knows, thinks, or believes about the environment, as a result of map perception (Olson, 1984). Every stimulus generated by map perception is enhanced and integrated with the existing knowledge from the *memory* of a person. The memory is generated by personal experiences, observations, sensory input, and learning. These mental processes are more intensive if the person maintains high attention which is stimulated by high motivation or desire. It was demonstrated that map memory and attention to a specific geographic object is much higher if a person believes that the object is important (Olson, 1984). So in the case of navigation, the location of the object plays only secondary role for remembering the object.

Generally, the form, the structure, and the quantity of knowledge represented on the map with a chosen visual graphic hierarchy should correspond with the intellectual hierarchy of the person to provide optimum map reading and understanding (Olson, 1984).

While the perceived viewing *scale* of environment is adapted smoothly and dynamically, the scale of the map stays the same. The navigator has to understand the difference between both representations: the view and the map. In a technically augmented navigation a navigation module can show the map of the same area at different scales and within several levels of detail. However, a cognitively consistent agreement with the viewing scale can not be sustained permanently as levels of detail are changing stepwise.

2.5.2. Large-scale and small-scale space

The part of environment which can not be visualy perceived from a single standpoint is frequently referred to as a *large-scale space*, while a *small-scale space* is the part, which can be seen at a single location (Kuipers, Levitt 1988). A typical small-scale space is a room. Only a minimum effort is necessary to navigate inside it. The natural environment, especially the forested areas, represent a large-scale space, where the destination and a great portion of the route is not perceivable (Pick et al. 1995).

To reveal the structure of a large-scale space, the navigator has to integrate local observations over time, rather than perceive the environment from a single vantage point (Kuipers 1983a, Kuipers, Levitt 1988, Gluck 1991). Mental imagery has the potential to store the large-scale space representations. Some authors also distinguish a *medium-scale space* which belongs to a close neighbourhood observable in short time interval but not from a single standpoint.

The cognitive aspect of the space scale is in opposition to a cartographic notion of representational scale, where a small scale corresponds to a heavily reduced representation of geographic area, and a large scale corresponds to a minor reduction (Gluck 1991). In cartography, the two adjectives small and large relate to a small and large value of scale fraction, respectively (Robinson et al. 1995, Kraak, Ormeling 2003).

The confusion becomes even greater, when the navigator relies upon paper map, which is a representation of large-scale space. Montello (1993) proposes that a large-scale space could be less ambiguily termed as a large-size space to express the size of space relative to a person. He also gives an extended classification of psychological spaces regarding their projective size relative to a human body (Montello 1993):

- *Figural space* is projectively smaller than human body. It is perceived by sight or even by touch from a single location without locomotion. It can contain 2D pictorial and small 3D object spaces.
- *Vista space* is projectively larger than the human body, but is still perceived from one single location.
- *Environmental space* is also projectively larger than the body, however one can apprehend it only after integration of information over longer period of time which possibly involves locomotion.
- *Geographical space* is projectively much larger than the body, so it must be learned from symbolic representations.

2.5.3. Commonsense geographic knowledge and knowledge in the world

People acquire *spatial knowledge* about locations they have visited. The experiences about the locations which are relevant to navigation include knowledge of locations, distances, and directions. The acquisition about the location starts immediately after arrival and persists over long time periods, even after the person leaves it (Montello 1998). Such learning can result in a cognitive map, and also in the enhancement of a general *commonsense geographic knowledge* about the properties of the kind of environment that has just been passed through. Most people have at least a basic commonsense geographic knowledge ie. the knowledge about geographic properties of real environment, which is acquired permanently from birth on, without concentrated effort, to find and follow routes from one place to another, and to use the relative position of places (Kuipers 1977).

The skill of reading maps is also a part of commonsense geographical knowledge. Map reading a is a skill that can be acquired by learning the use of maps, improvising the process of mapping, improving visual perception skills, and even by being exposed to a map (Castner 1981).

When people pass through unfamiliar environment, a cognitive map can not be created in advance. They depend on external information, or on the *knowledge in the world* (Norman 1988). Such external knowledge could be eg. textual or pictorial signs, symbols, signposts, and architectural clues in buildings (Raubal at al. 2002a, 2002c). In a natural environment, such explicit form of knowledge is usually absent, scarce, or unsufficient, so people have to rely on navigation tools and maps.

2.5.4. Naive geography

Much of the commonsense knowledge related to geography can be attributed under the term *naive geography* proposed by Egenhofer and Mark (1995). The basic suggestions why we observe that humans deal with the knowledge of geography naively are the following:

- geographic space is cognitively treated as two dimensional, such as shown on a map,
- the height and slope are treated separately from location, as its qualitative property, again as on a map,
- the earth is practically considered flat, ie. as a small-scale space within the horizon of a person, which visually does not exhibit sphericity,
- humans recall geographic configuration of objects as a cognitive map rather then as true experiental geographic representation; they have greater confidence in mental and symbolic representations than in experience,

- mental representation of geographic space appears generalized, incomplete and uncertain, but humans compensate for the lack of information with their commonsense knowledge combined with experiences, which both affect their behaviour and actions,
- humans conceptualize and (re)organize their representation of geographic space according to the scale of space appearing in cognitive process (large-scale, small-scale, and discrete intermediate stages),
- humans switch between scales of cognitive representations of space unevenly and abruptly, to adapt their behaviour to the environment,
- humans adapt the scale, generalization, and granularity of the object space according to the level of details needed to act rationally,
- cognition process first reconstructs topology of geographic space and afterwards refines it with often distorted metric information, and not vice versa,
- humans involved in wayfinding process prefer cardinal directions, like north-south, or east-west, and not accurate azimuths,
- distances are perceived asymmetrically, as inbound and outbound journeys are topologically, temporally, and visually different,
- humans infere about distances locally, relative between objects, and not in conventional global coordinate systems,
- very large and very short distances in a large-scale space are not added mathematically, but approximatelly; short distances are often neglected.

We can prolong this findings with *naive navigation* as in fact most of the principles of orientation come from a commonsense knowledge. Egenhofer and Mark (1995) revealed, that the principles of naive geography can be a cognitive tool to reform a system of GIS functions such as, eg. map algebra (Tomlin 1990), which will mimic human behaviour, actions, or needs. While naive geography is a commonsense explanation of cognition about geographic space, still one has to interprete it cautiously as it is culturally affected.

2.5.5. Knowledge about navigation space

Individuals learn about their environment incrementally (Gluck 1991). When they navigate through environment, they acquire the knowledge about geographical space in three levels (Siegel, White 1975):

- First they acquire *landmark knowledge* about important, particular, spatially discrete, and disconnected locations. It represents the facts about the spatial layout of destinations and landmarks along the way.
- Then they learn about the routes, connecting landmarks, and about the whole network comprising the route. This so called *route knowledge* provides the ability to keep track of the position and bearing.
- Further recognition leads to *survey* or *configurational knowledge* which builds the abstract topological and partly metric impression about distances and orientation. This is the knowledge about the environmental properties along the path and about the objects. When it is collected from a map, it extends to regions which are not directly perceived. It leads to a comprehension of the structural principles embedded in a navigable space.

Golledge and Stimson (1990) presented this structure from the perspective of complexity for learning, by decomposing spatial knowledge to:

- *declarative knowledge*, eg. the knowledge of place,
- *procedural knowledge*, eg. the wayfinding knowledge,

- *configurational knowledge*, eg. the integral knowledge of linked places, routes and areas.

The consequence of the three forms of knowledge is spatial orientation. *Spatial orientation* is the ability to relate personal location to environmental frames of reference (Cheesman, Perkins 2002). In this dominant framework of landmark, route, and survey knowledge, the spatial knowledge is followed by additional knowledge, and so on and on (Montello 1998).

The interpretation above is more simply expressed by Golledge (1992b); in the spatial domain, the system of geographical knowledge refers to places, lines, and areas, respectively. However, as practically shown by Hill (1998), not all three levels have to be developed, since eg. local inhabitants in villages can perfectly manage routes they know, without having survey knowledge about the entire surrounding environment, and about the spatial layout.

Route knowledge enables the person to form a narrow strip cognitive map, whereas survey knowledge helps to form broad comprehensive cognitive map. So, any description of environment usually takes one of the two perspectives (Tversky 1993):

- A *route perspective*, which takes the person to a mental tour of the environment through the mental locations of landmarks. It uses the person's internal reference frame in terms of front, back, left, and right.
- A *survey perspective*, which gives the person a bird's eye view of the environment, where landmarks are positioned relative to one another in an external reference frame in terms of cardinal directions north, south, east, and west.

Montello (1998) has argued, that the levels of knowledge are temporally overlapping, and cognitivelly parallel. In an extension to this framework, he proposed:

- The phases are not sharply distinct; eg. the metric route knowledge is acquired already on the first exposure to a novel place, ie. while landmarks are still identified.
- Quantity, accuracy and completeness of spatial knowledge continue to increase indefinitely with familiarity and repetitive exposure to the location.
- Knowledge about separately learned places is integrated into a complex, hierarchically organized knowledge.
- Individuals differ in spatial knowledge of the same navigable space with respect to the degree of knowledge integration.
- Topological non-metric knowledge is only supplement to metric knowledge, but not precursor or intrinsic part of it.

Additionally, some people use imaginative patterns, or *image schemata*, which topologically structure experiences in relation to the existing commonsense geographic knowledge. Image schemata show how to react in different environmental circumstances, eg. when travelling along the path, or entering a building (Raubal et al. 1997, 2002, Raubal, Worboys 1999, 2002).

On the other hand, the knowledge of a large-scale space is treated also from the perspective of computational theory of spatial knowledge, used in artificial intelligence and robotics (Kuipers 1983b, Kuipers, Levitt 1988). According to Kuipers' pioneering idea in this field, the knowledge is a cognitive map, which includes a four-level semantic hierarchy of *sensorimotor*, *procedural*, *topological*, and *metrical relationships*, the first two referring to the actions in space, and the other two representing the properties of space.

2.5.6. Frames of reference

When humans form cognitive maps, they align them with body axes, and use local frames of reference (Gluck 1991). Geographic orientation involves *frames of reference* that spatially relate the navigator to something concrete or abstract, and organize spatial knowledge about location, distance, and heading. As the navigator moves to a new position, he also maintains orientation with the actions of updating, using particular frame of reference (Montello 2005). To be able to distinguish and use different types, we present here the relevant terms.

The terminology and taxonomy of reference frames is a bit confusing, as they are applied in environmental, behavioural, philosophical, cognitive, linguistic, and neuroscience. Frank (1998) formalizes them regarding the three parameters that characterize them: origin, orientation, and handedness of the coordinate system. Besides, the meaning of terms for the reference frames in different languages differ in the semantics of spatial description, and in the possible representational states of reference frames. It seems that the options in language suggest which reference frames are feasible for navigation tasks (Levinson 1996). Namely, describing spatial relations with qualitative spatial expressions to move around home area has been one of the first early uses of language (Tversky 1993).

Frames of reference are directly or indirectly tied to a human body, usually to its sagittal plane (Figure 2.2.). The *sagittal plane* of a human body is any vertical anterior to posterior plane that passes through the body parallel to the *median plane* (or, *mid-sagittal plane*), which is cutting the body into two, more or less mirror-image halves (URL 1). The *axis of orientation* of a person is alligned with the sagittal plane. Hence, we can make the following distinction (Klatzky 1998):

- *heading* is the angle between the axis of orientation and some reference direction to external object;
- *bearing* from the current location to another external location is the angle between some reference direction and the line between the two locations.

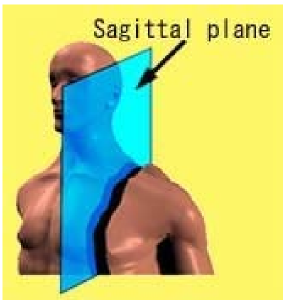


Figure 2.2. Sagittal plane
(source: www.tech.nite.go.jp)

If the reference direction is aligned with the axis of orientation, the bearing between the current and the external location is ego-oriented. The *course*, or the direction of travel, is defined with the past few locations that were occupied by a person.

Locations of objects in the environment are determined by distances and bearings (Figure 2.3.). If the they are measured from a person, ie. from an ego, they are called egocentric, otherwise they are allocentric (Klatzky 1998).

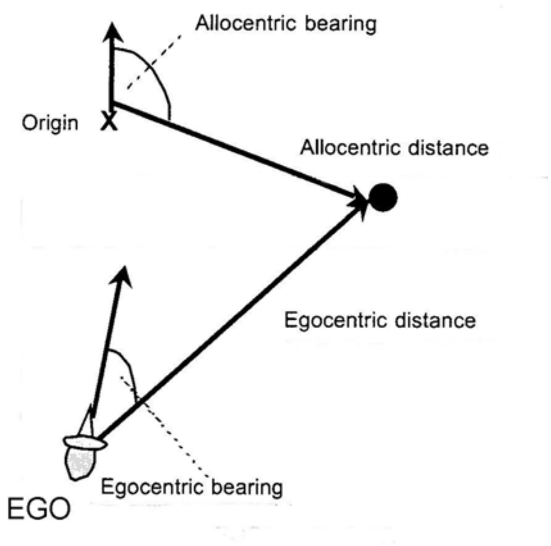


Figure 2.3. Egocentric and allocentric distances and bearings
(in adaptation to (Klatzky 1998))

Likewise, the *egocentric frame of reference* is centered in the person, ie. in the ego, with the ego's orientation (Frank 1998, 2005). In other words, the locations in an egocentric reference frame are represented from the perspective of the perceiver, whereas in an *allocentric frame of reference* they refer to arbitrary external perspective from a point independent of perceiver's location. Egocentric parameters can be encoded accurately, while allocentric parameters are usually encoded with substantial errors, as they require imagined translation and rotation to represent them (Klatzky 1998).

Allocentric reference frames can be fixed, where the location refers to a stable landmark, or, coordinated, where it refers to a system of abstract coordinate axes, possibly of global coverage (Montello 2005). In navigation with map and compass, fixed system is usually applied as it is feasible only locally. Coordinated reference frame, being geocentric by nature, can be used in a combinataion with fixed frames eg. in navy navigation at sea as described in (Hutchins 1995).

Humans are capable of transforming their perception of space into the perception from another vantage point, occupied by another person or an object. Such an ability is important both, in the planning and in the execution phases of navigation. When we describe the environment from the perspective of another objects or person (relatum), we term such reference system as *intrinsic* or *deictic* (Frank 1998, 2005).

Humans have also the ability to describe ego- or allocentric location in absolute and relative reference frames. *Absolute reference frames*, where the orientation is given from the outside, normally use cardinal directions (eg. north, west, south, east), or refer to geographic directions (eg. toward a landmark, down the valley, seaward) for qualitative spatial reasoning. *Relative reference frames* rather use the body-centered direction terms (eg. front, back, left, right) (Frank 1998).

Any computational model of reference frame, or transformation of reference frame from visual perception to the relative expression, must be specified with three characteristics (Frank 1998, 2005):

- the origin of coordinate system,
- the orientation of the coordinate system,
- the handedness of the coordinate system.

The three parameters formally characterize the reference frame, and can refer to an ego (ie. to the speaker, the perceiver), to another person, or to an external object. For example, the egocentric representation and understanding of space incorporates polar coordinate system with the ego in the origin, where the reference axis of the system is the ego's axis of orientation (Klatzky 1998).

The following example of qualitative measures in an egocentric reference system described in (Frank 2005) seems quite plausible in rough navigation procedures. It encodes distances into zones (here, near, far, and very far) and directions into eight cardinal direction values so that the horizon is evenly (ie. "equiangularly") covered with cones (Figure 2.4.).

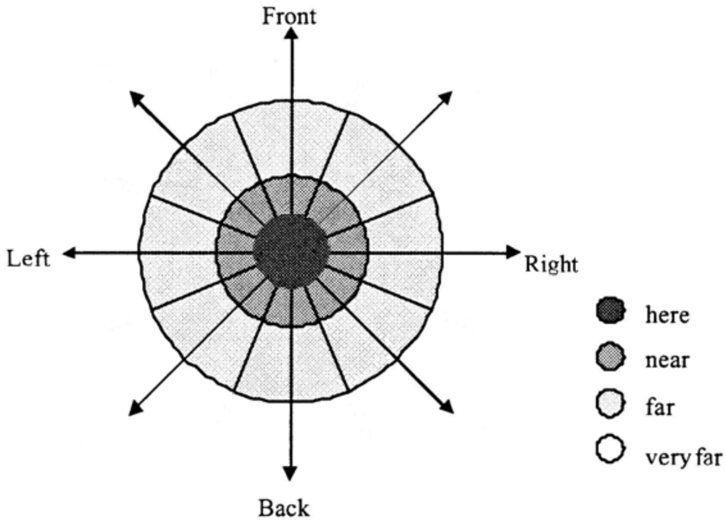


Figure 2.4. Qualitative distances and directions
(in adaptation to (Frank 1998))

Navigating humans orient themselves in terms of an egocentric spatial structure, which relies upon the basic egocentric directions, ie. ahead, behind, left and right. Human sensory modalities tend to integrate this information into a single coherent sensation, which allows successful navigation (Cheesman, Perkins, 2002).

3. OPTIMUM PATH IN A NATURAL ENVIRONMENT

In this chapter we start with the description of differentiation and complexity of a natural landscape to give a cue why so many optimum path conditions can exist. Then we introduce the idea of a least-cost path and consider the path selection from three general aspects:

- cognitive, with the recognition of individual bias in the process of path selection,
- physical, with the emphasis on the motoric abilities and limitations of the human body,
- theoretical, with the discussion of mathematical or physical background of optimum path selection criteria in idealized and generalized environment.

We also define the terms friction, risk, and cost, which are crucial for the definition of optimality.

3.1. Landscape differentiation, visual access, and spatial layout

Natural landscape is independent of human intervention. Three physical characteristics of natural environment affect orientation (Montello 2005): differentiation, visual access, and complexity of spatial layout.

Differentiation is the degree to which parts of environment look the same, or different in size, shape, colour, and other characteristics. Greater differentiation makes more useful landmarks (Lynch 1960), but on the other hand, too much differentiation may lead to disorientation and confused mental imagery. An example of extremely differentiated landscape are boulder fields and karstic forest gridded with sinkholes (Figure 3.1.).

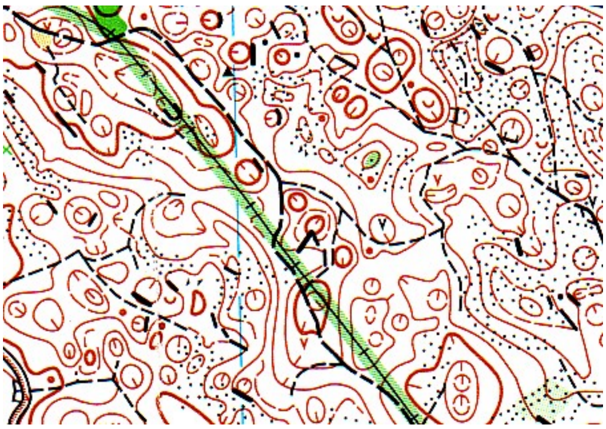


Figure 3.1. Highly differentiated karstic terrain
(source: map Lome, Slovenian Orienteering Federation, original scale 1:10.000)

Visual access is the degree of what can be seen from different vantage points in a certain type of landscape. It defines the horizon, ie. the limit between small-scale and large-scale spaces at each viewpoint, it denotes the perceivable landmarks, destinations, and sections of the planned path. Visual access in a natural environment can span from extremely low in a dense forests, to extremely high on top of barren mountain ridges. It can be computed with a *viewshed analysis*, which is a common function in raster geographic information systems, and a part of map algebra (Tomlin 1990). Fisher (1993) has shown that the implementation uncertainity and the computed viewshed variability depend on the viewpoint and target representation. The elevation data quality and the type of mathematical algorithm share much less influence.

Complexity of spatial layout is a vague expression of how much the structure and the distrubition of spatial objects are irregular, heterogene, visually cluttering, curved, and asymmetric. Complexity can have an impact on disorientation, but this depends on the type of terrain and on the experiences.

The three described qualitative characteristics of landscape are hard to assess individually. Since the reality is divided into objects, we can apply *categorical coverages* for a subdivision of navigable space according to the uniform qualitative properties of objects (Frank et al. 1997). The properties should be related to the potential (inter)actions in orientation, although geographical objects can also be vague, granular, and fuzzy, thus far from uniform. Interactions and properties jointly determine the affordances in the process of navigation (Frank 2005).

3.2. The concept of optimum path

The human wayfinding consists of *destination choice* and *path selection* (Golledge 1995b). In a real world, there is infinite number of paths between the origin and the destination. So, we first have to choose an arbitrary *optimum path condition*, and then apply it to plan and execute a single *optimum path*. The rudimental selection and planning of the optimum path comes just after the definition of origin and destination. The optimum path condition is indirectly defined by the intention of the travel, whilst the decision about the course of the optimum path is a complex spatial reasoning task.

The optimum path in idealized case, where the optimum path condition can be expressed mathematically, and, where a person is navigating on an analytically determined surface (possibly, with additional analytically defined isotropic or anisotropic properties) is uniquely defined curved line on this surface. However in reality, the landscape characteristics, described in the previous chapter cause, that the idealized or the planned optimum path is practically never the same as the executed.

3.3. Individual bias in optimum path selection

The navigator has to decide upon the optimum path by himself, based on the information available (Bratt 2002). The information can refer to the current environmental conditions, and to himself. Regarding the environment, the choice of optimum path condition is imposed by structural characteristics and differentiation of topography, ie. by the type of relief, topographic features, and landmarks encountered along the path.

Regarding the navigator, we observe, that the visual and cognitive processing during navigation is an interpretative and a person-specific task, where the navigation abilities must be harmonized with the physical abilities of the navigator, and also with his emotions. Each individual selects his own distinct optimum path so, that certain disability of the individual can be compensated with some ability. The navigator chooses the path which would optimally fit his:

- spatial abilities (eg. the navigator wishes to avoid the path over very differentiated terrain, if he is weak in map interpretation and reading),
- orientation abilities (eg. the navigator who has poorly developed sense of direction would prefer a variant of the path which demands faster and longer travel with few turns, over shorter and complicated orienteering),
- acceptance of risk (eg. the navigator prefers the variant of the path where the visibility through vegetation is not lower than 10 m, and where the terrain is not steep),

- physical strenght and condition (eg. the navigator prefers steep and short path, over flat and long one),
- psychophysical endurance and motivation (eg. the navigator prefers short variant of the path with dense wet shrubs, over longer path around it).

Golledge (1995b) has shown that the path selection depends on the current body orientation of the individual, so route heading in some direction is more acceptable than in the other. The strategy of navigation and the path selection criteria alter, when the destination is distant, and when the path penetrates through unfamiliar territories. The selected optimum path is asymmetrical as it is usually not retraced on returning to origin, especially when the visual perspectives of the environment in the forth and back direction of travel are remarkably different (Sadalla et al. 1980, Golledge 1995b).

3.4. Definition of friction, risk, and cost

Three general terms will be frequently used in the continuation: friction, risk, and cost. We provide here the definitions to explain their specific meaning in this thesis.

Friction is a physical impact of the surfaces of topographic features to a moving human body. For example, harsh ground, slope, and lush forest vegetation provoke a high friction of locomotion, while flat paved road has a minimal friction. A friction is caused by environmental circumstances. It affects energy consumption.

Risk is a concept that denotes a potential negative (unwanted) impact to the execution of travel along the path. Usually it expresses the possibility to prolong the distance or time of travel. It can arise from a present navigation process or from future actions. We will show in the simulation, that the risks are caused by real or potential tool performance, ie. by real or potential errors caused by the characteristics of the tool.

The concept of *cost* is necessary to express the different aspects of the travel (eg. speed, distance, height difference, energy, risk) with a single value. There is no single general formula for the cost as the impact of each aspect could be weighted and expressed in a different way. In the simulation part of the thesis, the term cost defines the worthiness of a path regarding the risks, the frictions, and the travelled distance:

$$Cost = \mathrm{f}\,(Distance, Frictions, Risks)$$

3.5. Optimum path interpreted as the least-cost path

In this thesis, we limit the description to the basic and the most common types of optimum paths in a natural environment, which are providing fast, short, or flat travel, with minimum risk or under optimal physical strain. For the purpose of computation, the optimum path can be interpreted as the *least-cost path*, where the cost of passage is measured in the values of distance, time, friction, height difference, risk, or energy spent (Douglas 1994). It can be computed with a relevant mathematical optimization criterium, expressed numerically (Lee, Stucky 1998). Since we search for the path with the *minimum costs*, the resulting path is sometimes termed the *minimum path* (Collischonn, Pilar 1999). In the simulation part of the thesis, we use the following definition of the least-cost path:

$$Cost = \mathrm{f}\,(Distance, Frictions, Risks) = \min$$

Regardless of the type of measure for the cost, the path has to adapt to the environmental conditions. For illustration, the following varieties of topographic data play the role in the description of friction and obstacles: relief forms, height differences, topographic slope, types of vegetation, man-made objects, hydrographic objects, existing footpaths, and roads.

Specific simulation models and algorithms for searching the optimum path have been developed, eg., in robotics (Kuipers 1977, Kuipers, Levitt 1988), in environmental monitoring for the solution of travelling salesman problem (Balstrøm 2002), and in urban studies for location-allocation, network optimization, commuting, and transport (Golledge 1995a). However, these methods are out of scope for this research as they are solving the tasks which are not directly related to the navigation in a natural environment.

Mathematically, the computation of the least-cost path is one of the problems found in the calculus of variations, where we search for the extremal solutions of the functional. Two examples of the least-cost path which show how people travel with the least effort, can be found in optics. According to the Snell's second law of refraction (from 1621), the rays of light bend, when they pass from one medium to another (Figure 3.2.). Secondly, Huygens (1629-1695) has explained that, when the rays pass through a narrow aperture, they form a spherical wavefront expanding from the aperture, and diffract in all directions as if coming from a new source of light (Figure 3.3.). Similarly, the topographic features which slow down, hinder, or deviate the locomotion, act as refraction lens, or a diffraction slit to the least-cost path (Tomlin 1990, Douglas 1994).

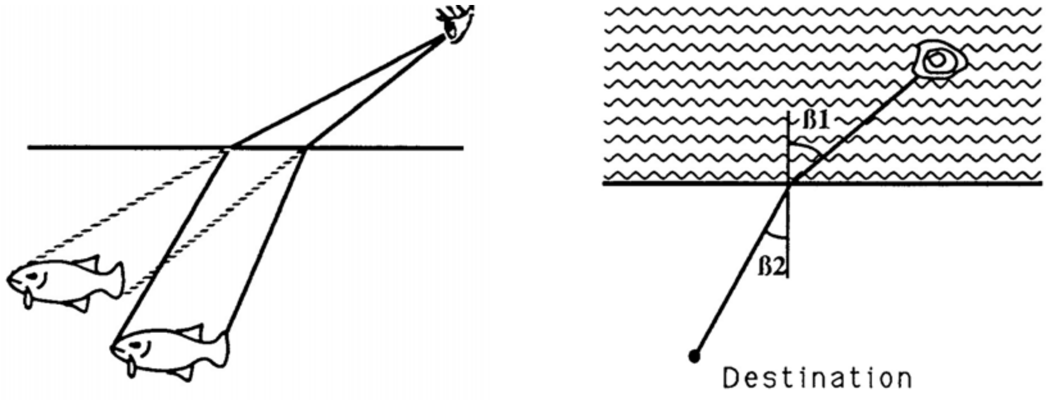


Figure 3.2. The analogy to refraction: the rays of light (left), the least-cost path (right) (source: (Douglas 1994))

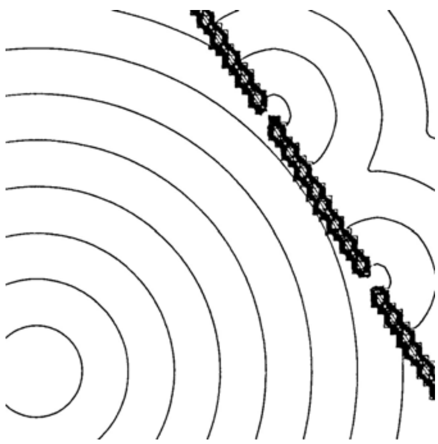


Figure 3.3. The analogy to diffraction: the wavefronts formed by the rays of light, or by the cost of path
(source: (Douglas 1994))

In the next chapters, we consider each type of the least-cost path condition in general, however cognitive path selection may not be so simple process as assumed in the computational algorithms (Golledge 1995b). The deficiency of many models is that they are best suited to mathematical criteria of which people are not aware, or are incapable of using (Golledge 1995b).

3.5.1. Straight traverse as the theoretically simplest path

We assume, that we navigate on a smooth horizontal plane without obstacles. Then the straight traverse from the origin to the destination is represented by a line with a constant bearing (Figure 3.4.). The constant bearing ensures the *maximum simplicity*, which means no changes of direction, and no intermediate waypoints. The straight traverse is also the shortest and the fastest path at the same time.

Figure 3.4. Straight traverse (red) with a constant bearing
(source: map Gora pri Komendi, Slovenian Orienteering Federation, original scale 1:10.000)

If flat terrain has heterogene properties, the straight traversal can become non-optimal because of friction, or even impossible because of obstacles. When we navigate with a constant bearing on an undulating surface, then the path is a curved line of profile, and only its horizontal projection is straight. We deal with more realistic surfaces in the next subchapters.

At any occasion when the destination becomes close, accesible, and clearly visible, the cognitive part of the navigation process is completed regardless of the optimum path condition. Namely, a locomotory act to reach the destination is then only pure coordination of the ambulatory motor system to patterns of optic flow in the environment (Warren et al. 1986, Montello 2005).

3.5.2. Shortest path

The selection criterion of the *minimum distance* is dominant in most planar network flow or routing computational models as it maximizes the economic utility or minimizes the cost or time expended in travel (Golledge 1995b). From differential geometry we know that the shortest path on a smooth mathematical surface is *geodesic* (or geodesic line) (Figure 3.5.). Geodesic on a flat or inclined plane is a straight line. Generally, a geodesic on an arbitrary curved surface is a complex curve which is not lying on a plane. Its bearing is continuously changing. A normal to the surface in each of its points is lying on an osculating plane to the curve in that point.

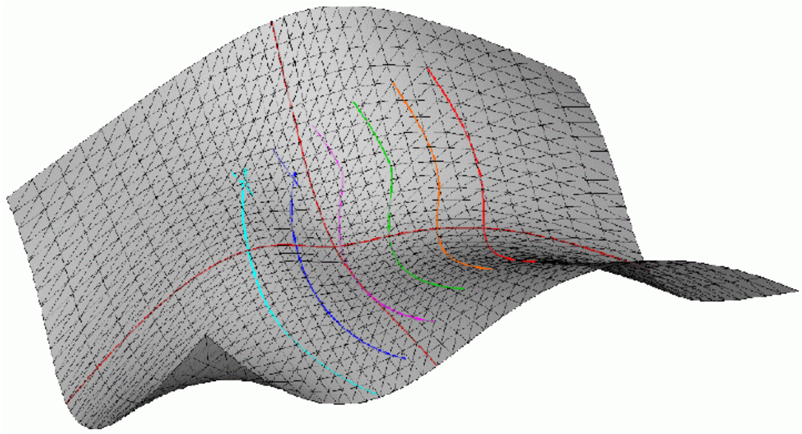


Figure 3.5. The shortest paths on a smooth surface shown with geodesics
(source: http://www.lems.brown.edu/vision/Presentations/Wolter/Figs/GeodOffNurbsCurv.gif)

In a real world, we usually locomote on the terrain with heterogene friction characteristics, so we are not searching for the shortest path. We rather assume some other optimality criterion, like the minimization of time, as described in the next chapter.

3.5.3. Fastest path

In most wayfinding tasks, the *minimum time* needed to reach the destination is the most important condition for the optimum path selection. In physics, the time of travel of a rigid body is proportional to the length of the path, and inversely proportional to its speed. Therefore, to reach the destination in the minimum time, we have to minimize the distance and maximize the speed. The human body is not rigid, nor passive. Humans use cognition to optimize (minimize) the distance, and internal locomotor abilities (eg. power, motoric properties) to optimize (maximize) the speed in certain environmental conditions which usually are heterogene.

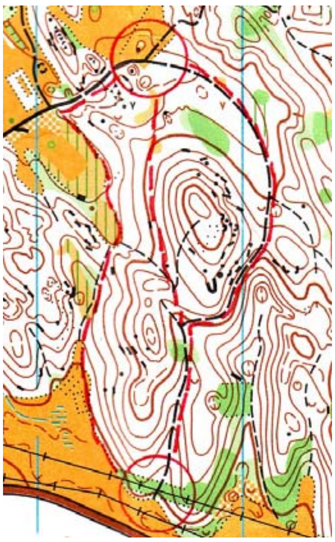


Figure 3.6. Three possible fastest paths between the origin and the destination
(source: map Predgriže, Slovenian Orienteering Federation, original scale 1:10.000)

On undulating terrain, the shortest path is generally not the fastest for a person with bad physical condition, or for someone who wants to avoid the risk of slipping on steep slopes. Likewise, the shortest path over areas of strong friction is not the fastest one, since it does not enable speedy travel (Figure 3.6.). Analytical or numerical consideration of the fastest path has to take into account various complex friction and risk parameters with abstraction from details which are irrelevant or less influencing.

This is the most important optimum path condition for the continuation of the thesis. Further numerical and formalized treatment of the fastest path with raster and vector type data will be given in the simulation chapter.

3.5.4. The path with optimum height difference or slope

The undulation of the surface can be expressed with several geometric measures, eg. with the *height difference* (the relative height), the *inclination angle*, or the *inclination gradient*, where the gradient is the height difference per distance unit, or, the tangent of the inclination angle. For a height difference, we need two arbitrary points on a surface, while the inclination angle and gradient are usually measured in a single point. We can pass the surface point in the direction of the maximum value upslope or downslope, or in any other direction. If we pass the surface point in the direction of isoline, the inclination is zero.

An undulated surface acts as a friction of the locomotion. It slows down the movement and refracts the path around steep slopes. People often prefer sinuous but less steep path (Figure 3.7.), instead of shorter and steeper ones (Collischonn, Pilar 2000). They often choose a path which locally has eg., a minimum inclination, a constant inclination, an inclination not greater than some value, or a minimum height difference. However, when we consider the entire path, we always deliberate about the steepness and the height difference with regard to the distance and other frictions. We can not express the optimum path condition with a simple and single statement involving only inclination or height difference.

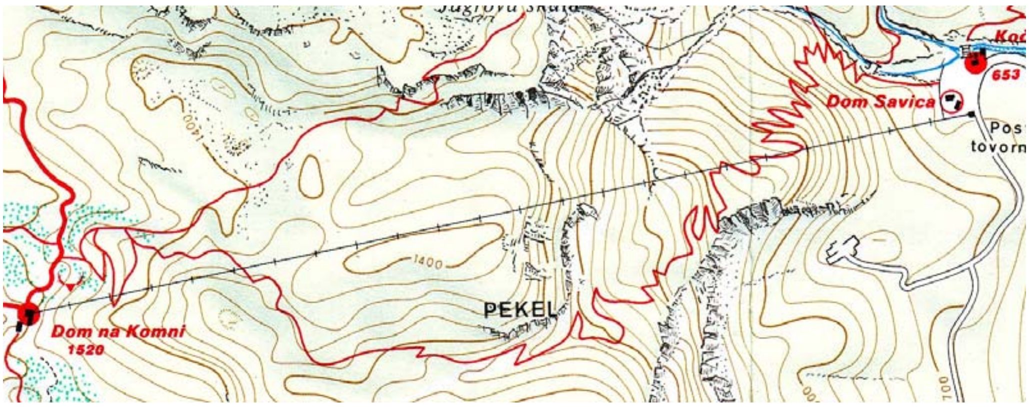


Figure 3.7. Sinuous mountain path
(source: map Julijske Alpe - Bohinj, Alpine Association of Slovenia, original scale 1:20.000)

3.5.5. The path with optimum energy consumption

For the locomotion of the human body two types of energy are important: kinetic and potential energy in the gravity field. Considering that the mass of the body and the gravity acceleration are constant, we know from physics, that we have to minimize the velocity and the height difference in order to minimize the kinetic and potential energy, respectively. However, as we have to consider both parameters in a connection with distance and frictions, we rather require optimum energy consumption instead of minimum. This condition is important in mountaineering and hiking on longer distances, and in the situations of being lost.

3.5.6. The path with minimum risk

In the conditions described above, the optimum path can be evaluated with the costs related to distance, time, slope, and energy. However, a navigation is challenged also by different risks. One of the most common is the *risk to get lost*, where the notion of being lost can mean:

- to lose the optimum path for a short time and locally,
- to lose the way substantially, which may cause serious, but solvable relocation troubles with a great loss of time,
- to become lost on unfamiliar terrain outside the range of map, where the problem may be unsolvable in a reasonable time limit, and where the situation in the extreme case may endanger personal safety, or even survival (eg. in the dusk, fog, in the wintertime, in a state of exhaustion).

The risk to get lost can be minimized with tracking distinguished linear objects (eg. roads, footpaths, power lines), or with choosing frequent, clear and visible landmarks. It can be expressed numerically with various measures, eg. with the distance from the roads, with the density of the potential waypoints, with the visibility of terrain due to vegetation, and with the quality of the footpaths.

The least risky path or the easiest path to find is often long and time consuming. We apply it only in the case, where the primary task of the wayfinding is to reach the target regardless of time and distance. Usually, we rather combine two optimality conditions: the fastest path with minimum risk.

We can distinguish two types of risks: the navigation risks, and the risks associated with dangers. While the navigation risks influence the quality and the strategy of navigation, the risks associated with dangers depend on environment and can have physical impact on a human body. Physical dangers jeopardize the navigator's locomotion, feeling, and health. To avoid them, the navigator slows down the motion or avoids certain topographic features. Such are the risks:

- to get injured on steep or rocky terrain,
- to become exhausted on steep or long sections,
- to get wet on passage of the waterflows or marshes,
- to get dirty or muddy on passage of scrubs or wet soil.

We will not deal with the risks related to dangers in the continuation of this thesis.

3.5.7. Other types of optimum paths

Regardless of the navigation task and intention of the navigator, the shortest or the fastest path traditionally dominates among the accepted criteria for the optimum path. Beside these two, other interesting, but quite subjective criteria applicable mainly in urban environment and in city parks, have been studied and tested. Golledge (1995b) has ranked some of them by preference of tested subjects walking between a common origin and not too distant destination in a university campus: (1) the path with fewest turns, (2) the most aesthetic path, (3) first noticed path, (4) the path with longest leg first, (5) the path with many curves, (6) the path with many turns, (7) the path different from previous, and, (8) the path with shortest leg first. It is interesting, that even in such simple and restricted environment high preference was given to curved paths and diagonals.

The same author is quoting plenty of other criteria from the perspective of normal daily activities undertaken by individuals (Golledge 1995a): the path with the least obstacles, the path with high landscape differentiation, the detouring path, the path with the minimum actual or perceived congestion, the path with the minimum number of segments, the path with fewest left turns, and the path with minimum non-orthogonal intersections.

In certain circumstances, the secondary condition can be also fulfilment of some additional task, such as eg. reconaissance, hiding, surveying, rescuing, or sightseeing (Lee, Stucky 1998). Such tasks impose special conditions for the optimum path, like the most safe path, the most hidden path, the most economic path, the most interesting path, or the path with the most beautiful vistas. None of the conditions from this chapter will be treated separately in the thesis.

3.5.8. Optimum path with multiple least-cost conditions

In the chapters above, we presented each least-cost condition separately, while in a real world wayfinding tasks we usually combine multiple conditions. For example, we often require that the optimum path must be the shortest, the fastest and the least risky at the same time.

When we are not familiar with the environment, we solve multiple least-cost conditions cognitively, by planning of the path in advance with the aid of a map. We simultaneously assess and compare the relevant parameters for each single least-cost path condition, eg. the distances, the frictions, the risks, and the number of waypoints. As some condition can be preferential, we have to weight each one cognitively, before we start the travel. In execution of the travel, we perform path integration as well as cognitive cost integration and prediction to follow the planned optimum path.

For the computation of the optimum path, we need quantitative assessment of the parameters. We numerically accumulate the costs of travel in different directions, compare them and decide which direction of travel offers the least costs. Therefore, the optimum path is a cognitive or a numerical compromise between the weighted multiple least-cost conditions where each of them requires minimization of cost regarding respective single parameter. We show such case in the simulation chapter.

4. NAVIGATION WITH MAP AND COMPASS

We start the chapter about navigation with map and compass with the description of both tools, and the principles of their use. While a compass is showing only the direction of the magnetic north, a map is a complex representation of multiple objects and phenomena, which depends on the scale and the rate of cartographic generalization. We explain this with the comparison of a topographic and an orienteering map.

Since the orienteering map contains all relevant topographic features for navigation, we introduce the technique and the competition rules of orienteering, which will serve as a showcase of navigation strategies in the continuation of the thesis. We also provide the arguments why we rather study the orienteering in a natural environment instead of the orienteering in a city. At the end, we explain the principles of navigation with incomplete tools: without a map, without a compass, or without any tool.

4.1. The principles of map and compass use

The navigation with compass and map is a usual type of navigation in a natural environment. It requires a paper map, which represents the environment from a bird's-eye view, and a compass, which is sensitive to the ubiquitously present Earth's magnetic field. In fact, a map is the principal tool, meanwhile a compass serves for a proper orientation of a map towards the north, and for the explicit route sketching. Several types of topographic maps and charts can be used, provided that the level of details is sufficient to recognize the origin, the destination, and the topographic features important for navigation. Likewise, several different kinds of compasses exist regarding the precision of the compass scale and the construction properties. In this thesis we focus on the orienteering technique of navigation with an orienteering map and compass.

Navigation with map and compass is cognitively demanding as we must create a cognitive map, and use cardinal directions which can not be sensed by humans. It demands from the navigator a substantial perceptual, interpretative, and cognitive skills. The technologically simple and cognitively complex navigation with compass and map will be later opposed to and compared with a technologically complex and cognitively more simple technically augmented navigation with a GPS receiver. We first decribe a compass and a map, and then the specific principles of orienteering.

4.2. Compass

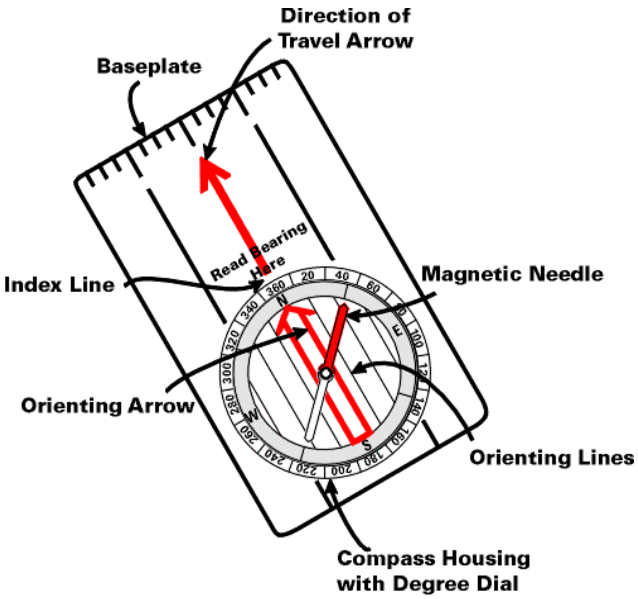
A compass is a tool, which shows the direction of the Earth's geomagnetic field on its location. Practically, it shows the direction of geomagnetic north, which is away from the geographic poles a coarse aproximation to the direction of the geographic north. It works in dynamic and stationary situations, indoors and outdoors, regardless of weather, daytime, and physical obscurity, does not need external power supply, nor any prior knowledge about the environment (Rainer et al. 2007). The Earth's geomagnetic field can not be switched off, like eg. a satellite system.

For humans, a compass is a surrogate for the magnetic sense of direction. In the past, the magnetic compass enabled cartographers to make maps and charts, and to explore the world (Hutchins 1995). In the time of invention, it provided much better accuracy of directions, than it was possible by visual estimation only by means of known landmarks or celestial bodies such as the sun and the Pole Star.

4.2.1. Orienteering compass

In orienteering, the two main functions of compass are the determination of cardinal compass directions, and the orientation of a map northwards. Precise angular measurements are never applied.

The orienteering compass contains a magnetic needle with red end pointing northward, and white end pointing southward (Figure 4.1.). The needle permanently tends to swing left-right and up-down to align itself along the line of geomagnetic force, or specifically, along its horizontal and vertical component. As the orienteer has to move fast, the compass has to be robust, simple, without angular scale, but with a sensitive needle which calms down quickly as the competitor changes the heading when running. Orienteers use a special ergonomic thumb variety of compass with plastic pointer to fix competitor's instant position on the map (Figure 4.1.). It helps to keep the map orientated correctly at all times.



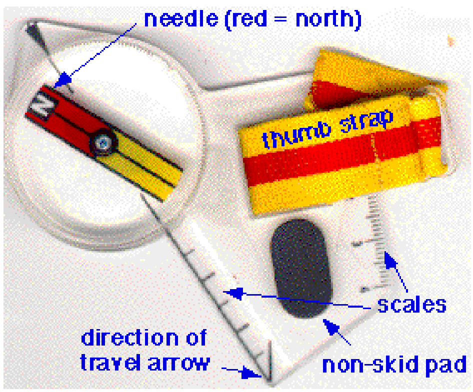


Figure 4.1. Orienteering compass (left) and thumb variant (right)
(sources: www.princeton.edu/~oa/manual/images/compass.gif;
http://www.williams.edu/Biology/Faculty_Staff/hwilliams/Orienteering/Images/lgthumb.gif)

4.2.2. Geomagnetic declination

As the geomagnetic north and south pole do not coincide with the geographic poles (or with the axis of Earth rotation), this fact is exhibited as horizontal angular discrepancy, called geomagnetic declination, which near geomagnetic poles reaches significant values (Figure 4.2.). This can happen also on some other regions on the world, as the declination and the geomagnetic field in general are not constant, neither static.

For navigation and orientation with a map, the declination has to be checked, and in many regions also accounted for. Most topographic maps have a diagram with the explanation of the amount of declination and its annual variation for the region being mapped. Commercial producers are offering compasses which allow for adjustment of declination with a rotating needle-capsule and azimuth ring, which can then be set with a screw to the right declination mark of the region printed on the bottom of the capsule.

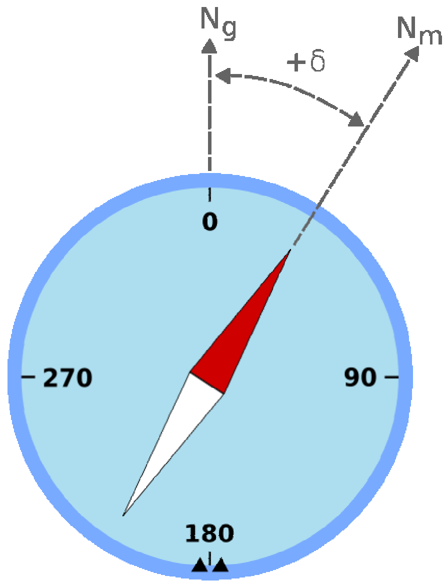


Figure 4.2. Magnetic declination δ between the geographic (Ng) and the magnetic north (Nm) (source: http://en.wikipedia.org/wiki/Image:Magnetic_declination.svg)

In orienteering sport, the problem of declination is surpassed by drawing the orienteering maps with the meridians pointing to magnetic north, not to true (geographic) north (IOF 2000, Bratt 2002). So, the declination adjustment is done at the time the map is drawn, rather than during navigation. If the cartographer uses magnetic north as a reference and if the navigator uses a magnetic compass to determine directions (presuming that the declination does not change locally), both tools will work together perfectly although the mapped positions do not refer to geographic grid (Hutchins 1995).

The magnetic meridians are considered straight and parallel within the confines of such a map. As the orienteering maps do not cover a territory wider than few kilometers in any direction, this is practically true. Also the declination is practically constant (and its variation is zero), until the next map update or reprint.

4.3. Map

The Western conception of navigation is based on maps, which encourage a conception of travel as a sequence of locations on a map (Hutchins 1995). They represent a spatial analogy to positions in a real world. The maps can be treated as computational devices, or analog computers (Hutchins 1995).

In the navigation with compass and map, we use exclusively different types of paper maps which are real, viewable, and permanently tangible. To understand the differences between them, we first introduce the role of map scale and cartographic generalization. Then we describe and compare two different maps: a topographic map at the scale 1:25.000, and a standard orienteering map at the scale 1:10.000.

4.3.1. Map scale and cartographic generalization

A paper map has a *map scale*. The map scale determines which level of details can be represented with a certain positional accuracy. It has important influence on psychologic perception of space and spatial behaviour because it directs spatial communication, and enables to test validity of simulations, among others (Montello 1993). We anticipate, that many types of errors of navigation are closely connected to the map scale, and that the map scale has a direct impact on the navigation strategy.

The navigation in a natural environment can be executed with a suitable map at a sufficiently large scale to show all relevant topographic details. Maps at small scales lack navigational information, and the risk to get lost becomes too big. For the discussion of orienteering only, the scale of 1:25.000 is referred by the term 'small scale' and the scale of 1:10.000 to the term 'large scale', although in cartography and cartometry we find other quite different categorizations of map scales (see eg. Maling 1989).

When we compare the topographic map to the orienteering one, we observe that the mapped information relevant for navigation is substantially generalized (Figure 4.3.). From the standpoint of visual map perception, the most obvious difference in representation at a small scale is increased smoothness of all linear cartographic symbols including contour lines, this being achieved by line simplification. There is also evident lack of minor topographic objects, which could serve as waypoints. Most of the footpaths are omitted, which can be very misleading when we are searching for a certain crossing among several others not being mapped. Neighbouring buildings are often represented with one single areal sign. Shorter, narrower, and intermitent streams are omitted. The density of vegetation is not shown, so the navigator can encounter impassable areas which are not designated on a map. Representation of vegetation types and their fuzzy boundaries is generally the least reliable and accurate of all features.

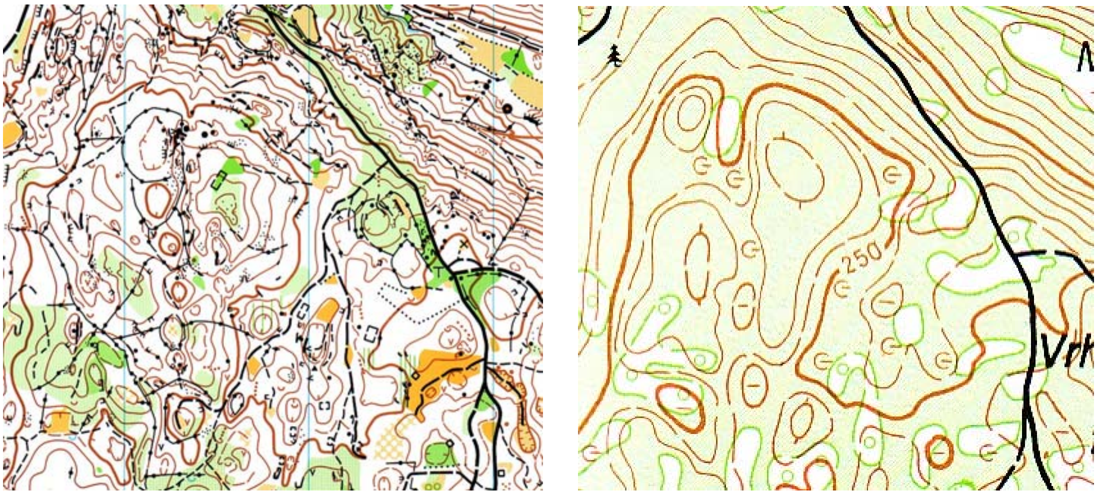


Figure 4.3. Orienteering map (left) and topographic map (right) showing the same area (sources: map Krajna vas 2, Slovenian Orienteering Federation, original scale 1:10.000; map sheet Branik, National Topographic Map 1:25.000, Surveying and Mapping Authority of the Republic of Slovenia)

At small scales the choice of waypoints is limited to either represented linear objects, or to larger and obvious point objects which afford longer spans between consecutive control features. Therefore, fine navigation on short (eg. few hundred meters) distances is less reliable, depending on the type of terrain. Distance judgement and sustaining the planned course is less accurate on longer distances, especially if the terrain is complex and undulated. However, such a map is good for a general topographic navigation on longer distances as the scale allows for representation of larger areas on a single sheet of paper with still satisfying detailness.

Maps at smaller scales omit important features for navigation (eg. the landmarks which are potential waypoints), causing deprivation of information for the navigator. The more is a map generalized, the more the navigator needs to interprete, reason, and inference from the existing information on the map. The lack of information is therefore replaced by vague inferencing and cognition.

4.3.2. Topographic map

Topographic maps are reference maps showing natural and man-made objects (Robinson et al. 1995). Maps at scales smaller than 1:200.000 are taxonomized as (general) geographic maps. In comparison to geographic maps, found in atlases, topographic ones are positionally highly accurate, usually official, and issued as a series of sheets, joined together without overlapping. Frequently, the fundamental topographic map scale is 1:25.000, followed by 1:50.000, 1:100.000, and 1:200.000. Thus 1 cm on a map represents 250 m, 500 m, 1 km, and 2 km, respectively. A topographic map is usually produced and interpreted from aerial survey photos, and terrain inspection.

Topographic maps are indispensable tool for orientation in unstructured space lacking artificial structures like routes (Pick et al. 1995). For pedestrian land navigation we practically use only the scales larger than 1:50.000 (Figure 4.4.).

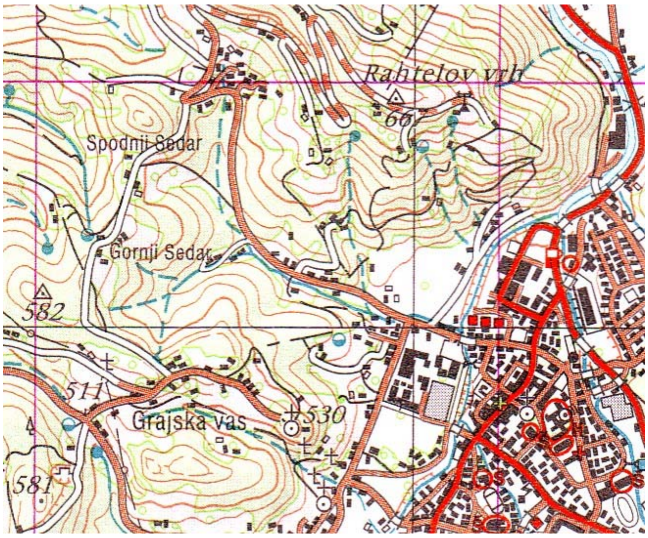


Figure 4.4. A topographic map at the scale 1:50.000
(source: map sheet Slovenj Gradec, National Topographic Map 1:50.000, Surveying and Mapping Authority of the Republic of Slovenia)

On a topographic map at the scale 1:25.000 all methods of cartographic generalization are applied, yet in lesser extent than on general geographic maps. Some objects are omitted, reduced, simplified, smoothed, exaggerated, displaced, joined, or reclassified (João 1998). Bearings can be slightly inaccurate in some cartographic projections, but without a major impact on the navigation process.

The highest positional accuracy, which is in principle achievable on a topographic map is around 0,2 mm times the map scale denominator (Maling 1989). Such is also the range of manual digitizing error (João 1998). The graphic accuracy of 0,2 mm is a consequence of mapping, reprographic, printing, and cartometric technologies. The value is valid for the paper maps since a human sight can distingush objects not nearer than 0,2 mm at the normal viewing distance of 20 cm. Thus graphic accuracy greater than the visual treshold makes no sense on a paper map.

Beside graphic inaccuracy, the position of individual objects can locally be altered also by generalization. João (1998) has shown, that the displacements caused by cartographic generalization are generally small, and below the cut-off limit of 0,3 mm, when we compare well-defined points on a map at 1:50.000 with a map at 1:10.000. Nevertheless, in some topographic situations the displacement can reach values over 0,5 mm, ie. well over the double value of graphic accuracy.

4.3.3. Orienteering map

Orienteering map is a very detailed topographic map with standardized map symbols according to IOF (IOF 2000). The mapping is accomplished exclusively by terrain works with the aid of the existing large scale topographic maps and aerophotogrammetric imagery, if available. A specialized computer software OCAD with a built-in signature catalogue for all object types is available for orienteering map design and finalization. For orienteering competitions, the map is overprinted with the course polygon. The feature type and the detailed location of an individual control point on it, is described on a separate graphic description of microlocations (ie. control descriptions, scheme of microlocations).

The map shows every topographic feature and detail, which could influence map reading or route choice. On an orienteering map virtually no significant cartographic generalization of topography is present. The precision and the spatial resolution allow for a high density of waypoints. Objects are mapped on true positions, therefore bearing determinations are true. At such a scale all linear, angular, and areal deformations owing to cartographic projection are negligible for navigation. Accompanied with a compass, it is an excellent tool for fine navigation on relatively short distances, from few hundred meters, up to few kilometers.

Regarding the map quality, the general rule for an orienteering map is that the competitor shall not perceive any inaccuracy in the map (IOF 2000). If the distance between the neighbouring features on the map does not deviate more than 5%, this satisfies accuracy requirements for orienteering. For example, if two footpath crossings (or, two boulders) are 100 m apart, the accuracy of mapped separation should always be better than 5 m. This rule implies that distance judgement by a skilled orienteer could be as precise as 5% of the distance passed. If such a discrepancy occurs laterally, ie. if the target object on a 100 m distance is shifted on a map for 5 m to the left, or to the right of the true line, it causes the deviation angle of around 3 degrees. This angle should be the minimal perceived angular deviation limit for a skilled orienteer.

4.3.3.1. Map scale, format, and contents

Orienteering map scale is often larger than the scale of the most detailed available topographic maps for countryside areas. The map is usually at the scale of 1:10.000, where 1 cm on the map is conveniently 100 m in the nature. For larger competition events a map scale 1:15.000 is used. For park and city orienteering, sprint orienteering, or training, the scale 1:5.000 is also used.

The map is insular and mute. The margins of topographic contents are mostly linear communications or other natural boundaries. No toponyms are present, nor any heights or coordinates. Map format is arbitrary, and the size is kept handy, frequently smaller than A4, and rarely larger than A3. The map is not folded. When on the course, the map format allows for simple folding to halves or quarters.

The map is drawn with the orientation towards the geomagnetic north. Thin parallel magnetic meridians are shown in blue on even spacings. Several magnetic north arrows at the edges of the map support orientation of a map at a glance to avoid mixing north and south.

Orienteering map is printed in black, brown, blue, yellow, and green. Similarly as on a topographic map, the map symbology is divided into five categories: landforms, water, rock, man-made features, and vegetation, each represented by a different colour.

The smooth part of the relief surface is most precisely shown with brown contour lines on a short equidistance, which is generally 5 m. The discontinuities are precisely shown with a special point, linear, or areal cartographic symbols for isolated terrain features. All footpaths are shown precisely, in different classes, in black colour.

Forests are shown on white paper background as this is the most common land cover on orienteering maps and should allow easy reading in obscured lighting. Patches of open areas without forest are shown in orange, so they are easily discernible on a predominantly white map background. The open areas allow fast running and are important relocation objects when a competitor is lost, or when he needs orientation to the next control point. The impassable or nearly impassable forests, and thick vegetation, are classified into several penetration grades, and shown in dark or light green colour, respectively.

4.3.3.2. Course overprint

The orienteering course is shown on a map with overprint in purple, as a polygon composed of marked control points, and connected with an open polygon of straight lines (Figure 4.5.):

- the middle of an equilateral triangle is representing the position of the starting control point of the course,
- the middle of a circle is representing the position of the intermediate control point,
- the number near the circle is representing the sequential number of the intermediate control point,
- the straight line is representing the shortest possible path between two successive control points,
- the middle of two concentric circles is representing the position of the destination control point of the course.

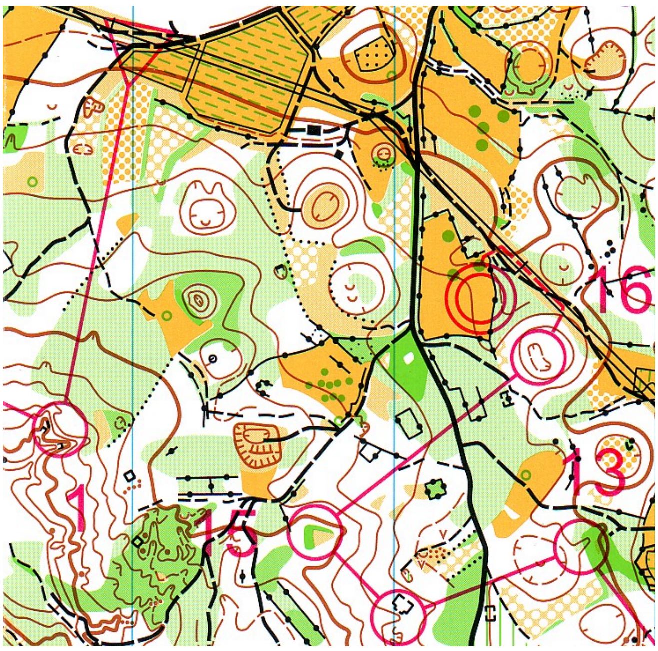


Figure 4.5. Orienteering map with a course overprint
(source: map Krajna vas 2, Slovenian Orienteering Federation, original scale 1:10.000)

4.3.3.3. Control point descriptions

The control point features are waypoints, which are usually not discernible from afar, either because they are obscured by vegetation, or because they are surrounded by other similar features. The control point flag is usually placed behind the feature regarding the expected direction of approach. When the competitor approaches the control point, he has to navigate precisely to find the control flag.

Although an orienteering map is very detailed and precise, it still can not depict the detailed characteristics of the feature, or of that part of the feature, where the control flag has been placed. Therefore, an additional explanatory graphic scheme of microlocations for all control points is overprinted in purple on the blank marginal part of a map, or printed separately on a piece of paper, called control description card or sheet (Figure 4.6.).

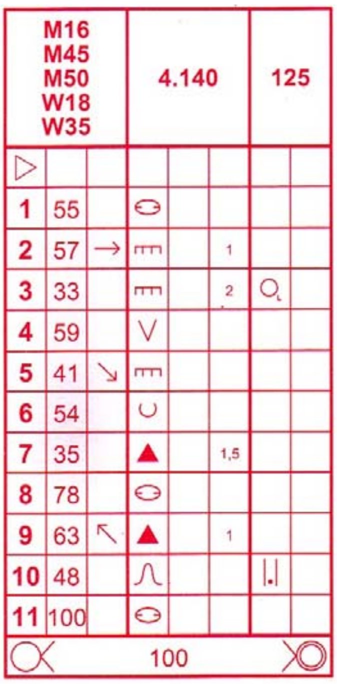


Figure 4.6. Control point descriptions

The scheme reduces the risk to miss the control point when already being in its vicinity. It describes the objects associated with the relevant starting, end, and intermediate control points more precise than cartographic symbols on the map. The scheme and the symbols used on the control description card are also standardized by IOF (IOF 2000). The individual columns of the scheme show (Figure 4.6., from left to right):

- the successive number of control point,
- the code of control point, which is written on the control flag (usually an arbitrary two digit number),
- the relative position of a feature with control point with respect to other features of the same type, which are also shown within the same control point circle (eg. the eastern, northern, lower, or middle feature in the circle),
- the feature type shown with a standardized symbol (possibly not the same symbol as that on the map),
- the special characteristic of individual feature (eg. deep, overgrown, rocky, ruined)
- the size of a feature (eg. the relative height in meters, the dimensions),

- the position of a control flag relative to the feature (eg. southern edge, peak, south-east corner, upper part, endpoint),
- the additional important information for the runner (eg. water stand, medical first aid).

4.3.4. Orienteering vs. topographic map

The distinction between cartographic representation techniques at different scales is influencing the errors and the strategy of navigation. The navigator has to adapt his actions to the utmost precision which can be achieved with a map at a given scale. There is no sense to apply fine navigation with a heavily generalized map. We summarize and compare the information contents of a topographic map relative to an orienteering one (Table 4.1.).

Characteristic	Orienteering map	Topographic map
Map users	only sport orienteers and occasionally pupils	civilians, professionals and army
Map purpose	precise orienteering on relatively short distances	general pedestrian orientation and navigation, terrain recognition and identification, mountaineering, trekking, long distance orienteering events, scout or army marches; other navigation; other civilian professional and amateur usage
Map scale	large; 1:10.000, 1:15.000	small; 1:25.000, 1:50.000, 1:100.000, 1:200.000
Map coverage	individual insular-type maps in various formats and scales	maps in series, uniformly covering the whole country in the same format and scale; the area is tiled by the edges of individual map sheets
Map format	smaller than A3, often less than A4	larger than A3; folded map
Mapping technique	complete terrain recognition and feature identification; aided by existing maps and aerial survey photos	aerial survey; terrain identification or control for a minority of objects
Detailness	very high; all significant objects shown	high; some object types which would be helpful in navigation are omitted
Positional accuracy	very high; eg. 3 m on a map at 1:10.000	high; locally degraded by generalization; eg. 15 m on a map at 1:25.000
Coordinate system	national cartographic projection (eg. Transverse Mercator type) without any indication of coordinates	national cartographic projection (eg. Transverse Mercator type) with explicit indication of planar and optionally geographic coordinates
Map grid	geomagnetic meridians	cartographic grid; optionally geographic meridians and parallels
Boundary of area	irregular; often linear features like roads, paths and river flows	map sheet edges are normally pairs of meridians and parallels; parallels to y and x axes of the national cartographic projection are also a possible choice

North diagram	only magnetic north arrows	detailed scheme with values; geomagnetic declination with annual variation, convergence of meridians between cartographic and geographic grid
Cartographic generalization	minor simplifications and line smoothing	moderate; line smoothing, feature shifting, elimination, aggregation of areas
Standardization of contents	world-wide; IOF symbolization	nation-wide; in some cases also international (eg. on NATO charts)
Relief depiction	detailed contour lines; details of discontinuities of relief surface; no terrain heights are shown	smoothed contour lines; schematic representation of discontinuities; significant terrain heights are shown
Vegetation depiction	detailed; classified according to transience and vegetation edge discernibility	moderately simplified; classified into few generic landcover categories
Footpaths depiction	detailed; classified according to width, importance and quality	generalized; some important or marked footpaths and tracks are shown; incomplete; often outdated
Toponymy depiction	none; only map title	most existing toponyms are present

Table 4.1. Characteristics of an orienteering map and a topographic map

4.4. Principles of orienteering competition

In this chapter, we quote the basic principles of orienteering sport competition. The competitive orienteering is applying the ordinary procedure of map and compas usage under the following presumptions.

Before the race, the competitor does not know the terrain, nor the map, nor the course. The competitor receives the map with the course printed on it when he starts off from the origin control point. The origin and the destination control points of the course are not the same. Before the race, the origin is reached by following a provisionally marked path. This rule suggests that orienteering is a typical *drop-off problem*, where the navigator is exposed to a substantial initial uncertainity in current situation (Pick et al. 1995).

The control points are physically marked with the red-white flags about 1 m above ground, on the selected terrain features. The competitor must find all control points in the same order as shown with the polygon on the map. The proof of finding the control point is electronic registration of visit with a personal identification chip attached on the competitor's finger, or a needle punch of the appropriate microlocation square on a control card (a small piece of cardboard). Mixing the order, or missing one single control point, means disqualification. To make the course more difficult, the person who traces it, places the control points:

- on a difficult terrain (eg. on a very detailed features, or on a very monotone terrain),
- on distances of alternating length (eg. from 100 m to more than 500 m),
- on the opposite side of obstacles (such as eg. high hills, deep valleys, dense bush, marsh, river),
- so that following the existing footpaths or roads would be too long, and too time consuming,
- so that the control points are not easily found, nor easily seen from afar.

The fastest competitor is the winner, therefore the optimum path condition is the fastest path. To avoid following other competitors, they start at different times, and in different course categories depending on the gender, age and abilities (routine) of the competitor. So, chasing somebody who might be lost as yourself, does not pay off. In principle, the natural phenomena are so irregular and differentiated that every runner can choose his own path, which fits his physical and cognitive abilities.

4.5. Orienteering disciplines

Various versions of orienteering exist on foot (cross-country events), skis, mountain bikes, or wheel-chairs for handicapped (trail orienteering). The foot orienteering courses are differentiated by:

- the distance of the course, to sprint (in parks and cities, winner's time about 15 minutes), middle (winner's time 30-40 minutes, 4-6 km), long (ie. classic, winner's time 60-80 minutes, distance 8-12 km), and marathon orienteering (ie. ultra distance, few hours to two days, possibly in a mountainous terrain, use of 1:25.000 map, individual legs up to few kilometers long),
- the course setting and the manner of visit of the control points, to sequential (in a prescribed order), or score orienteering (to find as many control points as possible within a time limit, regardless of the order),
- the time of competition, to day (in daylight) and night orienteering (in the dark),
- the nature of competition, to individual (individual performs independently), relay (two or more individuals run consecutive individual races), and team competitions (two or more individuals collaborate).

The orienteering principles are applicable for any other navigational purpose, recreative or professional. For example, Douglas (1994) has used the simulation of classic orienteering technique to demonstrate the performances of his raster-based algorithm for the computation of the least-cost path. Balstrøm (2002) has simulated a score orienteering principle in a hybrid least-cost path algorithm to plan a visit to meteorological rain gauges in the barren wilderness of Faeroe Islands. In the thesis, we will simulate the most common type of orienteering, ie. classic, sequential, daylight, individual, foot orienteering.

4.6. Orienteering in a natural environment and in a city

An important distinction which characterizes two extremes in terms of regularity and complexity of navigation strategies, is that between built and natural environments (Montello 2005). With the urbanization of environment, human knowledge of navigation skills had to be adapted to a new organizational structure of the environment. A pioneering and fundamental work on perception and organization of spatial information for navigation through cities is (Lynch 1960). Most people in the world (and also the scientists in the realm of wayfinding) live in urban areas. Consequently, abundant scientific literature exists for the navigation in artificial environments, and relatively few for natural countryside, even in the fields of military strategic planning, and orienteering (Balstrøm 2002).

The orienteering events are organized in both surroundings: in a pathless natural environment, and in the old city centres (which are usually closed to traffic). Although the competition and the orienteering principles in both cases are the same, the physical and the visual structures of environment, and the corresponding actions are completely different. The following description of differences between the two environments has the intention to explain, why we rather focus in a natural environment, when we use map and compass. Additionally, we compare orienteering in a natural environment and in a city, with an ordinary wayfinding in a city as practiced by pedestrians in everyday life.

A *natural environment* is relatively free of human impact, meanwhile *built environments* have large stationary objects, artificial regular patterns, straight lines, and right angles, so they lack variation. The table below compares the distinct attributes of both environments (Table 4.2.).

Characteristic	Natural environment	City
Structural organization of space	complex, fuzzy boundaries, irregular	high, systematic, crisp boundaries (street and house numbering system; separated pedestrian and traffic surfaces)
Symmetry of shapes	nearly none	moderate or high
Metric	curvilinear, irregular, differential turns	straight, Manhattan, right angle turns
Topology	relatively less important	very important
Surface macromorphology	varied	relatively monotone
Surface micromorphology	rough, detailed, highly differentiated	smooth, weak friction (allows vehicle traffic)
Slopes	rough, from flat to very steep, high variability possible	smooth, from flat to moderate, steep parts with staircases
Visual complexity of scene	moderate to very high	moderate
Landmarks	natural objects	built objects
Additional symbolic and semiotic information cues (knowledge in the world)	scarce or unavailable	abundant and systematic, essential for organization of a city and navigation
Physical obstacles to movement	irregular by shape and distribution (trees, steep slopes, deep water bodies)	large, regular and predictable (poles, buildings, fences)
Legal restrictions to movement	none or very rare (protected natural areas, sometimes with fences)	frequent (forbidden trespassing of roads, railways, private property)
Locomotion	difficult, physically tiring, slow	easy, fast

Table 4.2. Navigational characteristics of a natural environment and a city

When we oppose orienteering in a natural environment to the navigation in a city, we have to confront and consider substantially different cues for navigation (Figure 4.7.). A topographic type of map and a compass are generally obsolete in a city, where we rely on other orientation cues. In case we are not familiar with the city, we use a city map with street index.

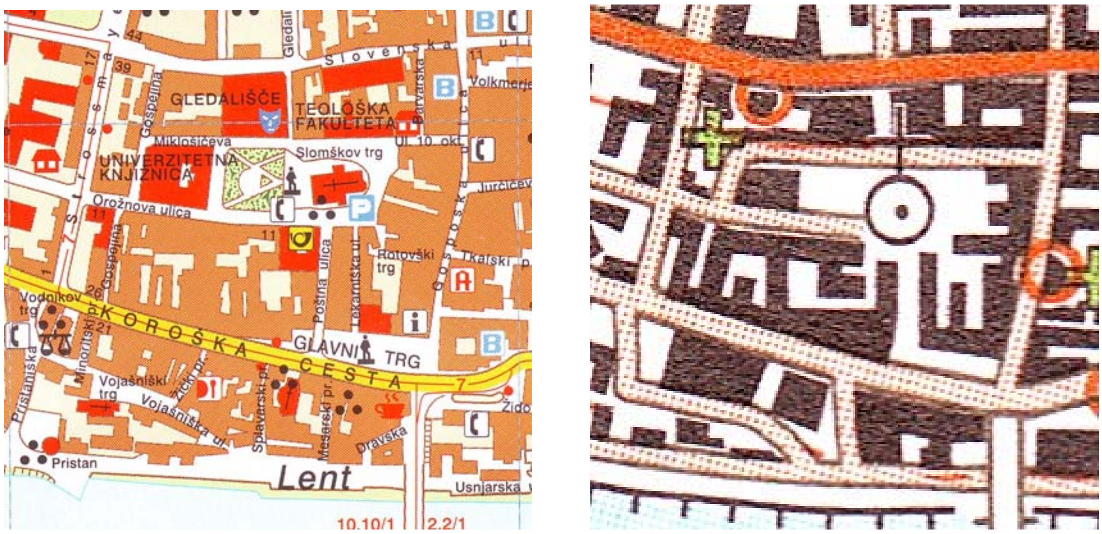


Figure 4.7. A city map (left) and a topographic map (right) of the same area
(sources: city map Maribor, Regional Surveying and Mapping Authority Maribor, original scale 1:10.000;
map sheet Maribor, National Topographic Map 1:50.000, Surveying and Mapping Authority of the Republic of Slovenia)

The navigator in a city in principle does not rely on navigation tools, but rather on symbolic and semiotic spatial information (eg. signs, posts, house numbers, information panels), which represent real features (Raubal 2002c). In areas without roads and paths, semiotic and man-made network infrastructure is missing (Balstrøm 2002). The second table compares orienteering in both environments, and pedestrian wayfinding in a city (Table 4.3.).

Characteristic	**Orienteering in a natural environment**	**Orienteering in a city**	**Wayfinding in a city**
Distances	hardly assessed in forests, more easily on open areas	easily assessed	easily assessed
Headings	differential changes in many sequences	constant on longer sections	constant on longer sections
Waypoints	related to natural objects	street crossings and house corners	distal landmarks, objects equipped with artificial semiotic labeling system (signs)
Execution of turns	smooth, non-orthogonal	(rect)angular, instantaneous	(rect)angular, instantaneous
Route selection	fuzzy fastest path	crisp fastest path along street network	crisp path along street network, various path conditions used
Route identification	relatively easy on footpaths, difficult when relying on terrain features	easy, but much care needed to follow the order of right angle turns	easy, if signs are abundant, and very difficult without them
Use of compass	essential	essential	unnecessary

Use of map	permanent use essential	permanent use essential	unnecessary when in a familiar city, city map needed in an unfamiliar city
Use of knowledge in the world	nil	nil	useful in a familiar city, essential in an unfamiliar city

Table 4.3. Characteristics of orienteering and wayfinding in a natural and a city environments

We conclude that the knowledge in the world, and the other physical and visual characteristics of a city usually do not require the use of orienteering strategies to execute wayfinding in a city. We continue the research with the study of orienteering with map and compass in a natural environment.

4.7. Navigation with incomplete tools

The principles of navigation with map and compass are universal in the Western culture. In this chapter we will shortly present, that we use a similar technique also in the situations where we have incomplete tools: no compass, no map, or no tools at all. We refer to them also later, in the simulation chapter. Beside not possessing a tool, the reason to use incomplete tools can be a malfunction of the tool, eg. because of local geomagnetic anomaly, or because the map is outdated.

Suppose we have to travel to the destination not too far away, eg. up to few kilometers. We know an approximate direction and distance to the destination, but we do not know the terrain configuration, nor any features in the area. There are no fixed control points on the way. We then consider the following three situations.

4.7.1. Navigation with map only

Navigation with map only can be a simple task if there is enough mapped orientation cues within the visual space. The navigator has first to orientate himself with the aid of surrounding features in order to match the north arrow on the map with the true north. After he starts the travel, he just has to execute feature matching permanently and direct his locomotion adequately. Normally, the travel must be executed slowly and precisely. So, the compass is substituted with more precise and more frequent feature matching.

Navigation with map only is a usual way of navigation in unknown urban environment, where abundant landmarks and knowledge in the world help to orient the map appropriately. In a natural environment, we try to stay on the footpaths. If this is impossible, the strategy is to select waypoints on shorter distances and to pay more attention on the surrounding objects. However, then we locomote slower then with map and compass.

4.7.2. Navigation with compass only

Navigation with compass only is much more risky. Theoretically, and supposing that we know the distance and the bearing to the destination, we can fix the bearing towards the destination on the point of origin, and then proceed along a straight line, hoping we will hit the destination. We can also count steps in between to roughly assess the distance, if the distance is not too long.

Usually, there are obstacles on the way, so we have to make turns left or right from the direction of travel. If we do not judge distances, read the exact bearings and reproduce the path with the origin, destination and all turns on a piece of paper at some deliberately chosen scale, we will soon lose the general geographic orientation, and we will be lost without any reasonable chance to relocate. We can not choose and use waypoints as we do not have a map. The precise navigation with compass only is inconvenient for longer distances.

The orientation with compass only is pure dead reckoning depending solely on the accuracy of bearings and approximate distances. The farther the destination is away, the bigger is the error in the final position. This technique is used on the high seas, since feature matching with nautical chart can be done only in the visual vicinity of coastline. Usually it is accompanied with additional navigation aids and tools.

4.7.3. Navigation without any tools

Navigating without map and compass, knowing only rough bearing and distance directly to the destination can be performed by dead reckoning, but with many chances to miss the target. We encounter the same problems as in the navigation with compass only. The absence of waypoints requires to rely only on environmental cues and personal feeling.

Navigation without any tools in unknown natural environment should be limited to survival adventures, where it must be combined with the observation of celestial bodies and topographic features. The environment can offer many cues for orientation to a mindful person. More or less they can be treated as a commonsense geographic knowledge, although this can vary substantially between individuals. We mention here some environmental cues briefly, without going into further details, since they have little relevance for the continuation of the thesis (Herlec at al. 1990):

- time and position of the sun,
- time and position of the North Star (Polaris) and some other celestial constellations,
- time, position and phases of the moon,
- direction of the wind and its consequences,
- certain characteristic of vegetation,
- noise caused by traffic,
- flow of rivers and direction of valleys,
- configuration of major mountain ridges,
- any kind of linear man-made objects (eg. footpaths, electricity power lines), which can lead to a settlement.

On the other hand, a rational and skillful navigator can manage navigation without any tools at hand, if he is familiar with the configuration of the terrain. This is possible, if the navigator has developed a cognitive map of the area beforehand, either by personal reconnaisance, or with the aid of anterior map perception, or from an exhaustive verbal description. Every adult person is able to develop a simple cognitive map and to navigate in familiar environment, eg. from home to a job, or through a city park, however such abilities differ accross persons of different intelligence, gender, age, cultural background, skills, experiences, knowledge, etc.

A special case of navigation startegy without tools is used in a city, where a person is heading to the destination, which is well seen distal landmark. Normally, the person would heuristically choose the road, which has the least deviation angle from the destination direction (Hochmair 2000). Some animals are able to combine recognition of landmarks with dead reckoning to update their orientation, and their knowledge of location and heading, as they move about (Wehner 1999, Montello 2005).

Navigation to an exposed landmark in a city may be disturbed by other high built objects. In such a case the least angle navigation is replaced by the directional navigation, where the person has to estimate approximately directions and distances, until the landmark is perceived again. Humans tend to execute and memorize turns linked to the body's orthogonal axes, favouring the least disorienting turns of around 0, 90 and 180 degrees from the direction of forward motion (Gluck 1991).

5. STRATEGY OF NAVIGATION WITH MAP AND COMPASS

In this chapter we first decompose the strategy into primitive actions to explain the sense-plan-act architecture of the navigation process. We hierarchically divide the optimum path (and the course) into legs, runs and segments. We show the fundamental method of orientation with map and compass, which is used throughout the navigation process.

Planning of the optimum path consists of rough optimum path selection, definition of waypoints, and detailed navigation techniques. To evaluate numerically the costs of the optimum paths in the simulation chapter, we need to distinguish separate processes beforehand.

We conclude the chapter with the argumentation why we have to know so the general as the particular characteristics of the strategy and planning. We argue the tight connection between the tool, the errors and the strategy, which is needed to demonstrate the hypothesis.

5.1. General structure of the navigation process

Humans often require extensive attentional resources to form an *explicit strategy* of navigation. Each navigation regardless of the type involves *planning* and *execution* of movements (Montello 2005). In orienteering, the strategy is chosen in short moments just before the navigator starts off the execution and locomotion. The planning phase includes three essential elements (Diagram 5.1.):

- the mental recall of the *method of orientation* with compass and map,
- the selection of intermediate *waypoints*, and,
- the selection of the *optimum path*.

The strategy phase is culminating in a mental construction of the optimum path. When the navigator begins to locomote, he tries to follow the planned optimum path in a real world. Since the tools, the planning, and the execution are not perfect, the navigator is making errors. We distinguish three types of errors: the *errors of tools*, the *errors of planning*, and the *errors of execution*. A single initial error can trigger another one, so they are accumulated and cascaded in the course of navigation. When they reach some treshold, they are recognized by the navigator. Usually this happens as late as in the execution phase. When the error is perceived, the navigator has to update his position and orientation, or even locally change the optimum path.

The navigation is planned and executed section by section. The strategy of updating by sections corresponds to the *plan-decide-move*, or, the *sense-plan-act architectures*, which are often used in the computational agent-based simulations (Krek 2002). The sense-plan-act architecture is applied at the waypoints, where beliefs are checked and revised. All three components are error prone. The execution of the sense-plan-act sequence must be iterative to avoid the negative effect of errors that have arisen from imperfect perceptions (Raubal, Worboys 2002).

If the errors are perceived too late, the updating procedure becomes inefficient, and the navigator gets lost. Now he has to perform the relocation procedures to return back to the optimum path, or to plan a new one. The trials and scenarios of unsuccessful navigation are not considered further in the thesis. After we have shown the general procedural structure of a successful navigation, and after the tools have been described above, we now proceed with the detailed explanation of each phase of the navigation process from the Diagram 5.1., starting with the method of orientation with map and compass.

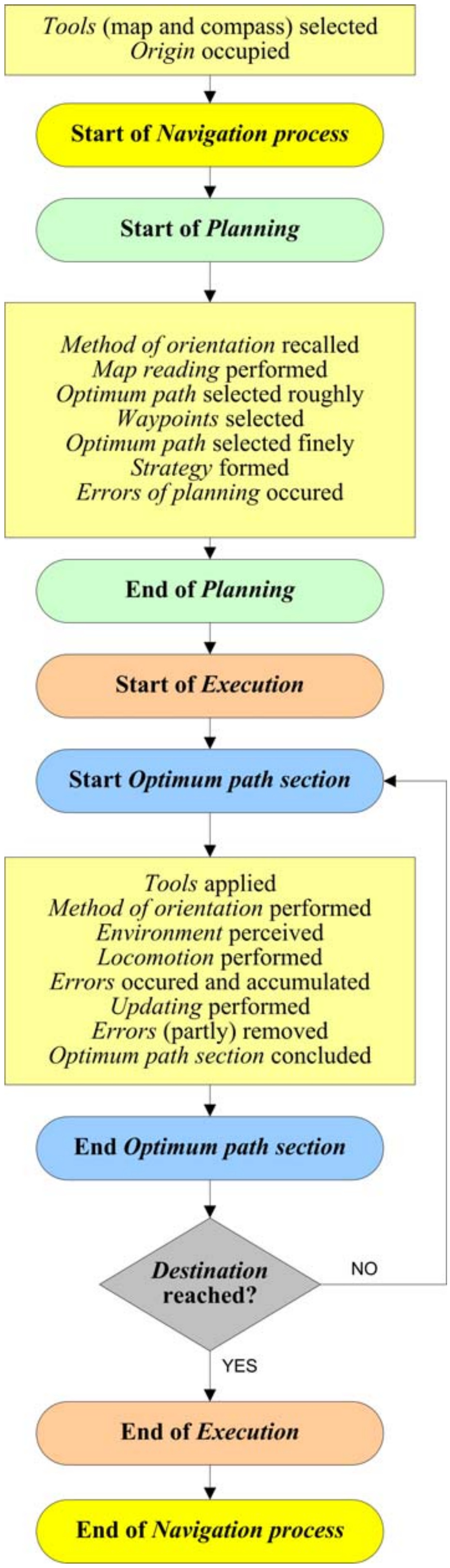


Diagram 5.1. The general structure of a successful navigation process

5.2. Method of orientation

In the beginning of navigation, the map requires an alignment of two directions: the one that the navigator is facing in the local environment, and the other which is the direction on the map toward its top (Montello 2005). Usually, the first is the *heading* towards the waypoint, and the second is the *north direction*. When the heading and the north direction on the map are aligned to the correspondent pair in a real world, the direction of each feature on the map matches the direction of the corresponding feature in the local environment. This simple rule is reflected in the method of orientation described below.

When we use a compass and a map, we always apply the same short and simple procedure to orientate the body in the direction of the optimum path. The *method of orientation* is realized by a succession of primitive acts, which go as follows (Figure 5.1.):

1. On the standpoint, take (or lift) the map and hold it horizontally.
2. Find the standpoint on the map.
3. Find the current destination on the map.
4. Rotate the map to align the straight line from the standpoint towards the destination so, that it will point forward.
5. Put the compass onto the map.
6. Turn the body (and the map with the compass on it) until the north point of the compass needle coincides with the (geomagnetic) north on the map.
7. Now, the map and the compass norths are coinciding. The current destination is exactly in the direction in front of us, and the map is oriented exactly as the terrain situation. The navigation can begin or resume.

Figure 5.1. The orientation completed: the person is heading to the destination, the map is oriented to the north

The method of orientation is first performed at the starting point and later repeated every time when we need to check or amend the orientation on the way, or at the intermediate waypoints. This simple operation can be performed standing still, or on the move. It fits only to the combination of map and compass. The other tools require profoundly different methods of orientation (or positioning).

We can observe that the method of orientation suggests *heading-up* (or *forward-up*, *track-up*) orientation of the map, and not the *north-up* alignement, which is normally prefered by people using traditional maps in ordinary map reading tasks (Montello 2005). When we navigate with a map in hands, we literally treat the forward-direction as the up-direction, as the landscape in fact visually rises in the visual field as it stretches out in front of us (Shepard, Hurwitz 1984). With the use of heading-up direction, we avoid mental rotation which would be otherwise necessary to orient the map from a perceived north-up to a heading-up position at every single realization of the method of orientation (McGranaghan et al. 1987).

5.3. Detailed structure of the navigation process

As an indicative example of navigation in a natural environment we take the orienteering with map and compass. The detailed strategy of orienteering involves appropriate structuring of the optimum path, and the actions pertaining to each part of the structured path.

5.3.1. Hierarchical division of the optimum path: course, leg, run, segment

Humans apply *hierarchical reasoning* for solving spatial situations (Timpf, Frank 1997). Hierarchical reasoning has economic foundations: it uses the least detailed strategy sufficient. Hierarchy is also a conceptual tool to structure the levels of detail of the environment (Tversky 1993, Timpf, Frank 1997). We apply it for the dividing and subdividing of the optimum path, hence the strategy and the execution of navigation incorporate the concept of hierarchical structuring.

In orienteering and in many other navigation tasks, the *course* between the origin and the destination is divided into legs (Figure 5.2.). Each *leg* is accomplished by a visit of control point, ie. of a waypoint marked by a control flag, and shown with a circle on the map. To reduce the risk of getting lost between two subsequent control points, each competitor further subdivides a leg to several *runs* with additional waypoints represented by well defined topographic features, which are shown with cartographic symbols on the map, but not marked extra. Each run traverses the terrain of different risk and friction characteristics. For a section of the optimum path, having more or less the same characteristics, we use hereafter the term *segment*. Thus, the orienteering course is a multi-stop travel, hierarchically composed of three types of routes:

- a course from the origin to destination,
- a leg from the current control point to the next one,
- a run from the current waypoint to the next one, and,
- a segment traversing the area of the same risk and friction.

Each run, leg, or course, is concluded at a waypoint. The last waypoint of a leg is the control point, and the last waypoint of a course is the destination control point. Segments are divided only by crisp or fuzzy risk and friction boundaries.

For an experienced and skilled orienteer, the planning phase in the competitive orienteering can last only few seconds between the arrival at each individual control point or waypoint, and the proceeding to the next one. Normally, the competitor does not perform any specific planning for the whole course, but only for the subsequent leg and run, however this fact does not alter the general procedure of navigation. It only breaks the course into a sequence of planning-execution cycles, where the navigator has to accomplish several cognitive and locomotoric tasks with map and compass, that provide intermediate solutions to the overall navigation task.

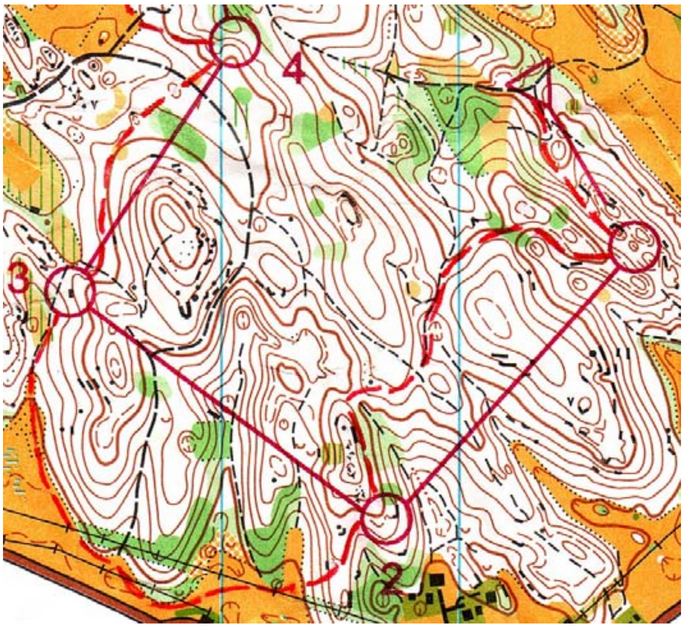


Figure 5.2. The course overprint (purple) with the planned optimum path (dashed red) for four legs
(source: map Predgriže, Slovenian Orienteering Federation, original scale 1:10.000)

5.3.2. Sense-plan-act architecture of the navigation process

The entire planning-execution cycle can be represented in a table as a sense-plan-act architecture (Table 5.2.). Each of the navigation tasks is described in detail in the next chapters.

STEP	PLANNING AND EXECUTION ACTIONS	BENEFITS GAINED OR QUESTIONS ANSWERED
1	**Start of the course**	Standing on the origin.
	Take the map and compass. Check general map data.	What is the map scale? What is the contour equidistance? What is the map production date?
	Check general course data.	What is the length of the course? What is the climbing of the course? How many control points are there?
2	**Start of the leg**	Standing on the control point.
	Perform the method of orientation.	On which control point am I now? Which control point must I visit next? Where is all that on the map? In which direction is that?
	Check the orientation. Compare the situation on the map with the real situation in front of you.	Did I turn correctly? How the terrain looks like?
	Interprete the contour lines and the topographic features along the leg.	What are the challenges along the leg?
	Select a rough optimum path for the leg, satisfying the fastest path condition.	Which optimum path fits my abilities best?
	Break the optimum path into runs. Select the waypoints.	The rough strategy is made. The waypoints are selected.
3	**Start of the run**	Standing on the waypoint.

	Perform the method of orientation.	On which waypoint am I now? Which waypoint must I visit next? Where is all that on the map? In which direction is that?
	Check the orientation. Compare the situation on the map with the real situation in front of you.	Did I turn correctly? How the terrain looks like?
	Interprete the contour lines and the topographic features along the run.	What are the challenges along the run?
	Cognitively construct the detailed optimum path for the run. Assess the risks, the frictions, and the segments of the run.	The detailed strategy is made. The navigation techniques are chosen. The segments are recognized.
	Construct a cognitive map of the run.	Memorize the relevant map details.
4	**Start of the execution**	Standing on the waypoint, or passing an arbitrary checking position.
	Move along the planned optimum path.	The navigation is physically executed.
	Update the position. Permanently observe the current small-scale space. Identify the contour lines and the topographic features on the map. Estimate the distance passed. Check the position. Assess the risks and the frictions.	Where do I navigate? What is around me? How far am I? What can I afford to do? Am I still on the optimum path?
	Update the orientation. Pause or slow down the movement. Roughly perform the method of orientation. In the vicinity of waypoint, precisely perform the method of orientation.	Where am I right now? Where do I want to proceed? Where is that on the map? In which direction is that?
	Go to the step 4 until the waypoint is reached.	The execution cycle of the run.
5	**End of the execution**	The waypoint is reached.
6	**End of the run**	The waypoint is reached.
	Go to the step 3 until the control point is reached.	The planning-execution cycle of the run.
7	**End of the leg**	The control point is reached.
	Go to the step 2 until the destination is reached.	The planning-execution cycle of the leg.
8	**End of the course**	The destination is reached.

Table 5.1. Planning and execution actions with a map and compass

5.4. Map recognition

In the planning phase, the navigator has to read the map and perform the following basic tasks:

- check the map scale and contour equidistance,
- check the overprinted course characteristics,
- read and interprete the portion of the map between two subsequent waypoints,
- construct a cognitive map of the area between two subsequent waypoints.

Map reading and memorizing enables planning and prepares the navigator for the execution phase.

5.4.1. Checking map scale, contour equidistance and overprinted course

Generally, orienteering is performed with an orienteering map at the scale 1:10.000 or 1:15.000, or with a topographic map at the scale 1:25.000 or 1:50.000. The two map types in each of the two pairs appear similar in cartographic design, and use the same legend of cartographic symbols. It is always recommendable to check the map scale, and the equidistance of contour lines in advance, if we are not sure about these basic facts.

Topographic and orienteering maps are generally accurate, but can differ locally from the situation in a real world. It is likely that after some time from the print of the map minor differences have occured in the condition of urban areas, paths and vegetation, however there is a low probability that the relief surface has changed, and that the environment does not resemble the map any more. If we suspect that minor mismatches would be encountered on the map due to the lack of cartographic updateness, we check the mapping date in advance.

The overprinted course on the map is divided into legs. The only checking of the course on the map is inquiry about the number and the type of the control points, and, the length and the total climb of the course. All these facts are usually known to orienteer in advance. The real length of the course is measured along the straight connections of control points regardless of the optimum path. It is informative and preliminary, serving only for a rough impression and preparation for the technical and physical difficulties waiting for the orienteer on the real course.

5.4.2. Map reading and interpretation

Map reading is a cognitive process related to the cartographic projection, abstraction and generalization of map contents. To interprete the map appropriately, the navigator has to acquaint with the map form and contents already in the planning phase. Map reading ability is a prerequisite for map interpretation. We distinguish and define that:

- *map reading* is the ability to semantically identify the map feature correctly (ie. which type of feature is represented with a certain cartographic symbol),
- *map interpretation* is the ability to pragmatically identify and match the map feature with the corresponding one in a real world (ie. where the map feature is in a real world).

We read and interprete two important spatial cartographical representations on the map:

- contour lines with accompanied relief fetaures, and,
- topographic features.

5.4.2.1. Interpretation of contour lines

The terrain is represented on the map with contour lines and additional symbols for distinctive landforms. As the terrain is a three-dimensional surface represented on a flat map, the interpretation of landforms from symbolic contour patterns is not trivial. It is the most demanding task in map reading, that has to be learned. It uses a specialized knowledge developed by experiences and practice (Montello et al. 1994).

An additional help for the orienteer are index contour lines drawn with a thicker line. Normally, this is every fifth contour. When the terrain is flat, the intermediate dashed contour lines shown on a half of equidistance are used, too. The orienteer has to imagine land morphology with the aid of contours and by inferencing about the valleys from hydrographic network.

Especially important is a correct interpretation of surface extremes. Peaks, hilltops, and ridges should be distinguished from depressions, sinkholes, and valleys, respectively (Figure 5.3.). The navigator has to find the innermost closed contour line along the planned path to reveal that. Such oval or round contour line has a perpendicular tag pointing into it, or a minus sign inside it, if the area is depression, otherwise (ie. on peaks) it is void. As the mapped sign is a minor detail surrounded by many others, it can be easily overlooked by the navigator in motion.

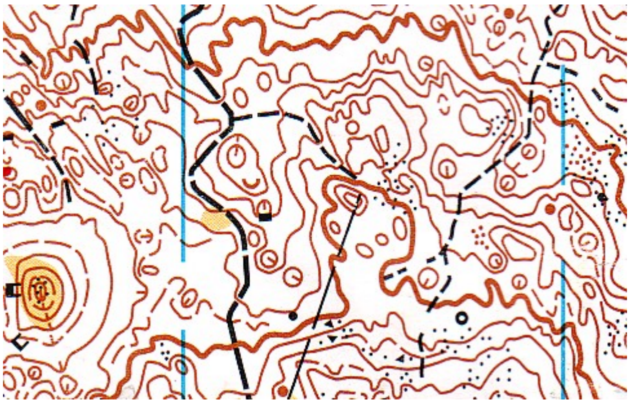


Figure 5.3. A confusing mix of small peaks and depressions
(source: map Soriška planina, Slovenian Orienteering Federation, original scale 1:10.000)

5.4.2.2. Interpretation of topographic features

Topographic features serve for general and detailed orientation. They afford checking of direction, relative positioning, speed and distance planning, optimum path delineation and tracing, and risk avoidance. Misinterpretation of topographic features leads to serious navigation errors.

Many topographic objects are vague which means that their crisp boundaries can not be determined, or even a definition of the object can not be expressed without vague assertions. Features with crisp points, lines, or closed boundaries are potential waypoints. Three specific navigation techniques related to crisp linear topographic features will be described in the forthcoming chapters: navigation to collecting feature, navigation along linear objects (ie. along "handrails"), and aiming off.

Distinct topographic features act as *attractors* (Golledge 1995b). Landmark features are positive attractors, and obstacles which have to be avoided or bypassed, are negative attractors. An open area in the middle of the forest is a typical positive attractor for the orienteer, especially if he is not experienced. On the other hand, an area sown with boulders represents a negative attractor as it distracts the navigator, even if the terrain is otherwise open and flat.

5.4.3. Construction of cognitive map

The ability to form a cognitive map of a large-scale space from observation of a paper map, which has to be mentally projected onto observations of environment, is used to select and follow a proper optimum path. Formation of a cognitive map is a usual consequence of map reading and interpretation before we start to navigate. A short-term cognitive map of the area which is just behind the visual space of the navigator, prevents from too frequent checks of the mapped features on a paper map. It should refer to the current section of the planned path only, usually between two consecutive waypoints. The best moment to update cognitive map is just before the run begins, while passing the waypoint, or the control feature. Experienced map reader needs a single glimpse to memorize key features. Considerable time savings can be achieved that way.

5.5. Planning of the optimum path

This chapter contains the core of the planning process and describes step by step how to delineate the optimum path. Beside the method of orientation this is the most important strategic action of the entire navigation process. The optimum path planning process activates the navigator's imagination, knowledge, memory resources, and other cognitive abilities.

The optimum path selection for planning of the entire course is a reiterating loop of a rough and detailed optimum path selection, which is concluded by the execution at the end of each cycle. Therefore, a single decision-making, or advance planning for the whole course, does not occur.

In the *rough optimum path selection* the navigator decides which type of optimum path (eg. shortest, fastest, least risky) will be used regarding the topographic configuration. He roughly figures out where the optimum path is on the map. In this phase, the entire leg is planned. Then the waypoints are selected.

In the *detailed optimum path selection* the navigator chooses the detailed navigation techniques between two consecutive waypoints. In this phase, the consecutive runs of the leg are planned.

5.5.1. Rough optimum path selection

On the beginning of the leg, the navigator has to roughly assess several reasonable paths leading from the current control point to the next one with observation of the map. He is percepting the respective portion of the map, representing the leg, at one glance. Models of human macrospatial behaviour suggest, that places farther away, or routes with stronger friction (eg. with obstacles, vegetation, slopes) invite less interaction by the navigator (Montello 1997). The navigator is deliberating and pondering eg. about choosing a straighter version of the path which is shorter but more difficult for navigation, or, a path along linear features such as footpaths or power lines which is longer but easier to navigate.

Different preference characteristics of the path are usually blended within a single leg: to move fast, on the shortest distance, with an optimum risk, and under a normal physical strain. The navigator has to balance between the loss of time and saving energy. He has to study the horizontal and vertical configuration of the terrain. Sloping path with varying relative heights will occur on undulated terrain. He may want to avoid slopy sections because of, eg. bad physical condition, slippery terrain, danger of injuries. Rarely he chooses the path which follows the maximum inclination. Rather a bit longer but less steep path is preferred. Once, the optimum path preferences have been chosen, they often persist in the mind even if orientational perspectives change (Golledge 1995b).

At the end of the process the navigator ought to select a single, but still rough (overview) optimum path to be able to fix the intermediate waypoints immediately after that. The detailed techniques to track the path are selected later, after the selection of waypoints, and just before the execution.

5.5.2. Selection of waypoints

In orienteering, waypoints are usually located at landmarks. Some of them are critical for navigation, the others may only reassure the navigator that he is on the optimum path, ie. they reassure his orientation. Landmarks may be complex and large configurations of objects. On a very differentiated terrain it is extremely difficult task to identify which landmark is critical for the decision making, and which feature is really suitable as a landmark (Presson, Montello 1988).

Waypoints are chosen continually along the path (Figure 5.4.). Some are defined sharply, the others are interpreted fuzzly and intermediately. Normally, they are visited by the navigator, or at least sighted from the immediate vicinity. They reduce the risk of erorrs as they prevent the navigator from relying on dead reckoning on too long distance. *Distal landmarks* in precise orienteering on short distances can not help the updating of direction. We rather apply continuous feature matching to nearby objects, especially in a forest, where the sight distance is limited.

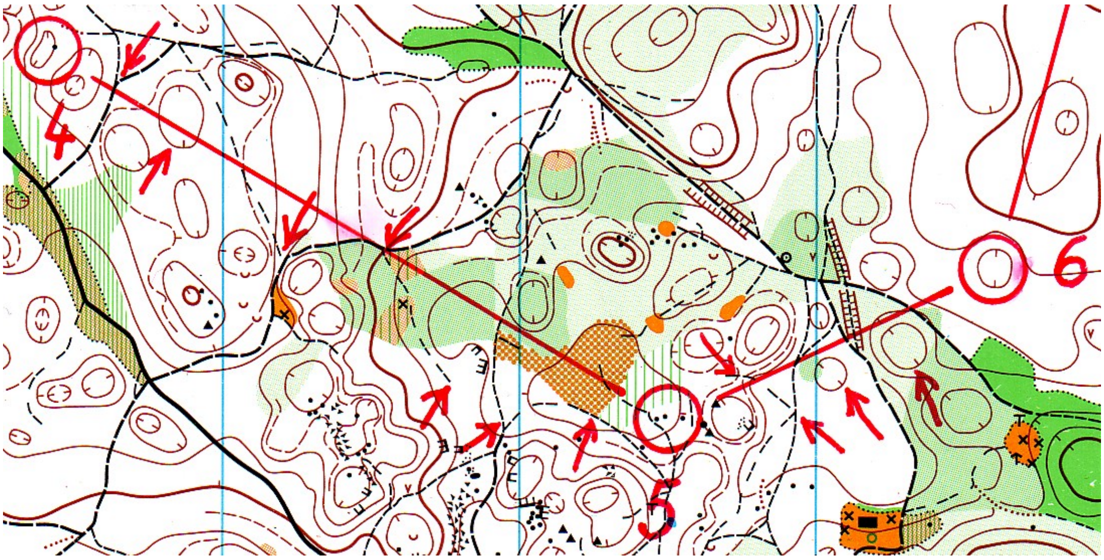


Figure 5.4. The consecutive runs between the waypoints (arrows) on the way from the control point 4 to 6
(source: map Fedranov gozd, Slovenian Orienteering Federation, original scale 1:10.000)

5.5.2.1. Strategy to select waypoints

The selection of waypoints depends on the optimum path, the topographic situation and the ability of the navigator. The strategy to select waypoints is a sensitive balance of several rules as follows.

- The waypoint is usually located at a discernible and easy-to-find landmark, which is a single topographic object, a group of objects, or a part of an object.
- The crucial waypoint should be located only at well defined point feature, a sharp-cornered detail or crossing on linear feature, a small areal feature, a corner on polygonally bounded area, or a geomorphological detail of the terrain surface. Generally, it should be on the place where topography changes in some way.
- The waypoint is normally located on an object or on the detail of an object, which is represented on the map with a cartographic symbol.
- The distance between two consecutive waypoints should be accomodated to the complexity of terrain.
- The waypoint should be selected along the planned optimum path, or vice versa, the optimum path should be selected so that affords good choice of waypoints.

- The waypoints should be selected so that they preserve the intended characteristics of the optimum path (eg. the minimum distance, the maximum speed).
- If it is in accordance with the optimum path condition, the waypoints should be chosen as much on the straight line between two control points, as possible. Beside short distance and speedy travel, this prevents from disorientation of the navigator at the waypoint, as he prefers to maintain more or less the same direction.
- The waypoints should be selected so that they prevent an excessive risk for errors, or a risk to get lost. The negative side effect is, that the insertion of waypoints to diminish the risk usually prolongs the path and deviates it from the straight line.
- To avoid missing the target, the navigator chooses the intermediate waypoint in the vicinity of the control point, ie. on the approach to the target.
- The waypoints which are not located at easily accessible or well sighted landmarks should be avoided.

Although the conditions above may seem simple to understand, the task of selecting the waypoints is far from easy and seldom optimal regarding the terrain and the navigator's abilities. Actually, the majority of errors are triggered by imprudent selection of waypoints or by relying too long solely to compass bearing. We observe, that the selection of waypoints is the core of the strategy.

The inclusion of intermediate waypoints can slightly lengthen the optimum path, and the availability of waypoints can even influence the planning of the optimum path (Bratt 2002). However, a good selection of them essentially reduces the probability of errors in navigation. The selection of waypoints supports the segmentation and the hierarchical chunking strategy, eases the cognitive path integration, and discretizes the process of dead reckoning.

5.5.2.2. Marked and unmarked waypoints

A waypoint can be unmarked on the terrain and selected by the navigator, or marked in advance by the course setter. In orienteering, four types of waypoints are used:

- the origin on the start of the course,
- the control points which divide the course into legs,
- the intermediate waypoints which divide the leg into runs,
- the auxiliary waypoints,
- the destination at the end of the course.

While the origin, the destination, and the control points are specially marked on the terrain and on the map, the *intermediate waypoints* are not. The later are just regular cartographic symbols on the map representing arbitrary features in a real world. The intermediate waypoints are also termed as *attack points*, since the competitor uses them to attack (approach) another waypoint, and, finally the control point at the end of the leg (Figure 5.5.).

When the navigator chooses a straight path along the azimuth, and if the length of sight (eg. in a forest) is short, the navigator can trace the optimum path by a sequence of objects lined in approximatelly the same azimuth, but not shown on the map (eg. distinguished trees or other minor details). Such *auxiliary waypoints* aid the navigator to sustain the bearing and to find the next intermediate waypoint (Figure 5.6.). That way the distance between the intermediate waypoints can be longer. As auxiliary waypoints are not marked, nor mapped, the navigator has to remember which feature is associated with the desired bearing (Hutchins 1995).

The determination of marked control points before the orienteering competition event is a process of topographic feature selection, marking on the map, and finally position checking in situ, either with direction and distance from several neighbouring objects, or with GPS coordinates.

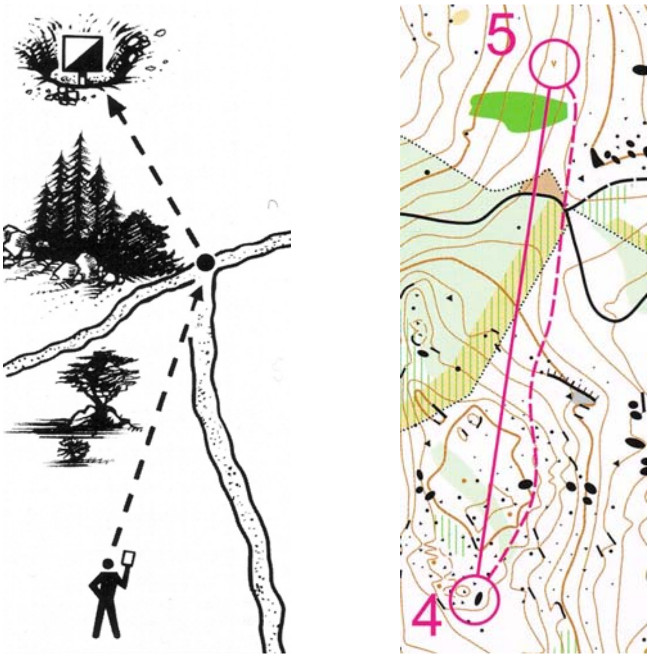


Figure 5.5. The waypoint (crossing) as an attack point
(source: (Bratt 2002))

Figure 5.6. Alligned trees (1,2) as auxilliary waypoints on the way to a regular waypoint (WP)

When the competitor then searches for the control point, he first has to find the object, and then reveal the control flag on the appropriate part of the object (Figure 5.7.). Therefore, the competitor is in fact searching for the object. If he finds the object and interpretes it properly, he has realistic chances to find the flag. Frequently and intentionally, neither object nor the control flag can be easily perceived from afar and from the expected direction of approach.

If the leg is short (eg. up to few hundred meters), and if the terrain does not have visual and physical obstacles or big height differences, the navigator can carefully execute wayfinding relying only to compass bearing towards the next control point. In this special case one leg consists of one single run where no additional waypoints are necessary.

Figure 5.7. The marked control point and the control point feature (rock)

5.5.3. Detailed optimum path selection

The detailed optimum path selection can be reinterpreted from (Montello 2005) as a behavioural problem involving planning and decision-making referring to creating shortcuts, avoiding obstacles, and scheduling trip sequences.

While the aim of a rough optimum path selection is a selection of waypoints, the main intention of the detailed optimum path selection is the reduction of risk. The navigator chooses specific navigation techniques based on a commonsense knowledge about orientating in the nature. Frequently, the configuration of terrain provides additional cues for navigation, like:

- line or areal objects, which lie along or in a similar direction as the optimum path,
- line or areal objects, which lie transversally to the optimum path,
- smooth relief forms represented on the map with contour lines along, or in a similar direction as the optimum path.

In the next subchapters, we describe the following navigation techniques used to avoid the risk in such advantageous circumstances:

- searching for a catching or collecting feature,
- moving along a linear object which serves as a "handrail",
- bypassing a hill or a depression by moving along the same contour line,
- aiming off from the direction to the waypoint intentionally to choose less risky direction of approach.

In general, the terrain usually lacks facilitating conditions. In this case, the navigator has to apply the technique, which represents the essence of detailed navigation: the navigation with constant bearing.

5.5.3.1. Navigation with constant bearing

Generally, the main task of rough optimum path selection is the division of the leg with waypoints into runs which can be navigated with a constant bearing and without risk to miss the target waypoint. The amount of acceptable risk is person-specific, so is the span between the chosen waypoints. Due to the physical, psychological and physiological reasons humans practically never locomote along ideal straight line, even if the destination is close and visible. After having the waypoints chosen in the rough optimum path selection, a straight traverse needs no detailed advance planning, but only careful execution, which is described in the chapter about the execution.

5.5.3.2. Navigation to catching or collecting feature

Catching or collecting feature is a large and easily recognizable object which lies in the direction of movement, but behind the control point (Figure 5.8.). This is the object which "catches" the navigator when he overshoots the control point, ie. when he runs too far. Encountering such a feature means that he has to turn back to find the control point.

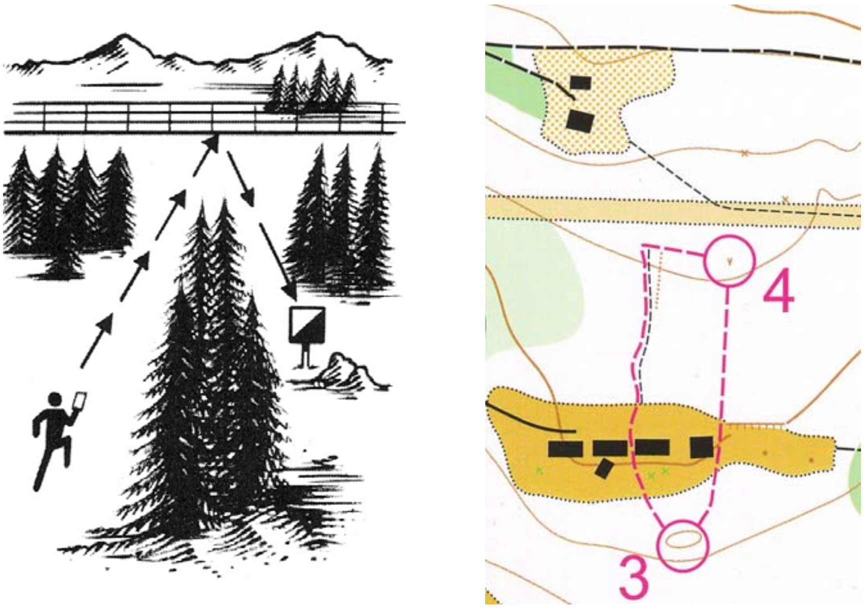


Figure 5.8. Navigation to catching or collecting feature
(source: (Bratt 2002))

Therefore, it is useful and more safe to plan the approach to the control point from the direction which will provide a relocation feature perpendicular to the final heading. Such objects are eg. roads, footpaths, trenches, brooks, forest clearings, etc. Planning the optimum path in this way means that the navigator avoids the danger of being lost after overshooting the control point.

5.5.3.3. Navigation with "handrail"

A "handrail" is a linear object which lies at least locally in the direction of the control point. Such a feature can ease and assist the navigation, as running along it does not involve frequent checks of direction (Figure 5.9.). However, the navigator must carefully judge the distance in order to leave in time the "handrail" for the final search of the control point. "Handrails" are frequently used by inexperienced orienteers. The beginner's courses are always set up along them to avoid complicated orienteering. Experienced navigators use "handrail" only if it enables faster run to the control point. Typical "handrails" are footpaths, fences, power lines, obvious vegetation edges, ridges, streams, and valleys. Generally, the man-made objects are convenient "handrails" which facilitate navigation.

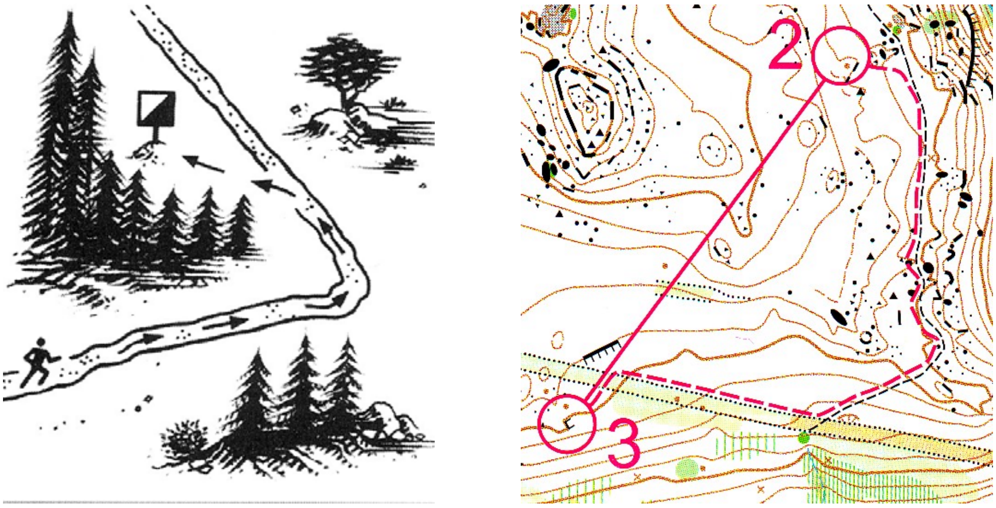


Figure 5.9. Navigation along "handrail" feature
(source: (Bratt 2002))

5.5.3.4. Contouring

In a physically rational navigation, the orienteer saves the energy by choosing the path without height difference, that is, a path on the same terrain height. Such a path can be interpreted from the map with the aid of contour lines. A contour line connects the points of the terrain surface sharing equal height (Figure 5.10.). It is shown on the map by a smooth curve. Thus, moving along a countour line (or drawing such a line on a map) can be termed contouring.

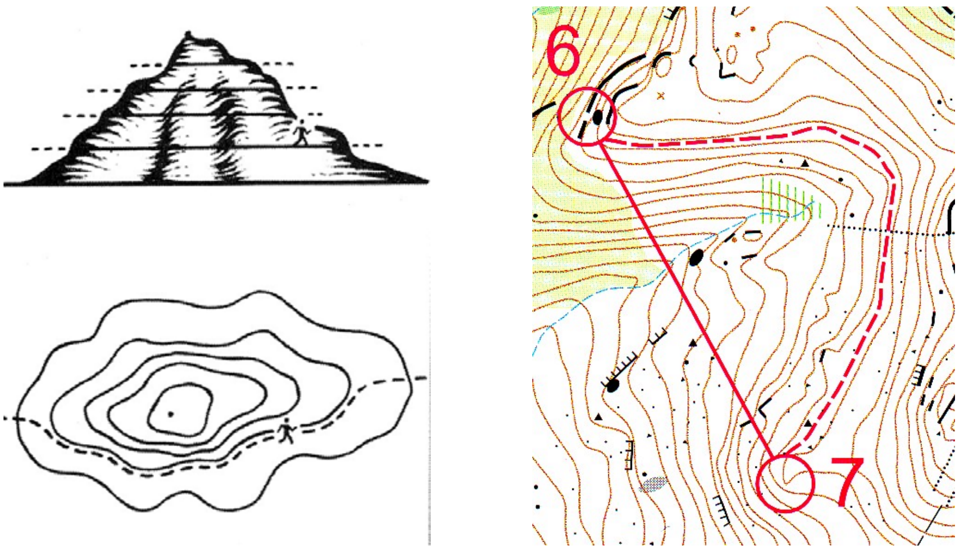


Figure 5.10. Contouring
(source: (Bratt 2002))

One has to be quite experienced to follow a contour in a real world, where it is invisible and not smooth, and where it can pass obstacles or steep slopes which gradually deviate the runner from the desired height. The direction of the contour line is represented by its tangent on the horizontal plane. Each point on the line has a different direction. It has been shown that oblique turns in navigation are more disorienting than orthogonal turns (Montello 2005), therefore it is not an easy task to run along the contour line and update the coincidence of north direction of the map and compass. The navigator has to rely rather on the surrounding objects and morphology of the terrain surface, than on some reference direction.

If the height difference between the origin and destination is not zero, experienced orienteers read it from the map and practise a similar but more risky technique of ascending with the same inclination, which is rewarding only on shorter distances and on monotone slopes.

5.5.3.5. Aiming off

Point control features are generally hard to find relying solely to bearing and distance judgement, especially when the distance is long. When such control feature is lying on a linear object, or when a linear attack object exists in the vicinity of the control feature, it is useful and safe to deliberately aim off a bit to the left or to the right (Figure 5.11.). Namely, when finally hitting that linear object, the navigator has only to turn right or left, respectively, and proceed along it to find the control object. The same technique can be applied when much bigger nonlinear object exists near the target.

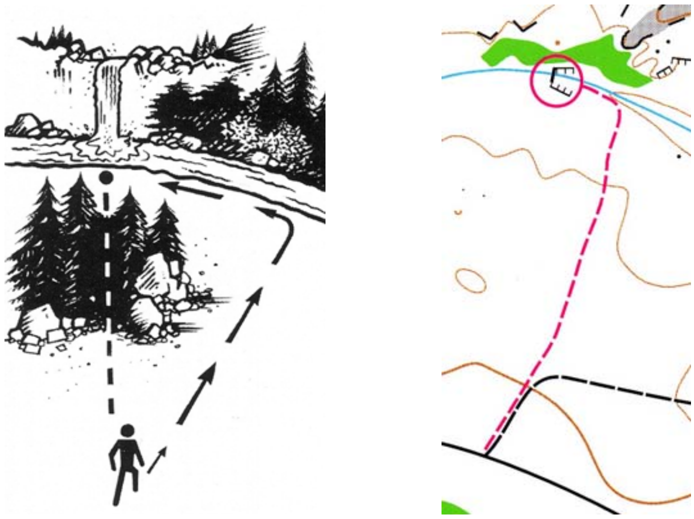


Figure 5.11. Aiming off
(source: (Bratt 2002))

This technique brings about a bit longer path, however the probability of error in the search of the control feature is minimized. In nautical navigation, it is often used when a seafarer is approaching the destination on the coast from the open sea. In case he does not see it, he intentionally heads to one side of it and then turns to reach it by sailing along the coastline (Hutchins 1995). The technique is called edge following as the last part of the manoeuvre is in fact navigation with a "handrail".

5.6. Argumentation of the central role of strategy

The description of the strategy of navigation above has shown the general and also the particular characteristics of the navigation process. The planned actions constraint the actions of the execution, and the errors of planning constraint the errors of execution. So, knowing and planning the strategy in advance, ie. before the start of locomotion, means to avoid as many errors as possible in advance. Since knowing the performances of the tool enables the navigator to predict the potential errors of navigation, he can choose the right strategy. We argue, that the navigator chooses that strategy, which offers least risks for the occurence of errors.

In the demonstration of the hypothesis, we will draw a map with different possible optimum paths between some origin and destination. To be able to define them, we need to know the detailed techniques of optimum path selection described above. Some of the optimum paths will fit better to a particular tool, while the other paths will be less risky with another tool. So we argue, that the risk of errors with some tool defines the strategy and consequently the choice of the optimum path.

6. EXECUTION OF NAVIGATION WITH MAP AND COMPASS

When the navigator concludes the planning cycle and starts to locomote, the execution begins. In this chapter we decompose the execution into primitive actions, considering separately:

- locomotion,
- updating of position and orientation,
- estimation of distances,
- rough and precise orientation procedures.

They result in the executed optimum path. In that way we explain, why the planned and the executed optimum paths are different.

6.1. Locomotion

Each execution of navigation is started with a motor action of the navigator. He can move along the planned optimum path in various modes of locomotion: by walking, climbing, running, or sprinting. The manner and the speed of locomotion are affected by personal and environmental factors. The mental imagery developed by sensing the environment also constraints the nature, type, speed, and direction of locomotion (Golledge 1992b).

6.1.1. Kinesiologic impacts on locomotion

Humans locomote bipedally by legs, making the optimum path stepwise discrete rather than smooth and continuous. Physical condition, strength and flexibility of the body determine the length and the frequency of the individual's step. The number of executed steps in principle influences the estimation of passed distance. In precise navigation, the orienteers often count steps on the approach from the attack waypoint to a hidden control point.

A natural ground is uneven. Consequently, the steps are unequally long, their frequence is changing, and the load of the individual leg is changing regarding the micro and macro terrain conditions. The step length is adjusted to strike the ground primarily by varying the vertical component of the motor impulse. In contrast, horizontal impulses contribute little to variation in step length (Warren et al. 1986). Geomorphologic irregularities require visual regulation of the step length to secure a proper footing. Excessive visual attention to footing can disturb feature matching along the path.

From anthropometry (and commonsense perception) we know, that the human body and extremities are not symmetrical. Consequently, in the course of running the individual parts of the body are compensating this asymmetry, eg. the legs with the adaptation of the force of the jump, the feet with the adequate positioning of the step on the ground, and even the spine with a scoliotic-type bending. For example, each slight rotation of the body resulting from such uneven transverse (ie. left-right) positioning of individual footsteps can disorient the navigator. Kinesiologically, even on a smooth flat surface a step with a leftleg and a rightleg can differ slightly, causing the deviation of the path or even long term circular motion from the point of origin back to itself. Accumulation of deviations caused by asymmetry of the individual human body can be completely avoided by careful feature matching, therefore we do not quantify this effect.

6.1.2. Environmental impacts on locomotion

Beside internal or personal motor and navigation skills, the environmental circumstances like *friction*, also play the role in efficient locomotion. A friction acts as unwanted *resistance to locomotion* and causes higher energy consumption. The only welcome friction is the friction of the feet (ie. the soles of the footwear) which keeps the body in a constant equilibrium during the locomotion.

Resistance to locomotion is composed from the air resistance, which is negligible in navigation, and the friction produced by penetrating the vegetation (eg. grass, bush, trees), passing uneven ground (eg. broken, sandy, rocky, marshy), or ascending the slopes. Some friction types can be assessed in advance by map reading, and evaluated qualitatively or quantitatively. This is shown in the simulation chapter.

6.2. Updating of position and orientation

The navigation with map and compass is a combination of the dead reckoning procedure with the updating of position, distance and direction between waypoints. Knowing permanently where you are, means knowing the way (Hill 1998), and, the question "Where am I?" is a question about correspondence between the surrounding world and some representation of that world (Hutchins 1995). The navigator must visually and semantically connect affordances and information, ie. what is seen in a real world and on the map (Raubal 2002c). In orienteering, affordances represent the possibilities for different routings through a complex environment.

The execution of a route consists of a series of *view-action pairs*, where a *view* is a set of sensory inputs at the current location of the navigator, and the *action* is a motor operation that changes the current view (McGranaghan et al. 1987). So following an action, the environment provides the next view, triggering the next action, etc. In orienteering, the actions are related to processes, which can be overlapping or separate, continuous or interrupted, ie. to:

- initial reconnaissance,
- feature matching, alignment, and mental rotation,
- localization,
- thumbing the map,
- estimation of distances.

6.2.1. Mental adaptation to the map

In the planning phase, the navigator identifies the map features and the optimum path by map reading and map interpretation, however he has still not began the locomotion. His map inspection produces certain beliefs about the characteristics and the configuration of topographic features.

When he starts the execution, he triggers the view-action iteration. His beliefs have to be revised, as he compares the map with the actual terrain. The navigator has to 'get used to the map', ie. he has to match his expectations about the terrain provided by map reading, to the perceived experiences (Ottosson 1996, Johansen 1997). This is gradually performed within the execution cycles of the first few runs of the course. Some orienteers reinforce reconnaissance as well as other actions of updating with the *thinking-aloud* technique on the move.

Adaptation to the map includes proper perception of the mapped topographic objects, relief surface, dimensions, forms, distances, height differences, map scale, directions, and positions, in comparison to the adequate real world situation. The *all-at-once nature of a map* is very effective for mental adaptation to the map and for the decision-making processes which follow afterwards (McGranaghan et al. 1987).

Although the catalogue of cartographic symbols for orienteering maps is standardized worldwide, individual cartographers interprete and map the same objects differently. Therefore, the orienteer has to adapt also to the mapper-specific interpretation on the map, which affected the quantity of represented objects and their categorization.

In the process of mental adaptation, the navigator establishes his true beliefs about the relation between the real environment and the mapped one. If he is well trained, this happens shortly after he has started off the origin of the course, otherwise it may last even until the first control points are visited. The longer he was not using a map (counted in weeks, months), the longer will be the adaptation process. Some extreme or uncommon types of terrain (eg. karstic) can require longer adaptation than the others.

Bad mental adaptation to the map on the beginning is a frequent source of serious errors, therefore in the beginning of the course, the navigator has to move a bit slower and sharpen his attention to get used to the map and to the type of terrain.

6.2.2. Feature matching, alignment, and mental rotation

To keep track of locomotion, the navigator observes and percepts the environmental scene, and receives a continuous optical flow. He activates proprioception of locomotion. To answer where, how far, how fast, and in what direction he locomotes, he additionally has to integrate observations with a symbolic representation of environment, and remember important scenes (Pick et al. 1995). These could be shown on a real map, or constructed as mental imagery. The perception of environment encoded on the map is also called a *mediated perception* (Gibson 1979).

A cognitive map is a *map in the head* (Kuipers 1983a). In orienteering, the navigator constructs a cognitive map for a short section of the optimum path at the end of the planning process. Usually, the memorized section is one run between two waypoints, however the amount and quality of cognitive map depends upon mental patterns and abilities of the navigator. With cognitive mapping, he avoids too frequent map reading which would cause slower locomotion.

As long as a cognitive map is sustained in the navigator's mind, his permanent task is observation of close topographic features and relief form details within the visual space, and matching the cognitive map to the observations. Since this is done by the selected features (which are not landmarks), the respective cognitive action is often called *feature matching*. Walsh and Martland (1994) have shown in a detailed analysis of the strategies in orienteering that at the beginning of the route, and at the control points, the method of orientation (chapter 5.2.) is used by orienteers strictly, whilst the orientation to prominent landmarks is used as a means of maintaining and reinforcing orientation along the route.

Feature matching facilitates generation of *viewpoint hypotheses* about self-orientation (Pick et al. 1995). The hypotheses are compared to expectations and evaluated with examination of additional scenes, and so on. After features are mentally matched, the navigator performs *alignment* relative to the surrounding objects, to maintain the direction of the optimum path.

The usual elements of cognitive maps are paths, edges, districts, nodes and landmarks (Lynch 1960). To properly execute the alignement, *mental rotation* of cognitive map has to be performed (Pick et al. 1995). In orienteering, a *heading-up orientation* of the map is used, so the cognitive map has practically the same orientation. A significant minority of people prefer more familiar, constant *north-up orientation* of the map as it facilitates acquiring knowledge of spatial layout (Montello 2005). However in competitive orienteering this is time consuming and error prone option, which is never applied, because it requires permanent and excessive mental rotation.

Mental alignment with topographic features demands high working memory as translations and especially rotations of mental representation can easily become incorrect even for otherwise intelligent people (Montello 2005). Therefore, the navigator neglects details and compares only the selected and distinguished objects perceived in his small-scale space, with his cognitive map.

The identification of an object or, a specific landmark, is an interactive process of matching the representation of object on a map, with expectations about its appearance and with the visual scene perceived (Hutchins 1995). According to the *first law of geography*, closer features are similar (Tobler 1970), and, according to the *first law of cognitive geography*, people believe that closer objects are similar (Montello et al. 2003). Such a belief of *distance-similarity* aids the matching of the perceived topographic features to the situation on the map.

6.2.3. Localization

The most basic task in navigation is determining one's location and direction in relation to the rest of environment. Traditional paper maps require from the navigator to infer a map location from environmental cues (McGranaghan et al. 1987). For locating the person's place we use the term *localization*.

Localization in orienteering is overlapping to some extent with feature matching and alignment, and comes as a final result of both. It can be performed mentally by combining several different general techniques that use observed or estimated angles, directions, and distances to the surrounding landmarks, being visible or out of sight, but usually close to the navigator (Golledge 1992b). They are used to locate or check landmark features and own position. Usually they are executed roughly and can be based on:

- *offset measure*, which uses an offset distance and a direction from a path linking two landmarks, to locate the destination,
- *triangulation*, which uses perceptual bearings on three prominent landmarks, to infer back-bearings defining the navigator's location,
- *projective convergence*, which uses imagined bearings from three landmarks to a landmark that is not seen,
- *trilateration*, which uses two or more distances to landmarks, to locate a landmark that is out of sight.

When a cognitive map for the respective area does not exist, or when it does not match any more with the navigator's beliefs and observations, the entire process of updating looks like that:

1. Slow down the locomotion.
2. Read and interprete the map.
3. Identify mapped contour lines and topographic features in a real world.
4. Match the current real situation to the corresponding map detail.
5. Create viewpoint hypotheses, and compare them with expectations.
6. Perform alignment to landmarks, and adapt accordingly the heading.
7. Fix own precise position on the map.
8. Roughly or precisely execute the method of orientation with compass and map, if necessary.
9. Create a cognitive map of the next section, and memorize it.

10. Proceed in the corrected heading until cognitive map persists, or until features do not match any more.

The above procedure is practically the most challenging skill of an orienteer which has to be performed in motion, and requires reasoning about the planned path in advance. Considering that on a several kilometers long course, the navigator has to make turns at least at every few hundred meters, pausing on each turn for eg. ten seconds can amount to several minutes at the end.

The procedure of localization and updating described above, is related to the tools, ie. to compass and map. If we change the navigation tools, the position has to be fixed by some other method. The navigator can vaguely inference about his position by means of specific environmental stimuli, like sunlight, noises from the nearby road, by seeing some distant familiar object (ie. distal landmark), or by reasoning about the properties of the encountered object. Such aids to navigation are mainly valuable in the relocation process and in the navigation without proper tools. To a limited extent, they are available only to sensitive, experienced and emphatic navigators.

6.2.4. Thumbing the map

Mental awareness about the environment and own position is one of the basic requirements to ensure precise navigation. Physically, it is realized by holding the appropriately oriented map in front of the body, and by pointing the map with a thumb on the current location, ie. by thumbing the map, as termed in orienteering literature (Bratt 2002). Since the navigator is constantly switching his head position, sight direction, and eye focus, from the map to the terrain and back again, he may loose his position on the map otherwise. Thumbing can be done by thumb itself or by pointing the corner of transparent compass base to the map.

Practically, every significant movement along the path, accompanied with the matching procedure, should be reflected in the shift of the thumb across the map. In complicated terrain situations with many similar objects, eg. with a grid of karstic sinkholes, thumbing the map is essential to prevent from getting lost.

6.3. Estimation of distances

Distance is fundamental to the prediction and explanation of spatial behaviour, so it has also a crucial role in all phases of updating and execution. Information about distance helps the navigator to orient himself, to locate places, and to efficiently utilize his resources, like energy or time, during navigation (Montello 1997). The knowledge of two types of distances are important for the execution of navigation: the environmental and the perceptual distance.

6.3.1. Environmental or cognitive distance

The theory of spatial behaviour explains that *environmental distance* is a distance, which is larger than the human body, but apprehendable by direct travel experience. As understanding of a large-scale space requires integration of knowledge over time, environmental distance is a part of cognitive map, and is frequently labeled as *cognitive distance* (Golledge, Stimson 1990). Experiments have shown that distances derived from cognitive maps are:

- asymmetric, as the length of the distance forward is perceived different than the length of the same distance backward,

- resolution dependent, as distances in a dense and information rich cognitive map appear longer,
- alignment dependent, as equal distances in various geographic directions appear different.

Generally, the distances estimated between nearby waypoints appear relatively larger than the distances between faraway landmarks, and the distances are judged longer when they have barriers, many turns or nodes (Tversky 1993, Freundschuh 1998).

6.3.2. Perceptual distance

The distance which is directly visually perceived from a single vantage point in a small-scale space is a *perceptual distance* (Montello 1997). As known from mathematics and as we practise in geodesy, we can determine the location of a point in a polar coordinate system, if we know its direction and radius-vector. In navigation we use a compass to determine bearings. On the other hand, we do not have an equivalent tool to determine distances. Although distances can be estimated or calculated from a map, every navigator additionally relies on his perception of reality and proprioception. Multiple, partially redundant direct sources of information provide a heuristic basis for judgement of distance (Montello 1997):

- the number of environmental features encountered, or route segments executed,
- travel time spent, and,
- the amount of effort and energy expended.

Such sensorimotor apprehension of information from the body, and perceptual estimation of distances usually subjectively tend to lengthen, or shorten the estimated distance and cause the errors of execution.

6.3.3. Quantitative and qualitative estimation of distances

The basic judgement of distances is a commonsense knowledge. It is learned from practice and experiences, thus trained orienteers, scouts, surveyors, geographers, and some other professionals can judge distances more precise than ordinary people. We usually estimate *egocentric distance*, ie. the relative distance between the navigator and the landmark (Loomis et al. 1992, Fukusima et al 1997). The egocentric distance can be related to the distance forward if we want to reveal how far is to the destination, or, to the distance backward if we want to assess the distance passed and the distance remained to the destination.

Frequently, a rough estimation of dimensions (ie. *absolute distances*) is also necessary when the navigator has to distinguish between the neighbouring objects of the same type, but of different size. If the objects are not in the vicinity of the navigator, he has to take into account also the distance to these objects. Objects can then be differentiated by comparison one to another, or to an object of known size.

Distances are usually estimated quantitatively in metric units, however there are notable exceptions. A simple but inaccurate means to judge the distance is also the time passed. For example, if we walk fast on a relatively flat and smooth surface, we can beat approximately one kilometer in about 10 minutes, or 6 kilometers in one hour (Herlec et al. 1990).

When the relief surface is undulated, humans normally do not estimate height differences separately from distance, but still tend to estimate flat distances, taking into account the delay caused by ascend, or, the advance casued by descend. Such mental behaviour is altered only when the distance is comparable to height difference. People living in flat countryside express distances of walking in kilometers, while those living in the mountains rather estimate them in hours.

The observations are accurate, if the visibility is good and if the route section is not too long. In certain situations, eg. in overgrown or very undulated terrain, and in the darkness, the assessment of distances becomes difficult. Then, the most promising technique is determination of distance by counting of steps. It is frequently used in orienteering to find a close but hidden control point. However it has several drawbacks:

- the navigator has to know the length of his step,
- the length of the step is not constant, but depends on a number of parameters, where the two most significant are the type of terrain and the level of tiredness of the navigator,
- the navigator has to convert the distances from the number of steps to the metric system,
- counting steps and calculating distances distracts the navigator's attention from other cognitive and physical operations, so experienced navigators use double pacing and count only leftleg or rightleg steps.

Distances in orienteering are also judged and compared qualitatively, especially when assessing the length of runs, legs, and courses. Several levels of granularity, usually not more than five, are distinguished. If names are given to levels, they are arbitrary, and denoting very far, far, commensurate, close, and very close features (Hernandez et al. 1995). Qualitative distance judgement is also applied in the vague estimation of the place where we switch between rough and precise navigation.

Many other objective, but inaccurate methods exist for assessment of distances, like the angle of sight, the "jump" of thumb, the recognition of details, the comparison with known distances, the duration of echo (Herlec et al. 1990). They are all out of the scope of this thesis.

6.4. Execution of the method of orientation

We have shown above that updating of position and orientation can include the execution of the method of orientation, if necessary. The frequency of performing the method should be optimal regarding the environmental circumstances. The navigator can not move fast and concentrate on locomotion, if he is permanently and strictly practising visual checking and amending the bearing for negligible values after each minor move. Instead of such a "differential" execution of the method of orientation, he rather switches between rough and detailed navigation, when this is reasonable.

6.4.1. Rough navigation

Geographic orientation in a natural environment is a composition of partial knowledge states about location, distance, and direction. The navigator can not cope with all possible aspects of orientation at all times, so he is permanently disoriented to some degree. Nevertheless, he is usually able to reach the destination with rough navigation instead of being completely oriented (Montello 2005).

The navigator is moving over harsh and overgrown terrain, which deviates him permanently from the optimum path. If he wants to move fast, he can not maintain precise bearing. He has to execute the method of orientation on certain intervals. In the middle of the run, it is not rational to execute the method of orientation precisely or frequently. Likewise, if the navigator is executing the aiming off technique, or a navigation to catching feature, sustaining accurate bearing is meaningless for success. When catching feature, or a waypoint is on a large object, or along a linear feature, the navigator can even omit the method of orientation, as he will find the feature easily.

Since the estimation of bearing is coupled with feature matching, a rough method with rough bearing normally suffices, unless the navigator is approaching the control point. It is much more important to avoid gross errors of direction determination (eg. 180° or 90° errors), which may lead to a total loss of orientation, to interruption of normal execution, and to relocation process. Small errors in the bearing can be easily corrected with the method of orientation, while gross errors demand relocation and belief revision.

6.4.2. Precise navigation

Rough navigation helps to find consecutive waypoints, while precise (or fine) navigation is used on approach to the control point. Normally, it is performed in the last 100 or 200 m, where the navigator also has to pay full attention to the mapped details. To avoid overshooting or undershooting the control point, the navigator has to switch from rough to precise execution of the method of orientation after passing the last waypoint, which is in fact the *attack point* for the control point. Generally, this occurs before the control point can be visually perceived.

Judging what is rough or precise navigation, and when to switch between both, is a cognitive process related to the navigator's experiences, feeling, self-consciousness, concentration, and many other factors. Only on extremely complex or monotone terrain, precise navigation is used along the entire leg between two consecutive control points, eg. in a labyrinth of boulders and sinkholes, or in a flat forest with no significant details.

In the course of precise navigation process, the navigator is faced with the *sorities paradox* (Hyde 2005). He is constantly balancing between vague assertions about the boundaries between objects. This can be most easily shown in the case of moving accross undulated terrain surface. The navigator is observing the terrain surface and comparing it with its contourline representation on the map. On rough terrain surface, he has to decide where one object (eg. a sinkhole) smoothly changes to another (eg. to a saddle). However, he can not denote a single step with which he would pass over the border between the two. Paradoxically, he will reach the second object step by step, logically keeping in mind as still being on the first object but actually already standing on the other. Nevertheless, he has to draw a decision where to amend and change the technique, ie. where to switch between rough and precise navigation.

6.4.3. Finding the control point

As mentioned above, we need to apply the method of orientation precisely, when approaching the control point. However, this is not the only requirement. The control feature is an object shown on a map, where the control flag is set up. Generally, the control flag is not visible on approach as the aim of orienteering is to find first the topographic object, not a red-white flag which only designates the object and serves as an instrument for registration of individual visit.

A recommended technique to find the control feature easier, is to choose such an attacking direction that the navigator will see the control feature from as far as possible. When the control feature is small, the control flag can be found easily. If the control feature is large or if it is surrounded by similar objects, the navigator must navigate precisely and inspect also the control point description carefully in order to attack the right part or side of the feature, eg. top or footpoint, north or south, upper or lower end. The description can show the dimensions or height of the feature, and distinguish between other qualitative characteristics, eg. between shallow and deep; open and overgrown; rocky, sandy and marshy.

Despite slowing down the pace because of feature matching, reading control point descriptions, and practising fine navigation, many competitors waste a lot of time in a search for the control flag or in a try to distinguish which of the similar objects is the control feature. Therefore, precise work with the tools is essential even though the remaining distance to the control point is very short.

7. ERRORS OF NAVIGATION WITH MAP AND COMPASS

The tools for navigation destine the affordances for the occurence of navigation errors. To assess the risks of navigation in the chapter about simulation, we first have to classify and study the errors of the tools, and the possible errors within the frame of the planning and execution cycles. We realize, that the tools can have own errors and failures, yet the most fatal errors in navigation are caused by human cognitive factors.

We shortly analyse also the impact of personal factors regarding physical ability, mental concentration, and emotions, although they are not the topic of this thesis. In the simulation chapter, we separate them from the navigation procedure and neglect them, since they can cause irrational outcomes of the navigation process. We also add the discussion about the consequences of errors, and about the relocation process.

7.1. Errors in technical sciences vs. errors of navigation

In the forthcoming discussion, numerous errors of navigation with map and compass are quoted. They can be described and classified in many different ways. Geodesy and other measuring technical sciences explain errors through three different types (Maling 1989, Thapa, Bossler 1992, Manning, Brown 2003):

- *Gross errors or mistakes* are blunders caused by human mistakes or equipment failure.
- *Systematic or cumulative errors* are the result of the influence of a scientifically explainable (though not necessarily understood) physical processes on the measurements.
- *Random or accidental errors* are the result of imperfect measurement technique or equipment.

Only the definition of gross errors directly incorporates human factor into measurements, while systematic errors concentrate on the study of physical processes without accounting for human influence. Random errors focus on techniques and equipment which have been invented, introduced and used by humans, so they involve human impact indirectly, but they are usually treated statistically.

Errors build up as a square root of the square of the component parts, thus the overall error is a combination of variances. While geodesy calculates errors, and then elliminates and repeats measurements burdened with gross errors, in navigation the tasks are not repeated, but rather indentified within the navigation process and accepted as a part of it. Each executed optimum path is unique and is never repeated. Besides, measurements are only a part of the navigation process. Planning and execution of navigation incorporate also cognitive and locomotoric processes which have to be treated in a different way.

7.2. Classification of errors of navigation

After we decomposed the navigation process with map and compass into planning and execution, both being influenced by the tools, we introduce a classification of errors which shows, that the errors of navigation are a consequence of the tools used. We can distinguish three basic types of errors:

- *errors of tools for navigation*,
- *errors of planning and strategy*, and,
- *errors of execution*.

The errors of tools are considered separately from the errors of processes. Several dilemmas could be clarified with the aid of such a classification of errors, like:

- What has caused the error? Could the role of the tools be explained directly or indirectly?
- In which phase or sub-phase of navigation has the error appeared?
- How was the error produced, and how it evolved? Which data were assessed erroneously while using the tools?
- What are the consequences? How do we relocate afterwards?

The question is, why explicit strategies are different, ie. why navigation with map and compass differs, eg. from navigation with a GPS receiver. We argue, that this is because the affordances offered by the tools to make various (harmless or fatal) errors, are different. Each tool affords certain kinds of errors. Some types of errors can just deviate the path, while the others lead to become lost. So indirectly, we try to show that the affordances to get some type and some quality of information with some tool, are different.

7.3. Errors of compass

A compass alone is a simple and reliable tool. Since it has few component parts there is low probability that it will fail to work correctly. However failures do occur and can have serious impact on navigation (Bratt 2002).

7.3.1. Needle not settled down

When the navigator is in the motion, the needle is swinging horizontally. High quality compasses compensate swinging of the needle very fast with a damping liquid inside the compass housing, and have special construction of the needle support. Nevertheless, the needle needs some moments to settle down, before the navigator estimates fine bearing. Therefore, erroneous reading which can amount up to ten or more degrees is the navigator's fault. This is far the most important and frequent error related to compass. It could also be treated as the error of execution.

7.3.2. Needle not pointing northward

Sometimes abrupt temperature differences or leaking liquid from the compass housing result in forming an air bubble, which slows down the settling of the needle and prevents it to point exactly to the north. Such an inconvenience can cause deviation of the bearing up to ten degrees. In case the bubble does not vanish, the compass should be replaced by another one.

7.3.3. Demagnetized needle

The needle can become temporarily or permanently demagnetized by metallic objects in close contact with the compass, like pocket knife, keys, car roof, or overhead power lines. The needle can then point randomly, or even the poles of the needle can become switched, so that the red end is pointing south. Such a compass can cause gross errors already in the beginning of navigation and is useless.

7.3.4. Needle affected by geomagnetic anomaly

Sometimes, but rarely, the needle can be affected by local anomalies in the geomagnetic field which unpredictably and substantially swing the needle off the north direction. This can happen near rocks containing metal, above subsurface iron ore bodies, or even on larger areas impacted by geomagnetic storm caused by increased solar activity. Reading a compass on such places is unreliable. If the anomaly is local, relocating to another position may help.

7.4. Errors of map

A map is a visual (carto)graphical representation of knowledge about topographic reality which has been acquired through observations. Like the knowledge about topography is imperfect, so inevitably is a map. Imperfection of the map comprises of two distinct components (Worboys, Duckham, 2004):

- *Inaccuracy*, which refers to a lack of correlation between observations or representations, and reality. A map is a result of both: inaccurate observations and inaccurate representations, in that order.
- *Imprecision*, which stems from a lack of specificity, or a lack of detail in an observation or representation. In the mapping process, imprecission occurs either intentionally, or as an inadvertent cartographer's blunder.

Maffini et al. (1989) provide general comments on the generation of errors in geographic information systems, which can be easily adopted also for maps, mapping, and navigation. The first potential generator of errors is due to the inherent properties of nature. Maps usually represent crisp objects and boundaries, while the nature is fuzzy, granular, and gradual. The second one is the nature of measurements resulting in a map. Any measurements in a real world are inherently prone to errors. The nature of the use of map, which encompasses cartometry, semantic interpretation, and spatial reasoning, can also cause errors in navigation. The third potential source of errors are all aspects of data model which were used to make a map, or to represent a map eg. within a mobile navigation device.

Measurement and observation (ie. reading) of imperfect map can be a cause of error in navigation, however most metric-quality maps like orienteering and topographic maps in developed countries are accurate enough for any kind of cartometric works (Maling 1989). In spite of simplicity of map and compass, their use paradoxically results in plenty of navigation errors, as we will show in the continuation.

On small scale maps, positional deterioration caused by map generalization prevents from practising precise cartometry. On the contrary, large scale maps at 1:10.000 practically do not exhibit any generalization effects, while at 1:25.000 they are generally weak. A large scale topographic map is only seldom the cause of a major navigation error except in the case when it was not updated for a longer period.

As mapping is a complex process involving methodological, technological and cognitive issues, many possible factors affect the accuracy and precision of a map. We will limit the discussion of map errors to those aspects which can have significant impact on navigation. Maling (1989) distinguishes three components of map accuracy and related errors, which are described in the following chapters:

- quantitative or positional accuracy,
- qualitative accuracy,
- completeness.

7.4.1. Quantitative or positional errors

Positional accuracy is the closeness of location of points of map detail to their true ground positions, measured in the same coordinate system (Maling 1989, Joao 1998). Often the term quantitative accuracy, which refers to the accuracy of horizontal and vertical position, is equaled to map accuracy. Such an explanation can be misleading because it neglects qualitative accuracy and completeness as integral parts of the total map accuracy. Both will also be treated later.

Positional errors of a map are measurable and computable, eg. with comparison of map detail with another source of higher accuracy, or with original ground positions. They can be expressed numerically and treated statistically (Maling 1989). Positional errors of a map are directly transformed into positional errors of navigation. However, if the topographic map is compiled according to some standard, they are usually not fatal for navigation.

7.4.1.1. Horizontal positional errors

Horizontal positional errors are due to the mapping process, or due to the generalization procedures (Maling 1989).

Errors caused by mapping
The impact of mapping errors on navigation depends on the map scale. The larger is the scale, the smaller will be its footprint on the optimum path on the ground. None of the mapping errors can directly cause a need for relocation. Mapping errors can be divided into two large groups (Maling 1989, Thapa, Bossler 1992, Joao 1998):

- *errors of topographic survey*, which includes also geodetic control, and,
- *errors of map production*, which includes drawing, compilation, colour registration, and reproduction.

The larger is the scale of a map, the greater is the share of survey errors and the smaller is the influence of map production errors. Specifically, the *draughting error* according to various topographic standards, is rated as a value between 0,1 mm and 0,2 mm and is constant regardless of map scale. Thus, if the accuracy of geodetic control point determination is just few centimeters on a global geocentric ellipsoid and though negligible at any mapping scale, still its draughting error can amount up to 0,2 mm on the map.

In the last decade satellite positioning and digital mapping techniques have contributed to even better map accuracy and to the fact that mapping errors became marginal when we compare them with positional impacts of map generalization. Mapping error of 0,2 mm on a map causes a shift of object detail for 2 m at 1:10.000, and 5 m at 1:25.000. This is negligible for navigation in nearly all cases, since taking a rough bearing with compass brings much larger errors.

Errors caused by generalization
There are three generalization procedures which affect the position of a mapped feature: *line simplification*, *displacement*, and *aggregation*. While at 1:10.000 none of them is significantly present, the topographic maps at 1:25.000 show moderate influence of all the three. In navigation with a map at 1:25.000, the aggregation of symbols for buildings in urban areas prevents fine navigation around them, identification of their corners, and estimation of their true linear dimensions and areas. Likewise, line simplification and smoothing at the same scale prevents from identification of details on footpaths or on contour lines. Displacement can only cause negligible shifts in position.

7.4.1.2. Height errors

Vertical positional errors, or height errors, are treated separately from planimetric accuracy as the measurements of horizontal position and height also pertain to separate procedures (Maling 1989). Unlike the positional error, the magnitude of the vertical one does not depend on map scale.

Wrong contour line labeling
Contours are labeled only on topographic maps for general use. Orienteering maps do not show labels, as there is little need to know the heights above sea level for precise orientation on relatively short distances, where only relative heights are of paramount importance. Wrong contour line label on a topographic map is cartographer's gross error which is very rare. When present, it can cause confusion about the direction of the slope and show apparently swapped hills and sinkholes. Such blunder results in a change of the navigator's belief about the morphology of terrain surface.

Contour line positionally shifted, or height inaccurate
In topographic mapping, the height of the entire contour line may float around the real value from about one third to a half of equidistance, where the allowable discrepancy depends on the topographic survey standard. Generally, the neighbouring countour lines are inaccurate for a similar value up or down. Orienteering maps at 1:10.000 usually have 5 m equidistance, and topographic maps at 1.25.000 have 10 m equidistance, which brings up to 2,5 m or 5 m discrepancy in the height above sea level, respectivelly. However, the relative heights do not alter significantly.

If vertical inaccuracy of a contour line originates from a local positional mapping error, the contour detail is shifted horizontally to a place with different height than nominal. Physical evidence of innacurate contour line height is a bit higher or lower height difference, or a slightly different slope than it is shown on a map. Larger horizontal shifts occur on flat terrain where photogrammetric contour line mapping is less reliable, locomotion is easier and height differences are not so important for navigation.

No matter what kind of vertical error occured, it virtually does not affect the quality of navigation, only if the form of the contour line reflects the relief form adequately. Any quantitative error which is not a gross one, being either horizontal or vertical, can be avoided by observing and following surrounding terrain features.

7.4.2. Qualitative errors

A qualitative error is a discrepancy between the situation on the ground and the category or meaning of a feature interpreted on a map. While positional errors follow mathematical and statistical cues, qualitative errors involve cartographer's judgement and interpretation of natural phenomena (Maling 1989). For the navigator they are much more influential for his assessment of integral map accuracy and general map quality than positional errors. The same holds for completeness.

Regarding the effects on navigation, we will deal with qualitative errors with respect to classification and map updatedness.

7.4.2.1. Misclassification errors

Natural objects are first semantically interpreted, categorized, and classified by the mapmaker. When the navigator uses a map, he interpretes topographic features in the planning phase and matches the features in the execution phase. If any of interpretations does not fit to the real situation, the navigator can have serious problems with orientation.

Map compilation involves a certain measure of judgement how to select an object or its boundary, which is not clearly defined on the ground (Maling 1989). Many natural features like vegetation, do not have crisp boundaries. This property hinders correct classification by both, the cartographer at work and by the navigator using the map.

The information about the type, presence, or absence of a specific obstacle on the way is much more important for optimum path selection than its exact position. Maling (1989) argues that the only exceptions are topographic maps for precise route finding. The larger is the scale, the more important should be the positional accuracy. However, larger scale brings also less reduction and more qualitative details important for navigation. The fact is that positional accuracy remains easily assessable, while qualitative accuracy requires interpretation rules.

7.4.2.2. Map contents not updated

This is the most unwanted and fatal error related to maps and mapping. Updating of map contents comprehends adding, deleting (erasing), and changing map symbolization in order to match map contents with the actual topographic detail. Most of map updating is related to new constructions or reconstructions of man-made objects, ie. to additions and sharp changes of map symbols. Such features like roads, footpaths, buildings and installations are often used as "handrails" or as catching and collecting features.

Natural phenomena change in a different way. Changes are often local and do not occur nor frequently, nor regularly. Vegetation cover (eg. forests, agricultural crops) can be cut down. Otherwise, the changes of overgrowing are slow and mainly affect vegetation borders which become fuzzy and unsharp. Generally, the vegetation borders are the least reliable elements of map contents. Hydrography can be altered temporary in the case of high or low waters, but rarely the river-bed or stream is modified, except if regulated. Relief apparently does not visually change at all in short term, except after natural hazards (eg. by erosion or earthquake).

A special type of temporary lack of updatedness is the difference between the seasonal statuses of objects (Joao 1998). Topographic maps which are compiled with photogrammetric imagery use photos taken in spring or autumn which enable interpreter to distinguish true ground surface under leafless trees. Orienteering maps show the status of vegetation and hydrography in the moment of detailed ground survey. This can be in the peak of growing or in a defoliating stadium, and in the time of low or high waters of hydrography. Thus, the season can influence the identification of these features, and can lead to discrepancies about the category of vegetation type, runability (penetration), or stream width. In extreme situations, the differences between seasonal statuses of object can have similar impact on navigation as a map which has not been updated.

7.4.3. Errors of completeness

A completeness can be related to the cartographic data model and to the data themselves. *Data completeness* is an error of omission or commission, and can be measured, while *model completeness* represents the exhaustiveness of the cartographic data model compared to a certain abstract model of reality (Brassel et al. 1995). When map objects are omitted by erroneous map compilation, the incompleteness has a similar effect for navigation as a lack of updatedness.

The completeness of a map is affected by measurements and sources for map compilation, yet the most important factor which restricts the cartographer from replicating natural phenomena in full detail, is map generalization triggered by map scale (Maling 1989). Among the generalization techniques, only reduction and selection alter the completeness of a map.

The model completeness can be interpreted as a *fitness of use*. It describes the ability to satisfy a certain set of application requirements. The complete representation of landmarks is of paramount importance for navigation as they define the optimum path.

The map contents can be too sparse or too dense for effective navigation. In the first case, the important objects and details are omitted. In the second, the objects are mapped too close, too small, and too many of them, so that they can not be distinguished or read appropriately. Visual clutter on a map prevents reliable and fast navigation.

7.5. Errors of strategy and planning

The errors of strategy and planning are directly or indirectly a consequence of the tools. Below we present the most important errors that occur in the planning phase (Tables 7.1., 7.2., 7.3.). They are treated in the same order of precedence as the actions in the chapter about planning. Some of them, like the errors of map reading and feature interpretation, apply also to the execution phase. Among the error types explained below, the errors related to waypoints seem to be the most important for efficient navigation.

General strategic errors	**Explanation**
The planning phase too short, or completely omitted.	The navigator begins with execution too soon without deliberation. The sense-plan-act architecture of navigation is not respected.
The planning phase too long.	The navigator is losing time with too much or too frequent reasoning.
The method of orientation not respected properly.	The navigator does not reproduce the work with map and compass properly.

Table 7.1. General strategic errors

Map recognition errors	**Explanation**
Errors of map checking	
Map scale and contour equidistance not checked.	The navigator will make wrong distance and height difference judgements.
Map updateness not checked.	The navigator is not prepared for the eventual mismatches between the reality and the mapping. He will have wrong beliefs about the terrain characteristics.
Course overprint not checked.	The navigator's judgements about the course distance, direction and difficulty will be wrong.
Errors of map reading and interpretation	
Bad map reading knowledge.	Misunderstanding of map symbolization and its meaning.
Wrong map interpretation.	The types, shapes and dimensions of features are not assessed appropriately.

Errors of contour line interpretation	
Misestimation of contour height or height difference.	The physical effort and energy that will be spent could be underestimated.
Hills mixed with depressions.	Misinterpreted oval closed contours create misbeliefs about the shape and slope direction of the terrain surface.
Misinterpretation of slope direction.	The mix of upward and downward directions affects the amount of energy spent.
Misestimation of slope steepness.	The error can cause a change of the optimum path in the execution phase, which influences the amount of energy spent.
Errors of topographic feature interpretation	
Misinterpretation of topographic feature types.	The error can cause misbeliefs about the topographic configuration.
Terrain runability misinterpreted.	Segments of the optimum path are assessed wrong.
Misinterpretation of dimensions, distances, positions and directions.	Judgements on a map are made without proper feeling or cartometric work.
Misinterpretation of fuzzy feature boundaries.	The navigator's interpretation of feature boundaries can be different than the cartographer's.
Errors of cognitive map construction	
Too distorted or no cognitive map developed in the planning phase.	The navigator is not concentrated enough or unable to create a cognitive map.

Table 7.2. Map recognition errors

Errors of optimum path planning	**Explanation**
Errors of rough optimum path selection	
The optimum path condition not clear.	The optimum path will not be chosen appropriately.
Bad choice of rough optimum path.	The chosen rough optimum path does not fit to the optimum path condition and to the terrain characteristics.
Errors of waypoint selection	
Too many, too few or no waypoints chosen.	The distances between waypoints are too short, too long, or the individual legs are not divided into explicite runs. The navigator will be losing time, or he can get lost.
Inappropriate feature chosen for a waypoint.	The feature may be too large, too small, obscured, too fuzzy, undistinct, too complex, or unaccessible.
The feature chosen for a waypoint is not a mapped landmark.	Features without distinct characteristics should not be chosen for waypoints.
A waypoint too deviated from the optimum path.	Even if a landmark is well distinct, it is inappropriately selected if it prolongs the optimum path.

Errors of detailed optimum path selection	
Planning a constant bearing when not appropriate.	Selection of a straight path along an azimuth is inappropriate in the case of low runability, obstacles and complex geomorphology.
Ignoring catching feature when suitable.	The executed path can become more risky.
Ignoring "handrails" and footpaths on complex terrain.	The executed path can become shorter, but slower and more risky.
Relying too much on "handrails" and footpaths.	The executed path can become less risky, but much longer.
Contouring not planned when suitable.	The executed path can become shorter, but more steep and risky.
Aiming off not planned when suitable.	The executed path can become shorter, but more risky.

Table 7.3. Errors of optimum path planning

7.6. Errors of execution

Most of the available navigation and orienteering literature deals with the errors of execution, and virtually none with the errors of planning. This can be due to the fact, that the errors of execution can be directly observed when locomoting along the optimum path. In certain real world situations, we can not sharply delimit both types of errors, since the actions of planning and execution are partly overlapping or exchanging in a fuzzy way.

Most errors of execution are related to position and orientation updating. Quick awareness of errors is paramount cognitive demand in orienteering (Seiler 1996). Some of the errors appear similar in consequences, however they can have different cognitive backgrouds. In the following tables they are taxonomized partly in relation to the tools, and partly to the actions of execution (Tables 7.4. to 7.12.). In between, we find extremely important errors of beliefs, which are the most common and fatal reason to become lost.

Locomotion errors	Explanation
Too fast or too slow locomotion.	The speed of locomotion is not harmonized with reading of the map, navigation skills, physical condition or environmental circumstances.
Lateral deviation of locomotion.	Systematic difference between the length of the left and the right leg step can cause deviation from the optimum path and even walking in a circle.

Table 7.4. Locomotion errors

Reconnaissance errors	Explanation
Bad mental adaptation in the beginning.	Not getting used to the map, compass and terrain conditions in the beginning can lead to false estimation of distances, directions and positions.
Unsufficient observations of visual space.	Too much attention is given to map and compass reading, or to the ground morphology in front, in comparison to visual perception of topographic features.
Unsufficient or no feature matching.	The comparison between map and environment is not performed appropriately.

Table 7.5. Reconnaissance errors

Errors related to the use of compass	Explanation
180 degrees error.	This error is caused by mistaking the north direction for south, or vice versa (Bratt 2002). Fatal and frequent error.
Map is held oriented north-up.	Having permanently map north in front of the navigator can provoke misalignment effect (Montello 2005).
Relying on a compass too much.	The navigator does not use a map, as he ignores the fact that the compass bearing is not enough accurate while running (Bratt 2002).
Map north not coincided with the compass north.	Too rough navigation is performed or too little attention is paid to a compass.
A compass is held non-horizontally.	The swinging of the needle can be obstructed. The north direction is shown wrong.

Table 7.6. Errors related to the use of compass

Errors related to the use of map	Explanation
Map reading errors.	Misunderstanding of map symbolization and its meaning. Misinterpretation of topographic features and relief forms. Visual perception of the mapped features can be limited by the sight ability of the navigator, or due to environmental conditions (eg. in the darkness and shadows in the forest, by exchanging of shady with sunny areas, in a rainy weather).
Reading too much detail or checking the map too frequently.	The navigator is losing time by zigzaging around, and possibly losing (natural feeling for) orientation temporarily (Bratt 2002).
Ignoring details on a map.	Feature matching is performed unprecisely.
Thumbing the map not performed or not following current position.	The navigator can lose information about his current position and can try to match wrong features.

Table 7.7. Errors related to the use of map

Errors related to beliefs	Explanation
The illusion of being oriented.	A false feeling about own orientation and about orientation of certain topographic features, like roads in a forest, can provoke the illusion of being oriented (Hill 1998).
The illusion of being on the right position.	This is a general misbelief, usually caused by temporary neglecting feature matching. It can range from a minor fault to a complete spatial disorientation.
The parallel error.	This error is caused by misleading terrain features, convincing oneself that the situation is correct. Namely, similar features often occur on the same type of terrain. The relocation process often starts with random roaming around the predicted location of the feature for which the navigator wrongly believes the location is true. The navigator has to change his belief and admit himself that he is lost. This is a frequent and most frustrating error. Most orienteers get lost because of it (Crampton 1988, Bratt 2002).
A belief that the map is locally wrong.	Sometimes the map is not updated, however in general the map is likely to be correct. Rather the navigator is probably lost (Bratt 2002).
Ignoring the compass.	The navigator does not admit the true bearing of the compass but rather believes his feelings that he is running in the right direction (Bratt 2002).
Relying too much on environmental cues for orientation.	Sometimes the navigator neglects the compass since he feels that some environmental cue is sufficiently evident about his true orientation. This can be the direction of the sun or sunlight, seing some distal landmark, or hearing the road noise near a major motorway which is making him feel safe.
90 degrees error.	When the navigator arrives from a forest to a crossing of a gridlike system of footpaths, he can choose the east instead of the north path, or the west instead of the south path (Bratt 2002). The error is frequent and fatal.
Disorientation after sharp turns on control points or at waypoints.	Neglecting care of compass north on leaving an important waypoint can cause disorientation.

Table 7.8. Errors related to beliefs

Errors of distance estimation	Explanation
Wrong environmental or cognitive distance estimation.	The distances that are apprehendable by direct travel experience can be over- or underestimated, so the target can be over- or undershooted, respectively.
Wrong relative distance estimation.	The navigator percepts wrong the passed or the remained distance to the destination. The result is over- or undershoot. A change of speed can produce this error of distance determination (Hutchins 1995).
Confusion in combining long and short distances.	Exchanging of short and long legs or runs can cause wrong perceptions and estimations of distance.

Table 7.9. Errors of distance estimation

Errors of rough navigation	Explanation
The method of orientation performed too rough.	The bearing towards the destination is determined inaccurately.
The method of orientation performed too precise.	Too accurate determination of the bearing towards the destination is too time-consuming.
The frequency of performing the method of orientation inadequate.	Too frequent is time-consuming, too infrequent can cause disorientation.
Changing mind about the optimum path in the middle of a leg.	After choosing one of the variants of the optimum path, the navigator withdraws his decision in the middle of a leg, and selects another set of waypoints. This can lead to a fatal confusion, prolongation of travel, and waste of time (Bratt 2002).

Table 7.10. Errors of rough navigation

Erros of precise navigation	Explanation
Navigation with a constant bearing unprecise.	If maintenance of the right bearing is not assured with the method of orientation, the target can be missed.
Catching feature mixed with other object.	When many similar catching features (eg. footpaths) are available transversally to the optimum path, over- or undershooting can occur.
Following a "handrail" too long or leaving it too soon.	Leaving a "handrail" should be executed at distinct feature details only.
Aiming off on the wrong side of the destination.	Imprecise method of orientation and feature matching can cause aiming off to the left instead to the right, or vice versa.
Aiming off too much.	This error causes a longer detour and a waste of time, or even missing the destination.
Ascending or descending while contouring.	This error causes a waste of energy and possibly miss of destination.
Contour line details misinterpreted while contouring.	The result can be over- or undershooting the target, or a loss of orientation as the contour line is meandering.

Table 7.11. Errors of precise navigation

Errors on approaching the control point	Explanation
Control point description ignored or misinterpreted.	The navigator does not understand the symbols used in microlocation descriptions, or does not interprete the feature detail correctly.
Being drawn by a similar feature or by an incorrect control point.	Seeing a waypoint feature resembling to the expected can draw the navigator's attention to check it for sure instead of following the chosen optimum path (Bratt 2002).
Looking for the control feature too soon or too late.	The navigator does not start the precise navigation manoeuvre at the right moment, so he searches for the control point at wrong place (Bratt 2002).

Fine navigation near the control feature not applied appropriately.	When hitting the vicinity of the control point, the navigator must slow down, raise attention, count steps, follow features details and determine precise bearing. If not, he can change his mind, wander about the feature or even start to follow other competitors.

Table 7.12. Errors on approaching the control point

7.7. Personal factors in navigation

A systematic review of the wayfinding literature published by Gluck (1991) has exposed that the affective concerns of navigation are unsufficiently explored. They involve human emotions including fear, need for novelty, need for connecting the others, etc.

This thesis does not pretend to deal with the psychological side of navigation, however we can not complete the chapter about errors without a mention of psychogene factors, which often crucially affect the entire process of navigation, consciously or unconsciously. Most errors occur when the navigator is physically tired and metally unprepared (Bratt 2002). The errors arising from mental deconcentration and weak emotional preparation for navigation are not limited to the tools, the strategy and the execution.

7.7.1. Personal constitution and abilities

The constitution and functioning of the human brain and body undoubtedly affect the navigation process. Virtually, there are as many planned and executed optimum paths, as is the number of navigators. Some people better master navigation, the others are faster runners. The navigation knowledge can be enhanced with learning and experience, while physical condition can be achieved by special athletic training. Trained navigators know how to distribute energy along the optimum path, do not fear to roam through the forest, and do not psychically collapse after getting lost.

Balstrøm (2002) shows, that realistic friction values could be estimated only after taking into account the personal physical condition, footwear and weather. Any kind of numerical cost estimate of the path beaten should be taken with care, as friction values are individual and person dependent.

7.7.2. Mental concentration and motivation

Cognitive processes in navigation require permanent mental concentration and motivation of the navigator. Kinesiological evidence of poor motivation is slower running with incomplete extension of legs in the knee and in the ankle, while navigational evidence can be any of the errors of planning and execution described above. Some general factors causing mental deconcentration can be, for example:

- weak concentration ability in general,
- weak self-confidence in general,
- extreme weather conditions,
- tight time limits for the travel,
- unharmonized locomotion and cognition during navigation,
- pursuing or disturbing by other persons (eg. in orienteering),

- false feeling of safety on open, simple, or familiar terrain, or when following obvious "handrail" feature,
- early relaxation and psychological decompensation just before the end of the course in the vicinity of still unvisible destination,
- loss of orientation or panic when getting lost,
- unclear goals and objectives of the travel,
- bad physical condition, physical fatigue, or exhaustion.

Distraction can also come from apparently tiny physiological reasons which accumulate over time, such as eg.:

- frequent switches in mind from pure physical motion which is basically a muscular action, to map interpretation which is a cognitive process,
- change of the visual focus from infinity to the map reading distance of around 20 cm,
- focusing the sight to a single spot on a map or compass instead of paying attention to objects and morphology of the landscape,
- partial disfunction and neglection of the lateral sight, which diminishes body equilibrium,
- twinkling image of the handheld map while observing it during locomotion,
- leaning and bending of the body and the head forward,
- bending a bit to the side of the hand, which is holding a map, thus diminishing body equilibrium and preventing optimum muscular activity,
- stiff forward position of the hand which is holding a map during map reading,
- change of the motive force, followed by altered muscular tonus and respiratory rhythm during map reading.

The awareness of the competitor that he is making errors, or that he is lost or tired, psychologically triggers the fall of motivation, attention, and self-confidence. Losing the idea of own location can cause apathy, fear, panic and instinctive irational behaviour (Herlec et al. 1990). A single misbelief about own abilities can not only affect the travel beween two consecutive control points, but can extend to the whole course. The initial exaggeration of speed and misestimation of runability of the terrain on one leg can also cause various cognitive and physical errors along the entire course.

7.7.3. Emotional traps in navigation

A fear of making errors in navigation and a fright of being lost is the main emotional barrier of technically untrained navigators. Ordinary people do not enter woods or unknown territories just because of fear. The fear rises adrenalin secretion, increases blood supply to the legs, and mobilizes self-preserving concentration by active problem solving (Hill 1998). Without going into further details, we count here some practical orienteering hints to avoid or to overcome the troubles with emotional deconcentration.

- Define beforehand what is the motivation: the fastest run, the shortest path, or simply finding the target. Prepare adequately for the navigation task and assess own abilities positively and objectively.
- Ignore other people on the route and don't follow them. They can be searching for some other target. Even if they are moving in the same direction, they can be lost.
- Don't deliberate about the past navigation errors. This can spoil self-confidence and cause further mistakes.
- Don't change mind about the choice of the optimum path in the middle of section.
- Don't prejudice that the route is too difficult. Instead of self-depressing with doubts, rather slow down the pace, rise attention and think rationally.

- Don't underrate the difficulties of the terrain and the course.
- When searching for some feature, don't wander out of the way to check every object which resembles to the target feature. Just proceed along the planned path carefully. There is high probability that similar objects appear on the same type of terrain.

7.8. Consequences of errors

The navigator has to recover from the error as soon as it is perceived. Whatever is the source of error, it is demonstrated as a set of logical contradictions between the map and the real situation. The discrepancies are reflected in:

- horizontal displacement (absolute position),
- false distances to nearby topographic objects (relative positions),
- vertical displacement (height differences),
- wrong arrangement of nearby topographic objects (topologic relations),
- false bearing (north direction).

The accumulation and the ignorance of discrepancies can generally have four consequences:

- *Slight deviation of movement.* It can be corrected by precise navigation techniques, since the navigator roughly knows his true position and direction all the time.
- *Local deviation off the optimum path.* It can be corrected by a local correction of the optimum path and by feature matching without relocation process.
- *Gross deviation off the optimum path and temporary loss of orientation.* It can be corrected only with a relocation procedure in which the navigator has to judge and rougly retrieve his approximate position and direction, revise his beliefs and find the least risky way back to the optimum path from the current approximate position onward.
- *Gross deviation off the optimum path and permanent loss of orientation (ie. getting lost).* The navigator has wandered out of the map area, or has become totally deconcentrated and confused. The navigation is terminated. The navigator can not rescue himself from the situation without a help from other persons. He is unable to identify or orient his present location with respect to known locations, and has no effective means to do this (Hill 1998).

The consequence of each deviation is a minor or substantial waste of time. In extreme situations, it can amount to physical exhaustion, and to threat to health and life. A systematic review of the wayfinding literature published by Gluck (1991) has exposed a complete lack of explicit treatment of two important situations found in navigation: being lost, and how to relocate (ie. to get "unlost").

7.9. Relocation

Most people with at least basic orienteering knowledge are able to navigate precisely but slowly, ie. unrisky. When the navigator masters the method of orientation and the planning of navigation, the main challenge of orienteering becomes fast locomotion with local switches between rough and precise navigation without major errors. In normal environmental conditions, skilled and phychophysically well prepared navigator never gets lost, and rarely needs to relocate. Nevertheless, everybody is making errors. When the navigator makes a gross deviation off the optimum path, he has to start the relocation procedure to return to the optimum path.

The detailed relocation techniques are not relevant for this thesis, therefore we only quote here the order of some recovery actions, which are frequently used by orienteers and scouts (Bratt 2002, Herlec et al. 1990).

1. When lost, stop immediately. Don't panic and don't wander around headless.
2. Check the coincidence of compass and map norths, and reconsider the situation and beliefs.
3. Try to find out what kind of error has been done.
4. Try to remember, which was the last known position on the map, how far was that, and what kind of features have been encountered since then.
5. Use the mehod of orientation with compass and map. Try to identify and match some obvious topographic features inside the visual space.
6. Regardless of how (un)precise, find out at least own approximate position.

Often it is impossible to figure out the position solely from the observations from the current standpoint. In this case, the lost navigator usually tries the technique of direction sampling by walking short straight distances from the standpoint and back in the most promising directions to discover additional orientation cues (Hill 1998). To prevent from getting lost again in such short sampling excursions, he can reasure his prompt return with the *look-back technique* by pausing, turning around, and memorizing the view behind (Cornell et al. 1992). The views of the path in both directions, forth and back, can be quite different. After consideration of situation, the navigator can then choose between the following three options:

- Try to return to the last visited and known location on the optimum path.
- Take the compass bearing towards an obvious linear feature (eg. a road), or to a large feature (eg. a clear-cut in the forest), and find it. Then reconsider the least risky way from some detail of this fetaure to the nearest planned or passed waypoint on the optimum path.
- Locate own position by the intersection of back-azimuths to at least two known distinguished features, if available. Then reconsider the least risky way from some detail of this fetaure to the nearest planned or passed waypoint on the optimum path.

8. TECHNICALLY AUGMENTED NAVIGATION

In order to study and simulate different tools, strategies and errors of navigation in the simulation chapter, we provide here the analysis of the technically augmented navigation, as an opposition to the classical orienteering with map and compass. We first give a generalized description of functionalities of an integrated navigation device, represented by a GPS receiver with a screen map, and by a GNSS receiver with a screen map. We use the second term for a satellite receiver, which works well also in the situations with obscured sky view, because it (hypothetically) uses also the additional interoperable satellite systems, such as eg. Galileo. Namely, in the GPS receiver case, the reception of GPS signals can vanish in the forests.

After the definition of the tools, we generally present the strategy, planning, execution and errors of technically augmented navigation in a comparable way to the orienteering navigation case. With this chapter we complete all the necessary arguments to deal numerically with the risks and costs of the use of each tool in the simulation chapter.

8.1. The concept of an integrated navigation device

Many new technically advanced aids to navigation and positioning have been invented in the last decades. Most of them need electric power supply or installation of some extra infrastructure to support the navigation system. As those systems are technically and functionally described elsewhere, the research is limited to a general comparison of navigation errors in a natural environment between two generic techniques of navigation, the first being orienteering with a map and compass, and the second being positioning and navigation with an integrated navigation device consisting of a GPS (or a GNSS) receiver with map display ability (Figure 8.1.).

Figure 8.1. Technically augmented navigation devices with a screen map (sources: http://www.fieldandtrek.co.uk/images/products/25464_m.jpg; http://www.outdoorsmagic.com/news/images/ViewRanger-1-web.jpg; http://www.actionoutdoors.co.uk/onlinestore/images/satmap/satmap1.jpg)

In the technically augmented land navigation, some of the human abilities to perform wayfinding are substituted and compensated by technology. Five distinct functions or components of automated navigation system have been proposed (McGranaghan et al. 1987):

- positioning,
- displaying position,
- storing geographic database,
- planning a route,

- steering along the route with a set of instructions.

The functions and the strategies of navigation are completely different from what is described in the chapters about classical navigation with map and compass. When a technically augmented navigation is performed nowadays in a complex natural environment, the planning of a route and the steering along the route are usually unavailable or in a very primitive form. In classic navigation, displaying the position and storing the geographic database are indirectly provided by a paper map, while the other three tasks have to be solved cognitively by the navigator with the additional aid of some traditional tools, if available.

8.2. GPS receiver

Currently, the most outstanding and widespread technical device used in technically augmented outdoor navigation is a GPS receiver. Depending on the type of instrument and the technique of surveying, we can in principle dynamically obtain our current position with the accuracy of several meters to few centimeters, which is sufficient for navigation. The details about the GPS system and receivers can be found in numerous textbooks and commercial materials (see eg. Grejner-Brzezinska 2004).

The GPS receiver as a stand-alone instrument is only a positioning tool. It can provide point coordinates in various forms, ie. as planar coordinates in a chosen cartographic projection, as threedimensional geocentric coordinates, or as geographic longitude and latitude on the Earth's sphere or ellipsoid, with the additional value for ellipsoidal height. In this thesis, we deal with a graphic display of the current GPS receiver position as a point symbol overlaid on a screen orienteering map which is geocoded in the same coordinate system. We also assume, that the GPS receiver can store waypoints, including the origin and the destination, and that it can show the current bearing with an arrow pointing straight to the next waypoint on the route.

8.2.1. The problem of GPS signal reception

For a reliable 3D position fix, the GPS receiver should receive at least the signals from four satellites. When the visibility of satellite constellation is weak, like in a forest or in urban canyons, the following situations can occur:

- The calculated position is only 2D, thus less reliable and without a fix for the elevation value.
- The position is wrong for several tens to even hundreds of metres. The whole track represented on the screen map is positionally displaced, or has sharp spikes of false fixes. The GPS receiver is worthless for precise navigation in such a situation. It is better not to use it.
- The initialization fails to complete, and the position is not fixed. The GPS positioning does not work at all.

If we suppose that in the monotonely dense forest the GPS receiver produces good fixes interchanged with bad fixes as we move, the risk is floating from normal to high, but we do not know where the risk changes. We can not realistically predict nor the extent of risk, neither the location of risk in the forest. Such a situation can be often experienced with handheld GPS devices. Without a careful selection of survey points, or without a sophisticated receiver and software, the user in a forest is not able to follow the accuracy of his GPS positioning in a real-time. (This paragraph refers to the state of affairs in the year 2004, when the thesis was started, however this is not relevant for the positive demonstration of the hypothesis.)

As the navigator should know the error characteristics of his tool, he ought to avoid navigation in forests on longer distances. The risk to get lost with a GPS receiver in a forest is very high, therefore we have to select paths over open areas. Any traverse of a forest area should be very short, since it is based on dead reckoning. In this case, only the screen map remains as a useful part of the device, so the strategy of navigation in a forest is the same as in classic *navigation with map without compass* (Chapter 4.7.1.).

8.2.2. Coupling a GPS receiver with auxilliary tracking devices

No single technique or sensor can provide complete and ubiquitous tracking information with continuously high performance and reliability (Grejner-Brzezinska 2004). The problem of satellite signal reception can be partly solved by combining the GPS receiver with other positioning and tracking devices, like inertial systems (INS), pedometers, odometers, or gyroscopes, which can sustain precise positioning mostly within short and not too frequent GPS outages. Still, such devices can not be widely used in forests. Robust hybrid devices also have own weaknesses. The structure and the appearance of risks in mixed instrumental configurations with integrated devices becomes more complex and unpredictable, so we omit hybrid systems from further treatment.

8.3. Interoperable GNSS receiver

The problem of navigation with a GPS receiver in densely forested areas and inside some buildings will be at least partially solved by the introduction of new satellite frequencies with increased penetration (ie. with increased signal power and longer codes), and with the densification of constellation with new satellites in the near future (Cross 2006). Yet, the risk of using a GPS receiver when the sky is physically obscured will not vanish completely.

The prospects of ubiquitous positioning are growing with the development of additional GNSSs, like European Galileo, Russian GLONASS, Chinese Beidou and Indian IRNSS. If the number of satellites in the sky will be substantially higher, and if the GNSSs will be interoperable (which is expected), we can reach practically permanent availabilty of signals even in critical situations. The user will even not be able to distinguish which satellite system has provided him the position.

For the sake of comparison of navigation strategies, we additionally deal also with the interoperable GNSS receiver with a screen map, which is presumably reliable in the forest. From now on, it is referred to as GNSS receiver. Beside the mentioned advantage, it has the same characteristics as the GPS receiver with a screen map.

8.4. Screen map

We presume that the GPS (and the GNSS) receiver has the ability to display a scanned, raster type orienteering map. Since the original paper orienteering map is overprinted with the orienteering course according to IOF rules, the scanned map also contains the course overprint. Map contents must be rotated adequately to compensate for magnetic declination. A screen map is a virtual type of map, having the following properties, which are different from a paper map:

- digital geolocated data base is used to portrait topographic features,
- the screen is handheld, pocket-size, thus quite small; only a part of the map can be displayed at a time,
- the map can be zoomed in and out; the screen map scale is variable,

- the map can be panned manually or automatically ("moving map") to show the current position, usually in the bottom centre of the map,
- the map is usually oriented north-up for map inspection, or track-up (ie. heading-up) when planning or following the route or track,
- the brightness of the screen can be adapted to the environmental circumstances,
- the currently used screens on the market can not be folded or unfolded,
- the display device needs a miniature electric battery to operate.

Physically, the display can be imbedded in a GPS receiver, or better vice versa - a GPS receiver is a part of a personal digital assistant (PDA). There is also a tendency to replace a PDA with a smart phone, equipped with a miniature keyboard, or with a touch screen stylus.

8.5. Integrated navigation device: the GPS (the GNSS) receiver with a screen map

The two integrated navigation devices which are used later in the simulation chapter are the GPS receiver with a screen map, and the GNSS receiver with a screen map. As the individual components have already been described, we need to specify only the typical functions of both tools, gained by the integration.

We treat the technically augmented navigation in the view of the current state-of-the-art in the commercial market of navigation devices. Usually, the functionalities include (see eg. Mio Technology Limited, Nav N Go Kft. 2006):

- display of a portion of scanned map in an arbitrary and variable scale,
- display of a graphic scale and the direction of north at the edge of the screen map,
- optional display of a GPS signal reception providing information about the quality of a full 3D position, or, of only 2D position when less than four satellites are used,
- digitizing, uploading and downloading a route, consisting of straight lines between waypoints (the route planning mode),
- digitizing, uploading and downloading a track, consisting of a curved line between the origin and the destination (the track planning mode),
- display of the current position at the bottom center of the screen map,
- display of the screen map with a north-up or track-up orientation,
- moving and rotating the screen map automatically as the position and orientation change,
- display of the direction to the destination or to the next waypoint,
- display of the straight distance to the destination, or to the next waypoint along the route, if the route planning mode is applied,
- display of the curved distance to the destination along the track, if the track planning mode is applied.

In orienteering tasks, the navigator has to define beforehand the route or the track to follow. For the route, he has to digitize discrete waypoints and the final destination, meanwhile for the track, he has to sketch a complete continuous curved line to the destination. As the track mode can be treated as a route with high density of waypoints, we deal only with routes in the continuation of this thesis. Route definition and following can be more flexible in rugged terrain conditions, where defining a track for the entire course in advance could be too vague without experiencing the respective environment visually and physically.

We could also speculate with the amended version of device, which would additionally display the automatically calculated optimum path and the bearing along the path, ie. the bearing as a tangent to the path, or better, a secant from the current position to some proximal position ahead, where the constant forward distance should be chosen and entered into device by the user. Such a device would be inefficient in a natural environment, since it needs very accurate topographic, friction, and risk data, which are too person-specific and too task-specific. This option is not treated further in the thesis.

8.6. The sense-plan-act architecture of technically augmented navigation process

In this chapter we deal only with a fully functioning GPS device on open areas, where we suppose that the signal reception is sufficient permanently and everywhere. The navigator does not bear any major risk about his position. His preferred strategic planning is tracing the optimum path without needing to introduce any intermediate waypoint at all. So, the method of orientation with the GPS receiver is solely observing the automatically displayed current position and direction to the destination on the screen map. However, when a straight passage on open areas is not possible because of obstacles (eg. hills, water bodies, rough surface), the navigator has to define waypoints digitally in advance.

Positioning with a GPS receiver has a considerable advantage in comparison to the navigation with map and compass, where the current position is not shown explicitely by the tools, and where we need carefully chosen additional waypoints to avoid the risk of dead reckoning errors, which are growing with the distance from the origin. As with map and compass, we need to address the three essential elements of navigation, the *method of orientation*, the selection of intermediate *waypoints*, and the selection of *optimum path*. We take the table of the sense-plan-act architecture from the navigation with map and compass, and accomodate it to a new device (Table 8.1.). We can observe below, that the procedural changes are virtually slight, caused by the fact that the method of orientation is substituted with a simple automatic positioning. However, the risk of navigation is much lower with a GPS, or even with an interoperable GNSS positioning.

The following comments are also important to explain the table of the sense-plan-act architecture:

- The waypoints still play a considerable role for navigation, and have to be digitized before the locomotion starts. This action slows down the navigation process. Before the travel begins, the navigator interactively chooses the position of the waypoints and the destination. The tool is then automatically measuring and displaying the current position, the direction to the next waypoint, and the remaining distance.
- The navigator can choose the waypoints which are not distinguished features, but just arbitrary GPS positions where the optimum path makes a turn. This is important difference with regard to the classical method. Very high density of waypoints leads to a replacement of the route with a continuous curved track.
- As the optimum path choice is not automatic, map recognition, reading and interpretation are still very helpful skills. So is the construction of a cognitive map. Planning of the optimum path has to be done cognitively, since the development of an automatic navigation procedure for the cross-country navigation would be possible only with extremely detailed spatial data, and with relatively sophisticated software.
- Navigation with a constant bearing directly to the destination is the most desired technique for the navigation with a GPS receiver, if the terrain characteristics allow for it. On undulated terrain, contouring can help, however other classic detailed optimum path selection techniques are obsolete for navigation with a GPS receiver (ie. navigation to a catching or collecting feature, navigation with "handrail", and aiming off).

STEP	PLANNING AND EXECUTION ACTIONS	BENEFITS GAINED OR QUESTIONS ANSWERED
1	**Start of the course**	Standing on the origin.
	Take the GPS-based device and switch it on. Wait for the GPS to initialize positioning and open the file with the geolocated orienteering map.	Is the GPS receiver working?
	The current position is shown with a point symbol on the north-up oriented screen map. Check general map data.	What is the original paper map scale and the level of presented detail? What is the contour equidistance? What is the map production date?
	Check general course data by panning the map, then reestablish the current position at the bottom centre of the screen map.	What is the length of the course? What is the climbing of the course? How many control points are there?
2	**Start of the leg**	Standing on the control point.
	Check the current position on the screen map.	On which control point am I now? Which control point must I visit next? Where is all that on the map?
	Select cognitively the rough optimum path for the leg, satisfying some optimum path condition.	Which type of optimum path fits my abilities best?
	Break cognitively the optimum path of the current leg into runs. Digitize waypoints on the screen map eg. with a screen cursor cross or with a touch screen stylus. The device will automatically orient the screen map in track-up direction towards the next waypoint.	The rough strategy is made. The waypoints are selected. The screen map is track-up oriented.
	Interprete contour lines and topographic features along the leg.	What are the challenges along the leg?
3	**Start of the run**	Standing on the waypoint.
	Check the position on the screen map. Check where and what is the next waypoint. Read from the screen what is the straight distance to it.	On which waypoint am I now? Which waypoint must I visit next? Where is all that on the map?
	Interprete contour lines and topographic features along the run.	What are the challenges along the run?
	Cognitively construct the detailed optimum path for the run. Assess the risks and the segments of the run.	The detailed strategy is made. The navigation techniques are chosen. The segments and the risks are recognized.
	Construct a cognitive map of the run.	Memorize the relevant screen map details.
4	**Start of the execution**	Standing on the waypoint or passing an arbitrary checking position.
	Assess cognitively the spatial orientation by feature matching. Start (or continue) locomotion in the likely direction of the next waypoint. The device will automatically (re)start showing the straight direction to the next waypoint.	Am I turned in the right direction?
	Check the straight direction to the next waypoint on the screen.	In which direction is the next waypoint?

	Turn yourself appropriately and follow the optimum path.	The navigation is physically executed.
	Compare the situation on the map with the real situation in front of you.	How the terrain looks like?
	Permanently observe the current small-scale space. Occasionally check the current position, the suggested direction, and the remaining distance to the next waypoint on the screen map. Identify contour lines and topographic features on the map. Assess the risks.	Where do I navigate? In which direction am I heading? How far from the waypoint am I? What is around me? What can I afford to do? Am I still on the optimum path?
	Pause or slow down to amend the direction. In the vicinity of the waypoint, observe the displayed position, direction, and distance. Perform feature matching carefully to hit the waypoint.	Where am I right now? Where do I want to proceed? Where is that on the map? In which direction is that?
	Go to the step 4 until the waypoint is reached.	The execution cycle for the run.
5	**End of the execution**	The waypoint is reached.
6	**End of the run**	The waypoint is reached.
	Go to the step 3 until the control point is reached.	The planning-execution cycle for the run.
7	**End of the leg**	The control point is reached.
	Go to the step 2 until the destination is reached.	The planning-execution cycle for the leg.
8	**End of the course**	The destination is reached.

Table 8.1. Planning and execution actions with technically augmented navigation device

8.7. Execution of technically augmented navigation

The execution of technically augmented navigation procedurally looks much the same as with map and compass, except that feature matching is partly replaced by observing the position on the screen map, while orientation checking is replaced by checking the direction arrow on the screen. As with a compass, the speed of locomotion can be obstructed by looking at the screen too frequently. The main difference to the map and compass navigation is substantially less risky wayfinding.

Localization and thumbing the map are not neccessary with a GPS device any more. The estimation of distances is also less important, as the passed and the remained straight distance to the target is permanently calculated from the current and the target positions. The distinction between rough and detailed navigation is blurred, as positioning provides exact positions. The attack points are not important any more. The executed path is prone only to minor errors and the risk to get lost is minimized, if the GPS signals are available. If the errors of navigation are perceived too late, the navigator does not get lost, but just wanders off the optimum path and prolongs the travel.

8.8. Errors of technically augmented navigation

Technically augmented navigation devices allow lay persons to navigate with much less errors, since the navigation and map reading skills are less important in comparison to the navigation with compass and map. If positioning works well, and if the navigator has at least basic map reading knowledge, it is practically impossible to get lost with a GPS receiver.

In this chapter we provide an overview of the errors of technically augmented navigation in a comparable way to the presentation of errors that can emerge with the use of map and compass. We again present three groups of errors: the errors of the tool, the planning, and the execution.

8.8.1. Errors of an integrated navigation device

We start with the errors of electronic tool, which consists of two components. The GPS positioning component uses completely different sophisticated technology than a compass, meanwhile the screen map component is, regarding the quality of the contents, practically equivalent to a paper orienteering map. Supposing that the GPS receiver is operating faultlessly, has adequate power supply, and initializes normally, the only remaining error of the GPS receiver is the positioning error. The positioning error can be generally expressed as a summation of four errors (Manning, Brown, 2003):

- global reference frame error,
- observation error,
- positioning system computation error,
- transformation to the local datum model error.

Each error can be subdivided into many impact factors, which are treated in detail in geodesy textbooks, however only the *observation error* is significantly and more or less unpredictably changing in the cross-country navigation, when we pass through different types of terrain and vegetation. With a handheld GPS receiver, the positioning accuracy on open terrain usually stays within 5 to 15 m range, and is sufficient in nearly every circumstance. Therefore, all error components can be treated together as a single positioning error. In fact, the navigator has very scarce opportunities to actively affect the positioning error, except the option to choose the more appropriate optimum path on open terrain instead in the forest.

The errors related to the cartographic design and contents of the screen map are practically the same as of its paper counterpart. The peculiarity of the screen map errors is related more to the specific way and medium of its use. A small screen can show only a window of the entire map, thus preventing from the use of advantageous all-at-once nature of a paper map (McGranaghan et al. 1987). The survey knowledge may not be acquired appropriately. Displaying the screen map at different scales and the moving window can also cause disorientation, false beliefs and erroneous estimation of positions, directions, and distances.

8.8.2. Errors of strategy and planning

In this chapter, we take the table of errors from the map and compass case, and comment only the differences, in case we use the GPS positioning. With an integrated navigation device we use essentially the same strategy of waypoint-based navigation technique as with map and compass, except that the orientation and positioning methods are different. Among the errors of strategy and planning, the errors related to waypoints still remain to be the most important. We can see from the tables below, that the errors related to map use stay the same, while positioning, waypoint selection, and detailed optimum path selection could be much easier and less error prone with integrated navigation device (Tables 8.2., 8.3., 8.4.).

General strategic errors	Explanation
The planning phase too short, or completely omitted.	Like map and compass.
The planning phase too long.	Like map and compass.
The method of orientation not respected properly.	The method of positioning and displaying the current position is simple and straightforward. However, to guess own orientation when standing still (when track-up map orientation is not working), we must perform feature matching, which is difficult cognitive task.

Table 8.2. General strategic errors

Map recognition errors	Explanation
Errors of map checking	
Map scale and contour equidistance not checked.	Like paper map.
Map updateness not checked.	Like paper map.
Course data not checked.	Like paper map.
Errors of map reading and interpretation	
Bad map reading knowledge.	Like paper map.
Wrong map interpretation.	Like paper map.
Errors of contour line interpretation	
Misestimation of contour height, or height difference.	Like paper map.
Hills mixed with depressions.	Like paper map.
Misinterpretation of slope direction.	Like paper map.
Misestimation of slope steepness.	Like paper map.
Errors of topographic feature interpretation	
Misinterpretation of topographic feature types.	Like paper map.
Terrain runability misinterpreted.	Like paper map.
Misinterpretation of dimensions, distances, positions, and directions.	Like map and compass, but the error is less important as with the GPS receiver we know precise position permanently.
Misinterpretation of fuzzy feature boundaries.	Like paper map.
Errors of cognitive map construction	
Too distorted or no cognitive map developed in the planning phase.	Like paper map, but less important since we can not get lost if the GPS positioning works well.

Table 8.3. Map recognition errors

Errors of optimum path planning	Explanation
Errors of rough optimum path selection	
The optimum path condition not clear.	Like map and compass.
Bad choice of rough optimum path.	Like map and compass.
Errors of waypoint selection	
Too many, too few or no waypoints chosen.	Like map and compass, but the waypoints can be selected on different places and in a different way.
Inappropriate feature chosen for the waypoint.	Like map and compass, but the error is largely irrelevant since the waypoints do not need to be distinguished features.
The feature chosen for a waypoint is not a mapped landmark.	Largely irrelavant error when we use a GPS positioning.
The waypoint too deviated from the optimum path.	Like map and compass.
Errors of detailed optimum path selection	
Planning constant bearing when not appropriate.	Like map and compass.
Ignoring catching feature when suitable.	Irrelavant error when we use a GPS positioning.
Ignoring "handrails" and footpaths on complex terrain.	Irrelavant error when we use a GPS positioning.
Relying too much on "handrails" and footpaths.	In principle we do not need to use "handrails" and footpaths with GPS positioning at all, but it is more confortable sometimes.
Contouring not planned when suitable.	Like map and compass.
Aiming off not planned when suitable.	Irrelavant error when we use a GPS positioning.

Table 8.4. Errors of optimum path planning

8.8.3. Errors of execution

Most errors of execution with map and compass are related to position and orientation updating. Position updating is done automatically with a GPS-based device, and orientation updating is largely eased by the display of the direction and distance to the next waypoint, and with a permanent automatic track-up screen map orientation. Temporary unawareness of errors can lead only to a waste of time and to a prolongation of travel, but not to situations of becoming completely lost (as far as the GPS positioning is functioning).

In the following tables, we presume again, that the navigation is performed on open area (Tables 8.5. to 8.13.). As in the previous chapter, we give only notions about the differences between classic and technically augmented navigation. We can realize that with a precise GPS positioning on open terrain, most of the errors which are present in map and compass navigation, become obsolete or have less devastating effects on the quality of navigation.

Locomotion errors	Explanation
Too fast or too slow locomotion.	Like map and compass.
Lateral deviation of locomotion.	Like map and compass, but less important, as we can immediately see the deviation of position from the optimum path on the screen and correct it.

Table 8.5. Locomotion errors

Reconnaissance errors	Explanation
Bad mental adaptation in the beginning.	Like map and compass, but less important, as an automatic GPS positioning and display of direction to the next waypoint compensate the error.
Unsufficient observations of visual space.	Like map and compass, but less important, as an automatic GPS positioning and display of direction to the next waypoint compensate the error.
Unsufficient or no feature matching.	Like map and compass, but less important, as an automatic GPS positioning and display of direction to the next waypoint compensate the error.

Table 8.6. Reconnaissance errors

Errors related to the use of compass vs. GPS receiver	Explanation
180 degrees error.	Irrelevant error when positioning with a GPS receiver.
The map is held oriented north-up.	The error becomes irrelevant immediately after the beginning of locomotion, when the GPS-based device establishes a track-up orientation of the screen map.
Relying on compass too much.	Relying on a GPS receiver too much on open areas is not a comparable error as the position is being permanently displayed over the screen map. However, it is extremely important that in a forest we do not rely on the GPS position.
The map north not coincided with the compass north.	Irrelevant error, since in locomotion a proper screen map orientation is provided automatically.
The compass is held non-horizontally.	Irrelevant error when positioning with a GPS receiver, however similar error is to hold a GPS receiver too close to the body and so obstructing the reception of the GPS signals.

Table 8.7. Errors related to the use of compass vs. GPS receiver

Errors related to the use of map	Explanation
Map reading errors.	Like paper map. Additionally, errors can occur because of a small screen, moving map and changing map display scale.
Reading too much detail or checking the map too frequently.	Like paper map, but even more important. Too much map reading is needless, since the positioning is accurate and permanent.
Ignoring details on the map.	Like paper map.
Thumbing the map not performed or not following current position.	Irrelevant error, as the current position is permanently displayed on the screen map.

Table 8.8. Errors related to the use of map

Errors related to beliefs	Explanation
The illusion of being oriented.	Largely irrelevant with a precise GPS position.
The illusion of being on the right position.	Irrelevant error, as the current position is accurately and permanently displayed on the screen map.
Parallel error.	Irrelevant error, as the current position is accurately and permanently displayed on the screen map.
A belief that the map is locally wrong.	Much less important than with a paper map as a precise position compensates for map inaccuracies. It is also likely, that the navigator will believe that the displayed position is wrong.
Ignoring the compass.	Ignoring the GPS position and relying rather to own feeling can occur, but not so likely.
Relying too much on environmental cues for orientation.	Can occur, but less likely.
90 degrees error.	Can occur, but the error can be resolved quickly with an observation of the displayed position and direction to the next waypoint.
Disorientation after sharp turns on control points or at waypoints.	Can occur, but the error can be resolved quickly with an observation of the displayed position and direction to the next waypoint.

Table 8.9. Errors related to beliefs

Errors of distance estimation	Explanation
Wrong environmental or cognitive distance estimation.	Largely irrelevant with a precise GPS position and display of the remained distance.
Wrong relative distance estimation.	Largely irrelevant with a precise GPS position and display of the remained distance.
Confusion in combining long and short distances.	Largely irrelevant with a precise GPS position and display of the remained distance.

Table 8.10. Errors of distance estimation

Errors of rough navigation	Explanation
The method of orientation performed too rough.	Irrelevant error with a GPS positioning.
The method of orientation performed too precise.	Irrelevant error with a GPS positioning.
The frequency of performing the method of orientation inadequate.	Irrelevant error with a GPS positioning, since the position is updated permanently.
Changing mind about the optimum path in the middle of a leg.	Less likely to occur. It causes only a waste of time, which can be considerable.

Table 8.11. Errors of rough navigation

Erros of precise navigation	Explanation
Navigation with a constant bearing unprecise.	Irrelevant error with a precise GPS positioning.
Catching feature mixed with other object.	Irrelevant error with a precise GPS positioning.
Following a "handrail" too long or leaving it too soon.	Irrelevant error with a precise GPS positioning.
Aiming off on the wrong side of the destination.	Irrelevant error with a precise GPS positioning.
Aiming off too much.	Irrelevant error with a precise GPS positioning.
Ascending or descending while contouring.	Can occur, but the error can be resolved quickly with an observation of the displayed position.
Contour line details misinterpreted while contouring.	Largely irrelevant error with a precise GPS positioning.

Table 8.12. Erros of precise navigation

Errors on approaching the control point	Explanation
Control point description ignored or misinterpreted.	Can occur, but the error can be resolved quickly with an observation of the displayed position. The exception is a very complex terrain.
Being drawn by a similar feature or by an incorrect control point.	Can occur, but the error can be resolved quickly with an observation of the displayed position.
Looking for the control feature too soon or too late.	Can occur, but the error can be resolved quickly with an observation of the displayed position.
Fine navigation near the control feature not applied appropriately.	Can occur, but the error can be resolved quickly with an observation of the displayed position. The exception is a very complex terrain.

Table 8.13. Errors on approaching the control point

9. SIMULATION OF NAVIGATION IN A NATURAL ENVIRONMENT

In this chapter, we simulate navigation with three different tools: (1) map and compass, (2) GPS receiver with screen map, and (3) GNSS receiver with screen map, in a natural environment. After we define the goal of simulation, we describe raster and vector approaches to simulation. An assessment of the methods shows why the cognitive vector procedure is used for simulation. To formalize the demonstration of the hypothesis, we need to define the variables and the parameters of navigation. Then we numerically simulate the risks and the frictions of navigation, and compute the resulting cost and time for the optimum paths. Together with the analysis of risks and strategies they provide all neccessary functional dependencies to construct the formal demonstration of the hypothesis, which follows at the end of the chapter.

9.1. The goal of simulation

Most cognitive research is grounded in a real world experience (Cheesman, Perkins, 2002). Hutchins (1995) argued that all known technical forms of human navigation can be described as single computational process. Therefore, a suitable computational method to simulate the navigation process is desired, which would imitate human problem-solving process. It should include tools, errors and strategies. We chose natural environment for simulation as precise navigation in the wild crucially depends on the tools.

The final goal of simulation is a demonstration of the hypothesis. According to the hypothesis, we have to show numerically that tools, as a consequence of their error characteristics, determine the strategy of navigation. We do this by calculating, experimenting and inferencing about the changing costs, frictions and risks along different optimum paths, with different tools, and different strategies. The costs, expressed by distance or time units, can show which tools are more suitable for some path than the other. Different risks and waypoint combinations will show why.

9.2. The methods of simulation

The physical result of the navigation process is the *optimum path*, being planned or executed. For the demonstration of the hypothesis, we choose to research the planned optimum path, so we could use some kind of a GIS-like computational method with the aid of a scanned orienteering map. Thus, we omit experimental testing of the executed optimum paths with human navigators on the real-world terrain.

In a real world, individual optimum path segments have different risk and friction values. The final cost of the optimum path is accumulated along the path, segment by segment. We can write generally:

$$CostSeg = \mathrm{f}\,(Distance, Risk1, \ldots RiskN, Friction1, \ldots FrictionN)$$

$$CostOptPath = \Sigma\ CostSeg$$

The risk and friction properties can have uniform (ie. *isotropic*) effect in all directions, and the properties can be uniformly dispersed over the surface, ie. they are *homogeneous* inside the area categorized uniformly. However, this is rarely the case in the nature.

Risks and frictions could be defined by a mathematical model, experimentally, or experientally. Their numerical values are unitless, and can be normalized. If all are equal to 1, than the cost is equal to distance. If they are larger than 1, they act as a virtual prolongation of distance. If they are lower than 1, the travel is faster than in normal conditions. Accordingly, costs are expressed in meters. We can use the following simple formula to calculate the cost, wherein the multiplication of all risks and frictions together is called the total factor:

$$CostSeg = Distance * Risk1 * ... * RiskN * Friction1 * ... * FrictionN$$

$$CostSeg = Distance * TotalFactor$$

From physics we know that:

$$Time = Distance / Speed$$

Therefore, if both sides of the equation for *CostSeg* are divided by *Speed* in normal or average circumstances, then *CostSeg* represents *Time* in actual circumstances:

$$Time = CostSeg / Speed = Distance * TotalFactor / Speed$$

Usually, *Speed* is expressed in m/s, or in km/h. Since in running sports we rather measure *Pace* in minutes per kilometer, we can calculate the actual *Time* as:

$$Time = CostSeg * Pace = Distance * TotalFactor * Pace$$

Execution of the simulation can be done by various raster or vector approaches. In the first case, the cost is calculated for a path segment traversing a grid cell, and in the second, for a linear path segment of arbitrary length having the same risk and friction properties. For an automatic least-cost path computation in a street network we usually use graph-based algorithms (eg. Dijkstra's) and vector data, whereas in a natural environment which lacks man-made network structures, raster based algorithms and raster data prevail (Krek 2002, Douglas 1994).

In the simulation, we will use the vector method. This decision was accepted after numerical tests by both methods. We describe and compare the methods in the continuation. We also present the arguments for acceptance of the vector method in spite of wide popularity of raster based method in the current GIS software solutions.

9.2.1. Raster based methods

In raster based methods the navigation area is represented by a grid lattice, and the definition of frictions requires topographic and other data layers in a raster form. The resulting optimum path is a chain of square grid cells (ie. pixels) in 2D space (Tomlin 1990, Douglas 1994, Eastman 2003, Zhan et al. 1993, Lee, Stucky 1998, Collischonn, Pilar 1999), cubic elements (ie. voxels) in 3D space (Scott 1994), and other tesselations in 2D space, 3D space, and on the Earth's sphere (Stefanakis, Kavouras 1995). The sequences of pixels can be converted to vectors and handled as a network for further analysis (Balstrøm 2002).

9.2.1.1. Douglas' algorithm

Although verbal formulations of the least-cost paths sound simple, the solutions are frequently not so straight forward. General steps of the Douglas' algorithm are presented here, since many references to orienteering practice can be drawn from it (Douglas 1994).

To prepare for the calculation of the least-cost path, first the cost of passage has to be estimated for every part of the navigation area. This can be realized by assigning a friction value to every area of homogene property, eg. to the vegetation area of the same kind and penetration grade.

The area of navigation is then completely covered by the cost-of-passage areas represented as a map of vector polygons. Since the concept of Douglas' least-cost path algorithm is raster based, the cost-of-passage areas have to be rasterized with a polygon-to-grid algorithm. The result is the cost-of-passage matrix, where the value in each grid lattice point represents the cost of passage in the vicinity of that point. The matrix can be refined by additional consideration of digital elevation model (DEM), and by anisotropic treatment of terrain slope.

Before the algorithm starts, we have to select the function or formula which defines, how the total cost will be calculated from the distances and the cost-of-passage matrix. Lengthier diagonal passages over grid cell have 1,41 times greater cost than direct traverses. The spreading algorithm applies the spreading function from the destination back to the origin in order to compute the accumulated cost surface. It assigns the accumulated cost to each grid lattice point in the navigation area, showing the cost of getting from that point to the destination.

Finally, the least-cost path is computed as a descending slope line on the accumulated cost surface going from the destination back to the origin. It is represented by that succession of grid lattice points, which gives the lowest accumulated cost. Therefore, the slope line is the line, which intersects isolines of the same cost at right angles. To find the descending chain of cost values, the algorithm performs a slope line tracking procedure, followed by a line generalization (reduction) algorithm, which smooths the slope line.

9.2.1.2. Raster data and friction layers

For the presentation of the algorithm, a single cost variable is often used. However, for realistic computations many cost variables should be collapsed simultaneously onto the accumulated cost surface. Frictions which produce the costs have to be defined as eg. dimensionless values of 1,00 for normal passability of the grid cell, of less than 1,00 for grid cell offering speedier travel, and of more than 1,00 for grid cell offering slower travel. Obstacles should have either very high values (eg. 9999), or symbolic negative values (eg. -1,00). The friction layers and the resulting costs in a natural environment usually account for the resistance to locomotion, obstacles, and slope. Navigation related risks are usually not taken into account.

Rationally accessible raster data layers from which the frictions and obstacles can be deduced, comprise the following themes, eg. digital elevation data, vegetation, hydrography, roads and footpaths, man-made built objects, and other topographic features. All raster layers should have the same coordinate system and resolution with a grid cell of eg. 10 m or finer. From the individual friction layers the total friction layer is computed cell by cell, usually by multiplication of anisotropic and isotropic friction values (Zhan et al. 1993, Eastman 2003).

After this extensive data management, the costs and the optimum path can be computed as already described. All computations can be numerically performed eg. with IDRISI raster GIS software (Eastman 2003) with many function steps using RECLASS, ASSIGN, DISTANCE, SCALAR, COST/COSTGROW, VARCOST, SURFACE/SLOPE, SURFACE/ASPECT, OVERLAY, PATHWAY, and some other auxiliary function modules.

9.2.2. Vector based methods

From a pure mathematical point of view, an automatic calculation of vector optimum path can be done with approximation of smooth surface from digital elevation model, and then with a calculation of geodesic curve on top of the surface, as it is known from differential geometry. Obviously, this is even without accounting for risks and surface frictions a tough job.

Balstrøm (2002) argues that for a difficult landscape it is easier to derive the path from a map and other resources, than to forge out blindly along the path calculated in a raster simulation. So, a reasonable solution would be to first derive the path cognitively, and than assign risks and frictions to its segments. For such vector simulation we do not need DEM, nor topography and friction layers. They are replaced by a colour-scanned orienteering map reading:

- DEM is replaced by reading and interpreting contours on a map,
- cardinal direction distances in a DEM grid cell are replaced by more realistic digital measurement of vector path segments,
- obstacle layer is replaced by simply avoiding obstacles while recognizing and mapping vector optimum path on a scanned map background,
- friction layers are replaced by map interpretation for each path segment.

Still, for each risk and friction, being isotropic or anisotropic, a separate experiental look-up table needs to be constructed and calculated in advance, and then applied on each optimum path segment. The costs of segments are obtained by a spreadsheet calculation. The relevant risk and friction values for a segment are multiplied by its distance, then the costs are summed up for each run and optimum path.

9.3. Assessment of simulation methods

We have described two substantially different aproaches to the study of optimum path costs. The raster approach is executed in the following order:

1. define the topographic and other data layers that provoke friction (the same for all tools),
2. define the topographic and other data layers that provoke risk (different for each tool),
3. compute the friction and risk layers from the data layers,
4. compute the accumulated cost of travel for all grid cells of the navigation area,
5. compute a single optimum path on the accumulated cost surface automatically,
6. repeat the steps 4 and 5 for different tools,
7. assess and compare the form and the costs of automatically calculated optimum paths.

The cognitive vector approach has the following steps:

1. define the spreadsheets with the reference friction values (the same for all tools),
2. define the spreadsheets with the reference risk values (different for each tool),
3. define cognitively and draw all rational optimum paths on the map,
4. for every single path segment calculate all risks, frictions and costs for each tool,
5. sum the costs of segments for all optimum paths and for all tools,
6. assess and compare the costs of cognitively derived optimum paths.

We can conclude the discussion about the methods with a short overview of advantages and disadvantages of each. Practical consideration was also done with a hand calculation for a small test case. It showed that the vector type method is more appropriate for further treatment in this thesis.

9.3.1. Assesment of Douglas' method

Advantages:

- The algorithm is automated and can be executed in several raster based commercial GIS application programs. It basically follows the principles of well known Douglas' least-cost path computation. The optimum path is automatically derived at the end of it.

Disadvantages:

- Details of the commercial GIS algorithm (eg. IDRISI) are unknown. It has many sensitive steps. Programming of an own adequate algorithm from the beginning would be too time consuming and complicated.
- The method requires a multilayered vector map where all point, line and areal topographic data must be resampled, reclassified with look-up tables, and generalized into pixel form to get frictions and obstacles (Stefanakis, Kavouras 1995).
- The calculation of frictions from generalized topographic data results in several data layers. The data processing is cumbersome and complicated.
- There is no possibility to introduce waypoints into calculation, and to imitate human navigation process.
- The result of a single execution of raster method is only one optimum path. To get different versions, we ought to change the input frictions. The expression of navigation related risks is not practiced with this method.
- The distances are calculated only in 8 cardinal directions from the grid cell. This is unrealistic and unprecise, but enough good eg. for hydrologic studies of water flow (Douglas 1994, Stefanakis, Kavouras 1995, Collischonn, Pilar 2000).
- There are big chances that the algorithm will perform unrealistic (Collischonn, Pilar 2000). For better results the cell size should be preferably smaller than the dimension of the smallest relevant object for navigation. This can result in enormous amount of raster data.

9.3.2. Assessment of vector method with cognitively defined optimum paths

Advantages:

- The optimum path is defined cognitively and drawn manually as vectors over a scanned orienteering map. It is shown detailed and realistic as a smooth line.
- Precise distances are measured cartometrically.
- The method allows to choose realistic waypoints.
- Assessment of risks and frictions is realistic and precise. Very complex terrain can be tested, as passability of the optimum path segments is evaluated experientally from the map, and not by some black box GIS application.
- The costs are computed only for the path itself, and not for the whole area of navigation.
- Vector procedure is computationally simpler than raster one because the optimum path is not calculated by a spreading algorithm. The computation of a descending slope line by a tracking procedure on the accumulated cost surface is not needed.

Disadvantage:

- The method is not fully automated, but needs experiental valuations and spreadsheet calculations.

9.4. Variables and parameters for simulation

From now on, we try to demonstrate the hypothesis with vector based data. As evident from the hypothesis, there are three interrelated *variables*, which we have to alter and simulate to demonstrate the hypothesis: the *tools*, the *strategy*, and the *errors*. By changing them, we get the *costs* for the optimum paths from which conclusions can be drawn.

There are also additional *parameters* which have to be fixed, when we vary the three variables: the *origin,* the *destination*, the *waypoints*, the *optimum path condition*, the *environment*, and the *navigator*. We show the assumptions about them, too. The results of simulation should show the functional dependences between variables and parameters, and demonstrate the hypothesis.

9.4.1. Tools

We simulate three general tool combinations:

- the orienteering map and compass (ie. tools for classic orienteering),
- the GPS receiver with screen orienteering map (ie. the first technically augmented device for satellite navigation),
- the GNSS receiver with screen orienteering map (ie. the second technically augmented device for satellite navigation).

The simulation of different tools can potentially answer the questions:

- Which risks are associated with the tools?
- How strong are the risks?
- How the weaknesses of each tool affect the costs?

9.4.2. Strategy

The strategy of navigation with map and compass is based on the determination of displacement to a future position by measuring and predicting the distances and directions from a known position. To avoid missing the desired future position, we support the basic strategy with feature matching and observation of distal landmarks when available. When we use the GPS or the GNSS receiver, we practice the strategy of positioning.

Variation of strategies can, for example, answer the following questions:

- Which strategy produces lower risks or costs for a certain tool?
- And vice versa: which tool fits best to a specific strategy?

9.4.3. Errors

In the simulation, the potential errors will be embedded within the values of risks, and in the cognitive construction of the optimum path. Each risk acts as an error threat, which can potentially cause to get lost, or to deviate from the optimum path. We will intentionally not deal with the situations where the navigator gets lost, since rational relocation procedures are often disturbed by irrational behaviour and emotions (Hill 1999).

We will not simulate every single error separately, since this would require:

- extensive field trials and experiments,
- local and extremely detailed data,
- several different computational approaches for error modelling,
- introduction of the concept of fuzziness into the calculation of costs.

The observation of risk values can potentially answer the following questions about the errors:

- Which risks are the highest for a cetrain tool, ie. which errors cause the highest costs?
- How the costs change, when the risk of errors change?
- Which optimum path gives the least costs with a chosen tool, if we alter the risks?

9.4.4. Origin, destination and waypoints

In the beginning of simulation, we will choose the origin and the destination, which are fixed locations throughout the simulation. Therefore, we indirectly limit the area of navigation, the distance to be traveled, and the rational distances between the waypoints.

Every strategy with each tool needs also waypoints. People visually and cognitively always rely to waypoints and landmarks at least as backup, and even with the most precise tools. In the simulation, we will just vary the number, the distribution, and the type of waypoints. On open area less waypoints are needed then in a forest. A risky navigation requires more waypoints. Waypoints will serve to divide and conquer the optimum path. The optimum paths will be realized with different permutations of waypoints between a common origin and destination.

9.4.5. Optimum path condition

To compare the costs of navigation along different optimum paths, we have to select an arbitrary optimum path condition. For all tools we choose the condition which is fundamental in orienteering: to run along the fastest path, ie. to select the path which is beaten in the minimum time. We assume that throughout the simulation, the optimum path condition stays the same.

9.4.6. Environment

When we choose the origin and destination, we get the environmental conditions in between, which have to be accounted numerically for each tool and optimum path. To ensure variability, we choose the navigation region with mixed feature types on open and forest area. Certain topographic features represent specific risks and frictions if they are traversed. They are related to:

- ground and vegetation, which cause a resistance to locomotion,
- visibility, terrain complexity (ie. geomorphology) and presence of linear features, which affect the amount of the navigation risk,
- obstacles, which deviate the optimum path,
- slope magnitude in the direction of passage, which is a kind of a friction,
- the available landmarks, which provide the appropriate choice of waypoints.

There is not any general and rigorous measure of environmental complexity which would encompass all the listed impact factors. As described in later chapters, we account for each friction and risk separately.

9.4.7. Navigator

Different humans navigate in different ways. Even by experienced orienteers the time to reach the same destination can differ for more than a factor of 2. Since we do not study human behavioural patterns, we assume, that the navigator is always the same regardless of the tool used. He has average navigation skills and physical condition. He locomotes at the highest speed allowed by the environmental friction, by his abilities and by the potential risk of errors. It is also assumed that the optimum path is followed permanently by rational decision making (ie. without disturbing emotions). He navigates at daylight in normal dry weather conditions.

Since the simulation method includes plenty cognitive decisions about risks, frictions, waypoints and optimum paths, and since the author of this thesis has average orienteering skills, it is reasonable to presume that the navigator is himself. So the optimum paths and the waypoints will be chosen on the map as he would navigate in a real world.

9.5. Simulation with cognitively defined vector optimum paths

To simulate the navigation, we select the navigation area on a scanned orienteering map. We select the origin and the destination. We interprete and draw several optimum paths on the map. For each path we choose and draw the waypoints. We cut each single run into homogene segments having the same risk and friction properties. We mark the segments on the optimum paths. We assess numerically the risks and the frictions per segment from the map in separate spreadsheets. We calculate the cost of each segment and sum the costs of segments to get the cost for each run. Then we sum the costs of runs to get the cost for each optimum path in another spreadsheet. Finally, we compare the costs and draw conslusions about the validity of the hypothesis. A vector type path is thus defined by human inferencing before the calculation phase.

Before the calculation phase, we preliminary construct three spreadsheets to numerically express the reference categories of various isotropic and anisotropic risks and frictions, which are needed to calculate the individual cost:

- the spreadsheet with isotropic resistance to locomotion and navigation risks,
- the spreadsheet with anisotropic slope friction,
- the spreadsheet with dead reckoning and waypoint discernibility risk.

As it will be shown in the continuation, the risks and frictions in the spreadsheets are modelled as a blend of the author's personal experiences and estimates, the theoretical backgrounds of navigation, and the approximate empiric equations. The mapping of the optimum paths and the waypoints was performed with OCAD mapping software. All spreadsheet calculations were made in MS Excell.

9.5.1. Spreadsheet with isotropic resistance to locomotion and navigation risks

The speed of running in orienteering is mainly influenced by areal and linear objects. Therefore, the spreadsheet where the reference frictions will be assessed numerically, will be based on the object types (Spreadsheet 9.1.). We use the IOF mapping standard for orienteering maps (IOF 2000) for this purpose. Initially, we make a list of all area and line object types that are allowed on a paper orienteering map.

In the standard, vegetation types, communications, and water object types are classified according to the resistance to locomotion, visibility and ease of navigation. As some object types in fact represent the land cover, they sometimes appear on the map combined pairwise, like eg. the undergrowth in a forest. Line object types always flow within some land cover or even divide two land covers, like eg. a footpath along the forest edge. Such combinations prevent us from quoting all possible frictions and risks exactly. We rather evaluate average conditions in average circumstances.

The name of the object type given by the IOF standard is shown in the first column of the spreadsheet and the description of IOF standard cartographic symbol is in the second one (Spreadsheet 9.1.).

RESISTANCE TO LOCOMOTION AND NAVIGATION RISKS						
AREA OBJECT TYPE	**CARTOGRAPHIC SYMBOL**	**Affordable speed**	**Resistance to loco.**	**Navigation risk-M&C**	**Navigation risk-GPS**	**Navigation risk-GNSS**
Open land (usually short grass)	Yellow area	95%	1,05	0,90	0,70	0,70
Rough open ground (eg. tall grass, moor, fell area)	Light yellow area	80%	1,25	0,95	0,75	0,75
Open land with scattered trees	Yellow dot screen area	92%	1,09	0,95	0,75	0,75
Forest: easy running (80-100% speed)	White area	90%	1,11	1,00	1,50	0,80
Forest: slow running (60-80% speed, low visibility)	Light green area	70%	1,43	1,10	1,65	0,90
Forest: difficult to run (20-60% speed, low visibility)	Medium green area	40%	2,50	1,20	1,80	1,00
Vegetation: very difficult to run (0-20% speed, barely passable)	Dark green area	10%	10,00	1,30	1,95	1,10
Additional land cover						
Undergrowth: slow running (60-80% speed, good visibility)	Sparse vertical green hatching	70%	1,43	(risk)*1,05	(risk)*1,05	(risk)*1,05
Undergrowth: difficult to run (20-60% speed, good visibility)	Dense vertical green hatching	40%	2,50	(risk)*1,10	(risk)*1,10	(risk)*1,10
LINE OBJECT TYPE					**Open / Forest**	
Minor road (paved, 3-5 m wide)	Double black line with brown fill	100%	1,00	0,75	0,70 / 0,80	0,70
Road (unsurfaced, less than 3 m wide)	Thick black line	100%	1,00	0,75	0,70 / 0,80	0,70
Vehicle track	Dashed thick black line	95%	1,05	0,80	0,75 / 0,90	0,75
Footpath	Dashed medium black line	95%	1,05	0,80	0,75 / 0,90	0,75
Small footpath (80-100% speed)	Dashed thin black line	90%	1,11	0,80	0,75 / 0,95	0,75
Less distinct small footpath	Dashed interrupted thin black line	80%	1,25	0,85	0,80 / 1,05	0,80
Crossable watercourse (crossing without bridge)	Blue line	20%	5,00	0,90	0,90 / 0,90	0,90

Spreadsheet 9.1. Resistance to locomotion and navigation risks

9.5.1.1. Resistance to locomotion

The third column shows the affordable speed over or along the object, as specified by the IOF standard. The values are given in a percentage of the maximum running speed. Where the standard quotes a range of values for an object type, the mid-value is chosen. Other values were assessed from the author's individual experience. In reality, they can vary more than 10-20% even for skilled orienteers, and much more for unskilled and untrained individuals.

The values of resistance to locomotion in the fourth column are reciprocal values of the affordable speed values. Thus, the maximum speed value of 100% gives the nominal resistance of 1,00 and, eg. 50% of the maximum speed gives the resistance of 2,00. If the optimum path is in a forest or on an open area which is covered with undergrowth, we take the value of the undergrowth (see the subtitle 'Additional land cover' in the Spreadsheet 9.1.). If the optimum path is on a footpath, we take the value of this object regardless of the vegetation cover along it.

All values for areal object types in the spreadsheet refer to a flat terrain with homogene coverage properties without interfering objects or obstacles. All values for linear objects types also refer to a flat terrain and straight lines without interfering objects or obstacles. The affordable speed values describe how fast can a person locomote. Therefore, the resistance to locomotion is limited to isotropical physical friction only. It is not taking into account any navigation risk, nor anisotropic effect of the terrain slope.

9.5.1.2. Navigation risk with map and compass

The fifth column contains the navigation risk when using map and compass. Navigation risk is a complex mixture of several impact factors. They can prolong the distance and time of travel, and in extreme case, even cause a loss of orientation. The values in this column are related to the approximate distance or time spent in comparison to the optimum path in a forest, which has the nominal value of 1,00.

For area object types, most of the navigation risk is contributed by the density of vegetation which reduces visibility and deviates the executed optimum path from the planned one. Geomorphologic complexity also contributes to this risk. Passage of such an area can have two possible consequences: (1) an intentional and additional slow down of locomotion to stay on the optimum path, or (2) an unintentional waste of time and prolongation of the travel after wandering off the optimum path.

If area object is covered with undergrowth, its risk value has to be multiplied by 1,05 or 1,10 depending on the density of undergrowth (see the subtitle 'Additional land cover' in the Spreadsheet 9.1.). If the optimum path traverses a terrain with a lot of details, like karstic sinkholes, rocks or boulders, the individual values should be additionally multiplied with the value 1,20 or 1,40 depending on terrain roughness.

For line object types, the risk values appear lower due to the ease of navigation along them, regardless of visibility and complexity of the surrounding terrain surface.

9.5.1.3. Navigation risk with the GPS receiver and screen map

The sixth column contains the navigation risk when using the GPS receiver with a screen map. The use of such a tool on open areas is very reliable, so the values of risk are the values for map and compass, reduced for 0,20.

In the forest areas, the GPS does not work. We have to rely on the screen map, but without a compass or any other tool. In average forested environment, the navigation risk is approximately 0,50 higher than with map and compass, ie. the value is 1,50. To get the risk of using the GPS in more dense forest conditions, we multiply the values for map and compass with 1,50. For areas with additional undergrowth, the rule is the same as for map and compass.

Navigation along line objects is easy task with all types of tools. However, navigating with the GPS on open areas is slightly easier than with map and compass, and slightly more difficult in forest areas. This is reflected in the two-fold navigation risk values. We propose, that the GPS receiver fails to operate immediately after we enter the forest, and resumes the operation immediately after we appear on the edge of the open area.

9.5.1.4. Navigation risk with the GNSS receiver and screen map

The seventh column contains the navigation risk when using the GNSS receiver and screen map. We propose that the use of such a tool is very reliable, so on the open as in the forest areas. Therefore, all values of the navigation risk for area object types are the values for map and compass, reduced for 0,20. For areas with additional undergrowth, the rule is again the same as for map and compass.

Navigation along line objects is as easy as for the GPS receiver on open area. We retain the first of the two values from the GPS column.

9.5.2. Spreadsheet with anisotropic slope friction

Anisotropic friction refers to the passage of various terrain slopes in different directions. It has the same effect for all simulated tools. We take three steps to express the slope friction. We first explain how we measure the slope magnitude and the direction of passage. Secondly, we calculate the speed of directly upslope and directly downslope passage with the aid of the hiking function and the hiking diagram (Spreadsheet 9.2.). Finally, we construct the spreadsheet with intermediate slope frictions for various directions of passage and slope magnitudes (Spreadsheet 9.3.).

9.5.2.1. Slope magnitude and direction of passage

Slope magnitude is defined with inclination in degrees. The perception of difficulty of slopes is individual (Balstrøm 2002). In this simulation we categorize the magnitude into 5 discrete classes: flat (0°), gentle (5°), medium (15°), steep (25°), and very steep (35°). The magnitude values represent the inclination in the direction of maximum upslope.

The direction of passage accross the slope is defined with the angle between the direction of maximum upslope and the direction of locomotion, expressed in degrees. The angle is categorized into 9 discrete classes: directly upslope (0°), nearly directly upslope (22,5°), askew up (45°), gently askew up (67,5°), horizontal (90°), gently askew down (112,5°), askew down (135°), nearly directly downslope (157,5°), directly downslope (180°). For the remaining semicircle from 180° to 360°, we use the same values symetrically.

9.5.2.2. Hiking function and diagram of upslope and downslope speed

For all five slope magnitudes (inclinations), we calculate the speed of locomotion directly upslope and directly downslope. We use the following empiric equation called the hiking function (Tobler 1993):

$$Speed = 6 * \exp(-3{,}5 * \text{abs}(\tan(Inclination) + 0{,}05))$$

where *Speed* is expressed in km/h and *Inclination* of slope is in degrees (Figure 9.1.). For flat terrain, it gives the speed of 5,04 km/h (*SpeedFlat*). Originally, the equation is valid for hiking on footpaths. For off-path hiking, it should be multiplied by 3/5, ie. by 0,6. Such a correction is too general for this research. Since we have already taken into account the resistance to locomotion related to specific object types in the spreadsheet with isotropic frictions, and as we need only relative frictions, not speed values, we keep using the original equation without the multiplication factor.

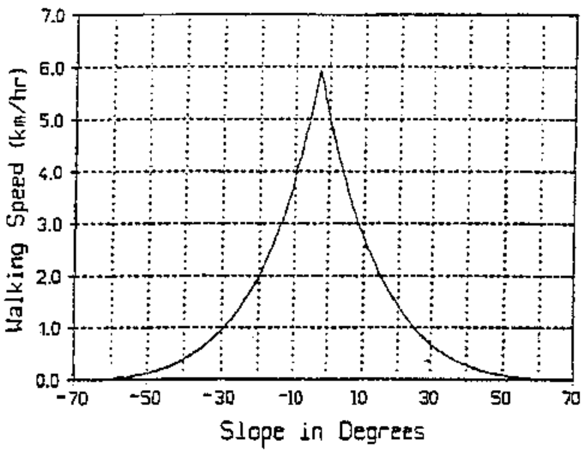


Figure 9.1. The hiking function graph
(source: (Tobler 1993))

The hiking function was originally derived from the experiental diagram (Imhof 1968), which is still used on many contemporary topographic and mountaineering maps (Figure 9.2.). From the diagram, a hiker can graphically interpolate the hiking time in ascent or descent with respect to horizontal distance and height difference.

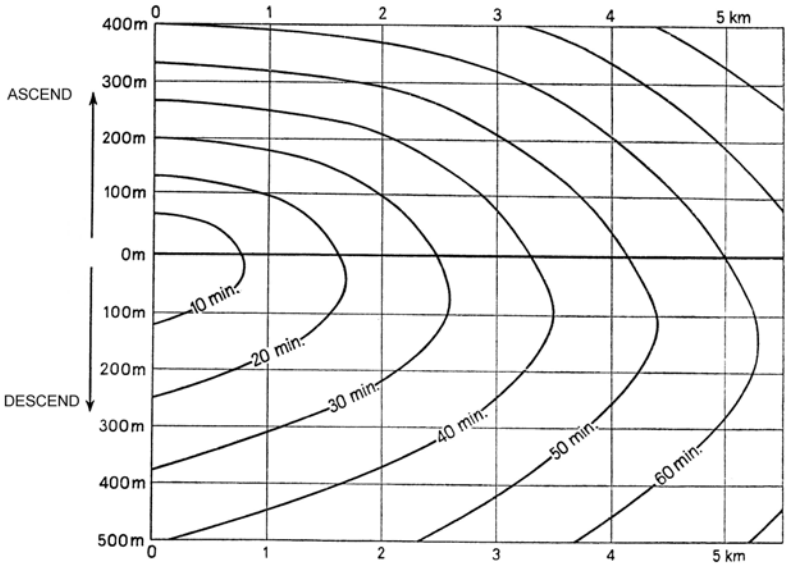


Figure 9.2. The diagram of hiking time
(source: (Imhof 1968))

The results of speed calculation for upslope and downslope locomotion are shown in the Spreadsheet 9.2.

UPSLOPE AND DOWNSLOPE SPEED						
Values represent speed (km/h)		SLOPE MAGNITUDE				
	Inclination (deg)	0	5	15	25	35
DIRECTION OF PASSAGE	Direction (deg)	flat	gentle	medium	steep	very steep
directly upslope	0,0	5,04	3,71	1,97	0,98	0,43
directly downslope	180,0	5,04	5,26	2,80	1,40	0,62

Spreadsheet 9.2. Upslope and downslope speed

9.5.2.3. Friction of askew passage

To get the values for other intermediate directions of passage, we make a new spreadsheet which contains 45 friction values, ie. 5 for inclinations multiplied by 9 for directions (Spreadsheet 9.3.). Before the calculation, we have to obtain normalized frictions instead of speeds, so we divide the speeds of upslope and downslope locomotion:

$$Friction = SpeedFlat \ / \ Speed$$

We insert the friction values into the first and the last row of the spreadsheet. On flat terrain, the friction has a nominal value of 1,00. Speedier travel has friction value lower than 1,00 and slower travel gets higher value than 1,00.

We use another empiric equation, which calculates the intermediate friction values separately for directions of passage between upslope and horizontal (22,5°; 45°; 67,5°) and, between downslope and horizontal (112,5°; 135°; 157,5°). For the horizontal passage of any slope, we get the same value as on flat terrain, ie. 1,00. The equation is (Eastman 2003):

$$EffectiveFriction = StatedFriction^{\,f}$$

where: $f = \cos^{k}(Direction)$

StatedFriction is *Friction* for directly upslope or directly downslope travel given by the hiking function. *EffectiveFriction* is the final value of friction accounted for the anisotropic effect of intermediate askew *Direction* value. *Direction* is given in degrees. With the user-defined coefficient *k* we choose the directional specificity of the function *f* (Figure 9.3.). The most appropriate value is $k = 2$.

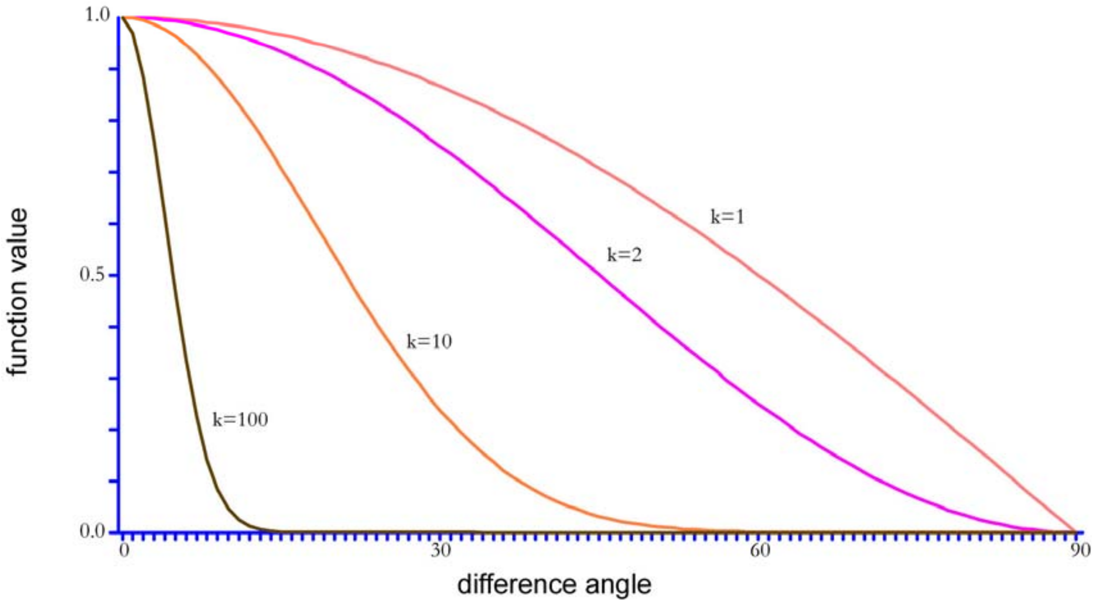


Figure 9.3. The diagram of impact of various coefficients k
(source: (Eastman 2003))

From the calculated spreadsheet, we learn that the fastest hiking is gently downslope, followed by either flat terrain or horizontal path accross the slope. All other frictions appear greater, also downslope where fast and steep descent can be dangerous.

When calculating the slope friction for individual optimum path, the category of slope magnitude and the category of direction of passage have to be estimated approximately, by the eye, from the map, without explicit cartometric measurement. At the bottom of the spreadsheet there is an auxiliary table of contour separations (ie. horizontal distances between consecutive contours) for easier estimation of slope magnitude on the map. They are expressed in milimetres on a map, and in meters in a real world, for each slope magnitude category, arbitrary map scale, and contour equidistance.

SLOPE FRICTION						
Values represent friction		SLOPE MAGNITUDE				
	Inclination (deg)	0	5	15	25	35
DIRECTION OF PASSAGE	**Direction (deg)**	flat	gentle	medium	steep	very steep
directly upslope	0,0	1,00	1,36	2,55	5,11	11,60
nearly directly upslope	22,5	1,00	1,30	2,23	4,03	8,10
askew up	45,0	1,00	1,17	1,60	2,26	3,41
gently askew up	67,5	1,00	1,05	1,15	1,27	1,43
horizontal	90,0	1,00	1,00	1,00	1,00	1,00
gently askew down	112,5	1,00	0,99	1,09	1,21	1,36
askew down	135,0	1,00	0,98	1,34	1,90	2,86
nearly directly downslope	157,5	1,00	0,96	1,65	2,99	6,01
directly downslope	180,0	1,00	0,96	1,80	3,60	8,17

Speed on flat area (km/h)	5,04
Power of Cos(Direction)	2

CALCULATION OF CONTOUR SEPARATION						
Map scale denominator	10000					
Contour equidistance (m)	5					
Slope magnitude (deg)		0	5	15	25	35
Contour separation (m)	**real-world**	-	57	19	11	7
Contour separation (mm)	**map**	-	5,7	1,9	1,1	0,7

Spreadsheet 9.3. Slope friction

9.5.2.4. Comments on empiric equations

The task of this thesis is not a real world testing, whether both empiric equations are fully suitable for orienteering, different individuals, or various environmental circumstances. Nevertheless, some critique is in place to point out potential uncertainities:

- The Imhof's diagram conforms to an average walking hiker in alpine type of environment, without presumtions about the meandering of the path. It supposes that the hiker needs 1 hour for an ascent of 400 m height difference, or for a descent of 700 to 800 m height difference, or for a 4,5 to 5 km long horizontal hike.
- While in the spreadsheet the speed of walking (ie. hiking) horizontally accross the slope is numerically the same on all slopes (ie. friction is equal to 1,00), full speed running is generally slower, if the slope is steep, because of the risk to sprain the ankle (ie. a real friction would be higher than 1,00). This effect has not been taken into account. Without extensive tests in real environment it is hard to estimate and very person specific.
- In contrast with walking, running on moderately steep downslope can be faster than on flat terrain (ie. friction would be less than 1,00). Numeric evaluation would also need real world trials.
- Balstrøm (2002) argues that the risks and frictions, when hiking in the wild, are highly individual, and that realistic values depend on physical condition, own size, weight and constitution, age, gender, footwear, weather, backpack, and other factors. Thus, the resulting cost estimates for optimum paths have to be taken with care of what has been presumed about the fixed parameters in simulation.
- Balstrøm (2002) also made a field test with inclinometer. If inclination is bigger than 30%, the hiker makes hairpin bends, or chooses another track. An inclination of 40% is the limit of direct upslope hiking for experienced mountaineers and orienteers.

9.5.3. Spreadsheet with dead reckoning and waypoint discernibility risk

In this chapter, we define the methodology to take into account two different risks in a single step:

- the risk of dead reckoning for various distances, and,
- the risk of waypoint discernibility for various object types representing waypoints.

9.5.3.1. Dead reckoning risk

The risk of navigation with map and compass generally increases by some factor with the distance from the starting waypoint, if there are no additional orientation cues present in the visual horizon of the navigator. When the navigator starts off the waypoint in some chosen straight direction, his method of navigation is dead reckoning, ie. he knows the approximate direction and distance to the next waypoint. His navigation suffers from the error of direction and distance determination. He can deviate from the true direction by some angle. Additionally, he can undershoot or overshoot the destination by some distance, where the second case is more likely and risky. Namely, when he undershoots the waypoint, he usually proceeds until he perceives it, or until he feels he has wandered too far, thus overshooting it. All such cases will be discussed bellow. The compound risk can refer to the entire leg, run, a sequence of run segments, or to a single run segment.

The combined risk is relevant only when using map and compass, or when using the GPS receiver with screen map within a forest, where it does not work appropriately. However, in a forest this risk is larger for the GPS receiver with screen map, than for map and compass, as the deviation angle in the first case is larger. Namely, when only screen map is functioning, the navigator can apply only navigation with feature matching, which is in most terrain conditions unreliable for position and direction determination.

The use of a fully functional GNSS receiver, or a GPS receiver on open areas, does not suffer from dead reckoning and waypoint discernibility risk, as the strategy of navigation is precise positioning, which is quite different from map and compass case. There exists a slight risk of unprecise position determination, ie. a positioning risk, however most receivers work within the absolute accuracy of several meters which is even on large scale maps close to the graphical accuracy of a map. For example, the positioning accuracy of 5 m reduced to the scale 1:10.000 is 0,5 mm, while the map drawing accuracy is around 0,3 mm. We can presume, that the actual microrelief details and map reading on the run usually cause larger discrepacies of locomotion than unprecise position itself.

In any case, the dead reckoning risk is also irrelevant for navigation along "handrail" objects like footpaths, other line objects, sharp area and vegetation boundaries, open corridors within forest, or along any objects which lie along the path and aid the orientation.

9.5.3.2. Waypoint discernibility risk

The consequences of dead reckoning errors depend on the discernibility of the destination waypoint. If the destination is obscured eg. by vegetation, the navigator can search for it longer or even miss it (Figure 9.4.). As he also has to navigate precisely and slowly to find it, he can lose time. The discernibility of waypoint can be numerically defined by the radius of visibility, ie. by the distance from which the waypoint object can be perceived.

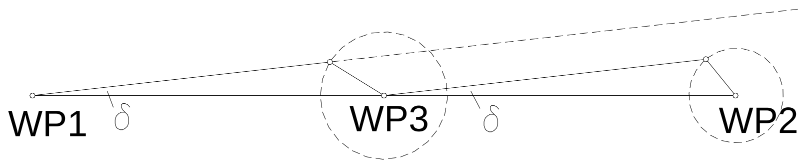


Figure 9.4. Prevention of overshooting the target waypoint WP2 by an intermediate waypoint WP3

In a real world, the radius is usually different for various directions of approach to the waypoint. Thus, the discernibility of waypoint is anisotropic. This anisotropy can be realistically assessed only by a terrain inspection. In cognitive vector procedure, we have to check microdetails around the waypoint shown on the orienteering map and estimate the radius only for the respective direction of approach. Therefore, we rather avoid here the declaration of iso- or anisotropy.

The risk regarding the discernibility of waypoint is tied to the entire run, to a sequence of run segments, or to a single run segment, ie. to the distance between two consecutive waypoints, or to a recognizable change of any risk or friction property along the run. When we plan the optimum path, we should choose well discernible waypoints on longer runs, and obscured or smaller features only on short runs. Namely, waypoints affect the selection of optimum path and the division into runs. Visible waypoints are eliminating or diminishing the risk of dead reckoning.

When navigating with a GNSS receiver, or with a GPS receiver on open areas, the discernibility of waypoint does not matter as the positioning procedure leads the navigator precisely to the waypoint. However, in a forest this risk is the same for a GPS receiver with screen map and for a map and compass.

9.5.3.3. Combined risk function

As we can observe from the Figure 9.4., the *Risk* factor as a reason of dead reckoning and discernibility of waypoint is a function of distance *D* between waypoints, deviation angle *δ* and waypoint discernibility radius *R*.

$$Risk = \mathrm{f}\,(D, \delta, R)$$

To derive the equation for this functional dependence and to construct the spreadsheet, we combine plane trigonometry with some experiental values and practical cognition of orienteer, as follows. We distinguish three cases:

- the case of no risk,
- the case of risk to deviate from the optimum path,
- the case of risk to overshoot the destination waypoint.

Beside the three cases, we also describe a special limiting situation between the deviation and the overshoot, where the *Risk* changes abruptly.

9.5.3.4. The case of no risk

When the discernibility radius of the destination waypoint is greater or equal to the distance to this waypoint, the navigator sees the destination waypoint from the origin waypoint (Figure 9.5.). He can travel directly to the destination without dead reckoning. Therefore:

$$R \geq D \Rightarrow Risk = 1$$

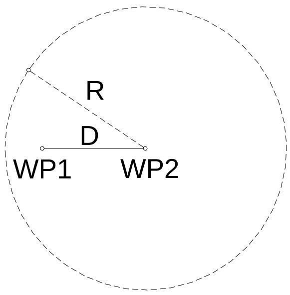


Figure 9.5. Target waypoint WP2 visible from the starting waypoint WP1

9.5.3.5. The case of risk to deviate from the optimum path

When the discernibility radius of the destination waypoint is shorter than the distance to this waypoint, the navigator does not see the destination waypoint from the origin waypoint. He wishes to beat the distance in least time, so he heads straight and practices dead reckoning with some unwanted angular deviation δ from the desired direction (Figure 9.6.). The orienteer is aware of possibility of errors, so he has to navigate slowly, or he can risk to miss the destination. This risk can be expressed with the additional distance, he has to travel, if he deviates off the optimum straight line.

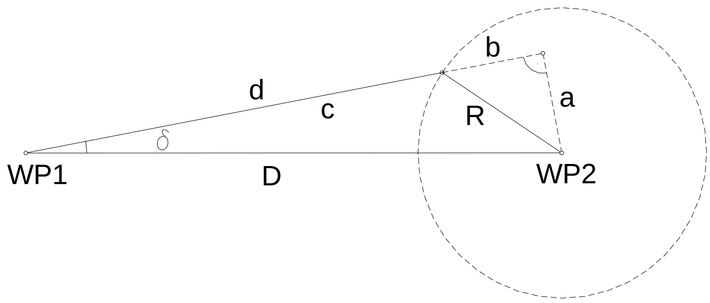


Figure 9.6. Deviated path without overshoot of the target waypoint WP2

Under this case, we explore the situation, where the waypoint discernibility radius is greater than the deviation distance (the perpendicular distance from the erroneously executed path to the destination waypoint):

$$R > D.\sin\delta$$

From the Figure 9.6. we see that in this case, the navigator's executed path *d* intersects the circle of visibility of waypoint. At the intersection point, the navigator perceives the waypoint and turns toward it without any further risk. The *Risk* factor is the ratio between the total executed path *Distance* and the planned straight path *D*:

$$Risk = Distance / D = (d + R) / D$$

where:

$Distance = d + R$

$d = c - b$

$c = D.\cos\delta$

$b = \mathrm{sqrt}(R^2 - a^2) = \mathrm{sqrt}(R^2 - D^2.\sin^2\delta)$

thus:

$Risk = (D.\cos\delta - \mathrm{sqrt}(R^2 - D^2.\sin^2\delta) + R) / D$

The above equation for *Risk* expresses the ratio of prolongation of the path, or the risk, if we know the values of *D*, *δ*, and *R*. We observe from personal experience, that for the average terrain conditions and for the orienteer using an orienteering type of compass, the deviation angle *δ* is 5°. The distance *D* can be measured cartometrically from the orienteering map. The radius *R* can also be estimated from the map approximately, from the type of waypoint object and its surrounding land cover. *Risk* in this case usually reaches the values which are just a bit greater than 1,00. The path is prolonged for few percents. If the prolongation turns out larger, the orienteer should have planned additional intermediate waypoints to avoid excessive risk.

If the navigator is using a GPS receiver in a forest, the deviation angle *δ* is much larger, eg. between 10° and 30°, as he uses only the screen map and the feature matching technique. We presume in the calculation that the average angle would be 20°. Such a significant angular deviation leads more often to the overshoot case as is described later, than only to a minor prolongation of the path.

We can oppose this simple method and argue that the orienteer can also miss the waypoint when he is inside the visibility radius, if he is looking into wrong direction. However, we just want to calculate the average potential risk, since mathematical prediction of all possible outcomes for complex human behaviour in a real environment is impossible.

9.5.3.6. The limiting situation between deviation and overshoot

For further explanation of this risk, we need to study also a special situation where the deviated path touches the circle of waypoint visibility (Figure 9.7.), ie. the situation where:

$R = D.\sin\delta$

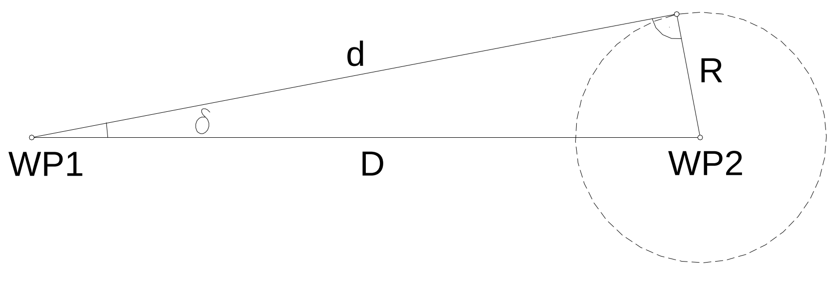


Figure 9.7. Deviated path at the waypoint discernibility limit

From the figure we see, that:

$b = 0 \Rightarrow Risk = (D.\cos\delta + R) / D = (D.\cos\delta + D.\sin\delta) / D$

$Risk = \cos\delta + \sin\delta$

If we fix the radius R and the deviation angle δ, we can calculate the limiting *Risk* from the equation above. Furthermore, we can also calculate the limiting distance D between the waypoints:

$Risk = (D.\cos\delta + R) / D \Rightarrow D = R / (Risk - \cos\delta)$

The touching situation is important as a limit, where the value of *Risk* abruptly changes to greater values, if the deviation angle δ becomes larger, or if we miss the circle of waypoint discernibility. This is shown below in the explanation of the fourth case.

9.5.3.7. The case of risk to overshoot the destination waypoint

In the last case, we research the situation, where the waypoint discernibility radius is shorter than the perpendicular deviation distance (Figure 9.8.), ie. the situation where the orienteer misses the circle of waypoint visibility:

$R < D.\sin\delta$

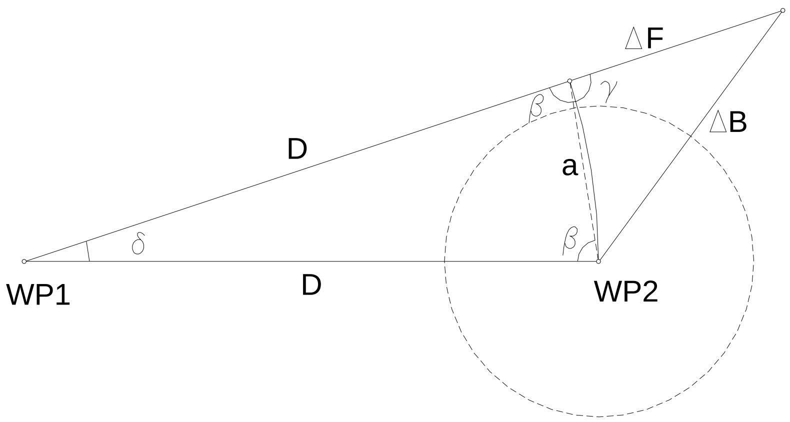


Figure 9.8. Deviated path with overshoot distance ΔF and relocation distance ΔB

This is the most risky situation of dead reckoning. When the navigator passes the expected and planned distance D between waypoints, he does not find the destination, nor can he see it. He is usually continuing in the same direction, until he feels he has gone too far, ie. until he overshoots the distance D for a certain amount, eg. for 20 or 30 %. This is quite a subjective feeling where the reactions of the orienteer highly depend on his experiences and the type of terrain. It appears the same for the map and compass case, and for the use of GPS receiver in a forest. After missing the destination, a rational navigator will stop for a moment and try to reorientate and relocate himself with the aid of the surrounding objects. After the relocation manoeuvre, he will turn into the direction of destination waypoint and run toward it.

For the sake of risk assessment, we suppose that he will now hit the visibility radius without further troubles and find the waypoint. This scenario is very common and the most probable outcome in orienteering sport. As mentioned in the previous case, there are also other scenarios which can span from immediate relocation before any significant overshoot, to a total loss of orientation with substantial overshoot and with confused unpredictable roaming about the forest.

To derive the *Risk* for this case, we first introduce the coefficient of overshoot k expressed as a share of the original distance D, where eg. a value of 0,20 would stand for 20% of D, and would mean that a distance of 1,20 * D has been traveled before relocation. With a fixed coefficient k, we can then obtain the extra distance forward ΔF and backward ΔB, and the total *Distance* traveled, as follows:

$$Distance = D + \Delta F + \Delta B$$

where the overshoot distance forward is:

$$\Delta F = k.D$$

and the way back to the waypoint can be derived from the cosine theorem:

$$\Delta B^2 = \Delta F^2 + a^2 - 2.\Delta F.a.\cos\gamma$$

where in the isosceles triangle:

$$\sin(\delta/2) = a/(2.D) \quad \Rightarrow \quad a = 2.D.\sin(\delta/2)$$

$$\gamma = 180^\circ - \beta = 180^\circ - (180^\circ - \delta)/2 = 90^\circ + \delta/2$$

and further:

$$\Delta B^2 = k^2.D^2 + 4.D^2.\sin^2(\delta/2) - 2.k.D.2.D.\sin(\delta/2).\cos(90^\circ+\delta/2)$$

$$\Delta B^2 = k^2.D^2 + 4.D^2.\sin^2(\delta/2) + 4.k.D^2.\sin^2(\delta/2) = D^2.(k^2 + 4.\sin^2(\delta/2) + 4.k.\sin^2(\delta/2))$$

$$\Delta B = D.\text{sqrt}(k^2 + 4.\sin^2(\delta/2).(1+ k))$$

Then the total distance is:

$$Distance = D + \Delta F + \Delta B = D + k.D + D.\text{sqrt}(k^2 + 4.\sin^2(\delta/2).(1+ k))$$

$$Distance = D.(1 + k + \text{sqrt}(k^2 + 4.\sin^2(\delta/2).(1+ k)))$$

Finally, the risk is:

$$Risk = Distance\ /\ D$$

$$Risk = 1 + k + \text{sqrt}(k^2 + 4.\sin^2(\delta/2).(1+ k))$$

From the equation we see, that the *Risk* is independent of the radius R and the distance D.

9.5.3.8. Spreadsheet construction

The spreadsheet for the calculation of risk has two parts (Spreadsheet 9.4.). The first contains an illustrative matrix of risk values for different sample distances D and radii R, after we define the deviation angle δ and the coefficient of overshoot k. The matrix contains the risks, calculated for all three cases (Figure 9.9.). Cells with the value of "1" pertain to the first case of no risk. The constant values lying approximately under the diagonal, pertain to the fourth, the overshooting case, while all others are close to 1,00 and pertain to the second, the deviation case. Below the matrix we see a row with limiting distances for the touching case. As mentioned, the spreadsheet is for illustration only, and is not used for the calculation of costs.

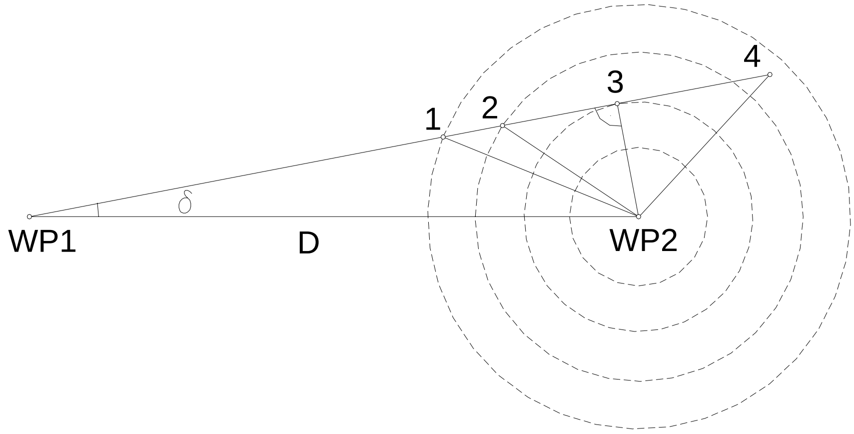


Figure 9.9. Deviated path with the circles of discernibility for different cases

The second part offers the possibility of individual calculation of risk when we deal with runs, sequences of segments, or individual segments in the phase of calculating the costs. To get the *Risk* value, we have to enter the distance *D*, the waypoint discernibility radius *R*, the deviation angle *δ* and the coefficient of overshoot *k*. The imbeded *if* clause decides which case has to be used.

DEAD RECKONING AND WAYPOINT DISCERNIBILITY RISK										
Deviation angle (deg)	5									
Coefficient of overshoot	0,20									
Values represent risk	**WAYPOINT DISCERNIBILITY RADIUS (m)**									
DISTANCE WP1-WP2 (m)	5	10	15	20	30	40	50	100	150	200
50	1,05	1,02	1,01	1,01	1,00	1,00	1	1	1	1
100	1,42	1,05	1,02	1,02	1,01	1,01	1,00	1	1	1
200	1,42	1,42	1,42	1,05	1,02	1,02	1,01	1,00	1,00	1
300	1,42	1,42	1,42	1,42	1,05	1,03	1,02	1,01	1,00	1,00
400	1,42	1,42	1,42	1,42	1,42	1,05	1,03	1,01	1,01	1,00
500	1,42	1,42	1,42	1,42	1,42	1,42	1,05	1,02	1,01	1,01
1000	1,42	1,42	1,42	1,42	1,42	1,42	1,42	1,05	1,02	1,02
Distance wp1-wp2 (m)										
(Deviation distance = Radius)	57	115	172	229	344	459	574	1147	1721	2295
Risk (Distance wp1-wp2 <= Radius)	1									
Risk (Deviation distance < Radius)	varies									
Risk (Deviation distance = Radius)	1,08									
Risk (Deviation distance > Radius)	1,42									
INDIVIDUAL CALCULATION OF RISK										
Distance wp1-wp2 (m) =	100									
Waypoint discernibility radius (m) =	10									
Deviation angle (deg) =	5									
Coefficient of overshoot =	0,20									
Risk =	1,05									

Spreadsheet 9.4. Dead reckoning and waypoint discernibility risk

9.5.4. Execution of simulation

After we have constructed the spreadsheets with the reference risk and friction values, we execute the simulation in the steps which have been introduced in the first paragraph of the chapter 9.5. Each step is described in a separate subchapter as follows.

9.5.4.1. Selection of map and navigation area

For the simulation, the Slovenian orienteering map no. 074 - Bloščica has been chosen, showing a part of dinaric plateau in the central-south part of Slovenia (Figure 9.10.).

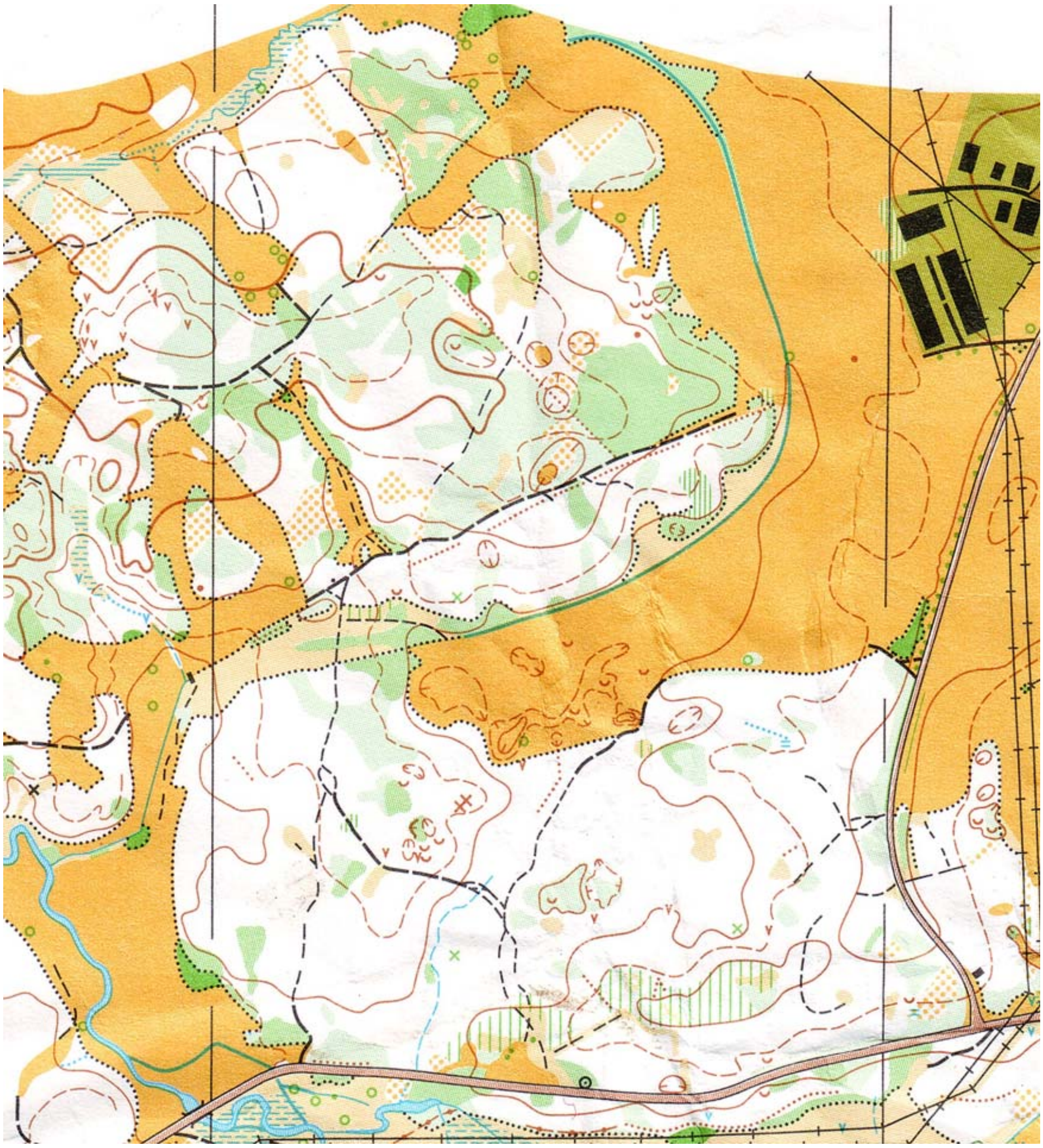


Figure 9.10. Orienteering map of the navigation area
(source: map Bloščica, Slovenian Orienteering Federation, original scale 1:10.000)

The map scale is 1:10.000, and the contour equidistance is 5 m. The selected navigation area has the size of approximately 1 km² in the north-east part of the map. It encompasses moderately detailed, flat and hilly terrain, partly open and partly forested, with few footpaths. The resolution of scanned map is adequate to clearly show the smallest relevant detail of every topographic feature shown with cartographic symbolization.

9.5.4.2. Selection of origin and destination

Large empty circles are used to show the origin (1) and the destination (33), with the feature or its detail in the middle (Figure 9.11.). Pink colour, as in orienteering course setting, is selected to contrast with the other map contents.

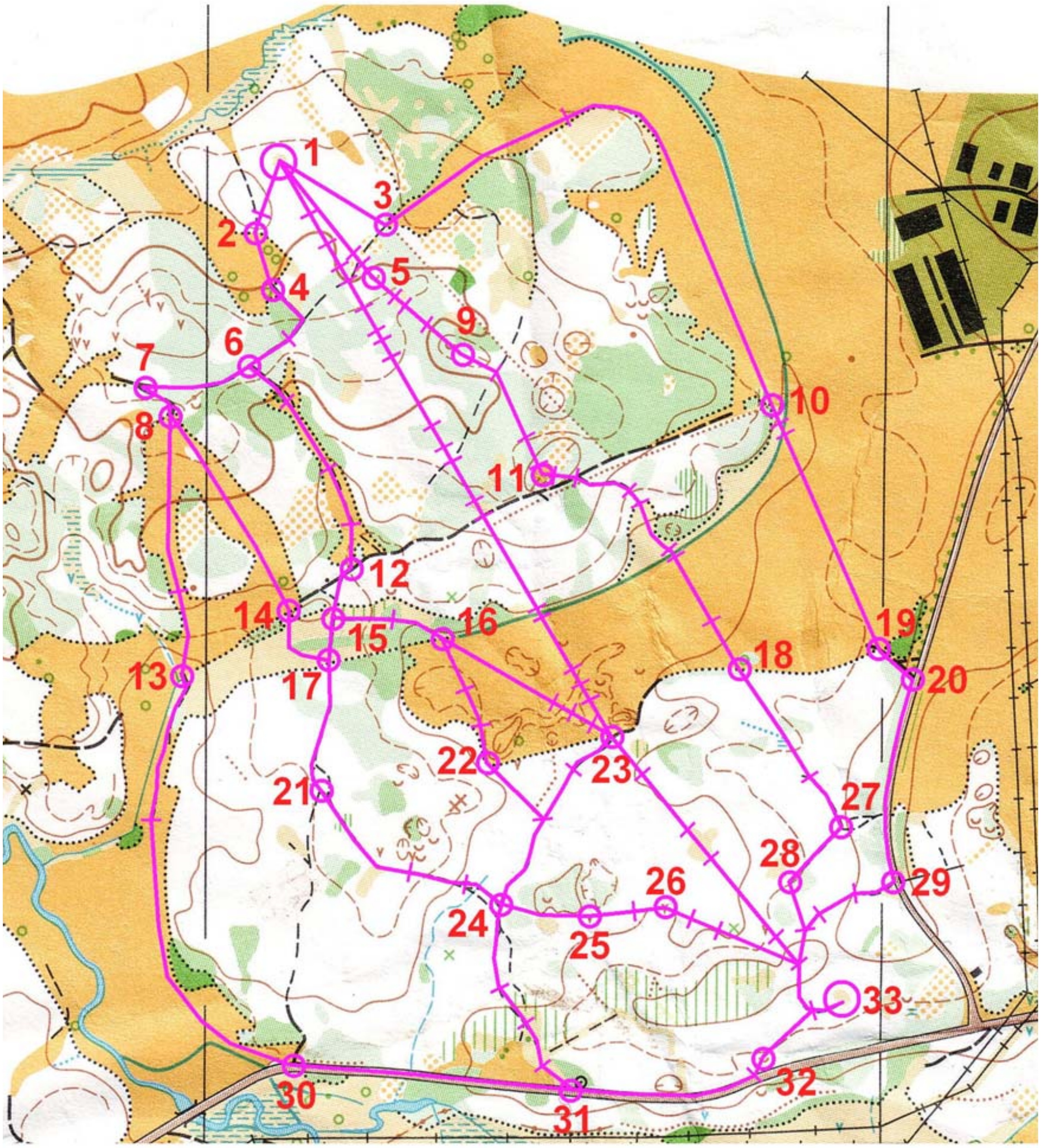


Figure 9.11. Orienteering map with origin (1), destination (33), waypoints (numbered), and segmented optimum paths
(source: map Bloščica, Slovenian Orienteering Federation, original scale 1:10.000)

OCAD 9.0.1 Professional desktop mapping software (ie. Orienteering CAD) was used to draw the optimum paths for simulation. It has predefined cartographic symbols for this purpose. One single leg of the course is shown, thus the origin and the destination would have been two marked control points if the course had really been set up on the terrain.

9.5.4.3. Cognitive interpretation of optimum paths

Between the origin and the destination we define and draw several versions of optimum paths which will be numerically tested for the risks, frictions and costs (Figure 9.11.). Each optimum path is found by experiental reasoning, ie. by cognitive "armchair orienteering".

Pink line is used, as in orienteering course setting. Wherever possible, the optimum path is straight, following some azimuth. Otherwise the path adapts to the terrain characteristics, or tracks the existing footpaths (compare the Figure 9.11. with the Figure 9.10., as the pink line symbol overlaps the footpath symbol).

9.5.4.4. Selection of waypoints

Each selected waypoint is a point object, or a point detail on a linear or areal object. It is identified on the map as a cartographic symbol or its detail. Waypoints delimit the leg into a succession of runs.

Small empty pink circle with a bold red number is used to show the waypoint in the middle of a circle (Figure 9.11.). The numbering of waypoints starts at the origin and ends at the destination, increasing along the optimum path in the direction of locomotion.

9.5.4.5. Segmentation of runs

A run virtually consists of one or more segments, wherein the risk and friction properties are approximately homogene. In the execution part of orienteering the location of risk or friction change can be perceived, but is usually not precisely defined or surveyed by the navigator, thus the segment endpoints are not waypoints.

For the simulation purposes, each run is further divided into segments on places where the optimum path crosses, touches, joins or branches off a linear object or a border of areal object, which alters one or more risk or friction property. We manually map short pink line ticks across the path to show the segmentation (Figure 9.11.). Segments are counted between two waypoints in the direction of locomotion, but are not numbered explicitely on the map. Each run has own local numbering of segments.

9.5.4.6. Assessment of risks and frictions per segment

After the optimum paths, the origin, the destination, the waypoints with numbering, and the segment tics have been drawn, we construct a spreadsheet to input the individual risks and frictions, and to calculate the cost for each run by summation of relevant segment cost values (Spreadsheet 9.5.). As the risks are tool-specific, the costs are calculated separately for the three selected types of tools (abbreviated by M&C, GPS, and GNSS).

COSTS OF RUNS													
RUN	Resistance to loco.	Navigat. risk-M&C	Navigat. risk-GPS	Navigat. risk-GNSS	Slope friction	DR & WP risk-M&C	DR & WP risk-GPS	Distance (m)	WP radius	DR distance	Cost M&C	Cost GPS	Cost GNSS
wp 1													
seg 1	1,11	1,00	1,50	0,80	1,17	1,01	1,63	41			53,9	130,2	42,6
seg 2	1,11	1,00	1,50	0,80	0,96	1,01	1,63	14	14	seg 1-2	15,1	36,5	11,9
wp 2								55			69,0	166,6	54,5
wp 1													
seg 1	1,11	1,00	1,50	0,80	1,00	1,01	1,63	90	20	seg 1	101,3	244,2	79,9
wp 3								90			101,3	244,2	79,9
wp 1													
seg 1	1,11	1,00	1,50	0,80	1,00	1,03	1,63	86			98,1	233,6	76,4
seg 2	1,09	0,95	0,75	0,75	1,00	1,03	1,63	**10**			10,6	13,3	8,2
seg 3	1,43	1,10	1,65	0,90	1,36	1,03	1,63	12	15	seg 1-3	26,3	62,6	21,0
wp 5								108			135,0	309,5	105,6
wp 1													
seg 1	1,11	1,00	1,50	0,80	1,00	1,01	1,63	45			50,6	122,1	40,0
seg 2	1,11	1,00	1,50	0,80	1,17	1,01	1,63	25			32,9	79,4	26,0
seg 3	1,11	1,00	1,50	0,80	0,99	1,01	1,63	19			21,2	51,0	16,7
seg 4	1,43	1,10	1,65	0,90	1,36	1,01	1,63	13			28,2	68,0	22,8
seg 5	1,11	1,00	1,50	0,80	1,30	1,01	1,63	22			32,2	77,6	25,4
seg 6	1,43	1,10	1,65	0,90	1,30	1,01	1,63	19			39,4	95,0	31,8
seg 7	1,11	1,00	1,50	0,80	1,00	1,01	1,63	8			9,0	21,7	7,1
seg 8	1,43	1,10	1,65	0,90	0,98	1,01	1,63	14			21,9	52,8	17,7
seg 9	1,11	1,00	1,50	0,80	0,96	1,01	1,63	58			62,7	151,1	49,4
seg 10	1,43	1,10	1,65	0,90	1,00	1,01	1,63	19			30,3	73,1	24,5
seg 11	1,11	1,00	1,50	0,80	1,00	1,01	1,63	11			12,4	29,8	9,8
seg 12	1,43	1,10	1,65	0,90	0,99	1,01	1,63	33			52,1	125,6	42,0
seg 13	1,25	0,95	0,75	0,75	0,99	1,01	1,63	**13**			15,5	19,7	12,1
seg 14	1,11	1,00	1,50	0,80	0,96	1,01	1,63	77			83,2	200,6	65,6
seg 15	5,00	0,90	0,90	0,90	1,00	1,01	1,63	**8**			36,5	58,7	36,0
seg 16	1,05	0,90	0,70	0,70	1,00	1,01	1,63	**43**			41,2	51,5	31,6
seg 17	1,05	0,90	0,70	0,70	0,98	1,01	1,63	**13**			12,2	15,3	9,4
seg 18	1,05	0,90	0,70	0,70	1,36	1,01	1,63	**14**			18,2	22,8	14,0
seg 19	1,05	0,90	0,70	0,70	1,00	1,01	1,63	**30**	108	seg 1-19	28,7	35,9	22,1
wp 23								484			628,4	1351,7	503,8
wp 2													
seg 1	1,05	0,90	0,70	0,70	1,00	1	1	**43**	43	seg 1/ir	40,6	31,6	31,6
wp 4								43			40,6	31,6	31,6
wp 3													
seg 1	1,05	0,90	0,70	0,70	0,98	1	1	**155**		ir	143,5	111,6	111,6
seg 2	1,05	0,90	0,70	0,70	1,00	1	1	**291**		ir	275,0	213,9	213,9
wp 10								446			418,5	325,5	325,5
wp 4													
seg 1	1,05	0,80	0,90	0,75	1,17	1	1	51		ir	50,1	56,4	47,0
seg 2	1,05	0,80	0,90	0,75	1,00	1	1	35		ir	29,4	33,1	27,6
wp 6								86			79,5	89,5	74,6
wp 5													
seg 1	1,43	1,10	1,65	0,90	1,30	1,00	1,05	22			45,0	70,8	36,7
seg 2	1,25	0,95	0,75	0,75	1,00	1,00	1,05	**10**			11,9	9,9	9,4
seg 3	1,43	1,10	1,65	0,90	0,96	1,00	1,05	21			31,7	49,8	25,8
seg 4	1,43	1,10	1,65	0,90	1,80	1,00	1,05	33	50	seg 1-4	93,6	147,2	76,4
wp 9								86			182,2	277,6	148,3
wp 6													
seg 1	1,05	0,80	0,90	0,75	1,00	1	1	82		ir	68,9	77,5	64,6
wp 7								82			68,9	77,5	64,6
wp 6													
seg 1	1,05	0,80	0,90	0,75	0,96	1	1	39		ir	31,4	35,4	29,5
seg 2	1,05	1,00	1,50	0,80	0,96	1	1	**58**		ir	58,5	87,7	46,8
seg 3	1,25	0,85	0,80	0,80	0,96	1	1	**43**		ir	43,9	41,3	41,3
seg 4	1,05	1,00	1,50	0,80	1,00	1	1	**32**		ir	33,6	50,4	26,9
wp 12								172			167,4	214,8	144,4

Spreadsheet 9.5. Costs of runs

wp 7													
seg 1	1,05	0,80	0,90	0,75	0,99	1	1	32		ir	26,6	29,9	24,9
wp 8								32			26,6	29,9	24,9
wp 8													
seg 1	1,05	0,90	0,70	0,70	0,96	1	1	**130**		ir	117,9	91,7	91,7
seg 2	1,05	0,90	0,70	0,70	1,65	1,00	1	**63**	35	seg 2/ir	98,5	76,4	76,4
wp 13								193			216,5	168,1	168,1
wp 8													
seg 1	1,05	0,90	0,70	0,70	0,96	1,00	1	**167**	120	seg 1/ir	151,7	117,8	117,8
wp 14								167			151,7	117,8	117,8
wp 9													
seg 1	1,11	1,00	1,50	0,80	1,30	1,01	1,63	29			42,2	102,3	33,5
seg 2	1,43	1,10	1,65	0,90	1,00	1,01	1,63	53			84,0	203,6	68,1
seg 3	1,43	1,10	1,65	0,90	1,34	1,01	1,63	22			46,8	113,4	37,9
seg 4	1,05	0,90	0,70	0,70	0,98	1,01	1,63	**7**	35	seg 1-4	6,5	8,2	5,0
wp 11								111			179,5	427,5	144,6
wp 10													
seg 1	1,25	0,95	0,75	0,75	0,98	1,01	1	**14**			16,4	12,9	12,9
seg 2	5,00	0,90	0,90	0,90	1,00	1,01	1	**9**			40,9	40,5	40,5
seg 3	1,05	0,90	0,70	0,70	1,05	1,01	1	**173**	60	seg 1-3/ir	173,2	133,5	133,5
wp 19								196			230,5	186,9	186,9
wp 11													
seg 1	1,05	0,90	0,70	0,70	1,17	1,02	1,63	**9**			10,1	12,6	7,7
seg 2	1,43	1,10	1,65	0,90	1,60	1,02	1,63	17	5	seg 1-2	43,4	104,4	34,9
seg 3	1,11	0,80	0,95	0,75	0,96	1	1	42		ir	36,0	42,7	33,7
seg 4	1,11	1,00	1,50	0,80	0,96	1,00	1	12			12,8	19,1	10,2
seg 5	1,25	0,95	0,75	0,75	0,96	1,00	1	**41**			46,7	36,8	36,8
seg 6	5,00	0,90	0,90	0,90	1,00	1,00	1	**7**			31,6	31,5	31,5
seg 7	1,05	0,90	0,70	0,70	1,00	1,00	1	**54**			51,3	39,8	39,8
seg 8	1,05	0,90	0,70	0,70	1,30	1,00	1	**40**	94	seg 4-8/ir	49,3	38,3	38,3
wp 18								222			281,2	325,2	232,9
wp 12													
seg 1	1,11	0,80	0,95	0,75	1,00	1	1	41		ir	36,4	43,2	34,1
wp 15								41			36,4	43,2	34,1
wp 13													
seg 1	1,25	0,85	0,80	0,80	1,00	1	1	**110**		ir	116,9	110,0	110,0
seg 2	1,05	0,90	0,70	0,70	1,00	1	1	**232**		ir	219,2	170,5	170,5
wp 30								342			336,1	280,5	280,5
wp 14													
seg 1	1,25	0,95	0,75	0,75	1,00	1	1	**58**		ir	68,9	54,4	54,4
wp 17								58			68,9	54,4	54,4
wp 15													
seg 1	1,11	0,80	0,75	0,75	0,96	1	1	**44**		ir	37,5	35,2	35,2
seg 2	1,11	0,80	0,95	0,75	0,99	1	1	39		ir	34,3	40,7	32,1
wp 16								83			71,8	75,9	67,3
wp 15													
seg 1	1,11	0,80	0,75	0,75	1,00	1	1	**32**		ir	28,4	26,6	26,6
wp 17								32			28,4	26,6	26,6
wp 16													
seg 1	1,05	0,90	0,70	0,70	0,99	1,01	1	**41**			38,6	29,8	29,8
seg 2	1,05	0,90	0,70	0,70	1,36	1,01	1	**30**			38,8	30,0	30,0
seg 3	1,05	0,90	0,70	0,70	1,00	1,01	1	**27**	35	seg 1-3/ir	25,7	19,8	19,8
wp 22								98			103,1	79,7	79,7
wp 16													
seg 1	1,05	0,90	0,70	0,70	1,00	1	1	**95**			89,8	69,8	69,8
seg 2	1,05	0,90	0,70	0,70	0,96	1	1	**15**			13,6	10,6	10,6
seg 3	1,05	0,90	0,70	0,70	1,30	1	1	**17**			20,9	16,2	16,2
seg 4	1,05	0,90	0,70	0,70	1,00	1	1	**15**	142	seg 1-4/ir	14,2	11,0	11,0
wp 23								142			138,4	107,7	107,7

Spreadsheet 9.5. Costs of runs (cont.)

wp 17													
seg 1	1,11	0,80	0,95	0,75	1,00	1	1	98		ir	87,0	103,3	81,6
wp 21								98			87,0	103,3	81,6
wp 18													
seg 1	1,11	1,00	1,50	0,80	1,00	1,42	1,63	94	5	seg 1	148,5	255,3	83,6
seg 2	1,25	0,85	1,05	0,80	1,05	1	1	45		ir	50,0	61,8	47,1
wp 27								139			198,5	317,1	130,6
wp 19													
seg 1	1,05	0,80	0,90	0,75	1,30	1	1	32		ir	34,9	39,3	32,8
wp 20								32			34,9	39,3	32,8
wp 20													
seg 1	1,00	0,75	0,70	0,70	1,00	1	1	**151**		ir	113,3	105,7	105,7
wp 29								151			113,3	105,7	105,7
wp 21													
seg 1	1,05	0,80	0,90	0,75	1,00	1	1	94		ir	79,0	88,8	74,0
seg 2	1,05	0,80	0,90	0,75	0,98	1	1	41		ir	33,8	38,0	31,6
seg 3	1,11	0,80	0,95	0,75	1,00	1	1	32		ir	28,4	33,7	26,6
wp 24								167			141,1	160,5	132,3
wp 22													
seg 1	1,11	1,00	1,50	0,80	1,00	1,02	1,63	59	10	seg 1	66,8	160,1	52,4
seg 2	1,11	0,80	0,95	0,75	1,00	1	1	71		ir	63,0	74,9	59,1
wp 24								130			129,9	235,0	111,5
wp 23													
seg 1	1,05	0,80	0,90	0,75	1,00	1	1	39		ir	32,8	36,9	30,7
seg 2	1,11	0,80	0,95	0,75	1,00	1	1	113		ir	100,3	119,2	94,1
wp 24								152			133,1	156,0	124,8
wp 23													
seg 1	1,11	1,00	1,50	0,80	1,30	1,42	1,63	37			75,9	130,5	42,7
seg 2	1,11	1,00	1,50	0,80	0,99	1,42	1,63	52			81,2	139,7	45,7
seg 3	1,25	0,95	0,75	0,75	1,00	1,42	1,63	**27**			45,6	41,3	25,3
seg 4	1,11	1,00	1,50	0,80	1,05	1,42	1,63	74			122,6	210,8	69,0
seg 5	1,11	1,00	1,50	0,80	1,30	1,42	1,63	26	7	seg 1-5	53,3	91,7	30,0
seg 6	1,11	1,00	1,50	0,80	0,96	1	1	39		ir	41,6	62,3	33,2
seg 7	1,11	1,00	1,50	0,80	1,00	1	1	23		ir	25,5	38,3	20,4
wp 33								278			445,8	714,7	266,4
wp 24													
seg 1	1,11	1,00	1,50	0,80	1,05	1,42	1,63	65	5	seg 1	107,7	185,2	60,6
wp 25								65			107,7	185,2	60,6
wp 24													
seg 1	1,25	0,85	1,05	0,80	1,00	1	1	63		ir	66,9	82,7	63,0
seg 2	1,25	0,85	1,05	0,80	1,17	1	1	31		ir	38,5	47,6	36,3
seg 3	1,25	0,85	0,80	0,80	1,00	1	1	**34**		ir	36,1	34,0	34,0
seg 4	1,25	0,95	0,75	0,75	1,00	1	1	**29**		ir	34,4	27,2	27,2
wp 31								157			176,0	191,5	160,5
wp 25													
seg 1	1,11	1,00	1,50	0,80	1,00	1,06	1,63	32			37,7	86,8	28,4
seg 2	1,11	1,00	1,50	0,80	1,05	1,06	1,63	23	5	seg 1-2	28,4	65,5	21,4
wp 26								55			66,1	152,4	49,9
wp 26													
seg 1	1,11	1,00	1,50	0,80	0,99	1,42	1,63	21			32,8	56,4	18,5
seg 2	1,11	1,00	1,50	0,80	1,05	1,42	1,63	22			36,5	62,7	20,5
seg 3	1,11	1,00	1,50	0,80	1,60	1,42	1,63	63	7	seg 1-3	159,1	273,5	89,5
seg 4	1,11	0,80	0,95	0,75	0,96	1	1	39		ir	33,2	39,5	31,2
seg 5	1,11	0,80	0,95	0,75	1,00	1	1	23		ir	20,4	24,3	19,1
wp 33								168			282,0	456,4	178,8

Spreadsheet 9.5. Costs of runs (cont.)

wp 27													
seg 1	1,25	0,85	1,05	0,80	0,98	1	1	55		ir	57,2	70,6	53,8
wp 28								55			57,2	70,6	53,8
wp 28													
seg 1	1,11	1,00	1,50	0,80	1,30	1,03	1,63	40	5	seg 1	59,5	141,1	46,2
seg 2	1,11	1,00	1,50	0,80	1,30	1	1	24		ir	34,6	51,9	27,7
seg 3	1,11	1,00	1,50	0,80	0,96	1	1	39		ir	41,7	62,6	33,4
seg 4	1,11	1,00	1,50	0,80	1,00	1	1	23		ir	25,6	38,3	20,4
wp 33								126			161,5	294,0	127,7
wp 29													
seg 1	1,11	0,80	0,95	0,75	0,96	1	1	31		ir	26,4	31,4	24,8
seg 2	1,11	1,00	1,50	0,80	0,98	1,02	1,63	30	5	seg 2	33,3	79,8	26,1
seg 3	1,11	0,80	0,95	0,75	1,00	1	1	16		ir	14,2	16,9	13,3
seg 4	1,11	0,80	0,95	0,75	1,30	1	1	24		ir	27,7	32,9	26,0
seg 5	1,11	0,80	0,95	0,75	0,96	1	1	39		ir	33,2	39,5	31,2
seg 6	1,11	0,80	0,95	0,75	1,00	1	1	23		ir	20,4	24,3	19,1
wp 33								163			155,3	224,7	140,5
wp 30													
seg 1	1,00	0,75	0,70	0,70	1,00	1	1	**205**		ir	153,8	143,5	143,5
wp 31								205			153,8	143,5	143,5
wp 31													
seg 1	1,00	0,75	0,80	0,70	1,00	1	1	136		ir	102,0	108,8	95,2
seg 2	1,11	1,00	1,50	0,80	1,17	1,01	1,63	15	5	seg 2	19,6	47,6	15,6
wp 32								151			121,6	156,4	110,8
wp 32													
seg 1	1,11	1,00	1,50	0,80	1,05	1,03	1,63	50	7	seg 1	59,8	142,5	46,6
seg 2	1,11	0,80	0,95	0,75	1,00	1	1	23		ir	20,4	24,3	19,1
wp 33								73			80,3	166,7	65,8

Spreadsheet 9.5. Costs of runs (cont.)

In the first column we define each run shown on the map (Figure 9.11.) by entering a succession of separate lines for the starting waypoint, each segment and the target waypoint.

The second column refers to the terrain specific resistance to locomotion (from the Spreadsheet 9.1.) for each segment. It is assessed from the type of area or line object type shown on the map. It has the same value for all three tools.

The third, the fourth and the fifth column represent the navigation risk for each tool separately. First, the object type is interpreted from the map, then its navigation risk is found in the Spreadsheet 9.1.

The sixth column contains the slope friction for each segment, assessed from the contours and the direction of the optimum path on the map, where the friction value is taken from the anisotropic slope friction Spreadsheet 9.3. The value is the same for all three tools.

So far, all risks and frictions referred to a single segment. The columns from the seventh to the eleventh refer to the combined dead reckoning and waypoint discernibility risk (Spreadsheet 9.4.), which can be related to a single segment, or to any succession of them within the same run. For some segments this risk can even not be applicable. This risk does not apply for the GNSS receiver with a screen map, so its value is exactly "1" everywhere.

To calculate this risk, we first measure the distance of each segment on the digital map and sum the distances for each run. We express the measurements in meters on the terrain. The distances for open area segments are written in bold numbers for later reference.

Then we assess which sequence of segments is navigated by dead reckoning (DR), and what is the radius of waypoint (WP) discernibility at the end of section. From the radius and from the sum of relevant segment distances, we then calculate the risk with the aid of the Spreadsheet

9.4. (see the title 'Individual calculation of risk' in the spreadsheet). The deviation angle of 5º for the map and compass case, and the angle of 20º for use of the GPS receiver with a screen map in a forest, were used. The coefficient of overshoot of 0,20 (ie. 20%) has been chosen. The resulting risk pertains to each relevant segment in the sequence.

When we navigate along footpaths, fences, sharp vegetation edges, electricity lines, or within open corridors in the forest, we do not use the dead reckoning technique, so the computation of the dead reckoning risk is not relevant. Such segments are marked with "ir" (irrelevant) in the "DR distance" column. The same holds for the use of the GPS receiver on open terrain. When sections are marked as "seg *i-j*/ir", the first part refers to the navigation with map and compass, and the second to the navigation with a GPS receiver. For each "ir" segment the value of dead reckoning risk is exactly "1".

9.5.4.7. Computation of cost for segments and runs

Finally, the last three columns of the Spreadsheet 9.5. represent the cost of each segment and run, for each tool being compared. The cost of travel along a segment is computed from the distance of the segment, multiplied by the values of all relevant risks and frictions. The total factor is the multiplication of all risks and frictions for a segment. For navigation with map and compass the cost is:

*CostSegMC = DistanceSeg * ResistToLoco * NavigRiskMC * SlopeFriction * DrWpRiskMC*

For a GPS receiver with screen map the cost is:

*CostSegGPS = DistanceSeg * ResistToLoco * NavigRiskGPS * SlopeFriction * DrWpRiskGPS*

For a GNSS receiver with screen map the cost is:

*CostSegGNSS = DistanceSeg * ResistToLoco * NavigRiskGNSS * SlopeFriction*

In this way, the costs are determined segment by segment for all the tools, then separately added together into costs of the run in the same three columns as:

$$CostRunMC = \Sigma\ CostSegMC$$
$$CostRunGPS = \Sigma\ CostSegGPS$$
$$CostRunGNSS = \Sigma\ CostSegGNSS$$

and for the distance:

$$DistanceRun = \Sigma\ DistanceSeg$$

9.5.4.8. Computation of cost for optimum paths

The final spreadsheet contains the costs of runs sequentially combined from origin to destination into several versions of optimum paths (Spreadsheet 9.6.). Each path is marked by capital letter and descriptive name. All reasonable combinations of runs indicated with the waypoint numbering were calculated. The values for distances and costs of the runs were taken from the previous Spreadsheet 9.5. and simply summed together to get the total distance and the cost for each optimum path.

$$DistanceOptPath = \Sigma \, DistanceRun$$
$$CostOptPathMC = \Sigma \, CostRunMC$$
$$CostOptPathGPS = \Sigma \, CostRunGPS$$
$$CostOptPathGNSS = \Sigma \, CostRunGNSS$$

Three additional columns show the average total factor for every tool, and for each run and optimum path, as:

$$AverageFactorRunMC = CostRunMC \; / \; DistanceRun$$
$$AverageFactorRunGPS = CostRunGPS \; / \; DistanceRun$$
$$AverageFactorRunGNSS = CostRunGNSS \; / \; DistanceRun$$

$$AverageFactorOptPathMC = CostOptPathMC \; / \; DistanceOptPath$$
$$AverageFactorOptPathGPS = CostOptPathGPS \; / \; DistanceOptPath$$
$$AverageFactorOptPathGNSS = CostOptPathGNSS \; / \; DistanceOptPath$$

COSTS OF OPTIMUM PATHS

A - THE MOST OPEN PATH

RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 3	90	101,3	244,2	79,9	1,13	2,71	0,89
wp 3 - wp 10	446	418,5	325,5	325,5	0,94	0,73	0,73
wp 10 - wp 19	196	230,5	186,9	186,9	1,18	0,95	0,95
wp 19 - wp 20	32	34,9	39,3	32,8	1,09	1,23	1,03
wp 20 - wp 29	151	113,3	105,7	105,7	0,75	0,70	0,70
wp 29 - wp 33	163	155,3	224,7	140,5	0,95	1,38	0,86
Optimum path A	1078	1053,8	1126,3	871,3	0,98	1,04	0,81

B - THE SHORT AND DEMANDING PATH

RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 5	108	135,0	309,5	105,6	1,25	2,87	0,98
wp 5 - wp 9	86	182,2	277,6	148,3	2,12	3,23	1,72
wp 9 - wp 11	111	179,5	427,5	144,6	1,62	3,85	1,30
wp 11 - wp 18	222	281,2	325,2	232,9	1,27	1,46	1,05
wp 18 - wp 27	139	198,5	317,1	130,6	1,43	2,28	0,94
wp 27 - wp 28	55	57,2	70,6	53,8	1,04	1,28	0,98
wp 28 - wp 33	126	161,5	294,0	127,7	1,28	2,33	1,01
Optimum path B	847	1195,1	2021,5	943,5	1,41	2,39	1,11

C - THE SHORTEST AND THE MOST STRAIGHT PATH

RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 23	484	628,4	1351,7	503,8	1,30	2,79	1,04
wp 23 - wp 33	278	445,8	714,7	266,4	1,60	2,57	0,96
Optimum path C	762	1074,2	2066,4	770,2	1,41	2,71	1,01

D - THE STRAIGHT PATH WITH A SAFE FINISH

RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 23	484	628,4	1351,7	503,8	1,30	2,79	1,04
wp 23 - wp 24	152	133,1	156,0	124,8	0,88	1,03	0,82
wp 24 - wp 31	157	176,0	191,5	160,5	1,12	1,22	1,02
wp 31 - wp 32	151	121,6	156,4	110,8	0,81	1,04	0,73
wp 32 - wp 33	73	80,3	166,7	65,8	1,10	2,28	0,90
Optimum path D	1017	1139,4	2022,3	965,7	1,12	1,99	0,95

E - THE SHORT PATH WITH A RISKY FINISH

RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 2	55	69,0	166,6	54,5	1,25	3,03	0,99
wp 2 - wp 4	43	40,6	31,6	31,6	0,94	0,73	0,73
wp 4 - wp 6	86	79,5	89,5	74,6	0,92	1,04	0,87
wp 6 - wp 12	172	167,4	214,8	144,4	0,97	1,25	0,84
wp 12 - wp 15	41	36,4	43,2	34,1	0,89	1,05	0,83
wp 15 - wp 16	83	71,8	75,9	67,3	0,87	0,91	0,81
wp 16 - wp 23	142	138,4	107,7	107,7	0,97	0,76	0,76
wp 23 - wp 33	278	445,8	714,7	266,4	1,60	2,57	0,96
Optimum path E	900	1048,9	1444,0	780,6	1,17	1,60	0,87

F - THE SHORTEST PATH ALONG FOOTPATHS

RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 2	55	69,0	166,6	54,5	1,25	3,03	0,99
wp 2 - wp 4	43	40,6	31,6	31,6	0,94	0,73	0,73
wp 4 - wp 6	86	79,5	89,5	74,6	0,92	1,04	0,87
wp 6 - wp 12	172	167,4	214,8	144,4	0,97	1,25	0,84
wp 12 - wp 15	41	36,4	43,2	34,1	0,89	1,05	0,83
wp 15 - wp 16	83	71,8	75,9	67,3	0,87	0,91	0,81
wp 16 - wp 22	98	103,1	79,7	79,7	1,05	0,81	0,81
wp 22 - wp 24	130	129,9	235,0	111,5	1,00	1,81	0,86
wp 24 - wp 31	157	176,0	191,5	160,5	1,12	1,22	1,02
wp 31 - wp 32	151	121,6	156,4	110,8	0,81	1,04	0,73
wp 32 - wp 33	73	80,3	166,7	65,8	1,10	2,28	0,90
Optimum path F	1089	1075,6	1450,9	934,8	0,99	1,33	0,86

Spreadsheet 9.6. Costs of optimum paths

G - THE SAFE PATH WITH A RISKY FINISH							
RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 2	55	69,0	166,6	54,5	1,25	3,03	0,99
wp 2 - wp 4	43	40,6	31,6	31,6	0,94	0,73	0,73
wp 4 - wp 6	86	79,5	89,5	74,6	0,92	1,04	0,87
wp 6 - wp 12	172	167,4	214,8	144,4	0,97	1,25	0,84
wp 12 - wp 15	41	36,4	43,2	34,1	0,89	1,05	0,83
wp 15 - wp 17	32	28,4	26,6	26,6	0,89	0,83	0,83
wp 17 - wp 21	98	87,0	103,3	81,6	0,89	1,05	0,83
wp 21 - wp 24	167	141,1	160,5	132,3	0,84	0,96	0,79
wp 24 - wp 25	65	107,7	185,2	60,6	1,66	2,85	0,93
wp 25 - wp 26	55	66,1	152,4	49,9	1,20	2,77	0,91
wp 26 - wp 33	168	282,0	456,4	178,8	1,68	2,72	1,06
Optimum path G	982	1105,2	1630,1	869,0	1,13	1,66	0,88

H - THE SAFE PATH WITH A SAFE FINISH							
RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 2	55	69,0	166,6	54,5	1,25	3,03	0,99
wp 2 - wp 4	43	40,6	31,6	31,6	0,94	0,73	0,73
wp 4 - wp 6	86	79,5	89,5	74,6	0,92	1,04	0,87
wp 6 - wp 12	172	167,4	214,8	144,4	0,97	1,25	0,84
wp 12 - wp 15	41	36,4	43,2	34,1	0,89	1,05	0,83
wp 15 - wp 17	32	28,4	26,6	26,6	0,89	0,83	0,83
wp 17 - wp 21	98	87,0	103,3	81,6	0,89	1,05	0,83
wp 21 - wp 24	167	141,1	160,5	132,3	0,84	0,96	0,79
wp 24 - wp 31	157	176,0	191,5	160,5	1,12	1,22	1,02
wp 31 - wp 32	151	121,6	156,4	110,8	0,81	1,04	0,73
wp 32 - wp 33	73	80,3	166,7	65,8	1,10	2,28	0,90
Optimum path H	1075	1027,3	1350,7	916,8	0,96	1,26	0,85

I - THE PATH WITH AN EASY DETOUR AND A RISKY FINISH							
RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 2	55	69,0	166,6	54,5	1,25	3,03	0,99
wp 2 - wp 4	43	40,6	31,6	31,6	0,94	0,73	0,73
wp 4 - wp 6	86	79,5	89,5	74,6	0,92	1,04	0,87
wp 6 - wp 7	82	68,9	77,5	64,6	0,84	0,95	0,79
wp 7 - wp 8	32	26,6	29,9	24,9	0,83	0,93	0,78
wp 8 - wp 14	167	151,7	117,8	117,8	0,91	0,71	0,71
wp 14 - wp 17	58	68,9	54,4	54,5	1,19	0,94	0,94
wp 17 - wp 21	98	87,0	103,3	81,6	0,89	1,05	0,83
wp 21 - wp 24	167	141,1	160,5	132,3	0,84	0,96	0,79
wp 24 - wp 25	65	107,7	185,2	60,6	1,66	2,85	0,93
wp 25 - wp 26	55	66,1	152,4	49,9	1,20	2,77	0,91
wp 26 - wp 33	168	282,0	456,4	178,8	1,68	2,72	1,06
Optimum path I	1076	1189,1	1625,1	925,7	1,11	1,51	0,86

J - THE PATH WITH AN EASY DETOUR AND WITH MANY FOOTPATHS							
RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 2	55	69,0	166,6	54,5	1,25	3,03	0,99
wp 2 - wp 4	43	40,6	31,6	31,6	0,94	0,73	0,73
wp 4 - wp 6	86	79,5	89,5	74,6	0,92	1,04	0,87
wp 6 - wp 7	82	68,9	77,5	64,6	0,84	0,95	0,79
wp 7 - wp 8	32	26,6	29,9	24,9	0,83	0,93	0,78
wp 8 - wp 14	167	151,7	117,8	117,8	0,91	0,71	0,71
wp 14 - wp 17	58	68,9	54,4	54,5	1,19	0,94	0,94
wp 17 - wp 21	98	87,0	103,3	81,6	0,89	1,05	0,83
wp 21 - wp 24	167	141,1	160,5	132,3	0,84	0,96	0,79
wp 24 - wp 31	157	176,0	191,5	160,5	1,12	1,22	1,02
wp 31 - wp 32	151	121,6	156,4	110,8	0,81	1,04	0,73
wp 32 - wp 33	73	80,3	166,7	65,8	1,10	2,28	0,90
Optimum path J	1169	1111,2	1345,7	973,5	0,95	1,15	0,83

K - THE LONGEST AND THE SAFEST PATH							
RUN	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
wp 1 - wp 2	55	69,0	166,6	54,5	1,25	3,03	0,99
wp 2 - wp 4	43	40,6	31,6	31,6	0,94	0,73	0,73
wp 4 - wp 6	86	79,5	89,5	74,6	0,92	1,04	0,87
wp 6 - wp 7	82	68,9	77,5	64,6	0,84	0,95	0,79

Spreadsheet 9.6. Costs of optimum paths (cont.)

9.6. Results of simulation and analysis

The result of computation are two spreadsheets with the costs of optimum paths sorted alphabetically and by the time of travel. Both are used to infer another two spreadsheets with the general characteristics of risks and strategies.

9.6.1. Optimum paths sorted by alphabetic order

The first spreadsheet with the results presents the summary of cost computations (Spredsheet 9.7.). Eleven optimum paths are ranked alphabetically from the path A to the path K. For each optimum path the following data are shown:

- the descriptive name,
- percentage of the distance on the open area (from 18% to 74%),
- the number of runs (from 2 to 12),
- the total distance (from 762 m to 1262 m),
- the cost for each of the three tools (from 770,2 to 2066,4),
- the average total factor for each of the three tools (from 0,81 to 2,71).

OPTIMUM PATHS IN ALPHABETIC ORDER									
OPTIMUM PATH	Open (%)	Runs	Distance (m)	Cost M&C	Cost GPS	Cost GNSS	Average fac.-M&C	Average fac.-GPS	Average fac.-GNSS
A - the most open	74	6	1078	1053,8	1126,3	871,3	0,98	1,04	0,81
B - short and demanding	21	7	847	1195,1	2021,5	943,5	1,41	2,39	1,11
C - the shortest and the most straight	19	2	762	1074,2	2066,4	770,2	1,41	2,71	1,01
D - straight with a safe finish	18	5	1017	1139,4	2022,3	965,7	1,12	1,99	0,95
E - short with a risky finish	43	8	900	1048,9	1444,0	780,6	1,17	1,60	0,87
F - the shortest along footpaths	35	11	1089	1075,6	1450,9	934,8	0,99	1,33	0,86
G - safe with a risky finish	21	11	982	1105,2	1630,1	869,0	1,13	1,66	0,88
H - safe with a safe finish	25	11	1075	1027,3	1350,7	916,8	0,96	1,26	0,85
I - easy detour and a risky finish	25	12	1076	1189,1	1625,1	925,7	1,11	1,51	0,86
J - easy detour and many footpaths	28	12	1169	1111,2	1345,7	973,5	0,95	1,15	0,83
K - the longest and the safest	62	10	1262	1192,9	1310,3	1018,9	0,95	1,04	0,81

Spreadsheet 9.7. Optimum paths in alphabetic order

Since the cost is by definition a function of distance, frictions and risks, the descriptive names reflect the distances (with adjectives: shortest, straight, longest), and the risks (with adjectives: safe, open, easy, demanding, risky). Some of the paths have notable characteristics, which have to be exposed before the analysis:

- Two paths, A and K, detour the forest in the middle of navigation area. They are safe, long, and run mostly on open area (74% and 62%, respectively).
- The entirely trackless path C is the most straight, the shortest, and has only two runs, thus only one intermediate waypoint. This would be the optimum path for experienced orienteers.
- The path E is short, runs 43% on open area, but has a risky trackless finish.
- The path F follows mainly footpaths in the shortest possible combination. 35% of the path passes open area.
- Other paths pass mostly through the forest. They are different compromises between distance, frictions and risks.

9.6.2. Optimum paths sorted by time

The second spreadsheet with the results shows the optimum paths sorted by the time of travel (Spreadsheet 9.8.). It includes the calculated time expressed in minutes and seconds, provided by the equation:

$$Time = Distance * AverageTotalFactor * Pace$$

The normal (or the average) value of *Pace* is given in the row above the spreadsheet for an average navigator. We usually choose values between 6 and 7 min/km, as practiced in normally passable forest. For the sake of comparison, we note that the average running pace on a flat and smooth surface is between 4 and 5 min/km, the jogging pace is 6 min/km, the slow running pace is 7 min/km, and the hiking (walking) pace is about 10 to 12 min/km (or 5 km/h).

OPTIMUM PATHS SORTED BY TIME							
Normal pace (min/km)	7						
OPTIMUM PATH	Tool	Open (%)	Cost	Average factor	Distance (m)	Time (min	sec)
C - the shortest and the most straight	GNSS	19	770,2	1,01	762	5	23
E - short with a risky finish	GNSS	43	780,6	0,87	900	5	28
G - safe with a risky finish	GNSS	21	869,0	0,88	982	6	5
A - the most open	GNSS	74	871,3	0,81	1078	6	6
H - safe with a safe finish	GNSS	25	916,8	0,85	1075	6	25
I - easy detour and a risky finish	GNSS	25	925,7	0,86	1076	6	29
F - the shortest along footpaths	GNSS	35	934,8	0,86	1089	6	33
B - short and demanding	GNSS	21	943,5	1,11	847	6	36
D - straight with a safe finish	GNSS	18	965,7	0,95	1017	6	46
J - easy detour and many footpaths	GNSS	28	973,5	0,83	1169	6	49
K - the longest and the safest	GNSS	62	1018,9	0,81	1262	7	8
H - safe with a safe finish	M&C	25	1027,3	0,96	1075	7	11
E - short with a risky finish	M&C	43	1048,9	1,17	900	7	21
A - the most open	M&C	74	1053,8	0,98	1078	7	23
C - the shortest and the most straight	M&C	19	1074,2	1,41	762	7	31
F - the shortest along footpaths	M&C	35	1075,6	0,99	1089	7	32
G - safe with a risky finish	M&C	21	1105,2	1,13	982	7	44
J - easy detour and many footpaths	M&C	28	1111,2	0,95	1169	7	47
A - the most open	GPS	74	1126,3	1,04	1078	7	53
D - straight with a safe finish	M&C	18	1139,4	1,12	1017	7	59
I - easy detour and a risky finish	M&C	25	1189,1	1,11	1076	8	19
K - the longest and the safest	M&C	62	1192,9	0,95	1262	8	21
B - short and demanding	M&C	21	1195,1	1,41	847	8	22
K - the longest and the safest	GPS	62	1310,3	1,04	1262	9	10
J - easy detour and many footpaths	GPS	28	1345,7	1,15	1169	9	25
H - safe with a safe finish	GPS	25	1350,7	1,26	1075	9	27
E - short with a risky finish	GPS	43	1444,0	1,60	900	10	6
F - the shortest along footpaths	GPS	35	1450,9	1,33	1089	10	9
I - easy detour and a risky finish	GPS	25	1625,1	1,51	1076	11	23
G - safe with a risky finish	GPS	21	1630,1	1,66	982	11	25
B - short and demanding	GPS	21	2021,5	2,39	847	14	9
D - straight with a safe finish	GPS	18	2022,3	1,99	1017	14	9
C - the shortest and the most straight	GPS	19	2066,4	2,71	762	14	28

Spreadsheet 9.8. Optimum paths sorted by time

Each path appears in three different rows for M&C, GPS and GNSS. We observe:

- The times are quite realistic, so the risks and frictions have been assessed realistically, too.
- All paths with GNSS have lower cost and time than the others, since all navigation risks with GNSS are much lower then with GPS and M&C (see the upper group in the spreadsheet).
- All paths with GPS, except the most open path A, have higher cost and time than the others, since GPS has very high navigation risks in a forest (see the lower group in the spreadsheet, and the shaded path A with GPS in the middle group). Second best path with GPS is the longest and the safest path K, which also predominantly lies on open area.
- Likewise, the average total factors are generally the lowest for GNSS, and the highest for GPS.
- The path with the minimum cost and time is the path C with GNSS, since this is the shortest path, and since GNSS works also in a forest.
- The path with the maximum cost is the path C with GPS, since GPS has very high navigation risks in a forest. The end of the sorted list occupy the paths with GPS in predominantly forest area.
- The cost with GPS in a forest is much higher than for M&C, while the cost with M&C on open is only a bit higher than for GPS, so the cost with GPS is in general higher than for M&C.
- The costs with GPS and M&C depend on the amount of forest areas. The costs for GNSS are only a compromise between distance, risks and frictions regardless of the amount of forests.

From the risk and friction Spreadsheets 9.1., 9.3. and 9.4., we observe that the frictions do not depend on the tools. According to the cost and risk characteristics, we can formally write:

For a chosen path on open area, it is always true:

$$CostOptPathGNSS = CostOptPathGPS$$
$$CostOptPathGNSS < CostOptPathMC$$
$$CostOptPathGPS < CostOptPathMC$$

For a chosen path on forest area, it is always true:

$$CostOptPathGNSS < CostOptPathGPS$$
$$CostOptPathGNSS < CostOptPathMC$$
$$CostOptPathGPS > CostOptPathMC$$

Therefore, for a chosen path on any type of terrain, it is always true:

$$CostOptPathGNSS \leq CostOptPathGPS$$
$$CostOptPathGNSS < CostOptPathMC$$

9.6.3. General characteristics of risks

It can be seen from the equations for cost in the chapter 9.5.4.7., that the costs for tools differentiate only in the navigation risk, and in the combined dead reckoning and waypoint discernibility risk values. For a chosen segment, the resistance to locomotion and the slope friction are the same for all tools. Therefore, the environmental fritions do not make the difference between the costs of a single optimum path travelled by the three tools. They make the difference only between the costs of different optimum paths, and contribute to more realistic calculations.

The GNSS tool does not suffer from dead reckoning (ie. the risk value is always 1). The GPS tool does not suffer from it on open terrain and on footpaths, meanwhile in the pathless forest it is very high. With the aid of risk Spreadsheets 9.1. and 9.4., we generally compare the risks for the tools on open area and in a forest (Table 9.1.).

GENERAL CHARACTERISTICS OF RISKS

	Map and compass	GPS and screen map	GNSS and screen map
Open area	**LOW** navigation risk **LOW** DR & WP risk	**VERY LOW** navigation risk **NO** DR & WP risk	**VERY LOW** navigation risk **NO** DR & WP risk
Forest area	**MEDIUM** navigation risk **HIGH** DR & WP risk	**VERY HIGH** navigation risk **VERY HIGH** DR & WP risk	**LOW** navigation risk **NO** DR & WP risk

Table 9.1. General characteristics of risks

9.6.4. General characteristics of strategies

We can also draw conclusions about the general riskiness of navigation with different tools on open area and in a forest (Table 9.2.). This table is a general picture of what has been described in the detailed discussion of strategies for each tool. Each table cell shows a general risk, followed by underlined prevailing strategy of navigation. Supplementary or backup strategy is quoted after this.

GENERAL CHARACTERISTICS OF STRATEGIES

	Map and compass	GPS and screen map	GNSS and screen map
Open area	**LOW RISK** <u>Dead reckoning</u> Feature matching Distal landmarks	**VERY LOW RISK** <u>Positioning</u> Direction to waypoint Feature matching	**VERY LOW RISK** <u>Positioning</u> Direction to waypoint Feature matching
Forest area	**MEDIUM RISK** <u>Dead reckoning</u> Feature matching	**VERY HIGH RISK** <u>Feature matching</u> Distance estimation	**VERY LOW RISK** <u>Positioning</u> Direction to waypoint Feature matching

Table 9.2. General characteristics of strategies

We observe that direct positioning with GNSS and GPS has very low risk, when it works. On the other side, map and compass navigation uses more risky dead reckoning aided by feature matching. This is in fact an indirect positioning. The highest risk has sole feature matching with GPS in a forest, ie. when we use only a screen map for indirect positioning. Both Tables 9.1. and 9.2. also explain the behaviour of costs with different tools in the Spreadsheet 9.8.

The table shows that the risks are systematically bound to the strategies with certain tools. Risks potentially develop into errors according to what errors are afforded by the tool. So the navigation risk and the dead reckoning risk stem from the error characteristics of the tools.

Tools essentially differ in their affordance of potential errors. According to the potential or actual performance of the tool within certain environmental circumstances, the appropriate strategy is chosen.

9.7. Demonstration of the hypothesis

From the results of simulation we demonstrate the hypothesis by substitution technique as follows.

First we assumed that we navigate in a natural environment with three different tools: the map and compass, the GPS receiver with a screen map, and the GNSS receiver with a screen map. We suppose that the navigator knows how to use the tools. The orienteering map at the scale 1:10.000 is used in each case, as it provides all necessary information for precise navigation. In the chapter 9.4.5. we assumed that the optimum path condition for the simulation is the fastest path:

***OptimumPathCondition* = 'fastest path'**

In the chapter 9.2. we showed that:

*Time = Cost / Speed = Cost * Pace*

therefore:

Time = f (*Cost*,*Pace*)

In the chapter 3.5.3. we described the fastest path, which requires that the travel is executed in the minimum time. If the specified optimum path condition is valid, then:

***Time* = min (*Cost*,*Pace*)**

In the chapter 9.4.7. we assumed that the navigator has average navigation skills and physical abilities, so his pace is average, hence:

***Pace* = average (*Navigator*)**

Since the time has to be minimum, and the pace is constant throughout the simulation (ie. 7 min/km), the optimum path is characterized by the minimum cost. In the chapter 9.2. we described the method for calculation of the cost. We showed that the cost is computed by the multiplication of distance with the applicable frictions and risks:

*Cost = Distance * Friction1 * ... * FrictionN * Risk1 * ... * RiskN*

which we can shorten to:

*Cost = Distance * Frictions * Risks*

Therefore we can generally write:

***Cost* = f (*Distance*,*Frictions*,*Risks*)**

With the theoretical and practical study of navigation in the chapter 5.5.2., we summarized in the chapter 9.4.4. that the optimum path is physically fixed by the origin, the destination and the waypoints:

***Distance* = f (*Origin*,*Destination*,*Waypoints*)**

In the chapter 9.5.4. we described the execution of simulation, where in the subchapters 9.5.4.2., 9.5.4.3., and 9.5.4.4., we explained how origin, destination, waypoints, and the optimum paths in between were cognitively defined on the orienteering map (Figure 9.11.). The distance was measured by digital cartometry along the chosen optimum paths.

In the chapters 9.5.4.6. and 9.5.4.7., where we described the execution of simulation, we showed that the frictions constitute of the compound effect of the resistance to locomotion, and the slope friction:

Frictions = f (*ResistToLoco*,*SlopeFriction*)

In the chapter 9.5.1. and subchapter 9.5.1.1., we constructed the Spreadsheet 9.1., where the resistance to locomotion is numerically and experientally assessed for each possible area and line object type shown on the map. According to the IOF mapping standard (IOF 2000), each object type allows to locomote by some share of the maximum affordable speed or pace, specified in per cents. Therefore, this friction type lowers the average pace of the navigator.

In the chapter 9.5.2. we provided the method of calculation for the slope friction. In the subchapter 9.5.2.2. we first showed, that the actual speed or pace of the navigator travelling directly upslope or downslope depends on the inclination of the slope. We used the hiking function by Tobler (1993), which was derived from the experiental diagram of hiking time by Imhof (1968).

In the subchapter 9.5.2.3. we showed that the anisotropic slope friction of askew passage depends on the direction of passage across the slope. We used the experiental equations by Eastman (2003). The reference slope friction values were calculated for different slope magnitudes (inclinations) and directions of passage in the Spreadsheet 9.3. The slope friction lowers or rises the average pace of the navigator.

To calculate the compound effect of both frictions, we multiply:

Frictions = *ResistToLoco* * *SlopeFriction*

From the explanation above we observe, that the frictions are related only to environmental conditions. When we travel along the chosen optimum path with a tool, they stay the same with each tool. They do not depend on the tool and the strategy of navigation, therefore:

***Frictions* = f (*Environment*)**

The actual effect of frictions in the simulation case was described in the subchapters 9.5.4.6. and 9.5.4.7., and taken into account in the Spreadsheet 9.5. They represent a portion of the cost of each optimum path.

As shown in the equations above, the cost is influenced also by the risks. They are represented by the navigation risk and by the combined dead reckoning and waypoint discernibility risk:

Risks = f (*NavigRisk*,*DrWpRisk*)

In the chapter 9.5.1. we described the navigation risk in general, while in subchapters 9.5.1.2., 9.5.1.3., and 9.5.1.4., we explained it for each type of tool separately. The navigation risk depends on the visibility through vegetation, and on the geomorphologic complexity of the terrain surface, therefore on the same area and line object types as the resistance to locomotion does. However, the risk can differ for each tool and strategy of navigation, since eg. satellite navigation with GPS depends on the quality of the received satellite signals. The reference values of the navigation risk were represented within the Spreadsheet 9.1. separately for each tool.

In the chapter 9.5.3. we described the combined dead reckoning and waypoint discernibility risk. The dead reckoning risk occurs when the navigator relies too long on the distance and direction estimation. If he deviates from the direction of the waypoint, which can be indiscernible from afar, the travelled path is prolonged. In the description of different cases of deviation, we showed with analytically derived experiental equations, that the combined risk depends on the distance between waypoints, the waypoint discernibility radius, and the potential deviation angle from the true direction. While the first two variables depend on the choice of the available waypoints, the deviation angle depends on the navigation tool and strategy. The dead reckoning and waypoint discernibility risk was computed with the aid of the Spreadsheet 9.4. in the chapter 9.5.3.8.

To calculate the effect of all risks, we multiply:

$$Risks = NavigRisk * DrWpRisk$$

In the execution of simulation, they are taken into account in the Spreadsheet 9.5. (subchapters 9.5.4.6. and 9.5.4.7.). If a particular risk is not applicable for some tool, it has the value of 1. The risks also represent a portion of the cost of each optimum path:

$$Cost = Distance * ResistToLoco * SlopeFriction * NavigRisk * DrWpRisk$$

From the inferencing above, we can further develop the following interesting conclusions. Since the pace is constant, we can write:

$$Time = \min (Cost, Pace) \Rightarrow Time = \min (Cost)$$

For a chosen optimum path, the distance and the frictions are constant for all tools, therefore:

$$Cost = \min (Distance, Frictions, Risks) \Rightarrow Cost = \min (Risks)$$

and consequently:

$$Cost = \min (Risks) \Rightarrow Time = \min (Risks)$$

From the discussion of variables in both types of risks, we have already concluded, that the risks along a particular optimum path depend on the tool and the strategy of navigation. To support this conclusion, we refer again to the numerically simulated navigation with the three tool types along eleven optimum paths. We assessed first the risks and frictions per segment, and we calculated the costs of each segment and run in the Spreadsheet 9.5. In the Spreadsheet 9.6. we calculated the costs of optimum paths. The results were summarized in the Spreadsheet 9.7. and sorted by time in the Spreadsheet 9.8. The general conclusion as a consequence of the use of different tools and strategies was:

$$CostOptPathGNSS \leq CostOptPathGPS$$
$$CostOptPathGNSS < CostOptPathMC$$

In the Table 9.1. (chapter 9.6.3.), we additionally evaluated the navigation risk, and the dead reckoning and waypoint discernibility risk for all three types of tools. We found that the risks crucially differ whether the tools are used on the open area or in the forest.

To explain why they differ, we constructed the Table 9.2. (chapter 9.6.4.), where we retained the treatment of risks regarding the use of tools on the open and in the forest area. For each situation we indicated the amount of risk and the strategy of navigation used. The risks were qualitatively ranked from very low to very high. The prevailing strategies were generally termed as dead reckoning, positioning, and feature matching, while the backup or the supplementary strategies are feature matching, distal landmarks, direction to waypoint, and distance estimation.

From both tables we can confirm positively, that the risks change substantially, if we use different tools and strategies, so we can resume this inferencing with the formal statement:

***Risks* = f (*Tool*,*Strategy*)**

From the theoretical and practical considerations throughout the thesis, and from the Table 9.2., we observe, that certain strategies are more suitable for a particular tool than the others. Moreover, for the use of a particular tool we can choose between different strategies regarding the environmental conditions (eg. open vs. forest area). The unanswered question remains: Which factor influences the choice of a specific strategy with a particular tool?

In the main chapters 5., 6., and 8., we described in detail how explicit strategies are formed in the planning and execution cycles with different tools. Strategies are needed to avoid getting lost, ie. to avoid the errors of navigation. In the chapters 7. and 8.8., we constructed the lists of potential errors which can occur when we use a chosen tool and strategy. In the chapter 5.5.3. we described some explicit techniques to avoid the errors of navigation, eg. aiming off, navigation with "handrail", and navigation to catching feature.

To conclude inferencing, we assume that the navigator chooses some particular tool. The potential errors provided by the use of this particular tool influence the choice of appropriate strategy. The performance (or better, misperformance) of this tool provides the affordance for the occurence of errors. The navigator chooses the strategy which provokes less errors than any other strategy used with this tool, thus the chosen strategy minimizes the occurence of errors:

***Strategy* = min (*Errors*)**

If this condition is fulfilled, the navigator will not get lost. He will complete the travel along the optimum path in the minimum time, as required by the optimum path condition. The statement also approves that the errors afforded by the tool are central to the navigation process.

The sequence of statements above positively demonstrates the hypothesis:

The *error* characteristics of the *tool*, define the *strategy* of navigation.

For a short and concise demonstration of the hypothesis, we connect all variables, parameters, and equations, as described above, into a logical and functional dependence, according the theoretical and practical findings of the thesis. Finally, we argument the hypothesis with the following formal statement:

IF
    ***OptimumPathCondition* = 'fastest path'**
THEN
    ***Time* = min (*Cost*,*Pace*)**
WHERE
    ***Pace* = average (*Navigator*)**
    ***Cost* = f (*Distance*,*Frictions*,*Risks*)**
        WHERE
            ***Distance* = f (*Origin*,*Destination*,*Waypoints*)**
            ***Frictions* = f (*Environment*)**
            ***Risks* = f (*Tool*,*Strategy*)**
                WHERE
                    ***Strategy* = min (*Errors*)**

Practically, the demonstration of the hypothesis shows that the navigator always chooses the strategy that will potentially cause least errors of navigation with the chosen tool.

10. CONCLUSIONS

In the conclusion we discuss the results of the thesis and the possible future work within the scope of navigation strategies, tools and errors. Finally, we provide a short summary of the thesis.

10.1. Results of the thesis

The thesis has shown, that the inherent characteristics of the navigation tool define, which errors are afforded by the tool. The functionality of the tool influences which strategy of navigation is chosen in a certain navigation task.

The major result of the thesis is a unique numerical and formal connection of nearly all aspects of navigation. We have qualitatively and quantitatively explained:

- what do we have to use to navigate (tools),
- how do we navigate (strategy),
- how do we organize the navigation (origin, destination, waypoints),
- where do we navigate (environment),
- who is navigating (navigator),
- what are the challenges of navigation (distance, frictions, risks),
- what is the goal of navigation (optimum path condition),
- how do we evaluate the navigation (cost, pace, time),
- how do we fail in the navigation (errors).

This clearly shows how complex the navigation process in a real world is. Any simulation that empirically or analytically imitates human behaviour in a natural environment has potential risk to be oversimplified on one hand, or too fragmented into details on the other. Regarding the simulation, the scientific achievements of the thesis are the following:

- The cognitive science was coupled with the geographic information science, instead of studying navigation only from the technical standpoint. The orienteering sport discipline was described with a spatial and cognitive approach. In this way, a scientific explanation of otherwise commonsense and naive navigation knowledge of orienteering was given.
- The cognitive aspects of human behaviour were explained numerically and realistically, instead of only verbally and theoretically.
- The complex real-world environment was used for the explanation of human behaviour, instead of artificial, simplified, or laboratory environment.
- The complex navigation was decomposed into planning and execution, and both further to primitive actions, instead of observing the navigation as a single black box process.
- The natural environment was used for simulation, instead of widely studied urban environment.
- The errors of navigation were treated cognitively, instead of using the standard least-squares adjustment, or any other statistical theory.
- The simulation was executed with generalized presumptions about the navigator and the tools, without introducing technical details of the currently popular navigation tools.
- The least-cost path problem was numerically approached by a nonstandard procedure, which is more realistic than the existing commercial raster type algorithms.
- The simulation was executed with a unique and scientifically complex method, which incorporates the use of geocoded data, a formalized mapping standard, spatial reasoning and cognitive interpretation, a numerical evaluation of risks and frictions, anisotropy and nonhomogeneity of natural environment, empirical findings and equations,

orienteering sport rules, economic evaluation of the optimum path by the cost, and numerical solution to an implicit variational problem.

10.2. Future research

The navigation in a natural environment is a part of everyday life. It is widely publicized in popular textbooks about orientation, however it has not been reaserched adequately in the scientific literature. Many questions remain unanswered and many simulations could be performed for each aspect of navigation. We here propose and group by topic some further research questions envisioned as a possible continuation of this thesis. We start each topic with a heading statement which gives an affordance to raise new ideas.

Tools and strategies
Usually, several similar strategies of navigation could be used with the same tool. Which strategy is optimal with some type of tool? What kind of strategy and tool should we use for different optimum path conditions? Usually, the tools are invented before the navigation strategies are formed (eg. as in the case of compass, map, GPS). Can we rather invent the optimum strategy first, and then develop the adequate tool?

Tool functionalities
The tool provides affordances for the use of different strategies of navigation. What is the optimum list of functionalities to avoid errors with some type of technically augmented navigation device? What kind of user interface is required?

Cartography and navigation
From the cartographic point of view, there is a wide field of topics for investigation. How do we navigate with significantly more generalized maps, ie. in a situation of information deprivation? What would be the required quality, quantity, and cost of geolocated data for an automatic navigation in a natural environment? How could we enhance the functionalities of the topographic or the city map in a digital format to match some type of navigation strategy, or some type of task (eg. some location based service)? How would the change of map design influence the navigation? How do we navigate, if we omit from the map all information except the isolines and the landforms? How does the limited size of screen map influence the navigation and the perceptions of the navigator? How does the lack of all-at-once characteristic of a map affect the survey knowledge of navigation?

Navigation skills
We have proposed in the thesis, that the navigator knows how to use the tool. How can we measure and assess the lack of navigation skills? What kind of errors are afforded then? How much of the necessary navigation skills can be replaced by the advanced and upgraded tool?

Waypoints
Most navigation techniques use waypoints to aid the navigator. Usually, the waypoints are crucial for the success of navigation. How could we grade and classify the quality and salience of waypoints by numerical measures for their use with different tools? What kind of measurable effect has the introduction of knowledge in the world (eg. marked footpaths, sign posts) into a natural environment?

Preferences in navigation
We have shown that the optimum path choice is person-dependent. How do we measure behavioural patterns of different navigators travelling from a common origin to a common destination? What kind of preferences could be distinguished when choosing own optimum path version in a natural environment regarding the risk and friction avoidance?

Errors of navigation
In the research, we have constructed and evaluated the planned optimum paths, where we have intentionally avoided the errors. How can we measure and assess the executed paths where the errors have occured? How do we measure the negative effects of each individual error? How could we profit from field tests? How are the errors generated, ie. which error comes first and which are only a consequence of it? What would be the strategy, if we succeed to completely elliminate the possibility of some kind of error?

Measures of navigation quality
We have measured the quality of the planned optimum path in terms of cost and time. What are the other measures for the assessment of quality of navigation (planned, or executed)?

Environment
We have shown, that the orienteering in a natural environment is quite different from the orienteering in an urban environment. Which tool and strategy is optimal for some type of environment? What are the strategies of navigation indoors?

Visual access and differentiation
The navigation process is a sequence of view-action pairs. The views are sensed by the navigator and the actions are chosen by him. Can we classify the navigability of some type of environment, eg. by measuring the differentiation and visual access? Which are the methods of small-scale space evaluation for navigation? Can we use adapted viewshed analyses from GIS for this purpose?

Environmental conditions
The strategy depends also on the environment. The conditions in the environment can change. We have proposed in the thesis that we navigate at daylight, that the weather is normal, and the ground is dry. How the strategy, the errors and the costs change if we navigate in extreme conditions, eg. in a snowcovered forest, in the darkness, in the extreme temperatures?

Emotions and beliefs
There is a lack of cognitive research about the role of emotions and beliefs in navigation. How could we numerically assess emotional effects in navigation? What is the measurable effect of fear? How and when does the belief revision occur, and what are the consequences if the process fails?

Being lost
Few researches deal with the situations of becoming lost, and being lost. The strategies of relocation with different tools, especially with technically augmented ones, are also neglected. How do people react to the situation? What is the measurable role of emotions? How can we numerically assess different situations of being lost? What is the cost of rescue and relocation (eg. in terms of time)? What are the optimum ways of relocation when we get lost with some tool (or with a tool which fails to work, or without a tool)?

Navigator
We have also assumed in this thesis, that the navigator is an average healthy and adult person. How do we answer to the questions above, if the navigator is blind or physically handicapped? How do the children cope with orientation problems? What is the influence of gender and age?

Optimum path condition
Finally, we were investigating the case, where the optimum path condition is the fastest path. The different purposes of the travel provide numerous other strategies of path selection. In military operations the most hidden way may be desirable. Transport planners may want to to find the layout of a future road providing the least construction cost. In the summertime, shady path may be preferred. In touristic travels, the path must follow only maintained or marked

footpaths, where also a nice view and an interesting sightseeing objects can be visited, possibly in a circular itinerary. In mountaineering, steep or climbing path is preferred on ascension, while gentle slope is recommended for downhill. Here, an adequate portion of risk may even be desirable for a well trained person. All the questions quoted above could be repeated for every type of optimum path condition.

New scientific field?
Only a selection of ideas has been presented above. Evidently, the navigation has gained special attention in the era of digital spatial data, telecartography, positioning systems, location based services, and mobile communications. Should we raise the whole new scientific discipline called *ubiquitous navigation* (Diagram 10.1.)?

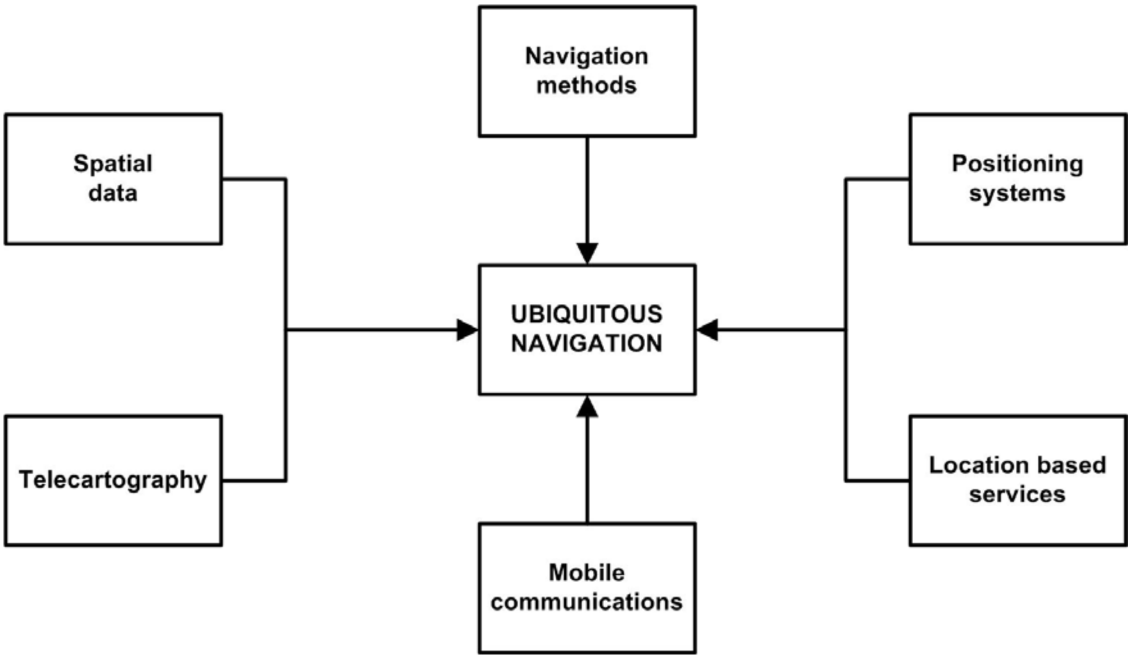


Diagram 10.1. Ubiquitous navigation

10.3. Summary

The goal of the thesis was to demonstrate the hypothesis, that the *error* characteristics of the navigation *tool* determine the *strategy* of navigation. The vector type of computation with a cognitive delineation of several optimum paths on an orienteering map was used to confirm this statement. The orienteering in a natural environment was chosen as the test navigation case. The technique of orienteering with map and compass was confronted with the positioning technique with a GPS receiver, and with a GNSS receiver which is hypothetically functioning everywhere. The numerically evaluated and calculated risks, frictions, and costs were compared, and the general conclusions which demonstrate the hypothesis, were formed.

The thesis starts with the definition of the basic terms which are used extensively throughout the thesis: topography, natural environment, map, cognitive map, locomotion, mobility, navigation, wayfinding, orientation, orienteering, waypoint, and, landmark. The navigation components (ie. position, distance, direction) are presented. The terms dead reckoning, path integration and updating are also explained as they are important for understanding of the topic. Since most of the errors in navigation are related to cognition, we describe the role of cognitive categories such as perceptions, senses, attention, experiences, knowledge, spatial abilities, navigation skills, mental imagery, affordances, and beliefs.

We usually use maps to navigate. Maps are symbolic representations of environment. They show a small-scale space perceived by the navigator from a certain standpoint, and a large-scale space behind the navigator's visual horizon. We also deal with other information sources: the commonsense geographic knowledge and the naive geography are presented to understand the behavioural patterns of human navigator. The role of knowledge in the world (like eg. informative signs) is also presented, however in a natural environment it is usually absent. Humans relate information and actions in space to the frames of reference. Their description is concluding the chapter about spatial cognition.

Then we introduce the idea of the optimum path and the least-cost path. We first start with the discusion of landscape differentiation, visual access, and spatial layout since the least-cost path depends on environmental parameters. Several types of least-cost paths are presented, including the most important, the fastest path, which is used in the simulation.

The next chapter describes the navigation with map and compass. Both tools are presented in detail: a simple orinteering compass, and a complex map. We compare the topographic and the orienteering maps as the levels of cartographic generalization are different. We introduce the technique of orienteering, and compare the orienteering in a natural environment and in a city. We add also the explanation of the principles of navigation with incomplete tools, since we use them later in the case when the GPS receiver fails to operate in a forest.

The strategic phase of navigation is a complex process, where we plan the forthcoming actions. We have to reaserch its connection with the tools and the errors of navigation. Therefore, we decompose the strategy into primitive actions in the framework of the sense-plan-act architecture. The optimum path is hierarchically divided into legs, runs, and segments, where legs as a part of the orienteering course lie between two control points, runs lie between two waypoints, and segments represent the chunks of optimum path with homogeneous friction and risk properties. We show the fundamental method of orientation with map and compass, which is used every time when the navigator wants to reassure his position and orientation. The planning of navigation is explained as a sequence of rough optimum path selection, definition of waypoints, and detailed navigation techniques.

The execution part of navigation is also decomposed into primitive actions, considering separately locomotion, updating of position and orientation, estimation of distances, and orientation procedures. The result is the executed optimum path. The detailed understanding of strategy and execution is fundamental for the research, as the planned and the executed paths are in principle different.

A separate treatment is dedicated to the errors in navigation, where we focus on the cognitive and physical background of errors, and not on the accuracy of navigation and position. We classify, study and quote the detailed errors of the tools, the planning, and the execution. We shortly analyse also the impact of physical ability, mental concentration, and emotions on navigation. The discussion about the consequences of errors, and about the relocation process is added.

To compare different tools and strategies, we describe another class of navigation technique: the technically augmented navigation, as an opposition to the classical orienteering with map and compass. Two devices are presented: a GPS receiver with a screen map, and a GNSS receiver with a screen map which is hypothetically functioning everywhere. The strategy, the planning, the execution, and the errors of the technically augmented navigation are analysed in a comparable way to the orienteering navigation case.

The most important part of the thesis is the simulation chapter, where we numerically demonstrate the hypothesis. We describe raster and vector approaches to simulation. The vector cognitive approach is then used. We take an orienteering map, we choose the origin and the destination of one single leg of the orienteering course, and we draw several optimum paths between both. Then we cognitively define the waypoints and segments along each of the different optimum paths. The optimum path condition requires, that the (fictive) navigator has to travel the distance in the shortest time.

We construct the spreadsheets of frictions and risks for each of the three tools. The values are either computed from empirical equations, or assessed by experiental reasoning and inferencing. The following frictions and risks are computed: the resistance to locomotion, the navigation risk, the anisotropic slope friction, and the dead reckoning and waypoint discernibility risk. From the distances, frictions, risks, and the average running pace of the navigator, we then compute the cost and time for each run, each optimum path, and each tool. We name each path by the prevailing characteristic, and sort them by the cost and time of travel. From the results we infer the general characteristics of risks and strategies regarding the three tools used on open areas and in a forest. The general strategy of navigation with map and compass is dead reckoning, aided by feature matching, while for the GPS and GNSS receiver cases the strategy is positioning aided by the display of straight direction to the next waypoint on a screen map.

In the conclusion, we summarize the results and the general observations which have been deduced from them. We observe, that the time of travel functionally depends on cost and pace, where the pace depends on the physical condition of the navigator, and the cost depends on distances, frictions and risks. The distance is influenced by the position of origin, destination and waypoints. The frictions depend on the environment, however the risks depend on the strategy, where the strategy depends on the tool. Finally, we conclude that the tool provides affordances for the emerging errors. The series of formal statments within an IF clause demonstrates the hypothesis.

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BIOGRAPHY OF THE AUTHOR

Dalibor Radovan was born in Ljubljana, Slovenia, on September 5, 1960. In 1979 he graduated with distinction at the secondary school Gimnasium Bežigrad, Ljubljana, with mathematical specialization. In 1984 he graduated with distinction from geodesy at the University of Ljubljana, Faculty of Architecture, Civil Engineering and Geodesy, with the thesis "Optimization of digital terrain model for the calculation of deflection of the vertical". In 1990 he received master degree with distinction in geodesy at the same faculty with the thesis "The problems of reagregation of spatial data in areal interpolation methods".

From 1984 on, he is employed at the Geodetic Institute of Slovenia. In 1992 he became the head of the Automated Cartography and GIS department. From 1995 to 2005, he was adviser to the director of the institute. From 2005 on, he is the head of research and development sector.

From 1990 he also holds a part time employment at the University of Ljubljana, Faculty for Civil Engineering and Geodesy, Department of Geodesy, as a university lecturer for Automated Cartography and Cartographic Projections. He was a supervisor or co-supervisor to 27 graduate students and one master od science. From 1984 he is a registered scientist at the Ministry of Higher Education, Science and Technology of Slovenia. He is also a regular reviewer for the projects of Slovenian Agency of Science.

In the Slovenian official cooperative bibliographic records (COBISS), his name has been entered in over 350 reference titles referring to projects, monographs, articles, lectures, etc. In the last decade he was the leading researcher or the coordinator in over hundred projects from the following fields of geomatics and geodesy:

- automated cartography - software development and database management,
- topographic, nautical, navigational (for LBS), thematic and tactile mapping,
- DTM production, quality control, modelling and software development,
- toponymic standardization and establishment of diverse databases and gazeteers,
- maritime hydrography and electronic navigation charting,
- numerical methods in cartography,
- education and training in geomatics, geodesy and cartography,
- 3D city and landscape modelling,
- higher geodesy - transition to a new national coordinate system (horizontal, vertical, gravimetric),
- GPS and GNSS - national GPS reference stations network and tide gauge installation,
- geomagnetism - development of national network of reference stations,
- digital land cadastre - metadata cataloguing, quality enhancement and digitizing of cadastral maps,
- geoinformation infrastructure - standardization, legal aspects and establishment,
- digital libraries and georeferenced multimedia,
- terminology and lexicography in GIS, cartography, toponymy and geodesy.

In the process of transition from classic to digital technology, many of these projects were pioneering works in Slovenia. Among them, he was the coordinator of the World Bank project (1998-2000) for the establishment of the Training Centre for Geomatics in Slovenia, which still permanently adresses geomatics experts and all registered surveyors in the state.

In 1996 he was leading the project for establishment of the national strategy of official topographic and cartographic system of Slovenia. He is a member of the Commission for Standardization of Geographical Names at the Government of Slovenia. He has written some of the fundamental publications of the commission.

In 2002 he was the national representative at the International Hydrographic Conference when Slovenia entered as a member state into the International Hydrographic Organization (IHO). He coordinated the production of the first nautical charts, bathymetric data and hydrographic documents of Slovenia.

From the year 2000 on, he was the coordinator of the installation of national permanent GPS network. He is cooperating in the preparation process for the transition to a new national coordinate system. In 2004 he co-authored the national strategy of development of the fundamental geodetic system which has been adopted by the Government of Slovenia.