Universal scaling laws for critical quantum sensing

Louis Garbe, TU Wien

IQFA’XII, ENS Lyon

TU WIEN

VCQ

Université de Paris

Outline

- Introduction: active quantum metrology
- Critical fully-connected models
- Quench in the Gaussian regime
- Non-Gaussian effects

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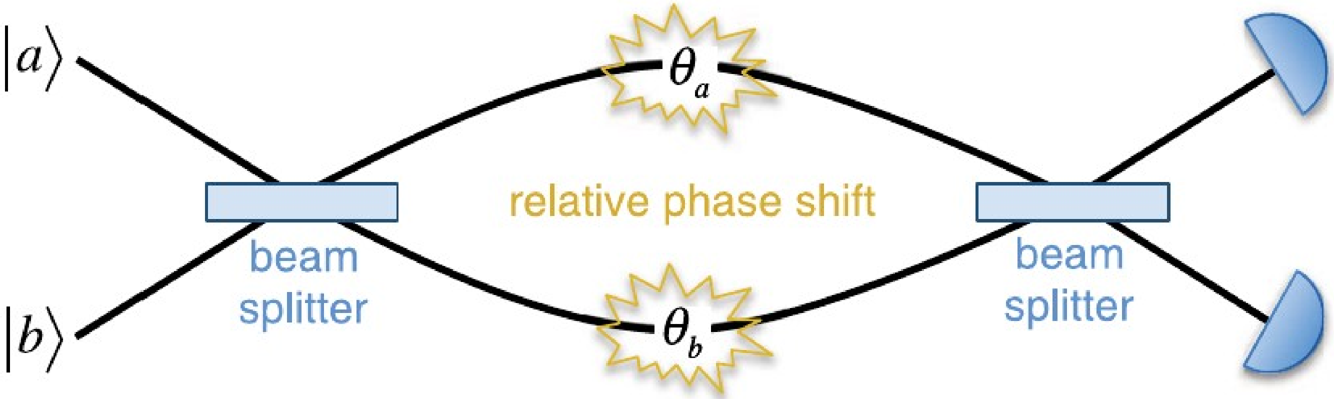
SQL and Heisenberg limit



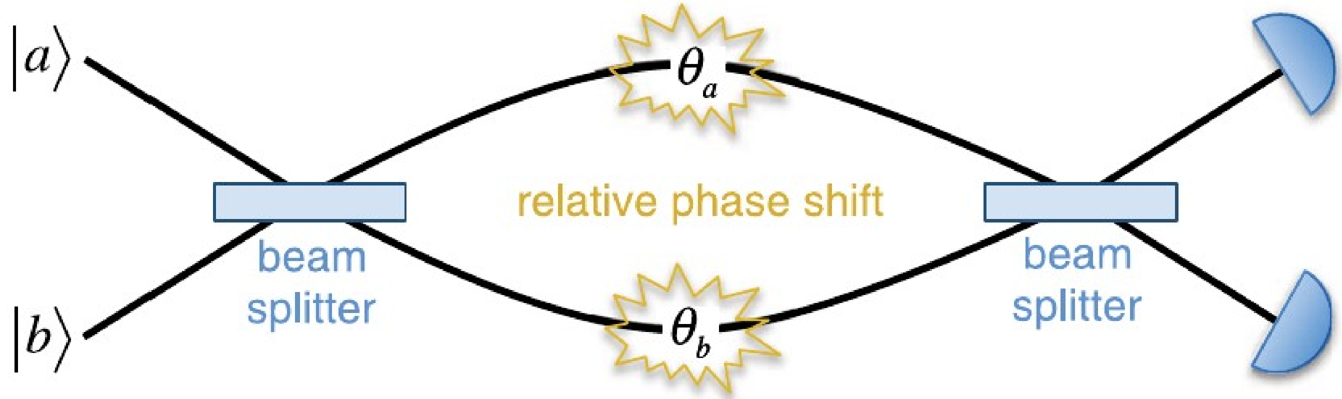
Pezzè et al., Rev.Mod. Phys. 90, 035005 (2018)

State preparation

Encoding

Measurement

SQL and Heisenberg limit



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State preparation

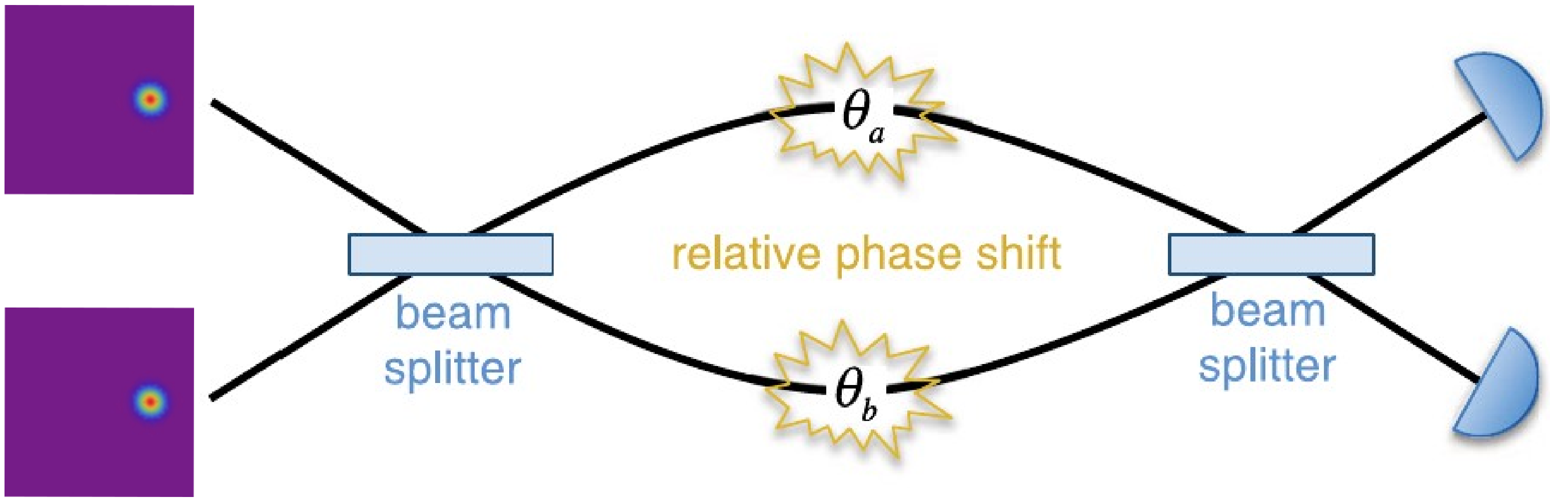
Encoding

Measurement

For optimal measurement: $\Delta x \geq \frac{1}{\sqrt{\mathcal{I}}}$

SNR: $Q = x^2 \mathcal{I} = \left(\frac{x}{\Delta x}\right)^2$

SQL and Heisenberg limit

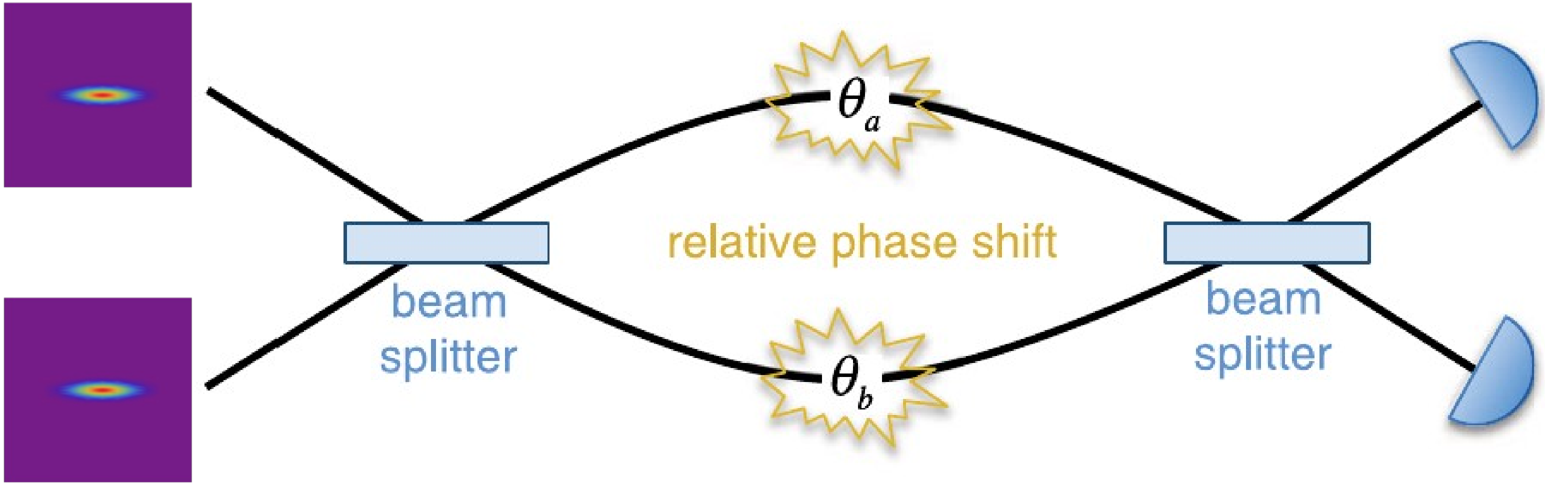


Standard Quantum Limit: $Q \sim nT^2$

$n = \langle \hat{a}^\dagger \hat{a} \rangle$

T : flight time

SQL and Heisenberg limit



Heisenberg Limit: $$Q \sim n^2 T^2$$

Beyond Heisenberg scaling

$$Q \sim n^2 T^2$$

PRL **98,** 090401 (2007) PHYSICAL REVIEW LETTERS week ending 2 MARCH 2007

Generalized Limits for Single-Parameter Quantum Estimation

Sergio Boixo, Steven T. Flammia, Carlton M. Caves, and JM Geremia

PHYSICAL REVIEW A **88**, 013817 (2013)

Quantum metrology with SU(1,1) coherent states in the presence of nonlinear phase shifts

K. Berrada

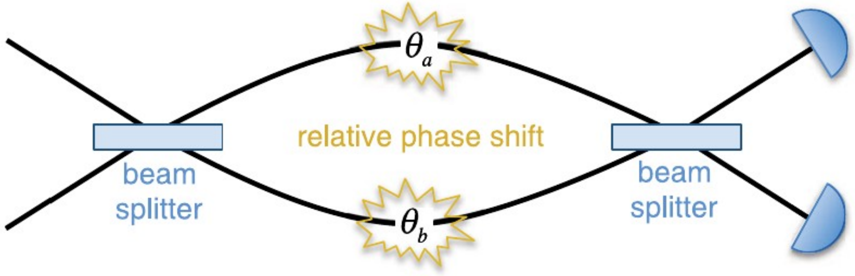
New Journal of Physics

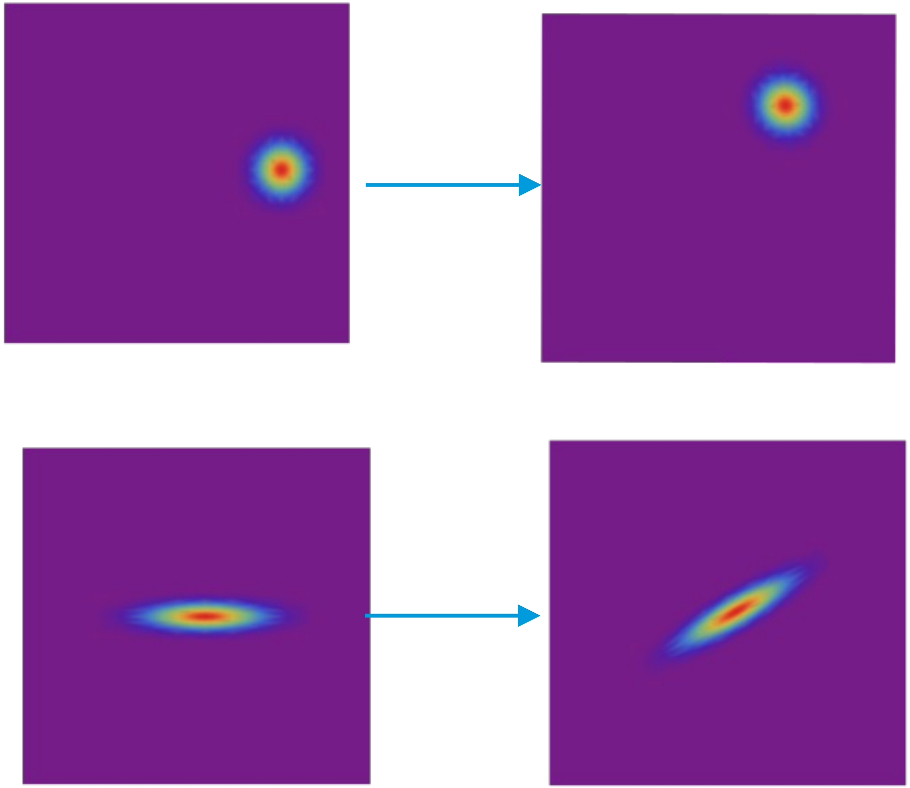
The open-access journal for physics

Sub-Heisenberg estimation of non-random phase shifts

Ángel Rivas[1] and Alfredo Luis[2,3]

Metrology with active interferometry

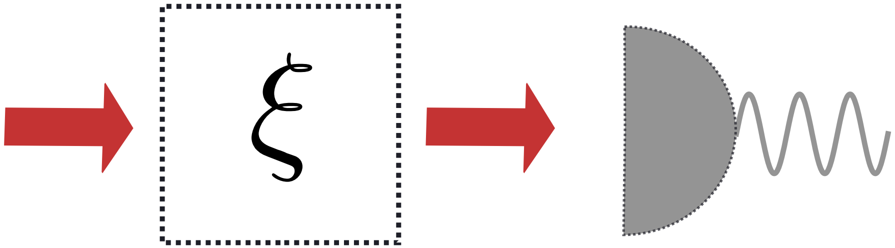




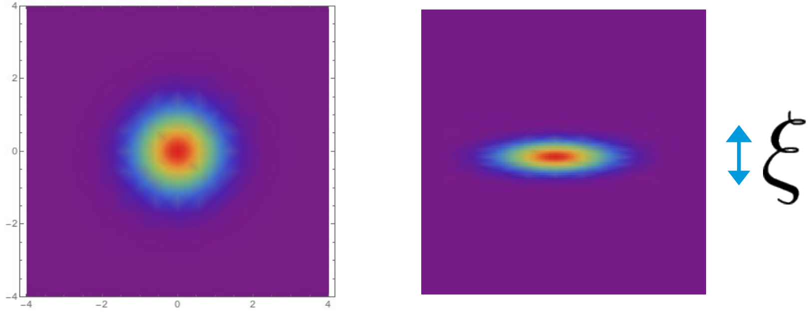
Passive interferometry

Excitation number and fluctuations profile remain constant in time

Metrology with active interferometry



The parameter is encoded in the change of fluctuation profile



n changes in time

Metrology with active interferometry

Given a number profile $n(t)$, can we get a time-scaling behavior beyond Heisenberg?

Metrology with active interferometry

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Which systems allow to implement these profiles?

Metrology with active interferometry

For ***active*** protocols with ***Gaussian*** states:

$$Q \leq \left[\int_0^T n(t)dt\right]^2$$

Metrology with active interferometry

For ***active*** protocols with ***Gaussian*** states:

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$$n(t) \sim cst \rightarrow Q \sim n^2T^2$$

Connects with previous results: Sparaciari et al., JOSA B 32, 1354 (2015)
Safranek et Fuentes PRA 94, 062313 (2016)

Metrology with active interferometry

$$n(t) \sim t^{\alpha} \rightarrow Q \sim T^{2\alpha+2}$$

We can go beyond quadratic scaling in time

Metrology with active interferometry

$$n(t) \sim t^{\alpha} \rightarrow Q \sim T^{2\alpha+2}$$

We can go beyond quadratic scaling in time

n evolves periodically between 0 and $n_{max} \rightarrow Q \sim n_{max}^2 T^2$

Heisenberg scaling is restored

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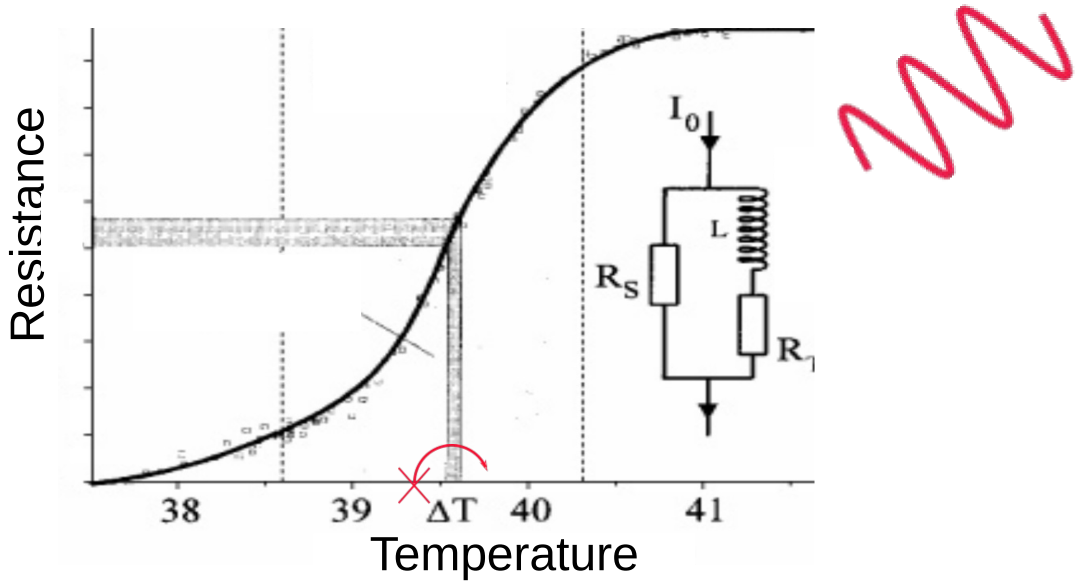

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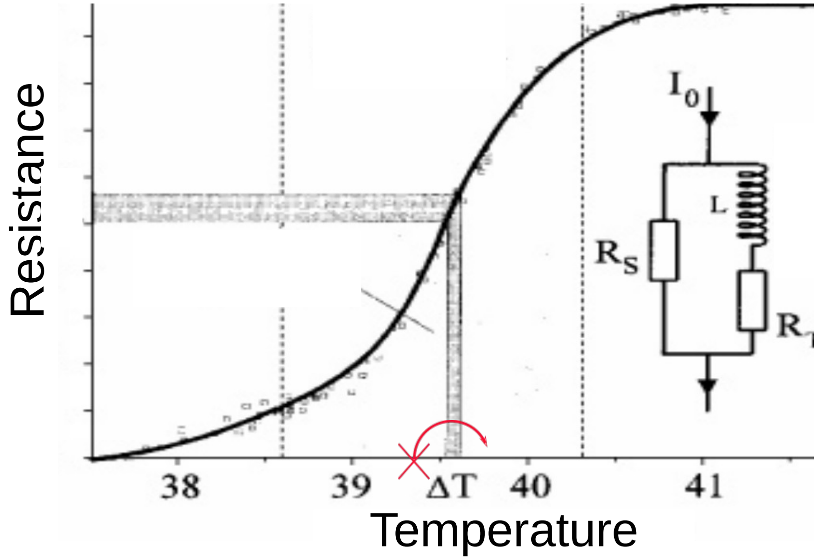

Small change of temperature -> Big leap in resistance

Critical quantum sensing

Quantum phase transitions

Driven by Q fluctuations, at zero T

Q correlations: squeezing, entanglement etc.

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Quantum metrology

Use Q correlations for sensing

Ex: light squeezing in GW detectors, entanglement in atomic clock

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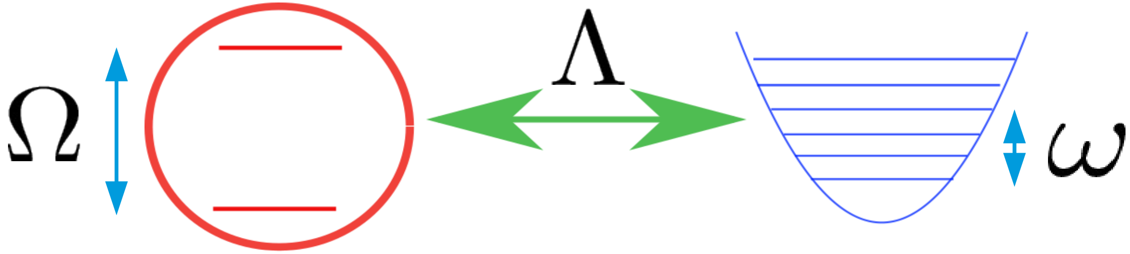
Critical Quantum sensing

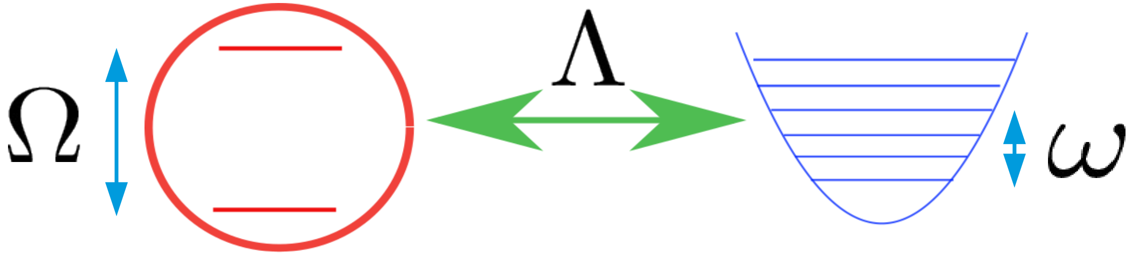
Exploit the full quantum fluctuations near a critical point for sensing



$$\frac{\Omega}{\omega} \gg 1$$

$$\hat{H} = \omega \hat{a}^\dagger \hat{a} + \Omega \hat{\sigma}_z$$
$$+ 2\Lambda \hat{\sigma}_x (\hat{a} + \hat{a}^\dagger)$$

Fully-connected models



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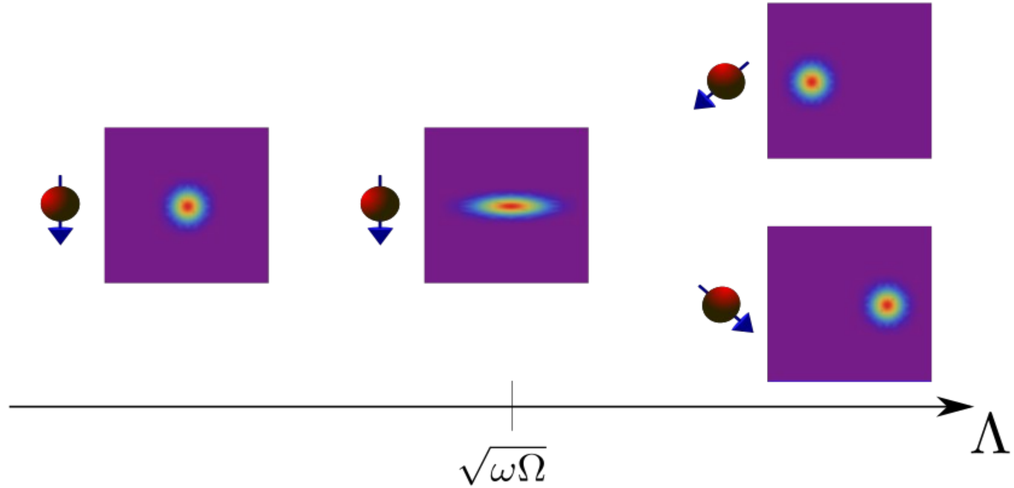

Hwang, Puebla and Plenio, PRL 115, 180404 (2015)

Non-linear oscillator

The qubit can be adiabatically eliminated:

$$\hat{x} = \frac{\hat{a} + \hat{a}^\dagger}{\sqrt{2}} \qquad g = \frac{\Lambda}{\sqrt{\omega\Omega}}$$

$$\hat{H} = \frac{\omega}{2}\left[\hat{p}^2 + (1 - g^2)\hat{x}^2\right] + \frac{g^4}{\eta}\hat{x}^4$$

$$\hat{p} = i\frac{(\hat{a}^\dagger - \hat{a})}{\sqrt{2}} \qquad \eta = \frac{\Omega}{\omega} \gg 1$$

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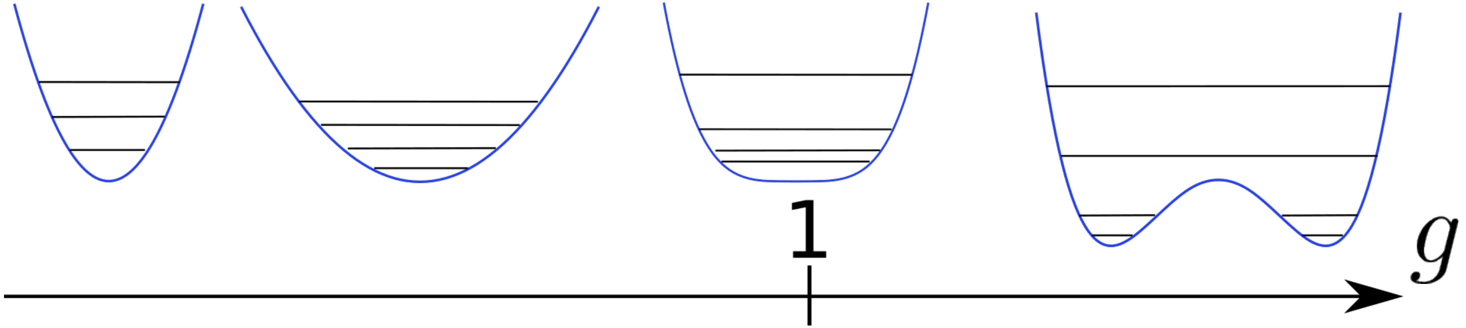
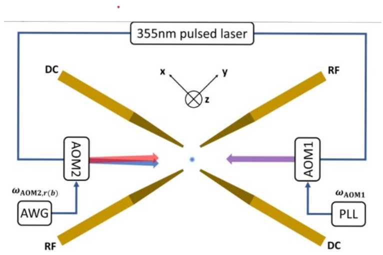
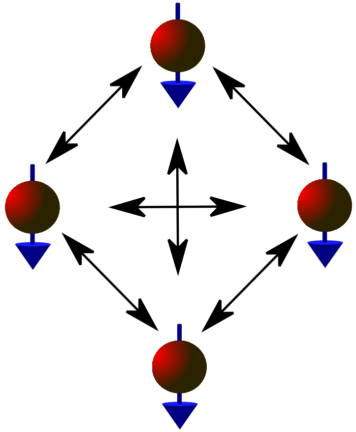


Link with fully-connected models

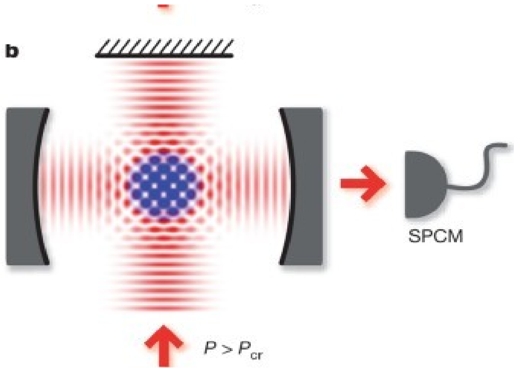
In general: all fully-connected systems
can be mapped to this non-linear oscillator



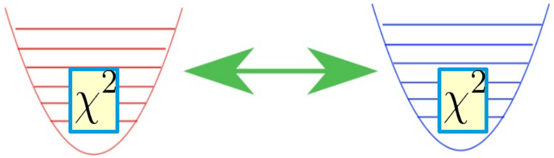
Rabi model:
Trapped ions



LMG model:
Cold gases



Dicke model:
Atoms in cavity



Coupled oscillators:
Levitating nanosphere

Our results apply to all of these models

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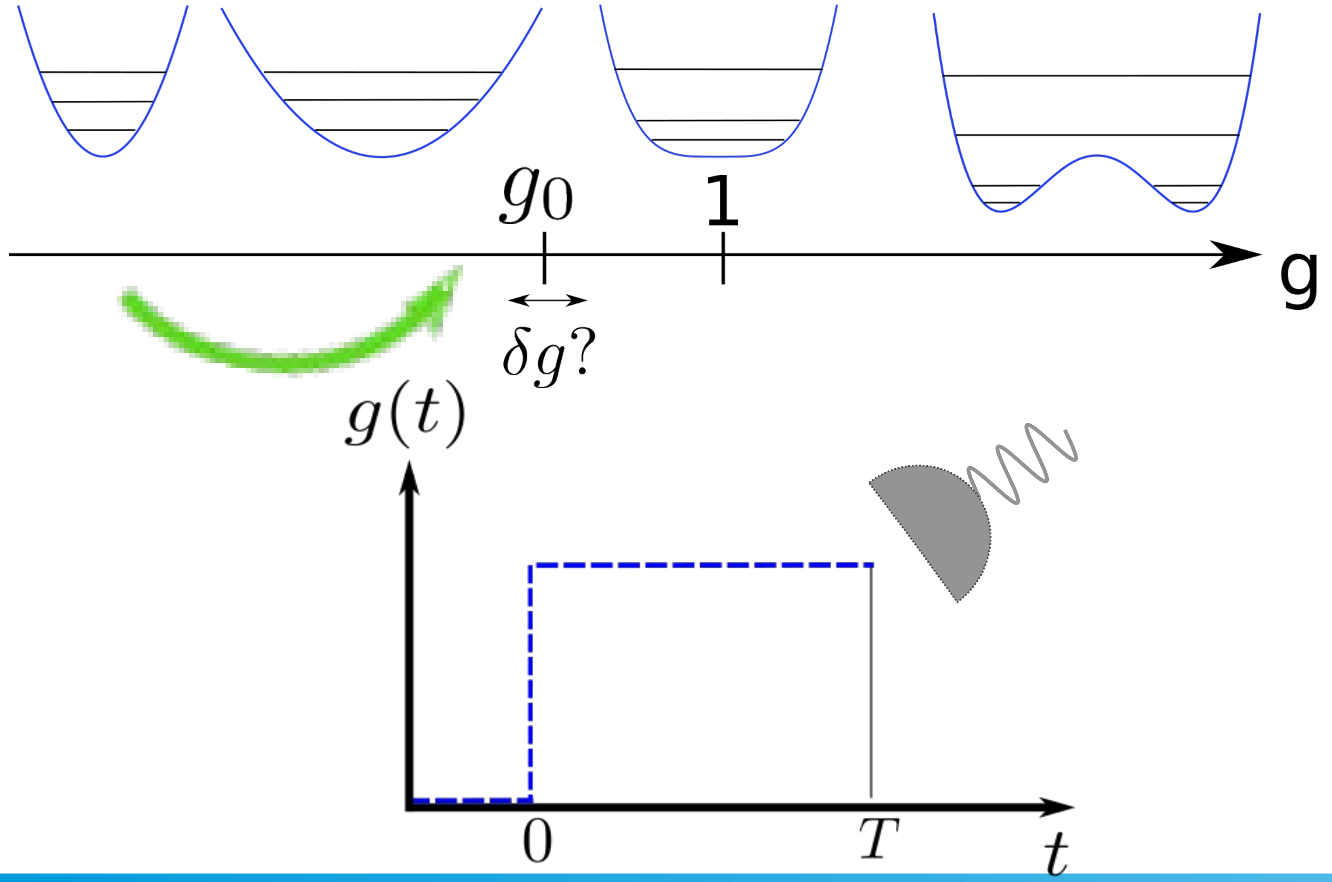
Sensing Protocol



Sudden quench:

$$g : 0 \to g_0 + \delta g$$

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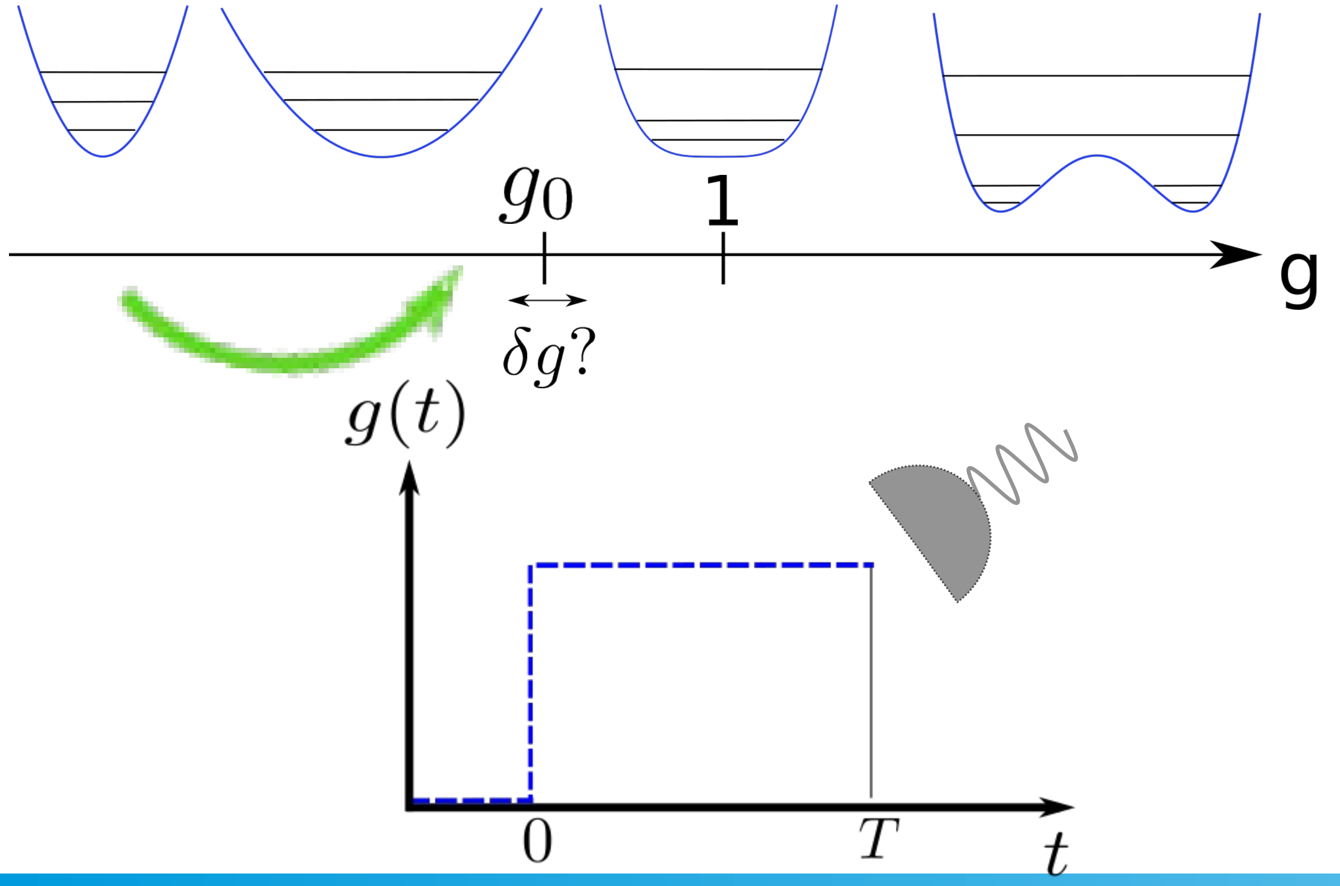

Sudden quench:

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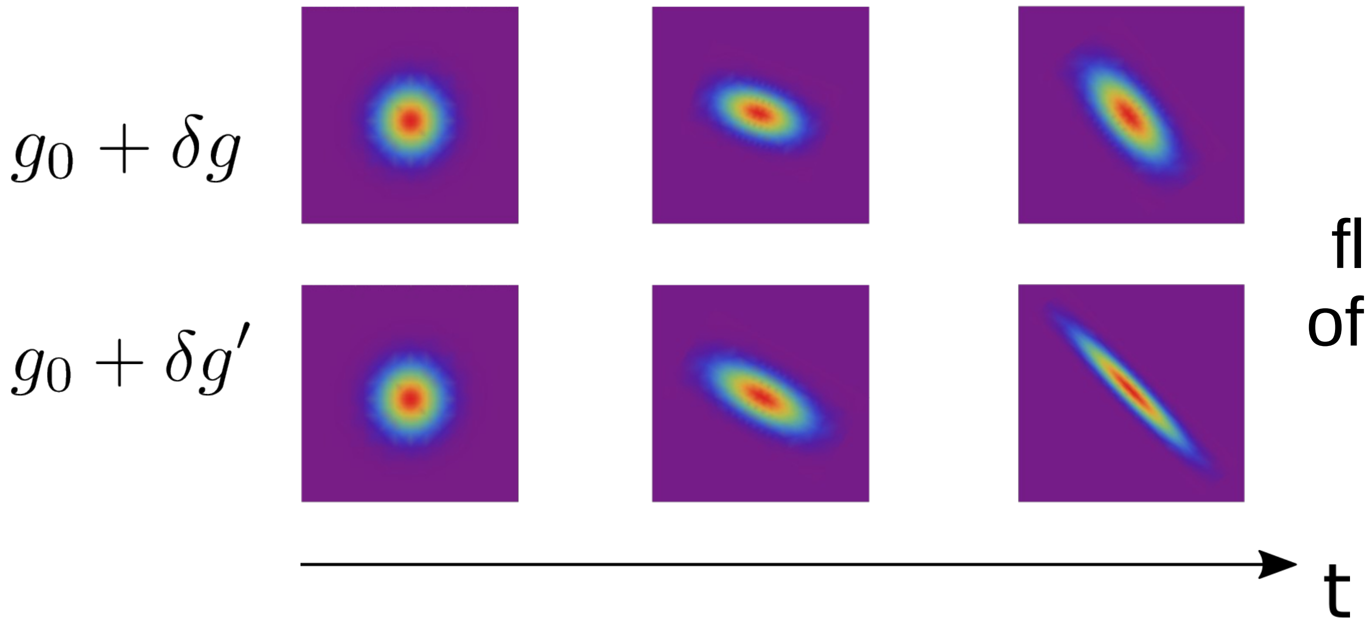
With Rabi model:

$$\Lambda = \Lambda_0, \quad \omega = \omega_0$$

$$\Omega = \Omega_0 + \delta\Omega$$

$$g = \frac{\Lambda}{\sqrt{\omega\Omega}} = g_0 + \delta g$$

Sensing Protocol



By measuring the fluctuations at the end of the evolution, we can reconstruct δg

Quench at the critical point

$$g_0 = 1, \;\; \eta \to \infty$$

$$\hat{H} = \frac{\omega}{2}\left[\hat{p}^2 + (1-g^2)\hat{x}^2\right] + \frac{g^4}{\eta}\hat{x}^4$$

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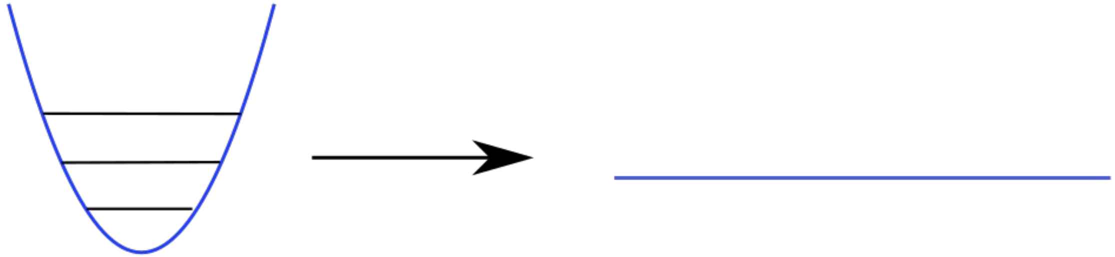

Flat potential, no gap.
Squeezing unbounded

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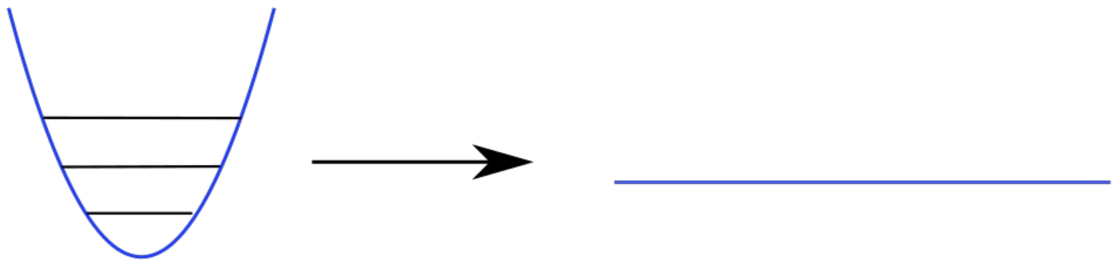
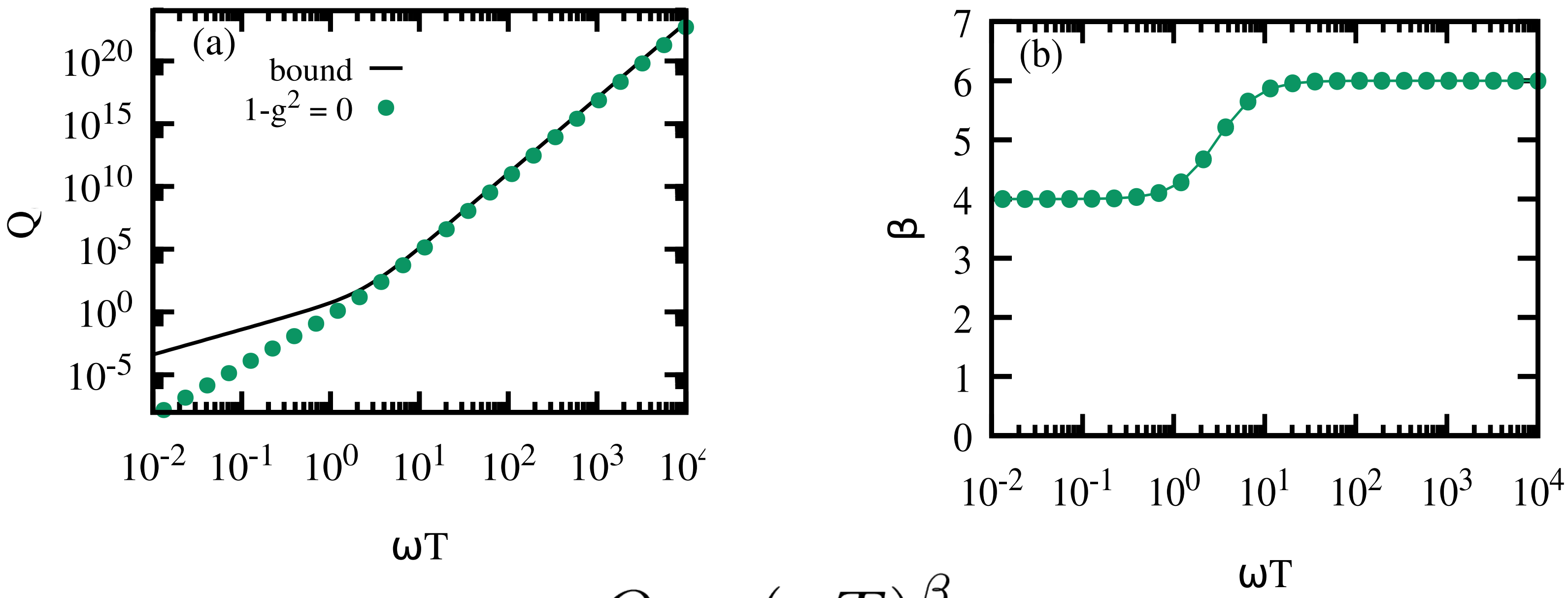


Flat potential, no gap.
Squeezing unbounded

$$n(t) \sim (\omega t)^2 \to Q \sim \left[\int_0^T n(t)\right]^2 = (\omega T)^6$$

Quench at the critical point

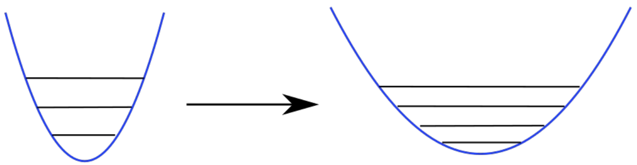


$$Q \sim (\omega T)^{\beta}$$

Our bound is saturated for $\omega T > 1$

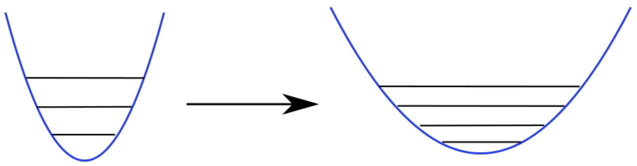
Quench away from the critical point

$g \neq 1$: bounded fluctuations.



Finite gap $\Delta \sim \omega\sqrt{1-g^2}$

Quench away from the critical point



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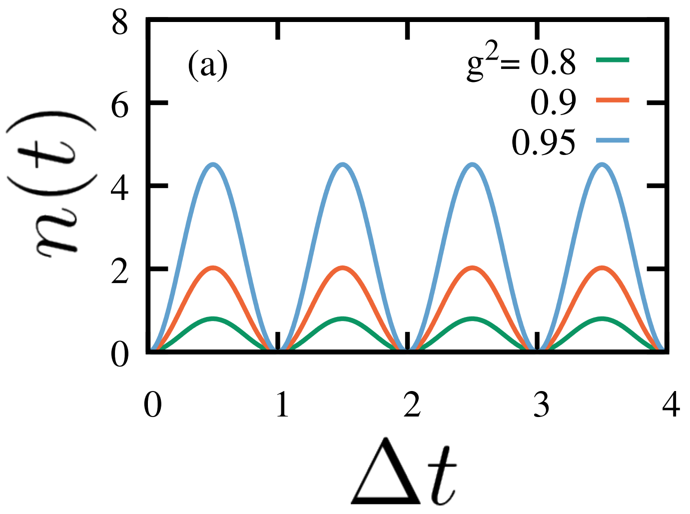
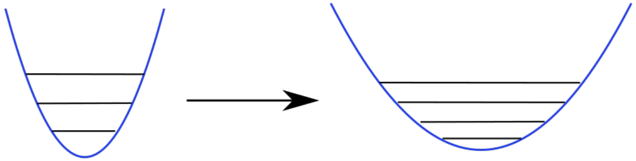
For $T \ll 1/\Delta : n(t) \propto (\omega t)^2$

For $T \gg 1/\Delta$: periodic oscillations



Quench away from the critical point

$g_0 < 1$: bounded fluctuations.



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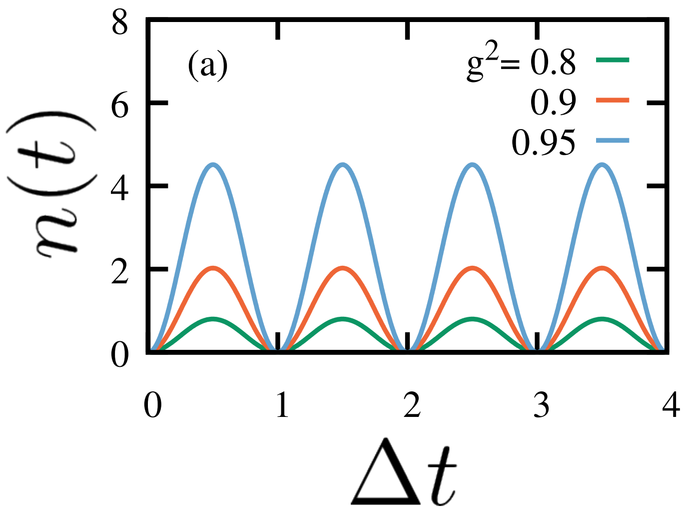
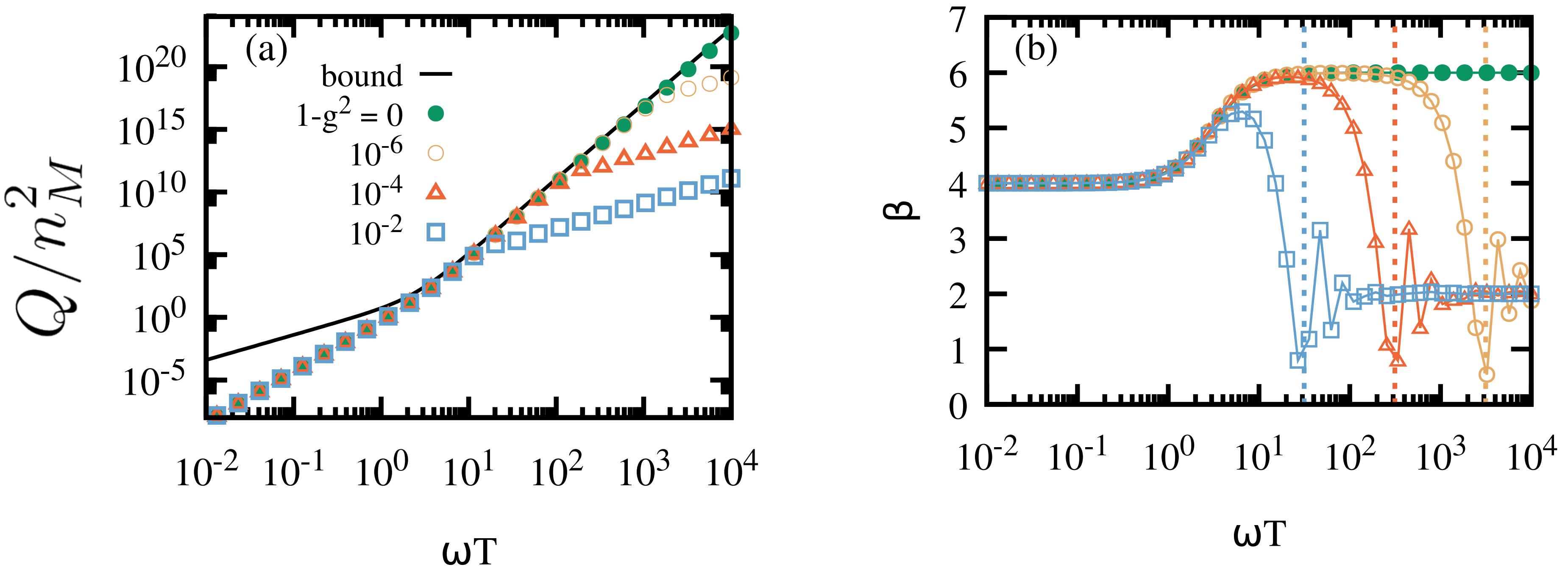


Bound prediction: T^6, then T^2

Quench away from the critical point



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Non-linear effects

So far: thermodynamic limit $\eta = \infty$-> system exactly solvable

$$\eta \neq \infty$$

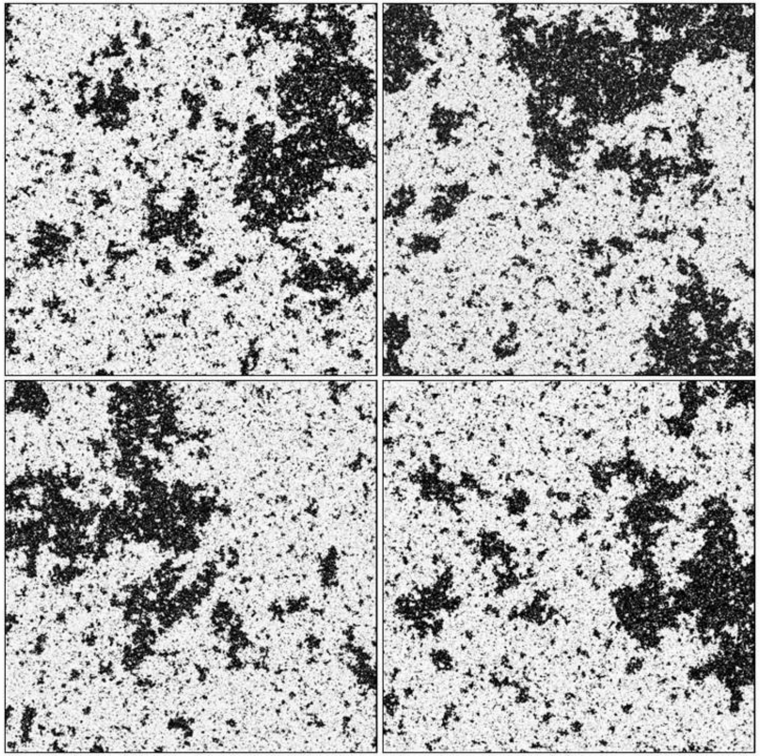
-The state is no longer Gaussian

-Dynamics not solvable exactly

-Our bound should no longer hold

RG-like approach

In systems with short-range interaction: scale-invariance close to the critical point



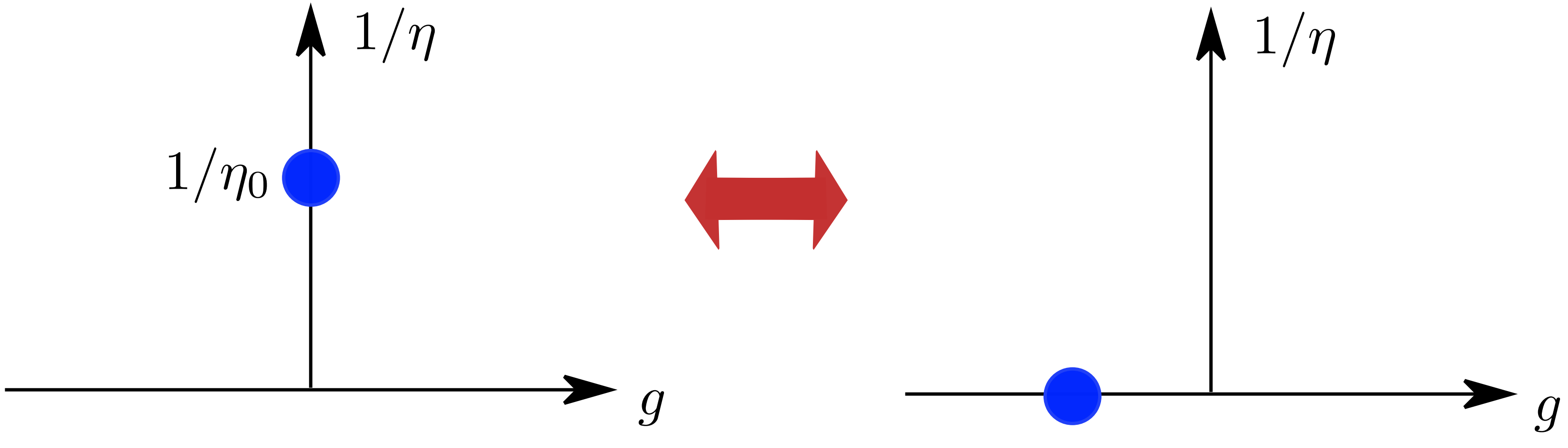
RG-like approach

The Hamiltonian is invariant under this transformation:

$$\begin{cases} \hat{p} \to \hat{p}' = \alpha \hat{p}, \\ \hat{x} \to \hat{x}' = \frac{1}{\alpha}\hat{x}, \\ \eta \to \eta' = \frac{1}{\alpha^6}\eta \\ \omega \to \omega' = \frac{1}{\alpha^2}\omega \\ 1 - g^2 \to (1 - g'^2) = \alpha^4(1 - g^2) \end{cases}$$

We rescale *quadratures* instead of physical position.

RG-like approach



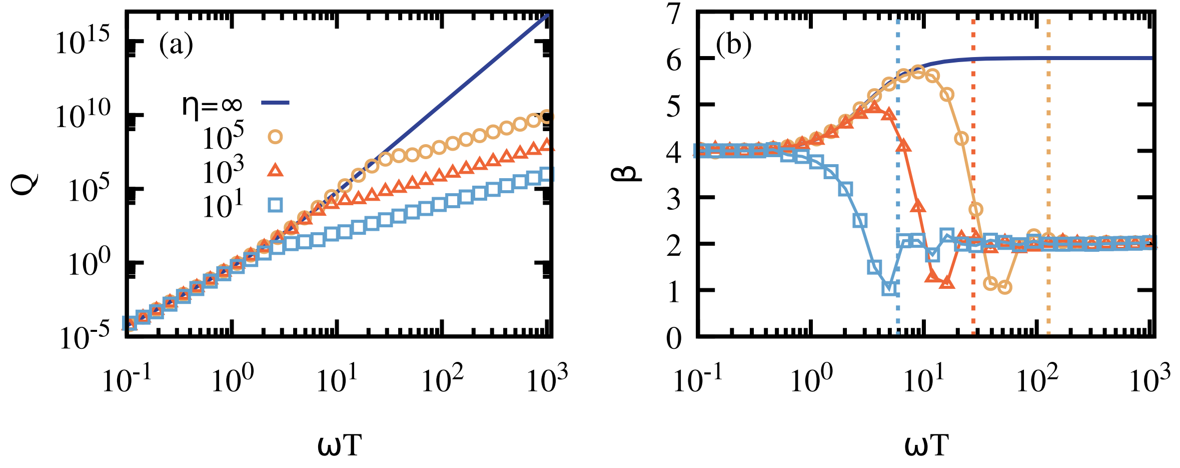
$$g* = \sqrt{1 - \eta_0^{2/3}}$$

We can map quantities for $g \sim 1$, and η finite with quantities for $g \neq 1$ in the thermodynamic limit.

RG-like approach

Minimal gap: $\Delta_m \sim \omega\eta^{-1/3}$

Same scaling regimes as before



We can apply a renormalisation-group-like approach in a 0-d system with no coherence length

Take-home messages

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- For periodic evolution, we retrieve quadratic scaling: more prevalent that we previously thought
- These scaling laws are independent of the model
- We can treat non-Gaussian corrections with a RG-like approach.

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- Experimental implementation: cold atoms, levitating nanosphere

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- Quench the system across the transition -> possible exponential increase in the number of photons
- Effects of decoherence
- General periodic evolutions

Thank you for your attention!

Ricardo Puebla
(IFF Madrid)

Simone Felicetti
(IFN Rome)

Obinna Abah
(Newcastle University)

arXiv: 2110.04144