Critical bosonic systems for quantum metrology

Louis Garbe

Atominstitut, Technische Universität Wien, Vienna, Austria.

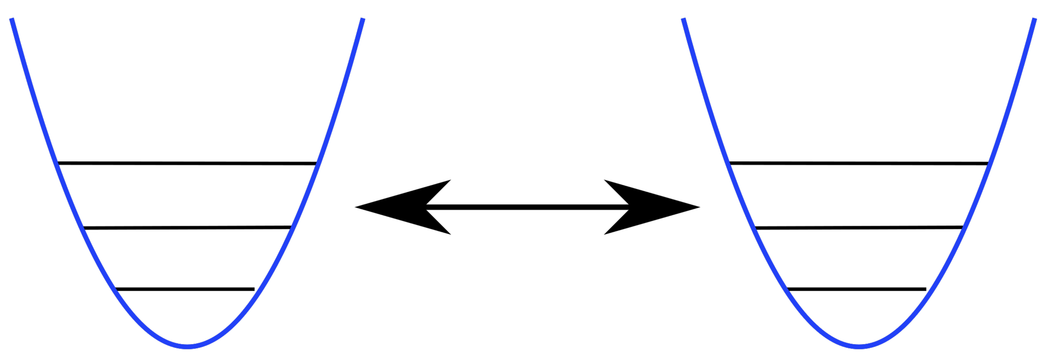
louis.garbe@tuwien.ac.at

TU WIEN ATOMINSTITUT

1. Introduction

Near a phase transition, a system is very sensitive to small perturbations. For instance, a small change in the temperature of a superconductor just below transition can result in a giant leap of resistivity, a phenomenon which is exploited in state-of-the-art photon detectors. In this work, we combine ideas from quantum metrology and quantum phase transitions, to show how the critical behavior of a bosonic system could be used for sensing. We study two coupled bosonic in the so-called ultrastrong-coupling regime, a toy model whose effective phenomenology can describe a vast range of quantum optical systems.

2. Coupled oscillators in USC



$$\hat{H} = \omega(\hat{a}^\dagger\hat{a} + \hat{b}^\dagger\hat{b}) + \frac{\Lambda}{2}(\hat{a}+\hat{a}^\dagger)(\hat{b}+\hat{b}^\dagger)$$

Ultrastrong coupling regime: $\Lambda \sim \omega$

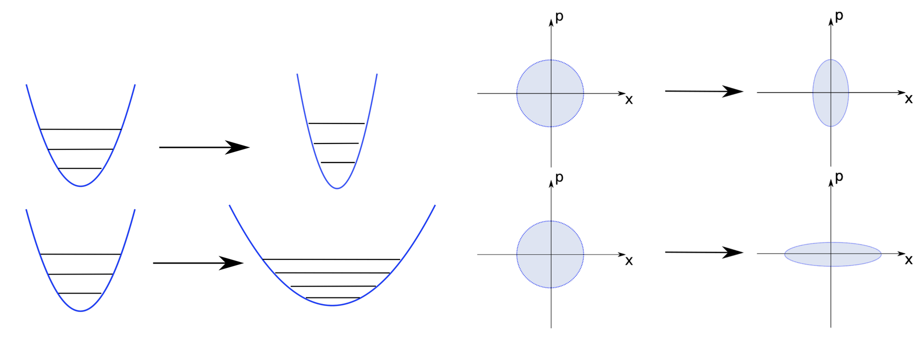
Ex: vibrational motion coupled with cavity mode, coupled superconducting resonators

3. Critical point

Polaritons: $\hat{c} = \frac{\hat{a}+\hat{b}}{\sqrt{2}}, \quad \hat{d} = \frac{\hat{a}-\hat{b}}{\sqrt{2}}$

$$\hat{H} = \frac{1}{2}[\omega\hat{p}_c^2 + (\omega+\Lambda)\hat{x}_c^2] + \frac{1}{2}[\omega\hat{p}_d^2 + (\omega-\Lambda)\hat{x}_d^2]$$

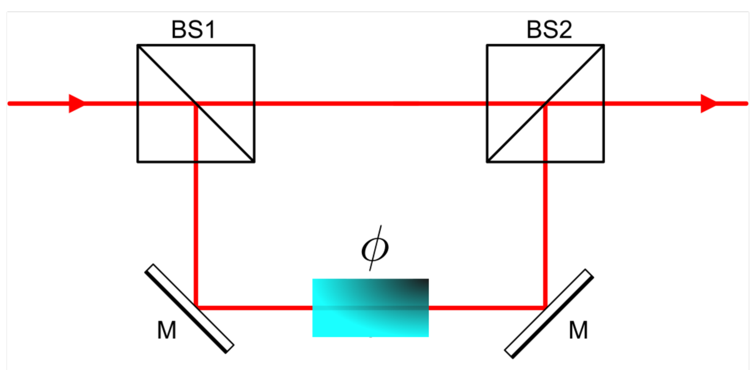
Flattened potential→ mode squeezing.



At $\Lambda = \omega$: the system becomes unstable. Properties of the critical point:

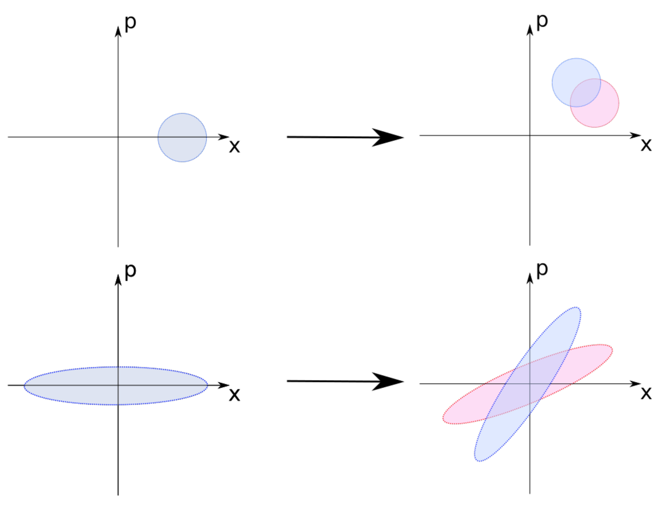
- Diverging quantum fluctuations, closed gap
- Critical exponents $\langle\hat{x}_c^2\rangle \propto (\Lambda-\Lambda_c)^\nu$, gap $\Delta \propto (\Lambda-\Lambda_c)^{z\nu}$
- Kibble-Zurek mechanism
- Renormalization group-like structure

4. Sensing protocol



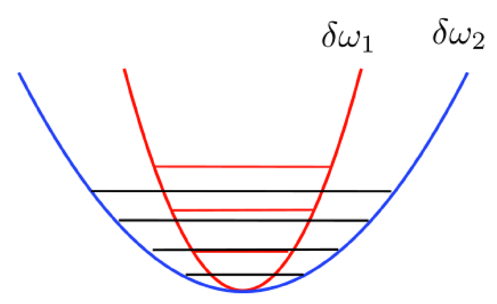
Phase-shift : *passive* channel. The number of probes (photons, atoms etc.) is fixed.

$$|\psi\rangle \to e^{i\phi\hat{a}^\dagger\hat{a}}|\psi\rangle$$



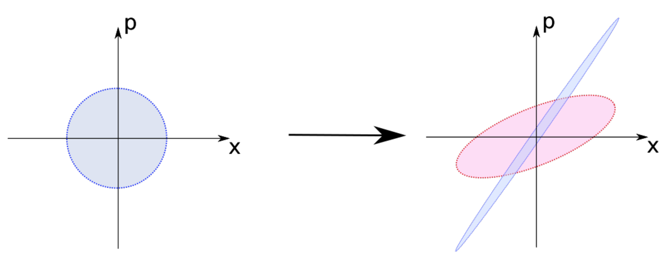
Sudden quench: $0 \to \Lambda \sim \omega_0$

$\omega = \omega_0 + \underbrace{\delta\omega}_{\text{unknown}}$



Both phase-shift and squeezing: *active* channel.

$$|\psi\rangle \to e^{i\phi_\omega\hat{a}^\dagger\hat{a}}e^{\frac{i}{2}(\xi_\omega^*\hat{a}^2 - \xi_\omega\hat{a}^{\dagger 2})}|\psi\rangle$$



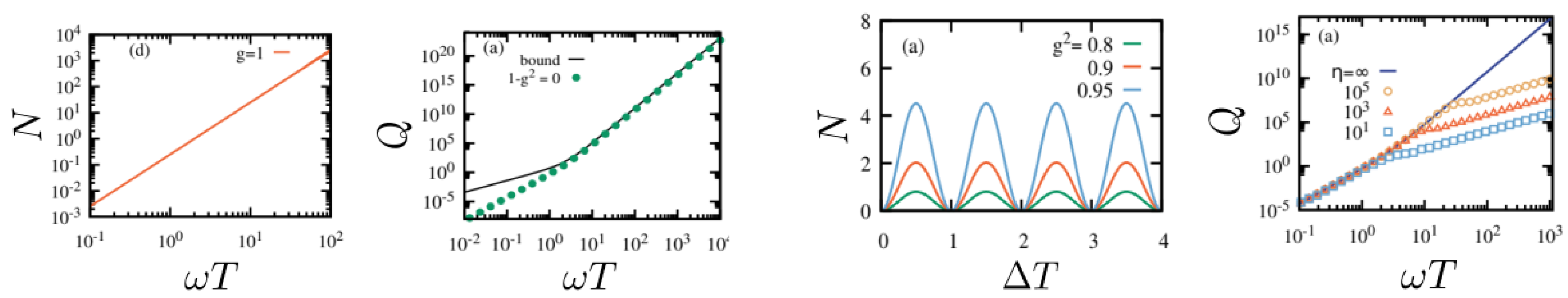
5. Results

Precision quantified by the **Quantum Fisher Information**: $\mathcal{I} = 4(\langle\partial_x\psi|\partial_x\psi\rangle + \langle\psi|\partial_x\psi\rangle^2)$
Scaling with the average number of probes N and the protocol duration T.

Passive protocols: Heisenberg scaling: $\mathcal{I} \leq N^2T^2$	Active protocols: $\mathcal{I} \leq \left(\int_0^T \hat{N}(t)\right)^2 \to T^\beta$

Quench at the critical point $\Lambda = \omega_0$:
$N(t) \sim t^2 \to \beta = 6 \to$ beyond quadratic scaling.

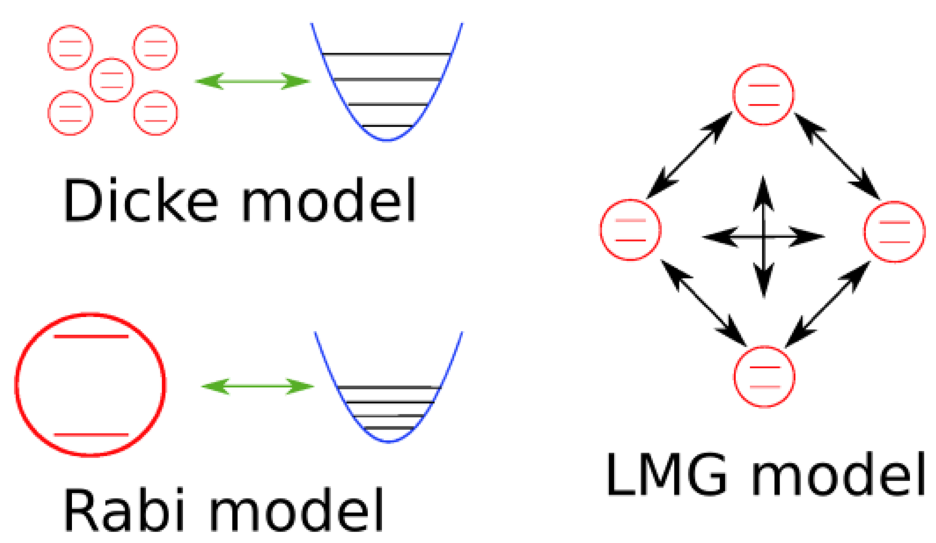
Quench below critical point $\Lambda < \omega_0$:
Finite gap, N shows periodic oscillations.



Further studies: Increasing Λ with a continuous ramp instead of a sudden quench
"Finite-size" effects $\hat{H}' = \hat{H} + \chi\left[(\hat{a}^\dagger\hat{a})^2 + (\hat{b}^\dagger\hat{b})^2\right]$

6. Conclusions

- We showed that critical bosonic systems could be used for sensing purpose, achieving non-trivial scaling for the precision.
- The boost in accuracy comes from an interplay between phase shift and power amplification.
- These results are not limited to two bosonic modes, but apply to any fully-connected systems.



7. Outlook

- Devise a protocol in a concrete experimental platform.
- Study in further details the effect of dissipation.
- Use these systems for the creation of highly non-classical states, such as photon-substracted squeezed states.

8. References

[1] L. Garbe, O. Abah, S. Felicetti and R. Puebla, *arXiv: 2110.04144* (2021)

[2] L. Garbe, O. Abah, S. Felicetti and R. Puebla, *arXiv:2112.11264* (2021)

[3] L. Garbe, M. Bina, A. Keller, M. Paris, and S. Felicettti, *Phys. Rev. Lett.* **124**, 120504 (2020)