Safe vehicle handling, in any situation? – a continuous topic for vehicle dynamics engineers over time

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Contents

温故知新
learning lessons from the past and developing new ideas

Motivation

How safe vehicle handing is retated to stability ?

Motivation

How a stable vehicle follows the desired route/path ?

https://www.youtube.com/watch?v=sR2PP9gUaUg

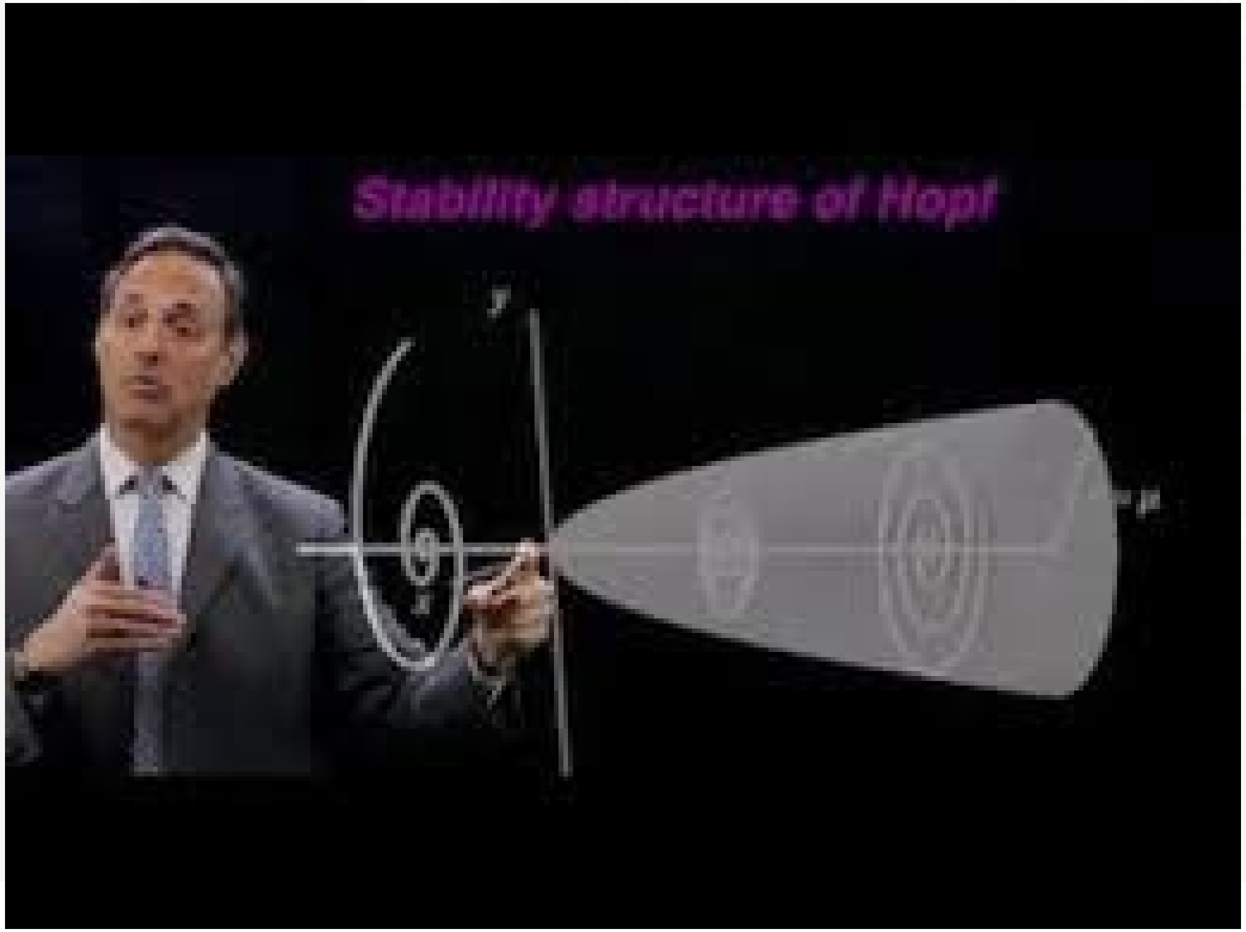
Motivation

Which are <u>all of the possible stable motions</u> of a car under driver's control and external disturbs ?

Which are the disturbs the driver is able to control ?

Motivation

Stability analysis and bifurcation theory provide an insight on big disturbs acting during steady state motion



Motivation

Are handling/stability problems taught to novice drivers ?

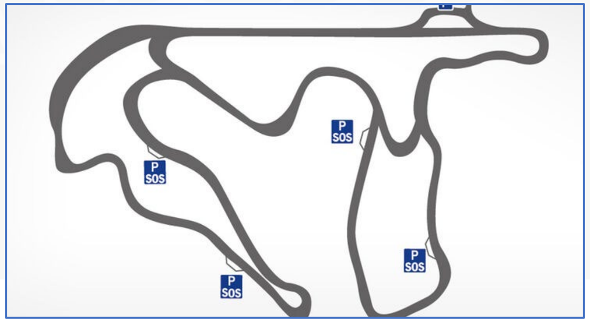
Terms, definitions, meanings related to vehicle handling

What is … ?

Vehicle handling behaviour* is the reaction of the vehicle
to the steering input,
to acceleration or deceleration from the accelerator or braking pedal input
of the driver during cornering and due to external disturbances

Aims: The vehicle
must be easy controllable (and not overchallenge the driver),
must not surprise the driver also at disturbances,
must indicate its limit behviour clearly, and
changes in handling behaviour e.g. due to payload, tyre and road conditions should be as small as possible

Testing: With simulation and testing on proving grounds;
open-loop and closed-loops;
objective criteria and subjective evaluation



Handling Track in Nardo, Italy

* M Mitschke, H Wallentowitz: Dynamik der Kraftfahrzeuge, Springer, 2014

Terms, definitions, meanings related to vehicle handling

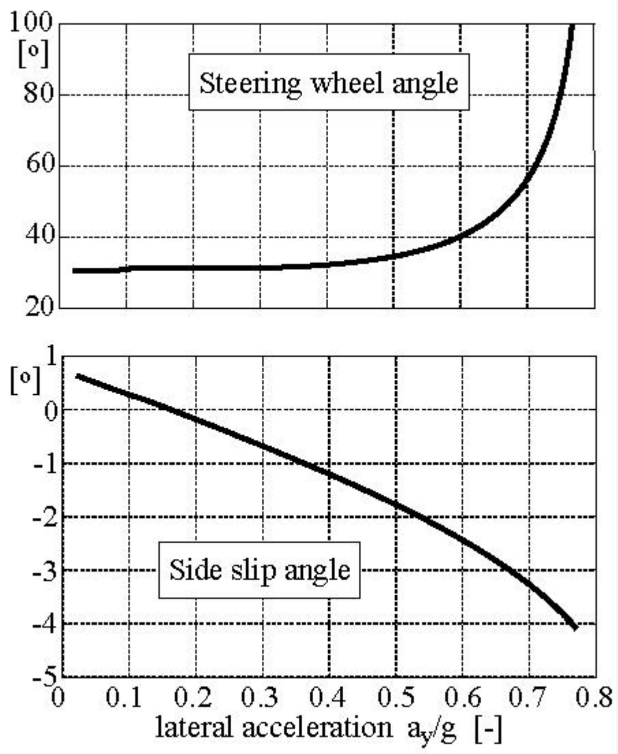
When is … ?

a vehicle motion stable or unstable?

Frequent answers:

- A vehicle is stable
  when the vehicle side slip angle is small
  when the vehicle side slip gradient is small
  when the side slip angle of the rear tyres are below saturation
  when the rear axle is strong/stable
  when there is enough yaw damping

- A vehicle is unstable
  when it slides
  when it rolls over
  when it spins in or out
  when it is oversteering



from G Rill: Road vehicle dynamics, Taylor & Francis, 2012

Terms, definitions, meanings related to vehicle handling

Steady-state cornering

Vehicle side slip angle:

$$\beta = \frac{l_H}{\rho} - \alpha_H = \beta_0 - \alpha_H$$

$$\beta = \frac{l_H}{\rho} - \frac{m l_V}{c_{\alpha H} l} \frac{v^2}{\rho}$$

Characteristic equation:

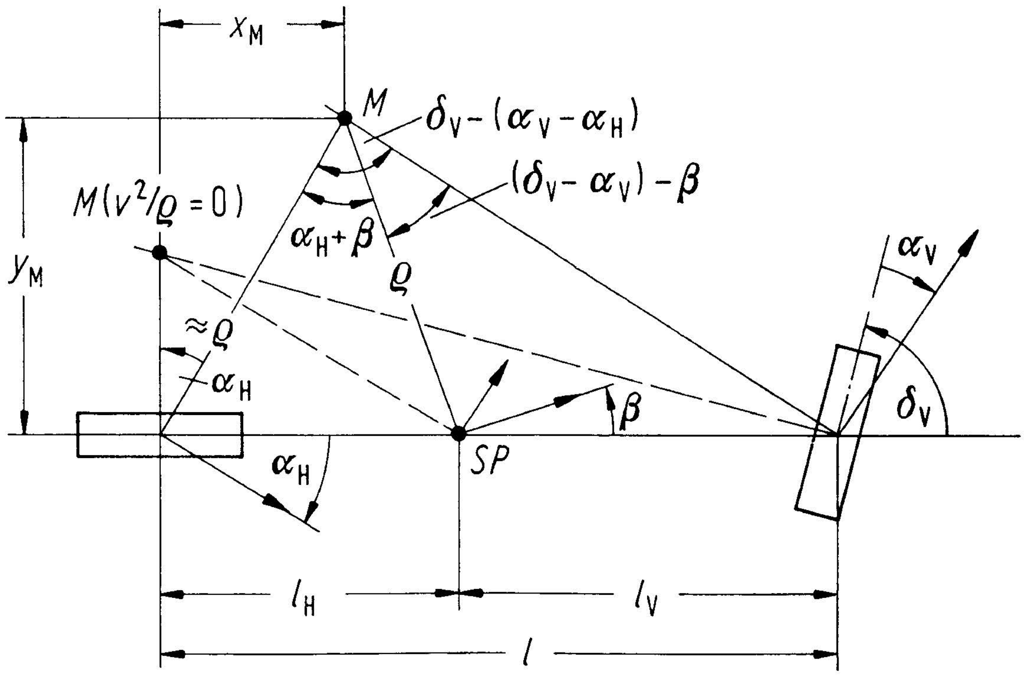
$$\lambda^2 + 2\sigma_f \lambda + \nu_f^2 = 0$$

with

$$2\sigma_f = \frac{m(c'_{\alpha V} l_V^2 + c_{\alpha H} l_H^2) + J_z(c'_{\alpha V} + c_{\alpha H})}{J_z m v}.$$

and

$$\nu_f^2 = \frac{c'_{\alpha V} c_{\alpha H} l^2 + m v^2 (c_{\alpha H} l_H - c'_{\alpha V} l_V)}{J_z m v^2}$$



from M Mitschke, H Wallentowitz:
Dynamik der Kraftfahrzeuge, Springer, 2014

Terms, definitions, meanings related to vehicle handling

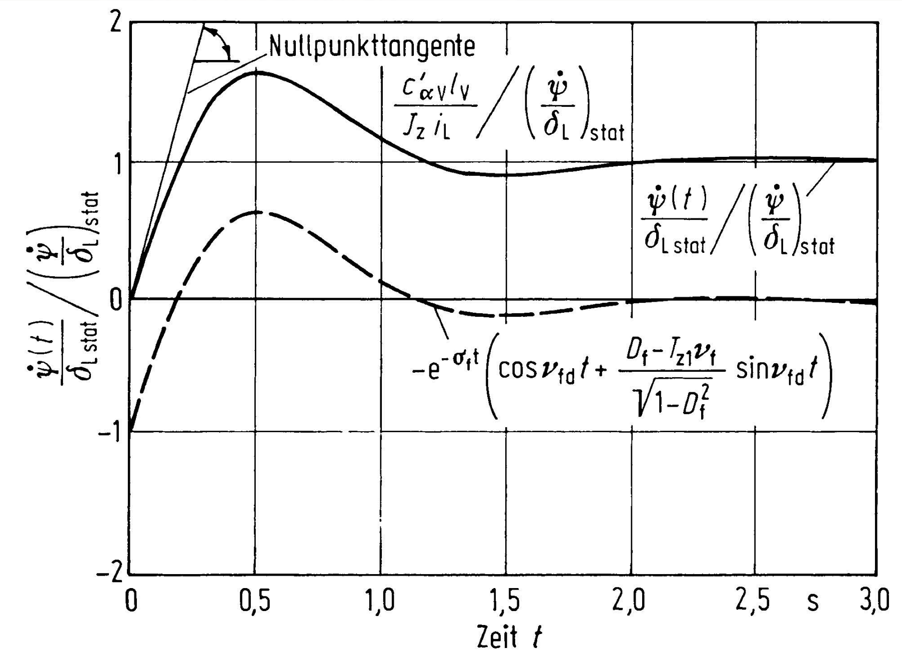
Step steer

Step steer response for conj. complex EV: $\lambda_{1,2} = -\sigma_f \pm \sqrt{\sigma_f^2 - \nu_f^2}$

$$\frac{\dot{\psi}(t)}{\delta_{Lstat}} = \left(\frac{\dot{\psi}}{\delta_L}\right)_{stat} \left[1 - e^{-\sigma_f t}\left(\cos \nu_{fd} t + \frac{D_f - T_{z1}\nu_f}{\sqrt{1-D_f^2}} \sin \nu_{fd} t\right)\right]$$

with

$$2\sigma_f = \frac{m(c'_{\alpha V} l_V^2 + c_{\alpha H} l_H^2) + J_z(c'_{\alpha V} + c_{\alpha H})}{J_z m v}$$

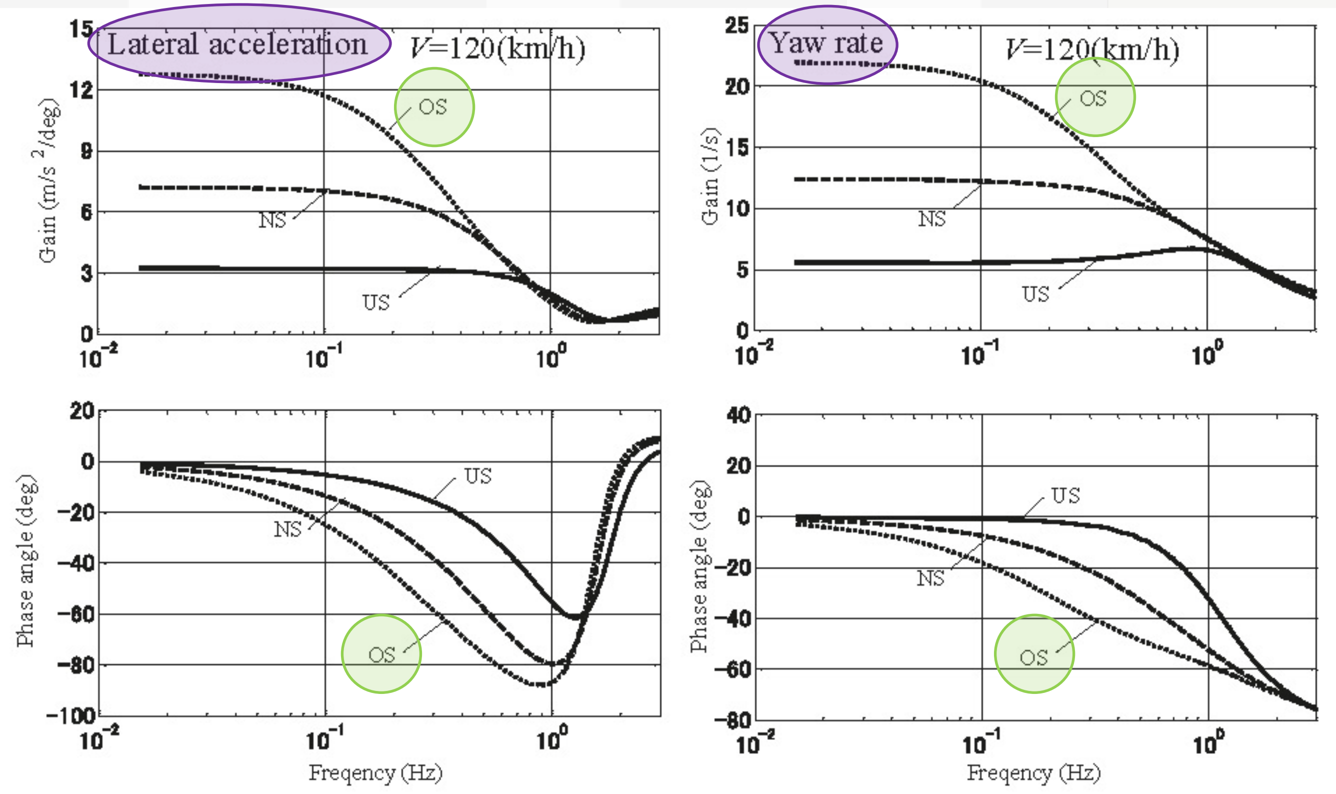


from M Mitschke, H Wallentowitz:
Dynamik der Kraftfahrzeuge, Springer, 2014

Terms, definitions, meanings related to vehicle handling

What is … ?

Vehicle responsiveness* is characterised by the time lag of the vehicle reaction to a steering input by the driver



$$G^{\ddot{y}}_{\delta}(s) = V\frac{s\beta(s)}{\delta(s)} + V\frac{r(s)}{\delta(s)}$$

$$= VG^{\beta}_{\delta}(0)\frac{s(1+T_{\beta}s)}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}} + VG^{r}_{\delta}(0)\frac{1+T_r s}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}}$$

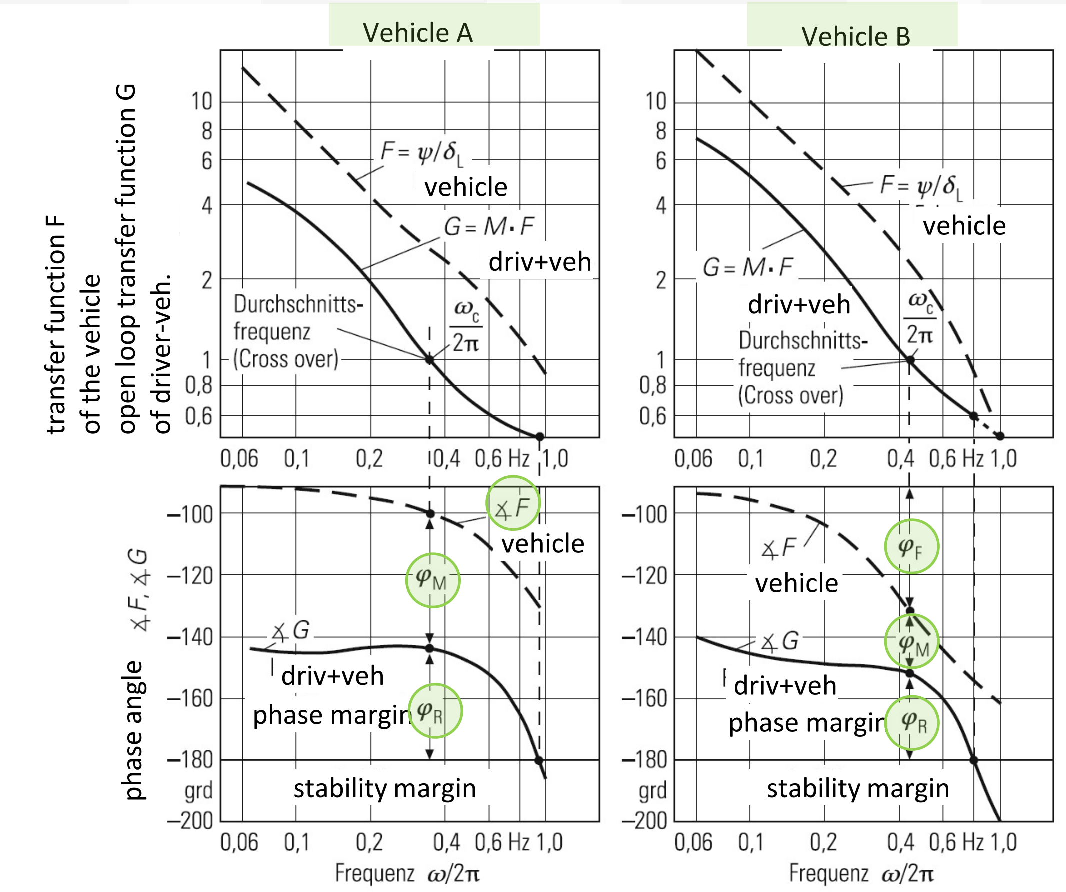
$$= \frac{1}{1+AV^2}\frac{V^2}{l}\frac{\frac{l_r}{V}s-\frac{m}{2l}\frac{l_f V}{K_r}s+\frac{I}{2lK_r}s^2}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}} + \frac{1}{1+AV^2}\frac{V^2}{l}\frac{1+\frac{m}{2l}\frac{l_f V}{K_r}s}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}}$$

$$= \frac{1}{1+AV^2}\frac{V^2}{l}\frac{1+\frac{l_r}{V}s+\frac{I}{2lK_r}s^2}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}} = G^{\ddot{y}}_{\delta}(0)\frac{1+T_{y1}s+T_{y2}s^2}{1+\frac{2\zeta s}{\omega_n}+\frac{s^2}{\omega_n^2}}$$

from M Abe: Vehicle Handling Dynamics, B-H, 2015

Terms, definitions, meanings related to vehicle handling

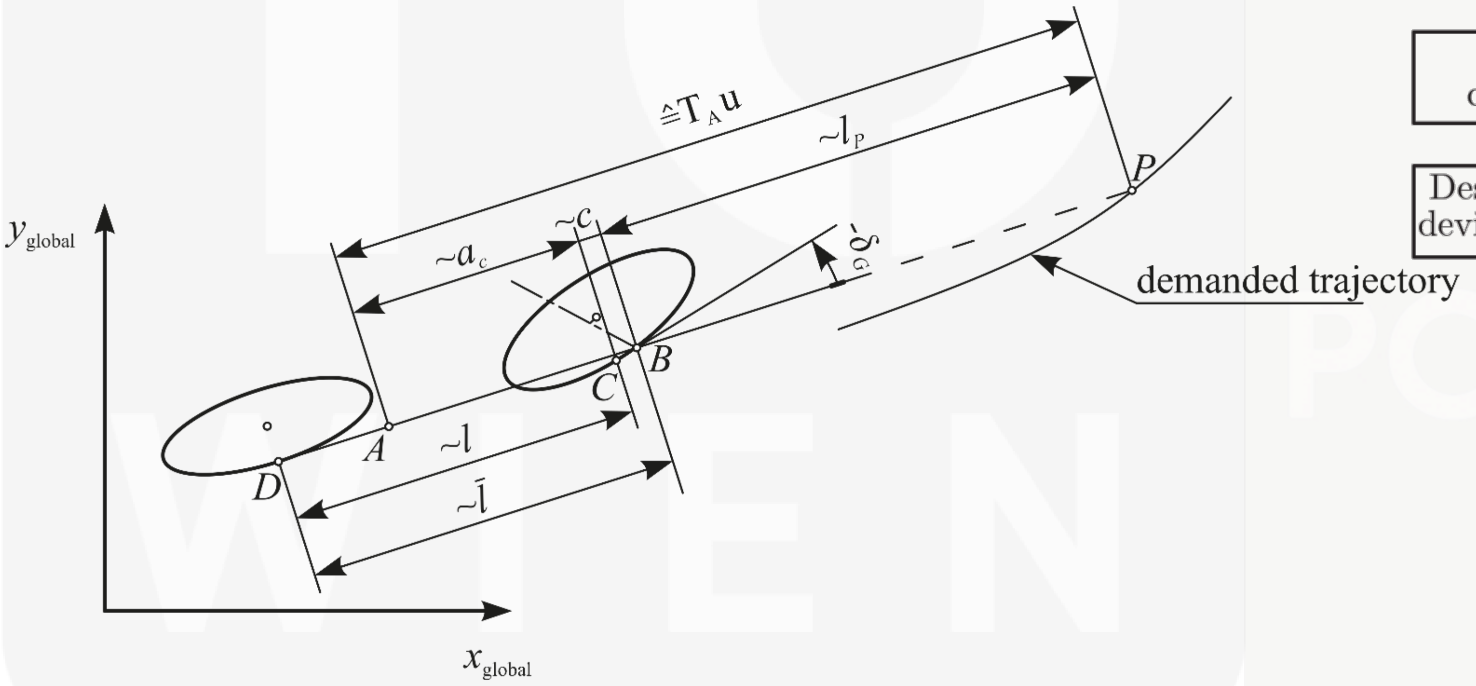
Adding a driver model* with preview, prediction, time lag and time delay
crossover behaviour: driver adapts to different vehicles

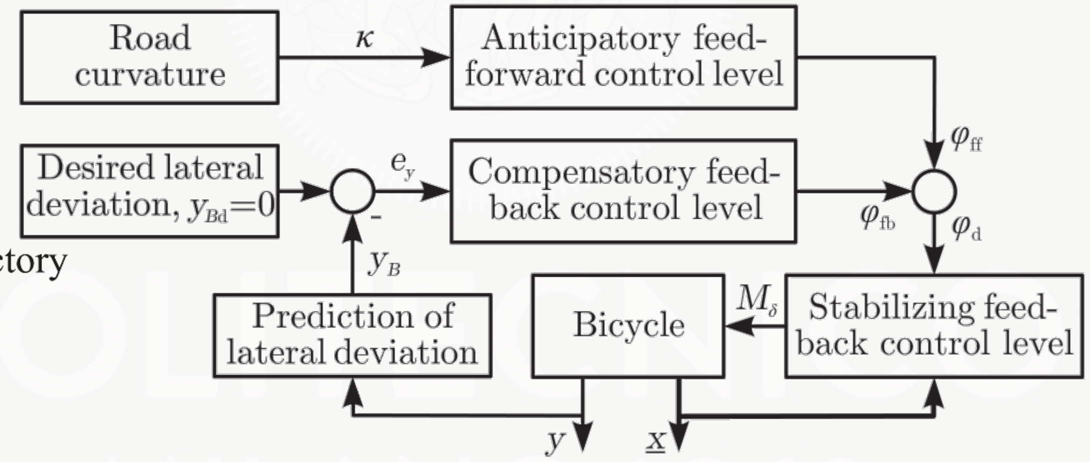


from M Mitschke, H Wallentowitz:
Dynamik der Kraftfahrzeuge, Springer, 2014

Terms, definitions, meanings related to vehicle handling

Example: bicycle rider has to simultaneously stabilise the weave mode and control the path with steering (and with lean?)
→ controllablity/stablity problem





from Thesis (M Haudum) at TU Wien

Terms, definitions, meanings related to vehicle handling

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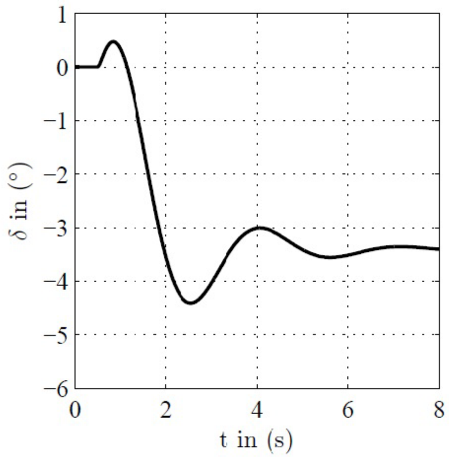
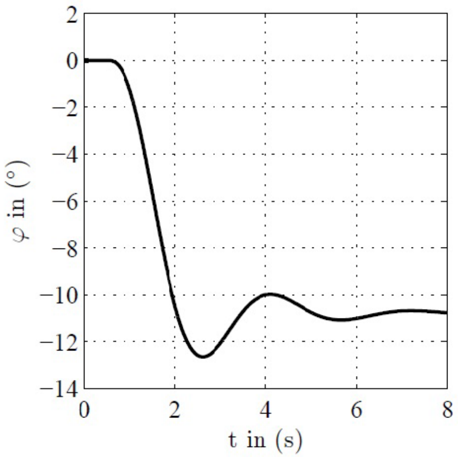
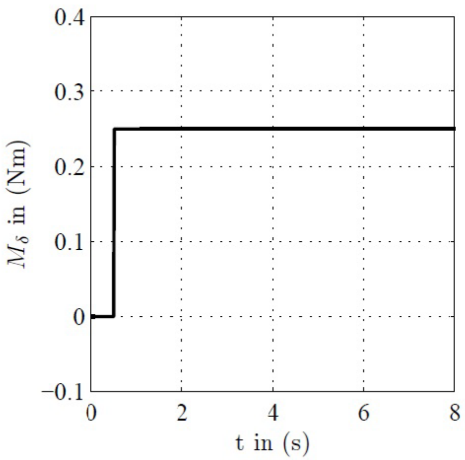


(a) Lenkwinkel δ



(b) Rollwinkel φ



(c) Lenkmoment M_δ

$$G_{\delta M_\delta}(s) = 4,7607 \frac{(s-2,842)(s+2,842)}{(s+23,58)(s+0,4586)(s^2+3,661s+10,76)}$$

conj. complex RHP: unstable motion (“weave mode”)
RHZ: “non-minimum phase system” (“counter steer”)

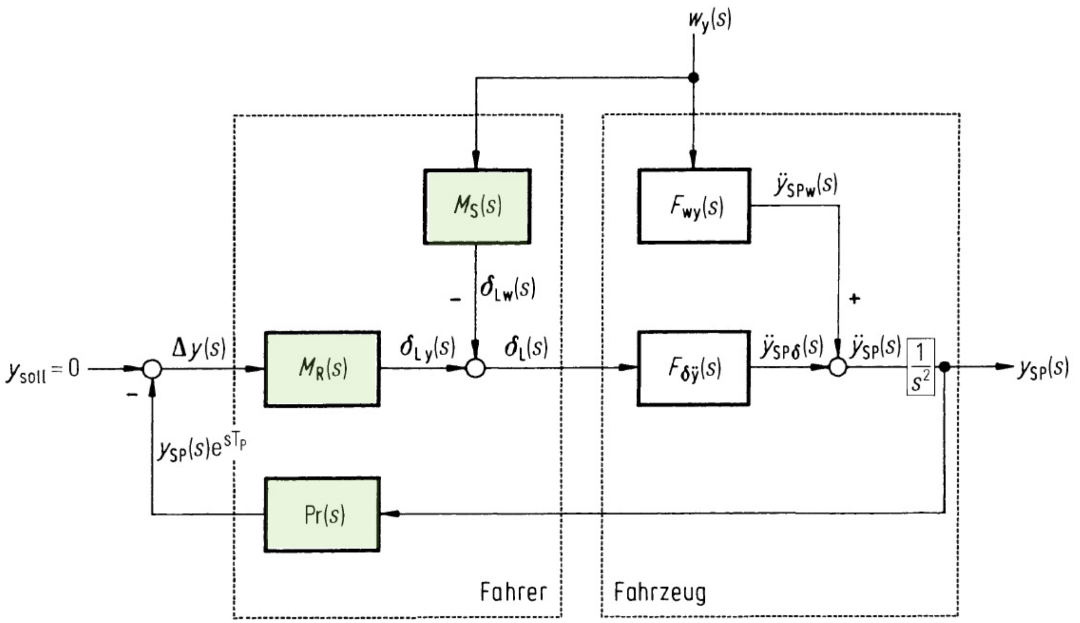
→ consequences: driver must learn to handle/control a bicycle
→ controllability/observability and stabilisation problem
rider model: RHP and RHZ determine bandwidth of the rider control input

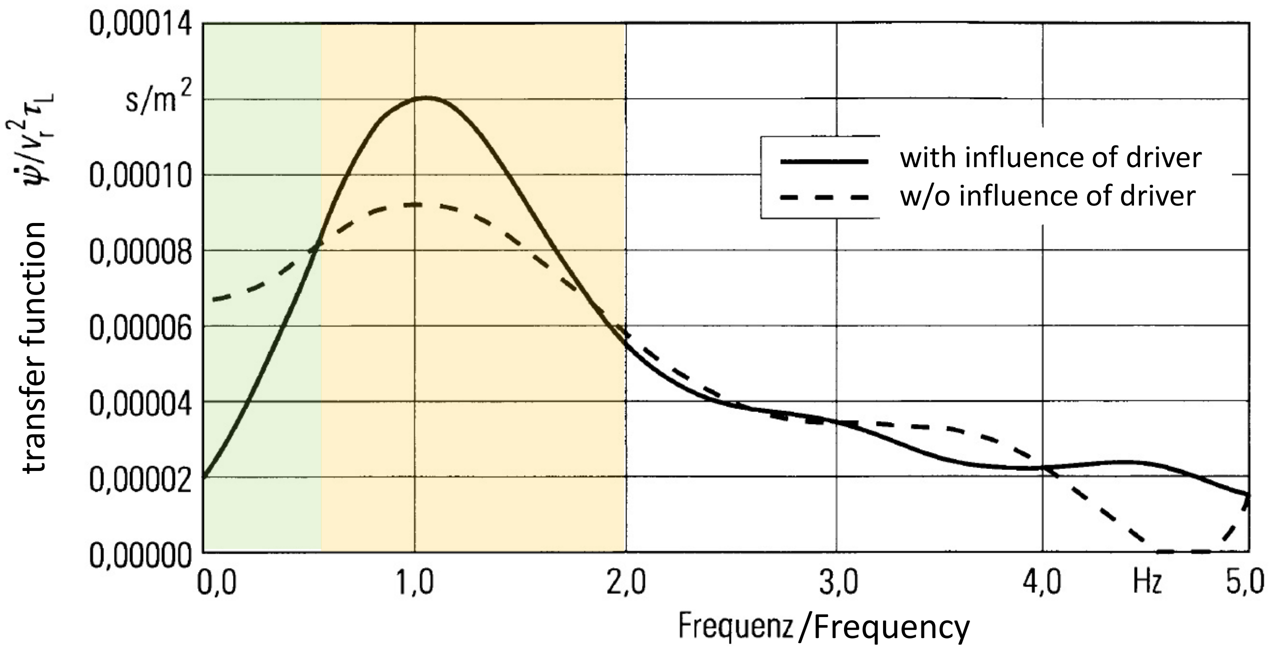
from Thesis (M Haudum) at TU Wien

Terms, definitions, meanings related to vehicle handling

External disturbances

open-loop behaviour: transfer functions, neutral steer point, centre of percussion, static margin
closed-loop behaviour:
Example: straight motion with side wind (ramp or sinusoidal or stochastic)



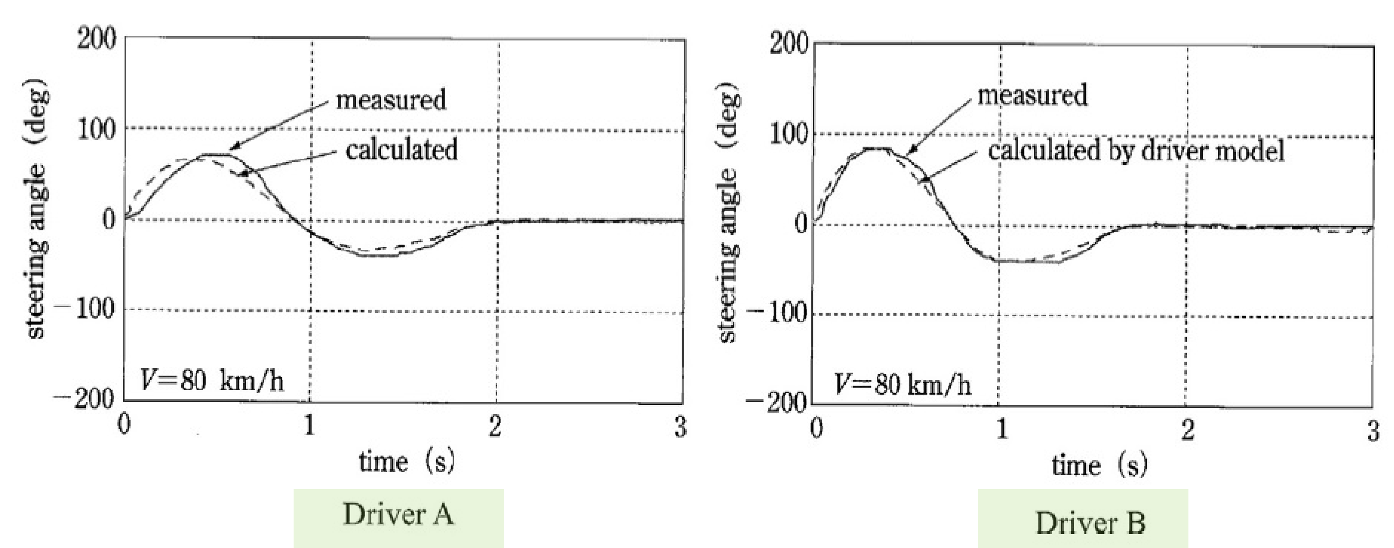


PhD-thesis of H Wallentowitz: Fahrer-Fahrzeug-Seitenwind, 1978

Terms, definitions, meanings related to vehicle handling

External disturbances

open-loop behaviour: transfer functions, neutral steer point, centre of percussion, static margin
closed-loop behaviour:
Example: lane change manoeuvre at 80 km/h
influence of change of tyre characteristics on driver (model parameters)



simplified driver model:

$$\delta(s) = -\frac{h}{1+\tau_L s}\{(1+\tau_h s)y(s) - y_{OL}(s)\}$$

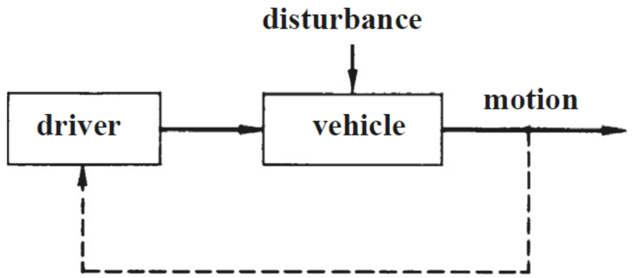
from M Abe: Vehicle Handling Dynamics, B-H, 2015

Terms, definitions, meanings related to vehicle handling

Vehicle handling includes aspects of

Now: driver/test engineer objective and subjective evaluation
Future: passengers/control design engineer evaluation?

- **stability**: "stability refers to the unwillingness of a vehicle to be deflected from the existing path, usually a desirable trait, in moderation" (Dixon); „stability refers to the tendency of a system to return to a previously established equilibrium when disturbed" (Milliken)
- **controllability**: "the ability of the driver to 'steer' the vehicle from an initial to a final state using admissible inputs"; the ability to initiate a change of the vehicle yaw motion, typically by steering, but also by application of the brake/accelerator pedal, in particular when close to the limits of handling



including effects of external disturbances from the environment

e.g. from side wind, road grade, road conditions, other road users, etc.

- **responsiveness:** "characterises the reaction of the vehicle to given input"
- **agility:** "the ease, speed and accuracy with which the operator can alter path curvature" (Blundell/Harty)
- **manoeuvrability:** "the ability of a vehicle to complete a manoeuvre as fast as possible and without exceeding existing physical limitations, like tyre adherence or road borders" (regardless of a driver's skill/evaluation, Sharp)

Terms, definitions, meanings related to vehicle handling

Stability in the sense of Ljapunov

Given: model of vehicle, w or w/o driver, subsystem, controller
described by $\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}, t)$ with state vector $\boldsymbol{y} = [y_1, y_2, \ldots, y_m]^{\mathrm{T}}$

and further: an undisturbed motion: driving straight, steady-state cornering, lane change, etc.
described by the partial solution $\tilde{\boldsymbol{y}}(t)$ of $\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}, t)$

Now, we observe the evolution of the disturbances $y_i(t_0) - \tilde{y}_i(t_0)$ starting from $t = t_0$ over time.

Definition: we call the undisturbed motion $\tilde{\boldsymbol{y}}(t)$ *stable* with respect to the variables y_i
if for any $\varepsilon > 0$ there exists $\eta > 0$ such that for any solution $\boldsymbol{y}(t)$, satisfying the condition $|\boldsymbol{y}(t_0) - \tilde{\boldsymbol{y}}(t_0)| < \eta$,
the inequality $|\boldsymbol{y}(t) - \tilde{\boldsymbol{y}}(t)| < \varepsilon$ holds for all t $> t_0$. ε is then a measure of stability.

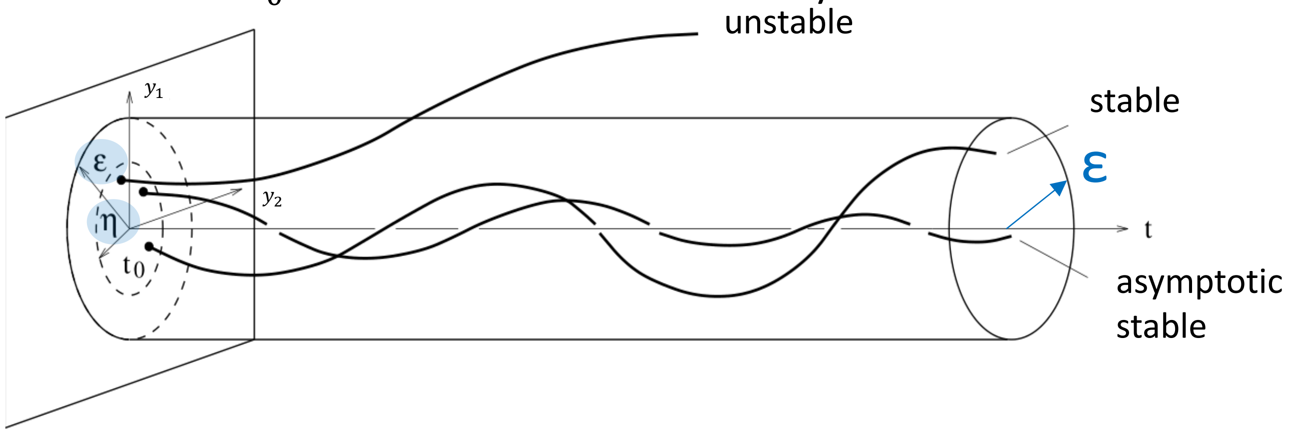


illustration for equilibrium $\tilde{y}_i$=0

Terms, definitions, meanings related to vehicle handling

Remarks

- A partial solution is called attractive if there exits a number $\eta > 0$ (domain of attraction) having the property:
  $\lim_{t\to\infty} |\boldsymbol{y}(t) - \tilde{\boldsymbol{y}}(t)|$=0, whenever $|\boldsymbol{y}(t_0) - \tilde{\boldsymbol{y}}(t_0)| < \delta$

- A solution that is simultaneously stable and attractive, is called asymptotic stable. A solution might be attractive, without being stable.

- The given definition characterises local stability.

- Stability in first approximation (Ljapunov): If all roots of the characteristic equation of a first approximation have negative real parts, then irrespective of terms of order higher than one ("nonlinearities"), the unperturbed motion is asymptotically stable.

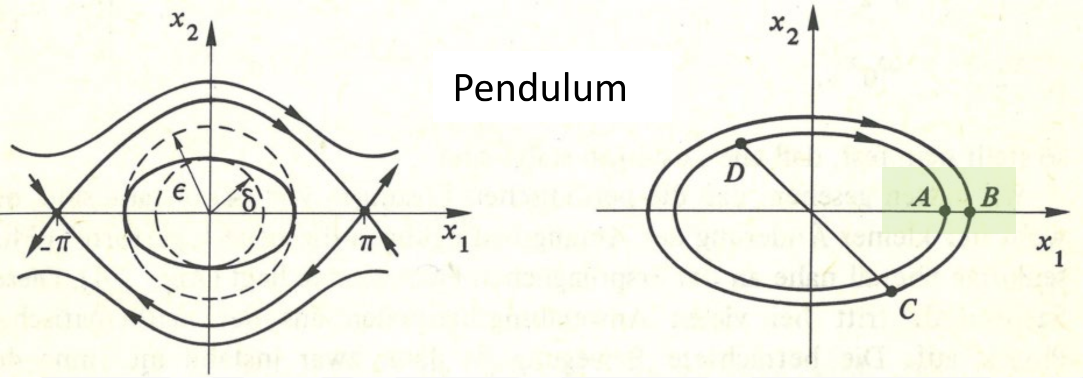
  - it addresses the local stability of a nonlinear system
  - it serves as fundamental justification of using linear control techniques in practice
  - but: what is the domain of stability, how large is the linear range, how large is η? → Lyapunov's direct method

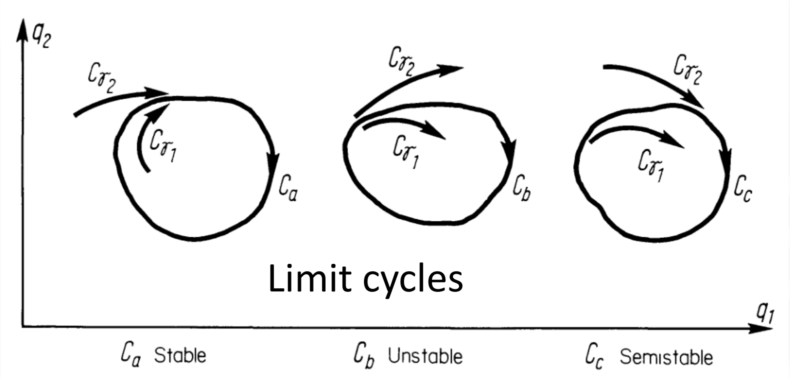
- The equilibrium of $\dot{\boldsymbol{x}} = \boldsymbol{A}(t)\boldsymbol{x} + \boldsymbol{f}(\boldsymbol{x}, t),\ |\boldsymbol{f}| = o(|\boldsymbol{x}|)$ is stable in the finite time interval $|t - t_j|$
  if all eigenvalues of $\boldsymbol{P}(t)$ at time t_j are strictly negative.

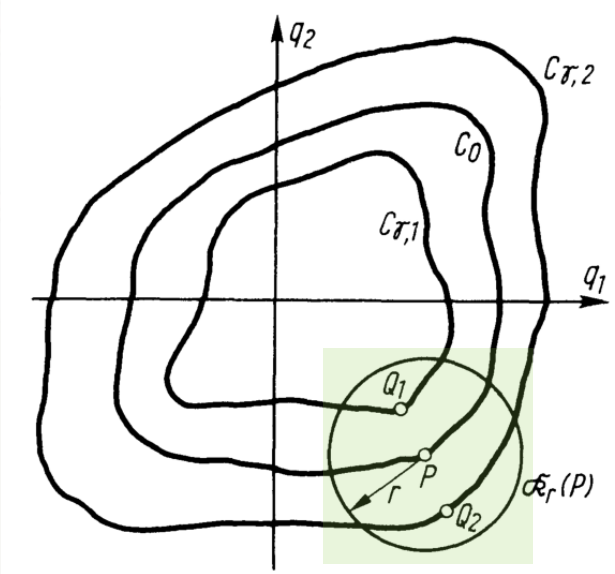
Terms, definitions, meanings related to vehicle handling

Other concepts of stability

- Orbital stability: “the undisturbed solution which corresponds to the phase curve C_0 , and the adjacent, disturbed solutions correspond to the phase curves $C_{\gamma,1}$ and $C_{\gamma,2}$, which remain entirely in the 'tube’, we have orbital stability” (geometrical concept)







- Input to state/output stability

- Practical stability

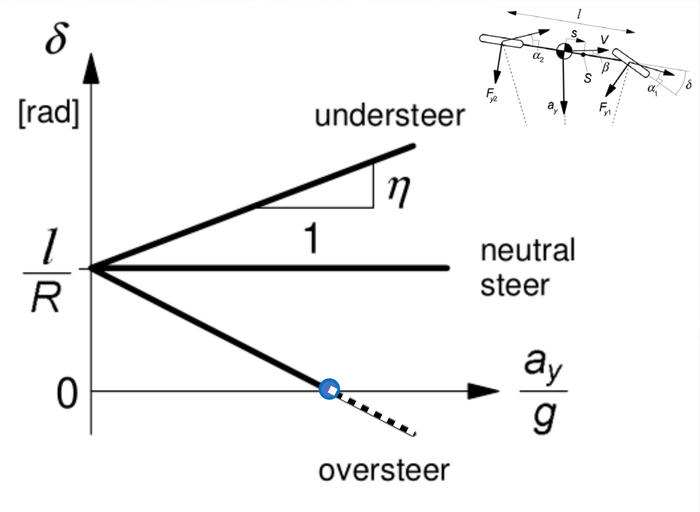
Methods in vehicle handling stability analysis

Simple linear vehicle model*

Stability of steady-state cornering

$$a_0\ddot{r} + a_1\dot{r} + a_2 r = b_1\delta$$

$$a_2 = C_1 C_2 l^2\left(1 + \eta\frac{V^2}{g\,l}\right) = C_1 C_2 l^2\left(\frac{\delta}{l/R}\right)_{ss} > 0$$



Divergent instability (spin-out without oscillations) for the oversteered vehicle exceeding the critical speed:

$$V < V_{crit} = \sqrt{\frac{g\,l}{-\eta}} \qquad (\eta < 0)$$

$$\eta = -\frac{mg}{l}\,\frac{aC_1 - bC_2}{C_1 C_2}$$

* HB Pacejka: Simplified Analysis of Steady-state Turning Behaviour of Motor Vehicles, Part 1-3, Vehicle System Dynamics, 1973

Methods in vehicle handling stability analysis

Basic vehicle + driver model

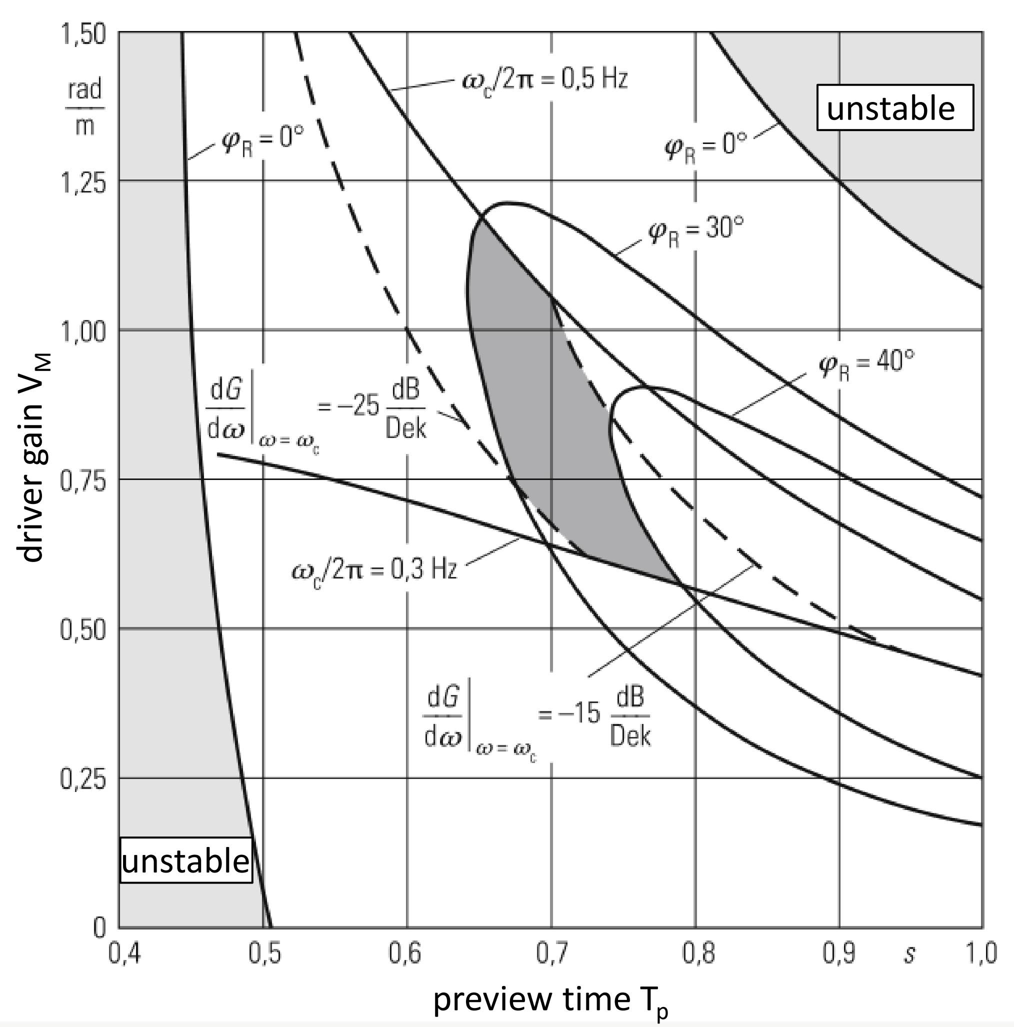
Basic driver model

$$M_R(s) = \frac{\delta_L(s)}{\Delta y(s)} = V_M \frac{1 + T_D s}{1 + T_I s} e^{-s\tau}$$

cross-over behaviour (vehicle-driver adaptation)

$$G(j\omega) = M_R \cdot F_\delta = \frac{\omega_c}{j\omega} e^{-j\omega\tau}$$

with
preview length $L = v\,T_p$
predicted path deviation Δy
neuromuscular delay time T_I, dead/reaction time τ

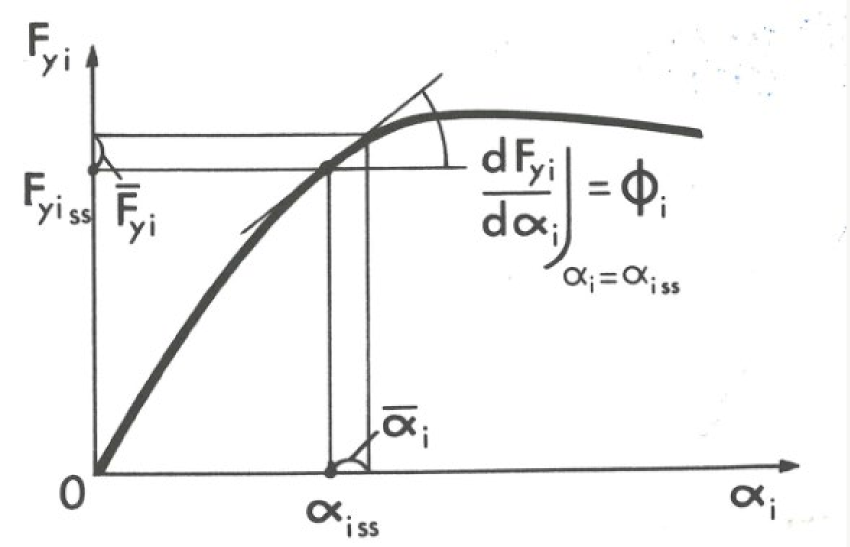


M Mitschke, H Wallentowitz: Dynamik der Kraftfahrzeuge, Springer, 2014

Methods in vehicle handling stability analysis

Extension to simple nonlinear vehicle model*

Stability of steady-state cornering



$$m^2k^2V^2\lambda^2 + m\left\{(a^2+k^2)\phi_1 + (b^2+k^2)\phi_2\right\}V\lambda + \ell^2\phi_1\phi_2 - m(\phi_1 a - \phi_2 b)V^2 = 0$$

Conditions for stability:

$$\ell^2\phi_1\phi_2 - m(\phi_1 a - \phi_2 b)V^2 > 0$$

and

$$(a^2+k^2)\phi_1 + (b^2+k^2)\phi_2 > 0.$$

↔

$$\phi_1\phi_2 \frac{\partial\delta}{\partial \ell/R} > 0$$

* HB Pacejka: Simplified Analysis of Steady-state Turning Behaviour of Motor Vehicles, Part 1-3, Vehicle System Dynamics, 1973

Methods in vehicle handling stability analysis

Extension to simple nonlinear model*

Stability of steady-state cornering

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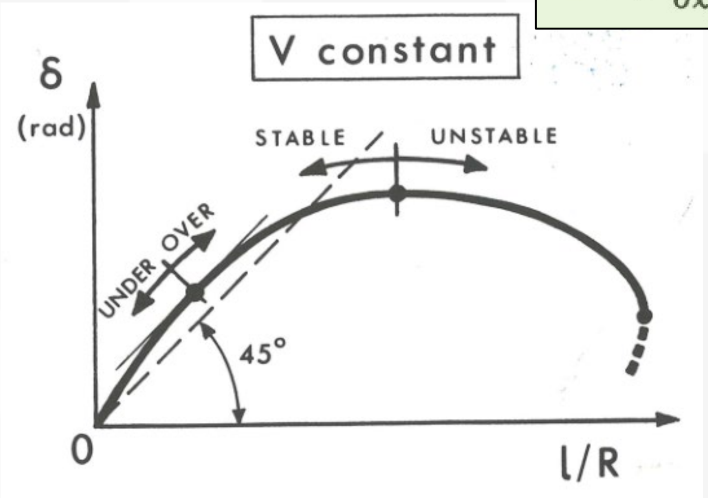
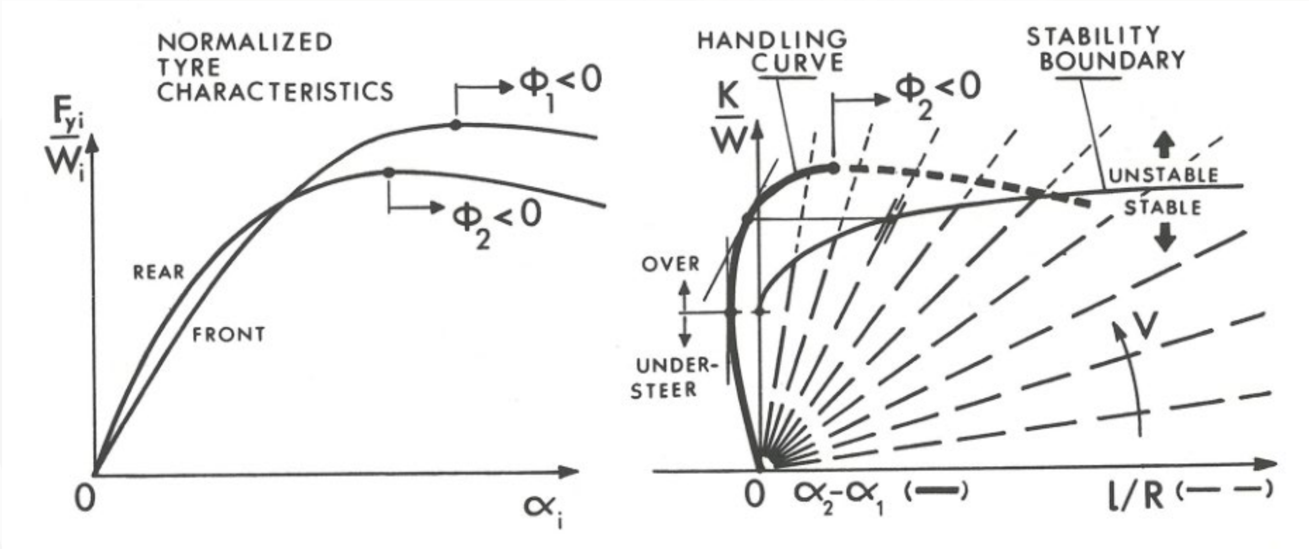




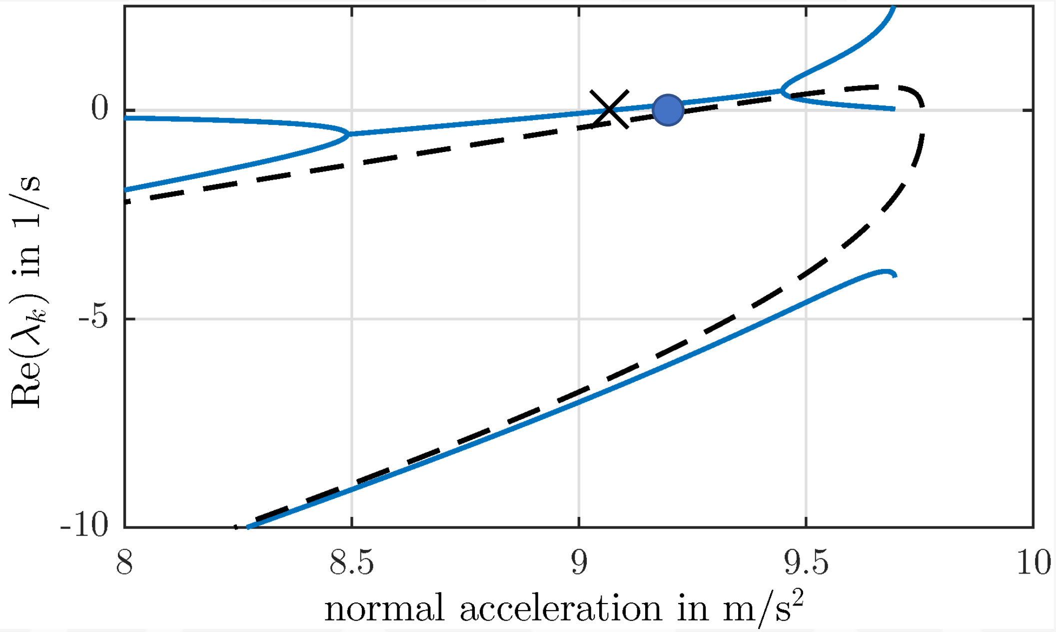
* HB Pacejka: Simplified Analysis of Steady-state Turning Behaviour of Motor Vehicles, Part 1-3, Vehicle System Dynamics, 1973

Methods in vehicle handling stability analysis

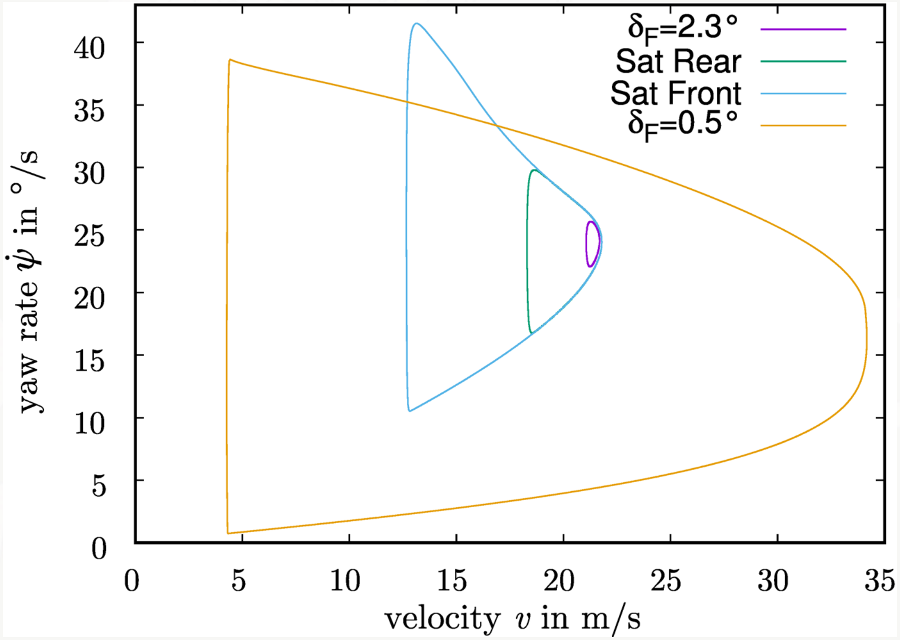
Extension to 4 DOF (v, ω_R, r, β) nonlinear 2-wheel vehicle model (RWD, OS)*

Stability of steady-state cornering

Hopf point at 4 DOF model (X), zero eigenvalue at 2 DOF model (●)



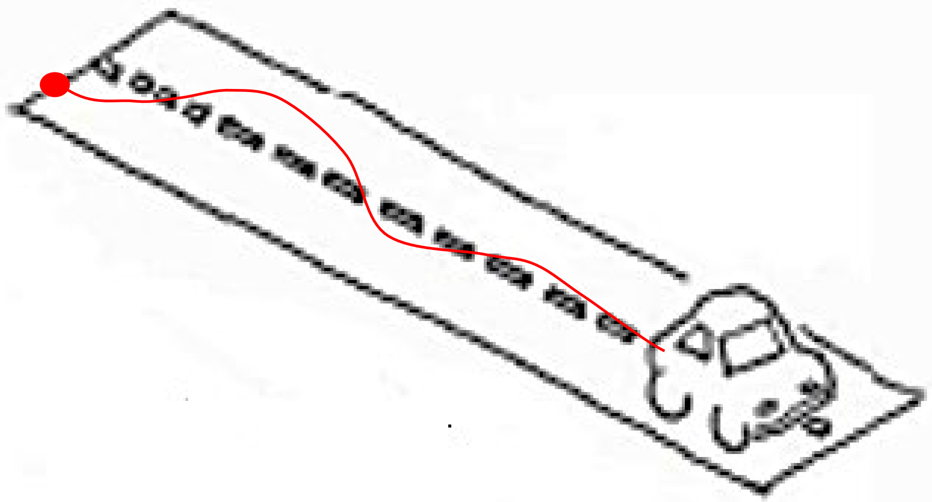
Periodic solutions: Bifurcation parameter δ_F



* J Edelmann, A Steindl, M Plöchl: Limit cycles at oversteer vehicle, *Nonlinear Dynamics,* 2019

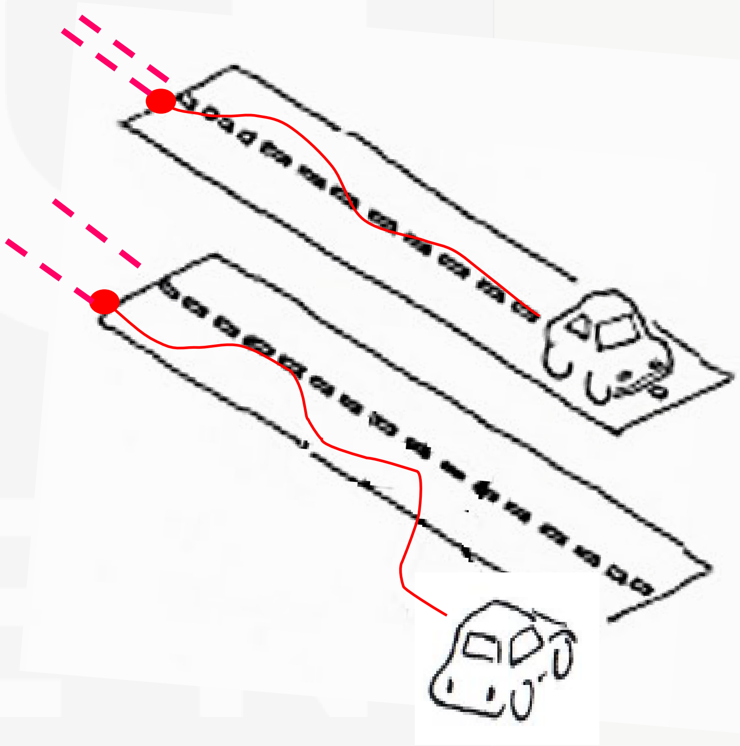
Methods in vehicle handling stability analysis

Small disturb



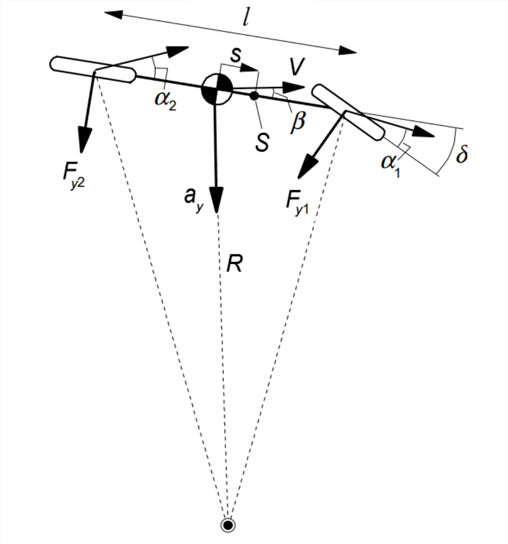
Methods in vehicle handling stability analysis

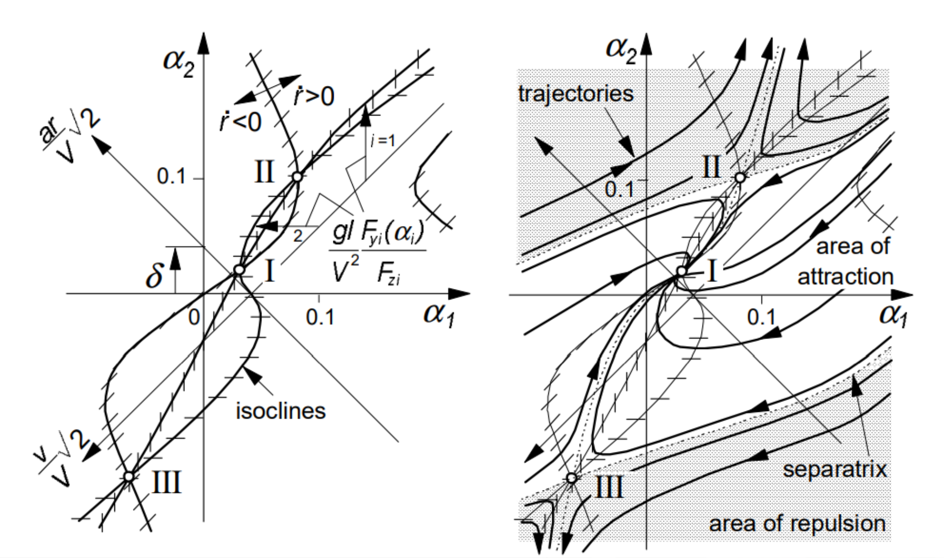
Big disturb

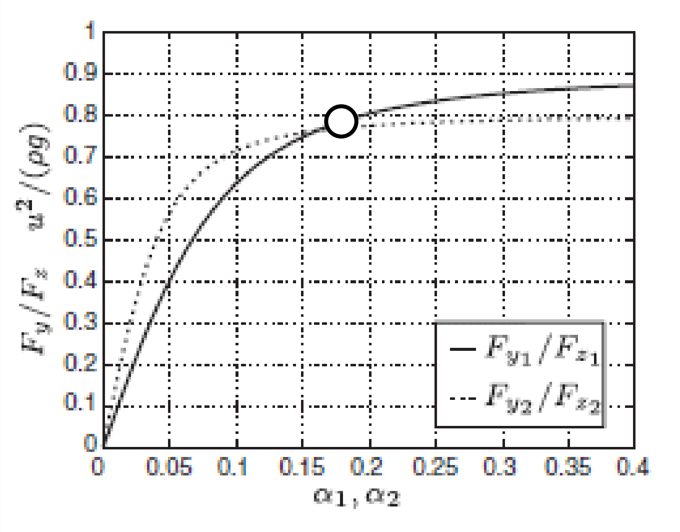


Methods in vehicle handling stability analysis

Response to big disturb





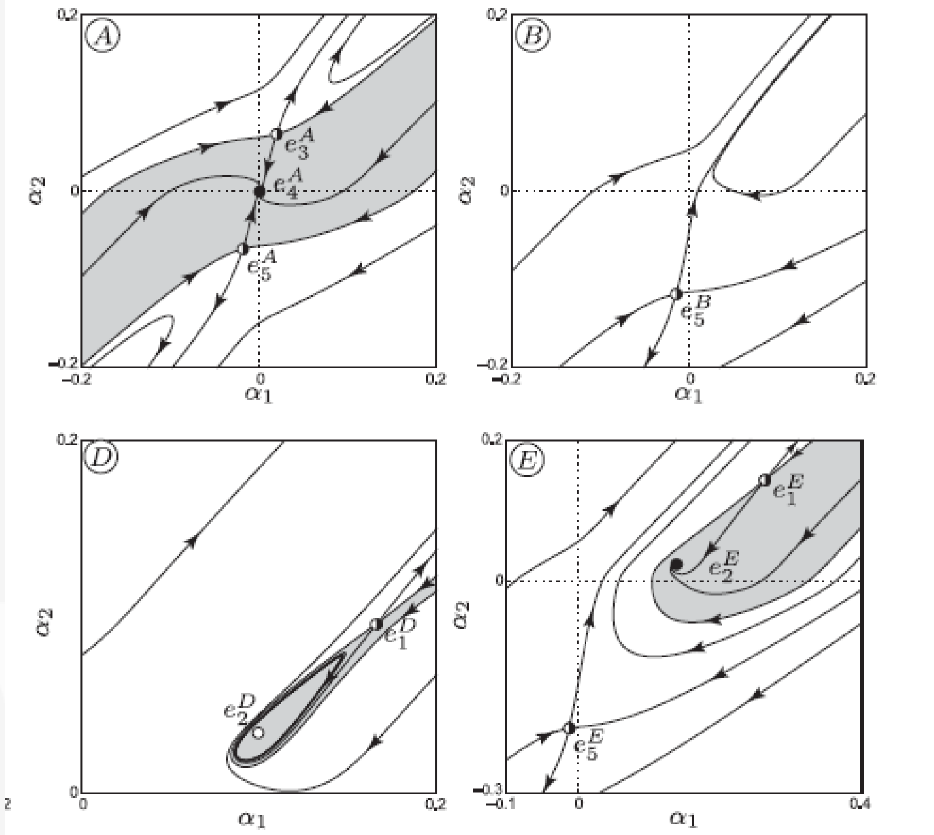


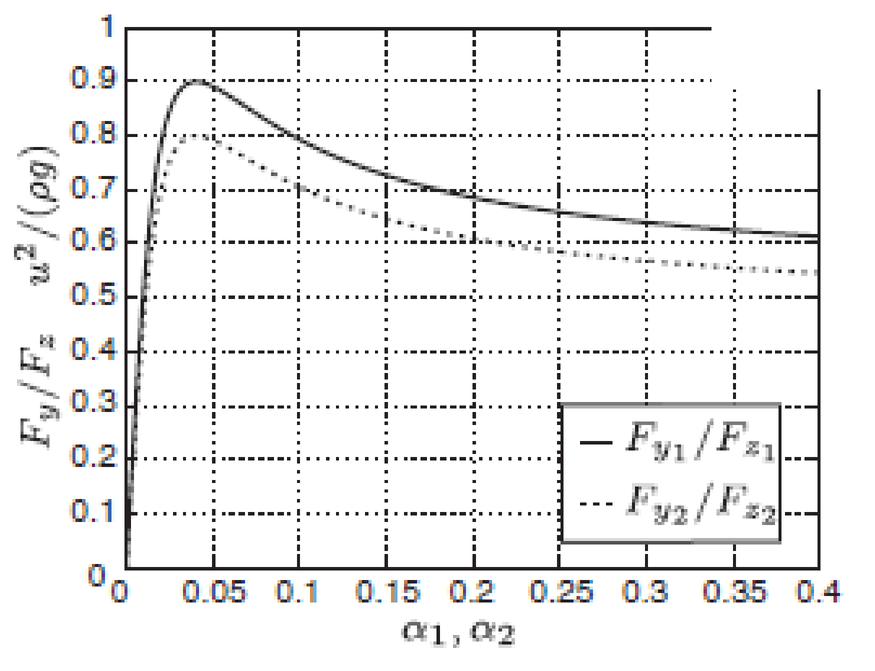
How big and what shape of the domain of attrction?

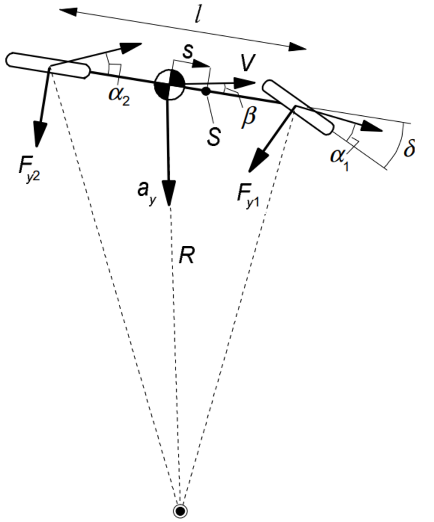
HB Pacejka: Simplified Analysis of Steady-state Turning Behaviour of Motor Vehicles, Part 1-3, Vehicle System Dynamics, 1973

Methods in vehicle handling stability analysis

Response to big disturbance – fixed control (oversteering vehicle)



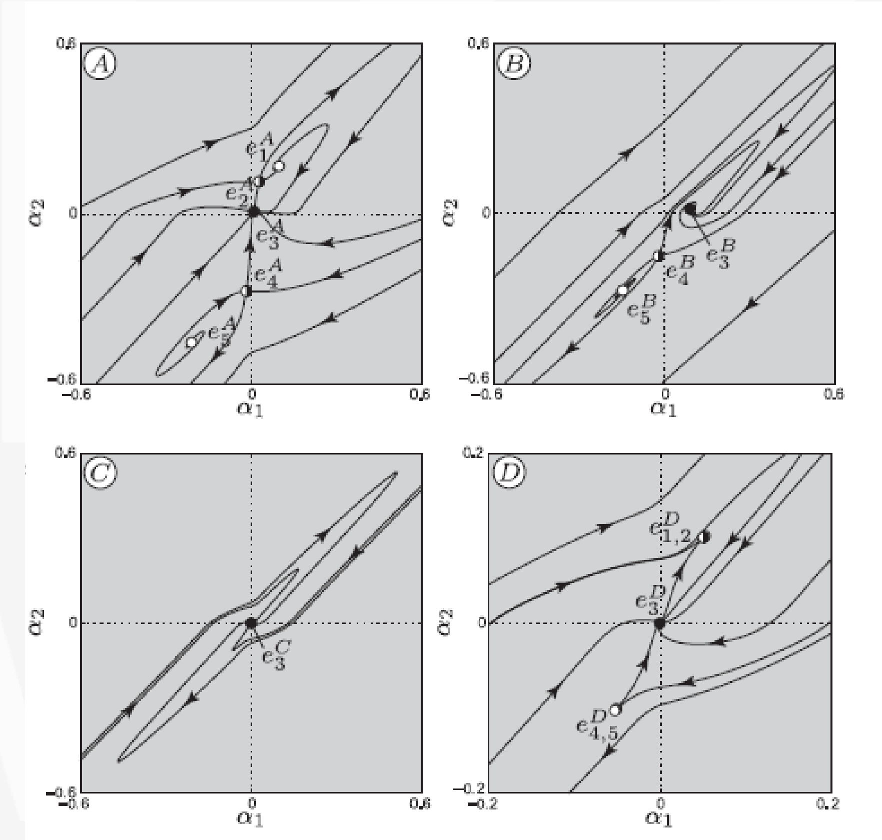


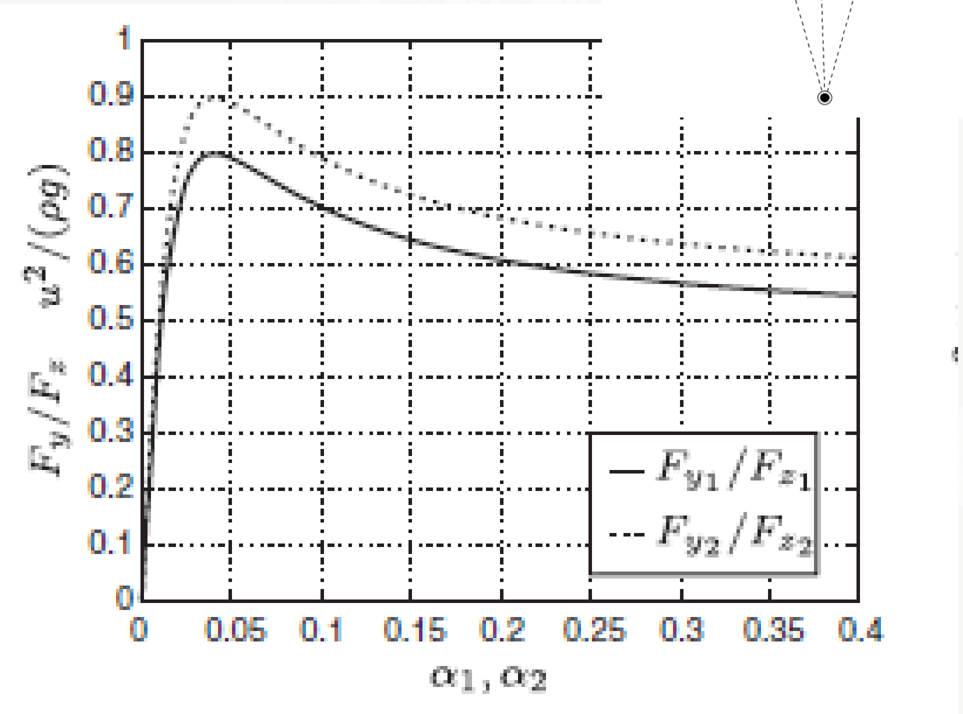


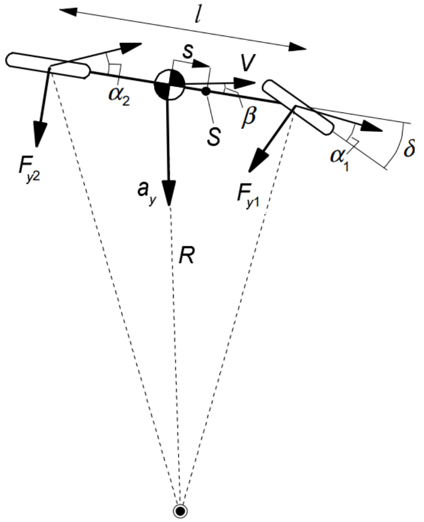
Della Rossa, F., Mastinu, G., Piccardi, C.: Bifurcation analysis of an automobile model negotiating a curve. Vehicle Syst. Dyn. 50(10), 1539–1562 (2012)

Methods in vehicle handling stability analysis

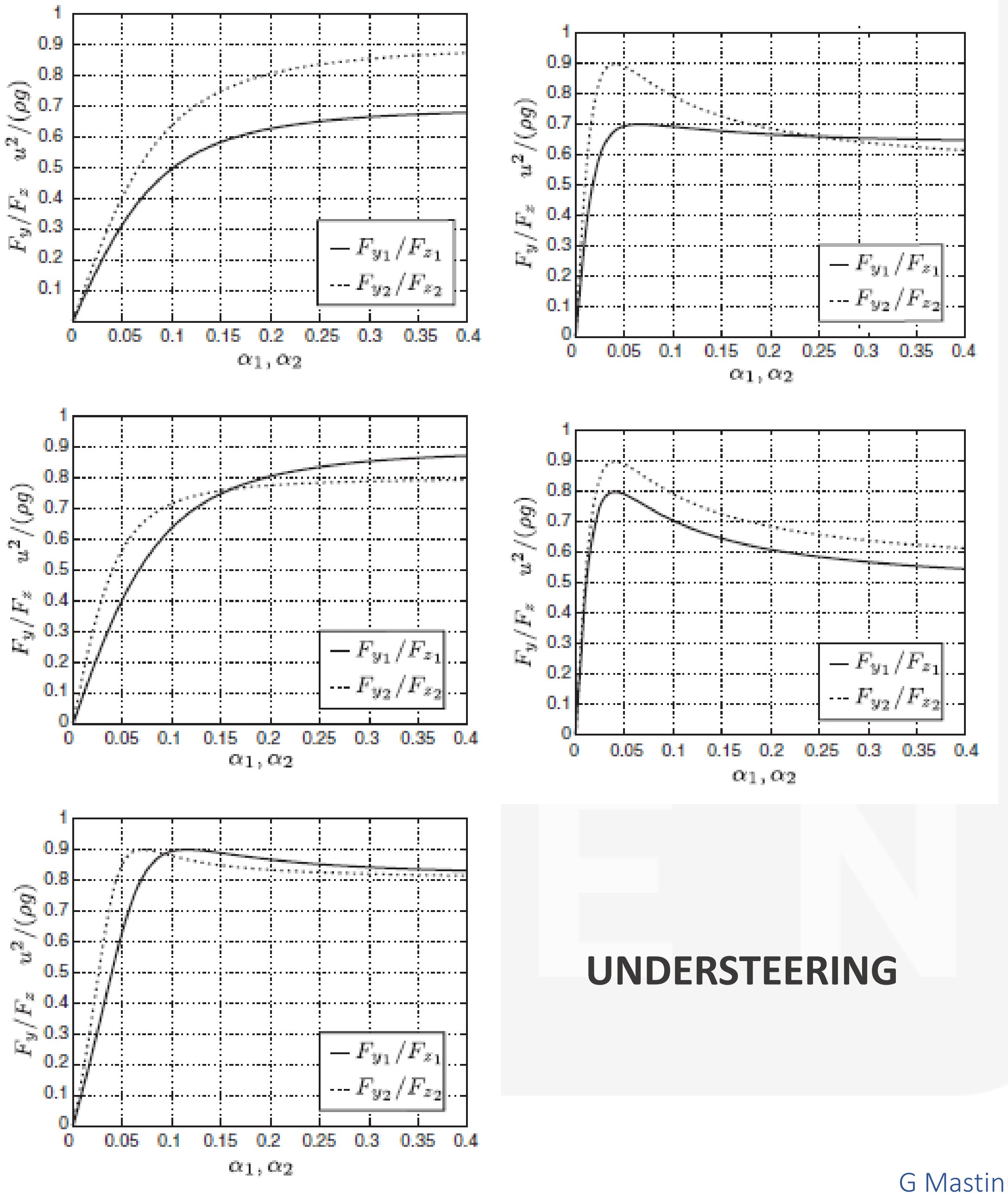
Response to big disturbance – fixed control (understeering vehicle)



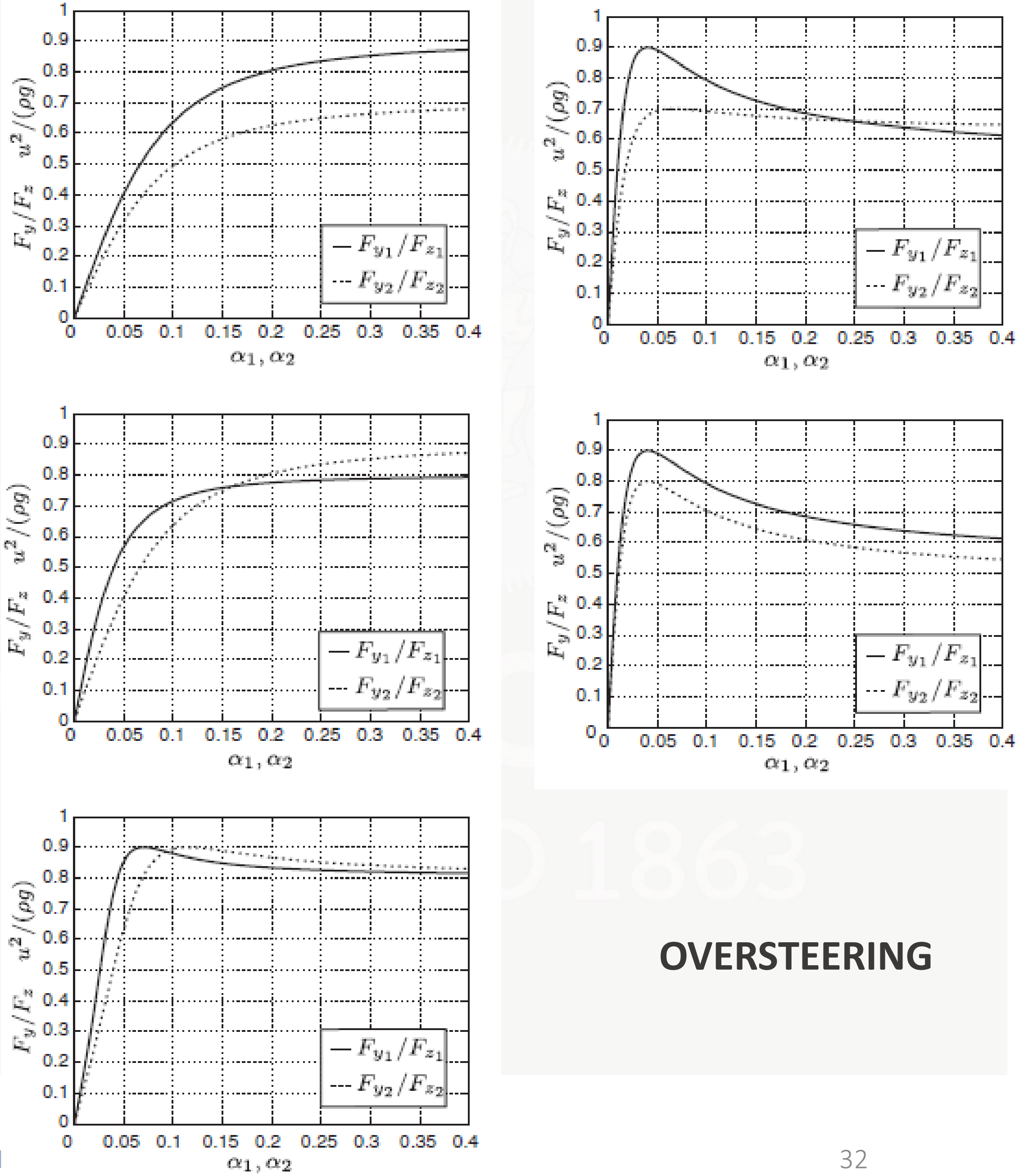




Della Rossa, F., Mastinu, G., Piccardi, C.: Bifurcation analysis of an automobile model negotiating a curve. Vehicle Syst. Dyn. 50(10), 1539–1562 (2012)



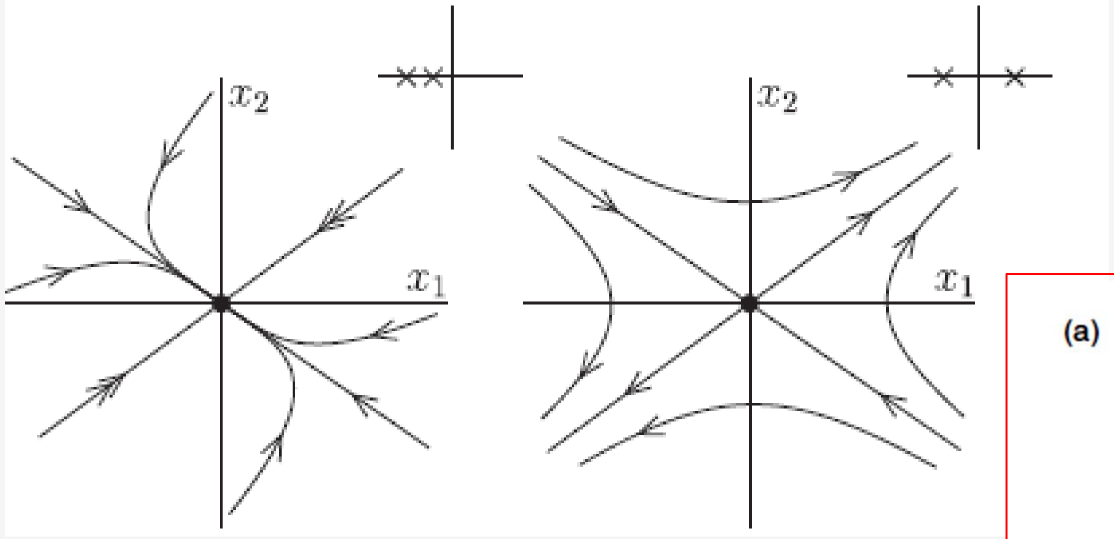
UNDERSTEERING



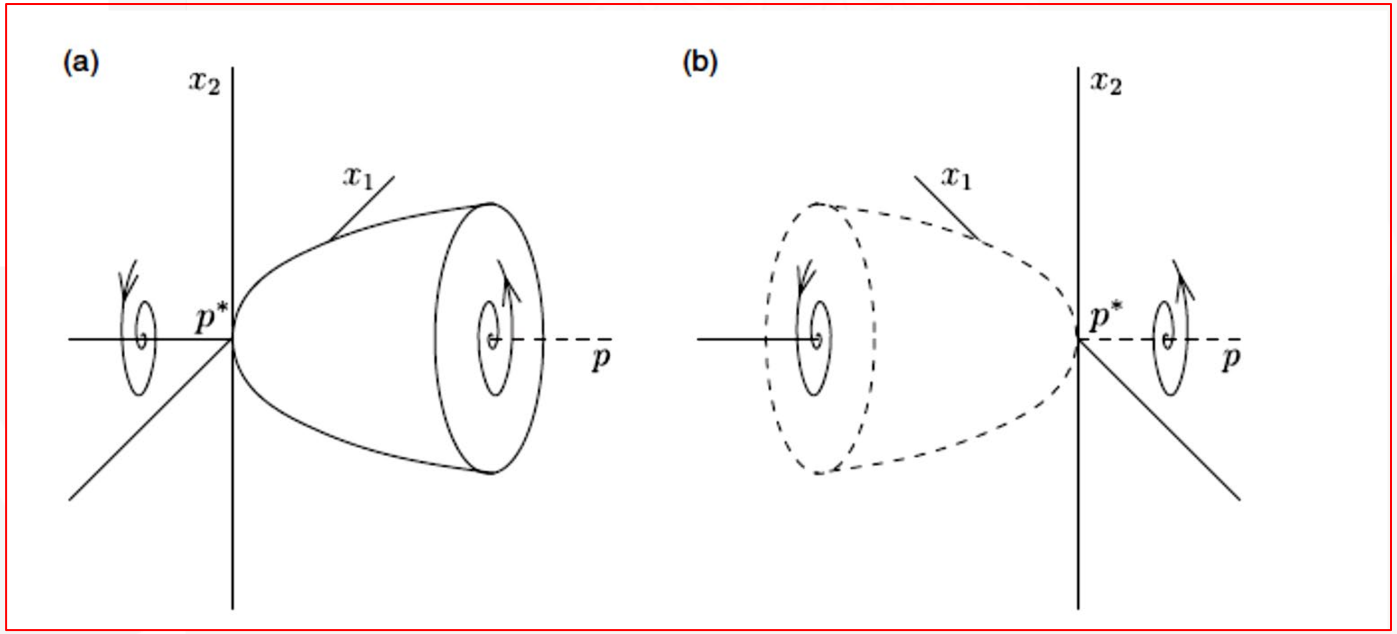
OVERSTEERING

Methods in vehicle handling stability analysis

Bifurcation analalysis = Stability analysis when system's parameters are varied



Hopf Bifurcations normal for vehicle and driver !



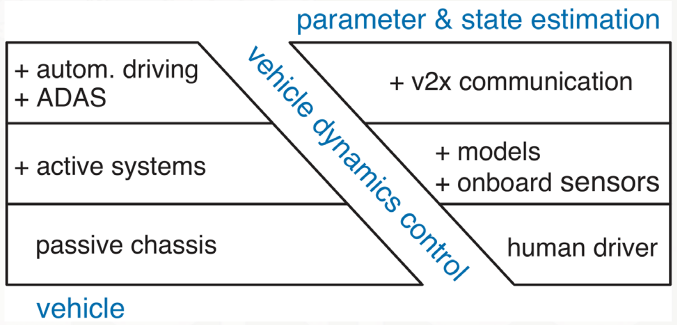
Associated challenges (in brief)

Increasing demands on model/control accuracy, calculation times, robustness

Models for appropriate references: what do we wish? what can we reach?
for internal control models, e.g. (n)MPC
for offline controller design
should be reliable until the limit of handling

New approaches (incorporating physics)
data driven methods, machine learning, NNs (e.g. PINNs)

Parameterisation of models and controllers
parameters from prototype testing
continuous onboard parameter estimation
integration of data from vehicle fleet, cloud, digital twins
machine learning
integration of updates



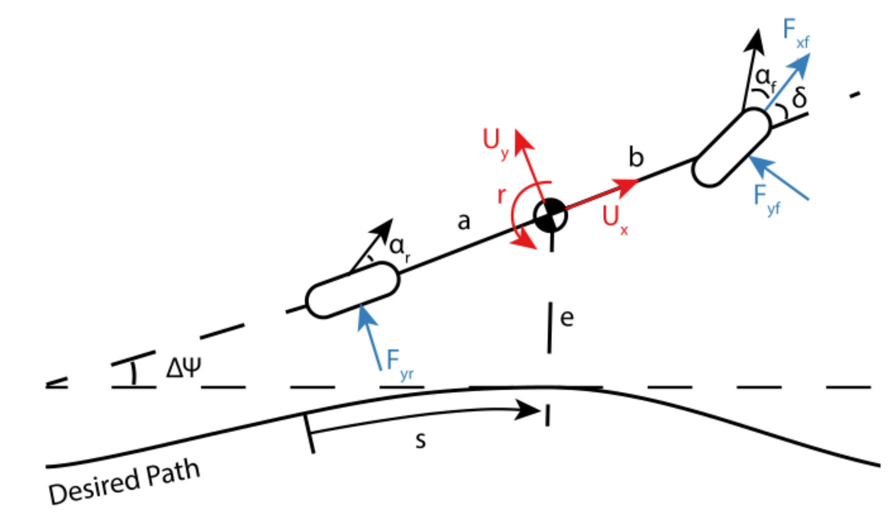
Example: Tyre–road contact (friction potential estimation):
- estimators and virtual sensors: small excitation -> sensors
- new sensors (wet road detection…)
- machine learning, NNs
- digital friction map / v2x / dynamic cloud
- ➢ sensor fusion

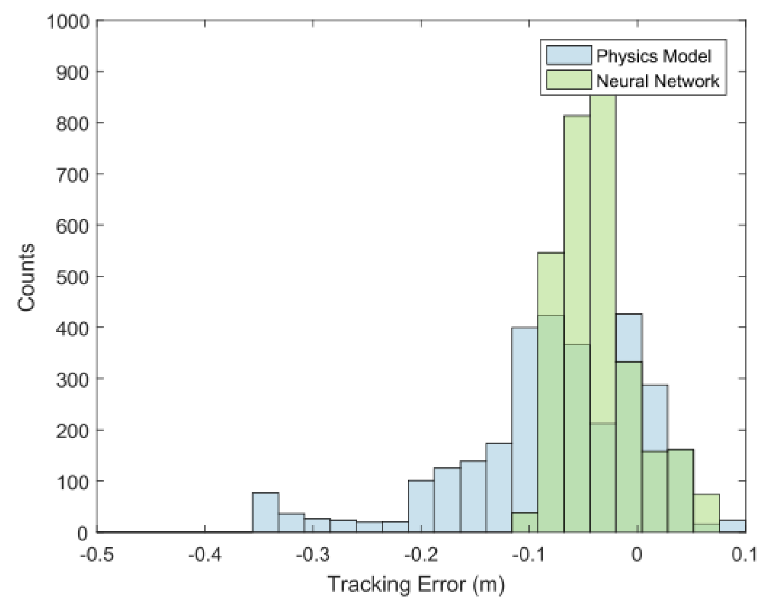
Associated challenges (in brief)

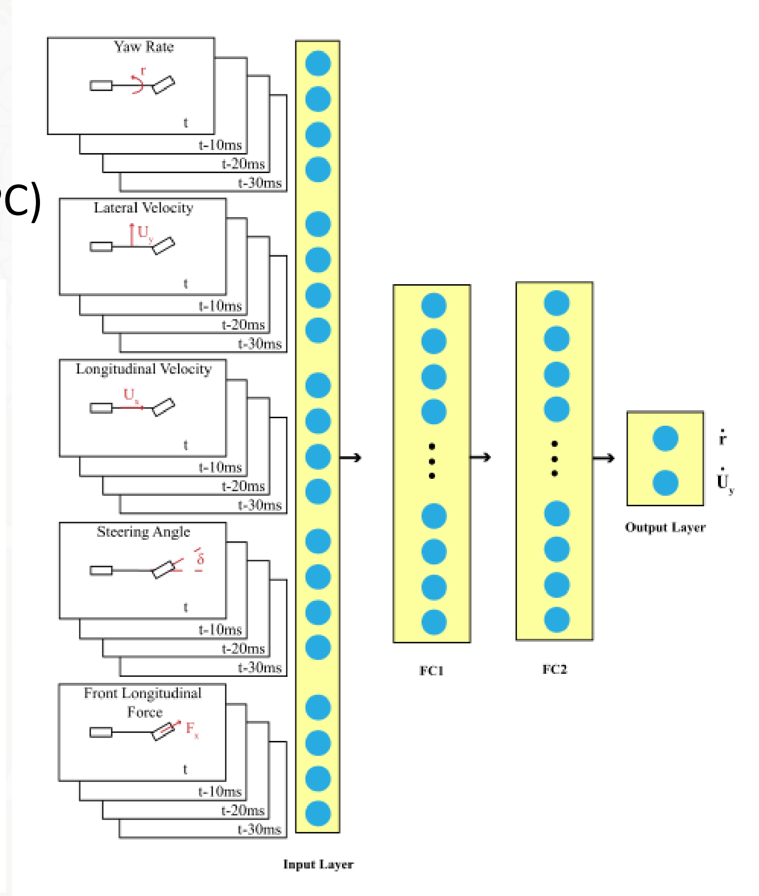
New approaches

Example: Neural Network Model Predictive Motion Control (NNMPC)*

Approach for constructing the vehicle model, selecting training data, and incorporating the learned neural network (NN) model in a nonlinear solver (MPC)







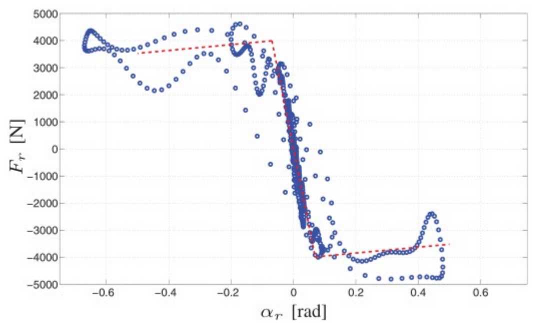
* NA Spielberg, M Brown, JC Gerdes: Neural Network Model Predictive Motion Control Applied to Automated Driving With Unknown Friction, IEEE Trans. Control Syst. Technol., 2022

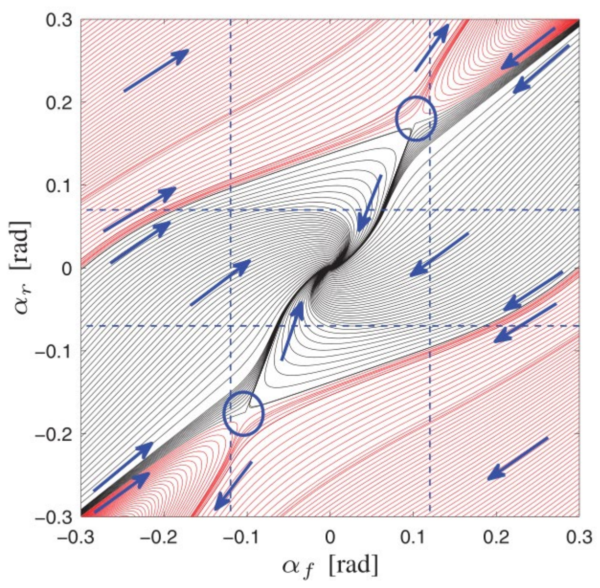
Associated challenges (in brief)

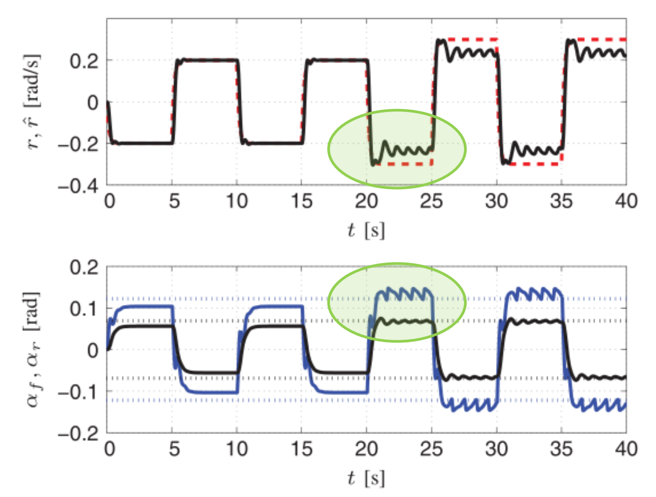
Integrated control, Control allocation

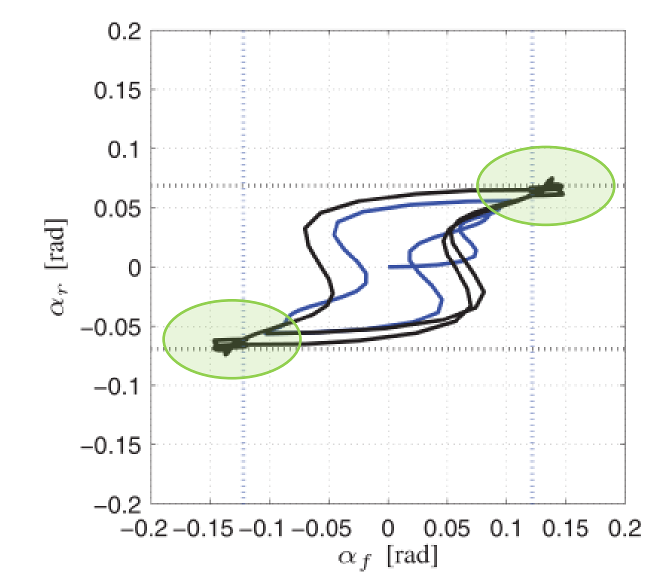
Example:

MPC based control strategy to improve yaw stability with
 steering input from the driver,
   and
 active front steering,
 and differential braking









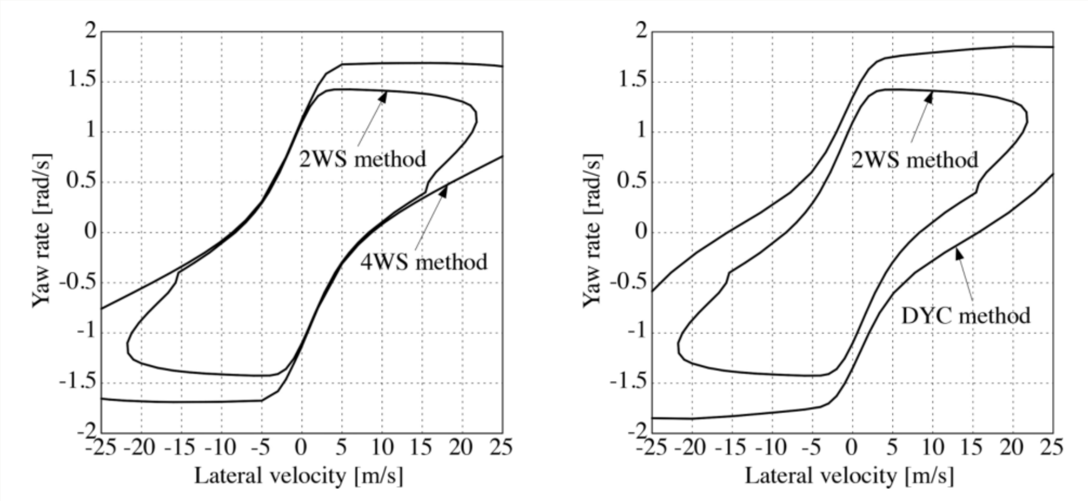
* S Cairano, E Tseng, D Bernardini, A Bemporad: Vehicle Yaw Stability Control by Coordinated Active Front Steering and Differential Braking in the Tire Sideslip Angles Domain, IEEE Trans. Control Syst. Technol., 2013

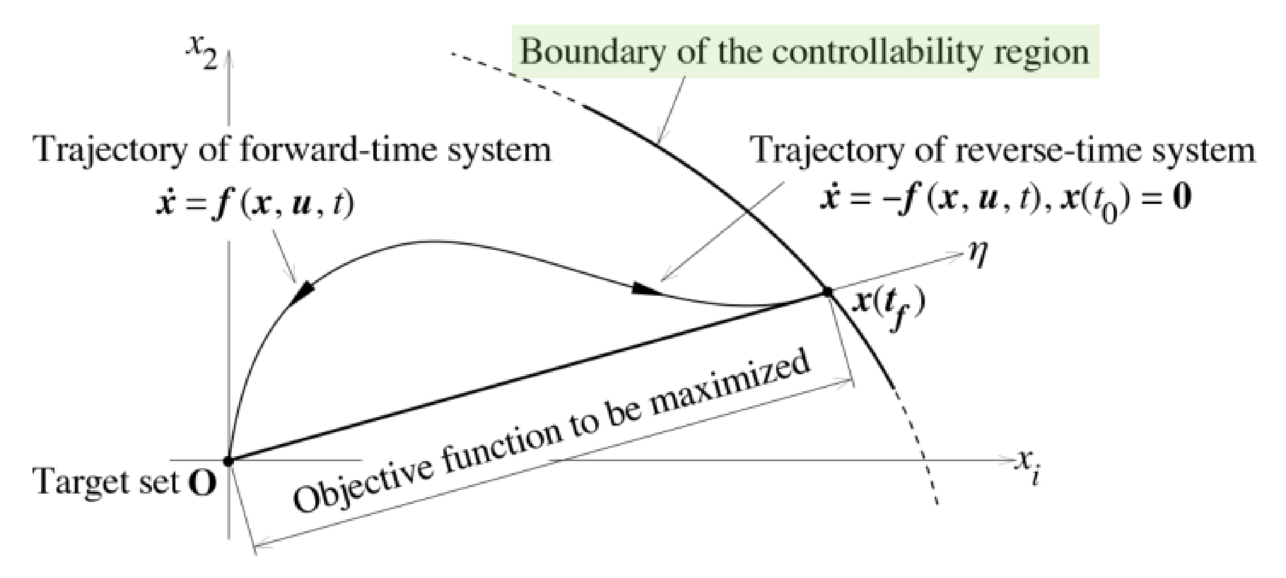
Associated challenges (in brief)

Controllability/reachability/observability (measures)
…related to stability

Is it possible to stablise the motion?
Is it possible to track a given path?
What reference states can the controller reach?
Which actuator (set) performs best?
Which sensors are required?

How to measure controllability/observability?





* S Horiuchi: Evaluation of chassis control algorithms using controllability region analysis. IAVSD 2016

Associated challenges (in brief)

Stable controller design: Time-delayed systems
Dynamics of vehicle stability control subjected to feedback delay*

Feedback control: $M_z(t) = K_v(v(t-\tau) - v_0) - K_r(r(t-\tau) - r_0)$

Linearized model with time delay: $\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{x}(t-\tau)$

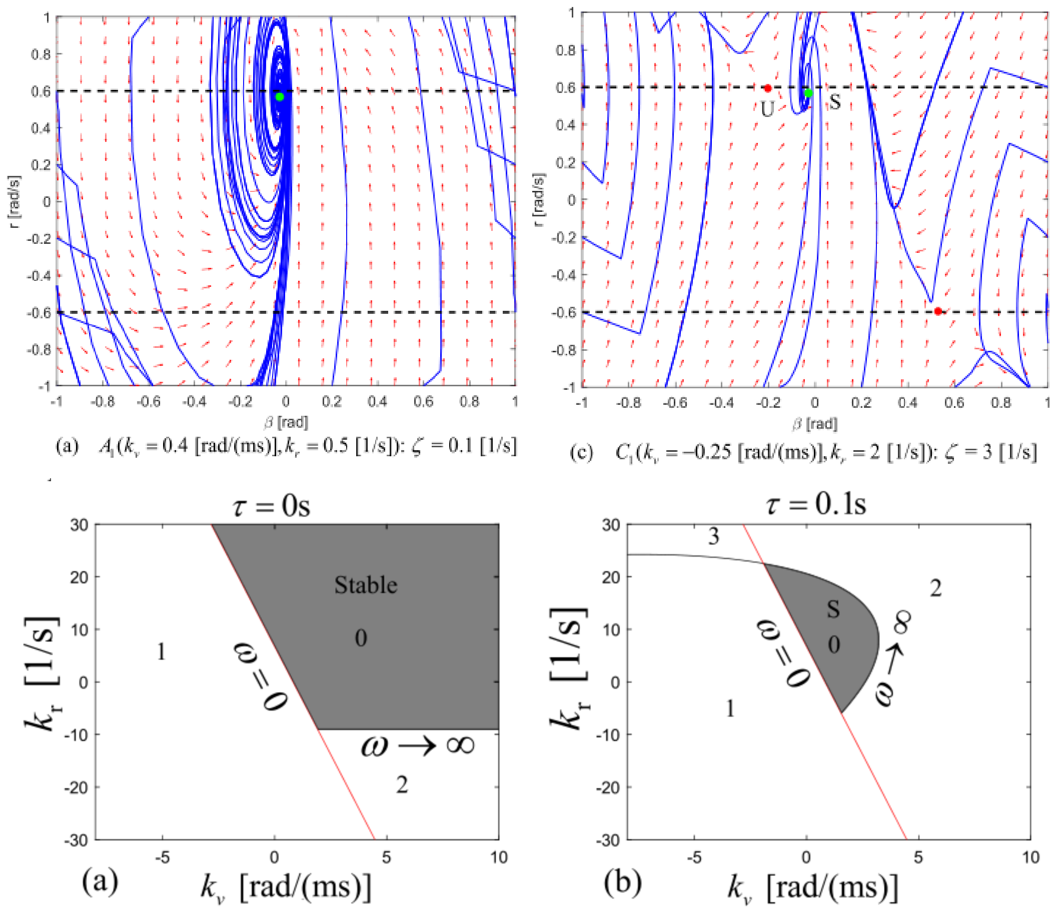
$\det(\lambda \mathbf{I} - \mathbf{A} - \mathbf{B}e^{-\lambda\tau})$

which gives

$$D(\lambda) := \lambda^2 - b_0\lambda - (a_{11}k_r + a_{12}k_v - \lambda k_r)e^{-\lambda\tau} + c_0 = 0$$

D-subdivision method: $\lambda = i\omega$

$$R(\omega) = -\omega^2 - (a_{11}k_r + a_{12}k_v)\cos(\omega\tau) + k_r\omega\sin(\omega\tau) + c_0$$
$$S(\omega) = -b_0\omega + (a_{11}k_r + a_{12}k_v)\sin(\omega\tau) + k_r\omega\cos(\omega\tau)$$



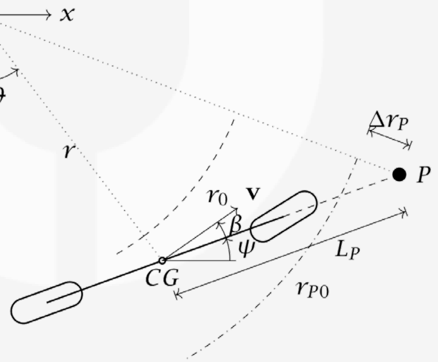
* H Lu, G Stepan, J Lu, D Takacs: Dynamics of vehicle stability control subjected to feedback delay, European Journal of Mechanics, 2022

Associated challenges (in brief)

Stable controller design: Time-delayed systems
Stability of a driver-vehicle system with steering & throttle control*

4 DOF (v, ω_R, r, β) nonlinear 2-wheel vehicle model (RWD, US)

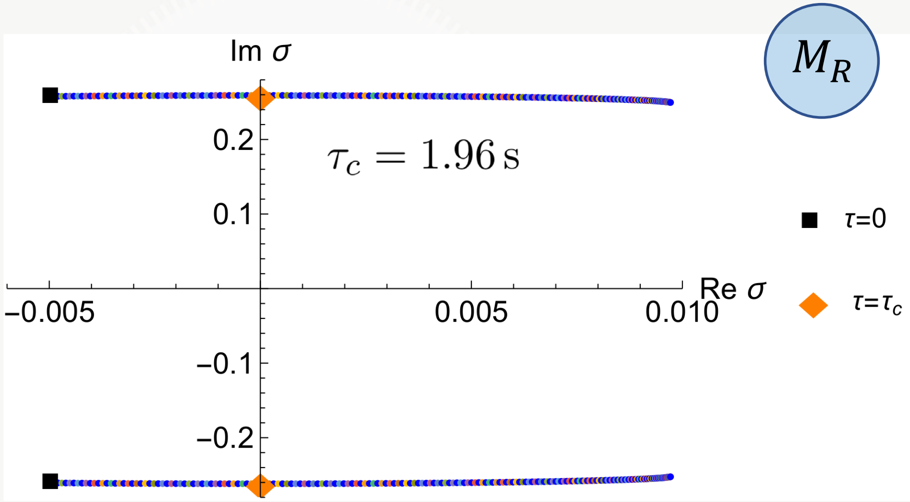
(Human) driver inputs
$u_i \in \{\delta_F, M_R\}$

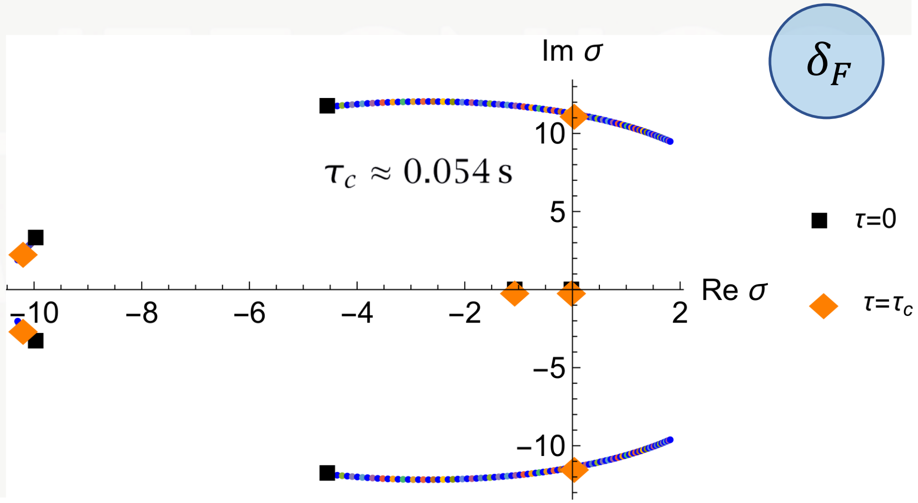


Simplified precision driver model

$$T_M \Delta \dot{u}_i + \Delta u_i = c_P \Delta r_P(t-\tau) + c_D \Delta \dot{r}_P(t-\tau)$$

with delay time T_M (fixed at 0.2 s)
human reaction time τ
and preview length L_P





* J Edelmann, M Plöchl, A Steindl: Stability of a driver-vehicle system with steering and throttle control, ENOC, 2022

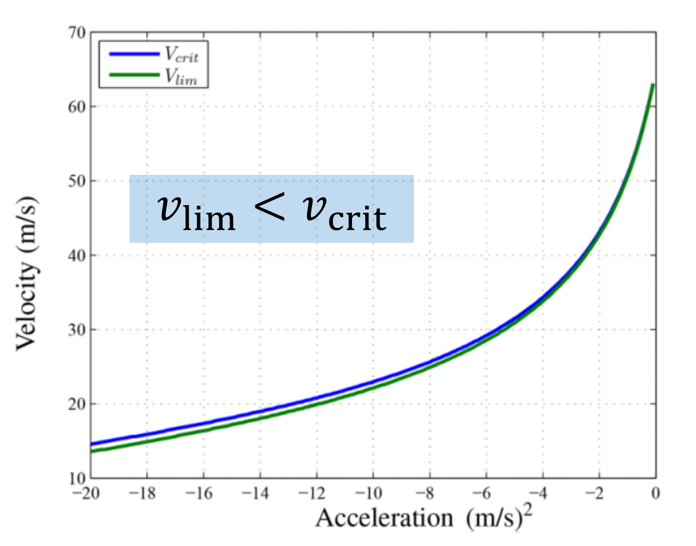
Associated challenges (in brief)

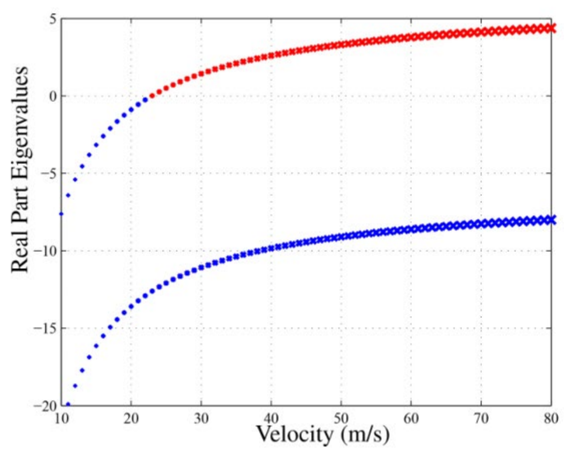
Stable controller design: Time-varying systems
Example: Stability in straight driving for braking conditions

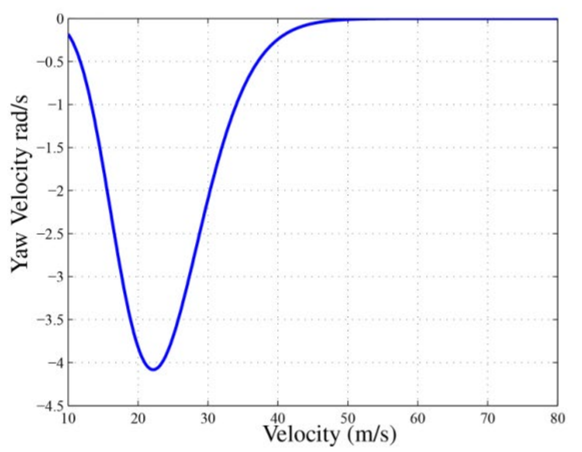
2-wheel OS vehicle model with linear* or locked front/rear tyres**:
time-varying linear system: $\dot{\boldsymbol{y}} = \boldsymbol{A}(t)$

→ "frozen eigenvalues" at time instant t_j
- stable (in the finite time interval $|t - t_j|$) below v_{crit}, unstable above v_{crit}

→ "frozen eigenvalues" for all t
- stable in the sense of Ljapunov
  (vehicle will return to straight motion)
- for OS vehicle* or locked rear tyres**
- "practically unstable" (large amplitude burst)





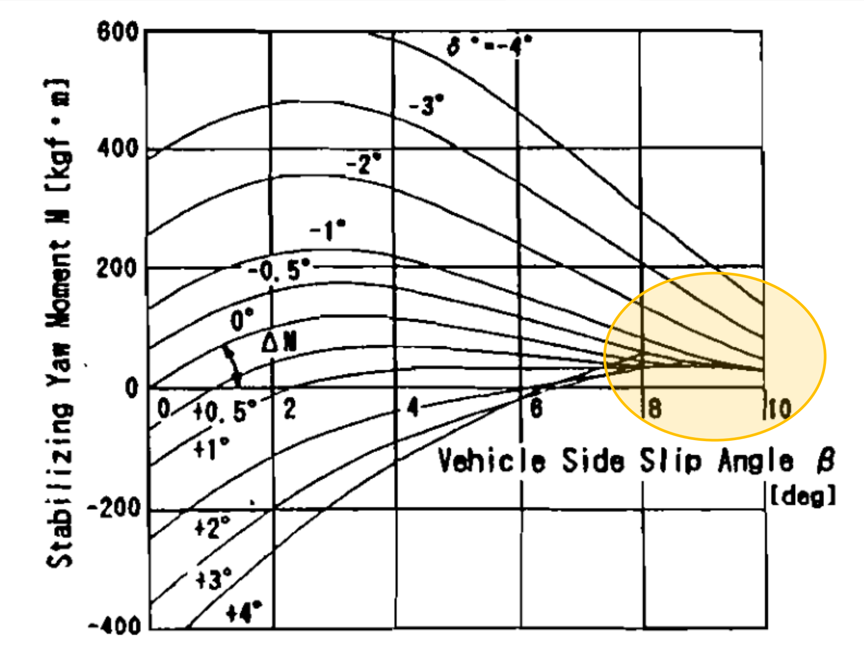


→ Ljapunov's Direct Method
→ check by simulation
→ H Rosenbrock: The stability of linear time-dependent control systems, 1963

* IG Salisbury, DJN Limebeer, A Tremlett, M Massaro: The unification of acceleration envelope and driveability concepts, IAVSD 2015
** HB Paceka, Tyre and vehicle dynamics, B.H. 2006

Open-loop, closed-loop(s) handling stability

Stability control: from ESC to autonomous driving



Y Shibahata, K Shimada, T Tomari: Improvement of Vehicle Maneuverability by Direct Yaw Moment Control, AVEC’92

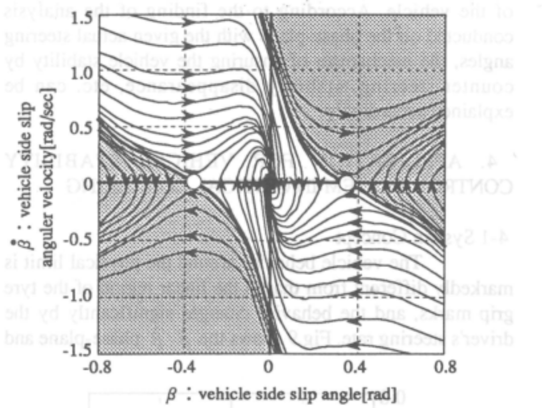


Fig.6 β - $\dot{\beta}$ phase-plane trajectory at V=100km/h, δ = 0rad

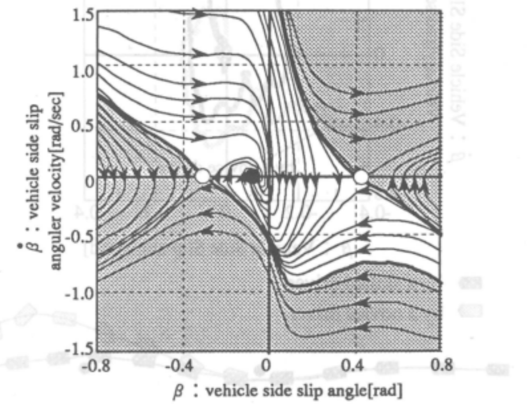


Fig.7 β - $\dot{\beta}$ phase-plane trajectory at V=100km/h, δ = 0.08rad

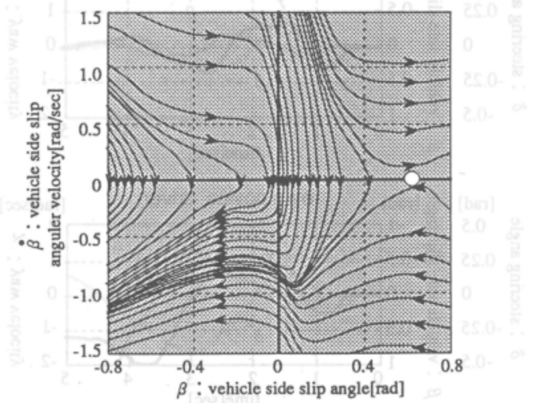
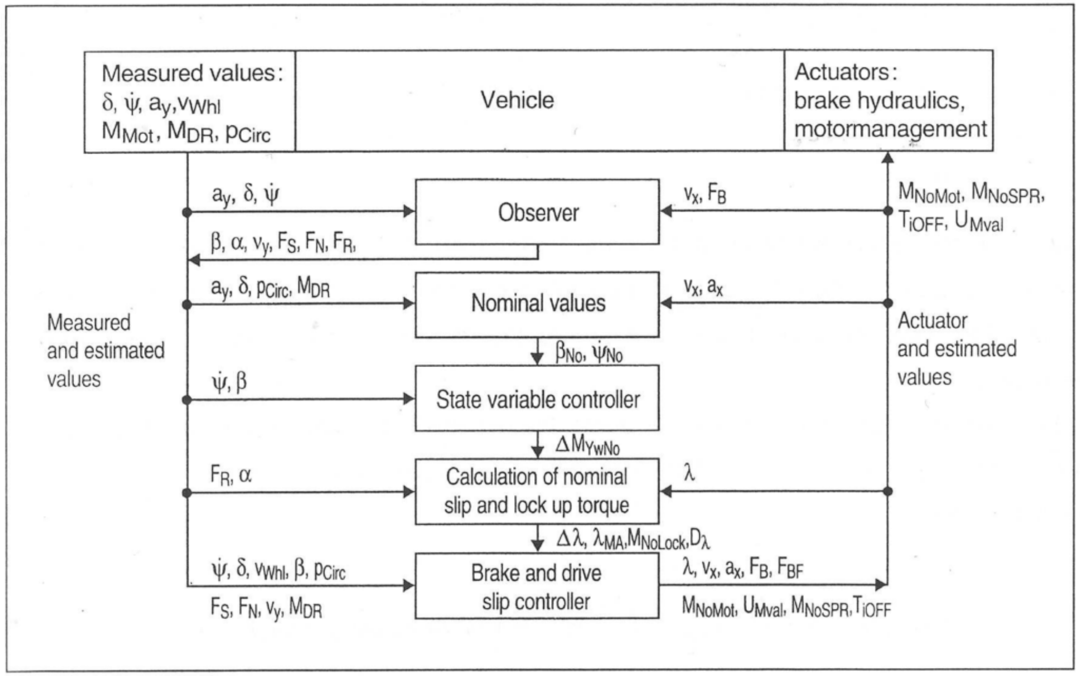


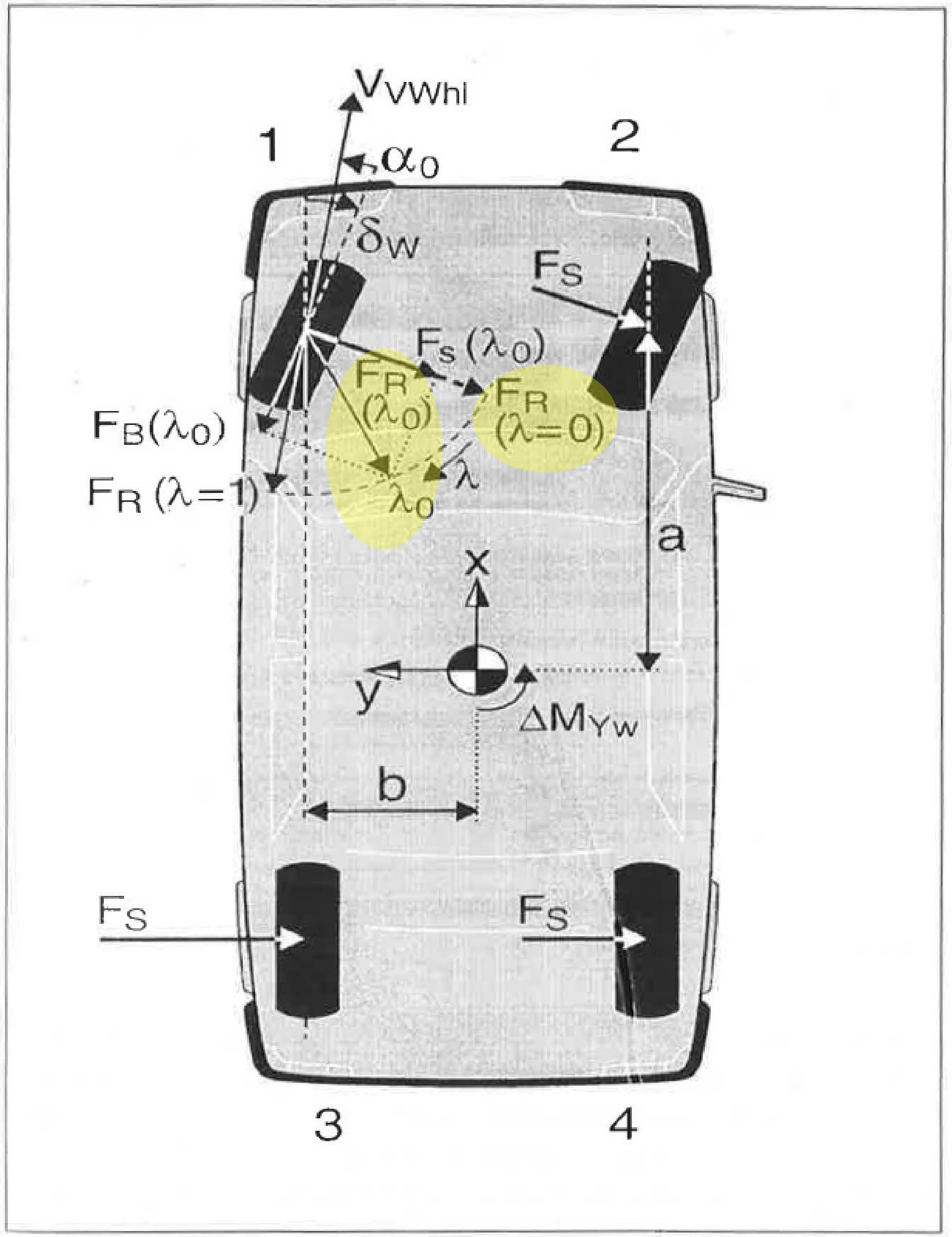
Fig.8 β - $\dot{\beta}$ phase-plane trajectory at V=100km/h, δ = 0.16rad

S Inagaki, I Kushiro, M Yamamoto: Analysis on vehicle stability in ctitical cornering using pahse-plane method, AVEC’94

Open-loop, closed-loop(s) handling stability

Stability control: from ESC to autonomous driving





A van Zanten, R Erhardt, G Pfaff, F Kost, U Hartmann, T Ehret: Control Aspects of the Bosch VDC, AVEC'96

Open-loop, closed-loop(s) handling stability

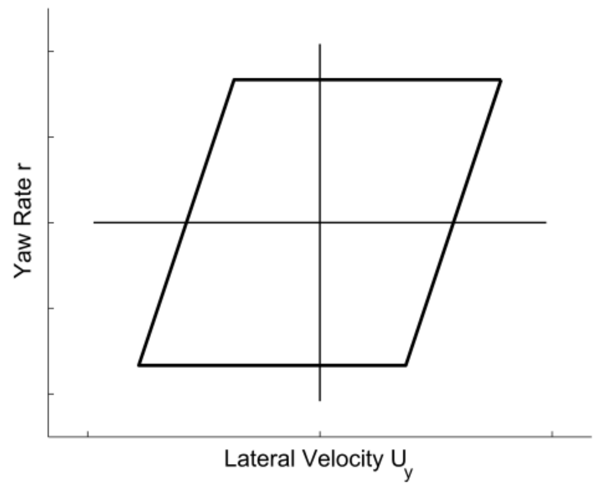
Stability control: from ESC to autonomous driving
Example: Combined path tracking, vehicle stabilization, and collision avoidance*

MPC optimisation calculates the front tyre force

$$\min \sum_{k=1}^{n} x^{k^T} Q^k x^k + v^{k^T} R^k v^k + \left(\sigma_{\text{veh}}^k\right)_+ W_{\text{veh}}^k + \ldots \left(\sigma_{\text{env}}^k\right)_+ W_{\text{env}}^k$$

$$\begin{aligned} H_{\alpha_r}^k x^k &\le G_{\alpha_r}^k \quad k = 1\ldots n \\ H_{\text{veh}}^k x^k &\le G_{\text{veh}}^k + \sigma_{\text{veh}}^k \quad k = 1\ldots n \\ H_{\text{env}}^k x^k &\le G_{\text{env}}^k + \sigma_{\text{env}}^k \quad k = 1\ldots n \\ \left|F_{yf}^k\right| &\le F_y f, \quad \max^k \quad k = 0\ldots n \\ |v^k| &\le v_{\max}^k \quad k = 1\ldots n \end{aligned}$$

- (Trust region on rear tyre slip angle)
- Stability Envelope
- Environmental Envelope
- Constraints on lateral force resp. steering angle (force)



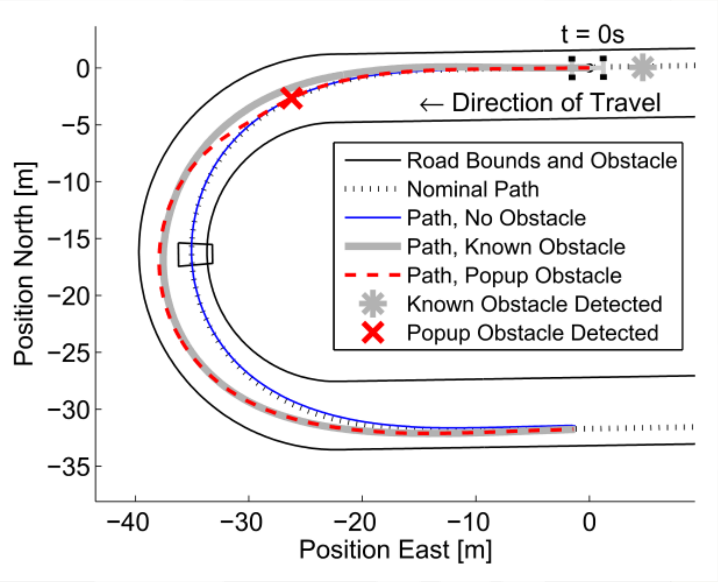
Stability envelope for vehicle stabilisation

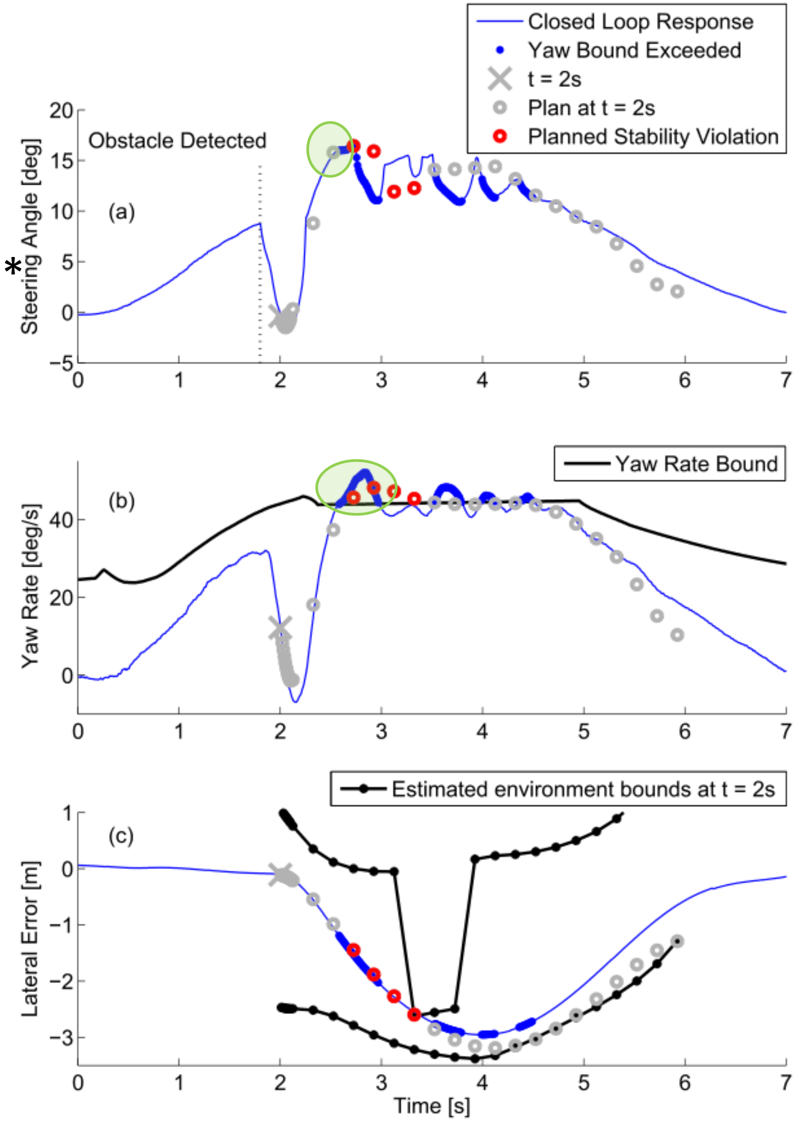
$W_{\text{env}} \gg W_{\text{veh}} \gg ||Q||_\infty$ encodes a priorisation of collision avoidance, then stability, and finally path tracking

* J Funke, M Brown, SM Erlien, JC Gerdes: Collision Avoidance and Stabilization for Autonomous Vehicles in Emergency Scenarios IEEE Trans. Control Syst. Technol., 2017

Open-loop, closed-loop(s) handling stability

Stability control: from ESC to autonomous driving
Example: Combined path tracking, vehicle stabilization, and collision avoidance*

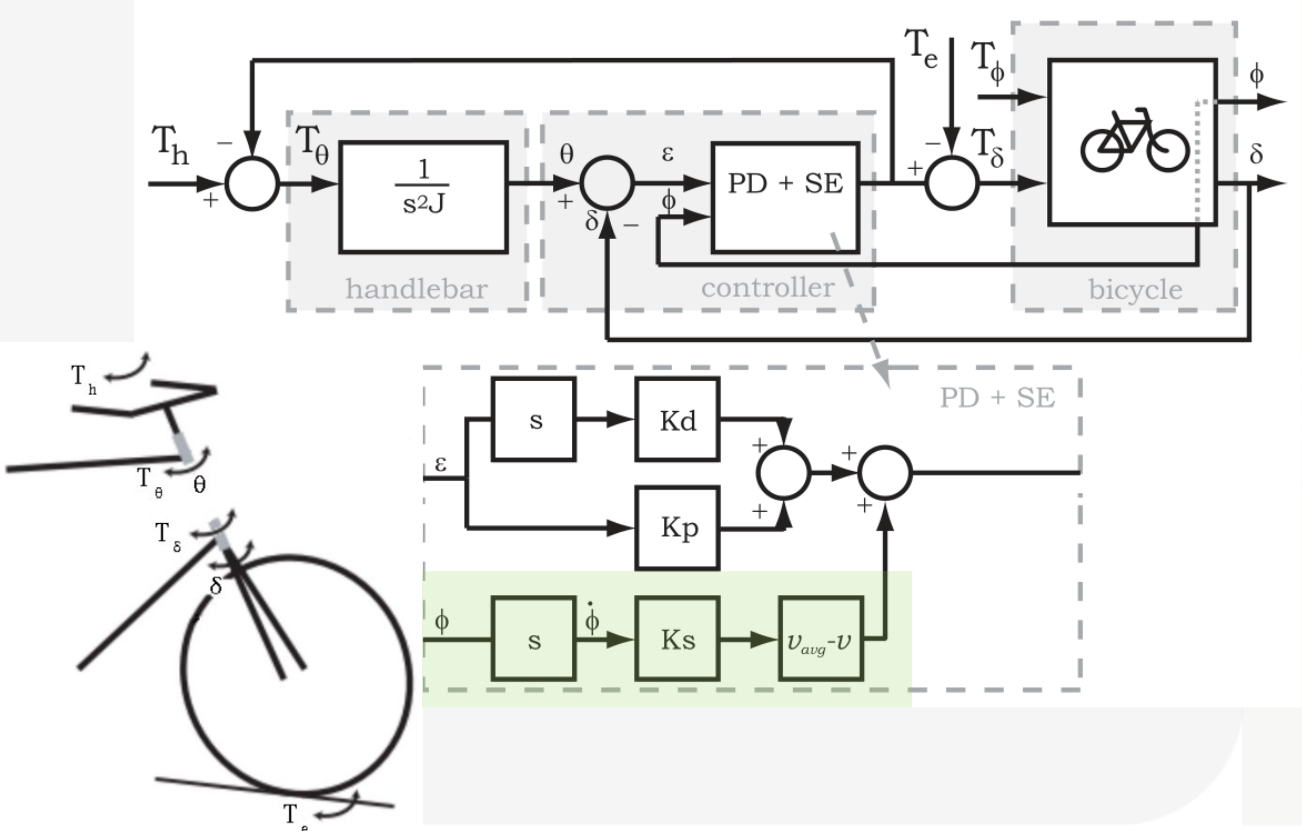


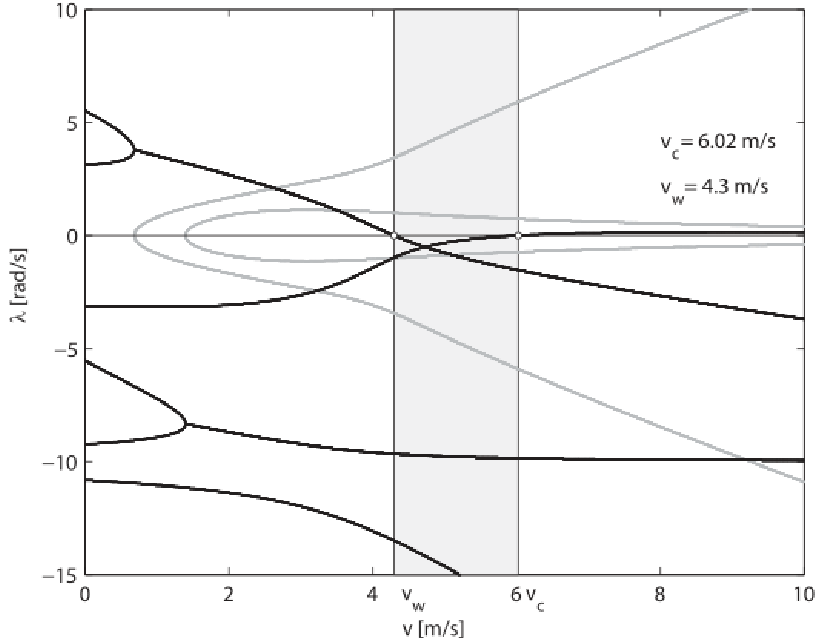


* J Funke, M Brown, SM Erlien, JC Gerdes: Collision Avoidance and Stabilization for Autonomous Vehicles in Emergency Scenarios IEEE Trans. Control Syst. Technol., 2017

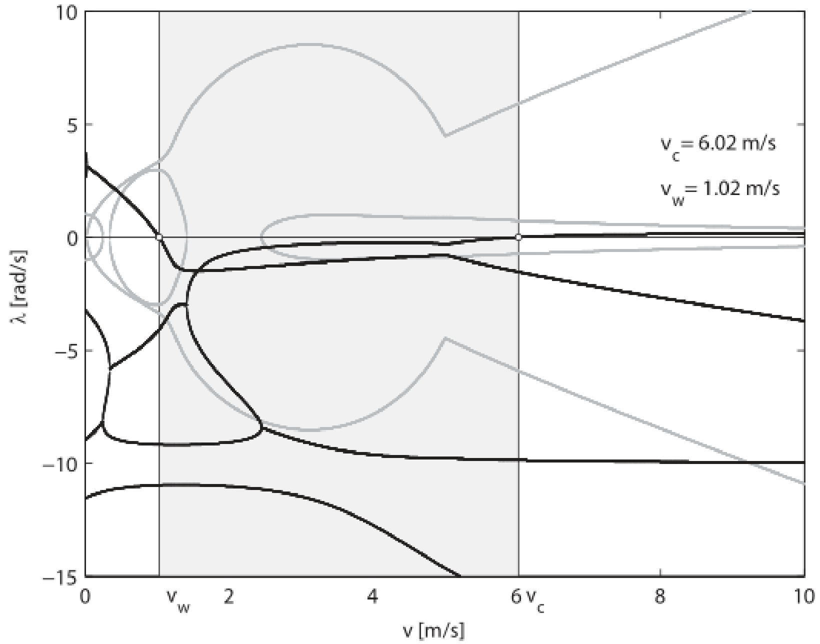
Open-loop, closed-loop(s) handling stability

Stability control: weave mode stabilization*





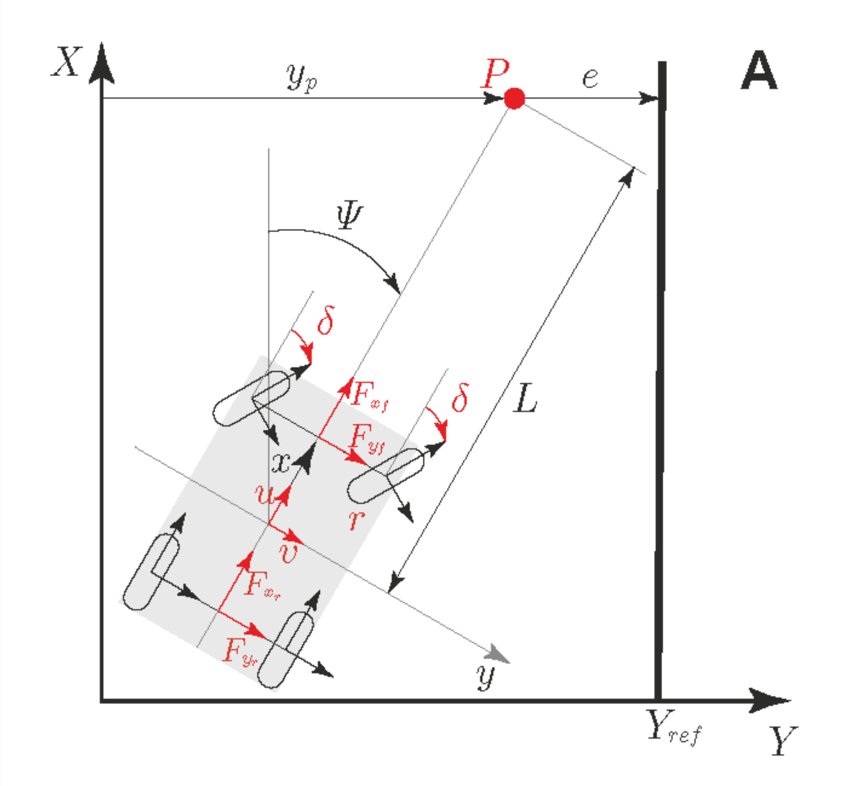
Eigenvalues of the steee-by-wire benchmark bicycle model



* AL Schwab, N Appleman: Dynamics and Control of a Steer-by-Wire Bicycle, BMD 2013

Open-loop, closed-loop(s) handling stability

Hopf bifurcation at high speed either for understeer or oversteer vehicles



$$\dot{v} = -\frac{1}{mu}\left(B_f C_f D_f + B_r C_r D_r\right)v - \frac{1}{mu}\left(mu^2 + \left(aB_f C_f D_f - bB_r C_r D_r\right)\right)\mathrm{r}$$

$$\dot{r} = -\frac{1}{Ju}\left(aB_f C_f D_f - bB_r C_r D_r\right)\mathrm{v} - \frac{1}{Ju}\left(a^2 B_f C_f D_f + b^2 bB_r C_r D_r\right)r$$

$$\dot{\delta} = \frac{1}{\tau}\left(-\delta - k\left(y_G + L\psi\right)\right)$$

$$\dot{y_G} = v + u\psi$$

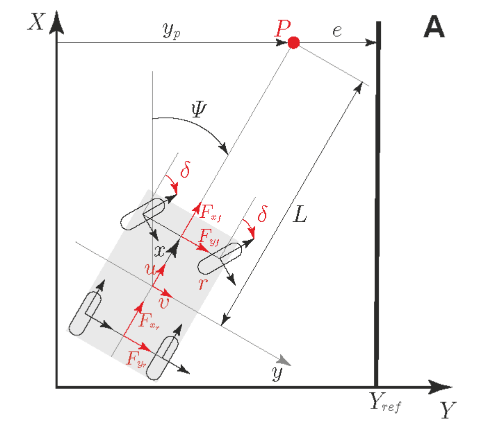
$$\dot{\psi} = r$$

Tousi S, Bajaj A.K., Soedel W (1991) Finite Disturb Directional Stability of Vehicles with Hman Pilot Considering Nonlinear Cornering Behavior, Vehicle System, Dynamics, 20:1, 21-55, DOI: 10.1080/00423119108968978, 1991

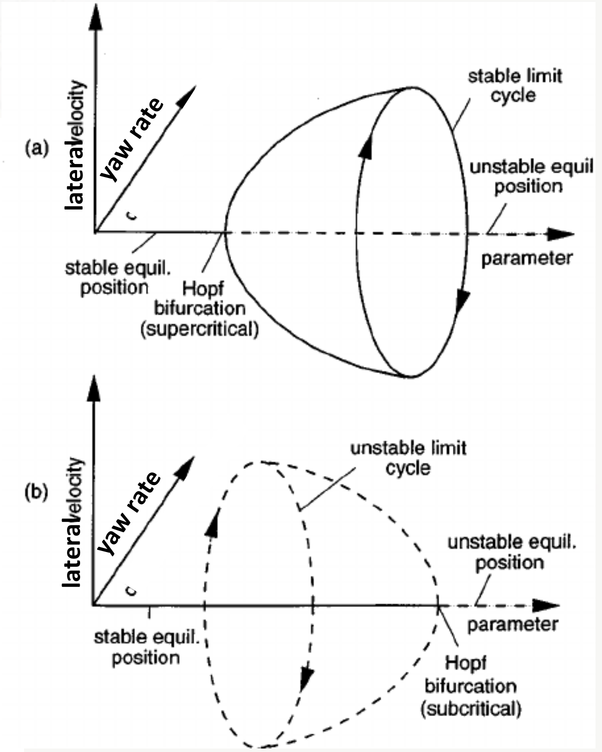
Liu, Z., Payre, G., Bourassa, P.: Nonlinear oscillations and chaotic motions in a road vehicle system with driver steering control. Nonlinear Dyn. 9(3), 281–304 1996

Open-loop, closed-loop(s) handling stability

Hopf bifurcation at high speed either for understeer or oversteer vehicles – Hurwitz criterion



$$\lambda^5 + \alpha_1\lambda^4 + \alpha_2\lambda^3 + \alpha_3\lambda^2 + \alpha_4\lambda + \alpha_5 = 0$$



$$\alpha_1 = \frac{1}{\tau} + \frac{B_f C_f D_f (a^2 m + J) + B_r C_r D_r (b^2 m + J)}{J m u},$$

$$\alpha_2 = \frac{b B_r C_r D_r - a B_f C_f D_f}{J} + \frac{u B_f C_f D_f (a^2 m + J) + B_r C_r D_r \left(\tau (a+b)^2 B_f C_f D_f + u (b^2 m + J)\right)}{J m \tau u^2},$$

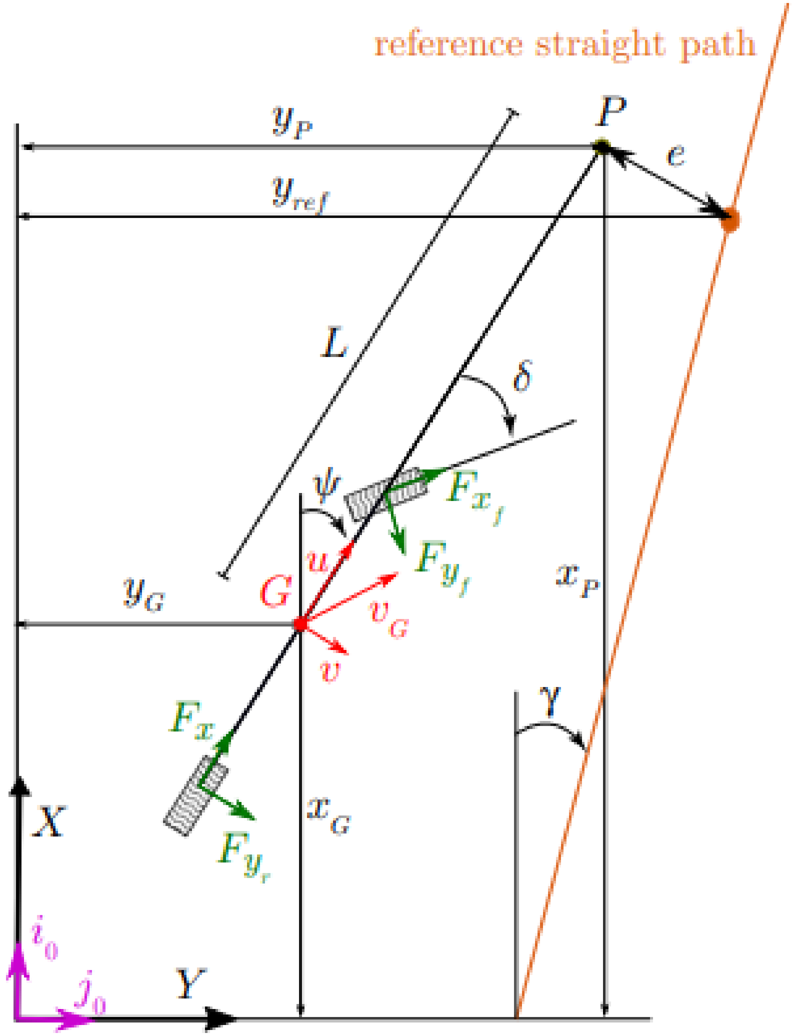
$$\alpha_3 = \frac{a k T_{prev} B_f C_f D_f}{J\tau} u + \frac{B_f C_f D_f (Jk - am) + bm B_r C_r D_r}{J m \tau} + \frac{(a+b)^2 B_r C_r D_r B_f C_f D_f}{J m \tau u^2},$$

$$\alpha_4 = \frac{k T_{prev} (a+b) B_r C_r D_r B_f C_f D_f}{J m \tau} + \frac{bk (a+b) B_r C_r D_r B_f C_f D_f}{J m \tau u},$$

$$\alpha_5 = \frac{k (a+b) B_r C_r D_r B_f C_f D_f}{J m \tau}$$

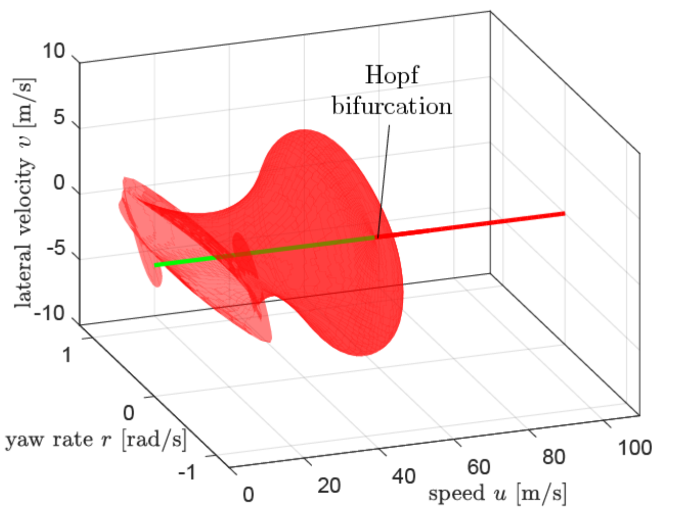
Open-loop, closed-loop(s) handling stability

Response to big disturbance with simple driver model

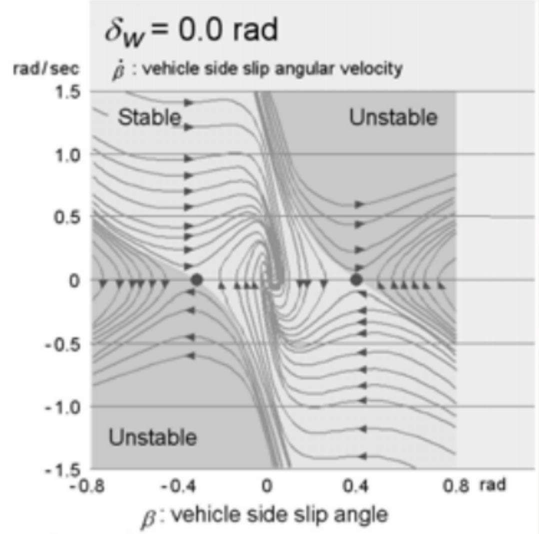


UNDERSTEERING

Vehicle and driver

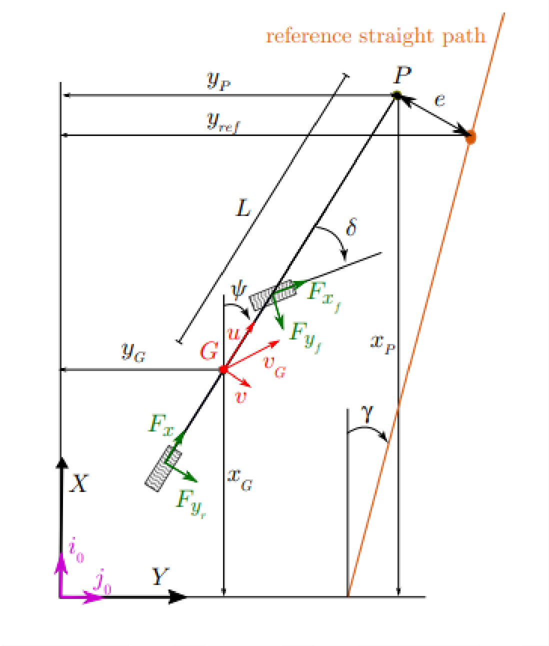


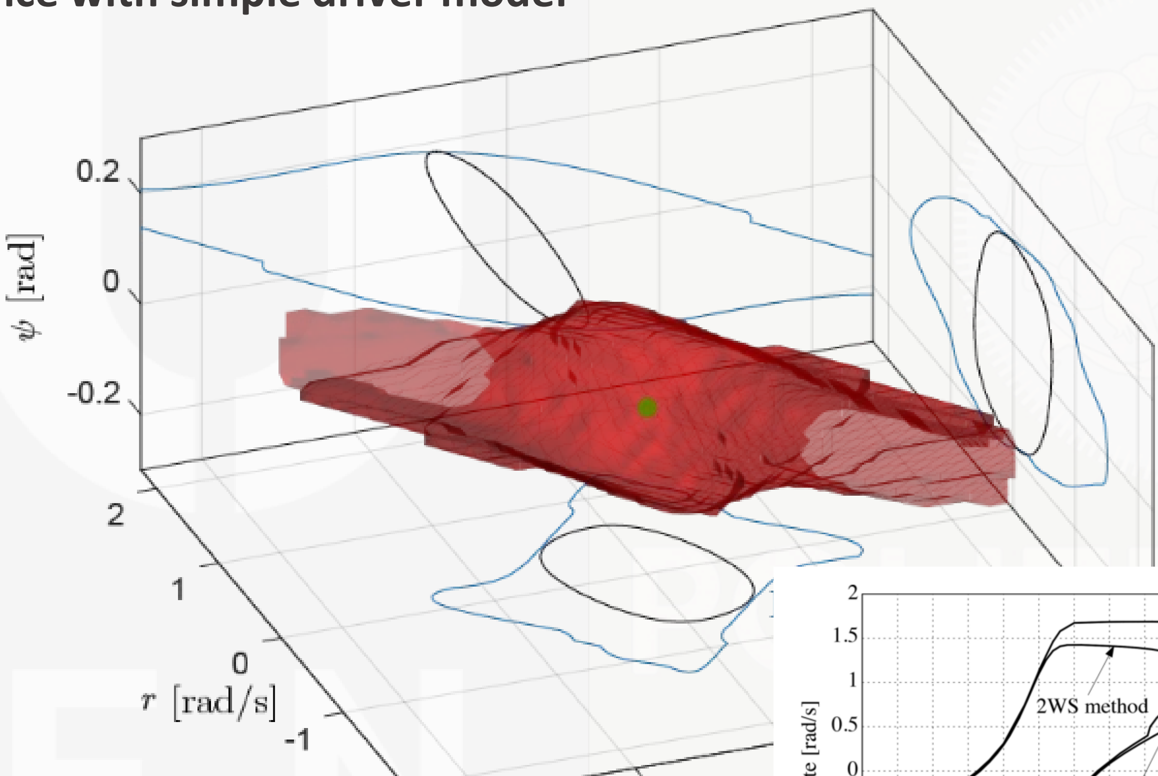
No driver – fixed control

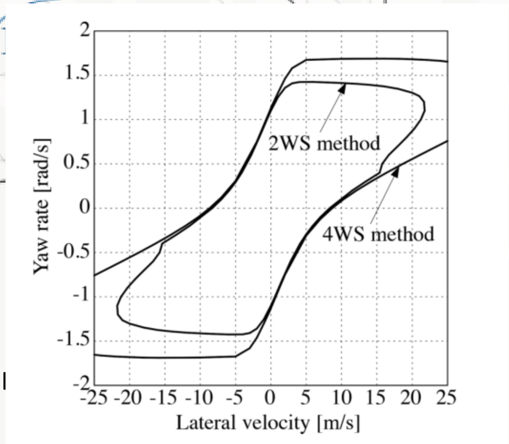


Open-loop, closed-loop(s) handling stability

Response to big disturbance with simple driver model







UNDERSTEERING

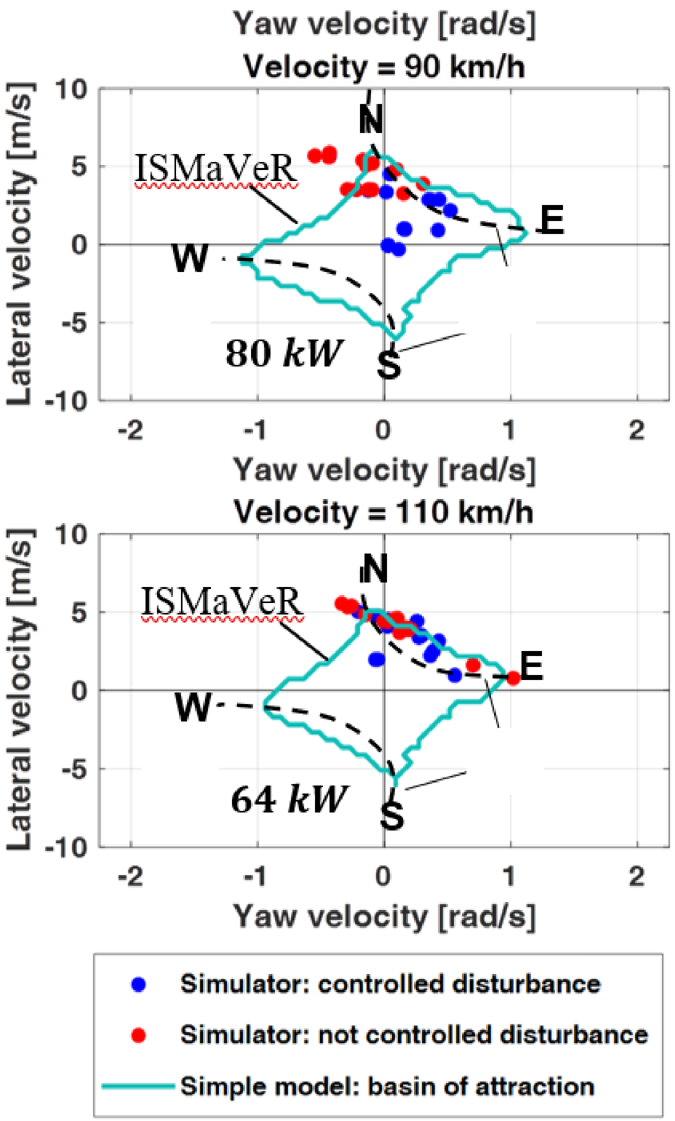
Vehicle and driver

F Della Rossa, G Mastinu: Straight ahead running of a nonlinear car and driver model–new r ... Vehicle System Dynamics56(5), pp. 753-768

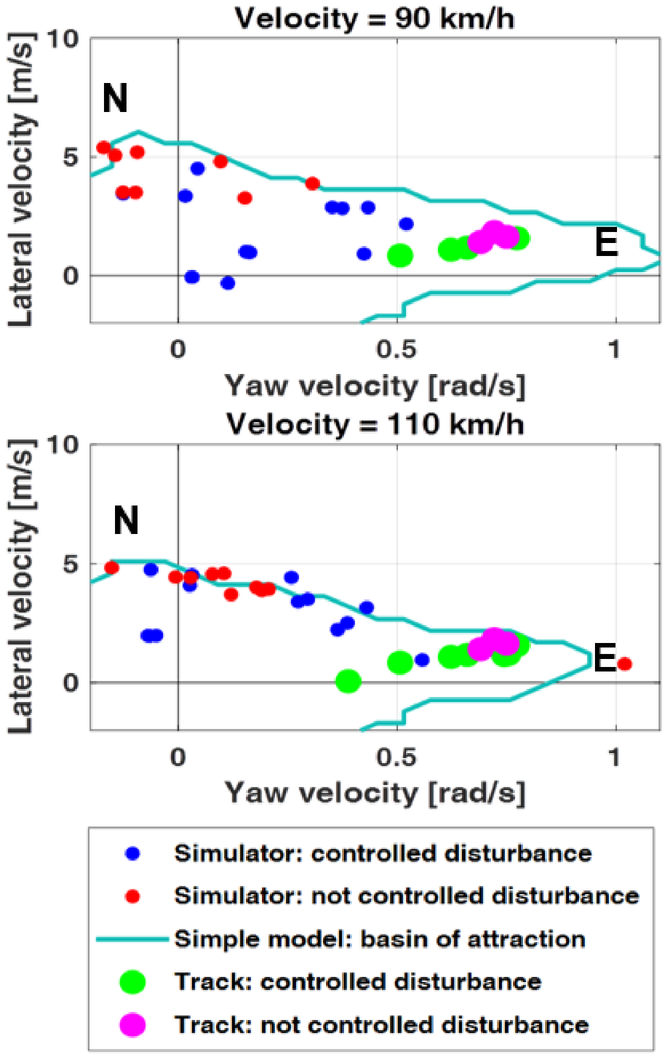
S Horiuchi: Evaluation of chassis control algorithms using controllability region analysis. IAVSD 2016

Open-loop, closed-loop(s) handling stability

Response to big disturbance with simple driver model – preliminary investigations



A



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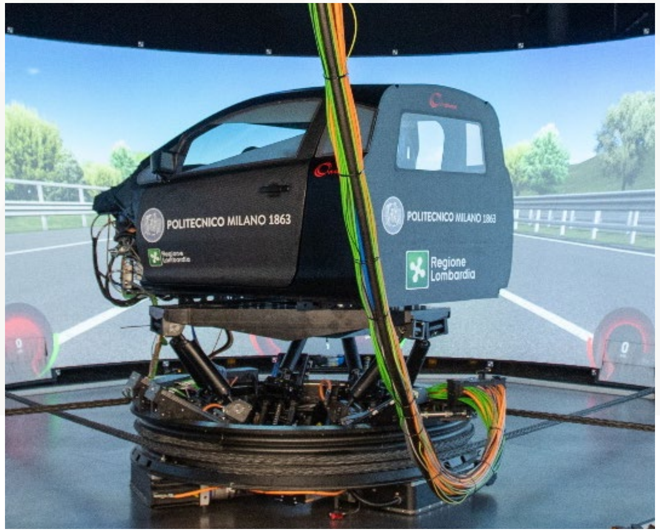

Open-loop, closed-loop(s) handling stability

Vehicle System Dynamics
International Journal of Vehicle Mechanics and Mobility
Taylor & Francis
Taylor & Francis Group
ISSN: 0042-3114 (Print) 1744-5159 (Online) Journal homepage: https://www.tandfonline.com/loi/nvsd20

Vehicle control synthesis using phase portraits of planar dynamics

Carrie G. Bobier-Tiu, Craig E. Beal, John C. Kegelman, Rami Y. Hindiyeh & J. Christian Gerdes

Vehicle System Dynamics
ISSN: 0042-3114 (Print) 1744-5159 (Online) Journal homepage: https://www.tandfonline.com/loi/nvsd20

Analysis of accelerating and braking stability using constrained bifurcation and continuation methods

Shinichiro Horiuchi , Kazuyuki Okada & Shinya Nohtomi

Proceedings of the ASME 2020
...al Design Engineering Technical Conferences
...rs and Information in Engineering Conference
IDETC/CIE2020
August 16-19, 2020, Virtual, Online

IDETC2020-22387

DRAFT: BIFURCATION ANALYSIS OF A LANE KEEPING CONTROLLER WITH FEEDBACK DELAY

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Open-loop, closed-loop(s) handling stability

ISO 1176

Road vehicles — Masses — Vocabulary and codes

ISO 2416

Passenger cars — Mass distribution

ISO 3833

Road vehicles — Types — Terms and definitions

ISO 8855

Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary

ISO 4138

Passenger cars — Steady-state circular driving behaviour — Open-loop test method

ISO 7401

Road vehicles — Lateral transient response test methods — Openloop test methods

Open-loop, closed-loop(s) handling stability

ISO 8725 **Road vehicles — Transient open-loop response test method with one period of sinusoidal input**

ISO/TR 8726 **Road vehicles — Transient open-loop response test method with pseudo-random steering input**

SAE J266 Steady-State Directional Control Test Procedures for Passenger Cars and Light Truck

ISO 19364**, Passenger cars — Vehicle dynamic simulation and validation — Steady-state circular driving behaviour**

ISO 19365 Passenger cars — **Validation of vehicle dynamic simulation — Sine with dwell stability control testing**

UN 157 **Automated Lane Keeping Systems (ALKS)**

ISO 22737 **Standard for Low-Speed Autonomous Driving (LSAD) systems.** (L4)

Open-loop, closed-loop(s) handling stability

BS ISO 8855:2011

BSI Standards Publication

Road vehicles — Vehicle dynamics and road-holding ability — Vocabulary

Open-loop, closed-loop(s) handling stability

- 12.2 **Equilibrium and stability**

- 12.2.3 **non-oscillatory stability** stability characteristic at a prescribed steady state (12.2.1) if, following any **small** temporary disturbance input (11.1.2) or control input (11.1.1), the vehicle returns to the steady state without oscillation
- 12.2.4 **non-oscillatory instability** stability characteristic at a prescribed steady state (12.2.1) if a small temporary disturbance input (11.1.2) or control input (11.1.1) causes an ever-increasing vehicle response (12.1.1) without oscillation See A.2.
- 12.2.5 **neutral stability** stability characteristic at a prescribed steady state (12.2.1) if, as a result of any small temporary disturbance input (11.1.2) or control input (11.1.1), the vehicle attains a new steady state
- 12.2.6 **oscillatory stability** stability characteristic at a prescribed steady state (12.2.1) if a small temporary disturbance input (11.1.2) or control input (11.1.1) causes an oscillatory vehicle response (12.1.1) of decreasing amplitude and a return to the original steady state
- 12.2.7 **oscillatory instability** stability characteristic at a prescribed steady state (12.2.1) if a small temporary disturbance input (11.1.2) or control input (11.1.1) causes an oscillatory vehicle response (12.1.1) of ever-increasing amplitude about the initial steady state See A.3.

E/ECE/TRANS/505/Rev.3/Add.156

4 March 2021

Agreement

Concerning the Adoption of Harmonized Technical United Nations Regulations for Wheeled Vehicles, Equipment and Parts which can be Fitted and/or be Used on Wheeled Vehicles and the Conditions for Reciprocal Recognition of Approvals Granted on the Basis of these United Nations Regulations*

(Revision 3, including the amendments which entered into force on 14 September 2017)

Addendum 156 – UN Regulation No. 157

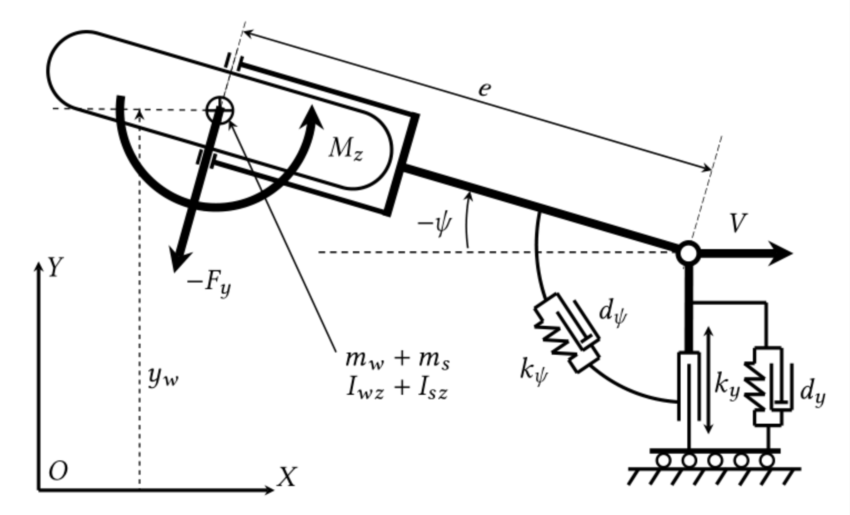
Date of entry into force as an annex to the 1958 Agreement: 22 January 2021

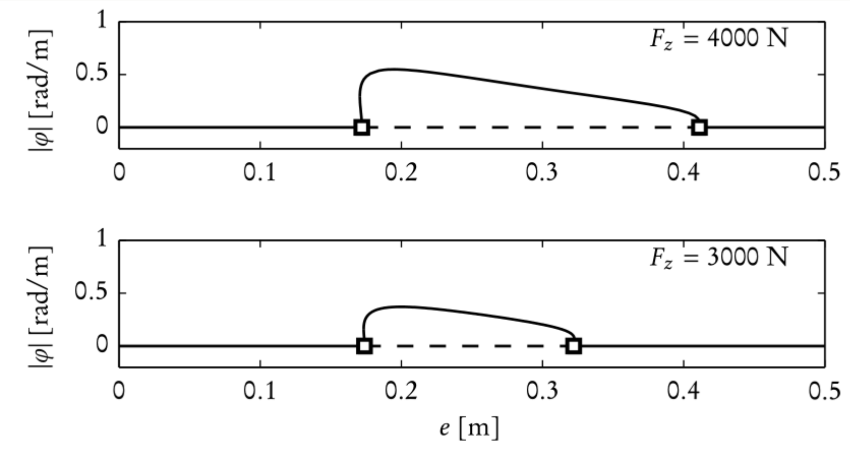
Uniform provisions concerning the approval of vehicles with regard to Automated Lane Keeping Systems

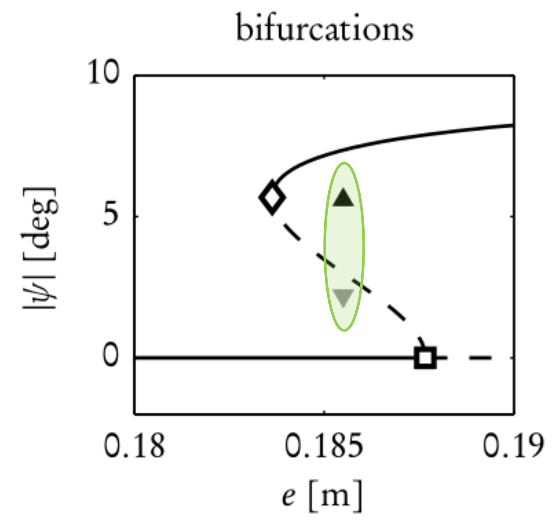
This document is meant purely as documentation tool. The authentic and legal binding text is: ECE/TRANS/WP.29/2020/81.

Examples for nonlinear stability, loss of stability, disturbance behaviour

Wheel shimmy: studied by Pacejka, Besselink and Ran
Nonlinear stability analysis of a trailing wheel suspension*





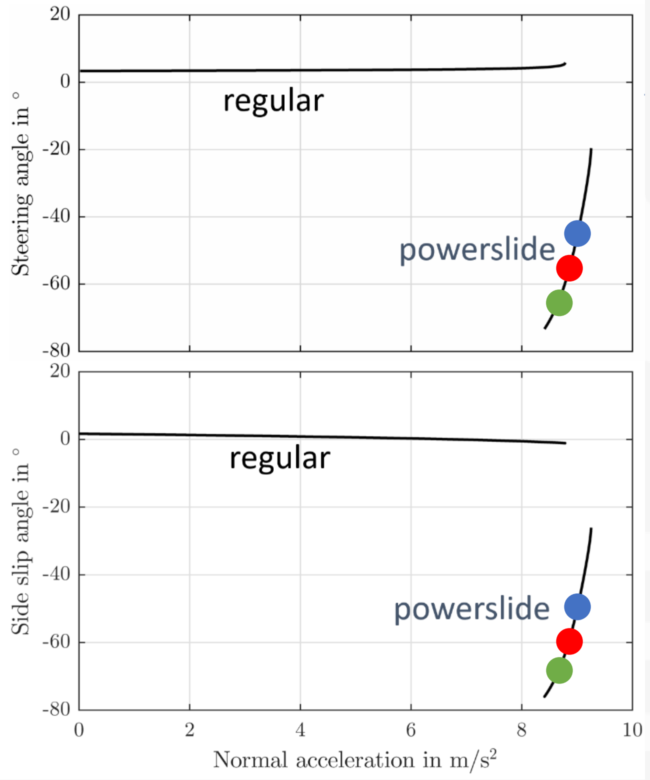


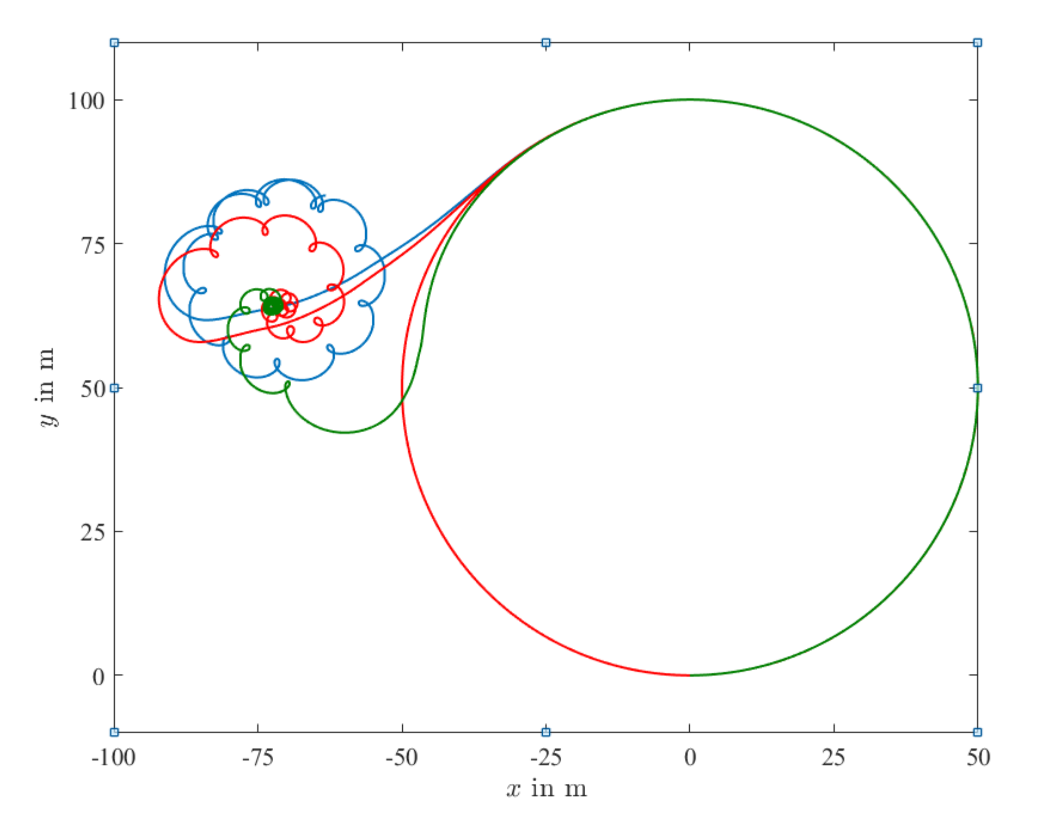
* S Ran: Tyre Models for Shimmy Analysis: from linear to nonlinear. PhD-Thesis, Tue, 2016
I Besselink: Shimmy of Aircraft Main Landing Gears, PhD-Thesis, TU Delft, 2000.
HB Pacjeka: The wheel shimmy phenomenon. PhD-thesis, TU Delft, 1966.

Examples for nonlinear stability, loss of stability, disturbance behaviour

Powerslide: Nonlinear (open loop) stability analysis

Loss of stability – fixed controls; trajectory of vehicle‘s CG



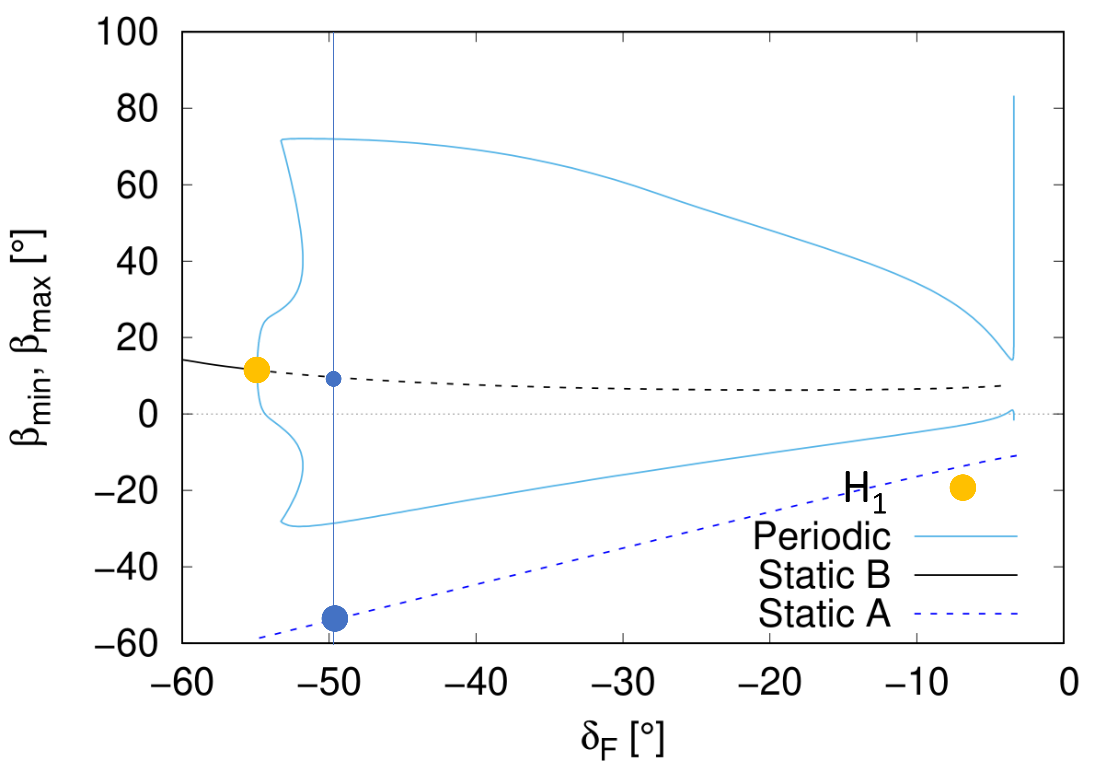


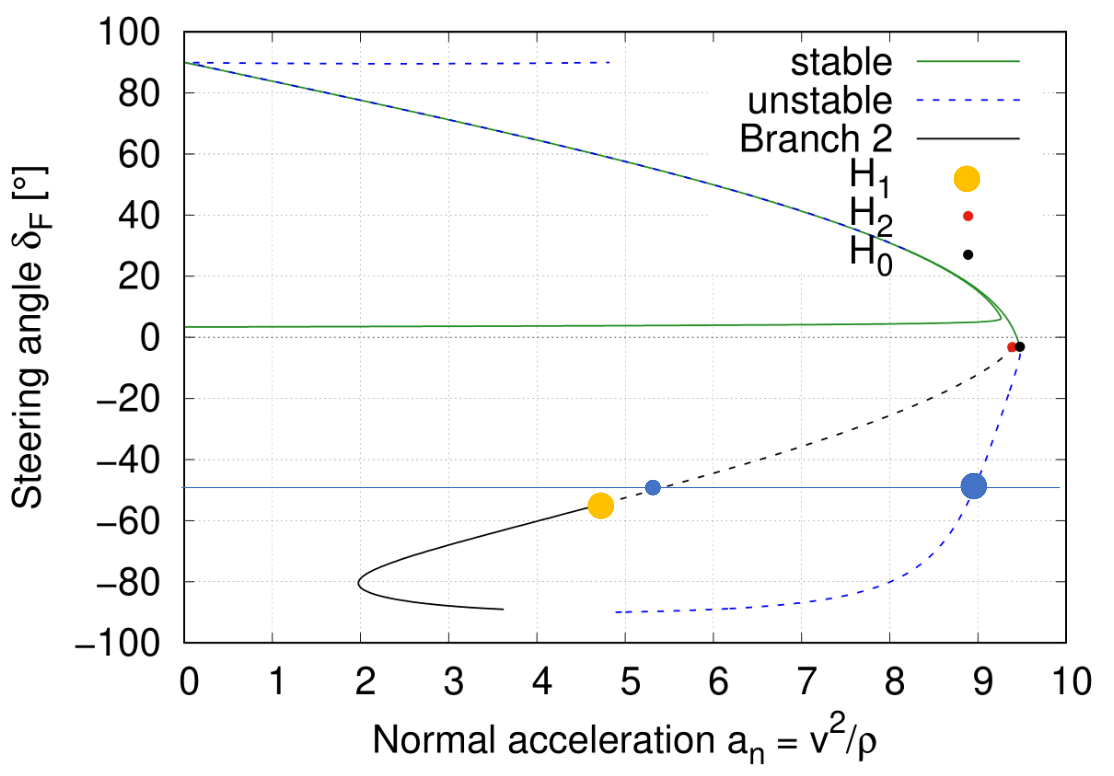
→ Powerslide is (divergent) unstable, resulting trajectory/solution is periodic

Examples for nonlinear stability, loss of stability, disturbance behaviour

Powerslide: Nonlinear (open loop) stability analysis *

Bifurcation and handling diagram



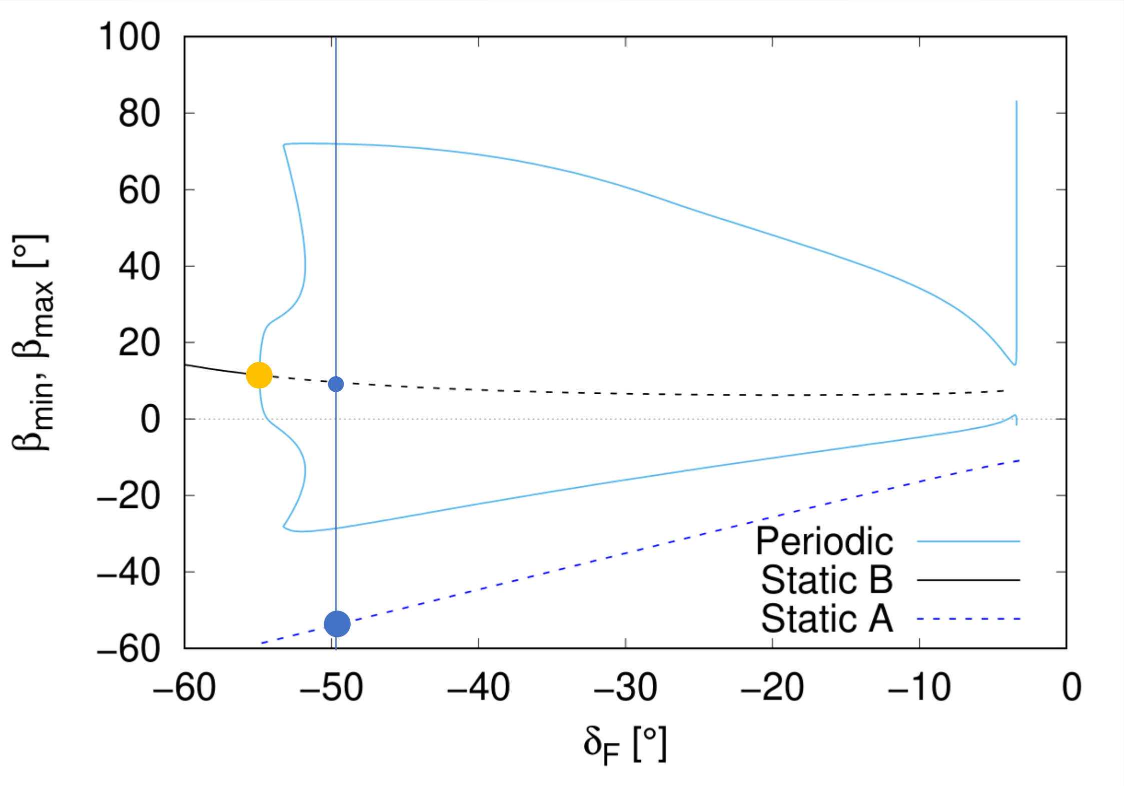


* J Edelmann, M Plöchl, A Steindl: Periodic motions for an understeering vehicle, ESMC, 2022

Examples for nonlinear stability, loss of stability, disturbance behaviour

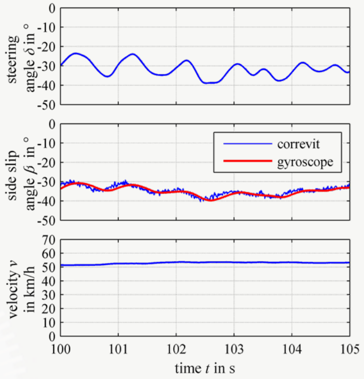
Powerslide: Nonlinear (open loop) stability analysis

Periodic motion with small radius of curvature (‚donuts‘)

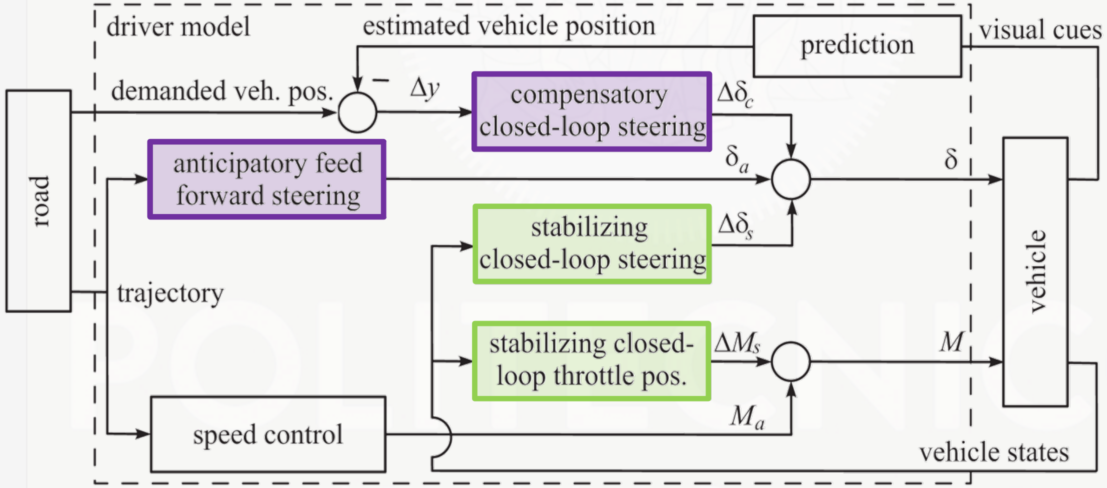


https://videohive.net/item/car-drifting-on-snow-aerial-view/15769463

Examples for nonlinear stability, loss of stability, disturbance behaviour



Powerslide*: Closed-loop human driver powerslide

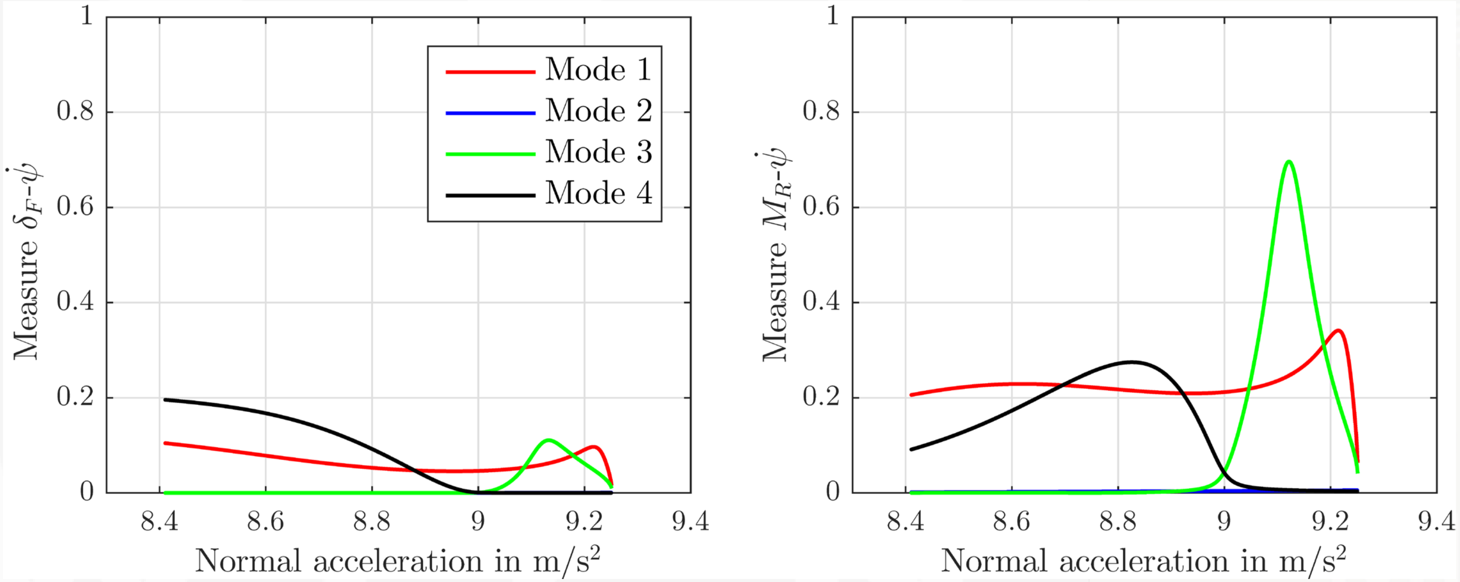


* J Edelmann, M Plöchl: Controllability of the powerslide motion of an automobile with different actuation inputs, PAMM, 16, 2016

Examples for nonlinear stability, loss of stability, disturbance behaviour

Powerslide*: Closed-loop human driver powerslide

Gross-measure of input $\Delta\delta_F$ / ΔM_R and output $\Delta\dot{\psi}$
(combined controllability/observability)



→ the sensor/actuator combination $\Delta\dot{\psi}$/ ΔM_R is very effective for stabilisation of unstable Mode 1

* J Edelmann, M Plöchl: Controllability of the powerslide motion of an automobile with different actuation inputs, PAMM, 16, 2016

Conclusions and Questions for discussion

温故知新
learning lessons from the past and developing new ideas

- The stability of the vehicle motion has continuously contributed to a better understanding of the behaviour and dynamics of the vehicle over the last decades.
- The development of handling control strategies made our vehicles safer and is becoming ever important.
- The combined closed-loop system of (robot/human) driver and vehicle including possibly large disturbances still deserves more attention.

➢ Safe vehicle handling – in any situation? Are we prepared?

➢ Do we have a clear understanding of the term stability in theory, practical application, and testing/homologation? Is there more effort required?

➢ What are the topics – related to handling stability – that are still of interest or may become future challenges for industry?

➢ What is missing to successfully address these issues? How can academia support?