Singular trajectories in incipient 3D laminar-turbulent transition

M. Kaczvinszki[a], S. Braun[a]

The Fisher eq. (1) plays a crucial role in nonlinear problems, like population dynamics or heat conduction. In our case it's the fundamental eq. to understand the early stage of 3D laminar-turbulent transition, where it represents the temporal evolution of the local wall shear stress and the displacement thickness for marginally separated flows[1],

$$u_t - u_{zz} = u - u^2. \quad (1)$$

In particular, solutions of the Fisher(-KPP) eq. may blow up in finite time t. This happens not according to a simple similarity law, but rather requires a logarithmic regulation on an exponentially short time scale regime[2], leading to a pair generation of moving singularities (localized infinities), which propagate away from their creation point[1]. We expect such singular trajectories to be associated to vortex-kernels of the well-known hairpin vortices, embodying the transition to turbulence.

In our work we managed to stepwise estimate the initial motion of these singularity pairs after the blow-up time, using the framework of matched asymptotic expansions in spanwise direction z. Essentially, in this initial phase a change of the spatial structure from a first to a second order pole occurs, best characterised by the asymptotic sublayer (stretched coordinate ζ) with leading order differential equation

$$g'' + g' - g^2 = 0, \quad (2)$$

and boundary conditions for $\zeta \to \pm\infty$. We achieve the tricky numerical computation of (2) with Chebyshev collocation after splitting off singular terms (with pole position ζ_s as degree of freedom) and hindering far field contributions appropriately.

In general, near the blow-up time t_s the complex symmetry $u(z, t_s - t) \sim -u(iz, t - t_s)$ holds. Hence, the numerically known path of singularities in the complex plane[3] before t_s corresponds to the real valued initial trajectories z_p after t_s and we can compare it with our analytic estimate

$$\frac{z_p}{\pm\sqrt{8(t-t_s)\tau}} \sim 1 - \frac{1}{8}\frac{\ln(\tau)}{\tau} + \frac{\zeta_s}{4}\frac{1}{\tau} + o\left(\frac{1}{\tau}\right). \quad (3)$$

While the values of the emerged constants depend on the global solution [i.e. initial condition of (1)], already simple guesses are in great correspondence, suggesting the convergence of expansion (3) in the temporal range $\tau = -\ln(t - t_s) \in (\infty, 1)$ and the start of a follow-up regime on which we are currently working.

We believe that understanding the dynamics of such moving singularities of the Fisher eq. will broaden the insight in hairpin vortex generation and the opportunities of even more complex nonlinear models.

This work is founded by the Austrian Science Fund (FWF) (grant no. P31873-N32).

[a] Institute of Fluid Mechanics and Heat Transfer, TU Wien, Getreidemarkt 9, 1060 Vienna, Austria

[1] Braun and Kluwick, *J. Fluid. Mech.* **514**, 121 (2004)

[2] Hocking et al., *J. Fluid. Mech.* **51**, 705 (1972)

[3] Weideman, *SIAM Journal on Applied Dynamical Systems* **2**, 171 (2003)