Revisiting Brandom's Incompatibility Semantics

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Abstract: We critically analyse Robert Brandom's incompatibility semantics for classical S5, developed in the context of analytic pragmatism. Among other problems, we point out that Brandom's claim that incompatibility semantics, although holistic and non-compositional, is nevertheless recursively projectible, rests on an assumption that is at odds with intended applications. We also explore an alternative approach that aims at a formal model of Brandom's concept of a 'game of giving and asking for reasons' (GOGAR).

Keywords: semantics, incompatibility, holism, Robert Brandom

1 Introduction

In his John Locke Lectures, published as *Between Saying and Doing: Towards an Analytic Pragmatism*, Brandom (2008) stakes out an ambitious program in the philosophy of language highlighting a pragmatist and inferentialist approach to meaning, that acknowledges the precedence of deontic normative over non-modal vocabulary in the elaboration of successful communication. This endeavour entails a new type of formal semantics for propositional modal logic. This *incompatibility semantics* features several aspects that distinguish it from traditional Tarski/Kripke-style semantics. Rather than truth in a model, the basic notion is that of incompatibility between or, more generally, *incoherence* among a set of sentences. This leads to a *holistic* account, in which the semantic status of particular sentences can only be asserted relative to a given context of other sentences. As a consequence the semantics is *non-compositional*: the meaning of a logically complex sentence is not determined by the semantic interpretants (incompatibilities) of just its parts and the connectives used to form it. It is often claimed that holistic, non-compositional semantics cannot account for the projectibility and systematicity of language, and hence also not for

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its learnability. Therefore it is at the core of Brandom's project to address such worries by showing that incompatibility semantics enjoys what he calls *'recursive projectibility'*.

The paper is organized as follows. After briefly discussing its main tenets and presenting its formal ingredients in Section 2, we take a critical look at some aspects of Brandom's presentation in Section 3 and suggest some corresponding amendments. In Section 4, we point out that Brandom's claim that incompatibility semantics admits recursive projectibility rests on an additional assumption, namely *'inferential conservativity'*, that is not entailed by his axioms and that is actually quite problematic with respect to the intended models.[2] Section 5 seeks to clarify the relation between incompatibility semantics and classical logic, including its extension to modal logic S5, in a manner that is more transparent than Brandom's own take. Since incompatibility semantics, as presented by Brandom (2008), is largely severed from its pragmatist and inferentialist context, we suggest in Section 6 an alternative approach by introducing a formal model of the *game of giving and asking for reason* (GOGAR), introduced by Brandom (1994), that interprets the inference rules of a particular sequent calculus for intutionistic logic as interactions between a proponent (of a claim) and a questioner. We conclude in Section 7 with some remarks about open issues and the relation to more recent work by Brandom and his collaborators.

2 Brandom's incompatibility semantics in a nutshell

In the following we are working with languages $\mathcal{L}$ for propositional modal logic. Each language is a set of atomic or logically complex sentences.[3] Such languages need not be syntactically closed "upwards", i.e. in terms of forming arbitrarily complex sentences by applying logical connectives. However, all languages are assumed to be closed "downwards", such that all subformulas of sentences in $\mathcal{L}$ are themselves members of $\mathcal{L}$. These languages are called *proper*.

The point of departure for Brandom's semantic theory is his "suggestion: represent the propositional content expressed by a sentence with the set of sentences that express propositions incompatible with it" (Brandom, 2008, p. 123). *Incompatibility* is a material, binary relation among sentences,

[2] The problem described in Section 4 has already been outlined, from a somewhat different perspective, in Fermüller (2010).

[3] We will use p, q, r, ... for atomic formulas and F, G, H, ... for arbitrary formulas.

a generalization of contradictoriness to the case of non-logical properties. However, as Brandom is quick to point out, such a semantic representation of propositional content would be too narrow, since it fails to take into account incompatibilities that hold between three or more sentences while any two of them are compatible. To overcome this limitation, Brandom generalizes incompatibility from a relation among sentences to a relation among sets of sentences. Formally, an *incompatibility function* I from the power set of $\mathcal{L}$, $\mathcal{P}(\mathcal{L})$, to $\mathcal{P}(\mathcal{P}(\mathcal{L}))$ relates to each set of sentences the set of sets of sentences that are incompatible with it. An ordered pair $\langle \mathcal{L}, \mathrm{I} \rangle$ is called an *standard incompatibility frame* on $\mathcal{L}$. In addition, Brandom introduces the (non-relational) notion of *incoherence*, which is a material generalization of inconsistency applying to (single) sets of sentences. Formally, $Inc \subseteq \mathcal{P}(\mathcal{L})$ comprises all and only the incoherent sets of sentences of a given language $\mathcal{L}$. Incoherence is a property of a set which is inherited by any superset of it, i.e. *Inc* satisfies the following axiom.

Axiom (Persistence) $\forall$ finite $X, Y \subseteq \mathcal{L}$, and $X \subseteq Y$, if $X \in Inc$ then $Y \in Inc$.

An ordered pair $\langle \mathcal{L}, Inc \rangle$ is called a *standard incoherence frame* on $\mathcal{L}$. For any given language $\mathcal{L}$, the standard incompatibility frame and the standard incoherence frame correspond to each other in virtue of jointly satisfying the following axiom.

Axiom (Partition) $\forall X, Y \subseteq \mathcal{L}, X \cup Y \in Inc$ iff $X \in \mathrm{I}(Y)$.

Since reference and truth are not among the resources of incompatibility semantics, (material) entailment is not to be defined in terms of truth-preservation. Rather, material[4] entailment can be viewed in terms of the preservation of compatibility or coherence from premises to conclusion, i.e. the entailment relation between premise set and conclusion holds iff every set of sentences that is compatible with the set of premises is also compatible with the conclusion. This idea is equivalently expressed by the notion of *(material) incompatibility-entailment*. A set X of sentences materially incompatibility-entails a sentence F iff every set Z that is incompatible with $\{F\}$, is also incompatible with X. Brandom generalizes this to a set Y in place of a single sentence F in the following way.

[4]Brandom is trying to articulate “the material (that is, non-, or better, pre-logical) sense of ‘good inference’ [in which sense, for instance] ‘Pedro is a donkey,’ incompatibility-entails ‘Pedro is a mammal’" (Brandom, 2008, p. 121).

Definition 1 (Incompatibility-Entailment) *Given an incompatibility function I, a (possibly infinite) set $X \subseteq \mathcal{L}$ and a finite set $Y \subseteq \mathcal{L}$,*

$$X \models_{\mathrm{I}} Y \textit{ iff } \bigcap_{F \in Y} \mathrm{I}(\{F\}) \subseteq \mathrm{I}(X).$$

According to this definition, the set Y is read disjunctively, i.e. the "heuristic meaning" of $X \models_I \{F_1, \ldots, F_n\}$ is that X entails F_1 *or* ... *or* F_n (Brandom, 2008, p. 42).

Definition 2 (Validity) $\forall X \subseteq \mathcal{L}$, X *is* valid *iff* $Y \in \bigcap_{F \in X} \mathrm{I}(\{F\}) \Rightarrow Y \in \mathit{Inc}$.

X is valid iff only incoherent sets are incompatible with X, read disjunctively. As a special case we have $\emptyset \models_I \{F\}$ iff $\{F\}$ is valid.

Now we are in a position to introduce, axiomatically, the logical operators, negation, conjunction and necessity.

Axiom (Negation Introduction; NI) $\forall X \subseteq \mathcal{L}, X \cup \{\neg F\} \in \mathit{Inc}$ iff $X \models F$.

Axiom (Conjunction Introduction; CI) $\forall X \subseteq \mathcal{L}, X \cup \{F \wedge G\} \in \mathit{Inc}$ iff $X \cup \{F, G\}$.

Axiom (□ Introduction; LI) $\forall X \subseteq \mathcal{L}, X \cup \{\Box F\} \in \mathit{Inc}$ iff $X \in \mathit{Inc}$ or $\exists Y \subseteq \mathcal{L}[X \cup Y \notin \mathit{Inc} \; \& \; Y \not\models \{F\}]$.

These axioms specify the content expressed by a logically compound sentence, according to incompatibility semantics, by specifying which sets of sentences are incompatible with it. However, it is a crucial feature of this semantics that the semantic interpretants of compound sentences are not determined by or computable from the semantic interpretants of their components alone, i.e. that this semantics is *not compositional*. For instance, it is easy to see that what is incompatible with F, on the one hand, and what is incompatible with G, on the other, do not fully determine what is incompatible with $\{F, G\}$. This non-compositionality makes incompatibility semantics *holistic*. It has been a common criticism of semantic holism that non-compositionality prevents it from accounting for the projectibility, systematicity and learnability of language. Brandom insists that the standard arguments to this effect are fallacious, and are refuted by his incompatibility semantics.

> ... although that semantics is not compositional, it is fully recursive. The semantic values of logically compound expressions are wholly determined by the semantic values of logically simpler ones. It is holistic,

> that is, non-compositional, [...] But this holism within each level of constructional complexity is entirely compatible with recursiveness between levels. And this is not just a philosophical claim of mine. The system I am describing allows us to prove it. (In this context, proof is the word made flesh.) (Brandom, 2008, p. 135)

In the rest of the paper we are going to critically evaluate Brandom's claim (and his 'proof' of it) as well as essential parts of the underlying machinery of incompatibility semantics.

3 A critical analysis of Brandom's account

To gain a better understanding of the essential features of the suggested semantic framework, we take a closer, critical look at Brandom's definitions and axioms outlined in Section 2. First, note that Brandom employs an unconventional concept of 'language'. While a language $\mathcal{L}$ may contain logically complex formulas, $\mathcal{L}$, in general, is not closed under forming complex formulas from given ones using the logical connectives. We may, however, assume that all languages considered here are *proper*, i.e., they are closed under taking sub-formulas. Moreover, for our purposes, it is sufficient to consider only *finite languages*. A more problematic feature of Brandom's notion of a logical language is that he only considers conjunction ($\wedge$), negation ($\neg$), and the modal operator ($\Box$) for necessity. Disjunction and implication are only introduced as operators 'abbreviating' certain formulas, built up from atomic formulas using conjunction and negation only. While one can, of course, treat all connectives of *classical logic* as defined from conjunction and negation alone, this is not the case for most other logics. If the aim of incompatibility semantics is to provide an independent approach to the meaning of logical connectives that does not exclude nonclassical logics from the outset, then it is certainly odd to declare that $F \vee G$ is to be understood as abbreviation for $\neg(\neg F \wedge \neg G)$ and that also $F \rightarrow G$ has no meaning beyond serving as abbreviation for $\neg(F \wedge \neg G)$. In particular, the decision to treat disjunction and implication not as first class citizens of the logical vocabulary, but as defined connectives, excludes the possibility to come up with a notion of logical validity that is able to distinguish between classical and intuitionistic logic, as demonstrated by the following well-known fact.

Fact 1 *Over the language fragment, where $\neg$ and $\wedge$ are the only logical connectives, classical tautologies coincide with intuitionistic tautologies.*

We will indicate in Section 5 how one can augment Brandom's account to accommodate a richer logical vocabulary. More importantly, we will introduce an alternative to incompatibility semantics in Section 6 that will treat the usual propositional connectives on a par with each other and that arguably remains closer to Brandom's own pragmatist and inferentialist perspective.

As pointed out in Section 2, Brandom presents incompatibility semantics in terms of 5 axioms and some accompanying definitions. They center around the notions of *incompatibility* and *incoherence*, the latter modeled as a set *Inc* of sets of formulas, i.e., $Inc \subseteq \mathcal{P}(\mathcal{L})$. The *Axiom of Persistence* postulates *incoherence* to be monotonic (with respect to the subset relation), which induces a monotonic notion of material inference.While there is nothing wrong with investigating a monotonic notion of (material) incoherence, it is somewhat strange that the *Persistence Axiom* explicitly restricts monotonicity to *finite* sets of sentences. Since Brandom hardly wants to claim that infinite sets of sentences may be coherent even if they contain incoherent subsets, we interpret this oddity simply as an indication that the presented version of incompatibility semantics is intended for finite language scenarios only.[5]

Brandom defines a *standard incoherence frame* as a pair $\langle \mathcal{L}, Inc \rangle$. Since this definition refers to the *material* level of an interpreted language and not to a logical frame in the sense of Kripke semantics for modal logic, it would be better to speak of an incoherence *model*, instead. Even more confusingly, there is a second definition of a *standard incompatibility frame* as a pair $\langle \mathcal{L}, \mathrm{I} \rangle$, where I is a function of type $\mathcal{P}(\mathcal{L}) \mapsto \mathcal{P}(\mathcal{P}(\mathcal{L}))$, called *incompatibility function*. Since $X \in \mathrm{I}(Y)$ iff $X \cup Y \in Inc$ (*Partition Axiom*), incoherence frames and incompatibility frames amount to just two different presentations of the same concept. The *Partition Axiom* can be understood as solely introducing convenient notation.

To amend the outlined infelicities in Brandom's presentation of incompatibility semantics, we will adopt the following alternative notion.

Definition 3 *An* incoherence model $Inc_{\mathcal{L}}$ *over the (finite) language* $\mathcal{L}$*, is a subset of* $\mathcal{P}(\mathcal{L})$*, such that*

(i) $\emptyset \notin Inc_{\mathcal{L}}$,

(ii) if $X \subseteq Y$ *and* $X \in Inc_{\mathcal{L}}$*, then* $Y \in Inc_{\mathcal{L}}$.

[5] There remains a minor lacuna in Brandom's account: the empty set should be declared to be coherent ($\emptyset \notin Inc$). While this follows from persistence if there are other coherent sets, one needs to make it explicit for models where all other sets are incoherent.

When there is no danger of confusion, we will drop the subscript referring to $\mathcal{L}$. The sets in *Inc* are called *incoherent* with respect to *Inc*. If $X \notin$ *Inc* then X is *coherent* with respect to *Inc*. We say that X is *incompatible* with Y (with respect to *Inc*) if $X \cup Y \in$ *Inc*. The set of sets that are incompatible with Y is denoted by $\mathrm{I}(Y)$. If $X = \{F\}$ we also say that the formula F is incompatible with Y (in *Inc*).

The remaining three axioms, called **CI**, **NI**, and **LI** in Brandom (2008), concern conjunction, negation, and necessity, respectively (see Section 2). Inspecting Brandom's formulations of these axioms, restated in Section 2, more closely, reveals that in each case one has to restrict the corresponding statements to refer to only those languages in which the exhibited complex formula actually occurs. (Recall that Brandom allows for languages that contain, e.g., F and G, but not $F \wedge G$, $\neg F$ or $\Box F$. For such languages, **CI**, **NI**, and **LI** are inappropriate without a restricting clause.) From now on, we will assume that in any statement that implicitly or explicitly refers to some language $\mathcal{L}$, $\mathcal{L}$ contains every formula that is mentioned in that statement.

The axiom for conjunction (**CI**) just states that, in judging incoherence, sets of formulas are treated as conjunctions of formulas. Since Brandom wants incoherence to serve as 'a generalization of inconsistency to the case of non-logical properties' (Brandom, 2008, p. 141) this is an obvious choice. The axioms for negation and necessity are much less straightforward, since they involve the notation of incompatibility-entailment, defined by $X \models_{\mathrm{I}} Y$ iff $\bigcap_{F \in Y} \mathrm{I}(\{F\}) \subseteq \mathrm{I}(X)$[6]. Brandom wants Y to be read disjunctively, rather than conjunctively, in order to be able to mimic Gentzen's classical sequent calculus **LK**. While understandable as a proof strategy this does not sit well with the general setup of incompatibility semantics as Brandom himself seems to admit in a footnote on page 123 of Brandom (2008), where he explicitly states that a 'very natural way' to generalize from a single-conclusion to a multiple-conclusion version of incompatibility entailment is to interpret the set Y on the right hand side as a conjunction. We therefore suggest to consider the following definition, that has the additional advantage of making transparent that one does not need to involve the function I:

[6]Right after the definition Brandom adds: 'When Y is empty we read $\bigcap_{F \in Y} \mathrm{I}(\{F\})$ as equivalent to $\mathcal{P}(\mathcal{L})$'. However, e.g., the proof of Claim **2.1 (Weakening)** (Brandom, 2008, p. 143) starts as follows: 'Suppose $X \models Y$. Then $\bigcap_{F \in Y} \mathrm{I}(\{F\}) \subseteq \mathrm{I}(X)$.' But this is clearly false if $\bigcap_{F \in Y} \mathrm{I}(\{F\})$ is to be read as $\mathcal{P}(\mathcal{L})$, as stipulated in the cited remark. Moreover, identifying $\mathrm{I}(\emptyset)$ with $\mathcal{P}(\mathcal{L})$ is already ruled out by the Partition Axiom.

Definition 4 *The set of formulas X* materially incompatibility-entails *the set of formulas Y with respect to an incoherence model $Inc_{\mathcal{L}}$, written $X \models_{Inc} Y$, iff for all $Z \subseteq \mathcal{L}$: $Y \cup Z \in Inc$ implies $X \cup Z \in Inc$.*

We write $F_1, \ldots, F_n \models G$, instead of $\{F_1, \ldots, F_n\} \models \{G\}$. Note that in this single-conclusion case Definition 4 coincides with Brandom's original definition. We observe that $X \models_{Inc} \emptyset$ holds for every X. This is also the case for Brandom's original definition, if one ignores his additional remark about identifying the empty intersection of sets of formulas with $\mathcal{P}(\mathcal{L})$. If one wants to express incoherence in terms of entailment, then one should introduce the logical constant $\bot$ (*falsum*). Moreover, it is useful to also add $\top$ (*verum*). Brandom's axioms for introducing connectives should consequently be augmented as follows.

(*Falsum* Introduction $\bot$I:) $\{\bot\} \in Inc$

(*Verum* Introduction $\top$I:) $X \cup \{\top\} \in Inc$ iff $X \in Inc$.

Note that $\bot$**I** guarantees that $X \in Inc$ is equivalent to $X \models_{Inc} \bot$.

Brandom also defines a notation of 'validity' that renders a formula F 'valid' iff $\emptyset \models_{Inc} \{F\}$. This piece of terminology is unfortunate, since validity traditionally does not refer to the material level of a given interpretation, but rather singles out what holds with respect to *all* interpretations. If one wants to preserve validity as a *logical* notion, one should call F valid iff $\emptyset \models_{Inc} \{F\}$ *for every incoherence model $Inc_{\mathcal{L}}$*. Note that only with respect to this latter, more traditional notion, does it make sense to claim, as Brandom does, that the 'intrinsic' logic of incompatibility is the classical modal logic S5.

Fortunately, neither Brandom's strange notion of validity, nor his problematic generalized entailment relation matter for the presentation of axioms **NI**, and **LI**, since only single-conclusion material entailment is used there. However, both axioms are highly problematic with respect to the claim that incompatibility semantics enjoys recursive projectibility, by which Brandom means that judgments about the incoherence of sets of logically complex sentences can be systematically reduced to judgments that only involve less complex sentences. To get a better view on the problem, we reformulate **NI** without making explicit use of the entailment relation.

(Negation Introduction NI:) $X \cup \{\neg F\} \in Inc_{\mathcal{L}}$ iff for all $Y \subseteq \mathcal{L}$: if $\{F\} \cup Y \in Inc_{\mathcal{L}}$ then $X \cup Y \in Inc_{\mathcal{L}}$.

This condition is *circular*, which is brought out most clearly by considering the instance in which $X = \emptyset$ and $Y = \{\neg F\}$. Since we may safely assume that $\{F, \neg F\} \in Inc_{\mathcal{L}}$, the statement boils down to $\{\neg F\} \in Inc_{\mathcal{L}}$ iff $\{\neg F\} \in Inc_{\mathcal{L}}$ in this case. [7]

The axiom **LI** for introducing the modal operator is plagued by the same problem as **NI**: the statement is circular. The scope of the quantifier in the statement refers to all sets of formulas in the given language and hence includes the set $X \cup \{\Box F\}$, the (in)coherence of which is to be settled.

Let us sum up our analysis of Brandom's presentation of incompatibility semantics, so far. Brandom's axioms refer to three basic notions: *Inc* (incoherence as a property of sets of formulas), the incompatibility function I, and incompatibility-entailment. In fact, each of these notions can be defined in terms of any of the two other notions. In particular, it is sufficient to consider just *Inc*. Since *Inc* refers to the material level, one should replace Brandom's talk of 'incoherence frames' and 'incompatibility frames' by references to *incoherence models*, as defined in Definition 3. This leaves only the axioms **CI**, **NI**, and **LI** to be considered. The fact that non-modal formulas are built up from atomic formulas using conjunction and negation only, spoils prospects to come up with a complete, independent semantic framework for (potentially) nonclassical logics. Most problematic, however, is the fact that the axioms for negation (**NI**) and for necessity (**LI**) are circular. In the next section, we investigate why Brandom nevertheless thinks that incompatibility semantics admits 'recursive projectibility' and characterizes the logic S5.

4 Problems with recursive projectibility

To get a better grip on the circularity problem outlined in the last section, we focus on negation and consider the following example.

[7] There seems to be a tension (an incoherence?) between the accounts, respectively, in the main text and the appendix of (Brandom, 2008) concerning the semantics of negated sentences. On the one hand, the main text (p. 127) suggests a recursive, step by step, extension of an incoherence frame in tandem with the corresponding incompatibility consequence relation in order to provide the incompatibility sets for more and more complex negated sentences. Such a stepwise construction of the semantics of negated sentences may avoid circularities. However, on the other hand, in the appendix (p. 142) the semantics of all logical connectives is clearly presented axiomatically and there is no indication of a semantic construction by successive extensions starting from an incoherence frame for a language containing only atomic formulas etc. Such a construction would also require more machinery, such as definitions of how to dovetail the respective extensions of the language/semantics by the three connectives.

Example 1 Let $\mathcal{L} = \{p, q\}$ and $Inc_{\mathcal{L}} = \emptyset$. Obviously $p \models_{Inc} p$ and $q \models_{Inc} q$. Since there are no incoherent sets, we also have $p \models_{Inc} q$ and $q \models_{Inc} p$.

Now let us consider $\mathcal{L}' = \{p, q, \neg p\}$. In Brandom's terminology, $\mathcal{L}'$ is a proper extension of $\mathcal{L}$. Since p entails itself, axiom **NI** yields $\{p, \neg p\} \in Inc'_{\mathcal{L}'}$ for any Inc' over $\mathcal{L}'$. We want to keep $\{p, q\}$ coherent, like in $Inc_{\mathcal{L}}$. But what about $\{q, \neg p\}$? Here we run into the circularity pointed out in Section 3: **NI** requires that $\{q, \neg p\} \in Inc'_{\mathcal{L}'}$ iff for all $Y \subseteq \mathcal{L}'$ either $Y \cup \{p\}$ is coherent or $Y \cup \{q\}$ is incoherent. Since $\{p\}$ and $\{p, q\}$ are coherent and $\{p, \neg p\}$ is incoherent this boils down to $\{q, \neg p\} \in Inc'_{\mathcal{L}'}$ iff $\{q, \neg p\} \in Inc'_{\mathcal{L}'}$. In other words, we are free to declare $\{q, \neg p\}$ to be either coherent or to be incoherent in $Inc'_{\mathcal{L}'}$ without violating Brandom's axioms for incompatibility semantics.

As we have seen in Section 2, a central claim of Brandom is that incompatibility semantics, although holistic, nevertheless is 'fully recursive'. More precisely, Brandom claims:

> The semantic values of all the logically compound sentences are computable entirely from the values of less complex sentences. (Brandom, 2008, p. 135)

But Example 1 refutes this claim. The semantic value of $\neg p$ in the model $Inc'_{\mathcal{L}'}$, i.e. its coherence or incoherence jointly with other sentences, is *not* determined by the coherence or incoherence of the sets of sentences in the language $\mathcal{L}$, where $\mathcal{L}' = \mathcal{L} \cup \{\neg p\}$. So why does Brandom think that he can maintain his claim, although (apparent?) counterexamples are readily specified? The answer to this question can (only) be found in Section 5 of Appendix I to Chapter 5 of Brandom (2008). There, it turns out that in extending an incoherence model *Inc* ('frame' in his terminology) from a language $\mathcal{L}$ to a model Inc' over a richer language $\mathcal{L}'$, just like in Example 1, Brandom does not consider it sufficient that Inc' and *Inc* coincide over sets of sentences in $\mathcal{L}$. He rather imposes another property, namely *inferential conservativity*, defined as follows.

Definition 5 *Let $\mathcal{L} \subseteq \mathcal{L}'$ and let Inc be an incoherence model over $\mathcal{L}$. Then an incoherence model Inc' over $\mathcal{L}'$ is* inferentially conservative with respect to *Inc iff, for all $X, Y \subseteq \mathcal{L}$, $X \models_{Inc} Y$ iff $X \models_{Inc'} Y$.*[8]

[8] In Brandom (2008) $\models_{Inc}$ and $\models_{Inc'}$ refer to the disjunctive generalization of the single-conclusion incompatibility entailment relation, whereas we prefer the conjunctive version of Definition 4. However, nothing in our criticism below depends on this choice.

Brandom shows that for every incoherence model *Inc* over $\mathcal{L}$ and any proper extension $\mathcal{L}'$ of $\mathcal{L}$, there exists a unique smallest incoherence model *Inc*$'$ over $\mathcal{L}'$, called *the model for* $\mathcal{L}'$ *determined by Inc*, that is inferentially conservative with respect to *Inc*.

Let us revisit Example 1 to see whether the insistence on inferential conservativity indeed settles the question whether $\{q, \neg p\}$ should be incoherent in $Inc'_{\mathcal{L}'}$, extending $Inc_{\mathcal{L}}$. Recall that $q \models_{Inc} p$. If we declare $\{q, \neg p\}$ to be coherent in *Inc*$'$ then we obtain $q \not\models_{Inc'} p$, since $\{p, \neg p\} \in Inc'_{\mathcal{L}'}$, but $\{q, \neg p\} \notin Inc'_{\mathcal{L}'}$. Therefore we have to set $\{q, \neg p\} \in Inc'_{\mathcal{L}'}$, if we want $Inc'_{\mathcal{L}'}$ to be inferentially conservative over $Inc_{\mathcal{L}}$.

We emphasize that Brandom's axioms for incoherence and incompatibility entailment do not constrain the models as indicated above. If incompatibility semantics is intended to apply only to models that are inferentially conservative over a given model over an atomic language, then one should add corresponding axioms. But this complaint should not eclipse a more urgent question, namely whether restricting attention to 'determined' models meshes with the intended meaning and use of incompatibility semantics. We argue that this is not the case, by instantiating the propositional variables p and q of Example 1 with concrete sentences as follows:

p**:** "It is raining in Vienna."

q**:** "It is raining in New York."

The incoherence model over the language $\mathcal{L} = \{p, q\}$ in Example 1 declared p to be compatible with q. This is certainly a reasonable choice also with respect to the given natural language interpretation of p and q. Once more, we expand $\mathcal{L}$ to $\mathcal{L}'$ by including also $\neg p$ = "It is not raining in Vienna". As we have seen above, insisting on inferential conservativity forces us to declare "It is not raining in Vienna" to be incompatible with "It is raining in New York". This looks very odd, indeed. Incompatibility semantics should allow us to formally model a situation where not only p is compatible with q, but also $\neg p$ is compatible with q (and where the three mentioned statements are the only ones under consideration). However, we can only accomplish this if we ignore inferential conservativity and consequently dispense with recursive projectibility.

5 Characterizing classical logic and S5

Brandom (2008) shows that the set of formulas that are coherent in all incoherence models coincides with the set of valid formulas in modal logic S5. The proof is quite involved and not without problems, because of the issue with Brandom's disjunctive version of the generalized incompatibility entailment relation that we have pointed out in Section 3. In any case, it is important to recognize that (material) incompatibility entailment behaves quite differently compared to classical material entailment, even in its ordinary, single-conclusion format. To get a better view of the issue, let's first review some basic notions of classical logic.

Definition 6 *A* (CL-)valuation *(or* interpretation*) is a function v that assigns either 1 (for 'true') or 0 (for 'false') to every propositional variable.*[9] *It is extended to propositional formulas, built up using negation and conjunction, as usual:*

$$v(\neg F) = 1 - v(F) \qquad\qquad v(F \wedge G) = \min(v(F), v(G))$$

We may add the atomic formula $\bot$ (falsum), stipulating $v(\bot) = 0$.

A set of formulas X materially CL-entails *a formula F* with respect to an interpretation *v (written $X \models_v F$) iff $v(G) = 0$ for some $G \in X$ or $v(F) = 1$.* *X* logically CL-entails *a formula F iff $X \models_v F$ for all interpretations v. F is* CL-valid *($\models_{CL} F$) iff $v(F) = 1$ for all valuations.*

We write $G_1, \ldots, G_n \models_{v/CL} F$ instead of $\{G_1, \ldots, G_n\} \models_{v/CL} F$.

Already on the atomic level, a difference between material CL- and incompatibility entailment emerges. Let p and q be propositional variables. For every valuation v, we clearly have

$$p \models_v q \text{ or } q \models_v p.$$

In contrast, there are incoherence frames *Inc*, such that

$$\text{neither } p \models_{\mathit{Inc}} q \text{ nor } q \models_{\mathit{Inc}} p.$$

To see the latter, consider a language containing also the propositional variables r and s, and let $\{q, r\} \in \mathit{Inc}$, $\{p, r\} \notin \mathit{Inc}$, $\{p, s\} \in \mathit{Inc}$, $\{q, s\} \notin \mathit{Inc}$.

[9] Usually, one assumes an infinite supply of propositional variables. But when the context fixes a finite language $\mathcal{L}$ (in the sense of Brandom) we may safely assume that only those propositional variables that occur in $\mathcal{L}$ are meant.

Even if we restrict attention to languages containing only those propositional variables that are explicitly mentioned in the entailment claim, incompatibility entailment behaves non-classically: if $\mathcal{L} = \{p, q, p \wedge q\}$ and $\{p, q\} \in \mathit{Inc}$, but $\{p\} \notin \mathit{Inc}$ and $\{q\} \notin \mathit{Inc}$ we have

$$p \not\models_{\mathit{Inc}} \perp \text{ and } q \not\models_{\mathit{Inc}} \perp, \text{ but } p \wedge q \models_{\mathit{Inc}} \perp.$$

In spite of the indicated differences, one can establish a clear and tight connection between incompatibility semantics and classical semantics in a manner that is simpler and more transparent than Brandom's approach. The idea is to simply declare a set of sentences to be incoherent iff the conjunction of its members is false. More formally, consider the following correspondence between incompatibility models and sets of CL-valuations.

Definition 7 *Let Inc be an incoherence model over a language $\mathcal{L}$. The* corresponding set of CL-valuations $\mathcal{V}_{\mathit{Inc}}$ *is defined as*

$$\mathcal{V}_{\mathit{Inc}} = \{v \mid \forall X \in \mathit{Inc}\, \exists F \in X \colon v(F) = 0\}.$$

Since we only consider finite sets of sentences, we have $X \in \mathit{Inc}$ iff all valuations in $\mathcal{V}_{\mathit{Inc}}$ evaluate the conjunction of formulas in X as false.

The inverse translation, from sets of valuations to incoherence models needs to be handled with some care, due to Brandom's unusual definition of a (proper) language $\mathcal{L}$.

Definition 8 *Let $\mathcal{L}$ be a proper language (in the sense of Brandom) and let $\mathcal{V}$ be a set of CL-valuations over the set $\mathcal{L}_{at}$ of propostional variables occurring in $\mathcal{L}$. Then the* incoherence model $\mathit{Inc}^{\mathcal{V}}_{\mathcal{L}}$ corresponding to $\mathcal{V}$ *is given by*

$$\mathit{Inc}^{\mathcal{V}}_{\mathcal{L}} = \{X \subseteq \mathcal{L} \mid \forall v \in \mathcal{V}, \exists F \in X \colon v(F) = 0\}.$$

Let us indicate why $\mathit{Inc}^{\mathcal{V}}_{\mathcal{L}}$, as just defined, indeed constitutes an incoherence model, complying with Brandom's axioms. Persistence, i.e. the fact that $X \in \mathit{Inc}^{\mathcal{V}}_{\mathcal{L}}$ implies $X' \in \mathit{Inc}^{\mathcal{V}}_{\mathcal{L}}$ for every $X \subseteq X' \subseteq \mathcal{L}$, is maintained since $\exists F \in X \colon v(F) = 0$ trivially implies $\exists F \in X' \colon v(F) = 0$ whenever $X \subseteq X'$.

To see that axiom **CI** holds for $\mathit{Inc}^{\mathcal{V}}_{\mathcal{L}}$, it suffices to recall that we are only dealing with finite languages here and thus may identify a set of formulas with the conjunction of its members. (The empty conjunction is identified with $\top$.)

To understand that also the negation axiom **NI** is satisfied, recall the reformulation of **NI** as stated in Section 3. Since there is no reference to the logical form of the involved formula, except for the negation sign preceding F, we may code X, Y, and F by propositional variables x, y, and f, respectively. In accordance with Definition 8, we may thus expresses the claim $X \cup \{\neg F\} \in \mathit{Inc}_{\mathcal{L}}^{\mathcal{V}}$ as the claim that $v(x \wedge \neg f) = 0$ for all corresponding valuations v. Proceeding analogously for $\{F\} \cup Y$ and $X \cup Y$ reduces **NI** to the following claim referring to classical logic:

$$v(x \wedge \neg f) = 0 \text{ iff } \forall y[v(f \wedge y) = 0 \text{ implies } v(x \wedge y) = 0].$$

That this statement is true for all CL-valuations v can be verified by eliminating the propositional quantifier and coding the whole claim as the propositional formula

$$\neg(x \wedge \neg f) \leftrightarrow [(\neg(f \wedge \bot) \to \neg(x \wedge \bot)) \wedge [\neg(f \wedge \top) \to \neg(x \wedge \top))$$

which is a classical tautology.

What about the modal operator $\Box$? We argue that the above analysis straightforwardly generalizes to a characterization of $\mathsf{S5}$. Recall that the standard (Kripke) semantics for modal logics refers to a model $\langle W, R, V\rangle$, where W is a non-empty set of *worlds*, R a binary *accessibility relation* over W, and where V associates a CL-valuation with every $w \in W$. The semantics of $\neg$ and $\wedge$ refers to the CL-valuations as usual, thus assigning a truth value $v_w(F)$ to every non-modal formula F in each world w. This extends to modal formulas via the condition $v_w(\Box F) = 1$ iff $v_{w'}(F) = 1$ for all w' such that $R(w, w')$. A formula F is called *valid* in $\mathsf{S5}$ if for all models, where R is reflexive, symmetric and transitive (i.e. an equivalence relation) $v_w(F) = 1$ in every world w. It is easy to see that only *connected* components of the graph (*frame*) $\langle W, R\rangle$ are relevant when evaluating formulas in a given world. Hence, in the case of $\mathsf{S5}$, we may focus on the special case where R is the *universal relation*, i.e. $R(w, w')$ for all $w, w' \in W$. But in such models duplicates of worlds associated with the same valuation are redundant. To sum up these observations: we may define $\mathsf{S5}$-validity with respect to an arbitrary set of CL-valuations $\mathcal{V}$, rather than with respect to Kripke models, by declaring for any $v \in \mathcal{V}$ that $v(\Box F) = 1$ iff $v(F) = 1$ for all $v \in \mathcal{V}$. Notice that this means that the correspondence between incoherence models and sets of CL-valuations, established by Definitions 7 and 8, carries over to languages that include modal formulas. In other words, we may still interpret $X \in \mathit{Inc}$ as expressing that for every $v \in \mathcal{V}_{\mathit{Inc}}$ we have

$v(F) = 0$ for at least one $F \in X$. An argument analogous to that for the axiom **NI**, above, shows that also the axiom **LI** for the introduction of the modal operator $\Box$ is satisfied for the suggested interpretation of incoherence.

6 A GOGAR model

Compared to Brandom's rather involved and indirect proof that S5-validity coincides with coherence in all incoherence models, the considerations in Section 5 provide a much more direct route to the understanding of the relation between incompatibility semantics and classical Tarski/Kripke semantics. However, one might complain that our analysis is at variance with Brandom's deliberate avoidance of reference to truth values and his intention to interpret 'incoherence' as a pragmatic notion, rather than as a purely semantic concept. However, as our above analysis reveals, the setup of the semantic machinery in Brandom (2008) is largely severed from Brandom's philosophical stance about logic: accepting incompatibility semantics does not directly support normative inferentialism. In fact we share Brandom's favoring of an approach to logic that gives preference to normative pragmatics over pure semantics and that consequently respects inferentialist insights. We also embrace the concept of logical expressivism regarding the meaning of connectives. Since incompatibility semantics itself hardly adequately meets corresponding demands, we finally want to explore, at least tentatively, an alternative approach that is more directly connected to Brandom's concept of a 'game of giving and asking for reasons' (GOGAR), introduced in *Making It Explicit* (Brandom, 1994) and that may well be classified as inferentialist, pragmatist, and logically expressivist.

We suggest to model GOGAR[10] as a formal game played by a *proponent* **P** and a *questioner* **Q**, reminiscent of the 'dialogical logic' of Lorenzen (1960). The game is not intended to cover the full range of possible interactions between two rational conversationalists, but rather restricts attention to a scenario where **P** seeks to defend a single claim questioned by **Q**. Let us first ignore the logical structure of sentences. The corresponding *atomic game* instantiates the following simple schema:

P: *asserts a claim* p
Q: *asks for reasons to accept* p
P: *offers corresponding reasons* $r_1, \ldots, r_n$

[10] Another formal model of GOGAR is presented in Porello (2012).

At this point **Q** may declare to be satisfied or not. If **Q** is not (yet) satisfied, the game continues by treating the asserted reasons as further claims made by **P** that may be questioned by **Q**. Deciding rationally whether **Q** should declare to be satisfied consists in making two different kinds of judgments: (1) judging whether the reasons $r_1, \ldots, r_n$ materially entail p and (2) judging whether to accept $r_1, \ldots, r_n$ (independently of the claim p). Note that we did not exclude the possibility that **P** offers the asserted claim itself among the reasons to accept it. Such a move by **P** clearly settles the entailment judgment (1), but has no bearing on the second judgment (2). Here, we are only interested in the 'intrinsic logic' of GOGAR and hence focus on the entailment question. At a first glance, this seems to trivialize to task of **P**: simply repeating the claim when questioned by **Q** settles the matter. But this is only the case if we understand the (atomic) game as strictly adversarial, i.e., formally, as a win-lose game. However, we may just as well assume that it is in the interest of the players to make non-trivial entailment claims explicit. In fact, rather than simply referring to an arbitrarily given atomic relation, we may connect the game with incompatibility semantics and stipulate that **P** and **Q** agree about an *atomic* incoherence frame $\mathit{Inc}_{\mathcal{L}}$, where $\mathcal{L}$ is the set of (atomic) sentences assertible as claims or reasons. In this scenario the above game may proceed as follows.

Q: *chooses* $q_1, \ldots, q_m$ *such that* $\{p, q_1, \ldots, q_m\} \in \mathit{Inc}_{\mathcal{L}}$
P: *replies by pointing out that* $\{r_1, \ldots, r_n, q_1, \ldots, q_m\} \in \mathit{Inc}_{\mathcal{L}}$

Depending on $\mathit{Inc}_{\mathcal{L}}$, **Q** may not be able to make the indicated move. In this case $r_1, \ldots, r_n \models_{\mathit{Inc}} p$ has been established. Similarly, **P** may not be able to make her move, which means that $r_1, \ldots, r_n \not\models_{\mathit{Inc}} p$. Furthermore, if **P** has a reply to every possible move of **Q**, then, again, $r_1, \ldots, r_n \models_{\mathit{Inc}} p$.

Let us now consider richer languages. Like in the atomic case, a general GOGAR instance starts with a claim F by **P**, followed by reasons $G_1, \ldots, G_n$ for accepting F, asserted by **P** after questioned by **Q**. This results in a state denoted by $G_1, \ldots, G_n \triangleright F$, corresponding to a material entailment claim $G_1, \ldots, G_n \models F$. Not only the claim (consequent) F, but also the stated reasons (premises) may be questioned by **Q**. Since we do not want to prevent **Q** from questioning the same sentence more than once during the run of the game, the collection $G_1, \ldots, G_n$ is formally modeled as a *multiset*, rather than a set of sentences.

In contrast to Brandom, we aim at an autonomous semantics for implication ($\rightarrow$) that does not define $F \rightarrow G$ as an abbreviation of $\neg(F \wedge \neg G)$. Rather, we want to come up with a GOGAR rule that matches the logically expressivist insight that $F \to G$ is just a syntactic device that allows one to express that G is materially entailed by F in a given context of further assertions. We therefore suggest the following rule that refers to state $H_1, \ldots, H_m \triangleright F \to G$.

Q: *asks for reasons to accept* $F \to G$ *if* $H_1, \ldots, H_m$ *are accepted*
P: *asserts that* $H_1, \ldots, H_m$*, augmented by* F*, are reasons for asserting* G

From now on we will denote such a rule more concisely as

$$\frac{H_1, \ldots, H_m \triangleright F \to G}{H_1, \ldots, H_m, F \triangleright G}$$

For conjunctive claims we stipulate the following rule.

$$\frac{H_1, \ldots, H_m \triangleright F \wedge G}{H_1, \ldots, H_m \triangleright F \ \text{ and } \ H_1, \ldots, H_m \triangleright G}$$

In words: when **Q** asks **P** for reasons to accept $F \wedge G$ given the sentences $H_1, \ldots, H_m$, **P** replies that $H_1, \ldots, H_m$ constitute reasons to accept F as well as reasons to accept G. Thus conjunction ($\wedge$) is treated as an expressive device to join two separate entailment claims. Note that this rule indicates that an overall state of GOGAR is given by a multiset $\{\Gamma_1 \triangleright F_1, \ldots, \Gamma_n \triangleright F_n\}$ of component states, where each Γ_i is a multiset of sentences currently offered as reasons for the claim F_i. To determine how the game is to proceed, we let **Q** choose the component state (entailment claim) to which the next round of interactions has to apply.

The GOGAR rule for disjunction involves a choice by player **P**:

$$\frac{H_1, \ldots, H_m \triangleright F \vee G}{H_1, \ldots, H_m \triangleright F \ \text{ or } \ H_1, \ldots, H_m \triangleright G}$$

In other words, in reply to **Q**'s questioning of the claim $F \vee G$, **P** reduces her disjunctive claim to claiming one of the disjuncts.

We still need rules for reducing states that involve logically complex sentences asserted as reasons. The case for conjunctive reasons is particularly simple: if **Q** questions a conjunctive sentence $F \wedge G$, asserted by **P** as a reason for accepting some claim, then **P** simply replaces $F \wedge G$ by F and G in the multiset of asserted reasons. In our concise notation this amounts to

$$\frac{F \wedge G, H_1, \ldots, H_m \triangleright I}{F, G, H_1, \ldots, H_m \triangleright I}$$

For disjunctive reasons we introduce the following rule:

$$\frac{F \vee G, H_1, \ldots, H_m \triangleright I}{F, H_1, \ldots, H_m \triangleright I \;\text{ and }\; G, H_1, \ldots, H_m \triangleright I}$$

In other words, if prompted by **Q**, **P** makes explicit that in order to establish the material entailment claim $F \vee G, H_1, \ldots, H_m \models I$ it is sufficient to establish the two entailment claims that result from replacing $F \vee G$ by either F or G, respectively, in the premises (i.e., in the multiset of reasons given so far for accepting I).

Finally, consider the following rule for the case, where **Q** questions $F \rightarrow G$ among the reasons given by **P** for accepting the sentence I.

$$\frac{F \rightarrow G, H_1, \ldots, H_m \triangleright I}{F \rightarrow G, H_1, \ldots, H_m \triangleright F \;\text{ and }\; G, H_1, \ldots, H_m \triangleright I}$$

Again, the rule can be understood as making explicit that establishing $F \rightarrow G, H_1, \ldots, H_m \models I$ can be reduced to establishing $G, H_1, \ldots, H_m \models I$ as well as $F \rightarrow G, H_1, \ldots, H_m \models F$. The presence of $F \rightarrow G$ in the latter entailment claim may appear redundant at first glance. However, it turns out that one should allow **Q** to question $F \rightarrow G$ again in this situation. (Alternatively, one may introduce a rule forcing **P** to provide another copy of the indicated implication, if prompted to do so by **Q**.)

No separate rules for negated sentences as reasons or claims are needed if $\neg F$ is treated as $F \rightarrow \bot$, where $\bot$ denotes a sentence that is incoherent in all interpretations.

To turn our GOGAR model into an ordinary two-person extensive form game, we still have to do three things: (1) define how the game ends, (2) specify pay-off values for both players at final states, and (3) settle a remaining indeterminacy about the possible continuation at non-final states: once **Q** has picked a component (entailment claim) to which one of the above game rules is to be applied, who gets to choose which non-atomic formula (claim or reason) is to be reduced?

Let us address task (1) first. Note that it is not reasonable to declare that the game ends if and only if all sentences in all entailment claims that constitute the current state game are atomic, since we keep non-atomic sentences asserted as reasons available for further questioning. Rather, we declare that the game ends as soon as, in every component (entailment claim) of the current state, the claimed sentence (conclusion) is atomic and also appears among the sentences asserted as reasons for it or else $\bot$

appears among the asserted reasons (premises). I.e., final states are multisets of component states, each of the form $p, H_1, \dots, H_n \triangleright p$ or of the form $\bot, H_1, \dots, H_n \triangleright p$, where p is an atomic sentence. Of course, there is still no guarantee that a given instance of the game ever ends: we have to take into account infinite runs as well.

Tasks (2) and (3) are not independent of each other. If we model GOGAR as a win-lose game, then we stipulate that **P** wins the game whenever it reaches a final state and **Q** wins if the game runs forever. In this case we should give **P** the right to choose the next non-atomic formula to be reduced (i.e., the rule to be applied) in the component of the current state that has been chosen by **Q**, since otherwise **Q** could trivially force each non-atomic instance of the game to run forever. An alternative option is to model GOGAR as cooperative game, where both players prefer to reach a final state, rather then to play forever. In the latter case, it does not matter whether **P** or **Q** chooses the sentence to be reduced next.

Claim 1 *Suppose that GOGAR starts with* ***P****'s claim that F and* ***P*** *offers the reasons $H_1, \dots, H_n$ for accepting F, when questioned by* ***Q****.*

In the win-lose version of GOGAR, ***P*** *has a winning strategy iff F is a logical consequence of $H_1, \dots, H_n$ according to intuitionistic logic ($H_1, \dots, H_n \models_{IL} F$).*

In the cooperative version of GOGAR, ***P*** *and* ***Q****, jointly have a winning strategy iff $H_1, \dots, H_n \models_{IL} F$.*

A detailed proof of Claim 1 is beyond the scope of this paper. However, we may indicate the essence of the proof by directing the reader to the sequent calculus **G3i** for intuitionistic logic in (Troelstra & Schwichtenberg, 2000). Reading our game rules from bottom to top and replacing $\triangleright$ by the sequent arrow turns them into the propositional rules of **G3i**. Moreover, the axioms of **G3i** match the definition of final game states. In this manner, our version of GOGAR emerges as an interpretation of this intuitionistic calculus.[11]

7 Conclusion

Our re-assessment of incompatibility semantics revealed a number of problems with Brandom's definitions and claims. Most importantly, the central

[11] It is easy to see that **G3i** proofs correspond to winning strategies in GOGAR. To complete the proof, one still has to establish that the above stipulations about the possible successions of moves amount, in both versions of the game, to a successful proof search strategy in **G3i**.

claim that, although holistic and non-compositional, incompatibility semantics admits recursive projectibility and hence refutes the claim that holistic semantics cannot account for the systematicity and learnability of language, has been shown to rest on the additional assumption of inferential conservativity. This assumption, however, is at odds with the intended application of incoherence models as pointed out in Section 4: recursive projectibility can only be obtained for a price that, arguably, is too high to pay.

Another complaint about Brandom's approach to logical semantics is the fact that it does not, at least not directly, amount to a pragmatist and inferentialist account that ties in with logical expressivism. In particular, Brandom's own insight that the logical connective in $F \to G$ can be seen as an expressive device for expressing that G is materially entailed by F is not reflected in a corresponding meaning postulate in incompatibility semantics. Therefore we suggested an alternative approach that seeks to define the meaning of logical connectives via rules of an idealized, formal 'game of giving and asking for reasons', in which a proponent **P** systematically reduces a logically complex entailment claim to an atomic entailment, prompted by systematic questioning by a second player **Q**. While this is similar to dialogical logic (Lorenzen, 1960), a main difference is that the role of **Q** need not necessarily be understood as antagonistic to that of **P**.[12] Moreover, we indicated how our GOGAR model can be connected to the concept of incompatibility-entailment at the atomic level. The game amounts to an interpretation of a cut-free sequent calculus that is sound and complete for intuitionistic logic. It remains to be investigated whether this model can be extended to rules for asserting sentences that feature a modal operator. Another line for further research is to explore connections to the notion of scorekeeping in language games by Lewis (1979) and observations about the necessity of postulating a common ground between effective conversationalists (Stalnaker, 2002). Finally, it should be mentioned that Brandom and his collaborators recently shifted attention to an account of material and logical entailment that, in contrast to the incompatibility semantics of Brandom (2008), embraces *non-monotonicity* and consequently relates to sequent systems that do not admit weakening (see Brandom, 2021). We plan to investigate whether our game semantic approach can be extended to cover also corresponding insights about non-monotonic inference.

[12] Another major difference is that in the GOGAR model **Q**, unlike the opponent to **P** in Lorenzen's game, does not assert sentences herself, but only questions those asserted by **P**.

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