TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

Research group
Mechanics of Solids
Mechanics & Mechatronics

DISSERTATION

Nonlinear modeling and analysis of thin dielectric elastomer structures as electro-elastic material bodies and surfaces

submitted in satisfaction of the requirement for the degree of Doctor technicae (Dr. techn.), at TU Wien, Department of Mechanical and Industrial Engineering

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Doctoral Thesis

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Vienna, June 2nd, 2020 *Signature*

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Preface

Prior to the scientific discussions, I would like to take this opportunity to express my gratitude to my supervisor Prof. Michael Krommer, for his scientific support, his engagement and the willingness to escort my academic career till the completion of this thesis. Thank you for the opportunity to work in such a close collaboration on this interesting topic, and all the best for the new duties and challenges awaiting you at JKU Linz.

Now, at the finishing phase of this thesis, I feel overwhelmed, when I think back to the fortunate coincidence that brought me to the institute when Prof. Krommer took on the lead of this group. Developing the foundation of this new group was as much part of the job as developing advancements in science and teaching; for his outstanding scientific support, I want to thank my research fellow Yury Vetyukov, and Alois Steindl for his warm and welcoming spirit.

I also want to thank my colleagues within the research group Mechanics of Solids, as well as the remaining colleagues from the fifth floor, thank you for the pleasant working environment, the support and all the great moments we shared. To all my friends, and former colleagues which became friends, thank you for the great time, but more important thank you for the support during difficult moments and the opportunity to share them.

Finally I want to thank my family for whom I switch to German:
Für die wundervolle Unterstützung, und die Möglichkeit mein Studium soweit voranzutreiben bedanke ich mich ganz herzlich bei meinen Eltern, Geschwistern und Paten, Danke das ich immer auf euch zählen kann!

Ein ganz besonderer Dank gilt meinem Ehemann, für deine immerwährende Geduld, die Motivation und Hoffnung die du mir schenkst meinen eingeschlagenen Weg beizubehalten, Danke für deine Liebe die du mir zuteil werden lässt!

Kurzfassung

Weiche Materialien die sich durch das Anlegen eines elektrischen Signals verformen können, bieten aufregende neue Möglichkeiten, hin zu einer technologisch neuen Generation von lasttragenden mechanischen Komponenten wie weiche Roboter, weiche Assistenztechnologien oder sogar Luftfahrzeuge mit weichen Flügeln. Kandidaten für die konkrete Umsetzung gehören zu den sogenannten "smarten" Materialien, wobei dielektrische Elastomere (DEs) besonders vielversprechend sind und eine Untergruppe der sogenannten elektro-aktiven Polymere (EAPs) darstellen. Verblüffend ist vor allem deren Eigenschaft sich umso leichter verformen zu lassen, je dünner die Materialstärke ist. Daher werden in der Praxis jene Komponenten die mit dielektrischen Elastomeren bestückt sind verhältnismäßig dünn ausgeführt, wofür sich bevorzugt eine strukturmechanischen Beschreibung als materielle Fläche anbietet.

Zu den Herausforderungen die es bei der Modellierung von dielektrischen Elastomer Aktuatoren zu meistern gilt gehören große Verformungen, das hyperelastische Materialverhalten und die verschiedenen Stufen der elektromechanischen Kopplung, wie elektrostatische Kräfte aber auch Effekte höherer Ordnung, wie z.b. Piezoelektrizität oder Elektrostriktion.

Ausgangspunkt dieser Arbeit stellt die Theorie elektro-elastisch gekoppelter Strukturen dar, wobei die grundlegenden Gleichungen sukzessive von der Kopplung mittels rein elektrostatischer Kräfte zu Effekten höherer Ordnung am Beispiel der Elektrostriktion, erweitert werden. In der konkreten Umsetzung wird das Materialgesetz schrittweise aus der drei dimensionalen Formulierung in eine ebene Formulierung übergeführt, wobei mehrere multiplikative Aufspaltungen des Deformationsgradiententensors eine intensivierte Kopplung zwischen dem mechanischen und dem elektrischen Feld ermöglicht.

Das direkte Verfahren zur Modellierung von Platten und Schalen als materielle Fläche erlaubt es auf elegante Weise eine vollständig strukturmechanische Theorie herzuleiten, wobei das widerspruchsfreie Entwickeln des hyperelastischen Materialgesetzes, aufgeteilt in einen Membran- und einen Biegeanteil als neuartig erachtet wird und eine eindeutige physikalische Interpretation der einzelnen Zusammenhänge zulässt. Die numerische Analyse von verschiedenen Beispielproblemen, bestätigt die Validität der beschriebenen Modelle, im Vergleich zu drei dimensionalen finiten Elemente Ergebnissen sowie zu veröffentlichten Ergebnissen anderer Modelle.

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Abstract

Soft materials, which deform under an electrical stimulation pose exciting new options for a technologically new generation of load bearing mechanical devices such as soft robots, assistant devices or solid state aircrafts. Promising candidates for such soft, smart materials are dielectric elastomers (DEs), a special type of electroactive polymers (EAPs). An intriguing property of DEs is the circumstance that they work the better the smaller the thickness gets. Taking advantage of this fact gives rise to the typical thin application designs for which a structural mechanics approach to the modeling of dielectric elastomer plates and shells as a material surface applies.

Modeling dielectric elastomers vouch for multiple challenges as there are large deformations, hyperelasticity and multiple levels of electro-mechanical couplings by means of electrostatic forces as well as higher order effects such as piezoelectricity and electrostriction. Taking the electro-elastic coupled theory into the center of this thesis, the governing equations get gradually extended from the incorporation of electrostatic forces to higher order effects by means of electrostriction. To this end, the constitutive relations are step by step extended from the three dimensional to the structural level and multiple multiplicative decompositions of the deformation gradient tensor enable an increased coupling between the mechanical and the electrical fields.

A direct approach for modeling plates and shells as a material surface enables the elegant development of a thorough structural theory. In particular, contradiction free two dimensional constitutive relations for a hyperelastic plate, with separate membrane and bending parts comprise a novelty and offer great physical insight. Numerical studies on different example problems confirm the validity of the developed models in comparison to three dimensional results as well as in comparison to different approaches found in literature.

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1 Introduction

1.1 Problem statement: State of the art

Soft and flexible materials e.g. certain polymer or gel type materials show the capabilities to react upon different types of stimulation e.g.: electric field, temperature, conductivity or different chemical field properties such as pH-value, or concentration. Polymers which are capable to react upon an applied electric field- so called electro-active polymers, promise a new generation of soft technological devices with a great range of possible future applications. Exemplar realizations within the laboratory frame are: soft and flexible sensors integrated to robotic skins [50] - possibly in form of textile fibers [40] or integrable soft actuators [8, 4, 10, 25] for among others position adjustment. Such soft components already pave the way for future soft robots see e.g. Hareesh et al. [21] and Henke et al. [22]. Entirely soft robots carrying a soft control unit were presented in [23], and also energy generators featuring soft transducers [11, 9] have already been built. Because of the simple function principle and its robustness the so called dielectric elastomers (DEs), which belong to the group of electronic electroactive polymers (EEAPs), have gained broader attention over ionic polymer metal composites (IPMCs), where ion movement and diffusion cause the specimen to deform. Although both types feature the electric field as actuation stimulus, however, the driving mechanisms in DEs are mainly due to attractive coulomb forces and ferroelectricity. The unique actuation property of DEs to actuate large strains at high electric fields provokes a muscular-like behavior acquiring the name *artificial muscle* for these materials. Applying an electric field to the electrodes mounted on top and bottom of a thin film elastomer squeezes the elastomer in field direction causing an extension of the area orthogonal to the field direction. This area change together with the thickness squeeze also causes a change in the capacitance and hence provides information about the deformed current state used for sensing. Material scientists in the 1990s [49] presented enhanced, so called electrostricitive elastomers with amplified actuation properties by featuring crystal inclusions embedded into the elastomer matrix. Still, the implementation to broader application fields is limited due to the high field strength needed for actuation that may cause an electric breakdown and hence the destruction of the devices see e.g. Xu et al.[56].

Apart from other limiting factors, such as saturation effects like polarization saturation studied in Liu et al.[35], strain stiffening in [54], or localization effects causing the formation of wrinkles, see Mao et al. [36] and Zurlo[60], a solid understanding of the inherent electro-elastic coupling is of great value in order to develop further enhanced materials as well as to circumvent the limitations by implementing proper control strategies. To this end, accurate mathematical models are vital to increase the knowledge and to develop predictive tools decreasing the need for prototyping and testing.

The challenging task of modeling large strains initiated by an electric field attracted already a growing number of scientists. Original modeling approaches date back to Toupin [51], more recent discussions are found in Prechtl [44, 45] and Eringen and Maugin [19], who characterized the generalized continuum theory for electro-magnetic elastic continua using a modern coordinate free notation, a notation distinctively shaped by Lurie [34]. Contemporary literature nowadays mainly focus on the incorporation of multiple fields, see Mehnert et al. [37] for thermo-electro elastic DEs, and on irreversible viscoelastic effects e.g. Ask et al. [6, 7] or Zäh and Miehe [58], as well as the incorporation of higher order nonlinear effects such as electrostriction, see Jiménez and McMeeking [26], Ask et al. [5], Zäh and Miehe [57], Zhao and Suo [59] and our own work [47]. Substantial testings and material parameter identification have been acquired by Wissler and Massa [55] for 3Ms VHB cohecive compound, and recently Mehnert et al. published extend material tests for VHB with respect to viscoelasticity [38], thermo-viscoelcasticity [31] and the effect of carbon grease electrodes on the elastic and viscous material parameter [39]. Material testings for electrostrictive polyurethane has been published in [14].

In contrast to the three dimensional approach pursued in the references so far, the focus within this dissertation lies in a more efficient modeling technique. Taking advantage of the typical thin, plate type, geometry of dielectric elastomers, a structural mechanics approach poses particular advantages with respect to great accuracy and efficiency. Models on generalized continua, have been first developed in the late 19*th* century by the Cosserat brothers [13]. Among the applications of micro-polar shell theories, we mention Libai and Simmonds [32], Eremeyev et al. [18], Altenbach and Eremeyev [2], and with respect to the numerical analyses Pimenta et al. [41, 42].

Relevant for this work are Kirchhoff-Love shells with five mechanical degrees of freedom as derived by Eliseev and Skoblikov [16]. Recent publications on the modeling of dielectric elastomers using different approaches are [43], [48] as well as [27] where a solid shell finite element formulation has been used. The present formulation follows the one developed by Eliseev and Vetyukov [17] and Vetyukov et al. [53] where a direct approach to model plates and shells as material surfaces was presented. The particular novelty of the published contents in this thesis are the electro-elastically coupled structural relations by means of coulomb force interaction, discussed in [46], directly derived two dimensional constitutive relations aiming for a thorough structural description as developed in [29] and [20], and the incorporation of electrostriction into the structural plate theory in [28], and [] (JIMSS 2019 not yet published).

1.2 Scope of this thesis

The cumulative thesis starts with a short review of the basic theories and principles used in the publications constituting the thesis. Subsequent to a review of the general three dimensional relations for electro-elastic continua, an asymptotic analysis justifies the presumptions used in the following direct approach to modeling electro-elastically coupled plates and shells as a material surface. Special emphasize in the introductory part is put on the multiplicative decomposition of the deformation gradient tensor, presenting the basic concept and its impact on the field of stresses and strains.

The geometric nonlinear theory for thin plates under the action of electrostatic force coupling is demonstrated in *Paper A*. A three dimensional approach to incorporate higher order couplings by means of electrostriction is demonstrated in *Paper B*. In *Paper C* electrostriction is implemented to the present plate theory by applying a plane stress assumption and using an electrostrictive material parameter found by correlating published measurement results to the present model. In *Paper D* a complete direct approach towards modeling thin plates under the action of electrostatic forces is demonstrated. Through the use of a polar decomposition of the surface deformation gradient tensor, a decoupling into separate membrane and bending energies is established accounting for electrostatic forces as a source of *self-stress*, a so-called *Eigenspannungsquelle*. Raising the geometric nonlinear plate theory to account for constitutive coupling, multiple multiplicative decompositions of the deformation gradient are discussed in *Paper E*. Extending the relations found in *Paper D*, the newly gained electrostrictive plate model shows a very good agreement with other models found in the literature, but remains free from any contradictory assumptions concerning the state of stress.

1.2.1 List of publications representing this Doctoral Thesis

E. Staudigl, M. Krommer, Y. Vetyukov (2017)
"Finite deformations of thin plates made of dielectric elastomers: Modeling, numerics and stability"
Journal of Intelligent Material Systems and Structures, 29(17), 3495-3513

E. Staudigl, M. Krommer, A. Humer (2018)
"Modeling of Dielectric Elastomers Accounting for Electrostriction by Means of a Multiplicative Decomposition of the Deformation Gradient Tensor" *Analysis and Modelling of Advanced Structures ans Smart Systems*
Altenbach H., Carrera E. and Kulikov G. (eds.), Springer, Vienna

E. Staudigl, M. Krommer,Y. Vetyukov, A. Humer (2018)
"Nonlinear electro-elastic modeling of thin dielectric elastomer plate actuators" *Proceedings of SPIE Smart Structures and Materials + Nondestructive Evaluation, and Health Monitoring*
edited by Yoseph Bar-Cohen, Vol. 10594, doi:10.1117/12.2295881

E. Hansy-Staudigl, M. Krommer, A. Humer (2019)
"A complete direct approach to nonlinear modeling of dielectric elastomer plates"
Acta Mechanica 230(11): 3923-3943
https://doi.org/10.1007/s00707-019-02529-1

E. Hansy-Staudigl, M. Krommer (2020)
"Electrostrictive polymer plates as electro-elastic material surfaces: Modeling, analysis and simulation"
Accept (22-May-2020) @ *Journal of Intelligent Material Systems and Structures*

Elisabeth Hansy-Staudigl takes the main responsibility for the preparation and writing of the published contents. In all of the listed papers the numerical implementation and analyses of the results were conducted by her, except of the three dimensional finite element results conducted for the sake of comparison. She gradually prepared an original draft of the published papers while the analytical analyses, model development and final editing has been conducted under collaboration with the co-authors.

1.3 Generalized electro-elastic continua

In the first part, the three dimensional balance equations (equilibrium conditions) of electro-elastic continua together with the corresponding boundary conditions are revised; for a detailed derivation we refer to *Paper B* of this cumulative dissertation. With the three dimensional relations at hand, an asymptotic analysis demonstrates the validity of the classical assumptions concerning the state of stress and strains in thin plates and shells. Based on these findings a complementary set of strain measures derived from the basic differential geometry of surfaces renders the gateway to the direct approach for thin plates and shells as a material surface. Having introduced the extended geometric nonlinear theory for thin plates and shells, the method of multiplicative decomposition is revisited and extended to a hybrid multiplicative and additive decomposition for the material surface in the final part, showing the enhanced action of the electrical field on the field of stresses and strains.

1.3.1 Three dimensional electro-elastic balance equations

Starting with the material independent electrical equations, in the form of the Maxwell equations of electrostatics, Faraday's law establishes the electric field vector as the negative gradient of the electric potential φ

$$\nabla_3 \times \boldsymbol{e} = \boldsymbol{0} \quad \rightarrow \quad \boldsymbol{e} = -\nabla_3 \varphi, \tag{1.1}$$

and Gauss law for conservation of charge in a non-conducting dielectric reads

$$V: \qquad \nabla_3 \cdot \boldsymbol{d} = 0 \tag{1.2}$$

$$S: \qquad \boldsymbol{n} \cdot \boldsymbol{d} = -\sigma, \tag{1.3}$$

with the surface charge density σ. $\boldsymbol{d} = \epsilon_0 \boldsymbol{e} + \boldsymbol{p}$ is the electric displacement vector, and ϵ_0 the vacuum permittivity. The polarization vector $\boldsymbol{p}$ represents the macroscopic density of electrical dipole moments acting within the body.

In a spatial continuum mechanics representation, the electric field contributes to the mechanical balance equations in form of an electrostatic volume force $\boldsymbol{f}^{em} = (\nabla_3 \boldsymbol{e}) \cdot \boldsymbol{p} + q\boldsymbol{e}$ and volume couple $\boldsymbol{c}^{em} = \boldsymbol{p} \times \boldsymbol{e}$. As no free charges are present in a non-conducting dielectric, $q = 0$ holds. Hence the balance of liner and moment of momentum are

$$\begin{aligned} \rho \dot{\boldsymbol{v}} &= \nabla_3 \cdot \boldsymbol{\sigma}_3 + \rho \boldsymbol{f}_3 + \boldsymbol{f}^{em} \\ \boldsymbol{c}^{em} &+ {}^3\boldsymbol{\varepsilon} \cdot\cdot \boldsymbol{\sigma}_3 = \boldsymbol{0}, \end{aligned} \tag{1.4}$$

where $\boldsymbol{v} = \dot{\boldsymbol{r}}_3$ represents the velocity field of the body's material points and $\rho \boldsymbol{f}_3$ are mechanical body forces. The balance of moment of momentum evidently shows that the mechanical Cauchy stress $\boldsymbol{\sigma}_3$ is not symmetric. Moreover, the presence of the electrical volume couples renders the definition of the symmetric stress tensor $\boldsymbol{\sigma}_3^S = \boldsymbol{\sigma}_3 + \boldsymbol{e}\boldsymbol{p}$. However, by using the relation $\boldsymbol{f}^{em} = \nabla_3 \cdot \boldsymbol{\sigma}_3^E = \nabla_3 \cdot (\boldsymbol{e}\boldsymbol{d} - 1/2\varepsilon_0(\boldsymbol{e}\cdot\boldsymbol{e})\mathbf{I}_3)$ the symmetric total stress tensor $\boldsymbol{\sigma}_3^{tot} = \boldsymbol{\sigma}_3 + \boldsymbol{\sigma}_3^E$ is introduced such the local balance of linear momentum

$$V: \qquad \rho\dot{\boldsymbol{v}} = \nabla_3 \cdot \boldsymbol{\sigma}_3^{tot} + \rho\boldsymbol{f}_3 \tag{1.5}$$

$$S: \qquad \boldsymbol{\sigma}_3^{tot} \cdot \boldsymbol{n} = \boldsymbol{t}_3 \quad , \quad \boldsymbol{u} = \bar{\boldsymbol{u}}, \tag{1.6}$$

is obtained, where $\boldsymbol{t}_3$ denotes mechanical boundary forces.

The fundamentals of finite strain elasticity shall not be repeated, we refer to [1], [19], [33]. Rather, we directly provide the material counterparts of the three dimensional field quantities, for which the three dimensional deformation gradient tensor $\mathbf{F}_3 = (\mathring{\nabla}_3 \boldsymbol{r}_3)^T$ must be introduced with the differential operator defined in the reference configuration $\mathring{\nabla}_3$ and the position vector of the material point in the actual configuration $\boldsymbol{r}_3$. The mechanical and total Piola-Kirchhoff stress tensors follow to

$$\mathbf{S}_3 = J_3 \mathbf{F}_3^{-1} \cdot \boldsymbol{\sigma}_3 \cdot \mathbf{F}_3^{-T}, \qquad \mathbf{S}_3^{tot} = J_3 \mathbf{F}_3^{-1} \cdot \boldsymbol{\sigma}_3^{tot} \cdot \mathbf{F}_3^{-T}. \tag{1.7}$$

$J_3 = \det \mathbf{F}_3$ determines the volume change. The electric displacement vector, as well as electric field vector and polarization vector in material coordinates become

$$\boldsymbol{\mathcal{D}} = J_3 \boldsymbol{d} \cdot \mathbf{F}_3^{-T}, \qquad \boldsymbol{\mathcal{E}} = -\mathring{\nabla}_3 \varphi = \boldsymbol{e} \cdot \mathbf{F}_3, \qquad \boldsymbol{\mathcal{P}} = J_3 \boldsymbol{p} \cdot \mathbf{F}_3^{-T}. \tag{1.8}$$

Finally, it remains to introduce the three dimensional Green-Lagrange strain tensor $\mathbf{G}_3 = 1/2(\mathbf{C}_3 - \mathbf{I}_3)$ with the aid of the right Cauchy-Green tensor $\mathbf{C}_3 = \mathbf{F}_3^T \cdot \mathbf{F}_3$.

1.3.2 Generalized constitutive relations

Multi-field problems are strongly tightened to the specific material properties, e.g. if the material has a pronounced capability to react on the mechanical field through elasticity, or the electrical field if it has conductive or dielectric properties. In order to incorporate these properties into the theory, a constitutive relation, capable to describe these properties must be constructed. With the aid of a thermodynamically consistent energy functional such as the internal energy e for electro-elastically coupled material bodies, the functional dependency of the internal energy follows from its local balance,

$$\rho\dot{e} = \boldsymbol{\sigma}_3 \cdot\cdot \nabla_3 \boldsymbol{v} + \ell^E, \quad \ell^E = \rho \boldsymbol{e} \cdot \frac{d}{dt}\left(\frac{\boldsymbol{p}}{\rho}\right). \tag{1.9}$$

More details on the derivation of the local electro-elastic theory are provided in *Paper B*. A Legendre transformation $\Phi = e - \boldsymbol{e} \cdot \boldsymbol{p}/\rho$ yields the Helmholtz free energy $\Phi = \Phi(\boldsymbol{e}, \mathbf{F}_3)$ with

$$\rho\dot{\Phi} = \boldsymbol{\sigma}_3 \cdot\cdot \dot{\mathbf{F}}_3 \cdot \mathbf{F}_3^{-1} - \boldsymbol{p} \cdot \dot{\boldsymbol{e}}, \tag{1.10}$$

where the relation $\nabla_3 \boldsymbol{v} = \dot{\mathbf{F}}_3 \cdot \mathbf{F}_3^{-1}$ has been used. A comparison of these relations to the rate of the Helmholtz free energy

$$\dot{\Phi} = \frac{\partial \Phi}{\partial \mathbf{F}_3} \cdot\cdot \dot{\mathbf{F}}_3 + \frac{\partial \Phi}{\partial \boldsymbol{e}} \cdot \dot{\boldsymbol{e}}, \tag{1.11}$$

yields the general constitutive relations for the non symmetric Cauchy stress tensor $\boldsymbol{\sigma}_3$ and the polarization vector $\boldsymbol{p}$

$$\boldsymbol{\sigma}_3 = \rho \frac{\partial \Phi}{\partial \mathbf{F}_3} \cdot \mathbf{F}_3^T, \qquad \boldsymbol{p} = -\rho \frac{\partial \Phi}{\partial \boldsymbol{e}}. \tag{1.12}$$

However, we rather use a material description, therefore the relations 1.7 and 1.8 are accounted for, $\Phi(\boldsymbol{e}, \mathbf{F}_3) = \psi(\mathbf{F}_3^{-T} \cdot \boldsymbol{e}, \mathbf{F}_3) \equiv \psi(\boldsymbol{\mathcal{E}}, \mathbf{F}_3)$, and by use of the right Cauchy-Green tensor $\mathbf{C}_3 = \mathbf{F}_3^T \cdot \mathbf{F}_3$ the Helmholtz free energy $\psi(\boldsymbol{\mathcal{E}}, \mathbf{C}_3)$ is established, such that the local form of the balance of the Helmholtz free energy in a material description is

$$\rho_0 \dot{\psi} = (\boldsymbol{S}_3 + \boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{E}} \cdot \mathbf{C}_3^{-1}) \cdot\cdot \frac{1}{2}\dot{\mathbf{C}}_3 - \boldsymbol{\mathcal{P}} \cdot \dot{\boldsymbol{\mathcal{E}}}, \tag{1.13}$$

from which the constitutive relations follow to

$$\mathbf{S}_3^S = 2\rho_0 \frac{\partial \psi}{\partial \mathbf{C}_3}, \qquad \boldsymbol{\mathcal{P}} = -\rho_0 \frac{\partial \psi}{\partial \boldsymbol{\mathcal{E}}}, \tag{1.14}$$

with $\mathbf{S}_3^S = \boldsymbol{S}_3 + \boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{E}} \cdot \mathbf{C}_3^{-1}$. In order to take the polarization in vacuum into account, see Dorfmann and Ogden [15], we augment the Helmholtz free energy with an augmentation term $\Omega = \psi(\boldsymbol{\mathcal{E}}, \mathbf{C}_3) + \psi^{aug}(\boldsymbol{\mathcal{E}}, \mathbf{C}_3)$ and compute the rate

$$\begin{aligned} \rho_0 \dot{\Omega} &= \left(\rho_0 \frac{\partial \psi}{\partial \mathbf{C}_3} + \rho_0 \frac{\partial \psi^{aug}}{\partial \mathbf{C}_3}\right) \cdot\cdot \dot{\mathbf{C}}_3 + \left(\rho_0 \frac{\partial \psi}{\partial \boldsymbol{\mathcal{E}}} + \rho_0 \frac{\partial \psi^{aug}}{\partial \boldsymbol{\mathcal{E}}}\right) \cdot \dot{\boldsymbol{\mathcal{E}}} = \\ &\frac{1}{2}\left(\boldsymbol{S}_3 + \boldsymbol{\mathcal{P}}\boldsymbol{\mathcal{E}} \cdot \mathbf{C}_3^{-1} + 2\rho_0 \frac{\partial \psi^{aug}}{\partial \mathbf{C}_3}\right) \cdot\cdot \dot{\mathbf{C}}_3 - \left(\boldsymbol{\mathcal{P}} - \rho_0 \frac{\partial \psi^{aug}}{\partial \boldsymbol{\mathcal{E}}}\right) \cdot \dot{\boldsymbol{\mathcal{E}}}, \end{aligned} \tag{1.15}$$

which yields the constitutive relations for the total second Piola-Kirchhoff stress tensor $\mathbf{S}_3^{tot}$ and the material electric displacement vector $\boldsymbol{\mathcal{D}}$

$$\mathbf{S}_3^{tot} = 2\rho_0 \frac{\partial \Omega}{\partial \mathbf{C}_3}, \qquad \boldsymbol{\mathcal{D}} = -\rho_0 \frac{\partial \Omega}{\partial \boldsymbol{\mathcal{E}}}. \tag{1.16}$$

1.3.3 Linear theory of electro-elasticity

Having defined the general constitutive relations, we introduce the common basis of all nonlinear considerations, the linear theory of electro-elasticity. To this end, the mechanical strains are considered to remain small $\mathbf{G}_3 \approx \varepsilon_3$, while the material field quantities need not be distinguished from the spacial ones

$$\mathbf{S}_3^{tot} \equiv \boldsymbol{\sigma}_3^{tot}, \quad \boldsymbol{\mathcal{E}} \equiv \boldsymbol{e}, \quad \boldsymbol{\mathcal{D}} \equiv \boldsymbol{d}. \tag{1.17}$$

Now, writing the liner counterpart of the augmented free energy $\Omega^{lin} = \Omega^{lin}(\varepsilon_3, \boldsymbol{e})$, the relation

$$\dot{\Omega}^{lin} = \frac{\partial \Omega^{lin}}{\partial \varepsilon_3} \cdot\cdot\, \dot{\varepsilon}_3 + \frac{\partial \Omega^{lin}}{\partial \boldsymbol{e}} \cdot \dot{\boldsymbol{e}} = \boldsymbol{\sigma}_3^{tot} \cdot\cdot\, \dot{\varepsilon}_3 - \boldsymbol{d} \cdot \dot{\boldsymbol{e}} \tag{1.18}$$

holds, which yields

$$\boldsymbol{\sigma}_3^{tot} = \frac{\partial \Omega^{lin}}{\partial \varepsilon_3}, \quad \boldsymbol{d} = -\frac{\partial \Omega^{lin}}{\partial \boldsymbol{e}}. \tag{1.19}$$

In the linear setting the augmented free energy is introduced per unit volume rather than per unit mass as it was the case in the nonlinear setting. In order to incorporate constitutive coupling, the mechanical strain tensor is additively decomposed into a purely mechanical part ε_3^{me} and an electrical part $\varepsilon_3^{el} = \varepsilon_3^{el}(\boldsymbol{e})$, hence $\varepsilon_3 = \varepsilon_3^{me} + \varepsilon_3^{el}(\boldsymbol{e})$ holds; also the augmented free energy is decomposed additively as

$$\Omega^{lin} = W^{lin}(\varepsilon_3^{me}) + W^{el}(\boldsymbol{e}) = W^{lin}(\varepsilon_3 - \varepsilon_3^{el}) + W^{el}(\boldsymbol{e}). \tag{1.20}$$

Linear constitutive relations follow by taking the augmented free energy as a quadratic form

$$\Omega^{lin} = \frac{1}{2} \varepsilon_3^{me} \cdot\cdot\, {}^4\mathbf{C} \cdot\cdot\, \varepsilon_3^{me} - \frac{1}{2} \boldsymbol{e} \cdot \boldsymbol{\epsilon} \cdot \boldsymbol{e}, \tag{1.21}$$

where ${}^4\mathbf{C}$ is the 4th rank tensor of elasticity and $\boldsymbol{\epsilon}$ is the rank 2 permittivity tensor; therefore, the specific constitutive relation for the total stress tensor and the electric displacement vector are

$$\boldsymbol{\sigma}_3^{tot} = \frac{\partial \Omega^{lin}}{\partial \boldsymbol{\varepsilon}_3} = {}^4\mathbf{C} \cdot\cdot (\boldsymbol{\varepsilon}_3 - \boldsymbol{\varepsilon}_3^{el}), \tag{1.22}$$

$$\boldsymbol{d} = -\frac{\partial \Omega^{lin}}{\partial \boldsymbol{e}} = \boldsymbol{\epsilon} \cdot \boldsymbol{e} + \frac{\partial \boldsymbol{\varepsilon}_3^{el}}{\partial \boldsymbol{e}} \cdot\cdot {}^4\mathbf{C} \cdot\cdot (\boldsymbol{\varepsilon}_3 - \boldsymbol{\varepsilon}_3^{el}) = \boldsymbol{\epsilon} \cdot \boldsymbol{e} + \frac{\partial \boldsymbol{\varepsilon}_3^{el}}{\partial \boldsymbol{e}} \cdot\cdot \boldsymbol{\sigma}_3^{tot}. \tag{1.23}$$

Discussing the linear relations in more detail, one observes the decoupling of the mechanical and electrical fields due to the specific form of 1.21; solely the presence of the electrical strain tensor $\boldsymbol{\varepsilon}_3^{el}$ is causing an electrical response due to the electrical field. Therefore, the electrical eigenstrain tensor $\boldsymbol{\varepsilon}_3^{el}$ allows to incorporate electrical couplings in case of e.g. electrostriction, where $\boldsymbol{\varepsilon}_3^{el} = \mathbf{M} \cdot \boldsymbol{ee}$ is a function of the second rank tensor $\boldsymbol{ee}$. $\mathbf{M} = \mathbf{M}^T$ is the fourth rank tensor of electrostrictive parameter with symmetry properties allowing to re-write $\boldsymbol{\varepsilon}_3^{el}$ as $\boldsymbol{\varepsilon}_3^{el} = \mathbf{M} \cdot\cdot \boldsymbol{ee} = \mathbf{M} \cdot \boldsymbol{e} \cdot \boldsymbol{e} = \boldsymbol{e} \cdot \mathbf{M} \cdot \boldsymbol{e}$. Therefore, the relations

$$\boldsymbol{\sigma}_3^{tot} = {}^4\mathbf{C} \cdot\cdot (\boldsymbol{\varepsilon}_3 - \mathbf{M} \cdot \boldsymbol{ee}), \quad \boldsymbol{d} = \boldsymbol{\epsilon} \cdot \boldsymbol{e} + 2(\mathbf{M} \cdot \boldsymbol{e}) \cdot\cdot {}^4\mathbf{C} \cdot\cdot (\boldsymbol{\varepsilon}_3 - \mathbf{M} \cdot\cdot \boldsymbol{ee}) \tag{1.24}$$

are established.

1.3.4 Multiplicative decomposition

Moving the attention back to the case of finite strain elasticity with large deformations, a multiplicative decomposition of the deformation gradient tensor $\mathbf{F}_3$, which replaces the additive strain decomposition in the linear problem, is used to incorporate constitutive couplings. This approach has already been adopted in different scientific fields e.g. large strain plasticity, thermoelasticity or biomechanics[33]. Further applications of this technique involving electro-elastic coupling phenomena were published in [24] for piezoelectric materials and for electrostrictive elastomers in *Paper B* of the present thesis. Therefore, the fundamental idea and properties of this technique are revised.

Splitting the deformation gradient tensor $\mathbf{F}_3$ into an electrical part $\mathbf{F}_{3,el} = \mathbf{F}_{3,el}(\boldsymbol{\mathcal{E}})$ and into a mechanical part $\mathbf{F}_{3,me}$, and applying a right, Lee-type [30], decomposition order to the deformation gradient, $\mathbf{F}_3 = \mathbf{F}_{3,me} \cdot \mathbf{F}_{3,el}$ holds, see Figure 1.1. Accordingly the Green-Lagrange strain tensor $\mathbf{G}_3 = 1/2(\mathbf{C}_3 - \mathbf{I}_3)$, with the right Cauchy-Green tensor $\mathbf{C}_3 = \mathbf{F}_3^T \cdot \mathbf{F}_3$ is

$$\mathbf{G}_3 = \frac{1}{2}\left(\mathbf{F}_{3,el}^T \cdot \mathbf{F}_{3,me}^T \cdot \mathbf{F}_{3,me} \cdot \mathbf{F}_{3,el} - \mathbf{I}_3\right) = \frac{1}{2}\mathbf{F}_{3,el}^T \left(\mathbf{F}_{3,me}^T \cdot \mathbf{F}_{3,me} - \mathbf{F}_{3,el}^{-T} \cdot \mathbf{F}_{3,el}^{-1}\right) \cdot \mathbf{F}_{3,el}. \tag{1.25}$$

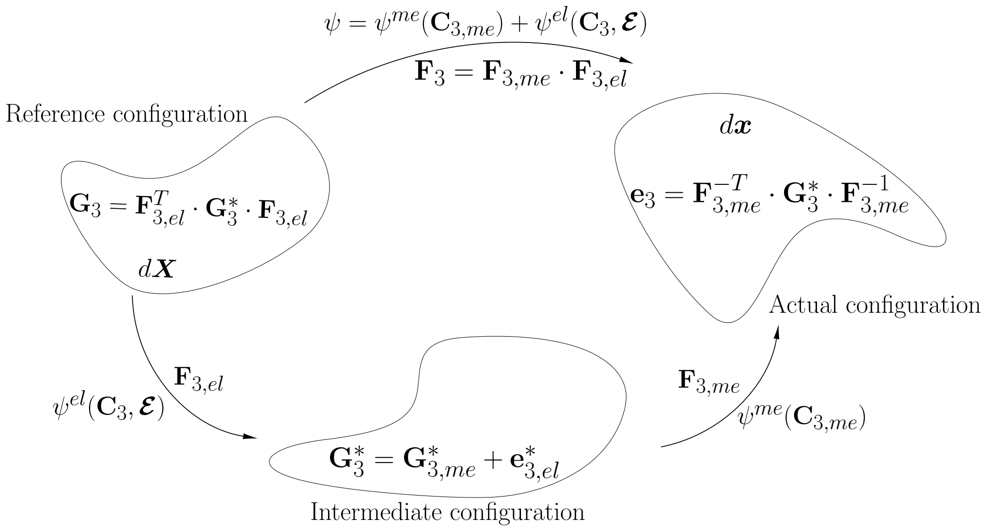

Figure 1.1: Strain tensors in the associated configuration.

Adding now the vanishing quantity $\mathbf{I}_3 - \mathbf{I}_3$ we obtain

$$\mathbf{G}_3 = \frac{1}{2}\mathbf{F}_{3,el}^T \cdot \left(\underbrace{\mathbf{F}_{3,me}^T \cdot \mathbf{F}_{3,me} - \mathbf{I}_3}_{2\mathbf{G}_{3,me}^*} + \underbrace{\mathbf{I}_3 - \mathbf{F}_{3,el}^{-T} \cdot \mathbf{F}_{3,el}^{-1}}_{2\mathbf{e}_{3,el}^*} \right) \cdot \mathbf{F}_{3,el}, \tag{1.26}$$

where the newly defined quantities $\mathbf{G}_{3,me}^*$ and $\mathbf{e}_{3,el}^*$ denote the respective Green-Lagrange and Almansi strain tensor with respect to an intermediate configuration denoted with *, visible in Figure 1.1. Hence, the push-forward of the Green-Lagrange strain tensor into the intermediate configuration is

$$\mathbf{G}_3^* = \mathbf{F}_{3,el}^{-T} \cdot \mathbf{G}_3 \cdot \mathbf{F}_{3,el}^{-1} = \mathbf{G}_{3,me}^* + \mathbf{e}_{3,el}^*, \tag{1.27}$$

showing an additive decomposition into an intermediate mechanical Green-Lagrange type and an intermediate electrical Almansi type strain tensor. As a consequence of the decomposition of the Green-Lagrange tensor, we propose an additive decomposition of the Helmholtz free energy as well. Taking in analogy to the linear relations the mechanical part to depend solely on the mechanical right Cauchy-Green tensor $\mathbf{C}_{3,me} = \mathbf{F}_{3,me}^T \cdot \mathbf{F}_{3,me} = \mathbf{F}_{3,el}^{-T} \cdot \mathbf{F}_3^T \cdot \mathbf{F}_3 \cdot \mathbf{F}_{3,el}^{-1} = \mathbf{F}_{3,el}^{-T} \cdot \mathbf{C}_3 \cdot \mathbf{F}_{3,el}^{-1}$, while keeping the general dependency for the electrical part, the Helmholtz free energy

$$\psi = \psi^{me}(\mathbf{C}_{3,me}) + \psi^{el}(\mathbf{C}_3, \boldsymbol{\mathcal{E}}) \tag{1.28}$$

is obtained, from which the extended symmetric second Piola-Kirchhoff stress $\mathbf{S}_3^S$ follows to

$$\mathbf{S}_3^S = \rho_0 \frac{\partial \psi^{me}}{\partial \mathbf{C}_{3,me}} \cdot\cdot \frac{\partial \mathbf{C}_{3,me}}{\partial \mathbf{C}_3} + \rho_0 \frac{\partial \psi^{el}}{\partial \mathbf{C}_3} = \rho_0 \mathbf{F}_{3,el}^{-1} \cdot \frac{\partial \psi^{me}}{\partial \mathbf{C}_{3,me}} \cdot \mathbf{F}_{3,el}^{-T} + \rho_0 \frac{\partial \psi^{el}}{\partial \mathbf{C}_3}, \quad (1.29)$$

where the electrical field appears through $\mathbf{F}_{3,el}$, but also through $\mathbf{C}_{3,me}$ and the derivative of the electrical part of the free energy. The latter contribution results in electrostatic forces as in contrast to the linear theory we have $\psi^{el} = \psi^{el}(\boldsymbol{\mathcal{E}}, \mathbf{C}_3)$. Incorporating specific models for the electrical deformation gradient allows to account for higher order effects such as piezoelectricity or electrostriction.

1.3.5 Asymptotic analysis of a plate

In contrast to three dimensional problems where the system of differential equations, eq. 1.31 remain fully coupled, problems involving a small thickness to length ratio, so called structural plate and shell problems, retain a reduced system of differential equations as the components relating the thickness direction degenerate. An asymptotic analysis of the linear, three dimensional relations applied to the case of thin plates, illustrates the general behavior and traces the field entities which dominate the behavior of the plate. For the detailed analysis including the boundary at the circumferential side edges, we refer to [53]. Introducing first the three dimensional position vector $\boldsymbol{r}$

$$\boldsymbol{r} = \boldsymbol{x} + z\boldsymbol{k}, \quad -h \leq z \leq +h, \quad \boldsymbol{x} \in S, \quad (1.30)$$

composed of $\boldsymbol{x}$, the position vector to points of a reference surface S with $z = 0$, the out-of plane unit normal vector $\boldsymbol{k}$ in thickness direction, and the distance z from the reference surface, of the plate. We start from the three dimensional equilibrium conditions and the stress free boundary conditions at the top and bottom surface

$$\nabla_3 \cdot \boldsymbol{\sigma}_3^{tot} + \rho \boldsymbol{f}_3 = 0, \quad \boldsymbol{k} \cdot \boldsymbol{\sigma}_3^{tot}\Big|_{z=\pm h} = 0. \quad (1.31)$$

Together with the linear kinematic relations and the compatibility conditions

$$\boldsymbol{\varepsilon}_3 = (\nabla_3 \boldsymbol{u}_3)^S, \qquad \nabla_3 \times (\nabla_3 \times \boldsymbol{\varepsilon}_3)^T = \mathbf{0}, \quad (1.32)$$

where $\boldsymbol{u}_3$ denotes the displacement vector and $()^S$ the symmetric part of the resulting tensor, respectively. In the asymptotic analysis we separate the tensor valued quantities into their in plane- and out of plane components

$$\mathbf{I}_3 = \mathbf{I} + \boldsymbol{k}\boldsymbol{k}, \quad \boldsymbol{f} = \mathbf{I}\cdot\boldsymbol{f}_3, \quad \boldsymbol{\sigma}_2^{tot} = \mathbf{I}\cdot\boldsymbol{\sigma}_3^{tot}\cdot\mathbf{I}, \tag{1.33}$$
$$\boldsymbol{\varepsilon}_3 = \varepsilon_3\boldsymbol{k}\boldsymbol{k} + \boldsymbol{\gamma}\boldsymbol{k} + \boldsymbol{k}\boldsymbol{\gamma} + \boldsymbol{\varepsilon}_2, \tag{1.34}$$
$$\boldsymbol{\sigma}_3^{tot} = \sigma_z^{tot}\boldsymbol{k}\boldsymbol{k} + \boldsymbol{\tau}\boldsymbol{k} + \boldsymbol{k}\boldsymbol{\tau} + \boldsymbol{\sigma}_2^{tot}, \tag{1.35}$$
$$\boldsymbol{u}_3 = u_z\boldsymbol{k} + \boldsymbol{u}_2, \tag{1.36}$$

with the out of plane shear strain $\boldsymbol{\gamma}$ and the shear stress vector $\boldsymbol{\tau}$. Following Vetyukov [52] the formal small, and dimensionless parameter λ is introduced, a quantity which indicates the thinness of the plate. Accounting for λ in the position vector $\boldsymbol{r}$ and the decomposed differential operator ∇_3

$$\boldsymbol{r} = \lambda^{-1}\boldsymbol{x} + z\boldsymbol{k}, \qquad \nabla_3 = \lambda\nabla + \boldsymbol{k}\partial_z, \tag{1.37}$$

and inserting 1.37 into the governing equations 1.31, the asymptotic analysis can be started by assigning the proper order of smallness to the three dimensional relations

$$\boldsymbol{\sigma}_3^{tot} = \lambda^{-2}\overset{0}{\boldsymbol{\sigma}}{}^{tot} + \lambda^{-1}\overset{1}{\boldsymbol{\sigma}}{}^{tot} + \dots, \quad \boldsymbol{\varepsilon}_3 = \lambda^{-2}\overset{0}{\boldsymbol{\varepsilon}} + \lambda^{-1}\overset{1}{\boldsymbol{\varepsilon}} + \dots, \tag{1.38}$$

which yields a solution that varies much slower in the plane than over the thickness. The analysis of the stress field has two important outcomes, the principle term is plane as $\overset{0}{\boldsymbol{\sigma}}{}_z^{tot} = \overset{1}{\boldsymbol{\sigma}}{}_z^{tot} = \mathbf{0}$ holds, and with the aid of the external forces the tensor of internal in-plane forces

$$\nabla\cdot\mathbf{T} = 0, \quad \mathbf{T} = \int_{-h}^{h}\overset{0}{\boldsymbol{\sigma}}{}_2^{tot}dz \tag{1.39}$$

and internal moments (stress couples) $\mathbf{M}(\boldsymbol{x})$, together with the resultant transversal shear force $\boldsymbol{Q}$

$$\nabla\cdot\boldsymbol{Q} + q = 0, \qquad \boldsymbol{Q} = \int_{-h}^{h}\overset{1}{\boldsymbol{\tau}}dz, \qquad q = \int_{-h}^{h} f_z dz$$
$$\nabla\cdot\mathbf{M} - \boldsymbol{Q} = 0, \qquad \mathbf{M} = -\int_{-h}^{h} z\overset{0}{\boldsymbol{\sigma}}{}_2^{tot}dz, \tag{1.40}$$

arise. Eliminating $\boldsymbol{Q}$ yields the classical equation of equilibrium known from linear plate theory $\nabla\cdot\nabla\cdot\mathbf{M} - q = 0$. Omitting further details on the asymptotic procedure, the compatibility condition 1.31 yields the through the thickness linear distributed strain field

$$\overset{0}{\varepsilon}_2 = \varepsilon - z\kappa, \tag{1.41}$$

where the coefficients $\varepsilon(\boldsymbol{x})$, and $\kappa(\boldsymbol{x})$, depend solely on the in-plane coordinates $\boldsymbol{x}$. Obviously, these strain coefficients comprise geometrical entities which underly a certain freedom in their specific definition. For the kinematic relation 1.32 the asymptotic analysis of the displacement field $\boldsymbol{u}_3$ shows that κ is the negative linearized curvature and ε the in-plane strain of the plate:

$$\varepsilon(\boldsymbol{x}) = (\nabla \boldsymbol{u})^S, \quad \kappa(\boldsymbol{x}) = \nabla\nabla w, \tag{1.42}$$

where $\boldsymbol{u}$ and w are the leading order terms in $\boldsymbol{u}_2$ and u_z.

We restrict the analysis of the electrostatic equations to the practically important case of a plate with electrodes mounted on top h and bottom $-h$. For this case, the single non-vanishing entry of the electric field vector acts in thickness direction, with the voltage v as a measure for the potential difference according to 1.1. Introducing the constitutive relation for the electric displacement vector to be purely electric assuming any coupling effect to be absent

$$\boldsymbol{d} = \boldsymbol{\epsilon} \cdot \boldsymbol{e} \tag{1.43}$$

where $\boldsymbol{\epsilon}$ is the permittivity tensor, an asymptotic analysis yields the remaining field quantities. Taking the constitutive relation in thickness direction poses a constant leading order term for the electric displacement vector $\partial_z \overset{0}{d}_z = 0 \rightarrow d_{z0} = const$. Replacing the boundary condition 1.3 for the surface charge density σ by an integral form involving the total charge Σ at an electrode occupying the two dimensional domain S_e

$$\int_{S_e} d_{z0} = -\Sigma, \tag{1.44}$$

and projecting the constitutive relation to the thickness direction

$$d_{z0} = -\boldsymbol{k} \cdot \boldsymbol{\epsilon} \cdot \boldsymbol{k} \partial_z \overset{0}{\varphi}, \qquad \int_{-h}^{h} \partial_z \overset{0}{\varphi} dz = v, \qquad \rightarrow \quad d_{z0} = -\sigma = cv \tag{1.45}$$

yields the voltage upon integration through the thickness. Prescribing the voltage v is used for actuation, where $c = \epsilon/(2h)$ is the capacity per unit area.

1.3.6 Direct approach to electro-elastic shells as material surface

Now, we focus on the extended theory for finite deformations of electro elastic plates using the direct approach for shells as material surface, as it has been introduced in *Paper D*. The direct approach comprises a powerful and elegant method to derive the generalized system of balance equations, alongside with the option to directly incorporate extended electro-elastic relations. A fundamental input to the present theory of generalized continua is the chosen set of degrees of freedom - a step, where the presented relations rely on the work of Vetyukov [52].

Geometry of a surface

First, the relevant differential geometric relations are revised. For an introduction into the fundamentals in differential geometry we recommend [12]. The shell is parameterized by the coordinates q^1, q^2, the position vector $\boldsymbol{r} = \boldsymbol{r}(q^\alpha)$ points to an arbitrary particle of the shell and defines the local basis $\boldsymbol{r}_\alpha$, the cobasis $\boldsymbol{r}^\alpha$ and the corresponding orthogonal vector of unit normal $\boldsymbol{n}$ of the surface

$$\boldsymbol{r}_\alpha = \frac{\partial \boldsymbol{r}}{\partial q^\alpha} \equiv \boldsymbol{r}^\alpha \partial_\alpha, \quad \boldsymbol{r}_\alpha \cdot \boldsymbol{r}^\beta = \delta_\alpha^\beta, \quad \boldsymbol{n} = \frac{\boldsymbol{r}_1 \times \boldsymbol{r}_2}{|\boldsymbol{r}_1 \times \boldsymbol{r}_2|}, \quad \nabla = \boldsymbol{r}^\alpha \partial_\alpha. \tag{1.46}$$

With aid of the differential operator ∇, the first and second metric tensor of the surface

$$\mathbf{a} = \nabla \boldsymbol{r} = \boldsymbol{r}^\alpha \boldsymbol{r}_\alpha = \nabla \boldsymbol{r} = \mathrm{a}_{\alpha\beta} \boldsymbol{r}^\alpha \boldsymbol{r}^\beta = \mathbf{I} - \boldsymbol{n}\boldsymbol{n}, \quad \mathbf{b} = -\nabla \boldsymbol{n} = -\boldsymbol{r}^\alpha \partial_\alpha \boldsymbol{n} = \mathrm{b}_{\alpha\beta} \boldsymbol{r}^\alpha \boldsymbol{r}^\beta, \tag{1.47}$$

with the covariant components $\mathrm{a}_{\alpha\beta} = \boldsymbol{r}_\alpha \cdot \boldsymbol{r}_\beta$ and $\mathrm{b}_{\alpha\beta} = \boldsymbol{r}_{\alpha\beta} \cdot \boldsymbol{n}$ define the length and angles on the surface, where $\boldsymbol{r}_{\alpha\beta} \equiv \partial_\alpha \partial_\beta \boldsymbol{r}$ holds. The first metric tensor $\mathbf{a}$ has the role of an identity tensor in the tangent plane and defines the elementary area of the surface by

$$dS = \sqrt{a}\, dq^1 dq^2, \qquad a = |\boldsymbol{r}_1 \times \boldsymbol{r}_2|^2 = \det(\mathrm{a}_{\alpha\beta}). \tag{1.48}$$

The second metric tensor $\mathbf{b}$ determines the curvature of the surface, the components of both metric tensors $\mathbf{a}$ and $\mathbf{b}$ have to satisfy the compatibility conditions $\partial_\alpha \partial_\beta \partial_\gamma \boldsymbol{r}$.

In order to proceed to the general case of arbitrary large displacements we distinguish between the actual configuration S denoted by $\boldsymbol{r}(q^\alpha), \boldsymbol{n}, \mathbf{a}, \mathbf{b}$, ∇ and a reference configuration S_0 with $\boldsymbol{R}(q^\alpha), \boldsymbol{N}, \mathbf{A}, \mathbf{B}, \mathring{\nabla}$. With aid of the deformation gradient $\mathbf{F} = \mathring{\nabla}\boldsymbol{r}^T = \boldsymbol{r}_\alpha \boldsymbol{R}^\alpha$ the geometric nonlinear Green-Lagrange type strain measures

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T \cdot \mathbf{F} - \mathbf{A}\right) = \mathrm{E}_{\alpha\beta}\boldsymbol{R}^\alpha \boldsymbol{R}^\beta, \qquad \mathrm{E}_{\alpha\beta} = \frac{1}{2}\left(\mathrm{a}_{\alpha\beta} - \mathrm{A}_{\alpha\beta}\right), \tag{1.49}$$

$$\mathbf{K} = \mathbf{F}^T \cdot \mathbf{b} \cdot \mathbf{F} - \mathbf{B} = \mathrm{K}_{\alpha\beta}\boldsymbol{R}^\alpha \boldsymbol{R}^\beta, \qquad \mathrm{K}_{\alpha\beta} = \mathrm{b}_{\alpha\beta} - \mathrm{B}_{\alpha\beta} \tag{1.50}$$

are defined.

It remains to mention that the notation in this introductory part is chosen in order to emphasize the difference between the linear strain measures $\boldsymbol{\varepsilon}, \boldsymbol{\kappa}$ and their geometric nonlinear counterparts $\mathbf{E}, \mathbf{K}$. However, in *Paper D+E* the notation $\boldsymbol{\varepsilon}, \boldsymbol{\kappa}$ is used for finite deformations as well.

Principle of virtual work

We use the principle of virtual work to introduce the direct approach for a Kirchhoff-Love shell, relying on the presentation in *Paper D*. We consider a shell as a material surface being composed of adjacent material particles with five mechanical degrees of freedom, three translations $\delta\boldsymbol{r}$ and two rotations $\delta\boldsymbol{n}$. The *drilling* rotation about the unit normal vector is prohibited as the shell can be considered as a two-dimensional continuum of "needles" in terms of a single director attached to each particle, hence the variation of $\boldsymbol{r}$ and $\boldsymbol{n}$ undergo the constraint

$$\delta(\boldsymbol{r}_\alpha \cdot \boldsymbol{n}) = 0 \quad \rightarrow \quad \nabla\delta\boldsymbol{r}_\alpha \cdot \boldsymbol{n} + \delta\boldsymbol{n} = 0, \tag{1.51}$$

which constitutes Kirchhoff's kinematic assumption. We introduce the principle of virtual work

$$\int_{S_0} \left(\delta A^i + (\boldsymbol{b} \cdot \delta\boldsymbol{r} + \boldsymbol{m} \times \boldsymbol{n} \cdot \delta\boldsymbol{n} - \sigma\delta V)\right) dS_0 + \delta A^{e,b} = 0. \tag{1.52}$$

with the purely mechanical external body forces $\boldsymbol{b}$ and body moments $\boldsymbol{m}$ per unit reference surface and the virtual work of the external surface charges $\sigma\delta V$ with the electric charge σ and the voltage V per unit reference surface. δA^i is the virtual work of internal forces, moments and charges, including electrostatic / ponderomotive forces as well; external mechanical forces and moments at the boundary, enter through the virtual work $\delta A^{e,b}$. Integration is performed per unit area of the reference surface S_0, with the area transformation factor $J = \sqrt{\mathrm{a}/\mathrm{A}}$ relating the actual and the reference area elements $dS = J dS_0$.

The shell strain measures $\mathbf{E}$ and $\mathbf{K}$ vanish, if and only if the material surface undergoes a rigid body motion with the electrical voltage held constant; hence, the virtual work of internal forces, moments and charges vanishes in case of a virtual rigid body motion

$$\delta A^i = 0 \quad \rightarrow \quad \delta\mathbf{E} = \mathbf{0}, \qquad \delta\mathbf{K} = \mathbf{0}, \qquad \delta V = 0. \tag{1.53}$$

The variations $\delta\boldsymbol{r}$ and $\delta\boldsymbol{n}$ are formally assumed to be independent allowing to account for Kirchhoff's kinematic assumption $\nabla\delta\boldsymbol{r}_\alpha \cdot \boldsymbol{n} + \delta\boldsymbol{n} = 0$ by introducing a vector valued Lagrange multiplier $\boldsymbol{q}$. Introducing as well tensor valued Lagrange multipliers $\boldsymbol{\tau}$ and $\boldsymbol{\mu}$, and a scalar valued multiplier q, the virtual work δA^i can be formally replaced by

$$-\boldsymbol{\tau}\cdot\cdot\,\delta\mathbf{E} - \boldsymbol{\mu}\cdot\cdot\,\delta\mathbf{K} + q\delta V + \boldsymbol{q}\cdot(\nabla\delta\boldsymbol{r}\cdot\boldsymbol{n} + \delta\boldsymbol{n}). \tag{1.54}$$

Now, the principle of virtual work can be rewritten as

$$0 = \int_{S_0} (\boldsymbol{\tau}\cdot\cdot\,\delta\mathbf{E} + \boldsymbol{\mu}\cdot\cdot\,\delta\mathbf{K} - \boldsymbol{q}\cdot(\nabla\delta\boldsymbol{r}\cdot\boldsymbol{n} + \delta\boldsymbol{n}) - \boldsymbol{b}\cdot\delta\boldsymbol{r} - \boldsymbol{m}\times\boldsymbol{n}\cdot\delta\boldsymbol{n})dS_0$$
$$- \int_{S_0} (q-\sigma)\delta V dS_0 + \delta A^{e,b}. \tag{1.55}$$

After some mathematical manipulations and the use of the divergence theorem on the surface, the relation in 1.55 yields

$$0 = \int_{S_0} ((...)_1)\cdot\delta\boldsymbol{r} + (...)_2)\cdot\delta\boldsymbol{n})dS_0 + \int_{\partial S_0} ...dl = 0, \tag{1.56}$$

where the bracketed terms define the equilibrium conditions

$$\mathring{\nabla}\cdot\bar{\mathbf{T}} + \boldsymbol{b} = \mathbf{0}, \qquad \mathring{\nabla}\cdot\boldsymbol{\mu}\cdot\mathbf{F}^T\cdot\mathbf{a} + \boldsymbol{q} - \boldsymbol{m}\times\boldsymbol{n} = \mathbf{0}, \tag{1.57}$$

using the abbreviation $\bar{\mathbf{T}} = \boldsymbol{\tau}\cdot\mathbf{F}^T + (\boldsymbol{\mu}\cdot\mathbf{F}^T)\cdot\mathbf{b} + \mathbf{F}^{-1}\cdot\boldsymbol{q}\boldsymbol{n}$. Since $\mathbf{E}$ and $\mathbf{K}$ are symmetric Green-Lagrange type strain measures, the inherent symmetry property of the double dot product implicates the symmetry of the Lagrange multiplier $\boldsymbol{\tau}$ and $\boldsymbol{\mu}$ which get identified as total second Piola-Kirchhoff type stress and couple stress tensor. $\boldsymbol{q}$ is the transverse force and q the internal charge per unit reference area. Therefore, the total external and internal charges

$$Q^{ext} = \int_{S_0} \sigma dS_0, \qquad Q^{int} = \int_{S_0} q dS_0 \tag{1.58}$$

are found, which have to balance each other $Q^{ext} - Q^{int} = 0$.

Replacing the negative virtual work of the internal forces with the variation of an energy function per unit reference surface $\delta\Psi$,

$$-\delta A^i = \delta\Psi = \boldsymbol{\tau} \cdot\cdot\, \delta\mathbf{E} + \boldsymbol{\mu} \cdot\cdot\, \delta\mathbf{K} - q\delta V, \tag{1.59}$$

yields the constitutive relations analogous to the three dimensional Helmholtz free energy. Consequently, as $\Psi = \Psi(\mathbf{E}, \mathbf{K}, V)$ holds, the variation

$$\delta\Psi = \frac{\partial\Psi}{\partial\mathbf{E}} \cdot\cdot\, \delta\mathbf{E} + \frac{\partial\Psi}{\partial\mathbf{K}} \cdot\cdot\, \delta\mathbf{K} + \frac{\partial\Psi}{\partial V}\delta V, \tag{1.60}$$

yields the constitutive relations of an electro-elastic material surface

$$\boldsymbol{\tau} = \frac{\partial\Psi}{\partial\mathbf{E}}, \qquad \boldsymbol{\mu} = \frac{\partial\Psi}{\partial\mathbf{K}}, \qquad q = -\frac{\partial\Psi}{\partial V}. \tag{1.61}$$

1.3.7 Constitutive framework for plates

For a plate with $\mathbf{A} = \mathbf{I}$ and $\mathbf{B} = \mathbf{0}$ we use the polar decomposition of the surface deformation gradient $\mathbf{F} = \mathbf{R} \cdot \mathbf{U}$, such that the strain measures are

$$\mathbf{C} = \mathbf{U}^2, \qquad \mathbf{K} = \mathbf{U} \cdot \mathbf{R}^T \cdot \mathbf{b} \cdot \mathbf{R} \cdot \mathbf{U}, \tag{1.62}$$

with $\mathbf{C} = 2\mathbf{E} - \mathbf{I}$. Altenbach and Eremeyev [3] have shown that a strain energy of the form

$$W = W(\mathbf{U}, \mathring{\nabla}\mathbf{n} \cdot \mathbf{R}) = W(\mathbf{U}, \mathbf{U} \cdot \mathbf{R}^T \cdot \nabla\mathbf{n} \cdot \mathbf{R}), \tag{1.63}$$

identically fulfills the frame indifference requirement; hence, this requirement holds for $W = W(\mathbf{C}, \mathbf{K})$ as well.

In order to bring the constitutive framework with $\Psi = \Psi(\mathbf{E}, \mathbf{K}, V)$ in this required form the relation $\mathbf{C} = \mathbf{F}^T \cdot \mathbf{F} = 2\mathbf{E} - \mathbf{I}$ is used yielding the stress measures

$$\boldsymbol{\tau} = 2\frac{\partial\Psi}{\partial\mathbf{C}}, \qquad \boldsymbol{\mu} = \frac{\partial\Psi}{\partial\mathbf{K}}, \tag{1.64}$$

with the variation

$$\delta\Psi = \boldsymbol{\tau} \cdot\cdot\, \frac{1}{2}\delta\mathbf{C} + \boldsymbol{\mu} \cdot\cdot\, \delta\mathbf{K} - q\delta V. \tag{1.65}$$

Introducing an additively decomposed Helmholtz free energy $\Psi = W + \Psi^{el}$ with a mechanical and an electrical part, each part must preserve this frame indifference property. By application of a multiplicative decomposition of the deformation gradient tensor and an additive decomposition of $\mathbf{K}$, the strain measures of the mechanical part get replaced $\Psi = W(\mathbf{C}_{\text{me}}, \mathbf{K}_{\text{me}}) + \Psi^{el}(\mathbf{C}, \mathbf{K}, V)$, which yields

$$\delta\Psi = \frac{\partial W}{\partial \mathbf{C}_{\text{me}}} \cdot\cdot\, \delta\mathbf{C}_{\text{me}} + \frac{\partial W}{\partial \mathbf{K}_{\text{me}}} \cdot\cdot\, \delta\mathbf{K}_{\text{me}} + \frac{\partial \Psi^{el}}{\partial \mathbf{C}} \cdot\cdot\, \delta\mathbf{C} + \frac{\partial \Psi^{el}}{\partial \mathbf{K}} \cdot\cdot\, \delta\mathbf{K} + \frac{\partial \Psi^{el}}{\partial V}\delta V. \tag{1.66}$$

Computing the variation of the elastic strain measures

$$\begin{aligned} \delta\mathbf{C}_{\text{me}} &= \mathbf{F}_{el}^{-1} \cdot \delta\mathbf{C} \cdot \mathbf{F}_{el}^{-T} - 2\text{sym}(\mathbf{C}_{\text{me}} \cdot \delta\mathbf{F}_{el} \cdot \mathbf{F}_{el}^{-1}), \\ \delta\mathbf{K}_{\text{me}} &= \delta\mathbf{K} - \frac{\delta\mathbf{K}_{el}}{\delta V}\delta V \end{aligned} \tag{1.67}$$

and performing some rearrangements, the variation

$$\delta\Psi = \left(\mathbf{F}_{el}^{-1} \cdot \frac{\partial W}{\partial \mathbf{C}_{\text{me}}} \cdot \mathbf{F}_{el}^{-T} + \frac{\partial \psi^{el}}{\partial \mathbf{C}}\right) \cdot\cdot\, \delta\mathbf{C} + \left(\frac{\partial W}{\partial \mathbf{K}_{\text{me}}} + \frac{\partial \psi^{el}}{\partial \mathbf{K}}\right) \cdot\cdot\, \delta\mathbf{K} + \left(\frac{\partial W}{\partial V} + \frac{\partial \psi^{el}}{\partial V}\right)\delta V \tag{1.68}$$

yields the stress measures

$$\boldsymbol{\tau}^{tot} \equiv 2\left(\mathbf{F}_{el}^{-1} \cdot \frac{\partial W}{\partial \mathbf{C}_{\text{me}}} \cdot \mathbf{F}_{el}^{-T} + \frac{\partial \psi^{el}}{\partial \mathbf{C}}\right), \tag{1.69}$$

$$\boldsymbol{\mu}^{tot} \equiv \left(\frac{\partial W}{\partial \mathbf{K}_{\text{me}}} + \frac{\partial \psi^{el}}{\partial \mathbf{K}}\right), \tag{1.70}$$

$$q^{el} \equiv -\frac{\partial W}{\partial V} + \frac{\partial \psi^{el}}{\partial V}, \tag{1.71}$$

showing the correspondence to the three dimensional total stress tensor $\mathbf{S}_3^{tot}$ and the electric displacement vector $\boldsymbol{\mathcal{D}}$, if Ψ is identified as the augmented free energy. Details of this approach are presented in *Papers D + E*.

1.4 Scientific impact

The core outcome of the present thesis is the constitutive modeling of electro-elastically coupled plates as material surface. Besides the clear identification of electrostatic stress measures including an unambiguous identification of the electrostatic stress tensor, the innovative part lies in the rigorous development of electro-elastic coupled two dimensional continua within a structural mechanics approach at finite strains. Contrary to different approaches published in literature, the focus within our work lies on the one hand in the rigorous application of a structural mechanics formulation with respect to both modeling and numerical implementation; hence, the formulation itself is completely free from any quasi-analytic ansatz functions or assumptions concerning the state of stress. On the other hand, and more important, the focus in our work lies in the physical reliability, offering not only numerical solutions but also a clear interpretation for the analytical relations with tight connections to the three dimensional fields.

1.5 Short summary of the scientific papers

Paper A introduces the pursued structural mechanics approach for electro-elastically two dimensional coupled continua, where solely electrostatic forces acting on the deformed configuration are accounted for. An extended purely two dimensional principle of virtual work yields the proper work conjugate stress and strain measures, while the incompressibility condition applied to the three dimensional constitutive relations yields the proper connection between the structural and the three dimensional field quantities, where the resulting two dimensional stress tensor obtained through the application of the plane stress condition and an integration through the thickness of the structure can be rather considered as a quasi three dimensional approach and no direct structural theory.

Paper B shows the rigorous derivation of the electrostatic field quantities entering the continuum mechanics approach. Starting from the three dimensional Maxwell equations for electrostatics, the action of the electrostatic field is successively incorporated into the mechanical field equations up to the respective constitutive relations. At the basis of a multiplicative decomposition of the deformation gradient tensor, the theory for constitutively coupled continua is derived and exemplar test bench results for electrostrictive elastomers are solved analytically.

Paper C presents the implementation of the coupled constitutive theory into the structural mechanics formulation at the basis of a plane stress assumption using an electrostrictive material parameter found by correlating published measurement results to the present model. The theory has been implemented into our in-house shell finite element solver *ShellFE*. The numerical results underline the effect of a constitutively coupled theory in comparison to coupling solely based on electrostatic forces.

In *Paper D* a complete direct approach towards modeling plates under the action of electrostatic force coupling is demonstrated. By applying a polar decomposition to the deformation gradient tensor, an additively decomposed constitutive relation separating a membrane and a bending energy that exhibit electrostatic forces as a source of *self-stress*, a so-called *Eigenspannungsquelle* is obtained. Each part is composed of separate strain measures along with specific material values identified as membrane and bending stiffness. A thorough comparison of the numerical results to results obtained with a three dimensional finite element solver, as well as to the literature confirms the discussed direct approach.

Paper E comprises the extension of the newly developed direct approach in *Paper D* to incorporate constitutive coupling through a hybrid multiplicative decomposition of the surface deformation gradient tensor and an additive decomposition of the surface curvature tensor. Knowing the three dimensional electro-mechanical stress tensors expressed in *Paper B*, the entangled electro-elastic stress measures of the generalized two dimensional continuum are identified showing a clear interpretation of these quantities. A comparison of the numerical results to different theories from the literature and from our previous contributions are very promising.

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Paper A

Finite deformations of thin plates made of dielectric elastomer: Modeling, numerics, and stability

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In this article, we present a nonlinear theory for thin plates, which are made of incompressible electroded dielectric elastomer layers. The layers are assumed to exhibit a neo-Hookean elastic behavior, and the effect of the electrostatic forces is taken into account by means of the electrostatic stress tensor. A plane state of stress is imposed on the total stress tensor, based on which two-dimensional constitutive relations for the plate are derived. A geometrically nonlinear formulation for the plate as a material surface is developed, and solutions are computed using nonlinear finite elements. The numerical results are compared to available results from the literature verifying our approach, and an additional nonsymmetric example problem is studied with respect to stability.

Published in: Journal of Intelligent Material Systems and Structures
Publisher: Sage

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Paper B

Modeling of Dielectric Elastomers Accounting for Electrostriction by Means of a Multiplicative Decomposition of the Deformation Gradient Tensor

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Nonlinear modeling of inelastic material behavior by a multiplicative decomposition of the deformation gradient tensor is quite common for finite strains. The concept has proven applicable in thermoelasticity, elastoplacticity, as well as for the description of residual stresses arising in growth processes of biological tissues. In the context of advanced materials, the multiplicative decomposition of the deformation gradient tensor has been introduced within the fields of electro-elastic elastomers, shape-memory alloys as well as piezoelastic materials. In the present paper we apply this multiplicative approach to the special case of dielectric elastomers in order to account for the electrostrictive effect. Therefore, we seek to include the two main sources of electro-mechanical coupling in dielectric elastomers. These are elastostatic forces acting between the electric charges and electrostriction due to intramolecular forces of the material. In particular we intend to study the significance of electrostriction for the particular case of dielectric elastomers, in the form of a thin layer with two compliant electrodes.

Editor: Altenbach H, Carrera E, Kulikov G
Book Title: Analysis and Modelling of Advanced Structures and Smart Systems
Publisher: Springer

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PAPER C

Nonlinear electro-elastic modeling of thin dielectric elastomer plate actuators

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Electro-active polymers undergo large deformations while being typically very thin, this encourages us to study the geometric nonlinear set up within the structural mechanics framework of thin plates and shells as a material surface. In this paper, the full set of three dimensional, geometric nonlinear field equations are incorporated to develop constitutive relations by introducing a generalized free energy function, which takes parts from a pure mechanical strain energy (e.g. neo-Hookean) and a mixed electro-mechanical free energy. The key feature is the multiplicative decomposition of the deformation gradient tensor, which allows for separate constitutive models for any electro-mechanic coupling phenomenon. We apply this model exemplary to the case of electrostriction and use Gauss law of electrostatics in order to incorporate charge controlled actuation, which has been reported to omit pull-in instability. In order to translate the resulting equations to their two-dimensional geometrically nonlinear counterparts for thin plates, plane stress condition is imposed on the total stress tensor and the effect of the electrostrictive coupling is investigated on voltage, as well as on charge controlled actuation, employing non-linear Finite Elements. Finally, results are compared to numerical as well as experimental results on electrostrictive coupling and charge controlled actuation.

Editor: Bar-Cohen Y
Publisher: Proceedings SPIE Smart Structures and Materials + Nondestructive Evaluation and Health Monitoring

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Paper D

A complete direct approach to nonlinear modeling of dielectric elastomer plates

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In this paper, we present a complete direct approach to nonlinear modeling of thin plates, which are made of incompressible dielectric elastomers. In particular, the dielectric elastomers are assumed to exhibit a neo-Hookean elastic behavior, and the effect of electrostatic forces is incorporated by the purely electrical contribution to the augmented Helmholtz free energy. Our approach does not involve any extraction-type procedure from the three-dimensional energy to derive the plate augmented free energy, but directly postulates the form of this energy for the structural plate problem treated in this paper. Results computed within the framework of this novel approach are compared to results available in the literature as well as to our own three-dimensional finite element solutions. A very good agreement is found.

Published in: Acta Mechanica
Publisher: Springer

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Paper E

Electrostrictive polymer plates as electro-elastic material surfaces: Modeling, analysis and simulation

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In this paper we discuss modeling of electrostrictive polymer plates as electro-elastic material surfaces. A complete direct approach is developed without the need to involve the three-dimensional formulation. Ponderomotive forces and couples as well as constitutive coupling by means of electrostriction are accounted for. We propose a rational formulation for the augmented free energy of electro-elastic material surfaces incorporating electrostriction by a multiplicative decomposition of the surface stretch tensor and an additive decomposition of the surface curvature tensor into elastic and electrical parts. Numerical results computed within the framework of this complete direct approach are compared to results computed with a method that requires the numerical integration of the three-dimensional augmented free energy through the thickness of the plate and to alternative formulations reported in the literature.

Published in: Journal of Intelligent Material Systems and Structures
Publisher: Sage

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1. Hansy-Staudigl, E., Krommer, M.: Electrostrictive polymer plates as electro-elastic material surfaces: Modeling, analysis and simulation. *Journal of Intelligent Material Systems and Structures*; accepted (22-May-2020).
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Conferencetalk

1. Modeling electro-elastic coupling phenomena of electroactive polymers in the context of structural mechanics; E. Hansy-Staudigl , M. Krommer, Y. Vetyukov, A. Humer; *GAMM 90th Annual Meeting of the International Association of Applied Mathematics and Mechatronics*, Vienna, Austria, March 2019
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