4TH IVS TRAINING SCHOOL ON VLBI FOR GEODESY AND ASTROMETRY

IVS Training School

L11: How do we model observations - Signal propagation

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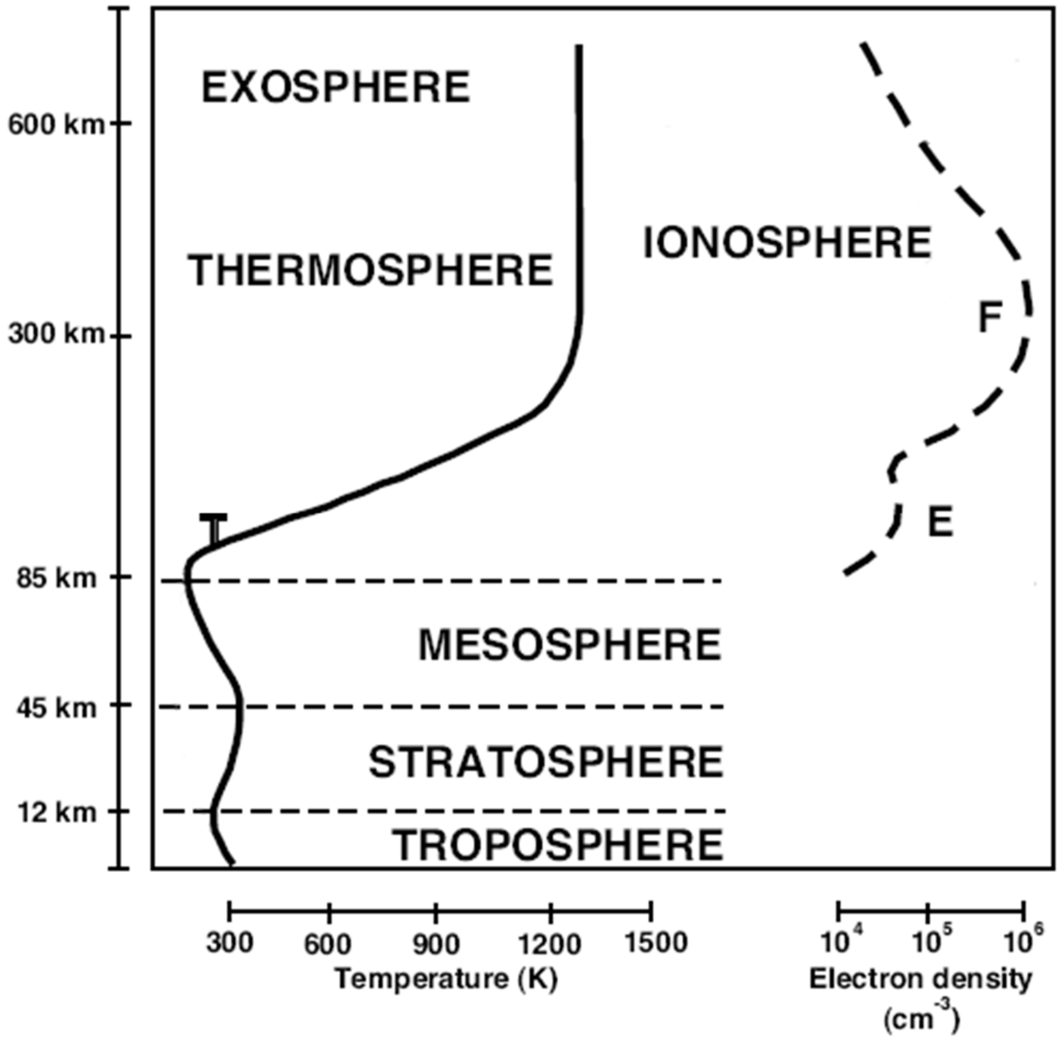
Atmosphere



Ionosphere:

Number of free electrons and ions is large enough to affect propagation of electromagnetic waves

Wikipedia.de

Ionosphere

- Dispersive medium
  - Propagation velocity of an electromagnetic wave is dependent on its frequency
  - Velocity of a sinusoidal wave and a wave group are different
    - phase vs. group velocity

$$\nu_{ph} = \frac{c}{n_{ph}} \qquad \nu_{gr} = \frac{c}{n_{gr}}$$

- Group refractive index

$$n_{gr}^{ion} = 1 + C_2 \frac{N_e}{f^2} = 1 + 40.31 \frac{N_e}{f^2}$$

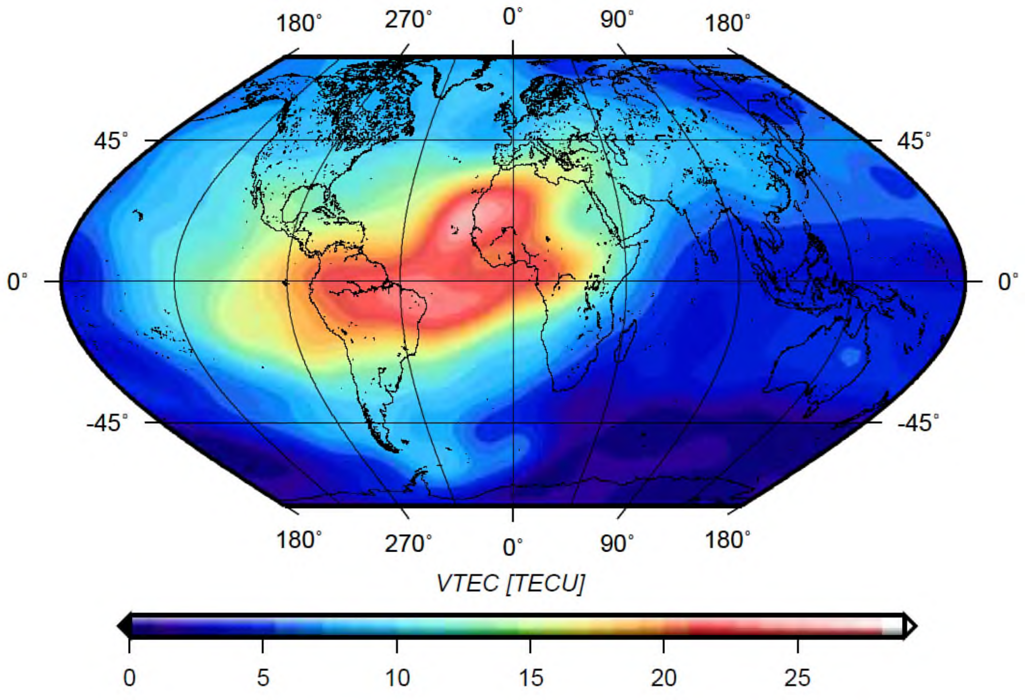
Ionosphere

- Total Electron Content (TEC) represents the total amount of free electrons in a cylinder with a cross section on 1 m^2 and a height equal to the slant signal path

$$STEC = \int N_e(s)ds$$

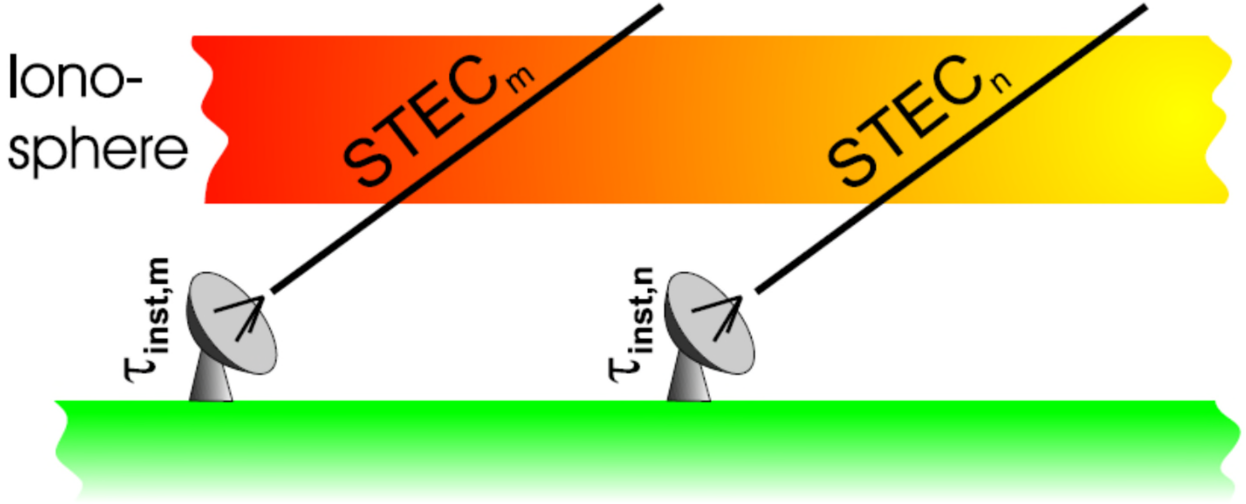
- Measured in TEC Units (TECU): 1 TECU is equivalent to 10^{16} electrons/m^2

- 1 TECU corresponds to
  - 7.6 cm at S-band (2.3 GHz)
  - 0.6 cm at X-band (8.4 GHz)



Ionosphere and X/S VLBI

- VLBI is affected by differential STEC values



Hobiger, 2005

$$\tau_{gx} = \tau_{if} + \frac{\alpha}{f_{gx}^2} \qquad \tau_{gs} = \tau_{if} + \frac{\alpha}{f_{gs}^2}$$

$$\alpha = \frac{40.31}{c}\left(\int N_e ds_1 - \int N_e ds_2\right) = \frac{40.31}{c}(STEC_1 - STEC_2)$$

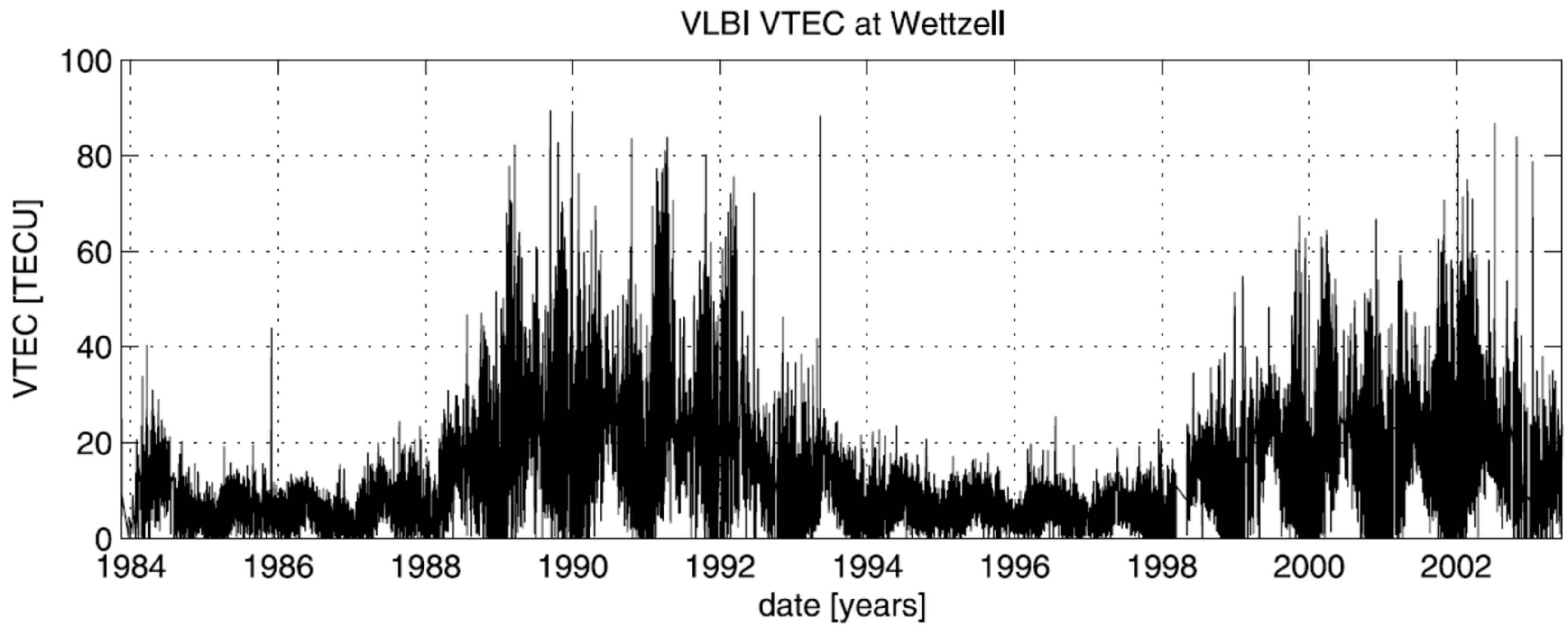
Ionosphere and X/S VLBI

- Ionosphere-free group delay based on effective frequencies

$$\tau_{if} = \frac{f_{gx}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gx} - \frac{f_{gs}^2}{f_{gx}^2 - f_{gs}^2}\tau_{gs}$$

Ionosphere and X/S VLBI

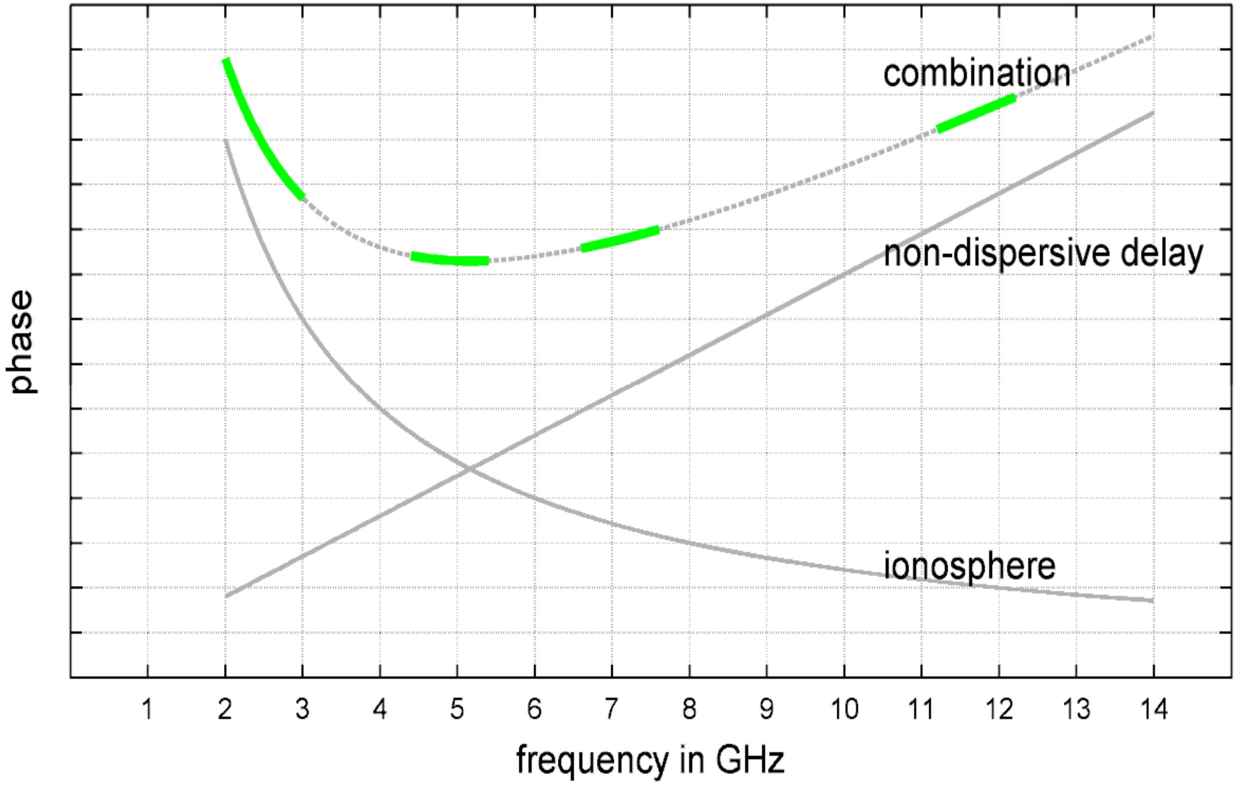
- Vertical TEC estimation from VLBI
  - only possible with appropriate use of constraints



Hobiger

Ionosphere and VGOS

- Phases are connected across the whole band
- Differential ionosphere delays are estimated together with the group delays in the fringe-fitting process

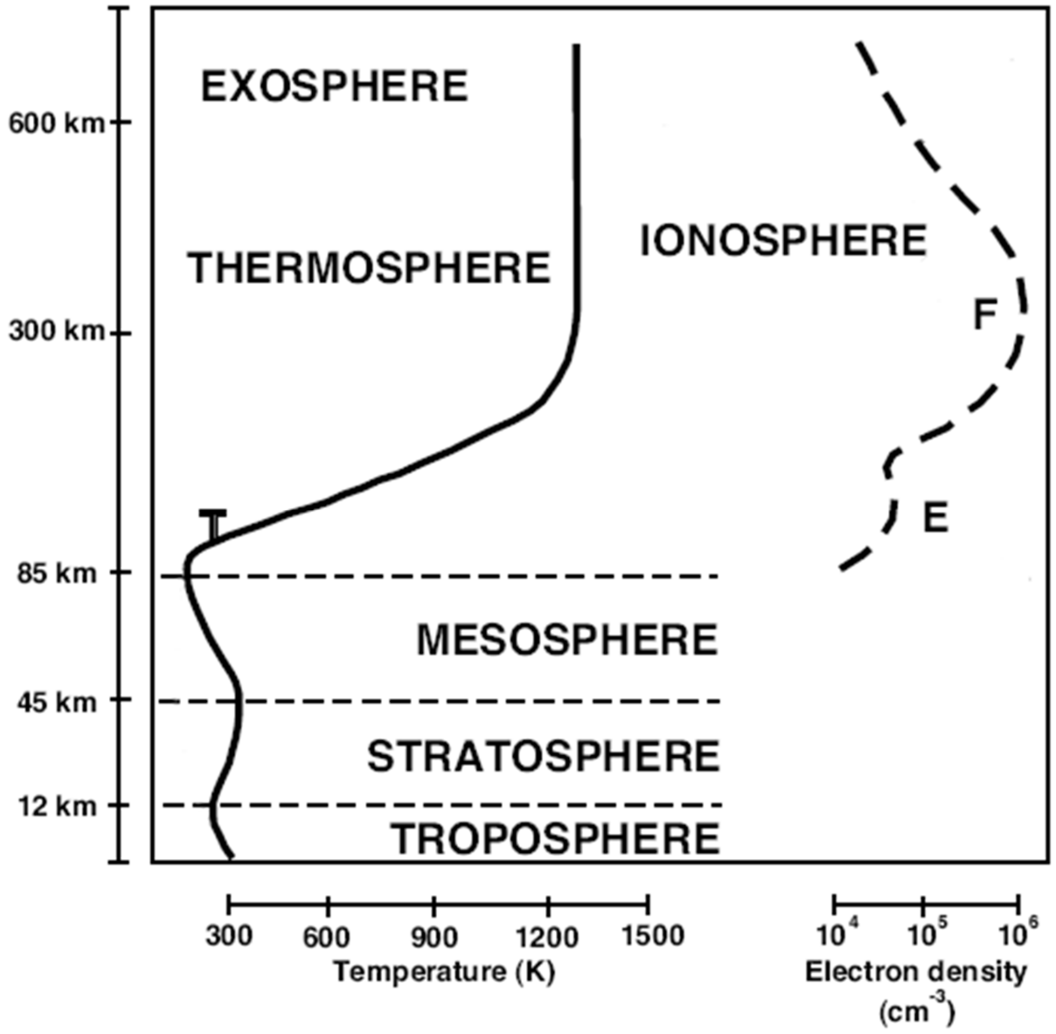


Neutral atmosphere

- "Troposphere delays"
  - strictly speaking delays in neutral atmosphere up to 100 km
  - Refractivity as function of pressure, temperature and humidity

$$N = (n - 1) \cdot 10^6$$

  - Separation into hydrostatic and non-hydrostatic ("wet") refractivity

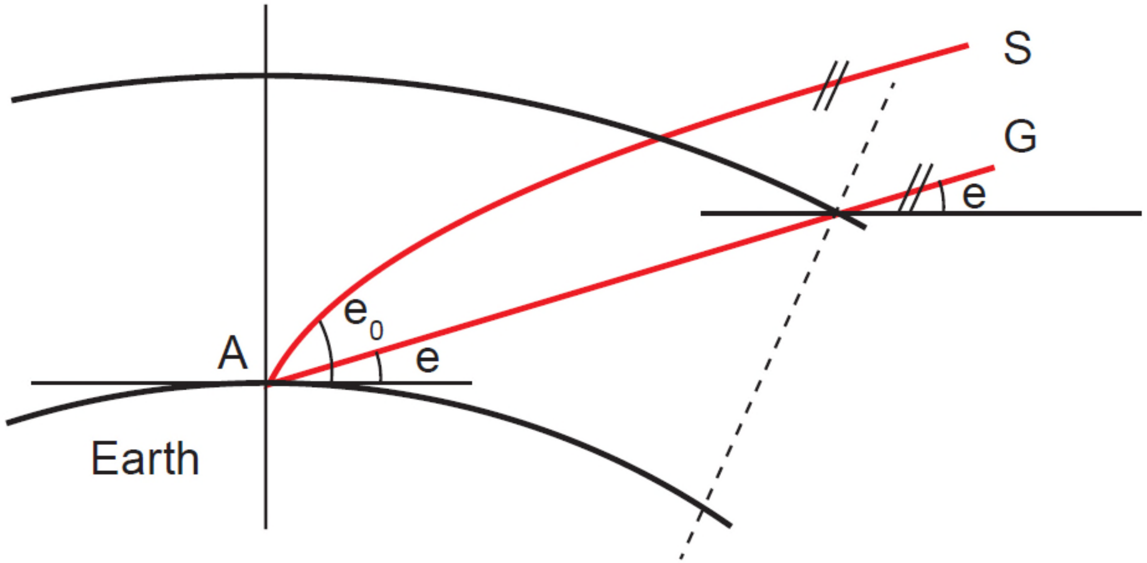


Wikipedia

Definition of tropospheric delay

- Electric path length L is minimized

$$L = \int_S n(s)\,\mathrm{d}s$$

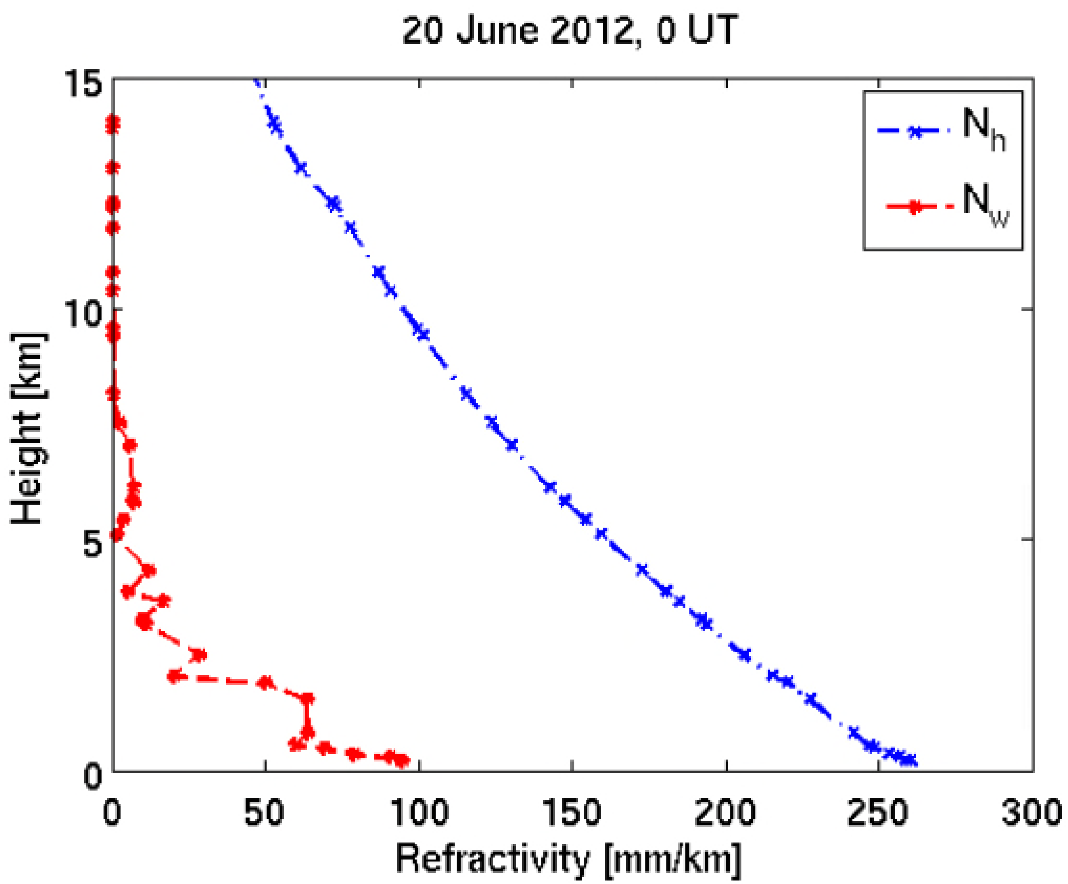


- Tropospheric delays: $\Delta L = L - G = \int_S n(s)\mathrm{d}s - G = \Delta L_h + \Delta L_w + S - G$

- Bending effect S – G about 2 dm at 5 degrees elevation (part of hydrostatic mf)

Hydrostatic versus wet refractivity

- Refractivity from radiosonde data



Hydrostatic refractivity N_h

Wet refractivity N_w

$$N = k_1 \frac{R}{M_d} \rho + k_2' \frac{e}{T} + k_3 \frac{e}{T^2}$$

hydrostatic — "wet"

Hydrostatic zenith delay

- Equation by Saastamoinen (1972)

$$\Delta L_h^z = 0.0022768 \frac{p_0}{f(\theta, h_0)}$$

≈ 2.3 m (8 nsec) at sea level

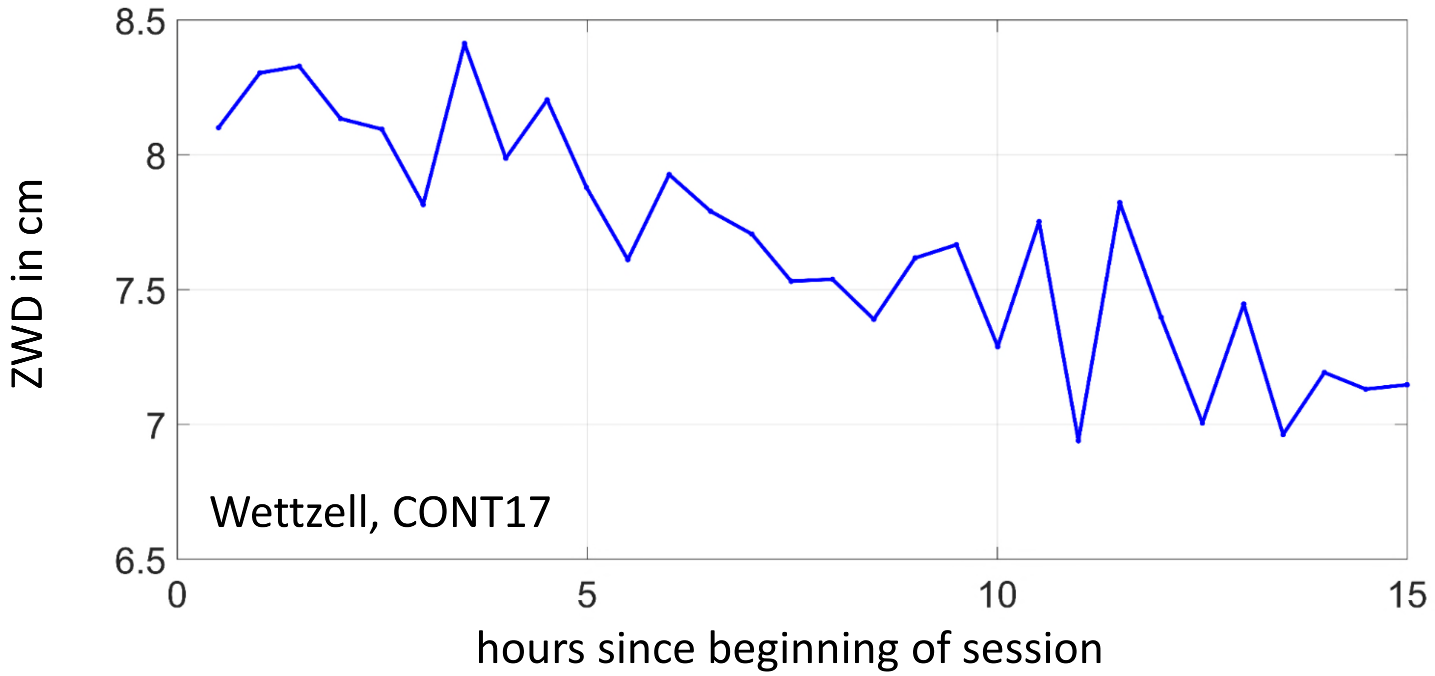
- We need the pressure at the site to determine the hydrostatic zenith delay very accurately
  - local recordings at the site (preferable if available)
  - gridded values from numerical weather models
  - empirical (blind) models like GPT2 etc
    - Caveat: do not use atmosphere loading corrections!

Troposphere delay modelling

- Mapping functions for a priori hydrostatic delay and estimating zenith wet delays

$$\Delta L(e) = \Delta L_h^z \cdot mf_h(e) + \Delta L_w^z \cdot mf_w(e)$$

- Zenith wet delays estimated every 20 to 60 minutes



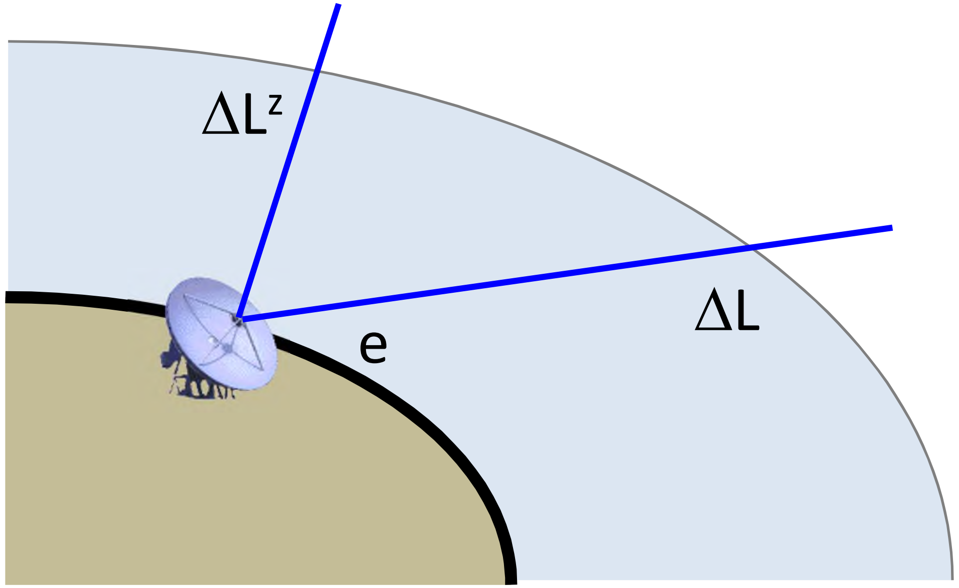
Mapping functions

- Correlation between height, clocks and zenith delays
- Partials are sin(e), 1, and mf(e)
  - Mapping function mf(e) not perfectly known, in particular at low elevations
  - Low elevations necessary to de-correlate heights, clocks, and zenith delays

- Mapping function errors via correlations also in station heights (and clocks)
- Trade-off $\rightarrow$ about 5 degrees cut off elevation angle
  - sometimes with down-weighting

Mapping functions

- The station height error is about 1/5 of the delay error at 5 degrees elevation

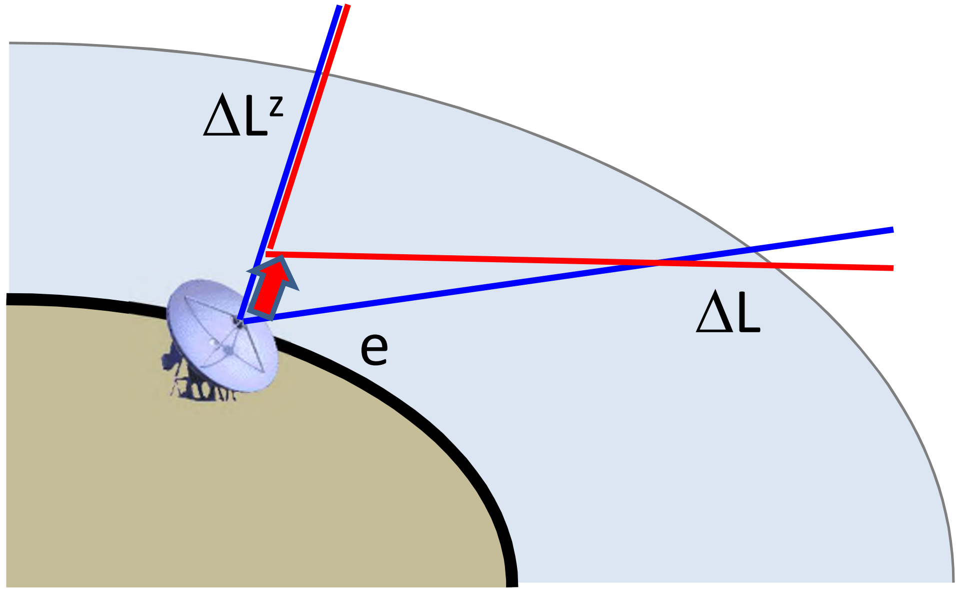


$\Delta L(e) = \Delta L^z \cdot mf(e)$

- The decrease of the zenith delay is about half of the station height increase

Mapping functions

- The station height error is about 1/5 of the delay error at 5 degrees elevation



$$\Delta L(e) = \Delta L^z \cdot mf(e)$$

$$\Delta L(e) = \Delta L^{z\prime} \cdot mf(e)'$$

- The decrease of the zenith delay is about half of the station height increase

Mapping functions

- Continued fraction form (Herring, 1992)

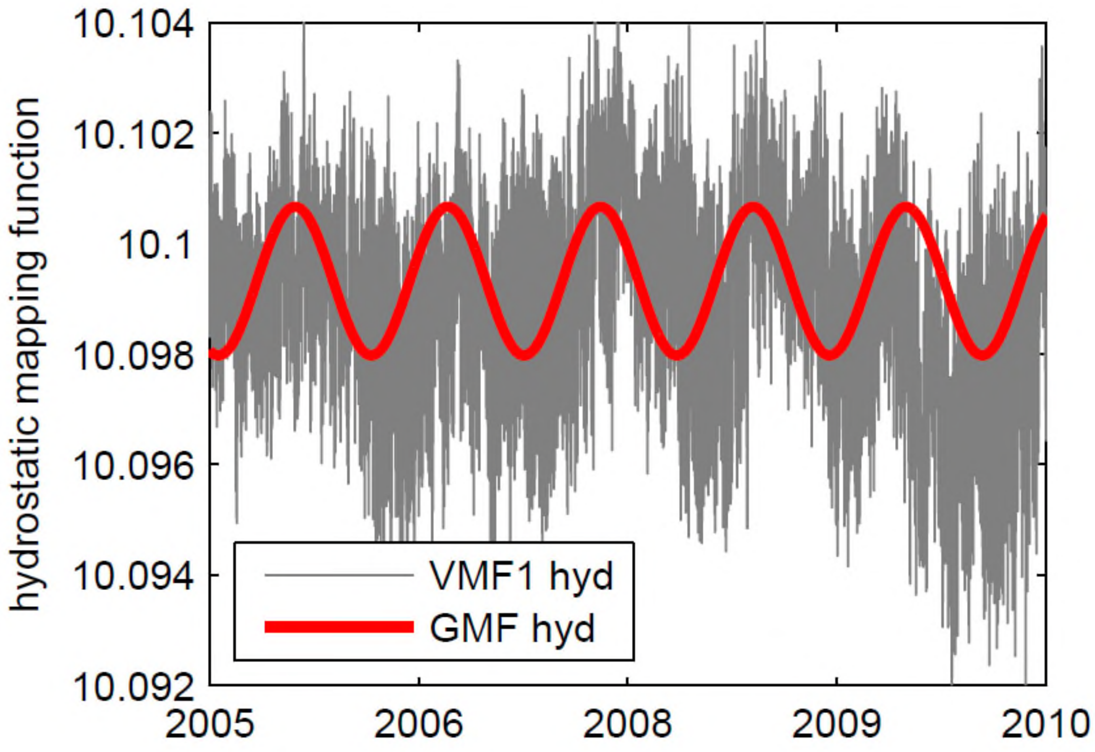
$$mf(e) = \frac{1 + \cfrac{a}{1 + \cfrac{b}{1 + c}}}{\sin(e) + \cfrac{a}{\sin(e) + \cfrac{b}{\sin(e) + c}}}$$

- Example Vienna Mapping Functions
  - Empirical functions for b and c coefficients
  - Coefficients 'a' by ray-tracing and inversion using 6h data of the ECMWF
  - Available for all IVS sites and on global grid

Mapping functions

- Global Mapping Functions GMF (GPT2, ..) is an averaged VMF

VMF1 versus GMF at Fortaleza (Brazil) at 5 deg. elevation



Mapping function data

- http://vmf.geo.tuwien.ac.at/
- Vienna Mapping Functions coefficients (from analysis and forecast data)
  - including zenith hydrostatic (and wet) zenith delay
- Empirical mapping functions, e.g. GPT3
- …

- ftp://ftp.gfz-potsdam.de/pub/GNSS/products/gfz-vmf1
- Potsdam Mapping Factors @ GFZ Data Server

- UNB-VMF1

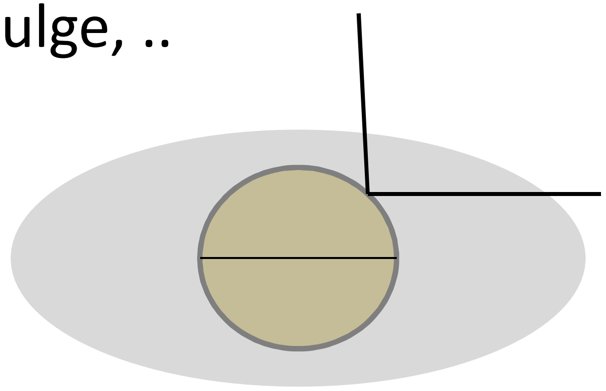
Tropospheric gradients

- Describe asymmetric delays

$$\Delta L(a,e) = \Delta L_0(e) + mf_g(e)(G_n \cos(a) + G_e \sin(a))$$

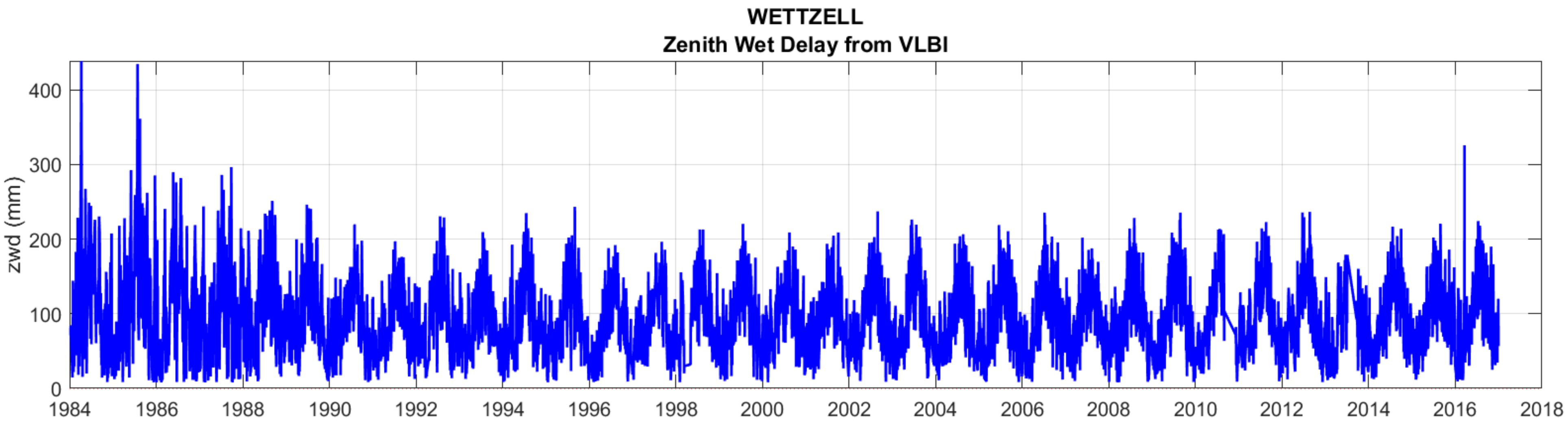
$$mf_g(e) = \frac{1}{\sin(e)\tan(e) + C},$$

- Typical gradient: 1 mm (corresponds to 10 cm at 5 deg. elevation)
- Estimated, e.g., every 3 hours
- Caused by weather fronts, coastal situations, atmospheric bulge, ..



Climate studies

- Zenith wet delays at Wettzell (Landskron, 2018)



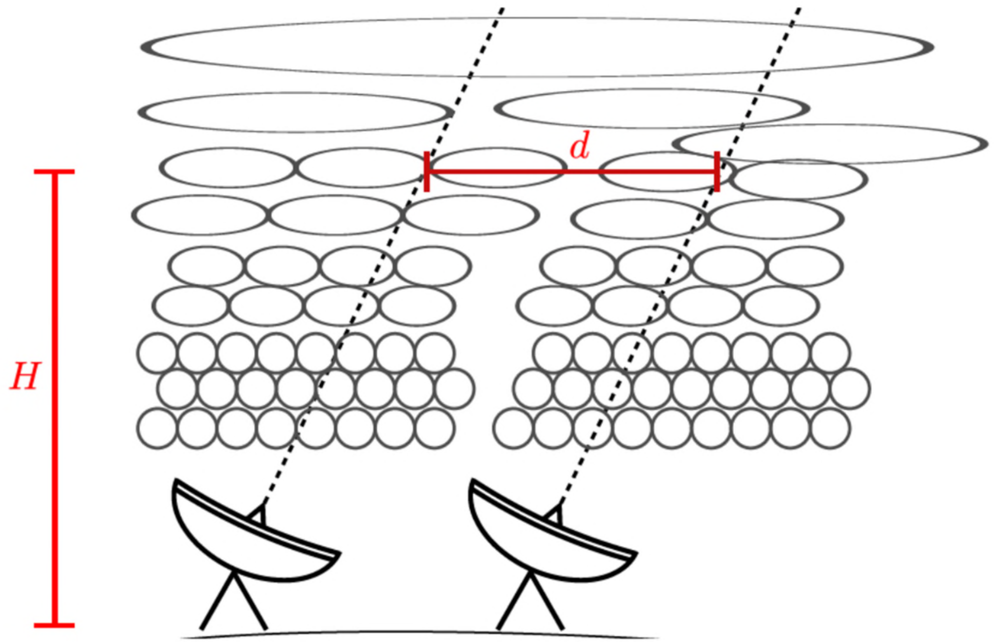
Questions?

Atmospheric turbulence

- Random fluctuations in refractivity distribution
- Structure function as modified by Treuhaft and Lanyi (1987)

$$D_n(\mathbf{R}) = \left\langle [n(\mathbf{r}) - n(\mathbf{r}+\mathbf{R})]^2 \right\rangle = C_n^2 \frac{\|\mathbf{R}\|^{2/3}}{1 + \left[\frac{\|\mathbf{R}\|}{L}\right]^{2/3}}$$

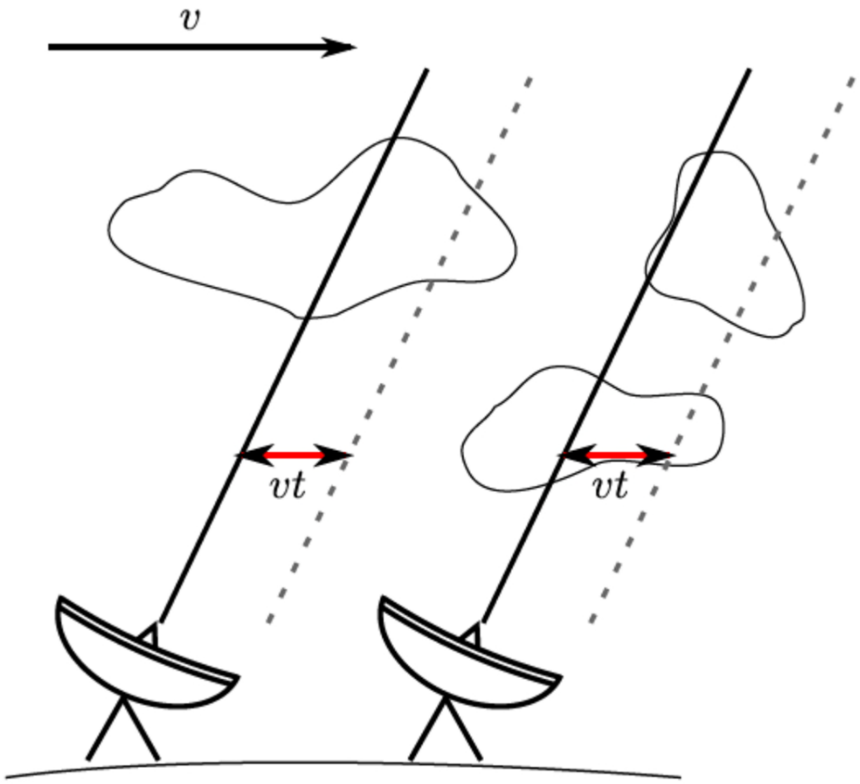
  - C_n^2 refractive index structure constant
  - L saturation length scale



Halsig, 2018

Atmospheric turbulence

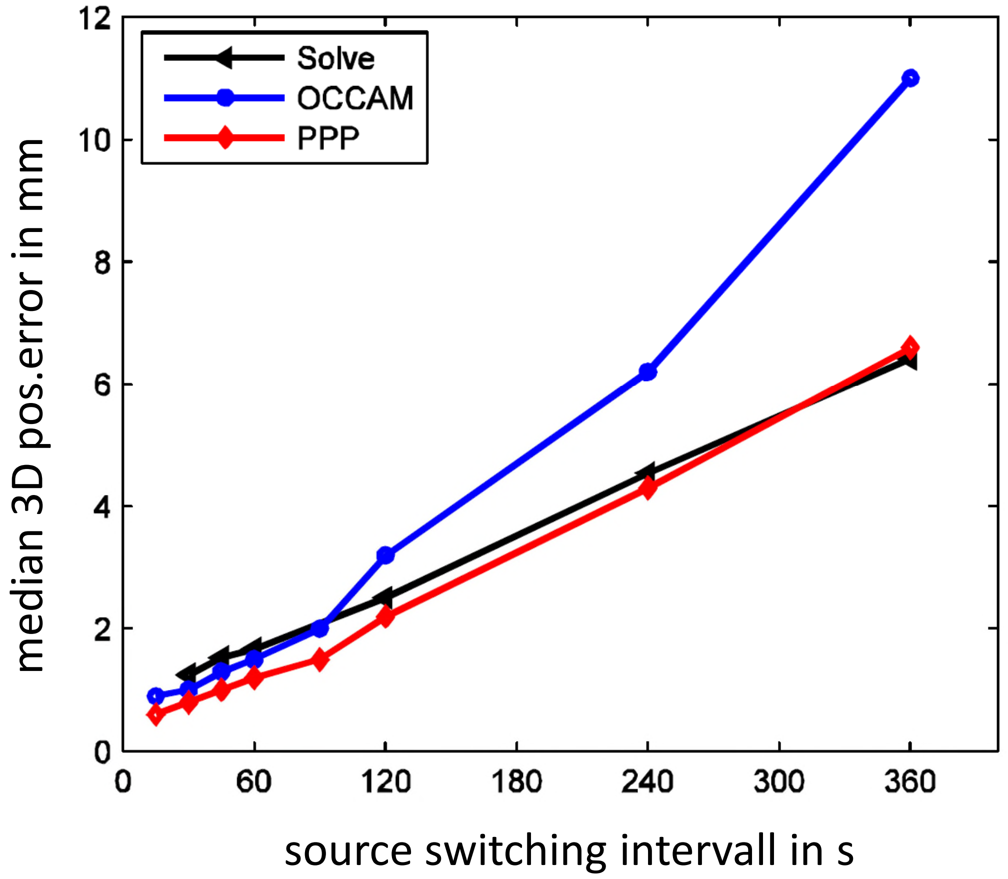
- Close observations in space and time are correlated
- Frozen flow theory for equivalence of correlation in space and time



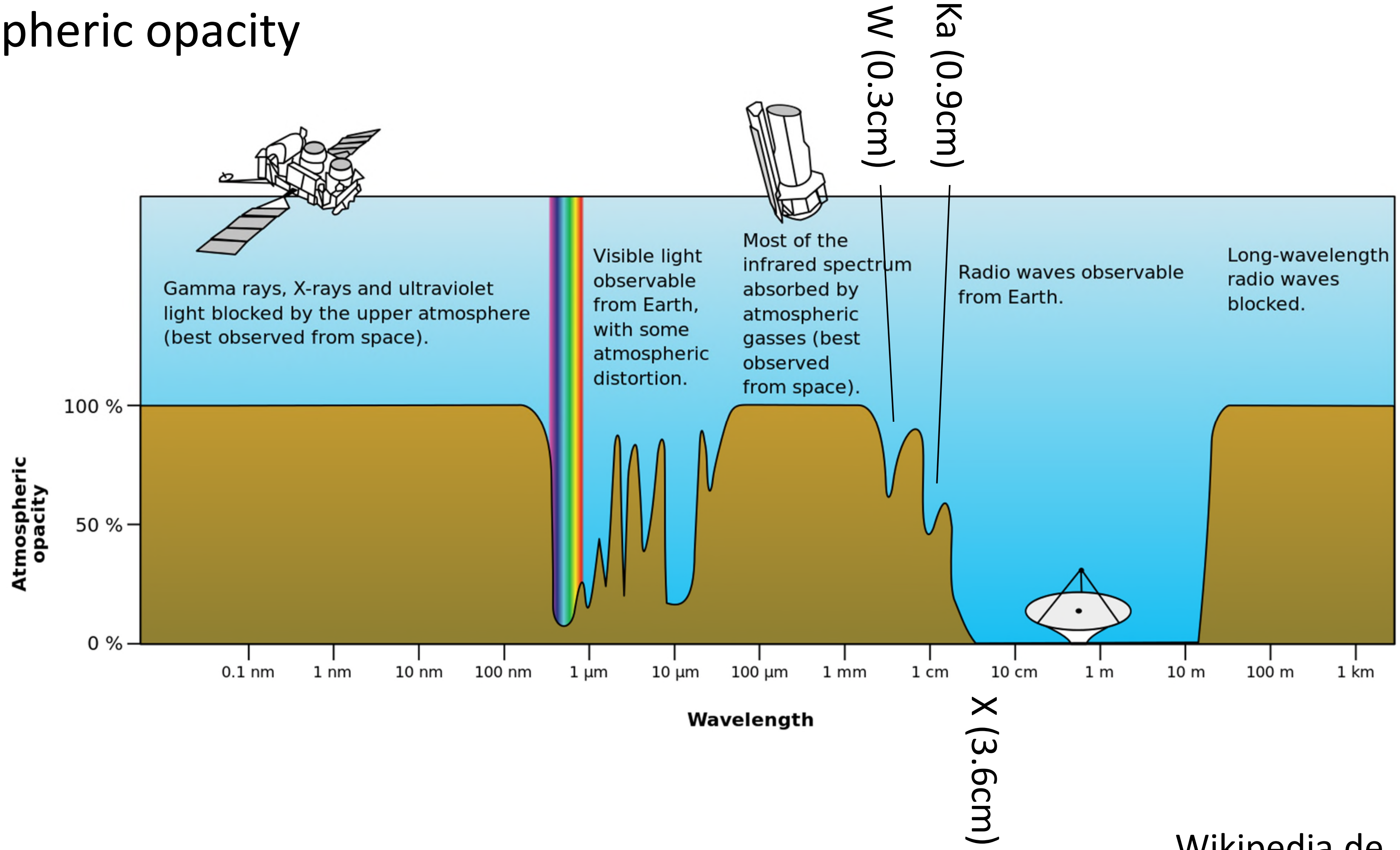
Halsig, 2018

Atmospheric turbulence

- Correlations can be used in
  - analysis (a priori correlation)
  - simulations (e.g. VGOS)



- Atmospheric opacity



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