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Portfolio Optimization with Factor Views

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Abstract

In this thesis, I have implemented the Black-Litterman model with factor views as forward-looking forecasts constructed according to the average value and momentum, which can be measured in sector portfolios of the European equity market index STOXX 600. The goal was to investigate whether it is possible to use factor views, constructed from the past pricing information, in combination with the BL model, and if this approach results in improved risk-return properties of the optimized portfolios. Additionally, it was investigated if such optimized portfolios, and in which parameter setting, deliver risk-adjusted returns in excess of the STOXX 600 Index as a benchmark. The portfolio optimization was conducted on 10 sector portfolios defined by the STOXX Europe 600 Index universe in the period from 1999 to 2019. The factor portfolios were constructed using best 2 and worst 2 performing sectors according to the 12-week momentum and the book-to-market ratio respectively. First 5 years of data have been used for estimating the first sector covariance matrix and for computation of the factor views. The historical simulations have been performed from 2004 to 2019 using 4-week rebalancing period. My empirical findings show that, over the investigated period, the Momentum factor has shown higher premia relative to the Value factor. This fact has also been reflected in the resulting Black-Litterman optimized portfolios. The BL approach with the Momentum factor has resulted in superior risk-return portfolios relative to the benchmark. The optimization with the Value factor shows close to no positive effect on the portfolio characteristics. Suprisingly, using both factors in combination, yields no benefits over the Black-Litterman optimization with the Momentum factor. Contrary to the efficient market theory developed in the 1970s, by using the approach described in this research, under no transaction-costs condition and by using only publicly available data, I was able to outperform the European equity market in risk-adjusted terms by using a wide range of Black-Litterman framework parameter settings. However, the performance inevitably comes with additional factor risk, which must be regarded in further analysis. The presented BL factor approach is suitable for tilting diversified portfolios towards factors that are known to be performance relevant [Fama and French, 1992].

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Part I
Introduction

1 Introduction

The mean-variance model introduced by Harry Markowitz in 1950 is a mathematical model for asset portfolio optimization. Before this revolutionary work, diversification was mostly based on the intuition of "not putting all of your eggs in one basket" [Hirani and Wallström, 2014, Olsson and Trollsten, 2018]. Based on the assumption that investors want the highest returns with the lowest possible risks, this model accurately chooses a combination of assets that maximize the return for a given level of risk [Markowitz, 1952, Olsson and Trollsten, 2018, Satchell and Scowcroft, 2003]. The model consists of two parts: the assets' mean expected return and the assets' variance. Both of these components are based on historical data, and hence, may not be the best predictors of future performance[Nielson et al., 2016]. Even though this model has laid out the basis for the Modern Portfolio Theory (MPT), and its wide academic acceptance for decades, for many reasons, it has had little to no acceptance in the investment management industry [Michaud, 1989, He and Litterman, 1999, Olsson and Trollsten, 2018]. Optimal portfolio weights are very sensitive to slight changes in expected returns, frequently resulting in portfolios with huge short positions [He and Litterman, 1999, Idzorek, 2005, Michaud, 1989, Olsson and Trollsten, 2018]. These optimal portfolios can hardly be implemented and prove highly risky out of sample.

In practice, investment managers would like to give more weight in the portfolio to the assets with a more promising outlook, or to assets they feel are undervalued at the moment [He and Litterman, 1999]. Unfortunately, the mean-variance framework requires expected returns to be specified for every asset in the portfolio, which may be inconvinient for the investors [He and Litterman, 1999, Idzorek, 2005].

An equity investor usually has some assumptions, or supposedly has superior information, about future performance for a particular stock, or set of stocks, which are typically only a small subset of the available investment universe [He and Litterman, 1999].

Let us assume an investor holds an equally weighted portfolio of world equity indices, and he has information that German equities are going to overperform French stocks by 5% per year [He and Litterman, 1999][1]. Knowing nothing about future returns of the other countries, he would like to incorporate that single piece of information into the mean-variance model [He and Litterman, 1999].

Intuitively, the investor could start by increasing the returns of German

[1]Example taken from [He and Litterman, 1999]

equity by 2.5%, and decreasing French equity returns by -2.5% [He and Litterman, 1999]. Due to the high sensitivity to changes in expected returns, the resulting mean-variance portfolio would become extreme, with huge swings, and the portfolio weight for French equity would become a large short position [He and Litterman, 1999]. This shortcoming, due to the absence of a neutral starting point for expected returns and the complex mapping between expected returns and portfolio weights, makes the Markowitz model inappropriate for an investor or portfolio manager to express their active opinions about future returns [He and Litterman, 1999].

The Black-Litterman (BL) model uses market implied expected returns as a neutral starting point for portfolio optimization. This idea was first expressed by Sharpe [1974], who inspired the discussions of making appropriate investment decisions in the optimization process [Litterman, 2003, Olsson and Trollsten, 2018]. The subjective views of an investor are afterwards applied on top of the returns implied by the market, i.e., equilibrium expected returns. Apart from the neutral starting point in the BL model, investors can specify subjective opinions in relative and absolute ways without having to specify the complete set of expected returns for all assets in the portfolio [He and Litterman, 1999, Xu et al., 2008, Black and Litterman, 1992].

In a relative view, an investor compares two assets and expects that asset one will outperform asset two by x% [Xu et al., 2008]. In an absolute view, investors specify their expectancy in terms of the percentage return that they believe one asset will provide [Xu et al., 2008]. These views are general statements about future expected asset performance.

In the case when an investor has no views about the market, the unconstrained optimal portfolio weights equal the equilibrium market capitalization weights, and the BL expected returns equal to the market implied equilibrium returns [Black and Litterman, 1992, Xu et al., 2008]. In the opposite case, when an investor has one or multiple views, the BL model combines them with the starting point implied market returns and tilts the optimal portfolio towards the expressed view [Black and Litterman, 1992, Xu et al., 2008].

In essence, an investor first invests in the market portfolio and then readjusts positions with views. The BL models' advantages and disadvantages depend on the user's ability to forecast the expected returns accurately [Olsson and Trollsten, 2018]. In the case of an absence of forecasts, the user is not motivated to deviate from the reference point, i.e., market equilibrium portfolio [Black and Litterman, 1992, Olsson and Trollsten, 2018]. In other words, in the case of absent forecasts, you stay invested in the market portfolio [Black and Litterman, 1992, Olsson and Trollsten, 2018].

The foundation of the Capital Asset Pricing Model (CAPM) was established in the mid-1960s by William Sharpe [Sharpe, 1964] and John Lintner [Lintner, 1965] [Polovenko, 2017]. The CAPM suggests that the expected returns of an asset are a function of its return's covariance with the returns on the overall market portfolio [Hirani and Wallström, 2014, Olsson and Trollsten, 2018]. However, studies by Fama and French [1992, 2004], and many others, e.g. Daniel and Titman [2012] and Lewellen and Nagel [2006], uncovered almost no relationship between market betas and expected returns, and instead found a strong cross-sectional relationship between the characteristics of assets like size or book-to-market ratio and expected returns [Daniel and Titman, 2012].

These deviations from the Efficient Market Hypothesis (EMH) can be considered as market anomalies that enable investors to generate excess returns. In the meantime, many would argue that the first factor, the CAPM beta, only measures an asset's sensitivity to market movements and does not explain excess returns over time [Lewellen and Nagel, 2006]. Investors that seek premia should consider other factors that have exhibited long-term market outperformance. The Value factor rests on a belief that assets that are cheap relative to some measure of fundamental value outperform those that are pricier [2]. This factor has been backed by the robust empirical findings that have shown that value investing can generate excess returns over long periods [Nielson et al., 2016].

There are many different ways to define the intrinsic value of stocks. Investors may, for example, examine cash flows, earnings and sales. Fama and French [1992, 2004] have shown how inexpensive stocks, i.e. assets with higher book-to-market ratios, have outperformed cheaper ones, i.e. assets with lower book-to-market ratios [Nielson et al., 2016]. In this work, I shall use book-to-market ratios of assets as a means for the (active) view generation in the BL model.

The momentum anomaly empirical evidence was first published in 1993 by Jegadeesh and Titman [1993] [Nielson et al., 2016]. They demonstrated that stocks that outperformed in the medium term would continue to perform well and that stocks that had lagged would continue to lag [Nielson et al., 2016]. Technical analysts have been doing momentum trading for decades now.

One of the explanations for the momentum anomaly is that investors

[2] Retrieved from https://www.risk.net/definition/value-factor (15.02.2021)

usually underreact to the improvements of company fundamentals, but tend to pile onto the trade when the stock price outperformance has caught the attention of investors [Nielson et al., 2016]. Such dynamics enable winners to keep winning and momentum investing to work [Nielson et al., 2016]. This tends to continue until a catalyst indicating a negative fundamental change causes it to stop [Nielson et al., 2016]. One common way to measure momentum in the equity market is to classify assets by 6 to 12-month price returns [Nielson et al., 2016]. Momentum is the second factor I intend to use as a means to generate subjective views to be applied in the BL model.

1.1 Motivation

According to the Efficient Market Hypothesis, consistent alpha generation is not possible. The EMH states that the current asset pricing contains all available information and that consistent risk-adjusted excess return generation is not possible by, for example, buying undervalued stock [3].

This cornerstone of modern finance suggests that stocks always trade at fair value, making it impossible for stock-picking experts and fundamental value analysts to beat the market in the long run [4]. Extensions to pure CAPM efficiency regard anomalies as factors which come with factor risk but also with a factor premium. Taking exposure to these factor risks is reported to help outperform the market. In this thesis, instead of using investors' subjective forecasts, I plan to use the Black-Litterman model with (active) views, which are systematically constructed by using Value and Momentum factors in order to analyze risk-return characteristics of such portfolios.

Furthermore, I will investigate if this combination of the BL model and the two factors, i.e. Momentum factor and Value factor, can tilt the market portfolio in such a way so that it overperforms the STOXX600 market portfolio returns benchmark. This approach, if successful, would result in:

- Systematic active view generation for the BL model without the need for a subjective input from an investor.
- Constructing portfolios that yield higher risk adjusted returns than the market portfolio.

[3]https://www.investopedia.com/terms/e/efficientmarkethypothesis.asp (10.06.2020)
[4]https://www.investopedia.com/terms/e/efficientmarkethypothesis.asp (10.06.2020)

1.2 Problem Statement

Can we detect a value premium and / or a momentum premium in the European stock market over the last 20 years? I analyze the equity universe defined by the STOXX 600 Index and investigate on the sector level whether Black Litterman (BL) optimized portfolios with factor forecasts can improve portfolio-return characteristics. I.e., forward-looking BL model views are constructed according to the average value and momentum which can be measured in sector portfolios. Additionally, which risk properties would the optimized portfolios have in terms of Sharpe ratio, information ratio and would this strategy deliver a positive Capital Asset Pricing Model (CAPM) Alpha, i.e. would these portfolios deliver risk-adjusted returns in excess of the STOXX 600 Index?

A factor can be thought of as any characteristic related to one group of securities that is important in explaining its return and risk [Nielsen, 2016, Bender et al., 2013, Ang, 2014]. The single factor CAPM states that the β is driving all assets' expected returns. This factor is the market return in excess of T-Bills, and the higher an asset's exposure to it, the higher the risk premium. The market itself is an example of the most important equity factor [Nielsen, 2016, Bender et al., 2013]. Further examples, among others, include fundamental factors like momentum, value, asset growth and low-volatility. Since factors affect asset returns and asset classes have different exposures to factor risks, would it be possible to integrate this information into portfolio optimization itself (see Hypothesis 1 in Section 1.3)? If a particular sector index has higher return momentum than others, why not overweight this sector relative to others and check if this portfolio strategy delivers consistent overperformance relative to the market over time (see Research Question in Section 1.3)?

Modern Portfolio Theory (MPT), or mean-variance analysis, is de facto the standard when it comes to financial risk management and empirical asset pricing (optimal asset allocation in a portfolio). This portfolio diversification strategy tool is highly appreciated among today's investors, risk managers, and investment institutions. MPT proves to be highly advantageous, because it allows an investor to select a portfolio with maximum return for a given level of risk or with minimal risk for a given level of return [5] [Satchell and Scowcroft, 2003, Idzorek, 2005]. There are, however, some disadvantages to this model. If an investor has some additional information about future returns of particular portfolio constituents, which they usually have, and makes small readjustments in expected returns, most of the portfolio positions and weights change in such a way that they become unreasonable [He and Litterman, 1999].

[5] https://www.investopedia.com/terms/b/black-litterman_model.asp

Furthermore, Markowitz's model uses only backward-looking information, which is inappropriate for the task of integrating return forecasts. Most investors know that past performance is no guarantee of future performance [Nielson et al., 2016]. Working with only historical data may lead to overlooking newer circumstances, since such may not have occurred in the past.

1.3 Hypothesis and Research Question

- Hypothesis 1 : Factor views can be used in BL portfolio optimization to tilt BL portfolios towards pronounced factor portfolios which inherit the factor-return characteristics of the view.
- Hypothesis 2 : The higher the confidence in the factor views the stronger the factor characteristics of the resulting BL portfolios.
- Hypothesis 3 : Combined factor views offer diversification benefits.

Research Question : Is it possible to outperform the European equity market by just using publicly available information contrary to the efficient market theory developed in the 1970s (see Section 1.1)?

1.4 Research Methodology

This paper relies on a quantitative approach where financial time series data is used for evaluating the performance and behaviour of portfolios constructed using market anomalies in combination with the BL framework. The outcomes of the quantitative analysis are used to draw conclusions about the model and the parameters applied in construction of the portfolios.

To be able to answer the research question and to accept or reject the hypothesis, it is necessary to apply the appropriate research methodologies. Portfolio optimization in finance is a quantitative process of selecting assets in such a way to maximize the expected return while minimising the financial risk [Markowitz, 1952, He and Litterman, 1999]. According to Chen et al. [2010], quantitative finance and risk management research requires finance theories, finance policies, and methodology.

The BL framework builds on top of the Markowitz Portfolio Theory in finance [Black and Litterman, 1992, Satchell and Scowcroft, 2003]. Another prominent theory in finance, the CAPM [Sharpe, 1964, Lintner, 1965], is used in this thesis to evaluate the ability of the proposed method to yield the excess returns relative to the market and to measure the riskiness of the same approach.

CAPM alpha is a measure of excess return relative to the market over some period of time. It often describes a strategies or managers ability to "beat the market".

The authors of Chen et al. [2010] classify mathematics and statistics as methodologies used in quantitative finance and risk management research. Variance has a central role in statistics and probability theory and it is used as a measure of dispersion of portfolio returns throughout this paper.

The Sharpe ratio [Sharpe, 1964], as a mathematical methodology in quantitative finance and risk management, according to Chen et al. [2010], is used to measure the reward-to-volatility trade-off.

Another mathematical methodology according to Chen et al. [2010], the Information ratio (IR), is used as a measure to evaluate the residual excess returns, i.e. returns not explained by the market, per unit of volatility in relation to the market benchmark. This measure is similar to the Sharpe ratio, with the difference that the Sharpe ratio uses risk-free rate as a benchmark and the IR uses a risky index, such as market portfolio, as a benchmark [Grubjesic and Orhun, 2007].

Hypotheses proofs will be done by applying standard time-series statistics on CAPM alpha, Sharpe ratio, Information ratio and historical backtesting by using success criteria discussed in Section 1.5.

The data set in this research consists of weekly returns (including dividends) in a 30-year period between the 3. September, 1989 and 8. February, 2019, totaling to 1571 weekly observations. The source of data is Thomson Reuters Datastream, which can be classified as a high quality data source. The total data collection consists of six data frames (matrices), each containing a part of the information necessary to compute the indices, benchmark and factors. Section 4 gives a detailed overview of the data sample used in this master thesis.

The Black-Litterman portfolio optimization model has been extensively covered and used over decades, as well as Value and Momentum factor investing strategies. However, empirical research of the two-method combination has, to my knowledge, never been performed on the STOXX 600 sector indexes. The starting point for a literature review and as a primary source of information were the classical references on momentum and value anomalies by Jegadeesh and Titman [1993] and Fama and French [1992], the journal paper where the Black-Litterman model was first published [Black and Litterman, 1992], the theoretical framework consisting of established contemporary theories about active portfolio management by Grinold and Khan [1999] and a primer on factor investing by Ang [2014]. All further resources and references are retrieved from highly popular and reliable sources of academic research and information such as JSTOR and Google Scholar by using the major search keywords extracted from the primary sources of literature and based on the research question.

The motivation for the layout and texts in the introductory and theoretical parts of this thesis is drawn from Black and Litterman [1992], He and Litterman [1999], Jegadeesh and Titman [1993], Ang [2014], Fama and French [1992], Idzorek [2005], Polovenko [2017], Olsson and Trollsten [2018], Skeie-Larsen et al. [2018], Schepel [2019], Bender et al. [2013] and Nielson et al. [2016].

1.5 Success Criteria and Aim of the Work

The aim of this work is to check the hypotheses (see Section 1.3) by implementing the combined Black-Litterman and Factor-Investing model. The goal is to test and determine if value and momentum factors can be used separately, and in combination, as forecasts in the BL portfolio optimization model.

More accurately, it will be investigated if these two factors can be used as forecasts instead of the subjective investor views usually used in the BL model, and if this approach results in any market portfolio diversification improvement.

Furthermore, it will be investigated if such portfolio optimization yields any risk-return premia compared to the market, or in other words, if this approach delivers excess returns without taking too much risk. This will be done by building portfolios using sector market data from STOXX600 in the period from 1999 to 2019 and comparing with the market-weighted portfolio of the same universe of assets.

The most important metric in answering the Research Question (see Section 1.3) will be the Information ratio (IR). This metric can be defined as follows:

$$IR_p = \frac{mean(r_p - r_b)}{std(r_p - r_b)}$$

Where,

- IR_p represents the Information ratio on the given portfolio p,
- r_p is a vector of returns of the portfolio p,
- r_b is a vector of returns of the benchmark portfolio b.

IR measures the performance of the portfolios if the value is added through under- or overweighting of assets relative to the benchmark portfolio given the same market risk [Goodwin, 1998, Skeie-Larsen et al., 2018]. Generally, a higher IR indicates higher returns in excess of the benchmark. Grinold and Khan [1999] state that an IR of 0.5 is “good ”, 0.7 is “very good”, and 1 and above is “exceptional” [Skeie-Larsen et al., 2018].

The CAPM alpha is used to describe the strategie's, or portfolio's, ability to outperform the market benchmark on a risk-adjusted basis.

It can be defined as follows:

$$\alpha_p = mean(r_p - r_f) - \beta_p * mean(r_m - r_f)$$

Where,

- α_p denotes the CAPM alpha on the given portfolio p,
- r_f is a vector of the risk-free returns f,
- r_m is a vector of returns of the market portfolio m,
- β_p is a regression coefficient of the portfolio p defined as $\beta_p = \frac{cov(r_p, r_m)}{var(r_m)}$.

CAPM alpha describes the value added, in excess of the returns from the benchmark, from choices a portfolio manager makes [Skeie-Larsen et al., 2018]. If a portfolio manager can successfully forecast the expected returns, the portfolio's alpha will be positive. On the contrary, if a portfolio is underperforming the benchmark without considerably lower market-risk exposure, the alpha will be negative. The higher the CAPM alpha, higher the market-risk adjusted out-performance. This metric will be used to test the Hypothesis 1 (see Section 1.3). Furthermore, it will be used in addition to the IR to answer the Research Question (see Section 1.3).

The Hypothesis 2 (see Section 1.3) will be rejected or approved by performing historical simulations, i.e. backtesting, with broad range of BL models' investor view confidence values (see Section 3.1.2) and by comparing the risk-return characteristics of the resulting portfolios.

The Sharpe ratio describes the average return earned in excess of the risk-free rate per unit of volatility. It can be defined as follows:

$$SR_p = \frac{mean(r_p - r_f)}{std(r_p - r_f)}$$

Where,

- SR_p represents the Sharpe ratio of the portfolio p.

The Sharpe ratio measures the risk-adjusted returns. In this metric, the standard deviation of the excess returns is used as a measure for the volatility. If a particular portfolio setup has a higher Sharpe ratio compared with the benchmark, this means that this strategy is either less volatile (less risk through higher diversification) or delivers higher returns for the same amount of risk. The higher the Sharpe ratio, the better is the portfolio. This ratio will be used to accept or reject the Hypothesis 3 (see Section 1.3).

1.6 Relevance to the Curricula of Business Informatics

The portfolio optimization process is a process of selecting an optimal portfolio out of (infinitely) many possible portfolio alternatives. This optimization is subject to given constraints (mainly on portfolio weights and desired portfolio characteristics) and constitutes a sub-branch of convex optimization. Mathematical optimization problems arise in all quantitative disciplines from computer science and engineering to economics and operations research.

This thesis contributes to the fields of corporate finance and optimization, which are an integral part of the curriculum of Business Informatics. The optimization and corporate finance section of this thesis is mostly related to the following curriculum courses:

- VU Optimization in Business and Economics
- VU Advanced Financial Planning and Control
- VU Project and Enterprise Financing
- VU Model-based Decision Support
- VU Machine Learning

Part II
Main Part

2 State of the Art

Currently, there is a vital discussion about the long-term stability of factor returns in equity-portfolio management. There is a branch of literature, most notably the work of the 2013 Nobel Laureate Eugene Fama and his coauthor Kenneth French, which consider factor premia as constant and stable over time [Fama and French, 2015]. A second branch focuses on the time-variation of factor premia and tries to characterize their dynamics [Dangl and Halling, 2012, Daniel et al., 2019]. This thesis is intended to contribute to this discussion by combining Black-Litterman portfolio optimization [Dangl and Aussenegg, 2018, He and Litterman, 1999, Black and Litterman, 1992] with (dynamic) factor views [Jegadeesh and Titman, 1993, Fama and French, 1992, 2015].

The Black-Litterman portfolio optimization model has been extensively covered and used over decades, as well as value and momentum factor investing strategies. However, to my knowledge, empirical research of the combination of these two methods has never been performed on the STOXX 600 sector indexes.

Within this master thesis, the idea of incorporating return forecasts into the portfolio optimization model makes the mean-variance theory inappropriate for the task, which is shown by He and Litterman [1999]. They discuss the disadvantages of modern portfolio theory in detail when trying to incorporate return forecasts. They demonstrate the advantages of the BL model when working with investor views of future returns.

Contrary to the MPT, which uses historical expected returns, the BL model uses market returns that are implied by the market portfolio. Black and Litterman [1992] discuss how CAPM equilibrium returns can be used as a starting point for a portfolio optimization process. Furthermore, they indicate how the portfolio weights are tilted according to the investor views, and the benefits of the implied returns when investors have no views at all or have no confidence in their forecasts. They also show how the investors can control the portfolio tilt between the CAPM prior distribution and their subjective forecasts by using parameter τ. This parameter, which characterizes the investors' confidence in their forecasts, or investor skepticism regarding the implied CAPM returns, plays a crucial role in the BL portfolio optimization. This paper argues and shows how this new method in portfolio optimization, according to the investor's certainty in their own subjective forecasts, controls the magnitude and tilts the portfolio from the market neutral starting point. The higher the confidence in investor views, the stronger the tilt toward the investor forecast. Conversely, the low investor confidence in the views keeps the portfolio weights close to those implied by the market. In this paper, I shall derive the implied CAPM prior and use it as a starting point for the portfolio optimization. Additionally, I will use a broad interval of investors' forecast confidence values in the backtesting process in order to find out which specific value adjusts the neutral weights, according to the investors' views, that deliver the highest premia.

Bender et al. [2013] offer an overview of factor investing based on the existence of factors grounded in the academic literature that have earned a premium over extended periods. They argue that factor indexes should not be taken as a replacement for market-cap indexes, but on the contrary, that they rebalance away from the neutral market cap starting point. For this reason, they can be viewed as the result of an active view or decision and the investor has to form his own belief about what explains historical premia and if it is going to persist in the future.

Jegadeesh and Titman [1993] and Fama and French [1992] discuss Momentum and Value factors respectively, and document how these two factors have made abnormal returns over long periods. The future return forecasts for the BL model views within this thesis is going to be derived using these two factors. The difference in average returns of the winning (top) and loosing (bottom) portfolios will be used as performance predictors for the next period. This top/bottom portfolio performance difference, as a predictor of future returns, shall be used as (active) views in the BL portfolio optimization model.

Fabozzi et al. [2006] present how to incorporate trading strategies in the Black-Litterman model. Specifically, they discuss how to incorporate factor models and cross-sectional rankings in this framework. In their example, by using MSCI World data from 1980/1/1 to 2004/4/31 consisting of 23 developed market country indices, they demonstrated how to combine momentum strategy with the market equilibrium in a portfolio optimization framework by using the BL model. By rebalancing the portfolio according to the momentum factor at the end of each month, this approach resulted in significant outperformance over the MSCI World Index in a period of about 25 years.

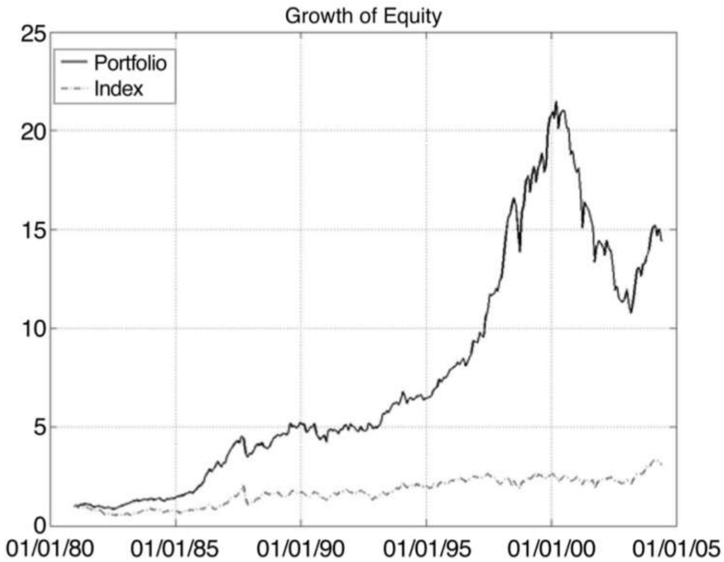

Figure 1: *Momentum Optimized Strategy and the MSCI World Index [Fabozzi et al., 2006]*

The key difference of my master thesis to the work of Fabozzi et al. [2006] is that my paper concentrates on the aggregation to 10 European industry portfolios, which is not done by Fabozzi.

3 Theory

3.1 The Black-Litterman Model

Based on ideas from the CAPM and the mean-variance model, the Black-Litterman model was developed in 1990 by Fischer Black and Robert Litterman at Goldman Sachs [Black and Litterman, 1992, Polovenko, 2017, Walters, 2014]. Before this model, investors had to input expected returns for every asset in the mean-variance model in order to be able to compute the optimal portfolio weights. This resulted in unintuitive portfolios and weights that made no sense to investors, especially when investors tried to incorporate their subjective forecasts for future returns. The BL model was developed to provide a neutral starting point weights or point of gravity, to which investors would retreat in the absence of subjective opinions of future returns. In this case, investors would simply invest in the market capitalization weighted portfolio.

The BL model uses a Bayesian approach to infer the posterior probability distribution of the expected returns using the CAPM and the additional subjective investor views [Idzorek, 2005, Polovenko, 2017]. In this model, the CAPM equilibrium implied returns constitute the prior information, and the investor views are the additional information [Polovenko, 2017]. The blend of the prior and the additional information forms the posterior distribution, i.e., the posterior expected returns.

3.1.1 The Starting Point - CAPM

The CAPM theory builds on the earlier work of the mean-variance analysis from Markowitz [1952], and was separately introduced by Sharpe [1964], Lintner [1965], Mossin [1966] and Treynor [1961] [6].

One of the BL model's basic assumptions, if an investor has no views for the future returns, is that expected returns should be following the market equilibrium returns. Therefore, if an investor has no views, he should hold the market portfolio [Idzorek, 2005].

A starting point for the BL model is the CAPM implied returns [Sharpe, 1974, Idzorek, 2005]. The CAPM assumes that expected returns of all assets will converge in direction of an equilibrium in such a way that, given all investors hold the same belief, demand will be met by supply [Olsson and Trollsten, 2018, He and Litterman, 1999].

$$E(r_i) = r_f + \beta_i(E(r_m) - r_f) \tag{1}$$

[6] https://en.wikipedia.org/wiki/Capital_asset_pricing_model

Where,

- $E(r_i)$ is the expected return on security i,
- r_f is the risk-free rate,
- $E(r_m)$ is the expected return of the market portfolio,
- $(E(r_m) - r_f)$ is the market risk premium,

and β_i is a regression coefficient defined as $\beta_i = \frac{\sigma_{im}}{\sigma_m^2}$ where σ_{im} is the co-variance between returns of asset i, and the returns of the market portfolio m, and σ_m^2 is the market portfolio variance. The parameter β_i can also be interpreted as the sensitivity of the asset returns to the market returns [Polovenko, 2017].

The basic linear relationship between risk and return is modeled by the CAPM [Olsson and Trollsten, 2018]. According to the CAPM, the only risk that cannot be eliminate through diversification is the assets' β [Olsson and Trollsten, 2018]. In other words, investors get rewarded for taking more risk that cannot be diversified away, i.e. for taking on more systematic risks [Olsson and Trollsten, 2018]. The compensation that an investor requires for bearing the risk, the risk premium, is a function of investors' risk aversion coefficient [Olsson and Trollsten, 2018].

CAPM claims that the tangency portfolio is the market portfolio:

$$w_m = \frac{1}{\delta}\Sigma^{-1}(\mu - r_f\mathbf{1})$$

Expected market risk premium:

$$\begin{aligned} E(r_m) - r_f &= w_m'(\mu - r_f\mathbf{1}) \\ &= \frac{1}{\delta}(\mu - r_f\mathbf{1})'\Sigma^{-1}(\mu - r_f\mathbf{1}) \end{aligned}$$

Market variance:

$$\begin{aligned} \sigma_m^2 &= w_m'\Sigma w_m \\ &= \frac{1}{\delta^2}(\mu - r_f\mathbf{1})'\Sigma^{-1}\Sigma\Sigma^{-1}(\mu - r_f\mathbf{1}) \\ &= \frac{1}{\delta}\left[E(r_m) - r_f\right], where \\ I &= \Sigma\Sigma^{-1} \end{aligned}$$

$$E(r_m) - r_f = \delta \sigma_m^2 \tag{2}$$

Where,

- δ represents the average risk aversion coefficient,
- μ represents the expected return on security i for $i = 1, ..., n$, with n assets in the market,
- w_m is a vector of market portfolio capitalization weights,
- σ_m^2 is the variance of market portfolio expected returns, and
- Σ is the covariance matrix of excess returns for the assets.

According to the CAPM, any asset's expected risk premium $(E(r_i) - r_f)$ is the product of the market risk premium $(E(r_m) - r_f)$ and the β_i (2). Furthermore, the CAPM claims that all investors should hold the same tangency portfolio, i.e., the market portfolio [Polovenko, 2017]. If this holds, then the market capitalization of assets will determine their weight w_i in the CAPM portfolio at the equilibrium w_m [Polovenko, 2017, Walters, 2014]. The CAPM portfolio is optimal in the sense that it has a higher Sharpe ratio than any other market portfolio on the efficient frontier [Polovenko, 2017].

3.1.2 Reverse Optimization

The BL model starts with the process of reverse optimization. This process derives the implied equilibrium expected returns, which serve as a neutral starting point in the process of optmization [Olsson and Trollsten, 2018, Hirani and Wallström, 2014]. These, implied by the market, returns serve as the gravitational point for the vector of weights that will converge to the equilibrium [Olsson and Trollsten, 2018, Sharpe, 1974]. This approach enables investors without any subjective views to invest in the market-capitalization weighted portfolio automatically. Contrary to the mean-variance optimization, the BL model's reverse optimization uses weights of the market portfolio as input, and computes returns that are implied by the market [Polovenko, 2017]. There are two primary purposes for this being a good way to forecast equilibrium implied returns [Polovenko, 2017]. First, weights as input tend to be easier to predict [Polovenko, 2017]. Second, the weights as parameters are easier to be interpreted by practitioners [Polovenko, 2017]. A neutral starting point is calculated using reverse optimization, which finds the single global maximum of the convex quadratic utility function (U) [7].

[7] following example from [Polovenko, 2017, Olsson and Trollsten, 2018, Skeie-Larsen et al., 2018]

$$U = w_m'\Pi - (\frac{\delta}{2})w_m'\Sigma w_m \tag{3}$$

Where,

- U is the investor's utility,
- Π is a $N \times 1$ vector that represents the implied excess equilibrium return for all assets in the market portfolio.

Maximization of the utility function U, without constraints, with respect to the weights, is done by taking the first derivative of (3) with respect to the weights w_m, and setting it to 0 [Polovenko, 2017, Walters, 2014]:

$$\nabla U = \Pi - \delta\Sigma w_m = 0 \tag{4}$$

By rearanging the Equation (4), we obtain a matrix-vector representation of the market-implied excess returns [Polovenko, 2017]:

$$\Pi = \delta\Sigma w_m$$

The covariance matrix Σ can be estimated by using the historical returns of assets [Idzorek, 2005]. The market capitalization weights vector w_m is directly obtained using the market capitalization of the portfolio constituents [Idzorek, 2005]. The risk aversion coefficient in (5) can either be an informed guess taken from the literature, or it can be mathematically derived from (1),(2) or (4).

$$\delta = \frac{E(r_m) - r_f}{\sigma_m^2} \tag{5}$$

Where,

- σ_m^2 is the market portfolio variance defined as $\sigma_m^2 = w_m'\Sigma w_m$.

The idea behind the risk aversion coefficient δ is to describe human investment behavior [Idzorek, 2005]. This parameter scales the estimate of the reverse optimization, i.e., it is a scaling factor for the implied excess returns [Idzorek, 2005].

δ in Equation (5) is the rate at which more return is required for more risk, i.e., a reward that investors require for bearing the market risk [Idzorek, 2005]. By rearranging the formula from (5), we can derive the Equation in (6).

$$\Pi = E(r_m) - r_f = \delta\sigma_m^2 \tag{6}$$

Equation (6) illustrates the close relationship between the BL model's implied equilibrium expected return and the CAPM equilibrium [Olsson and Trollsten, 2018]. Furthermore, it shows that the CAPM market equilibrium can be utlilized to derive the market risk premium [Olsson and Trollsten, 2018].

The risk aversion parameter from Equation (5) can be used in Equation (4). By plugging the Σ, w_m, and δ into Equation (4), the prior equilibrium returns for the assets in the portfolio are obtained.

One of the main BL model attributes is the assumption that the expected returns are not observable fixed values but rather stochastic variables [Idzorek, 2005]. This model assumes that the covariance matrix of the implied returns Σ_Π is proportional to the covariance matrix of the returns $\tau\Sigma$ [Idzorek, 2005, Polovenko, 2017]. The parameter τ is used as the constant of proportionality, such that $\Sigma_\Pi = \tau\Sigma$ [Polovenko, 2017, Walters, 2014]. Considering that the uncertainty in the mean of the returns is considerably smaller than the variability in the return itself, τ is usually given a low value close to zero [Idzorek, 2005, Polovenko, 2017].

Black and Litterman [1992] and He and Litterman [1999] choose values of τ between 0.025 and 0.05, which implies using 40 to 20 return observations for estimating the expected returns respectively. Equation (7) gives the prior distribution of the BL model, which is the estimate of the mean with a proportional variance [Polovenko, 2017].

$$E(r) \sim N(\Pi, \tau\Sigma), r \sim N(E(r), \Sigma) \tag{7}$$

The τ is a scalar allowing investors to specify the level of uncertainty they have in the market-implied returns [Polovenko, 2017]. The lower the τ, the higher the confidence in the market-implied returns. With low values of τ investors express uncertainty in their views and stay invested close to the market portfolio. Conversely, high τ values indicate high confidence in investor opinions, and it tilts the portfolio toward investor forecasts.

3.1.3 Views

Typically, investors have specific opinions regarding the expected returns of the assets in their portfolio, which they would like to express in the process of portfolio formation [Idzorek, 2005, Polovenko, 2017]. These opinions represent statements expressed in respect to the vector of expected returns [Polovenko, 2017]. Adding a view to the BL model creates a positive or negative portfolio tilt in the direction of the asset for which the prediction is provided [He and Litterman, 1999]. If a view is more bullish then the expected return implied by the market, the tilt is positive [He and Litterman, 1999]. Conversely, if a prediction is more bearish than the implied return, a negative tilt is created.

The Black-Litterman model allows investors to express as many views as they want, or none at all. In the case of no views, the investor stays invested in the market portfolio.

To merge the implied equilibrium returns with k subjective views, investors need to specify [8]:

1. Q a $k \times 1$ vector expressing the expected returns on assets estimated by investors.
2. P a $k \times n$ matrix expressing k views on n assets in terms of asset weights. It further specifies if views are absolute or relative.
3. Ω a $k \times k$ diagonal covariance matrix of the views. This matrix is diagonal because the BL model assumes views to be independent and uncorrelated. ω_i is the diagonal element of the omega matrix.

Black and Litterman [1992] defined the investor's views as follows[9]:

$$PE(R) = Q + \epsilon \tag{8}$$

Where,

- $E(R)$ is the posterior return vector that is unknown and to be estimated,
- ϵ is a $k \times 1$ error vector with a mean of 0, and a variance of $\Omega(\epsilon \sim N(0, \Omega))$.

Uncertainty in views is represented by the error term ϵ. Without the error term, the investor would be 100% confident in his subjective forecast. The error terms do not enter the BL formula directly. Ω contains the variances of the error terms that are connected to the views [Idzorek, 2005]. The algebraic representation of P and Q is as follows:

$$P = \begin{bmatrix} P_{1,1} & . & . & . & P_{1,k} \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ P_{n,1} & . & . & . & P_{n,k} \end{bmatrix} \qquad Q = \begin{bmatrix} q_1 \\ . \\ . \\ . \\ q_n \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ . \\ . \\ . \\ \epsilon_n \end{bmatrix}$$

The BL model views can be absolute or relative with row sums in the P matrix equal to 1 and 0, respectively. The Q vector contains the expected

[8] following example from [Polovenko, 2017]

[9] [Idzorek, 2005, Polovenko, 2017]

returns that investors estimated for a certain asset to yield (absolute view), or the expected over or underperformance of one asset relative to another (relative view) [Olsson and Trollsten, 2018].

One possible absolute view could be "The European energy sector equities will return 10%". A relative view example is "The European energy sector equities will overperform the European utility sector by 5%".

The variance of the view portfolio Ω specifies the certainty of the investor's views [Idzorek, 2005]. Many different studies estimate omega differently. In this work, the views are generated from past pricing information, and thus, the certainty of the predictions will also be generated from the past as follows:

$$\begin{gathered} \Omega = P\Sigma P', \text{where} \\ \omega_{i,j} = P\Sigma P', \forall i = j \\ \omega_{i,j} = 0, \forall i \neq j \end{gathered} \tag{9}$$

Having defined the expected returns and the variance for the views, we can define the conditional distribution as:

$$PE(R)|E(R) \sim N(Q, \Omega)$$

3.1.4 The Posterior Returns

The Black-Litterman master formula is an approach of combining the CAPM prior with the investor views using the Bayesian framework. This merging of the prior distribution with the view distribution results in BL returns, i.e., posterior distribution expected returns [Olsson and Trollsten, 2018]. The posterior returns are distributed as [10]:

$$R \sim N(E(R), M) \tag{10}$$

With the mean of the distribution $E(R)$ given by:

$$E(R) = \Pi + \tau\Sigma P'(P\tau\Sigma P' + \Omega)^{-1}(Q - P\Pi) \tag{11}$$

And the covariance matrix of posterior expected returns M given by:

$$M = ((\tau\Sigma)^{-1} + P'\Omega^{-1}P)^{-1} \tag{12}$$

In the case that the investor has no confidence in his views, or has no views at all, he will end up holding the market portfolio [Polovenko, 2017]. In this case, the expected returns of the BL model become Π [He and Litterman, 1999, Polovenko, 2017].

[10] following example from [Polovenko, 2017, Skeie-Larsen et al., 2018, Olsson and Trollsten, 2018]

$$E(R) = \Pi$$

On the contrary, if an investor is 100% certain in his predictions, i.e. by setting Ω to equal 0 in Equation (11), the posterior returns become:

$$E(R) = \Pi + \tau\Sigma P'(P\tau\Sigma P')^{-1}(Q - P\Pi)$$

Additionally, if an investor provides a view for every asset in the portfolio with a 100% forecast confidence, then:

$$E(R) = P^{-1}Q$$

The uncertainty regarding the prior distribution, denoted by the scalar τ, regulates the convergence of means either to the prior distribution or to investors' views [Idzorek, 2005]. With the greater confidence level in one's own expressed views, the closer the new posterior return vector will converge to the views [Idzorek, 2005]. Conversely, with lower confidence in expressed views, the new posterior return will be closer to the implied equilibrium return Π [Idzorek, 2005].

To calculate the posterior weights, one needs to take the additional source of uncertainty into account. By using only Σ derived from the historical data, one assumes that the posterior expected returns are constants. Since the posterior return itself is a random variable, an investor needs to take the uncertainty of the future holding period into the calculation as well. He and Litterman [1999] have shown how to appropriately estimate the distribution of the posterior covariance matrix $\Sigma*$ [Walters, 2014]:

$$\Sigma* = \Sigma + M \tag{13}$$

By substituting the M from (12), we get the following posterior covariance matrix:

$$\Sigma* = \Sigma + ((\tau\Sigma)^{-1} + P'\Omega^{-1}P)^{-1}$$

With the new covariance matrix, one can calculate the new posterior optimal portfolio weights, using Equation (4) with the posterior covariance $\Sigma*$ and solving for w_m instead of Π this time [Polovenko, 2017, Xu et al., 2008].

Figure 3.1.4 summarizes all steps performed for deriving the new combined return vector.

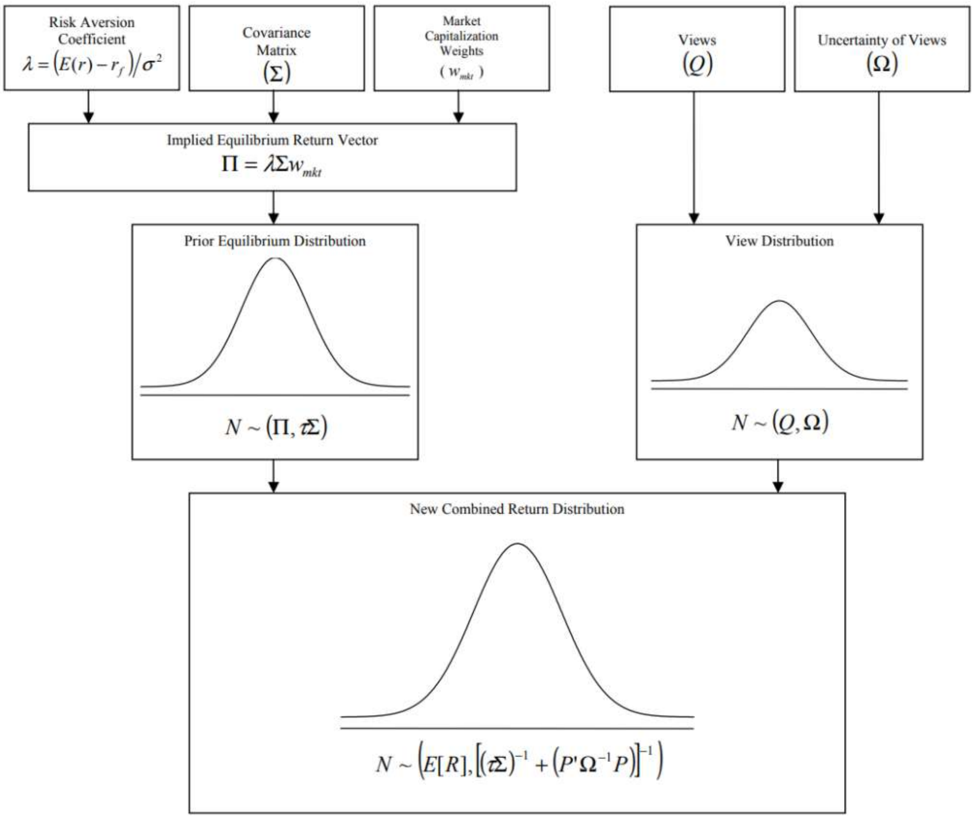

Figure 2: *Graphical Summary of the Black-Litterman Master Formula [Idzorek, 2005])*

3.2 Factor Views

3.2.1 Momentum Factor

One of the most popular ways to successfully predict returns of assets, identified by practitioners and researchers, is the Momentum factor [Jegadeesh and Titman, 1993]. The implementation of this factor is fairly simple. It is usually based on choosing assets according to their performance P over the previous J months and holding these for a certain period K in the future. The formula for computing the momentum (MOM) of an asset at time point t, without excluding the most recent period return, is defined as follows:

$$MOM_t = \frac{P_t - P_{t-J}}{P_{t-J}} \tag{14}$$

Although Jegadeesh and Titman [1993], Clifford [1994], Fama and French [1996] and Grinblatt and Moskowitz [2004] argue that excluding the most recent period from the calculation ($\frac{P_{t-1}-P_{t-J}}{P_{t-J}}$) due to short-term mean reversal issues is advantageous , I have decided to use Equation (14) as a proxy for momentum calculation in this master thesis [Lilloe-Olsen, 2016]. This decision, and the choice of momentum parameter settings, is elaborated in more details in Section 4.3.

Let us assume for a moment; we have only ten assets in the investable universe. An investor would like to go long (buy with the expectation of the price going up, and sell at a higher price) with 2 best-performing asset according to the momentum, and short 2 (borrow and sell, with the intent to repurchase the stock at a lower price) worst performing one based on the same criteria.

At the beginning of a time period t, the assets are ranked in ascending order based on their returns in the last J months [Jegadeesh and Titman, 1993]. The "winners" momentum portfolio is constructed from top N, equally weighted, assets based on this ranking. The bottom N assets would form a "losers" portfolio. Jagadeesh and Titman's [1993] classic reference on momentum has shown that by constructing long-short portfolios of winners and losers for different choices of J and K provide significant abnormal returns.

3.2.2 Value Factor

A concept that cheap stocks, relative to some measure of fundamental value, outperform pricier stocks over a long horizon is a foundation of value investing [11] [12], which dates back to Graham et al. [1934] [13] [Lilloe-Olsen, 2016]. Fama and French [1998] have shown that the difference in returns of global portfolios between high and low book-to-market stocks has been 7.68% per year.

Investors and academics differ on the best representation for the value of a company. High book-to-market equity B/M, earnings to price E/P, or cash flow to price C/P are the usual classifications for "value" firms [Fama and French, 1998]. Strong value premium in average returns for the U.S. equity has been shown by Fama and French [1992, 1996] and by Lakonishok et al. [1994].

A company's book value of equity, or shareholder's equity, represents the total equity of the business available for distribution to common shareholders [Dangl, 2018]. More accurately, a company's book value of equity B is derived from its last balance sheet by subtracting the total liabilities and preferred stocks from the firm's total assets. Thus, the book value is a backward-looking measure of the company's equity value. The market value M of the equity of a publicly-traded company is calculated through the multiplication of the current share price by the number of outstanding shares. This measure is determined by the current supply and demand of a company's stock on the stock market and can be regarded as a forward-looking.

The book-to-market ratio of sector s at time t is calculated by dividing the sector's book value $B_{s,t}$ by its market value $M_{s,t}$.

$$book - to - market_{s,t} = \frac{B_{s,t}}{M_{s,t}} \tag{15}$$

Where,

- $B_{s,t}$ is a sector's book value of equity according to the last balance sheet,
- $M_{s,t}$ is the current market value of the sector, according to the stock market.

This ratio provides insight into the company's or sector's market value relative to its actual capital. In practice, undervaluation or overvaluation of a stock in terms of the book-to-market ratio is determined by comparison to direct competitors from the same or similar industry and sector.

[11] https://www.risk.net/definition/value-factor
[12] https://www.msci.com/documents/1296102/1339060/Factor+Factsheets+Value.pdf
[13] https://en.wikipedia.org/wiki/Security_Analysis_(book)

3.2.3 Generating Subjective Views using Factors

The BL framework blends the subjective views of investors with the prior information from the market equilibrium. In this research, the investor's views are derived from the Momentum and Value factors. To arrive at the relative views, which are applicable to the BL model, the factor information must be converted to equity returns. This will be done by tracking the average historical returns of the top and bottom portfolios according to a factor.

For every rebalancing point in investable time t, the top portfolio and the bottom portfolio contain the two best and the two worst performing sectors, according to a factor. For example, assuming we use the Value factor at the time point t, the two sectors with the highest book-to-market ratio, each weighted with 50%, will represent the top portfolio. Conversely, at the same time, the two sectors with the lowest book-to-market ratio are contained in the bottom value portfolio. The top and bottom portfolio average returns from the beginning of the data sample are then subtracted at time $t+1$. This difference between the average returns of the top and the average returns of the bottom portfolio is treated as a relative view of future performance in the BL model as over- or underperformance of the sectors.

For example, if the top portfolio historically, i.e. from the start of the data sample, returned 1.5% on average, and the bottom portfolio yielded an average of 0.5% in the same time window, then the BL relative view Q (see Section 3.1.3) would express 1% in relative performance between the top and bottom portfolios. The BL model tilts the portfolio towards the sectors contained in the top portfolio, i.e., it puts more weight to the top portfolio, and less weight to the bottom portfolio. For every time point t, the average return of the top and bottom portfolio is recalculated using the returns from the beginning of the data sample up to the current time t.

This section demonstrates the approach for building the two-factor view, i.e., the combination of Momentum and Value factors in one view. The one-factor view strategy uses the Q vector and the P matrix with one dimension less, i.e. 1×1 and 1×10 respectively.

In this research, the portfolio rebalancing is performed on a four-week basis. The average historical returns of the factor portfolios are measured on a four-week basis as well. For every rebalancing point, two new relative views are created. The Q 1×2 vector will contain the future expected over-performance, which could also be negative sometimes, of the top portfolio vs. the bottom portfolio for the next four weeks. Thus, the Q vector can be mathematically expressed as follows:

$$Q = \begin{pmatrix} mean(R_{p,MT}) - mean(R_{p,MB}) \\ mean(R_{p,VT}) - mean(R_{p,VB}) \end{pmatrix}$$

Where,

- $R_{p,MT}$ are historical returns of the momentum top portfolio,
- $R_{p,MB}$ are historical returns of the momentum bottom portfolio,
- $R_{p,VT}$ are historical returns of the value top portfolio,
- $R_{p,VB}$ are historical returns of the value bottom portfolio.

$R_{p,MT}$, $R_{p,MB}$, $R_{p,VT}$ and $R_{p,VB}$ represent the returns of factor portfolios within the market portfolio returns r_m. Historical returns of a portfolio in this context refer to the past returns from the beginning of the data sample up to the current time point t.

The average returns of the top and the bottom portfolio difference ($mean(R_{p,T}) - mean(R_{p,B})$) represents the systemic forecast, or the expected return, for the performance of these market sub-portfolios for the next four weeks, i.e. up to the next rebalancing time point.

The P matrix defines the assets in the market portfolio on which the views from the Q vector are applied (see Section 3.1.3). As defined in the BL model (see Section 3.1.3), the relative views are applied where the row sum of the weights in a P matrix are zero [Black and Litterman, 1992]. The P is a 2×10 matrix, because there are two views and ten assets in the portfolio. The relative forecasts of the Momentum and Value factors are assigned to stocks by setting 1/2 to the assets included in the top, and −1/2 to the assets included in the bottom portfolio.

Let us assume for a moment that assets one and seven are in the top and assets three and six are in the bottom momentum portfolio. Ones are assigned to the first asset in the *momentum* top portfolio MT_1, and to the second asset in the momentum top portfolio MT_2. −1/2 is assigned to both assets in the momentum bottom portfolio, i.e., to MB_1 and to MB_2. The same procedure is applied to assets in both sub-portfolios (VT_n and VB_n, where n equals 1 or 2) of the Value factor. A possible mathematical representation of a P matrix example is presented below:

$$P = \begin{bmatrix} MT_1 & 0 & MB_2 & 0 & 0 & MB_1 & MT_2 & 0 & 0 & 0 \\ 0 & VB_1 & VB_2 & 0 & 0 & 0 & 0 & 0 & VT_1 & VT_2 \end{bmatrix}$$

Consequently, for the example described above, the P matrix has the following layout:

$$P = \begin{bmatrix} 1/2 & 0 & -1/2 & 0 & 0 & -1/2 & 1/2 & 0 & 0 & 0 \\ 0 & -1/2 & -1/2 & 0 & 0 & 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix}$$

The first row of the P matrix above expresses a relative view which states that the portfolio constituents at positions one and seven will outperform the assets at positions three and six by the amount defined at position one in the vector Q. The same logic is applied to assets of the value sub-portfolios in the second row of matrix P above (VT_1,VT_2,VB_1,VB_2). A zero value in the P matrix indicates an asset not included in any of the sub-portfolios. The posterior weight for such an asset will be computed by the BL model without any view.

The variance of the factor portfolios is represented by $P\Sigma P'$ (see Equation (9)). The computation of Ω is following methods applied by He and Litterman [1999]. This view confidence is proportional to the variance of the factor portfolio returns. This allows the BL model to put more weight on assets in regimes when the volatility is low, and to deviate more from the market equilibrium. On the contrary, in times when the uncertainty in stock returns is higher, the framework puts less weight on views and stays closer to the market equilibrium weights.

Given the views for the factor sub-portfolios, the new posterior expected returns in Equation (11) are calculated. With the new updated expected returns including the forecasts of the factor portfolios, the new optimal weights, i.e., the posterior weights, are computed for assets in the portfolio.

Period for period, the new updated weights at rebalancing time t are used to calculate the returns of the optimized portfolio at $t+1$ up to the next rebalancing time point at $t+4$. This enables strict out-of-sample simulation of historical performance of the new BL model's optimized portfolio which is used for evaluation of characteristics and benchmarking against the market portfolio.

4 Methodology and Data

This section presents data, methods and procedures performed in order to obtain the empirical findings of this research. Additionally, a detailed overview of the parameters used for the BL model and the setup configuration for the historical simulations is given. This chapter is divided into eight parts, where every section is described in a systematic and detailed manner.

- Data and Indices Reconstruction

  Describing the data set, methods for computing financial indices and the reconstruction process of the market portfolio benchmark.

- Properties of Factor Forecasts (Q Vector)

  Describes the statistical sample properties of factor views.

- Momentum and B/M Ratio of Sector Portfolios

  Discusses the methods of momentum horizon selection and the portfolio formation timing. The descriptive statistics of sectors' momentum and book-to-market ratio is presented.

- The Black-Litterman Model Parameter Setup

  Argues the methods and choices of the parameter setup used in the Black-Litterman model.

- The Investment Process

  A detailed overview of the method applied for constructing the new portfolios through the investment period.

- Factor Portfolios (P Matrix)

  Presenting the sector allocation of the factor portfolios throughout the investment period.

- The Author's Expectations

  Summarizing the performance of the factor portfolios.

- The Evaluation Characteristics

  Description of the evaluation methods for the empirical findings of this thesis.

4.1 Data and Indices Reconstruction

My research design applies the BL portfolio optimization model to sector portfolios defined by the STOXX Europe 600 Index universe (SXXP). As of February, 2019, this index represented large, mid and small capitalization companies across 17 countries of the European region: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Norway, Poland, Portugal, Spain, Sweden, Switzerland and the United Kingdom [14]. The number of components in this index is fixed to 600. The assets in the STOXX 600 Index are grouped by similar business spheres into ten different sectors listed in Table 1 below.

Table 1: *STOXX Europe 600 - Sector Indices*

Sector Name	Symbol
Oil & Gas	SXEP
Basic Materials	SXBSCP
Industrials	SXIDUGR
Consumer Goods	S3000P
Health Care	SXDP
Consumer Services	S5000R
Telecommunications	SXKP
Utilities	SX6P
Financials	SXFINP
Technology	SX8P

The data set in this research consists of weekly returns (including dividends) in a 30-year period between the 3. September, 1989 and 8. February, 2019, totaling to 1571 weekly observations. The source of secondary data is the Thomson Reuters Data stream. The total data collection consists of six data frames (matrices), each containing a part of the information necessary to compute the indices, benchmark and factors. The content of the six data matrices is summarized below.

- *A* - The three month EURIBOR risk-free rate of return in percentages for all 1571 weeks.
- *B* - The corporate information data, including the companys' stock ticker, sector and industry identification, etc.
- *C* - The weekly returns data frame for all 1.544 stocks in the European equity universe.
- *D* - The weekly market capitalization for all 1.544 securities.

[14] https://qontigo.com/, https://stoxx.com/

- E - The weekly book-to-market equity ratio for all 1.544 securities.
- F - The Boolean matrix indicating whether an asset was in the STOXX 600 Index in a given week.

Matrices C, D and E are 1571×1544, i.e., each of them has 1.571 rows (weeks) for 1.544 columns (stocks).

The STOXX Europe 600 Index was first introduced in 1998 and the data for index reconstruction in Matrix F is available starting September, 3, 1999. However, the first 260 weeks of data, or five years, will be used to estimate the input parameters for the BL model. The backtesting was conducted from September, 2004 to February, 2019. The total 752 weekly samples in the period of over 13 years are sufficient for this kind of research design.

As already mentioned, the STOXX 600 Index has a fixed number of components. However, the constituents of this index may change on a daily basis. The data in C contains weekly returns for all 1.544 securities and in order to reconstruct the index, we need to consider only the companies that were included in this aggregate index, i.e., STOXX Europe 600 Index, in a given week. Filtering the relevant stocks is accomplished by applying a Boolean filter F, to asset returns C, market capitalization D and book-to-market ratio E matrices.

The total market capitalization of the STOXX 600 Index at the time t is the sum of its individual component market capitalizations at the same time point. The market capitalization of a sector at time t ($mv_{S,t}$) is calculated by summing up the individual inter-sector market valuations of securities at time t. The sector-weighted market capitalization of a stock at time t ($wmv_{i,t}$), or sector weighted value of a stock, is the relative market capitalization of a stock within the sector $\frac{mv_{i,t}}{mv_{S,t}}$. The sum of sector-weighted market valuations of stocks that belong to the same sector equals one. To obtain the sector-weighted return of a stock at time t ($wr_{i,t}$), the market value of a stock from the previous time point ($wmv_{i,t-1}$) is multiplied with that stock's return at time t ($r_{i,t}$). The return of a sector at time t ($r_{S,t}$) is the sum of the sector-weighted stock returns for the same point in time. The performance of a sector through time $cumulativeR_S$ is obtained by the cumulative multiplication of weekly sector performances for the whole investment period.

This process of sector indices reconstruction can be defined as follows:

$$mv_{S,t} = \sum mv_{i,t}, i \in S$$
$$wmv_{i,t} = \frac{mv_{i,t}}{mv_{S,t}}$$
$$wr_{i,t} = wmv_{i,t-1} r_{i,t}$$
$$r_{S,t} = \sum_{i=1}^{n} wr_{i,t}$$
$$cumulativeR_S = \prod_{u=1}^{t} (1 + r_{S,u})$$

Where,
$mv_{S,t}$ is a market value of a sector S at time t,
$mv_{i,t}$ is a market value of a stock i at time t,
$wmv_{i,t}$ is a sector-weighted market value of stock i at time t,
$wr_{i,t}$ is a sector-weighted return of a stock i at time t,
$r_{S,t}$ is a return of sector S at time t and
$cumulativeR$ is a cumulative return of a sector S from the beginning of the investment period ($t = 1$) to the last week of investment period m.

This method of sector index reconstruction is performed for each sector separately.

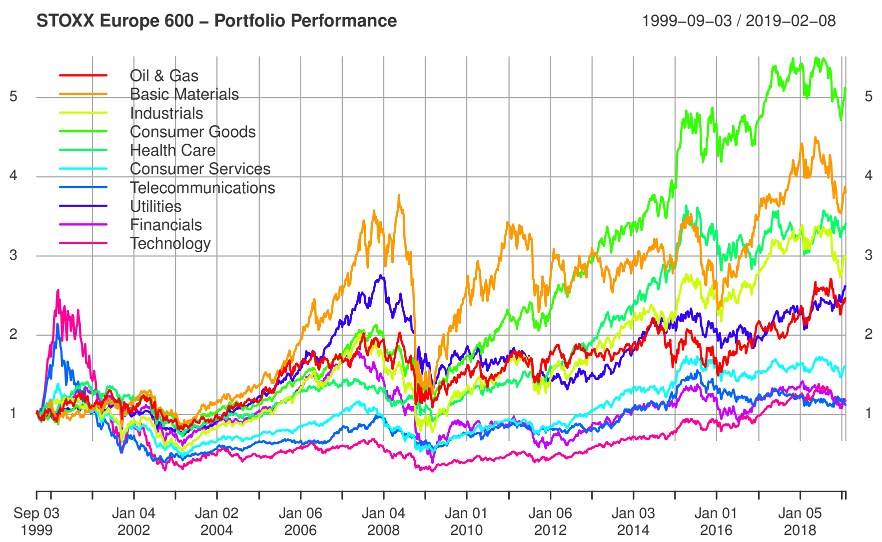

Figure 3: *STOXX Europe 600 - Sector Performance from 1999 to 2019*

The performance summary of all ten STOXX Europe 600 portfolio sectors from 1999 to 2020 is illustrated in the Figure 3.

The distribution of returns per sector from 1999 to 2019 is shown in the Figure 4.

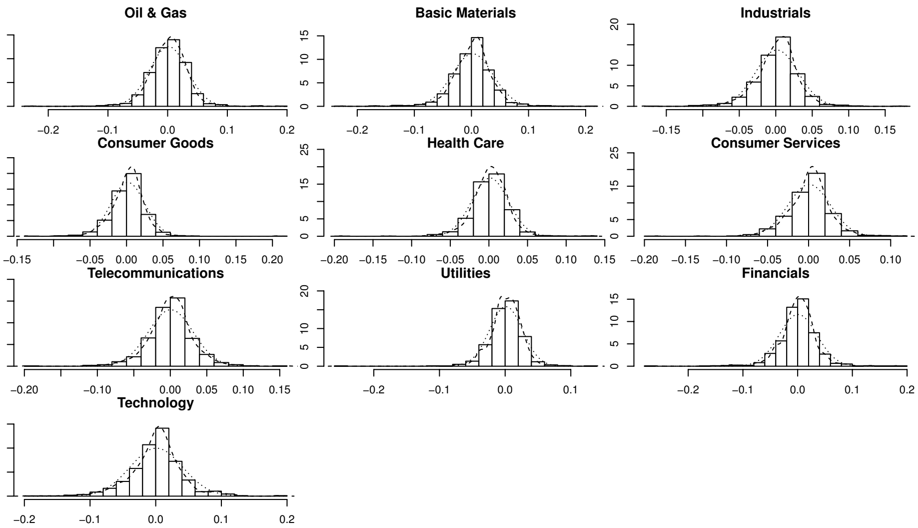

Figure 4: *STOXX Europe 600 - Distribution of Sector Returns from 1999 to 2019*

4.1.1 Reconstructing the Benchmark

An investor without subjective views about the future performance of assets in a portfolio is not motivated to deviate from the market equilibrium [Black and Litterman, 1992, Olsson and Trollsten, 2018].

The reconstruction process for the market benchmark, i.e., the performance of the market equilibrium portfolio, is similar to the method of sector-index reconstruction described in the previous section.

The only difference is that the weighting of stock returns is not in relation to the sectors, but to the market as a whole. The market value of each stock is weighted relative to the total market valuation of all assets combined, i.e., the stock market value at time t is divided by the total market value of all stocks for the same point in time.

Next, the market weighted value of stocks at time $t-1$ is multiplied by the returns at t of the same assets, which results in the market weighted returns of assets. The cumulative product, across the whole investment period, of the sum of all the market weighted returns in the STOXX 600 asset universe is the benchmark used to evaluate the results in this research.

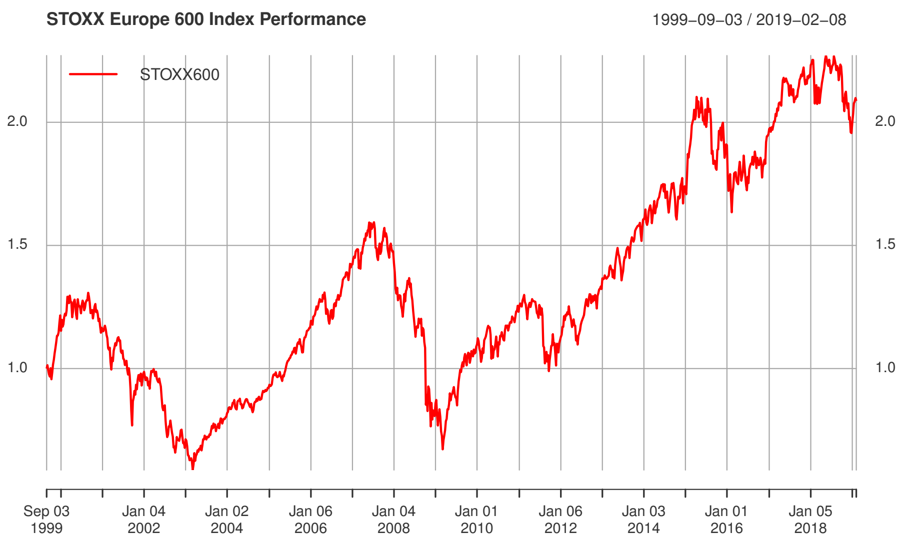

Figure 5: *STOXX Europe 600 - Index (the Benchmark) Performance from 1999 to 2020*

The market portfolio is fixed in the number of components it holds. In this case, the STOXX 600 Index consists of 600 stocks and 10 sectors. However, this index is dynamic in terms of constituent weights, which are not constant and change over time.

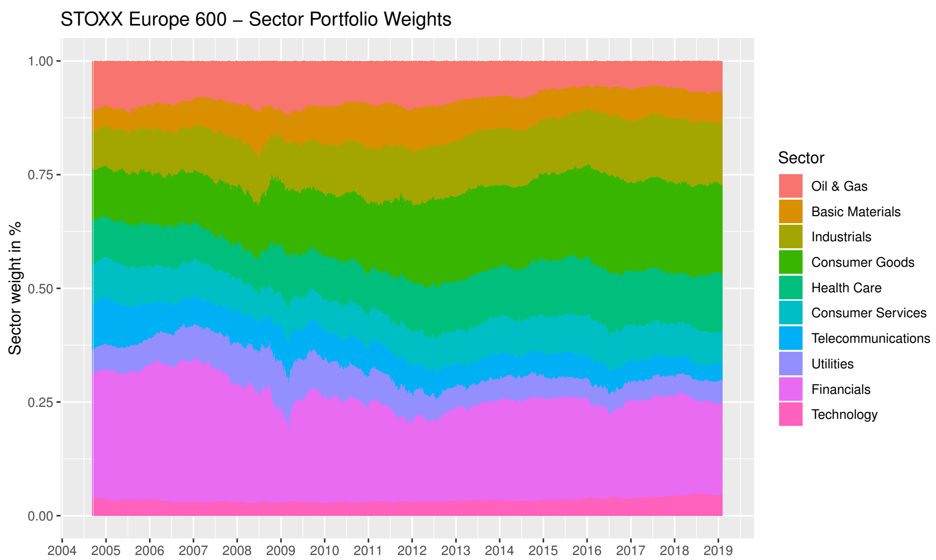

Figure 6: *STOXX Europe 600 - Sector Market Weights from 2004 to 2019*

Figure 6 illustrates the sector market weights in the STOXX 600 portfolio from January 1., 2004 to January 1., 2019. These (see Figure 6) would be the weights of sectors as assets over time if you were just a passive investor only holding the market portfolio. The sector market weights are derived from the market capitalization of single stocks, relative to the market value of the equilibrium portfolio. The BL model uses these weights as one of the inputs in order to compute the market-implied returns of the same.

The statistical properties of STOXX Europe 600 sector weights are illustrated in Figure 7 below:

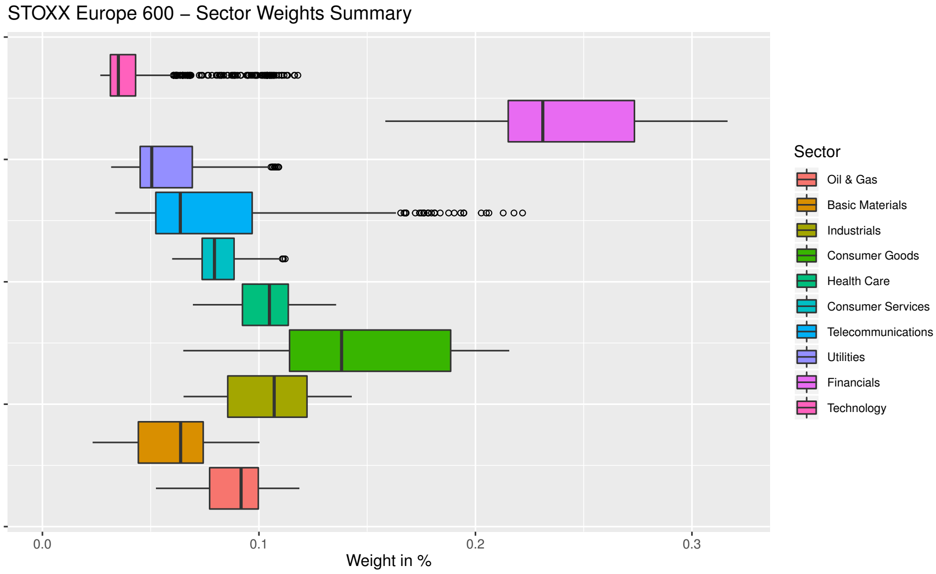

Figure 7: *STOXX Europe 600 - Sector Market Weights Summary from 2004 to 2019*

4.1.2 The Covariance Matrix

Litterman and Winklemann [1998] show how the choice of a covariance matrix is essential for forecasting the returns and emphasize its importance in the portfolio management industry. In this work, the covariance matrix is estimated based on the previous 260 weekly returns.

The first covariance matrix estimated at t_{260} is derived from weekly returns in a period from t_0 to t_{259}. The second covariance matrix at t_{261} is calculated from previous 260 weekly returns from t_1 to t_{260}, and so on. This rolling window has no time decaying weighting, meaning that the first weekly-return in the window has the same weight as the last one.

The BL model uses the historical correlations of asset returns, among other parameters, to estimate the market implied returns. The correlation coefficients between the sector returns are exhibited in Table 2 above the diagonal elements. The lower intersector correlation coefficients indicate the diversification potential that may provide gains through reduction of non-systematic risk in the portfolio. Table 2 shows that the correlation for the most of sector combinations are far from one in the period from 1. January, 2000 to 1. January, 2019.

Table 2: *STOXX Europe 600 - Sector Correlation from 2000-01-01 to 2019-01-01*

	Oil & Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecommunications	Utilities	Financials	Technology
Oil & Gas	1.00	0.76	0.70	0.67	0.60	0.63	0.43	0.69	0.67	0.47
Basic Materials		1.00	0.86	0.73	0.55	0.71	0.46	0.65	0.76	0.58
Industrials			1.00	0.79	0.64	0.88	0.62	0.67	0.86	0.77
Consumer Goods				1.00	0.73	0.77	0.51	0.70	0.74	0.56
Health Care					1.00	0.67	0.54	0.64	0.63	0.53
Consumer Services						1.00	0.71	0.65	0.81	0.80
Telecommunications							1.00	0.53	0.58	0.70
Utilities								1.00	0.69	0.47
Financials									1.00	0.65
Technology										1.00

Figure 8 displays the annualized variances of STOXX 600 sector weekly returns for period from 1. January, 2000 to 1. January, 2019.

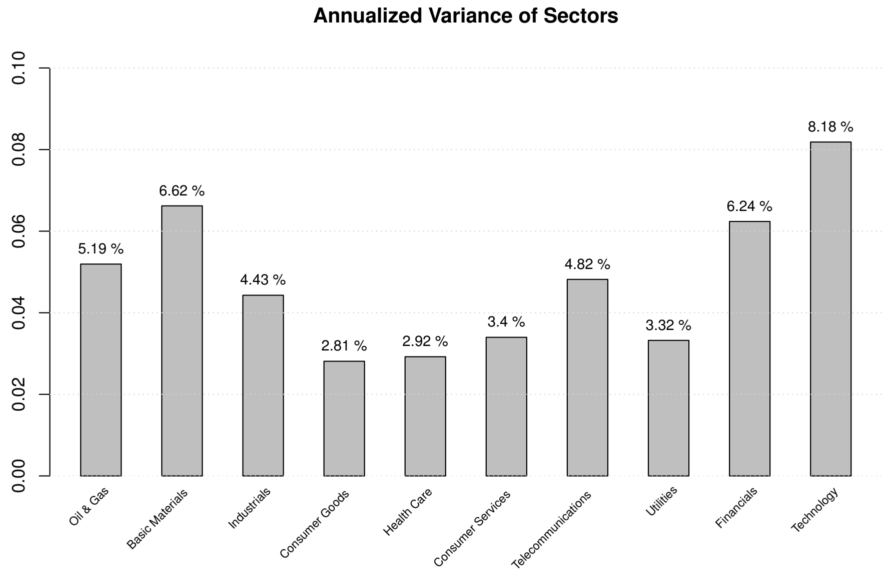

Figure 8: *STOXX Europe 600 - Sector Annualized Variance from 2000-01-01 to 2019-01-01*

4.1.3 The Risk-free Rate

The three-month EURIBOR weekly return is used as the risk-free proxy for the BL parameter estimation, as well as a risk-free rate used throughout this research. Figure 9 illustrates the development of the risk-free returns through time.

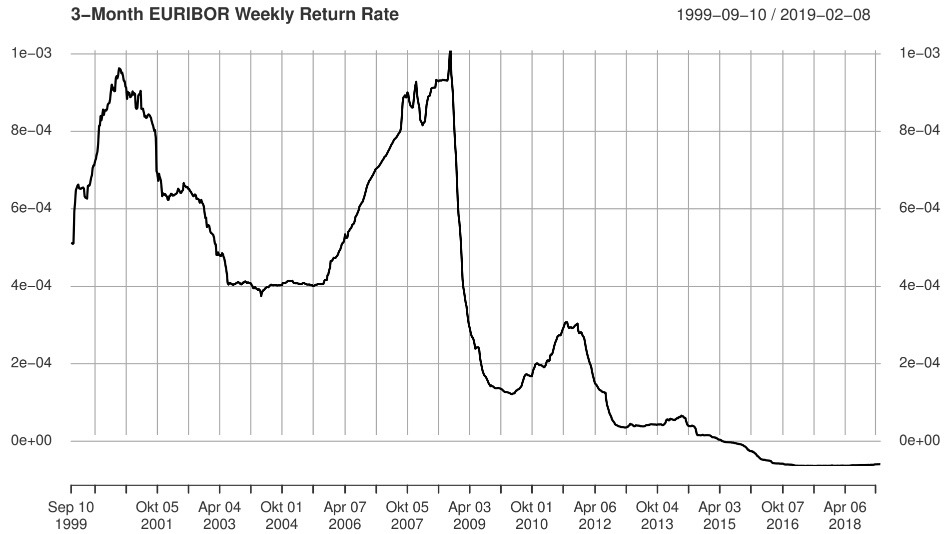

Figure 9: *3-Month EURIBOR Weekly Return-Rate from 1999 to 2019*

4.1.4 Additional Data

More interested readers and researchers can find the additional data about STOXX Europe 600 sector indices in terms of B/M ratios (Table A.1), 12-week momentum (Table A.2) and sector portfolio weights (Table A.3) in Appendix A.

4.2 Properties of Factor Forecasts

In this section, I want to give an overview of the statistical sample properties of factor views used in the BL model as subjective investor forecasts. Section 3.2.3 in the theory part of this thesis discusses the details of forecasts computation.

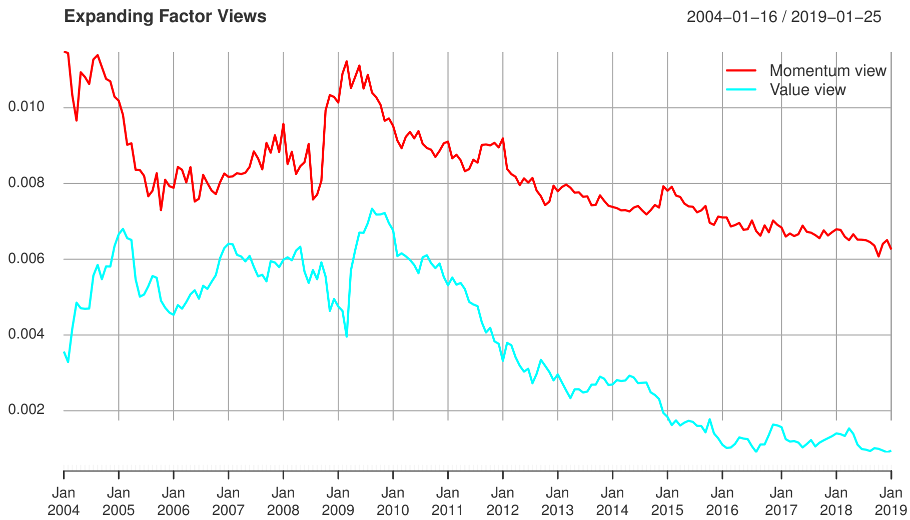

Figure 10: *Comparison of Expanding Factor Views from 2004 to 2019*

Figure 10 shows that the expanding momentum view values are significantly higher than those of the Value factor. This indicates that the difference in returns between the top and bottom portfolios of the two factors has been higher within the Momentum sub-portfolios.

The values of the value view are in a strong decline since mid 2009, and have not shown signs of a recovery since. In 2019, the Value view is almost 0%. One could interpret this as a disappearance of the Value factor.

From the Figure 10 can also be seen that the momentum view has also been steadily declining in the period between mid 2009 to 2019 from over 1% to about 0.6% in 4-week interval. Nevertheless, the momentum view stayed significantly stronger then the value forecast in the same period.

Table 3 shows statistical sample properties of the factor views.

Table 3: *Statistical Properties of Factor Views*

	Momentum view	Value view
Min.	0.61%	0.09%
1st Qu.	0.73%	0.17%
Median	0.80%	0.41%
Mean	0.82%	0.38%
3rd Qu.	0.90%	0.56%
Max.	1.15%	0.73%

At each rebalancing time point, the average forecasted overperformance, for the period of the next four weeks, of the top momentum portfolio relative to the bottom one of the same factor is 0.82% (see Table 3). The average 4-weekly forecast throughout the investment period for the Value top portfolio is 0.38%. This indicates that the undervalued companies, or cheaper ones in terms of the B/M ratio, have on average yielded higher returns in this time period relative to the ones in bottom value portfolio.

The first question that pops up when having forecasts is: How well do they fit the actual or realized returns. This question will be tackled at the end of this chapter in Section 4.7.

4.3 Factors of the Sector Portfolios

As suggested by Jegadeesh and Titman [1993], the typical momentum horizon is three to twelve months, with the same holding period of three to twelve months respectively. However, Jagadesh and Titman's [1993] paper has investigated different equity universes, namely the equity returns of the New York Stock Exchange (NYSE), and of the American Exchange (AMEX) in the period from 1965 to 1989. The empirical data in this thesis is based on weekly returns data, which shall not be compounded to returns over longer periods. Thus, the momentum horizon will be expressed on a weekly basis.

By performing a few backtests with the BL framework using only the momentum factor, i.e., by constructing top and bottom momentum portfolios based on four-week returns using the described BL model parameters from Section 4.4, and by comparing the total returns from this simple strategy against the benchmark, I discovered that the best momentum horizon that yielded best performance for this asset class using the four-week holding period is 12 weeks. All further simulations and results will assume the moment horizon of 12 weeks. Figure 11 illustrates the 12-week momentum of STOXX 600 sector portfolios.

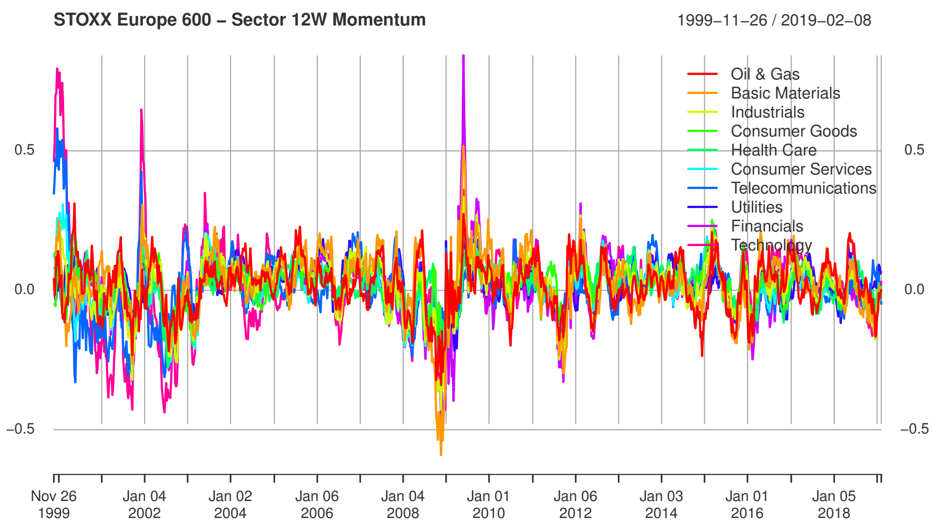

Figure 11: *STOXX Europe 600 - 12-Week Momentum of Sector Portfolios*

Contrary to the work of Jegadeesh and Titman [1993], whose best performing momentum horizon of 12 months included a one month period lag between the portfolio formation and the holding period, I found out that this lag period had no significant impact on the factor sub-portfolio returns. Therefore, I decided to leave out this lag period in the portfolio formation process in order to keep this work's complexity as low as possible.

The calculation of sector B/M ratios is presented in the theory part of this research, in Section 3.2.2. Figure 12 shows the development of sector portfolios' book-to-market ratios in the period between 1999 and 2019.

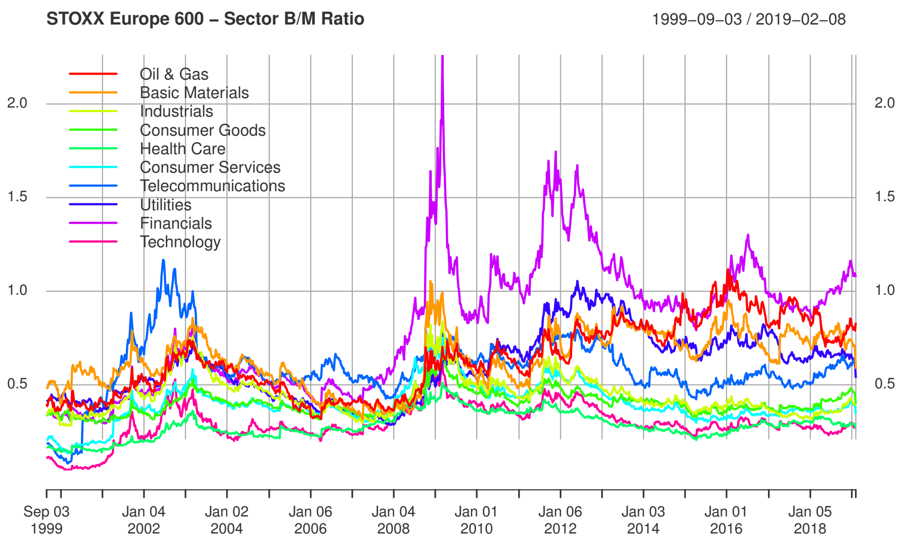

Figure 12: *STOXX Europe 600 - Book-to-Market Ratio of Sector Portfolios from 1999 to 2019*

The descriptive statistics of the STOXX600 sector portfolio factors can be found in Tables A.1 and A.2 in Appendix A.

4.4 The Black Litterman Model Setup

4.4.1 The Choice of Tau (τ)

The scalar τ regulates the convergence of means either to the prior or to the posterior distribution [Idzorek, 2005]. It has an inverse relationship to the weights of the implied equilibrium returns [Idzorek, 2005]. A higher tau implies a higher divergence from the market equilibrium portfolio. Black and Litterman [1992], Lee [2003] and Meucci [2010] argue that, for most applications, the value of this scalar should be close to zero [Schepel, 2019, Skeie-Larsen et al., 2018]. He and Litterman [1999] describe the value of τ of 0.05 as an estimate of the CAPM equilibrium returns using 20 years of annual return data [Polovenko, 2017, Idzorek, 2005].

For consistency with earlier applications, in this work I choose τ equal to 0.05 as an estimate of the CAPM equilibrium returns using 20 weeks of weekly return data.

4.4.2 Risk Aversion (δ)

The risk aversion coefficient determines what proportion of the capital is invested in the risky vs. the risk-free assets. He and Litterman [1999] use 2.5 as a default level for the parameter δ [Olsson and Trollsten, 2018]. For the European indices, Drobetz [2001] uses the risk aversion of 3 [Skeie-Larsen et al., 2018]. Following the research of Drobetz [2001], I will use a level of the risk aversion equal to 3.

4.5 The Investment Process

At the first rebalancing time point t_0, the market capitalization weights vector w_m, the Σ and the δ are computed. Using these estimates (w_m, Σ, and δ), the implied equilibrium excess return vector Π is calculated (see Sections 3.1.2 and 4.1.2).

Next, the average four-week returns of the factor portfolios are calculated from the beginning of the data-set t_{0-260} up to the current rebalancing time. The view information is formed by calculating the vector Q and matrices P and Ω (see Sections 3.1.3 and 3.2.3).

The master formula of the Black-Litterman model (see Section 3.1.4) computes the vector of posterior expected returns $E(R)$ and the posterior covariance matrices (M and $\Sigma*$) using the information from priors (Π and Σ) and the additional information in (Q, P and Ω).

The optimization process calculates the new optimal weights vector using the posterior covariance matrix $\Sigma*$ and the market-implied return vector Π at t_0. This output vector of the optimizer is a set of the new weights of assets, which represent the new portfolio at the current prices in t_0. This portfolio constituent weighting scheme will be held from t_{0+1} until the next rebalancing period in t_{0+4}.

The optimization of weights is performed in steps of four, i.e., the portfolio is rebalanced every fourth week. The covariance matrix of weekly returns Σ is recalculated in a four-week step as well. Aditionally, the average four-week returns of the top and the bottom portfolios of both factors are calculated in a four-week step too.

4.6 Factor Portfolios (P Matrix)

After having defined the Q vector in Section 3.1.3, the factor properties in Section 3.2, the BL model default parameter setup in Section 4.4, and the investment process in Section 4.5, in this section, I present the P Matrix used throughout the investment period. For the exact calculation of the P matrix please refer to Section 3.2.3.

The P Matrix links the factor views, i.e. Q vector values, with the corresponding sectors. 1/2 is assigned to a sector if it is in a top portfolio of any of the two factors for the next holding period. Conversely, -1/2 is assigned to a sector if it is in any bottom portfolio for the period of next four weeks after portfolio positions rebalancing.

As an aid for inspecting the top/bottom portfolio positions of sectors according to a factor respectively, I made a visual summary of the values in P Matrices presented in Figures 13 and 14.

Figure 13: P Matrix - *Sum of Momentum Factor's Positions in 4-Week Interval per Year*

Figures 13 and 14 show the sum of the sector assignments to factor sub-portfolios throughout the investment period for Momentum factor and Value factor, respectively. Please note that the data sample ends in February, 2019. Therefore, the last row does not reflect the positions for the whole year of 2019.

Figure 14: P Matrix - *Sum of Value Factor's Positions in 4-Week Interval per Year*

Some authors exclude Financials sector from their calculations, since their B/M ratios are misleading (trade at a $B/M < 1$ over a long period) in the aftermath of the 2007-08 crisis. Figure 14 shows that Financials is included in the Value factor portfolio in this research, especially after it became "cheap" (see Section 3.2.2) in 2007.

4.7 The Authors' Expectations

To demonstrate the potential of the proposed method, I have measured and summarized the average four-weekly returns of the top and bottom portfolios of both factors from 1999 to 2019. The results are presented in the table below:

Table 4: *Average 4-Weekly Returns per Year of the Factor Portfolios*

Year	Momentum Top	Momentum Bottom	Value Top	Value Bottom	Market
2000	-0.30%	-4.52%	-0.13%	-2.46%	-1.58%
2001	-1.16%	-1.71%	-0.06%	-1.65%	-1.23%
2002	-0.98%	-2.43%	-1.58%	-4.09%	-2.22%
2003	1.19%	0.75%	1.40%	1.39%	1.14%
2004	1.02%	0.58%	1.39%	0.21%	1.02%
2005	1.43%	1.67%	1.41%	1.88%	1.88%
2006	1.96%	0.95%	2.19%	0.49%	1.64%
2007	1.24%	0.00%	0.28%	0.06%	0.36%
2008	-2.66%	-4.76%	-4.69%	-4.49%	-4.12%
2009	3.24%	2.74%	4.09%	1.53%	2.25%
2010	1.35%	1.06%	0.39%	1.30%	1.26%
2011	-0.03%	-0.81%	-1.44%	0.17%	-0.70%
2012	0.94%	1.29%	1.06%	1.97%	1.48%
2013	1.51%	1.42%	1.72%	1.61%	1.29%
2014	1.62%	0.14%	0.59%	1.44%	1.10%
2015	0.31%	0.75%	0.34%	1.24%	0.77%
2016	0.94%	0.56%	0.74%	0.06%	0.13%
2017	1.05%	0.70%	0.71%	1.07%	0.89%
2018	-0.34%	-0.63%	-1.01%	-0.32%	-0.79%
2019	2.07%	7.40%	7.31%	6.12%	6.54%
Min	-2.70%	-4.80%	-4.70%	-4.50%	-4.10%
Average	0.60%	-0.10%	0.40%	0.10%	0.20%
Max	3.20%	2.70%	4.10%	2.00%	2.30%
Variance	0.02%	0.04%	0.03%	0.04%	0.03%

It is worthwile mentioning that the average 4-week returns for year 2019 in Table 4 are not representative, since the data sample ends in February, 2019. Therefore, the 2019 returns of the respective portfolios are excluded from the computation of statistical summary in Tables 4 and 5.

Over the course of about 20 years, the top two sectors, according to the 12 week momentum, have outperformed the bottom two sectors, according to the same criterium, by an average of 0.70% (0.60%-(-0.10%)) per 4-week interval (see table 4). It is important to note that the returns of the top momentum portfolio are far superior relative to the market portfolio returns and have smaller variance as well. The second factor, i.e., the Value factor, shows less promising results. The top two value sectors have, on average, overperformed the bottom two value sectors by 0.30% (0.40% - 0.10%) on a 4-week basis. The top portfolios of the Momentum and Value factors have, on average, outperformed the market portfolio performance which yielded an average of 0.20% four-week return.

Table 5 shows the statistical properties of the top and bottom factor portfolio 4-week returns in the period from 2004 to 2018.

Table 5: *Average 4-Weekly Returns per Year of the Factor Portfolios in the Investment Period*

	Momentum Top	Momentum Bottom	Value Top	Value Bottom	Market
Min	-2.70%	-4.80%	-4.70%	-4.50%	-4.10%
Average	0.90%	0.40%	0.50%	0.50%	0.60%
Max	3.20%	2.70%	4.10%	2.00%	2.30%
Variance	0.02%	0.03%	0.04%	0.02%	0.02%

We can now compare the factor forecasts from Table 3 with the realized returns in Table 5. The average 4-week momentum view of 0.82% (see Table 3) is relatively close to 0.50% (0.90%-0.40%) difference in 4-week returns between the top and the bottom momentum portfolios. Interestingly, the average value view of 0.38% (see Table 3) is relatively far off from the 0.00% (0.50%-0.50%) in realized average 4-week returns difference between the value sub-portfolios (see Table 5) in the same time period.

The main reason for the divergence in predicted and relized returns is that the factor views are backward-looking and slowly adapt the new information.

One of the approaches to potentially make the factor views more accurate could be the implementation of a rolling window with a lookback period fixed in length, e.g., using only the last 5 years of data to compute the factor views for the next period.

Figure 15 displays the cumulative performance of the factor portfolios and the cumulative performance of the STOXX 600 market portfolio in the period from 2000 to 2019.

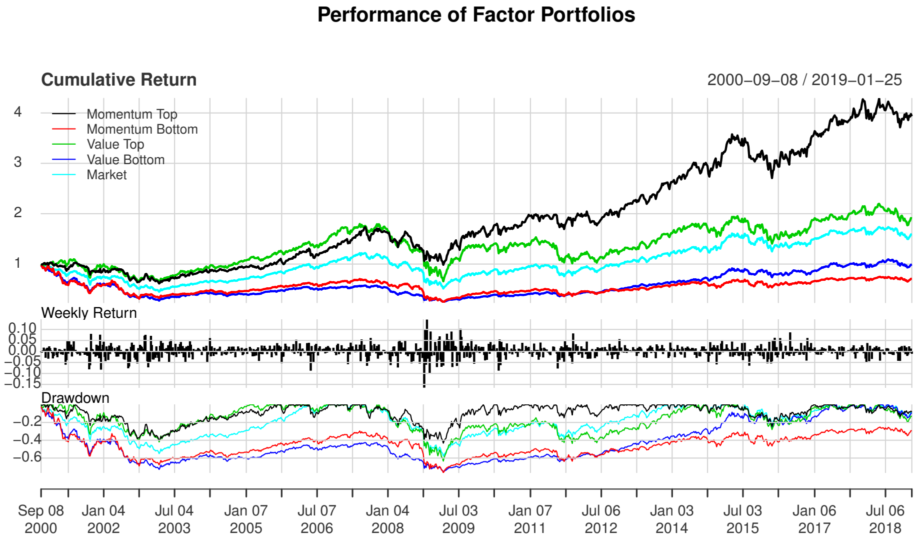

Figure 15: *Performance of Factor Portfolios*

As it can be observed in Figure 15, 1$ invested in Momentum Top portfolio in September, 2000 would result in almost 4$ in January, 2019. Figure 15 clearly illustrates how factor portfolios show different characteristics over time and how both top portfolios have outperformed the bottom portfolios of respective factors in this time period. Interestingly, the Value Top portfolio has been outperforming the Momentum Top portfolio up to the beginning of 2008, than has experienced a hard drawdown of about 60% during 2009, and has never cought up with the performance of the Momentum Top portfolio within the range of this data sample.

Table 6: *Risk-Return Properties of Factor Portfolios in the Period from 2000-09-08 to 2019-01-25*

	Market	Momentum Top	Momentum Bottom	Value Top	Value Bottom
Total return	59.39%	293.53%	-29.31%	90.61%	-0.51%
Average return*	4.34%	9.81%	0.93%	6.20%	1.94%
Average excess return*	2.61%	7.98%	-0.75%	4.44%	0.24%
Sharpe ratio*	0.14	0.39	-0.03	0.19	0.01
IR*		0.52	-0.29	0.20	-0.25
CAPM beta		0.92	1.12	1.12	0.94
CAPM alpha*		5.30%	-3.64%	1.47%	-2.16%
Volatility*	18.45%	19.65%	23.53%	22.35%	19.63%

(*) - Annualized

Table 6 summarizes the characteristics of factor portfolios and the market portfolio for the same period as in Figure 15.

Table 7: *Risk-Return Properties of Factor Portfolios in the Period from* 2004-09-03 to 2019-01-25

	Market	Momentum Top	Momentum Bottom	Value Top	Value Bottom
Total return	142.80%	346.71%	64.16%	96.11%	157.41%
Average return*	8.10%	12.98%	5.99%	7.53%	8.36%
Average excess return*	6.75%	11.57%	4.67%	6.19%	7.01%
Sharpe ratio*	0.36	0.57	0.21	0.26	0.39
IR*		0.48	-0.22	-0.06	0.03
CAPM beta		0.94	1.10	1.18	0.85
CAPM alpha*		4.81%	-2.65%	-1.68%	1.22%
Volatility*	18.03%	19.25%	21.73%	22.66%	17.17%

(*) - Annualized

Table 7 shows the properties of the factor portfolios during the investment period, i.e., during period where the backtesting has been performed using BL-optimized portfolios. This table may be useful in checking the Hypotheses and to answer the Research Question (see Section 1.3).

4.8 The Evaluation Characteristics

The scope of this thesis is to investigate the portfolio performance, achieved by a combination of the BL model with the systematically generated subjective views from the publicly available information by using Momentum and Value factors. This approach will be evaluated using a long period of real market data. The performance is to be evaluated in terms of risk-return characteristics against the market equilibrium portfolio as a benchmark. The research methodology and the sucess criteria are discussed in Sections 1.4 and 1.5 respectively.

5 Results and Analysis

This section presents and discusses the risk-return characteristics of the resulting BL-optimized portfolios with factor views. Additionally, in order to analyze how different parameters influence the properties of the resulting portfolios, the sensitivity analysis is performed on forecast confidence and investor's risk-aversion parameters and on factor view lookback horizon. In further text, the BL-Momentum and BL-Value portfolios represent the BL-optimized portfolios using momentum and value views respectively. Additionally, the BL-Factors portfolio represents the BL-optimized portfolio using views of both factors, i.e., momentum and value views combined as discussed in Section 3.2.3.

5.1 Risk-Return Characteristics

With default key parameter settings, Table 8 illustrates both risk and return figures for each portfolio in the period between September 3, 2004 and January 25, 2019.

Table 8: *Risk-Return Properties of the BL-Optimized Portfolios*

	Market	BL-Momentum	BL-Value	BL-Factors
Total return	142.80%	154.67%	143.45%	151.14%
Average return*	8.10%	8.42%	8.13%	8.32%
Average excess return*	6.75%	7.06%	6.78%	6.97%
Sharpe ratio*	0.36	0.38	0.36	0.38
IR*		0.40	0.09	0.21
CAPM beta		0.99	1.00	0.99
CAPM alpha*		0.37%	0.01%	0.27%
Volatility*	18.03%	17.84%	18.09%	17.90%
Max. Drawdown	57.85%	56.61%	57.78%	56.64%
Tracking Error*		0.73%	0.33%	0.99%

(*) - Annualized

In terms of performance, all BL-optimised portfolios tend to behave similarly. This comes as a surprise, since all portfolios have been optimized using different factors or a combination thereof. The only reason for the resulting portfolios to stay so close to the benchmark is that the volatility of factor portfolios is measured to be enormous, i.e. the Ω was huge (see Section 3.1.3). All three optimized portfolios have yielded a positive alpha either with equal or marginally higher Sharpe ratio relative to the market. This first piece of empirical evidence shows that the factor views can be used in BL portfolio optimization and supports the Hypothesis 1 (see Section 1.3).

As expected, in terms of Sharpe ratio, average returns, maximum drawdown and the return volatility, the BL-Value portfolio is closest to the benchmark portfolio. The tracking error of only 0.33% indicates a weak divergence from the Market portfolio returns. Small average value premia, i.e. low values of the value view (see Section 4.2), have contributed to this fact and have not supported a strong departure from the market weights as a reference point in the optimization process. On the contrary, the average momentum premia has been higher during the investment period (see Section 4.2), and has contributed to a higher tracking error and a stronger departure from the equilibrium, hence has enabled more active portfolio management in case of this factor.

Over the investigated period, the BL-Momentum portfolio showed a positive annualized alpha of 0.37% and has yielded 11.87% compounded relative to the benchmark. Additionally, the BL-Momentum portfolio has a significant annualized Information ratio of 0.40, which, in this parameter setting, is not sufficient to be classified as “good” according to Grinold and Khan [1999]. Noteworthy, the BL-Momentum portfolio has a lower maximum drawdown and a lower annual return volatility compared to the benchmark.

Surprisingly, throughout the investment period, the BL-Factors portfolio shows no benefits over the BL-Momentum portfolio in risk-adjusted terms. This portfolio yields slightly lower total return, lower IR, higher return volatility, and an equal Sharpe ratio with higher annual tracking error relative to the BL-Momentum portfolio. This may suggest that, in this setting, the combination of the two views in BL-optimization does not result in a better diversification, which gives no support for the Hypothesis 3 (see Section 1.3).

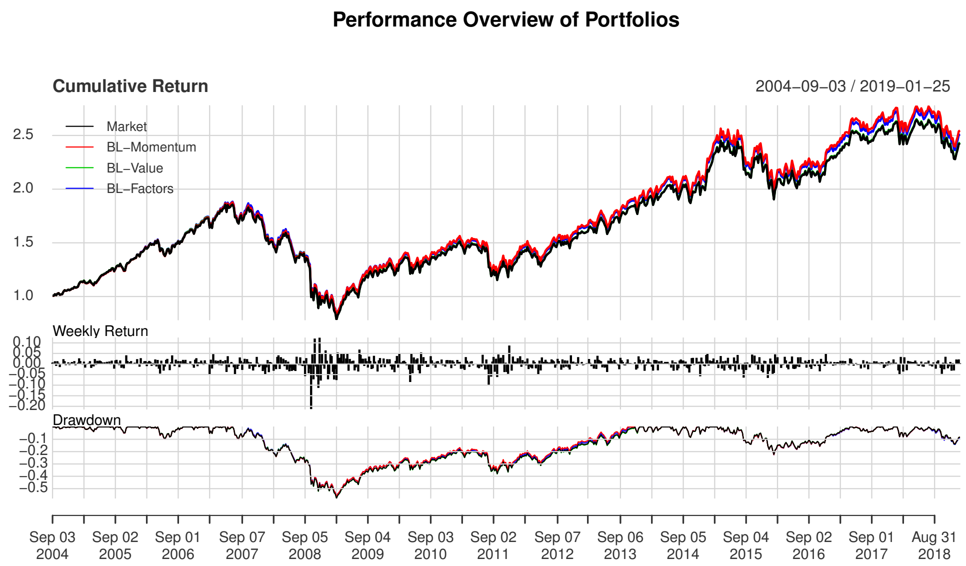

Figure 16: *Cumulative Returns of the BL-Optimized Portfolios*

Graphic illustration 16 shows the performance and drawdowns of the optimized portfolios and the benchmark portfolio throughout the investment period. It is clear that all four portfolios react in a very similar manner. One possible explanation for the similarities is the fact that all four portfolios are based on the market capitalization portfolio in the BL model [Litterman, 2003, Olsson and Trollsten, 2018, Xu et al., 2008]. However, it is astonishing to see the differences in performance of the BL-Momentum and BL-Factors with the Market portfolio, despite the fact that all optimized portfolios are fundamentally derived from the same market capitalization weights [Olsson and Trollsten, 2018]. The over- or underperformance of the BL-optimized portfolios relative to the benchmark is a result of integration of the appropriate subjective views, i.e. factor views, in these portfolios, which further signifies the impact of the factor forecasts in the optimization process.

5.2 Sensitivity Analysis

5.2.1 Lookback Horizon of Factor Views

In order to identify and analyze how the factor view horizon may influence the risk-return properties of the optimized portfolios, I have performed back-testing with different lookback periods for estimating forecasts as suggested by Table 9. The third column in Table 9 ("expanding"), simulates an expanding factor view window from the beginning of the data sample (see Sections 3.2.3 and 4.2). Columns 4, 5 and 6 represent the outcomes of historical simulations using a rolling window for computation of the factor forecasts with lengths of 156, 52 and 24 weeks respectively.

The motivation behind the use of a fixed-length window for calculating factor views is that the new information may be more relevant for the immediate future, and possibly make the backward-looking forecasts more accurate. As illustrated in Table 9, fixing and shortening the factor view lookback period tends to impact the risk-return characteristics of the BL-Momentum portfolio mostly.

Table 9: *Simulating Different Lengths for View Calculation with:* $\tau = \frac{1}{20}$ *and* $\delta = 3$

Factor view lookback		expanding	156 weeks	52 weeks	24 weeks
Total Return	BL-Momentum	154.67%	157.57%	156.43%	155.05%
	BL-Value	143.45%	143.29%	143.72%	143.37%
	BL-Factors	151.14%	153.99%	153.43%	151.66%
CAPM alpha*	BL-Momentum	0.37%	0.44%	0.41%	0.38%
	BL-Value	0.01%	0.00%	0.01%	0.00%
	BL-Factors	0.27%	0.34%	0.33%	0.28%
Sharpe ratio*	BL-Momentum	0.38	0.39	0.39	0.38
	BL-Value	0.36	0.36	0.36	0.36
	BL-Factors	0.38	0.38	0.38	0.38
IR*	BL-Momentum	0.40	0.46	0.47	0.43
	BL-Value	0.09	0.08	0.12	0.09
	BL-Factors	0.21	0.27	0.27	0.23
Volatility*	BL-Momentum	17.84%	17.87%	17.85%	17.86%
	BL-Value	18.09%	18.10%	18.11%	18.11%
	BL-Factors	17.90%	17.93%	17.91%	17.92%
Tracking Error*	BL-Momentum	0.73%	0.82%	0.73%	0.72%
	BL-Value	0.33%	0.33%	0.34%	0.34%
	BL-Factors	0.99%	1.07%	1.00%	1.00%

(*) - Annualized

As shown in Table 9, the Information ratio of all BL-optimized portfolios tends to increase with the computation of factor views in a rolling window.

Table 9 also suggests that the BL-Momentum portfolio prefers and benefits from the rolling factor view calculation most, especially in range from 156 to 52 weeks, as it achieves an increase in CAPM alpha and a slight increase in Sharpe ratio.

Interestingly, in terms of the total return, Sharpe ratio and the CAPM alpha, the BL-Value portfolio stays indifferent across the whole range of factor view lookback settings. Therefore, it is not exactly clear if this portfolio shows any improvements by using factor views computed in rolling windows.

Again, BL-Factors portfolio shows no diversification benefits over the

BL-Momentum portfolio in terms of Sharpe ratio, which is another argument against the Hypothesis 3.

5.2.2 Confidence of Factor Views

In order to obtain a better understanding of the extent how the confidence in factor views influences the properties of the optimized portfolios, Table 10 illustrates the backtested performance of portfolios when altering values for τ while holding other parameters fixed.

Overall, increased view confidence is followed by an increase in tracking error and return volatility in all optimized portfolios. This comes as no surprise, as higher view confidence allows stronger departure from the equilibrium. Conversely, lowered belief confidence values, make the optimized portfolios converge closer towards the equilibrium weight vector.

Furthermore, Table 10 shows that the higher confidence in factor views tends to results in stronger factor characteristics, in terms of total return, Sharpe ratio, IR and the CAPM alpha (see Table 7), of the resulting BL optimized portfolios, what fuels the Hypothesis 2.

With more confidence in factor views, the BL-Momentum portfolio shows a strong increase in total return, CAPM alpha and in Sharpe ratio. In the case of the BL-Value portfolio, with maximum view confidence of 1, the CAPM alpha turns negative followed by a slight decrease in Sharpe ratio and by a small increase in IR relative to the default parameter setting.

Table 10: *Simulating Varying View Confidence with: $\delta = 3$ and Expanding Factor View*

View confidence		$\tau = 1$	$\tau = 1/10$	$\tau = 1/40$	$\tau = 1/80$
Total Return	BL-Momentum	377.53%	166.53%	148.64%	145.60%
	BL-Value	142.53%	144.17%	143.02%	142.79%
	BL-Factors	300.16%	162.19%	145.38%	142.44%
CAPM alpha*	BL-Momentum	5.88%	0.72%	0.18%	0.09%
	BL-Value	-0.14%	0.02%	0.00%	0.00%
	BL-Factors	4.82%	0.60%	0.09%	0.00%
Sharpe ratio*	BL-Momentum	0.60	0.40	0.37	0.37
	BL-Value	0.35	0.36	0.36	0.36
	BL-Factors	0.49	0.39	0.37	0.36
IR*	BL-Momentum	0.41	0.40	0.38	0.31
	BL-Value	0.08	0.11	0.05	0.02
	BL-Factors	0.29	0.27	0.09	-0.03
Volatility*	BL-Momentum	18.95%	17.69%	17.94%	17.99%
	BL-Value	19.81%	18.15%	18.07%	18.06%
	BL-Factors	21.72%	17.82%	17.96%	18.00%
Tracking Error*	BL-Momentum	11.74%	1.44%	0.38%	0.23%
	BL-Value	4.38%	0.54%	0.24%	0.21%
	BL-Factors	14.46%	1.79%	0.64%	0.51%

(*) - Annualized

Once again, the BL-Factors portfolio, as a hybrid of BL optimization with momentum and value views together, by judging based on the annualized Sharpe ratio, shows no signs of diversification improvements relative to the BL-Momentum portfolio across the whole range of τ values, which is yet another argument against the Hypothesis 3. One possible explanation for this could be that, during the investment period, the Value factor portfolios have higher volatility and lower Sharpe ratios relative to the respective portfolios of the Momentum factor (see Table 7).

5.2.3 Investors' Risk-Aversion

By performing *ceteris paribus* on delta, i.e. by altering the values of δ while holding other parameters fixed, Table 11 illustrates the impact of change of the future risk-aversion coefficient on the performace of the BL portfolios.

Table 11: *Simulating Varying Risk-Aversion with:* $\tau = \frac{1}{20}$ *and Expanding Factor View*

Risk aversion		$\delta = 1$	$\delta = 2$	$\delta = 5$	$\delta = 10$
Total Return	BL-Momentum	178.68%	160.54%	150.03%	146.60%
	BL-Value	141.88%	143.07%	143.75%	143.97%
	BL-Factors	169.91%	155.81%	147.42%	144.64%
CAPM alpha*	BL-Momentum	1.06%	0.54%	0.23%	0.13%
	BL-Value	-0.07%	-0.01%	0.02%	0.04%
	BL-Factors	0.81%	0.40%	0.16%	0.07%
Sharpe ratio*	BL-Momentum	0.42	0.39	0.37	0.37
	BL-Value	0.36	0.36	0.36	0.36
	BL-Factors	0.40	0.38	0.37	0.37
IR*	BL-Momentum	0.42	0.41	0.37	0.28
	BL-Value	0.03	0.07	0.12	0.13
	BL-Factors	0.27	0.24	0.15	0.06
Volatility*	BL-Momentum	17.66%	17.79%	17.89%	17.93%
	BL-Value	18.33%	18.15%	18.05%	18.02%
	BL-Factors	17.95%	17.90%	17.91%	17.92%
Tracking Error*	BL-Momentum	2.10%	1.07%	0.48%	0.31%
	BL-Value	0.85%	0.45%	0.26%	0.23%
	BL-Factors	2.62%	1.38%	0.72%	0.57%

(*) - Annualized

Table 11 clearly demonstrates how the higher values for the risk-aversion parameter lower the influence of the factor views on the vector of posterior expected returns on which the optimization is performed. On the contrary, we can observe a consistent increase in tracking errors by lowering the risk-aversion parameter, allowing BL-optimized portfolios to diverge away from the equilibrium in direction of the investor beliefs. The higher the risk-aversion, the lower is the effect of differences in expected returns (coming from the factor views) on the portfolio holdings. This is so since high risk-aversion moves attention of investors to return variance and away from the return expectations.

5.3 The Choice of Extreme Parameters

In this Section, I want to demonstrate the performance of the BL portfolio optimization with factor views using the most extreme setup of parameters, i.e. $\tau = 1$ and $\delta = 1$ with a factor view lookback horizon of 52 weeks, since the BL-optimized portfolios deliver no significant alpha in the default parameter setting ($\tau = 1/20$ and $\delta = 3$). This new set of parameters (high tau means high confidence in factor views, low delta means high relevance of expected returns in portfolio selection) will show us portfolios of investors who will tilt their portfolios aggressively towards factor views. I.e., it helps us to assess the premium (if any) that comes with a factor tilt.

This portfolio setup may be hard, or even impossible, to achieve in practice for most investors, since it employs relatively high degree of leverage with large short positions.

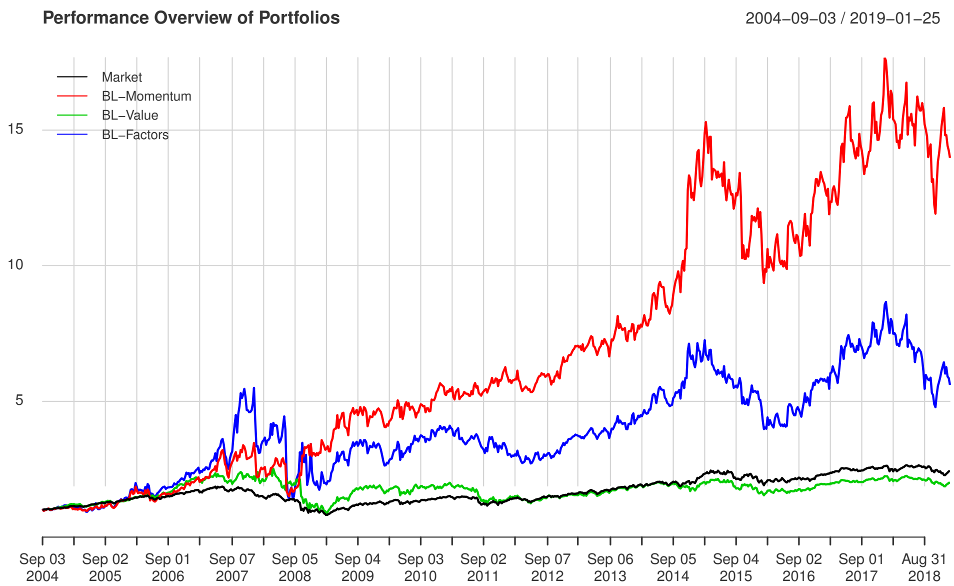

Figure 17: P*erformance Summary of the BL-Portfolios in Extreme Configuration*

Figure 17 and Table 12 illustrate the performance and the risk-return characteristics of the BL portfolios in extreme parameter setup. In this configuration and over the investment period, the BL-Momentum overperforms the market portfolio by 1154.77% in raw return with the total return of 1297.57%. Furthermore, the BL-Momentum portfolio achieves annualized IR of 0.49, which is very close to be classified as "good" according to Grinold and Khan [1999], and a relatively high annual Sharpe ratio of 0.67 what indicates clear improvement in risk-adjusted terms.

Table 12: *Risk-Return Characteristics of the BL-Portfolios in Extreme Configuration*

	Market	BL-Momentum	BL-Value	BL-Factors
Total return	142.80%	1297.57%	99.47%	460.73%
Average return*	8.10%	27.77%	9.11%	25.60%
Average excess return*	6.75%	26.18%	7.75%	24.04%
Sharpe ratio*	0.36	0.67	0.27	0.47
IR*		0.49	0.06	0.35
CAPM beta		0.60	1.33	0.84
CAPM alpha*		19.42%	-1.22%	16.09%
Volatility*	18.03%	35.05%	27.88%	45.84%
Max. Drawdown	57.85%	56.63%	66.42%	74.62%
Tracking Error*		34.16%	15.41%	43.35%

(*) - Annualized

Figure 18 illustrates the weights of the BL-Momentum portfolio in extreme parameter setup. This Figure well demonstrates the active portfolio management with Black-Litterman framework using Momentum factor view with an extreme confidence.

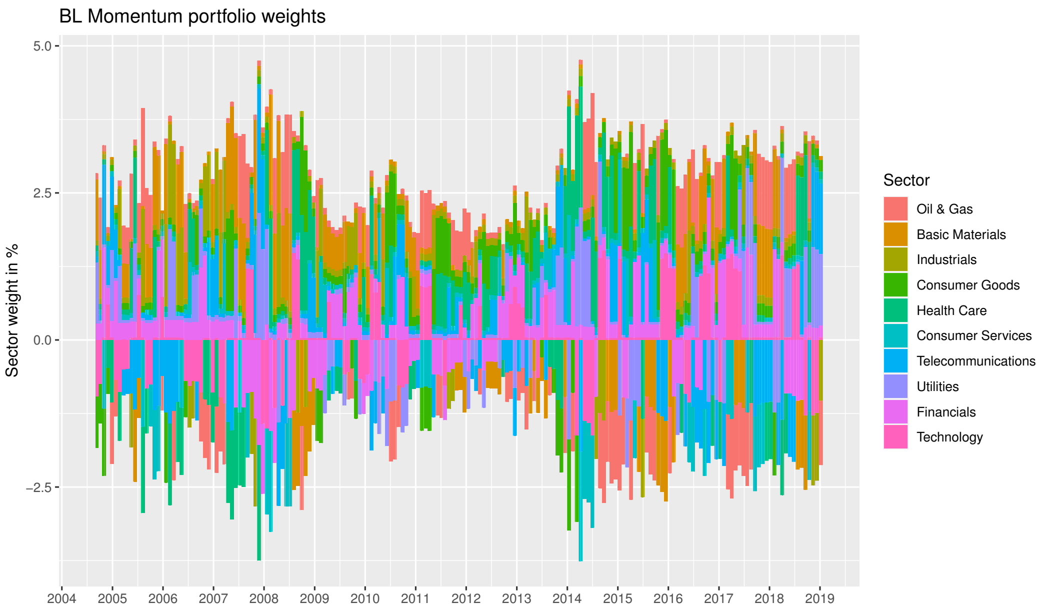

Figure 18: *Weights of the BL-Momentum Optimized Portfolio with Overconfident Beliefs*

6 Conclusion

Using factor views in Black-Litterman portfolio optimization with European STOXX 600 sector indices in the period from 2004 to 2019 can be summarized in four key pillars. First, factor views can be used as investors' subjective forecasts of future returns in combination with the Black-Litterman framework. Second, portfolio optimization with the momentum view delivers superior risk-adjusted returns relative to the benchmark. Suprisingly, the Value factor shows close to no positive effect on the portfolio characteristics. Third, combining the value view with the momentum view in the optimization process yields no diversification benefits relative to the optimization with only momentum view. Lastly, factor view lookback horizon and the view confidence play an important role in risk-return characteristics of the optimized portfolios.

All three BL-optimized portfolios yield a positive CAPM alpha already in default parameter settings. In accordance with the factor views, the optimized portfolio with the Momentum factor deviates significantly from the market benchmark compared to the BL-portfolio with the value view. The Value factor adds a lot of volatility with only a slight increase in average return. This causes the Sharpe ratio to go down, i.e. there is no positive effect on the characteristics of the resulting portfolio. Based on these empirical findings, I conclude that the Hypothesis 1 can be accepted.

The empirical findings have shown that varying view confidence plays an important role on the resulting characteristics of the BL-optimized portfolios. The higher the confidence in factor views, the more similar are the resulting BL-optimized portfolios with the factor portfolios in terms of the Information and Sharpe ratios. Therefore, I accept Hypothesis 2.

In risk-adjusted terms, based on Sharpe ratio, the BL-optimized portfolio using two factor views, i.e. momentum view and value view combined, has not resulted in diversification improvements relative to the BL-Momentum portfolio in any performed historical simulation. I suppose this is the case because the Value factor is not able to improve portfolio characteristics over the analyzed horizon. Thus, combining Value and Momentum factor and having low risk aversion and high confidence in these views leads to extreme portfolio positions which increase portfolio volatility a lot but only moderately improve average returns. Combination of factors improves the Sharpe ratio compared to the benchmark. It is, however, dominated by the pure Momentum factor, since it works so well over the analyzed horizon. Hypothesis 3 can neither be clearly accepted nor rejected based on the presented analysis. I suggest for future work to expand the set of factors in a broader analysis to gain better information about the validity of Hypothesis 3.

Additionally, in this thesis I have shown how the BL optimized portfolios benefit, in risk-adjusted terms, from the use of rolling windows for the computation of the factor views, and how the varying values of the risk-aversion parameter influence the risk-return characteristics of the resulting portfolios.

Contrary to the efficient market theory developed in the 1970s, by using the approach described in this research, under no transaction-costs condition and by using only publicly available pricing data, I was able to outperform the European equity market in terms of Information ratio by using a wide range of BL framework parameter settings. The presented BL factor approach is suitable for tilting diversified portfolios towards factors that are known to be performance relevant [Fama and French, 1992]. Thus the research question can be answered with "yes", it is possible to outperform the European equity market index STOXX 600 by employing the BL factor approach. However, the performance inevitably comes with additional factor risk, which must be regarded in further analysis.

7 Further Research

In this research, I layed out some evidence of premium added from using Momentum and Value factors in the BL optimization process using 10 sector indices from STOXX 600 Index with 4-week rebalancing policy. Further research could include broader investment universe including more assets in the portfolio composition and for the computation of factor views.

Additionally, other style factors such as Quality or Volatility can be included in the research. The covariance matrix could be estimated using a time-decay approach. It would be interesting to see how the different rebalancing periods together with the transaction costs influence the risk-return characteristics of the resulting optimized portfolios. Furthermore, it might be interesting to inspect if the introduction of the no-shorting constraint would improve the risk-adjusted returns due to lower risk taking.

Part III
Summary

In this research, I have implemented the combination of Black-Litterman model with factor views as this models' forward-looking forecasts constructed according to the average value and momentum, which are measured in sector portfolios of the European equity market index STOXX 600. The aim of this research was to investigate whether it is possible to use these factor views, constructed from the past pricing information, as inputs to the Black-Litterman model so that the resulting portfolios are tilted towards the pronounced factor portfolios, and whether this approach results in improved risk-return properties of the optimized portfolios. Additionally, it was investigated if a combination of the two Factors in the portfolio optimization process delivers any diversification benefits over the use with only one Factor, and if the varying view confidence affects the characteristics of the resulting portfolios. Finally, it was investigated if such optimized portfolios deliver risk-adjusted returns in excess of the STOXX 600 Index as a benchmark.

The portfolio optimization was performed on 10 sector portfolios defined by the STOXX Europe 600 Index universe (SXXP) in the period from September, 2004 to January, 2019. As of February, 2019, this index represented large, mid and small capitalization companies across 17 countries of the European region: Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, the Netherlands, Norway, Poland, Portugal, Spain, Sweden, Switzerland and the United Kingdom [15]. The number of components in this index is fixed to 600. The empirical data in this thesis is based on weekly returns data, which shall not be compounded to returns over longer periods. The factor portfolios were constructed using best 2 and worst 2 performing sectors according to the 12-week momentum and the book-to-market ratio respectively. First 5 years of data have been used for estimating the first sector covariance matrix and for computation of the factor views. The historical simulations have been performed from September, 2004 to January, 2019 using 4-week rebalancing period.

My empirical findings show that, over the investigated period, the Momentum factor has shown higher premia relative to the Value factor. This fact has also been reflected in the resulting BL-optimized portfolios. The BL approach with the Momentum factor has resulted in superior risk-return portfolios relative to the benchmark. The empirical findings also show that varying view confidence plays an important role on the resulting characteristics of the BL-optimized portfolios. The higher the confidence in factor

[15] https://qontigo.com/, https://stoxx.com/

views, the more similar are the resulting BL-optimized portfolios with the factor portfolios. The optimization with the Value factor shows close to no positive effect on the portfolio characteristics. Suprisingly, using both factors in combination, yields no benefits over the BL optimization with the Momentum factor. This combination of factors improves the Sharpe ratio compared to the benchmark. It is, however, dominated by the pure Momentum factor, since it works so well over the analyzed horizon. Contrary to the efficient market theory developed in the 1970s, by using the approach described in this research, under no transaction-costs condition and by using only publicly available data, I was able to outperform the European equity market in risk-adjusted terms by using a wide range of BL framework parameter settings. However, the performance inevitably comes with additional factor risk, which must be regarded in further analysis. The presented BL factor approach is suitable for tilting diversified portfolios towards factors that are known to be performance relevant [Fama and French, 1992].

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A Appendix

Table A.1: *STOXX 600 - Average weekly sector book-to-market ratio per year*

Year	Oil & Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecommunications	Utilities	Financials	Technology
1999	0.39	0.49	0.34	0.35	0.16	0.20	0.15	0.43	0.34	0.09
2000	0.40	0.54	0.32	0.32	0.15	0.18	0.24	0.40	0.33	0.06
2001	0.47	0.59	0.42	0.34	0.18	0.32	0.61	0.46	0.41	0.20
2002	0.57	0.64	0.53	0.38	0.25	0.42	0.94	0.57	0.57	0.33
2003	0.64	0.72	0.61	0.46	0.28	0.46	0.70	0.63	0.64	0.32
2004	0.54	0.60	0.49	0.40	0.24	0.40	0.58	0.50	0.55	0.25
2005	0.44	0.53	0.42	0.37	0.25	0.38	0.53	0.43	0.50	0.26
2006	0.38	0.41	0.36	0.37	0.25	0.36	0.59	0.38	0.46	0.25
2007	0.39	0.37	0.32	0.34	0.29	0.34	0.50	0.32	0.49	0.26
2008	0.48	0.53	0.50	0.46	0.38	0.56	0.60	0.41	0.92	0.41
2009	0.64	0.73	0.61	0.54	0.43	0.61	0.69	0.59	1.18	0.45
2010	0.63	0.56	0.49	0.44	0.36	0.46	0.64	0.65	1.02	0.38
2011	0.66	0.66	0.52	0.47	0.37	0.51	0.69	0.78	1.28	0.39
2012	0.74	0.76	0.54	0.45	0.34	0.52	0.74	0.95	1.39	0.41
2013	0.82	0.81	0.45	0.42	0.29	0.41	0.61	0.89	1.07	0.31
2014	0.79	0.77	0.40	0.41	0.26	0.37	0.55	0.72	0.93	0.29
2015	0.91	0.73	0.38	0.36	0.23	0.32	0.48	0.70	0.90	0.26
2016	0.96	0.79	0.40	0.38	0.28	0.34	0.54	0.75	1.13	0.29
2017	0.89	0.73	0.34	0.37	0.29	0.36	0.52	0.67	0.93	0.28
2018	0.78	0.76	0.36	0.42	0.30	0.37	0.58	0.67	0.99	0.26
2019	0.81	0.69	0.40	0.45	0.29	0.39	0.63	0.62	1.10	0.28
Min.	0.38	0.37	0.32	0.32	0.15	0.18	0.15	0.32	0.33	0.06
Avg.	0.63	0.64	0.44	0.40	0.28	0.39	0.58	0.60	0.82	0.29
Max.	0.96	0.81	0.61	0.54	0.43	0.61	0.94	0.95	1.39	0.45
Variance	0.04	0.02	0.01	0.00	0.01	0.01	0.03	0.03	0.11	0.01

Table A.2: *STOXX 600 - Average weekly 12-week momentum of sectors per year*

Year	Oil & Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecommunications	Utilities	Financials	Technology
1999	6.60%	10.55%	10.52%	-1.07%	9.13%	15.11%	45.97%	-1.16%	9.44%	66.12%
2000	3.91%	0.71%	1.69%	3.91%	6.15%	1.91%	0.18%	3.35%	4.85%	7.65%
2001	-1.51%	1.78%	-4.75%	-1.57%	-2.35%	-5.62%	-9.92%	-2.07%	-4.36%	-12.53%
2002	-2.94%	-1.30%	-5.30%	-0.67%	-6.68%	-6.67%	-7.57%	-4.73%	-6.49%	-11.93%
2003	0.38%	3.10%	5.78%	0.70%	1.05%	2.79%	3.42%	1.63%	3.78%	6.15%
2004	5.46%	4.07%	3.55%	2.15%	1.52%	2.29%	3.82%	7.09%	3.36%	-0.10%
2005	7.33%	7.33%	6.52%	5.95%	6.01%	2.94%	1.10%	5.93%	5.73%	3.50%
2006	2.00%	8.15%	6.59%	4.74%	2.24%	5.01%	4.15%	8.12%	6.21%	1.64%
2007	2.91%	7.46%	3.21%	4.48%	-1.09%	0.86%	5.48%	6.33%	-1.50%	1.56%
2008	-8.76%	-14.48%	-13.96%	-8.33%	-4.26%	-11.93%	-9.06%	-9.99%	-15.60%	-12.92%
2009	5.11%	12.93%	8.84%	4.59%	3.19%	5.95%	4.17%	1.63%	9.44%	5.15%
2010	1.52%	7.20%	5.32%	6.66%	3.34%	5.25%	3.37%	-0.41%	-0.04%	3.77%
2011	1.68%	-4.05%	-2.45%	0.46%	1.57%	-2.05%	-0.92%	-3.05%	-6.06%	-0.51%
2012	1.77%	3.20%	5.13%	6.07%	5.67%	4.68%	-0.08%	0.51%	6.11%	4.97%
2013	1.06%	0.17%	5.61%	4.00%	5.07%	6.13%	6.62%	4.59%	6.01%	6.06%
2014	-0.10%	0.20%	0.65%	2.49%	4.86%	0.57%	2.71%	4.09%	1.88%	1.52%
2015	0.03%	-0.20%	3.65%	5.79%	3.96%	5.49%	4.10%	-0.04%	2.54%	5.24%
2016	2.89%	4.41%	1.42%	-0.52%	-2.05%	-2.13%	-3.58%	-1.71%	-1.67%	0.99%
2017	2.81%	5.14%	4.75%	3.99%	2.44%	1.88%	1.16%	4.32%	4.43%	6.62%
2018	2.51%	-1.97%	-2.20%	-1.74%	0.67%	-0.60%	-1.06%	1.07%	-2.94%	-1.11%
2019	-3.50%	-2.67%	-2.15%	-1.17%	-0.68%	-0.69%	1.50%	7.13%	-2.47%	-2.47%
Min.	-8.80%	-14.50%	-14.00%	-8.30%	-6.70%	-11.90%	-9.90%	-10.00%	-15.60%	-12.90%
Average.	1.50%	2.50%	2.00%	1.90%	1.90%	1.50%	2.60%	1.60%	1.10%	3.80%
Max.	7.30%	12.90%	10.50%	6.70%	9.10%	15.10%	46.00%	8.10%	9.40%	66.10%
Variance	0.10%	0.40%	0.30%	0.10%	0.10%	0.30%	1.20%	0.20%	0.40%	2.40%

Table A.3: *STOXX 600 - Average weekly sector portfolio weights per year*

Year	Oil & Gas	Basic Materials	Industrials	Consumer Goods	Health Care	Consumer Services	Telecommunications	Utilities	Financials	Technology
1999	9.06%	3.42%	7.96%	9.15%	9.44%	9.17%	15.23%	3.66%	26.55%	6.36%
2000	8.03%	2.69%	7.11%	7.79%	9.24%	10.09%	16.04%	4.01%	24.96%	10.04%
2001	9.44%	3.35%	7.09%	9.44%	11.13%	9.89%	11.13%	4.84%	27.58%	6.12%
2002	10.41%	4.18%	7.61%	11.65%	11.11%	9.40%	9.43%	4.84%	26.83%	4.53%
2003	10.02%	4.28%	7.81%	11.67%	10.48%	8.89%	10.93%	4.79%	26.88%	4.25%
2004	9.98%	4.41%	8.50%	11.40%	9.71%	8.97%	10.01%	5.19%	27.64%	4.19%
2005	10.33%	4.69%	9.21%	11.27%	9.23%	8.58%	9.05%	5.72%	28.40%	3.51%
2006	9.23%	5.88%	9.62%	11.32%	8.89%	7.92%	6.55%	6.97%	30.41%	3.21%
2007	8.50%	6.96%	10.53%	12.10%	7.57%	7.72%	6.34%	8.03%	29.12%	3.11%
2008	10.18%	7.92%	10.30%	12.96%	8.42%	6.69%	6.83%	9.36%	24.40%	2.94%
2009	10.60%	7.19%	10.38%	13.90%	9.67%	7.04%	7.00%	9.01%	22.21%	2.99%
2010	9.41%	8.83%	11.25%	14.67%	9.37%	7.25%	6.33%	7.37%	22.55%	2.97%
2011	9.77%	9.16%	11.84%	16.14%	9.70%	7.16%	6.18%	6.79%	20.14%	3.13%
2012	9.84%	8.93%	11.84%	18.49%	10.85%	7.34%	5.45%	5.36%	18.76%	3.14%
2013	8.16%	7.62%	12.31%	18.75%	11.11%	7.77%	5.09%	4.70%	21.10%	3.41%
2014	7.68%	6.95%	12.26%	18.04%	11.52%	7.85%	4.88%	5.12%	22.37%	3.31%
2015	5.94%	6.05%	12.21%	19.29%	12.47%	8.35%	5.31%	4.41%	22.43%	3.56%
2016	5.74%	6.08%	13.15%	20.72%	12.48%	8.42%	5.00%	4.28%	20.21%	3.91%
2017	5.73%	6.62%	13.83%	19.89%	11.68%	7.59%	4.33%	4.37%	21.80%	4.16%
2018	6.58%	6.60%	13.86%	19.90%	11.76%	7.34%	3.71%	4.45%	21.15%	4.65%
2019	6.89%	6.77%	13.46%	19.64%	12.88%	6.97%	3.65%	4.98%	20.20%	4.56%
Min.	5.73%	2.69%	7.09%	7.79%	7.57%	6.69%	3.65%	3.66%	18.76%	2.94%
Avg.	8.64%	6.12%	10.58%	14.68%	10.41%	8.11%	7.55%	5.63%	24.08%	4.19%
Max.	10.60%	9.16%	13.86%	20.72%	12.88%	10.09%	16.04%	9.36%	30.41%	10.04%
Variance	0.03%	0.04%	0.05%	0.17%	0.02%	0.01%	0.12%	0.03%	0.12%	0.03%