Predicting Capillary Ripples and Nonlinear Squire–Taylor Modes by Viscous–Inviscid Interaction past a Trailing Edge

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We consider the steady developed flow of a liquid layer having just passed the trailing edge of horizontal flat plate in the asymptotic limit of large Reynolds number. The body/interface forces at play are gravity acting vertically and constant surface tension on both sides of the layer. Two-tiered (internal) viscous-inviscid interaction accounts rigorously for the interplay of these and the viscous forces in the long-wave limit. Here a suitably formed reciprocal Weber number (T) and reciprocal Froude number (G) form the sole control parameters. The solutions of the nonlinear interaction problem disclose a surprising richness of phenomena, distinguished by the range of values of T: antisymmetric ($0 \leq T \leq 1/2$) and symmetric ($T > 1$) capillary wave trains represent a nonlinear extension of classical Squire modes, where the latter ones can be seen as planar Rayleigh–Plateau modes; if $1/2 < T < 1$, either gross flow reversal far downstream or a blow-up at some finite distance from the edge terminates the interactive flow description. Both scenarios are found to be separated by an unstable flow represented by a curve in the (T, G)-plane. Amongst others, we present a weakly nonlinear WKBJ analysis capturing the waves and briefly address the types of choking for T being close to $1/2$ or 1 as well as the surprising intricacies associated with the Euler stage regularising the blow-up. Finally, the analogies and differences to the impact of capillarity on the developed flow through the orifice of a circular pipe are highlighted.

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