TECHNISCHE UNIVERSITÄT WIEN Vienna | Austria

Master Thesis

Target Scores: Factor Investing with Orthogonalized Mimicking Portfolios

carried out for the purpose of obtaining the degree of Master of Science (MSc), submitted at TU Wien, Faculty of Mechanical and Industrial Engineering, by

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Diplomarbeit

Target Scores: Factor Investing mit Orthogonalisierten Faktor-Portfolios

ausgeführt zum Zwecke der Erlangung des akademischen Grades eines Diplom-Ingenieurs (Dipl.-Ing.) / Master of Science (MSc), eingereicht an der TU Wien, Fakultät für Maschinenwesen und Betriebswissenschaften, von

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Wien, Mai 2023

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Affidavit

I declare in lieu of oath, that I wrote this thesis and performed the associated research myself, using only literature cited in this volume. If text passages from sources are used literally, they are marked as such.

I confirm that this work is original and has not been submitted elsewhere for any examination, nor is it currently under consideration for a thesis elsewhere.

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Wien, Mai 2023 ………………………………

Matthias Hochsteiner

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Abstract

Factor-based investment approaches have gained popularity in the world of equity-portfolio management in recent years. These portfolio strategies overweight stocks with high exposure and underweight stocks with low exposure to selected firm characteristics. A common problem in factor investing is *unintended exposure*: Increasing the portfolio exposure to a certain factor unintendedly changes the exposure to other factors. This kind of cross-contamination of factor portfolios makes performance attribution and the analysis of factor-return characteristics difficult or even impossible. In this thesis, I investigate a portfolio approach that implements an orthogonality feature with respect to the factor exposure, i.e., factor-mimicking portfolios are orthogonalized, such that they implement pure single-factor exposure, coming without any unintended exposure to other factors. For comparative purposes, I examine a second portfolio approach that uses a classic methodology to build factor portfolios. The analyses in this research work are based on backtests over a 20-year time horizon. I put a main focus on the return characteristics of orthogonalized factor-mimicking portfolios and find an unexpected interdependence between factor exposure and returns. In fact, orthogonality in the space of firm characteristics seems to translate into uncorrelated factor returns. On the contrary, I observe a high level of cross-contamination in the factor returns resulting from the second approach without the exposure orthogonality feature. Also, it becomes apparent that the orthogonal portfolio approach yields the highest information ratio which classifies the approach as most attractive when investing benchmark-oriented. Further, I study the effects of the three factors size, value and momentum on the portfolio performance. I find that size is the only factor that realizes a positive premium with either of the two approaches.

Kurzfassung

Faktorbasierte Investmentansätze konnten in den letzten Jahren im Bereich des Aktien-Portfoliomanagements an Popularität gewinnen. Diese Portfoliostrategien übergewichten Aktien mit hohem Exposure und untergewichten Aktien mit niedrigem Exposure gegenüber ausgewählten Unternehmensmerkmalen. Ein häufiges Problem bei Factor Investing ist *unbeabsichtigtes Exposure*: Das Erhöhen des Portfolio-Exposure gegenüber eines bestimmten Faktors verändert unbeabsichtigt das Exposure gegenüber anderen Faktoren. Diese Art der Kreuzkontamination von Faktorportfolios erschwert die Performance-Interpretation sowie die Analyse von Faktorrendite-Eigenschaften oder macht diese sogar unmöglich. In dieser Forschungsarbeit untersuche ich einen Portfolioansatz, der eine Orthogonalitätseigenschaft hinsichtlich dem Faktorexposure realisiert. Das wird erreicht, indem Faktorportfolios orthogonalisiert werden, sodass diese reines Single-Faktorexposure, ohne einhergehendes unbeabsichtigtes Exposure gegenüber anderen Faktoren, umsetzen. Zum Vergleich analysiere ich einen zweiten Portfolioansatz, der eine klassische Methodik zur Bildung von Faktorportfolios verwendet. Die Analysen in dieser Forschungsarbeit basieren auf Backtests über einen Zeitraum von 20 Jahren. Mein Hauptfokus liegt auf den Rendite-Eigenschaften von Faktorportfolios mit orthogonalem Exposure. Dabei entdecke ich eine unerwartete Wechselbeziehung zwischen Faktorexposure und Renditen. Tatsächlich scheint die Orthogonalität innerhalb der Unternehmensmerkmale zu unkorrelierten Faktorrenditen zu führen. Hingegen weisen die Faktorrenditen, die aus dem zweiten Ansatz ohne Orthogonalitätseigenschaft resultieren, ein hohes Maß an Kreuzkontamination auf. Außerdem zeigt sich, dass der orthogonale Portfolioansatz die höchste Information-Ratio liefert, was ihn zum attraktivsten Ansatz macht, wenn benchmarkorientiert investiert wird. Darüber hinaus untersuche ich die Auswirkungen der drei Faktoren Size, Value und Momentum auf die Portfolio-Performance. Es stellt sich heraus, dass lediglich der Size-Faktor mit beiden Ansätzen eine positive Prämie erzielt.

Nomenclature

Symbols

x	A scalar
$\mathbf{x}$	A vector
$\mathbf{x}'$	The transpose of vector $\mathbf{x}$
$\mathbf{X}$	A matrix
$\mathbf{X}^{-1}$	The inverse of matrix $\mathbf{X}$
$E(x)$	The expected value of x

Acronyms

APT	Arbitrage pricing theory
BTP	Book-to-price ratio
CAPM	Capital asset pricing model
(LO)CFA	(Long-only) correlated factor approach
D/E	Debt-to-equity ratio
ETF	Exchange-traded fund
HML	High minus low
IR	Information ratio
ISIN	International securities identification number
LOMTR	Long-only minimum tracking-error approach
MCAP	Market capitalization
MOM	Momentum
MPT	Modern portfolio theory
OFA	Orthogonal factor approach
P/E	Price-to-earnings ratio
SMB	Small minus big
SMM	Shrinkage towards a single-factor market model estimator
TE	Tracking-error
WML	Winners minus losers

Contents

Chapter 1

Introduction

1.1 Overview

The explanation of portfolio returns and corresponding risk has been topic of famous papers in financial research. Many of those publications build on the principles of modern portfolio theory (MPT) initiated by Markowitz (1952). MPT highlights the importance of diversification and suggests investors should use mean-variance analysis for constructing portfolios, with the mean and the variance of the portfolio's returns serving as measures of return and risk. On that basis, a portfolio is considered efficient when it provides the highest expected return for a given risk level or the lowest risk for a given expected return. With the capital asset pricing model (CAPM), one of the most popular models in that field was introduced by Treynor (1961), Sharpe (1964) and Lintner (1965). The CAPM is based on the assumption that the overall market portfolio is efficient and puts returns and the systematic risk[1] into relation. In fact, it states that a stock's expected return is determined by the stock's behavior in relation to the market (measured by the stock's market beta). Following, extended versions of the CAPM had been emerging. For example, Black (1972) published a modified version with more restrictive assumptions. The main idea of the CAPM, that beta drives the expected return as only factor, persisted. Previous empirical tests of the CAPM, such as Jensen et al. (1972) and Fama and MacBeth (1973), pointed out certain discrepancies but agreed on the simple positive relation between beta and average stock returns. Accordingly, the CAPM played an important role in the way of thinking about risk and return for a long time. However, as time went on, various studies revealed empirical problems of the CAPM and the explanatory power got more and more called into question. The identification of additional firm characteristics that seemed to affect expected returns invalidated the market beta as sole predictor for stock returns.

Basu (1977) found that price-to-earnings (P/E) ratios[2] could help to explain returns as low P/E stocks had higher returns than high P/E stocks on average. Also, low P/E portfolios showed higher returns than justified according to their risk. Banz (1981) first

[1] The systematic risk (also called market risk) represents the non-diversifiable risk.

[2] The price-to-earnings ratio puts a firm's stock price and its annual earnings per share into relation.

reported the so-called *size effect*.[3] He discovered that smaller firms did earn superior risk-adjusted returns than larger firms after controlling for market beta. Indeed, the returns of small stocks were higher than their market betas would have predicted while the opposite was the case with the returns of large stocks. Further, Rosenberg et al. (1985) documented a positive relation between a firm's average stock returns and its book-to-price ratio[4]. Similarly, Bhandari (1988) observed a positive relation between a stock's debt-to-equity (D/E) ratio and its expected returns.

A popular work on the cross-section of returns was published by Fama and French (1992). They concluded that the two attributes size and book-to-market[5] are strongly related to average stock returns. Following, Fama and French (1993) used the time-series regression approach of Jensen et al. (1972) to test a three-factor asset-pricing model. They regressed stock returns on the returns to a market portfolio and to mimicking long-short portfolios for size and value. The long-short portfolios are supposed to mimic the underlying risk factors in stock returns. They are formed based on sorts of stocks on size and book-to-market. In such regressions, the slopes indicate if the utilized factors are able to capture variation in stock returns while the resulting intercepts show how well the model explains the cross-section of average returns. A proper model leads to intercepts that are close to 0 which is the case with the three-factor regressions. Therefore, Fama and French (1993) found in their study that a market factor plus a factor for each size and value are suitable to explain the cross-section of average stock returns. The authors' three-factor model (see Section 2.2) has gained great popularity in research and laid the foundation for further studies in that field.

The momentum factor was initially documented by Jegadeesh and Titman (1993) who observed investment strategies that buy past winners and sell past losers.[6] The authors recognized that these strategies result in excess returns which are not attributable to their systematic risk.

The discovery of these relations between certain firm characteristics (labeled as factors) and returns provided the basis for new explanations regarding the cross-section of expected stock returns. Accordingly, the theory of different factors as return drivers shaped the way not only for further empirical studies but also for a new investment approach, known as *factor investing* (see Ang, 2014).

The fact that specific firm attributes appeared to contribute to the understanding of the cross-section of expected stock returns has obviously inspired many researchers. That became apparent in a review by Harvey et al. (2016). The authors tested more than 300 proposed factors with the result that the majority of them may be attributable to data mining and inappropriate statistical significance levels. However, a few factors have been validated by different scientific works. For example, Dimson et al. (2017) stated that the five factors size, value, momentum, income and volatility should be

[3]The size is measured by the firm's market capitalization.
[4]The book-to-price ratio puts a firm's book value and its stock price into relation.
[5]The book-to-market ratio puts a firm's book value and its market value into relation.
[6]Stocks with high returns in the past are considered as past winners while stocks with poor returns in the past are considered as past losers.

monitored by investors. They seem to carry a premium which is not due to systematic risk and hence contradicts the principles of market efficiency[7]. Therefore, researchers have been interested in explaining the factor premiums and how these phenomena come into existence. Indeed, opinions differ on that topic. Ang (2014) broadly discusses factor theories and provides a fundamental overview on factor investing. He represents the camp which considers the observed factor premiums as risk premiums. Investors with factor exposure would be compensated with high returns over longer terms if they accepted the corresponding factor risk. On the other hand, there is a camp which regards the observed phenomena rather as mispricings, e.g. Daniel and Titman (2006). They relate a stock's expected return to the *intangible return* which embodies the element of the past return that cannot be explained by any fundamental reason.

These findings have been particularly interesting for investment professionals. Portfolio managers use the exposure to factors to influence risk and return expectations of their portfolios. Various popular investment strategies, such as *value investing*, *momentum investing* or *low-volatility investing*, are based on the theory of factors (see Ang, 2014). Dimson et al. (2017) mention that exchange-traded funds (ETFs) have had a crucial role in making factor investing (also called *smart-beta* investing) available to a broad range of investors. According to a report by ETFGI[8], assets of $1.13 trillion were invested in equity-based smart-beta ETFs at the end of June 2022. This volume was distributed over about 1250 ETFs from about 200 providers.

1.2 Research topic and approach

This thesis deals with factor-based investment approaches in the field of equity-portfolio management. These portfolio strategies overweight stocks with high exposure and underweight stocks with low exposure to selected firm characteristics. A common problem in factor investing is *unintended exposure*: Increasing the portfolio exposure to a certain factor unintendedly changes the exposure to other factors. This kind of cross-contamination of factor portfolios makes performance attribution and the analysis of factor-return characteristics difficult or even impossible.

In this thesis, I study a factor-based portfolio approach with a special feature regarding the factor exposure. The aim of the approach is to utilize factor-mimicking portfolios that implement pure single-factor exposure and hence come without any unintended exposure to other factors. This is achieved by imposing orthogonality on the exposure of the factor-mimicking portfolios. Moreover, I examine another factor-based approach that follows a more classic way to create factor portfolios without making use of any special characteristics. Throughout the analyses, I consider the three factors size, value and momentum. In the course of this thesis, I want to address the following research

[7]A market is considered as efficient when stock prices fully reflect all available information. For that reason, firms are always fairly valued and investors cannot beat the market on a risk-adjusted basis. Fama (1970) provides an early review of the theory. Ang (2014) concludes that markets are not perfectly efficient. Accordingly, he refers to studies that consider markets as near-efficient.

[8]`https://etfgi.com/news/press-releases/2022/07/etfgi-reports-smart-beta-etfs-listed-globally-gathered-net-inflows-788`, last accessed 03-10-2022

questions:

- What are the return characteristics of factor portfolios that implement orthogonal exposure to selected firm characteristics? Does orthogonality in the space of firm characteristics translate into uncorrelated factor returns?

- What are the effects of the three considered factors size, value and momentum on the portfolio performance?

For that purpose, I set up an analysis model with the programming language Python. This model allows me to apply the aforementioned construction approaches and build factor-based stock portfolios. Here, the exposure to the three considered factors can be adjusted to desired levels. Further, the model lets me calculate retrospective portfolio returns and performance. Eventually, I use these functions to perform backtests over a multi-year time horizon. In this way, I can then analyze and compare both construction approaches on different aspects to find anwers to the stated research questions.

The analyses in this research work are based on a comprehensive dataset from the data provider MSCI which contains all relevant stock information.[9] In fact, the dataset provides firm and index data related to the stock market index MSCI USA[10] in form of weekly observations. Besides stock and index returns, several firm characteristics of index constituents are available. In the backtests, required data is retrieved from the dataset which is then further processed for the retrospective implementation of the considered investment strategies. The strategy implementation is basically done by determining portfolio weights on a weekly basis according to the latest information at the respective point in time. This leads two weekly portfolio returns and to a certain performance over a selected time period. The factor portfolios are constructed in a two-step process. In the first place, the market-weighted index portfolio is created. Then, a factor-based long-short portfolio is added to the market portfolio. In this way, the original weights of the market portfolio are adjusted and the factor exposure is modified. Therefore, the determination of the factor-based long-short portfolios represents the key step in the construction procedure.

As already mentioned above, I examine two different portfolio construction approaches in this thesis. The two approaches build on the same general concept where rank scores are used as given firm characteristics that represent the exposure to a factor (see Dangl, 2022). The rank scores are assigned to the firms based on a factor-related firm characteristic. However, the two approaches differ significantly in the way the factor-based long-short portfolios are determined. The first approach makes use of an optimization method presented by Dangl (2022). The aim is to receive a long-short portfolio with orthogonal exposure to the considered factors. Additionally, this long-short portfolio should lead to an overall factor portfolio with minimum tracking-error relative to the index portfolio as reference. The second approach simply assigns equal positive

[9]The dataset was provided by the IQAM Research Center in the course of an analysis performed for IQAM Research. I want to acknowledge the support of IQAM Research Center in my research.
[10]`https://www.msci.com/our-solutions/indexes/developed-markets`, last accessed 31-10-2022

weights to the high-scoring firms representing the long side of the long-short portfolio. Likewise, it assigns equal negative weights to the low-scoring firms representing the short side of the long-short portfolio. In this case, no further special condition is sought.

The thesis is outlined as follows. In this first chapter, I give a brief overview of the emergence of factors in the world of portfolio management over time. I introduce some popular scientific publications on the role of factors in explaining average stock returns. Further, I mention how these findings have been used by investment professionals to make factor investing available to the public. In Chapter 2, I deal with theories behind the occurrence of factors and I present a few famous models that combine certain factors to explain stock returns. Moreover, I specifically cover observations and theories related to the three mainly considered factors throughout this thesis, namely size, value and momentum. In Chapter 3, I describe the two portfolio construction approaches that are examined in the course of this research work. At that stage, the focus lies on the mathematical backgrounds of both approaches. In Chapter 4, I then explain how the analysis model written in Python is actually set up. I address the dataset used for the backtests as well as the key steps required for constructing the factor portfolios and calculating performance. In Chapter 5, I discuss the backtest results realized by selected factor-based investment strategies over a 20-year time period. I analyze certain characteristics related to the two construction approaches and I compare the approaches to each other on different aspects. Also, I look at the effects of short-sale constraints on the considered strategies. Eventually, Chapter 6 concludes the main findings.

Chapter 2

Factor theories

In this chapter, I introduce theories behind factor premiums that try to explain why these phenomena occur in the market. Also, I present a few factor models that attracted attention in research due to their ability to explain average stock returns. Further, I cover the mainly considered factors throughout this thesis in more detail by bringing up important observations on each of them.

2.1 Factor premiums

As already mentioned in Section 1.1, the identification of different factors has played an important role in explaining the cross-section of expected stock returns. The fact that other factors than the systematic risk (market beta) seem to carry a premium and influence average returns led to various questions. Particularly, researchers were interested in how the factor premiums come about. In this regard, I briefly mentioned two camps in Section 1.1 representing factor risk theories on the one hand and mispricing theories on the other hand. Ang (2014) provides a thorough argumentation of this question by relating the premiums to factor risk. Accordingly, I will put the focus on risk theories and I will summarize essential aspects of them below.

Ang (2014) states that factor risks are the underlying drivers of the risk premiums of assets. If the risk premium of a factor is positive, then a higher factor exposure results in a higher expected return of the asset. An important term that the author often uses in his explanations is '*bad times*'. According to him, each factor represents a different set of bad times and investors, who are able to bear the losses in bad times, will be compensated with factor risk premiums. In order to illustrate that with an example, the author uses the CAPM initially formulated by Treynor (1961), Sharpe (1964) and Lintner (1965). The CAPM, which can be seen as the most basic theory of factor risk premiums, is written as

$$E(r_i) - r_f = \beta_{i,m} * [E(r_m) - r_f], \tag{1}$$

with r_i, r_f and r_m as the returns of the considered stock i, the risk free asset and the market. The market beta, which is calculated by $\beta_{i,m} = \text{cov}(r_i, r_m)/\text{var}(r_m)$, represents the factor exposure of stock i. It measures the co-movement of stock i with the market portfolio. Thus, the CAPM declares that there is a single factor driving the stock returns. The factor is the market portfolio and the exposure to the factor is given by the market beta. In this case, Ang (2014) interprets the factor risk as follows. Times of low or negative market returns correspond, so to speak, to the bad times defined by the CAPM. Stocks have different exposure to the market factor and accordingly, the model states that the factor risk premium increases with a higher market exposure. In other words, this means that stocks, which plunge when the market declines, are considered risky and hence investors are compensated with risk premiums. Assuming that the average investor is risk averse, stocks with high market betas need to have high expected returns in order to be held. If a stock pays off in bad times on the other hand, its risk premium is low as it is beneficial to hold.

Ang (2014) further argues that the general consideration of the CAPM, that assets' risk premiums due to underlying factors are the rewards for the losses during bad times, is still valid. However, many studies have found that the CAPM as single-factor explanation for the cross-section of expected stock returns is not valid. Stocks are exposed to different factor risks which can be described with the aid of multifactor models. These models consider various definitions of bad times across many factors and can be written in the form (see Ang, 2014)

$$E(r_i) - r_f = \beta_{i,1}E(f_1) + \beta_{i,2}E(f_2) + \ldots + \beta_{i,k}E(f_k), \tag{2}$$

with $\beta_{i,k}$ as the exposure of stock i regarding factor k. $E(f_k)$ represents the expected risk premium of factor k. For example, f_1 could be again the risk premium of the market factor. In addition, f_2 could be the risk premium of a long-short investment strategy that buys value stocks and sells growth stocks. Studies have shown that value stocks outperform growth stocks in the long term (see Section 2.3.2). In this case, the factors would then define bad times as times with low market returns and (or) times where value stocks underperform growth stocks. Accordingly, in a multifactor model the stock's risk premium is determined by capturing multiple sources of factor risk instead of only the market risk in the CAPM. For enduring the various risk sources, investors need to be compensated (see Ang, 2014). I present a few multifactor models which attracted attention in research in Section 2.2. Furthermore, Ang (2014) provides a breakdown of a few key lessons regarding both the CAPM and the multifactor world which are illustrated in Table 1 below.

2.2 Multifactor models

The arbitrage pricing theory (APT), introduced by Ross (1976) as alternative to the CAPM, is known as the first multifactor model. APT describes that expected stock returns follow a linear factor structure where each factor is weighted with a beta coefficient if arbitrage opportunities have been exhausted. Since then, researchers have developed multifactor models that consider specific attributes.

Table 1: Key lessons of CAPM vs. multifactor models (taken from Ang (2014)).

	CAPM	**Multifactor Models**
Lesson 1	Diversification works. The market diversifies away idiosyncratic risk.	Diversification works. The tradeable version of a factor diversifies away idiosyncratic risk.
Lesson 2	Each investor has his/her own optimal exposure of the market portfolio.	Each investor has his/her own optimal exposure of each factor risk.
Lesson 3	The average investor holds the market.	The average investor holds the market.
Lesson 4	The market factor is priced in equilibrium under the CAPM assumptions.	Risk premiums exist for each factor assuming no arbitrage or equilibrium.
Lesson 5	Risk of an asset is measured by the CAPM beta.	Risk of an asset is measured in terms of the factor exposures (factor betas) of that asset.
Lesson 6	Assets paying off in bad times when the market return is low are attractive, and these assets have low risk premiums.	Assets paying off in bad times are attractive, and these assets have low risk premiums.

A popular model is the three-factor model formulated by Fama and French (1993). Essentially, they modified the CAPM with two additional factors resulting in

$$E(r_i) - r_f = \beta_{i,m}[E(r_m) - r_f] + \beta_{i,SMB}E(SMB) + \beta_{i,HML}E(HML), \tag{3}$$

with SMB and HML as factors that include both the size and the value effect. SMB stands for *small minus big* and represents a long-short portfolio based on the market capitalization of the stocks. The portfolio is meant to be constructed through buying small firms and selling large firms. Therefore, it captures the outperformance of small firms relative to large firms over the long term. HML stands for *high minus low* and likewise represents a long-short portfolio. In this case, the portfolio is formed on the book-to-market ratio[1] of the stocks. By buying stocks with a high book-to-market ratio and selling stocks with a low book-to-market-ratio, the outperformance of value stocks relative to growth stocks is captured. Thus, both SMB and HML are so-called *factor-mimicking* portfolios that carry a positive risk premium. Eventually, Fama and French (1993) found that the factor combination applied in their three-factor model serves as a proper indicator for average stock returns.

Carhart (1997) studied another multifactor model. He extended the above three-

[1] The book-to-market ratio is the book value of a stock divided by its market capitalization.

factor model by a momentum factor which yields a four-factor model written as

$$E(r_i) - r_f = \beta_{i,m}[E(r_m) - r_f] + \beta_{i,SMB}E(SMB) + \\ + \beta_{i,HML}E(HML) + \beta_{i,WML}E(WML), \tag{4}$$

with WML now representing the momentum factor and standing for *winners minus losers*. Analogous to the other two added factors, it is constructed as a factor-mimicking portfolio. To capture the momentum effect, it goes long past winners and short past losers based on a one-year return momentum.[2] The author recognized that the four-factor model further improves on the explanatory power of factors compared to the three-factor model.

2.3 Factors

As already addressed previously, factors carry risk premiums and can hence be seen as return drivers in the long term. For that reason, various factors have gained popularity in both the financial research and in the investment industry. In some literature, factors are generally categorized into different groups (see e.g. Ang, 2014). In this thesis, the focus is put on the so-called *investment-style* factors as they can be utilized to implement factor-based strategies in the stock market. According to e.g. Dimson et al. (2017), the five factors size, value, momentum, income and volatility can be seen as reputable and should be monitored by investors. Within the group of investment-style factors, Ang (2014) further differentiates between static and dynamic factors. The market is a static factor as investors only need to go long to earn a risk premium. On the other hand, the premiums of dynamic factors, like the five aforementioned, can only be collected by trading stocks on a regular basis. To give an example, the size effect is captured by implementing the SMB strategy over a longer term. In the analysis part of this thesis, I only consider the three factors size, value and momentum. Therefore, I want to solely cover these factors in more detail below.

2.3.1 Size

The size factor was first recognized by Banz (1981) as well as Reinganum (1981) and later employed by researchers, e.g. Fama and French (1993), to explain the cross-section of average stock returns. The related size effect refers to the observations that small firms performed better than large firms over long terms, adjusted for their market risk.

Figure 1 shows the market-adjusted performance of the corresponding SMB strategy from 1965 until 2011. The SMB strategy is represented by the solid line and plots the value of \$1 invested over the course of the period. Apparently, SMB reached a maximum short after Banz (1981) published his study. Then, from around 1985 until the end of the displayed period, the risk-adjusted size effect is not visible. Hence, small stocks did not carry a premium in that particular time period. Similarly, Fama and French (2012)

[2] More precisely, the author used eleven-month returns lagged one month. Hence, the most recent month was not considered for calculating the return momentum.

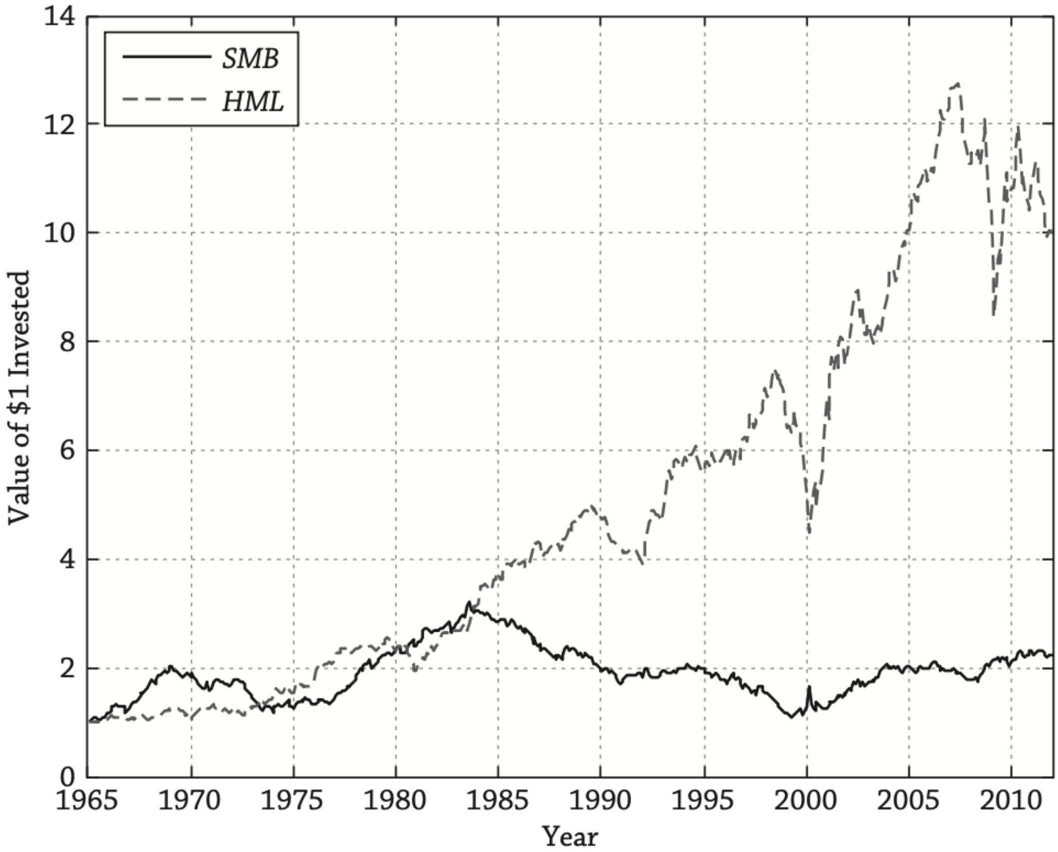

Figure 1: Performance of the market-adjusted SMB and HML strategies from 1965 until 2011 (taken from Ang (2014)).

examined international stock returns from 1990 until 2011 and they did not find a size premium in any region.[3] In their study, average risk-adjusted returns resulting from the SMB strategy are all close to zero. Accordingly, Ang (2014) concludes that there are two potential explanations for the size effect dropping out. First, the size effect was originally found due to data mining and hence the existence might have never been valid (see also Harvey et al., 2016). Second, the effect was present indeed, but was then removed by rational investors who wanted to benefit from the findings and forced up the prices of firms with rather small market capitalization. Schwert (2003) discusses that case and mentions that investment vehicles, set up by practitioners at about the time of the initial discoveries, implemented corresponding strategies. Anyway, important to add here is that the above approached pure size effect is based on market risk-adjusted returns. In general, small firms actually have higher returns than large firms on average (see Ang, 2014).

2.3.2 Value

Research on the value factor goes back to Basu (1977) who found that low P/E portfolios had higher returns than their market exposure would have suggested. Rosenberg et al. (1985) discovered that an HML strategy based on the book-to-price ratio leads to an

[3] The examined regions are North America, Europe, Japan, Asia Pacific.

abnormal performance. Similarly to the size factor, the value factor was then used by researchers, e.g. Fama and French (1993), as indicator for average stock returns. Further, Zhang (2005) provided a study in which he argues the value premium. He highlights that, especially in bad times, having assets in place is much riskier than cutting back on growth options. Accordingly, value firms have to deal with unproductive assets and have more difficulties to reduce capital stocks than growth firms do.

Besides the SMB strategy, Figure 1 also shows the market-adjusted performance of the HML strategy. The HML strategy is represented by the dashed line and likewise plots the value of $1 invested from 1965 until 2011. Compared to size, the value premium persisted throughout the displayed period. Also, the chart illustrates certain periods where the HML strategy had significant losses. Two such periods are particularly noticeable. First, value performed poorly during the late 1990s in the internet bull market. Second, a similar picture becomes apparent during the financial crisis in 2007 and 2008. However, the HML strategy recovered quite quickly from those big losses and value stocks clearly outperformed growth stocks over the displayed period.

According to Ang (2014), a vast number of explanations that emerged for the value effect can be categorized into two groups: rational theories that support the idea of a risk premium and behavioral theories that assume mispricings as the cause. Rational theories describe that value stocks, adjusted for market exposure, tend to perform well or poorly together with other value stocks. Further, it is declared that a portion of value risk can be diversified away. The remaining risk amount cannot be diversified away and must therefore be priced which results in a value premium. On the other hand, behavioral theories often argue the value premium as a consequence of overreaction to recent news (see Daniel and Titman, 2006). The rationale is that value stocks are cheap as their growth prospects are underestimated by investors. At the same time, investors overestimate the prospects of growth stocks and hence they are expensive.

2.3.3 Momentum

The momentum factor was introduced by Jegadeesh and Titman (1993). They recognized that WML strategies lead to significant high returns which are not explained by their systematic risk. The WML strategy refers to an investment approach that buys past winners and sells past losers. A few years later, Carhart (1997) added the momentum factor to the three-factor model by Fama and French (1993) and tested the resulting four-factor model. In fact, the extension had a positive impact on the model's explanatory power. The momentum effect follows the assumption that stocks with high returns in the past months will continue to rise. Likewise, stocks with low or negative returns in the past months will continue to perform poorly. For that reason, momentum strategies are associated with terms like *trend investing* or *the trend is your friend* (see Ang, 2014).

Figure 2 shows the performance of the WML strategy in addition to the SMB and HML strategies over the period from 1965 to 2011. Here, the dashed line plots $1 invested with the momentum strategy. Apparently, momentum outperformed size and value over the displayed period significantly. At the same time, the figure indicates that momentum

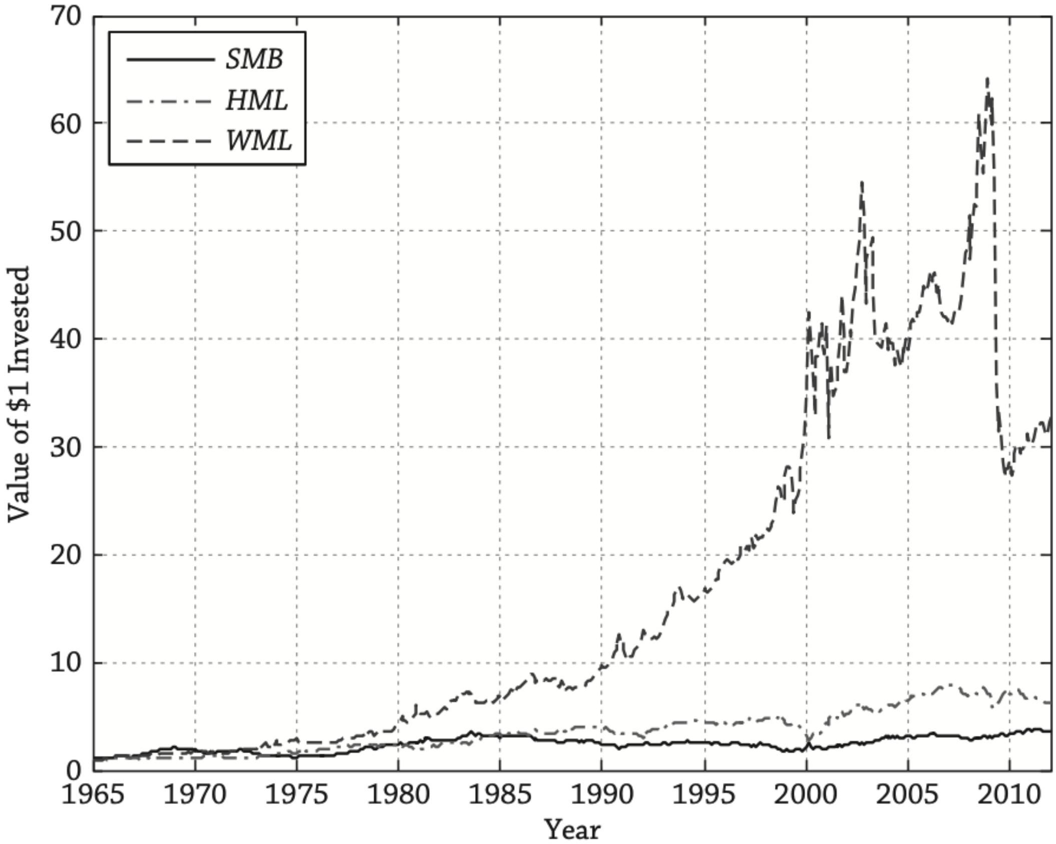

Figure 2: Performance of the market-adjusted SMB, HML and WML strategies from 1965 until 2011 (taken from Ang (2014)).

underlay bigger crashes at certain stages. This was the case, for example, in some periods between 2000 and 2010. From around 2003 until the end of the displayed time window in the figure, momentum realized a negative performance.

Daniel et al. (2012) documented that these sharp losses are due to leverage dynamics in the long-short momentum portfolio. Eventually, high leverage of the portfolio's short side (past losers) drives the tail-risk of the WML strategy. Consistent with that, also Daniel and Moskowitz (2016) investigated the so-called *momentum crashes*. They conclude that the momentum premium is robust in normal environments. In insecure states, however, past losers carry a high premium. The authors mention periods following multi-year market declines or periods of high market volatility as such insecure environments. When the conditions become better and the market recovers, past losers often earn high returns. Since momentum strategies short past losers, a momentum crash is the consequence. Thus, the momentum effect happens in reversed form in these times. During good times, this effect does not apply equally to past winners. Therefore, the authors ascertain an asymmetry regarding the relation of winner and loser exposure and returns in extreme times. As explanations for their findings, Daniel and Moskowitz (2016) studied options such as compensations for crash or volatility risk, with none of these fully accountable. Ang (2014) describes the phenomenon by comparing momentum and value strategies on a specific aspect. Value investors buy stocks that have fallen far enough to an attractive level where they have high expected returns. Hence, value investing generally has a stabilizing effect. On the contrary, momentum investors consider stocks with high past returns as attractive. They buy these stocks which then continue to move up. Eventually, momentum investing is destabilizing which goes along with periodic crashes. Ang (2014) concludes that most of the theories are behavioral based on investors' overreaction or underreaction on stock news.

Chapter 3

Portfolio construction

In this chapter, I describe the two different portfolio construction approaches which are investigated by the backtests in the analysis part of the thesis. In the first instance, I introduce the general idea behind both approaches as well as the used notation.

3.1 Idea and notation

The factor-based portfolios in this thesis are constructed by following the idea of the *target scores* method (see Dangl, 2022). In this method, rank scores with respect to a certain factor are used as given firm characteristics. The weighted portfolio score is defined as the exposure to that factor. Eventually, the objective of the target scores method is to enable the construction of factor portfolios featuring desired exposure levels. This is achieved by combining a market-weighted index portfolio and a factor-based long-short portfolio. The market-weighted index portfolio represents the core component of the resulting factor portfolio while the long-short portfolio is added to adjust the factor exposure according to the target. Therefore, the factor-based long-short portfolios play a key role in both considered construction approaches. Throughout this chapter, I use the following notation (see Dangl, 2022):

n	number of assets
m	number of different factors (firm characteristics)
$\mathbf{S}$	matrix $(n \times m)$ of factor scores, with full column rank
$\mathbf{\Sigma}$	covariance matrix $(n \times n)$ of assets returns, positive definite
$\mathbf{w}$	column vector $(n \times 1)$ of asset weights
$\mathbf{b}$	column vector $(m \times 1)$ of target scores

The rank scores are assigned to the firms based on a factor-related firm metric. Thus, the ranking according to the corresponding firm characteristic is done for each considered factor. Obviously, the respective score follows from the rank. Since the number of relevant firms generally varies, the scores are normalized on a range from 0 to 100.

Eventually, matrix $\mathbf{S}$ contains these scores for each factor in its columns. The respective factor exposure is then represented by the weighted portfolio scores given by $\mathbf{S}'\mathbf{w} = \mathbf{b}$.

Both construction approaches build on this general concept just explained. They differ in the methodology to determine the particular factor-based long-short portfolios. The first approach is based on a tracking-error minimization method and utilizes a factor-mimicking long-short portfolio with orthogonal exposure to selected firm characteristics. In the second approach, the factor-based long-short portfolio weights are simply assigned according to the firm scores. This can be seen as a classic way which does not come with any special properties in terms of return volatility or factor exposure.

3.2 Orthogonal Factor Approach (OFA)

In this approach, a portfolio is being searched that meets the target exposure $\mathbf{b}$ to a selected set of factors and has minimum tracking-error to a certain reference portfolio $\mathbf{w}_0$. In this case, the reference portfolio is a market-weighted index portfolio with weights summing to 1. I describe the construction of $\mathbf{w}_0$ in more detail in Section 4.2.4. Moreover, the long-short portfolios in this approach come with a special feature. In fact, they implement orthogonal exposure to selected factors. The effects of this characteristic represent a main focus point in this thesis.

In the following, I summarize the essential aspects of this construction approach which is presented by Dangl (2022). As already mentioned, the minimum tracking-error weights $\mathbf{w}$, which lead to the target scores $\mathbf{b}$ for a set of factors, should be found. Therefore, the optimization problem can be written as

$$\begin{gathered}\min_{\mathbf{w}} \left\{ \frac{1}{2}(\mathbf{w}-\mathbf{w}_0)'\mathbf{\Sigma}(\mathbf{w}-\mathbf{w}_0) \right\}, \\ \text{such that } \mathbf{S}'\mathbf{w} = \mathbf{b}. \end{gathered} \tag{5}$$

The constraint is used to set a target for the weighted portfolio scores which represent the exposure to the respective factors. Generally, this condition would be satisfied by many portfolios. However, there is only one portfolio with minimum tracking-error weights meeting the target scores. Using the Lagrangian of the above problem

$$L(\lambda, \mathbf{w}) = \frac{1}{2}(\mathbf{w}-\mathbf{w}_0)'\mathbf{\Sigma}(\mathbf{w}-\mathbf{w}_0) + \lambda'(\mathbf{b}-\mathbf{S}'\mathbf{w}), \tag{6}$$

with λ as column vector $(m \times 1)$ of Lagrangian multipliers, leads to the first order optimality criteria

$$\frac{\partial L}{\partial \mathbf{w}} = \mathbf{\Sigma}(\mathbf{w}-\mathbf{w}_0) - \mathbf{S}\lambda = 0, \tag{7}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{b} - \mathbf{S}'\mathbf{w} = 0. \tag{8}$$

Rearranging (7) leads to

$$\mathbf{w}^* = \mathbf{w}_0 + \mathbf{\Sigma}^{-1}\mathbf{S}\lambda, \tag{9}$$

and substituting this optimal $\mathbf{w}^*$ into (8) results in

$$\begin{aligned} \mathbf{S}'(\mathbf{w}_0 + \boldsymbol{\Sigma}^{-1}\mathbf{S}\lambda) &= \mathbf{b} \\ \mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S}\lambda &= -\mathbf{S}'\mathbf{w}_0 + \mathbf{b} \\ \lambda^* &= -(\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S})^{-1}\mathbf{S}'\mathbf{w}_0 + (\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S})^{-1}\mathbf{b}. \end{aligned} \tag{10}$$

As the score matrix $\mathbf{S}$ has full rank and it is assumed that the inverse covariance matrix $\boldsymbol{\Sigma}^{-1}$ exists, also $(\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S})^{-1}$ exists. Substituting the resulting λ^* back into (9) gives the optimal weight vector

$$\begin{aligned} \mathbf{w}^* &= \mathbf{w}_0 - \boldsymbol{\Sigma}^{-1}\mathbf{S}(\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S})^{-1}\mathbf{S}'\mathbf{w}_0 + \boldsymbol{\Sigma}^{-1}\mathbf{S}(\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S})^{-1}\mathbf{b} \\ &= \mathbf{w}_0 - \mathbf{B}\mathbf{S}'\mathbf{w}_0 + \mathbf{B}\mathbf{b} \\ &= \mathbf{w}_0 + \mathbf{B}(\mathbf{b} - \mathbf{b}_0) \\ &= \mathbf{w}_0 + \mathbf{B}\Delta_{\mathbf{b}}, \end{aligned} \tag{11}$$

using the weighted portfolio score of $\mathbf{w}_0$, defined by $\mathbf{S}'\mathbf{w}_0 = \mathbf{b}_0$, and the target score difference $\Delta_{\mathbf{b}} = \mathbf{b} - \mathbf{b}_0$. Further, the matrix $\mathbf{B}$ is introduced which is defined by

$$\mathbf{B} = \boldsymbol{\Sigma}^{-1}\mathbf{S}(\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S})^{-1}, \quad (n \times m). \tag{12}$$

Equation (11) represents the result of this optimization method and yields the minimum tracking-error portfolio $\mathbf{w}^*$. It consists of the market-weighted index portfolio $\mathbf{w}_0$ and a factor-based long-short portfolio determined by $\mathbf{B}\Delta_{\mathbf{b}}$. Hence, $\mathbf{w}^*$ is a linear function in the deviation of the desired target scores $\mathbf{b}$ from the scores $\mathbf{b}_0$ attributable to the reference portfolio.

The matrix $\mathbf{B}$ plays an important role in this construction approach. It contains a set of minimum-variance basis weight vectors with certain factor-mimicking score features in its columns. The minimum-variance property of the columns of $\mathbf{B}$ follows from (11) in the initial form with the reference portfolio $\mathbf{w}_0 = 0$. This results in

$$\mathbf{w}^* = \mathbf{B}\mathbf{b} \tag{13}$$

and represents a solution to the original minimization problem. Also, the factor-mimicking property can be illustrated briefly by considering the first factor of a set of factors. The related basis weight vector is found in the first column of $\mathbf{B}$. So, the vector $\mathbf{w}_{\mathrm{B}_1} = \mathbf{B}_{.,1}$ is defined and the corresponding factor exposure is then given by

$$\mathbf{w}'_{\mathrm{B}_1}\mathbf{S} = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix}. \tag{14}$$

This shows the orthogonality characteristic with respect to the factor exposure of the long-short portfolios in this approach. In this case, $\mathbf{w}_{\mathrm{B}_1}$ leads to an exposure of 1 to the first factor and to an exposure of 0 to all other factors. Essentially, this applies to each basis weight vector $\mathbf{B}_{.,j}$ related to factor j and its resulting factor exposure. This allows to modify the exposure to a certain factor in a controlled way without influencing the exposure to other factors. Moreover, performance implications caused by exposure adjustments regarding a selected factor can be attributed to only that factor. Consequently, this enables an appropriate interpretation of the performance of factor portfolios constructed by using this approach.

In the following, I address another important aspect relevant for the OFA (see Dangl, 2022). Ultimately, the construction procedure must result in a proper overall portfolio $\mathbf{w}^*$ with weights summing to 1. The market-weighted index portfolio $\mathbf{w}_0$ already has that property. This means that the long-short portfolio weights, which are added to $\mathbf{w}_0$, must sum to 0 in order that the overall weight condition is met. However, so far the weights in the factor-mimicking basis vectors of $\mathbf{B}$ do not sum to 0 since no weight constraint has been considered yet. To actually receive this property, the portfolio constraint can be used as a special characteristic which is implemented by putting an additional column of 1s to the first position of $\mathbf{S}$, such that

$$\mathbf{S} = \begin{pmatrix} s_{11} & s_{12} & \dots & s_{1m} \\ s_{21} & s_{22} & \dots & s_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ s_{n1} & s_{n2} & \dots & s_{nm} \end{pmatrix} \rightarrow \begin{pmatrix} 1 & s_{11} & s_{12} & \dots & s_{1m} \\ 1 & s_{21} & s_{22} & \dots & s_{2m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & s_{n1} & s_{n2} & \dots & s_{nm} \end{pmatrix}. \tag{15}$$

This changes the shape of both $\mathbf{S}$ and $\mathbf{B}$ to $(n \times (m+1))$. The first column of $\mathbf{B}$ now contains a portfolio with weights summing to 1 that leads to zero exposure regarding all other considered characteristics. Furthermore, the factor-mimicking weight vectors in the subsequent columns of $\mathbf{B}$ now result in long-short portfolios with weights summing to 0. This is important since then $\mathbf{w}^*$ eventually represents a proper portfolio with weights summing to 1. The weight vector in the first column of $\mathbf{B}$ does not have further impact on the final portfolio.

In terms of the applicability of this portfolio construction approach, a further essential issue has to be clarified. Throughout the explanation of the OFA so far, it was assumed that the inverse of the covariance matrix $\mathbf{\Sigma}^{-1}$ exists. However, this may not be the case in practice. The properties of the sample covariance matrix, based on a history of past stock returns, cause problems which have been documented by, e.g., Jobson and Korkie (1980).

Dangl and Kashofer (2013) explain that issue as follows.[1] A sample covariance matrix $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ is based on a matrix $(T \times n)$ of stock returns over the horizon from $t-(T-1)\Delta t$ to t. The number of considered historical periods T depends on the incremental Δt over which returns are calculated (weekly, monthly, etc.) and the overall time window.[2] The number of observed stocks is represented by n. When dealing with portfolios that contain many different stocks and hence $T < n$, the sample covariance matrix $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ is exposed to large estimation errors and it is singular. The authors state that the rank of $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ is bounded from above by $\min\{n, T-1\}$ and they provide the

[1] The notation has been adapted to be consistent with other parts of the thesis.

[2] According to Dangl and Kashofer (2013), it is common to determine the covariance matrix based on two years of weekly returns.

following example. The weekly returns over two years of the 500 stocks in the S&P 500 are considered for estimating the sample covariance matrix. This gives $n = 500$ and $T = 104$ (observations per stock) which means that $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ has at most rank 103. In such a case, the sample covariance matrix does not have full rank and is therefore not invertible. Consequently, an alternative way is required to get a covariance matrix which can be applied for this portfolio construction approach.

For that purpose, a so-called *shrinkage* method can be used. Shrinkage is a statistical technique that goes back to Stein (1956). Efron and Morris (1977) present a general introduction about shrinkage by addressing real-life examples. Eventually, Jorion (1986) highlights the relevance of shrinkage for portfolio selection problems. With the aid of shrinkage, the sample covariance matrix $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ is combined with the covariance matrix from a structural estimation $\hat{\mathbf{\Sigma}}_{\text{A}}$ (see Dangl and Kashofer, 2013). As such estimations are based on rigid assumptions, shrinking goes along with ignoring information, comprised in the sample covariance matrix, about individual stocks. The balance between losing information contained in $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ and increasing estimation error robustness through the target $\hat{\mathbf{\Sigma}}_{\text{A}}$ can therefore be controlled by the shrinking intensity. The general approach can be written as (see Ledoit and Wolf, 2004; Dangl and Kashofer, 2013)

$$\hat{\mathbf{\Sigma}} = \delta\hat{\mathbf{\Sigma}}_{\text{A}} + (1-\delta)\hat{\mathbf{\Sigma}}_{\text{SMP}}, \tag{16}$$

with the weight $\delta \in (0, 1]$ representing the shrinkage intensity. The sample covariance matrix $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ is, so to speak, shrinked towards the shrinkage target $\hat{\mathbf{\Sigma}}_{\text{A}}$.

There are different ways to determine the target (or estimated) covariance matrix $\hat{\mathbf{\Sigma}}_{\text{A}}$. Dangl and Kashofer (2013) present some of these methods. In the OFA, the method called *shrinkage towards a single-factor market model estimator (SMM)* is used. Therefore, this procedure is introduced in the following. The method builds on the idea of a single-factor market model which is advocated by the CAPM (see Section 2.1). In this case, the covariance matrix receives a structure which assumes that pairwise stock covariances are induced by a single market factor. Thus, the target covariance matrix is determined by (see Dangl and Kashofer, 2013)

$$\begin{aligned}\hat{\mathbf{\Sigma}}_{\text{MM}} &= \hat{\boldsymbol{\beta}}\hat{\boldsymbol{\beta}}'\hat{\sigma}^2_{\text{M}} + \hat{\mathbf{\Sigma}}_{\text{I,M}} \\ &= \begin{pmatrix} \hat{\beta}_1^2 & \hat{\beta}_1\hat{\beta}_2 & \dots & \hat{\beta}_1\hat{\beta}_n \\ \hat{\beta}_2\hat{\beta}_1 & \hat{\beta}_2^2 & \dots & \hat{\beta}_2\hat{\beta}_n \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\beta}_n\hat{\beta}_1 & \hat{\beta}_n\hat{\beta}_2 & \dots & \hat{\beta}_n^2 \end{pmatrix} \hat{\sigma}^2_{\text{M}} + \begin{pmatrix} \hat{\sigma}^2_{\text{I},1} & 0 & \dots & 0 \\ 0 & \hat{\sigma}^2_{\text{I},2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{\sigma}^2_{\text{I},n} \end{pmatrix},\end{aligned} \tag{17}$$

with $\hat{\boldsymbol{\beta}}$ as the column vector $(n \times 1)$ containing each stock i's estimated market beta $\hat{\beta}_i$ $(i = 1, \dots, n)$. $\hat{\sigma}^2_{\text{M}}$ is the sample estimate of the market index variance while $\hat{\sigma}^2_{\text{I},i}$ represents the sample estimate of the idiosyncratic variance of each stock's returns. These estimates vary over time as they depend on a historic window of returns up to a certain point in time t. At that time t, the values $\hat{\beta}_i$ and $\hat{\sigma}^2_{\text{I},i}$ are estimated by using a linear regression of each stock i's most recent T returns $r_{i,s}$ on the corresponding market index returns $r_{\mathrm{M},s}$ in the form

$$r_{i,s} = \alpha_i + \beta_i r_{\mathrm{M},s} + \epsilon_{i,s}, \quad s = t, \dots, t-T+1. \tag{18}$$

The number of observations per stock T depends on the data frequency and the size of the rolling return window. The idiosyncratic variance[3] is determined by $\hat{\sigma}^2_{\mathrm{I},i} = \frac{1}{T-2}\sum \hat{\epsilon}^2_i$. This regression is performed for all considered stocks ($1 \leq i \leq n$).

The estimated target covariance matrix $\hat{\mathbf{\Sigma}}_{\mathrm{MM}}$ and the sample covariance matrix $\hat{\mathbf{\Sigma}}_{\mathrm{SMP}}$ can then be applied according to the above mentioned shrinkage form (16) to get the final estimated covariance matrix (see Dangl and Kashofer, 2013)

$$\hat{\mathbf{\Sigma}}_{\mathrm{SMM}} = \delta_{\mathrm{SMM}}\hat{\mathbf{\Sigma}}_{\mathrm{MM}} + (1-\delta_{\mathrm{SMM}})\hat{\mathbf{\Sigma}}_{\mathrm{SMP}}, \tag{19}$$

with δ_{SMM} being the shrinkage intensity in this method. As already mentioned previously, selecting the shrinkage weight means a tradeoff between increasing the imposed structure of the target matrix and losing information included in the sample matrix. Consequently, researchers have dealt with the right choice of the weight and investigated procedures to optimize the shrinkage intensity (see Ledoit and Wolf, 2003, 2004). In this thesis, however, the fixed value $\delta_{\mathrm{SMM}} = 0.5$ is used for the shrinkage intensity instead of an optimized value.

To sum up, the OFA is based on two essential parts. First, the tracking-error minimization method represents the overall approach to determine the weights of the factor portfolio. Second, the shrinkage method is required to get a covariance matrix which is invertible and can therefore be applied in the overall calculation.

3.3 Correlated Factor Approach (CFA)

This approach follows a simpler procedure and can be seen as a classic way to construct factor portfolios. As mentioned in Section 3.1, the difference between the two approaches lies in the construction of the factor-based long-short portfolios. Ultimately, the long-short portfolios in this approach consist of equal positive and equal negative firm weights. For every factor, these weights are assigned only according to the related normalized rank scores contained in the matrix $\mathbf{S}$. In contrast to the OFA, the portfolios resulting from this approach do not feature any special characteristics in terms of return volatility or factor exposure.

In the following, I show how this construction procedure is actually set up. Let $\mathbf{s}_j$

[3]Greene (2003) discusses properties of a linear regression model and explains how to choose an unbiased estimator of the variance term.

be a column vector containing the rank scores of n firms regarding factor j in the form

$$\mathbf{s}_j = \mathbf{S}_{.,j} = \begin{pmatrix} s_{1j} \\ s_{2j} \\ \vdots \\ s_{nj} \end{pmatrix}, \tag{20}$$

with s_{ij} $(i = 1, \dots, n)$ as the score of firm i on a normalized range from 0 to 100. Based on the scores, a long-short portfolio is created. Firms with scores ≥ 50 receive equal positive weights while firms with scores < 50 receive equal negative weights. For illustration purposes, the long-short portfolio $\mathbf{w}_{j,c}$ can be separated in a long component

$$\mathbf{w}_{j,lo} = \begin{pmatrix} w_{1j,lo} \\ w_{2j,lo} \\ \vdots \\ w_{nj,lo} \end{pmatrix}, \quad (n \times 1), \quad w_{ij,lo} = \begin{cases} 1/n_{lo} & s_{ij} \geq 50 \\ 0 & s_{ij} < 50 \end{cases} \tag{21}$$

with n_{lo} equal to the number of firms with a score ≥ 50, and a short component

$$\mathbf{w}_{j,sh} = \begin{pmatrix} w_{1j,sh} \\ w_{2j,sh} \\ \vdots \\ w_{nj,sh} \end{pmatrix}, \quad (n \times 1), \quad w_{ij,sh} = \begin{cases} 0 & s_{ij} \geq 50 \\ -1/n_{sh} & s_{ij} < 50 \end{cases} \tag{22}$$

with n_{sh} equal to the number of firms with a score < 50. The numbers n_{lo} and n_{sh} are not necessarily the same. However, the weights in $\mathbf{w}_{j,lo}$ always sum to $+1$ while the weights in $\mathbf{w}_{j,sh}$ always sum to -1. These two portfolio components added together represent the overall long-short portfolio

$$\mathbf{w}_{j,c} = \mathbf{w}_{j,lo} + \mathbf{w}_{j,sh} = \begin{pmatrix} w_{1j,lo} + w_{1j,sh} \\ w_{2j,lo} + w_{2j,sh} \\ \vdots \\ w_{nj,lo} + w_{nj,sh} \end{pmatrix} = \begin{pmatrix} w_{1j,c} \\ w_{2j,c} \\ \vdots \\ w_{nj,c} \end{pmatrix}, \quad w_{ij,c} = \begin{cases} 1/n_{lo} & s_{ij} \geq 50 \\ -1/n_{sh} & s_{ij} < 50 \end{cases} \tag{23}$$

with a resulting sum of weights equal to 0. Eventually, the final weights in the CFA are determined similarly to the weights in the OFA (see resulting formula in (11)), such that

$$\mathbf{w} = \mathbf{w}_0 + \mathbf{C}\Delta_{\mathbf{b}}, \tag{24}$$

with $\mathbf{w}_0$ as the index portfolio and $\Delta_{\mathbf{b}}$ representing the desired differential score. Here, the basis weight vectors of the factor-based long-short portfolios are contained in the matrix $\mathbf{C}$ which is defined by

$$\mathbf{C} = \begin{pmatrix} \frac{1}{\bar{s}_1} w_{11,c} & \frac{1}{\bar{s}_2} w_{12,c} & \cdots & \frac{1}{\bar{s}_m} w_{1m,c} \\ \frac{1}{\bar{s}_1} w_{21,c} & \frac{1}{\bar{s}_2} w_{22,c} & \cdots & \frac{1}{\bar{s}_m} w_{2m,c} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\bar{s}_1} w_{n1,c} & \frac{1}{\bar{s}_2} w_{n2,c} & \cdots & \frac{1}{\bar{s}_m} w_{nm,c} \end{pmatrix}, \quad (n \times m). \tag{25}$$

These basis weight vectors build on the aforementioned construction of the factor-based long-short portfolios $\mathbf{w}_{j,c}$. However, there is still another criterion that has to be considered. In order to use (24) in an appropriate way, the basis weights in $\mathbf{C}$ with respect to factor j have to lead to an exposure of 1 to that respective factor (compare with (14)). This is not yet the case with the illustrated long-short portfolio $\mathbf{w}_{j,c}$ since the exposure $\mathbf{w}'_{j,c}\mathbf{s}_j$ generally results in an average portfolio score $\bar{s}_j \neq 1$. Therefore, the weights in $\mathbf{w}_{j,c}$ have to be normalized with that average score such that $\mathbf{C}_{.,j} = \frac{1}{\bar{s}_j}\mathbf{w}_{j,c}$ eventually suits as basis weight vector regarding factor j. In this way, the corresponding factor exposure is equal to the portfolio score $\mathbf{C}'_{.,j}\mathbf{s}_j = 1$ which is a prerequisite for the basis weights in $\mathbf{C}$.

Thus, the exposure to a selected factor can be controlled properly using a desired differential score as it has been the case in the previous approach. Nevertheless, there is an essential difference between the OFA and the CFA in terms of factor exposure. Due to its orthogonality characteristic, the basis weights in $\mathbf{B}$ yield an exposure of 0 to all factors other than the considered factor j (see (14)). This does not apply to the basis weights in $\mathbf{C}$ which may also have a certain exposure to every other factor. This implies for the application of (24) that the differential score $\Delta_{\mathbf{b}_j}$ allows to control the exposure to factor j but that will go along with uncontrolled influence on the exposure to the other factors.

Chapter 4

Analysis model

In this chapter, I describe how the analysis model, which is programmed in Python, is set up. The model is based on various functions that enable backtesting through retrospective portfolio construction and return calculation. In the programmed functions, I make use of a few quite helpful Python packages. The *pandas*[1] library provides powerful data analysis tools while the *numpy*[2] package is used for matrix calculations. In terms of visualizations, the libraries *matplotlib*[3] and *seaborn*[4] are applied. Throughout the explanations in the following sections, I want to demonstrate the main functionalities of the analysis model by providing concrete application examples. First of all, I introduce some aspects regarding the used data.

4.1 Data and notation

The analyses in the backtest part of the thesis are done for US equities. The used dataset is received from the data provider MSCI and contains various historic information on the stock market index MSCI USA and its constituents on a weekly basis. According to MSCI, the index is supposed to cover the large and mid cap segments of the US equity market.[5] The dataset provides data from January 1995 to June 2022.[6]

Generally, each row of the MSCI dataset represents the observation of a specific firm at a certain point in time and contains various qualitative and quantitative stock characteristics. In addition, a data row includes the weekly index return and the return of the risk-free asset for that respective point in time. The following columns of the dataset are used in the analysis model. The column '*isin*' contains the international securities identification number (ISIN) of a stock which also serves as identifier in the model. The observation date is found in the column '*date*'. Since plenty of firms are covered, there are usually many rows referring to the same observation date. The column

[1] `https://pandas.pydata.org/docs/`, last accessed 03-11-2022
[2] `https://numpy.org`, last accessed 03-11-2022
[3] `https://matplotlib.org`, last accessed 03-11-2022
[4] `https://seaborn.pydata.org`, last accessed 03-11-2022
[5] `https://www.msci.com/our-solutions/indexes/developed-markets`, last accessed 17-10-2022
[6] The dataset was provided by the IQAM Research Center in the course of an analysis performed for IQAM Research.

'*IsinIX*' indicates if a stock was constituent of the index at that point in time. Here, the value is either 1 for constituents or 0 for firms which were not part of the index. Further, three return columns are essential for the analyses. The column '*R*' contains the weekly simple total stock return (full reinvestment of dividends is assumed) while the column '*MSCI_ US*' contains the simple total index return and the column '*Rf*' contains the return of the risk-free asset. These values are required to determine the excess stock and index returns which are used for the calculations. In terms of factors, I already mentioned that three different firm characteristics are considered. The underlying attribute of the size factor is the market capitalization which is represented by the column '*MCAP*' in the dataset. For the value factor, the book-to-price ratio in the column '*BTP*' is used and the momentum factor is quantified by the 11-month momentum value[7] contained in the column '*Mom11m*'. In the upcoming sections, I may use the notations MCAP, BTP and MOM to refer to the three factor-related firm characteristics.

So, the dataset points out whether a firm was an index constituent at an observed point in time or not. Based on this information, the index portfolio can be constructed. Figure 3 shows the number of index constituents over time according to the given data.

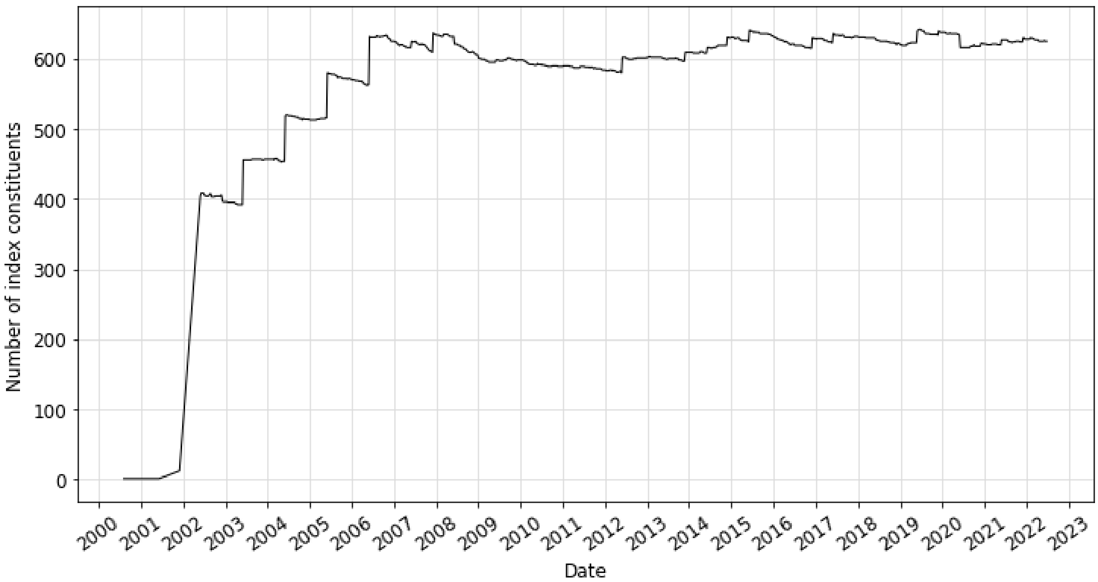

Figure 3: Number of constituents of the stock market index MSCI USA according to the given dataset from August 2000 to June 2022.

Although the overall data history in the set goes back to 1995, the figure demonstrates that analyses, which require information on index constituents, cannot be done in the first several years of the time horizon. Only from around the middle of 2002, a stable and suitable number of constituents is given. This means that proper backtests can be implemented over the period limited from June 2002 to June 2022. Therefore, a 20-year data history is available for analysis.

In the analysis model, covariance matrices are estimated on the basis of simple total

[7] The 11-month momentum is equal to the simple total return over 11 months in the period from $t-12$ to $t-2$ since the return of the previous month is not considered by this value.

returns. In terms of return history and data frequency, 2-year rolling time windows with weekly observations are utilized. For calculations that require stock or index returns, the respective excess returns are generally applied. The excess return is determined by subtracting the return of the risk-free asset from the simple total return of a stock or the index. The rolling windows of excess returns play an essential part in the OFA as they are eventually used for estimating the covariance matrices. Therefore, the structure of these windows is presented in the following, exemplary for a set of n stocks. At time t, the return window

$$\mathbf{R} = \begin{pmatrix} r_{t-T+1,1} & r_{t-T+1,2} & \cdots & r_{t-T+1,n} \\ r_{t-T+2,1} & r_{t-T+2,2} & \cdots & r_{t-T+2,n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{t,1} & r_{t,2} & \cdots & r_{t,n} \end{pmatrix}, \quad \text{with } r_{t,n} = r_{t,n}^{total} - r_{t,n}^{risk\text{-}free} \tag{26}$$

is retrieved from the dataset. As already mentioned before, the rolling window always contains weekly returns over two years which results in 104 weekly return observations for each stock ($T = 104$). The number of stocks n_t depends first on the index constitution at time t. Second, constituents with missing values (returns, firm characteristics), which are relevant for any calculation in the model, are removed from the considered set of stocks. The exact way how relevant stocks are selected is described in Section 4.2.1. In the case of the index returns, the rolling window is generally built in the same way. Obviously, it has only one instead of n_t columns. Throughout this chapter, I use the following notation:

n	number of stocks
m	number of different factors (firm characteristics)
T	number of observations (weekly)
$\mathbf{R}$	matrix ($T \times n$) of excess stock returns
$\mathbf{R}_m$	column vector ($T \times 1$) of excess index returns
$\mathbf{S}$	matrix ($n \times m$) of factor scores (rank scores)
$\boldsymbol{\Sigma}$	covariance matrix ($n \times n$) of stock returns
$\mathbf{w}$	column vector ($n \times 1$) of stock weights
$\mathbf{b}$	column vector ($m \times 1$) of target scores

All the variables listed above generally change with time and should thus carry a subscript t. For brevity reasons, however, I omit the subscript at some points in the sections below.

4.2 Portfolio construction

In this section, I describe how the portfolio construction is set up in the analysis model. I divide the procedure in its key parts by making use of a few subsections. If necessary, I certainly distinguish between the two construction approaches presented in Chapter 3.

4.2.1 Stock selection

The selection of all relevant stocks at a considered point in time t mainly follows a two-step process. First, stocks that were part of the index at time t, which is indicated by the mentioned data field, are pre-selected. In a second step, it is examined for each pre-selected stock whether all required data is available in the dataset. The following conditions have to be met by a stock i:

(i) Every value in the rolling window of stock returns $\mathbf{R}_{\cdot,i}$ has to be contained for the observed stock (see (26)). Additionally, the stock return at the time $t+1$ has to be available as this value is required for the performance calculation (see Section 4.3).[8]

(ii) All three characteristic values MCAP, BTP and MOM have to be available at the time t. Further, the two values MCAP and BTP must be greater than 0. These values are required for assigning the firms with rank scores (see Section 4.2.2).

If any of the conditions is not met by stock i, it is removed from the stock set. Stocks that fulfill all conditions are considered as relevant and are part of the remaining set of stocks. Consequently, all further steps in the portfolio construction procedure are based on this set of stocks. Ultimately, these firms will be found in the resulting portfolio for the regarded point in time t. In order to simplify matters, this set of relevant firms is labeled as F_t.

In the model, lists and sets are applied to carry out the mentioned operations and to store the items. Below, a few short application examples of the model are provided. Table 2 contains the number of firms in certain stages of the stock selection procedure for three different observation dates. The numbers show that there is usually a significant

Table 2: Application example: Number of firms pre-selected, excluded and finally remaining during the stock selection procedure. Three different dates distributed over the available data history are observed.

	2002-06-07	2012-06-01	2022-06-03
Pre-selection	408	602	626
Violation of (i) or (ii)	38	42	58
Remaining	370	560	568

amount of firms where relevant values are missing. In these three cases, the exclusion rate related to the pre-selected amount of stocks ranges from about 7% to about 10%.

[8]Actually, a more detailed analysis would be necessary to determine the respective reason for the missing return at $t+1$ (delisting, default, etc.).

4.2.2 Scoring

As explained in Section 3.1, rank scores serve as foundation for implementing factor-based investment strategies in this thesis. The score of a firm regarding a certain factor j quantifies the exposure of the firm to that factor. Essentially, the scoring itself is done quite intuitively. At time t, the characteristic values with respect to factor j of all n_t firms in F_t are retrieved from the dataset (e.g. MCAP figures for the size factor). Then, the firms are ranked based on their value on a range from 0 to $n_t - 1$. Eventually, the ranks of the firms represent their initial scores. This is done for each factor in the following fashion:

- MCAP figures for size are ranked in descending order. Thus, the firm with the smallest market capitalization in F_t receives the highest score. This may not be intuitive at first glance. In fact, it was set due to the empirical findings regarding the size factor. Since in this case firms with small values carry a premium, they get the higher scores.
- BTP figures for value are ranked in ascending order. Thus, the firm with the highest book-to-price ratio in F_t receives the highest score.
- MOM figures for momentum are ranked in ascending order. Thus, the firm with the highest 11-month return in F_t receives the highest score.

In the model, the scoring is implemented with the rank() function which is integrated in the pandas library. For the ranking, the *average* method is used which becomes relevant when dealing with ties. If ties occur, the respective firms receive a score which is equal to the average rank of those values. For example, there are two firms with equal values for the subsequent ranks 250 and 251. Both firms then receive the average rank $(250 + 251)/2 = 250.5$. More than two equal values are handled analogously.

Due to the fact that the number of firms n_t in F_t generally varies over time (see Table 2), the initial scoring range from 0 to $n_t - 1$ would constantly change either. Therefore, the scoring range is normalized to a scale from 0 to 100. This means that each score s is normalized by $s/(n_t - 1) * 100$. Finally, the normalized scores for each factor can be found in the columns of matrix $\mathbf{S}_t$.

In the following, the scoring applied in the model is illustrated by providing two different tables. Table 3 shows the scoring of a small set of firms regarding the size factor for a specific date. The firms are sorted by their scores which are already contained in normalized form. Hence, the three firms on the top are assigned with the three lowest scores while the three firms at the bottom are assigned with the three highest scores in F_t. Furthermore, the descending ranking order regarding the size factor becomes obvious in this instance. Table 4 presents the normalized scores of another small set of firms within F_t for all three factor-related characteristics. In this case, the firms are sorted by their ISIN in alphabetical order which leads to the structure that is applied in the scoring matrix $\mathbf{S}_t$. In fact, the three columns of Table 4 containing the scores exactly represent the matrix columns, such that

$$\mathbf{S}_t = \begin{pmatrix} 20.105820 & 44.797178 & 94.003527 \\ 66.490300 & 89.594356 & 79.188713 \\ \vdots & \vdots & \vdots \\ 39.329806 & 36.507937 & 1.410935 \end{pmatrix}, \quad (568 \times 3).$$

Until this point, there was no differentiation between the OFA and the CFA necessary. The stock selection and the scoring procedure are implemented in the same way for both portfolio construction approaches. However, the OFA requires an additional characteristic as portfolio constraint (see Section 3.2) which adjusts the matrix to

$$\mathbf{S}_t = \begin{pmatrix} 1 & 20.105820 & 44.797178 & 94.003527 \\ 1 & 66.490300 & 89.594356 & 79.188713 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 39.329806 & 36.507937 & 1.410935 \end{pmatrix}, \quad (568 \times 4).$$

4.2.3 Shrinkage

As presented in Section 3.2, shrinkage enables to estimate a covariance matrix which is invertible and can therefore be used for the tracking-error minimization. Since the method is only applied in the OFA, this subsection is relevant for this particular approach but not for the CFA. According to Section 3.2, shrinkage mainly follows a two-step procedure.

First, the target covariance matrix $\hat{\mathbf{\Sigma}}_{\mathrm{MM}}$ has to be determined by using (17) for the observed point in time t. In order to do this, a linear regression (see (18)) is performed for all stocks in F_t to estimate each stock i's sample estimates for the market beta $\hat{\beta}_i$ and for the idiosyncratic variance $\hat{\sigma}^2_{\mathrm{I},i}$. The regression is based on stock i's most recent return window $\mathbf{R}_{.,i}$ (see (26)) as well as the return window of the index $\mathbf{R}_m$. Additionally, the sample estimate of the market index variance $\hat{\sigma}^2_{\mathrm{M}}$ is needed. $\hat{\beta}_i$ and $\hat{\sigma}^2_{\mathrm{M}}$ are calculated by applying the integrated pandas functions cov() and var(), such that

$$\hat{\beta}_i = \frac{\mathrm{cov}(\mathbf{R}_{.,i}, \mathbf{R}_m)}{\mathrm{var}(\mathbf{R}_m)}$$

leads to the sample estimate of stock i's market beta and

$$\hat{\sigma}^2_{\mathrm{M}} = \mathrm{var}(\mathbf{R}_m)$$

results in the sample estimate of the market index variance. In order to determine $\hat{\sigma}^2_{\mathrm{I},i}$, a few intermediate steps are necessary. Referring to (18), the sample estimate $\hat{\alpha}_i$ is calculated by

$$\hat{\alpha}_i = \bar{r}_i - \hat{\beta}_i \bar{r}_m,$$

with $\bar{r}_i$ and $\bar{r}_m$ being the means of the returns of stock i and the market index. In a next

Table 3: Application example: An extract of firms including their market capitalizations and their corresponding scores. The MCAP values are given in million $.

$t = $ 2022-06-03

ISIN	Name	MCAP	Score
US0378331005	Apple Inc.	2,281,430	0.000000
US5949181045	Microsoft Corporation	1,914,359	0.176367
US02079K1079	Alphabet Inc.	1,444,781	0.440917
...	...	...	...
US22266L1061	Coupa Software Incorporated	5,275	99.647266
US70614W1009	Peloton Interactive, Inc.	4,142	99.823633
US1468691027	Carvana Co.	2,965	100.000000

Table 4: Application example: An extract of firms including their assigned scores regarding all three considered factors.

$t = $ 2022-06-03

ISIN	MCAP-Score	BTP-Score	MOM-Score
AN8068571086	20.105820	44.797178	94.003527
BMG0450A1053	66.490300	89.594356	79.188713
BMG169621056	68.253968	78.483245	91.710758
...	...	...	...
US98980F1049	69.488536	21.340388	71.428571
US98980G1022	54.497354	3.527337	60.317460
US98980L1017	39.329806	36.507937	1.410935

step, the difference between stock i's real return and the return modeled by the linear regression is required for each date considered in the return window. This differential return is represented by the residual value $\hat{\epsilon}_{i,s}$ which is determined by rearranging (18) into

$$\hat{\epsilon}_{i,s} = r_{i,s} - \hat{\alpha}_i - \hat{\beta}_i r_{m,s}, \quad s = t, \dots, t - T + 1,$$

with T being the number of observations for each stock. Eventually, stock i's sample estimate for the idiosyncratic variance can be calculated by[9]

$$\hat{\sigma}^2_{\mathrm{I},i} = \frac{1}{T-2} \sum \hat{\epsilon}^2_i.$$

As already mentioned above, this procedure is performed for each stock in F_t which then allows to use (17) and receive the target covariance matrix $\hat{\boldsymbol{\Sigma}}_{\mathrm{MM}}$.

In the second main step of the shrinkage process, the estimated covariance matrix

[9] Greene (2003) discusses properties of a linear regression model and explains how to choose an unbiased estimator of the variance term.

Table 5: Application example: Rolling window of weekly excess stock returns over two years for an extract of firms.

$t = 2022\text{-}06\text{-}03$

	AN8068571086	BMG0450A1053	⋯	US98980L1017
2020-06-12	-0.136000	-0.124957	⋯	0.057389
2020-06-19	0.037338	-0.041847	⋯	0.109117
2020-06-26	-0.115199	-0.055603	⋯	0.054678
...	...	...	...	...
2022-05-20	0.009131	-0.013234	⋯	-0.053971
2022-05-27	0.172226	0.044718	⋯	0.230247
2022-06-03	-0.011968	-0.015798	⋯	-0.009262

$\hat{\mathbf{\Sigma}}_{\text{SMM}}$ is determined by using (19). Thus, the sample covariance matrix $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ and the shrinkage intensity δ_{SMM} are required in addition to $\hat{\mathbf{\Sigma}}_{\text{MM}}$. The sample covariance matrix is given by $\hat{\mathbf{\Sigma}}_{\text{SMP}} = \text{cov}(\mathbf{R})$ where the integrated pandas function cov() is applied again. For the shrinkage intensity, the fixed value $\delta_{\text{SMM}} = 0.5$ is used throughout the analysis part.

In the following, a few illustrations from applying the model are provided. Table 5 represents a small extract of the 2-year return window for a given date. It shows the weekly returns of three different stocks at the three first dates and the three last dates in the window. The date at the bottom of the table is the considered date t for the estimation of the covariance matrix. The table contains 104 dates because of the rolling window parameters and a set of 568 relevant firms (compare with Table 2) sorted in alphabetical order. These returns in matrix form represent $\mathbf{R}$ (104×568) at time t. Proceeding from this return window, the sample covariance matrix $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ can be calculated by

$$\hat{\mathbf{\Sigma}}_{\text{SMP}} = \text{cov}(\mathbf{R}) = \begin{pmatrix} 0.004762 & 0.001296 & \dots & -0.000682 \\ 0.001296 & 0.001211 & \dots & -0.000417 \\ \vdots & \vdots & \ddots & \vdots \\ -0.000682 & -0.000417 & \dots & 0.006130 \end{pmatrix}, \quad (568 \times 568).$$

As mentioned in Section 3.2, the sample covariance matrix is singular and subject to large estimation errors. For that reason, the rank of the matrix can be checked which results in $\text{rank}(\hat{\mathbf{\Sigma}}_{\text{SMP}}) = 103$. In fact, $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ does not have full rank and is therefore not invertible. This example shows that the sample covariance matrix cannot be used for the minimization method and shrinkage has to be performed.

Accordingly, the sample covariance matrix $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ is shrinked towards the target $\hat{\mathbf{\Sigma}}_{\text{MM}}$

by applying (17), (19) and the shrinkage intensity $\delta_{\text{SMM}} = 0.5$. This leads to

$$\hat{\mathbf{\Sigma}}_{\text{SMM}} = \delta_{\text{SMM}}\hat{\mathbf{\Sigma}}_{\text{MM}} + (1-\delta_{\text{SMM}})\hat{\mathbf{\Sigma}}_{\text{SMP}}$$

$$= \begin{pmatrix} 0.004783 & 0.000870 & \dots & -0.000045 \\ 0.000870 & 0.001215 & \dots & 0.000063 \\ \vdots & \vdots & \ddots & \vdots \\ -0.000045 & 0.000063 & \dots & 0.006157 \end{pmatrix}, \quad (568 \times 568).$$

Comparing the values in $\hat{\mathbf{\Sigma}}_{\text{SMP}}$ and $\hat{\mathbf{\Sigma}}_{\text{SMM}}$ demonstrates that shrinkage goes along with losing information contained in the sample covariance matrix. However, the rank of the estimated covariance matrix now results in $\text{rank}(\hat{\mathbf{\Sigma}}_{\text{SMM}}) = 568$. This is the full rank which means that $\hat{\mathbf{\Sigma}}_{\text{SMM}}$ is invertible. It can thus serve as appropriate covariance matrix for the tracking-error minimization method in the OFA.

4.2.4 Weight allocation

The last essential step of the portfolio construction procedure in the model is the allocation of the portfolio weights. As explained in Chapter 3, in general a factor-based long-short portfolio is added to the market-weighted index portfolio at time t in order to adjust the factor exposure. Here, the two approaches differ in how the long-short portfolio is determined. The approaches have already been thoroughly explained in Section 3.2 and Section 3.3. Eventually, this subsection presents the main steps including illustrations for each of the approaches.

Both approaches build on the calculation of the market-weighted index portfolio $\mathbf{w}_0$ consisting of all n_t firms in F_t (see Section 4.2.1). At time t, the index portfolio is received by

$$\mathbf{w}_0 = \frac{1}{V}\begin{pmatrix} \text{MCAP}_1 \\ \text{MCAP}_2 \\ \vdots \\ \text{MCAP}_n \end{pmatrix}, \quad V = \sum_{i=1}^{n} \text{MCAP}_i,$$

with MCAP_i representing the market capitalization of stock i. The weights in $\mathbf{w}_0$ sum to 1. Applied in the model, Table 6 shows an extract of the weights in $\mathbf{w}_0$ for a selected date in tabular form. The subset is sorted by the weight in order to illustrate the weight range within $\mathbf{w}_0$. For the portfolio, the average factor scores $\mathbf{b}_0$ can be calculated by

$$\mathbf{b}_0 = \mathbf{w}_0'\mathbf{S} = \begin{pmatrix} 1 & 17.187506 & 38.884204 & 58.315269 \end{pmatrix}.$$

In this calculation, the scoring matrix $\mathbf{S}_t$, which has been modified to meet the OFA requirements at the end of Section 4.2.2, is applied. Thus, the first column contains the value-weighted average of $\mathbf{w}_0$ which equals 1. Apart from the first column in $\mathbf{b}_0$, the result states that $\mathbf{w}_0$ has significant exposure to large firms (note scoring order of size)

Table 6: Application example: An extract of firms including their weights in the market-weighted index portfolio $\mathbf{w}_0$, sorted by the weight.

$t = 2022\text{-}06\text{-}03$	
ISIN	Weight
US0378331005	0.063039
US5949181045	0.052896
US02079K3059	0.039921
...	...
US22266L1061	0.000146
US70614W1009	0.000114
US1468691027	0.000082

and perceptible exposures to rather highly-valued firms[10] as well as firms with rather high momentum. Applying the OFA or the CFA now allows to adjust these exposures to a desired direction given by the differential scores $\Delta_{\mathbf{b}}$.

In the OFA, the scoring matrix $\mathbf{S}$ and the inverse of the estimated covariance matrix $\boldsymbol{\Sigma}^{-1} = \hat{\boldsymbol{\Sigma}}^{-1}_{\mathrm{SMM}}$ are required to determine the factor-mimicking basis weights $\mathbf{B}$ for the long-short portfolios with (12). Then, (11) in the form

$$\mathbf{w}_{\mathrm{OFA,j}} = \mathbf{w}_0 + \mathbf{B}_{.,\mathrm{j}}\Delta_{\mathrm{b_j}} \tag{27}$$

can be used with a selected differential score $\Delta_{\mathrm{b_j}}$ to calculate the final portfolio $\mathbf{w}_{\mathrm{OFA,j}}$ for factor j. Applying the model at time t, the basis weights

$$\begin{aligned}\mathbf{B} &= \boldsymbol{\Sigma}^{-1}\mathbf{S}(\mathbf{S}'\boldsymbol{\Sigma}^{-1}\mathbf{S})^{-1} \\ &= \begin{pmatrix} -0.001424 & -0.000003 & -0.000020 & 0.000044 \\ -0.020185 & 0.000113 & 0.000103 & 0.000059 \\ \vdots & \vdots & \vdots & \vdots \\ 0.001033 & 0.000031 & 0.000027 & 0.000056 \end{pmatrix}, \quad (568 \times 4).\end{aligned}$$

are received. The first column of $\mathbf{B}$ contains the portfolio, resulting from the portfolio constraint as special characteristic, which will not be analyzed any further. Each other column represents a factor-mimicking long-short portfolio with respect to a certain factor. These portfolios consist of weights that lead to one additional score of factor exposure towards the related factor and to zero exposure towards the other factors (see Section 3.2). Last but not least, the final portfolio $\mathbf{w}_{\mathrm{OFA}}$ for each factor is determined

[10] An average value score of below 50 indicates a higher exposure to firms with low BTP ratios. Such firms have rather high prices relative to their book values which makes them highly-valued. At the same time, they are considered as low-value firms since they provide less value for their price compared to firms with high BTP ratios.

using the differential score $\Delta_{\mathrm{b_j}} = 20$. Table 7 shows all three resulting portfolios.

Table 7: Application example: An extract of firms including their final weights in every considered OFA portfolio ($\Delta_{\mathrm{b}} = 20$).

$t = 2022\text{-}06\text{-}03$	$\Delta_{\mathrm{b}} = 20$	$\Delta_{\mathrm{b}} = 20$	$\Delta_{\mathrm{b}} = 20$
ISIN	Size	Value	Mom
AN8068571086	0.001634	0.001293	0.002560
BMG0450A1053	0.002709	0.002515	0.001636
BMG169621056	0.001701	0.002139	0.003943
...	...	...	...
US98980F1049	0.000692	-0.000322	0.000646
US98980G1022	0.000437	0.001259	0.001449
US98980L1017	0.001475	0.001396	0.001992

In the CFA, only the scores in $\mathbf{S}$ are needed to determine the basis weights $\mathbf{C}$ for the long-short portfolios (see Section 3.3). Then, similarly to above, (24) in the form

$$\mathbf{w}_{\mathrm{CFA,j}} = \mathbf{w}_0 + \mathbf{C}_{.,\mathrm{j}}\Delta_{\mathrm{b_j}} \tag{28}$$

can be used in this case to calculate the final portfolio $\mathbf{w}_{\mathrm{CFA,j}}$ for factor j. Applying the model at time t again, the basis weights $\mathbf{C}$ result in

$$\mathbf{C} = \begin{pmatrix} -0.00007 & -0.00007 & 0.00007 \\ 0.00007 & 0.00007 & 0.00007 \\ \vdots & \vdots & \vdots \\ -0.00007 & -0.00007 & -0.00007 \end{pmatrix}, \quad (568 \times 3).$$

Each column in $\mathbf{C}$ represents a long-short portfolio with respect to a certain factor. According to the purpose, the creation of $\mathbf{C}$ has been defined in a way that the weights in the portfolios always sum to 0. Same as in the OFA, the portfolio weights lead to one additional score of factor exposure towards the related factor. Unlike in the OFA, the portfolios are not only exposed to the related factor but also to the other factors (see Section 3.3). The next step is again to determine the final portfolio $\mathbf{w}_{\mathrm{CFA}}$ for each factor. The resulting portfolios, based on the differential score $\Delta_{\mathrm{b_j}} = 20$ for every factor, are illustrated in Table 8.

Important to mention is that so far no short-sale constraints have been implemented in the portfolio construction procedures. Therefore, negative weights can be found in both Table 7 and Table 8.

In the following, the difference between the basis weights $\mathbf{B}$ and $\mathbf{C}$ in terms of their implications on factor exposure is addressed. As already mentioned, the basis weight vector in $\mathbf{B}$ with respect to factor j leads to the exposure of 1 towards factor j and to

Table 8: Application example: An extract of firms including their final weights in every considered CFA portfolio ($\Delta_{\text{b}} = 20$).

$t = $ 2022-06-03	$\Delta_{\text{b}} = 20$	$\Delta_{\text{b}} = 20$	$\Delta_{\text{b}} = 20$
ISIN	Size	Value	Mom
AN8068571086	0.000280	0.000280	0.003092
BMG0450A1053	0.001863	0.001863	0.001863
BMG169621056	0.001856	0.001856	0.001856
...	...	...	...
US98980F1049	0.001848	-0.000964	0.001848
US98980G1022	0.001990	-0.000821	0.001990
US98980L1017	-0.000543	-0.000543	-0.000543

the exposure of 0 towards other factors. The basis weight vector in $\mathbf{C}$ with respect to factor j results in the exposure of 1 towards factor j too. Nevertheless, it is also exposed to the other factors. Accordingly, the effects on the final exposure caused by the two approaches will differ. In order to illustrate the differences, the average factor scores given by the index portfolio $\mathbf{w}_0$ (see Table 6), which have been calculated above,

$$\mathbf{b}_0 = \begin{pmatrix} 1 & 17.187506 & 38.884204 & 58.315269 \end{pmatrix},$$

can be observed in the first instance. Eventually, Table 9 shows the final factor exposure of the portfolios resulting from both approaches. The first three rows of the table show the special feature of the OFA. Each of the three OFA factor portfolios are related to one certain factor j. More precisely, the factor-mimicking long-short portfolio with respect to factor j has been used to modify the index portfolio $\mathbf{w}_0$. According to the table, the final exposure to factor j is different to the corresponding average factor score of $\mathbf{w}_0$ which is contained in $\mathbf{b}_0$. In fact, the exposure is exactly increased by $\Delta_{\text{b}} = 20$. The final

Table 9: Application example: Factor exposure of the final OFA and CFA portfolios ($\Delta_{\text{b}} = 20$).

$t = $ 2022-06-03			
Portfolio	Size	Value	Mom
$\mathbf{w}_{\text{OFAsize}}$	37.187506	38.884204	58.315269
$\mathbf{w}_{\text{OFAvalue}}$	17.187506	58.884204	58.315269
$\mathbf{w}_{\text{OFAmom}}$	17.187506	38.884204	78.315269
$\mathbf{w}_{\text{CFAsize}}$	37.187506	41.974365	53.494797
$\mathbf{w}_{\text{CFAvalue}}$	20.612422	58.884204	58.413960
$\mathbf{w}_{\text{CFAmom}}$	11.433936	37.679086	78.315269

exposure to the other factors apart from j is equal to the corresponding average factor scores in $\mathbf{b}_0$. Apparently, the implemented weight adjustments did not influence the exposure to the other factors at all. This is achieved by the orthogonality characteristic of the OFA factor-mimicking basis weight vectors represented by the matrix $\mathbf{B}$. While the basis weights with respect to factor j have an exposure of 1 to the factor j, they have an exposure of 0 to the other factors. In this way, the factor exposure of the final portfolios can be fully controlled which is demonstrated by Table 9. **Also, this characteristic makes it possible to analyze the performance of considered factor portfolios properly. Since implemented differential scores do not lead to unintended exposure changes, performance differences can be clearly assigned to an observed factor.** This feature does not apply for the CFA which can be seen in the bottom three rows of the table. Here, the final factor portfolios also show an exposure to factor j which is increased exactly by $\Delta_\text{b} = 20$. However, the final exposure to the other factors apart from j are not equal to the corresponding average factor scores in $\mathbf{b}_0$. In contrast to before, they have obviously been influenced by the implemented weight adjustments. This occurs due to the missing orthogonality feature of the basis weight vectors in the matrix $\mathbf{C}$. Similar to the OFA, the CFA basis weight vectors with respect to factor j have an exposure of 1 to the factor j. The essential difference is that they do not have an exposure of 0 to the other factors. That is why the final factor exposure cannot be totally controlled by the CFA like it is the case with the OFA. Moreover, this causes troubles in the performance interpretation of CFA factor portfolios. As implemented differential scores generally come along with unintended exposure changes, performance differences cannot be clearly assigned to an observed factor.

4.3 Performance calculation

Performance backtests regarding the discussed investment approaches play a key part in this thesis. The implementation of the factor strategies in the model are set up in a way that a constructed factor portfolio generates a return over one week before a new portfolio is constructed and invested again. This procedure is performed on a weekly basis and results in weekly portfolio returns. The portfolio construction has been described in Section 4.2. The return and performance calculation are presented in the following by observing a general factor portfolio $\mathbf{w}$ over K periods. Basically, this factor portfolio $\mathbf{w}$, which could represent any other of the discussed portfolio strategies, is constructed and invested first at t_0. From then on, the strategy is executed until t_K where the investment period of the last portfolio ends.

At each time $t = t_0, \ldots, t_{K-1}$, the portfolio $\mathbf{w}_t$ is constructed from scratch according to the methodology explained in Section 4.2. In doing so, the construction is based on the latest data at the respective point in time. In general, this leads to a varied set of stocks F_t and modified portfolio weights every week. Then, the constructed portfolio $\mathbf{w}_t$ is invested and generates the return

$$r_{p,t+1} = \mathbf{w}_t * \mathbf{r}_{t+1},$$

over the course of one week until $t+1$. $\mathbf{r}_{t+1}$ contains the weekly returns from t to $t+1$ for all portfolio constituents. The weekly portfolio return $r_{p,t+1}$ can also be written as

$$r_{p,t+1} = r_{m,t+1} + r_{ls,t+1} = \mathbf{w}_{0,t} * \mathbf{r}_{t+1} + \mathbf{w}_{ls,t} * \mathbf{r}_{t+1},$$

where it is separated between the return generated by the index portfolio $\mathbf{w}_{0,t}$ and the return generated by the long-short factor portfolio $\mathbf{w}_{ls,t}$. $\mathbf{w}_{ls,t}$ is equal to $\mathbf{B}_t\Delta_\mathbf{b}$ using the OFA and $\mathbf{C}_t\Delta_\mathbf{b}$ using the CFA. This return is determined weekly at each time $t = t_1, \ldots, t_K$ which leads to K portfolio returns over the observed period, illustrated by

$$\mathbf{r}_p = \begin{pmatrix} r_{p,t_1} \\ r_{p,t_2} \\ \vdots \\ r_{p,t_K} \end{pmatrix}, \quad (K \times 1)$$

These returns can then be used to calculate the performance of the strategy by

$$p_{t+1} = p_t * (1 + r_{p,t+1}), \quad p_0 = 100,$$

with p_0 serving as starting value for the performance backtest. Applying this equation successively gives the weekly performance values

$$\mathbf{p} = \begin{pmatrix} p_0 = 100 \\ p_1 = p_0 * r_{p,t_1} \\ p_2 = p_1 * r_{p,t_2} \\ \vdots \\ p_K = p_{K-1} * r_{p,t_K} \end{pmatrix}, \quad ((K+1) \times 1),$$

resulting in the value p_K. Eventually, p_K represents the performance which would have been achieved retrospectively by implementing the strategy over the selected time horizon. The model allows to carry out performance backtests for all discussed factor strategies $\mathbf{w}_{\text{OFAsize}}$, $\mathbf{w}_{\text{OFAvalue}}$, $\mathbf{w}_{\text{OFAmom}}$, $\mathbf{w}_{\text{CFAsize}}$, $\mathbf{w}_{\text{CFAvalue}}$, $\mathbf{w}_{\text{CFAmom}}$ as well as for the simple index strategy $\mathbf{w}_0$.

In the following, again some examples are provided to illustrate outcomes from applying the model. In doing so, the implementation of the index strategy and the factor strategies are examined over four periods. Table 10 shows the generated returns for all considered strategies related to the OFA. As the strategies are started with the initial portfolio construction and investment on 2022-05-06, there are no returns available for this date. Table 11 shows the achieved performance of these strategies throughout the observed time horizon. The strategies related to the CFA are implemented in the same way. Thus, only the performance values of those strategies are provided in Table 12 for comparative purposes.

In this case, the performance of all the strategies does not differ that much because of the short horizon. The tables should rather serve as quick outcome examples in order to demonstrate the aforementioned procedure. In the actual analysis part of the thesis, the backtests are certainly performed over multi-year time horizons to examine the strategies in more detail.

Table 10: Application example: Portfolio returns of the index strategy and the OFA strategies ($\Delta_b = 20$) over four periods from 2022-05-06 to 2022-06-03.

$\Delta_b = 20$

Date	$\mathbf{w}_0$	$\mathbf{w}_{\text{OFAsize}}$	$\mathbf{w}_{\text{OFAvalue}}$	$\mathbf{w}_{\text{OFAmom}}$
2022-05-06	-	-	-	-
2022-05-13	-0.022981	-0.020960	-0.022143	-0.026571
2022-05-20	-0.032368	-0.031759	-0.026236	-0.038967
2022-05-27	0.063737	0.061836	0.060469	0.062181
2022-06-03	-0.010569	-0.011320	-0.011594	-0.012046

Table 11: Application example: Portfolio performance of the index strategy and the OFA strategies ($\Delta_b = 20$) over four periods from 2022-05-06 to 2022-06-03.

$\Delta_b = 20$

Date	$\mathbf{w}_0$	$\mathbf{w}_{\text{OFAsize}}$	$\mathbf{w}_{\text{OFAvalue}}$	$\mathbf{w}_{\text{OFAmom}}$
2022-05-06	100.000000	100.000000	100.000000	100.000000
2022-05-13	97.701948	97.903973	97.785745	97.342915
2022-05-20	94.539501	94.794629	95.220199	93.549751
2022-05-27	100.565200	100.656349	100.978068	99.366786
2022-06-03	99.502341	99.516954	99.807357	98.169777

Table 12: Application example: Portfolio performance of the index strategy and the CFA strategies ($\Delta_b = 20$) over four periods from 2022-05-06 to 2022-06-03.

$\Delta_b = 20$

Date	$\mathbf{w}_0$	$\mathbf{w}_{\text{CFAsize}}$	$\mathbf{w}_{\text{CFAvalue}}$	$\mathbf{w}_{\text{CFAmom}}$
2022-05-06	100.000000	100.000000	100.000000	100.000000
2022-05-13	97.701948	97.774365	97.544734	97.709500
2022-05-20	94.539501	94.801499	95.104281	94.198838
2022-05-27	100.565200	100.606865	101.111078	100.328961
2022-06-03	99.502341	99.331945	99.804140	99.349463

Chapter 5

Backtests

In this chapter, I document the outcomes generated by implementing various backtests. The analyses are based on the model which I explain in Chapter 4. Particularly, the focus of the backtests lies on the return characteristics of the addressed factor-based investment strategies. I also investigate the impact of modified factor exposure through the OFA and the CFA on the portfolio performance. Moreover, I observe the performance as well as the average portfolio scores of the market-weighted index portfolio. Further, I am interested in the consequences of short-sale constraints on the portfolio approaches. For the backtests, I use the 20-year time period from the beginning of June 2002 until the beginning of June 2022. The following investment strategies (portfolios) are analyzed in the sections below:

$\mathbf{w}_0$	Market-weighted index portfolio
$\mathbf{w}_{\text{OFAsize}}$	Factor portfolio with adjusted size exposure built by the OFA
$\mathbf{w}_{\text{OFAvalue}}$	Factor portfolio with adjusted value exposure built by the OFA
$\mathbf{w}_{\text{OFAmom}}$	Factor portfolio with adjusted momentum exposure built by the OFA
$\mathbf{w}_{\text{CFAsize}}$	Factor portfolio with adjusted size exposure built by the CFA
$\mathbf{w}_{\text{CFAvalue}}$	Factor portfolio with adjusted value exposure built by the CFA
$\mathbf{w}_{\text{CFAmom}}$	Factor portfolio with adjusted momentum exposure built by the CFA

In these cases, the increased size exposure means the overweighting of small firms while the increased value and momentum exposure indicate the overweighting of low-valued firms and firms with high momentum. Below, I also observe the long-short parts of the portfolios only that are responsible for adjusting the factor exposure. There, I use the suffix 'LS' for the portfolio notations above (e.g. $\mathbf{w}_{\text{OFAsizeLS}}$).

5.1 Market-weighted index portfolio

The market-weighted index portfolio $\mathbf{w}_0$ represents the foundation of each of the mentioned strategies when it comes to the weight allocation in the final portfolios at a certain point in time t. As mentioned previously, $\mathbf{w}_0$ is constructed based on the underlying index data of the MSCI USA. However, $\mathbf{w}_0$ has not exactly the same constellation as the original index. This is due to the stock selection procedure where all firms with missing data are excluded from the relevant set of stocks F_t (see Section 4.2.1). Eventually, this leads to less portfolio constituents and in further consequence to a little performance deviation compared to the original index. This consequence is illustrated by Figure 4. In this case, the adjustments in the index portfolio result in a slight overperformance of

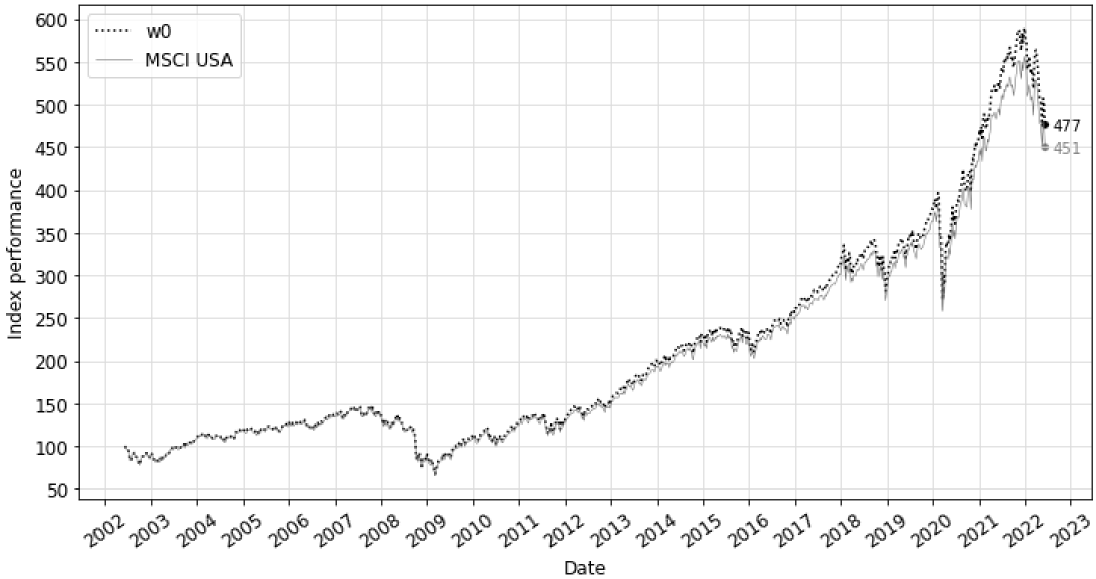

Figure 4: Performance deviation between the market-weighted index portfolio $\mathbf{w}_0$ applied in the analysis model and the original index MSCI USA over the time period from 2002-06-07 to 2022-06-10.

$\mathbf{w}_0$. That matter will not be analyzed any further. However, since $\mathbf{w}_0$ serves as reference portfolio in the analysis model, I wanted to provide this information at that point.

In the following, I look at another interesting aspect of the index portfolio $\mathbf{w}_0$. In Section 4.2.4, the average factor scores of $\mathbf{w}_0$ are calculated exemplary for a certain point in time. As explained previously, those average scores $\mathbf{b}_0$ represent the exposure of the index portfolio to the considered factors. Figure 5 shows the factor exposure of $\mathbf{w}_0$ over the available 20-year time period.

The blue line shows the exposure to the size factor which is constantly on a rather low level close to 20 throughout the entire period. As already mentioned in Section 4.2.4, the low exposure is caused by the scoring order of the size factor. While small firms receive high scores, large firms get low scores in the analysis model. Since $\mathbf{w}_0$ is a market-weighted portfolio, the firm weights are allocated in reversed order to the scores, with large firms receiving high weights. Therefore, $\mathbf{w}_0$ is exposed to large firms leading to low average size scores. Compared to the others, the blue line runs quite stable. This is mainly because of the direct relation between the portfolio weights and the size scores.

The value exposure is represented by the green line which starts at an average score of about 36 and ends at a score of about 39. So, the value exposure at the beginning of the period is similar to the exposure at the end of the window. However, in this case obvious fluctuations can be observed in between. Mostly throughout the time period, the

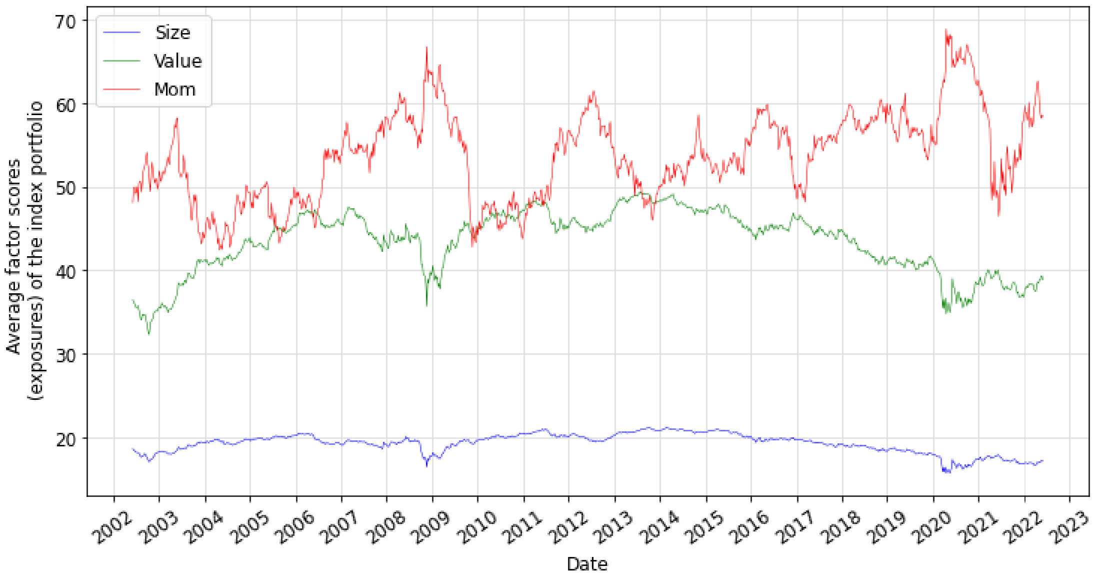

Figure 5: Average factor scores (exposure) of the market-weighted index portfolio $\mathbf{w}_0$ over the time period from 2002-06-07 to 2022-06-03.

line moves between an exposure of about 35 and 50. Thus, the constituents of $\mathbf{w}_0$ have rather been highly-valued. Temporarily, the exposure was close to 50 which indicates a balanced value exposure of the portfolio.

Last but not least, the red line shows the momentum exposure of $\mathbf{w}_0$. In the 20-year period, the exposure increased from an average score of about 48 to about 58 amid significant fluctuations. At some points the exposure was at the lower levels of a bit under 45. On the other hand, the exposure nearly reached a value of 70 mid-April 2020. Most of the time the factor score is above 50 which points out that $\mathbf{w}_0$ is exposed to firms with rather high momentum.

5.2 Factor-based long-short portfolios

In this section, I put the focus only on the factor-mimicking long-short component of the portfolios. Since the respective weights in the long-short parts always sum to 0, they do not represent proper portfolios. Nevertheless, the long-short components play a key part in the portfolio construction as they enable to adjust the factor exposure in a desired direction. Therefore, I examine the returns and the performance attributable to them independent from the portfolio core component $\mathbf{w}_0$. Furthermore, it is interesting to observe differences between the basis weights determined by the OFA and the CFA.

Accordingly, Table 13 shows the correlation coefficients between the returns generated by the basis weight vectors of both the OFA and the CFA. Returns within the available 20-year period have been used for the calculation. The table points out a few considerable relations. First of all, it can be seen that there is hardly any correlation between the returns of the OFA basis long-short portfolios. This is actually quite interesting since the OFA construction framework is supposed to impose orthogonality

Table 13: Correlation coefficients between the returns generated by the OFA and CFA basis weight vectors ($\Delta_b = 1$) over the time period from 2002-06-07 to 2022-06-10.

LS	OFAsize	OFAvalue	OFAmom	CFAsize	CFAvalue	CFAmom
OFAsize	1.000000	-0.024242	0.040262	0.442367	0.098093	-0.070243
OFAvalue	-0.024242	1.000000	-0.086432	0.167255	0.459340	-0.211546
OFAmom	0.040262	-0.086432	1.000000	-0.472630	-0.558175	0.769475
CFAsize	0.442367	0.167255	-0.472630	1.000000	0.732061	-0.719029
CFAvalue	0.098093	0.459340	-0.558175	0.732061	1.000000	-0.748767
CFAmom	-0.070243	-0.211546	0.769475	-0.719029	-0.748767	1.000000

on the factor exposure. Apparently, this leads to barely correlated returns which further indicate a considerable degree of return orthogonality. With respect to the OFA, the data therefore suggest an unexpected interdependence between the factor exposure and the returns. On the other hand, the CFA factor returns show a different picture. In fact, they seem to be highly cross-contaminated given the corresponding correlation coefficients. In contrast to the OFA, the CFA does not implement the orthogonality feature regarding the factor exposure in the portfolio construction. The data points out a significant positive correlation between the CFAsizeLS and the CFAvalueLS returns. Further, there are significant negative correlations between the CFAmomLS and both other CFA returns. In terms of performance interpretation, these relations cause troubles which will be addressed on the basis of Table 14 below.

Across the two construction approaches there is a low correlation between the OFAsizeLS and the CFAsizeLS returns according to Table 13. Similar to that, also the OFAvalueLS and the CFAvalueLS returns are weakly correlated. At the same time, the OFAmomLS and the CFAmomLS returns show a significant correlation between each other. Additionally, there is a low to moderate negative correlation between the OFAmomLS returns and both the CFAsizeLS and CFAvalueLS returns.

Next, I also want to address the retrospective performance realized by the long-short factor portfolios throughout the 20-year time period. For that purpose, Table 14 summarizes the backtest results at the end of the period for varied differential scores. Obviously, the size of the differential score only has an amplifying effect on the realized performance. Whether a long-short portfolio influences the performance in a positive or negative way is already determined by the corresponding basis weight vectors. Anyway, the table shows some interesting backtest outcomes for the observed period. For example, an increased exposure to the size factor comes with a premium with both construction approaches. On the other hand, the momentum factor has negative performance implications in both approaches. Considering the value factor, the impact depends on the particular approach. While the factor causes an underperformance with the OFA, the factor generates a premium with the CFA. With the CFAsizeLS and the CFAvalueLS portfolio not only the two best-performing long-short portfolios are based on the CFA.

Table 14: Performance of the factor-based long-short portfolios with varying Δ_b over the time period from 2002-06-07 to 2022-06-10 ($p_0 = 100$).

LS	OFAsize	OFAvalue	OFAmom	CFAsize	CFAvalue	CFAmom
$\Delta_b = 1$	100.3948	99.6002	99.5094	100.8836	100.7722	99.3436
$\Delta_b = 5$	101.9815	98.0078	97.5380	104.4498	103.8385	96.6451
$\Delta_b = 10$	103.9819	96.0332	95.0565	108.9751	107.6110	93.1230
$\Delta_b = 15$	106.0008	94.0770	92.5601	113.5693	111.3005	89.4590
$\Delta_b = 20$	108.0378	92.1395	90.0530	118.2251	114.8905	85.6790

Likewise, the worst-performing long-short strategy, represented by the CFAmomLS portfolio, originates from the CFA.

When interpreting the performance displayed by Table 14, the correlation data contained in Table 13 has to be taken into account. Especially with respect to the CFA, the high cross-contamination in the factor returns causes troubles in the performance assessment. More precisely, the relations make it difficult to attribute performance to a certain factor. For example, the CFAvalueLS returns have a correlation of about 73% to the CFAsizeLS returns. Therefore, it is difficult to assess whether the relatively good performance of the CFAvalueLS portfolio comes from the value factor itself or rather from the fact that the portfolio levers on the size factor which in turn enables a good performance. Similarly, this interpretation issue follows from the correlation of about -72% between the CFAmomLS and the CFAsizeLS returns. Again, it cannot be clearly evaluated whether the underperformance of the CFAmomLS portfolio is caused by negative performance implications of the momentum factor or by the situation that the CFAmomLS portfolio takes disadvantage from the size effect in reversed form. In contrast to that, the overperformance of the OFAsizeLS portfolio as well as the underperformance of the OFAvalueLS and the OFAmomLS portfolio can be attributed to the particular factors. This is possible due to the orthogonality feature in the factor exposure.

Another point becomes apparent looking at Table 14. In fact, the extent of the performance implications seems to be generally greater in the CFA than in the OFA. This makes sense since the OFA portfolios are constructed to be minimum tracking-error portfolios. In turn, this means that the corresponding factor-based long-short portfolios are the minimum-variance portfolios meeting the desired score characteristics. The CFA portfolios are constructed without taking the return volatility into account. Thus, they show a behavior with greater effects on performance both in a positive and a negative sense.

In addition to Table 14, the aforementioned findings can be observed in Figure 6. Moreover, the line charts show the performance over time. For the purpose of illustration, I used the differential score $\Delta_b = 20$ in the figure. In this way, the differences in the behavior of the long-short strategies are presented more clearly than it would be the case

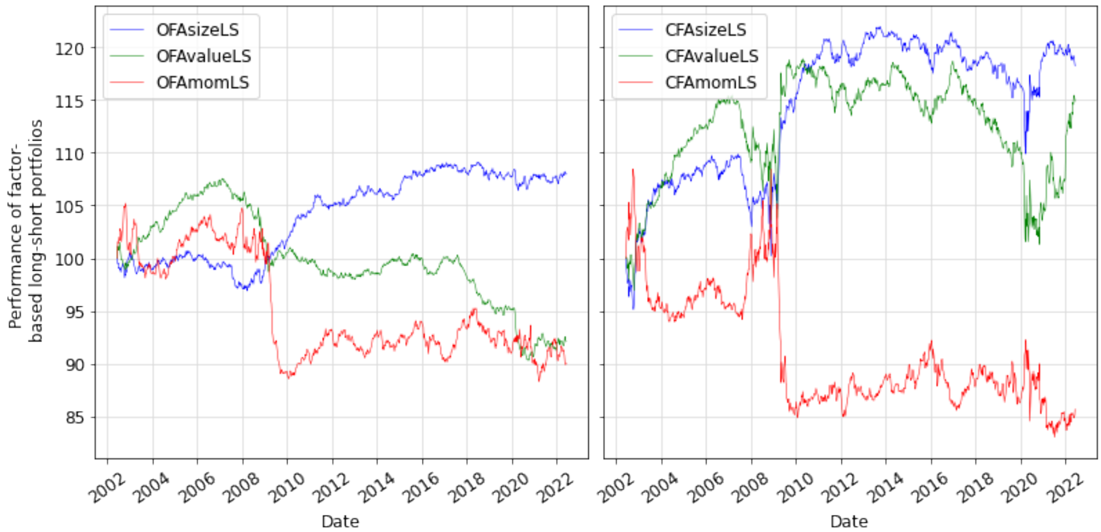

Figure 6: Performance of the factor-based long-short portfolios ($\Delta_\mathrm{b} = 20$) over the time period from 2002-06-07 to 2022-06-10 ($p_0 = 100$).

with lower differential scores. Indeed, the figure shows some interesting phenomena. In general, both sides of the figure make obvious that the time around the financial crisis marks a key period in terms of consequences for the total performance. In fact, different trend-setting developments with respect to the long-short portfolios took place in the years from about 2007 until 2010 according to the backtests.

Considering the left-hand side with the OFA charts first, it becomes apparent that the OFAvalueLS portfolio was the best-performing long-short strategy in the first few years followed by the OFAmomLS portfolio. Both of them had a positive performance in the beginning in contrast to the OFAsizeLS portfolio which slightly moved downward until 2008. However, from then on a trend reversal is recognizable. The OFAsizeLS portfolio started into a multi-year upward period before moving rather sidewards in the more recent years to end up as the best-performing OFA long-short strategy. At the same time, the OFAvalueLS portfolio passed through an overall downward trend following the positive performance from the first few years. Furthermore, the OFAmomLS portfolio was subject to a significant performance drop around 2009. After that, there has not been any serious recovery and it more or less moved sidewards until the end of the observation period.

On the right-hand side of Figure 6, the development of the CFA long-short portfolios is displayed. Likewise, the value factor had the most positive performance impact over the first few years until about 2007. The size factor also had positive influence while the momentum factor was the one with negative performance impact over that particular period. The time following is characterized by significant movements. In fact, the CFAsizeLS and the CFAvalueLS portfolio had a short but steep performance decrease before they entered a positive trend from around 2009 again. On the other hand, the CFAmomLS portfolio had a clear performance increase before passing through a sharp performance drop in 2009. Eventually, the CFAsizeLS portfolio outperformed the CFAvalueLS portfolio from then on. However, neither of them showed a real positive performance from about 2010 until the end of the considered period. Especially, the CFAvalueLS portfolio started into a significant downward trend in 2017 that lasted until about 2021. This trend is followed by a sharp recovery. The CFAmomLS portfolio essentially moved sidewards from 2010 going through certain up and down trends until the end of the window.

Actually, some of the relations discussed based on the correlation coefficients in Table 13 can be found again in Figure 6. For example, the line charts on the left behave quite independent from each other throughout the observation period. In contrast to that, a pattern is recognizable in the right-hand charts. While the CFAsizeLS and the CFAvalueLS portfolio move similar to each other, the path of the CFAmomLS portfolio seems to be opposed to the others in many cases. These occurrences correspond to the correlation data in Table 13. Comparing the two size portfolios OFAsizeLS and CFAsizeLS, similar trends can be found at least at some stages over the observation period. This is also valid for the two value portfolios across both approaches featuring similar-looking behavior at times. Accordingly, moderate correlation coefficients are stated in the table. Table 13 highlights a significant correlation between the two momentum portfolios. This is also underlined by the respective line charts in red which behave quite similarly in many stages.

With respect to the momentum factor, Figure 6 shows another phenomenon which has already been mentioned in Section 2.3.3. There, I referred to the sharp losses of the momentum long-short portfolio that have occurred from time to time (see Daniel et al., 2012; Daniel and Moskowitz, 2016). As explained above, these momentum crashes usually happen during periods of general market recoveries after bigger declines. Consistent with that, such crashes can be detected in the momentum line charts in Figure 6. The first one is visible around 2003 where the market started to recover from the downturn in the early 2000s. The second and most significant one occurred around 2009. Here, the market found itself in a rebound following the financial crisis 2007-2008. Therefore, the figure makes obvious that these momentum crashes represent serious setbacks for the performance of the long-short momentum strategies.

5.3 Overall factor-based portfolios

Eventually, I analyze the performance of the overall factor-based portfolios which consist of the index portfolio $\mathbf{w}_0$ combined with the long-short factor portfolios. As already mentioned previously, $\mathbf{w}_0$ represents the core of the overall portfolios while the additional long-short components adjust the factor exposure. In contrast to Section 5.2 before, this section deals with proper portfolios featuring weights summing to 1.

First of all, I am interested in the total performance of the overall factor portfolios. For that purpose, Table 15 summarizes the retrospective performance of the index as well as all considered OFA and CFA strategies over the 20-year time period. Same as in Table 14, the backtests have been done for varied differential scores in order to get an

Table 15: Performance of the overall factor-based portfolios with varying Δ_b over the time period from 2002-06-07 to 2022-06-10 ($p_0 = 100$).

	$\mathbf{w}_0$	OFAsize	OFAvalue	OFAmom	CFAsize	CFAvalue	CFAmom
$\Delta_b = 1$	476.9233	478.7938	474.9184	474.9566	479.9917	479.3688	475.0733
$\Delta_b = 5$	476.9233	486.3110	466.9387	467.0162	492.2487	488.8948	467.1996
$\Delta_b = 10$	476.9233	495.7854	457.0555	456.9367	507.5122	500.1901	456.3383
$\Delta_b = 15$	476.9233	505.3445	447.2759	446.7043	522.6793	510.7513	444.4236
$\Delta_b = 20$	476.9233	514.9862	437.6025	436.3386	537.7151	520.5234	431.5469

idea of the impact of the exposure level. According to the expectations, all three strategies where the long-short components generated a positive performance, were able to outperform the index portfolio $\mathbf{w}_0$. This applies to the OFAsize, CFAsize and CFAvalue portfolio. Analogously, the other three strategies performed worse than $\mathbf{w}_0$. Further, the performance ranking in Table 15 is consistent with the one in Table 14. As already figured out in Section 5.2, the CFAsize strategy turns out to be best-performing retrospectively over the observed 20-year period. It outperforms $\mathbf{w}_0$ by about 61 percentage points overall. In contrast, the CFAmom strategy underperforms $\mathbf{w}_0$ by about 45 percentage points.

At that point, I want to follow up on an observation made in Section 5.2. I mentioned the smaller performance implications of the OFA compared to the CFA which I attributed to the minimum-variance characteristics of the OFA long-short portfolios. In order to also examine this based on the backtest data, I determined the annualized tracking-error of the overall factor portfolios towards $\mathbf{w}_0$ over the considered time period.[1] Table 16 contains the values which were again calculated for varied differential scores. Indeed, the table shows that the values of the OFA portfolios, which are supposed to be

Table 16: Annualized tracking-error of the overall factor-based portfolios with varying Δ_b towards $\mathbf{w}_0$ over the time period from 2002-06-07 to 2022-06-10.

TE	OFAsize	OFAvalue	OFAmom	CFAsize	CFAvalue	CFAmom
$\Delta_b = 1$	0.000624	0.000675	0.001294	0.001499	0.001992	0.002439
$\Delta_b = 5$	0.003121	0.003374	0.006469	0.007494	0.009959	0.012197
$\Delta_b = 10$	0.006242	0.006747	0.012938	0.014989	0.019918	0.024394
$\Delta_b = 15$	0.009364	0.010121	0.019406	0.022483	0.029876	0.036592
$\Delta_b = 20$	0.012485	0.013494	0.025875	0.029977	0.039835	0.048789

minimum tracking-error portfolios, are smaller than those of the CFA portfolios for all factors. Therefore, the determined tracking-errors underline the expectations. Also, the table illustrates that the values increase the higher the applied differential score which is

[1] I used the formula TE $= \sqrt{(\mathbf{w}-\mathbf{w}_0)'\boldsymbol{\Sigma}(\mathbf{w}-\mathbf{w}_0) * 52}$ for the annualized tracking-error. The used weekly return history comprises the entire 20-year time period.

obvious. Based on the performance and the tracking-error, the information ratio (IR) of each factor strategy can be calculated.[2] In order to do that, I annualized the performance figures from Table 15. The annualized values are presented by Table 17.[3] The difference in annualized performance between each factor portfolio and $\mathbf{w}_0$ divided by the related tracking-error in Table 16 results in each portfolio's information ratio. Eventually, all information ratios are displayed by Table 18. The values allow to compare the perfor-

Table 17: Annualized return of the overall factor-based portfolios with varying Δ_b over the time period from 2002-06-07 to 2022-06-10.

	$\mathbf{w}_0$	OFAsize	OFAvalue	OFAmom	CFAsize	CFAvalue	CFAmom
$\Delta_\text{b} = 1$	0.081241	0.081452	0.081013	0.081017	0.081588	0.081517	0.081031
$\Delta_\text{b} = 5$	0.081241	0.082295	0.080098	0.080107	0.082952	0.082582	0.080128
$\Delta_\text{b} = 10$	0.081241	0.083340	0.078943	0.078929	0.084607	0.083819	0.078858
$\Delta_\text{b} = 15$	0.081241	0.084375	0.077777	0.077708	0.086205	0.084952	0.077432
$\Delta_\text{b} = 20$	0.081241	0.085400	0.076599	0.076443	0.087746	0.085980	0.075849

Table 18: Information ratio of the overall factor-based portfolios with varying Δ_b based on annualized returns and with $\mathbf{w}_0$ as benchmark.

IR	OFAsize	OFAvalue	OFAmom	CFAsize	CFAvalue	CFAmom
$\Delta_\text{b} = 1$	0.339032	-0.337513	-0.172655	0.231351	0.138843	-0.086123
$\Delta_\text{b} = 5$	0.337794	-0.338879	-0.175343	0.228340	0.134667	-0.091255
$\Delta_\text{b} = 10$	0.336243	-0.340584	-0.178701	0.224571	0.129442	-0.097672
$\Delta_\text{b} = 15$	0.334690	-0.342285	-0.182056	0.220796	0.124214	-0.104090
$\Delta_\text{b} = 20$	0.333133	-0.343983	-0.185407	0.217015	0.118981	-0.110509

mance of the factor portfolios by also taking their risk, measured by the tracking-error, into account. The effects can be seen quite well by considering the three factor portfolios that outperform the benchmark $\mathbf{w}_0$. Table 17 shows again that the CFAsize portfolio had the best overall performance ahead of the CFAvalue and the OFAsize portfolio. Adding risk as additional evaluation criterion, the situation changes however. In fact, on a risk-adjusted basis, the OFAsize portfolio can be seen as the most attractive factor strategy ahead of the CFAsize and the CFAvalue portfolio (see Table 18). This fact highlights the idea behind the OFA properties.

Additionally, Figure 7 shows the performance behavior of the overall portfolios over time in comparison to the index portfolio. The top part of the figure contains the performance charts of the OFA strategies while the performance charts of the CFA strategies are displayed in the bottom part. Again, I used the differential score $\Delta_\text{b} = 20$ for illustration purposes. Basically, the line charts now combine the behavior of both

[2]The information ratio puts the performance difference between a portfolio and a benchmark in relation to the corresponding tracking-error.

[3]Since the performance figures are based on a 20-year time period, I calculated the annualized performance for each portfolio by using the formula $p_y = (\frac{p_K}{p_0})^{1/20} - 1$.

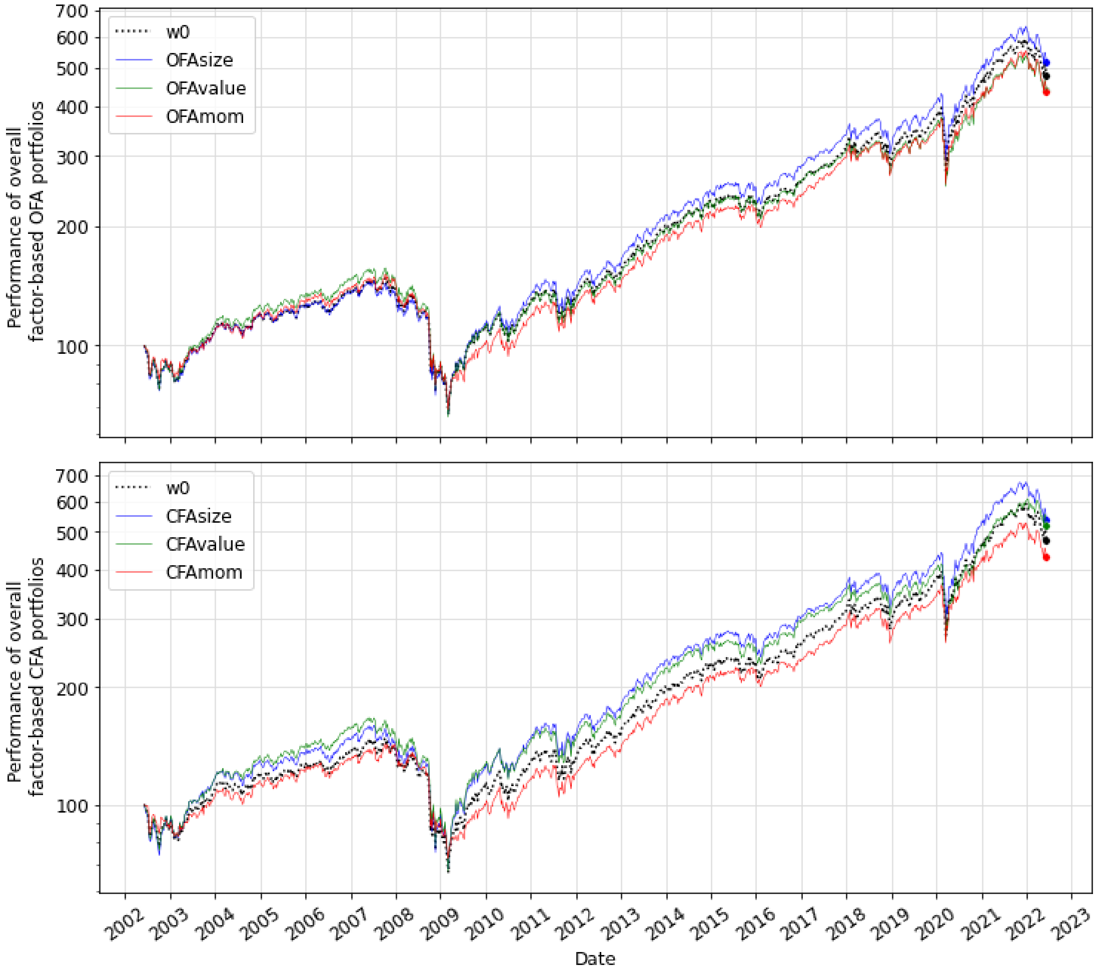

Figure 7: Performance (log scale) of the overall factor-based portfolios ($\Delta_b = 20$) over the time period from 2002-06-07 to 2022-06-10 ($p_0 = 100$).

the market component and the long-short components. All OFA and CFA line charts show a significant similarity to the performance chart of $\mathbf{w}_0$. This is due to the fact that the index portfolio represents the core of each of the factor-based portfolios. The deviations of the factor strategies from $\mathbf{w}_0$ are caused by overlaying the related long-short portfolios. In the displayed cases, the deviations from $\mathbf{w}_0$ correspond to the long-short behavior discussed in Section 5.2 and illustrated in Figure 6. Like already ascertained before, the performance charts in Figure 7 exhibit that the size portfolio generated the highest performance with both approaches over the 20-year period. At the same time, the momentum portfolios had the worst performance.

In the following, I want to address the resulting factor premiums with respect to both the OFA and the CFA. Thus, Table 19 comprises the premium generated by each factor-based portfolio over the considered 20-year time period. Important to mention here is that the values are obtained from dividing the absolute performance differences between the factor portfolios and the index portfolio by the applied differential score. Accordingly, all factor strategies that perform better than $\mathbf{w}_0$ lead to positive premiums. In turn, the factor premiums of the portfolios, which underperform $\mathbf{w}_0$, are negative. Further, the

Table 19: Premium generated by the overall factor-based portfolios with varying Δ_b over the time period from 2002-06-07 to 2022-06-10. The values are given in percent per additional factor score.

	OFAsize	OFAvalue	OFAmom	CFAsize	CFAvalue	CFAmom
$\Delta_\mathrm{b} = 1$	1.870494	-2.004908	-1.966691	3.068394	2.445533	-1.849967
$\Delta_\mathrm{b} = 5$	1.877533	-1.996920	-1.981419	3.065072	2.394303	-1.944746
$\Delta_\mathrm{b} = 10$	1.886210	-1.986786	-1.998659	3.058885	2.326681	-2.058502
$\Delta_\mathrm{b} = 15$	1.894748	-1.976492	-2.014599	3.050402	2.255198	-2.166646
$\Delta_\mathrm{b} = 20$	1.903142	-1.966043	-2.029238	3.039587	2.180003	-2.268820

table shows that the exact magnitude of the premiums depends on the differential score. The highest factor premium overall is achieved by the CFAsize portfolio and results in around 3%. The best-performing OFA strategy, which is represented by the respective size portfolio, yields a factor premium of around 1.9%. The three underperforming factor strategies lead to negative premiums in the region of -2%.

5.4 Balanced OFA portfolio

In this section, I put the focus on a specific factor portfolio that is based on the OFA. Essentially, this exemplary strategy should demonstrate an interesting application opportunity of the OFA. As already mentioned at some prior stages of the thesis, the OFA is special in terms of its implications on the factor exposures. In fact, through the orthogonality characteristic, the exposure to a certain factor can be influenced without any consequences on the exposure to the other factors. Thus, the factor exposure can be fully controlled which is not possible with the CFA (see Chapter 3 and Section 4.2.4).

In the following, I address a factor portfolio where the constellation is enabled through the OFA. I examine the performance of that strategy, denoted as OFAbal50, which features balanced exposure to all three considered factors. In that case, this means that the average portfolio score at each point within the observation period results in 50 for every factor. In order to reach the target scores, the required differential scores depend on the average factor scores of the index portfolio $\mathbf{w}_0$ which vary over time (see Figure 5). Accordingly, the differential score regarding each factor j has to be determined each point in time t, such that $\Delta^j_{\mathrm{b,t}} = \mathrm{b} - \mathrm{b}^j_{0,t}$ ($\mathrm{b} = 50$). This distinguishes this strategy from the strategies discussed above where the differential scores were fixed. Moreover, in the OFAbal50 strategy all three factor exposures are modified simultaneously in one portfolio. This is also different to the factor strategies above where only the exposure to one factor has been adjusted.

Eventually, Figure 8 shows that OFAbal50 outperformed $\mathbf{w}_0$ clearly in retrospect. Moreover, it seems that there are hardly any periods where the strategy lost significantly against the index portfolio. The strategies behaved quite similarly until the end of the

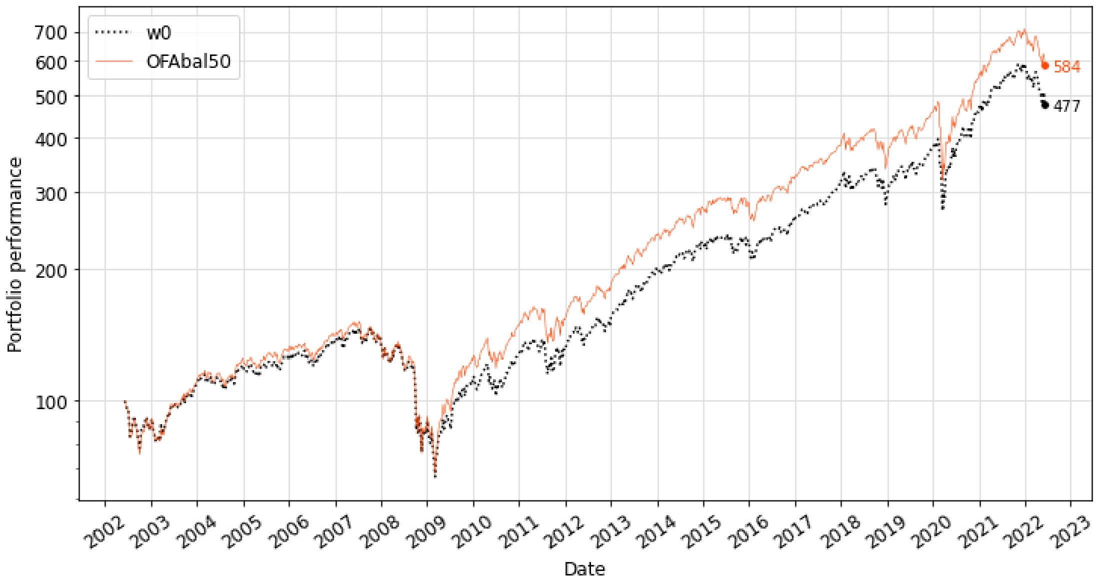

Figure 8: Performance (log scale) of a factor-based portfolio with balanced exposure (b = 50) over the time period from 2002-06-07 to 2022-06-10 ($p_0 = 100$).

financial crisis around the beginning of 2009. From then on, the OFAbal50 started to perform better than $\mathbf{w}_0$. In interim stages, the line charts moved a little closer again. However, in many periods the factor strategy managed to outperform the index. Ultimately, the performance difference at the end of the entire observation period results in 107 percentage points in favor of the OFAbal50 portfolio.

5.5 Short-sale constraints

In the above sections, the factor strategies have been examined simply by definition. Whether the resulting portfolio weight of a company is positive or negative has not played a role so far. Depending on the applied differential score, overlaying the index portfolio $\mathbf{w}_0$ with a long-short portfolio can obviously lead to negative weights in the overall portfolio. In this section, I want to take that aspect into account and test the limitations of the original factor strategies by considering short-sale constraints.

First of all, I am interested in the particular differential score levels that can be implemented without having any company with negative weights in the portfolios. Briefly speaking, the maximum exposure change has to be figured out where the overall factor portfolio would lose its long-only attribute. This has to be examined for both positive and negative Δ_b values since negative weights can be caused by either increasing or decreasing factor exposure. Furthermore, the calculations have to be done at each point in time separately as the portfolio constellations vary over time. The respective backtest results for all OFA and CFA factor portfolios are displayed in Figure 9 and Figure 10.

In general, the figures show that already the application of small differential scores yield negative weights in the overall factor portfolios. The maximum score decreases, which are represented by the line charts in the bottom parts of the figures, result in

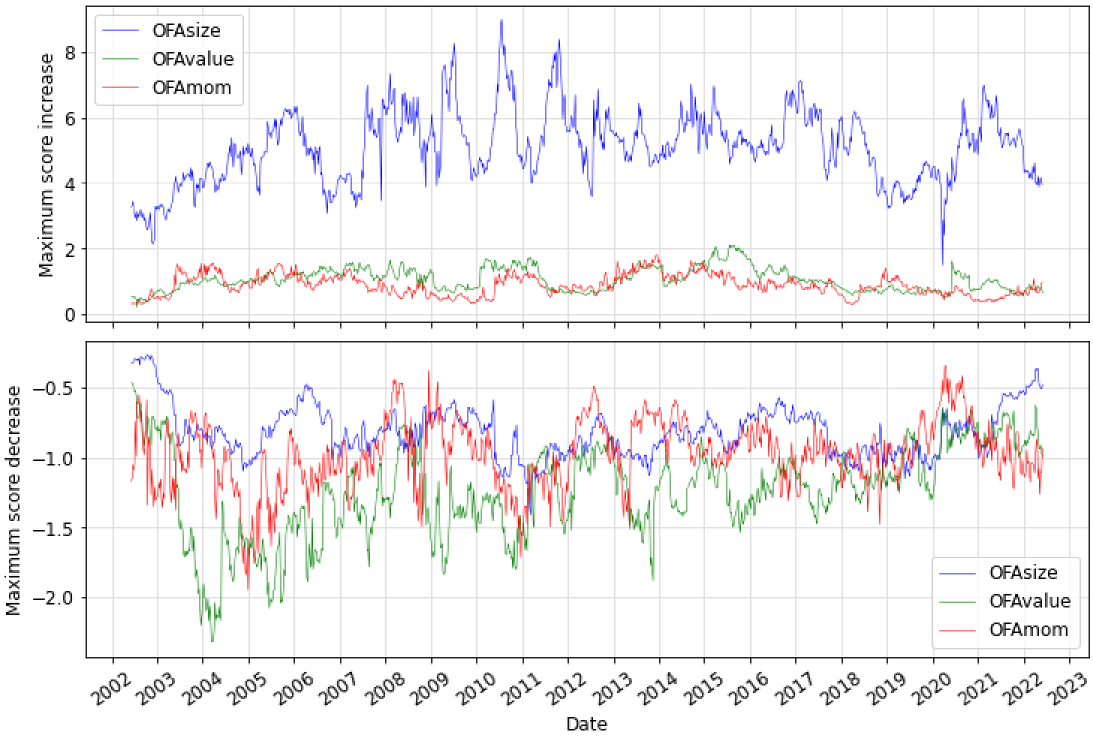

Figure 9: Maximum applicable differential score Δ_b before the first occurrence of a negative weight in the OFA space over the time period from 2002-06-07 to 2022-06-03.

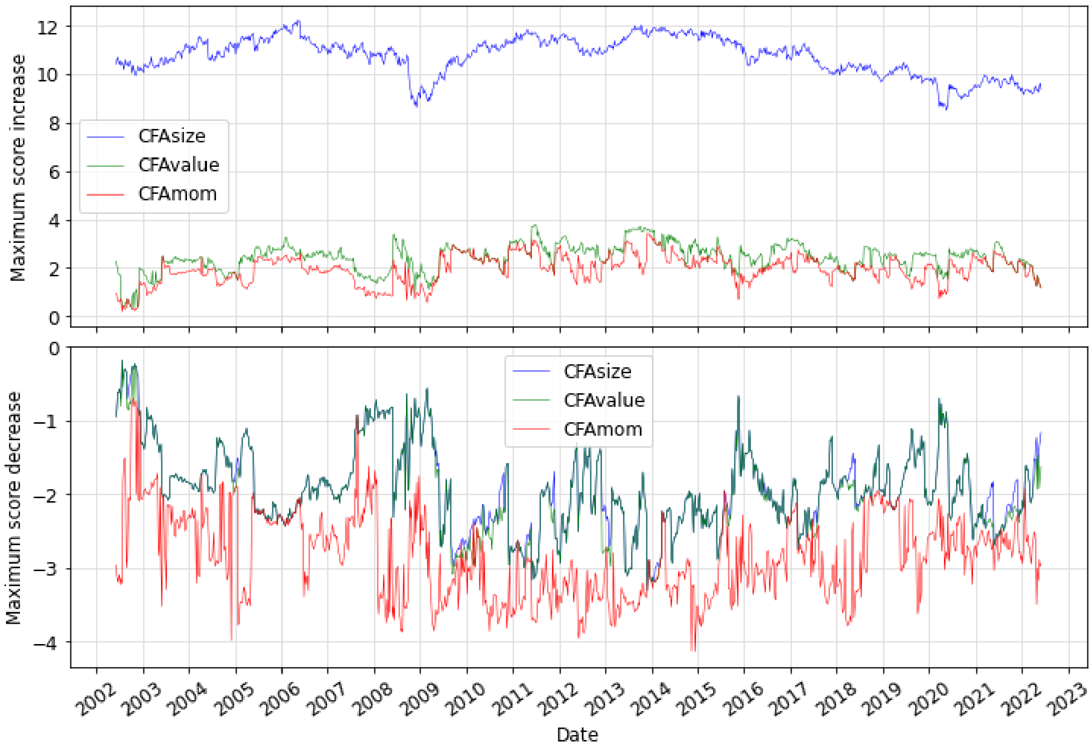

Figure 10: Maximum applicable differential score Δ_b before the first occurrence of a negative weight in the CFA space over the time period from 2002-06-07 to 2022-06-03.

relatively low absolute values in basically all cases. The negative differential scores of the OFA portfolios mainly move within a range of 0 and -2. Analogously, the differential score range for the CFA portfolios lies between 0 and -4. The top parts of the figures contain the maximum score increases. Here, the picture is a little different. In fact, the maximum differential scores for the value and momentum portfolios also remain on low levels within ranges between 0 and 2 as well as 0 and 4 respectively. At the same time, the maximum differential scores for the size portfolios step out of line and reach higher levels. While the OFAsize line chart moves in a quite volatile manner within a range between 2 and 9, the CFAsize line chart moves much more steadily in a range between 9 and 12. The higher absolute values in these two cases are due to the relation of $\mathbf{w}_0$ and the size factor. On one side, large firms receive high weights in $\mathbf{w}_0$ as the portfolio is market-weighted. On the other side, large firms tend to have negative basis weights in the corresponding long-short portfolio in consequence of the scoring order related to the size factor. Hence, the constellations of $\mathbf{w}_0$ and the basis weights more likely allow higher differential scores in these cases. The difference of the OFAsize and the CFAsize line charts in terms of fluctuations comes from the particular calculation method of the basis weights (see Chapter 3). In the CFA, the basis weights are determined only according to the factor scores. Therefore, the negative weights in the long-short portfolio are always calculated against the portfolio weights in $\mathbf{w}_0$ which are related to the larger companies. Since the long-short portfolio contains equal negative weights, always the smallest company that only just receives negative basis weights is decisive for the maximum positive differential score for CFAsize. Typically, that differential score should not be subject to unsteady short-term movements. In contrast, the basis weights in the OFA are determined by the optimization method described in Section 3.2. Here, the allocation procedure is not that straightforward and hence the decisive company cannot be anticipated in such an obvious manner. Thus, the blue line charts in the top parts of Figure 9 and Figure 10 correspond to expectations.

As already mentioned above, the differential score ranges where the overall factor portfolios consist of long-only weights are exhausted quickly. Typically, already little modifications in the factor exposure yield negative weights in the final portfolio. A differential score of e.g. 4 in positive or negative direction leads to the loss of the long-only attribute in most of the cases. Only the size portfolios usually allow somewhat greater margins in positive direction. Here, the maximum score increases reach levels of about 9 and 12 respectively according to the provided figures. Essentially, this means that long-only factor strategies can only be implemented with the model in a very limited form. In order to enable backtests of long-only strategies in a more flexible way, I integrated the portfolio construction under short-sale constraints in the analysis model.

With respect to the OFA, the long-only portfolio construction is again based on the above-introduced minimization problem which is given by (5) in Section 3.2. The first difference compared to above is that this problem is here extended with two weight conditions. They should enable the construction of proper long-only portfolios $\mathbf{w}$ where each weight w_i is greater or equal to 0. Moreover, all weights in $\mathbf{w}$ have to sum to 1.

Thus, the adapted minimization problem can be written as

$$\min_{\mathbf{w}} \left\{ \frac{1}{2}(\mathbf{w} - \mathbf{w}_0)' \mathbf{\Sigma} (\mathbf{w} - \mathbf{w}_0) \right\},$$
$$\text{such that } \mathbf{S}'\mathbf{w} = \mathbf{b},\ \sum_{i=1}^{n} w_i = 1 \text{ and } w_i \geq 0.$$

The second difference from above lies in the problem solving. While above the problem is solved analytically, the Python solver package *cvxpy*[4] is used at this point to determine the long-only minimum tracking-error (LOMTR) weights. Important to mention is that the orthogonality feature of the factor portfolios is gradually lost here.[5] Accordingly, I will denote the corresponding factor strategies with the term LOMTR.

With respect to the CFA, short-sale constraints have been integrated directly in the construction framework described in Section 3.3. In terms of the general weight calculations, nothing has been changed to the basic framework from above. The main difference is that the construction framework is applied within a loop to eliminate negative weights. Initially, the CFA factor portfolio $\mathbf{w}$ is determined as ever by using (28) on the basis of the relevant set of stocks F_t. Then, the loop comes into play which is configured as follows:

(i) All negative weights in $\mathbf{w}$ are set to 0 and concerned firms are excluded from further calculations.

(ii) Remaining firms with non-negative weights in $\mathbf{w}$ are subjected to the construction procedure again. This means that both the market weights $\mathbf{w}_0$ and the basis weights contained in $\mathbf{C}$ are determined from scratch based on the remaining set of firms (see Section 3.3 and Section 4.2.4). Then, (28) is applied again to receive the portfolio $\mathbf{w}$ with modified weights.

(iii) Above steps (i) and (ii) are repeated until $\mathbf{w}$ only consists of weights greater or equal to 0. Thus, $\mathbf{w}$ represents the long-only CFA (LOCFA) portfolio. All firms from the initial set of stocks F_t are now contained in $\mathbf{w}$ with weights that are either positive or equal to zero.

In the following, I analyze the performance implications of the construction frameworks just introduced which take short-sale constraints into account. Similar to the original factor strategies discussed in the sections above, I implemented backtests for the LOMTR and the LOCFA portfolios. In particular, I am interested in the effects of the short-sale constraints on the total performance and in the performance differences from the original factor strategies. Therefore, Table 20 summarizes the total performance over the 20-year time period of the index as well as all mentioned long-only factor strategies. As before, the backtests have been done for varied differential scores.

[4] `https://www.cvxpy.org`, last accessed 08-03-2023

[5] Whenever a further short-sale constraint becomes binding, the direction of the portfolio adaptation changes. This change in direction keeps the marginal adaptation orthogonal regarding only those stocks that have still positive weights. However, the overall adaptation loses orthogonality with Δ_{b}.

Table 20: Performance of the overall factor-based long-only portfolios with varying Δ_b over the time period from 2002-06-07 to 2022-06-10 ($p_0 = 100$).

LO	$\mathbf{w}_0$	MTRsize	MTRvalue	MTRmom	CFAsize	CFAvalue	CFAmom
$\Delta_b = 1$	476.9233	478.3981	471.9322	469.0348	479.9917	479.3978	475.2205
$\Delta_b = 5$	476.9233	488.7889	463.6364	462.9831	492.2487	488.8543	470.5781
$\Delta_b = 10$	476.9233	496.3305	448.3306	458.7680	508.5100	499.2958	450.7730
$\Delta_b = 15$	476.9233	502.7171	435.6431	440.1097	557.6296	538.3368	426.9710
$\Delta_b = 20$	476.9233	508.8964	421.6237	436.5819	661.4936	627.7021	453.5997

The table shows that the performance differences between the LOMTR portfolios and the original OFA portfolios (see Table 15) are basically not higher than a couple of percentage points. The performance of the long-only portfolios is typically a bit worse than the OFA performance without considered short-sale constraints. In a few cases, especially at lower differential scores, the LOMTR performance actually turns out slightly better or almost the same as the OFA performance. At $\Delta_b = 20$, the LOMTRsize portfolio lies about 6 percentage points behind the OFAsize portfolio over the entire period. Overall, the LOMTRvalue portfolio performs about 16 percentage points worse than the OFAvalue portfolio which represents the biggest occuring difference between the LOMTR and the OFA portfolios. The LOMTRmom performance is nearly equal to the OFAmom performance. It seems that, also at higher differential scores, the minimum tracking-error attribute ensures that the impact of the short-sale constraints are kept within reasonable limits. Nevertheless, in the case of the LOMTR portfolios, the short-sale constraints appear to reduce performance, at least from certain Δ_b levels.

Considering the LOCFA portfolios, Table 20 makes obvious that the level of Δ_b influences the relative performance to the CFA portfolios (see Table 15) significantly. While the performance differences are rather limited at lower differential scores, the deviations become quite large at $\Delta_b = 15$ and $\Delta_b = 20$ especially. At $\Delta_b = 20$, the LOCFAsize portfolio outperforms the CFAsize portfolio by about 124 percentage points over the full horizon. The LOCFAvalue performance lies about 107 percentage points above the CFAvalue performance. Also, the LOCFAmom performance is better than the CFAmom performance by about 22 percentage points overall. The implementation of short-sale constraints in the CFA leads to substantial portfolio modifications at higher differential score levels. Since increasing Δ_b results in a greater number of negative portfolio weights that are set to 0, fewer and fewer firms remain that are actually considered in the final stages of the portfolio construction loop. Apparently, this has quite positive performance implications in particular with respect to the LOCFAsize and the LOCFAvalue strategy.

Last but not least, I want to put the focus briefly on the LOCFAsize strategy which shows the best retrospective performance over the considered 20-year time period. Actually, it outperformed $\mathbf{w}_0$ by about 185 percentage points. Since that is quite a considerable number, I am also interested in the performance behavior of that particular long-only factor strategy over time. For that purpose, Figure 11 provides the corre-

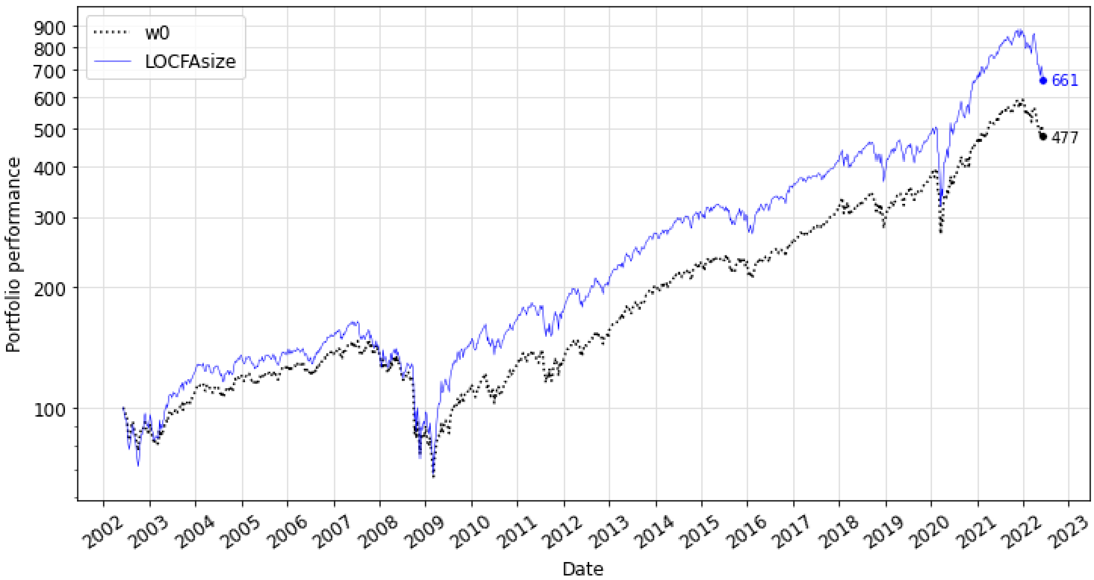

Figure 11: Performance (log scale) of the LOCFAsize portfolio ($\Delta_\mathrm{b} = 20$) over the time period from 2002-06-07 to 2022-06-10 ($p_0 = 100$).

sponding illustration. It shows the performance of the LOCFAsize portfolio applied with $\Delta_\mathrm{b} = 20$ over the 20-year time period in comparison to the $\mathbf{w}_0$ performance. Observing both line charts, three stages stand out in terms of relative performance. In fact, the blue line (LOCFAsize performance) features steeper increases than the black dotted line ($\mathbf{w}_0$ performance) in the stages following the market downturns of 2002, 2008 and 2020. Apparently, the strategy enables to generate superior returns in the periods of major market recoveries. The outperformance in those stages seems to be the main driver of the strong overall LOCFAsize performance.

Chapter 6

Conclusions

In this thesis, I investigate two factor-based portfolio construction approaches. The approaches build on the idea that factor exposure is quantified by weighted portfolio scores. For that purpose, rank scores with respect to factor-related firm characteristics are assigned to portfolio constituents in the first place. Both approaches combine a market-weighted index portfolio as reference component and a factor-mimicking long-short portfolio as adjusting component. In fact, the long-short portfolio is utilized with desired differential scores to modify the exposure to the considered factors. The two approaches differ in the methodology applied to construct the long-short portfolios. The orthogonal factor approach (OFA) is based on a tracking-error minimization method and further utilizes factor-mimicking portfolios with orthogonal exposure to selected firm characteristics. Therefore, the factor-mimicking portfolios in the OFA implement pure single-factor exposure and come without any unintended exposure to other factors. The correlated factor approach (CFA) represents a classic methodology to build factor portfolios without coming with any special property regarding return volatility or factor exposure. Thus, the CFA comes along with unintended factor exposure which causes troubles in the performance interpretation of corresponding factor portfolios.

The aim of this research work is to learn more about the return characteristics of orthogonalized factor-mimicking portfolios. Moreover, the thesis should provide information about the performance implications of the three factors size, value and momentum. In order to address these research topics, I create an analysis model with the programming language Python that comprises functions for both portfolio construction approaches. The MSCI USA index serves as reference portfolio. The model is set up to retrieve required stock and index information from a comprehensive dataset and to determine portfolio returns on a weekly basis. With the aid of this model, I perform backtests for factor portfolios based on both the OFA and the CFA over a 20-year time period from June 2002 to June 2022. Eventually, I use the backtest results to analyze and compare the observed factor portfolios with respect to return and risk aspects.

Research question 1: According to the first research question, '*What are the return characteristics of factor portfolios that implement orthogonal exposure to selected firm characteristics? Does orthogonality in the space of firm characteristics translate into uncorrelated factor returns?*', I study the return characteristics of factor portfolios that implement orthogonal exposure to selected firm characteristics. Actually, I find a considerable degree of orthogonality in the returns generated by the factor-mimicking long-short portfolios originated from the OFA. This is illustrated by the respective correlation coefficients. It is particularly interesting since the actual purpose of the OFA is to orthogonalize the factor exposure. The fact that the orthogonality property in the space of firm characteristics leads to uncorrelated factor returns represents a key finding of this thesis. The corresponding research subquestion of whether exposure orthogonality translates into uncorrelated factor returns can therefore be answered with yes. In contrast, the correlation coefficients between the CFA factor returns point out a high cross-contamination. In terms of return volatility, I observe that the OFA portfolios show significantly lower tracking-errors relative to the reference portfolio compared to the CFA portfolios. This is according to expectations as the OFA construction framework makes use of a tracking-error minimization which is not the case with the CFA. The effects of the OFA characteristics become apparent from taking the information ratios related to the different factor strategies into account. In the space of the OFA portfolios, only the size factor realizes a positive information ratio. The value and the momentum factor lead to negative ratios. Furthermore, the OFAsize portfolio yields the highest information ratio of any considered factor portfolio. This is the case despite the fact that both the CFAsize and the CFAvalue portfolio generate a higher overall performance. However, this outperformance comes along with a higher risk measured by the tracking-error. Therefore, the OFA turns out as the most attractive approach when investing benchmark-oriented.

Research question 2: According to the second research question, '*What are the effects of the three considered factors size, value and momentum on the portfolio performance?*', I observe the impact of the three factors size, value and momentum on the portfolio performance. I find that size is the only factor that realizes a positive premium with either of the two approaches. In contrast, the momentum factor leads to a negative premium in both cases. The value factor yields a negative premium in the OFAvalue portfolio and a positive premium in the CFAvalue portfolio. The positive premium of the CFAvalue portfolio has to be treated with caution however. Due to the high cross-contamination in the CFA factor returns, it is difficult to assess whether the CFAvalue premium comes from the value factor itself or rather from the fact that the portfolio levers on the size factor.

List of Figures

List of Tables

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Appendix A

Python implementation

Below, I provide the key functions of the analysis model written in Python.

A.1 Packages and basic input parameters

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.dates as mdates
import matplotlib.ticker as ticker
import seaborn as sns
import cvxpy as cp

window_yrs = 2
weight_smm = 0.5
```

A.2 Stock selection

```
# Retrieve stock returns
def get_returns_stocks(df_firmchar, ref_date, window_yrs):
    firms_inix = df_firmchar[(df_firmchar['date'] == ref_date)\
        & (df_firmchar['IsinIX'] == 1)]['isin'].tolist()
    firms_inix.sort()
    df_returns_temp = df_firmchar.pivot_table(values='R',
        index='date', columns='isin', dropna=False)
    df_returns = df_returns_temp[firms_inix] / 100
    df_returns.index.name = None
    df_returns.columns.name = None
    end = df_returns.index.get_loc(ref_date) + 1
    start = end - (window_yrs*52)
    return df_returns.iloc[start:end+1,:]
```

```
# List firms that do not meet selection criteria
def list_ret_nan(df_firmchar, ref_date, window_yrs):
    df_returns = get_returns_stocks(df_firmchar, ref_date, \
        window_yrs)
    ser_returns_count = df_returns.count()
    lst = ser_returns_count[ser_returns_count < \
        len(df_returns)].index.tolist()
    return lst

def list_mcap_nan(df_firmchar, ref_date):
    df_nan = df_firmchar[(df_firmchar['date'] == ref_date) & \
        (df_firmchar['IsinIX'] == 1) & \
        ((df_firmchar['MCAP'] > 0) == False)]
    lst = df_nan['isin'].tolist()
    return lst

def list_btp_nan(df_firmchar, ref_date):
    df_nan = df_firmchar[(df_firmchar['date'] == ref_date) & \
        (df_firmchar['IsinIX'] == 1) & \
        ((df_firmchar['BTP'] > 0) == False)]
    lst = df_nan['isin'].tolist()
    return lst

def list_mom_nan(df_firmchar, ref_date):
    df_nan = df_firmchar[(df_firmchar['date'] == ref_date) & \
        (df_firmchar['IsinIX'] == 1) & \
        ((df_firmchar['Mom11m'].isna()) == True)]
    lst = df_nan['isin'].tolist()
    return lst

# Create overall firm exclusion list
def get_firms_excl(df_firmchar, ref_date, window_yrs):
    set1 = set(list_ret_nan(df_firmchar, ref_date, window_yrs))
    set2 = set(list_mcap_nan(df_firmchar, ref_date))
    set3 = set(list_btp_nan(df_firmchar, ref_date))
    set4 = set(list_mom_nan(df_firmchar, ref_date))
    lst = list(set1 | set2 | set3 | set4)
    return lst

# Create list with relevant firms
def get_firms_rel(df_firmchar, ref_date, window_yrs):
```

```
    set_orig = set(df_firmchar[(df_firmchar['date'] == \
        ref_date) & df_firmchar['IsinIX'] == 1]['isin'])
    set_excl = set(get_firms_excl(df_firmchar, ref_date, \
        window_yrs))
    lst = list(set_orig - set_excl)
    lst.sort()
    return lst
```

A.3 Scoring

```
# Assign normalized scores based on firm characteristics
def get_mcap_scores(df_firmchar, ref_date, list_firms):
    n = len(list_firms) - 1
    df_mcap = df_firmchar[df_firmchar['date'] == \
        ref_date][['isin', 'MCAP']].set_index('isin')
    df_scores = df_mcap.loc[list_firms].rank(ascending= \
        False) - 1
    df_scores.index.name = None
    df_scores.rename(columns={'MCAP': 's_mcap'}, inplace=True)
    df_scores = (df_scores/n) * 100
    return df_scores

def get_btp_scores(df_firmchar, ref_date, list_firms):
    n = len(list_firms) - 1
    df_btp = df_firmchar[df_firmchar['date'] == \
        ref_date][['isin', 'BTP']].set_index('isin')
    df_scores = df_btp.loc[list_firms].rank() - 1
    df_scores.index.name = None
    df_scores.rename(columns={'BTP': 's_btp'}, inplace=True)
    df_scores = (df_scores/n) * 100
    return df_scores

def get_mom_scores(df_firmchar, ref_date, list_firms):
    n = len(list_firms) - 1
    df_mom = df_firmchar[df_firmchar['date'] == \
        ref_date][['isin', 'Mom11m']].set_index('isin')
    df_scores = df_mom.loc[list_firms].rank() - 1
    df_scores.index.name = None
    df_scores.rename(columns={'Mom11m': 's_mom'}, inplace=True)
    df_scores = (df_scores/n) * 100
    return df_scores
```

```
# Create pandas dataframe with firm scores
def get_scores_df(df_firmchar, ref_date, list_firms):
    s_mcap = get_mcap_scores(df_firmchar, ref_date, \
        list_firms)['s_mcap']
    s_btp = get_btp_scores(df_firmchar, ref_date, \
        list_firms)['s_btp']
    s_mom = get_mom_scores(df_firmchar, ref_date, \
        list_firms)['s_mom']
    list_ones = [1 for i in range(len(list_firms))]
    d_scores = {'ones': list_ones, 's_mcap': s_mcap, \
        's_btp': s_btp, 's_mom': s_mom}
    df_scores = pd.DataFrame(data=d_scores)
    return df_scores

# Create numpy array with firm scores
def get_scores_m(df_firmchar, ref_date, list_firms):
    df_scores = get_scores_df(df_firmchar, ref_date, \
        list_firms)
    m_scores = df_scores.to_numpy()
    return m_scores
```

A.4 Shrinkage

```
# Perform linear regression
def get_betas(df_returns_stocks, df_returns_index):
    df_returns = pd.concat([df_returns_index, \
        df_returns_stocks], axis=1)
    df_cov = df_returns.cov().iloc[1:,0]
    var_index = df_returns_index['MSCI_US'].var()
    ser_betas = df_cov / var_index
    df_betas = ser_betas.to_frame()
    df_betas.rename(columns={'MSCI_US': 'beta'}, inplace=True)
    return df_betas

def get_alphas(df_returns_stocks, df_returns_index):
    mean_r_stocks = df_returns_stocks.mean()
    mean_r_index = df_returns_index['MSCI_US'].mean()
    df_calc = get_betas(df_returns_stocks, df_returns_index)
    df_calc['mean_ri'] = mean_r_stocks
    df_calc['mean_rm'] = mean_r_index
    df_calc['alpha'] = df_calc['mean_ri'] - df_calc['beta'] \
        * df_calc['mean_rm']
```

```
    df_alpha = df_calc['alpha'].to_frame()
    return df_alpha

def get_epsilons(df_returns_stocks, df_returns_index):
    list_firms = df_returns_stocks.columns.tolist()
    list_dates = df_returns_stocks.index.values.tolist()
    df_indexret = pd.concat([df_returns_index]* \
        len(list_firms), axis=1)
    df_indexret.set_axis(list_firms, axis=1, inplace=True)
    df_alphas_temp = get_alphas(df_returns_stocks, \
        df_returns_index).T
    df_alphas = pd.concat([df_alphas_temp]*len(list_dates), \
        axis=0)
    df_alphas.set_axis(list_dates, axis=0, inplace=True)
    df_betas = get_betas(df_returns_stocks, df_returns_index)
    df_betaXindexret = df_indexret.copy()
    for i in range(len(list_firms)):
        df_betaXindexret.iloc[:, i] = \
            df_betaXindexret.iloc[:, i] * df_betas.iloc[i,0]
    df_epsilons = df_returns_stocks - df_alphas - \
        df_betaXindexret
    return df_epsilons

def get_idiovars(df_returns_stocks, df_returns_index):
    df_epsilons = get_epsilons(df_returns_stocks, \
        df_returns_index).T
    n = len(df_epsilons.columns.tolist())
    df_calc = df_epsilons.copy()
    df_calc['sum_squares'] = (df_epsilons**2).sum(axis=1)
    df_calc['idiovar'] = df_calc['sum_squares'] / (n-2)
    df_idiovar = df_calc['idiovar']
    return df_idiovar

# Determine the target covariance matrix
def get_cov_mm(df_returns_stocks, df_returns_index):
    var_index = df_returns_index['MSCI_US'].var()
    m_cov_sample = df_returns_stocks.cov().to_numpy()
    v_betas = get_betas(df_returns_stocks, \
        df_returns_index).to_numpy()
    m_idiovar = np.diag(get_idiovars(df_returns_stocks, \
        df_returns_index))
    m_cov_mm = (v_betas @ v_betas.transpose()) * \
```

```
        var_index + m_idiovar
    return m_cov_mm

# Shrink the sample covariance matrix towards the target
def get_cov_smm(df_returns_stocks, df_returns_index, \
    weight_smm):
    m_cov_sample = df_returns_stocks.cov().to_numpy()
    m_cov_mm = get_cov_mm(df_returns_stocks, df_returns_index)
    m_cov_smm = weight_smm * m_cov_mm + (1 - weight_smm) * \
        m_cov_sample
    return m_cov_smm
```

A.5 Weight allocation

```
# Determine market weights
def get_weights_market(df_firmchar, window_yrs, start_date, \
    end_date):
    df_mcap_all = df_firmchar.pivot_table(values='MCAP', \
        index='date', columns='isin', dropna=False)
    df_mcap_all.index.name = None
    df_mcap_all.columns.name = None
    start = df_mcap_all.index.get_loc(start_date)
    end = df_mcap_all.index.get_loc(end_date)
    df_mcap = df_mcap_all.iloc[start:end+1,:].copy()
    firms = df_mcap.columns.values.tolist()
    d_weights = {}
    for i in range(1, len(df_mcap)+1):
        date = df_mcap.iloc[i-1:i].index.values[0]
        firms_rel = get_firms_rel(df_firmchar, date, \
            window_yrs)
        for j in range(len(firms)):
            if firms[j] not in firms_rel:
                df_mcap.iloc[i-1, j] = 0
            else:
                continue
        sum_mcap = df_mcap.iloc[i-1:i].sum(axis=1)[0]
        df_mcap.iloc[i-1:i] = df_mcap.iloc[i-1:i] / sum_mcap
        df_weights = df_mcap.iloc[i-1:i].T
        df_weights = df_weights[df_weights.iloc[:,0] > 0]
        df_weights.rename(columns={date: 'w_mkt'}, \
            inplace=True)
        d_weights[date] = df_weights
```

```
    df_final = pd.concat(d_weights)
    return df_final

# Determine OFA basis weights
def get_bweights_ofa(df_firmchar, df_excreturns_st, \
    df_excreturns_ix, df_mktweights, ref_date, \
    window_yrs, weight_smm):
    list_firms = \
        df_mktweights.loc[ref_date].index.values.tolist()
    end_st = df_excreturns_st.index.get_loc(ref_date) + 1
    start_st = end_st - (window_yrs*52)
    end_ix = df_excreturns_ix.index.get_loc(ref_date) + 1
    start_ix = end_ix - (window_yrs*52)
    df_exc_st = \
        df_excreturns_st[list_firms].iloc[start_st:end_st,:]
    df_exc_ix = df_excreturns_ix.iloc[start_ix:end_ix,:]
    m_cov_smm = get_cov_smm(df_exc_st, df_exc_ix, weight_smm)
    m_scores = get_scores_m(df_firmchar, ref_date, list_firms)
    m_bweights = np.linalg.inv(m_cov_smm) @ m_scores @ \
        np.linalg.inv(m_scores.T @ np.linalg.inv(m_cov_smm) @ \
        m_scores)
    df_bweights = pd.DataFrame(m_bweights, \
        columns=['w_special', 'w_OFA_size', 'w_OFA_value', \
        'w_OFA_mom'], index=list_firms)
    return df_bweights

# Determine CFA basis weights (initial)
def get_bweights_cfa_initial(df_firmchar, df_mktweights, \
    ref_date):
    list_firms = \
        df_mktweights.loc[ref_date].index.values.tolist()
    # Mcap
    df_mcap = get_mcap_scores(df_firmchar, ref_date, \
        list_firms)
    df_mcap['w_cat'] = np.where(df_mcap['s_mcap'] < 50, -1, 1)
    n_pos = df_mcap[df_mcap['w_cat'] > 0]['w_cat'].count()
    n_neg = df_mcap[df_mcap['w_cat'] < 0]['w_cat'].count()
    df_mcap['w_mcap'] = np.where(df_mcap['w_cat'] < 0, \
        df_mcap['w_cat']/n_neg, df_mcap['w_cat']/n_pos)
    # Btp
    df_btp = get_btp_scores(df_firmchar, ref_date, list_firms)
    df_btp['w_cat'] = np.where(df_btp['s_btp'] < 50, -1, 1)
```

```
    n_pos = df_btp[df_btp['w_cat'] > 0]['w_cat'].count()
    n_neg = df_btp[df_btp['w_cat'] < 0]['w_cat'].count()
    df_btp['w_btp'] = np.where(df_btp['w_cat'] < 0, \
        df_btp['w_cat']/n_neg, df_btp['w_cat']/n_pos)
    # Mom
    df_mom = get_mom_scores(df_firmchar, ref_date, list_firms)
    df_mom['w_cat'] = np.where(df_mom['s_mom'] < 50, -1, 1)
    n_pos = df_mom[df_mom['w_cat'] > 0]['w_cat'].count()
    n_neg = df_mom[df_mom['w_cat'] < 0]['w_cat'].count()
    df_mom['w_mom'] = np.where(df_mom['w_cat'] < 0, \
        df_mom['w_cat']/n_neg, df_mom['w_cat']/n_pos)
    # Combined
    df_weights = pd.concat([df_mcap['w_mcap'], \
        df_btp['w_btp'], df_mom['w_mom']], axis=1)
    return df_weights

# Determine CFA basis weights (normalized)
def get_bweights_cfa(df_firmchar, df_mktweights, ref_date):
    df_weights = get_bweights_cfa_initial(df_firmchar, \
        df_mktweights, ref_date)
    list_firms = \
        df_mktweights.loc[ref_date].index.values.tolist()
    m_scores = get_scores_m(df_firmchar, ref_date, \
        list_firms)[:,1:]
    m_s_avg = df_weights.to_numpy().T @ m_scores
    df_weights['w_CFA_size'] = \
        df_weights['w_mcap']/m_s_avg[0,0]
    df_weights['w_CFA_value'] = \
        df_weights['w_btp']/m_s_avg[1,1]
    df_weights['w_CFA_mom'] = df_weights['w_mom']/m_s_avg[2,2]
    df_weights.drop(columns=['w_mcap', 'w_btp', 'w_mom'], \
        inplace=True)
    return df_weights
```

A.6 Return calculation

```
# Returns generated by market portfolio
def calc_returns_mkt(df_mktweights, df_excreturns_st, \
    start_date, end_date):
    df_mktweights_adj = \
        df_mktweights.loc[start_date:end_date,:]
    df_excreturns_adj = \
```

```
        df_excreturns_st.loc[start_date:end_date,:]
    list_dates = df_excreturns_adj.index.values.tolist()
    list_mkt = [np.nan]
    for i in range(len(list_dates)-1):
        inv_date = list_dates[i]
        r_date = list_dates[i+1]
        df_mkt = df_mktweights_adj.loc[inv_date]
        w_mkt = df_mkt.to_numpy()
        list_firms = df_mkt.index.values.tolist()
        r_firms = df_excreturns_adj[list_firms].loc[r_date, \
            :].to_frame().to_numpy()
        r_mkt = (w_mkt.T @ r_firms)[0,0]
        list_mkt.append(r_mkt)
    d_returns = {'r_mkt': list_mkt}
    df_returns = pd.DataFrame(data=d_returns, index=list_dates)
    return df_returns

# Returns generated by OFA long-short portfolios
def calc_breturns_ofa(df_firmchar, df_excreturns_st, \
    df_ofabweights, df_mktweights, start_date, end_date, \
    delta_size, delta_value, delta_mom):
    df_excreturns_adj = \
        df_excreturns_st.loc[start_date:end_date,:]
    list_dates = df_excreturns_adj.index.values.tolist()
    list_size = [np.nan]
    list_value = [np.nan]
    list_mom = [np.nan]
    list_comb = [np.nan]
    delta_b = np.array([[delta_size], [delta_value], \
        [delta_mom]])
    for i in range(len(list_dates)-1):
        inv_date = list_dates[i]
        r_date = list_dates[i+1]
        list_firms = \
            df_mktweights.loc[inv_date].index.values.tolist()
        r_firms = df_excreturns_adj[list_firms].loc[r_date, \
            :].to_frame().to_numpy()
        w = df_ofabweights.loc[inv_date]
        B = w.to_numpy()
        w_size = (w[['w_OFA_size']]*delta_size).to_numpy()
        w_value = (w[['w_OFA_value']]*delta_value).to_numpy()
        w_mom = (w[['w_OFA_mom']]*delta_mom).to_numpy()
```

```
        w_comb = B @ delta_b
        r_size = (w_size.T @ r_firms)[0,0]
        r_value = (w_value.T @ r_firms)[0,0]
        r_mom = (w_mom.T @ r_firms)[0,0]
        r_comb = (w_comb.T @ r_firms)[0,0]
        list_size.append(r_size)
        list_value.append(r_value)
        list_mom.append(r_mom)
        list_comb.append(r_comb)
    d_returns = {'br_OFA_size': list_size, \
        'br_OFA_value': list_value, 'br_OFA_mom': list_mom, \
        'br_OFA_comb':list_comb}
    df_returns = pd.DataFrame(data=d_returns, index=list_dates)
    return df_returns

# Returns generated by CFA long-short portfolios
def calc_breturns_cfa(df_firmchar, df_excreturns_st, \
    df_cfabweights, df_mktweights, start_date, end_date, \
    delta_size, delta_value, delta_mom):
    df_excreturns_adj = \
        df_excreturns_st.loc[start_date:end_date,:]
    list_dates = df_excreturns_adj.index.values.tolist()
    list_size = [np.nan]
    list_value = [np.nan]
    list_mom = [np.nan]
    list_comb = [np.nan]
    delta_b = np.array([[delta_size], [delta_value], \
        [delta_mom]])
    for i in range(len(list_dates)-1):
        inv_date = list_dates[i]
        r_date = list_dates[i+1]
        list_firms = \
            df_mktweights.loc[inv_date].index.values.tolist()
        r_firms = df_excreturns_adj[list_firms].loc[r_date, \
            :].to_frame().to_numpy()
        w = df_cfabweights.loc[inv_date]
        B = w.to_numpy()
        w_size = (w[['w_CFA_size']]*delta_size).to_numpy()
        w_value = (w[['w_CFA_value']]*delta_value).to_numpy()
        w_mom = (w[['w_CFA_mom']]*delta_mom).to_numpy()
        w_comb = B @ delta_b
        r_size = (w_size.T @ r_firms)[0,0]
```

```
        r_value = (w_value.T @ r_firms)[0,0]
        r_mom = (w_mom.T @ r_firms)[0,0]
        r_comb = (w_comb.T @ r_firms)[0,0]
        list_size.append(r_size)
        list_value.append(r_value)
        list_mom.append(r_mom)
        list_comb.append(r_comb)
    d_returns = {'br_CFA_size': list_size, \
        'br_CFA_value': list_value, 'br_CFA_mom': list_mom, \
        'br_CFA_comb':list_comb}
    df_returns = pd.DataFrame(data=d_returns, index=list_dates)
    return df_returns
```

A.7 Performance calculation

```
# Calculate performance
def calc_performance(df_returns):
    df_perf = df_returns.copy()
    df_perf.iloc[0,:] = 100
    for i in range(1, len(df_perf)):
        df_perf.iloc[i,:] = df_perf.iloc[i-1,:] * \
            (1 + df_perf.iloc[i,:])
    return df_perf
```