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Quantifying the welfare gains from an optimal tax reform in Austria

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1. Introduction

Taxes are part of our daily life. While we are consuming, working or buying assets we hand over a little bit of our money to the government. But can we, as a society, benefit from various changes to the current tax system in Austria?

The aim of this thesis is to identify the optimal tax system for Austria and decompose the welfare gains in great detail. We therefore take the overlapping generation model from Conesa, Kitao and Krueger (2009), calibrate it for Austrian data and analyze the impacts of different income taxes on society. The model is determined by heterogeneous agents with varying abilities and possibilities of facing an idiosyncratic income shock. Households cannot insure themselves against these idiosyncratic income realizations but can hold risk free assets as a compensation for the possible loss in earnings. With labor income taxes allowing for a wide range of progressivity, the policymaker can decide to redistribute the tax burden across low and high earners. The life cycle structure combined with incomplete markets indicates for positive capital taxes in recent literature.

Crucially, most of recent literature is based on US data. However, in comparison to the US tax system the Austrian one stands out due to high personal and high consumption taxes. The US tax system in contrast taxes firms higher than the Austrian tax system. These circumstances lead to the hypothesis that for both nations different optimal tax systems as well as life cycle paths are appropriate.

An optimal tax system is measured by the welfare for the whole society. But how do we determine the welfare of different tax systems? The social welfare is measured ex ante in a stationary equilibrium. For the overlapping generation model this means that before the ability type of an agent is known neither of the agent faces a positive nor a negative income shock. Furthermore, the policymaker optimizes the expected lifetime utility of a newborn agent and uses taxes as a tool to redistribute wealth. For instance, taking one Euro ceteris paribus from the high able and give it to the less able can have a social welfare effect for the whole society. This is due to the strict concavity of the value function comparing labor income between households with low and high abilities. Here, the policymaker acts as an insurer against low ability for a newborn household. However, if the tax burden is too high it can result into declining working hours since the agents may then value leisure more than one extra Euro of labor income. The same does apply for holding assets as they are less attractive to hold if the capital tax is too high.

Our calculations result in an optimal capital income tax of 29 % with a labor income tax schedule close to the benchmark tax system. The latter is a flat tax of 34 % with a deduction of about 5700 € (relative to an average household income of 48 373 €). With a capital income tax of 25 % in the benchmark model the optimal capital income tax increases by four percentage points. Surprisingly the aggregate capital stock rises despite the fact that holding assets is less attractive. The growth of the economy, defined by the aggregate output Y, rises by around 0.26 %. These results combined with higher aggregate consumption in the optimal tax model explains the welfare gains for the whole society. One interesting finding is the distribution of welfare between different productivity states. Households with positive income realizations benefit more than average while households with negative income realizations cannot increase their level of welfare.

The distribution of taxes is dominated by the labor income taxes. While more than half of the tax revenue stem from labor income taxes only one third are consumption taxes. The smallest share belongs to capital income taxes with only just over 10 %. The optimal tax system increases the amount of capital income taxes while lowering the labor income taxes for both ability types. This result accurately reflects the change from the benchmark to the optimal tax model.

The average labor income tax rate is slightly lower for all income levels compared to the benchmark. Therefore all households have to pay less labor income taxes. Although, higher capital income taxes increase the households tax burden. So how can we argue that this tax system is the optimal one?

The age dependency of the tax system takes a crucial part in the interpretation of a non zero capital tax. If taxes are age dependent the policymaker can adjust the labor income taxes in the way that households pay more when they are more productive or work more. This helps to smooth the tax burden over the life cycle. However, in absence of age dependent taxes, as in our case, the government can use progressive labor income taxes in combination with capital income taxes to better distribute the tax burden across the life cycle. On the one hand, high labor income taxes hamper young agents to acquire assets as an insurance. On the other hand, too high capital income taxes discourage savings. Conesa et al. (2009) calculated an optimal labor income tax of 23 % with similar progressivity compared to our model and a capital income tax of 36 %. Our results coincide with their outcome as our higher labor income taxes are balanced with a lower capital income tax.

Related Literature

Early studies from Judd (1985) and Chamley (1986) yet obtained an optimal capital income tax of zero in the long run. However, they used a neoclassical growth model with infinitely lived agents. Later Aiyagari (1995) showed that including heterogeneous agents, incomplete markets and tight borrowing constraints leads to positive capital income taxes, even in the long run. This results from precautionary acquiring of too much assets as an insurance for a potential income shock where positive capital income taxes can redistribute these asset holdings back to the efficient level. Erosa and Gervais (2002) and Garriga (2019) also find positive capital income taxes in a standard life cycle growth model as long as the tax rates cannot be made dependent on the age of the household. Conesa et al. (2009) calculated an optimal capital income tax of 36 % for US data with a model including incomplete markets, uninsurable income risk and a progressive labor income tax schedule. There are further studies analyzing the impact of various tax reforms on the optimal labor income tax code with a wide range of progressivity (Altig et al., 2001; Conesa & Krueger, 2006). However, in contrast to Conesa et al. (2009) these studies did not differentiate between labor and capital income taxes. Therefore, the comparability between their results is limited. Another study from Kindermann and Krueger (2014) differentiating between labor and capital income taxes found that the optimal marginal labor income tax is close to 90 % for 1 % of the top earners.

There are several other approaches adding further assumptions and elements. Krueger and Ludwig (2013) for example assumed endogenous wages and households to be able to decide their amount of education. These assumptions lead to a much more progressive tax system compared to the benchmark tax system from the US. Nakajima (2020) added housing to the model as a second asset that is taxed differently from capital income. He found that a capital income tax of 1 % is optimal in this model framework.

We start the thesis with formulating the model in the second section. As Conesa et al. (2009) used US data for their calculations, we have to adapt the model parameters to match with Austrian data. The model calibrations will be presented in section three. Then we optimize for the optimal tax system and present the results in section four. We focus our analysis on the welfare change for all ability types and present life cycle profiles for the benchmark as well as the optimal tax system. Another part presented in section four will be the distribution of the tax burden. Section five provides a sensitivity analysis with special focus on the consumption tax. The last section concludes this thesis.

2. The Economic Environment

2.1. Demographics

Every period of time t a new cohort of households is born and lives up to a maximum of J years. There are no personal bequests for a newborn household, so all individuals start their lives with zero assets. Time is discrete and for simplicity there is no population growth added to the model. Each household retires from work when they are Jr years old and receives a social security payment SS_t during retirement. These payments are financed by a social security tax τ_{SS} from labor income.

The survival probability for all representatives in period j who are still alive in period $j+1$ is denoted by ϕ_j. At the age J, one faces a certain death ($\phi_j = 0$). Accidental bequests Tr_t, which results from unpredictable deaths during the life cycle, are transferred back to the population alive via a lump sum payment.

2.2. Endowment and Preferences

In each period households have one unit of time which they can arbitrarily spend between labor and leisure. If they decide to work they can spend any amount of time working in a competitive market.

If agents decide to work they differ across three dimensions. There are two ability types $\alpha_i, i \in \mathcal{I}$ with equal population mass in the economy. One ability type represents households with compulsory school education. The other represents agents with a university degree. Another characteristic of households is a changing labor productivity ε_j over the life cycle. While reaching the age of Jr the labor productivity drops to zero and the agents retire.

Finally, each household of the same age and ability type faces the possibility of an idiosyncratic labor productivity shock. These shocks are modeled with a discretization of a continuous AR(1) process by using the method from Tauchen (1986).

The AR(1) process is given by $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$ where $\epsilon_t \sim N(0, \sigma_\epsilon^2)$. We approximate the AR(1) process by a discrete Markov chain with N states and a scaling variable λ. The first and last state are set to

$$\ln z_1 = -\lambda \left(\frac{\sigma_\epsilon^2}{1-\rho^2} \right)^{1/2} \text{ and } \ln z_N = \lambda \left(\frac{\sigma_\epsilon^2}{1-\rho^2} \right)^{1/2}.$$

The remaining states are equidistantly distributed between $\ln z_1$ and $\ln z_N$

$$\ln z_i = \ln z_{i-1} + \Delta \ln z, \quad \text{for } i = 2, \dots, N-1,$$
$$\text{with } \Delta \ln z = \left(\frac{\ln z_N - \ln z_1}{N-1} \right).$$

The transition matrix $P = (p_{ij})$ is then calculated by

$$p_{ij} = \begin{cases} \theta\left(\frac{\ln z_1 - \rho \ln z_j + (\Delta \ln z)/2}{\sigma_\epsilon}\right) & , i = 1, \\ \theta\left(\frac{\ln z_i - \rho \ln z_j + (\Delta \ln z)/2}{\sigma_\epsilon}\right) - \theta\left(\frac{\ln z_i - \rho \ln z_j - (\Delta \ln z)/2}{\sigma_\epsilon}\right) & , i = 2, \dots, N-1, \\ 1 - \theta\left(\frac{\ln z_N - \rho \ln z_j - (\Delta \ln z)/2}{\sigma_\epsilon}\right) & , i = N, \end{cases}$$

where θ is the cumulative distribution of the standard normal distribution. Now we apply the exponential function and receive $\mathcal{E} = [z_1, \dots, z_N]$ strictly positive states and a transition matrix P that should represent the cross sectional income realizations over the life cycle. The Markov chain has a stationary distribution which we denote with $\pi_t(\eta, \mathcal{E}) = \mathbb{P}(\eta' \in \mathcal{E}|\eta) = \pi(\eta, \mathcal{E})$ for $\eta \in \mathcal{E}$. Each household starts their live with an initial productivity level $\bar{\eta}$, with $\bar{\eta} \in \pi(\eta)$. $\pi(\eta)$ represents the probability of η under the stationary distribution π.

For a given time t we characterize the status of the actual labor productivity by η. A household can then be summarized by (a, η, i, j), where a denotes the asset holdings, η the labor productivity status, i the ability type and j the age. The pretax labor earnings for a household (a, η, i, j), who decides to work l_j hours, are denoted by $w\varepsilon_j \alpha_i \eta l_j$, where w is the wage per efficiency unit of labor. Additionally, the measure of agents characterized by (a, η, i, j) at time t are defined by $\Phi_t(a, \eta, i, j)$.

Preferences over consumption and leisure $\{c_j, 1 - l_j\}_{j=1}^J$ are described by the time separable utility function

$$\mathbb{E}\left(\sum_{j=1}^{J} \beta^{j-1} u(c_j, 1 - l_j) \right),$$

where β denotes the time discount factor and the expectations are formed over the idiosyncratic labor productivity and the mortality risk.

2.3. Technology

The technology is represented by a standard Cobb-Douglas production function $Y_t = K_t^\alpha N_t^{1-\alpha}$ and the following resource constraint

$$C_t + K_{t+1} - (1 - \delta)K_t + G_t \leq Y_t,$$

where C_t, K_t, N_t and G_t are the aggregate variables of consumption, capital stock, labor supply and government budget in period t. The value α denotes the capital share and δ the depreciation rate of physical capital.

2.4. Government

The government runs a social security pension system, levies taxes and balances the budget constraint. The pension system is financed by a social security tax τ_{SS} on labor earnings up to a maximum of $\bar{y}$. During retirement, each household receives a benefit SS_t which is the same for everyone regardless of the amount of taxes one has paid before. In this model we take the pension system as exogenously given and not part of the optimization process.

Moreover, the government levies three different taxes. Consumption is taxed by a proportional tax τ_c, which is exogenously given. Capital income $r_t(a + Tr_t)$ is taxed by a constant tax τ_k, where r_t denotes the risk free interest rate, a the households asset holdings and Tr_t the accidental bequests. Third, the government taxes labor earnings with a certain tax function T. The pretax labor earnings are defined by $yp_t = w_t\varepsilon_j\alpha_i\eta l_t$, where w_t represents the wage per efficiency unit of labor in period t. The first tax, that has to be paid by the employer, is the social security tax $ess_t = \tau_{SS}\min\{yp_t, \bar{y}\}$, which depends on personal pretax labor income yp_t up to a maximum threshold of $\bar{y}$. This tax does not count as taxable labor earnings for our tax function T and therefore we define taxable labor income as $y_t = yp_t - ess_t$.

For our analysis we want to find the optimal tax function $T(y_t)$ and capital income tax τ_k that maximizes the social welfare. Therefore each household faces the same tax schedule, where τ_k is constant over time and labor income taxes are described by a function T depending on the personal amount of taxable labor income y_t.

2.5. Market Structure

The capital market is incomplete in the sense of insurance against any uncertainties. Households fail to make optimal allocation of assets over the life cycle because of the possibility of an unpredictable productivity shock or mortality risk. Any kind of insurance contracts cannot be acquired to cover an income shock. However, they can hold risk free assets to compensate their loss in earnings. In case of an idiosyncratic income shock and non previously acquired assets there is no possibility to borrow assets though. This means that there is no kind of debt present in this model. Due to the possibility to die in each period, it is therefore impossible for agents to bequest any debt.

2.6. The Household Problem

We now want to define the households optimization problem for our model. The value function V_t describes the current value of period t compared with the future value, while acting optimally in the future. With a certain policy function $\{a'_t, c_t, l_t\}$, which represents the optimal choices of asset holdings, consumption and labor supply in period t the value function V_t solves the following equation

$$V_t(a, \eta, i, j) = \max_{a',c,l} \left\{ u(c, 1-l) + \beta\phi_j \int V_{t+1}(a', \eta', i, j+1)\pi(\eta, d\eta') \right\}, \qquad (2.1)$$

subject to

$$(1+\tau_{c,t})c + a' = w_t \varepsilon_j \alpha_i \eta l - \tau_{SS,t} \min\{w_t \varepsilon_j \alpha_i \eta l, \bar{y}\} + (1 + r_t(1-\tau_{k,t}))(a + Tr_t) - T_t(y_t),$$

for $j < Jr$,

$$(1+\tau_{c,t})c + a' = SS_t + (1 + r_t(1-\tau_{k,t}))(a + Tr_t),$$

for $j \geq Jr$,

$$a' \geq 0,\ c \geq 0,\ 0 \leq l \leq 1.$$

2.7. Stationary Equilibrium

Before we want to define our equilibrium we first need to specify the measure. The joint measure Φ contains the information about asset holdings, labor productivity status, ability type and current age of all individuals. Therefore the aggregate state of the economy is completely described by Φ.

Let $a \in \mathbb{R}_+, \eta \in \mathcal{E} = \{z_1, \ldots, z_N\}, i \in \mathcal{I} = \{1, \ldots, M\}, j \in \mathcal{J} = \{1, \ldots, J\}$ and let $\mathcal{S} = \mathbb{R}_+ \times \mathcal{E} \times \mathcal{I} \times \mathcal{J}$. Let $B(\mathbb{R}_+)$ the Borel σ-algebra of $\mathbb{R}_+$ and $P(\mathcal{E}), P(\mathcal{I}), P(\mathcal{J})$ the power sets of $\mathcal{E}, \mathcal{I}, \mathcal{J}$. Let $\mathcal{M}$ be the set of all finite measures over the measurable space $(\mathcal{S}, B(\mathbb{R}_+) \times \mathcal{E} \times \mathcal{I} \times \mathcal{J})$.

A stationary competitive equilibrium is described as a competitive equilibrium where all aggregate variables as well as per capita variables, function, prices and policies are constant over time.

Definition: Given a consumption tax τ_c, a social security tax τ_{SS} and government expenditures G, a stationary equilibrium is then described by household realization $\{V, c, a', l\}$, a firm production plan $\{N, K\}$, a labor income tax function $\{T : \mathbb{R}_+ \to \mathbb{R}_+\}$, a capital income tax τ_k, a social security benefit SS, prices $\{w, r\}$, a transfer Tr and a measure Φ with $\Phi \in \mathcal{M}$ such that:

(i) Taking prices as given, V solves the household problem (2.1) with the associated policy function $\{a', c, l\}$.

(ii) Prices w and r satisfy

$$r = \alpha \left(\frac{N}{K}\right)^{1-\alpha} - \delta,$$
$$w = (1-\alpha)\left(\frac{K}{N}\right)^{\alpha}.$$

(iii) The social security measures satisfy

$$\tau_{SS}\int \min\{w\varepsilon_j\alpha_i\eta l,\bar{y}\}\ \Phi(da\times d\eta\times di\times dj) = SS\int \Phi(da\times d\eta\times di\times\{Jr,\dots,J\}).$$

(iv) Lump-sum transfers are given by

$$Tr\int \Phi(da\times d\eta\times di\times dj) = \int (1-\phi_j)a'(a,\eta,i,j)\ \Phi(da\times d\eta\times di\times dj).$$

(v) The government budget balance satisfies

$$G = \tau_k\int r(a+Tr)\ \Phi(da\times d\eta\times di\times dj) + \tau_c\int c(a,\eta,i,j)\ \Phi(da\times d\eta\times di\times dj) + \\ + \int T(y)\ \Phi(da\times d\eta\times di\times dj).$$

(vi) Market clearing is given by

$$N = \int \varepsilon_j\alpha_i\eta l(a,\eta,i,j)\ \Phi(da\times d\eta\times di\times dj),$$

$$K = \int a\ \Phi(da\times d\eta\times di\times dj),$$

$$\int c(a,\eta,i,j)\ \Phi(da\times d\eta\times di\times dj) + K + G = K^\alpha N^{1-\alpha} + (1-\delta)K.$$

(vii) Law of motion

$$\Phi = H(\Phi),$$

where the function $H:\mathcal{M}\to\mathcal{M}$ is described by

$$\Phi(\mathcal{A}\times\mathcal{E}\times\mathcal{I}\times\mathcal{J}) = \int P((a,\eta,i,j);\mathcal{A}\times\mathcal{E}\times\mathcal{I}\times\mathcal{J})\ \Phi(da\times d\eta\times di\times dj),$$

for all $\mathcal{J}$ with $1\notin\mathcal{J}$ and

$$\Phi(\mathcal{A}\times\mathcal{E}\times\mathcal{I}\times\{1\}) = \begin{cases} \sum_{i\in\mathcal{I}} p_i & \textit{if}\ \ 0\in\mathcal{A},\bar{\eta}\in\mathcal{E} \\ 0 & \textit{else} \end{cases},$$

where

$$P((a,\eta,i,j);\mathcal{A}\times\mathcal{E}\times\mathcal{I}\times\mathcal{J}) = \begin{cases} \pi(e,\mathcal{E})\phi_j & \textit{if}\ \ a'(a,\eta,i,j)\in\mathcal{A}, i\in\mathcal{I}, j+1\in\mathcal{J} \\ 0 & \textit{else} \end{cases}.$$

3. Calibration

With the aim to determine the optimal tax code we first need to identify the model parameters to match with Austrian data. The following section describes the calibration process and summarizes the selected parameters in Table 3.1 at the end of this chapter.

3.1. Demographics

We assume that all households start working with an age of 20 years and are employed to a maximum age of 65. Therefore, the first period in the model represents a twenty year old agent who retires from work after $Jr = 46$ years. People live up to a maximum age of 100 years which yields to a model parameter $J = 81$.

The survival probability $\phi_j = 1 - \nu_j,\ j = 1, \ldots, 80$ is calculated with the data from the Human Mortality Database (2021) by taking the death rates ν_j for individuals between 20 and 100 years of age. With the age of 100 households face a certain death which leads to $\phi_{81} = 0$. We can now calculate the number of agents alive with $\upsilon_1 = 1$ and $\upsilon_j = \phi_{j-1}\upsilon_{j-1},\ j = 2, \ldots, 81$. The upper panel of Figure 3.1 displays the survival probability and the number of agents alive for every age. The survival probability is close to one during the first 70 years of age and then start to fall to zero. The number of agents alive starts to fall earlier but increases its gradient around the age of 70.

3.2. Preferences

The preferences over consumption and leisure are described by a time separable Cobb-Douglas function

$$u(c, 1-l) = \frac{(c^{\gamma}(1-l)^{1-\gamma})^{1-\sigma}}{1-\sigma}.$$

The households risk aversion σ is set to 4 following Conesa et al. (2009). The parameter γ denotes the share between consumption and leisure and is set to around 1/3 of endowed time in recent literature. We target a capital output ratio K/Y of 3 in the stationary equilibrium and average hours worked should result into 1/3 of endowed time.

3.3. Labor Productivity

Households labor productivity is determined by three different characteristics. First, the two ability types $\alpha_i,\ i = 1, 2$, with equal population mass $p_i = 0.5$, are set to $\alpha_1 = e^{-\sigma_\alpha}$ and $\alpha_2 = e^{\sigma_\alpha}$, which results in $\mathbb{E}(\ln(\alpha_i)) = 0$ and $\mathbb{V}\mathrm{ar}(\ln(\alpha_i)) = \sigma_\alpha^2$. We take the value $\sigma_\alpha^2 = 0.14$ from Conesa et al. (2009).

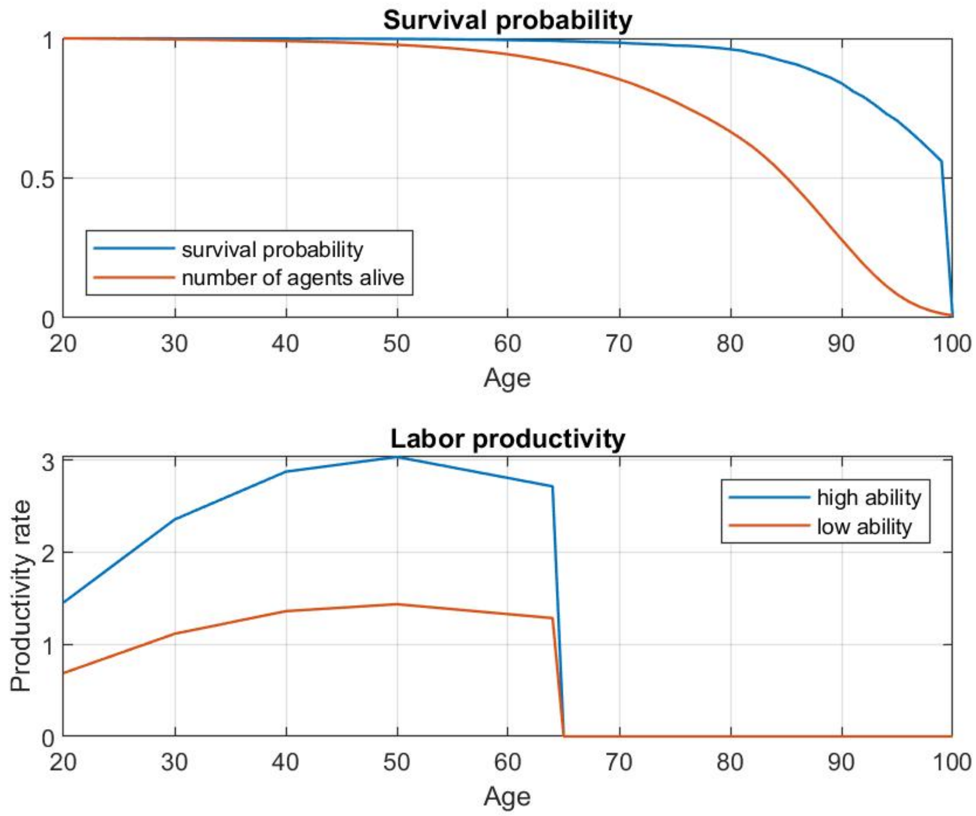

Figure 3.1.: Survival probability and labor productivity

The age dependent profile $\varepsilon_j, j = 1, \dots, Jr-1$, is modeled from gross annual income data by age group compared with the employment rates by age and gender out of 2019 from Statistics Austria (2020, 2021). The age dependent labor productivity profile is displayed in the lower panel of Figure 3.1. The slope increases in the beginning very fast and reaches its peak with the age of 50. Afterwards it slightly reduces back to the level reached with 40 years of age. The difference between the two ability types is clearly visible as the high able are more than twice as productive.

The stochastic income shocks are approximated by a $N = 7$ - state Markov chain from a simple AR(1) process. The scaling variable λ is set to $\lambda = 3$, which is recommended by Tauchen (1986). The AR(1) process is defined by a persistence parameter ρ and a variance σ_η^2. Fuchs-Schündeln et al. (2010) calculated a value of $\rho = 1$ and $\sigma_\eta^2 = 0.016$ to match with German income data. As the Austrian data is similar to the German data we take their variance parameter σ_η^2 and adapt the parameter ρ to $\rho = 0.99$ to allow computational feasibility in the discretization procedure of the AR(1) process.

3.4. Technology

We set the capital share parameter α to $\alpha = 0.36$, which is suitable to recent literature. The capital depreciation rate δ is taken from Conesa et al. (2009) and set to $\delta = 0.0833$.

3.5. Government

First, the pension system is financed by a social security tax τ_{SS}, which is between 15 % and 18 % of pretax labor earnings in Austria (Agenda Austria, 2021). We set $\tau_{SS} = 0.16$. The social security tax is paid up to a maximum value of $\bar{y}$. This threshold is set to $\bar{y} = 1.2075$, which is computed by taking the maximum value of pretax income for a single person, under the current Austrian tax system, multiplied by 1.5. This value should represent one full time employed person and one part time employed person with 50 % of full time per household.

Second, the households consumption is taxed by a proportional tax τ_c. There are three value added tax rates present in Austria, which is 10 % and 13 % for discounted goods and 20 % for standard products according to WKO (2021-b). We are only using one tax rate for the consumption tax and set τ_c to $\tau_c = 0.15$.

Third, capital and labor income taxes are part of the optimization process and of special interest. The capital tax τ_k is proportional while the labor income tax is specified by a three parametric function. The tax function $T(y_t)$ is taken from Gouveia and Strauss (1994)

$$T(y_t; \kappa_0, \kappa_1, \kappa_2) = \kappa_0(y - (y^{-\kappa_1} + \kappa_2)^{-1/\kappa_1}), \tag{3.1}$$

with the parameters $(\kappa_0, \kappa_1, \kappa_2)$. The parameter κ_0 controls the level of the tax rate, κ_1 specifies the progressivity of the tax rate and κ_2 balances the governments budget. The tax rate reduces to a flat tax for $\kappa_1 \to 0$. Figure 3.2 displays the average tax rates from Gouveia and Strauss (1994) with various combinations of the parameters $(\kappa_0, \kappa_1, \kappa_2)$ to sensitize for the shape of the function. The dashed blue line shows a flat tax with an average tax rate of 35 %. The solid red line presents the same flat tax as the dashed blue line but with positive progressivity κ_1. The higher the progressivity parameter is, the greater is the downward shift. This effect can be seen if we compare the solid red line with the dotted magenta line. The last combination of parameters, displayed as the dash-dotted green line, should indicate the role of the parameter κ_2. The line shifts downward the smaller the parameter κ_2 is but the slope also changes its curvature.

The tax function (3.1) can only display positive taxes. If the government assists low earners by paying them additional money to their wage instead of charging them any taxes, the tax function cannot represent this situation because negative taxes would be incurred. Here, the tax function for low income earners remain at zero.

Conesa et al. (2009) used the estimated parameters $(\kappa_0, \kappa_1, \kappa_2)$ from Gouveia and Strauss (1994) as their benchmark. These computations were made for the US and therefore we need to calibrate them for the Austrian tax system. We use the *Tax-benefit web calculator*[1] and their associated data from the OECD (2020). As an input to the calculator, we choose

[1]https://www.oecd.org/els/soc/tax-benefit-web-calculator/

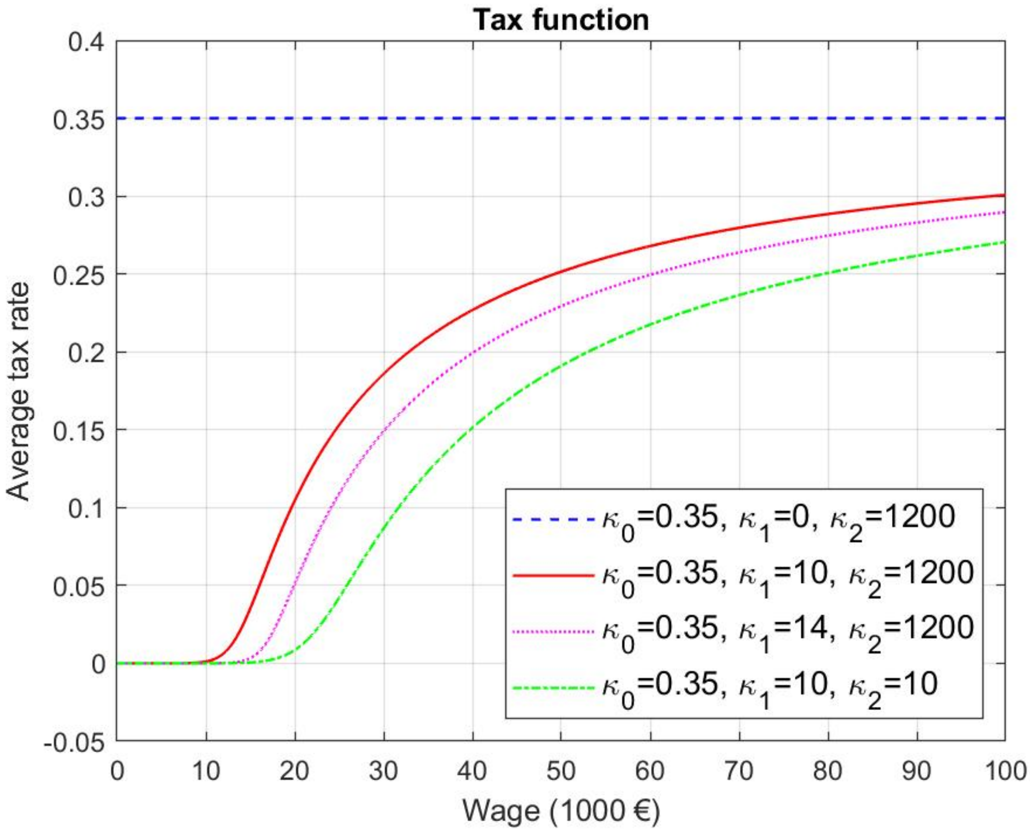

Figure 3.2.: Tax function from Gouveia and Strauss (1994) with different values of $(\kappa_0, \kappa_1, \kappa_2)$

a family with two children and one adult as a full time worker and the other adult as a part time employee with 50 % of full time. The parents are 35 years old and the children are 5 and 13 years of age. With a labor income from 10 % to 200 % of average wage (48 373 €) the tax burden is then calculated. Now the parameters $(\kappa_0, \kappa_1, \kappa_2)$, specifying the labor income tax function, are calibrated to match with the results from the *Tax-benefit web calculator*. The calibrated parameters are $(0.35, 10, 1200)$ and can be seen in Figure 3.2 as the solid red line. The benchmark capital tax is set to $\tau_k = 0.25$ taking the tax rate of savings and current accounts from WKO (2021-a).

3.6. Computational Experiment

The optimization process is specified by the four parameters $(\kappa_0, \kappa_1, \kappa_2, \tau_k)$. The parameter κ_2 is determined to assure the government budget balance and therefore the parameters $(\kappa_0, \kappa_1, \tau_k)$ are restricted to the existence of a suitable κ_2. The government wants to optimize the ex-ante lifetime utility of a newborn household entering the society in a stationary equilibrium. The government optimizes the following tax functions

$$\mathcal{T} = \{T_l(y_l), T_k(y_k);\; T_l(y_l) = T(y_l) \text{ and } T_k(y_k) = y_k\tau_k\},$$

where y_l defines taxable labor income and y_k taxable capital income. Labor income taxes are flexible with a wide range of progressivity while the capital income tax is held proportional to assure computational feasibility.

We measure the ex-ante expected lifetime utility for a given set of tax parameters with the following social welfare function

$$SWF\,(\kappa_0, \kappa_1, \tau_k) = \frac{1}{2}\sum_{i=1}^{2}\int V\;(a = 0, \eta = \bar{\eta}, i, j = 1)\;d\pi,$$

where each newborn household entering the economy with $a = 0$ assets, initial labor productivity status $\bar{\eta}$ and ability type i. In summary, welfare is determined for all households before the ability type as well as the initial labor productivity status of each individual household is known. All households start with zero assets and especially at young ages the desire to acquire assets, to insure yourself against productivity shocks, is great.

Parameters	**Value**	**Target**
Demographics		
Retirement age Jr	46 (65)	Compulsory retirement (assumed)
Maximum age J	81 (100)	Certain death (assumed)
Survival probability ϕ_j	Data	Human Mortality Database (2021)
Preferences		
Discount factor β	1.0124	K/Y = 3
Risk aversion σ	4	IES = 0.5
Consumption share γ	0.37	Average hours 1/3
Labor Productivity Process		
Variance types σ_α^2	0.14	Conesa et al. (2009)
Persistance ρ	0.99	Fuchs-Schündeln et al. (2010)
Variance shock σ_η^2	0.016	Fuchs-Schündeln et al. (2010)
Technology		
Capital share α	0.36	Conesa et al. (2009)
Depreciation rate δ	0.0833	Conesa et al. (2009)
Government policy		
Consumption tax τ_c	0.15	WKO (2021-b)
Capital tax τ_k	0.25	WKO (2021-a)
Marginal tax κ_0	0.35	*Tax-benefit web calculator*
Tax progressivity κ_1	10	*Tax-benefit web calculator*
Budget balance κ_2	1200	*Tax-benefit web calculator*
Payroll tax τ_{SS}	0.16	Agenda Austria (2021)

Table 3.1.: Calibrated parameters for Austria

4. Results

4.1. The Optimal Tax System

The optimal capital income tax is $\tau_k = 0.29$ with a labor income tax of $\kappa_0 = 0.34$ and $\kappa_1 = 8.5$. The parameter κ_2 yields to $\kappa_2 = 88.7$ to assure the government budget balance.

4.2. Comparison with the Benchmark

First we want to analyze the change of the aggregate variables and then go deeper in comparing the life cycle profiles of households with an optimal tax system against the benchmark tax system. Table 4.1 displays the changes of the aggregate variables for labor supply, capital, output and consumption as well as average hours worked and factor prices.

Variable	Benchmark	Optimal	Change in percent
Average hours worked	0.333	0.335	0.66
Total labor supply N	60.8	60.9	0.10
Capital stock K	338.7	340.4	0.53
Output Y	112.9	113.1	0.26
Aggregate consumption C	55.6	56.5	1.58
Wage per efficiency unit of labor w	1.23	1.23	0.00
Interest rate r	0.03	0.03	0.07
Social welfare (CEV)			0.54

Table 4.1.: Changes in the aggregate variables from the benchmark to the optimal tax system

On the one hand, we can observe that the change in percent of all aggregate variables is positive. That means under the new tax system aggregate consumption rises by around 1.6 % and with more consumption comes higher welfare.

On the other hand, average hours worked increase by 0.66 % and households have less time on leisure. This effect decreases the welfare of an agent. But overall, the increase in consumption overweights the slightly more invested time for labor supply.

Surprisingly, with a rise of the capital income tax from 25 % to 29 % the aggregate capital stock K is still growing, even if only slightly. Households tend to hold less assets the higher the capital income tax is. It seems that the growth of the economy exceeds the decline in the desire to hold assets.

The marginal tax rate κ_0 falls by 1 percentage point compared to the benchmark. This encourages households to supply more of their endowed time to the labor market, as they have to pay less taxes. Therefore average hours worked as well as total labor supply N rises. The greater increase of average hours worked than the total labor supply N results in a shift of labor supply from the high able to the less able households. The factor prices w and r remain approximately the same. We now want to take a look at the average tax rates and discuss the changes under the optimal tax system.

The average tax rate for the benchmark as well as the optimal tax system is presented in Figure 4.1. The average tax rate starts to rise by around 11 000 € and then strongly increases up to around 25 %. On this level the average tax rates stay constant with small gains.

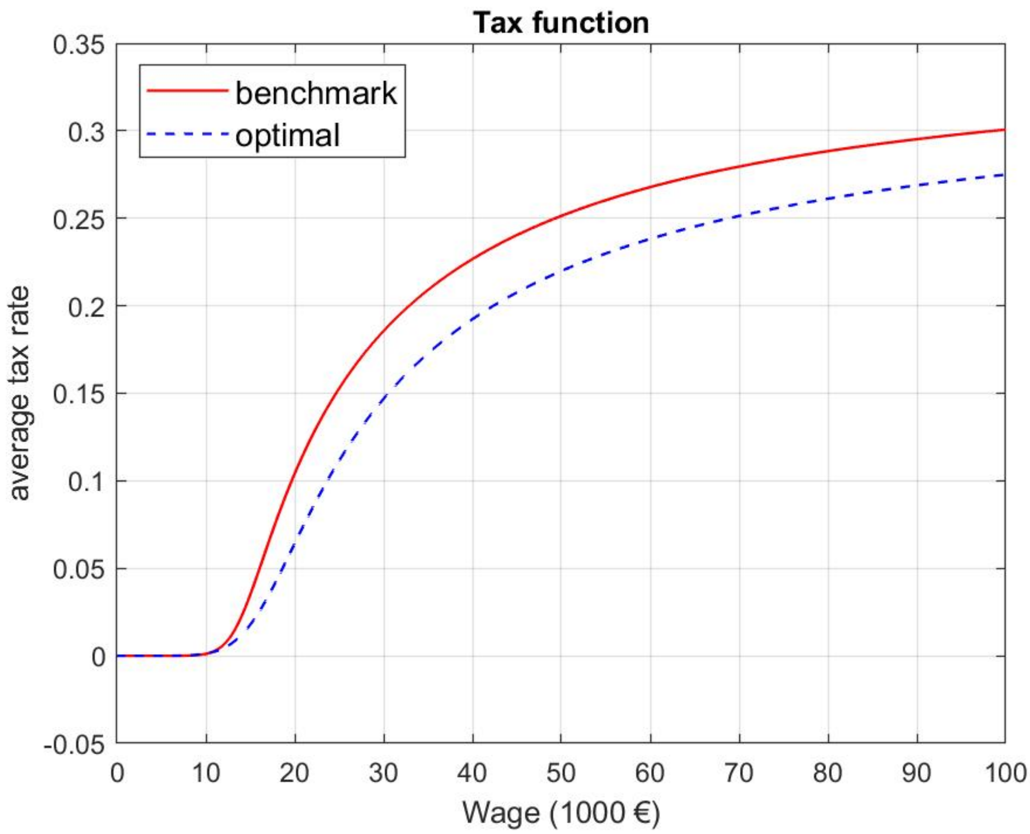

Figure 4.1.: Average tax rate for the benchmark and the optimal tax model

The reduction of the marginal tax rate κ_0 reduces the average tax rate for all income levels. Therefore all households benefit under the new tax system as they have to pay less income taxes. The next step in our analysis is to determine the welfare effect.

A value to measure the change in welfare is the consumption equivalent variation (CEV) which is defined by

$$CEV = \left(\frac{W(c_*, l_*)}{W(c_0, l_0)} \right)^{1/\gamma(1-\sigma)} - 1,$$

where $W(c, l)$ is the expected lifetime utility of a newborn household with a certain tax system. With the CEV we can calculate the welfare gains from switching between two steady state labor consumption allocations (c, l). Here we start with the benchmark allocation (c_0, l_0) and compare it to the consumption labor allocation (c_*, l_*) from the optimal tax system. Table 4.1 displays a CEV value of 0.54 %, which is a suitable increase when we take the rise in consumption as well as the higher working hours into account. The CEV value measures the welfare of a household with mean productivity level. But we also want to know how the welfare differ across the distribution of initial productivity levels. We display the change in the social welfare of all productivity states η from the benchmark to the optimal tax model in Figure 4.2.

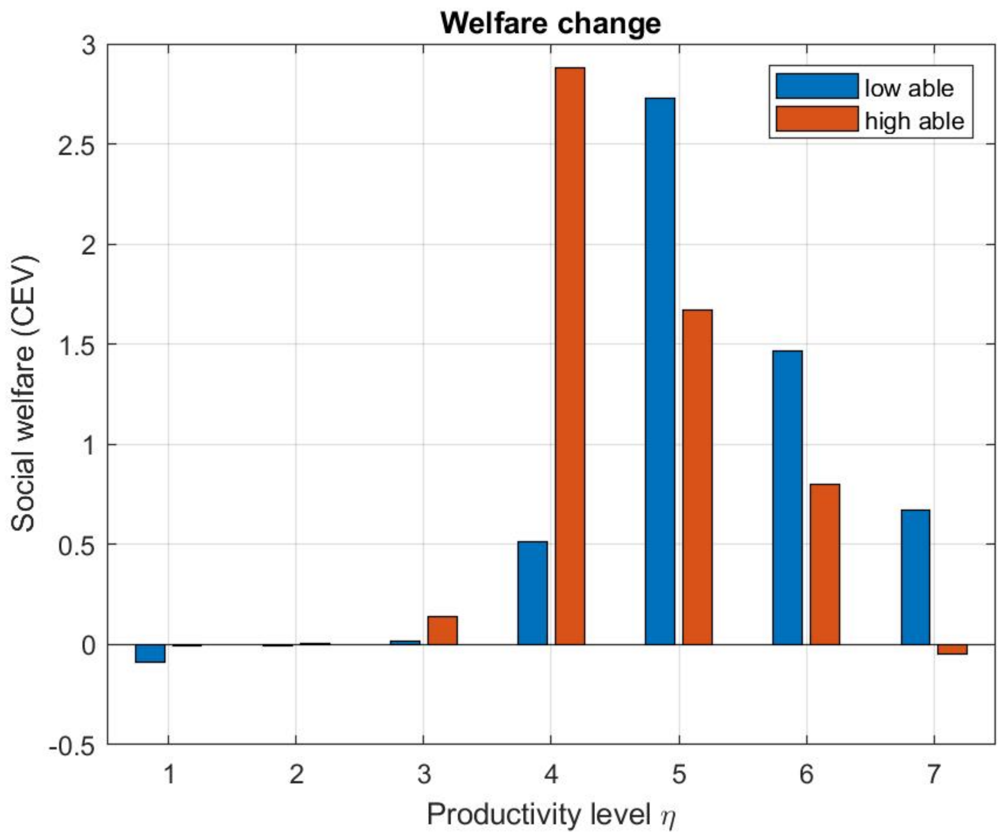

Figure 4.2.: Welfare change of all productivity states η

It is surprising that households with negative income realizations are equally well or even worse off under the optimal tax system. Only households with a positive or with no income shock benefit. Agents with low abilities and a positive income realization profit the most. High able households with average productivity have the highest welfare increase with nearly 3 %. The distribution of the productivity states is symmetrical and the outer states are less likely. The stationary distribution of the productivity states is given by approximately $\pi = [0.03, 0.10, 0.23, 0.30, 0.23, 0.10, 0.03]$. Therefore around 76 % of the households are in one of the inner three states. Extreme shocks (state 1,2,6 and 7) appear rarely and do not affect the average welfare that much. However, why do only households with positive income realizations benefit?

The first intuition is that households have to pay less labor income taxes. But that does apply for households with negative income realizations as well. Presumably, the time factor explains the welfare gain. Household are more productive and pay relatively less taxes over the years. This encourages agents to work more and allows them to acquire little more assets in each period. After their working period they benefit from higher consumption.

4.3. Life Cycle Profiles

We now want to analyze the welfare effects of the optimal tax system in more detail. Therefore, we plot the households life cycle profiles for assets, consumption, labor supply and income taxes. These life cycle profiles are displayed in Figure 4.3 for both ability types.

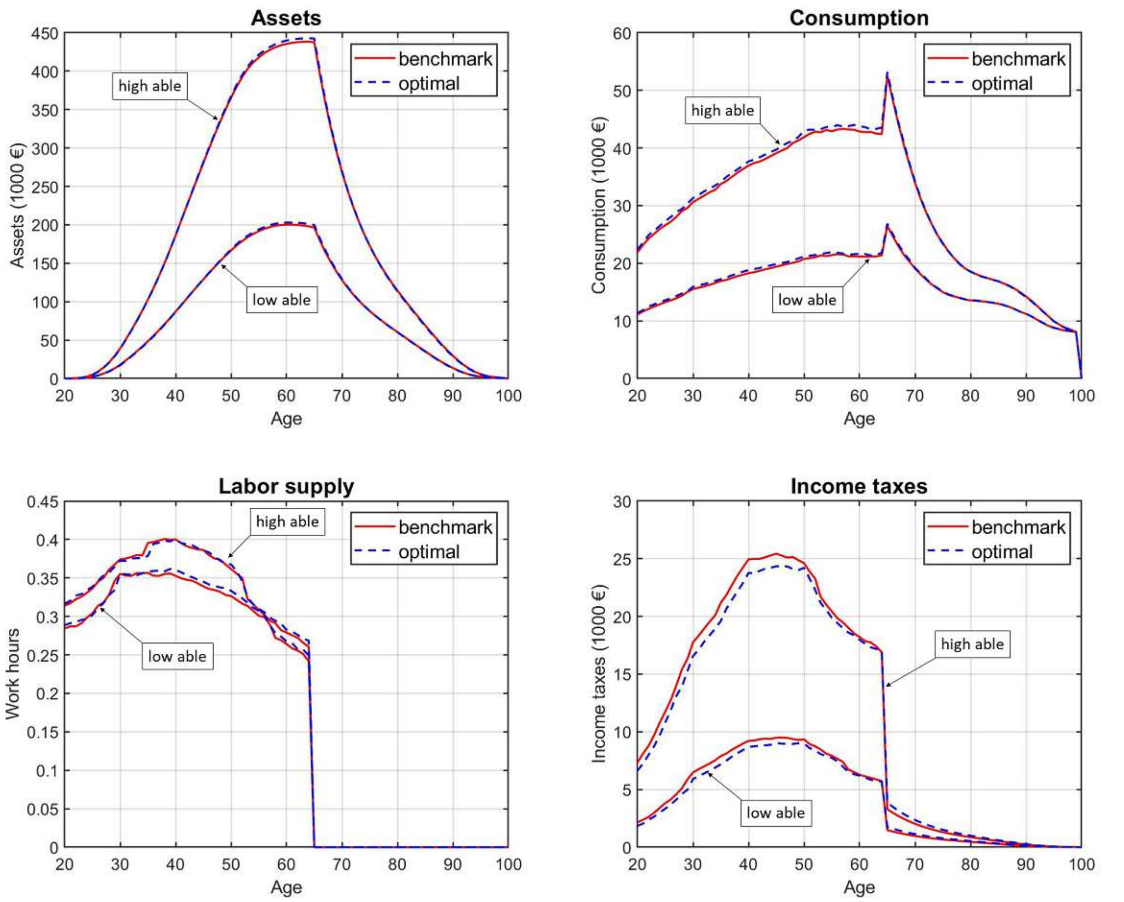

Figure 4.3.: Profiles of assets, consumption, labor supply and taxes over the life cycle

The current assets holdings for a certain age are presented on the upper left panel of Figure 4.3. The households asset holdings slightly increase from the age of 50 until retirement under the optimal tax system. This observation is larger for households with high ability, while it is almost the same for households with less ability. The hump shaped form of asset holdings is typical for a life cycle structure because agents save parts of their labor income for their pension. Moreover, the majority of the capital income tax burden have to be paid by households from 45 to 70 years of age.

On the upper right panel we can see the life cycle consumption path. The shape indicates that households consume more during their working period when they are more productive. After retirement agents start to dispose their collected assets for consumption. While getting older it is less likely to live that long and therefore households consume more at the beginning of their retirement period. The optimal tax model indicates more consumption during the working period but remains mostly the same for the retirement period.

The lower left panel displays the average hours worked for both ability types. Households start with around 30 % of their endowed time and increase their labor supply by around 8 percentage points until their forties. Afterwards they reduce it back to under 30 % until they reach their compulsory retirement age. The shape reflects the age dependent labor productivity over the working period. In other words, agents work more when they are more productive. The 0.66 % increase of hours worked in the model with the optimal tax code can be seen at the end of the working period. Because of the lower labor income taxes compared with the higher capital income tax, it is more efficient to invest more of your disposable time in the labor market as long as you can and postpone leisure to the retirement ages.

Finally, the income taxes are presented on the lower right panel of Figure 4.3. The income taxes include both capital and labor income taxes. The shape is dominated by labor income taxes which are the majority of the tax burden. The capital income tax is around one third of the total amount at the retirement age where the ratio of the capital tax reaches its peak. With a lower rise and fewer progressivity of the labor income tax function all income levels, especially lower earners, benefit. This can be seen in the reduced income taxes that has to be paid under the optimal tax system. Only at the end of the working period the income taxes of the benchmark compared to the optimal model are nearly the same. This is because of the higher capital income tax burden on the one hand and the increased working hours on the other hand.

4.4. Difference in the Ability Type

In the last section we saw how the aggregate variables changed from the benchmark to the optimal tax system and now we want to decompose these effects between the two ability types. The results are presented in Table 4.2.

Average hours worked rise for both ability types but the increase is much higher for the low able because they benefit more from less labor income taxes. The increase in the total labor supply N is only caused by less skilled households. High able agents value leisure more than any additional work because they would acquire more assets and in addition with higher capital income taxes, their tax burden would remain approximately the same or might even increase.

The capital stock K rises for both ability types with similar percent. All households can

Variable	Benchmark	Optimal	Change in percent
Low ability type			
Average hours worked	0.318	0.322	1.1
Total labor supply N	38.1	38.3	0.37
Capital stock K	215.1	216.5	0.66
Aggregate consumption C	38.3	39.0	1.76
Government budget G	13.9	13.8	-0.81
High ability type			
Average hours worked	0.348	0.349	0.25
Total labor supply N	83.5	83.5	-0.01
Capital stock K	462.2	464.4	0.47
Aggregate consumption C	73.0	74.0	1.48
Government budget G	35.4	35.5	0.32

Table 4.2.: Changes in the aggregate variables from the benchmark to the optimal tax system for both ability types

acquire more assets under the optimal tax system.

The aggregate consumption C is one of the main welfare enhancing forces and it is not surprising that this value increases for both skill levels. The combination of higher capital income taxes and lower labor income taxes supports the households with low ability and therefore their increase of 1.76 % is higher than the overall increase in consumption of 1.58 % (see Table 4.1).

As we hold the total amount of the government budget G constant over the optimization process, only the share between the two ability types changes. While the high able have to pay more taxes, the less able benefit from a tax reduction. To make further and more detailed statements about the tax burden we want to analyze the distribution of the taxes in the following section.

4.5. Tax Distribution

The government levies taxes on labor income, capital income and consumption to balance the government budget. We now want to analyze which tax accounts for the biggest share of the budget and determine the differences between the benchmark and the optimal tax model.

The distribution of the government budget is displayed in Figure 4.4. We can see that the labor income taxes add up to more than half of the government budget, while the capital income taxes are comparatively small with only 10.4 %. Taxes on consumption account for about one third of total taxes. This means that households pay most of their taxes during their working period and less taxes in their pension. If we compare the benchmark with the optimal tax distribution, plausible changes in the allocations of the tax burden emerge. On the one hand, lower labor income taxes decrease the ratio of the labor income taxes by

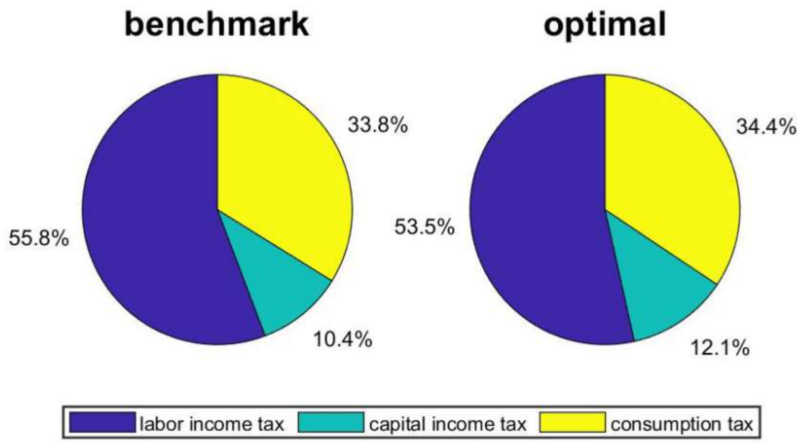

Figure 4.4.: Tax distribution

more than 2 percentage points. On the other hand, higher capital income tax increases the share by nearly 2 percentage points. The remaining part of the budget stem from higher aggregate consumption and thus higher consumption taxes.

We also want to take a closer look at the tax distribution for both ability types. Figure 4.5 presents this tax distribution for the benchmark model as well as the optimal tax model.

Before we discuss the two ability types in greater detail we can see that the same interpretation from Figure 4.4 applies if we compare the benchmark with the optimal tax model for both types.

The low able households benefit from less labor income taxes and therefore the share is smaller while consumption taxes take a bigger piece of the pie. The other way round applies for the high able households. Their amount of taxes on consumption are relatively small compared to the labor income taxes and therefore the ratio from the whole is smaller. The capital income taxes are only a small part of the whole tax burden, while again the ratio is even smaller for the high able. One necessary point that has to be mentioned is that the high able households pay around 70 % of the government budget while the low able are only responsible for 30 % of the tax burden (see Table 4.2). For example, if we want to calculate the share of the capital income taxes of 10.4 % from Figure 4.4, the low able are responsible for around 3.5 %, while the high able are accountable for approximately 6.9 %. This calculation should assist the statement between the share and the total amount of taxes.

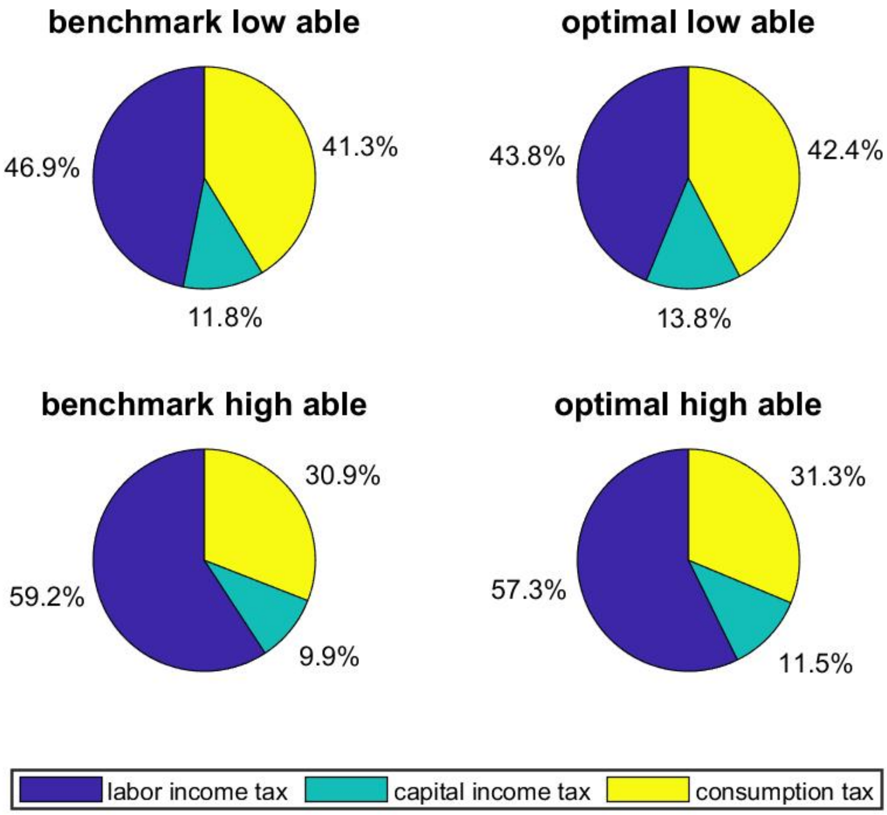

Figure 4.5.: Tax distribution for both ability types

5. Sensitivity Analysis, the Consumption Tax

In the last chapter of this thesis we want to analyze another aspect between a central European country as Austria compared to the US. We have already seen the difference in the optimal tax system with higher taxes on labor income and less taxes on capital income for Austria in relation to the US. We have learned that for Austrian data one third of the tax burden is financed by consumption taxes. What results can we expect if we change the exogenously given consumption tax?

In comparison to our calibrated consumption tax of $\tau_c = 0.15$, Conesa et al. (2009) used a consumption tax of $\tau_c = 0.05$ for their calculations. We want to determine the driving forces for their results by adapting the consumption tax to $\tau_c = 0.05$ and solve the model again. Due to the lower consumption tax the government budget change its amount and reduces the possible spendings of the government.

We solved the model again with a consumption tax of $\tau_c = 0.05$. All parameters are recalibrated to the same set of moments as in the baseline model. The results are as follows. The optimal capital income tax is $\tau_k = 0.29$ with a labor income tax schedule of $\kappa_0 = 0.33$, $\kappa_1 = 7$ and $\kappa_2 = 43.8$. The latter is a flat tax of 33 % with a deduction of about 5500 € (relative to an average household income of 48 373 €). The results are similar to the previous results from section 4.1. The capital income tax remains the same whereas the level of the tax schedule κ_0 falls slightly. The progressivity parameter κ_1 also decreases by 1.5 points. To compare these results with our previous ones we display the average tax rate for both models with different consumption taxes in Figure 5.1.

The difference between the two models is not that severe. The dash-dotted magenta line has little less progressivity which can be seen at the beginning of the increase. Afterwards the shape is more or less identical while at the end the impact of the one percentage point higher marginal tax rate of the dashed blue line is visible. In summary, the optimal tax system with a consumption tax of $\tau_c = 0.05$ taxes low earners slightly more while high income levels benefit from little less income taxes.

In a next step, we want to take a look at the change of welfare. The CEV value yields to 0.98 %. Therefore the welfare increase is almost twice as big compared to the model with $\tau_c = 0.15$. Considering the much lower consumption tax, this outcome is not very surprising. To analyze how welfare differ across the distribution of initial productivity states we again display the welfare changes for all income realizations and both ability types in Figure 5.2.

First, we can observe a welfare increase for all productivity states and ability types. No one is worse off under the optimal tax model. Second, the size of the increase is again significantly higher for households with positive income shocks. If we take a separate look on the two ability types we can detect a higher increase in the social welfare for the high able agents. Their social welfare increases by around 1.8 percentage points while the low able households only rise by around 1.3 percentage points. This result is surprising as the

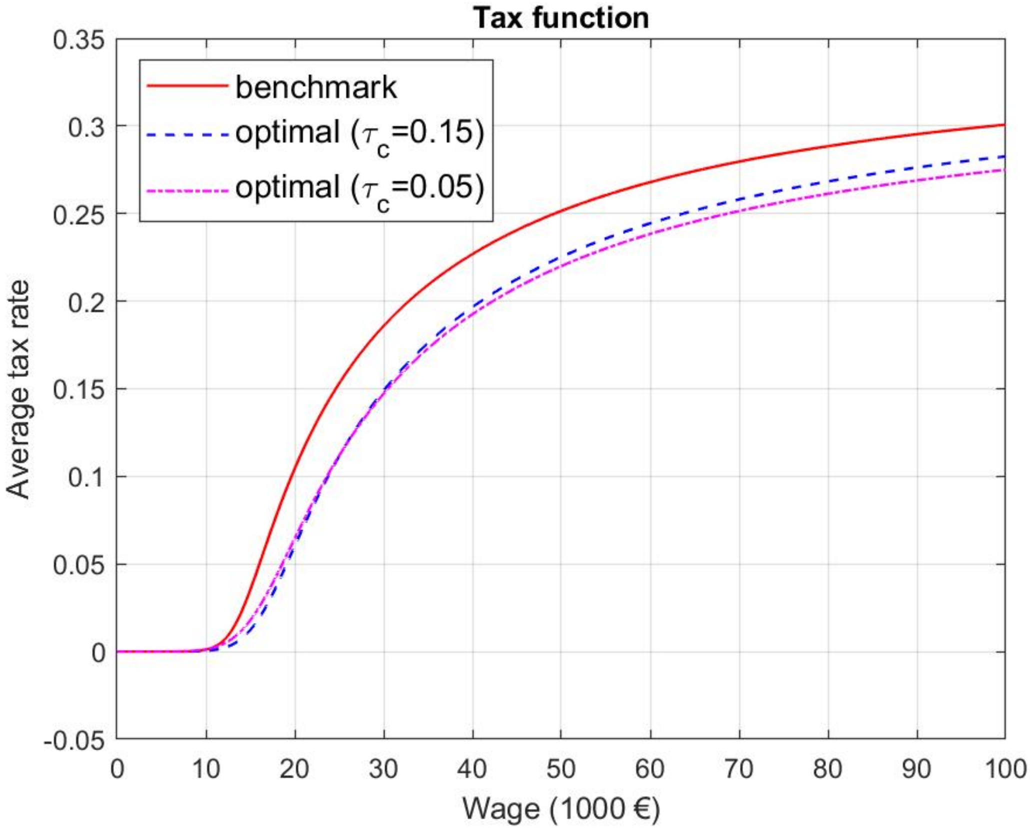

Figure 5.1.: Average tax rate for the benchmark and the optimal tax model with $\tau_c = 0.15$ and $\tau_c = 0.05$

higher capital income tax affects the high able more because they hold much more assets than the low able. In order to specify these results we now want to take a look at the households life cycle profiles.

5.1. Life Cycle Profiles

Following the same layout from section 4.3 we display the life cycle profiles of assets, consumption, labor supply and income taxes with a consumption tax of $\tau_c = 0.05$ in Figure 5.3.

We want to start with the assets life cycle profile on the upper left hand side. The shape of the slope looks similar to the profile with a higher consumption tax, only the peak seems more symmetrical. The more interesting observation is the y-axis where the amount of assets is displayed. The amount of assets at the peak rises by around 25 to 30 percentage points for both ability types. It follows that households use their extra money not only to consume more but also to buy more assets.

The upper right panel of Figure 5.3 shows the consumption path over the life cycle.

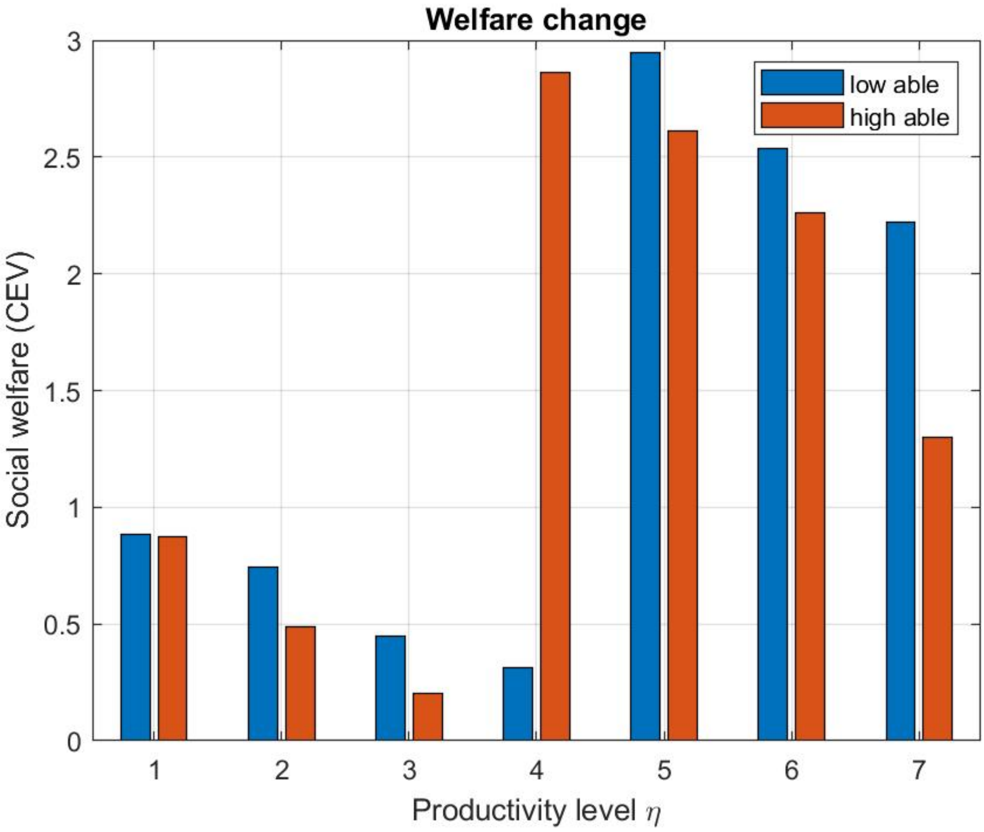

Figure 5.2.: Welfare change of all productivity states η with $\tau_c = 0.05$

The first finding is that the slope changes its shape while reaching the retirement period. Households consume more during their working period and when they retire their consumption drops significantly. These characteristics coincide with the results from Conesa et al. (2009). The reason is that households tend to smooth their consumption over the life cycle and this goal is easier to achieve with a lower consumption tax. The lower the consumption tax is, the lesser assets you need to provide for your pension to hold a certain level of consumption. Therefore households benefit during their working period because they can consume more for the same money and also during their pension where less assets are needed. If we compare the two consumption paths from Figure 4.3 and Figure 5.3 we can observe that the average consumption level is higher with a consumption tax of 5 %. While this difference is not that significant during the working period, it is clearly visible during the retirement period. The less able households have an average consumption level of about 22 000 € with $\tau_c = 0.05$ and around 16 000 € with $\tau_c = 0.15$. Another impact of the different shape at the time of retirement is the nonseparability between consumption and leisure during the working period. With only consumption present in the utility function households value consumption differently after retirement.

The shape of the life cycle profile of the labor supply looks quite similar compared to the profile with a higher consumption tax. The difference is again the amount of working

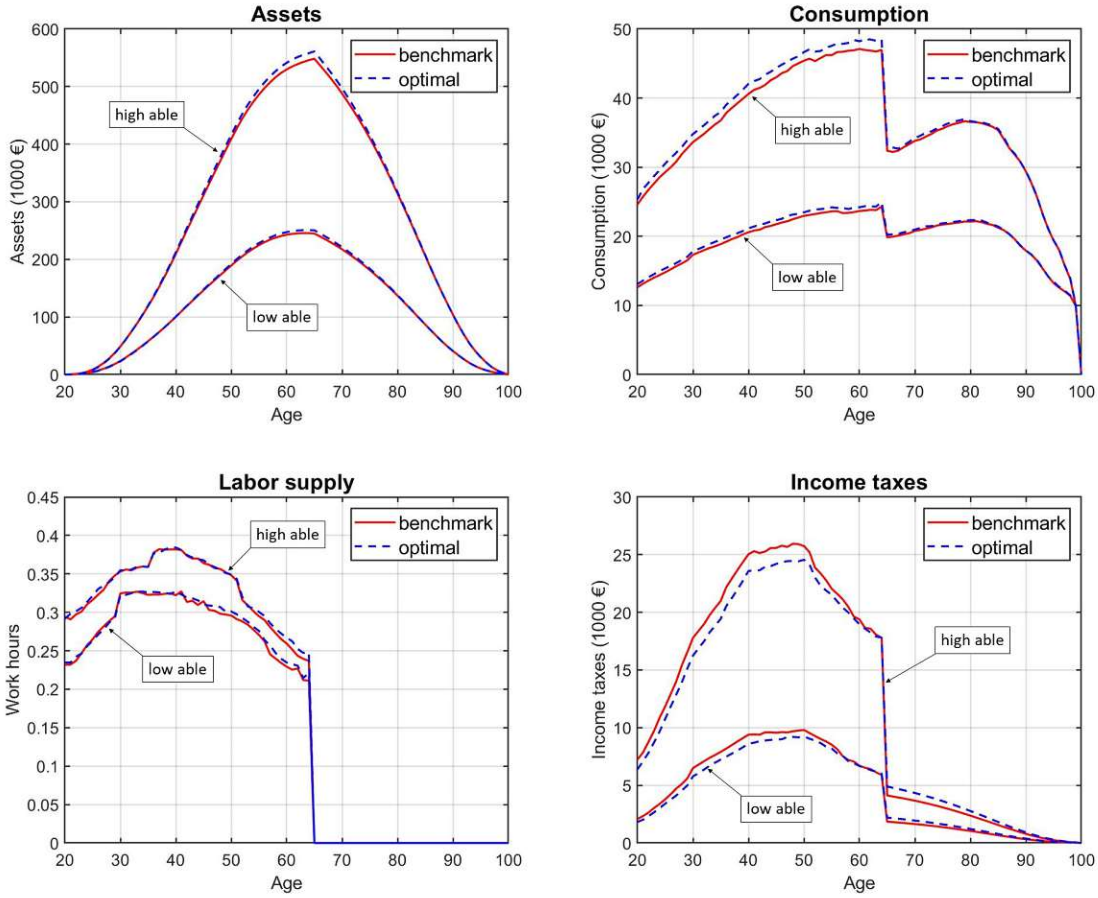

Figure 5.3.: Profiles of assets, consumption, labor supply and taxes over the life cycle with $\tau_c = 0.05$

hours. The average hours worked fall by around 45 minutes which is approximately 0.03 percentage points. Therefore households work less hours because they have to pay fewer taxes and can spend more time on leisure.

The lower right panel presents the income taxes over the life cycle. The shape as well as the amount of income taxes remain roughly the same. Because of the reduced working hours the labor income taxes decrease while the capital income taxes increase due to higher asset holdings.

5.2. Tax Distribution

The distribution of taxes supposedly change with a lower consumption tax. We now want to determine the size of the change. Figure 5.4 presents the benchmark and the optimal allocation of the tax distribution.

The ratio of consumption taxes declines significantly while labor and capital income

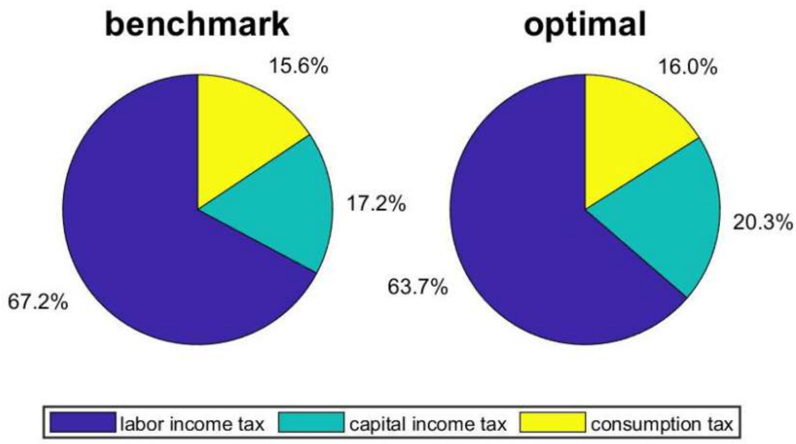

Figure 5.4.: Tax distribution with $\tau_c = 0.05$

taxes increase their share. Labor income taxes add up to two third of total taxes while capital income taxes almost double their share. Again capital income and consumption taxes increase in the optimal tax model while labor income taxes decrease. These results fit to our previous findings.

In summary, the lower consumption tax allows households to better smooth their consumption over the life cycle because they can acquire more assets during their working period and therefore consume more in their retirement period.

6. Conclusion

In this thesis we analyzed how changes to the current Austrian tax system can have a welfare gain for the whole society. We optimized an overlapping generation model with heterogeneous agents, different abilities and the possibility of facing an idiosyncratic income shock. The policymaker takes the households desire to hold assets as an insurance into account while determining the optimal tax system that redistributes wealth and income risk across society.

The main finding is that the current tax system in Austria is close to the optimal one found in this model framework. The slightly lower labor income tax schedule eases the acquiring of assets especially at young ages and increases the consumption during the working period. The higher capital income taxes affect older households more. However, they are more productive at the end of their working period and therefore this negative effect is not that severe.

This model framework leaves room for many interesting model extension for future research. The pension system used in this model does not distinguish between households with low and high labor income. Therefore, richer households acquire more assets to maintain a certain level of consumption during the retirement period. A pension system that reflects the labor income with an adapted social security payment could have serious impacts on households asset holdings and as a consequence also on the optimal tax system.

Another point neglected in the analysis is the possibility of altruism across generations. Lump sum payments to their children as well as agents entering the society with positive asset holdings are not present in this model. Further research might extend the model with one or two sided altruism and analyses the impacts on the optimal tax system.

In addition, the households structure could be considered and taken into the model. We focused only on average households with two parents and two children. Other household structures, such as single households or families with more or less than two children, could have an impact on the optimal labor income taxes.

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Appendix

A. Coding Procedure

The software used for the calculations in this thesis is `Matlab R2021a`. To determine the optimal tax system, the first step is to define all calibrated parameters from Table 3.1. Afterwards the asset grid as well as the labor grid are specified. The asset grid goes from 0 to 100 and contains 41 grid points. The length of steps is 0.5 at the beginning up to a step length of 10 at the end. This is due to the increase of the value function. At the beginning the increase in the value function is very high while at the end the slope remains with small gains. Therefore, a fine grid is optimal at the beginning while a coarse grid is sufficient at the end. The labor grid start at 0 and afterwards 29 grid points are equally distributed between 0.16 and 0.5. The computation time would significantly increase if we would use more grid points.

Before we can start the optimization process we need to calculate the idiosyncratic labor income shocks. Therefore we create a discrete time Markov chain and calculate the states of labor productivity and the transition matrix (see section 2.2). Thereafter, we can start the optimization process by defining the initial conditions for N, K, r, TrB, SS. We then run a while loop as long as we reach a steady state. A steady state is reached when the capital stock and the labor supply are not changing anymore. Each iteration calculates the aggregate variables and updates them for the next loop and so on. One iteration has the following procedure.

At the beginning the capital stock, the aggregate output and the wage per efficiency unit of labor are calculated. Thereafter we calculate the value function via backward iteration for the retirement period. This is done by determining the possible asset holdings for the previous period and use the golden search method to find the optimal policy. For each point determined by the golden search we calculate the utility and interpolate over the value function at this point. This is done until the maximum is found. The same procedure is done twice for both ability types to find the optimal policy for the working period but with additionally optimizing over the labor grid.

The value function iteration is done with interpolation because of speed reasons. The value function iteration without interpolation requires ten times more computation time.

We now have determined the optimal policy for all states of productivity, ability type and for every age. The next step is to calculate the households life cycle allocations for all variables. The aggregate variables determining the whole society are then calculated with the Monte Carlo method. We take 1000 household for each ability type as representatives of the whole society. The mean over the households then determines the aggregate variables. Afterwards the interest rate is updated to match capital supply with capital demand. If the capital stock and the labor supply are not changing anymore a steady state is reached.

Now we can calculate the optimal policies for different tax allocations and search for the optimal combination of labor and capital income taxes that maximizes welfare to society. The following pages display the Matlab code of the procedure described above.

B. Matlab Code

```
%% Parameters

% Demographics
Jr=46; J=81;

% Preferences
beta=1.0124; sigma=4; gamma=0.37;

% Labor productivity process
sigma_alpha=0.14; rho=0.99; sigma_nu=0.016;

% Technology
alpha=0.36; Delta=0.0833;

% Survival probabilities: surv(i)=prob(alive in i+1|alive in i)

dea_rate=[0.00033,0.00044,0.00038,0.00023,0.00038,0.00046,0.00032,0.00045,
  0.00041,0.00036,0.00045,0.00053,0.00053,0.00062,0.00053,0.00056,0.00063,
  0.00060,0.00066,0.00084,0.00090,0.00109,0.00092,0.00102,0.00109,0.00158,
  0.00163,0.00164,0.00193,0.00184,0.00213,0.00225,0.00273,0.00282,0.00317,
  0.00369,0.00406,0.00465,0.00463,0.00543,0.00607,0.00694,0.00739,0.00828,
  0.00849,0.01026,0.01097,0.01271,0.01337,0.01450,0.01554,0.01806,0.01957,
  0.02146,0.02256,0.02600,0.02645,0.02877,0.03154,0.03490,0.03921,0.04402,
  0.05360,0.06111,0.07320,0.08404,0.09418,0.10899,0.12550,0.14070,0.16095,
  0.18862,0.21028,0.23788,0.26917,0.29449,0.32818,0.36390,0.40136,0.44018,
  0.47993]';

surv=1-dea_rate; surv(81)=0;

% Number of agents alive
Nu=zeros(J,1); Nu(1)=1;
for i = 2:J; Nu(i)=surv(i-1)*Nu(i-1)/(1+nn); end

% Age productivity profile
inc=[1,1.62,1.977,2.087,1.93]';
ep1 = interp1(1:5,inc,0.9+(1:45)/10,'linear','extrap');
ep(1,1:Jr-1)=exp(-sqrt(sigma_alpha))*ep1(1:Jr-1);
ep(2,1:Jr-1)=exp(sqrt(sigma_alpha))*ep1(1:Jr-1);

% Government policy
tau_c=0.15; tau_ss=0.16; maxSS=0.805*1.5;

% Austrian tax system (benchmark)
kap_0=0.35; kap_1=10; kap_2=1200; tau_k=0.25;

```

```
%% Model calibration

% Asset grid
a_min=0; a_max=100; na=41;
agrid1=linspace(0,10,21); % 0.5 steps
agrid2=linspace(11,15,5); % 1 steps
agrid3=linspace(17.5,25,4); % 2.5 steps
agrid4=linspace(30,55,6); % 5 steps
agrid5=linspace(60,100,5); % 10 steps
agrid=horzcat(agrid1,agrid2,agrid3,agrid4,agrid5);

% Labor grid
nl=30;
gridl=linspace(0.15,0.5,nl); gridl(1)=0;
gridl1=(1-gridl).^((1-gamma)*(1-sigma));

% Number of different ability types
nty=2; % two ability types

% Idiosyncratic labor income shock
sigma_l=sqrt(sigma_nu); ns=7; mu1=0;
[log_grid, Pi] = mytauchen(mu1,rho,sigma_l,ns);
epsgrid=exp(log_grid);
Pi2=Pi';

mc = dtmc(Pi); % creating discrete time Markov chain
di = asymptotics(mc); % ergodic distribution of the Markov chain
di_sum = cumsum(di);

% Calculating random idiosyncratic shocks
Pc=cumsum(Pi,2); num_HH = 1000; labor=ones(num_HH,J);
rng(11111), U=rand(num_HH,J);
for h=1:num_HH
    if(U(h,1)≤di_sum(1)), state=1; elseif(U(h,1)≤di_sum(2)), state=2;
    elseif(U(h,1)≤di_sum(3)), state=3; elseif(U(h,1)≤di_sum(4)), state=4;
    elseif(U(h,1)≤di_sum(5)), state=5; elseif(U(h,1)≤di_sum(6)), state=6;
    else, state=7;
    end
    labor(h,1)=epsgrid(state);
    for t=2:J
        if(U(h,t)≤Pc(state,1)), state=1;
        elseif(U(h,t)≤Pc(state,2)), state=2;
        elseif(U(h,t)≤Pc(state,3)), state=3;
        elseif(U(h,t)≤Pc(state,4)), state=4;
        elseif(U(h,t)≤Pc(state,5)), state=5;
        elseif(U(h,t)≤Pc(state,6)), state=6;
        else, state=7;
        end
    labor(h,t) = epsgrid(state);
    end
end

% Golden search coefficient
phi1=(3-sqrt(5))/2;
phi2=(sqrt(5)-1)/2;
```

```
% Initialisation of life cycle profiles
a_bench=zeros(nty,J); l_bench=zeros(nty,J); c_bench=zeros(nty,J);
t_bench=zeros(nty,J); wages_bench=zeros(nty,J);

% Initialisation of aggregate variables
N_new=zeros(1,nty); K_new=zeros(1,nty); C_new=zeros(1,nty);
Tfun_new=zeros(1,nty); Tc_new=zeros(1,nty); Tk_new=zeros(1,nty);
TSS_new=zeros(1,nty); G_new=zeros(1,nty); TrB_new=zeros(1,nty);
ah=zeros(1,nty); Trn=zeros(1,nty); SS_new=zeros(1,nty); SWFh=zeros(1,nty);
summary=zeros(20,15);

% *** solving the model *** (113-375)

% Initial conditions for N,K,r,TrB,SS
N=60; N_old=0; K=330; K_old=0;
TrB=0.053; SS=0.28; r=0.0301;

n=1;
while((abs(N-N_old)>0.05 || abs(K-K_old)>0.5 || n<5) && n≤100)
n=n+1;
N_old=N;
K_old=K;

K = N*((alpha) / (r+Delta))^(1/(1-alpha));  % capital stock
Y = (K^alpha)*(N^(1-alpha)); % aggregate output
w = (1-alpha)*Y/N; % wage

valfun=zeros(na,ns,J); astar=zeros(na,ns,J);
util=zeros(na,ns,J); indl=ones(na,ns,Jr-1); cons=zeros(na,ns,J);
con=zeros(4,1); ut=zeros(4,1);

for ss=J-1:-1:Jr % retirement period
    vv=valfun(:,1,ss+1); % same for each j

    for i=1:na
        X(1)=a_min;
        X(4)=(SS+(1+r*(1-tau_k))*(agrid(i)+TrB))/(1+tau_c);
        X(2)=X(1)+phi1*(X(4)-X(1));
        X(3)=X(1)+phi2*(X(4)-X(1));

        F=zeros(4,1);
        for mm=1:4
            con(mm)=(SS+(1+r*(1-tau_k))*(agrid(i)+TrB)-X(mm))/(1+tau_c);
                if con(mm)<0
                    con(mm)=NaN;
                end
                ut(mm)= 1/(1-sigma)*con(mm)^(gamma*(1-sigma));

            F(mm)=ut(mm)+surv(ss)*beta*interp1q(agrid,vv,X(mm));
        end
        while(X(4)-X(1)) > 0.01
            if(F(2) > F(3))
                X(3:4)=X(2:3); con(3:4)=con(2:3);
                F(3:4)=F(2:3); ut(3:4)=ut(2:3);
```

```
                X(2)=X(1)+phi1*(X(4)-X(1));

                con(2)=(SS+(1+r*(1-tau_k))*(agrid(i)+TrB)-X(2))/(1+tau_c);
                if con(2)<0
                    con(2)=NaN;
                end
                ut(2)= 1/(1-sigma)*con(2)^(gamma*(1-sigma));

                F(2)=ut(2)+surv(ss)*beta*interp1q(agrid,vv,X(2));
            else
                X(1:2)=X(2:3); con(1:2)=con(2:3);
                F(1:2)=F(2:3); ut(1:2)=ut(2:3);
                X(3)=X(1)+phi2*(X(4)-X(1));

                con(3)=(SS+(1+r*(1-tau_k))*(agrid(i)+TrB)-X(3))/(1+tau_c);
                if con(3)<0
                    con(3)=NaN;
                end
                ut(3)= 1/(1-sigma)*con(3)^(gamma*(1-sigma));

                F(3)=ut(3)+surv(ss)*beta*interp1q(agrid,vv,X(3));
            end
        end
        [valfun(i,:,ss),index]=max(F);
        astar(i,:,ss)=X(index);
        cons(i,:,ss)=con(index);
        util(i,:,ss)=ut(index);
    end
end

con=zeros(nl,1); ut=zeros(nl,1); consh=zeros(4,1);
uth=zeros(4,1); indh=zeros(4,1);

for type=1:nty

for ss=Jr-1:-1:1 % working period

    for j=1:ns
        vv=valfun(:,:,ss+1)*Pi2(:,j);

        X=zeros(4,1); % X(1)=0
        earnl0=w*epsgrid(j)*gridl(nl)*ep(type,ss); % pretax labor income
        y=earnl0-tau_ss*min(maxSS,earnl0); % social security tax
        y_start=y-kap_0*(y-(y^(-kap_1)+kap_2)^(-1/kap_1));

        for i=1:na
            X(4)=min((y_start+(1+r*(1-tau_k))*(agrid(i)+TrB))/(1+tau_c),...
                a_max);
            X(2)=X(1)+phi1*(X(4)-X(1));
            X(3)=X(1)+phi2*(X(4)-X(1));

            F=zeros(4,1);
            for mm=1:4
                for k=1:nl
                        earnl0=w*epsgrid(j)*gridl(k)*ep(type,ss);
```

```
                    y=earnl0-tau_ss*min(maxSS,earnl0);
                    con(k)=(y+(1+r*(1-tau_k))*(agrid(i)+TrB)-...
                        kap_0*(y-(y^(-kap_1)+kap_2)^(-1/kap_1))...
                        -X(mm))/(1+tau_c);
                  if con(k) <0
                      con(k)=NaN;
                  end
                    ut(k)= 1/(1-sigma)*(con(k).^(gamma*(1-sigma))...
                        *gridl1(k));
            end
        [uth(mm),indh(mm)]=max(ut);
        consh(mm)=con(indh(mm));

        F(mm)=uth(mm)+surv(ss)*beta*interp1q(agrid,vv,X(mm));
        end

        while(X(4)-X(1)) > 0.01
            if(F(2) > F(3))
                X(3:4)=X(2:3); consh(3:4)=consh(2:3);
                indh(3:4)=indh(2:3); F(3:4)=F(2:3); uth(3:4)=uth(2:3);
                X(2)=X(1)+phi1*(X(4)-X(1));

                for k=1:nl
                    earnl0=w*epsgrid(j)*gridl(k)*ep(type,ss);
                    y=earnl0-tau_ss*min(maxSS,earnl0);
                    con(k)=(y+(1+r*(1-tau_k))*(agrid(i)+TrB)-...
                        kap_0*(y-(y^(-kap_1)+kap_2)^(-1/kap_1))...
                        -X(2))/(1+tau_c);
                if con(k) <0
                    con(k)=NaN;
                end
                    ut(k)= 1/(1-sigma)*...
                        (con(k).^(gamma*(1-sigma))*gridl1(k));
                end
                    [uth(2),indh(2)]=max(ut);
                    consh(2)=con(indh(2));

            F(2)=uth(2)+surv(ss)*beta*interp1q(agrid,vv,X(2));

            else
                X(1:2)=X(2:3); consh(1:2)=consh(2:3);
                indh(1:2)=indh(2:3); F(1:2)=F(2:3); uth(1:2)=uth(2:3);
                X(3)=X(1)+phi2*(X(4)-X(1));

                for k=1:nl
                    earnl0=w*epsgrid(j)*gridl(k)*ep(type,ss);
                    y=earnl0-tau_ss*min(maxSS,earnl0);
                    con(k)=(y+(1+r*(1-tau_k))*(agrid(i)+TrB)-...
                        kap_0*(y-(y^(-kap_1)+kap_2)^(-1/kap_1))...
                        -X(3))/(1+tau_c);
                if con(k) <0
                    con(k)=NaN;
                end
                    ut(k)= 1/(1-sigma)*...
                        (con(k).^(gamma*(1-sigma))*gridl1(k));
```

```
                    end
                        [uth(3),indh(3)]=max(ut);
                        consh(3)=con(indh(3));

                F(3)=uth(3)+surv(ss)*beta*interp1q(agrid,vv,X(3));
                end

            end
            [valfun(i,j,ss),index]=max(F);
            astar(i,j,ss)=X(index);
            cons(i,j,ss)=consh(index);
            util(i,j,ss)=uth(index);
            indl(i,j,ss)=indh(index);
        end
    end
end

% Calculating life cycle allocations for all households
assets=zeros(num_HH,J); % starting asset = 0
C=zeros(num_HH,J); wages=zeros(num_HH,J); hours=zeros(num_HH,J);
SocWhh=zeros(num_HH,J); TT=zeros(num_HH,J); LL=zeros(num_HH,Jr-1);
Taxl=zeros(num_HH,Jr-1); Taxk=zeros(num_HH,J); Taxc=zeros(num_HH,J);
for h=1:num_HH
  for t=1:J-1
      for j=1:ns
          if(labor(h,t)==epsgrid(j))
              [gap,ind]=min(abs(assets(h,t)-agrid));
              % find nearest gridpoint of current asset
              if(gap==0 || assets(h,t)<0)
                  current_ind=ind;
              else,if(agrid(ind)-assets(h,t)>0 || ind==na)
                     % find second nearest grid point
                     ind2=ind-1; else, ind2=ind+1; end
                  prob=gap/abs(agrid(ind)-agrid(ind2));
                  % probability of going to the nearest neighbour
                  % otherwise go to the second nearest
                  if(prob>U(h,t)) % draw randon number
                      current_ind=ind2; else, current_ind=ind; end
              end
              if t<Jr % working period
              earnl0=w*epsgrid(j)*gridl(indl(current_ind,j,t))*ep(type,t);
              y=earnl0-tau_ss*min(maxSS,earnl0);

              assets(h,t+1)=y+(1+r*(1-tau_k))*(assets(h,t)+TrB)-...
                  kap_0*(y -(y^(-kap_1)+kap_2)^(-1/kap_1))-...
                  (1+tau_c)*cons(current_ind,j,t);

              TT(h,t)=kap_0*(y-(y^(-kap_1)+kap_2)^(-1/kap_1));
              LL(h,t)=epsgrid(j)*gridl(indl(current_ind,j,t))*ep(type,t);
              Taxl(h,t)=min(earnl0,maxSS)*tau_ss;
              hours(h,t)=gridl(indl(current_ind,j,t));
              wages(h,t)=earnl0;
              C(h,t)=cons(current_ind,j,t);
              elseif t==J-1 % last period
              assets(h,t+1)=0;
```

```
              C(h,t)=cons(1,j,t);
              else % retirement period
              assets(h,t+1)=SS+(1+r*(1-tau_k))*(assets(h,t)+TrB)-...
                  (1+tau_c)*cons(current_ind,j,t);
              C(h,t)=cons(current_ind,j,t);
              end
              if assets(h,t+1)<0 && t+1≥Jr
                  assets(h,t+1)=0;
                  C(h,t)=(SS+(1+r*(1-tau_k))*(assets(h,t)+TrB))/(1+tau_c);
              elseif assets(h,t+1)<0 && t+1<Jr
                  assets(h,t+1)=0;
                  C(h,t)=(y+(1+r*(1-tau_k))*(assets(h,t)+TrB)-...
                      kap_0*(y-(y^(-kap_1)+kap_2)^(-1/kap_1)))/(1+tau_c);
              end
              Taxk(h,t)=r*tau_k*(assets(h,t)+TrB);
              Taxc(h,t)=tau_c*C(h,t);
          end
      end
  end
end

% Calculating aggregate variables for both ability types
N_new(type) = sum(Nu(1:Jr-1)'.*mean(LL));
K_new(type) = sum(Nu'.*mean(assets(:,1:J)));
C_new(type) = sum(Nu'.*mean(C));
TSS_new(type) = sum(Nu(1:Jr-1)'.*mean(Taxl));
Tfun_new(type) = sum(Nu(1:Jr-1)'.*mean(TT(:,Jr-1)));
Tk_new(type) = sum(Nu'.*mean(Taxk));
Tc_new(type) = sum(Nu'.*mean(Taxc));
G_new(type) = Tfun_new(type) + Tk_new(type) + Tc_new(type);
% TrB
Trn(type)=sum((1-surv)'.*(Nu'.*mean(assets(:,1:J))));
TrB_new(type)=(Trn(type))/sum(Nu);
% SS
SS_new(type)=TSS_new(type)/sum(Nu(Jr:J));
SWFh(type)=valfun(1,:,1)*di';
ah(type)=sum(Nu(1:Jr-1)'.*mean(hours(:,1:Jr-1)))/sum(Nu(1:Jr-1));
end % type

% Calculating aggregate variables
N=mean(N_new); TrB=mean(TrB_new); SS=mean(SS_new); K=mean(K_new);
G=mean(G_new);

if(K_old<K)
   if(abs(K_old-K)<2), r=r-0.00001; else, r=r-0.0001; end
else
   if(abs(K_old-K)<2), r=r+0.00001; else, r=r+0.0001; end
end

Y = (K^alpha)*(N^(1-alpha));
SWF=mean(SWFh);
ahours=mean(ah);
summary(n,:)=[N,K,K_old,r,mean(C_new),SS,TrB,Y,G,kap_0,kap_1,kap_2,SWF,...
    K/Y,ahours];
end % reached a steady state
```

```
%% Optimizing for the optimal tax system

% Initialisation of life cycle profile
a_lifecycle=zeros(nty,J); l_lifecycle=zeros(nty,J); ...
    c_lifecycle=zeros(nty,J);
t_lifecycle=zeros(nty,J); wages_lifecycle=zeros(nty,J);

% Initialation of aggregate variables
K_new=zeros(1,nty); C_new=zeros(1,nty); Tfun_new=zeros(1,nty);
Tc_new=zeros(1,nty); Tk_new=zeros(1,nty); TSS_new=zeros(1,nty);
G_new=zeros(1,nty); TrB_new=zeros(1,nty); Trn=zeros(1,nty);
SS_new=zeros(1,nty); SWFh=zeros(1,nty); ah=zeros(1,nty);
profile=zeros(500,16); Gkap2=zeros(4,1);

for tk=1:10 % loop for tau_k

tau_k=0.25+0.01*(tk-1);

for gg=1:10 % loop for kap_0

kap_0=0.30+0.01*(gg-1);
kap_2=0.01;

for ii=1:20 % loop for kap_1
    kap_1=4+0.5*(ii-1);

% Solve for kap_2
kap2(1)=kap_2;
if ii==1, kap2(4)=10000; else, kap2(4)=kap_2*2+2; end
kap2(2)=kap2(1)+phi1*(kap2(4)-kap2(1));
kap2(3)=kap2(1)+phi2*(kap2(4)-kap2(1));

for k2=1:4
    kap_2=kap2(k2);

    % *** solve the model *** (113-375)

    Gkap2(k2)=abs(G-GS);
end

while(kap2(4)-kap2(1)>0.1)
    if min(Gkap2)==Gkap2(1) || min(Gkap2)==Gkap2(2)
        kap2(3:4)=kap2(2:3); Gkap2(3:4)=Gkap2(2:3);
        kap2(2)=kap2(1)+phi1*(kap2(4)-kap2(1));
        kap_2=kap2(2);

        % *** solve the model *** (113-375)

        Gkap2(2)=abs(G-GS);
    else
        kap2(1:2)=kap2(2:3); Gkap2(1:2)=Gkap2(2:3);
        kap2(3)=kap2(1)+phi2*(kap2(4)-kap2(1));
        kap_2=kap2(3);

        % *** solve the model *** (113-375)
```

```
            Gkap2(3)=abs(G-GS);
        end
    end
    % Found optimal kap_2
    [¬,index]=min(Gkap2);
    kap_2=kap2(index);

    % *** solve the model *** (113-375)

    % Save the profile
    profile(ii+10*(gg-1)+10*20*(tk-1),:)=[N,K,K_old,r,mean(C_new),SS,TrB,Y,...
        G,tau_k,kap_0,kap_1,kap_2,SWF,K/Y,ahours];

    end % kap_1

    end % kap_0

    end % tau_k
```