TECHNISCHE UNIVERSITÄT WIEN

DIPLOMARBEIT

In perfekter Balance? - Über die optimale Steuerprogressivität für Paare und Singles

zur Erlangung des akademischen Grades

Diplom-Ingenieur/in

im Rahmen des Studiums

Statistik-Wirtschaftsmathematik

eingereicht von

Viola Garstenauer
Matrikelnummer 01525485

ausgeführt am Institut für Stochastik und Wirtschaftsmathematik
der Fakultät für Mathematik und Geoinformation der Technischen Universität Wien

Betreuung
Betreuer/in: Assistant Prof. Dipl.-Vw. PhD Nawid Siassi

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Chapter 1

Introduction

In the US a progressive tax system is in place, which means that high-income households have to pay relatively more taxes with respect to their income and low-income households relatively less. This sort of tax has two different implications. On the one hand, a progressive tax has insurance effects against negative income shocks, because if labor income is lower, at least tax rates are also lower, so the difference between pre-tax income and after-tax income is smaller when earning less. On the other hand, higher taxes on higher income also work as negative incentives to working and saving. To find the degree of progressivity where these two opposing effects are balanced, is essential.

In this work I want to find the welfare-optimizing degree of tax progressivity for labor income. To reach this ambitious goal I develop a general equilibrium model in the context of an overlapping generations life-cycle economy with heterogeneity. The economy consists of three different household types, namely married couples, single males and single females. The income is determined by an age- and gender-dependent productivity and a stochastic component. Women can decide between working not at all, part-time or full-time while all men in the economy are assumed to work full-time until they retire. There are utility costs of working for women and transfers for non-working women, both depend on marital status. There are different tax functions of the same form for singles and married couples. I solve the model using Bellman equations and only look at the model in stationary equilibrium.
I calibrate the model to US data from PSID to match labor market decisions of single and married women.
Then tax progressivity is optimized in two different settings. At first, the optimal tax progressivity for each household type is determined separately by only allowing his tax parameters to change. Couples would prefer a degressive tax system, because their income in my economy is always high enough for them to profit from this tax reform. The lower tax rate for higher incomes sets an incentive for less productive females to join the labor market. This reaction already finances the tax, because even though couple households are taxed less, there is more labor income to tax. The optimal degree of tax progressivity for female singles is larger than in the tax function of the calibrated model. They want to insure themselves better against negative income shocks and more single women are able to work part-time in their optimum opposed to the calibrated model. Labor participation of single females rises overall and their wealth shrinks a little as expected, because in more progressive tax systems the incentives to save are smaller.
For single males a corner solution on a restricted range for optimal tax progressivity is found - the most progressive tax system allowed seemingly is the best option for male singles, because they have no way of reacting to changes in labor income taxes and the only goal they have is to insure themselves agains negative income shocks as good as possible.
Next, I look for the optimal tax progressivity for singles in general, combining female and male singles. Here the same scenario occurs as in optimizing single males' tax progressivity - a corner solution on the high end of the restricted area of tax progressivities is found to be optimal. The

intuition behind the solution is different in this scenario. Single women earn a lot less than single males in the base scenario and the very progressive tax leads to a shift of after-tax labor income from men to women and a huge increase of part-time working single females, because the progressive tax of course also applies to labor income of part-time work. Overall, labor participation is 100% after this tax reform.
Finally, the optimal tax progressivity for all types of households together is identified. The restriction of progressivity degree being the same for married couples and singles is imposed. The optimal tax progressivity is very close to the real one, but the difference in tax levels for singles and married couples is huge. The tax level of singles is way lower than the one of married couples, which leads to significant shifts in after-tax labor income from couples to singles. Basically all singles get transfers for working in this system - financed by married couples. As a consequence, a large share of married women switches from working part-time in the benchmark model to working full-time in the optimum. In contrast, more single women prefer to work part-time in the optimum.
Because the solution presented above obviously is not Pareto improving, also a set of Pareto improving combinations of progressivity levels is visualised.

In the US and many other countries, taxation is based on marital status. However, when looking for optimal tax systems, a large amount of literature does not differentiate between married couples and singles. This distinction would be essential though, as singles and couples act differently in many ways. The work of Borella et al. (2018) provides great insights on how large the effects on gender and marital status are on labor market decisions, savings and consumption. The authors use a life-cycle economy with heterogeneous agents and develop two models - one with one type of households, where the only heterogeneity is introduced by age and a second model where people can differ with relation to gender, marital status, wages and life expectancy. They calibrate their first model only on men and their second on married couples, single women and single men. Then it is demonstrated that the second model outperforms the first one when it comes to modelling savings, labor supply, earnings as well as consumption.

The introduction of three different household types does add a new dimensionality and provides new insights to the existing literature. Petersen (2001) investigates in his paper on "The Optimal Level of Progressivity in the Labor Income Tax in a Model with Competitive Markets and Indiosyncratic Uncertainty" if consumers actually profit from a progressive tax system. His conclusion is that households do prefer progressive taxation to proportional taxes. The title of his article already suggests how close his model is to mine, however, he does not include different households types while I allow for single males, single females and married couples to have different characteristics and preferences.

In order to understand the effects of tax changes, one can develop a model and interpret simulated results like I did, but Storesletten et al. (2010) provide an analytical approach to the search for optimal tax progressivity. They can decompose the social welfare function and for each component determine how they influence the optimal degree of tax progressivity. They also use PSID data among others and calibrate their model to the US. Their model differs from mine in many ways, but learning the effects preferences, market structure etc. can have on optimal tax progressivity was very helpful.

As already mentioned, I also take a look at optimal tax progressivity for each household type. This includes single females and single males, of course. Alesina et al. (2007) present the bold idea of a gender-based taxation system. The authors argue that higher marginal tax rates on men would be optimal in a setting where society wants to tackle distribution problems with gender-dependent lump-sum transfers. They work with gender-specific linear taxes and their model includes many elements mine does not, like a marriage market, intrahousehold bargaining and of course gender-based taxes. However, the paper inspired me to add additional optimization experiments that differentiate between the taxes of single males and single females. The results for the two genders of single households differ substantially which reinforces the idea that gender-based taxation might be an option in battling inequality. However, some of my results are also driven by not modeling labor market decisions of men.

There is also a number of papers who deal with taxes and distinguish between between household types. Bronson and Mazzocco (2018) also develop a model with singles and married couples and calibrate it to US data. They explore the effects of three tax policies - shifting from joint to separate filing for married couples, implementing a deduction system for secondary earnings in a joint filing setting and introducing child care subsidies to both tax systems. Their results are clear - each of the aforementioed changes in the tax system has positive effects on labor market participation and labor supply, higher human capital and levels of welfare.

Also dealing with the optimization of income taxation for couples are Kleven et al. (2006). In their paper, they also assume that the primary earner always participates in the labor market while the secondary earner has a choice of whether to participate or not. However, their primary earner has the option to choose how many hours to work. They apply a fully general nonlinear tax system and find that their optimal tax system pays subsidies for secondary earnings, which decrease with primary earnings.

The paper closest to my thesis probably is the one by Leung (2019), who also uses a life cycle general equilibrium model with overlapping generations, differentiates between singles and couples and determines optimal tax progressivity maximizing welfare while applying the same form of tax functions as me.
He additionally includes the option for couples to file their taxes separately, which I do not include in my thesis. Shing divides his time periods in 5-year intervals while I am working with time periods of one year. My households face a risk of dying as soon as they survived the first year of their life, Shing assumes all households survive working age and only are at risk of dying once they retire. Shing's households can leave accidental bequests just like mine, but in his model the bequests are equally distributed between all households of working age, in my model, the newborn household who replaces the dead household, receives the whole accidental bequest. This leads to more heterogeneity in initial conditions of households. We also model the income process differently and calibrate our models to different moments. Shing's focus is on matching general moments like annual interest rate, female participation rates and the capital labor ratio, while the main goal of my calibration is to match women's labor decisions. What is more, Shing and I follow different approaches when it comes to optimization. He optimizes welfare under the constraint that the ratio of tax levels of singles and couples remains constant. I conduct several optimization experiments, but only one of them uses the same restriction. Our results also differ as a consequence of our different approaches and optimization processes.

The work is structured as follows. In chapter 2 the model is described and the stationary equilibrium of the modelled economy defined. Then US data is used to calibrate the model in chapter 3, followed by a description of the calibration and the implementation. In chapter 4, all of the optimizations are carried out and their results analysed. Finally, chapter 5 concludes and discusses ideas for future research.

Chapter 2

The Model

In order to solve the question which tax progressivity would be optimal in an economy which consists of singles and couples, I develop a general equilibrium model in the context of an overlapping generations life-cycle economy.

Let us quickly recall what this combination of models entails.

Firstly, working with a general equilibrium model implies that there are at least two different actors in my economy - households and firms. Firms need labor and capital input to produce output. These are exactly the two things households can offer. Households supply their labor to firms, which in return pay them wages. Furthermore, households lend firms capital, which in my economy is the only way they can save. Firms pay the money that households lent them back with an interest rate. Households themselves are interested in consumption, which they finance through their wages and the assets they have. In order to smooth consumption they can save money, as already mentioned, by lending it to firms and receiving their money with interest back at the end of the period. These are the two channels through which households and firms interact. General equilibrium theory suggests there exists an equilibrium, I will characterize it later on. In my model there will be more than one type of household, but the interaction process between firms and households remains the same. I will also add a government which makes social security payments and taxes labor income.

The fact that the model includes overlapping generations could have different implications. In the easiest and most often applied overlapping generations model the households live for two periods. In the first period they are alive, they are working, receive income according to their working time and can decide how much of their income they want to consume and how much they want to save. They only gain utility from consumption, but in the second period households are retired and therefore cannot create any income. As a result in this second life period they can only consume - and gain utility from - what they saved during their first period of life. How much households want to save during their first period depends on how much they discount future utility - how much they prefer consumption when they are young to consumption in their retirement. Now, why is it called the *overlapping* generations model? Because in each period there are two different generations alive at the same moment. There is the old generation, who was young last period and will die after this period as well as a new generation, that was just born in this period and will be the old generation of next period. Of course, there are many possible ways to make this model a little more complicated and life-like, but this is the basis of the model. One stochastic element of uncertainty is often added to put a more realistic spin to the consumption in the first period: e.g. possibility of death before the second period of life. This ensures that consumption during the first life period is even more valued, because the utility from consuming in the second period of life is not guaranteed.
The big difference between the overlapping generations model and other models in economics is

that most of the other models consider infinitely-lived households, often called dynasties, where everyone is altruistic and cares about the following generation's utility as much as theirs; so much so, that there is no distinction between them. These dynasties also discount future utility, but not because a certain household is not alive anymore at this point. In my model, there will be more than two generations, a detailed description of the generations follows in the next subsection.

The definition of a life-cycle economy is quite general.There have to be households who differ in their age and who will, just like in the overlapping generations model, die at some point. An important characteristic of this model is that it allows to introduce features that in an economy with infinitely-lived households do not make much sense, like a pension system, productivity profiles that depend on age or the possibility to have children. This already shows why it makes so much sense to look at an overlapping generations life-cycle economy. The life-cycle economy could theoretically only consist of one household who goes through life, but in combination with overlapping generations, there are households of every age alive at the same time who go through different phases of life, just like in the real world.

The last important feature of the model developed here is heterogeneity of agents. There will not just be different types of households, but different households of the same type. They will start their life under similar conditions and over their life time develop more and more heterogeneity.
To give just a quick overview over the topic of heterogenous agents, let us, like in Krueger (2018), take a look at an economy with stochastic income fluctuations. In this setting, households, who always want to smooth consumption, are at risk of a negative (or positive) income shock at any time. There are different ways to handle this situation.
There is a set of models where households can not do anything to insure themselves against the risks by saving or investing, even though they do know about them. In this setting, households will always only have available what they earn in this period and that sum might be high at some points in time, but lower in others.
The other possibility to deal with income fluctuation shocks in a model is to allow households to fully insure against them. This can be done by letting households trade a full set of state-contingent bonds to ensure their consumption tomorrow for all possible states they could be in tomorrow.
The third option is the middle way of the two models just described. Now households can partially self-insure themselves against future income shocks. Households in my economy will only have one way of doing so - by lending their assets as capital to firms. And here we are again, back at the equilibrium model we discussed at the beginning of this chapter.

2.1 Demographics

As already stated, my thesis focuses on exploring welfare-optimizing tax progressivity degrees. Since the tax system in the US works differently for couples and singles and I want to calibrate my model to the US, I need to differentiate between three types of households in my model economy: single women, single men and married couples, which for simplicity each consist of a female and a male. I could theoretically also only consider single and married households without differentiating between male and female singles, but as has been shown in Borella et al. (2018) there are huge differences in wages, hours worked, labor income and assets between single men and single women, to just name a few.

Households live in an overlapping generations environment, like explained above, but with 75 generations. They enter the economy when they are 25 and after surviving the first period for sure they are at a gender- and age-specific risk of dying before each period. Time periods are set to one year in order to capture as many age-dependent changes in working hours, productivity, etc. as possible without the data getting too noisy because of looking at too short periods of time.

The maximum age a household in my model economy can reach is 99, so at 99 the probability of dying for all female and male individuals is set to 1. All households retire once they are 65, which means after having been in the work force for 40 years. While retired the households receive a pension b_c for couples, b_f for female singles or b_m for male singles from the government. How large this pension is depends on the household type, but not on the individual income history. Even though this is not a very realistic setting, it serves the purpose in this case, because the huge advantage of this approach is that there is no need to calculate mean income of each household over their working age periods in order to set their individual pension. The pension is set to a certain fraction of mean income of the respective household type in equilibrium. The downside is that for some poor, unproductive households, this fraction of mean income of their household type will still be relatively high in comparison to their mean income over their lifecycle, which might not set the right incentives for them to save or work more. However, this will simultaneously lead to the well-earning households saving more, because for them the pension is a lot less than what they used to earn. These two mechanisms could weigh each other out, but even if they do not, the most important quality of the pension system in this model is that every retired household receives reliable payments for the remainder of their life.
The maximum age is set relatively high. It would be possible to work with a lower one too, as long as there are enough time periods left in which the individuals are retired to get the households to care about their consumption after they have exceeded working age. The behaviour of the households in my economy of saving for retirement during their working age, because they know the benefits will usually be lower than their income during their working age, is a very important feature of the model. This is why the households in this model know that if they are lucky and get to reach maximum age, they will have been retired for 35 years and can spend their savings over many years. Most of the households will die a long time before that, though. It should be noted that the saving process is more important than the dissaving process and my model also matches this one better. In reality people tend to save more, because they want to leave something behind for their children. In my model, families do not have children and for this reason, they do not want to leave behind any of their assets - they will involuntarily though, because they have a chance of dying every period and as already mentioned, it is highly unlikely that they reach the maximum age. As my households do not derive any utility from dying with assets, once the household is 99, it will spend anything that is left in assets, because they know they will not live to see the next period. This only applies to the very small percentage of people who survive until maximum age.

The proportion of married households to single households is fixed. Both individuals in a married couple are of the same age, enter the economy married, stay married all their life and die together with the probability of them dying being the mean of the male and female dying probability of the respective age. If a married household dies, a new married household enters the economy in the same time period, just like a dead single individual is instantaneously replaced by a new (25-year old) single person of the same gender.
There are many interesting papers that make excellent points for a marriage market and/or matching based on compatibility, for example in Siassi (2019) single households randomly meet

another single household of opposite gender and then assess, if they would be better off married or staying single. In Wu (2021), "high-ability males are matched with high-ability females".

However, the households in this model have no choice who they marry and, as I will explain in more detail later, the productivity processes of married individuals are completely independent. Not giving the couples the choice if they want to stay married and not giving the singles a choice of marriage makes the model computationally a lot easier.

2.2 Utility, Choices and Endowments

Households in my economy differ in their age (25 until 99), gender (male m or female f), type (married couple c or single s), asset holdings a and productivity stage k. Married couples are assumed to make all their decisions together and therefore have a joint utility function.

The working choice for single and married female individuals is discrete. The respective households can choose whether the woman is to work full-time, part-time or not at all separately for each time period. This significant simplification of the intensive margin is useful here. I am not so much interested in the precise extent of labor supply of women and the effects of taxes on it, but merely on their decision whether to work or not to work in a given environment. However, it is a well-known fact that part-time working women make up a significant part of the female labor force and therefore this distinction is included in my analysis as well. Computationally the discretization and the resulting much smaller grid of possible working decisions are very valuable.

In contrast, all men in my economy work full-time during their whole working life. Of course, this is a simplification as well, but in this context it is viable. The female labor supply elasticity is known to be usually higher than the male labor supply elasticity. Therefore female labor supply is more likely to adjust to a change in taxes than male labor supply. Especially in two-person households it is usually still the male part who is working full-time, because he is earning more, while women more often choose to work less or not at all. There are countless motives for this behaviour, but the most prominent are for sure that despite us living in the 21st century, women still tend to end up doing most of the unpaid work like child-care work, household work or caring for other sick or old relatives. These tasks prevent them from spending as much time doing paid work as men.

Households experience disutility from a female working in intensity n, with n denoting the discrete choice of working time: not at all, part-time or full-time. The disutility is summarized into the variable ϕ_c^n for couples and ϕ_s^n for singles. The disutility from not working is obviously zero, but the disutility from working part-time is not necessarily half of the disutility of working full-time, because utility and disutility are only measured on an ordinal scale.
The variable ϕ captures all utility costs women or couples have to pay if women are working full-time. There are as already mentioned the child-care responsibilities, household chores and other care work that often would be the task of the female part of the household. If the woman goes to work, someone else has to watch the kids, clean the house or care for the grandparents. If the woman enjoys all of these tasks, she gets less utility if she works, because then she cannot do what she enjoys doing. When considering couples, husband and wife might also love spending time together and if the wife goes to work, there is less time to do that, so the couple experiences disutility from the woman going to work. Maybe the woman also does not have kids or anything to do at all and just appreciates staying at home. Then she also would not gain, but lose utility

if she were to work. This can also be applied more generally - women could value leisure and therefore gain disutility from working.

All of the above arguments would of course also be good and sound arguments for men to dislike working, however they do not have a choice in this model. There is no need for modelling disutility from working for men, all men are assumed to work full-time anyway.

Each household can decide how much of their assets they want to consume and how much they want to save each period. There are no other options for spending their assets. Additionally, single females and couples have to make one more decision than single males - if the woman works full-time, part-time or not at all.

This leads to the following utility function for each individual in a couple. Bear in mind that consumption is consumption of the whole household. The working decision can only be made for the female part of the household,

$$u_c(c,n) = \frac{c^{1-\gamma}-1}{1-\gamma} - \phi_c^n.$$

where $u(c)$ is the utility of couples gained from consumption, c denotes consumption itself, γ is risk aversion and ϕ_c^n is the disutility of the woman in the couples c working with intensity n. Note that implicitly the assumption was made that both members of the household have the same utility function, which means the husband also experiences disutility if his wife is working. This is a valid approach, as couples usually value the time they can spend together. Once the household is retired, the term ϕ_f^n vanishes. This holds true for all types of households.
The utility function of single women reads as follows

$$u_f(c,n) = \frac{c^{1-\gamma}-1}{1-\gamma} - \phi_f^n,$$

with ϕ_s^n being the disutility of single women f working with intensity n.
The utility function of male households is the simplest, because there are no utility costs for working as men have no option but to work full-time.

$$u_m(c) = \frac{c^{1-\gamma}-1}{1-\gamma},$$

Consumption is only allowed to be positive, there is no borrowing in this economy. The utility function is strictly monotonic increasing in consumption. For females and couples, who can only decide on how much to work, there are thresholds at which the working decision switches and more utility can be gained by working less or more.
As explained above, households are at risk of dying every period and therefore a lot of them will die with assets. There are no children present in my model, therefore it would not make sense to give my households a utility benefit if they die with assets. Actually, the assets are transferred from the dead household to the newborn household who takes their place in the first period the newborn household is alive, but I would not see that as something the dead household benefits from but rather an easy way to create some heterogenity across the newborn generation. Why should an 80-year old couple suddenly have 25-year old children? However, it makes sense to assume wealth is unevenly distributed from the beginning.

As the author in De Nardi (2004) explores, it is possible to get much more realistic saving behaviours especially during old age if the dying household gains utility from leaving bequests, she even introduces a second intergenerational link that makes sure children of productive parents are also well-earning. However, I am not so much interested in the saving behaviour of the retired, and the model gets a lot more complex with bequest motives and realistic endowments, especially because the children would have to be alive at the same time as their parents. The Bellman equations would change, too, because there would be an additional stochastic element - the bequest children might receive from their parents once they die at some point in time. This effort does not outweigh the benefits in this case, so I will not include any intergenerational relationships.

2.3 Labor Productivity and Income

The labor productivity of the individuals of all three household types in each time period consists of two factors. Firstly, labor productivity depends on a deterministic component that varies with age (t) and gender (f or m), I will denote that part with $d_{t,f}$ for females f of age t and $d_{t,m}$ for males m of age t. Here no distinction between married and single individuals is made, because even though there is evidence that married couples earn more than single individuals (Siassi (2019)), my model lacks two of the most important features for recreating this marriage gap - intergenerational ties and selection into marriage. It would not be feasible to just create different productivity profiles for single and married males and females separately to create the right disparity in income therefore I refrain from doing anything like this.

The second component of labor productivity is stochastic and evolves according to an AR(1) process

$$\ln e_{s,i} = \rho \ln e_{s-1,i} + \epsilon_{s,i}, \ \epsilon_{s,i} \overset{iid}{\sim} N(0, \sigma_\epsilon^2),$$

where s denotes time and i denotes the individual household. The actual productivity shock is $e_{s,i}$ while $e_{s-1,i}$ is the shock from last period, ρ a persistence parameter and $\epsilon_{s,i}$ a normal and independent, identically distributed random variable with variance σ_ϵ^2. Remember that the households in the model are individual. There are not different household types with different ages, but of each type of household there are many agents of the same age, who possibly differ in asset holdings, working decision etc. and the i denotes the individual household. I will suppress the index i from now on and will only put the household type and member into the index of the productivity shock. I will only look at the economy in stationary equilibrium, therefore I will not use any time indizes, the only time dimension important in the model is the age of the households.

The income of a married couple of each age t during working age is the sum of the incomes of the two indivduals and a transfer that depends on the working hours of the female and looks like this

$$y_c(n, w, e_{cf}, e_{cm}) = nwe_{cf}d_{t,f} + we_{cm}d_m + \mathbb{1}_{\{n=0\}}\kappa_c,$$

where y_c is income, n again is the work intensity of the female (not at all, part-time or full-time), w is the wage rate in equilibrium, e_{cf} and e_{cm} stand for the stochastic productivity shocks of the female respective the male individual belonging to the household. I want to point out that those shocks are independent of each other, but of course each depends on their productivity shocks of the periods before. The non-stochastic factor of the productivity process is as already explained denoted as $d_{t,f}$ for females and $d_{t,m}$ for males. The last term describes the transfer κ_c the couple recieves if the woman does not work at all at that age. There is no transfer if the woman works part-time.

The income of single females is calculated as follows

$$y_f(n, w, e_f) = nwe_f d_{t,f} + \mathbb{1}_{\{n=0\}}\kappa_f.$$

Notice that the deterministic part of the income process $d_{t,f}$ does not depend on the household type, but only on gender and age. The productivity shock e_f is independent of the one for the couples, of course.

The income function for single males aged t differs from the single female one only in the applied deterministic productivity profile $d_{t,m}$ instead of $d_{t,f}$ and the absence of transfers,

$$y_m(w, e_m) = we_m d_{t,m}.$$

2.4 Taxes

The government taxes labor income of couples and singles differently in the US. There exists a range of papers on how to approximate the tax functions for married and unmarried households in the US. I will be working with one of the specifications estimated in Guner et al. (2014), who call this function the HSV specification following Bénabou (2002) and Storesletten et al. (2010). The tax functions for single and married households are of the same form, but the two parameters characterizing the tax function differ. The mean household income is normalized to one. The married couple gets taxed together and has to pay taxes following the function

$$T_c(y) = y_c - \lambda_c y_c^{(1-\tau_c)},$$

which means the disposable income of couples is

$$y_c^d = \lambda_c y_c^{(1-\tau_c)}.$$

The tax for single individuals looks like

$$T_s(y) = y_s - \lambda_s y_s^{(1-\tau_s)},$$

which leads to a disposable income for singles of

$$y_s^d = \lambda_s y_s^{(1-\tau_s)}.$$

Let us take a look at these tax functions now. If $\tau > 0$ holds, the tax system is progressive, so the tax rate is higher for households with high income and lower for households with low income. It even allows for transfers, because for a certain low level of income y the tax rate becomes negative. If τ was smaller than 0, the tax system would be regressive, which would make lower-income households pay more taxes than high-income households relative to their respective incomes. The parameter τ sets the progressivity degree of the tax and the parameter λ defines the tax level. This clear distinction of the roles of the two parameters is important, because I am mainly interested in tax progressivity τ.

2.5 Bellman Equations

After having introduced all elements of the model, we can now formulate the problems different household types have to solve. We will proceed as follows. At first, we will discuss the Bellman equations for the working age together with their corresponding budget constraints. Then the Bellman equations for the retired households and their budget constraints are presented.
Recall that Bellman equations express the value for the household of age t of making a decision this period in terms of the maximal instant utility at this and the discounted expected value at age $t+1$ conditional on the decision taken at age t.
In our model, there will not only be included a discount factor β which discounts future utility, but also an age-dependent survival probability ξ_f for females, ξ_m for males and married couples ξ_c.

2.5.1 The Couple Household Decision Problem

Married couples who are of working age t have to solve the following maximization problem

$$V_c^t(a, e_{cf}, e_{cm}) = \max_{c,a',n}\{u_c(c,n) + \beta\xi_c\mathbb{E}[V_c^{t+1}(a', e'_{cf}, e'_{cm})]\}$$

under the budget constraint

$$c + a' = (1+r)a + \lambda_c y^{1-\tau_c}.$$

The state variables of married households at age t are the assets saved until this period a and the random part of the productivity state of the female e_{cf} and the male e_{cm}. The control variables are the variables households can make a decision about at age t. They are consumption in period t, assets a' in the next period and the working decision of the female n. The budget constraint ensures that consumption c and saved assets in period t, a', equal the saved assets from the period before with interest rate, $(1+r)a$, and disposable labor income.

The Bellman equation for retired married couples of age t reads as follows

$$V_c^t(a) = \max_{c,a'}\{u_c(c) + \beta\xi_c\mathbb{E}[V^{t+1}(a')]\},$$

with the budget constraint

$$c + a' = (1+r)a + b_c.$$

Once retired, households only have one state variable, the assets saved for age t, called a. There are also less control variables, consumption c at age t and savings at age t, a'.

2.5.2 The Single Female Household Decision Problem

During working age single female households encounter the decision problem

$$V_f^t(a, e_f) = \max_{c,a',n}\{u_f(c,n) + \beta\xi_f\mathbb{E}[V_f^{t+1}(a', e'_f)]\},$$

under the budget constraint

$$c + a' = (1+r)a + \lambda_s y^{1-\tau_s}.$$

Female single households have the state variables assets a and random productivity status e_f and can choose their consumption c, their savings a' and how much they want to work n. Retired households of single females have to solve

$$V_f^t(a) = \max_{c,a'}\{u_f(c) + \beta\xi_f \mathbb{E}[V_f^{t+1}(a')]\}$$

under the budget constraint

$$c + a' = (1+r)a + b_f.$$

The state variable for single retired females is assets a at age t. The control variable is the savings decision a'.

2.5.3 The Single Male Household Decision Problem

Male households of age t during working age have to solve the following Bellman equation

$$V_m^t(a, e_m) = \max_{c,a'}\{u_m(c) + \beta\xi_m \mathbb{E}[V_m^{t+1}(a', e_m')]\}$$

under the budget constraint

$$c + a' = (1+r)a + \lambda_s y^{1-\tau_s}.$$

Retired single male individuals solve this maximization problem

$$V_m^t(a) = \max_{c,a'}\{u_m(c) + \beta\xi_m V_m^{t+1}(a')\},$$

subject to

$$c + a' = (1+r)a + b_m.$$

2.6 Firms

The output Y of our economy is produced by a representative firm via a Cobb-Douglas production function $Y = AK^\alpha N^{1-\alpha}$ with K being aggregate capital, N being aggregate effective labor supply and A being total factor productivity. The output elasticity of production with respect to capital input is denoted as α. Firms pay wages w per effective labor unit and hire capital at rate r. In our economy capital depreciates at rate δ. Therefore the firm in factor market equilibrium wants to find the solution to the following expression in order to maximize its profits

$$\max_{K,N} AK^\alpha N^{1-\alpha} - (r+\delta)K - wN.$$

The above postulated problem is easily solved by differentiating by capital K respective labor N and solving for interest rate r respective wage rate w. This leads to the following wage and interest rates in factor market equilibrium

$$w = (1-\alpha)AK^\alpha N^{-\alpha}, \tag{2.1}$$
$$r = \alpha AK^{\alpha-1}N^{1-\alpha} - \delta. \tag{2.2}$$

2.7 Government

The government provides the retired households with a pension whose level depends on the household type and the respective mean income during working age and to tax labor income of households depending on their marital status. The government holds foreign assets outside of the modeled economy and receives returns from these investments equal to R, they balance the government budget.

2.8 Equilibrium

The stationary equilibrium of the considered economy is a collection of value functions $V_c^t(a, e_{cf}, e_{cm})$, $V_f^t(a, e_f)$ and $V_m^t(a, e_m)$, individual policy rules for consumption $c_c(a, e_{cf}, e_{cm})$, $c_f(a, e_m)$, $c_m(a, e_m)$, assets $a'_c(a, e_f)$, $a'_f(a, e_f)$, $a'_m(a, e_m)$ and working time $n_f(a, e_f)$, $n_c(a, e_{cf}, e_{cm})$, price functions $\{r, w\}$ and a distribution of married couples $\lambda_c(t, a, e_{cf}, e_{cm})$, single females $\lambda_f(t, a, e_f)$ and single males $\lambda_m(t, a, e_m)$ such that:

- Given $\{r, w\}$ and the pensions b_c, b_f and b_m, households solve their maximization problem and $c_c(a, e_{cf}, e_{cm})$, $c_f(a, e_m)$, $c_m(a, e_m)$, $a'_c(a, e_f)$, $a'_f(a, e_f)$, $a'_m(a, e_m)$, $n_f(a, e_f)$ and $n_c(a, e_{cf}, e_{cm})$ are the associated policy functions.
- Prices $\{r, w\}$ satisfy the firm's first order conditions 2.1 and 2.2.
- Individual and aggregate labour and capital are consistent

$$\begin{aligned} N &= \int n_c e_{cf} d_{t,f} + e_{cm} d_{t,m} \; d\lambda_c(t, a, e_{cf}, e_{cm}) + \int n_f e_f d_{t,f} \; d\lambda_f(t, a, e_f) \\ &+ \int e_m d_{t,m} \; d\lambda_m(t, a, e_m), \\ K &= \int a_c \; d\lambda_c(t, a, e_{cf}, e_{cm}) + \int a_{sf} \; d\lambda_f(t, a, e_f) + \int a_{sm} \; d\lambda_m(t, a, e_m). \end{aligned}$$

- The government budget is balanced

$$\begin{aligned} \int b_c + \mathbb{1}_{\{n=0\}}\kappa_c \; d\lambda_c(t, a, e_{cf}, e_{t,cm}) \; &+ \; \int b_f + \mathbb{1}_{\{n=0\}}\kappa_f \; d\lambda_f(t, a, e_f) + \int b_m \; d\lambda_m(t, a, e_m) = \\ \int T_c(y_c) \; d\lambda_c(t, a, e_{cf}, e_{cm}) \; &+ \; \int T_s(y_f) \; d\lambda_f(t, a, e_f) + \int T_s(y_m) \; d\lambda_m(t, a, e_m) + R. \end{aligned}$$

- The distributions of couple households $\lambda_c(t, a, e_{cf}, e_{cm})$, single females $\lambda_f(t, a, e_f)$ and single males $\lambda_m(t, a, e_m)$ is consistent with the policy functions and stationary.

Chapter 3

Fitting the Model to US Data

This chapter will describe the process of fitting the model to US data. The structure is as follows. First, the data source will be discussed. Then a description of the calibration and its targeted moments is given.

3.1 Preparing PSID Data

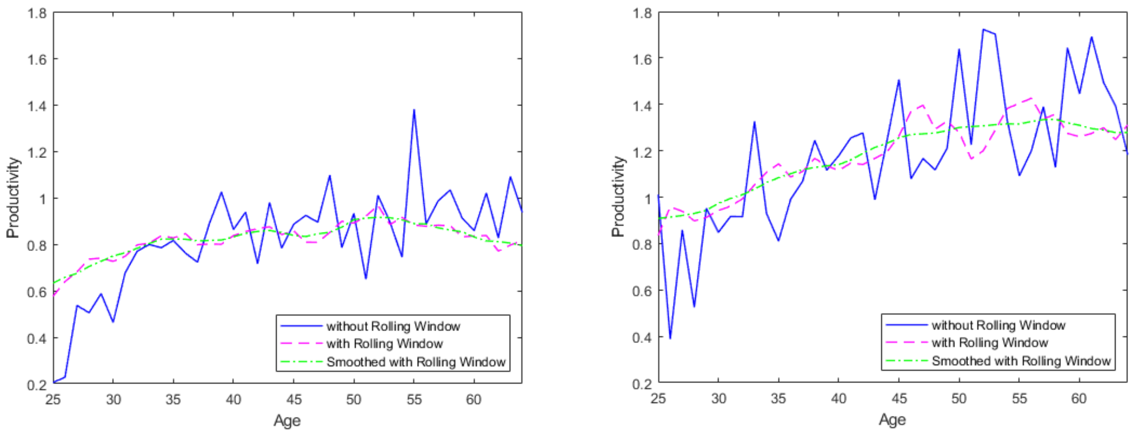
I am working with data from the Panel Study of Income Dynamics (2021) that I will from now on refer to as PSID. This study is a longitudinal household survey in the US that started collecting data in 1968 according to the *PSID Main Interview User Manual: Release 2021* (2021). Originally designed to gather data on low-income households, nowadays they follow a representative part of US households as well as the household's offspring leaving the household to form a new one. Every two years, the household's reference person is interviewed about their income situation, wealth, education, health and many more socioeconomically relevant topics. This makes the data valuable for observing e.g. trends over time or multigenerational economic conditions. However, I will not be using the panel dimension of the data, but only the results of the interviews that took place in 2019.

There are two sets of information that need to be extracted from PSID data in order to calibrate the model. First, the age- and gender-dependent productivity profiles need to be generated. Secondly, the shares of single and married women working not at all, part-time or full-time have to be calculated.

The PSID data is obtained by interviewing one person per household who also gives information about her spouse and other possible household members. This leads to a structure of the data with possibly two interesting data points per household. I always try to work with the biggest data set available for the variables in question. Consequently I use different data sets for different variables of interest. I will always specify the data used.

The first step to generating productivity profiles is to figure out what data is needed to construct this series. At first, a unit to measure productivity is needed. Hourly wage is a good indicator for productivity as it shows how much firms are willing to pay for an hour worked. Therefore wage per hour needs to be determined for men and women of each age during working age in the model economy. The working age in the model is 25 to 64. There is a variable called *hourly regular rate* in the PSID data, but it is only available for the interviewed person and not their spouse. Therefore I decided against using this variable. Instead, I use salary per year and hours worked which are available for both parts of the households. Salary per year is not always directly given, but calculating it when salary per week or month is given is obviously easy. Hours worked are always given per week, I also have to account for this by multiplying the hours by 52 to obtain an approximation of yearly hours worked. For the spouse, yearly salary is always

given directly. The biggest data set I can use is one without any missing values for gender, age, hours worked, salary and, for the reference person, also how often this salary is paid per year. Given all this, a person's yearly salary only has to be divided by its hours worked last year. This is done for all individuals for which the data is available and then the mean of each age cohort differentiated by gender is calculated. This is followed by normalizing the productivity profiles to one. This results in productivities for women that are below one for many age groups and productivities above one for most male age groups. Note that there is no need to know anything about the marital status of the individuals to calculate the gender- and age-dependent productivity profiles.
Figure 3.1a and figure 3.1b display the different steps in getting from the data to a smooth productivity profile. Because the number of observations of persons of a specific age and gender is limited and differs a lot, two methods to smooth the data are applied. The first one is using a rolling window of five years. This means also including data from persons two years younger and two years older than the relevant age to calculate the mean for that age. The original data without any smoothing is plotted as the blue line. The need for smoothing the data is very obvious, there are many jumps in the productivity profile that I would have trouble explaining. When using the rolling window the data appears to be a lot smoother already. However, to get rid of the remaining jumps, I smooth the data once more after normalizing the productivity profiles to one. In order to present the three steps visually, I normalized the productivity profiles in the intermediary stages as well. Some authors approximate the productivity profiles through polynomials in age, my method ensures I use productivity profiles close to the data.



(a) Women **(b)** Men

Fig. 3.1: Age-Dependent Productivity Profiles

After having explained how we get to nice smooth lines for the age- and gender-dependent productivity profiles, we can finally take a look at the profiles in figure 3.2. Male productivity is way higher than female productivity for all age cohorts. In both male and female productivity profiles it becomes clear that it is important to consider age as an indicator for productivity - women and men start at different levels, but neither ever gets back to this low starting point. This evolution can be interpreted as an experience premium. I have not included experience premia in my model, but through the productivity profiles a naive version of them is included implicitly. The male productivity profile rises monotonically until nearly the age of 60 before it starts to drop. The difference in productivity between the initial level and the level at age 64 equals more than 40 percent, which is large. Especially in contrast to the difference in female productivity between the age of 25 and the age of 64, which only seems to rise less than 30 percent. Keep in mind that they also start from a much lower level. Also notice that female productivity starts decreasing before the age of 55. There are more kinks in the female productivity profile. They could be explained by child-care obligations during the age of 35 to 50, because they might lead to women taking lower-paid part-time jobs to spend time with their children instead of higher-paid full-time jobs.

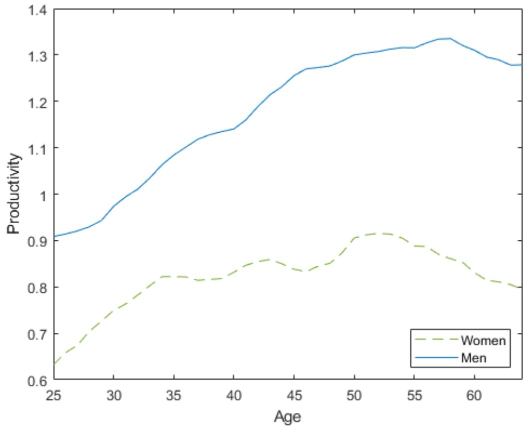
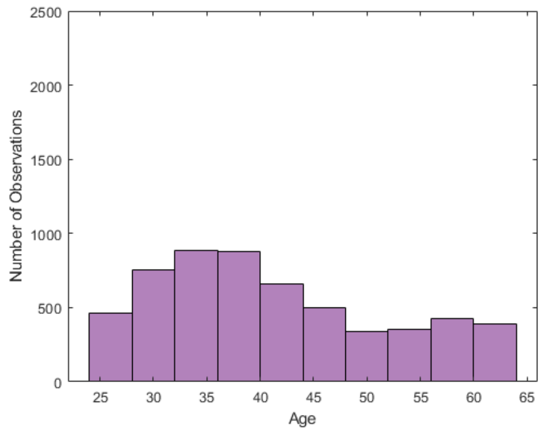


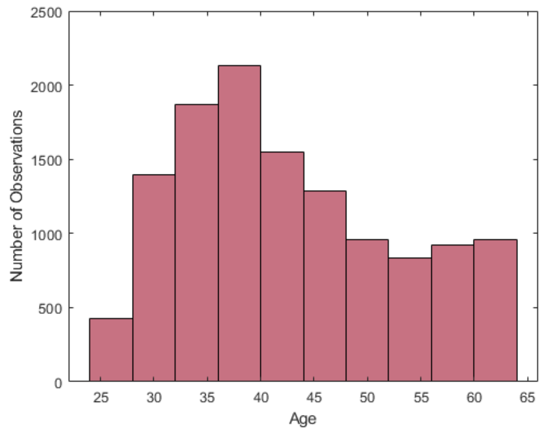
Fig. 3.2: Age-Dependent Productivity Profiles

The second information set for which the PSID data is required is the share of single and married women of each age working not at all, part-time and full-time. The calculation is straightforward. The biggest data set I can use contains no missing values for age, gender, marital status and working time. To be sure not to include women who live with a partner, but are not married, I exclude these women as well. The distinction between women who live alone and women who live with a spouse without being married is important, because a woman with a partner might act more like a woman in a married couple than an actual single woman. The financial safety is usually better with more than one person in a household, because the other partner can balance out potential negative income shocks. Child care options are also increased for couples. Even though women still tend to do more child care work, the male partner can look for the child better if he is existent than when he is not. This constraint leads to not excessively less data points.

The difference in number of observations of single and married women used for calculating the respective percentages of working not at all, part-time and full-time has to be mentioned. Figure 3.3a and figure 3.3b show how unevenly distributed the data is divided according to age. For the age of 30 to 45 there are significantly more observations of working time available than for the other age groups. What is more, the relevant data is available for way more married women than single women.



(a) Single Women



(b) Married Women

Fig. 3.3: Observations Used to Calculate Working Time Shares

The actual shares over the life-cycle will be presented alongside the shares simulated in the model.

3.2 Calibration

To calibrate the model, at first a set of parameters is fixed endogenously. Common values from the literature are used. The list of fixed parameters is presented in table 3.1.

Tab. 3.1: Fixed Parameters

Parameter	Value	Name
Risk Aversion	2	γ
Discount Factor of Future Utility	0.95	β
Age of Entry in Working Force	25	Ta
Retirement Age	65	T
Maximum Age	99	TT
Capital Share of Output	0.36	α
Depreciation Rate of Capital	0.06	δ
Level of Tax Rate for Singles	0.987	λ_s
Level of Tax Rate for Married Couples	0.913	λ_m
Degree of Tax Progressivity for Singles	0.034	τ_s
Degree of Tax Progressivity for Married Couples	0.06	τ_m

Risk aversion and the discount factor of future utility are chosen as usual. The age of entry into the working force is set to 25, because it is reasonable to assume most people will have finished their education by that age and therefore few distortions of the labor market should be visible from that age on. Additionally, by using the rolling window method described in the last subsection, there is no data available for a much younger starting age. The retirement age of 65 is taken from reality. To simulate a maximum age of over 99 would not provide any economic insights. Capital share of output and depreciation rate of capital are chosen as suggested in Heer and Maussner (2009). The level and progressivity degree of tax rates for married couples and singles is taken from the estimations of the HSV specification in Guner et al. (2014). In order to be able to use their tax functions, mean household income in the economy has to be normalized to one, so an income of 1 equals mean household income in the data. Therefore income in the model is normalized to 1, which would equal a little less than \$60.000.

AR(1) processes for the productivity shocks are modelled with a persistence parameter of 0.95 and a variance of 0.05. These values are close to the ones used in Heer and Maussner (2009). For the computation of discrete states, the Tauchen method from Tauchen (1986) is applied. Single and married males as well as married females have 5 possible productivity states, single females can be in 9 different productivity states. Married couples have a common utility function, so there are 25 different combinations of productivity states of male and female productivity states. A variety of numbers of productivity states were experimented with, this combination was found to be a good compromise between computation time and heterogeneity.

The share of married households of overall households in the economy is set to 50%, according to data from the *Historical Household Tables by United States Census Bureau* (2020) this is very close to the share of married households in the data. It is assumed that there are as many female as male singles, so their share is 25% each.

There is one more exogenous age-dependent variable that has not been mentioned yet. The survival probabilities of females and males are taken from the *Actuarial Life Table produced by The United States Social Security Administration* (2016). The Period Life Table for 2016 is the newest one available and contains the probability of dying within a year, the number of survivors out of 100.000 born people born alive and life expectancy, given you have reached the age you are at. I will only use the death probabilities and convert them into survival probability. Like explained in the model description, married couples die together. Therefore I set the survival probability of couple households as the mean of male and female survival probabilities.

Having decided for all of the pre-set parameters, the moments to be matched and the parameters to be calibrated have to be identified. The 10 moments to match are the shares of married women working full-time, part-time and not at all as well as the shares of single women working full-time, part-time or not at all. The seventh moment is mean household income, which needs to be normalized to one. The remaining three moments are that retirement benefits for all three types of households should each amount to 40% of mean income of the respective household type. The calibration parameters are transfers κ_c for not working married women, transfers κ_s for not working single women, utility costs of married women working part-time $\phi_c^{\frac{1}{2}}$ and full-time ϕ_c^1 as well as utility costs of single women working part-time $\phi_f^{\frac{1}{2}}$ and full-time ϕ_f^1. Total factor productivity A is the calibration parameter used to normalize mean household income to one. Furthermore, the retirement benefits b_c for couples, b_f for females and b_m for males are calibrated.
I define working not at all as working less than or equal to 5 hours per week, part-time work as working more than 5 hours, but less than including 30 hours per week. Full-time work is defined as working more than 30 hours per week. There are different ways to calculate these shares. The results of two approaches are shown in table 3.2. The first option of calculating the shares is by simply taking the mean share of women working not at all (by my definition), part-time or full-time over all women aged 25 to 64. However, like already discussed above, women aged 30 to 45 are over-represented in the sample. Therefore I prefer the second approach of first calculating the shares of women working not at all, part-time or full-time for each age from 25 to

64 separately and then taking the mean. This is of course equal to taking a weighted mean of the shares of all different age classes.

Tab. 3.2: Shares of Women Working Not at All/Part-Time/Full-Time in the Data

Time worked	**Mean Share over all relevant age groups**	**Mean of Shares in each relevant age group**
married women not working	22.9	23.75
married women working part-time	15.3	15.08
married women working full-time	61.79	61.17
single women not working	18.19	21.52
single women working part-time	12.75	12.52
single women working full-time	69.05	65.97

The results from using the two methods differ more for single women. For married females there is nearly no difference. The mean share over all relevant age groups of single women working full-time is more than 3 percentage points higher than the mean of shares in each relevant age group. This might result from the over-representation of the age groups 30 to 45, because even though during this time many women have child-care responsibilities, they do not necessarily for this whole 15-year long period of their life and therefore the share of women working full-time still might be higher than when they are over 50.
To sum up, the difference in the two approaches suggests that married women are more consistent in their working choice regarding age. In general, less than two thirds of single and married women seem to be working full-time. The share of married women working part-time is slightly higher than the share of female singles working part-time. The same holds true for the shares of not working women, which results in a higher share of single women than married women working full-time.

To get close to the shares in the right column of table 3.2, the following list of parameters was calibrated accordingly.

Tab. 3.3: Calibrated Parameters

Parameter	**Value**	**Name**
Transfers for not Working Married Women	0.125	κ_c
Transfers for not Working Single Women	0.205	κ_f
Utility Cost of Married Women Working Part-Time	0.075	$\phi_c^{\frac{1}{2}}$
Utility Cost of Married Women Working Full-Time	0.222	ϕ_c^1
Utility Cost of Single Women Working Part-Time	0.276	$\phi_f^{\frac{1}{2}}$
Utility Cost of Single Women Working Full-Time	1.15	ϕ_f^1
Total Factor Productivity	0.74	A
Retirement Benefits of Couples	0.26	b_c
Retirement Benefits of Single Females	0.18	b_f
Retirement Benefits of Single Males	0.35	b_m

Keep in mind that mean household income is calibrated to one. This information is necessary to set the transfers for not working single and married women into context. Couples receive less transfers than single women. This might seem odd, but a lot more single women than married couples receive transfers because of the bad economical state they are in. Transfers can be seen as a stand-in for different financial support programs of low-income households. They are necessary to get a share of single as well as married females to refrain from working, without transfers for not working women and utility costs for working, there is no incentive for women not to work. In order to match the shares in the data, these two channels of influencing the women's working decision are needed.
The utility costs cannot be compared, because utility is only measured on an ordinal scale. Retirement benefits are calibrated to match

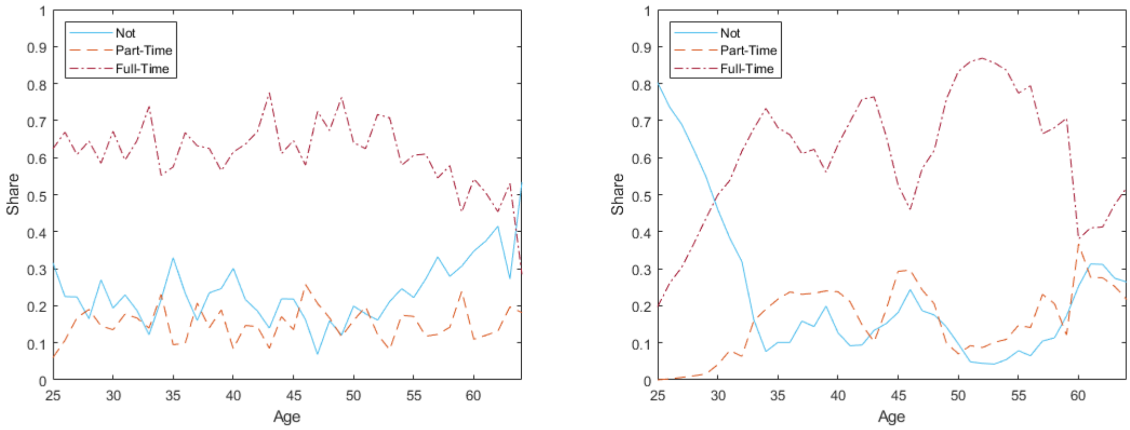
As can be seen in table 3.4, the data is matched pretty well.

Tab. 3.4: Moments

Moment	**Model**	**Data**
Share of Married Women not Working	23.92	23.75
Share of Married Women Working Part-Time	14.78	15.08
Share of Married Women Working Full-Time	61.3	61.17
Share of Single Women not Working	21.45	21.52
Share of Single Women Working Part-Time	12.45	12.52
Share of Single Women Working Full-Time	66.10	65.97

Figure 3.4a shows the shares of married women working not at all, part-time and full-time over their life-cycle according to PSID data. Meanwhile figure 3.4b contains the same information about the working decisions of married women in the model. Apart from the first five years, during which the share of non-working women is a lot higher in the model economy than in

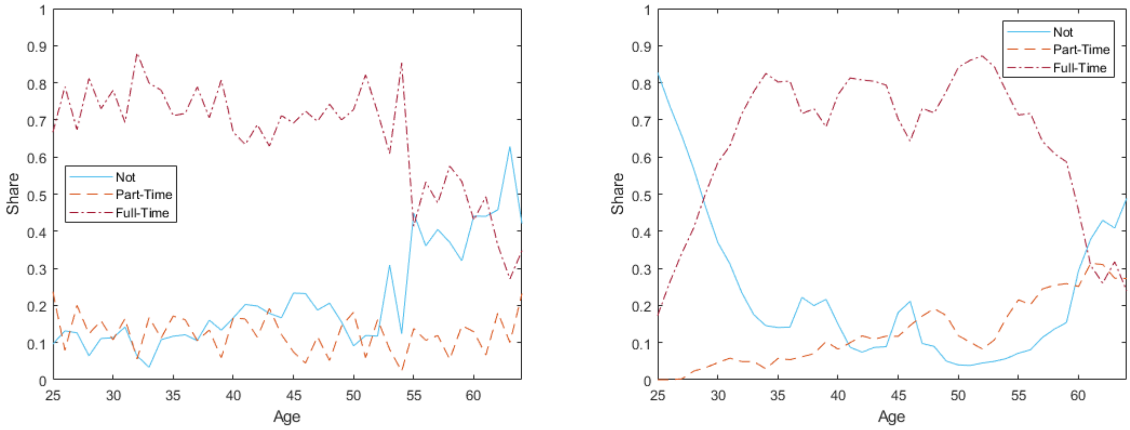
the data, the process looks very similar. The fact that many women stop working even before they reach the age of 65 is also well captured in the model. Why does a large share of women stop working before reaching the obligatory retirement age? The answer lies in the productivity profiles. There we saw how productivity of women develops dependent on their age. After the age of 55, productivity does not rise anymore, but instead starts falling.
The life-cycle data from PSID is not smoothed, so the kinks in the left figure are not representative, they are results from the not too large sample size. Therefore, it was not a goal to match all of these kinks. The discrepancy during the first five years can be explained by how the endowments are modeled. Households of age 25 receive whatever the household of the same household type left them when dying. This accidental bequest will often be zero, but more often than not it will be a sum that makes it more desirable for the married household to let the men work alone for the first few years and the women to only join after the bequest is used up.



(a) Data **(b)** Model

Fig. 3.4: Working Time of Married Women over their Life-Cycle

The situation is similar with the life-cycle working decisions of single women in the data (figure 3.5a) and the model (figure 3.5b). Again the first five years are not matched well for reasons we just discussed above. The rest of the life-cycle however seems to be matched well, the retirement process starts a little later in the model than in the data.



(a) Data **(b)** Model

Fig. 3.5: Working Time of Single Women over their Life-Cycle

Last but not least, the development of mean wealth and labor income over the life-cycle is of interest. Keep in mind that mean income before taxes is normalized to 1.
Income is, as described in detail in the last chapter, the product of the deterministic age- and gender-dependent productivity, random productivity shocks, wage rate and - for women - work intensity. After having taken a look at the deterministic productivity profiles of men and women it is therefore not surprising to see the mean single female life-cycle income to be below the ones of the other two household types. The same holds true for mean male income being lower than mean income of married couples, because couples in the model economy are designed to have exactly the same mean income as men if all women were not to work. As we have already seen that a huge share of women works at least part-time, the mean income of couples has to be higher than that of male singles. All these observations only hold true for the average, of course. The small kinks in life-cycle income of couples and females stems from the drops in labor force participation at these ages. Note that for women who are not working, the value of zero income is still included in the calculation of mean income, the transfers they receive are not.

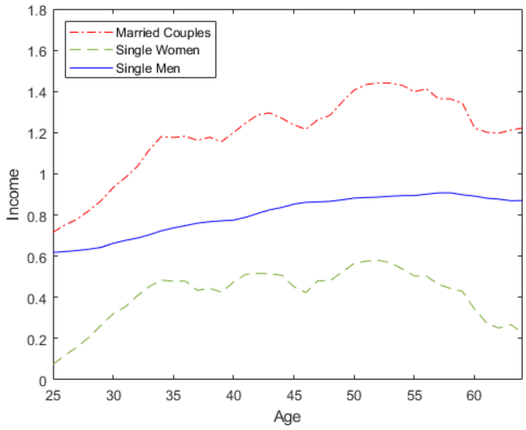


Fig. 3.6: Evolution of Labor Income over the Life-Cycle in the Model

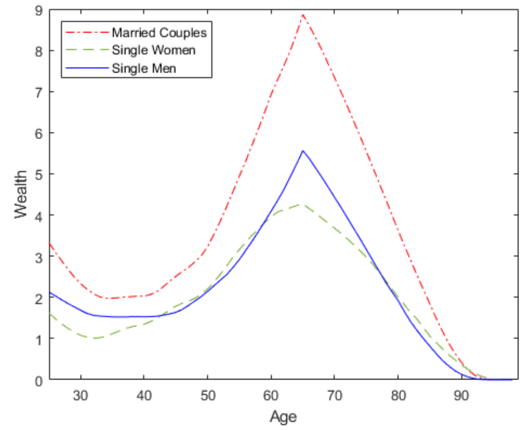


Fig. 3.7: Evolution of Wealth over the Life-Cycle in the Model

Wealth develops as could be expected. At the age of 25, bequests are received and the next few years dissaving seems to be the most attractive option. Starting at age 40, however, retirement is close enough for households to incentivize saving. Wealth rises for all three household types and only starts to fall once retirement age is reached. From then on, pensions and capital income are the only two forms of income. Because pensions are substantially smaller than income during working age for most households and households want to smooth their consumption, dissaving again is the best strategy. However, the peak of wealth at retirement age is much more distinctive for single males and married couples. The reason for the less visible peak in wealth of women at 65 is that a large amount of single women already leaves the labor force before reaching the obligatory retirement age, as depicted in figure 3.5b. Married women only show a milder version of the same behaviour and are not the sole earners of their households, which is why the peak is visible for married couples.
Note that most households will die before they reach the age of 99. Consequently, after the age of 90, it becomes viable to hold no assets and only live from the pension payments, because the probability to survive another period is just too small.
The richest households are married households. That is not surprising, as they are the only household type with potentially two incomes. However, even though the difference in the productivity profiles for men and women is substantial, the wealth they build up is very similar. This implies a higher savings rate for women, because we already observed how much higher mean income of single men is in comparison to mean income of single women. Note that non-working women receive transfers that do not show up in the depicted evolution of labor income.
The explanation for the higher incentive to save for single women is straightforward. They have to insure themselves better against productivity shocks to ensure they are capable of consumption smoothing in the future. Also, the option of not working for a certain number of time periods encourages women to save even more while working and dissave when they take a break from working. Note that these breaks can last for a different number of time periods.

3.3 Implementation

I implement the model explained above in MATLAB.
The Bellman equations are solved by backwards value function iteration.
The optimal working decision for each period is determined outside of the Bellman equations and with the already confirmed best option interpolation of the value function is applied to get better results for the policy functions.
For solving the Bellman equations, an asset grid of 80 points ranging from 0 to 20 is used. The spacing of the asset grid is chosen smaller closer to zero and equidistant from 1 on.
For creating the random productivity states the Tauchen method with 5 states for single men, married females and married males and 9 states for single females is used. The persistence parameter is set to 0.95 and the variance to 0.05 in all cases.
The simulation of household behaviour is carried out with Monte Carlo simulation. The asset grid for the Monte Carlo simulation is set to 200 grid points, also ranging from 0 to 20 and with more points close to zero.
The number of simulated time steps is 2000 and the number of households is 10.000 overall, split into 5.000 married households, 2.500 households of single males and 2.500 households of single females.
To find the equilibrium the fixed point problems of aggregate capital being equal to the sum of individual household assets and aggregate labor being equal to the sum of effective time worked

per household have to be solved. To find a solution to these fixed point problems, I turn them into one and search for the right capital-labor ratio, $\frac{k}{n}$ by guessing a ratio, solving the model for this ratio (as it implies values for wage rate r and interest rate r) and using the result $\frac{k^r}{n^r}$ as well as the last guess $\frac{k^l}{n^l}$ to determine the next guess by calculating $\frac{k^n}{n_n} = \psi \frac{k^l}{n^l} + (1-\psi)\frac{k^r}{n^r}$. This procedure is repeated until the last guess and the resulting outcome for $\frac{k}{n}$ are closer than 0.05. During optimization a similar approach is applied when looking for the right λ_c respective λ_s to fulfill the budget constraint. The tax level λ_s or λ_c is at first guessed, then the implied government budget is computed (which involves all of the above mentioned steps, of course) as well as the difference to the budget constraint in the benchmark model. Following this comparison, the tax level would be increased (decreased) if the government budget was too low (high) until the government budget is the same as in the calibrated model.
To find the optimal combination of degrees of tax progressivity τ_c and τ_s, grid search is applied. Which of the four tax parameters stay fixed and which are adjusted to fulfill the government budget constraint in which optimization experiment will be discussed in the next chapter.

Chapter 4

Tax Optimization

4.1 Setup

After finding a satisfactory calibration in the last chapter, I now want to use the obtained values for finding optimal tax progressivities under a number of different assumptions. The first one, which will apply to all scenarios is that I will only look at economies in stationary equilibrium. The first step is to characterize optimality in the given setting. I define the optimal tax system in this model as the one which maximizes welfare. Whose welfare is to maximize will depend on the circumstances.

In the first few scenarios I will only be looking at maximizing welfare of one household type while ignoring the well-being of the other household types. To maximize welfare under these circumstances means maximizing the welfare of a person entering the economy and only knowing their gender and marital status about themselves. They do not know the sum of their bequests, their productivity shocks, or their time of death, but are informed about the probabilities of each possible scenario. Thankfully, the value function at age 1 contains a lot of this information already. However, two transformations need to be applied to the value functions. The first one is to weight the possible starting values of assets (equaling inheritance) with the probability distribution of inheritances *inh* for the considered household type. Note that this information is not contained in the value function because of the assumption of inheriting at birth. Wealth is inequally distributed among household types. Therefore the probability distributions will differ. The second transformation is the one accountig for the stationary distribution of productivity shocks, *stat*. This leads to the following formulas for calculating the expected life time utility of a newborn couple

$$V_c^0 = inh_c V_c^1 stat_c,$$

with V_c^1 being a matrix of the results of the value function at age 1 for all different combinations of assets and productivity states.
The expected lifetime utilities of single females respective single males look like

$$V_f^0 = inh_f V_f^1 stat_f,\ V_m^1 = inh_m V_m^1 stat_m.$$

Optimal tax progressivity for each of the three household types will be determined under the assumption that the taxes stay the same for all other household types. For example, when searching for the optimal tax progressivity of couples, all tax parameters for singles will stay unchanged.

Optimal tax progressivity for Singles is characterized as the degree of τ_s which maximizes welfare for a person who knows she is born single, but does not know of which gender he or she will be. Expected lifetime utility is calculated just like above, but the results for females and males have to be weighted by the probability of a single person being a woman respectively a man, which is 50% in this economy. Expected lifetime utility of a person born single of not yet known gender is

$$V_s^0 = \frac{V_f^0 + V_m^0}{2}.$$

In the last scenario I will be looking at expected welfare of a person entering my economy before knowing anything about himself or herself - the person has no idea if they are going to be male or female, married or single and they also do not know how much they will accidentally inherit, the productivity shocks they will experience and when they will die. They know the chances of every possibility, so they are fully aware of all their options and what might become of them with what probability. Thanks to the structure of the model and its Bellman equations, after having solved the model, we again know exactly what we need to calculate expected welfare as we are aware of the value functions of all household types when they are "born" into the economy at age 25.

The value functions of each household type already contain nearly all of the knowledge explained above, death probabilities are accounted for as well as all possible combinations of productivity shocks. The only thing that is not already taken into account for in the value functions is the probability for the household type the newborn 25-year old is born in. To calculate these probabilities, the share of married persons, female singles and male singles has to be taken into account. Considering 50% of households are married couples, married couples always consist of two people in my economy and the value function for couples is the same for each individual in the couple, the formula for calculating expected welfare of a newborn given the value functions V_m^0 of single males, V_f^0 of single females as well as V_c^0 of couples, reads as

$$V^0 = \frac{V_m^0 + V_f^0 + 4V_c^0}{6}.$$

The calibrated model is considered the benchmark model from now on. Government budget of the benchmark model is calculated and has to stay the same in order for an optimization to be acceptable. Otherwise it would probably be optimal to let the government deficit rise which is not reasonable.

In the first three scenarios optimal tax progressivity for each household type is determined without any change in the tax parameters or consideration of the expected lifetime utility of the other household types. For the fourth scenario, single females and single males are put into one category, singles. To look for the optimal tax progressivity under these assumptions, a grid of values for the respective tax progressivity parameter τ is created. For each of the grid points the tax level λ of the considered household type is used to ensure government budget remains the same as in the benchmark model. Note that while for couples, the parameters for tax progressivity and tax level can be altered without changing the parameters for singles, when optimizing for single males and females, this cannot be done. When searching for the best tax progressivity for single women households, the tax parameters for single male households must not be changed. Of course, the same holds true for optimizing tax progressivity of male households. During the optimization of tax progressivity for singles, the tax parameters of married couples are fixed.

The search for optimal tax progressivity for singles and couples together was a little more difficult. The adventures in the search for the Holy Grail of tax progressivity are therefore described further on.

4.2 Results

4.2.1 Optimal Tax Progressivity for Couples

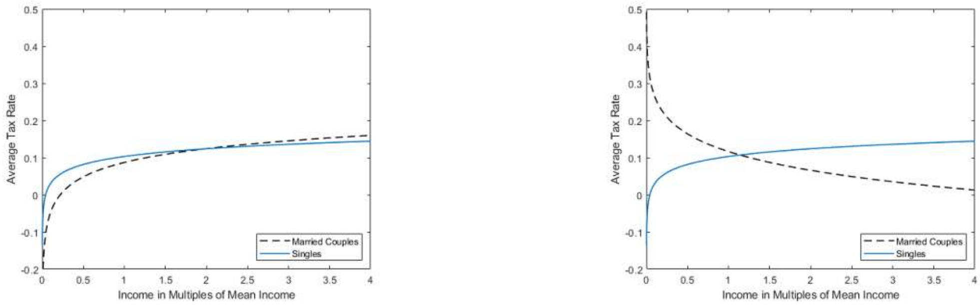
The approach to optimizing tax progressivity for couples has just been described, so I will only go over the essentials one more time.
The goal is to maximize expected lifetime utility of couples when tax parameters for singles stay constant. This is achieved by fixing τ_s and λ_s to their values in the benchmark model and calculating lifetime utility over a grid of possible values for τ_c. The tax level λ_c is used to keep government budget at benchmark level.

The results of the optimization are presented in table4.1. As can be seen, the optimal progressivity degree τ_c is negative. Thus, apparently a more degressive instead of progressive tax system would be optimal for couples if the tax parameters of singles remained unchanged. The corresponding tax level λ_c in this solution is smaller than in the benchmark model. Before interpreting this outcome, a look at the tax functions in the benchmark model and couples optimum should be taken.

Tab. 4.1: Tax Parameters

	Benchmark Model	**Couple's Optimum**
Degree of Tax Progressivity for Couples τ_c	0.06	-0.08
Level of Tax Rate for Couples λ_c	0.913	0.887
Degree of Tax Progressivity for Singles τ_s	0.034	0.034
Level of Tax Rate for Singles λ_s	0.897	0.897

This can be done in figure 4.1. On the left side the average tax rate of the benchmark is displayed while the optimal average tax rate for couples can be seen to the right. In the regressive tax system of the benchmark, married households with lower income have to pay relatively less taxes while households with higher labor income have to pay more in relation to their labor income. Couples would prefer to be taxed the other way around, though. For them, it is optimal to have high tax rates for low-income households and decreasing tax rates the higher household labor income is. Always keep in mind that the taxes are constructed for an economy with mean household income of one and mean income of married couples is higher than one in most age groups, so most couple households earn significantly more than mean income. It is understandable they wish to be taxed less. A lot is to be gained for high-income couple households, take a look at a couple earning 1.4 multiples of mean income, for example. Were they to be taxed after the benchmark tax scheme, they would end up with a disposable income of 1.24 multiples of mean income. However, when applying the new, optimal tax system, they would earn 1.275 multiples of mean income, which as 3 % of mean income more is quite substantial.



(a) Benchmark Model **(b)** Couples Optimum

Fig. 4.1: Tax Functions

Having learned more about the optimal tax function, a comparison of characterizing moments between the benchmark model and couples optimum seems like a good next step in getting to know this optimum better. Capital labor ratio $\frac{k}{n}$ did not change with respect to the benchmark value, because capital and labor increased with the same rate. Why do both rise, though? Couple households have higher incentives to save, because they have to insure themselves better against negative income shock than in the setting with a progressive tax system. This means more asset accumulation and higher savings, which build up the capital stock. As already discussed, couples have a relatively high income, mainly because they consist of two potential earners. When couples get taxed degressively, it becomes more attractive to earn more which couples actually can do by increasing the women's labor force participation. This leads to the higher effective time worked n in couples optimum in contrast to the benchmark model.
The interpretation of the increase in the gini coefficient for wealth is straightforward. High-income couple households are taxed less, which leads to higher after-tax income and more assets to save. This leads to more wealth for them and this mechanism also inspires many households to become high-income households by deciding the woman in the couples should work full-time, too. We will explain this phenomenon in more detail when looking at the working decision shares of women. The gini coefficient of gross labor income basically stays the same, which is partly because the taxes do not show up here directly and partly because the just explained effect on working time of married women leads to less productive women joining the labor force and even though more women decide to work, the inequality in in pre-tax income does not change that much. The situation is a different one for after-tax income. Here, the disparity enlarged with respect to the benchmark model. This is an expected change, degressive taxes are per definition increasing inequality. The rise in inequality in income including transfers can also be explained the same way.

Tab. 4.2: Moments

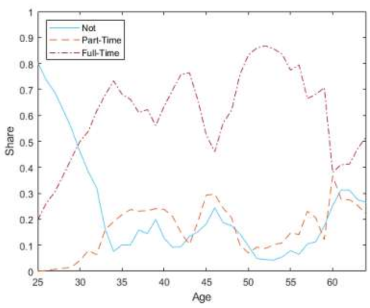
	Benchmark Model	**Couples Optimum**
Capital-Labor Ratio $\frac{k}{n}$	3.5	3.5
Gini Coeff. for Wealth	0.455	0.469
Gini Coeff. for Pre-Tax Labor Income	0.291	0.292
Gini Coeff. for After-Tax Labor Income	0.281	0.292
Gini Coeff. for Income (incl. transfers)	0.263	0.276

The topic of married women's labor force participation already came up. In table 4.2, the shares of women working full-time, part-time and not at all are listed for the benchmark and couples' optimum. The shares of single women did not change, because for them nothing changed. The only ones adapting are married women and they do so by increasing the share of women working full-time significantly by over 10 percentage points. Interestingly enough, the share of part-time working women remains nearly unchanged, but the share of non-working women decreased by the 10 percentage points the full-time working women share rose. The intuition behind that was already hinted at above. With the degressive tax system for couples in couples optimum, earning a lot of labor income is very attractive for couples, because they get taxed less the more they earn. This makes working the best option even for less-productive women who would have stayed at home in the benchmark model. Part-time work remains a good option for relatively productive women who earn enough to make the household income high enough to profit from the degressive tax system, but not enough for full-time work to outweigh the utility costs of doing so. This whole phenomenon reminds of the Laffer curve. The lower tax rates encourage more married women to enter into the labor force and the tax reform becomes self-financing.

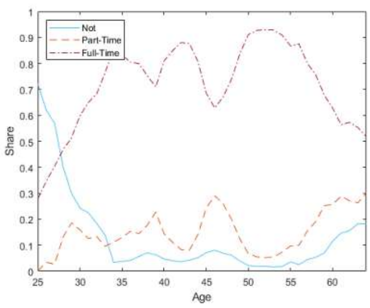
Tab. 4.3: Shares of Women's Working Decisions

	Benchmark Model	**Couples Optimum**
Married Women Working Full-Time	61.30	72.34
Married Women Working Part-Time	14.78	14.55
Married Women not Working	23.92	13.11
Single Women Working Full-Time	66.10	66.98
Single Women Working Part-Time	12.45	12.32
Single Women not Working	21.45	20.70

In figure 4.2, the just presented shares are depicted over the life-cycle of married women. The more degressive tax system manages to bring married women into the labor force earlier on, the bequests have a smaller effect on labor supply here than in the benchmark model. Like already discussed productivity also plays a smaller role, which is why more women continue to work until retirement in couples optimum. In between receiving bequests and retirement, the shares are also constantly higher than in the benchmark model.



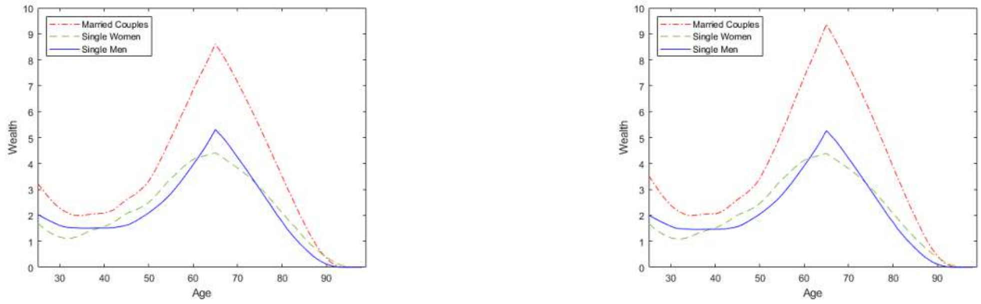
(a) Benchmark Model



(b) Couples Optimum

Fig. 4.2: Working Time of Married Women over the Life-Cycle

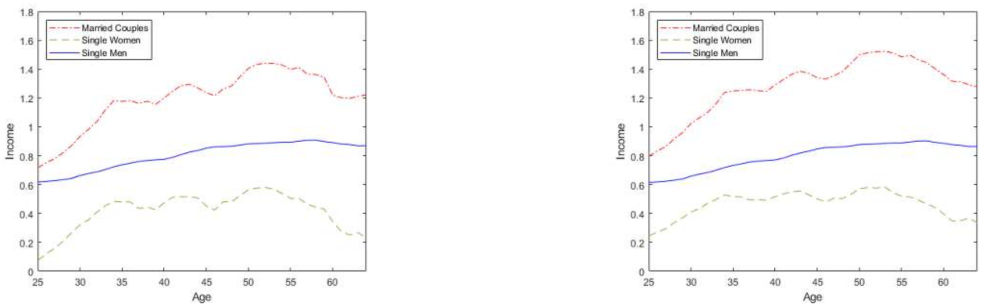
This leads us to the next variable of interest - wealth over the life-cycle. Again couples optimum is compared to the benchmark model in figure 4.3. Of course, the curves for singles look the same, but the peak in wealth for couples is visibly higher in couples optimum. Therefore mean bequests are higher than in the benchmark model, too. Married couples still dissave for the first years of their life, though, the bigger inheritance does not change this behaviour. However, couples wealth curve in couples optimum has a steeper increase than in the benchmark model from age 50 onwards. This also is a good illustration of the higher capital stock in the stationary equilibrium of couples optimum.



(a) Benchmark Model **(b)** Couples Optimum

Fig. 4.3: Wealth over the Life-Cycle

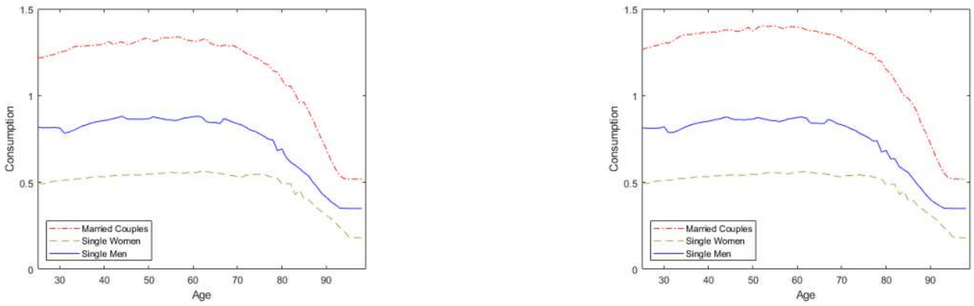
In figure 4.4, after-tax labor income in the benchmark model is compared to labor income in couples optimum. The shift upwards for married couples is visible for all age groups. How can they earn more labor income in their optimum, if it was stated before that the government budget has to stay the same as in the benchmark? The answer was already given by the plots above - higher labor market participation.



(a) Benchmark Model **(b)** Couples Optimum

Fig. 4.4: Labor Income over the Life-Cycle after taxes, without transfers

Last but not least, consumption is visualised in figure 4.5. Again it can be seen that consumption is higher in couples optimum than in the benchmark.



(a) Benchmark Model **(b)** Couples Optimum

Fig. 4.5: Consumption over the Life-Cycle

4.2.2 Optimal Tax Progressivity for Single Females

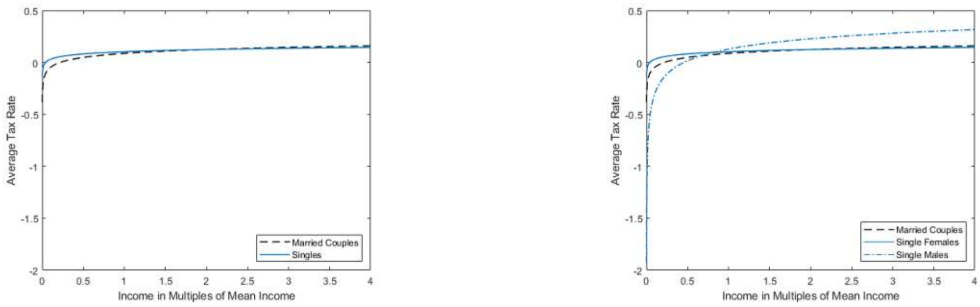
Next, the optimal tax progressivity for single females is searched for, again under the constraint that all tax parameters of other household types remain fixed. The procedure is the same as above for couples, a grid over possible values for tax progressivity degree $\tau_{s,f}$ is created and the tax level $\lambda_{s,f}$ works as a force to keep the government budget at the same level as in the benchmark equilibrium. However, in the benchmark, the tax function for single males and single females is the same. For the following optimization, there will be a differentiation between taxes imposed on single females and single males. The tax system of married couples as well as the tax system of males will remain untouched, while the optimal single female tax progressivity under these circumstances is searched for. I am basically applying a gender- and marital-status based tax here. This might seem politically unthinkable, but Alesina et al. (2007) show in their paper on gender-based taxation that a higher tax on men than women would be optimal under certain conditions.

Table 4.4 reveals that the optimal tax progressivity $\tau_{s,f}$ for single females is 0.175, so substantially more progressive than 0.034 like in the benchmark model. The corresponding $\lambda_{s,f}$ is smaller than the benchmark tax level.

Tab. 4.4: Tax Parameters

		Benchmark Model	**Single Females Optimum**
Degree of Tax Progressivity	τ_c	0.06	0.06
Level of Tax Rate	λ_c	0.913	0.913
Degree of Tax Progressivity	$\tau_{s,f}$	0.034	0.175
Level of Tax Rate	$\lambda_{s,f}$	0.897	0.872
Degree of Tax Progressivity	$\tau_{s,m}$	0.034	0.034
Level of Tax Rate	$\lambda_{s,m}$	0.897	0.897

When looking at the tax functions displayed in figure 4.6, it becomes clear how much more progressive the suggested optimal tax scheme for single women is. When applying the same axis for both plots, the lines in the benchmark model are hard to distinguish, while in females optimum the curve of average tax function of single females lies a lot below the taxes of single males and married couples until approximately a pre-tax income of around 0.8 multiples of mean household income, which implies a smaller average tax rate than for the other two household types. After that point, females in females optimum prefer to be taxed higher than any other household type. Recalling the female income development over the life-cycle, mean female labor income is well below that threshold. The new tax system therefore leads to substantially more redistribution.



(a) Benchmark Model **(b)** Females Optimum

Fig. 4.6: Tax Functions

Table 4.5 shows that also in females optimum, the capital-labor ratio $\frac{k}{n}$ remains untouched. The more interesting insight is that females optimum does not decrease the gini coefficient for wealth, but manages to decrease pre-tax labor income, after-tax labor income and income including transfers. That the gini coeffient for after-tax labor income would decrease when a more progressive taxation is applied, is expected behaviour as progressive taxation seeks to lessen inequality. However, the impact of the tax reform on pre-tax labor income is surprising as well. We will discover the reason for the improvement in the following paragraphs.

Tab. 4.5: Moments

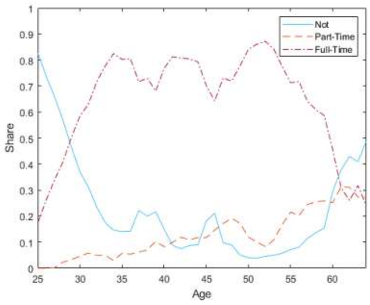
	Benchmark Model	**Single Females Optimum**
Capital-Labor Ratio $\frac{k}{n}$	3.5	3.5
Gini Coeff. for Wealth	0.455	0.454
Gini Coeff. for Pre-Tax Labor Income	0.291	0.285
Gini Coeff. for After-Tax Labor Income	0.281	0.268
Gini Coeff. for Income (incl. transfers)	0.263	0.260

At first, the shares of women working full-time, part-time and not at all presented in figure 4.6 show the expected result of no change in labor supply of married women as they are not affected by the tax change for single females. There is a huge change in labor market decisions of single women, though. The share of single women working full-time decreases by 10 percentage points alongside the shares of non-working single women who drop from over 20% to 6%. This means the share of single women who work part-time increases a lot and it does, in single females optimum the share is 25 percentage points higher than in the benchmark model. The reason for this development is the much higher progressivity in females optimum. Pre-tax part-time wage is half of full-time wage and is taxed as any other wage. In a less progressive system, this leads to substantially lower income than working full-time. Meanwhile in the more progressive optimum, the after-tax labor income of single females is high enough to motivate the 15 percent of non-working single women who are relatively productive to enter into the labor force and even attracts some of the less-productive single women working full-time who can gain more utility by working less and still getting paid relatively well in comparison to their full-time wage. Note that overall labor force participation of women rises due to the tax reform.

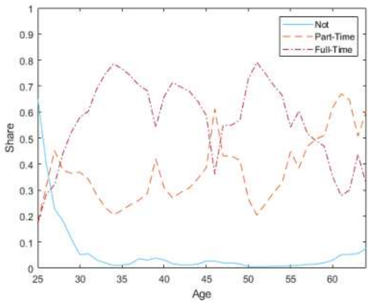
Tab. 4.6: Shares of Women's Working Decisions

	Benchmark Model	**Single Females Optimum**
Married Women Working Full-Time	61.30	61.06
Married Women Working Part-Time	14.78	14.71
Married Women not Working	23.92	24.23
Single Women Working Full-Time	66.10	56.32
Single Women Working Part-Time	12.45	37.57
Single Women not Working	21.45	6.1

In figure 4.7 the evolution of working time of single women over their life-cycle in single females optimum can be compared to the benchmark development. As we already knew from table 4.6, labor force participation is a lot higher in single females optimum, but a much larger share of women decides to work part-time. Just like in the optimum of couples, single females in single females' optimum are less influenced by bequests in their working choice, the delay in starting to work is a lot shorter than in the benchmark model.



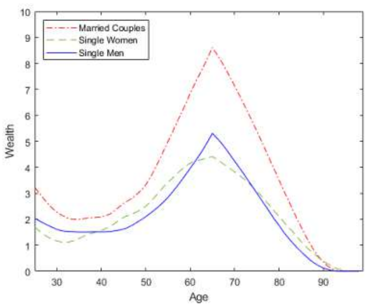
(a) Benchmark Model



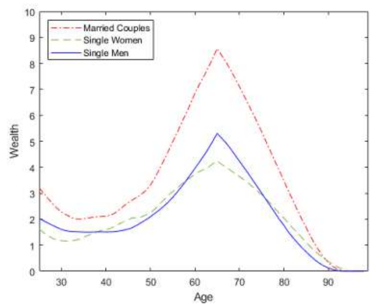
(b) Single Females Optimum

Fig. 4.7: Working Time of Single Women over their Life-Cycle

Studying figure 4.8, there does not seem to be a large difference in wealth accumulation over the life-cycle between the benchmark model and single females optimum. Single females even save less during working age than in the benchmark model. That makes sense, because more progressive taxation is a better insurance against negative income shocks and therefore works as a negative incentive to save.



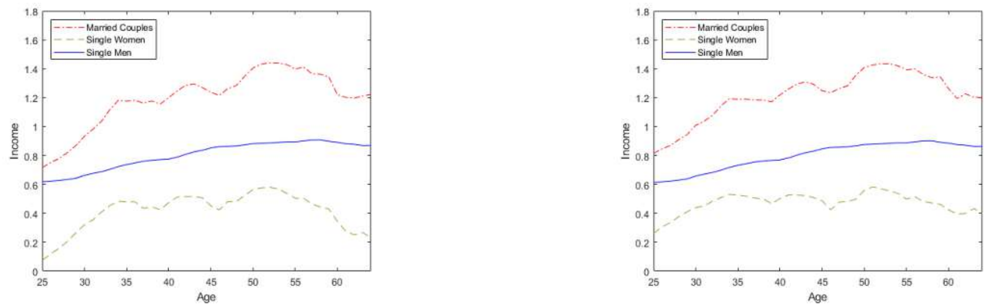
(a) Benchmark Model



(b) Single Females Optimum

Fig. 4.8: Wealth over the Life-Cycle

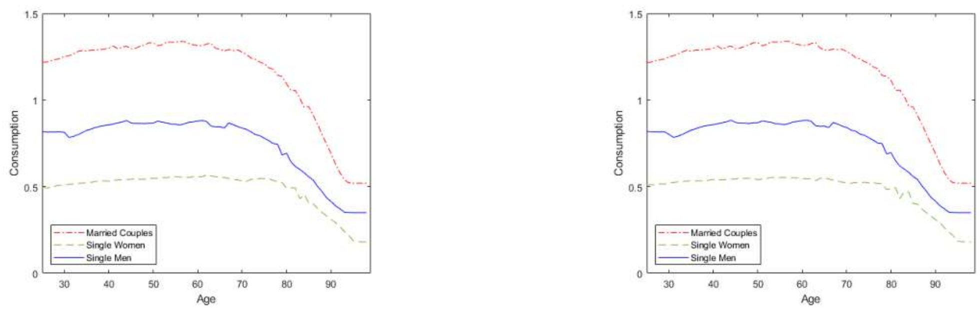
What is more, figure 4.9 shows that there is an increase in after-tax labor income of single females during their early and late years in single females optimum with respect to the benchmark model. As these are the years during which single women would earn least in the benchmark model, the higher progressivity of the new tax system is responsible for this mechanism. It is also the reason for the more flat-looking after-tax labor income curve. The question that already arose when studying the couples optimum is: How can single females mean labor income be higher if the government budget has to stay constant? The answer is again higher labor market participation.



(a) Benchmark Model **(b)** Single Females Optimum

Fig. 4.9: Labor Income over the Life-Cycle after taxes, without transfers

The difference between 4.10a and 4.10b might not be visible with the bare eye, but consumption is ever so slightly higher in single females optimum than in the benchmark.



(a) Benchmark Model **(b)** Single Females Optimum

Fig. 4.10: Consumption over the Life-Cycle

4.2.3 Optimal Tax Progressivity for Single Males

By now the procedure for figuring out which degree of tax progressivity is optimal for an individual who knows he is born as single male and is aware of all probabilites of his possible future states, but nothing more, is clear. The tax parameters of couples and single females stays fixed while the $\lambda_{s,m}$ holds the budget constant so that the expected lifetime utility maximizing degree of tax progressivity for single males $\tau_{s,m}$ can be found.

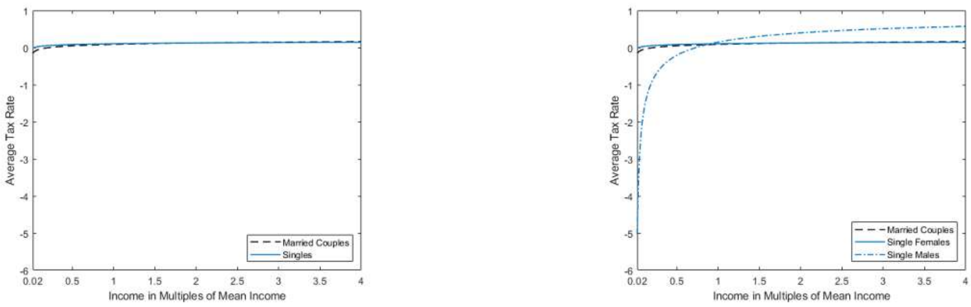
However, by applying this method for single males, I encountered troubles. As explained in the model description, males are assumed to work full-time during their whole working age. The only choice male singles can make is how much they want to save. As a consequence, male single households prefer the highest possible degree of progressivity - they are unable to react to tax changes via labor market participation, so their pre-tax labor income will stay the same no matter what the taxation is. This leads to men wanting to insure themselves against negative income shocks as good as possible. As good as possible can in this context be translated to "the highest degree of progressivity tax". To keep the results of the optimization from becoming too unreasonable, I restricted the grid points of $\tau_{s,m}$ to at least 0.5, which will also be the presented solution of the single males optimization considering only single male expected lifetime utility and not changing any of the other tax parameters than the ones of single males.

I want to emphasize again that the values presented as optimal for single male households in table 4.7 are a corner solution of the maximization problem with tax progressivity degree $\tau_{s,m}$ restricted to be greater than or equal to 0.5.

Tab. 4.7: Tax Parameters

		Benchmark Model	Single Males Optimum
Degree of Tax Progressivity	τ_c	0.06	0.06
Level of Tax Rate	λ_c	0.913	0.913
Degree of Tax Progressivity	$\tau_{s,f}$	0.034	0.034
Level of Tax Rate	$\lambda_{s,f}$	0.897	0.897
Degree of Tax Progressivity	$\tau_{s,m}$	0.034	0.5
Level of Tax Rate	$\lambda_{s,m}$	0.897	0.854

How high this degree of tax progressivity really is, is portrayed in 4.11. Note that the axes are exactly the same in both plots, but while with these choice of axes one can barely see that there are two different tax functions depicted in the figure of the benchmark model, the tax function of single male households is very distinctive. Up until an income of about 70 percent of mean income, taxes are negative - this means, the tax is not a tax, but a transfer. This is the best insurance against negative income shocks and as men have no possibility to change their working time dependent on their current productivity status, the best strategy for them is to insure themselves against this risk via a very progressive labor income tax. Note that well-earning households have to pay more taxes in this optimum.



(a) Tax Functions **(b)** Single Males Optimum

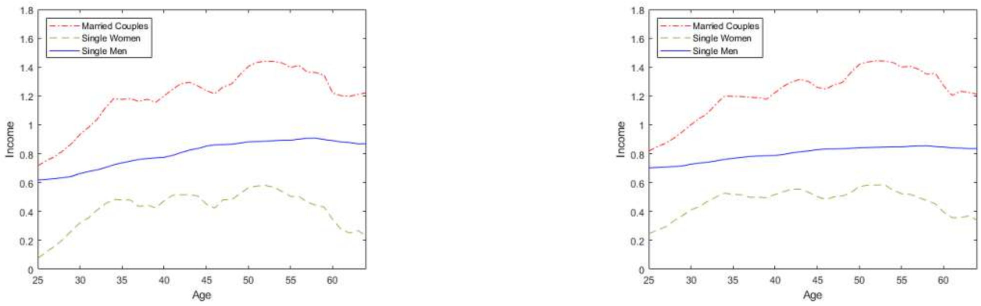
Fig. 4.11: Tax Functions

As always, capital-labor ratio presented in table 4.8 does not change from the benchmark to the optimum for single males. In contrast, the gini coefficient for wealth is lower in single males optimum. Of course, the gini coefficient for pre-tax labor income does not change, male labor supply cannot possibly change and the labor supply of the other household types is not influenced by the tax reform for single males. However, after-tax income decreases when switching from the benchmark to the much more progressive single males optimum. Consequently, disparity of income with transfers also falls, but note that the effect solely stems from changes in labor income, because men do not receive any transfers and the other household types receive as many transfers in single males optimum as in the benchmark model.

Tab. 4.8: Moments

	Benchmark Model	**Single Males Optimum**
Capital-Labor Ratio $\frac{k}{n}$	3.5	3.5
Gini Coeff. for Wealth	0.455	0.440
Gini Coeff. for Pre-Tax Labor Income	0.291	0.291
Gini Coeff. for After-Tax Labor Income	0.281	0.277
Gini Coeff. for Income (incl. transfers)	0.263	0.260

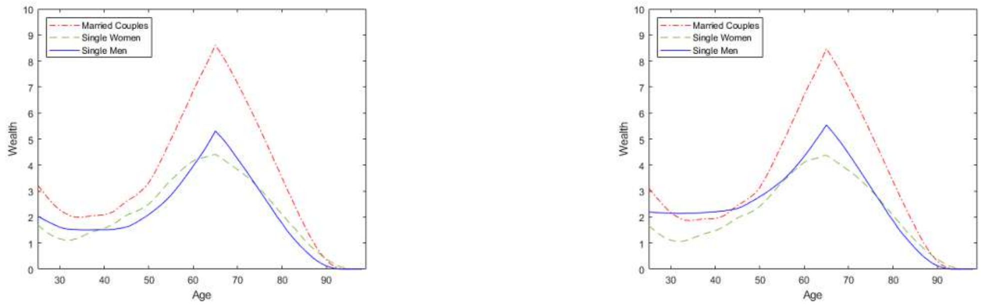
As already mentioned a few times, the change in single male taxes does not affect the other household types, therefore, I refrain from showing the table with the shares of women's working decision as there is essentially no difference between the optimum and the benchmark.
Labor income over the life-cycle as depicted in 4.12 is a perfect example of how a highly progressive tax scheme works. When comparing the labor income curve for men in the benchmark model to single males optimum, it becomes evident that the curve in the optimum is more of a flat line with nearly no positive slope, but starts a lot higher than in the benchmark model. This is exactly what the progressive tax scheme for single men accomplishes. The low-income households get transfers to increase their after-tax income while the well-earning households have to pay a high share of their labor income in taxes. This naturally leads to a much slower and overall less increase in mean labor income for single males over the life-cycle in the single males optimum.



(a) Labor Income in Benchmark Model **(b)** Labor Income in Single Males Optimum

Fig. 4.12: Labor Income over the Life-Cycle

I already discussed how high progressivity is a negative incentive for saving. However, single men do not want to dissave during their first ten years, their wealth during that time is higher in their optimum than in the benchmark model. The reason for this behaviour can be found in the male productivity profiles. Usually, men would dissave during the first few years because they know they will earn significantly more later in their life to save enough for retirement. In the more progressive setting we experience in the single males optimum, this expectation to earn so much more later on is not there anymore, because high labor income is heavily taxed and becomes less high after-tax labor income.



(a) Wealth in Benchmark Model **(b)** Wealth in Single Males Optimum

Fig. 4.13: Wealth over the Life-Cycle

4.2.4 Optimal Tax Progressivity for Singles

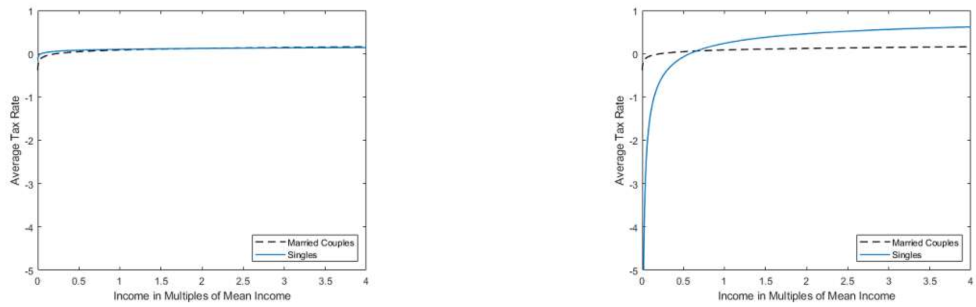
The optimal tax progressivity of single males and single females ceteris paribus already has been discussed. Now it is time to compute optimal tax progressivities for single and males together. This means the tax function is not split into a single female and a single male one, but the same one is used for both, just like in the benchmark model. Like before, the tax parameters of married households are fixed.
As explained above, to optimize tax progressivity for singles, expected lifetime utility of a newborn who only knows he or she is gonna be single, is maximized and weighted with the probabilities of being female or male when single.
Remember the two different optimal tax policies obtained by the exercises above. When optimizing for single females only, the optimal tax progressivity was 0.175, which is higher than in the benchmark model. In contrast, single males would prefer tax progressivity to be as high as possible, which under my constraints was 0.5. I set the same constraint again when solving for the optimal tax progressivity for singles. Apart from that, the method is the same as described in the last experiments.

Table 4.9 contains the optimized tax parameters for singles as well as the fixed ones for couples. Again, the optimum at $\tau_s = 0.5$ is at the lower bound of allowed degrees of tax progressivity τ_s under my constraints. The level of tax rate λ_s is also significantly lower than in the benchmark model.

Tab. 4.9: Tax Parameters

	Benchmark Model	**Singles Optimum**
Degree of Tax Progressivity for Couples τ_c	0.06	0.06
Level of Tax Rate for Couples λ_c	0.913	0.913
Degree of Tax Progressivity for Singles τ_s	0.034	0.5
Level of Tax Rate for Singles λ_s	0.897	0.759

In figure 4.14, average tax rates in the benchmark model are compared to the tax functions in singles' optimum. Note that the axes are the same in both figures. While in the benchmark average tax rates of singles and couples do not seem to differ when looking at them from that far away, the tax function of singles is a lot more degressive in the singles' optimum. Note that until an income of more than half of mean household income, single's average tax rates are negative, which translates into transfers. What is more, mean labor income of women in the benchmark model is less than 0.4 multiples of mean household income, which means most women will receive transfers no matter how much they work. This already implies that the tax burden will mainly fall on the men, who do not have a way to react to the change in taxes. I want to point out that in the male's optimum, men wanted to be taxed as progressive as possible. However, with the singles females exposed to the same tax function as single males, males would prefer to be taxed less progressive, because their income will be shifted towards single women. Single women on the other hand prefer it of course that the men pay taxes high enough for them to receive transfers nearly no matter how much they work. The increase in expected lifetime utility with higher tax progressivity of female singles is higher than the decrease of expected lifetime utility of single men. Consequently, this optimum is found.



(a) Benchmark Model **(b)** Singles Optimum

Fig. 4.14: Tax Functions

Looking at table 4.10, it becomes evident that the optimal tax function for singles is ambivalent in its consequences. Inequality in wealth rises when applying this tax policy, also pre-tax labor income and, more importantly, income including transfers are less equally distributed when applying the single's optimal tax function. After-tax income is the only of the presented values whose gini coefficent falls actually quite a bit in the single's optimum. The reasons for this interesting changes in inequality in the economy will be discovered shortly.

Tab. 4.10: Moments

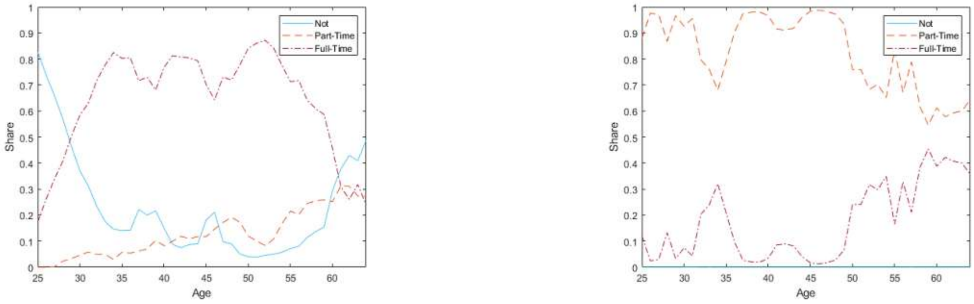
	Benchmark Model	**Singles Optimum**
Capital-Labor Ratio $\frac{k}{n}$	3.5	3.5
Gini Coeff. for Wealth	0.455	0.456
Gini Coeff. for Pre-Tax Labor Income	0.291	0.305
Gini Coeff. for After-Tax Labor Income	0.281	0.267
Gini Coeff. for Income (incl. transfers)	0.263	0.265

The first possible explanation for the above results is the effect of the tax change on female single's working decisions. As one can see, the effect is huge. The share of women working full-time drops from 66% in the benchmark to well below 20% in the new optimum. Not a single woman wants to stay at home anymore, but more than 80% of single women work part-time. Note that in the benchmark model, part-time working single women make up only 12% of single women. All single women want to work in the given scenario, because the taxes are so progressive that even the least productive women earn more when they go to work part-time than when they stay at home and receive benefits. Part-time work is the choice of most single females here, because the utility costs are lower, but the after-tax income of part-time work is not that much lower than the after-tax income of full-time work, because of the high tax progressivity.

Tab. 4.11: Shares of Women's Working Decisions

	Benchmark Model	Single's Optimum
Married Women Working Full-Time	61.30	60.64
Married Women Working Part-Time	14.78	14.91
Married Women not Working	23.92	24.45
Single Women Working Full-Time	66.10	17.54
Single Women Working Part-Time	12.45	82.45
Single Women not Working	21.45	0.00

The same message is received from figure 4.15. Tax progressivity is so high that even during the first few years, when single females used to live off the bequests they received and did not want to participate in the labor market, 100% of single women are working - most of them part-time for the reasons given above.

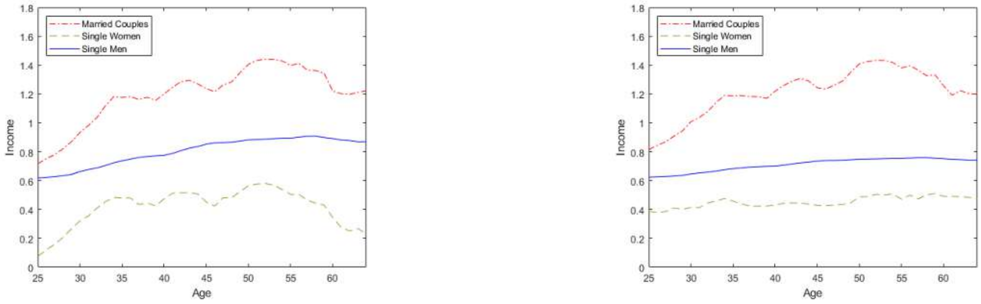


(a) Benchmark Model

(b) Singles Optimum

Fig. 4.15: Working Time of Single Women over their Life-Cycle

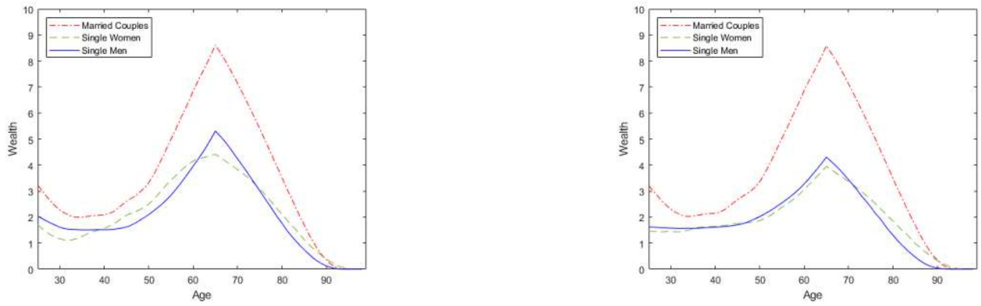
After-tax labor income develops as depicted in figure 4.16. There are no surprises here, after-tax mean labor income of single women rises to 0.4 multiples of mean income and stays at that level for the whole life-cycle of single females. Meanwhile mean after-tax labor income of men is lower than in the benchmark model and stays also pretty much the same over the whole life-cycle of single males, with a light upwards trend towards old age. This shows beautifully how the high tax progressivity shifts income from single males to single females and thus decreases inequality after taxes. Now I am also able to explain why income inequality before taxes is higher now than it was in the benchmark model. The reason is the changed labor participation of female singles. The mean labor income before taxes of women is lower in the optimum than in the benchmark model, because so much more women work part-time now. Yes, there are no women who do not work at all anymore, but the change from full-time to part-time is much more significant.



(a) Benchmark Model **(b)** Singles Optimum

Fig. 4.16: Labor Income over the Life-Cycle after taxes, without transfers

Last but not least, evolution of wealth over the life-cycle in figure 4.17 shows how close mean wealth of single females and single males lie together now. Two opposing effects on inequality are at work here. On the one hand, single males and single females are much more equal now, but the couples' wealth was unaffected by the changes in the tax system, so the inequality between couples and singles increased. Overall, this leads to a nearly unchanged gini coefficient of wealth.



(a) Benchmark Model **(b)** Singles Optimum

Fig. 4.17: Wealth over the Life-Cycle

4.2.5 Optimal Tax Progressivity for Couples and Singles

To determine optimal tax progressivity for couples and singles together requires a different strategy than the one applied to the cases above. I already described how expected lifetime utility of a newborn household is calculated, what remains is to analyze the optimization process itself.

Note that when optimizing for couples and singles simultaneously, none of the tax parameters stay fixed. Until now, always one parameter could be chosen freely and the other one would keep the government budget constant. However, with the four tax parameters $\tau_m, \tau_s, \lambda_c$ and λ_s, three parameters could be set freely and again only one is needed to ensure government budget is the same as in the benchmark model. I decided to explore how a tax system would look that did not distinguish between singles and males regarding the progressivity parameter. Also, a grid of three parameters is computationally very expensive, so the assumption was made that tax progressivity for couples and singles has to be the same, $\tau_c = \tau_s$. Keep in mind that tax levels τ_c and τ_s are still allowed to differ, so the resulting optimum will likely consist of two different tax functions.

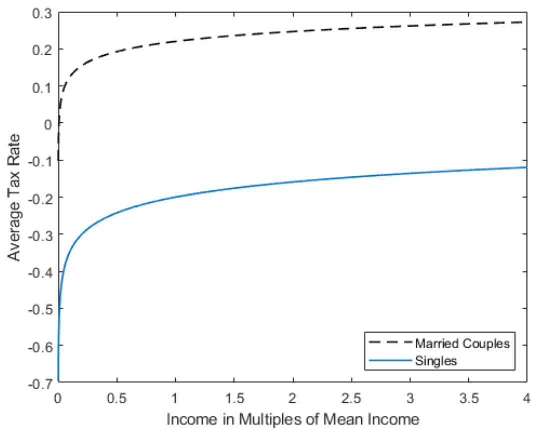
Anyway, this leads to a reduction of the dimensions of the optimization problem by one, which leaves three dimensions. However, only two of the remaining parameters can be chosen freely then, the third one needs to keep government budget stable. Of course, the tax progressivity parameter should not be the one to balance the budget, I am interested in knowing the optimal tax progressivity, not the best tax level. It does not matter whether tax level τ_c of married couples or τ_s of married females is used for balancing government budget, I decided to use a grid of tax progressivity degree $\tau_c = \tau_s$ and couple's tax level τ_c, so single's tax level τ_s will be used to balance the budget.

After applying grid search for this problem, the optimal tax system as presented in table 4.12 turned out to actually include a lower progressivity parameter for couples, but a higher one for singles than in the benchmark. However, the huge difference from the optimal tax system to the benchmark is the high level of tax rate λ_s for singles and the low level of tax rate λ_c for couples.

Tab. 4.12: Tax Parameters

	Benchmark Model	**Optimum**
Degree of Tax Progressivity for Couples τ_c	0.06	0.05
Level of Tax Rate for Couples λ_c	0.913	0.78
Degree of Tax Progressivity for Singles τ_s	0.034	0.05
Level of Tax Rate for Singles λ_s	0.897	1.2

For better understanding of the implications of the proposed optimal tax scheme, take a look at figure 4.18a. Remember that tax level of singles, λ_s, is significantly greater than 1, which is the reason for the single's average tax rate to be negative for all income levels in the plot. Negative taxes equal transfers, so no matter how much singles earn in this economy (income does never exceed 4 multiples of mean income in the model simulation), they always get transfers added to their labor income and do not have to pay taxes at all. However, someone has to pay money to the government - married couple households. The couples' average tax rate becomes positive with barely more than 0 income and is less progressive than in the benchmark model. Note that there is low tax progressivity in the optimum, so the redistribution within household types is small, but the redistribution between household types is enormous. Even though 50 % of all households are married couples, so individuals in couples make up about two thirds of the economy's population, it is still optimal to tax all these households at a very high level in order to redistribute labor income to single households.



(a) Tax Functions

Table 4.13 shows the most interesting moments of the optimum in comparison to the benchmark model. Now, the capital-labor ratio differs a little bit and is a little higher, which means there is a higher share of capital with respect to labor in the optimum. More capital implies a smaller interest rate. The consequently lower share of labor results in a higher wage rate. The differences are marginal, though.
What is more, the gini coefficient for wealth is smaller in the optimum, which was to be expected after seeing how much transfers single-households receive and how couple households pay for them with their high wages. The role savings play in this result will be discussed later. The gini coefficient of pre-tax labor income in contrast is a little higher in the optimum than in the benchmark, to determine where this stems from, the effects of the tax reform on labor market participation needs to be analyzed. This will happen below. As anticipated, after-tax labor income is a lot more equally distributed in the optimum than in the benchmark model, the same holds true for income including transfers.

Tab. 4.13: Moments

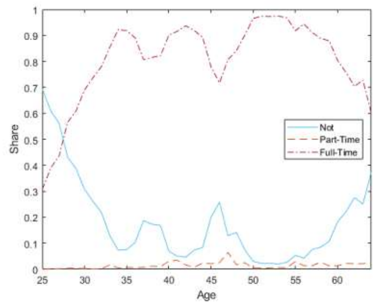
	Benchmark Model	**Optimum**
Capital-Labor Ratio $\frac{k}{n}$	3.5	3.6
Interest Rate r	5.93%	5.86 %
Wage Rate w	0.74	0.75
Gini Coeff. for Wealth	0.455	0.438
Gini Coeff. for Pre-Tax Labor Income	0.291	0.297
Gini Coeff. for After-Tax Labor Income	0.281	0.227
Gini Coeff. for Income (incl. transfers)	0.263	0.218

In table 4.14, the effects of the optimal tax reform on labor market decisions of women are displayed. Married women working full-time increase by nearly 20 percentage points because of the reform. Basically no married women work part-time, but there are still about 18 % of married women who refrain from working. These consequences of the change in tax functions are due to the lower insurance for couples against income shocks in the optimum. Their benchmark tax prgoressivity was higher and their tax level lower, so they have to pay a lot more taxes now without getting the same sense of security, because the tax system shifts income away from them towards the singles. This leads to couples having a lot less pre-tax labor income if the woman does not work full-time. Remember that the tax system is not very progressive, so higher labor income will be taxed more than lower labor income, but not high enough to make it not worth working full-time, which doubles married women's pre-tax labor income.
The interpretation of single women choosing to work a lot less full-time (around 20 percent decrease with respect to the benchmark model) and not at all, but working part-time significantly more often, is more straightforward. There are now transfers paid when earning labor income and the smaller the pre-tax income, the more transfers are paid. This makes it more desirable for single women to only work part-time as this is the lowest amount of working time they can invest while still profiting from the - for them - income-enlarging tax system. Note that non-working women do get transfers, but they are independent of the tax system.

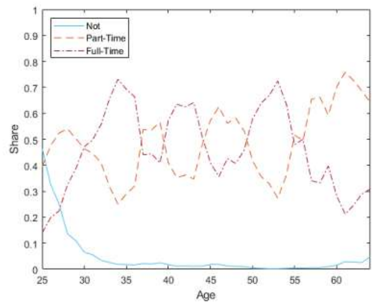
Tab. 4.14: Shares of Women's Working Decisions

	Benchmark Model	**Optimum**
Share of Married Women Working Full-Time	61.30	80.36
Share of Married Women Working Part-Time	14.78	1.41
Share of Married Women not Working	23.92	18.22
Share of Single Women Working Full-Time	66.10	46.30
Share of Single Women Working Part-Time	12.45	48.92
Share of Single Women not Working	21.45	4.78

In figure 4.19 married and single women's working time over the life cycle are displayed. The above discussed issues present themselves here as well - a higher share of married women is working full-time while about half of single women are working part-time. The higher shares of non-working women in the first few years of their working age persist, but to a much lower degree than in the benchmark model. Single women of old age do not switch from working to non-working anymore. The switches of single women between working part-time and full-time probably can be explained party by their productivity profiles. There are age groups of higher and age groups of less productive women, who decide accordingly whether they are productive enough to gain more utility from working full-time than part-time. Remember that what was said about the relatively low level of progressivity also holds true for transfers. If households earn a little more, they will get less transfers, but not by that much.



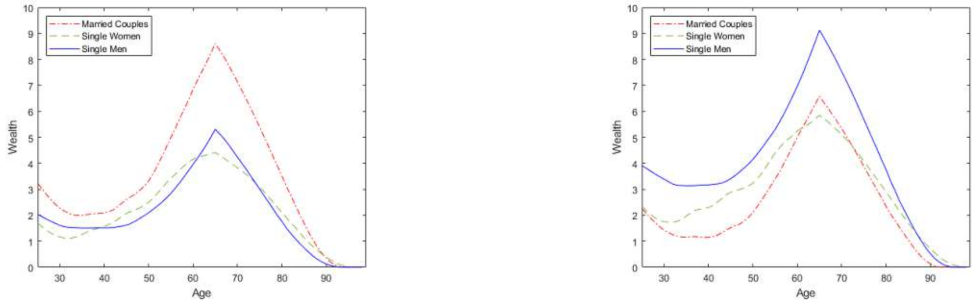
(a) Married Women



(b) Single Women

Fig. 4.19: Working Time of Women over their Life-Cycle

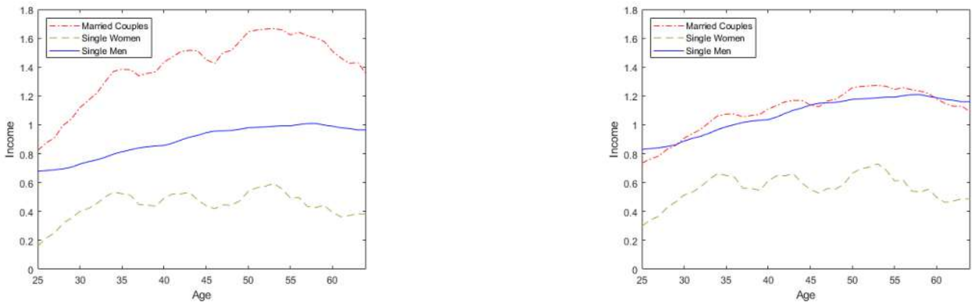
Figure 4.20 displays the wealth accumulation over the life-cycle in the benchmark model compared to the optimum. In the optimum, couples save a lot less than in the benchmark. They have a lot less income than in the benchmark and generally have a lower incentive to save than single households, because they can balance out the husband's negative income shock with the working decision of the female. Meanwhile single females now are very close in wealth to couples due to having more income and therefore having the opportunity of saving more. Single men profit from the tax reform as well, their mean wealth at 65 years of age even exceeds couples' mean wealth at the same age in the benchmark model.



(a) Benchmark Model **(b)** Optimum

Fig. 4.20: Wealth over the Life-Cycle

To once again prove that couples are really worse off in the optimum than in the model while the other two household types profit, figure 4.21 shows the changes from pre-tax to after-tax income. Couples theoretically make by far the most money, also because their households consist of two possible earners, but their after-tax income is not even above male singles' mean income in all age groups. This is also due to male singles substantial increase from pre-tax to after-tax income. The same is true for single females, but because their income before taxes is very low, even this extreme tax scheme does not get them close to after-tax income of males or married couples. However, they make a lot more money in the optimum than they would in the benchmark model - and all of this while working more part-time and less full-time.



(a) Before Taxes **(b)** After Taxes

Fig. 4.21: Labor Income over the Life-Cycle

4.2.5.1 Pareto Improving Tax Progressivity for Couples and Singles

The above obtained optimum clearly is not a Pareto optimum. In order to be one, all households have to be better off after the reform, which clearly was not true for married couples. They were worse off in every possible aspect, but because single women and single men profited over-proportionally much, the above presented tax system still was optimal under the assumptions made.

As an experiment, I therefore looked for a Pareto improving tax progressivity, but I changed the assumptions a little. Tax progressivity of couples and males is allowed to differ, instead I reduce the dimensionality of the optimization problem by restricting the parameters of tax level λ_c and λ_s to have the same ratio as in the benchmark model, so $\frac{\tau_c}{\tau_s}$ has to remain fixed. This enables me to create a grid over the two tax progressivity parameters τ_c and τ_s and use the tax level of singles λ_s to balance the budget. Tax level of couples λ_c is set to fulfill the constraint to hold

the ratio constant.
Figure 4.22 shows a heatmap for all combinations of tax progressivity parameters ranging from 0.5 to 1.2 in steps of 0.05. The purple area shows which combinations of tax progressivity are better than the benchmark for none of the three household types, married couples, single females and single males. The pink area highlights the combinations of τ_c and τ_s which are better than the benchmark for one of the household types. Orange is the code color for tax progressivity combinations where two out of three household types gain expected lifetime utility in relation to the benchmark. And finally, the small yellow area marks the Pareto improving options. Here all three household types have a higher expected lifetime utility than in the benchmark model.

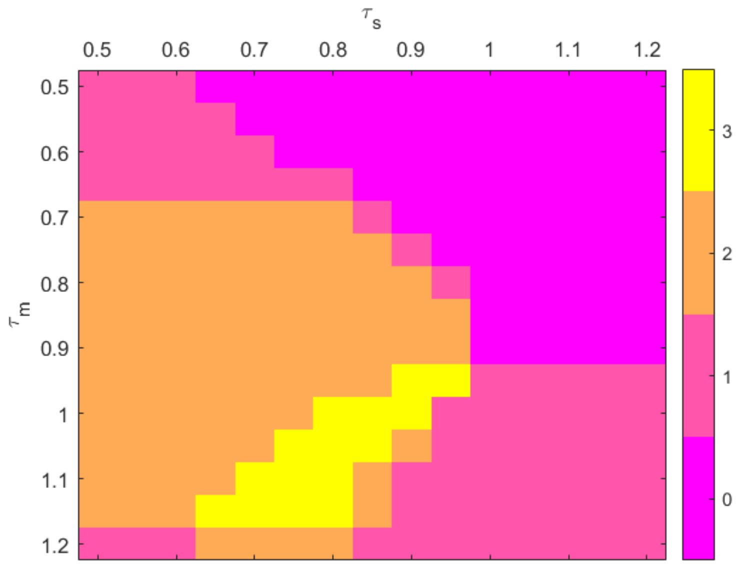


Fig. 4.22: Pareto Improving Tax Progressivity

Note that even though the combination $[\tau_c, \tau_s] = [0.95, 0.95]$ appears in the yellow Pareto improving zone, this does not imply that the solution obtained in the step above was Pareto improving, because different restrictions on the tax level parameters are imposed here.

Chapter 5

Conclusion and Ideas for Future Research

There are several lessons to be learned from this work. One of the main findings is that optimal tax progressivity depends a lot on the assumptions made and on what is seen as optimal. The results of the optimizations differed a lot for each household type, which proves once more how important it was to include different household types in the model. The optimal degree of tax progressivity obtained when looking at all household types at the same time is surprisingly close to the real one. However, the tax rate on married couples is probably too high to be implemented in reality without repercussions. If the obtained results for the optimal tax parameters were implied in real life, the divorce rate would probably rise moderately, but with less couples the tax system would not work anymore. While the missing divorce opportunity in my model may be criticized, the more important factor is the huge difference in pre-tax labor income between my household types which leads to these extreme results. The intuition that pre-tax labor income should be shifted from couples to singles is still right - married couples have more possibilities to insure themselves against negative income shocks. This is already implemented in the tax system, couples are taxed more progressively in the US at the moment.

The model used in this thesis could be expanded into many different directions.

First, one could introduce an extensive margin in the labor decisions for males or an intensive margin for females as well as for males. When searching for optimal tax progressivity, having at least an extensive margin for all participants in the labor market makes the results more realistic and scenarios like men wanting to be taxed extremely progressively would not occur anymore.

Introducing education levels could also prove very interesting, because if income was somehow dependent on education level, heterogeneity in the model would automatically increase. However, it would also mean an additional state variable at least during working age, so computation time would increase together with the heterogeneity of the model.
Especially a combination of education levels with some sort of marriage market or match-making depending on characteristics like education and wealth would add a lot of value to the model. A marriage market of course could also be a valuable addition on its own, because in reality a large share of couples has similar educational background and also shares other characteristics like wealth or preference for leisure. The difficulty with this approach would be to identify the

Another promising addition would be intergenerational links. The probably easiest part of that would be to include utility of bequests. If the dying household leaves behind assets for his descendants, he also derives some utility from them receiving his assets. A more interesting aspect would be to also allow for parents to have an effect on their child's education or productivity in general.

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