Transmission of rotation by a geometrically imperfect flexible shaft in a curved channel

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Abstract. Geometric imperfection of a flexible shaft transmitting rotary motion along a curved path results into a nonlinear relation between the rotation angles at the driving and the driven ends. Even the snap-through instability is possible if the intrinsic curvature of the shaft exceeds a certain threshold. In dynamics, this results into oscillatory response at the driven end when the angular velocity at the driving one is held constant. In the present contribution we analytically consider the case of small imperfection. A closed form expression for the profile of the vibration amplitude along the length coordinate follows after the linearization of the sine-Gordon equation in the vicinity of the stationary rotation regime. A comparison to the numerical finite element solution justifies the analytical result, allows to conclude on its domain of applicability and provides insights regarding the influence of nonlinearity and damping.

Introduction

Flexible shafts are used for delivering rotary motion, torque and and power from a motor to a distant point in space in various situations. The particles of the shaft undergo purely rotary motion if the shaft is confined in a borehole or in a stiff housing, such that its axis assumes a shape prescribed by the channel. A geometric imperfection of the shaft in the form of natural (or intrinsic) curvature along with the curvature of the constraining channel may result into highly irregular rotation of the tool even at slow motion and in the absence of friction [1]. A simple explanation is that the configurations, in which the directions of the curvatures coincide are energetically advantageous, and the opposite directions of the curvatures require higher degree of bending of the shaft to fit into the channel. As a result, the rotation angle at the driven end θ_{exit} becomes a nonlinear or even not a single valued function of the rotation angle at the driving end θ_{entry} even in statics, see Fig. 1. The equilibrium paths in the right part of the figure depend on the non-dimensional parameter combination $p\ell$, in which ℓ is the length of the shaft and p^2 is the product of the curvatures of the shaft and of the channel multiplied with the ratio of the bending and the torsional stiffness coefficients of the shaft, see [2].

Analysis

The dynamics of the structure is governed by the boundary value problem for the rotation angle $\theta(s,t)$ as a function of the spatial coordinate s and time t with c being the wave velocity:

$$\theta'' - \frac{1}{c^2}\ddot{\theta} = p^2 \sin\theta, \quad \theta(0,t) = \theta_{\text{entry}}(t), \quad \theta'(\ell,t) = 0. \tag{1}$$

Re-scaling the time and the spatial coordinate, this nonlinear hyperbolic partial differential equation may be transformed to the dimensionless sine-Gordon equation known for its soliton solutions [3]. We are, however, interested in the effect of the nonlinear term on the steady-state dynamics when the driving end of the shaft rotates with a given angular velocity, $\theta_{\text{entry}} = \omega t$, and seek the solution in the form

$$\theta = \omega t + \varphi(s,t), \quad \varphi(0,t) = 0, \quad \varphi'(\ell,t) = 0, \quad \varphi(s,t) = \varphi(s, t + 2\pi/\omega). \tag{2}$$

After the decay of the transient stage, the time period of the perturbation φ coincides with the time needed for a single rotation. At small p, a closed-form solution featuring a simple approximation $\varphi = \tilde{\varphi}(s)\sin\omega t$ becomes possible. A comparison against the results of the transient finite element analysis shows the range of applicability of the obtained analytical expression of the profile of the vibration amplitude $\tilde{\varphi}$.

References

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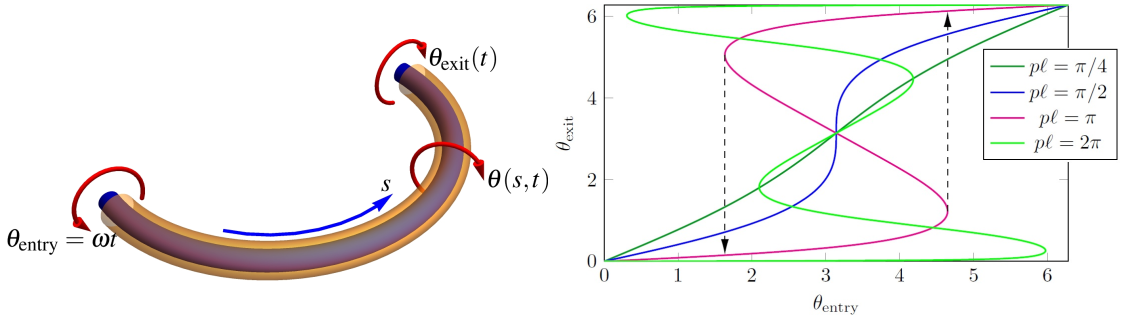

Figure 1: *Left*: Flexible shaft confined in a curved channel. *Right*: Static response of the elastic structure $\theta_{\text{exit}}(\theta_{\text{entry}})$ for various values of non-dimensional parameter combination $p\ell$; snap-through instability is possible at $p\ell > \pi/2$