TECHNISCHE UNIVERSITÄT WIEN

DIPLOMARBEIT

Costs of Gender Inequality. An economic analysis

zur Erlangung des akademischen Grades

Diplom-Ingenieur/in

im Rahmen des Studiums

Statistik und Wirtschaftsmathematik

eingereicht von

Stefani Rivić

Matrikelnummer 01427640

ausgeführt am Institut für Stochastik und Wirtschaftsmathematik
der Fakultät für Mathematik und Geoinformation der Technischen Universität Wien

Betreuerin: Univ.Prof.in Dipl.-Ing.in Dr.in techn. Alexia Fürnkranz-Prskawetz

Wien, 20. September 2023

(Unterschrift Verfasser/in) (Unterschrift Betreuer/in)

Abstract

Gender-based inequality is a phenomenon observed to a greater or lesser extent all over the world. In this thesis, we investigate the interplay between gender-based inequality and economic performance. First, we compare and synthesize different approaches in modelling the effect of gender-based discrimination on economic performance, applying an overlapping generations (OLG) framework. Since gender-based discrimination as well as its consequences are multidimensional, we analyze how gender-based inequality can be measured and what the crucial mechanisms in the models are. The thesis provides a literature review of the models in order to offer a comprehensive understanding of the feedback mechanism among female empowerment, economic development and the variable that measures gender inequality. We find that female empowerment, both within the private and the public sphere, has a positive effect on the economic development by stimulating human capital accumulation. Second, the practical application of the reviewed models is demonstrated by setting up a simple OLG bargaining model on the issue. The aim is to is to understand key features in modelling the interplay of gender inequality and economic growth. The model is analyzed analytically as well as numerically via simulations. We show that female empowerment leads to higher investments in education and lowers fertility. Finally, we compare our model and the models presented in the literature review, considering the effectiveness and performance. The thesis concludes with a discussion of the results and a summary of the implications of female empowerment for the economy.

Zusammenfassung

Geschlechterspezifische Ungleichheit ist heutzutage noch immer ein weltweites Phänomen. In dieser Masterarbeit wollen wir erforschen, wie sich die Ungleichheit der Geschlechter auf die Wirtschaft auswirkt. Zuerst synthetisieren und vergleichen wir verschiedene Modelle, die geschlechterspezifische Diskriminierung und wirtschaftliche Leistung in Zusammenhang stellen. Dabei beschränken wir uns auf Modelle, die den Ansatz überlappender Generation (OLG) verwenden. Da geschlechterspezifische Diskriminierung sowie dessen Auswirkungen multidimensional sind, analysieren wir dabei, wie die Ungleichheit der Geschlechter gemessen werden kann und welcher Mechanismus in der Modellierung entscheidend ist. Die Masterarbeit bietet hierbei einen Literaturüberblick, um dem Lesenden ein umfassendes Verständnis von dem Zusammenspiel zwischen weiblicher Emanzipation (*female empowerment*), Wirtschaft und der Variable, die geschlechterspezifische Ungleichheit misst, zu geben. Es stellt sich heraus, dass female empowerment die Bildung von Humankapital stimuliert und somit die Wirtschaft fördert. Im zweiten Teil der Masterarbeit werden die beschriebenen Modelle praktisch angewandt. Wir stellen ein einfaches OLG-Model zur beschriebenen Problematik auf. Das Ziel ist es, die wesentlichen Merkmale eines Models, welches die Rolle von weiblicher Emanzipation in der Wirtschaft darstellt, zu verstehen. Das Model wird analytisch sowie numerisch durch Simulationen diskutiert. Die Resultate zeigen, dass weibliches Empowerment zu geringerer Fertilität und höheren Investitionen in Bildung führt. Die Ergebnisse des eigenen Models sowie dessen Vor- und Nachteile werden mit den zuvor diskutierten Modellen verglichen. Die Masterarbeit schließt mit einer Zusammenfassung der Auswirkungen und Bedeutung von female empowerment auf die Wirtschaft ab.

Acknowledgement

First and foremost, I would like to thank Prof. Alexia Fürnkranz-Prskawetz for supervising my thesis, for her frankness and trust, for the excellent guidance and constant support during the whole research work. Your enthusiasm and expertise have motivated me during the course of my studies and master thesis. Thank you for the inspiration you are.

A million thanks to my sister, Ines, who was an emotional support in times when I doubted in myself. Thank you for being my greatest cheerleader!

I am grateful for my partner, who appreciates all my interests and endless talking on nerdy topics, who pays attention to my well-being, when I push too far and work too much. I am beyond grateful that you have my back.

Million thanks to all my friends: Thank you for not being upset for all the missed calls, unread text messages and cancellations. I appreciate every discussion and endless talking we have, every of your perspectives and insights, your knowledge and opinions. Thank you for celebrating all mile stones and sharing good times with me.

I would like to especially thank my parents, Ana and Zoran. I know that you did not have the chances you provided me in my life. I appreciate it more than you will ever know. I love you.

Stefani Rivić
Vienna, September 20, 2023

Eidesstattliche Erklärung

Ich erkläre an Eides statt, dass ich die vorliegende Diplomarbeit selbstständig und ohne fremde Hilfe verfasst, andere als die angegebenen Quellen und Hilfsmittel nicht benutzt bzw. die wörtlich oder sinngemäß entnommenen Stellen als solche kenntlich gemacht habe.

Wien, 20. September 2023

Stefani Rivić

Inhaltsverzeichnis

1 Introduction

According to an article by PWC, increasing female labor force participation to match the one of Sweden would account for 6 trillions$ GDP gain in OECD countries (PWC, 2023). A question: Are there more statues of men named John than statues of women? According to the UK Public Monuments and Sculptures Association the answer is "Yes", at least for the UK. The point of these random examples is that sexism does not only start with violence. Sexism or in other words gender-based discrimination is a phenomenon observed to a greater or lesser extent all over the world.

Even though gender equality has gone a long way and a lot of progress has been made in the past century, the effects of gender-based inequality can be still noted. For example, the gender wage gap closed only minimally over the past decade. Women earn 13 percent less on average per hour in the European Union, in Austria the pay gap is even higher with nearly 20 percent (European Comission, 2023). Roughly one could say women work one fifth of their time for free compared to their male colleagues in Austria - and this share does not even account for actual unpaid work!

The United Nations estimated that the total value of unpaid child care in the US in 2012 was 2.9 trillions $, which is approximately 20 percent of the GDP of that year (UN Women, 2023). Astonishingly, unpaid child care accounted for 309 billions $ in Australia in 2011. According to an Australian study, this would set unpaid child care as the number one industry of Australia, being even higher as the finance and insurance sector, which is officially in first place (PWC-Australia, 2023).

Thus, gender-based discrimination is an issue with a global sphere. One could argue, that unpaid child-care has not to be sexist as children have mothers *and* fathers in general. However, different studies show that unpaid child care is still a job mostly undertaken by women. For example, women spend between 251 (in the richest country) and 354 (in the poorest country) minutes per day on unpaid care and domestic work in Latin America, while men spent only between 139 and 141 minutes per day (UN Women, 2023). Worldwide, without exception, women perform the majority of unpaid care work, namely 76.2 per cent of the total of hours provided. In no country in the world provide men and women an equal share of unpaid care work (Addati et al., 2018). Even in high income countries, men still don't do their fair share even though the gender gap in unpaid care and domestic work has narrowed over the past decades.

The point is, that the amount of time spent on those kind of work has reduced for women mainly because they could outsource it, e.g. to paid workers, or due to the usage of modern domestic machines which help in routine tasks. And yes, men have increased their unpaid contributions, but they still spend comparatively little time on routine housework, much less time than women on childcare (though men's allocated time is increasing) and concentrate their unpaid work on less routine chores such as shopping and house repairs (UN Women, 2023).

It is often argued that the reason behind the unfair division of unpaid labor are the different earning abilities of men and women. Nevertheless, even in relationships where women's earnings exceed that of their husband, women tend to contribute more housework than their husbands. Social norms plays a huge role in explaining these difference: Women 'neutralize' the 'deviance' of their husband's financial dependence by adopting a behavior which is thought to be "typical" for a woman, for instance doing house work (Addati et al.,

2018). In fact, there is a lot of evidence that a fair part of economic differences regarding gender are grounded on cultural beliefs and norms.

For instance, a cross-country study of 25 OECD countries revealed a strong correlation between gender role attitudes and labor market outcomes (Fortin, 2005). The importance of culture becomes even more clear in a study by Fernández and Fogli (2009). They investigated labor force participation rates of immigrant women in the US and showed that it is strongly correlated with the labor force participation rate in the country of their origin. This is a surprising result as the women face the same economic and institutional constraints as American women but still act like the women in their original country, suggesting that culture plays a huge role in explaining economic behavior (Fernández and Fogli, 2009). Despite the difficulty in establishing causality between cultural norms and beliefs and economic circumstances regarding gender differences, the topic gained importance in research.

As one can see, gender-based discrimination is very complex and wide-spread, not only geographically but also in the sense of its emergence. Unfortunately, a naive approach to gender-based inequality is to think that it does not affect the society as a whole and is solely a "female issue". What is even worse, 50 percent of young men in the UK think that "feminism has gone too far" (Leah Rodrigues, 2020). The situation is not any different in Austria: An Austrian study from 2023 showed that 46 percent of Austrian men believe that the "Equality between men and women has progressed so far, that men are already discriminated". Moreover, half of the interviewed persons consider gender equality to be already reached (Ipsos-Austria, 2023). This opinion conflicts with a series of gender-based inequalities and shows that gender-based discrimination is not always obvious, but a system of complex mechanisms. In fact, gender-based discrimination is often considered as a personal issue but the consequences of gender-based discrimination take effect on the micro-social and macro-economic level.

As a simple illustration, think of a woman which earns less than her male colleague even though they are in the same position, have the same experience, same educational background and are equivalent co-workers, i.e. a woman which is affected by the (harmonized) gender pay gap. That woman does not only have to carry the personal burden -like psychological drawbacks, a disadvantage in her further career or a worse negotiation position aside to her weakened personal financial situation- but taking into account that "40 percent of women are their family's primary breadwinner" in the United States, her family will have widespread (financial) difficulties too (Holly Corbet, 2022).

Moreover, it is found that the financial drawback for mothers is higher than for women without children. There are different aspects that should be taken into account in explaining this circumstance. From a rational economic point of view, one would argue that mothers are endowed with less human capital due to the interruptions on the labour market or reductions in working time, in which they miss job-relevant training. From a sociological perspective, stereotypical expectations with respect to the mother‘s time and energy may impose the hiring and promotion decisions of some employers. Lack of child-care and flexible working arrangements intensify the situation for mothers. In that context, the terms "motherhood pay gap" or "motherhood penalty" are used. The motherhood pay gap measures the pay gap between mothers and non-mothers as well as mothers and fathers. Even though there are measurement issues, the results are clear: mothers face wage penalties worldwide (Grimshaw and Rubery, 2015). For example, in

the United States mothers make 58 cent for every Dollar paid to fathers (Holly Corbet, 2022). In Austria, a mother earns 33 percent less on average than her colleagues without children. Actually, the majority of female income drawbacks can be traced back to maternity leave and the subsequent part-time work in Austria. Some studies have even shown that 80 percent of the gender pay gap are made by the motherhood pay gap (Köppl-Turyna, 2019).

As already mentioned, especially families where mothers are the primary breadwinner are affected by the pay gap. Indeed, there is a lot of empirical evidence that mothers spend more on children than fathers in general. One possible explanation dates back to ground-breaking work of Becker, who argued that women and men have different intrinsically comparative advantages regarding the number (quantity) of children and the time and goods ("quality") they spend on their children (Becker, 1960). As a result of female preferences towards child quality, they spend more on their children, including investments in their human capital. Actually, there is a lot of empirical evidence that women invest more in human capital too. To mention only some, Doepke and Tertilt (2019) show that capital in the hands of women promotes human capital formation and has the potential to boost the economy if the economy is already developed enough to have human capital as the main driver of economic growth. Hornset and Soysa (2020) prove empirically in a study from 1960 to 2018 that female empowerment is significant in establishing equal access to education and health. We will see in the literature review, that the tendency of women to stimulate human capital formation is crucial in explaining why gender-based discrimination is harmful to the economy.

Speaking of human capital, we could ask whether the situation is any different among researchers themselves. Are they spared from gender-based discrimination or can we find differences along the gender lines? Long time it was rumored that not Albert Einstein himself, but his wife Mileva Maric was the secret visionary of the relativity theory. While this could be disproved (Weiss, 2019), there are various cases where women's achievements were dismissed in favor of men. The most famous example is Rosalind Franklin: She explored with different experiments that the DNA is made up of double strands and a phosphate group. Nevertheless, James Watson and Francis Crick got the Nobel price for the "discovery" of the DNA. Even though, the situation is not that tragic nowadays, there are still structural problems with gender equality. In twelve countries and regions, mainly from the Western world except for Brazil, it is found that women comprise 40 percent of researchers on average. Unfortunately, the situation is worse in the field of science, technology, engineering and maths (STEM). Here the share of women almost cuts half with under 25 percent of researchers in the STEM area being female (Elsevier-Amsterdam, 2017). Even though this share varies among different studies, the results are always clearly indicating that women are strongly underrepresentated in STEM. Moreover, women tend to be underrepresented in the finance sector too. Women account only for 25 percent of board seats in traditional banks and bank supervision agencies. Additionally, only 10 percent of leaders in the digital finance industry, called fintech, are female. But why does it matter which gender the leader of a firm has? A positive relationship between more women on executive boards and the revenue earned by the respective fintech firm as well as the funding they receive for future investments is found. A 10 percent higher share of women on executive boards is associated with roughly 13 percent higher revenue and funding earned by a firm (Khera et al., 2021). There is a documented positive relationship

between gender diversity in a firm and the firm's performance. Firms with a higher share of women executives earn higher revenue and receive more funding. Better performing firms in turn stimulate economic growth and thus benefit society (Khera et al., 2022). In times of digitalization, fintech firms play an important role. But generally speaking, a special attention has to be put on the access to digital services as gender-based disadvantages do not stop there either. The "typical" aspects like education, financial situation and/or social norms impede females and cause the digital gender gap which divides in three components: (1) access and use of digital technologies and the internet; (2) development of the skills needed to use digital technologies and to participate in their design and production; and (3) advancement of women to visible leadership and decision making roles in the digital sector (Kuroda et al., 2019). Moreover, the Covid-19 pandemic intensified the importance of information and communication services (ICTs) which are a great part of digital technology and thus enhanced the digital gender gap.

The Covid-19 pandemic has indeed put us back with respect to gender equality. For example, women reduced their hours in the labor market in order to take care of their children four to five times more than fathers. Thus the gender gap in hours worked did increase by 20 to 50 percent in the United States. This highlights how the Covid-19 pandemic challenged women´s employment (Collins et al., 2021). Furthermore, it is empirically shown that a lot of the extra household work which had to be done during the Covid-19 pandemic was unequally divided among partners, allocating more work to the woman (Del Boca et al., 2020).

Another subject with worldwide impact is sustainability. The climate crisis is considered to be one of, if not to say the biggest danger of our times. A potential improvement regarding this issue would be establishing more gender diverse boards of directors and more policies and practices that enable or reinforce gender diversity throughout the organization of a firm. Demographic and structural gender diversity are significant predictors of a firm's environmental sustainability initiatives (Kassinis et al., 2016). Moreover, it is found that women are more affected by the consequences of climate crisis than men in Subsaharan Africa. In addition, an empirical study of 141 countries from 1981 to 2002 shows that climate change lowers the life expectancy for females more than for males. Hence, they value resources more and have more incentives to preserve them. Gender-based inequality is often a product and source of the considered surrounding, in this case the natural environment itself. However, there are many more indices for the tendency of women to value environmental sustainability more than men (Meinzen-Dick et al., 2014). As we can see in all those examples, gender-based inequality comes with a series of consequences that affect more or less all members of a society. Gender equality is a valuable goal in itself. Nevertheless, there are costs associated with discrimination. If we speak of costs of sexism, we consider all the foregone output an economy could have generated if it would have well-established equal chances for women and men and gender-based discrimination would not occur. For instance, developing countries, which tend to have greater gender inequality, would increase economic output by an average of 35 percent by closing the gap in women's labor force participation (Georgieva et al., 2023). Another example to take into account, is unequal access to education. Until the 1960s women were restricted to study medicine or law in the United States. If we consider talent to be equally distributed among children -and there is no scientific indication that strikes to believe the opposite-, this would mean that a part of the US population was not able to allo-

cate their talents optimally due to barriers. Despite that, the removal of those obstacles between 1960 and 2010 is examined to explain up to 40 percent of aggregate growth of the US market! Vice versa, this means that the US has been losing revenue for years due to the effects of gender-based discrimination in the educational sector (Chang-Tai Hsieh and Klenow, 2019).

Not only the consequences of gender-based discrimination are widespread, but also the mechanisms how they work and channels through which they impact the economy, environment and society are complex and heterogeneous. As we have seen there are different triggers like the gender wage gap, gender digital gaps, gender hours worked gap and so on. Now the question is, whether there exists something that all those triggers have in common? Something like a basic mechanism, through which gender-based discrimination impacts the economy? The aim would be to understand the mechanisms and find the common factor. If we could do so, we could set up something like a baseline model, which sets gender-based discrimination and economic output in relation. Moreover, we could take gender-based inequality as an independent variable in explaining economic performance. Beside from the precious goal of understanding the impact of gender-based discrimination on the economy and other corresponding variables, we could find -at least theoretically- solutions to the problem of gender-based discrimination. A solution describes a set of feasible actions policy-makers could take in order to stimulate gender-equality and oppose gender-inequality.

Before we can answer the question whether there is such a model or not, we first need to understand what gender-based discrimination is. Following the definition of EIGE, the European institute of gender equality, we find that sex- and gender discrimination is

> *"Discrimination occurring due to interaction between sex (as the biological characteristics of women and men) and their socially constructed identities, attributes and roles and society's social and cultural meaning for biological differences between women and men."* European Institute for Gender Equality (2022b)

In simpler words, gender discrimination is a form of discrimination where a person is treated differently because of her:his gender. This can happen to men as well as to women. Despite that, as mentioned before women are treated unequal to men in their education, career, economic advancement and political influences more often. Knowing what discrimination exactly means, we can start analyzing the systems in which and how it occurs. We will do this in two parts. First, we will review the literature. The aim here is to synthesize different approaches in explaining the interplay between gender inequality and economic performance. Moreover, we will look for the common factor of the different models. Second, we construct a model on the interplay of gender inequality and economic performance which aims to be simpler than the models reviewed and conduct experiments with it. The goal is to check whether a simplified model is able to replicate well-known behavior and what limitations it has.

The thesis is structured as follows: in section 2 we review the literature. We will start with a discussion of the origins of gender inequality in section 2.1, which will be followed by a detailed description of the baseline model in section 2.2. After that, we will focus on female bargaining power in the private sphere (section 2.3), the impact of female emancipation in the public sphere (section 2.4) and on the influence of female health (section 2.5). In section 3.1 we analyze what could be the common factor of the

previously discussed papers and set up a baseline model. Next, we focus on a simple macroeconomic model as well as variations of the model and conduct various simulations in section 3.2. Finally, section 4 discusses the results and draws conclusions.

2 Literature Review

In this section, we review different papers that link gender-based inequality and economic growth. More specifically, we compare and synthesize different approaches in the field of gender-based discrimination. The goal is to find a common factor in the variety of models and extract the crucial mechanism of how gender-based inequality affects macroeconomic performance in those models. Moreover, the question will be how discrimination can be quantified. Gender-based discrimination is something that happens primarily on a social level and describes the prejudicial treatment of a group of people on the grounds of their gender. While this would be really difficult to measure, discrimination against women has such a long tradition, that it is already manifested and internalized in various aspects of life, such as earnings which are very easy to measure. Hence, different gender gaps are not only cause but also the product of gender-based inequality and will be a crucial part in the presented papers. On the other side of the discrimination are the consequences it causes. We are especially interested in economic consequences of gender-based discrimination. As there are different ways of measuring discrimination, the first question is whether the consequences are different as well. Or do all those channels disadvantage the economy in the same way? From the mathematical point of view, it is interesting to ask whether different ways of quantification lead to different results. In fact, we will see that all models and numbers indicate that the consequences are clear and unambiguous. Second, we want to discuss the consequences and their scope. We will see, that inequality between the sexes, no matter on how the discrimination is realized, lead to worse economic performance. Furthermore, there is a clear interplay between discrimination against women and education: all models indicate that female empowerment leads to more education of children and higher human capital investments.

As the topic is widespread, we focus on the interplay between gender-based discrimination and economic performance. Furthermore, we restrict ourselves to models with an overlapping generations (OLG) approach. To ensure a comprehensive review, we discuss the origin of gender-based discrimination in section 2.1. This is preliminary work for our baseline model, which will be discussed in section 2.2. The next three sections, 2.3 - 2.5, review different aspects of gender equality: female empowerment in the private sphere, female empowerment in the public sphere and female health. Note, that the models will always be discussed in comparison to our baseline model.

2.1 The Origin of Gender Inequality

Over hundred of years, the remains of the so-called “Birka warrior”, a viking skeleton from the 10th century, were thought to be male. Why is this surprising? Because, from a scientific, forensic view it was clear that the Birka warrior had to be female as the pelvis of the found skeleton is obviously female. Despite that, researchers argued the findings had to be male, as there were weapons and two sacrificed horses in the tomb. Even though, there were clear scientific arguments, they didn’t want to believe a female could be a worshiped

warrior. The situation gets worse, as DNA findings prove that the skeleton belonged to a woman and researchers still argue that in that case, the tombs must have been mixed-up (Perez, 2019). This shows, how sexism is deeply rooted in our society and does not even stop at science. The reasons behind have emerged over centuries in a complex, historically grown system. One of the main drivers of unequal treatment between the genders is the question on the appropriate role of women in society. As we have seen in the example before, even nowadays persons tend to replicate "old" behavioural patterns and reinforce stereotypes in that way. Not only within but also across different societies the opinion on that differs a lot. For instance, the World Values Survey tries to examine human values and beliefs. In the fourth so called wave of the World Value Survey participants were asked if they agree with the statement that "when jobs are scarce, men should have more right to a job than women". The results were ranging from 3.6 percent of the participants in Iceland to 99.6 percent in Egypt agreeing.

But why is that? Where do those beliefs come from and how can these cultural differences be explained? Ester Boserup argued that the differences in gender roles have their origins in the form of agriculture traditionally practiced in the pre-industrial period (Boserup, 1970). Keeping in mind, that agriculture has been the most important sector accounting for 58 percent to 80 percent of the labor force until the industrial revolution (Our World in Data, 2022), it seems natural or even indispensable that today's beliefs are consequences of behavior, systems and patterns of the pre-industrial agriculture.

There were two main practices to prepare the soil, the shifting and the plough cultivation. Shifting cultivation is labour and capital intensive. An important task in the shifting cultivation is weeding, which is primarily done by women and children. Plough cultivation requires high levels of strength in the upper body since handheld tools like the hoe are applied. Moreover, grip strength is needed to pull and control the animals that pull the plough. Women have a disadvantage in plough cultivation as they have significantly less upper body strength. In fact, men have 40 to 60 percent more strength in the upper body, and 41 percent more grip strength than women (Lewis et al., 1986). Thus, men have a natural advantage relative to women (and children) when using the plough. Conversely, shifting cultivation is beneficial for child rearing, a task that is mostly performed by women. The reason behind is that shifting cultivation is less dangerous than plough cultivation. Furthermore, shifting cultivation includes tasks that can be stopped and resumed easily, which is not the case for shifting cultivation. One has to take into account, that one third of the total time spent in agricultural tasks was used to prepare the soil.

Combining these facts, one can easily see why production was specialized along gender-lines in agriculture using the plough. This means, men tended to work on the fields while women were specialised in tasks at home. Thus, Boserup's hypothesis can be summarized as following: Societies where the plough was used primarily tend to have more sex-specific belief regarding the "natural" role of men and women. This is a consequence of the labor division between the sexes based on the fact that men have more physical strength and thus can steer the plough easier. Vice versa, regions where shifting cultivation was dominant did not specialize along gender lines. As a consequence, regions with shifting cultivation prevail more gender equality than those, where plough cultivation was historically dominant. Summarized, different farming practices have formed historically different beliefs and norms about the appropriate role of women in society (Boserup, 1970).

Now, one could ask for evidence on this hypothesis. A work by Alberto Alesina, Paola Giuliano and Nathan Nun turns the hypothesis into theory with their paper "On the origins of gender roles: Women and the plough" Alesina et al. (2013). They test Boserup's theory with an econometric model in which they put pre-industrial ethnographic data on traditional plough use in agriculture in relation with contemporary measures of gender equality. This measures contain information on beliefs about gender roles, female participation rates in the labour market, in politics and the number of female firm owner-ships.

In the following we will review the paper by Alesina et al. (2013). Simple ordinary least square regression (OLS) is performed on cross country-, within country- and individual level data to examine the long-term impact of traditional plough use. To do so, the authors link data on historical plough use, measured on the ethnicity level, to current variables of interest, e.g. real GDP and cultural beliefs on female labour force participation, measured on the spatial level, which can be either countries (for the cross country analysis) or regions within a country (for the within country level). On the individual level, the authors examine variation in cultural beliefs among the children of immigrants from diverse cultural backgrounds and different histories of ancestral plough in the United States and Europe, where they face the same environment regarding labour market opportunities, institution, policies, etc. By doing the analysis on the individual level, the authors control for the possibility that the results may be biased on the cross-country level by the characteristic of the country. The authors find similar impacts of traditional plough use at all three levels.

The procedure is the following: First, an indicator variable I_e^{plough} is introduced. I_e^{plough} is equal to 1 if the ethnic group e used the plough traditionally in farming. The data to define the variable is from the Ethnographic Atlas, which offers information about 1265 ethnic groups worldwide including historical plough use. Given the total number of individuals of ethnicity e nowadays, one can calculate the fraction of population who's ancestors used the plough, denoted by $\text{Plough}_{d,c}$. Let $N_{e,i,d,c}$ denote the number of persons of ethnicity e, living in grid-cell i located in district d in country c and $N_{d,c}$ the total number of individuals living in district d in country c . The grid-cells help to link ethnicity to district or country, using the data from "Ethnologue: Languages of the World" which offers data on the geographic distribution of over 7000 languages worldwide based on 1km-by-1km big grid-cells. The authors matched the languages to the ethnicities manually then. Hence, $\text{Plough}_{d,c}$ is given by

$$\text{Plough}_{d,c} = \sum_e \sum_i \frac{N_{e,i,d,c}}{N_{d,c}} I_e^{\text{plough}}$$

Let y_c denote an outcome of interest in country c. That could be female labour force participation in 2000, the share of firms with female ownership from 2003 to 2010 or the share of political positions held by women in 2000. Boserup's hypothesis is then tested (on the country level) with the equation

$$y_c = \alpha + \beta \text{Plough}_c + \mathbf{X}_c^H \Gamma + \mathbf{X}_c^C \Pi + \epsilon_c$$

where $\mathbf{X}_c^H$ and $\mathbf{X}_c^C$ are ethnographic respectively contemporary control variables. The historical control variable $\mathbf{X}_c^H$ is intended to capture historical differences between societies using the plough and not using the plough, including difference in the presence

of domesticated animals, economic development measured by the density of settlement, levels of political authority in the society and agricultural suitability. The contemporary control variable $\mathbf{X}_c^C$ captures the natural log of country´s real per capita GDP as well as a square of it.

The results indicate that countries with traditional plough use have a lower female participation rate, women are less likely to own firms and are less likely to take part in national politics. When controlling for GDP per capita, the magnitude of the relationship between plough use and the other variables doubles and the effect on participation in politics becomes significant. If GDP per capita is not controlled for, one does not take into account the positive correlation between female participation rate and wealth of a country. The point is, that countries with historical plough use tend to be richer nowadays and thus have more female labour force participation ceteris paribus. This means, that even if historical plough use indicates a negative relationship to female participation rate in politics, the negative effect is outbalanced by the positive effect of the wealth of a country. Thus, contemporary income has to be controlled and introducing $\mathbf{X}_c^C$ is inevitable. The positive relationship between GDP per capita and female participation rate is also the reason why the analysis cannot be applied to Western Europe and its offshoots. Moreover, the lack of variation in traditional plough use within Europe implies that Boserup's hypothesis and our empirical analysis cannot explain existing differences in gender role beliefs within Western Europe.

Nevertheless, the OLS estimates indicate that historical plough use is statistically significant in explaining female labour participation rate, female firm ownership and female participation in politics. Even the R-Squared, which explains how much of total variation in the data is explained by the regression model, increases by 0.06 if traditional plough use is introduced. This means that 6 percent of the total variation in female participation rate is explained by traditional plough use, confirming Boserup‘s theory.

To conduct the experiment on the individual level, the authors examine the variation in cultural beliefs. To do so, Alesina et al. (2013) rely on data from the World Value Survey. There, participants with different demographic characteristics were interviewed on different topics, including preferences and opinions. The measure on individual attitudes are based on the responds of the interview participant to the statement "When jobs are scarce, men should have more right to a job than women." and "On the whole, men make better political leaders than women do.". The responds could vary from "agree strongly" to "disagree strongly" in five levels, which the authors capture with an integer value from 1 to 5.

The econometric model on the individual level takes the following form

$$y_{i,d,e} = \alpha_c + \beta \text{Plough}_d + \mathbf{X}_e^H \Gamma + \mathbf{X}_d^H \Pi + \mathbf{X}_i \Phi + \epsilon_{i,d,c}$$

where i denotes an individual, d a district and c a country. $y_{i,d,e}$ is on of three outcomes, either female labour force participation, or attitude about female employment, or attitude about female leadership. α_c denotes country fixed effects. $\mathbf{X}_d^H$ captures historical differences between societies using the plough and not using the plough analogously to $\mathbf{X}_c^H$ but on district level. $\mathbf{X}_i$ denotes individual level controls like age, marital status and education.

Like on the district and country level, the authors continue to find a positive and

significant relationship between historical plough use and attitudes reflecting gender inequality.

What is even more interesting, is that this mechanism holds true if the external environment is held fixed. One could argue that traditional plough use had also an impact on policies, laws, markets and development of institutions and impacted female participation outside the domestic area not through the belief and norms but through these channels. To control for this theory, the authors conduct the tests on individual level, focusing on children from immigrants.

To do so, the authors examine the variation in cultural beliefs among the children of immigrants from diverse cultural backgrounds and different histories of ancestral plough in the United States and Europe. By doing so, they can hold country and even subnational effects, like policies, labour market opportunities or institution, fixed. To be more precise, the effect of historical plough use is isolated by examining variation among children of immigrants with different cultural background and different ancestral plough use but same environment.

The model is given by

$$y_{i,s,c} = \alpha_s + \beta \text{Plough}_c + \mathbf{X}_c^H \Gamma + \mathbf{X}_c^C \Pi + \mathbf{X}_i \Phi + \epsilon_{i,s,c}$$

where i denotes the daughter of an immigrant parent who currently lives in state s (USA or a Western European country) with country of ancestry c. The objective variable $y_{i,s,c}$ is an indicator variable, being one if the woman i takes part in the labour market. α_s describes country fixed effects, $\mathbf{X}_i$ captures individual level controls like age, marital status and education. $\mathbf{X}_c^H$ and $\mathbf{X}_c^C$ denote again historical and contemporary control variables.

The regression is estimated for unmarried and married women separately. Even though the effect is smaller than in the baseline model, the results are clear: cultural beliefs seem to be persistent over time, indicating a positive relationship between traditional plough use and beliefs about gender inequality.

To sum up, different farming practises cause different beliefs of the appropriate role of women in society. Ancestral plough use leads to lower female participation rates in the labour force and politics as well as fewer female firm owner-ships, indicating higher gender inequality. Even though the results must be read with caution, Boserup's theory holds true and gives some insight in why gender unequal beliefs are formed (Alesina et al., 2013).

2.2 The Baseline Model (Galor and Weil, 1996)

Our further review and research is based on a paper by Galor and Weil (1996). This model forms the basis for our review, with all other papers discussed building on the same basic mechanisms. The model introduces gender-based discrimination into an economic growth model by the gender wage gap. Measuring discrimination via the gender wage gap has two advantages: First, it is easy to quantify. Second, the gender wage gap is well reported empirically. The model is built on the framework of overlapping generations (OLG) where finitely-lived agents of two sexes couple in order to form households. We will discuss the model by Galor and Weil (1996) in detail. All papers reviewed in the following sections

will follow the benchmark model more or less and incorporate the mentioned elements in the same framework.

To analyze the effect of gender-based inequality on economic growth, Galor and Weil (1996) combine two well-known results from economic growth theory and family economics. The first result used in the paper is about the effect of population growth on the level of capital per worker, which is a standard result of most economic growth models. The interplay of population and economic growth has a long history and dates back to Malthus, who discussed the link between population growth and resource scarcity. However, it was Solow who explained the negative correlation between population growth and capital per worker due to capital dilution and who´s model became a standard model of economic growth theory Solow (1956). Since then, the effect of capital dilution is cited frequently to explain the negative coefficient on the rate of population growth in regressions of either the level or the growth rate of income.

The second result origins in family economics and discusses the relationship of fertility and (relative) wages. Becker (1960) suggested that fertility choice relies on an economic mechanism. To do so, he modelled children as a durable good which are part of the parent´s utility function. He showed that the effect of an increase in household income on fertility is dependant on whether the male or female wage rises. If the costs for children are fixed, an increase in household income results in an increase in the demand for children. Now assume all child rearing is done by the woman. On the one hand, a rise in male wage induces a pure income effect. Thus, the demand for children rises too. On the other hand, a rise in female wages does not only increase the household income but also opportunity costs for the woman and hence the price for children. This results in offsetting income and substitution effects in the demand for children. Historical data indicates that fertility declines as countries become richer. Therefore, one has to choose an utility function in Becker´s model, in which the substitution effect dominates the income effect. Moreover, it could be argued that the effect of an increase in women's *relative* wage is a decrease in fertility. This approach is also used by Galor and Weil (1996) in their model.

To link these two facts, it is argued that an increase in capital intensity translates into an increase in women´s wages. The reason behind this mechanism is the different factor endowment of the two sexes. As already discussed in section 2.1, men tend to have a comparative advantage in physical strength while this is not the case with mental strength. In other words, men and women have the same endowments of "brain" but different endowments of "brawn". As a country develops from pre-industrial to an industrialized economy, the rewards to brain versus brawn change in favor of the brain. As a result, the economy rewards the attribute more in which women have an advantage. This theory is also supported by data. To mention only one example, in the United States, full-time earnings of women increased by 46 to 67 percent of men's earnings over the period 1890-1988 due to industrialization and the referred higher demand for "brain" as for instance fine motor skills, in which women have an absolute and comparative advantage (Goldin, 1990).

Combining these pieces of the model generates a positive feedback as depicted in Figure 1: increase in capital per worker leads to a rise in female relative wages, and in turn closes the gender pay gap. Increasing female relative wages results in higher costs of having children and in turn reduces fertility. Decreasing fertility is related to lower population growth and causes an increase in the level of capital per worker which again

leads to a rise in female relative wages and so on. The element that captures gender-based inequality is women´s relative wages. Despite that, women's wages are not only a product of and causal factor in economic growth, women´s relative wages give a hint on how far an economy is in establishing equality and equity between the sexes. In summary, Galor and Weil (1996) use two important results of "standard" economic theory, namely the capital dilution effect by Solow and the fertility model by Becker, and link these with the "brain versus brawn" argument.

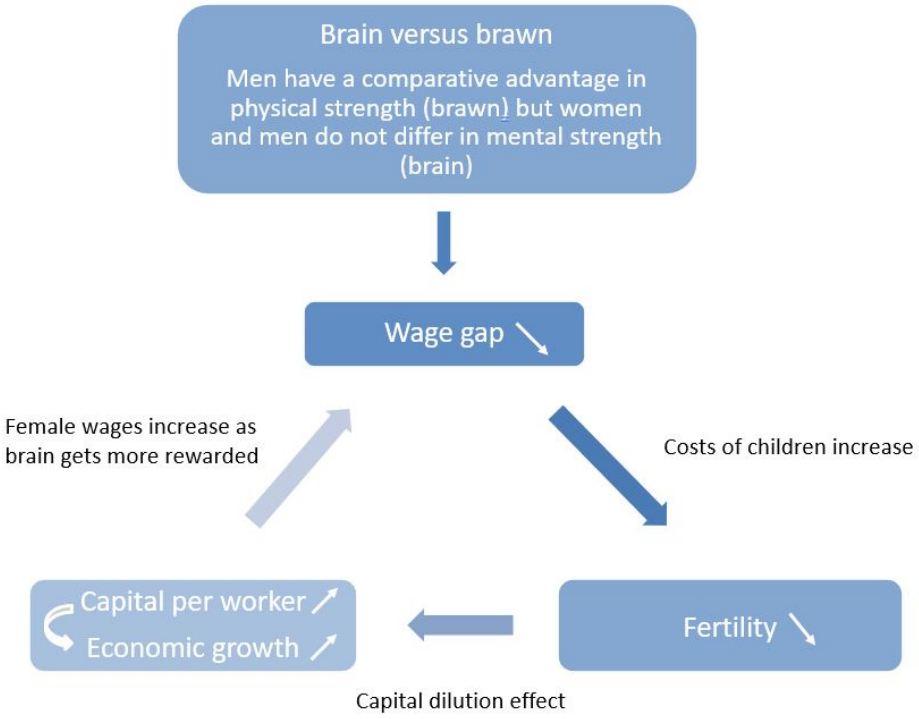

Abbildung 1: The feedback-loop mechanism between gender inequality (measured by the wage gap), fertility and economic growth (own illustration)

After we have discussed the crucial pieces of the model by Galor and Weil (1996), we want to take a closer look on the mathematics behind. The model describes an economy populated by overlapping generations of agents living for three periods. Agents can be either male or female. In the first period, agents are children and consume a fixed quantity of time from their parents. All children are identical and do not differ referring their sex. In the second period, the agents share their time between child-rearing labor market work, earning a wage. It is assumed that they do not consume in this period. Thus, couples can either spend their income on children or save it. In this period, men and women differ since it is assumed that they have different earning abilities. One has to take into account that all adults can supply physical or mental labour input on the market, but men and women differ in their endowments of these. While both sexes have equal quantities of mental input, women lack in physical labour supply causing the difference in earning wages. In the third and last period of life, agents do not work, but they consume their savings from the adulthood-period along with accrued interest regardless of their gender. The considered unit in the paper are couples. Women and men are born in couples,

they have joint consumption and utility. The capital stock in each period is equal to the aggregate savings in the previous period.

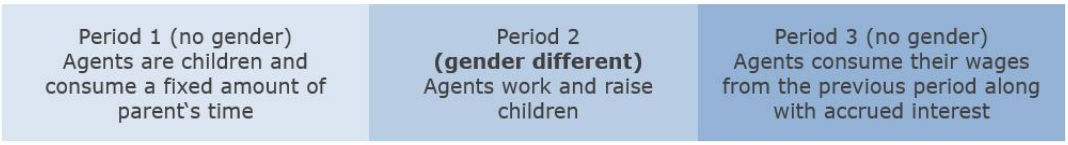

Abbildung 2: The OLG model in Galor and Weil (1996) (own illustration)

Production

Labour input factors are physical labour supply L^p, mental labour supply L^m and physical capital K. To incorporate the fact, that women and men supply the same amount of mental labour whereas women supply less physical labor, it is assumed that women are not endowed with physical strength at all. Men supply one unit of physical and one unit of mental labour, while women supply between zero and one unit of mental labour and zero physical labour.

Moreover, mental labour is relatively more rewarded than physical labour the more physical capital there is in an economy. This means physical capital complements mental labour and is incorporated in the production function by a CES-function, while physical labour is neither a substitute nor a complement for both of those inputs and is depicted by an additive term in the production function:

$$Y_t = a[\alpha K_t^\rho + (1-\alpha)(L_t^m)^\rho]^{\frac{1}{\rho}} + bL_t^p \tag{1}$$

with $a, b > 0$, $\alpha \in (0,1)$ and $\rho \in (-\infty, 1), \rho \neq 0$, which implies that the elasticity of substitution between physical capital and mental labour is smaller than 1 and implies the complementarity of those goods.

As only men supply physical labour, the number of couples is equal to L^p. The per-couple production function is thus

$$y_t = a[\alpha k_t^\rho + (1-\alpha)(m_t)^\rho]^{\frac{1}{\rho}} + b \tag{2}$$

with $k_t = K_t/L_t^p$ denoting per-couple capital and $m_t = L_t^m/L_t^p$ per-couple mental labour input at time t. All factor inputs are compensated according to their marginal products. I.e. the return of one unit of physical and mental labour input is

$$w_t^p = b \tag{3}$$

$$w_t^m = a(1-\alpha)m_t^{\rho-1} \times [\alpha k_t^\rho + (1-\alpha)(m_t)^\rho]^{\frac{1-\rho}{\rho}} \tag{4}$$

Equation (3) shows that wage for physical labour is constant, while the wage for mental labour depends on capital per couple and mental labour input. In particular, we can see in equation (4) that "mental" wage is increasing if physical capital per couple k_t increases. As already explained, men supply one unit of mental and physical input each and hence they earn the sum of equation (3) and (4). In contrast, women supply only "mental"

labour and earn the wage from equation (4).

Household Decision Problem

Couples gain utility from the number of offspring they have, denoted by n_t, and the consumption when they retire, denoted by c_{t+1}. Child quality is not taken into account. Raising children depends only on time. In addition, time cannot be spent on work and on raising children simultaneously. Hence, households decide on the number of children subject to the total amount of time they can spend on the market or on child-raising. The opportunity costs of having children are equal to foregone wages. Consequently, the cost for raising children are higher for men than for women and can lead to specialization including high wage gaps.

Depending on whether households have children or not the household income differs. A woman without children can spend 100 percent of her time on the labour market and will earn a wage of w_t^m for that. A man without children can supply physical and mental labour and will hence earn $w_t^m + w_t^p$. Therefore, households without children have a total income of $w_t^p + 2w_t^m$, i.e. two units of the wage for mental labour, which both spouses earn, and one unit of wage for physical labour, which only the male spouse earns.

Let z express the fraction of time one parent needs to raise one child. Remember, that parents can not do child rearing and labour market work at the same time, thus z is the fraction of time a parent can not supply their labour on the market and in turn will forego wages. As costs of children are considered to be foregone wages, we can estimate the marginal costs of children for both genders directly. If the woman raises the child, the marginal cost of one child is $z \cdot w_t^m$. Respectively the marginal cost of one child raised by a man is $z \cdot (w_t^m + w_t^p)$. Each spouse of the couple is endowed with one time unit. Therefore, if $zn_t < 1$, only the wife will spend time on raising the children. This is because men earn more than women and hence the cost for having children is lower for women than for men. It is therefore a rational decision for the couple to let the mother raise the children instead of the father. If $zn_t > 1$ both partners of the couple have to devote time to raising children, the wife full- and the husband part-time.

Note that the couple does not consume in period t. Consequently, the income is split between expenditures on children and savings s_t for the next period. Those savings accrued with interest will be consumed in period $t+1$. As a result, couples derive intertemporal utility from having children in period t or from future consumption in period $t+1$. In summary, the household faces the following maximization problem, deciding on the number of children and on future consumption

$$\max u_t = \gamma \ln n_t + (1-\gamma) \ln c_{t+1} \tag{5}$$

$$\text{s.t. } w_t^m z n_t + s_t \leq w_t^p + 2w_t^m \qquad \text{if } zn_t \leq 1 \tag{6}$$

$$w_t^m + (w_t^p + w_t^m)(zn_t - 1) + s_t \leq w_t^p + 2w_t^m \qquad \text{if } zn_t \geq 1$$

$$c_{t+1} = s_t(1 + r_{t+1}) \tag{7}$$

where γ is the utility weight of having children. Having children and saving are complementary goods, thus $1-\gamma$ is the utility weight regarding savings.

The second restriction, i.e. equation (7) follows from the fact that couples simply consume their savings from adulthood in their retirement. Equation (6) represents the budget constraint. In both cases, the right side of equation (6) is total income. The left

side depends on whether both parents need to spend time on child rearing or if the mother's time is enough. As already explained, both partners are endowed with one time unit. Thus, if the time spent on child rearing is smaller than one, i.e. $zn_t \leq 1$, only the mother will raise the couple´s offspring. The associated costs for having children are the foregone wages of the mother, i.e. $w_t^m \cdot zn_z$.

Vice versa, if child rearing time exceeds the total endowment of one spouse, i.e. $zn_t \geq 1$, both spouses need to spend time on child rearing. Hence, the mother will spend her whole wage on child rearing, i.e. w_t^m and the father will spend the remainder part, i.e. $zn_t - 1$, of his wage $w_t^p + w_t^m$. In both cases, the spending on children and savings, i.e. the left side of the budget constraint, is not allowed to exceed the total income of the couple, i.e. the right side of the budget constraint in equation (6).

The budget constraint is depicted in figure 3. On the y-axis the spending on children in terms of time is denoted. The maximum here is 2, this is the case of both parents spend all their time on child rearing since each parent has an total time endowment of 1. The x-axis depicts the other good, namely savings, in terms of income. The maximum income the couple has is $w_t^p + 2w_t^m$. The feasible points for any children-saving pair are given by the convex set defined by the inequalities $zn_t \leq 2$ and $s_t \leq w_t^p + 2w_t^m$.

The budget constraint has a kinked shape, as the marginal costs of having children depends whether one parent or both parents are needed for child rearing. Remember, that if one parent is enough, i.e. $zn_t \leq 1$, then the marginal costs of children are foregone female wages $-w_t^m$. Hence, the slope is given by $-1/w_m$ up to the point where $zn_t = 1$. For $zn_t > 1$ we need to take a look on the second case of the budget restriction. Rearranging terms such that zn_t is on the left side, we find $-1/(w^m + w^p)$ to be the slope.
The utility function is concave. Thus, the optimal pair of number of children and savings

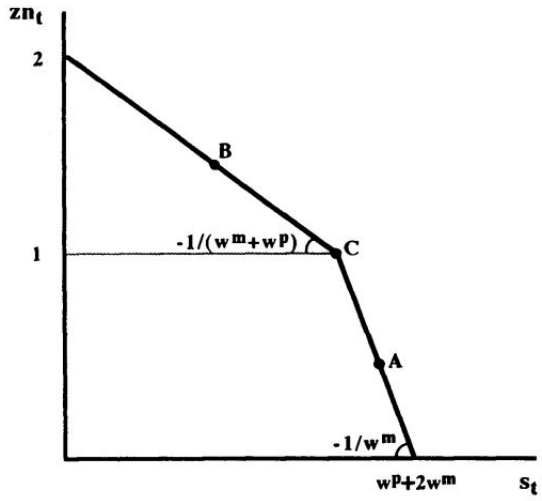

Abbildung 3: Budget constraint of a household in the baseline model and three possible optima A, B, C (Galor and Weil (1996); Figure 1, page 329)

will be attained at the edges of the convex set of feasible points. Depending on where the indifference curve is tangent to the budget constraint, the optimum is A, B or C. In A, a woman will work part-time and raise the children part-time while the man works full-time. This follows from the fact that the lower part of the budget constraint refers

to the case where $zn_t < 1$. Respectively in point B we face a situation where the woman raises the children full-time but also the man will raise the children part-time and work part-time. If neither of the conditions hold, i.e. the optimum is in the corner at point C, then we have complete specialization, which means $zn_t = 1$. In other words, the woman raises the children full-time and the man works full-time.

To derive the analytical solution to the household decision problem, we use the Karush-Kuhn-Tucker (KKT) theorem for optimization with equality and inequality constraints. We first solve the problem for the case $zn_t \leq 1$. First we use the equation (7) and substitute c_{t+1} with $s_t(1+r_t)$ in the utility function. Then, the (extended) Lagrange function is given by

$$\mathcal{L} = \gamma \ln n_t + (1-\gamma)\ln\left(s_t(1+r_t)\right) + \lambda(w_t^m z n_t + s_t - w_t^p - 2w_t^m)$$

Taking derivations after the decision variables n_t and s_t yields the KKT-system

$$\mathcal{L}_{n_t} = \gamma\frac{1}{n_t} + \lambda w_t^m z = 0 \tag{8}$$

$$\mathcal{L}_{s_t} = (1-\gamma)\frac{1}{s_t} + \lambda = 0 \tag{9}$$

$$\lambda(w_t^m z n_t + s_t - w_t^p - 2w_t^m) = 0 \tag{10}$$

$$w_t^m z n_t + s_t - w_t^p - 2w_t^m \leq 0 \tag{11}$$

$$\lambda \geq 0 \tag{12}$$

From equation (9) we get that $\lambda = -(1-\gamma)\frac{1}{s_t}$. Plugging this identity in equation (8) we get

$$\begin{aligned} \gamma\frac{1}{n_t} - (1-\gamma)\frac{1}{s_t}\cdot w_t^m z &= 0 \\ \Rightarrow n_t &= \frac{\gamma s_t}{(1-\gamma)w_t^m z} \end{aligned}$$

Now we have λ and n_t expressed in terms of s_t. Inserting these identities in equation (10) and rearranging terms, we can derive the optimal level of savings

$$\begin{aligned} w_t^m z\frac{\gamma s_t}{(1-\gamma)w_t^m z} + s_t &= w_t^p + 2w_t^m \\ \Leftrightarrow (\frac{\gamma}{1-\gamma} + 1)s_t &= w_t^p + 2w_t^m \\ \Rightarrow s_t &= (1-\gamma)(w_t^p + 2w_t^m) \\ \Rightarrow n_t &= \frac{\gamma(w_t^p + 2w_t^m)}{w_t^m z} \end{aligned} \tag{13}$$

Note, that we divide equation (10) by λ as the case $\lambda = 0$ can be excluded. If λ would be equal to zero, then we would have $\gamma\frac{1}{n_t} = 0$ which is a contradiction as $\gamma = 0$ can be excluded since the model would not be well-defined in that case.

However, as we are interested in time spent on children, we get from equation (13) by multiplying with z

$$zn_t = \gamma(2 + \frac{w_t^p}{w_t^p}), \qquad \text{for } zn_t \leq 1 \Leftrightarrow \gamma(2 + \frac{w_t^p}{w_t^p}) \leq 1$$

.

The latter case, $zn_t > 1$ is solved analogously. The corresponding KKT-system is

$$\mathcal{L} = \gamma \ln n_t + (1-\gamma) \ln (s_t(1+r_t)) + \lambda(w_t^m + (w_t^p + w_t^m)(zn_t - 1) + s_t - w_t^p - 2w_t^m)$$

$$\mathcal{L}_{n_t} = \gamma \frac{1}{n_t} + \lambda(w_t^m + w_t^p)z = 0$$

$$\mathcal{L}_{s_t} = (1-\gamma)\frac{1}{s_t} + \lambda = 0$$

$$\lambda(w_t^m + (w_t^p + w_t^m)(zn_t - 1) + s_t - w_t^p - 2w_t^m) = 0$$

$$w_t^m + (w_t^p + w_t^m)(zn_t - 1) + s_t - w_t^p - 2w_t^m \leq 0$$

$$\lambda \geq 0$$

Following the same steps as before, we obtain the optimal level for savings and number of children:

$$s_t = 2(1-\gamma)(w_t^p + w_t^m) \tag{14}$$

$$n_t = \frac{2\gamma}{z} \tag{15}$$

$$\Rightarrow zn_t = 2\gamma, \qquad \text{for } zn_t \geq 1 \Leftrightarrow 2\gamma \geq 1 \tag{16}$$

In summary, we find the following optimal amount of time spent on raising children

$$zn_t = \begin{cases} \gamma[2 + (w_t^p/w_t^m)], & \gamma[2 + (w_t^p/w_t^m)] \leq 1 \\ 2\gamma, & 2\gamma > 1 \\ 1, & \text{otherwise} \end{cases} . \tag{17}$$

Again, in equation (17) we can see that there is only one case, namely the first one, where women will devote some part of their time to the labour market. This is because individuals are endowed with one unit of time and the first case is the only one where time spent with children is less than one. What we can see in equation (17) as well, is that for sufficiently high relative wages of physical labour w_t^p/w_t^m or for high values of utility weights for children γ, women will always raise children full-time. In contrast, for high relative wages of mental labour, women will join the labour force. Moreover, as relative wages of mental labour increase, women will increase gradually their time on the labour market as well. The reason behind such an increase would be economic development in which "brain" gets rewarded more than "brawn" as explained before.

However, the two latter cases of equation (17) do not contain any form of relative wage at all and are constant functions. In these cases, women will always spend all of

their time with child-rearing. Moreover for $\gamma > 1/2$, zn_t will always be greater or equal than one which refers to the situation where women do not join the labour force at all. Hence, the authors exclude this case and assume $\gamma < 1/2$ in order to guarantee that there are situations in which women also supply work on the market. As a result, the second case of equation (17), i.e. $zn_t = 2\gamma$ for $2\gamma > 1$ will never hold true. Consequently, the first or third case of equation (17) will always hold which is equivalent to $zn_t \leq 1$. Another consequence of this assumption is that men will work always full-time in the model since one parent has enough time to raise the children and per assumption this parent will be the mother as her foregone wages, which are the price for having children, are lower.

In summary, taking into account the assumption $\gamma < 1/2$ the optimal time spent on raising children is

$$zn_t = \min\{1, \gamma[2 + (w_t^p/w_t^m)]\} \tag{18}$$

and the optimal levels for future consumption or equivalently for savings are

$$s_t = \begin{cases} (1-\gamma)[w_t^p + 2w_t^m], & zn_t \leq 1 \\ w_t^p + w_t^m & zn_t = 1 \end{cases} . \tag{19}$$

We have already discussed the interdependence of relative wages and female labour force participation. The next step is to combine the results from the production side and from the household side in order to analyze the impact of capital on fertility, or equivalently on time devoted to child raising.

The development of n_t and k_t

The aim of the paper by Galor and Weil (1996) is to examine the interplay between gender-based inequality and economic development. As discussed before, the increase in capital per worker is important for economic growth. Moreover, the increase in capital per worker is the reason why "brain" wins over "brawn" over time and hence closes the gender wage gap. Therefore, we analyze how fertility and capital per worker evolve over time.

Remember that the total number of couples L_t in period t is equal to L_t^p as only men supply physical labour and the amount of mental labour per couple is denoted by m_t. Both partners in the couple supply mental labour, males supply one unit and females $1 - zn_t$ units. Thus, the total amount of supplied mental labour by a couple is $L_t^m + (1 - zn_t)L_t^m = (2 - zn_t)L_t^m$. Combining these pieces, we obtain

$$m_t = \frac{L_t^m}{L_t^p} = \frac{L_t(2 - zn_t)}{L_t} = 2 - zn_t \tag{20}$$

Next, we use the identities defining the wages in equation (3) and (4) and insert them in equation (18). Then, the optimum zn_t can be rewritten as

$$zn_t = \min\{1, \gamma\left[2 + b/\left(a(1-\alpha) \times (2 - zn_t)^{\rho-1}[\alpha k_t^\rho + (1-\alpha) \times (2 - zn_t)^\rho]^{\frac{1-\rho}{\rho}}\right)\right]\} \tag{21}$$

Now, we have the optimal amount of time to be spend on child rearing expressed in terms of capital per couple and parameters. However, we can see that the right side depends on

zn_t. Hence, we need to find a function ψ such that $zn_t = \psi(k_t)$. To overcome this issue, we will apply the implicit function theorem.

Let $G(k_t, zn_t)$ be a function defined as

$$G(k_t, zn_t) := zn_t - \gamma \left[2 + b/\left(a(1-\alpha) \times (2 - zn_t)^{\rho-1}[\alpha k_t^{\rho} + (1-\alpha) \times (2 - zn_t)^{\rho}]^{\frac{1-\rho}{\rho}}\right)\right]$$

The subtrahend of $G(k_t, zn_t)$ is the optimal level for zn_t and the solution to our optimization problem, see equation (21). The optimum exists and hence, there exists a pair (zn_t, k_t) such that $G(k_t, zn_t) = 0$. Additionally, the derivative of G with respect to n_t, i.e. $\frac{\partial G(k_t, zn_t)}{\partial n_t}$ is continuous for $k_t \geq 0$ (see Appendix A). Therefore, we are allowed to use the implicit function theorem and know that there exists a unique, differentiable function ψ, such that

$$G(k_t, zn_t) = 0 \Leftrightarrow zn_t = \psi(k_t)$$

As a result, we can rewrite equation (21) and see that

$$zn_t = \min\{1, \psi(k_t)\}, \qquad \forall k_t \geq 0 \tag{22}$$

From the implicit function theorem, or better said from a corollary of it (the theorem of the inverse function), we know that there exists a level of per-couple capital k^* such that

$$k^* = \psi^{-1}(1)$$

$G(k_t, zn_t)$ is decreasing in k_t, thus $\psi'(k_t) < 0$, which means that time devoted to child rearing is decreasing with regard to capital per couple. As a result, we can conclude that

$$zn_t = \begin{cases} \psi(k_t), & k_t \geq k^* \\ 1, & k_t \leq k^* \end{cases} \tag{23}$$

with $\psi(k_t) \in (0, 1] \forall k_t \geq k^*$. Therefore, k^* can be interpreted as the highest level of capital per couple for which women raise children full-time.
We have shown mathematically that an increase in relative wage of mental labour will lower the amount of time parents devote to child rearing. The same effect is proven for an increase in capital per couple. Furthermore, we have seen that a rise in wages is associated with an increase in female labour force participation.

The next step is to investigate the evolution of capital per worker over time. First, the capital stock in period $t+1$ is equal to the savings of the previous period period t, i.e.

$$K_{t+1} = L_t s_t. \tag{24}$$

Second, the total number of working-age households at time $t+1$ is given by

$$L_{t+1} = n_t L_t \tag{25}$$

As a consequence, using the optimal solutions for s_t and n_t and apllying basic rearrangements, as well as the definition of k^*, the per-couple capital stock is given by

$$k_{t+1} = \frac{K_{t+1}}{L_{t+1}^p} = \frac{s_t L_t}{n_t L_t} = \frac{s_t}{n_t} = \tag{26}$$

$$= \begin{cases} = \frac{(1-\gamma)(w_t^p + 2w_t^m)}{\frac{\gamma(w_t^p + 2w_t^m)}{w_t^m z}} = z(\frac{1-\gamma}{\gamma})w_t^m, & k_t \geq k^* \\ \\ = \frac{w_t^p + w_t^m}{1/z} = z(w_t^p + w_t^m) & k_t \leq k^* \end{cases}.$$

We want to rewrite k_{t+1} like we did with zn_t by defining it as a function of k_t. To do so, we need to reformulate the wages from the production side in equation (4) and equation (3).

Wage for physical labour is constant, i.e. $w_t^p = b$ (equ. (3)) and hence we do not have to change anything here. The wage for mental labour depends on m_t which we defined in equation (20). We use equation (23) of optimal time spent on child rearing and get for the case $k_t \geq k^*$

$$\begin{aligned} w_t^m &= a(1-\alpha)m_t^{\rho-1} \cdot [\alpha k_t^\rho + (1-\alpha)(m_t)^\rho]^{\frac{1-\rho}{\rho}} \\ &= a(1-\alpha)(2-\psi(k_t))^{\rho-1} \cdot [\alpha k_t^\rho + (1-\alpha)(2-\psi(k_t))^\rho]^{\frac{1-\rho}{\rho}} \\ &= a(1-\alpha)(2-\psi(k_t))^{-1(-\rho+1)} \cdot [\alpha k_t^\rho + (1-\alpha)(2-\psi(k_t))^\rho]^{\frac{1-\rho}{\rho}} \\ &= a(1-\alpha)\frac{[\alpha k_t^\rho + (1-\alpha)(2-\psi(k_t))^\rho]^{\frac{1-\rho}{\rho}}}{(2-\psi(k_t))^{1-\rho}} \end{aligned}$$

Using this reformulation of w_t^m and inserting it in (26) we obtain

$$k_{t+1} = \begin{cases} za(1-\alpha)(\frac{1-\gamma}{\gamma})\frac{\{\alpha k_t^\rho + (1-\alpha)[2-\psi(k_t)]^\rho\}^{\frac{1-\rho}{\rho}}}{[2-\psi(k_t)]^{1-\rho}}, & k_t \geq k^* \\ z\{b + a(1-\alpha)[\alpha k_t^\rho + (1-\alpha)]^{\frac{1-\rho}{\rho}}\} & k_t \leq k^* \end{cases} \tag{27}$$

Now k_{t+1} is only dependent on parameters and k_t. Thus, we can define a function $k_{t+1} = \phi(k_t)$ where $\phi(k_t)$ is equal to equation (27) and the initial per-couple capital stock k_0 is historically given.

In order to obtain further insights, we take the derivations of $\phi(k_t)$ with respect to k_t and find

$$\phi'(k_t) = \begin{cases} \alpha z a(1-\alpha)(1-\rho)\frac{1-\gamma}{\gamma}k_t^{\rho-1}\frac{2-\psi(k_t)+k_t\psi'(k_t)}{[2-\psi'(k_t)]^2\{\alpha k_t^\rho+(1-\alpha)[2-\psi(k_t)]^\rho\}^{2-1/\rho}} & k_t \geq k^* \\ \alpha z a(1-\alpha)(1-\rho)k_t^{\rho-1}[\alpha k_t^\rho + (1-\alpha]^{1/\rho-2} & k_t \leq k^* \end{cases} \tag{28}$$

Since $\psi(k_t)$ defines time devoted to child raising and is hence always smaller or equal to one and the parameters α and ρ are smaller than one as well, the derivation will be always positive, indicating that capital is growing over time.

In summary, we find that $\psi(k_t)$ is decreasing and $\phi(k_t)$ is increasing with respect to k_t. This means, that fertility will in fact decrease as capital per worker will increase over time, proving the feedback loop between higher capital per worker, higher female relative wages, lower fertility and back to higher capital per worker which is exactly the effect of interest we wanted to show.

To summarize, we refer to Galor and Weil (1996) as our baseline model. Galor and Weil (1996) present a general equilibrium model with overlapping generations that incorporates two effects: First, the positive effect of capital accumulation on female relative wages; remember, that capital accumulation channels the comparative advantage of "brain" versus "brawn" in production and thus stimulates female labour force participation and wages. Second, the negative effect of women´s relative wages on fertility. These two effects generate a positive feedback from low fertility to higher capital per worker and output per

couple, higher relative female wages and back to lower fertility. Female relative wages or equivalently the wage gap are the measure of gender-based discrimination in this model. Therefore, the model does not only offer insights into linking fertility and economic growth but also the consequences of gender-based discrimination or vice versa the positive effect of gender equality and female empowerment. The described feedback loop is crucial for explaining the interplay between gender-based inequality and economic performance and will be referred to in the following literature review.

2.3 Female empowerment in the private sphere

In the baseline model of the last section gender-based inequality was manifested in the wage gap. It was modelled as the product of labour market structures and the different physical characteristics of women and men. Despite that, it was assumed that the household has one utility function collectively. However, what if women and men have different preferences? Which preference will "win" in the fight of two spouses?

In this section, we want to consider female empowerment within the household and its effects on the household as well as economic development. First of all, we want to emphasize one point: Female empowerment is a valuable goal in and of itself. To cite Kofi Annan, the Secretary General of the United Nations in the United Nation's 2005 report on the Millennium Development Goals (MDG): "The full participation of women to all levels of decision-making is a basic human right." (of Public Information, 2005). However, there are also quantifiable effects of female empowerment on the economy. Several studies have investigated the role of women in the family outcome and there is clear evidence of a correlation between female empowerment and the family outcome. Especially the aspect of education should be pointed out in this context. In fact, various papers, models and empirical findings suggest that there is a positive link between female empowerment and human capital.

Before we start with the literature review, we want to define some terms which will be often used in the context of intra-household bargaining. When modelling household behavior, there are different approaches. The following descriptions are based on a paper by Donni and Ponthieux (2011).

- Unitary approach: the household is treated as an entity and the individual behavior model is simply applied to the household. "Individual behavior model" describes a single agent, who rationally maximizes it's utility under a common budget constraint. The income of all household members is pooled and one utility function for all members is applied.
  This approach fits if the family is led by "head" of the family who takes over all decisions or all household members are identical. However, empirically there is not much evidence for using an unitary approach. In contrast, there is much evidence that goods, decisions and demand are very unequally distributed within the family.

- Non-unitary approach: The household is not modelled like an individual but as a composition of several individuals. The well-being of an individual depends on his or her own consumption as well as the consumption of others. There are several reasons why the consumption of other members has an influence on an agent´s own consumption. For instance, goods can be rival. Another thing to take into account

is that agents can behave altruistically, i.e. they care for the other members and hence for their consumption. Agents can either pool their income and share it, or they face individual budget constraints. In contrast to the unitary approach, the model does *not* consist of one common utility function and one common budget constraint.

- Cooperative model: the household members act as a unit in order to maximize the welfare of all members.

- Non-cooperative model: the household members make decisions in order to maximize their own utility. The ability to do that depends on their own bargaining power within the household.

  The terms of non-unitary approach and non-cooperative model are related and often used synonymous.

Having defined these concepts, we proceed with the literature review.

2.3.1 Gender equity and the escape from poverty (Prettner and Strulik, 2016)

Prettner and Strulik (2016) contribute to the literature by showing that female empowerment has a striking impact on the onset and speed of the demographic transition and the take-off to modern growth if women lean towards child quality in the Beckerian child quantity–quality trade-off, while men lean towards child quantity. The authors consider gender-specific preferences regarding their offspring:

1. Quantity-quality preference differential: fathers are assumed to want a large number of children, while mothers emphasize more education per child.

2. Daughter–son education preference differential: both parents prefer education of boys more than of girls

These two assumption on gender-specific preferences are in line with data for less developed countries. Moreover, the authors analyze the impact of female bargaining power within the household. To do so, they set up a non-cooperative household model. This means, they consider a household utility function u_t^h that captures gender-specific differences in tastes with respect to consumption c, fertility n, and education e of daughters and sons as well as (potentially endogenous) gender-specific differences in the bargaining power of the male θ and female spouse $1-\theta$. The household utility function U_t takes the form

$$U_t = \theta u_{t,m} + (1-\theta) u_{t,f} \tag{29}$$

where $u_{t,m}$ and $u_{t,f}$ denote the male respectively female utility functions. As in the baseline-model by Galor and Weil (1996), the costs of children are measured by foregone wages. As in the Beckerian framework, costs of children are measured by forgone wages due to the time-loss on the labour market. Let w_t denote the wage rate per unit of effective labour. Individuals supply their labour on the market and their efficiency depends positively on human capital $h_{t,i}$ and negatively on the time requirements due

to child rearing, which are denoted by ψ_i. Therefore, the total income of a household is $w_t[(1-\psi_m n_t)h_{t,m} + (1-\psi_f n_t)h_{t,f}]$. Households can spend their income on consumption c_t and expenditures on their children's education. Hence, the budget constraint is given by

$$w_t[(1-\psi_m n_t)h_{t,m} + (1-\psi_f n_t)h_{t,f})] = (e_{t,m} + e_{t,f})\frac{n_t}{2} + c_{t,m} + c_{t,f} \tag{30}$$

The household maximizes the utility-function given the budget constraint and derives optimal levels for (gender-specific) consumption, education of sons, education of girls and fertility. The production sector involves linear technological progress and human capital accumulation. In contrast to the baseline-model, the production function is not specified along gender-lines, but the utility function does by sex-specific variables and introducing different utility weights for daughters and sons. Additionally, female empowerment is endogenous in the model by incorporating that the female bargaining power of the next period depends on the relative income of women versus men, i.e. the wage gap. This creates a feedback-loop, where female empowerment is the result as well as the cause of (economic) development.

The authors show that development goes through a minimum of two and a maximum of three stages. In the first stage, neither boys nor girls get educated. If education of sons is more valued compared to the education of girls, there is a second stage in which only boys get educated. At the third and final stage daughters and sons get educated. Another striking result, is that if preferences regarding fertility and education among the spouses differ, female empowerment lowers the threshold levels for education of boys and girls and thus stimulates economic development. This is due to the fact, that women prioritize quality over quantity of children and thus increasing ceteris paribus their bargaining power leads to faster education of their children. Interestingly, this means that female empowerment has a positive inter-temporal effect on men too as it increases education of sons confirming that both sexes are better off by enforcing gender-equity.

Nevertheless, the authors show that if the desires on fertility and education of the children are the same for both parents, then a larger female bargaining power has no impact on the take-off of male or female education implying that the development is largely driven by the extent to which female preferences differ from male ones. However, this does not mean that female empowerment has no positive effects in this case, but that there is no effect on the timing of the take-off of the transition. Moreover, this result explains why studies can fail in explaining the effects of female empowerment. Especially in western countries, the preferences among the parents do not differ much while they do in developing countries according to data from the Demographic and Health Surveys. Furthermore, it is shown that a preference on education for boys over girls drives development, as it decreases the threshold levels for education of boys and girls while this is not true vice versa. Intuitively the reason behind is that if there is a daughter–son education preference differential, then the take-off of male education occurs before the take-off of female education. Consequently, household income is increasing earlier, fertility is decreasing and development is happening faster which in turn has positive effects on education of girls too. These results suggest that especially in countries, where both differentials, i.e. quantity-quality preference differential as well as the daughter–son education preference differential are large, female empowerment would be a successful strategy to enhance economic development and escape from the poverty trap.

In section 3 we will go into further details as we will rely on the model by Prettner and Strulik (2016) to derive our own model.

2.3.2 Does female empowerment promote economic development? (Doepke and Tertilt, 2019)

In contrast to Prettner and Strulik (2016), who put emphasis on the preference differential, the role of division of labor in household production, the so-called "specialisation hypothesis" is discussed by Doepke and Tertilt (2019). In their paper, gender inequality is measured by the wage gap as in the model by Galor and Weil (1996, see section 2.2). However, the spouses act non-cooperatively and the wage gap leads to household specialisation. By introducing gender-specific transfers, female empowerment is promoted.

Various female empowerment policies go beyond gender equality, and explicitly favor giving resources to women instead of men. For instance, many family cash transfer programs such as "Oportunidades" in Mexico pay out benefits to mothers instead of fathers. Further, in 2008 the World Bank committed $100 million in credit lines specifically to female entrepreneurs. Today, the majority of micro credit programs around the world are available exclusively to women. The reason behind is not only to overcome higher barriers for women in accessing financial markets, but founded on the belief that with money in the hands of women, expenditures on children increase and that more spending on children promotes human capital accumulation as shown in Prettner and Strulik (2016).

However, what if the studied economy does not rely on human capital like in advanced countries but physical capital accumulation is the engine of growth? Doepke and Tertilt (2019) shed light on that question by establishing a non-cooperative model of the household like in the model by Prettner and Strulik (2016). Despite the similarities between the papers, in Doepke and Tertilt (2019) the gender wage gap leads to endogenous household specialization. It is assumed, that the partner with less income specializes in household production as for example in child rearing. Additionally, there is no preference differential, i.e. women and men value public and private goods, including number and education of children, in the same way. The results of the two models coincide, suggesting that female empowerment leads to an increase in human capital. To point out here the difference to the last model once again: parents do not differ in their preferences but in the tasks they take over in the household production.

Furthermore, the effect of female bargaining power is analyzed in terms of mandated transfers women receive. In other words, the income of women is improved by transfers and cause a better bargaining position at home. In the household decision problem, both spouses maximize their utility functions which depend on the consumption of public and private goods. While private goods are gender-specific, the public goods represent all final or intermediate goods that the spouses jointly care about, such as shelter or goods related to children. Moreover, they are produced within the household using household production functions that combine purchased inputs and time. The spouses differ in their market wages and both split their time between household production and time spent on the labour market.

Another important assumption of the model is that each spouse has to combine her or his labour with her or his own good contribution to produce a public good. This means, that spouse A can not put effort into a task and spouse B purchases the goods which are

needed for that task. For example, let us assume that the wife is responsible for preparing the family's dinner. Then she has to buy all ingredients that are needed for the dinner and she has to spend time on cooking the dinner and her spouse can not contribute to neither of both since we assume that time and input can not be separated. This assumption leads to gender-specific budget constraints as they consume different goods depending on the task they are responsible for in the household. The fact, that women and men have different purchasing decisions are also widely investigated in marketing literature. For instance, Wolgast (1958) finds that wives are more often responsible for household goods, while men are more likely to be in charge for car decisions.

The utility functions are symmetric between men (m) and women (f) and each spouse takes the action of the other spouse as given. The utility function is given by

$$u_g = \log(c_g) + \int_0^1 \log(C_i)di$$

where $g \in f, m$ denotes the gender, c_g private-good consumption and C_i a continuum of public goods for the household, indexed from 0 to 1.

Let $C_{g,i}$ denote intermediate goods an agent of sex g needs to produce a public good C_i. Since time requirement $T_{g,i}$ and input $C_{g,i}$ for producing a good C_i can not be separated, each spouse has own, gender-specific constraints as already mentioned. Additionally, these gender-specific budget constraints include the gender wage gap, as each spouse receives the wage w_g and the gender-specific transfer x_g. The household faces the following budget and time constraints

$$\begin{aligned} C_i &= C_{m,i} + C_{f,i}, \forall i \\ C_{g,i} &= E_{g,i}^{1-\alpha(i)} T_{g,i}^{\alpha(i)} \forall i, \qquad (31) \\ c_g + \int_0^1 E_{g,i} di &= w_g(1 - T_g) + x_g \qquad (32) \\ \int_0^1 T_{g,i} di &= T_g \end{aligned}$$

where $E_{g,i}$ denotes good spending on good i by spouse g. Goods are produced according to a Cobb-Douglas production technology with $\alpha(i)$ denoting the production elasticity of time and respectively $1 - \alpha(i)$ of the intermediate good for producing good i. Equation (32) represents the budget constraint of spouse g. x_g denotes wealth, for example the initial endowment of spouse g or a lump-sum transfer. Since all individuals are endowed with one time unit, each spouse can supply $1 - T_g$ of her or his time on the labour market in order to earn the wage w_g if T_g is the total amount of time the spouse spends on public goods production.

The result of the utility maximization problem shows that if a gender wage gap exists, there is an unique equilibrium in which the spouse with less wage, per assumption the wife, specializes in producing time-intensive goods, including child rearing and human capital, while the better earning spouse, per assumption the husband, specializes in producing goods-intensive goods, for example housing.

Additionally, if the gender-specific transfer rises or equivalently if the policy makers introduce a gender-specific transfer, the results show that female consumption increases

while male consumption decreases. In other words, a lump-sum transfer to women will increase investments in human capital and spending on children. It is noteworthy, that the positive effect of female empowerment on human capital is the product of household specialisation and not of gender-specific preferences, i.e. the quantity-quality preference differential of men and women.

Furthermore, it is shown that a voluntary transfer of the husband to the wife does not change the equilibrium compared to the equilibrium with a mandated transfer. However, as female consumption rises if there is any transfer towards the wife, there exist wage gaps for which the husband does not want to provide any transfers to his wife voluntarily. Another extension of the model that is discussed is the provision of goods of one spouse that are used by the other spouse. In other words, the assumption that action and inputs can not be separated is relaxed and the richer spouse can buy that inputs which are needed by the poorer spouse. A typical example would be that the man does grocery shopping and the wife cooks the meals. The result of this extension are in line with the previous results, showing that household specialisation is happening in any case.

In summary, Doepke and Tertilt (2019) set up a model with symmetric preferences towards child quantity, child quality and consumption goods in contrast to the previous model by Prettner and Strulik (2016). The spouses face different budget constraints as they specialise in different household‘s good production. While females specialise in time-intensive tasks like child rearing and educating, men tend to take over good-intensive tasks as they earn better than their wives. The result is that empowering women, in this case via wealth transfers, stimulates spending on children and human capital accumulation. However, if the society of interest has low human capital and is primarily physical capital driven, then empowering women with wealth transfers will put harm on the economy since female consumption increases and vise versa savings, which accumulate physical capital, decrease. With being this said, we can answer the initial question: Empowering women promotes economic development if the considered economy has human capital as the key driver.

2.3.3 Would empowering women initiate the demographic transition in least developed countries? (De la Croix and Vander Donckt, 2010)

Last, we want to present a paper that does not only account for the gender wage gap, but includes different dimensions of gender inequality. De la Croix and Vander Donckt (2010) consider a variety of gender gaps to examine the effect of change in female bargaining power onto the economy and related variables as human capital. They distinguish explicitly between the survival gap, the wage gap, the social and institutional gap, and the educational gap. The different measures are based on World Economic Forum's Global Gender Gap (GGG) index, which provides a concise measure of gender equality. The educational gap as well as fertility are endogenous in the model, while the first three gender-based concepts are exogenous. By including different gender gaps, the authors want to examine which dimension of gender inequality is crucial for growth.

Note that the survival gap could also be interpreted as a health gap. We will take a closer look on gender health inequalities in section 2.5. Nevertheless, while women tend to have a higher life expectancy in most parts of the world, there is a special threat to women during pregnancy and giving birth. Moreover, there are gender-biased prenatal

sex selective practices favoring boys over girls in some parts of the world.

As in the baseline model, they set up a bargaining household OLG model. Agents are assumed to live for two periods, one in which they are children and just accumulate human capital and a second period which is adulthood, in which they mate, consume and raise children. Additionally, agents are identical except for the fact that women differ from men regarding child-rearing time.

Again, a non-unitary household approach is used. Thus, the spouses have individual utility functions. The utility function of an adult male (m) or female (f) individual is given by

$$V_t^i = u(c_t^i) + b(n_t)n_t \frac{(1-\theta_{t+1})V_{t+1}^f + \theta_{t+1}V_{t+1}^m}{2}, \quad i = f, m \tag{33}$$

θ_t measures male bargaining power, $u(c_t^i)$ captures the utility gained through consumption and $b(n_t)$ characterizes some degree of altruism towards children. The weights θ_t are assumed to depend on the earning abilities of the spouses which in turn depend on spouses' relative levels of human capital.

As can be seen in the rightmost term in equation (33), the utility of an individual in period t depends on the utility of an individual in period $t+1$, V_{t+1}^i. Remember, that we assume agents to live for two periods, i.e. they die after adulthood. Hence, V_{t+1}^i refers to the utility of the offspring and means that parents care about the well-being of their children. By involving V_{t+1}^i in the utility function, parents account for the future bargaining position of their children, which in turn depends on the earning abilities and hence on human capital. Thus, each generation cares for the human capital of the next generation automatically.

The welfare function of the household V^h is represented as a weighted sum of individual utilities of the couple, where the weights can be interpreted as the bargaining power of the spouses in the decision-making process.

$$V_t^h = \theta_t V_t^m + (1-\theta_t)V_t^f \tag{34}$$

Note, that the welfare function of the household depends on the bargaining power of the spouses directly, i.e. θ_t, but also on the bargaining power of the offspring, i.e. θ_{t+1} (compare equation (33)). The household decides on the human capital of their children and their own consumption in order to maximize household's utility. The couple pools their income and faces jointly a budget constraint. Human capital accumulation is dependant on the education parents provide to their children and the human capital of the parents themselves.

The wage gap is manifested in the bargaining power θ. In fact, a standard approach to endogenize bargaining power, is to model it as a function of $\frac{w_f}{w_m}$ where w_i denotes the wage of an individual of gender i. However, in the paper by De la Croix and Vander Donckt (2010) θ also depends on the institutional and social gap. The survival gap is defined as the ratio of the total time endowments and hence is part of the time constraint in the utility maximization problem. The education gap results directly from the model as a consequence of the gender-specific accounting of V_{t+1}^i since households decide on the education and in turn on the human capital of the children.

The authors test the impact of the different gender gaps by including them first separately and then jointly as constraints in the optimization problem. I.e. depending

on which condition(s) are binding, different equilibria can be observed. By doing so, the authors do not only allow to examine the effect of the different gender gaps, but are also able to simulate the pathways from the "corner regime", where fertility is high and female human capital non-existent, to an economy with low fertility, where education is available for girls and boys, by involving and resizing different gender gaps step-by-step.

The results show that a decrease in the wage gap has a negative effect on fertility and parents substitute quantity for quality in the quantity-quality trade-off of children, which is in line with the results from the previous papers. Furthermore, an increase in female survival probability has the same consequences as a lower wage-gap: Fertility decreases, female human capital investments increase and lead to a closing educational gap and economic growth is enhanced. Reducing the gender wage gap as well as the gender survival gap by improving mother's survival probability helps countries that are trapped in the corner regime to stimulate economic growth and demographic transition. Noteworthy, this is not the case with decreasing the social and institutional gap. In the corner regime, reducing that gap has no effect at all. However, countries which are already out of the corner regime can promote economic development by closing institutional gender gaps.

We have analyzed female empowerment within the private sphere and its impact on the economy in chapter 2.3 and want to summarize the key take-away. The crucial feedback mechanism operates the following way: A typical measure for gender inequality, namely a gender gap (for instance the gender wage gap), impacts the female bargaining position. The smaller the gender wage gap, the better the female bargaining position. The female bargaining position has a direct effect on education of the couples offspring. Empirical findings show that a better female bargaining position improves human capital accumulation. As a result of the increase in human capital, economic output rises and hence closes the wage gap a little more. This feedback-loop is presented in Figure 4. In contrast to the baseline model by Galor and Weil (1996), the element of female bargaining power is added to the models discussed. Hence, the wage gap does not affect the economy directly, but kind of detours via the female bargaining power. Another difference of the models in this chapter in comparison to the benchmark model is the focus on education instead of fertility. However, fertility and education -i.e. quantity versus quality- can be seen as the two sides of the same coin. All other key facts of the baseline model as well as the general structure are applied successfully more or less directly (compare Figure 1).

2.4 Female bargaining power in the public sphere

Formerly, the models of interest have discussed female bargaining power within the private sphere as the family or the household. In this section, we want to focus on the power of women outside their homes, i.e. in the public sphere. A great aspect of female power in public sphere is female franchise, female representation in politics and access to public goods and institutions. While at first glance it seems that women and men have the same chances to take part in politics, the truth is that there is still a number of inequalities: across OECD countries, only 31.6 percent of the seats in the lower/single houses of their parliaments were held by women in 2021 on average. A gender-balanced cabinet is a strong indicator of a government's commitment to gender equality. And while the trend is positive overall in OECD countries, in fact the number of women in cabinet posts increased by 6 percentage points, we still have not reached gender equality **?**oecd2023.

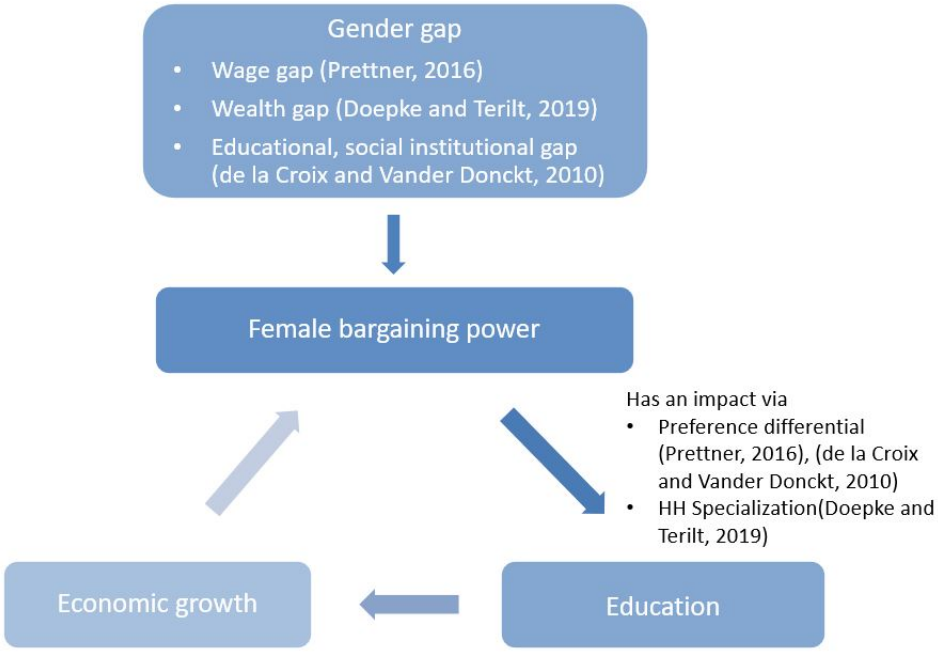

Abbildung 4: The feedback mechanism between female bargaining power and economic development in the private sphere. Note: HH denotes household. (own illustration)

The World Economic Forum published different metrics of gender gaps including political empowerment, economic participation, education and health worldwide. According to their studies, the greatest disparities are seen among political empowerment, a measure of gender parity within politics which takes into account political representation from the parliamentary level to heads of state (Crotti et al., 2021). Motivated by news like this, we want to understand how women gain power in the public sphere.

First of all, we want to review the history of female franchise briefly. Only 200 years ago women had no rights at all. Keeping in mind, that the enfranchisement of women took off at the end of the 19th century, it seems like a little wonder that we have female presidents nowadays. The first country that introduced the right for women to vote in national elections was New Zealand in 1892, followed by Australia in 1902. The first country in Europe was Finland, introducing female enfranchisement in 1907[1]. The famous British "suffragettes" movement, which intensified its action around 1905, struggled to achieve the right to vote until after World War I. As in the United states, the right to vote was granted to women in 1918. Austria and most Eastern European countries did so in the same year. Spain and Portugal followed in the 1930s while in Italy, France and Greece women were disfranchised until after World War II. The last country to introduce female rights to vote was Switzerland in 1971.

Outside Europe, the situation was quite similar. Japan, China and India introduced female franchise between 1945 and 1950. Latin America went through the same pattern

[1] An interesting side remark here: Finland and New Zealand were just recently in the center of media as the two female prime ministers Sanna Marin and Jacinda Ardern responded smart to a reporter´s sexist question. See for example https://www.cbsnews.com/news/jacinda-ardern-sanna-marin-prime-ministers-new-zealand-finland-historic-meeting/

as Europe, which means that there were waves of legislation in the 1930s and after World war II. In Africa most countries introduced female franchise after World War II except for South Africa who granted (white) women the right to vote in 1930. Nowadays, most countries offer equal political rights to men and women. Exceptions are countries in the Persian gulf who did not introduce female franchise yet (Goldin, 1990).

To understand how women gained political rights we have to go one step back in history. Until 200 years ago women were seen as men‘s property. All authority was owned by men: Women had no property rights, they were not allowed to enter contracts, had no rights on her earnings, moreover no parental rights over her children and could not obtain a divorce. Political rights for women were far away until the 20th century and all the power was with men. Yet, roughly 100 years before women were allowed to have a political voice, namely the right to vote, a series of political reforms was carried out. These political reforms targeted essentially areas of marriage like divorce, child custody and marital property. In fact, these reforms represent a shift of power from men to women. Having in mind, that this movement took place before the suffrage movement of women, moreover in an all-male legislature where only men had the chance to vote for such reform laws, the question arises where those reforms came from. Spoken differently: Why would men ever agree to grant women some of their own power and take the risk to weaken their own rights?

2.4.1 Women's Liberation: What's in it for Men? (Doepke and Tertilt, 2009)

Doepke and Tertilt (2009) try to answer that question by arguing that from a man´s perspective, there is a trade-off between his own wife and everybody else´s wife. While a man would naturally prefer that his wife has no rights at all, this does not hold true for other wives. First, fathers would prefer their daughters to have a strong bargaining position in their marriages. Second, as women value child quality their power translates into increasing human capital investment. A father prefers his child to have "high-quality" mates and therefore benefits from a strong bargaining power of his child´s future mother-in-law. Technological progress and the growing importance of human capital versus physical capital led to an shift of this trade-off and caused ultimately the expansion of women´s rights.

Similar to the models on female bargaining power in the private sphere of section 2.3, the utility functions U_i differ along gender lines. Men (m) and women (f) couple via marriage and decide on the allocation of consumption, c_i, between husband and wife as well as number of children, n, and the welfare of their children, , denoted by U_i', which is in particular the (sex-specific) education of children.

$$U_i(c_i, c_{-i}, n, U_m', U_f') = u(c_i, c_{-i}, n) + \gamma_i \frac{U_m' + U_f'}{2}$$
$$u(c_i, c_{-i}, n) = \log c_i + \sigma \log c_{-i} + \delta \log n$$

where i refers to an individual and $-i$ refers to the individual's spouse, γ_i is the weight that an individual puts on their children's welfare. σ measures the individual's spouse's consumption and is smaller than 1 (people value their own consumption more than their

spouse's one). Moreover, it is assumed that women value the welfare of their children more than men, i.e. $\gamma_f > \gamma_m$.

Households face a budget and a time constraint similar to the baseline model. Let H_i denote human capital endowment of an individual of gender i and H_i' the human capital endowment of the respective offspring of gender i. The household produces consumption goods according to a Cobb-Douglas production function, i.e.

$$c_m + c_f = A(t_f H_f)^{\alpha}(t_m H_m)^{1-\alpha}$$

where $\alpha \in (0,1)$ and t_i is the time spent on household production. The total amount of time equals to 1. Men spend all their time in production, whereas women divide their time on production and child-rearing (compare to baseline model). Thus, $t_m = 1$.The time constraint for women is given by

$$t_f + (\phi + e_m + e_f)n \leq 1$$

where ϕ is a fixed time cost per child and e_i is time devoted to educate a child of gender i. Human capital accumulates according to

$$H_i' = \max\{1, (Be_i)^{\theta} H_f^{\theta} H_m^{1-\beta}\}$$

where $B \geq 0$, $\theta \geq 0$ and $\beta \in (0,1)$. Note that there will be always some positive level, namely at least $H_i' = 1$ of human capital in the model. Intuitively, this describes the situation that people without any education still have some basic skills. Human capital is endogenous in the model and gender-specific, which is crucial here.

Women's rights are represented as the relative bargaining power of the husband and the wife in the decision making of the household. Additionally, women's rights are endogenous; in particular, men can vote on whether to extend rights to women. The goal is to examine how the economic changes affect men´s incentives to extend rights to women. To incorporate economic and political choices, the utility function is considered with different weights.

First, female utility gets the weight of zero and only the male utility function is maximized. This situation represents patriarchy P and all decisions are taken by men. In this situation, female and male consumption, fertility and education of the offspring is determined by

$$(c_m, c_f, n, e_m, e_f) = \text{argmax}\, U_m(c_i, c_{-i}, n, U_m', U_f') \tag{35}$$

Second, women´s and men´s utility is weighted equal. In this alternative, so-called empowerment regime E, decisions are made through efficient bargaining power between husband and wife. The choices on consumption, fertility and education are given by

$$(c_m, c_f, n, e_m, e_f) = \text{argmax} \frac{U_m(c_i, c_{-i}, n, U_m', U_f') + U_f(c_i, c_{-i}, n, U_m', U_f')}{2} \tag{36}$$

The maximization problems are solved recursively. The state variable is gender-specific human capital. Before decisions are taken, men decide which regime they prefer in a once-and-for-all vote in the initial period. They do so by maximizing the value function of both variants and choosing the one where they are better off. If the value function

of patriarchy and empowerment are the same, they choose patriarchy. This means, the regime evolves through maximizing the value function V_i^j, where $j \in \{P, E\}$

$$V_i^j(H_m, H_f, \bar{H}) = u(c_i, c_{-i}, n) + \gamma_i \frac{V_m^j(H'_m, H'_f, \bar{H}') + V_f^j(H'_m, H'_f, \bar{H}')}{2}$$

where the economic choices are determined by equation (35) and (36). The children's utilities are given by

$$U'_m = V_m^j(H'_m, \bar{H}'_f, \bar{H}')$$
$$U'_f = V_f^j(H'_f, \bar{H}'_m, \bar{H}')$$

where $\bar{H}'_i$ is the average human capital of individuals of sex i in the economy.

Solving the system results in different regimes and which regime is chosen depends on the assumptions made about the return to human capital, i.e. θ. θ is the elasticity parameter in the production function of human capital and indicates the increase in children's human capital, or in other words their income and hence their welfare, if time spent on their education increases by one unit. The elasticity parameter can be interpreted as the level of technological progress in the economy.

As already explained, the recursive system is solved for both regimes, the patriarchal and the empowerment regime, and then the male value functions are compared. If the male welfare is higher under the empowerment regime, men vote for female rights. This shows on the one hand, how male utility is affected by women's rights and on the other hand, how female rights evolve. The results depend on technological progress:

- No human capital technology: θ is sufficiently low and the optimal education is zero. Thus, parents do not care about the education of their offspring. As a result, the only incentive for a man to empower women is the utility of their daughters. Noteworthy, men´s concerns on their wives´ and daughters´utility are quite asymmetric: men would have to care so little for their wives and treat them so poorly that the fear of the same treatment to their daughters made them prefer empowerment. However, women had no rights before the start of mass education, indicating that the altruism effect is not strong enough to promote the empowerment regime.

- With human capital technology: θ is sufficiently productive and thus investments in education pay off. In contrast to the first case, human capital accumulation is in the center of the family. The results of the two regimes differ substantially; while in the patriarchy regime women consumed less than men, it is balanced under the empowerment regime. Moreover, education of boys and girls is greater under the empowerment regime than in the patriarchy. At the same time, fertility is lower in the empowerment regime. The relative amount of education of boys to girls does not change between the regimes. However, it is crucial that in the empowerment regime human capital accumulation is faster and hence is economic growth.

The main result here is that if returns to human capital are sufficiently high, men do not only empower women for altruism reasons but also because they want human capital to be high for their children due to mating reasons as discussed before. Therefore, with sufficiently high human capital technology men will vote for the empowerment regime. Particularly, if the economy starts in a patriarchy regime but there exists technological

progress, parents will at some point start to value the education of their children. Consequently, human capital growth as well as economic growth will accelerate over time, while fertility will decline. The spread of education will be accompanied with the expansion of economic and civil rights for women. In fact, these results are reflected by historical data.

2.4.2 The enfranchisement of women and the welfare state (Bertocchi, 2011)

Obviously, civil rights like property rights or the right of child custody emerged before political rights in the sense of being allowed to vote for women. However, it was a long way from no political voice to equal rights for men and women. This historical step-by-step process, i.e. the extension of political rights for women, can be understood as female empowerment.

Bertocchi (2011) considers this perspective of female empowerment by modelling women's disenfranchisement as a societal cost. The cost is determined by a country's culture, especially with it´s family culture and the referred "appropriate" role of a woman. The main driver of the observed cross-country differences in such attitudes is considered to be religion, with Catholicism being associated with a more traditional women's role and thus a lower cost of disenfranchisement. This assumption is tested empirically and confirmed by historical data.

In the model, the production side is modelled exactly like in our baseline model: Women and men differ in their brain vs. brawn endowment, explaining the gender wage gap. The production function depends on mental labour input, offered by men and women, physical labour input, offered solely by men and physical capital and has a Cobb-Douglas functional form. As in the baseline model, the wage gap closes as mental labour gets rewarded more and more over time. The wage rates are defined in equation (3) and (4), and men earn a sum of both while women earn only w_m.

The household side is modelled with a preference differential with women favoring public goods, including human capital investments. An individual gains utility from private good consumption and public goods, which are supplied by the government. The utility function reads as

$$u_t^i = c_t^i + \gamma^i g_t$$

where c_t^i are private consumption goods, consumed by an individual of gender i and g_t are public goods with γ^i as the preference weight of an individual with gender i. Income depends on earned wage and return to capital. However, individuals can not spend their whole income since they have to pay taxes to the government. Moreover, disposable income is reduced by the "cost of disenfrachisement", which is denoted by δ. Hence, the budget constraint takes the form

$$c_t^i \leq (1 - \tau_t) y_t^i - \delta$$

where τ is the tax on income y_t^i. Note, that in the first variant of the model, intra-household bargaining power is not modelled. Nevertheless, introducing this part yields the same results.

The government side is modelled explicitly which is novel compared to the models presented thus far. The government finances public goods, including education, by levying a proportional income tax. The tax rate depends on the relative income of men and women

as well as the preference for public goods. It is given by

$$\tau_t^i = 1 - \frac{y_t^i/\gamma^i}{y_t}$$

where y_t is the mean income of all individuals. The optimal tax rate τ_t^*, which is derived by the "median voter theorem" is ceteris paribus increasing in γ^i as well as increasing according to the gender wage gap. The optimal tax rate in a "patriachatic" regime is given by $\tau_t^* = 1 - \frac{y_t^m/\gamma^m}{y_t}$ while it is given by $\tau_t^* = 1 - \frac{y_t^*/\gamma^*}{y_t}$ under full suffrage, where y^* and γ^* are the median values of y and γ^i respectively.

At each period every enfranchised individual casts a vote on the temporary level of the tax. The level of the tax chosen by the median voter is affected by the wage inequality, the preference differential and the degree of enfranchisement. Similar to the last model, two economies and the according preferred tax rates, are compared: under full male suffrage with full female disenfranchisement and under universal suffrage.

To analyze how the economy evolves over time, and especially, how female enfranchisement develops over time, the model begins with a state where only men had the political right to vote. If only men are permitted to vote, the induced tax rate is smaller compared to the tax rate which would be levied if women were enfranchised too. The intuitive reason is the same as always: Women value public goods more and thus they would vote for governments which invest more into public goods. Mathematically, this translates into $\gamma^f > \gamma^m$. On the other hand, the disenfranchisement of women bears costs and hence lowers male utility. Thus, men compare where they are better off: either in the situation with lower taxes but higher costs of disenfranchisement, or in the situation with higher taxes but no penalty for disenfranchisement.

As the economy experiences technological progress, mental labour gets relatively more rewarded, thus public goods should be emphasized more- and a low tax rate is not optimal anymore. Moreover the gender wage gaps closes as the economy develops from physical capital dependant to human capital driven (compare to the baseline model), inducing a lower optimal tax rate. At some point in time, the cost for female disenfranchisement becomes higher than accepting the higher tax rate. As a result, men would be better off under universal suffrage and start to extend political rights to women. The franchise is extended sooner, the smaller the gender wage gap and the preference differential is and the higher the corresponding societal cost is.The theoretical results are tested and supported empirically.

In summary, closing the gender wage gap and in turn gender based inequality leads to higher female suffrage. Here we can see, that gender-based inequality in one sphere is not only the product but also the cause of gender-based inequality in another sphere, proving how complex the referring systems are.

2.4.3 Endogenous gender power: The two facets of empowerment (Hiller and Touré, 2021)

So far, female empowerment in the public and private sphere have been discussed separately, focusing either on female bargaining power within the household or on political and civil rights for women in the public. Possible feedback mechanisms between the public and the private sphere have not been analyzed yet. For instance, an improvement of women´s

position within the household and their emancipation there do not necessarily lead to a better negotiating position and higher influence of women in politics or the society. In turn, even though if women gain voice in politics it does not automatically mean that they emancipate in private life. Investigation on how women´s emancipation within the private and public sphere influence each other is provided by Hiller and Touré (2021).

To do so, the baseline OLG model is extended by a variation of human capital formation. In this model, human capital is the product of decisions in the private sphere, namely the choices of parents, and in the public sphere, namely of the policy makers. Hence children´s human capital depends on two substitutable goods - public spending by the government and parents' rearing time. In both spheres women´s bargaining power is crucial as it is assumed that women value the education of children more than men do.

Female bargaining power in the private sphere is modelled like in section 2.3, see for example the model by Prettner and Strulik (2016). Households consist of two spouses and two children of different sexes. Women and men have individual utility functions depending on consumption and their children´s future human capital, i.e. $u_t^i = \gamma^i \ln c_t^i + (1-\gamma^i) \ln h_{t+1}$ where c_t^i is private good consumption by an individual of gender i and h_{t+1} denotes investments in education of the offspring. Personal consumption is not sex-specific, it is assumed that the spouses have consume the same goods but put different weights γ^i on it. Men are considered to value personal consumption more than their children's human capital, i.e. $\gamma^m > \gamma^f$, while "women are more oriented towards the well-being of their family" (Hiller and Touré, 2021). The collective utility function of the household is a weighted average of the parents' individual utility functions, i.e.

$$H_t = \theta u_t^m + (1-\theta) u_t^f \tag{37}$$

where θ is male bargaining power. Bargaining power of women vs. men is dependant on the relative wage of the sexes as discussed in the baseline model. The more power the husband has, the less is spent on children's education and the more on consumption.

So far, the model is very similar to the models from section 2.3. However, a novel part of the model is the fact that the human capital of the next generation depends on time devoted to education of children, x_t and a tax levied by the government that is used to finance public goods for children, τ_t (compare the last model by Bertocchi (2011)). To be more precise, human capital is formed by

$$h_{t+1} = \alpha \tau_t + (1-\alpha) x_t \tag{38}$$

where $\alpha \in (0,1)$ is the relative weight of public spending versus parental time spend on child-education. The households face the following budget constraint

$$c_t = (1-x_t)(1-\tau_t)(w_t^f + w_t^m)$$

where w_t^i denotes the wage of an individual of gender i. Adults are endowed with one time unit of which they spend $1-x$ on the labour market.

Next, we continue with the description of the public sphere. In the model, a social planner, e.g. the government, elects public policy in order to maximize an utilitarian social welfare function W_t. The social planner accounts for the different political groups in the society. Here, the political groups are males and females The weight associated with the male institution is denoted by β_t, thus $1-\beta_t$ is an estimate of the institutional

power of women. A decrease of β_t translates into more voice of females in society and thereby a larger influence on public policy. Hence, the social welfare function reads as:

$$W_t = \beta_t u_t^m + (1 - \beta_t) u_t^f \tag{39}$$

with u_t^m and u_t^f denoting male and female utilities respectively.

A static equilibrium in this model is defined as the pair (x_t, τ_t) where x_t maximizes the private household utility in equation (37) and τ_t the social welfare function in equation (39). The problem is solved inserting the budget constraint and definition of human capital accumulation into the household utility function and the welfare function and then optimizing (for example by using the Lagrange theorem, compare to section 2.3.1) under the restriction, that the household take τ_t as given and the government takes x_t as given. This equilibrium can be interpreted as a Nash equilibrium in a game played between social planner and a representative household.

Deriving the equilibrium indicates that x_t is decreasing if either θ is increasing or if the preference differential $\gamma_m - \gamma_f$ is big (compare to Prettner and Strulik, 2016, section 2.3.1. We find analogue results for τ_t: the optimal tax rate is decreasing in β_t. Moreover, a large preference differential enforces the negative effect on the tax rate. However, female empowerment in both spheres has a positive effect on human capital formation, i.e. h_{t+1} is increasing in $1 - \theta$ and $1 - \beta$.

Despite that, substitution effects cause an increase of child rearing time and tax rates if β_t respectively θ_t are increasing. Intuitively, if public spending on education is too low, households try to compensate this imbalance by increasing child rearing time. Similar reasoning explains the increase of taxes in θ_t: if the government notices that household do not invest enough in human capital, they spend more on public schooling in order to compensate for the missing parental investments. Remember, that the solution is a Nash equilibrium and that decisions of the other side are taken as given.

But how can the private or the public sphere increase spending on human capital? The answer is female empowerment, or mathematically speaking by increasing the weight on the female utility. In other words, low female institutional power will impede female bargaining power in the privates sphere and vice versa. In the long run, female bargaining power in the public and private sphere will converge, as always one side tries to balance out the other side. This co-dependency is a crucial result of the paper by Hiller and Touré (2021). Hence, it is suggested that policies should target female empowerment in the private and public sphere jointly to promote human capital formation and economic development.

In summary, the emancipation of women depends mostly on two factors: First, the economic development from a physical to a human capital driven country. As discussed in the baseline model, this development implies a higher esteem of education. Second, the gender preference differential: It is assumed that women prefer public goods more than men. Especially mothers value the quality, i.e. the education, of their offspring more and hence have a direct effect on the next generation. Now the idea is that including women in politics will have a positive impact on human capital and thus men "donate" some of their power to women. A positive feedback-loop evolves in which the gender gap closes, the female bargaining power rises and hence investments in human capital increase, again positively influencing gender equality. The described feedback loop between female

empowerment, human capital, economic development and female suffrage is depicted in Figure 5.

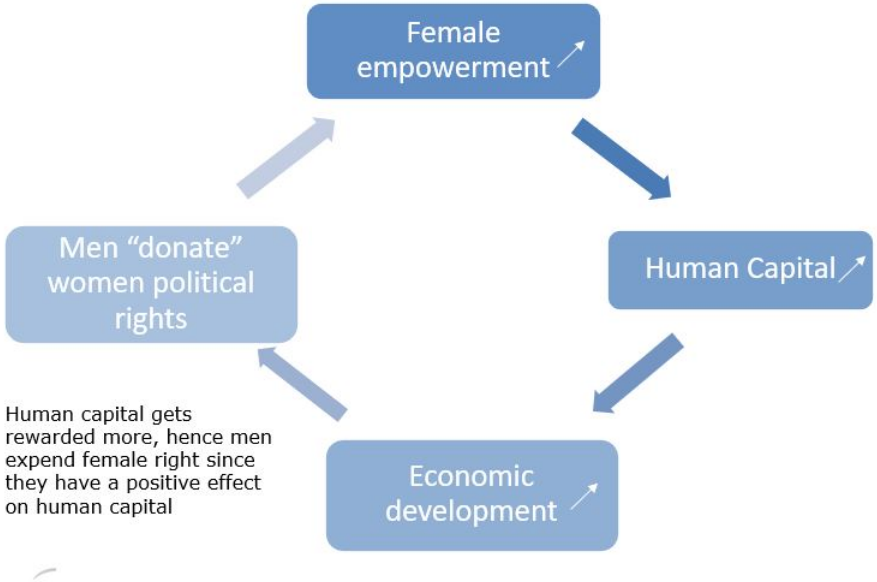

Abbildung 5: The feedback-loop mechanism in the public sphere: Female empowerment relies mostly om the development of the economy and the women's preferences towards public goods as human capital (own illustration)

2.5 Female Health

In this section we want to emphasize the role of female health in the economy. Gender differences and inequalities are not only limited to economic and socio-cultural differences between men and women, they result also in sex-specific differences in health status and healthcare. Moreover, equal access for women to healthcare facilities is often not available. Reasons could be socio-cultural norms which hinder women to even seek health care. Additionally, women receive on average less income than men and thus they are less likely to afford health care. Furthermore, women are more exposed to violence and discrimination, leading to a negative effect on their health. Another aspect to mention here is the sex-bias in pharmacological studies and treatments which in general disadvantage women (Perez, 2019). The list of gender-based inequality in health related issues is long. For instance, Bloom et al. (2020) show that on average men spend more time of their life span in good health than women do. Now one could argue that women have a higher life expectancy on average than men. However, if one is looking at "healthy life expectancy", which measures the "number of years during which an average person would consider themselves healthy" (World Population Review, 2023), the difference between men and women becomes significantly smaller. Especially in less developed countries, the healthy life expectancy turns out to be in male favor. Not surprisingly, the referring countries tend to have high gender inequality (Nagarajan, 2021). Since it could be argued that

good health is a requirement to take part in society, including socio-cultural and socio-economic constructs, men are definitely better off. However, as already discussed feedback-mechanisms show that the whole society is worse off, if a significant part -i.e. women- are discriminated.

In this section we want to analyze how these feed-back mechanisms work if the channel of discrimination is health-related. The models follow the same idea and structure as the baseline model: they set up OLG models where agents mate into couples and aim to maximize their utilities. A special focus is set on health. Health is treated as an asset which can either have an impact on wealth or is part of wealth respectively income. Interestingly, we will see that a better health situation of women goes hand in hand with higher human capital investments and has a positive effect on the education of children. Actually, a strong relationship between (female) health and education can be observed empirically and historically. Hence, the effect of female health fits perfectly into the spectrum of our analysis so far.

2.5.1 The contribution of female health to economic development (Bloom et al., 2020)

The first model on the contribution of female health to economic development that we will present first is by Bloom et al. (2020). They set up a model in which they examine the interplay between gender inequality in health and economic development. Additionally, Bloom et al. (2020) quantify the costs of gender-based discrimination in health care. The following channels are identified by which health affects the economy and how the demographic transition depends on gender-specific health.

1. Healthy women are able to participate productively in the labour market with direct consequences for effective labour supply and hence the level and growth of economic output.

2. In families with healthier mothers, child labour tends to occur less and the educational attainment of children tends to be higher.

3. Better female health lowers fertility. As already discussed, lower fertility is deeply entangled with female labour force participation and educational investments as women tend to prefer quality over quantity in the quality-quantity trade-off for children.

4. Better health increases the return on educational investment: first, through lower morbidity, allowing for greater labour market participation at the intensive margin; second, through lower mortality, affecting labour market participation at the extensive margin.

5. Better health of mothers affects the health of children directly through “in utero” effects and mothers’ ability to breastfeed and nourish their children in other ways . Thereby, female health improves development prospects over the long run through direct inter-generational transmission of resources.

The first channel is obvious, the second and third channel have already been discussed and incorporated in the presented models. The latter two are quite special, but will not

be discussed in detail in this thesis. For further information, we advice to refer to the original paper by Bloom et al. (2020).

In line with the baseline model, Bloom et al. (2020) set up an OLG model where female and males pair randomly and decide jointly on the number of children, education and consumption to maximize the couple´s utility which is similar to equation (5) of the baseline model except for a further term capturing education e_t [2]. The variable $\bar{e}$ captures education that children would get for free without any parental educational investments. The utility function[3] takes the form

$$u = \log(c_t) + \delta \log(n_t) + \gamma \log(e_t + \bar{e}) \tag{40}$$

Health is sex-specific and enters the model via the budget constraint

$$\xi_m w_t + \xi_f w_t(1 - \psi n_t) = c_t + e_t n_t \tag{41}$$

where w_t denotes the wage rate per unit of time and depends on human capital, c_t is consumption, n_t is number of children. ψ is the fraction of time women require to bear and rear children. Note that like in the baseline model, it is assumed that men are not involved in the process of child rearing. This is true for most developing countries, which are targeted by this study. ξ_m and ξ_f are parameters which capture male and female health, respectively.

We can see in the budget constraint (41) that the health status impacts the income: It is assumed that individuals need good health in order to be able to supply their labour on the market. Hence, if each individual is endowed with one time unit, in which he or she earns w_t and ξ_i is the amount of time he or she can spend in good health, then $\xi_i w_t$ is the individual‘s income. Moreover, the female income is discounted by the time a mother needs to bear and raise children, i.e. the costs of children are equal to her foregone loans as in the baseline model. Both, poor health status and child-bearing as well as child-rearing have a negative effect on female labour supply. As a result, there is an income gap in the model which is induced by unequal sex-specific health and child-rearing and -bearing.

If we assume that individuals can only supply labour in good health condition and females can only bear in good health condition, equation (41) shows that male labour force participation is equal to ξ_m while women have to share their healthy time between time devoted to their children and supplying labour onto the market. In addition, the budget constraint (41) indicates that an unit decrease in male health decreases labour supply by one, whereas an unit decrease in female health decreases female labour supply by $(1 - \psi n_t) < 1$. This is in line with empirical findings showing that poor health lowers male hours worked more than female hours worked. The other side of this medal is that the opportunity costs of having children is higher for healthier women rather then for unhealthier women.

[2]There is also an extension of the model, where the utility function takes a similar form to equation (34). Introducing endogenous bargaining utility form as previously, where the bargaining power depends on wage and thus on health, just reinforces the results.

[3]The utility function takes the same form as in the model by Prettner and Strulik (2016) but without a distinction between boys and girls. A similar approach is used in our own models in section 3.1 (compare Model 2, equation 8.

The optimization problem is solved by using the Lagrange method.[4] The only difference is the budget constraint. Solving the optimization problem yields the optimal level of fertility and the optimal level of human capital investment

$$n_t = \begin{cases} \frac{\gamma(\xi_m+\xi_f)}{(1+\gamma)\xi_f\psi} & , \hat{w}_t \leq \frac{\gamma\bar{e}}{\delta\xi_f\psi} \\ \frac{(\gamma-\delta)(\xi_m+\xi_f)\hat{w}_t}{(1+\gamma)(\xi_f\psi\hat{w}_t-\bar{e}} & , \text{else} \end{cases}$$

$$e_t = \begin{cases} 0 & , \hat{w}_t \leq \frac{\gamma\bar{e}}{\delta\xi_f\psi} \\ \frac{\delta\xi_f\psi\hat{w}_t-\gamma\bar{e}}{\gamma-\delta} & , \text{else} \end{cases}$$

First of all, we see that for low levels of wages, the couple divides their income between consumption and quantity of children without child quality, i.e. educational investments remain zero. Second, for sufficiently high household income, education turns positive. Additionally, for increasing income (and human capital being positive) fertility declines, which exactly replicates the demographic transition from a high- to a low-fertility regime. It is noteworthy that this threshold level of wage depends solely on female health as opposed to male health. The results according to health are

- Consumption increases with male and female health.
- Fertility increases/decreases with female/male health.
- Human capital investments increase with female health (in the modern growth regime, before that they are zero) and are unaffected by male health. Intuitively, income and substitution effects with respect to fertility cancel out for increasing male income, while the substitution effect dominates for increasing female income. Thus female health improvements stimulate educational investments at the expense of fertility.
- The income threshold for the demographic transition decreases with female health and is unaffected by male health.

Despite these positive effects of female health onto the economic development, especially in the long run, there is one negative side effect: Greater female health tends to depress wages in the short run. Consequently, there are dynamic trade-offs between short-run gains and long-run goals. As well, this explains why societies favor male over female health as they have static utility boosts by doing so. Most importantly, this dilemma between short-run and long-run gains causes the potential of a development trap. However, it is well established that female health accelerates demographic transition while male health improvements have the opposite effects. Hence, policies should target female health.

2.5.2 On gender and growth: the role of intergenerational health externalities and women's occupational constraints (Agénor et al., 2010)

We have already identified the impact of female health on education. Hence, there is an inter-generational feedback between mother´s health and children´s human capital. In

[4] How to solve a problem with the Lagrange method is presented in the Appendix, where we solve a similar problem (see section 3.1) step-by-step.

fact, there are many ways how women´s health affects their future children´s health and education. First, there is an obvious effect of mother´s health on her fetus. Children of malnourished women are at higher risk to have low birth weight, stunted growth, susceptibility to disease, and intellectual impairment. Moreover, it is well documented that poor health of mothers has an impact on their children even before birth via "in uterus" effects as shortly mentioned before (Lim, 2002). Furthermore, poor health and low life expectancy for mothers may adversely affect their daughters' health and education prospects since lower life expectancy decreases the value of educational investments (Jayachandran and Lleras-Muney, 2009). Another aspect to take into account is the mother-to-child transmission of HIV/AIDS. This issue is especially acute for Sub-Saharan Africa where the prevalence of HIV/AIDS is high.

Second, a mother's level of education also affects her children's health. A number of studies found that mothers with higher levels of education have healthier children. The intuitive explanation is that well-educated mothers have better knowledge on diet, hygiene and medical care. Additionally, an educated mother is more likely to send her daughters and sons to school which in turn has a positive effect on their health and their children´s health as they learn about healthy practises. Empirical evidence is provided by Sinha et al. (2007) or McGuire (2006).

Agénor et al. (2010) take these aspects in their model on inter-generational health externalities into consideration. Women (endogenously) share their time between child rearing, market work and home production work. In contrast to the last papers, the aspect of homework production is crucial here. The reason behind is that various studies have shown that women suffer disproportionately more from lack of infrastructure than men. Therefore, they need to spend a remarkable fraction of their time to compensate the related issues and allocate a greater portion of their time in household tasks. The huge difference in unpaid work between men and women has already been mentioned in the introductory motivation and plays a significant role here. For example, the WHO estimates that 40 billion woman-hours are spent annually in Africa to collect water. Moreover, infrastructure influences child rearing in terms of "efficiency" as it helps to save time if it is abundant. But where is the context to female health? Women have to share their time between child-rearing, market work and home production. Moreover, mothers have a direct effect on their children's health as described before. In addition, the time mothers can devote to their children are crucial for their health condition. As a result, if mothers are hindered on allocating time on their children because of unsatisfactory infrastructure, this has a direct effect on health of the offspring which in turn is a determinant for the health in adulthood and which will thus have an impact on their children's health. Here we can see that there is a transgenerational trade-off of health. In other words, the "opportunity costs of poor infrastructure for women include wage labor, acquiring an education, and investing in their own health and the health of their children" (Agénor et al. (2010), page 15).

Following the approach of our benchmark-model by Galor and Weil (1996), an OLG model is set up where women and men couple randomly and live for three periods - childhood, adulthood and retirement. They work and rear children only in adulthood, in retirement they live from the savings of adulthood, the assumptions on childhood are the same as usual. The couple decides on the number and health of children, consumption and savings for retirement together. Wives and husbands differ in their time allocations,

as men are assumed not to take part in child rearing and home production. All individuals are endowed with one unit of time. Men spend the total amount of time on labour market work and earn a wage for it. Women face the time constraint

$$\epsilon_t^{f,w} + \epsilon_t^{f,p} + p^c n \epsilon_t^{f,r} = 1$$

where $\epsilon_t^{f,j}$ is the time devoted to labour market work (w), home production (p) and child rearing (r), p^c is the probability to survive to retirement and n is fertility. The female time constraint has two implications: First, if time allocated to home production is high, time spent on the labour market and time devoted to children will be small. Second, if health of children or female labour force participation should be improved, time spent on home production has to be reduced which is only possible by improving infrastructure. Moreover, it is assumed that men have privileged access to the labour market in accordance with data on infrastructure. Both factors introduce the gender wage gap into the model. The couple gains utility from consumption in adulthood and retirement, from the health of their children and from home produced goods. The utility function has the same form as in the baseline model and is the weighted sum of the logarithms of consumption, children's health and home produced goods.

Children´s health depends on effective amount of time allocated to child rearing by the child's mother, on the mother's health, and the provision of health services by the government, which is congested by the aggregate stock of capital. In addition, health outcomes exhibit serial dependence, in the sense that health status in adulthood depends on health outcomes in childhood. Another feature of the model is the inclusion of child survival probabilities as well as the probability to survive to old age. The health of children is given by

$$h^C = \theta^r (h^f)^\kappa [(\zeta^r \epsilon^{f,r})^{\nu_C}] \cdot [(H^G / K^{P,T})^{1-\nu_C}]$$

where $K^{P,T}$ is private capital stock, H^G are health services by the government, ζ and ν are efficiency parameters, κ measures the impact of the mother's health on the child's health and θ^r measures effectiveness of time allocated on child rearing. Here we can see that time spent on children is crucial for their health. However, lack of infrastructure has a negative effect on time spent on child rearing and hence on their health.

Note that as the model is quite complex due to it's wide range of incorporated elements, we will dismiss from details here and restrict to the key take-aways.

The solution of the problem replicates the described mechanisms and data well. In the equilibrium, wages depend positively on productivity and thereby on health. Furthermore, time spent in home production decreases with availability of infrastructure. Consequently, women have more time to allocate on child rearing and market work, which in turn has a positive effect on income and thus on health. If the freed time is spent on child-rearing, the model suggests that the health of children improves, benefiting the health of the next adult generation. Better health is associated with higher life-expectancy and hence higher human capital investment and thus promoting growth. The main result here is that inter-generational transmission of health between mothers and children, and the persistence of health in the course of an individual's lifetime are essential concepts to understand the link between gender and growth. Moreover it may give a rationale on the existence of child-work and unemployment in early life due to parental decisions that have been made based on health issues. Another crucial take-away is that policies, that

aim to promote female labour force participation should be combined with reductions in infrastructure-related constraints and provide time-burden-reducing public goods.

2.5.3 The empowerment of women, fertility, and child mortality: Towards a theoretical analysis (Eswaran, 2002)

Eswaran (2002) introduces child mortality into the context of economic development and gender related health issues. To be more precise, the link between female bargaining power, investment in child healthcare and child mortality is examined. In the OLG model, each individual is assumed to live up to three periods. The first period refers to childhood. The second period is adulthood and is described as usual: males and females pair in order to have children. They decide jointly on fertility and healthcare expenditure per child. Healthcare provision determines the survival probability of the child. The only mortality considered here is child mortality, i.e. the event to fail to survive to period 2, adulthood. Infant mortality is not taken into account. The third period is old age and one assumption on retirement is crucial for the paper: retired people live from the income of their children. Namely each adult transfers a specific share of her or his income to the parents. This assumption is especially true for countries where capital markets are absent or governmental pension systems fail. Thus, there is another incentive for parents to invest in the health of their children, namely old age support for themselves. Note, that each child transfers income to the parents. Thus it could be argued that the more children a couple has the more old age support is guaranteed. In other words there are two ways to get old age support: Either by having few children, who are likely to survive due to their good health or many children, where the chance to survive for a single child are low but the probability that one of all children will survive is high. However, the quantity-quality trade off should not be dismissed here.

Furthermore, it is assumed that women experience a damage of their own health in childbearing. Therefore, the higher fertility the greater is the impairment to a mother´s health. While income is assumed to be pooled among spouses, which means that the cost of children in foregone income is the same for father and mother, the personal cost in loss of health is exceptionally higher for mothers. It should be remarked here, that the gender wage gap is not modelled in the paper by Eswaran (2002) in contrast to all the papers before. Nevertheless, there is a difference in the wealth between the sexes, as women have income losses due to their harmed health through childbearing. However, the structure of the model stays the same.

Individuals gain utility from their personal consumption, number and well-being of their children and old-age consumption which in turn depends on the transfers of their children. To capture the "depletion" of a mother´s health with fertility a multiplicative term is added to the female utility function. If women would not have children, their utility would be the same as for men, $u^f = u^m$. Introducing a "maternal depletion" function $D(n)$ which depends only on fertility, the female utility fulfills

$$u^f = D(n) \cdot u^m, \quad D(0) = 1 \quad D(n) > 0 \quad \text{and} \quad D'(n) < 0 \quad \forall n$$

In the model a Nash bargaining problem is solved where the intra-household female bargaining power is exogenous but discussed to be determined by labour force participation,

education and the wealth gap as well as cultural norms and systems[5]. Comparative analysis shows that an increase in the female bargaining power lead to a decrease in fertility and an increase in healthcare expenditure per child. This is in line with the intuitive reasoning that mothers have a higher burden with fertility and hence prefer better health care for their children in order to compensate foregone transfers in old age of hypothetical further children with a high survival probability. Again women have an emphasis on child quality in contrast to their husbands who favor child quantity, even though the reasons behind this result are different to the explanations before. This shows that female empowerment has a manifold of perspectives, facets and channels through which it takes effects. As already discussed, empowerment of mothers has the potential to spur the demographic transition. Especially in countries who suffer from unfavourable population growth policies should target female empowerment.

A side effect of a higher provision of health to children is a lower child mortality. This result is in line with empirical findings from India. Child mortality and fertility is high in Northern India, where female autonomy is limited more than in Southern India, which in turn is characterized by low fertility and child mortality rates. Additionally, incorporating educational investments for children reinforces the results. The reason behind is that education and health have positive feedback effects on each other as argued in Bloom et al. (2020). From the old age perspective, parents invest in the children´s education in order to improve their earning abilities. To protect these "investments" they provide even more health to their children.

Another feature of the paper is the explanation behind sex-specific survival rates. In many south-Asian countries, higher child mortality rates and lower school enrolment rates are evaluated for girls in contrast to boys. The most extreme form of this discrimination against female children is found in the sex ratio in births in favor of boys. In the basic form of the model, all children are assumed to provide transfers to their parents in old age. The reality is different to that as in most south Asian countries men are expected to provide old age support. As a result, if children are valued from parents only as "pension plans" and daughters do not provide any old age security, it is clear that parents will not invest in their daughter´s health or education. Therefore, if female bargaining power increases, health investment increase- but only for boys! Strikingly, discrimination against girls and female disadvantage (defined as excess of female child mortality over male child mortality) increases with increasing female empowerment. Unfortunately, also this pattern is observed empirically in different Indian areas (Gupta, 1987). More evidence is found in China (Hull, 1990), Taiwan and South Korea (Park and Cho, 1995) in the mid-eighties. However, these findings do not constitute any argument against female empowerment, but emphasizes that any reduction in fertility should be accompanied by altering preference of boys versus girls. For example, girls should get the chance to provide old age transfers too. To be able to do so, the status of women in society, including labour force markets, educational sectors and cultural norms, has to change. In other words, empowerment of women has entirely benign effects in developing countries, only if daughters are enabled to provide old age assistance to parents too, that is, daughters too must be rendered economically valuable to parents. The main point to take away here is

[5]Compare to the model by Bertocchi (2011), section 2.4.2. There we reviewed a model, which discusses female franchise. The cost for female disenfranchisement relied on the culture, indicating that more conservative societies have low costs for female disenfranchisement.

that under these cultural circumstances as observed in south Asia, the empowerment of women should be emphasized in the private and the public sphere simultaneously.
To end the section on female health, we want to point out not only the beneficial effects of health equality on the economy but especially on human capital and education. Health is beside income a significant part of wealth. Healthy mothers have a direct effect on the health and education of their children and hence on the level of human capital in the economy. In turn, increasing human capital promotes economic development which in turn closes the wealth gap and thus improves the health of mothers, inducing the feedback-loop again as depicted in Figure (6). A higher level in female health leads to higher educational investments, promotes faster demographic transition and stimulates economic growth. Again, gender equality is not only a goal in and on itself but has positive effects on manifold aspects in society and economy.

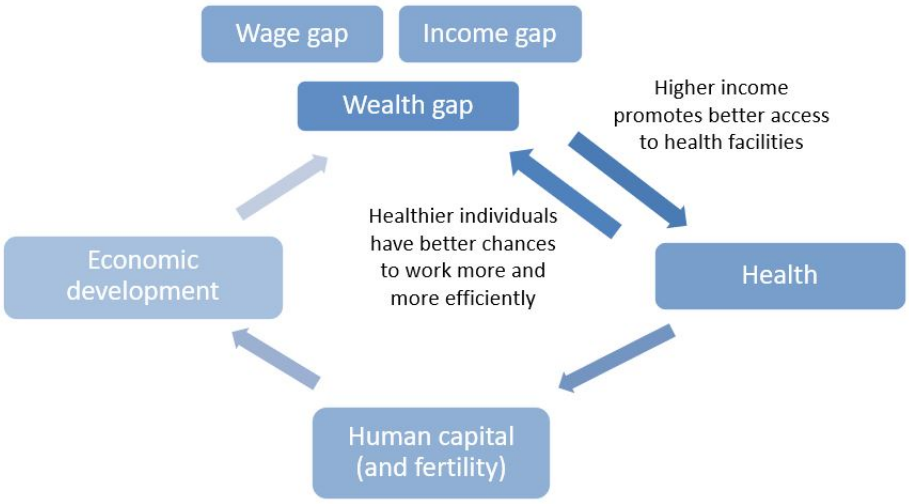

Abbildung 6: The feedback-loop mechanism between female health, economic development and human capital (own illustration)

The goal of equal rights, in particular the goal of equity between men and women justifies the variety of models presented in the literature review. Several studies highlight the importance and benefits of gender equality. We have seen that there is a clear, measurable link between gender-based inequality and economic growth. In fact, higher female empowerment can help developing countries to escape the poverty trap since female empowerment accelerates the demographic transition. In this context, in does not matter whether the empowerment happens within the private or the public sphere. Moreover, it is found that women value education more than men do and thus have a positive effect on public consumption and human capital investments. Even though the preference of mothers for child quality plays a huge role in this context, it is shown that even without this preference human capital accumulation profits from equal rights for women. Moreover, we have seen that not only equal rights but also equal chances especially regarding health issues promote economic growth and stimulate human capital investments - with or without counting for the preference differential between men and women. So to say,

the positive effect of equal rights and chances for women on human capital comes "naturally" without necessarily assuming that women put emphasis on human capital. Hence, particularly countries where human capital accumulation is the main driver of economic growth should target female empowerment. In this context, it is noteworthy that the drift from a capital intensive economy to a human capital intensive economy is one explanation why women emancipated over time. The advantage of "brain" over "brawn" led to a convergence of female wages to the level of male wages, in turn closing the gender gap and promoting the empowerment of women. As a result, one can conclude that there is a two-way causal feedback between female empowerment and economic growth, arguing that female emancipation is not only the result of but also the reason for economic growth.

3 Key features of the feedback between female bargaining power and economic performance: A model based on Prettner and Strulik (2016)

The aim of this thesis is to understand key features in modelling the interplay of gender inequality and economic growth. In other words, the reader should have a clue of the crucial points when modelling gender inequality and it's consequences. Hence, we want to break down the complexity and set up a very simple model on the issue. To do so, we choose an exemplary model among those presented in section 2 and try to simplify it. These insights will be compared to the analytical results of the original model. The set-up of our model as well as the analytical results will be presented in section 3.1.

Finally, we will execute simulations of the main variables. The goal is to see whether the simplified model is capable of replicating well-known and documented behavior, for example the demographic transition. The results will be described in section 3.2.

3.1 Model set-up and analytical results

We aim to set up a model that is complex enough to replicate the effects of gender-based inequality on economic performance from the literature review but that is simple enough to replicate the main dynamics discussed in section 2. We choose the model by Prettner and Strulik (2016) as our starting point. Since we want to offer a comprehensive description of our model, we will review the model by Prettner and Strulik (2016) shortly.

The model by Prettner and Strulik (2016) uses an OLG approach, in which agents live for two periods: in the first period, they are children and just consume their parents' time and receive education; in the second period, agents are adults. They mate randomly in pairs of different gender and decide on their own consumption, $c_{t,i}$, the number of children, n_t, they want to have and the education of their offspring, $e_{t,i}$, where the index $i = f, m$ denotes a female or male agent and t denotes the time period.

Female and male spouses differ in their preferences and hence have different individual utility functions they want to maximize. Prettner and Strulik (2016) included two assumptions on gender-related preferences, namely the quantity-quality preference differential, which describes that men favor a higher number of children, and the daughter-son preference differential, which characterizes the preference of both parents for the educa-

tion of boys over girls. The individual utility function $u_{t,i}$ of a spouse with gender i in period t takes the form

$$u_{t,i} = \ln c_{t,i} + \alpha_i \ln n_t + \gamma_i \ln e_{t,m} + \bar{e} + \delta_i \ln e_{t,f} + \bar{e}$$

where $\bar{e}$ is the baseline level of human capital that children can receive costlessly, by just observing parents and peers (Prettner and Strulik, 2016). As already mentioned, $c_{t,i}$, $e_{t,i}$ and n_t denote personal consumption, investments in education of children of gender i and fertility in period t. α_i is the preference weight for fertility, γ_i for education of boys and δ_i for education of girls of a spouse with gender i. The household maximizes the household utility function which is given by

$$U_t = \theta u_{t,m} + (1-\theta) u_{t,f}$$

where θ denotes the bargaining power of men and $1-\theta$ the bargaining power of women, and faces the budget constraint

$$w_t[(1-\psi_m n_t)h_{t,m} + (1-\psi_f n_t)h_{t,f})] = (e_{t,m} + e_{t,f})\frac{n_t}{2} + c_{t,m} + c_{t,f}$$

w_t denotes the wage rate per unit of effective labour, ψ_i is the time an agent of sex i devotes to child rearing and $h_{t,i}$ is the level of human capital an individual of sex i has in period t. The problem is solved by using the Lagrange theorem[6].

The solution of the problem indicates that fertility as well as education goes through three stages, depending on household's income. $w_{t,i}$ denotes the threshold levels for wage above which education for boys respectively education for girls turns positive. The interpretation of the results as well as further details have been presented in section 2.3.1. We will continue now with the set-up of our reduced models. The goal is to find out the crucial elements in modelling the interplay between gender inequality and economic performance.

3.1.1 Simplified Model 1: Without $\bar{e}$

Our starting point is the model by Prettner and Strulik (2016). Following their work, we use a household bargaining OLG model approach where agents live for two periods. In the second period, agents couple and get children. The spouses maximize their household utility function, which depends on fertility, personal consumption and education of their children.

We will abstract from daughter-son preference differential and assume that parents do not favor education of one gender over the other. Thus, we do not have two sexes regarding education, i.e. $e_{t,m}$ vs. $e_{t,f}$, but only e_t. The first reason to do so is the goal of simplification as mentioned already in the beginning. Hence, it will be interesting to compare our results with the results of the model by Prettner and Strulik (2016). The second reason is the difficulty to prove that parents favor boys over girls. Actually, early literature, for example by Deaton (1997), failed to show the gender bias by parents even though it is known to exist. In fact, Prettner and Strulik (2016) are aware of that fact

[6] A similar problem is solved for the reduced model (see section 3.1.1 and 3.1.2). A step-by-step derivation of the solution for the reduced model can be found in the Appendix.

and discuss this aspect in their paper as well. Nevertheless, we will use the empirical evidence to underpin our simplification. As a result, our reduced model will include only the quantity-quality preference differential, which have already been discussed sufficiently in the last section.

Another discrepancy to the original model is that we will not include $\bar{e}$ in our model. $\bar{e}$ is the baseline level of human capital that children can receive costlessly, by just observing parents and peers (Prettner and Strulik, 2016). We go a step further in the simplification of the model and do not differentiate between male and female consumption, $c_{t,m}$ vs. $c_{t,f}$. This means that we will consider that the spouses consume jointly and share the goods for consumption equally. Sure, this is another restrictive assumption but remember that the goal is to find the crucial parts of the model in order to get the same results qualitatively in respective to the feedback between gender inequality and economic performance.

The utility function in period t of an individual i, who can be either of female (f) or male (m) gender, is given by

$$u_{t,i} = \ln c_t + \alpha_i \ln n_t + \gamma_i \ln e_t \tag{1}$$

We use the standard approach of logarithmic utility functions which are often used in micro-economics to model intra-household trade offs as well as in the model by Prettner and Strulik (2016). c_t, n_t and e_t denote parental consumption, number of children, i.e. fertility, and education of children respectively. α_i and γ_i are the utility weights an individual has for the respective variable. To model the quantity-quality preference differential, we assume that $\alpha_m > \alpha_f$ and $\gamma_f > \gamma_m$. This formalizes the circumstance that women favor education of their children, i.e. quality, over fertility, i.e. quantity, while the opposite holds true for men.

The household utility U_t is a weighted sum of the individual utilities. The weights given to male and female utility reflect the bargaining power of the respective gender. In fact, θ denotes the bargaining power of the male spouse and $1-\theta$ the one of the female spouse with $\theta \in [0,1]$.

$$\begin{aligned} U_t &= \theta u_{t,m} + (1-\theta) u_{t,f} \\ &= \theta \left(\ln c_t + \alpha_m \ln n_t + \gamma_m \ln e_t\right) + (1-\theta)\left(\ln c_t + \alpha_f \ln n_t + \gamma_f \ln e_t\right) \qquad (2) \\ &= \ln c_t + \theta \left(\alpha_m \ln n_t + \gamma_m \ln e_t\right) + (1-\theta)\left(\alpha_f \ln n_t + \gamma_f \ln e_t\right) \qquad (3) \end{aligned}$$

Each member of the household is endowed with one time unit. The adult household members spend their time either on the labour market or child rearing. Every child needs some amount of time ψ, hence the total time spent on child care is given by ψn_t. Note, that at this point we simplify again the original model by Prettner and Strulik (2016) as we do not distinguish between ψ_f and ψ_m but consider that women and men share the time spent on child rearing equally. Though this is unrealistic, we restrict to that assumption for simplicity. As in the Beckerian framework, costs of children are measured by forgone wages due to the time-loss on the labour market. If w_t represents the wage rate per unit of effective labour, and the efficiency depends positively on human capital $h_{t,i}$ and negatively on the time requirements due to child rearing, then the total income of a household is $w_t[(1-\psi n_t)(h_{t,m} + h_{t,f})]$. Households can spend their income on consumption c_t and expenditures on their children's education. As a result, they face the following budget

constraint

$$w_t[(1-\psi n_t)(h_{t,m}+h_{t,f})] = e_t n_t + c_t \tag{4}$$

Households maximize (2) subject to the budget constraint in (4) and non-negativity constraints on all variables. The solution to all three variables reads the following

$$c^*_{1,t} = \frac{w_t(h_{t,m}+h_{t,f})}{\theta\alpha_m + (1-\theta)\alpha_f + 1} \tag{5}$$

$$e^*_{1,t} = \frac{w_t\psi(h_{t,m}+h_{t,f})\cdot[\theta\gamma_m + (1-\theta)\gamma_f]}{\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f)} \tag{6}$$

$$n^*_{1,t} = \frac{\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f)}{\psi(\theta\alpha_m+(1-\theta)\alpha_f+1)} \tag{7}$$

Note that the first position j of the subscript (j,t), $j \in \{1,2\}$ denotes the model, while the second position t denotes the period. A step-by-step derivation using the Lagrange method can be found in the Appendix.

Now, the first difference to the model solution of Prettner and Strulik (2016) is the nonexistence of a threshold level for wages. To be more precise, in the work by Prettner and Strulik (2016) education and fertility go through three stages of development which depend on the wage rate per unit of effective labour w_t as described in section 2.3.1: first, a stage with high fertility and no educational investments at all; second, a phase in which boys enjoy education but girls do not as a result of the daughter-son preference differential; third, the post-transition phase in which sons and daughters receive education and fertility is low. $\hat{w}_{t,m}$ denotes the threshold level, for which education for boys turns positive and $\hat{w}_{t,f}$ for girls. Obviously, $\hat{w}_{t,m} < \hat{w}_{t,f}$ holds true.

If there is no daughter-son preference differential, the second phase does not exist, i.e. $\hat{w}_{t,m} = \hat{w}_{f,m}$, which could be assumed to be also the result of our model. However, as $\alpha_m > \alpha_f > \gamma_f > \gamma_m$ always holds and we have non-negativity constraints on all parameters and variables, investments in child education e^*_t are always positive in our model! As long as the wage rate is not equal to zero and human capital h_t does exist (for at least one gender), investments in education will be positive in our model. In contrast to the model of Prettner and Strulik (2016), this means that in our model there do not occur three phases of development, at least not intrinsic by the model on household behavior including sex-specific preferences and levels of bargaining power. Actually, an increase in education over time would be the result of the increase in wages and/or human capital.

Another difference is that -at least for exogenous bargaining power θ- fertility does not change over time in our model. Interestingly, the solution for number of children $n^*_{1,t}$ is solely parametric in our model and depends on the child-rearing time ψ and the utility weight on fertility α_i. We assume that α_i and ψ do not change over time and are parameters. Hence, the optimal number of children is constant over time. This means that for exogenous bargaining power, our model is not capable of replicating the demographic transition regarding fertility.

A feature of our simplified model that coincides with the model by Prettner and Strulik (2016) is that for equal preference between the genders, i.e. $\alpha_m = \alpha_f$ and $\gamma_m = \gamma_f$, the positive effect of female bargaining power vanishes. The intuitive explanation behind this

result is that for equal preferences, the quantity-quality differential is turned off. Thus, female empowerment has no effect neither on education investments nor on number of children [7].

Vice versa, if we take a closer look onto equation (6) we can see that for a large quantity-quality preference differential, i.e. $\alpha_m >> \alpha_f$ and $\gamma_f >> \gamma_f$, between fathers and mothers, female empowerment has a great impact on the optimal level of children's education investments. To understand the positive effect of female empowerment on education mathematically, we take the derivative of equation (6) with respect to θ:

$$\frac{\partial e^*_{1,t}}{\partial \theta} = A \cdot \frac{(\gamma_m - \gamma_f)[\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)] - [\theta\gamma_m + (1-\theta)\gamma_f](\alpha_m - \gamma_m - \alpha_f + \gamma_f)}{(\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f))^2}$$

$$= \frac{A}{(\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f))^2} \cdot$$

$$[(\gamma_m - \gamma_f)(\theta\alpha_m + (1-\theta)\alpha_f) - (\gamma_m - \gamma_f)(\theta\gamma_m + (1-\theta)\gamma_f)$$

$$- (\alpha_m - \alpha_f)(\theta\gamma_m + (1-\theta)\gamma_f) + (\gamma_m - \gamma_f)(\theta\gamma_m + (1-\theta)\gamma_f)]$$

$$= \frac{A \cdot [(\gamma_m - \gamma_f)(\theta\alpha_m + (1-\theta)\alpha_f) - (\alpha_m - \alpha_f)(\theta\gamma_m + (1-\theta)\gamma_f)]}{(\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f))^2} < 0$$

where $A = w_t\psi(h_{t,m} + h_{t,f})$. The last inequality holds since $\alpha_m > \alpha_f > \gamma_f > \gamma_m$ implies that $\gamma_m - \gamma_f < 0 \wedge \alpha_m - \alpha_f > 0$ and the denominator is always positive. This proves the negative effect of male bargaining power θ on the education and consequently the positive effect of female bargaining power as measured by $1 - \theta$ on the education of the offspring. Moreover, we can see her that if the preferences of the parents are identical, i.e. $\alpha_m = \alpha_f$ and $\gamma_m = \gamma_f$, then the numerator is equal to zero and the positive effect of female empowerment on education vanishes.

The same effect of preference differential in combination with high female empowerment can be found on consumption. Consumption depends positively on households income in the numerator $w_t(h_{t,m} + h_{t,f})$ but negatively on the utility weights on fertility α_i. Nevertheless, if the male preference for child quantity and male bargaining power are large, the negative effect on consumption is even stronger. Contrariwise, female empowerment stimulates consumption as women have a smaller preference for fertility. To prove this, we take the derivativative of equation (5) with respect to θ:

$$\frac{\partial c^*_{1,t}}{\partial \theta} = -\frac{w_t(h_{t,m} + h_{t,f})}{(\alpha_m - \alpha_f)^2} < 0$$

The denominator is always positive. Income, defined by $w_t(h_{t,m} + h_{t,f})$ is positive as well, thus we find that the last inequality holds always.

If we take a closer look onto the optimal level of fertility in equation (7), we can clearly see the rivalry between education and fertility. Increasing the utility weights referring to education γ_i ceteris paribus decreases fertility as the term γ_i occurs negatively in the numerator. Moreover, an increase in the combination of preference differential, i.e. the

[7] See Prettner and Strulik (2016), Proposition 3 on page 62

difference between α_m and α_f gets larger, plus female bargaining power has a negative effect on the number of children in contrast to the positive effect it would have on education.

To prove this, we note that the quantity-quality preference differential implies $\alpha_m - \alpha_f > 0$ and $\gamma_f - \gamma_m > 0$. Since $\alpha_m > \alpha_f > \gamma_f > \gamma_m$ holds, it follows that $|\alpha_m - \gamma_m| > |\alpha_f - \gamma_f|$. Taking the derivative of equation (7) with respect to θ yields

$$\frac{\partial n^*_{1,t}}{\partial \theta} =$$

$$= \frac{[(\alpha_m - \gamma_m) - (\alpha_f - \gamma_f)]\psi(\theta\alpha_m + (1-\theta)\alpha_f + 1) + [\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)]\psi(\alpha_m - \gamma_f)}{(\psi(\theta\alpha_m + (1-\theta)\alpha_f + 1))^2}$$

$$= \frac{[(\alpha_m - \alpha_f) + (\gamma_f - \gamma_m)]\psi(\theta\alpha_m + (1-\theta)\alpha_f + 1) + [\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)]\psi(\alpha_m - \gamma_f)}{(\psi(\theta\alpha_m + (1-\theta)\alpha_f + 1))^2}$$

$$> 0$$

We can see that the positive effect of male bargaining power on the fertility would be significantly smaller, if the preferences among the spouses would be the same since in this case the first part of the numerator would be zero ($(\alpha_m - \alpha_f) + (\gamma_f - \gamma_m) = 0 \Leftrightarrow \alpha_m = \alpha_f \wedge \gamma_f = \gamma_m$). This clearly emphasizes the quantity-quality preference differential between mothers and fathers.

Finally, we want to summarize the key features of model 1. In model 1, expenditures on personal consumption and child expenditures are opposing which is in line with the original model by Prettner and Strulik (2016) as well as data. Moreover, our model is capable of pointing out the effect of the quantity-quality preference differential onto the optimal levels of consumption, fertility and education. Another feature, which our simplified model has, is the positive effect of female bargaining power on education if the preferences between the genders differ substantially which is again in line with the original model by Prettner and Strulik (2016).

Now, if the goal is to investigate the effect of female bargaining power on investment in education, which is crucial as we have already discussed in section 2, our model does the job and it is not necessary to consider the features of the original model by Prettner and Strulik (2016). In other words, one could dispense with the daughter-son education preferential, the baseline level of human capital $\bar{e}$ and the distinction of the two sexes in education $e_{t,m}$ vs. $e_{t,f}$, in child-rearing time ψ_m vs. ψ_f and in personal consumption $c_{t,m}$ vs. $c_{t,f}$ in the model by Prettner and Strulik (2016) in order to find the effect of the preference differential and female bargaining power on education qualitatively. Despite that, the model does not meet the expectations regarding fertility and fails to simulate the demographic transition.

3.1.2 Simplified Model 2: With $\bar{e}$

As mentioned before, our simplified model is not capable of replicating the demographic transition regarding fertility. Hence, we decide to take one step back in simplifying the model and include $\bar{e}$ again, which represents the basic skills children learn by observing parents. In contrast to the original model by Prettner and Strulik (2016) this model will

not include sex-specific education, consumption or child-rearing times. The individual utility function reads as

$$u_{t,i} = \ln c_t + \alpha_i \ln n_t + \gamma_i \ln(e_t + \bar{e}) \tag{8}$$

Again, considering that each spouse has the power of $1-\theta$ or respectively θ to force her or his individual preferences in the collective decision making, the household utility function can be written as

$$\begin{aligned} U_t &= \theta u_{t,m} + (1-\theta) u_{t,f} \\ &= \theta \left(\ln c_t + \alpha_m \ln n_t + \gamma_m \ln(e_t + \bar{e})\right) + (1-\theta)\left(\ln c_t + \alpha_f \ln n_t + \gamma_f \ln(e_t + \bar{e})\right) \end{aligned} \tag{9}$$

The budget constraint the family faces does not change in comparison to our first model, i.e.

$$w_t[(1-\psi n_t)(h_{t,m} + h_{t,f})] = e_t n_t + c_t$$

The derivation of the optimal solutions works analogously as before in model 1. In the following, the subscript 2 denotes the solution for model 2. The optimal paths for personal consumption, denoted by $c^*_{2,t}$, is given by

$$c^*_{2,t} = \frac{w_t(h_{t,m} + h_{t,f})}{\theta\alpha_m + (1-\theta)\alpha_f + 1} \tag{10}$$

However, if fertility or parental investments in education are of interest, different stages of development have to be considered. Following the approach of Prettner and Strulik (2016), let $\hat{w}$ denote the threshold level of wage rate per unit of effective labor above which investment in education becomes positive. It is given by

$$\hat{w} = \frac{\bar{e}(\theta\alpha_m + (1-\theta)\alpha_f)}{\psi(h_{t,m} + h_{t,f})(\theta\gamma_m + (1-\theta)\gamma_f)} \tag{11}$$

The derivation of $\hat{w}$ can be found in the Appendix in section D. Hence, we obtain the following solution for fertility and parental investments in education of their offspring:

$$n^*_{2,t} = \begin{cases} \frac{(h_{t,m}+h_{t,f})(\theta\alpha_m+(1-\theta)\alpha_f)}{\psi(h_{t,m}+h_{t,f})(\theta\alpha_m+(1-\theta)\alpha_f+1)}, & w_t \leq \hat{w} \\ \frac{w_t(h_{t,m}+h_{t,f})(\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f))}{(\psi w_t(h_{t,m}+h_{t,f})-\bar{e})(\theta\alpha_m+(1-\theta)\alpha_f+1)}, & w_t > \hat{w} \end{cases} \tag{12}$$

$$e^*_{2,t} = \begin{cases} 0, & w_t \leq \hat{w} \\ \frac{\psi w_t(h_{t,m}+h_{t,f})(\theta\gamma_m+(1-\theta)\gamma_f)-\bar{e}(\theta\alpha_m+(1-\theta)\alpha_f)}{\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f)}, & w_t > \hat{w}, \end{cases} \tag{13}$$

The Lagrange system for model 2 as well as an explanation of the referring solution are stated in the Appendix, section C.

At first sight, we recognize that model 2 resembles much more the original model by Prettner and Strulik (2016) than model 1 at least by it's structure. Above all, fertility and education go through different stages of development depending on the effective wage

rate similar to the original model by Prettner and Strulik (2016). As already mentioned in their paper, if one does not include the daughter-son education preference differential, then the threshold level of wage for which education turns positive is the same for boys and girls. This is exactly the situation we face in model 2.

Let's take a closer look at the optimal investment in education. If we compare the two optimal investments in education given in equation (6) and (13), we can see that the denominator is the same in both models while the numerator in equation (13) differs from the numerator in model 1 by the term $-\bar{e}(\theta\alpha_m+(1-\theta)\alpha_f)$. Intuitively, parents have to spend more on education in model 1 to compensate $\bar{e}$. The other way around, parents who can rely on some free education $\bar{e}$ for their children have not to spend so much on education e_t and hence the optimal amount of investment reduces by $\bar{e}(\theta\alpha_m+(1-\theta)\alpha_f)$.

Female bargaining power has not only a positive impact on the level of investment in education but also on its take off. To be more precise, we find that if preferences towards fertility and education differ among spouses, then female empowerment lowers the education threshold ($\hat{w}$) [8]. To proof this, take the derivation of $\hat{w}$ with respect to θ

$$\frac{\partial\hat{w}}{\partial\theta}=\bar{e}\frac{(\theta\gamma_m+(1-\theta)\gamma_f)(\alpha_m-\alpha_f)+(\theta\alpha_m+(1-\theta)\alpha_f)(\gamma_f-\gamma_m)}{\psi(h_{t,m}+h_{t,f})(\theta\gamma_m+(1-\theta)\gamma_f)^2}$$

Since $\theta<1$ and $\alpha_m>\alpha_f>\gamma_f>\gamma_m$ holds, and we have non-negativity constraints on all parameters and variables, we see that the derivation is always strictly positive under the constraint of the quantity-quality preference differential, which implies that $\alpha>\alpha_f$ and $\gamma_f>\gamma_m$. This means, that *male* bargaining power increases the threshold level of wage for which education turns positive. Thus, *female* bargaining power, measured by $1-\theta$ lowers the threshold. Hence, our result coincides here with the original model by Prettner and Strulik (2016).

In comparison to model 1, the introduction of $\bar{e}$ does not change the optimal level of consumption. Moreover, the resulting optimal consumption of both models, model 1 and 2, does not differ substantially from the original model by Prettner and Strulik (2016). The slight difference is that in the original model, female and male consumption has a positive factor of $(1-\theta)$ and θ respectively in the numerator. This is a consequence of the sex-specific set up in regard to consumption from which we dismissed.

Bringing the optimal levels of fertility of model 1 and 2 into comparison, we find the biggest change. Model 1 resulted in a constant number of children over time. In model 2, we find fertility to change over time and go through different stages which is much more in line with the original model than the first model.

If we consider equation (12) we see that the optimal number of children depends on the utility weights and negatively on child rearing time like model 1 but also on the household's human capital and household income $w_t(h_{t,m}+h_{t,f})$ which seems to be plausible. Moreover, we find the discretionary level of education $\bar{e}$ to have a positive effect on fertility in the second stage, i.e. in the case $w_t>\hat{w}$. $\bar{e}$ occurs negatively in the denominator of the second case of equation (12). Hence, increasing $\bar{e}$ ceteris paribus, decreases the denominator and in turn increase fertility. The intuitive reason is that the level of education children can obtain costlessly, $\bar{e}$, frees some amount of household income which parents can invest in fertility.

[8] Compare Prettner and Strulik (2016), Proposition 2, page 61.

We summarize the analytical part on model 2: Model 2 confirms the positive effect of female bargaining power on investments in education as well as take off of investing in education at all. Moreover, fertility is considered to go through different stages of development. Nevertheless, we have to check via simulation whether fertility is decreasing over time as historical data would suggest. Additionally, the importance of female bargaining power on this issue should be investigated. In fact, we will see in section (3.2) that fertility is decreasing over time in model 2.

3.1.3 Human capital accumulation in Model 1 and Model 2

In both models, model 1 and 2, human capital accumulation follows exactly the approach by Prettner and Strulik (2016). Teachers produce human capital of the next generation, i.e. period $t+1$, with a productivity level of B. They earn the prevailing wage w_t. The nominal expenditures on education are divided by the wage rate in order to get real expenditures. Thus, the average human capital formation per child is estimated by

$$h_{t+1} = \begin{cases} \bar{e}, & w_t \leq \hat{w} \\ \frac{Be_t}{w_t} + \bar{e}, & w_t > \hat{w} \end{cases} \tag{14}$$

The average human capital employed per household is given by $\bar{h}_t = h_{t,f}(1-\psi_f n_t) + h_{t,m}(1-\psi_m n_t)$. The production function in the economy is given by

$$Y_t = A_t \bar{h}_t L_t \tag{15}$$

where A_t denotes the state of technology and L_t is labour input in the production. Half of the total labour input is male and the other half is female. Technological progress is driven by education and by population size through the scale effect. Then technological development has the following form

$$A_{t+1} = \frac{\bar{h}_t}{2} \cdot \min\{\eta_1 N_t, \eta_2\} \cdot A_t + A_t \tag{16}$$

where N_t denotes population size, η_1 measures the scale effect and η_2 marks the upper bound of the scale effect. Note that equation (16) is a result from Lagerlöf (2006).

From the production function in equation (15), it follows that the wage rate per unit of effective labour is given by $w_t = A_t$. Household income can be estimated by $w_t \bar{h}_t$. As already mentioned, we follow the approach from Prettner and Strulik (2016).

To define sex-specific human capital in the original model by Prettner and Strulik (2016), we first rewrite equation (14) as

$$h_{t+1} = \max\{\bar{e}, \frac{Be_t}{w_t} + \bar{e}\}$$

The level of human capital of a child of gender i is produced with the same technology as the average human capital. The input are the investments in education his or her parents made for him or her. Thus we obtain

$$h_{i,t+1} = \max\{\bar{e}, \frac{Be_{i,t}}{w_t} + \bar{e}\}$$

As we can see here, the difference in female and male human capital is a consequence of the different amounts of investment in education of girls and boys. As we abstract from that, i.e. we do not consider the daughter-son preference differential, we do not have diversity in human capital. Therefore, individual human capital formation is independent of sex and takes the same form as average human capital formation, i.e.

$$h_{i,t+1} = h_{t+1} = \max\{\bar{e}, \frac{Be_t}{w_t} + \bar{e}\} \tag{17}$$

where e_t is the level of investment in education we obtained in equation (6) for model 1 respectively equation (13) for model 2. As a result, we have to substitute $h_{t,m} + h_{t,f}$ by $2h_t$ in the equations (10), (12) and (13).

The reason we derived the model using $h_{t,m} + h_{t,f}$ instead of $2h_t$ in the first place was to point out the crucial differences between the original model and our adaptations. Unifying human capital does not change the model in it's analytical derivations. With being this said, the description of both models is completed now.

3.2 Model behaviour and simulation

In order to investigate the behavior of our models even further, we simulate them with the statistical package **R**. To do so, we use the parameter values given in table 1. We apply the same parameter estimates as Prettner and Strulik (2016) in order to be able to compare the results from the original model by Prettner and Strulik (2016) with the results of both reduced models.

Parameter	Value	Parameter	Value
$\bar{e}$	1.5	θ	0.7
α_m	0.8	α_f	0.6
γ_m	0.3	γ_f	0.5
ψ	0.06	B	1.1
η_1	0.007	η_2	0.3

Tabelle 1: Parameter values for the simulation. The table is based on Table 1 by Prettner and Strulik (2016) on page 64.

We start our simulations with a female bargaining power of 0.3. According to UNDP (2012), that value is a rough estimate for the gender-gap on the subject of labour force participation and education in low-income countries in 2012. Prettner and Strulik (2016) consider that starting point because of very that reason. The alternative scenario will be the one of perfectly equal-righted genders. Specifically, we will set the bargaining power of both genders to 0.5.

Note that an exemplary piece of code, that is used to simulate the variables of interest can be found in the Appendix, section F.

3.2.1 Simulation of the simplified Model 1

We start with model 1. In Figure 7 the baseline scenario with female bargaining power at 0.3 is depicted with a blue, solid line. In contrast to that, the alternative scenario of a "perfect" world, in which both genders share their bargaining powers equally at a level of 0.5 is represented by a red, dashed line. Human capital in the left panel is an application of equation (17), using equation (6) for investments in education. The simulation of fertility follows equation (7). We see that a higher female bargaining power contributes to higher

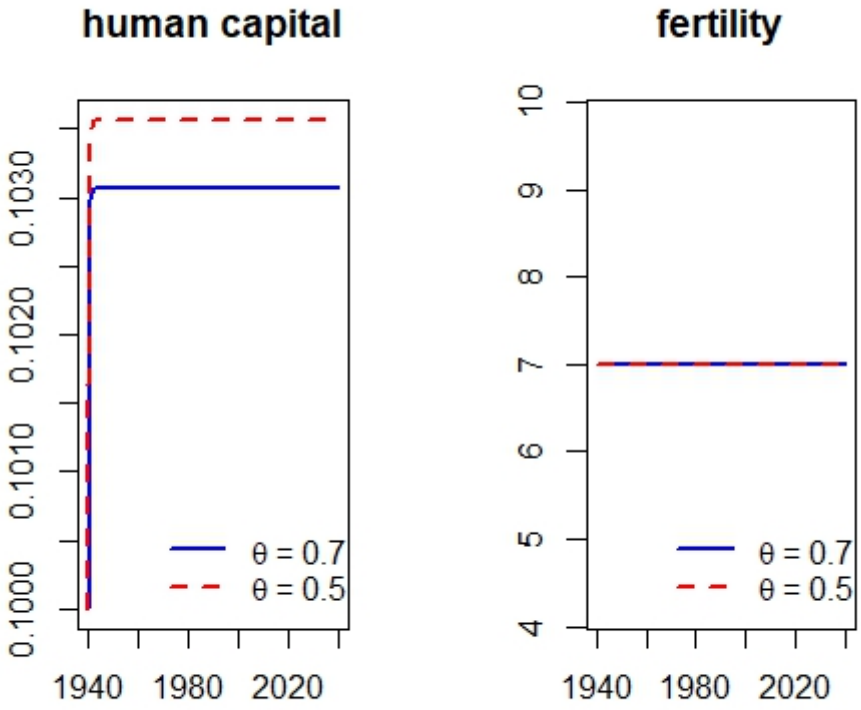

Abbildung 7: Comparison of different levels of female bargaining power in Model 1 (own illustration).

levels in human capital as the literature and data suggests. Anyway, there appears to be no impact of female bargaining power on fertility at all as we have already conducted from the analytical solutions. Moreover, the modification of the bargaining powers does not appear to alter the take-off of increase in human capital which contradicts scientific results. This is the consequence of the fact that investments in education are always positive in our model.

Let's take a closer look at the human capital on the left panel of Figure 7. The graph of human capital seems to jump after a short period of time and then remains constant over time. To test whether the stock of human capital does not change over time, we execute the simulation over the short time span of five years. There are two possible outcomes. Either human capital is constant and we should see a jump in human capital

rather than a continuous rise. Or human capital does undergo a continuous, but steep rise and truly evolves over time. The result of that experiment is presented in Figure 8 which depicts all time-dependent variables of model 1, i.e. human capital, fertility, investments in education and wages, at a female bargaining power of 0.3.

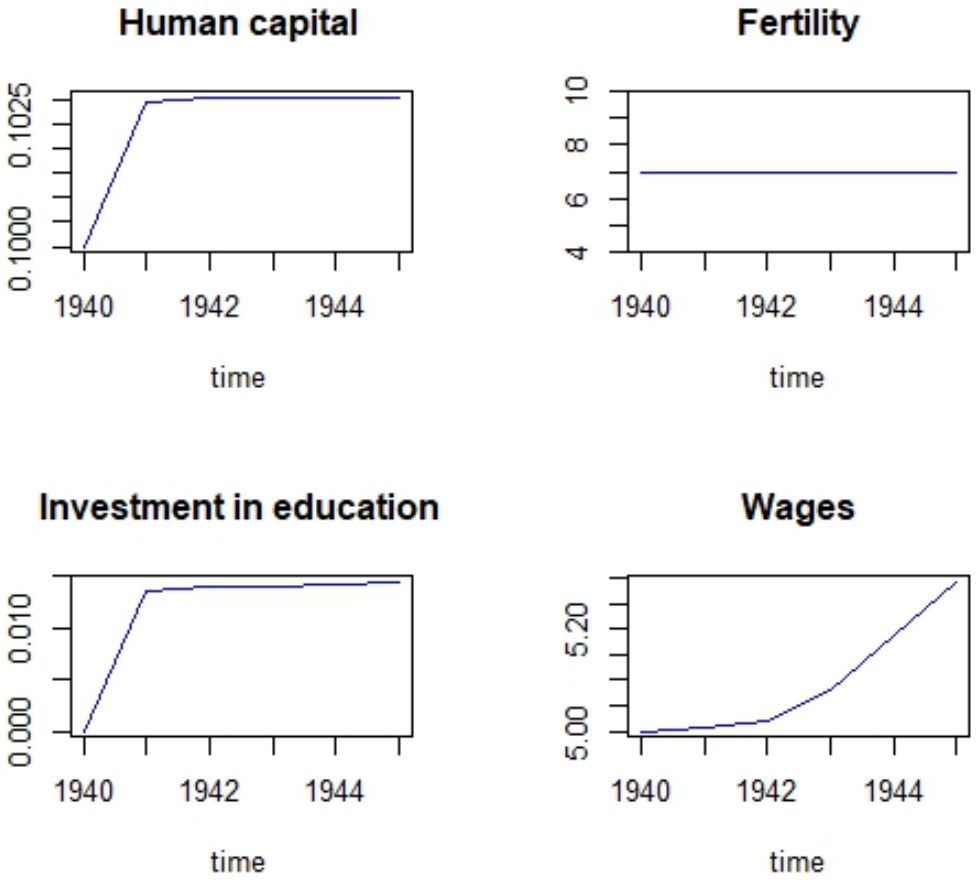

Abbildung 8: The evolution of human capital, fertility, investments in education and wages of Model 1 in the short run (own illustration).

Except for fertility all variables grow continuously over time. Hence, the jump in Figure (7) appears to be a steep rise. Additionally, we can see exponential growth of wages and from 1941 on exponential growth of parental investments in education as well, which both is not realistic. While this implies that model 1 is capable of reflecting development and effects of female bargaining power on human capital qualitatively, it does not reflect the reality in reference to size and speed. In summary, model 1 succeeds in proving the importance of female emancipation in the private sphere in human capital accumulation but fails in all other regards.

3.2.2 Simulation of the simplified Model 2

Next, we investigate model 2. As previously, the baseline scenario with a female bargaining power of 0.3 is illustrated by a blue full line and the alternative scenario by a red,

dashed line. The equations that are simulated are equation (17), using equation (13) for investments in education and equation (12) for fertility.

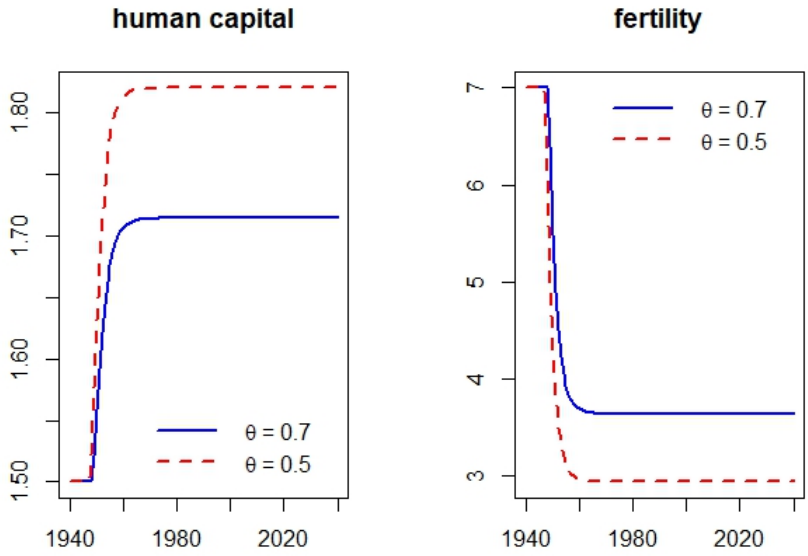

Abbildung 9: Comparison of different levels of female bargaining power in Model 2 (own illustration).

Again we can observe the positive influence of female bargaining power onto human capital in the left panel of Figure 9. Additionally, the effect is amplified in comparison to model 1 if we take a close look onto the scales of the graphics.

Moreover, we detect a negative effect of female bargaining power on fertility as can be seen in the right panel of Figure 9. The number of children converges to a level slightly below under 4 children per women in the baseline scenario, but reduces to a level of 2.9 in the alternative scenario of equal rights. Even though that exceeds the replacement fertility rate of 2.1 as estimated by OECD (2016), it is still a much better performance than in model 1.

Another feature of the model, is the negative effect of female bargaining power onto the threshold level of wage. As we depicted in Figure 9, we see that the red line decreases before the blue one. That indicates that with a higher level of female emancipation the take-off of the demographic transition happens earlier and proves the positive effect of female bargaining power in escaping the poverty trap.

To conclude the simulation of model 2, we finally explore the variables in more detail, using a female bargaining power of 0.3 as in model 1. To do so, we simulate human capital, fertility, investments in education and wages. Remember that household income is the product of the human capital employed by the household member and wages, hence we dismiss from depicting that as well.

In contrast to the simulation from model 1 (compare Figure 8), we consider a time-span of 25 years instead of 5 years since investments in education are not always positive in model 2, simply put they are non-existent for wage rates below the threshold level.

The results are represented in Figure (10).

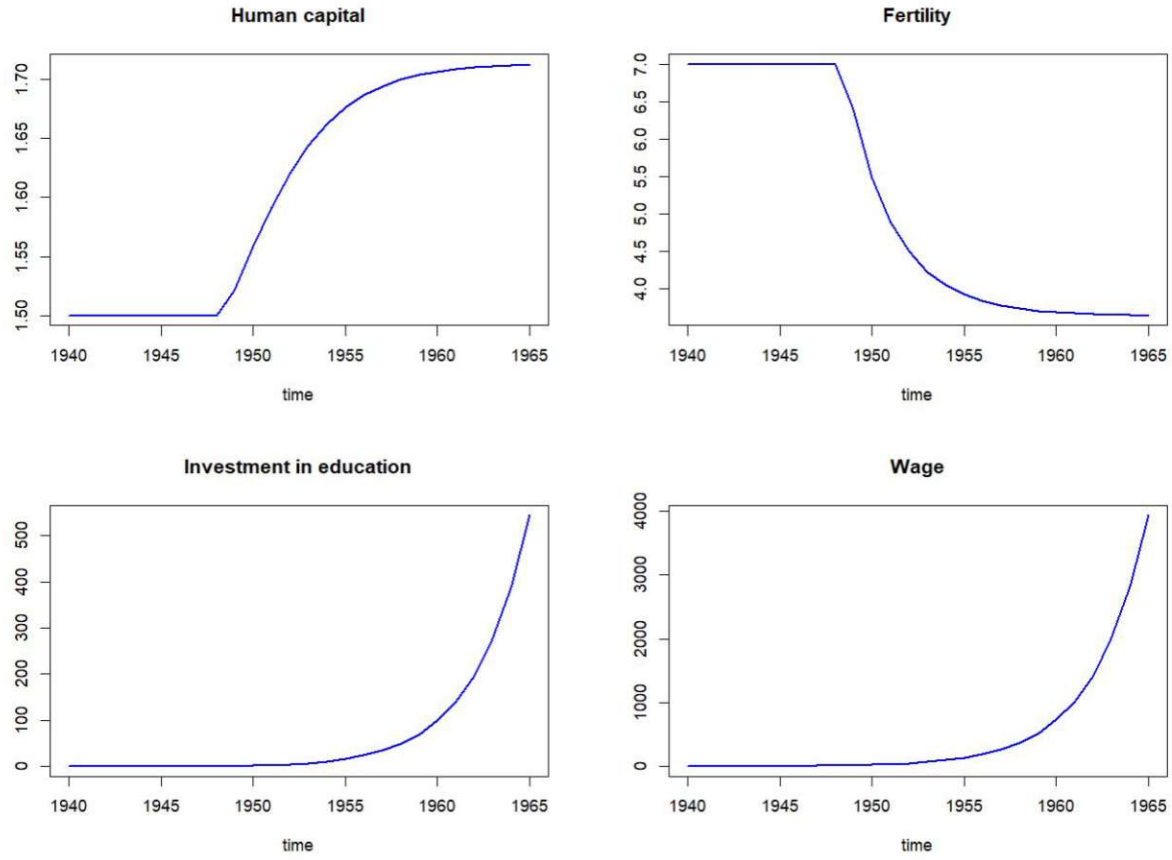

Abbildung 10: The evolution of human capital, fertility, investments in education and wages of Model 2 over 25 years (own illustration).

What we can see very well in Figure (10) is the take-off of the demographic transition regarding fertility as well as human capital. Furthermore, we can see the co-dependence of human capital and investments in education as they rise simultaneously. Until that point, it is in the late 1940's, income is so low, that the marginal utility of personal consumption outperforms the marginal utility of providing education to the children. Despite that, household income grows due to technological progress which causes wages to rise, which can be seen in the bottom-right corner of Figure (10). As a result, at some point wages will surpass the threshold-level and parents will start to invest in their offspring's education. Consequently, human capital increases and the growth of household income accelerates. While wages (and hence income) and investments in education increase exponentially, human capital growth flattens over time. Since human capital is produced by teachers who earn some wage too, real education expenditures stagnate even though nominal expenditures increase (compare equation (14)).

To sum up, model 2 does a much better job than model 1. Hence we can conclude that including some basic level of education ($\bar{e}$) is substantial in modelling the interplay between female bargaining power, human capital accumulation and fertility.

Finally, we want to finish this section by comparing the performances of our models and

the original model by Prettner and Strulik (2016). To do so, we depict human capital and fertility over time for all three models. The equations that are applied in the simulation are equation (17) using equation (6) for human capital in model 1 respectively (13) in model 2 and equations (7) and (12) for fertility in model 1 respectively in model 2. The equations for human capital and fertility from the original model are taken directly from Prettner and Strulik (2016)[9] The results are displayed in Figure (11).

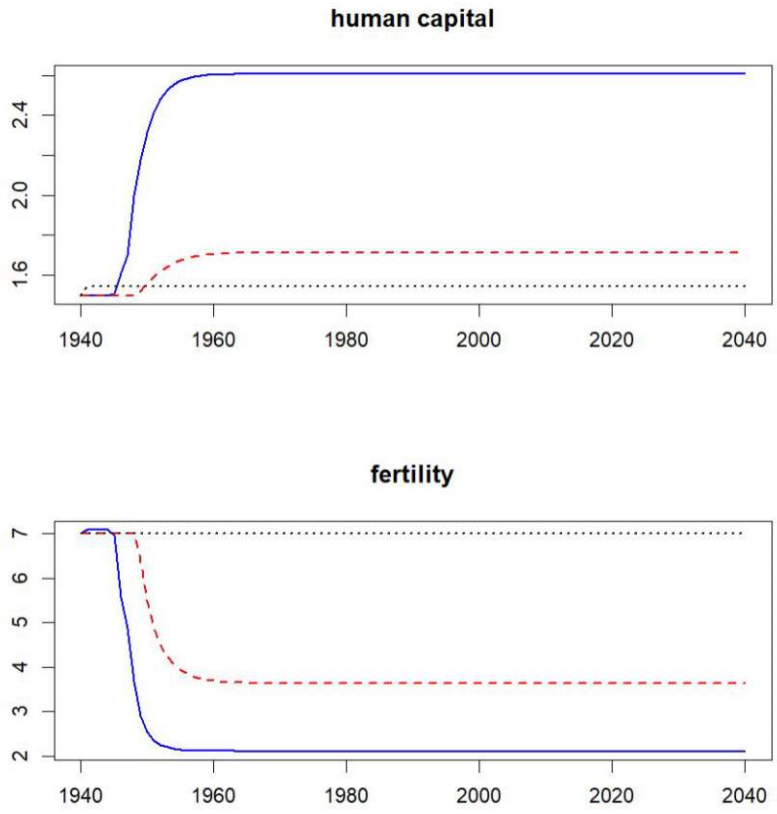

Abbildung 11: Comparison of Model 1 *(black, dotted line)*, Model 2 *(red, dashed line)* and the original model by Prettner and Strulik (2016) *(blue, full line)*. All models are run in the baseline set-up with a level of female bargaining power of 0.3. The human capital of the original model is the average of female and male human capital. (own illustration).

We have already discussed the differences between the models, their advantages and their failures. Particularly model 1 has the worst performance with it's constant fertility and the low growth in human capital in the direct comparison of all three models.

In the competition between model 2 and the original model, the original model is better off quantitatively. Moreover, if one is interested in further features like the difference in schooling of boys and girls, the original model offers the best results and insights. However, model 2 seems to display the same qualitative behavior as the original model. Especially the interplay of our variables of interest, namely female bargaining power, fertility and human capital can be clearly understood by the simpler set-up of model

[9]The optimal level of consumption, fertility and education from Prettner and Strulik (2016) are stated in the Appendix, section E.

2. Hence, we suggest that the simplification of the original model through model 2 is sufficient for our purposes and displays the role of female bargaining power well.

4 Conclusion

Despite the progress that has been made during the past century, gender equality is still not reached. The gender equality index reports that even the country with the highest level of gender equality, which is Sweden, reaches a score of 83,9 from 100 and hence can not provide full gender equality (European Institute for Gender Equality, 2022a). In this thesis, we have discussed different ways by which gender-based inequality may effect the economy. Particularly, the aim of the thesis was to discuss how the interplay between gender-based inequality and the economy can be formalized mathematically.

The literature review of the master thesis starts with a detailed discussion of the paper by Galor and Weil (1996), which was referred to as the baseline model. In the baseline model, gender inequality is measured by the gender wage gap. Two economic facts are used to model the feedback mechanism between gender inequality and economic performance: First the capital dilution effect, which explains the negative effect of population growth on capital per worker (Solow, 1956). Second, the fact that the cost for having children are foregone female wages and as a consequence, an increase in female wages lowers fertility which is in line with empirical data (Becker, 1960). The third piece of the model is the "brain versus brawn" theory. It argues that men have a comparative advantage to women in physical strength but both sexes can supply the same amount of mental labour input (Boserup, 1970). As the economy develops from pre-industrial to an industrialized economy, brain gets rewarded more than brawn and the wage gap closes as women have better earning opportunities. In turn, the opportunity costs for having children increase and fertility decreases. Consequently, capital per worker increases and stimulates economic development, which again closes the gender wage gap. This feedback loop is found to be crucial in modelling the impact of gender inequality and economic development. Mathematically, the feedback loop is formalized by a household model within an OLG framework. Households decide on the number of children and their consumption in order to maximize their utility but have to face a budget constraint. Their income is determined in the labour market and the economy is modelled with a CES production function for physical capital and mental labour, while physical labour is neither a substitute nor a complement. The optimization problem is solved with the Karush-Kuhn-Tucker theorem.

The literature review continues with studies that model the interplay between gender-based discrimination and the economy. Based on the feedback mechanism of the benchmark model, household bargaining models with an OLG approach are discussed. The bargaining power of a spouse is considered to depend on his or her wages or respectively his or her wealth. We focused on female bargaining power in the private sphere and in the public sphere. Moreover, we put special emphasis on female health, considering that a healthy woman has better earning possibilities respectively that health is part of wealth and hence puts women into a better bargaining position. The results show that a higher female bargaining power, both in the private and in the public sphere as well as through better health condition of the woman, leads to an increase in the human capital of the offspring and a decrease in fertility. As a result, if the economy is developed enough to be

human capital driven, then a rise in female bargaining power stimulates economic growth and has the potential to spur the demographic transition. Additionally, we have shown how the emergence of female franchise can be explained and modelled. The basic idea is that men "donate" women political rights since women have a positive influence on the human capital of their children.

In the third part of the thesis, the aim was to define a simple model that was capable of replicating stylized facts and was in line with the data. Following the work of Prettner and Strulik (2016), we set up two reduced models and compared them with the original model. The model is a household bargaining model in which spouses have sex-specific preferences and utilities. The household tries to maximize the household's utility, which is a weighted sum of the sex-specific utilities and the weights represent the bargaining power of the spouses. The spouses decide jointly on fertility, consumption and education of their children. The optimization problem is solved by using the Lagrange theorem.

We found that introducing a baseline level of education into the model, i.e. a minimum level of education children can get by just observing their parents, is crucial in order to simulate empirical data. The study of the models has shown analytically and numerically that female empowerment increases ceteris paribus investments in education and personal consumption while it lowers fertility. The effect of female empowerment is enforced if the preferences between the spouses differ substantially which is the case for many developing countries.

In summary, the thesis provides a sufficiently detailed report and discussion of models that explain the impact of gender-based inequality or vice versa of female empowerment on the economic performance. The positive effect of female empowerment is found to stimulate human capital accumulation and lower fertility. The same effect is found if female health conditions are improved. Particularly developing countries, which already rely on human capital, could accelerate the demographic transition by empowering women. Hence, political measures should target to increase female bargaining power within the private and the public sphere as well as through better health conditions for women.

An interesting question for future research would be to model the effects and causes of gender inequality in countries where a high degree of gender equality is already observed. For example, more research is needed on the unequal division of unpaid labour between men and women and its consequences since in no country in the world men and women perform the same amount of unpaid work (Addati et al., 2018) even though unpaid work would account for one fifth of the total GDP in the United States (UN Women, 2023). In addition, the Covid pandemic exacerbated the situation (Collins et al., 2021). Another topic that should be taken into account is the gender inequality in the STEM field since women are strongly underrepresented there (Elsevier-Amsterdam, 2017). Especially in times of digitalization, the influence of gender inequality on the digital, social and economic world should be investigated. Finally, the effect of female empowerment on the environment could be worthwhile to trace since women are found to value environmental sustainability more than men (Meinzen-Dick et al., 2014). In times of climate crisis, we see great potential for further research on the interplay of female empowerment and environmental sustainability.

Appendix

A The derivative of $G(k_t, zn_t)$ in section 2.2

We want to use the implicit function theorem in order to know that there exists an unique, differentiable function ψ such that

$$G(k_t, zn_t) = 0 \Leftrightarrow zn_t = \psi(k_t)$$

Let k^* and zn_t^* denote the level of capital and child-rearing time for which $G = 0$. In order to be allowed to apply the theorem, the function G has to be continuously differentiable and it's derivative with respect to n_t has to be invertible at (k_t^*, zn_t^*) .

The derivative of G with respect to n_t is given by

$$\frac{\partial G}{\partial n_t} = \frac{\partial}{\partial n_t}\left(zn_t - C \cdot \frac{(2-zn_t)^{1-\rho}}{[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}}\right)$$

$$= z - (C\frac{-z(1-\rho)(2-zn_t)^{-\rho}[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}}{([\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}})^2}$$

$$- \frac{-z(1-\alpha)\rho(2-zn_t)^{\rho-1}(2-zn_t)^{1-\rho}[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}-1}}{([\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}})^2})$$

$$= z + z \cdot C \cdot (1-\rho) \cdot \frac{(2-zn_t)^{-\rho} - (1-\alpha)[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{-1}}{[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}}$$

$$= z + z \cdot C \cdot (1-\rho) \cdot \left(\frac{1}{(2-zn_t)^\rho[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}} - \frac{(1-\alpha)}{[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1}{\rho}}}\right)$$

where $\frac{\gamma b}{a(1-\alpha)} := C$.

To prove whether the derivative is continuous, we use the the fact that for real-valued, continuous functions f and g, and scalars $\lambda \in \mathbb{R}$ it holds that $f+\lambda g$ and fg are continuous functions. Moreover, for $g(x) \neq 0$ it holds that $x \to \frac{f(x)}{g(x)}$ is a continuous function.

This means, we have to show that $(2-zn_t)^\rho[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho] \neq 0$ and $[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho] \neq 0$. We note that $zn_t \leq 1$ which implies that $2-zn_t > 0$, i.e. strictly positive.
α is a strictly positive parameter between zero and one. Thus, $\alpha k_t \geq 0$ if and only if $k_t \geq 0$. As a result, the denominators of $\frac{1}{(2-zn_t)^\rho[\alpha k_t^\rho+(1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}}$ and $\frac{(1-\alpha)}{[\alpha k_t^\rho+(1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1}{\rho}}}$ are the sum of one strictly positive and one positive term each and hence not equal to zero for any levels of $k_t \geq 0$ and n_t. Therefore, the derivative of G with respect to n_t is the combination of continuous functions and thus continuous itself.

Next, we want to prove that the derivative of G with respect to n_t is invertible. Let the

function $\frac{\partial G}{\partial n_t}$ be denoted as $g(k_t, zn_t)$. In order to prove that $g(k_t, zn_t)$ is invertible at (k_t^*, zn_t^*), we have to show that is not equal to zero at (k_t^*, zn_t^*).
We distinguish between the following cases:

- Case 1: $k_t = 0$
  We plug $k_t = 0$ into $g(k_t, zn_t)$:

$$g(0, zn_t) = z + z \cdot C \cdot (1-\rho) \cdot \left(\frac{1}{(2-zn_t)^\rho [(1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}} - \frac{(1-\alpha)}{[(1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1}{\rho}}} \right)$$

$$= z + z \cdot C \cdot (1-\rho) \cdot \left(\frac{1}{(1-\alpha)^{\frac{1-\rho}{\rho}} \cdot (2-zn_t)} - \frac{1-\alpha}{(1-\alpha)^{\frac{1}{\rho}}(2-zn_t)} \right)$$

$$= z + z \cdot C \cdot (1-\rho) \cdot \left(\frac{1}{2-zn_t} \cdot \left(\frac{1}{(1-\alpha)^{\frac{1-\rho}{\rho}}} - \frac{1}{(1-\alpha)^{\frac{1-\rho}{\rho}}} \right) \right)$$

$$= z + z \cdot C \cdot (1-\rho) \cdot 0 = z > 0$$

  where the last inequality holds since z is a positive parameter. Hence, $g(k_t, zn_t)$ is strictly positive for any zn_t from our definition set and for $k_t = 0$.

- Case 2: $k_t \to \infty$, $\rho \in (0,1)$
  First, we remark again that $2 - zn_t$ is bounded and thus a negligible scalar in taking the limits. For any $zn_t \in (0,1]$ we find that

$$\lim_{k_t \to \infty} g(k_t, zn_t) = z + z \cdot C \cdot (1-\rho) \cdot \Big(\lim_{k_t \to \infty} \frac{1}{(2-zn_t)^\rho [\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1-\rho}{\rho}}}$$

$$- \lim_{k_t \to \infty} \frac{(1-\alpha)}{[\alpha k_t^\rho + (1-\alpha)\cdot(2-zn_t)^\rho]^{\frac{1}{\rho}}} \Big)$$

$$= z + z \cdot C \cdot (1-\rho) \cdot (0-0)$$

$$= z > 0$$

We conclude that g is strictly positive for any $k_t \geq 0$ and for any zn_t in our definition set.
In particular, it holds that $g(k_t^*, zn_t^*) \neq 0$. Thus, all assumptions made in the implicit function theorem are fulfilled and we are allowed to apply the theorem.

B Derivation of Optima: Model 1

We want to maximize the household's utility function U_t in equation (2) restricted to a budget constraint in equation (3) by deriving the optimal levels of consumption c_t, number of children n_t and investment in human capital per child e_t. α and γ are the utility weights

of number and education of children, respectively. To rule out, that education is positive even if the number of children is zeor, we assume $\alpha > \gamma$. θ denotes the bargaining power of men m, hence $1-\theta$ the bargaining power of women f within the household. Note that here bargaining power is not endogenous yet. The time requirement that is needed for child rearing is given by ψ. The household's utility function is a weighted sum of the individual's utility functions $u_{t,i}$.

$$u_{t,i} = \ln c_t + \alpha_i \ln n_t + \gamma_i \ln e_t, \quad \alpha_i > \gamma_i, \quad i \in \{m, f\} \tag{1}$$

$$\max \; U_t = \theta u_{t,m} + (1-\theta) u_{t,f} \tag{2}$$

$$\text{s.t.} \; w_t[(1-\psi n_t)(h_{t,m} + h_{t,f})] = e_t n_t + c_t \tag{3}$$

We do so by applying Lagrange theorem. The Lagrangian function is given by

$$\begin{aligned}\mathcal{L} =& \theta \left(\ln c_t + \alpha_m \ln n_t + \gamma_m \ln e_t\right) + (1-\theta)\left(\ln c_t + \alpha_f \ln n_t + \gamma_f \ln e_t\right) + \\ &+ \lambda \left(w_t[(1-\psi n_t)(h_{t,m} + h_{t,f})] - e_t n_t - c_t\right)\end{aligned} \tag{4}$$

The derivations with respect to the decision variables are

$$\mathcal{L}_{c_t} = \frac{1}{c_t} - \lambda = 0 \tag{5}$$

$$\mathcal{L}_{n_t} = \theta \alpha_m \frac{1}{n_t} + (1-\theta)\alpha_f \frac{1}{n_t} - \lambda w_t \psi (h_{t,m} + h_{t,f}) - \lambda e_t = 0 \tag{6}$$

$$\mathcal{L}_{e_t} = \theta \gamma_m \frac{1}{e_t} + (1-\theta)\gamma_f \frac{1}{e_t} - \lambda n_t = 0 \tag{7}$$

$$\mathcal{L}_{\lambda} = w_t[(1-\psi n_t)(h_{t,m} + h_{t,f})] - e_t n_t - c_t = 0 \tag{8}$$

Solving the equations system (5) to (8) yields the optimal paths of n_t, c_t and e_t with respect to the parameters α, γ and ψ, and the given levels of wage w_t and human capital of parents $h_{t,i}$.
First of all, we get from (7)

$$n_t = \frac{1}{\lambda e_t}\left(\theta(\gamma_m - \gamma_f) + \gamma_f\right)$$

Plugging this identity into (6):

$$\theta \alpha_m \frac{\lambda e_t}{(\theta(\gamma_m - \gamma_f) + \gamma_f)} + (1-\theta)\alpha_f \frac{\lambda e_t}{(\theta(\gamma_m - \gamma_f) + \gamma_f)} - \lambda w_t \psi (h_{t,m} + h_{t,f}) - \lambda e_t = 0$$

We divide through λ, put $\lambda w_t \psi (h_{t,m} + h_{t,f})$ on the right side and rewrite some terms to get

$$e_t \frac{\theta(\alpha_m - \alpha_f) + \alpha_f}{\theta(\gamma_m - \gamma_f) + \gamma_f} - e_t = w_t \psi (h_{t,m} + h_{t,f})$$

We define $q := \frac{\theta(\alpha_m - \alpha_f) + \alpha_f}{\theta(\gamma_m - \gamma_f) + \gamma_f}$ and find

$$e_t(q-1) = w_t \psi (h_{t,m} + h_{t,f})$$

Hence, we get that the solution for the investment in human capital e_t is

$$e_t = \frac{w_t \psi (h_{t,m} + h_{t,f})}{q - 1}$$

Next, we rewrite n_t with the solution of e_t, i.e.

$$n_t = \frac{1}{\lambda} \frac{q-1}{w_t \psi (h_{t,m} + h_{t,f})} \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right)$$

Moreover, we see from equation (5) that $c_t = \frac{1}{\lambda}$. Thus, every variable is either written in terms of λ and parameters or has already a solution that is given solely by parameters and given variables like e_t. Hence, we can substitute all target variables in equation (8) to find the solution to λ

$$w_t[(1 - \psi \frac{1}{\lambda} \frac{q-1}{w_t \psi (h_{t,m} + h_{t,f})} \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right))(h_{t,m} + h_{t,f})]$$
$$- \frac{w_t \psi (h_{t,m} + h_{t,f})}{q-1} \frac{1}{\lambda} \frac{q-1}{w_t \psi (h_{t,m} + h_{t,f})} \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right) - \frac{1}{\lambda} = 0$$

A lot of terms cancel out. Additionally, we substract $w_t(h_{t,m} + h_{t,f})$ to be on the right side, multiplicate the equation by -1 and receive the equivalent equation

$$\frac{1}{\lambda} \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right)(q-1) + \frac{1}{\lambda} \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right) + \frac{1}{\lambda} = w_t(h_{t,m} + h_{t,f}) \tag{9}$$

$$\iff \frac{1}{\lambda} [q \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right) + 1] = w_t(h_{t,m} + h_{t,f}) \tag{10}$$

We know that

$$q \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right) = \frac{\theta(\alpha_m - \alpha_f) + \alpha_f}{\theta(\gamma_m - \gamma_f) + \gamma_f} \cdot \left(\theta(\gamma_m - \gamma_f) + \gamma_f\right) = \theta(\alpha_m - \alpha_f) + \alpha_f$$

Using this in equation (10) we get as a result

$$\frac{1}{\lambda} = \frac{w_t(h_{t,m} + h_{t,f})}{\theta(\alpha_m - \alpha_f) + \alpha_f + 1}$$

Hence, the optimal level of consumption in period t is given by

$$c_t = \frac{w_t(h_{t,m} + h_{t,f})}{\theta\alpha_m + (1-\theta)\alpha_f + 1}$$

Last, we derive the solution for optimal number of children in period t by using all results

$$
\begin{aligned}
n_t &= \frac{1}{\lambda}\frac{q-1}{w_t\psi(h_{t,m}+h_{t,f})}\left(\theta(\gamma_m-\gamma_f)+\gamma_f\right) \\
&= \frac{w_t(h_{t,m}+h_{t,f})}{\theta(\alpha_m-\alpha_f)+\alpha_f+1}\cdot\frac{q-1}{w_t\psi(h_{t,m}+h_{t,f})}\cdot\left(\theta(\gamma_m-\gamma_f)+\gamma_f\right) \\
&= \frac{\left(\theta(\gamma_m-\gamma_f)+\gamma_f\right)}{\psi(\theta(\alpha_m-\alpha_f)+\alpha_f+1)}\cdot(q-1) \\
&= q\frac{\left(\theta(\gamma_m-\gamma_f)+\gamma_f\right)}{\psi(\theta(\alpha_m-\alpha_f)+\alpha_f+1)} - \frac{\left(\theta(\gamma_m-\gamma_f)+\gamma_f\right)}{\psi(\theta(\alpha_m-\alpha_f)+\alpha_f+1)} \\
&= \frac{\theta(\alpha_m-\alpha_f)+\alpha_f}{\theta(\gamma_m-\gamma_f)+\gamma_f}\cdot\frac{\left(\theta(\gamma_m-\gamma_f)+\gamma_f\right)}{\psi(\theta(\alpha_m-\alpha_f)+\alpha_f+1)} - \frac{\left(\theta(\gamma_m-\gamma_f)+\gamma_f\right)}{\psi(\theta(\alpha_m-\alpha_f)+\alpha_f+1)} \\
&= \frac{\theta(\alpha_m-\alpha_f)+\alpha_f-\theta(\gamma_m-\gamma_f)-\gamma_f}{\psi(\theta(\alpha_m-\alpha_f)+\alpha_f+1)} \\
&= \frac{\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f)}{\psi(\theta\alpha_m+(1-\theta)\alpha_f+1)}
\end{aligned}
$$

Finally, we want to rewrite e_t without using the definition of q in order to have it in style of the other target variables. This helps us to point out the role of the bargaining power θ and $(1-\theta)$

$$
\begin{aligned}
e_t &= \frac{w_t\psi(h_{t,m}+h_{t,f})}{q-1} \\
q-1 &= \frac{\theta(\alpha_m-\alpha_f)+\alpha_f}{\theta(\gamma_m-\gamma_f)+\gamma_f} - 1 = \\
&= \frac{\theta\alpha_m+(1-\theta)\alpha_f-\theta\gamma_m-(1-\theta)\gamma_f}{\theta\gamma_m+(1-\theta)\gamma_f} = \\
&= \frac{\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f)}{\theta\gamma_m+(1-\theta)\gamma_f} \\
\Rightarrow e_t &= \frac{w_t\psi(h_{t,m}+h_{t,f})\cdot[\theta\gamma_m+(1-\theta)\gamma_f]}{\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f)}
\end{aligned}
$$

Summed up, we find the following optimal levels for fertility n_t, education per child e_t and parental consumption c_t in period t

$$n_t = \frac{\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)}{\psi(\theta\alpha_m + (1-\theta)\alpha_f + 1)}$$

$$e_t = \frac{w_t\psi(h_{t,m} + h_{t,f}) \cdot [\theta\gamma_m + (1-\theta)\gamma_f]}{\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)}$$

$$c_t = \frac{w_t(h_{t,m} + h_{t,f})}{\theta\alpha_m + (1-\theta)\alpha_f + 1}$$

C Lagrange system of Model 2 and explanation of the solution

We want to solve the following optimization problem

$$\begin{aligned} \max \;\; & U_t = \theta\left(\ln c_t + \alpha_m \ln n_t + \gamma_m \ln(e_t + \bar{e})\right) + (1-\theta)\left(\ln c_t + \alpha_f \ln n_t + \gamma_f \ln(e_t + \bar{e})\right) \\ \text{s.t.} \;\; & w_t[(1 - \psi n_t)(h_{t,m} + h_{t,f})] = e_t n_t + c_t \end{aligned}$$

where the decision variables are c_t, e_t and n_t.

The Lagrange system of model 2 is given by

$$\begin{aligned} \mathcal{L} =& \theta\left(\ln c_t + \alpha_m \ln n_t + \gamma_m \ln(e_t + \bar{e})\right) + (1-\theta)\left(\ln c_t + \alpha_f \ln n_t + \gamma_f \ln(e_t + \bar{e})\right) + \\ & + \lambda\left(w_t[(1 - \psi n_t)(h_{t,m} + h_{t,f})] - e_t n_t - c_t\right) \end{aligned}$$

The derivatives of the Lagrange function with respect to the decision variables are

$$\begin{aligned} \mathcal{L}_{c_t} &= \frac{1}{c_t} - \lambda = 0 \\ \mathcal{L}_{n_t} &= \theta(\alpha_m - \alpha_f)\frac{1}{n_t} + \alpha_f \frac{1}{n_t} - \lambda w_t \psi(h_{t,m} + h_{t,f}) - \lambda e_t = 0 \\ \mathcal{L}_{e_t} &= \theta(\gamma_m - \gamma_f)\frac{1}{e_t + \bar{e}} + \gamma_f \frac{1}{e_t + \bar{e}} - \lambda n_t = 0 \\ \mathcal{L}_{\lambda} &= w_t[(1 - \psi n_t)(h_{t,m} + h_{t,f})] - e_t n_t - c_t = 0 \end{aligned}$$

Following the same steps as in section B, we find the following solutions

$$c^*_{2,t} = \frac{w_t(h_{t,m} + h_{t,f})}{\theta\alpha_m + (1-\theta)\alpha_f + 1}$$

$$n^*_{2,t} = \frac{w_t(h_{t,m} + h_{t,f})(\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f))}{(\psi w_t(h_{t,m} + h_{t,f}) - \bar{e})(\theta\alpha_m + (1-\theta)\alpha_f + 1)}$$

$$e^*_{2,t} = \frac{\psi w_t(h_{t,m} + h_{t,f})(\theta\gamma_m + (1-\theta)\gamma_f) - \bar{e}(\theta\alpha_m + (1-\theta)\alpha_f)}{\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)}$$

Now we want to explain intuitively, why we distinguish between different cases in the final solution from section 3.1.2. $e^*_{2,t}$ is negative for some values of w_t. To be more precise, if the wage is below the threshold level $\hat{w}$ (see section D), investments in education are negative, i.e. $w < \hat{w}_t \Leftrightarrow e^*_{2,t} < 0$. We therefore set $e^*_{2,t} = 0$ for $w < \hat{w}_t$. As a result, we obtain

$$e^*_{2,t} = \begin{cases} 0, & w_t \leq \hat{w} \\ \frac{\psi w_t(h_{t,m}+h_{t,f})(\theta\gamma_m+(1-\theta)\gamma_f)-\bar{e}(\theta\alpha_m+(1-\theta)\alpha_f)}{\theta(\alpha_m-\gamma_m)+(1-\theta)(\alpha_f-\gamma_f)}, & w_t > \hat{w}, \end{cases}$$

Since investments in education are zero for wages below the threshold level, households share their income between personal consumption and fertility in this case. Consumption does not depend on the stage of development as described in section 2.3.1. Therefore, all income that is not spent on consumption is invested in fertility as long as educational investments are zero. In other words, investments in the number of children is not reduced by investments in the education of children. Intuitively this means that we do not have the discounting factor of γ_i, which denotes the preference of parents towards the education of their offspring, in the solution of n_t and explains the case distinction in equation (12).

D Derivation the threshold level for wage $\hat{w}$

The threshold level for wage $\hat{w}$ is defined as the minimum level of wage that is needed to start investing in the education of children. Solving the Lagrange system for model 2 yields the optimal level of education as given by

$$e^*_{2,t} = \frac{\psi w_t(h_{t,m} + h_{t,f})(\theta\gamma_m + (1-\theta)\gamma_f) - \bar{e}(\theta\alpha_m + (1-\theta)\alpha_f)}{\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)}$$

In order to find the level of wage for which $e^*_{2,t}$ turns positive, we solve

$$\frac{\psi \hat{w}_t(h_{t,m} + h_{t,f})(\theta\gamma_m + (1-\theta)\gamma_f) - \bar{e}(\theta\alpha_m + (1-\theta)\alpha_f)}{\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)} = 0$$

We multiply the equation with $\theta(\alpha_m - \gamma_m) + (1-\theta)(\alpha_f - \gamma_f)$, which is strictly positive as we assume that $\alpha_m > \alpha_f > \gamma_f > \gamma_m$. Doing simple transformations we find

$$\psi \hat{w}_t(h_{t,m} + h_{t,f})(\theta\gamma_m + (1-\theta)\gamma_f) - \bar{e}(\theta\alpha_m + (1-\theta)\alpha_f) = 0 \Leftrightarrow$$

$$\psi \hat{w}_t(h_{t,m} + h_{t,f})(\theta\gamma_m + (1-\theta)\gamma_f) = \bar{e}(\theta\alpha_m + (1-\theta)\alpha_f) \Leftrightarrow$$

$$\hat{w}_t = \frac{\bar{e}(\theta\alpha_m + (1-\theta)\alpha_f)}{\psi(h_{t,m} + h_{t,f})}$$

E Optimal levels of consumption, fertility and investments in education in the model by Prettner and Strulik (2016)

Here we state the solution to the optimization problem by Prettner and Strulik (2016), which is described in section 2.3.1 and section 3.1. The solutions are from Prettner and Strulik (2016), page 59 and 60.

$$c_{t,m} = \frac{\theta(h_{t,m} + h_{t,f})w_t}{1 + (1-\theta)\alpha_f + \theta\alpha_m}$$

$$c_{t,f} = \frac{(1-\theta)(h_{t,m} + h_{t,f})w_t}{1 + (1-\theta)\alpha_f + \theta\alpha_m}$$

$$n_t = \begin{cases} \frac{[(1-\theta)\alpha_f+\theta\alpha_m](h_{t,m}+h_{t,f})}{[1+(1-\theta)\alpha_f+\theta\alpha_m](\psi_m h_{t,m}+\psi_f h_{t,f})} & w_t \leq \hat{w}_{t,m} \\ \frac{2w_t(h_{t,m}+h_{t,f})[(1-\theta)\alpha_f-\gamma_f+\theta(\gamma_f+\alpha_m-\gamma_m)]}{[1+(1-\theta)\alpha_f+\theta\alpha_m][2w_t(\psi_m h_{t,m}+\psi_f h_{t,f})-\bar{e}]} & w_t \leq \hat{w}_{t,f} \\ \frac{w_t(h_{t,m}+h_{t,f})[(1-\theta)\alpha_f-\gamma_f-\delta_f+\theta(\gamma_f+\delta_f+\alpha_m-\gamma_m-\delta_m)]}{[1+(1-\theta)\alpha_f+\theta\alpha_m][w_t(\psi_m h_{t,m}+\psi_f h_{t,f})-\bar{e}]} & \text{otherwise} \end{cases}$$

$$e_{t,m} = \begin{cases} \frac{\bar{e}[\theta\alpha_m+(1-\theta)\alpha_f]+2w_t[(\theta-1)\gamma_f-\theta\gamma_m](\psi_m h_{t,m}+\psi_f h_{t,f})}{(\theta-1)\alpha_f+\gamma_f-\theta(\gamma_f+\alpha_m-\gamma_m)} & w_t \leq \hat{w}_{t,f} \\ \frac{\bar{e}[\theta(-\gamma_f+\delta_f+\alpha_m+\gamma_m-\delta_m)+(1-\theta)\alpha_f+\gamma_f-\delta_f]+2w_t[(\theta-1)\gamma_f-\theta\gamma_m](\psi_m h_{t,m}+\psi_f h_{t,f})}{(\theta-1)\alpha_f+\gamma_f+\delta_f-\theta(\gamma_f+\delta_f+\alpha_m-\gamma_m-\delta_m)} & w_t \leq \hat{w}_{t,f} \end{cases}$$

$$e_{t,f} = \begin{cases} 0 & w_t \leq \hat{w}_{t,m} \\ 0 & w_t \leq \hat{w}_{t,f} \\ \frac{\bar{e}[\theta(\gamma_f-\delta_f+\alpha_m-\gamma_m+\delta_m)+(1-\theta)\alpha_f-\gamma_f+\delta_f]+2w_t[(\theta-1)\delta_f-\theta\delta_m](\psi_m h_{t,m}+\psi_f h_{t,f})}{(\theta-1)\alpha_f+\gamma_f+\delta_f-\theta(\gamma_f+\delta_f+\alpha_m-\gamma_m-\delta_m)} & w_t \leq \hat{w}_{t,f} \end{cases}$$

F Exemplary Code used for the simulations in R

In this section we want to show an exemplary piece of code that is applied to simulate the fertility and human capital from model 2. Figure 10 is produced within this code.

```
#Parameters
alpham <-0.8 #children weight (male)
alphaf <-0.6 #children weight (female)
gammam <-0.3 #education weight male
gammaf <- 0.5  #education weight female
theta <- 0.7 #male bargaining power

psi <-0.06 #child rearing time
B <- 1.1 #schooling technology
ebar <- 1.5 #baseline education

eta1=0.007; #weight of population size for technological growth
eta2=0.3; #maximum technological growth

period <- c(1940:2040)

parameters <- c(alpham, alphaf,  gammam, gammaf,
                theta, psi, B, ebar, eta1, eta2)

#initial values

ninit=7;
hi.init=ebar; #hm
#hfinit=ebar; #hf
winit=5; #initial wage
hinit=2*hi.init #inital human cap per HH
incomeinit=hinit*winit #wh also income
popinit <- 2 #population size
einit <- 0

initval <- c(ninit, hi.init, winit, hinit, incomeinit, popinit, einit)

# calculation of optimal solution

model2 <- function(parameters, n.old, hi.old, w.old, pop.old){
  if (w.old <= (ebar*(theta*alpham+(1-theta)*alphaf)/(psi*(2*hi.old)*
     (gammam*theta+(1-theta)*gammaf)))){
    w = hi.old*(1-psi*n.old)*min(eta1*pop.old, eta2)*w.old + w.old
    n <- (theta*(alpham)+(1-theta)*(alphaf))*(2*hi.old)/
         (psi*2*hi.old*(theta*alpham+(1-theta)*alphaf+1))
    e <- 0
    hi <- ebar
    hbar = 2*hi.old*(1-psi*n)
    income = hbar *w
    pop = pop.old* n.old/2
    # solution <- list(c,n, hi.new, w.new, hbar.new, income, A.new)
    solution <- list(n,hi, w, hbar, income, pop, e)
  }
  else {
    w = hi.old*(1-psi*n.old)*min(eta1*pop.old, eta2)*w.old + w.old
```

```
    #c <- w*(hm+hf)/(theta*alpham + (1-theta)*alphaf+1)
    n <- w.old*2*hi.old*(theta*(alpham-gammam)+(1-theta)*(alphaf-gammaf
       ))/
         ((theta*alpham+(1-theta)*alphaf+1)*(-ebar+psi*w.old*(2*hi.old)
            ))
    e <- (psi*w.old*2*hi.old*(theta*gammam + (1-theta)*gammaf)-
          ebar*(theta*alpham+(1-theta)*alphaf))/
          (theta*(alpham-gammam)+(1-theta)*(alphaf-gammaf))
    hi <- max(B*e/w.old+ebar, ebar)
    hbar = 2*hi.old*(1-psi*n)
    income = hbar *w
    pop = pop.old* n.old /2
    # solution <- list(c,n, hi.new, w.new, hbar.new, income, A.new)
    solution <- list(n,hi, w, hbar, income, pop, e)
  }
  return(solution)
}

# initialize dataframe
solution2 <- data.frame(matrix(ncol = 7, nrow = length(period)))
colnames(solution2) <- c("fertility", "human capital", "wage", "hbar",
                         "income", "pop size", "investment in education")
solution2[1,] <- initval # set initial values

# filling the matrix with values;
#rows refer to time points, columns to variables
for (i in 2:nrow(solution2)){
  n.old <- solution2[i-1,1]
  hi.old <- solution2[i-1,2]
  w.old <- solution2[i-1,3]
  pop.old <- solution2[i-1,6]
  solution2[i,] <- model2(parameters, n.old, hi.old, w.old, pop.old)
}

### PLOTS
par(mfrow=c(2,2))
plot(period,solution2$`human capital`, type ='l',col='blue',lwd=1.5,
    xlab = "time", main  = "Human capital", ylab="")
plot(period,solution2$fertility, type ='l',col='blue',lwd=1.5,
    xlab = "time", main="Fertility",ylab="")
plot(period,solution2$`investment in education`, type ='l',col='blue',
    lwd=1.5, xlab = "time", main = "Investment in education",ylab="")
plot(period,solution2$wage, type ='l',col='blue',lwd=1.5,
    xlab = "time", main = "Wage",ylab="")
par(mfrow=c(1,1))
plot(period,solution2$income, type ='l',col='blue',lwd=1.5,
    xlab = "time", main = "Wages", ylab="")
```

Model 1 is implemented analogously. The referring equations are simpler as there is no case distinction. Hence the code that applies model 1 is shorter.

Literatur

Addati, L., Cattaneo, U., Esquivel, V., and Valarino, I. (2018). Care work and care jobs. for the future of decent work. International Labour Organisation Report.

Agénor, P.-R., Canuto, O., and Pereira da Silva, L. A. (2010). On gender and growth: the role of intergenerational health externalities and women's occupational constraints. World Bank Policy Research Working Paper, (5492).

Alesina, A., Giuliano, P., and Nunn, N. (2013). On the origins of gender roles: Women and the plough. The quarterly journal of economics, 2:469–530.

Becker, G. S. (1960). An economic analysis of fertility. demographic and economic change in developed countries. In NBER conference series, volume 11, pages 209–231.

Bertocchi, G. (2011). The enfranchisement of women and the welfare state. European Economic Review, 55(4):535–553.

Bloom, D. E., Kuhn, M., and Prettner, K. (2020). The contribution of female health to economic development. The Economic Journal, 130(630):1650–1677.

Boserup, E. (1970). Woman's Role in Economic Development. Routledge.

Chang-Tai Hsieh, Erik Hurst, C. I. J. and Klenow, P. J. (2019). The allocation of talent and u.s. economic growth. Econometrica, 87:1439–1474.

Collins, C., Landivar, L. C., Ruppanner, L., and Scarborough, W. J. (2021). Covid-19 and the gender gap in work hours. Gender, Work & Organization, 28:101–112.

Crotti, R., Pal, K. K., Ratcheva, V., and Zahidi, S. (2021). Global Gender Gap Report 2021. World Economic Forum.

De la Croix, D. and Vander Donckt, M. (2010). Would empowering women initiate the demographic transition in least developed countries? Journal of Human Capital, 4(2):85–129.

Deaton, A. (1997). The analysis of household surveys: a microeconometric approach to development policy. World Bank Publications.

Del Boca, D., Oggero, N., Profeta, P., and Rossi, M. (2020). Women's and men's work, housework and childcare, before and during covid-19. Review of Economics of the Household, 18:1001–1017.

Doepke, M. and Tertilt, M. (2009). Women's liberation: What's in it for men? The Quarterly Journal of Economics, 124(4):1541–1591.

Doepke, M. and Tertilt, M. (2019). Does female empowerment promote economic development? Journal of Economic Growth, 24(4):309–343.

Donni, O. and Ponthieux, S. (2011). Economic approaches to household behavior: from the unitary model to collective decisions. Travail, genre et sociétés, 26(2):67–83.

Elsevier-Amsterdam (2017). Gender in the Global Research Landscape: Analysis of Research Performance Through a Gender Lens Across 20 Years, 12 Geographies, and 27 Subject Ares. Elsevier.

Eswaran, M. (2002). The empowerment of women, fertility, and child mortality: Towards a theoretical analysis. Journal of Population Economics, 15(3):433–454.

European Comission (2023). The gender pay gap situation in the eu.

European Institute for Gender Equality (2022a). Gender equality index.

European Institute for Gender Equality (2022b). sex- and gender-based discrimination.

Fernández, R. and Fogli, A. (2009). Culture: An empirical investigation of beliefs, work, and fertility. American economic journal: Macroeconomics, 1(1):146–177.

Fortin, N. M. (2005). Gender role attitudes and the labour-market outcomes of women across oecd countries. oxford review of Economic Policy, 21(3):416–438.

Galor, O. and Weil, D. N. (1996). The gender gap, fertility, and growth. The American Economic Review, 86:374–387.

Georgieva, K., Sayeh, A. M., and Sahay, R. (2023). How to close gender gaps and grow the global economy.

Goldin, C. (1990). Understanding the gender gap: An economic history of American women. Number gold90-1. National Bureau of Economic Research.

Grimshaw, D. and Rubery, J. (2015). The motherhood pay gap: A review of the issues, theory and international evidence. Conditions of Work and Employment Series, (57).

Gupta, M. D. (1987). Selective discrimination against female children in rural punjab, india. Population and development review, pages 77–100.

Hiller, V. and Touré, N. (2021). Endogenous gender power: The two facets of empowerment. Journal of Development Economics, 149:102596.

Holly Corbet (2022). Mother's equal pay day 2022: How the pay gap impacts families.

Hornset, N. and Soysa, I. (2020). Does empowering women in politics boost human capital formation? an empirical analysis, 1960-2018.

Hull, T. H. (1990). Recent trends in sex ratios at birth in china. Population and Development Review, pages 63–83.

Ipsos-Austria (2023). Internationaler frauentag: Gleichstellung noch in weiter ferne.

Jayachandran, S. and Lleras-Muney, A. (2009). Life expectancy and human capital investments: Evidence from maternal mortality declines. The Quarterly Journal of Economics, 124(1):349–397.

Kassinis, G., Panayiotou, A., Dimou, A., and Katsifaraki, G. (2016). Gender and environmental sustainability: A longitudinal analysis. Corporate Social Responsibility and Environmental Management, 23(6):399–412.

Khera, P., Ng, M. S. Y., Ogawa, M. S., and Sahay, M. R. (2021). Is digital financial inclusion unlocking growth? International Monetary Fund.

Khera, P., Ogawa, S., Sahay, R., and Vasishth, M. (2022). The digital gender gap.

Kuroda, R., Lopez, M., Sasaki, J., and Settecase, M. (2019). The digital gender gap. W20 Japan, EY-GSMA.

Köppl-Turyna, M. (2019). Kinder machen den unterschied. Agenda Austria.

Lagerlöf, N.-P. (2006). The galor–weil model revisited: A quantitative exercise. Review of Economic dynamics, 9(1):116–142.

Leah Rodrigues (2020). Young men oppose feminism.

Lewis, D. A., Kamon, E., and Hodgson, J. L. (1986). Physiological differences between genders implications for sports conditioning. Sports medicine, 3:357–369.

Lim, H.-S. (2002). Low intakes of energy, folate, iron, and calcium of child-bearing korean women. Ecology of food and nutrition, 41(5):401–413.

McGuire, J. W. (2006). Basic health care provision and under-5 mortality: a cross-national study of developing countries. World Development, 34(3):405–425.

Meinzen-Dick, R., Kovarik, C., and Quisumbing, A. R. (2014). Gender and sustainability. Annual Review of Environment and Resources, 39:29–55.

Nagarajan, R. (2021). Women live longer, but don't have healthier lives.

OECD (2016). Fertility. OECD Factbook 2015-2016: Economic, Environmental and Social Statistics.

of Public Information, U. N. D. (2005). The Millennium Development Goals Report 2005. United Nations Publications.

Our World in Data (2022). Share of the labor force employed in agriculture.

Park, C. B. and Cho, N.-H. (1995). Consequences of son preference in a low-fertility society: Imbalance of the sex ratio at birth in korea. Population and development review, pages 59–84.

Perez, C. C. (2019). Invisible women: Data bias in a world designed for men. Abrams.

Prettner, K. and Strulik, H. (2016). Gender equity and the escape from poverty. Oxford Economic Papers, 69(1):55–74.

PWC (2023). Women in work index 2023.

PWC-Australia (2023). Australia in transition.

Sinha, N., Raju, D., and Morrison, A. (2007). Gender equality, poverty and economic growth. World Bank Policy Research Working Paper, (4349).

Solow, R. M. (1956). A contribution to the theory of economic growth. The quarterly journal of economics, 70:65–95.

UN Women (2023). Progress of the world's women 2019-2020.

UNDP (2012). Human development indices: a statistical update 2012, gender inequality index.

Weiss, M. (2019). Im Schatten Einsteins. Süddeutsche Zeitung.

Wolgast, E. H. (1958). Do husbands or wives make the purchasing decisions? Journal of Marketing, 23(2):151–158.

World Population Review (2023). Life expectancy by country.