Numerical investigation of mixed convection flow over a heated horizontal plate

AIC Project

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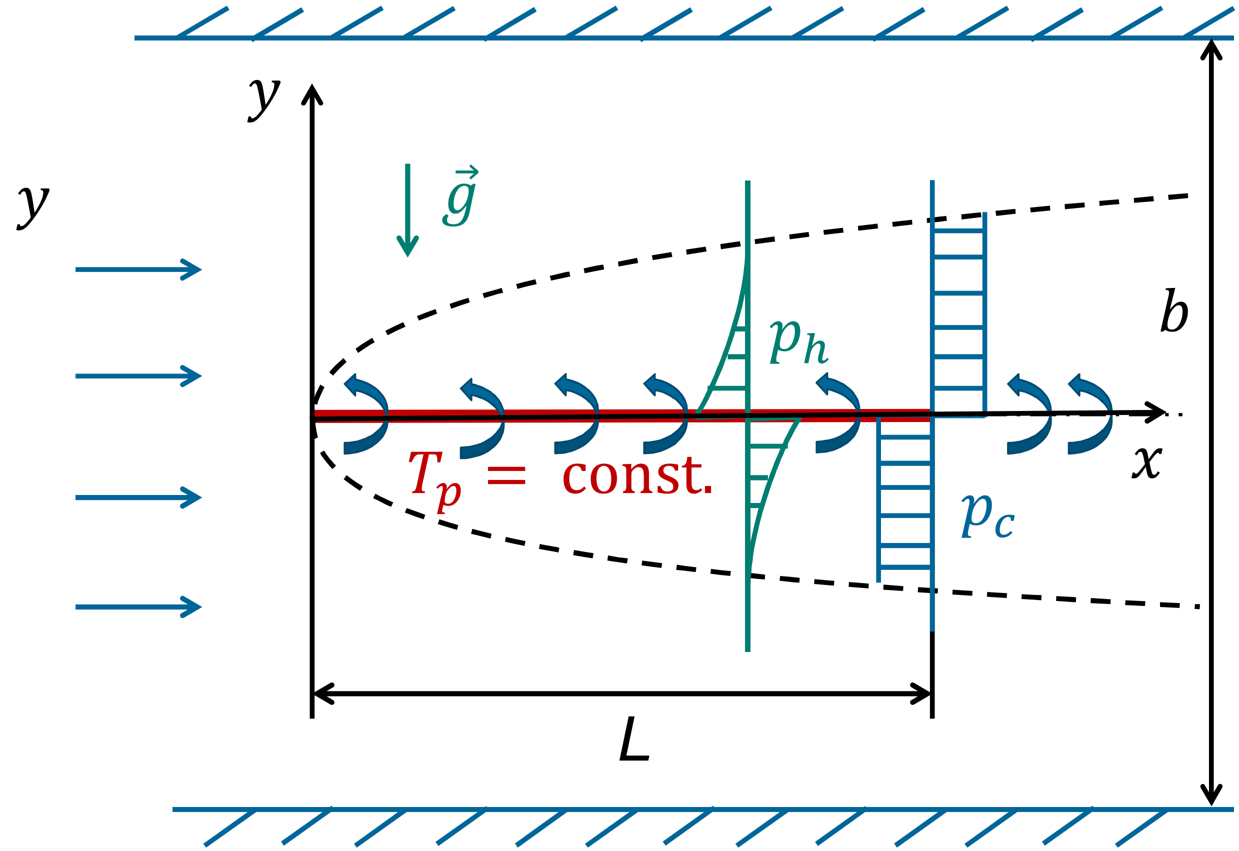
PROBLEM DESCRIPTION

- Kutta condition
- Wake
- Semi-infinite channel

M. Müllner and W. Schneider (2010),
Heat Mass Transf. **43**: 1097-1110



DIMENSIONLESS PARAMETERS

$$\mathrm{Pe}_L = \frac{u_\infty L}{\kappa} \gg 1$$

$$\mathrm{Pr} = \frac{\nu}{\kappa} \ll 1 \rightarrow \text{ inviscid}$$

$$\mathrm{Ri} = \frac{4}{\sqrt{\pi}} \frac{g\,\beta(T_p - T_\infty)\sqrt{\kappa L}}{\sqrt{u_\infty^5}}$$

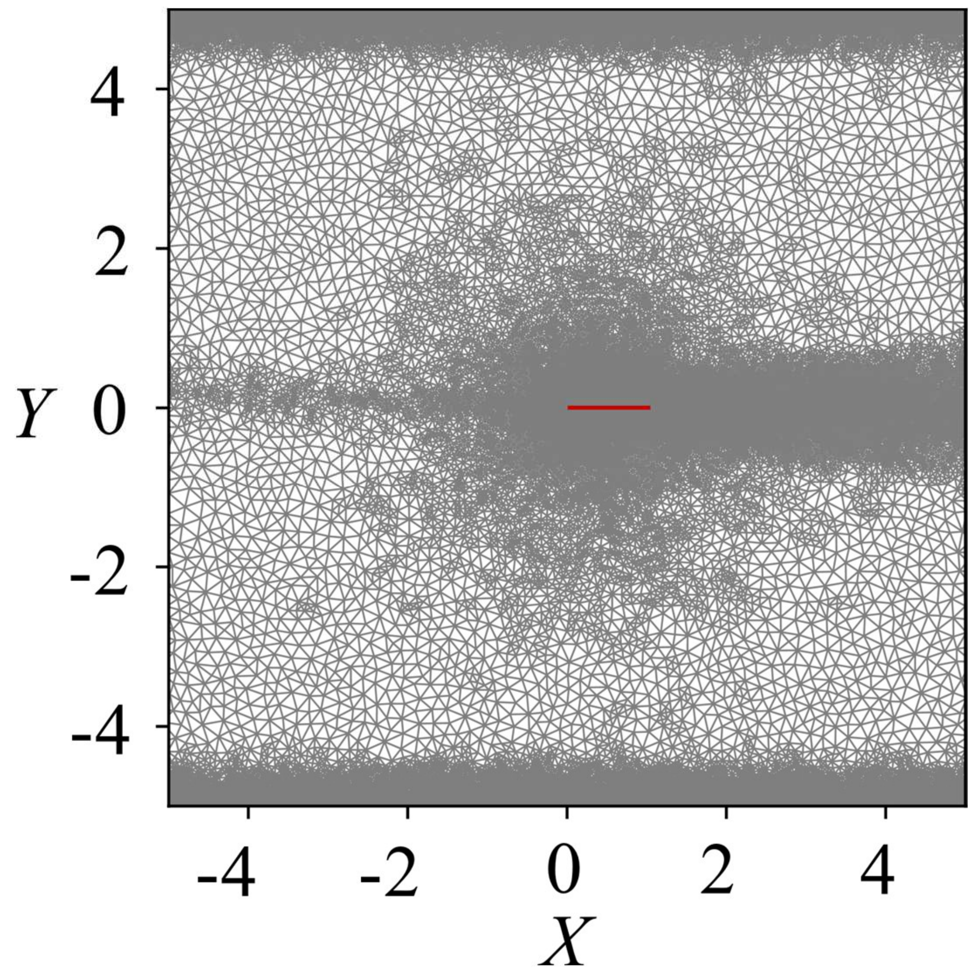
SCALING

$$(X, Y) = \frac{(x, y)}{L} \qquad P = \frac{p - p_\infty}{\rho u_\infty^2}$$

$$(U, V) = \frac{(u, v)}{u_\infty} \qquad \theta = \frac{T - T_\infty}{T_p - T_\infty}$$

NUMERICAL SOLUTION

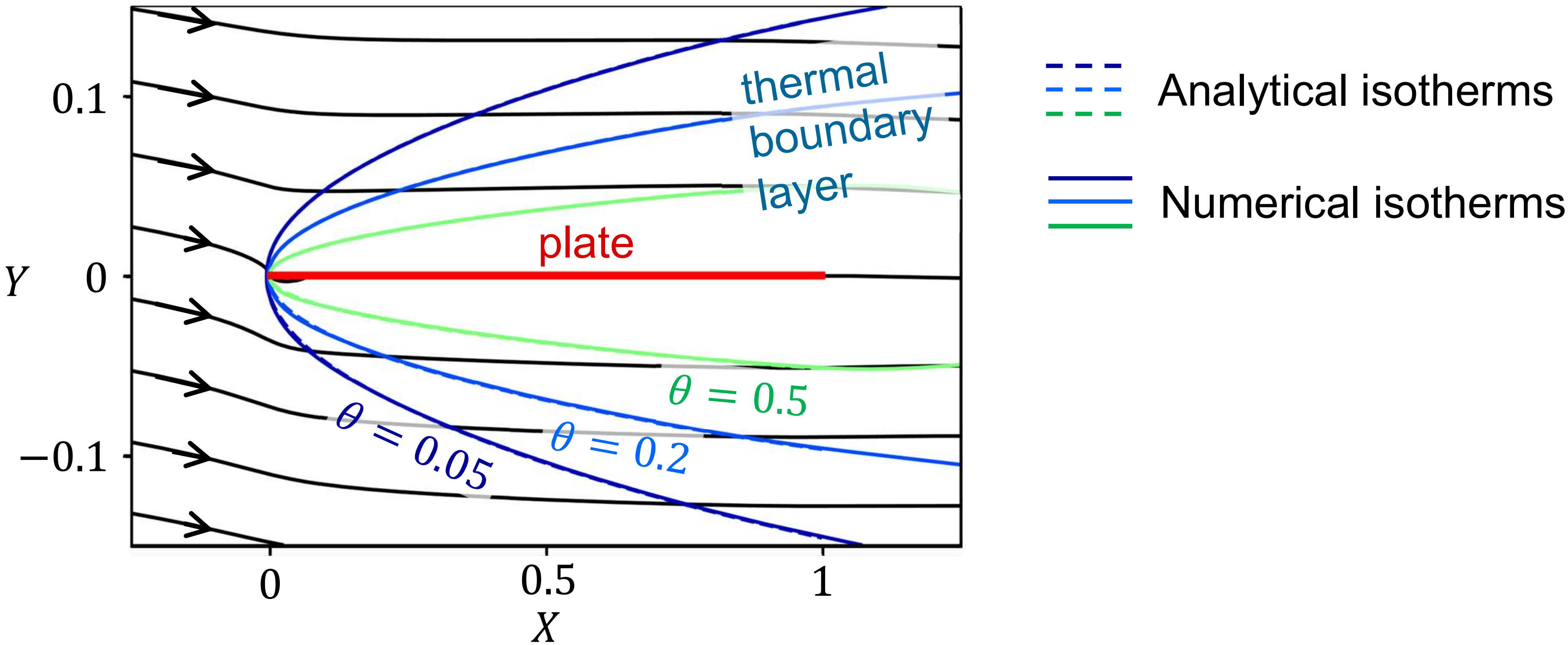
- 2D steady Navier-Stokes
- FEM-solver FEniCS
- 3rd order Taylor-Hood
- Plate thickness $10^{-3}L$, rounded edges
- Finite domain: $(-30, 30) \times (-b/2, b/2)$
- Outflow B.C.:
  - Start from standard
  - Adjust iteratively
- Adaptive mesh refinement



RESULTS

Streamlines and isotherms

$\mathrm{Pe}_L = 400, \mathrm{Pr} = 0.02, \mathrm{Ri} = 0.05, b = 20$



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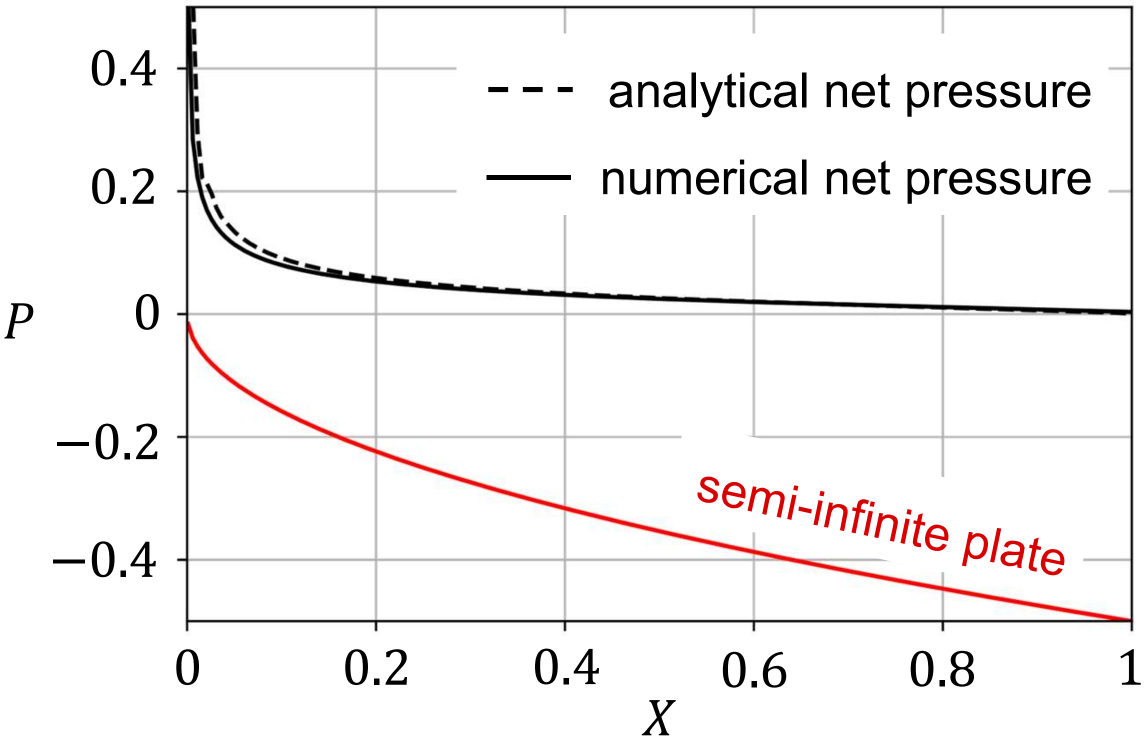
RESULTS

Pressure at the upper surface

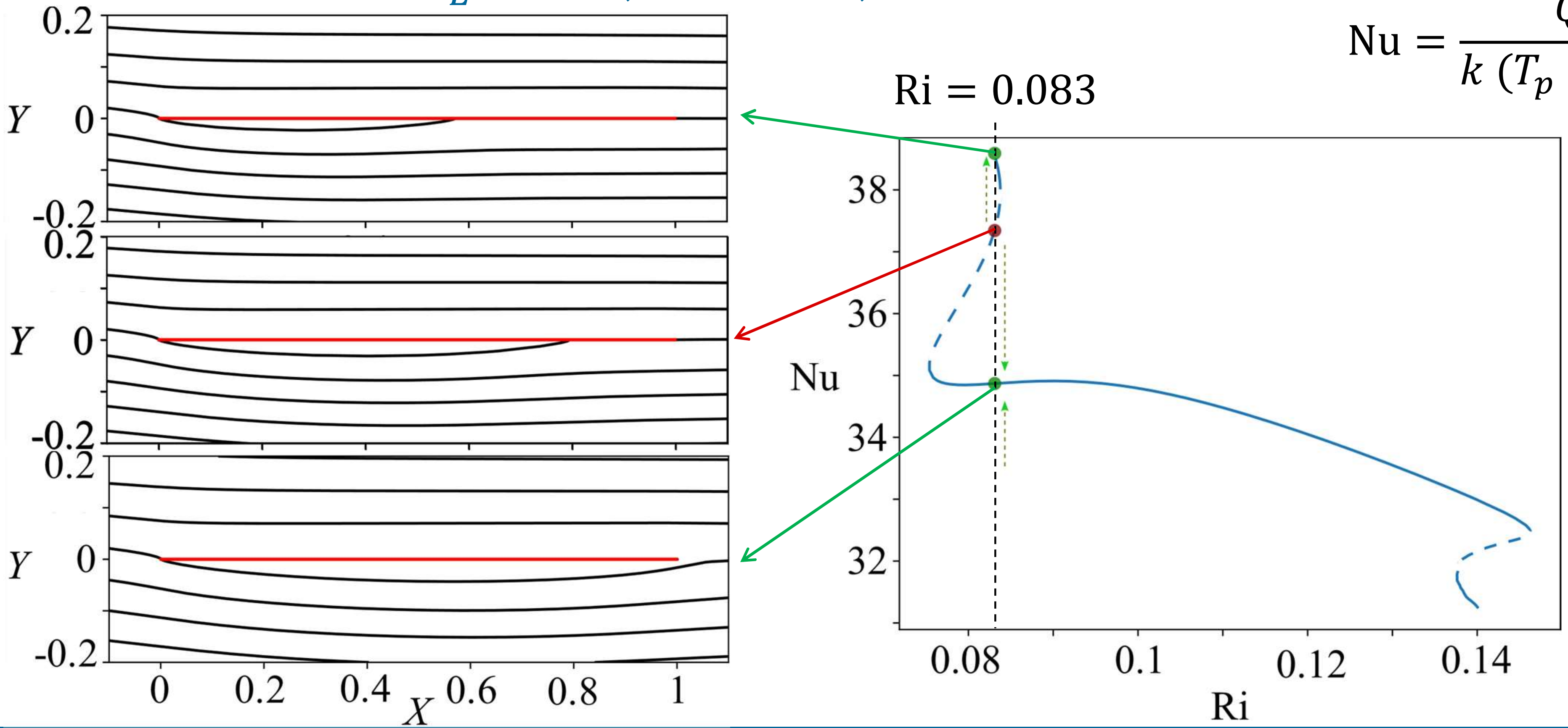
$\mathrm{Pe}_L = 400, \mathrm{Pr} = 0.02, \mathrm{Ri} = 0.05, b = 20$



NON-UNIQUENESS

$\mathrm{Pe}_L = 400, \mathrm{Pr} = 0.02, b = 10$

$$\mathrm{Nu} = \frac{\dot{Q}}{k\,(T_p - T_\infty)}$$



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CONCLUSIONS

- Lift force in the opposite direction than buoyancy
- Excellent agreement between numerical and analytical results for small Ri = 0.05
- Flow separation for higher Ri
- Non-unique solution in some ranges of Ri
- Stability to be investigated

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THANK YOU FOR YOUR ATTENTION!

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ITERATIVE OUTFLOW BOUNDARY CONDITION

- Goal: approximate a continuation of an infinite channel
- Start from the standard outflow boundary condition:
  - Free-slip
  - Far-field pressure p_∞
  - Homogeneous Neumann for temperature
- Problem:
  - Unphysical upward flow
  - Inconsistent pressure profile
- Iteratively adjust pressure to account for buoyancy:

$$p(y) = \frac{\sqrt{\pi}}{4} \mathrm{Ri} \sqrt{\mathrm{Pe}} \int_{-\frac{b}{2}}^{y} \theta(\bar{y}) \, \mathrm{d}\bar{y}$$

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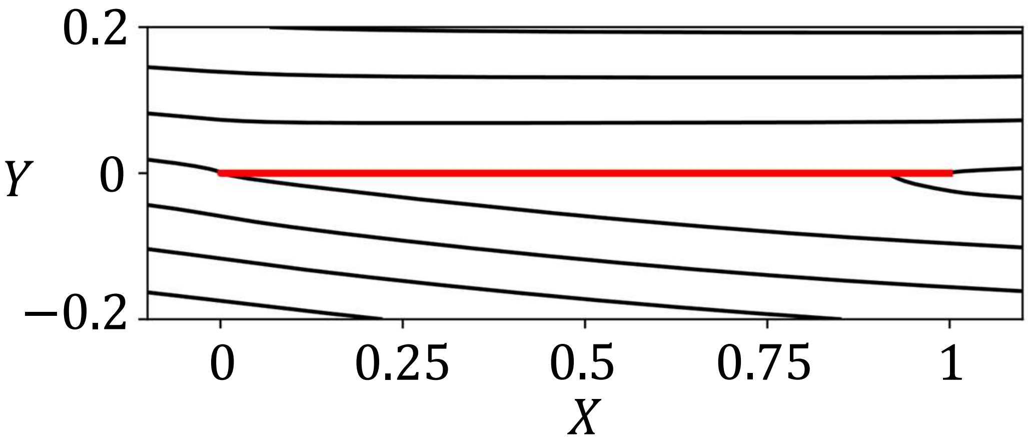
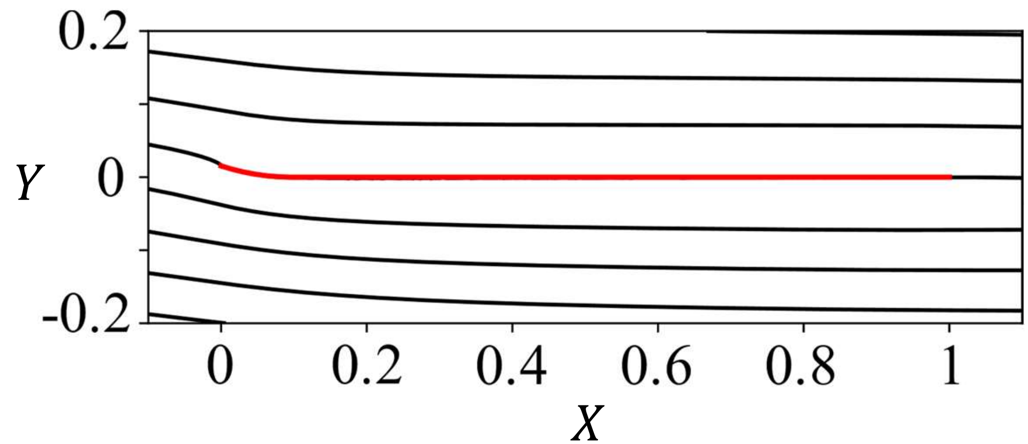
SEPARATION CONTROL

Bent leading edge

$\mathrm{Pe}_L = 400, \mathrm{Pr} = 0.02, \mathrm{Ri} = 0.143, b = 10$, $\alpha = 18°$, $L_b = 0.1$

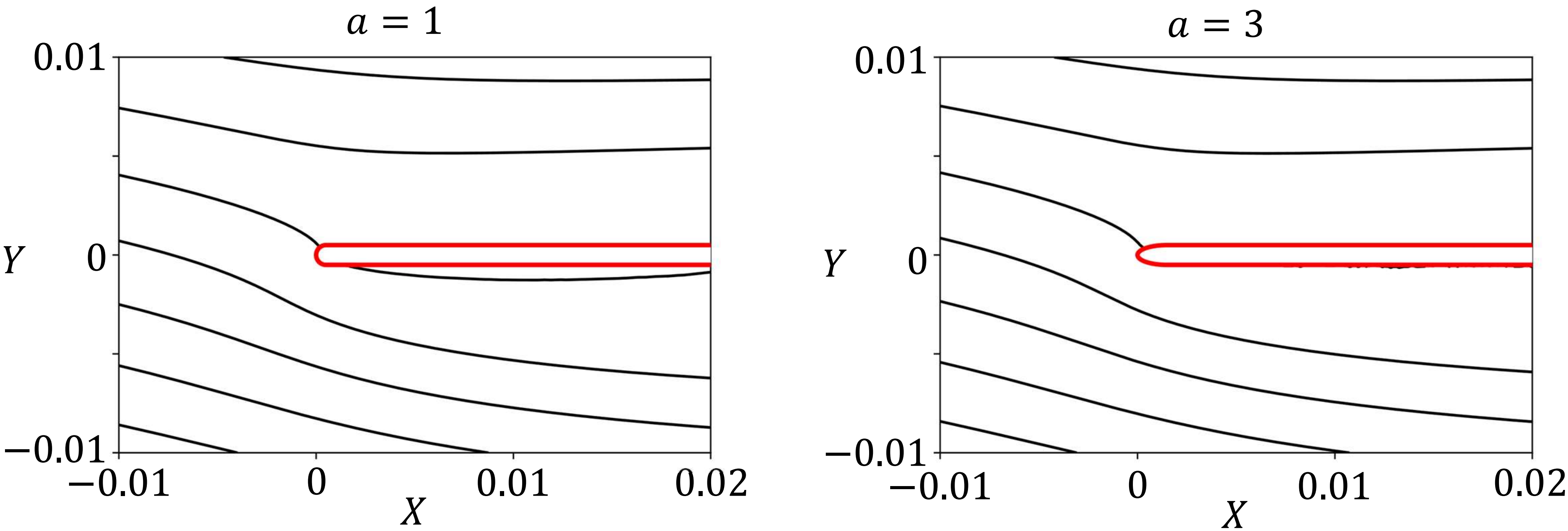




SEPARATION CONTROL

Elliptical leading edge

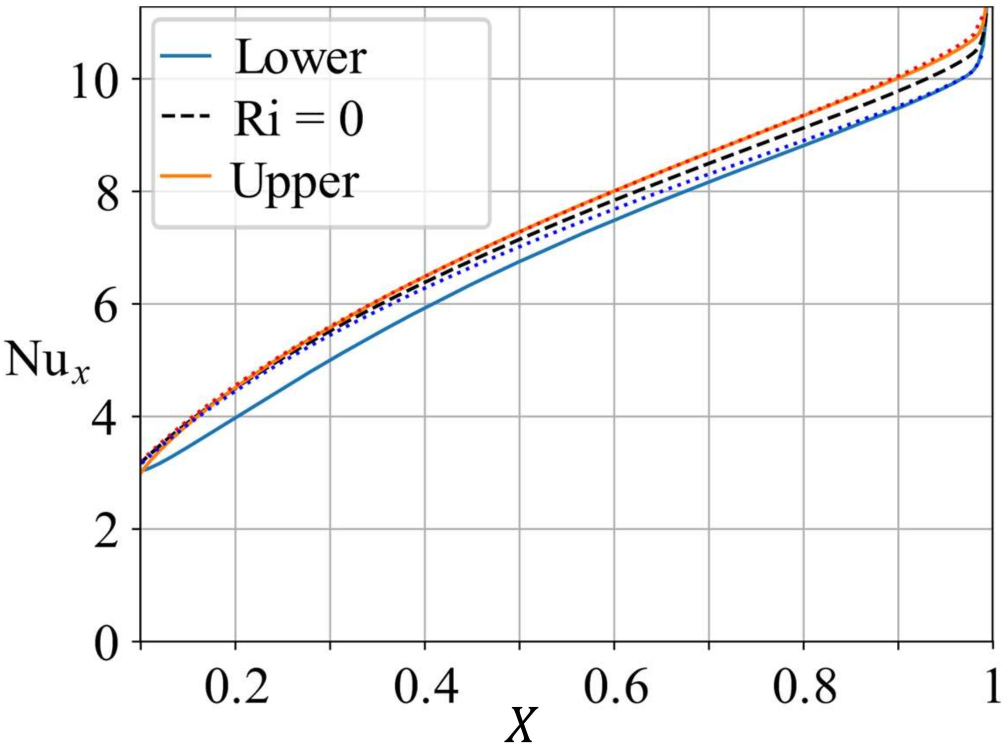
$\mathrm{Pe}_L = 20000, \mathrm{Pr} = 1, \mathrm{Ri} = 0.05, b = 10$



LOCAL WALL HEAT FLUX

Bent leading edge

$\mathrm{Pe}_L = 400, \mathrm{Pr} = 0.02, \mathrm{Ri} = 0.143, b = 10, \alpha = 18°, L_b = 0.1$



$$\mathrm{Nu}_x = \frac{\dot{q}\, L\, X}{k\,(T_p - T_\infty)}$$