TECHNISCHE UNIVERSITÄT WIEN

# DISSERTATION

## Cooperative and Multirate Simulation: Analysis, Classification and New Hierarchical Approaches

ausgeführt zum Zwecke der Erlangung des akademischen Grades einer
Doktorin der technischen Wissenschaften
unter der Leitung von

Ao.Univ.Prof. Dipl.-Ing. Dr.techn. Felix Breitenecker
E101
Institut für Analysis und Scientific Computing

und

Dipl.-Ing. Dr.techn. Nikolas Popper
E194
Institut für Information Systems Engineering

eingereicht an der Technischen Universität Wien
Fakultät für Mathematik und Geoinformation

von

**Dipl.-Ing. Irene Hafner, BSc**

Matrikelnummer: 00626250
[redacted]

Wien, im Oktober 2021

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Für meinen Vater, der jedem Menschen wertschätzend begegnete.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## Kurzfassung

In dieser Dissertation werden hierarchische Co-Simulationsmethoden vorgestellt und hinsichtlich Genauigkeit, Fehlerakkumulation und Stabilität untersucht.

Co-Simulation bezeichnet die Kopplung von zwei oder mehr Simulationen, die sich in dem verwendeten Simulationstool, Lösungsalgorithmus oder zumindest der Solverschrittweite unterscheiden. Methoden dieser Art haben sich mittlerweile als unentbehrliches Instrument zur gesamtheitlichen Abbildung komplexer Systeme aus unterschiedlichen Anwendungsgebieten etabliert. Der vielseitige Ursprung und das breite Spektrum an wissenschaftlichen Lösungsansätzen für Problemstellungen dieser Art haben dazu geführt, dass einige Begriffe unterschiedlich aufgefasst oder äquivalente Methoden verschieden bezeichnet werden. Dieser Umstand gab den Anstoß für die Zusammenführung, Klärung und Vereinheitlichung des Vokabulars in diesem Forschungsbereich zu Beginn dieser Arbeit.

Anschließend folgt ein Überblick über gängige Multirate- und Co-Simulationsmethoden, begonnen bei Verfahren für gewöhnliche Differentialgleichungen über gekoppelte differential-algebraische Gleichungssysteme bis hin zur Zusammenführung stark unterschiedlicher Ansätze, wie etwa diskret und kontinuierlich dargestellten Teilsystemen. Zudem wurde eine empirische Studie mit über fünfzig Teilnehmenden in Zusammenarbeit mit Kollegen nationaler und internationaler Forschungsgruppen ausgearbeitet und durchgeführt, deren Ergebnisse gängige Standards, Herausforderungen und Forschungsbedarf im Bereich der Co-Simulation aufzeigen.

Ob ihrer Vielfalt können Multirate- und Co-Simulationsmethoden anhand unterschiedlicher Gesichtspunkte strukturiert werden. Diese werden gleichzeitig mit der Klassifikation ausgewählter Literatur ebenfalls in dieser Arbeit präsentiert. Ein Nebenprodukt letzterer stellt unter anderem das Netzwerk an AutorInnen der betrachteten Publikationen dar, in dem deutlich wird, dass einige von ihnen für internationale Zusammenarbeit offen sind, während andere ihre Forschungstätigkeit bevorzugt innerhalb der eigenen Institution vornehmen.

Das Hauptaugenmerk dieser Dissertation liegt auf der Entwicklung und Untersuchung hierarchischer Co-Simulationsmethoden. Diese bezeichnen Co-Simulationen, die unter sich weitere Co-Simulationen, gegebenenfalls auf mehreren Ebenen, koordinieren. Fehlerschätzungen zeigen, dass durch die Einführung weiterer Levels keine zusätzlichen Fehler hinzukommen und Nullstabilität, gesondert auf jeder Ebene, analog zu herkömmlicher Co-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Simulation untersucht werden kann. Benchmark-Tests anhand gekoppelter Dahlquist-Gleichungen zeigen, dass numerische Stabilität sogar erhöht werden kann, sofern Systeme, die untereinander in höherem Ausmaß von Werten der jeweilig anderen abhängen, die Möglichkeit haben, gesondert an weiteren Kommunikationszeitpunkten Daten auszutauschen. Dadurch kann die Genauigkeit erhöht und qualitatives Verhalten erhalten werden, ohne die gesamte Co-Simulation zu verlangsamen.
Insgesamt stellt hierarchische Co-Simulation einen innovativen Ansatz dar, der vielversprechende Ergebnisse hinsichtlich Genauigkeit und Stabilität liefert und zudem Potential für weiterführende Studien bereithält.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## Abstract

In this thesis, hierarchical structures in cooperative simulation (abbreviated *co-simulation*) are introduced and investigated with regard to consistency and stability.

Co-simulation, understood as the coupling of two or more simulations which differ in their simulation tools, solver algorithms, or at least solver step sizes, has become an important instrument in the holistic representation of complex systems which arise in different fields of application. The variety of origins and multitude of scientific methods within this area have led to different perceptions of certain terms. This has motivated the presentation and unification of possible inconsistencies in terminology in the first part of this thesis.

The next chapter provides an overview of existing methods in this area, which range from multirate schemes for ordinary differential equation systems to gluing algorithms for high-index differential algebraic systems and coupling of highly contrastive approaches like continuous time and discrete event systems. Moreover, a two-stage Delphi study with over fifty participants has been developed and conducted in cooperation with colleagues from national and international research groups, whose results have further highlighted promising standards and present challenges in the area of co-simulation.

Due to their diversity, multirate and co-simulation methods can be structured according to various aspects which are presented along with the corresponding classification of selected literature. In addition, clusters of co-authorships illustrate how certain authors are conducting research by cooperating internationally while others seem to restrict themselves to working with colleagues from the same institution.

The main focus of this thesis lies on the presentation and investigation of hierarchical co-simulation approaches, meaning co-simulations which may coordinate further co-simulations beneath on several levels. Estimates regarding consistency show no additional errors despite the introduction of further co-simulation levels, and investigations on zero-stability can be conducted in analogy to those for traditional co-simulation approaches. Benchmark tests on coupled Dahlquist equations even show enhanced numerical stability if more closely linked subsystems can communicate at additional points in time and thus increase accuracy and maintain qualitative behavior without slowing down the whole co-simulation process.
All in all, hierarchical co-simulation is an innovative method with promising results regarding accuracy and numerical stability properties, and potential for further developments.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## Danksagung

Zuerst möchte ich allen Chefs, Kollegen und Kolleginnen der Arbeitsgruppe Modeling and Simulation der TU Wien und der dwh GmbH danken, die mir dieses Werk ermöglicht und die Zeit unterhaltsamer gestaltet haben, ebenso meinen Kommilitoninnen und Kommilitonen, ohne die das Studium nicht denkbar gewesen wäre und den vielen Studierenden, die Vertiefung und Perspektivenwechsel erlaubt haben.
Vielen Dank an meine wunderbar große Familie und meinen bunten Freundeskreis, die mich in unterschiedlichster Weise unterstützen und meinen Blick auf wesentlichere Dinge lenken.

Besonderer Dank an:
Felix, der meine akademische Laufbahn seit der ersten Seminararbeit begleitet. Danke für alles, was du mir ermöglicht hast – von Summerschools über Lehre und Konferenzteilnahmen mit so manchem Sprung ins kalte Wasser bis hin zu dieser Arbeit.
Niki, der meinen ausschweifenden Abhandlungen Fokus gab und ihnen den Startschuss ermöglichte.
Die österreichische Forschungsförderungsgesellschaft, die durch das Programm “Forschungspartnerschaften” die ersten umfangreichen Arbeiten an dieser Dissertation ermöglicht hat und den Sozialstaat Österreich, mithilfe dessen durch die Möglichkeit einer Bildungskarenz die Fertigstellung erfolgen konnte.
Gerald, Cláudio und alle weiteren Mitwirkenden und Teilnehmenden an der empirischen Studie, ohne die deren Ausarbeitung und Durchführung nicht denkbar gewesen wäre.
Martin, Matthias, Motzi, Petra, Štefan, Stuffy und Vera, für essentielle Inputs und neue Blickwinkel auf meine Arbeit.
Christoph, ohne den ich auf die meisten Prüfungen nicht einmal halb so gut vorbereitet gewesen wäre und Stuffy, die mir den Spaß am Studieren zeigte.
Martin, für Gespräche, in denen Mathematik, Philosophie und Nonsens verschwimmen.
Das Kleeblatt der Freundschaft, mein unerschütterliches Fundament.
Hannes – gna, für endlosen Tech Support und einen alternativen Zugang zu vielem.
Motzi und Hise, die Auffangnetz und Kontrapunkt vereinen.
Stefan, der mir zeigt, was Pragmatismus bedeutet. Für jeden Tag, den er bei mir ist.
Meine Kinder, die mich täglich wichtigere Dinge lehren als ich sie.
Meine Mutter, die auch jetzt noch alles uns unterordnet. Ich bewundere dich.
Jesus, ständigen Wegbegleiter meines behüteten Lebens, der mir erlaubt, nicht alles verstehen zu müssen.
Alle, die diese Zeilen lesen und denen namentlich zu danken ich verabsäumt habe – für eure Unterstützung und jegliches Interesse.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

# Contents





Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

CHAPTER 1

# Introduction

The importance of modeling and simulation as a means to carry out prototypical experiments or to approximate long-time behavior of systems in various fields of application is commonly known. As the complexity of considered problems and the requirement on the level of detail increases permanently, the demand for methods of system decomposition and coupled simulation approaches has also gained interest.

The first methods in this area have been developed for large systems of ordinary differential equations. These have been separated for parallelization motivated by challenges of fast computation (see f.i. Jackson (1991)) or into system parts showing stiff and non-stiff behavior in order to apply partitioned integration schemes (Günther and Rentrop 1994; Rice 1960). While the latter is still a present topic of interest (Striebel et al. 2009), nowadays co-simulation is also used for the overall simulation of heterogeneous systems which are per se divided due to different modeling approaches or differently suitable simulation software for system parts (cf. Chapter 3). These typically occur in complex applications such as Smart Grids, production facilities, health systems, or even social sciences. There, expertise in multiple areas on the structure of the real system parts and suitable modeling and simulation approaches is required, which can often not be met by one company. Co-simulation allows system parts to be implemented by experts in the respective areas and combined without detailed knowledge on the individual modeling paradigms as long as interfaces are properly defined and coupling requirements met.

The development of co-simulation methods has emerged from different fields of application and been approached from different theoretical points of view. This has led to different per-

ceptions of the same terms on the one hand and different words for the same methods on the other hand, which has made a comprehensive research on the state of the art in this area challenging. Therefore, in the course of the search for open research questions, the establishment of a comprehensible definition of relevant terms while acknowledging different interpretations, a historical as well as topical overview of related work, and methods for structuring existing approaches have all emerged as small research topics by themselves.

In addition, an empirical survey with over fifty experts has highlighted present challenges in the area of co-simulation. Therein, the most pressing subject seems to be the combination of discrete event and continuous time systems in a hybrid simulation, which is also reflected by the insights from the state of the art. However, this complex topic is already investigated by several large research groups with promising preliminary results, see Section 3.4.2. Also mentioned in the experts' assessment of current challenges in co-simulation are communication problems in cross-company projects and among theorists and practitioners, which may be mitigated with the aid of the terminology established in Chapter 2.

Furthermore, guidelines for the choice of a suitable macro step and numerical stability issues are named, both of which are addressed by the hierarchical co-simulation approach that constitutes the main research topic in this thesis:
In contrast to split methods where hierarchical schemes are occasionally applied, in co-simulation (for the distinction of these terms see Chapter 2 and Section 3.5), hierarchical structures are barely even mentioned let alone investigated with regard to convergence properties. This has motivated the investigations in Chapter 6. These show that while consistency is maintained and zero-stability can be determined similar to conventional co-simulation approaches, the introduction of further co-simulation levels and with them, additional communication points for selected subsystems, can even improve accuracy and numerical stability. Thus, hierarchical co-simulation presents an innovative method that moreover allows the utilization of improvement techniques of existing approaches.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

CHAPTER 2

# Terminology: Present Perceptions and Unification

Every researcher who has ever cooperated with project partners of different scientific fields knows that although people often seem to talk about the same things, they actually have quite different perceptions of terms and need to establish a glossary before eventually being able to start working together. However, even within Mathematics and therein research on coupled simulations, some expressions are used with different meaning and on the other hand, topics with the same meaning are referred to differently. This makes it hard to do research on related work or communicate own ideas. With regard to that, the present chapter intends to clarify the vocabulary used in this dissertation and also tries to cover most terms in the area of research on co-simulation in general. Excerpts of this chapter have been published in (Hafner and Popper 2017).

## 2.1 Objective

There are several publications presenting and reviewing investigations on common co-simulation methods. Still, most of these do not aim at the merging of terms but adopt the terminology used by certain previous studies in the respective research area while especially in application driven work, knowledge on similar research seems to be missing due to the lack of common consent.

A thorough overview of existing co-simulation methods is given by Busch (2012), who for some of the presented terms also provides information about alternatively used words, of

which several will be addressed in this chapter. Although focusing on a specific application, the work of Trčka (2008) also raises awareness of different terminology for the same approach. The Modelica Association has introduced the FMI (Functional Mockup Interface) standard, a wide-ranging project for the standardization of co-simulation and model exchange on implementation level. It provides a specification for tool-independent interfaces, albeit with an original focus on physical systems represented by differential-algebraic equations (DAEs), see (Modelica Association 2021) and Section 3.3.2. The standard documentation (Modelica Association 2014) also provides a glossary for basic terms such as *model*, *simulation*, and *co-simulation*, which will be revisited below.

Throughout this chapter, literature references as to where specific terms are used are given exemplary, not exhaustively. Further, I want to clarify that its main purpose is not the determination of a universally valid terminology by the consolidation of different concepts but raising the readers' awareness of the various origins and hence differences in meanings and definitions, which is important to prevent misunderstandings and advance early development phases of interdisciplinary cooperative projects. With this objective in mind, different meanings or understandings on terms used in research on co-simulation are illustrated, explained and further harmonized to the extent deemed possible.

## 2.2 Basic terms

Even within the modeling and simulation community, several basic terms are used with slightly different meaning. The following definitions raise no claim to completeness or generality but intend to clarify the usage within this work.

Fritzson (2004) provides an abstract definition of a *model* as follows: "A model of a system is anything an 'experiment' can be applied to in order to answer questions about that system." Here, even conceptional models such as mental or verbal ones – which usually precede and induce mathematical or logical models -, are included.
Breitenecker (1992) assumes a mathematical formulation in his definition of a model: "A model (MO) is the description of a process using a mathematical formulation and a certain language." A similar, although slightly more general definition that still anticipates later usage in simulation can be found in the glossary of the documentation on the FMI Standard:

**Definition 2.1** (Model (Modelica Association 2014))**.** "A model is a mathematical or logical representation of a system of entities, phenomena, or processes.(. . . )"

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

This, however, requires clarification of the meaning of the word *system*. Fitzgerald et al. (2014) define a *system* as follows: "an entity that interacts with other entities, including hardware, software, humans and the physical world". They describe a *model* as "an abstract description of the reality of a putative system" (with *abstract* meaning that it excludes (for the current purpose) irrelevant details), which is mostly consistent with the description above but already hints that a model can never represent the entirety of a real system.

Definition 2.1 is still rather too general for this work, but also encompasses *simulation models*. These are models prepared in a way ready to be simulated by a simulator (see Definition 2.5). Nonetheless, other kinds of models – such as mathematical models aiming at representing physical systems – are essential in earlier development phases and can influence the choice of further modeling and simulation approaches.

To be able to describe the progress of a model's properties, *solvers* are required.

**Definition 2.2** (Solver)**.** A solver is a solution algorithm that can be applied to specific simulation models.

For continuous systems represented by non-stiff ordinary differential equations (ODEs), an example for a solver would be a Runge-Kutta method, see also Definition A.6. For models described by differential-algebraic equations (DAEs), an index reduction (e.g. pantelides algorithm) or regularization method (transformation, projection) in combination with an implicit solution method (Implicit Euler, for example) is representing the solver. An event-handling algorithm (*scheduler*) can be regarded as solver for discrete event systems.

**Definition 2.3** (Simulation (Fritzson 2004))**.** "A simulation is an experiment performed on a model."

In this context, an *experiment* may be understood as "the performance of a certain method with a certain model where all aspects of execution control are included." (see Breitenecker (1992), where a *method* is defined as "a procedure, an algorithm, which does anything with the model (data base) in a much more general way").
This implies that *how* this experiment is performed depends on the chosen solution algorithm, which has to be defined before each simulation run.

*Remark* 2.4. Once more: these definitions do not claim universal validity or generalizability but only clarify the use in this thesis. In other disciplines, modeling and simulation may be understood differently: Brailsford et al. (2019) acknowledge that "computer scientists

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

talk about 'modelling and simulation', where modelling means building a model and simulation means running it to conduct experiments, whereas operational researchers tend to describe the process holistically as 'simulation modelling"'. Even within research designated to computer-aided simulation, a distinction between modeling and simulation may not be made according to above definitions, which admittedly are attributed to a physical modeling background. Considering mainly physical models described by equation systems and solved with numerical regularization and integration algorithms, the dissociation of model and simulation comes naturally and can easily be drawn. In system dynamics (SD) or agent based (AB) approaches, the model, understood as representation of the system, often inevitably incorporates parts of the solution algorithm, making it hard to regard them as separate entities.

**Definition 2.5** (Simulator)**.** A simulator is a tool allowing the implementation and simulation of models.

This definition already implies that in this work, the terms *simulator* and *simulation tool* are understood to have the same meaning, which conforms to the definition by the FMI (Modelica Association 2014): "A simulator can include one or more simulation programs, which solve a common simulation task.", where a *simulation program* is defined as "Software to develop and/or solve simulation models. The software includes a solver, may include a user interface and methods for post processing (see also: simulation tool, simulation environment)". Note that even though the "or more" part is not mentioned explicitly in Definition 2.5, neither is it ruled out. Other researchers, however, use *simulator* equally to the term *solver*, see for example (Gomes et al. 2018b): "A *simulator* (or solver) is an algorithm that computes the behavior trace of a dynamical system." Further, Gomes et al. ". . . use the term *simulation unit* for the composition of a simulator with a dynamical system."

There are numerous examples of a simulator or simulation tool. On the one hand, the choice of a specific tool can depend on the area of application and the representation of the model, on the other hand, some systems are modeled with an approach that allows the resulting model to be simulated by a previously chosen tool. Both ways are more or less commendable depending on the given case, but one should be aware that choosing a tool or modeling approach only because the user is already familiar to it may lessen the suitability and further the integrity of the whole modeling and simulation process. In some cases, allowing different approaches for specific parts of the regarded system and combining them thereafter in a co-simulation can simplify this process while maintaining the integrity.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Further, frequently used in terms of simulation are *variable*, *parameter* and *constant*, which will be understood as follows: A variable is, according to Fritzson (2004), “a quantity in the model that varies with time”, a constant “a quantity in the model that does not vary with time” and a (design) parameter “ remains constant during a simulation"(Fitzgerald et al. 2014). In contrast to a constant (as f.i. the gravitational constant), a parameter may, however, be changed before every simulation run (like the length of a pendulum).

## 2.3 Co-simulation: definition and demarcation from related terms

The probably most important and obvious term that shall be discussed is *co-simulation* itself. Co-simulation is used in various different areas. As many of these have developed individually and independently, co-simulation is on the one hand defined differently depending on the origin; on the other hand, other names are used to describe methods which, from an outer perspective, can be regarded as co-simulation.

Conventionally, co-simulation is used to describe a simulation within which at least two simulations are coupled, but this is the furthest extent to which different definitions concur. The manner of coupling and the differences in the participating sub-simulations are various, but taken as part of the definition by some researchers nevertheless. The word co-simulation can be regarded as an abbreviation for *cooperative simulation* or *coupled (system) simulation* (Busch 2012; Felippa et al. 2001).

The following terms are sometimes used synonymously although some of them do not quite comprise the same methods, as the subsequent sections will explain:

- simulator coupling (Busch 2012; Kübler and Schiehlen 2000b)
- coupled simulation (Busch 2012)
- solver coupling (Schmoll 2015)
- modular time integration (Busch 2012)
- distributed time integration (Arnold and Günther 2001)
- modular simulation (Kübler and Schiehlen 2000a)
- coupling of models in behavioral model description (Kübler and Schiehlen 2000a)

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

- (multidisciplinary) collaborative simulation (Liang et al. 2011; Zhang et al. 2011)
- parallelism across the system (Jackson 1991)
- parallelism across space (Jackson 1991)
- hybrid simulation (Mustafee et al. 2017)
- weak coupling (Kübler and Schiehlen 2000b)

### 2.3.1 Coupling concepts

To dissociate the understanding of *co-simulation* from other means of coupling, we consider the following concepts (see Brecher et al. (2009) and Thiede et al. (2016)):

- offline coupling
- model integration
- co-simulation
  - direct coupling
  - model synchronization

*Offline coupling* is used to describe the separate execution of systems with the exchange of results after a simulation run. Clearly, as no coupling of *simulations* takes place, this is not considered as co-simulation.

Further, it is important to emphasize the difference between co-simulation and the concept called *model integration* or also *model coupling* (Busch et al. 2007), which stands for export of model-code without solver and thence coupling several models in one software environment, where data exchange is realized by one common solution algorithm at each time step resp. event. Here, in contrast to co-simulation, the coupling takes place on a much deeper level – the model itself, which is then simulated on the whole. Figure 2.1 illustrates this difference.
An example for model coupling would be the combination of several discrete event based models via the DEV&DESS formalism (see Zeigler et al. (2000)). Similar, so-called *multi-method-modeling* combines different modeling approaches, f.i. an agent-based with a system dynamics approach (cf. Swinerd and McNaught (2012) and Wang et al. (2019)), while their simulation is still executed by one algorithm for the overall system.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

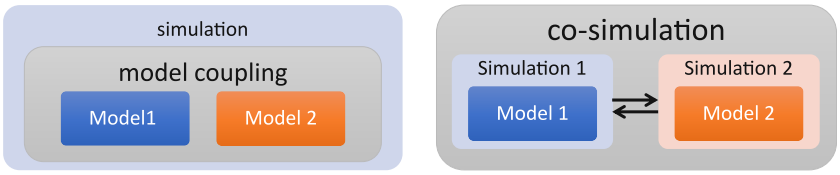


Figure 2.1: Model coupling (coupling of models, one simulation) vs. co-simulation (coupling of multiple simulations).

Co-Simulation, on the other hand, couples *simulations*, or, as Kübler and Schiehlen (2000a) would put it: models in the behavioral model description, not the mathematical model description. For more information see also (Benedikt et al. 2010), who explain that in case of considering the dynamic behavior of the overall system, all subsystems have to be “modeled on the behavioral description level with individual model description languages in their associated simulation tools”. This tells us that the distinction between model coupling and co-simulation depends on the definition of the terms “model" and “simulation", which may differ, as addressed in Section 2.2.

With this in mind, the definitions from Fitzgerald et al. (2014) will also be mentioned here: they introduce the term *co-model* as collaborative model consisting of a discrete event model, a continuous time model and a *contract* defining shared information, and further *co-simulation* as simulation of a co-model. Interestingly enough, this does not imply a different understanding of co-simulation (in the sense of model integration explained before) but a different understanding of the term *(co-)model*, which includes the (co-)simulation algorithm (“contract"). This is a wonderful example for the importance of the clarification of terms and need for the reader to be aware of these different possibilities of interpretation.

The above distinction within co-simulation of *direct coupling* (where, for the example of two coupled systems, one system handles the synchronization) versus *model synchronization* (via middleware or orchestrator) is revisited in Section 5.7.1.

The perspective of Geimer et al. (2006), where coupling concepts (in their words, variants of modeling, which again can be slightly misleading) are distinguished according to the number of modeling tools and integrators, respectively, leads to the division shown in Figure 2.2.
While category IV, which in this work (and numerous others) will be considered a subset of co-simulation, could instinctively be called “solver coupling”, this term is used differently by Schmoll (2015), who differs between co-simulation, understood as coupling of two or more dynamic subsystems, and solver coupling as “coupling between one dynamic and one or

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

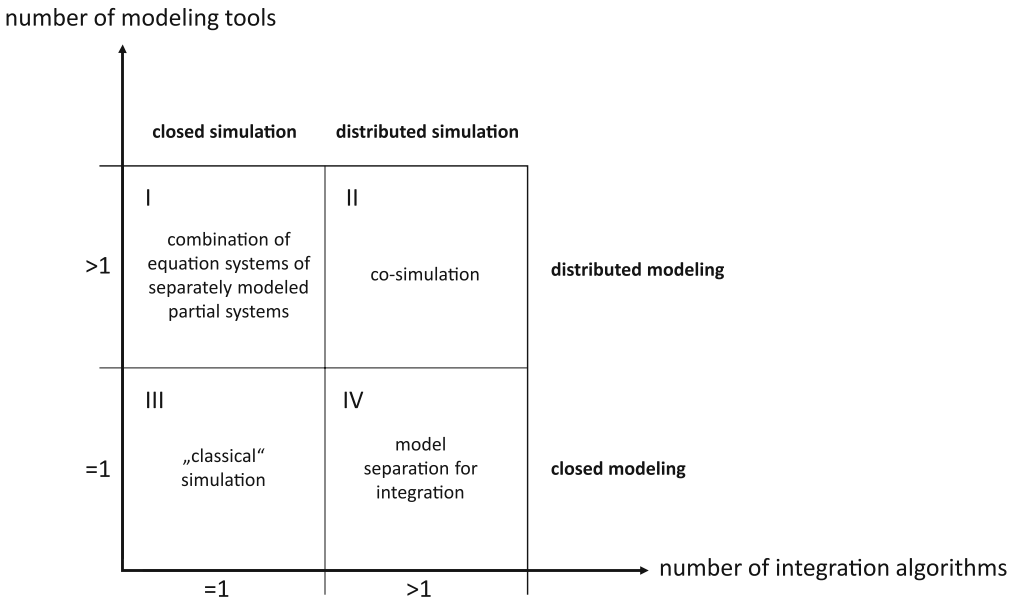


Figure 2.2: Coupling concepts according to Geimer et al., depending on the number of integration algorithms and modeling tools (after Geimer et al. (2006)).

more static subsystems". Busch et al. (2007) defines solver coupling as export of code including the integration method and uses the term *process coupling* for the case of parallel simulation processes.

Parallelism, however, can also be interpreted in several manners. Friedrich (2011), Gear (1988), Jackson (1991), and Jia and Leimkuhler (2003) distinguish different means of parallelism in solvers as follows:

- parallelism across the method
- parallelism across the system

Parallelism across the system (also called parallelism across space) describes the partitioning of the regarded problem into subsystems which can then be computed in parallel by interconnected processors, while by parallelism across the method (also called parallelism across time), "the several stages involved in the method can be mapped to distinct processors"(Jia and Leimkuhler 2003). Thus, the distinction defines whether the regarded *system* or the *solution algorithm* (resp. its calculations) is divided with the purpose of parallelization. Cases where parallelism across the method is applied exclusively are not regarded as co-simulation in this thesis. On the other hand, parallelism across the system and with

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

it separate execution of system parts, i.e. their simulation, implies co-simulation (without excluding additional parallelism across the method).

### 2.3.2 Simulator coupling

As mentioned before, *simulator coupling* is sometimes used equivalently to *co-simulation*. The Modelica Association (2014), for example, defines co-simulation as "Coupling (in other words dynamic mutual exchange and utilization of intermediate results) of several simulation programs including their numerical solvers in order to simulate a system consisting of several subsystems." However, if two systems are simulated by the same simulator using only different step sizes, *multirate simulation* (i.e. the coupled simulation of systems using different time scales) takes place, which is often considered a part of co-simulation (as two *simulations* are coupled). Still, if the simulators of those systems are the same, this example would not be a part of simulator coupling, which makes the latter a real subset of co-simulation.

### 2.3.3 Multirate simulation

For an initial value problem (IVP) that can be divided into one stiff and one nonstiff subsystem, Gomm (1981) describes a multirate method as the subsequent application of a linear multistep method on the stiff system with step size $h$ (and extrapolated values of the non-stiff system's states) and the non-stiff system with step size $kh$ (with $k \in \mathbb{N}$). This property of slower and faster varying variables (which is a property of the system, not the solution) is named *multirate behavior* by (Verhoeven et al. 2006b). Striebel (2006) uses a more general description of *multirate schemes* as "numerical integrators tailored to that property". In this thesis, we will regard multirate simulation as defined in the following:

**Definition 2.6** (Multirate simulation)**.** *Multirate simulation* describes a simulation where different time steps are used for the solution of different system parts.

Note that not every co-simulation necessarily falls under multirate simulation: even if all participating simulations use the same time step in their solution algorithm and different simulation tools are used (see f.i. Wetter (2011)), simulations are coupled, hence co-simulation without multirate behavior exists. Even the union of simulator coupling and multirate simulation cannot be found to cover all of co-simulation: imagine one simulator and two different one-step algorithms using an equal time step. What is more is that there exist *partitioned* multirate schemes, which comprise one solution algorithm that solves different system parts with different step sizes, but incorporated in this one algorithm, so no separate simulations

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

are discernable, see also Section 3.5. This suggests that partitioned multirate methods may not be seen as part of classic co-simulation. For better understanding, these relations are depicted in Figure 2.3.

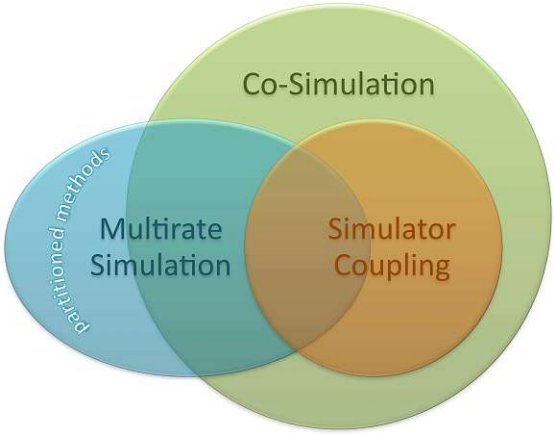


Figure 2.3: Relation of co-simulation, multirate simulation and simulator coupling.

### 2.3.4 Hybrid simulation

*Hybrid simulation* is a very delicate term; hybrid per se meaning (according to the Oxford Dictionary of English (Stevenson 2010)) “of mixed character; composed of different elements”, or more purposefully described by Mustafee et al. (2017) as “the result of merging two or more components of different categories to generate something new, that combines the characteristics of these components into something more useful”, it may represent various things: in the automotive industry, vehicles using a motor driven by electric energy as well as some other fuel, in biology, the offspring of different species, and even within simulation, the usage varies.

Some use *hybrid simulation* to describe any combination of heterogeneous models (note that this would fall under model coupling, not co-simulation) or simulations without necessarily specifying how the individual parts may differ (Mustafee et al. 2017), thus potentially being a superset of co-simulation; others use it to describe the combination of two specific simulation approaches, hence being a subset of co-simulation, such as discrete and continuous simulation approaches (Awais 2015) or Agent Based simulation with System Dynamics (Farsi et al. 2019; Lättilä et al. 2010), Lattice Boltzmann and Finite Difference methods (Ge et al. 2019), analytical components or real-time simulation with physical components (MTS Systems 2017; Murray et al. 2015), microscopic with mesoscopic simulations

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

(PTV AG 2017) or digital with analogue simulators (Ni and Broenink 2012; Troch and Breitenecker 1990). Brailsford et al. (2019) and Mustafee et al. (2017) state that hybrid simulation stands for the combination of two or all three techniques out of Discrete Event simulation, System Dynamics and Agent Based simulation. In addition, Mustafee et al. present terms introduced by others "which, arguably, have the same meaning, e.g., multi-method simulation, multi-paradigm modeling, cross-paradigm simulation, mixed-modeling and combined simulation." – listed here for reasons of completeness with emphasis on *arguably*, considering the jumble of usage of the words modeling and simulation, see Section 2.3.1, and vagueness regarding the kind of combined approaches.

Breitenecker and Popper (2007) use the term *hybrid decomposition* to describe structural-dynamic systems where state events initiate switches between different models that describe the same system, but possibly with a different and even potentially differently dimensioned state space.

Zhang et al. (2008) describe a hybrid (dynamic) system as "the mathematical representation of models with continuous-time behavior and discrete-event behavior". Correspondingly, nowadays *hybrid simulation* is used mostly to describe the combination of discrete event (DE) with continuous time (CT) simulation, which is the definition we will stick with in this work from now on. Note that this does not specify whether the hybrid is established on model level or simulation level. In the latter case, we will talk decidedly of *hybrid co-simulation* to prevent misunderstandings.

**Definition 2.7** (Hybrid simulation, hybrid co-simulation)**.** *Hybrid simulation* describes the combination of discrete event with continuous time representations. The co-simulation of discrete event and continuous time simulations is called *hybrid co-simulation*.

### 2.3.5 Modular and distributed time integration

The terms *modular time integration* and *distributed time integration* both imply the partaking of subsystems using continuous time simulation involving integration and thus a differential part in the system equations. Larsson and Krus (2003) even define co-simulation as "case when two or more numerical integrators collaborate in solving an initial-value problem". This, however, excludes systems which consist solely of partial systems using a discrete event, agent-based or cellular automaton approach. Therefore, modular or distributed time integration can only be seen as subset, not synonym, of co-simulation as defined in the following.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 2.3.6 Definition of co-simulation

All in all, an attempt at the most general definition for co-simulation that can be concluded from the information above is given as follows:

**Definition 2.8** (Co-Simulation)**.** Co-simulation is the coupling of two or more simulations which differ in at least one of the following aspects:

- simulation tool
- solver algorithm
- step size

So co-simulation – as understood in this thesis – includes simulations carried out for example with the same simulator but different solver algorithms or even the same solver algorithm but different time steps, but also simulations where all subsystems use the same algorithm and time step but different simulators.

I want to make clear that this is only one interpretation, as there are authors (compare Kübler and Schiehlen (2000b)) who speak of co-simulation only if weak coupling (see Section 2.5) is used and therefore inevitably communication between all solvers does not take place at every time step of the individual solvers but only at certain common and usually larger synchronization time steps, resulting in general in a multirate approach. Here, however, co-simulation is used as an hypernym for loose as well as strong coupling methods.

Definition 2.8 further implies that co-simulation can also happen in one simulation tool, but using different solver algorithms (e.g. possible in Simscape[1] with the *Solver Configuration* block). On the other hand, if the same solution algorithm and even the same step size is used (for example, a system simulated with a fixed-step algorithm such as explicit Euler in Dymola coupled with a simulation also using the explicit Euler algorithm but implemented in MATLAB), this is also considered co-simulation. Multirate simulations which differ not in the tool or the solver algorithm but only in the step size are also commonly known, as are of course combinations of these three points: two or more systems that are simulated with different tools requiring different solver algorithms that also use individual step sizes. Figures 2.4-2.8 show examples of different co-simulation structures that can be realized.

[1] references to this and other mentioned software and tools are found in the Appendix, Section A.3

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

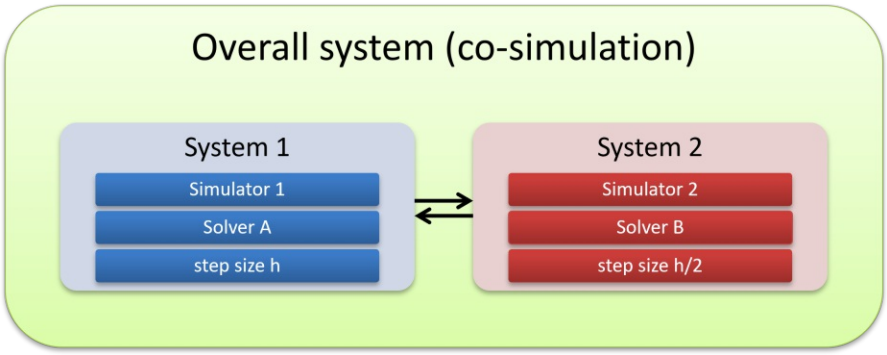


Figure 2.4: Schematic illustration of a co-simulation of two systems implemented in different simulators using different solvers and individual step sizes (Hafner and Popper 2017).

A classical case of two systems simulated in two different simulation tools using different solvers and individual step sizes is depicted in Figure 2.4. Although the step sizes in Figure 2.4 are given as $h$ and $h/2$, it is also possible for the solvers to use not fixed but individually controlled, adaptive step sizes.

However, as explained above, we also talk of co-simulation if the two regarded systems differ in only one of these aspects.

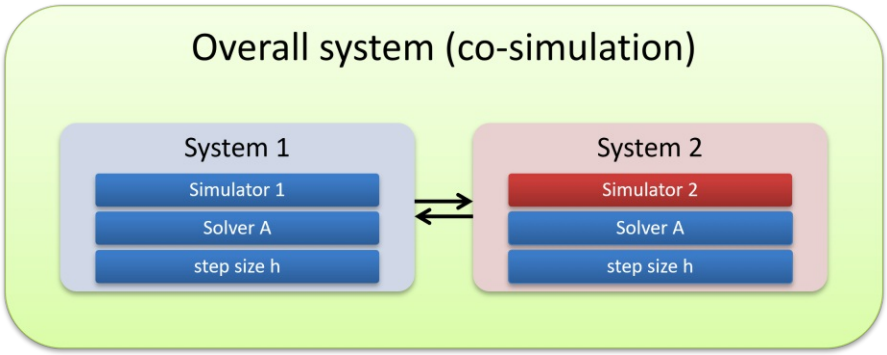


Figure 2.5: Schematic illustration of a co-simulation of two systems implemented in different simulators but using the same solver and step size (Hafner and Popper 2017).

The two co-simulated systems in Figure 2.5 are implemented in two different simulation tools but those use the same solver and even the same step size. Of course, this might not be an optimal solution if both systems (and hence the overall system respectively) are known and implemented by one person or team, as in this case the development of an integral monolithic simulation might provide better results. However, if the partial systems are developed and implemented independently and both solutions allow no information exchange

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

apart from values needed by the other system at certain synchronization references, this might be the only solution to couple these systems and achieve a holistic simulation (see also Section 5.4).

A special case is the one where both systems are implemented in the same simulator but differ in their solver algorithms. There are two possibilities for the realization of this problem via co-simulation: on the one hand, the way it is depicted in Figure 2.6, a co-simulation middleware (acting as *orchestrator*, see Section 5.7.1) can call two instances of the simulator, thus starting two processes which exchange data at some synchronization references and not necessarily sharing any information with the other process directly but only with the middleware. This always has to be the case if the simulators differ (as in Figures 2.4 and 2.8), although of course one of the participating systems can act as master and call the others.

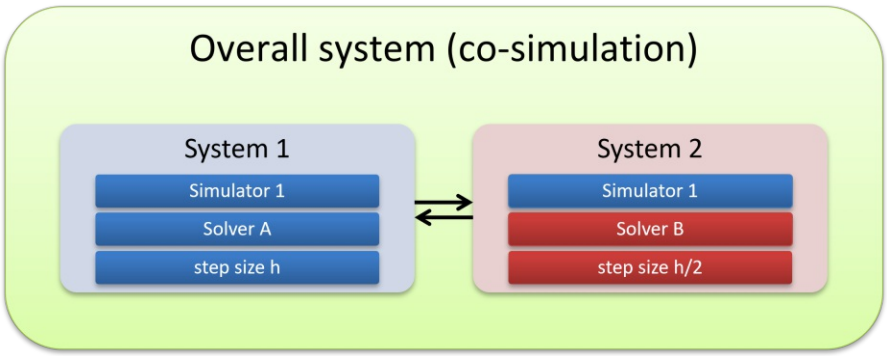


Figure 2.6: Schematic illustration of a co-simulation of two systems via a middleware calling two instances of the same simulator (Hafner and Popper 2017).

If, on the other hand, the same simulator is used and this simulator allows the usage of different solution algorithms (or differently parameterized realizations of the same solver) in one simulation, the actual calculation can take place in one instance of the used simulator which then organizes the communication between the partial systems and their respective solution algorithms, see Figure 2.7.

Multirate simulations which differ neither in the tool nor the solver algorithm but only in the step size are also commonly known, as are of course arbitrary combinations of the structures above and the extension to more than one system. This is exemplarily illustrated in Figure 2.8 for three systems of which some use the same simulator or solver and others differ in all respects.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

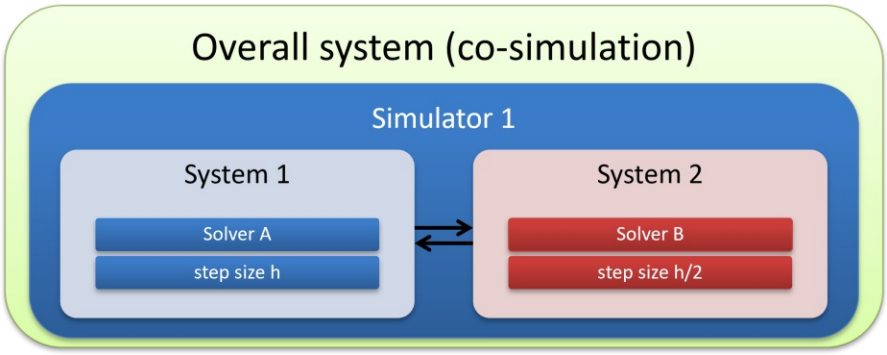


Figure 2.7: Schematic illustration of a co-simulation of two systems in the same simulator using different solvers and individual step sizes (Hafner and Popper 2017).

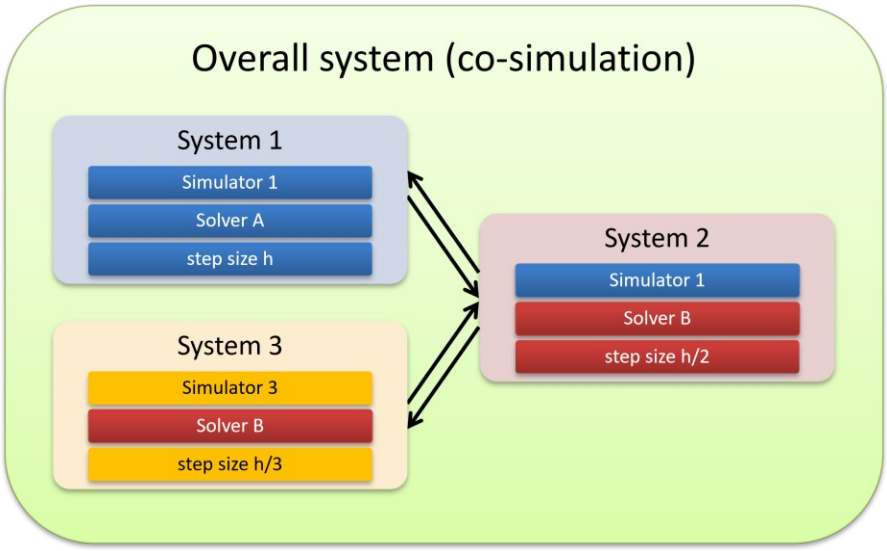


Figure 2.8: Schematic illustration of a co-simulation of three systems with some individual and some common simulators, solvers, and step sizes (Hafner and Popper 2017).

## 2.4 Multirate vocabulary

In the following, a few terms that are frequently used the context of multirate simulation are addressed. If all participating subsystems have a communication time step in common, this step is called *macro step* while the individual steps used by the partial system solvers are called *micro steps* (see e.g. Striebel (2006)). Synonyms for *macro step* are

- major step (Liang et al. 2011)
- global step (Liang et al. 2011)

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

- exterior step (Liang et al. 2011)
- time slab (Savcenco et al. 2007)
- communication step (Arnold et al. 2013)
- communication interval (Breitenecker et al. 1993)
- synchronization reference (Rumsey and Watkinson 2004)
- synchronization time step (Wetter 2011)
- synchronization point (Modelica Association 2014)
- sampling point (Modelica Association 2014)

Verhoeven et al. (2008) call the entirety of macro steps *coarse time grid*, the total of micro steps *refined time grid*. In case of constant macro and micro step sizes $H$ and $h$ respectively, the ratio $m := H/h$ is called *multirate factor* (Kübler and Schiehlen 2000a) or *stepsize ratio* (Striebel 2006). This makes sense especially if the overall system is divided into two parts, an *active* and a *latent* one (compare (Striebel 2006) and Section 5.8.1) where all latent parts use the macro step, the active ones the micro step and the communication between active and latent parts takes place at the macro steps (see also Figure 5.32). Thereby, *latency* is explained by Verhoeven et al. (2006b) as the case when "parts of the circuit are constant during a certain time interval".

Kvaernø¸ and Rentrop (1999) present two different strategies for multirate integration of systems divided into active and latent parts:

- *fastest first* and
- *slowest first*

This is extended by Günther et al. (2001) to a

- *compound step*.

Both fastest and slowest first approaches require sequential execution: Following a fastest first approach, the fastest, i.e. most active, fast varying part with the smallest time step, is integrated first, using extrapolated values from the latent, slower varying subsystems. The next most active system follows (or the latent one in case of only two subsystems) with interpolated values from the first system and extrapolated ones from all others and so

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

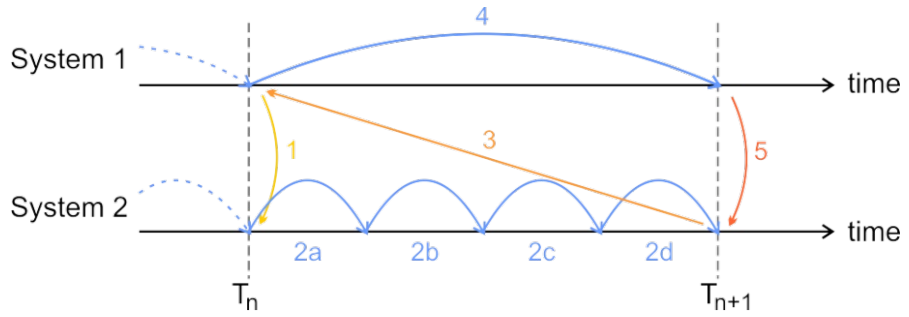


Figure 2.9: Illustration of the fastest first multirate method for one macro step $[T_n, T_{n+1}]$. Numbers indicate the sequence of solver steps (blue lines) and data exchange (yellow and orange lines) (Hafner and Popper 2017).

on until the most latent system is executed, using interpolated values for other systems' variables. Figure 2.9 illustrates this method for two participating subsystems.
The slowest first approach, on the other hand, executes the most latent system first, followed by the system with the next largest time step etc., as depicted in Figure 2.10 for the division in two systems.

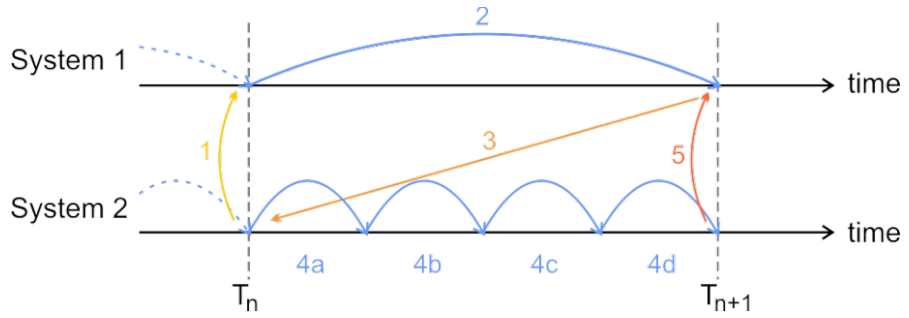


Figure 2.10: Illustration of the slowest first multirate method for one macro step $[T_n, T_{n+1}]$ (Hafner and Popper 2017).

In a compound step, parts are executed in parallel: One step with the respective micro step in all subsystems is followed by sequential calculation of the remaining micro steps in the active parts, see Figure 2.11.
Extending this, Verhoeven et al. (2006b) present variants of the compound step (compound, mixed compound, general compound), which are described in detail in Section 3.5.

Verhoeven et al. (2008) provide the term *single-rate* for solutions where all subsystem solvers use the same time step to the contrary of multirate simulation, which will also be the convention for this work.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

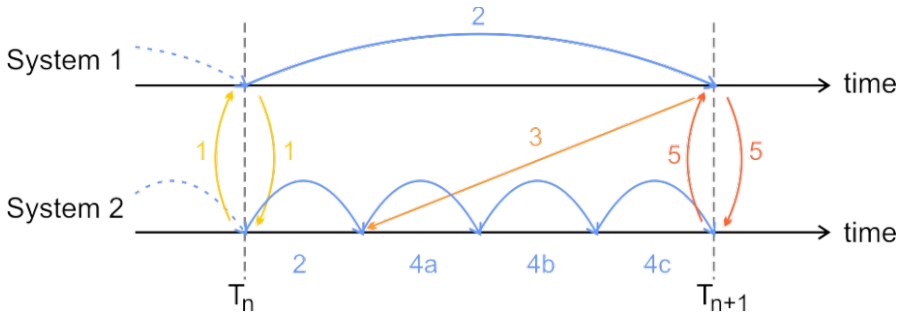


Figure 2.11: Illustration of a mixed multirate approach for one macro step $[T_n, T_{n+1}]$ (Hafner and Popper 2017).

## 2.5 Loose and strong coupling

When speaking of loose and strong coupling, we have to distinguish whether this refers to the coupling approach of the overall simulation or the intensity in which the participating subsystems depend on values from one another. We will start by the latter consideration, i.e. loosely coupled systems, which are defined by Striebel (2006) and Verhoeven et al. (2007) in case

$$\left\|\frac{\partial f_1}{\partial y_2}\right\| \ll \left\|\frac{\partial f_1}{\partial y_1}\right\| \text{ and } \left\|\frac{\partial f_2}{\partial y_1}\right\| \ll \left\|\frac{\partial f_2}{\partial y_2}\right\| \tag{2.1}$$

when regarding two coupled ODE systems

$$\begin{aligned} \dot{y}_1 &= f_1(y_1, y_2, t), \quad y_1(t_0) = y_{1,0} \\ \dot{y}_2 &= f_2(y_1, y_2, t), \quad y_2(t_0) = y_{2,0} \end{aligned} \quad . \tag{2.2}$$

Loose coupling of *simulations* – characterized by independent time steps in all sub-simulations which necessitates extrapolation in between – is sensible mainly for loosely coupled equation systems to avoid the occurrence of large splitting errors or the need for very small synchronization time steps alternatively.
With respect to simulations, loose coupling is also called

- weak coupling (Busch 2012, Striebel 2006)
- quasi-dynamic coupling (Trčka et al. 2009)
- ping-pong coupling (Trčka et al. 2009)
- solver coupling (Friedrich 2011)
- process coupling (Friedrich 2011).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Due to the necessary extrapolation (cf. Trčka (2008) and Wetter (2011) and Section 5.7), loose coupling methods are more prone to error accumulation and stability issues than strong coupling methods. Particular loose coupling algorithms vary from plainly sequential or parallel to iterative ones with or without common synchronization points and are illustrated in Section 2.7.

Strong coupling, on the other hand, describes the coupling of two or more simulations with iterations in every time step within every subsystem to fulfil given tolerances (see Matthies and Steindorf (2003) and Tseng and Hulbert (2001)). In this case, the subsystems do not use individual time steps between two macro steps (although this may be seen differently by some authors – see below). Such a strong coupling procedure is illustrated in Figure 2.12 for two subsystems.

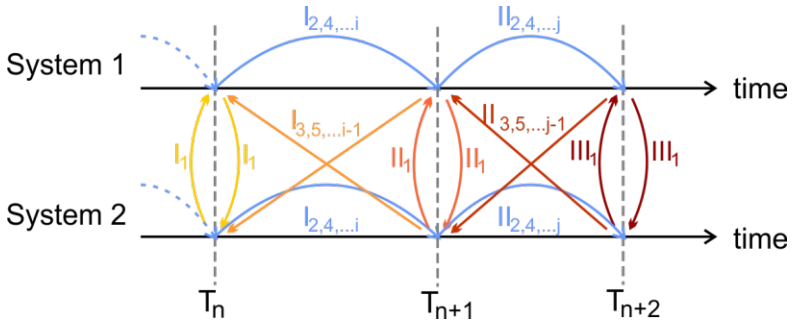


Figure 2.12: Illustration of the data exchange between two strongly coupled simulation algorithms (Hafner et al. 2016).

Strong coupling is motivated by high demands on accuracy where co-simulation is necessary due to different modeling approaches and requirements regarding implementation, not highly differing time constants. The gain in accuracy, however, comes with high computational costs due to the more frequent data exchange and permanent iterations, which makes strong coupling approaches unsuitable for real-time simulation. Other terms used for *strong coupling* are

- monolithic methods (Busch 2012)
- gluing methods (Busch 2012)
- fully dynamic coupling (Trčka et al. 2009)
- onion coupling (Trčka et al. 2009)
- tight coupling (Awais 2015).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Not altogether surprisingly, the perception of loose and strong coupling is not always the same throughout literature. Some authors refer to strong coupling as the intensity by which the two systems depend on one another (Viel 2014) while in the implementation, the realization of the coupling of the simulations might still be loose (in the sense of loosely coupled simulations explained above).

Regarding the example of two participating subsystems, Busch (2012) and Völker (2011) define strong coupling as code export of one system into the other system applying only one solver, which can be understood as model coupling as defined in Section 2.3.1 or analogous to the FMI for model exchange (Blockwitz et al. 2012). As soon as both subsystem solvers are used, Busch talks of weak coupling, which is further divided into the *embedded function approach* and what he calls *classical co-simulation*. In the *embedded function approach*, discretized equations of one system obtained by the solution algorithm of the respective solver are transferred to the other system. In so-called *classical co-simulation*, the coupled simulations run in separate simulation tools, thus allowing (and requiring) only the exchange of state values (and possibly derivatives) but no equations, consequently preserving the individual implementations and solution algorithms.

Pühringer (2017) regards weak coupling as the opposite of dynamic iteration (see Section 2.7), which interestingly corresponds to the definition for strong coupling by Trčka et al. (2009), where this is understood to mean iterations of the macro step. Tomulik and Fraczek (2011) and Wang et al. (2003) use the name *gluing algorithms* for coupling algorithms in general – in the sense of "gluing" initially separated system parts together. González et al. (2011), however, even speak of multirate strong coupling, hence clearly allowing individual steps of subsystem solvers.

Attention may again be paid to the fact that some authors refer to co-simulation only when loose coupling is applied (see for example Kübler and Schiehlen (2000b)), which means that strong coupling is not considered a part of co-simulation by those authors.
In the further course of this thesis, we will follow Definition 2.9:

**Definition 2.9** (Strong and loose coupling)**.** *Strong coupling* allows different solvers and simulators but requires the same time steps in all subsystems, permanent exchange of coupling data and iteration in every time step while *weak* or *loose coupling* allows different, individual time steps in the partial systems.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 2.6 Monolithic simulation

Monolithic simulation describes a non-partitioned approach to simulate the whole system of interest, consequently representing the opposite of co-simulation: no multiple rates, no co-simulation, just an “ordinary simulation” in one simulator with one solution algorithm and one time step.

Unfortunately, the term *monolithic methods* is by some (according to Busch (2012)) used for strong coupling methods or, vice versa, sometimes *strong coupling* is used to describe monolithic approaches (assembly of all equations and simulation in one environment (González et al. 2011)). This does make sense to some extent as at least strong coupling methods exclude multirate simulation – which is, as explained before, sometimes understood to be equivalent to co-simulation, hence in this understanding, a monolithic method would again be the opposite of co-simulation.

Wang et al. (2005), on the other hand, explain monolithic approaches to be the opposite of distributed simulation coupled by gluing algorithms, which could, contradictorily, be understood as the opposite of strong coupling, if no attention is paid to the fact that Wang et al. define gluing algorithms as “a class of algorithms that can be used to couple distributed component models for use in dynamics simulations".

A meaning outside the area of co-simulation is found in (Breitenecker and Popper 2007), where a *monolithic model description* stands for the representation of structural-dynamic systems in one model with a maximal, static state space in contrast to a so-called *hybrid decomposition*, confer Section 2.3.4.

In this thesis, the term *monolithic simulation* will be used for the opposite of co-simulation (in the sense of Definition 2.8).
Synonyms are

- mono-simulation (Trčka et al. 2009)
- monolithic approach (Sicklinger et al. 2014)
- uniform modeling (González et al. 2010)
- unified approach (Samin et al. 2007)
- unifying approach (Samin et al. 2007).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 2.7 Coupling algorithms

Coupling methods can be further divided into Gauß-Seidl[2] and Jacobi[2] type methods depending on the sequence of subsystem computation (see also Section 5.7.3). Both Gauß-Seidl and Jacobi type methods are referred to in various ways in literature.
*Gauß-Seidl type* methods are also called:

- staggered algorithms (Schierz and Arnold 2012)
- sequential algorithms (Schierz and Arnold 2012)
- staggered partition (Felippa et al. 2001)
- staggered solution (Felippa et al. 2001)
- conventional serial staggered procedure (Farhat and Lesoinne 2000)

*Jacobi type* methods also go by the names

- parallel algorithms (Schierz and Arnold 2012)
- conventional parallel staggered procedures (Farhat and Lesoinne 2000)

Figure 2.13 illustrates a Gauß-Seidl type algorithm for one macro-step in an overall system consisting of two partial systems.

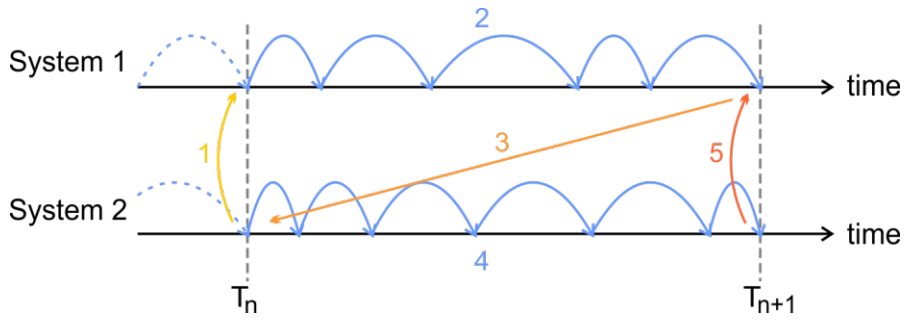


Figure 2.13: Gauß-Seidl type loose coupling co-simulation of two partial systems between two synchronization references $T_n$ and $T_{n+1}$ (Hafner et al. 2016).

One of the subsystems, w.l.o.g. System 1, is integrated first for one macro step, using individual steps and extrapolated values from System 2. As soon as System 1 has reached the next synchronization point, the current values of its states are transferred to System 2,

[2]The terms Jacobi scheme and Gauß-Seidl scheme respectively refer to the iterative matrix methods used to solve linear equation systems, see (Varga 1999).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

which is subsequently executed for the same macro step. At its micro steps in between, it can use interpolated values for variables from System 1.

A sequential algorithm without common macro steps is shown in Figure 2.14.

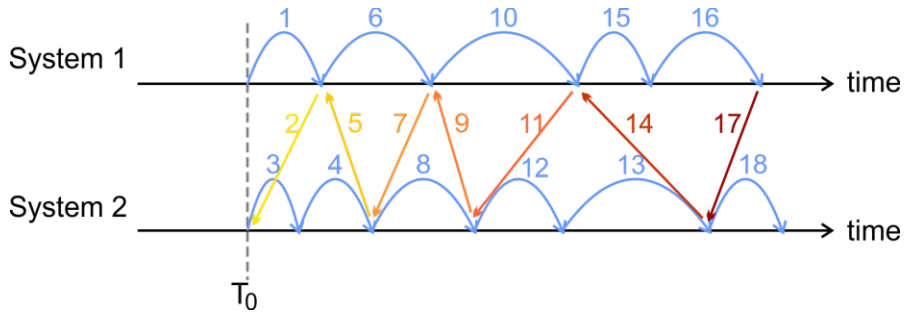


Figure 2.14: Asynchronous co-simulation algorithm, illustrated for two partial systems (Hafner et al. 2016).

Here, simulation times in all subsystems are compared after every step. The currently slowest system, i.e. the one with the smallest simulation time, is executed next for one of its micro steps. This is repeated and values are exchanged after every step. Note that this implies that the micro steps can be set completely independently of the other systems without even requiring an additional step at given synchronization points.

The procedure of a Jacobi type approach for two subsimulations is depicted in Figure 2.15.

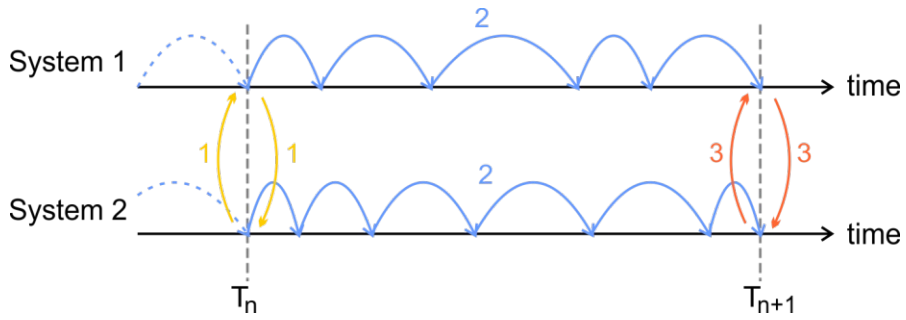


Figure 2.15: Jacobi type loose coupling co-simulation of two partial systems between two synchronization references $T_n$ and $T_{n+1}$ (Hafner et al. 2016).

With this method, both systems are executed in parallel (allowing also computational parallelization, rem.). This requires the use of extrapolated values of the respective other system in every subsimulation.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Iterative algorithms, starting from Gauß-Seidl or Jacobi type methods, are called

- waveform iteration (Busch 2012)
- waveform relaxation (Lelarasmee et al. 1982) or
- dynamic iteration (Miekkala and Nevanlinna 1987).

Such an iterative approach, starting from a Jacobi type method, is illustrated in Figure 2.16.

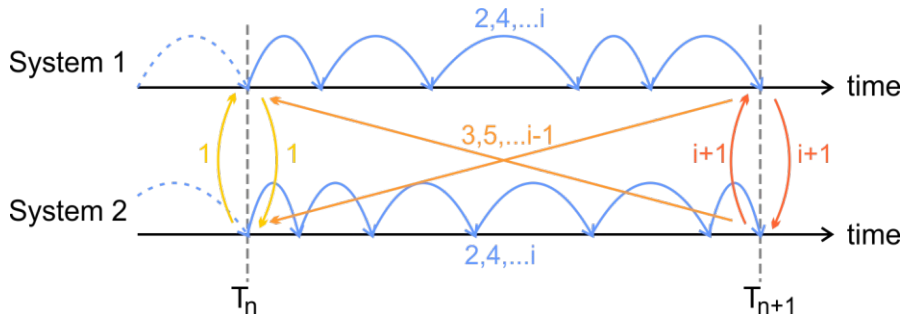


Figure 2.16: Waveform iteration of Jacobi type loose coupling co-simulation of two partial systems between two synchronization references $T_n$ and $T_{n+1}$ (Hafner et al. 2016).

After the first Jacobi type execution, the current macro step is repeated - using interpolated values in all subsystems, enabled by the states calculated in the prior execution – until a given tolerance is reached.

It shall be noted here that non-iterative methods are often called *explicit*, while iterative ones are also referred to as *implicit* (Schweizer and Lu 2014b). Therein, *fully implicit* is sometimes used to describe strong coupling methods (Matthies and Steindorf 2003). Methods with a predictor-corrector step – thus including rollback but no further iterations – are also called *semi-implicit* (Schweizer and Lu 2014a).

## 2.8 Orchestration

The term *master algorithm* is frequently used in the context of co-simulation (Arnold et al. 2011; Friedrich 2011; González et al. 2010; Völker 2011). It describes the algorithm acting on top level by organizing the communication with and between the participating subsystems, which in this respect are called *slaves*.

In recent years, the terms *master* and *slave* have by convention been progressively replaced to avoid association with slavery. Alternative terms are proposed for instance by the Open

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Compute Project (2020), therein *controller*, *main*, *primary*, *active*, *writer*, *source*, *control*, *local*, *parent*, *manager*, *superior*, or *original* instead of *master*, and *responder*, *secondary*, *replica*, *stand-by*, *reader*, *target*, *remote*, *agent*, *child*, and *subordinate* to replace the word *slave*. In other projects, the term *master* has been maintained while *slave* has been replaced by *replica* (Engine Yard 2017; Redis Ltd n.d.), *minion* (SaltStack 2021), or *puppet* (Linietsky et al. n.d.). Unfortunately, some of the presented alternatives may be mistaken as certain terms are already in use with different meaning (like *controller* in control theory or *agent* in Agent Based simulation). This could explain why no common convention within the modeling and simulation community has been established up to now. In this work, the *master/minion* terminology will primarily be adopted. In direct quotes from previously published work, however, *master/slave* will not be replaced.

With regard to electrical circuits, Striebel (2006) explains these terms as follows: "(. . . )we call a process *slave* if it works on a subcircuit's system and *master* if it involves the coupling system." The Modelica Association (2014) defines a master/slave relation as “A method of communication, where one device or process has unidirectional control over one or more other devices. Once a master/slave relationship between devices or processes is established, the direction of control is always from the master to the slaves. In some systems a master is elected from a group of eligible devices, with the other devices acting in the role of slaves.”

By some authors, a master is also called *orchestrator*, see f.i. the work of Gomes et al. (2017), who explain that an orchestrator is necessary to couple simulation units: “The orchestrator controls how the simulated time progresses in each simulation unit and moves data from outputs to inputs according to a co-simulation scenario. A co-simulation scenario is the information necessary to ensure that a correct co-simulation can be obtained. It includes how the inputs of each simulation unit are computed from other outputs, their experimental frames, etc.”. Depending on the interpretation, an orchestrator might only be characterized as such if the coupling is handled outside one of the subsystems, while a master can be represented by one of the subsystems calling the others. This topic will be addressed again in Section 5.7.1.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 2.9 Partitioning of mechanical systems

Depending on the kind of exchanged variables and corresponding coupling equations, mechanical systems can be divided by *force-force coupling*, *force-displacement coupling*, or *displacement-displacement coupling* (Schmoll 2015; Schweizer and Lu 2014b). Wang et al. (2003) declare kinematic and force information respectively as X and T vectors, thus replacing the above terms by the synonymically used *T-T method*, *T-X method*, and *X-X method* (see also Rustin et al. (2009)). Figure 2.17 illustrates these different approaches for a linear two-mass oscillator.

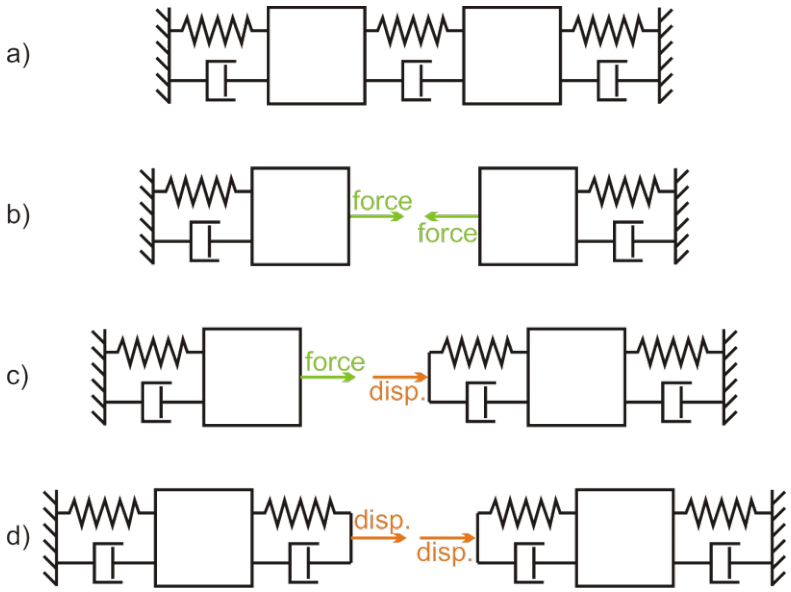


Figure 2.17: Sketch of a) linear two-mass oscillator coupled by b) force-force coupling, c) force-displacement coupling, d) displacement-displacement-coupling (Hafner and Popper 2017).

When the system is divided into two subsystems, the kind of coupling defines where the equations for the spring-damper elements in the middle are considered. In the force-force coupling approach, both systems exchange the force as coupling variable for which an equation is given in a separate coupling condition. If the force-displacement method is applied, the coupling variable for one system is again the force while the other system takes the displacement (i.e. position and velocity) as coupling variable leading to different coupling equations in comparison to the force-force approach. In case of displacement-displacement coupling, both systems use displacement variables as coupling variables.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 2.10 Varia

In the context of co-simulation, several other terms are frequently (and not always conterminously) used and will thus be addressed in this section.

Lelarasmee (1982) defines *internal variables* as variables of a subsystem that are calculated in this system and, correspondingly, *external variables* as internal variables from other subsystems which are needed in the current subsystem (and whose values therefore need to be extrapolated or interpolated).
Gu and Asada (2004) name variables used by all subsystems as *boundary variables*, also called *coupling variables* according to Schmoll and Schweizer (2012).

Depending on the development level (cf. Section 5.4) on which the coupling is considered, Tseng (2000) distinguishes between *divide-and-conquer* algorithms to partition complex systems into subsystems and *integrate-and-collaborate* approaches to couple already distributed submodels. Partitioning is also called *decomposition* by Wang et al. (2003), who refer to coupling of separate systems as *gluing*. Matthies et al. (2006) describe divide-and-conquer approaches as partition on the (mathematical) model level and integrate-and-collaborate methods as partition on the (real) system level.

*Splitting errors* are errors occurring due to system decomposition and the consequently required extrapolation (not due to discretization), see e.g. (Bartel et al. 2013; Schöps 2015). *truncation errors* are used to describe the error of integration methods (Gear and Wells 1984) as well as errors in the coupled procedure including both discretization and extrapolation errors (Zhang et al. 2011).

A definition of *state events* vs. *timed events* is found in (Gheorghe 2009): “discrete events are timed events scheduled by the discrete simulator. The events sent by the discrete simulator can be signals update events that are caused by the change of its input discrete signals or sampling events that are pure events (defined only by their time stamps) and indicate the sampling events time stamps”. On the other hand, “state events are unpredictable events generated by the continuous simulator. Their time stamp depends on the values of state variables (e.g. a zero-passing or a threshold crossing).”

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 2.11 Nexus of methods within and related to co-simulation

According to the – for this thesis – unified terminology, different sets can be built and are given below intending to clarify dependencies:

$$\begin{aligned}
\text{simulator coupling} &\subset \text{co-simulation}\\
\text{strong coupling} \cup \text{loose coupling} &\subseteq \text{co-simulation}\\
\text{modular time integration} &\subset \text{co-simulation}\\
\text{simulator coupling} &\not\subseteq \text{loose coupling}\\
\text{loose coupling} &\not\subseteq \text{simulator coupling}\\
\text{simulator coupling} \cap \text{loose coupling} &\neq \emptyset\\
\text{multirate simulation} &\not\subseteq \text{loose coupling}\\
\text{loose coupling} &\not\subseteq \text{multirate simulation}\\
\text{multirate simulation} \cap \text{loose coupling} &\neq \emptyset\\
\text{multirate simulation} &\not\subseteq \text{co-simulation}\\
\text{multirate simulation} \setminus \text{co-simulation} &= \text{partitioned multirate schemes}\\
\text{strong coupling} &\subset \text{singlerate simulation}\\
\text{multirate simulation} \cap \text{co-simulation} &\subset \text{loose coupling}\\
\text{hybrid simulation} &\not\subseteq \text{co-simulation}\\
\text{co-simulation} &\not\subseteq \text{hybrid simulation}\\
\text{hybrid simulation} \cap \text{co-simulation} &= \text{hybrid co-simulation} \neq \emptyset
\end{aligned}$$

Figure 2.18 aims to illustrate an overview of these relations.

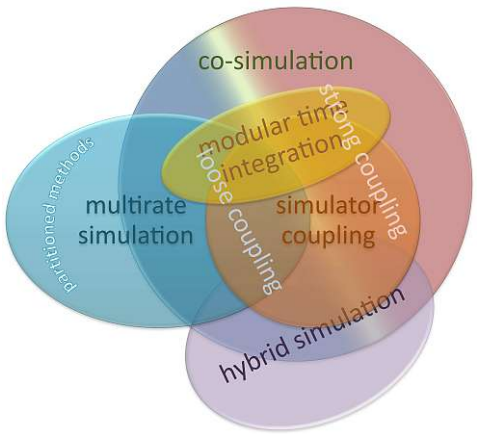


Figure 2.18: Relations of sets in co-simulation terminology.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 2.12 Co-simulation: what is it and what is it not?

Above deliberations show that careful definition of terms and awareness for different interpretation is crucial in order to be able to start investigations and understand related approaches in the area of co-simulation and multirate simulation.

For some terms, the definition is quite clear – f.i., simulation in one simulator with one solver and one time step is definitely no co-simulation. In case of physical models in mathematical representation, the line between model coupling and co-simulation is also easily drawn. This does not apply if methods such as Agent Based (AB) approaches or Cellular Automata (CA) are combined: with the coalescence of model and simulation in the individual approaches, the borders between model coupling and co-simulation may also become indistinct and perceptions may differ from researcher to researcher. Nguyen et al. (2017) present a "multi-agent approach for co-simulation", thus definitely classifying the approach as co-simulation. Ferreira et al. (2008) develop a multi-agent framework where human interaction is enabled in between several simulation runs. While indisputably cooperative, it remains hard to define whether this can be seen as co-simulation.

Another example for controversial interpretation is the case of Quantized State Systems (QSS, Kofman and Junco (2000)), where not time, but states are discretized and accordingly, "steps" in the solution algorithm are defined by states, not time. Therefore, whether simulations with different time steps are coupled (see Definition 2.8) cannot even be defined for these approaches. On the other hand, if in this condition, "time steps" were replaced by "steps", a coupled DESS of two systems solved with QSS solvers would always represent a co-simulation. Whether DEVS-based approaches (DEVS, DESS, DEV&DESS, P-DEVS, hyPDEVS, see Section 3.3.3) can universally be considered as co-simulation presents a topic for an almost philosophical discussion. Regarding the original DEVS, I would regard a coupled DEVS as model coupling of two or more Discrete Event systems. In case of DESS, on the other hand, the solution algorithm is incorporated in every atomic DESS itself, which would imply fulfillment of "differing in solver algorithms" in Definition 2.8.

What is more is that for some distinctions, the borders become blurred, as in the case of partitioned multirate methods and non-partitioned ones. Original partitioned Runge-Kutta schemes, for example, where the solution algorithm is partitioned and system parts solved with a smaller time step by utilization of stage values, are in my opinion coupled on such a deep level that no separate simulations can be discerned. However, there also exist par-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

titioned approaches incorporating even different integration schemes for stiff and non-stiff parts respectively, which makes it hard to decide where the line to co-simulation is crossed. In terms of loose and strong coupling, some speak of strong coupling as soon as inter-dependencies between systems are high (see Viel (2014) and compare the definition of loosely coupled systems in (2.1), others speak of the coupling method itself, meaning an iterative exchange in every synchronization step, which differs to a monolithic simulation in the manner that every subsystem is allowed to use its own simulator as long as communication takes place in the way the orchestrator demands it. As Definition 2.9 does only allow, not require, individual micro-steps in loose coupling co-simulation approaches, single-rate methods combining different simulators with rollback but no further iteration could be considered as grey area between strong and loose coupling methods, whereby strict criteria for distinction remain wishful thinking.

We can conclude that even though a general definition of some terms cannot be found without some contradictory description in literature, in this chapter the most common meanings have been pointed out with the aim at bringing the readers' attention to possible misunderstandings which are valuable for further pursuits in this area. Moreover, the usage in this thesis has been clarified, while co-simulation terminology will in general remain a topic for discussion.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

CHAPTER 3

# State of the Art in Co-Simulation and Related Methods

After the clarification of certain terms in the previous chapter, the reader is now introduced to an overview of existing and ongoing developments in multirate and co-simulation. While structured into thematical sections for better clarity, their contents are for the most part arranged chronologically to give an impression on the “history of co-simulation”, in particular, which developments are based on which, in addition to general information on the state of the art.

The first sections cover research on co-simulation of ODE and DAE systems, including different coupling methods along with comparisons and stability studies. This is followed by a description of standards for co-simulation, specific developments such as frameworks and investigations on hybrid co-simulation or partitioned methods. In conclusion, general strategies for the development and validation of coupling methods and summarized information on methods and challenges are given.

Each section starts with an overview of developments and references to the respective literature, for which more details are given afterwards for the interested reader (and can easily be skipped by the not-so-interested one).

## 3.1 Beginnings in classical co-simulation: coupling of ODEs

The first investigations on multirate and co-simulation –without use of these names, cf. Chapter 2– have started on ODE systems, motivated by differing stiffness properties and time constants in system parts (Andrus 1979; Gear and Wells 1984; Hofer 1976) or aiming at faster computation by parallelization (Jackson 1991). Some of these first developments, which are mostly specific to a given problem, are summarized below. The presented approaches vary between solutions with the same, yet adaptive step size (Hofer 1976), an adaptive approach with order control (Gear and Wells 1984) and the introduction of waveform iteration (White et al. 1985). Propositions regarding consistency depending on the used extrapolation order are found in (Andrus 1979; Gear and Wells 1984; Knorr 2002). What these investigations have in common is that the considered ODE IVP can be divided into two (or more) partial systems as depicted in (3.1):

$$\dot{\boldsymbol{y}}_1 = \boldsymbol{f}_1(t, \boldsymbol{y}_1, \boldsymbol{y}_2), \quad \boldsymbol{y}_1(t_0) = \boldsymbol{y}_{1,0} \tag{3.1a}$$

$$\dot{\boldsymbol{y}}_2 = \boldsymbol{f}_2(t, \boldsymbol{y}_1, \boldsymbol{y}_2), \quad \boldsymbol{y}_2(t_0) = \boldsymbol{y}_{2,0} \tag{3.1b}$$

As one of the first developments in this respect, Hofer (1976) presents a partially implicit method for the solution of ODE systems which can be partitioned into stiff and nonstiff parts. The formula can be seen as combination of a modified midpoint rule and the implicit trapezoidal rule which both operate with the same integration step size. Accuracy is improved by the introduction of automatic step size control which makes use of local extrapolation.

Andrus (1979) presents an algorithm for the numerical integration of partitioned ODE systems of the form (3.1). Assuming w.l.o.g. that (3.1a) is the slower reacting system, it can be integrated with a larger step size in comparison to (3.1b) and hence with a larger step size than if the whole system (3.1) were integrated as one.
The method is described in Algorithm 3.1.

**Algorithm 3.1** (Andrus 1979)**.** Given $t_0$, $\boldsymbol{y}_{1,0}$, and $\boldsymbol{y}_{2,0}$, integrate the equations

$$\boldsymbol{y}_1' = \boldsymbol{f}_1(t, \boldsymbol{y}_1, \boldsymbol{y}_2^*(t, \boldsymbol{y}_1)) \tag{3.2}$$

from $t_0$ to $t_0 + h$, where $\boldsymbol{y}_2^*(t, \boldsymbol{y}_1) = \boldsymbol{y}_2(\Delta t)$, $\Delta t = t - t_0$, and $\boldsymbol{y}_2(\tau)$ is the solution to the equations

$$\frac{d\boldsymbol{y}_2}{d\tau} = \boldsymbol{f}_2(t_0 + \tau, \boldsymbol{y}_{1,0} + \tau\dot{\boldsymbol{y}}_{1,0} + \left(\frac{\tau}{\Delta t}\right)^2 (\boldsymbol{y}_1 - \boldsymbol{y}_{1,0} - \Delta t\dot{\boldsymbol{y}}_{1,0}), \boldsymbol{y}_2) \tag{3.3}$$

with initial conditions $\tau = 0$ and $\boldsymbol{y}_2(0) = \boldsymbol{y}_{2,0}$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

In the algorithm, (3.2) is integrated with a fourth-order Runge-Kutta scheme while the method for (3.3) can be chosen freely as long as sufficiently accurate for the desired purpose. Andrus states that computational speedup can be achieved by the application of this algorithm "if (3.1b) can be integrated in closed form (assuming $\boldsymbol{y}_1$ is replaced with a given polynomial function of time) or if an evaluation of $\boldsymbol{f}_1$ is several times more time-consuming than an evaluation of $\boldsymbol{f}_2$."
Regarding the solution of $\boldsymbol{y}_1$, the method is proven to converge of order four, too, as long as the approximation of $\boldsymbol{y}_2$ does not add lower order terms to the solution of $\boldsymbol{y}_1$.

Gear and Wells (1984) are the first to apply a multirate algorithm with step size and order control for loosely coupled ordinary differential equation systems. Linear multistep methods are used for all subsystems but vary in the order and explicit/implicit approaches: to stiff parts, BDF methods are applied while non-stiff components are solved with an Adams method (see Section A.2 for background information). The presented method is restricted to nonstiff systems or systems that have their stiffness isolated in the separate components. The system is (non-automatically) partitioned according to the micro step size of subsystems.
To minimize rejections in the faster parts, a slowest first method is applied. In case of rejections, the step size is decreased by a power of 2. The step sizes of faster methods are required to be integer divisors of the step sizes of slower methods to minimize additional computational effort.
It is stated that stability and consistency of the integration methods and interpolation with errors no larger than $o(1)$ are necessary and sufficient for convergence of Lipschitz continuous problems. Further, "if the minimum order of accuracy of the integration formulas is $p$ (local errors $\mathcal{O}(h^{p+1})$) and of the interpolation formula is $q$ (local errors $\mathcal{O}(h^q)$) then the global error for suitably differentiable problems is $\mathcal{O}(h^{min(p,q)})$." It is shown that errors from interpolation and extrapolation are small in comparison to the truncation error, so the step size is adapted in order to keep the estimate for the truncation error below a certain tolerance. Stability is investigated for a linear system of two coupled ordinary differential equations. They state that in case of weakly coupled systems, stability properties of the coupled system can be assumed to meet the stability properties of the partial systems.
The developed method is applied to a test system for which it shows significantly better performance compared to a mono-simulation for all given scenarios as long as the coupling is weak, in particular if the slow components do not depend strongly upon the fast components.
Regarding accuracy, the mono-simulation naturally shows smaller errors for most variables.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

However, whereas in the multirate simulation all errors are in the same order of magnitude, in the mono-simulation great differences between the errors of the individual variables can be observed.

White et al. (1985) explain the general idea of waveform iteration (cf. Sections 2.7 and 5.7.3.2) for ODE systems along with detailed proof of uniform convergence first of the basic, stationary method for systems with strictly dominant and Lipschitz continuous mass matrix, and further of non-stationary methods, Newton-Raphson algorithms, and discretized methods. The numerical investigations are followed by techniques for the implementation of waveform relaxation methods along with examples.

Jackson (1991) presents small-scale parallelization methods for initial value problems of ordinary differential equation systems. Due to huge challenges regarding fast computation at this point in time, the motivation for the parallelization is the separation of large systems of ordinary differential equations to enable faster computation. For applications in real time or hardware-in-the-loop control systems, the separation of given systems for computational purposes is still an up-to-date problem. The paper focuses on the possibilities of parallelizing predictor-corrector Runge-Kutta schemes.

Knorr (2002) investigates consistency for parallel multirate schemes depending on the order of extrapolation. It is shown that for the multirate simulation of two systems of differential equations with a one-step method of order $p$ and extrapolation orders $q_1$ and $q_2$ the consistency error can be found as $min\{p, q_1 + 1, q_2 + 1\}$ for sufficiently small multirate factor and Lipschitz constants of the two right sides of the differential equations. For extrapolation orders greater than zero, however, stability issues can occur so the respective method has to be investigated further to ensure stability and hence convergence. Stability can be enhanced by consideration of gradients instead of former values for the extrapolation. These theoretical investigations are applied to a multirate vehicle simulation implemented in CASCaDE[1] and co-simulation using different simulators (DADS and MATLAB/Simulink) for the applications of an inverse pendulum and an Active Body Control system.

[1] references to this and other mentioned software and tools are found in the Appendix, Section A.3

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 3.2 Coupling methods for DAEs

Owned in particular to applications in mechanical systems, research on co-simulation has soon extended to differential-algebraic equation systems. These can be represented either as systems of ODEs which are coupled by algebraic constraints (see f.i. Gu et al. (2000)), or systems of DAEs coupled by output-input dependencies (as in Kübler and Schiehlen (2000b)) which means that the algebraic part can be restricted to the coupling equations or be part of every subsystem.

A range of developments found in the literature are summarized in this section. First, various coupling methods are presented, ranging from methods to regularize high-index DAEs (Gu 2001; Gu and Asada 2004; Gu et al. 2000) to automatic algorithms for the calculation of calling sequence (Glumac and Kovacic 2018) and linking of models (Stecken et al. 2019). This is followed by sections on certain kinds of coupling methods, therein iterative approaches (Section 3.2.2), master algorithms with different choices of macro step size (Section 3.2.3) and methods specialized in the decomposition and coupling of mechanical systems (Section 3.2.4). This arrangement may not be seen as classification (which follows in Chapter 5) but simply as means to provide a better overview owned to the multitude of referenced publications. In Section 3.2.5, works comparing two or more coupling approaches with respect to stability, accuracy or performance are presented. Investigations on stability and error estimates for co-simulation are found in Section 3.2.6.

### 3.2.1 Varia

Gu et al. (2000) present a method for the co-simulation of systems of ordinary differential equations coupled via algebraic constraints. The coupled system equations are interpreted as being controlled by the coupling variables to drive the value of the constraint to zero by iteration. The method described in this article uses the Singularly Perturbed Sliding Manifolds (SPSM) approach. This approach regularizes the DAE of the overall system by introducing a manifold defined by a weighed combination of the first $r-1$ derivatives of the constraint, where $r$ is the differential index of the DAE. Perturbation assures the asymptotic stability of the manifold and hence the possibility to extract the derivative of the coupling variables from the system.
Maple V is used to symbolically differentiate the algebraic constraints and return a C function incorporating the ODE formulation of the DAE. A middleware coordinates the provision of each subsystem simulator with the respective boundary conditions while they run simul-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

taneously. Every subsystem has to deliver a system description and the state variables as output to the coordinator. Maple then transforms the DAE into ODEs, which are solved in parallel and updated by synchronization at every integration step (which is the same for all subsystems at this state of development).
For a given test system of two connected robot arms, results show that even for unfulfilled constraint equations, the method lets them converge to zero (by differing speed for varying perturbation).

Gu (2001) considers incompatible boundary conditions between algebraically coupled subsystems of differential-algebraic equations caused by causal conflicts. This incompatibility is faced by a Boundary Condition Coordinator. Discrete Time Sliding Mode Control is applied to regularize high-index DAEs and extended to a Multi-Rate Sliding Mode Control method.

In (Gu and Asada 2004), modifications of the algorithm introduced in (Gu et al. 2000) are presented. Initially, two systems coupled by one algebraic constraint (corresponding to Kirchhoff's laws) and one boundary variable, resulting in a DAE of finite differential index, are considered. As described above, the basic Discrete Time Sliding Mode Control algorithm uses a weighted linear combination of the constraint equation describing a sliding manifold instead of the original constraint to reduce the index of the overall system. The sliding manifold is controlled by a boundary condition coordinator to approach zero. This method is modified to restrict the information from the subsystems required by the controller to the state outputs and their derivatives, as any additional subsystem information can in general not be assumed to be accessible in actual applications. Instead, the value of the sliding variable at every time step and the Jacobian of the boundary variable are calculated by using the values of the subsystem output state variables. The differential index can be determined by the minimum of the coupled subsystem's relative orders, so every subsystem can calculate the maximally needed order from the derivatives of its variables itself.
Additionally, criteria for parameter choices and nominal control are given. To avoid instabilities, the control itself may require small time steps compared to the subsystem integrators. This motivates a multirate algorithm where the control takes smaller steps while the subsystems maintain larger steps. Constraint error bounds as well as detailed convergence criteria are stated and proven in the paper while further information on the choice of the parameter for the sliding manifold can be found in (Gu 2001). Numerical examples demonstrate the performance improvement as well as the necessity of the given criteria. Finally, it is explained how the algorithm can be applied to more than two subsystems as long as they are coupled by linear algebraic constraints.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Glumac and Kovacic (2018) introduce an algorithm for the automatic calculation of the calling sequence for sequential co-simulation based solely on the system topology. The algorithm follows the following guidelines:

- all subsystems delivering inputs to a specific subsystem should either all precede the subsystem in the calling sequence or all come after it.
- the number of communication delays in the co-simulation should be minimized.

The algorithm is implemented in the co-simulation platform AVL Model.CONNECT following the FMI standard. Since the FMI does not require information on (algebraic) input-output dependencies of the FMUs (Functional Mockup Units), a further guideline taking these dependencies into account is not yet implemented but envisaged. The method is applied to simulate a hybrid electric vehicle.

Stecken et al. (2019) aim at the automatic linking of models in the co-simulation of production systems by extending the dynamic continuous engineering (DCE) approach which focuses on Industry 4.0 components or cyberphysical systems. The method is based on data classifications. According to Stecken et al.'s research, existing approaches only address partial aspects of automatic linkage, which they aim to improve. First, participating models and variables are assigned to newly defined classes. In the course of this, physical and logical constraints for the integration of new components have to be considered. Within the next step, *linking of models*, data type compatibility is taken into account as well. The obtained information is then stored in an AutomationML file to allow proprietary simulation tools that, f.i., do not support the FMI standard which would already offer such a description within an additional FMU. The algorithm also enables the selection of different "views" where individual models within one component (e.g., models specific for energy simulation) could be used. Further, they offer an implementation of a configurator as a GUI. Exemplarily, the method is applied to the co-simulation of a robotic cell with regards to energy. Advantages of automatic linkage are the reduction of errors due to manual connections in complex systems and easier adaption in case of alterations to existing systems.

### 3.2.2 Iterative methods

Iterative methods, *waveform relaxation*(WR) in particular seem to have been introduced by Lelarasmee et al. (1982) for DAEs, while the first mention including convergence theorems for certain methods applied to ODEs is found in (White et al. 1985) (cf. Section 3.1).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Throughout the years, dynamic iteration occurs time and again in different variations and improvements: The iterative approach presented by Rathinam and Petzold (2002) utilizes reduced order models, Arnold and Günther (2001) and Ebert (2004) introduce preconditioning to counter instabilities while Tomulik and Fraczek (2011) present an iterative algorithm showing similarities to the sliding mode control method (cf. Gu (2001)) and the algorithm of Sicklinger et al. (2014, 2015) uses interface Jacobians for stabilization. Schöps (2011) extends the application on PDAEs and discusses emerging stability issues.
Further information on the mentioned methods can be found below.

Lelarasmee et al. (1982) introduce different waveform iteration methods for both Gauß-Seidl and Jacobi type approaches. For the verification of the consistency of the partitioning, see (Lelarasmee 1982) and the theorem in (Lelarasmee et al. 1982) on page 134.
The iterations converge uniformly as long as the equations describing the dynamic behavior are Lipschitz continuous.

Arnold and Günther (2001) investigate a dynamic iteration method with a finite number of iteration steps in each macro-step for DAEs of index 1. It is found that a contractivity condition has to be fulfilled to guarantee stability and convergence. If this condition is violated, preconditioning is introduced to enforce convergence. Innovative about this approach is that the contractivity condition is found to ensure stable error propagation from window to window.

Rathinam and Petzold (2002) present a dynamic iteration method that makes use of reduced order models. They combine waveform relaxation, which is known to improve convergence properties of coupled systems (Lelarasmee et al. 1982; Miekkala and Nevanlinna 1987), and model reduction, which presents an entirely different approach for handling complex systems. The method of Rathinam and Petzold simulates every subsystem in turn while it is connected to reduced models of the other subsystems. The results of this simulation are then used to update the reduced model of this subsystem. For the model reduction, *proper orthogonal decomposition* (see f.i. Holmes and Holmes (2012)) is used. Convergence of the method is investigated and proven for two coupled ODE systems with certain restrictions. Tests of the method on a nonlinear power grid model and a discretized linear reaction-convection-diffusion type PDE show that iterations can be drastically reduced in comparison to common WR methods while at the same time reducing the maximum error. However, difficulties can occur for approximately equal eigenvalues of the covariance matrix of the fixed point trajectory of any subsystem. For this case, a slightly modified approach is

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

presented which improves convergence while decreasing accuracy. It should be noted that the method presumes unrestricted knowledge of all sub-models.

Ebert (2004) presents different relaxation methods for coupled ordinary differential equations as well as coupled differential-algebraic equations. First, it is shown that relaxation using Jacobi as well as Gauß-Seidl type iterations always converges towards the solution of the overall system in case it consists of two coupled ODEs. In case of DAEs, depending on the algebraic coupling equations, two coupled DAEs of index one can yield a coupled system of higher index, which can lead to divergence of the numerical solution of the coupled system. However, although for couplings of Jacobi and Gauß-Seidl type the index is shown to remain equal to one if both subsystem indices are 1, instabilities can still occur depending on the given system (or even just the coupling sequence for Gauß-Seidl type coupling, see also Arnold and Günther (2001)). To handle these instabilities, a preconditioning method for linear implicit DAEs of index 1 is presented. The idea is similar to the one developed in (Arnold and Günther 2001), substituting algebraic variables by a linear combination of the current values and those of prior iterations. The paper also presents a lemma with conditions to determine whether suitable parameters for the linear combinations can be found to guarantee convergence of the modified system. The linear error approximations for the respective methods which are used to prove convergence only cover the errors introduced by the relaxation, not errors due to the numerical integration.
Note further that although this work presents conditions for the determination of convergence properties and ways to improve stability, the preconditioning method requires detailed knowledge of the participating subsystems as well as intrusion in the subsystem solution algorithms.

Ebert (2008) investigates the coupled simulation of partitioned electrical circuits. He presents convergence criteria for coupled DAE systems in general and focuses on semi-explicit systems afterwards. For these, modified dynamic iteration methods are presented which can accelerate convergence. In the case of partitioned circuits, a method for partitioning and representation as graphs is introduced. In addition, tests regarding efficiency of dynamic iteration with respect to the macro step size and a possibility for step size control are presented and tested on benchmark examples of rectifier circuits including lumped elements and diodes. For these, the dynamic iteration method sometimes proves less efficient than a monolithic simulation but Ebert states that in case of PDE device models, dynamic iteration becomes very efficient and is easy to implement.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Tomulik and Fraczek (2011) present an iterative (gluing) algorithm showing some similarities to the sliding mode control method (cf. Gu (2001), Gu and Asada (2004), and Gu et al. (2000)). It is applied to a case study of a double pendulum where each pendulum represents one participating system. Integration is carried out with `ode45` in MATLAB. It is shown that consideration of velocity constraints in addition to displacement in the coupling equations significantly reduces oscillations. Due to the iterations, computational costs are rather high compared to other coupling techniques. At the synchronization steps, unknown constraint values are extrapolated either by cubic extrapolation based on consecutive points or separate of bottom and top envelopes. The latter increases efficiency, but Tomulik and Fraczek state that for more general systems, e.g. with lower frequency oscillations, more elaborate extrapolation techniques should be used.

Schöps (2011) discusses the multirate simulation of electric circuit (DAE) and field models (PDE). Error propagation for several macro time steps is investigated for DAEs that are coupled by Lagrange multipliers. In addition to the used multirate method, Schöps describes upcoming stability issues and gives iteration estimates (see Lemma 5.4 and Proposition 5.5 of his thesis). Applications of the method in field-circuit coupling, mechanical-electromagnetic coupling and thermal-electromagnetic coupling are presented in (Schöps 2015).

Sicklinger et al. (2014) describe an implicit Jacobi-type co-simulation method for multiphysics simulations: the IJCSA – Interface Jacobian-based Co-Simulation Algorithm, an iterative algorithm based on Jacobi-Type co-simulation and using interface Jacobians for stabilization. They consider an arbitrary number of subsystems described by input-output equations. Dependencies between these are defined by interface compatibility constraint equations which are iterated via a Newton algorithm. Sicklinger et al. propose additional enhancements for efficiency by using approximations for subsystem solution iterations. Information from the interface Jacobians of all subsystems is used for a global Jacobian to stabilize the overall co-simulation. Stability properties are compared with fixed-point Jacobi and Gauß-Seidl relaxation by several examples where IJCSA performs better than both, but the decomposition of the underlying system is crucial for nonlinear problems. Algebraic loops can also be handled due to the formulation of the method in residual form.
The IJCSA is applied in (Sicklinger et al. 2015) to co-simulate the emergency brake of a wind turbine model taking into account the interaction of blades, generator, control, and fluid. The air flow is modeled with OpenFoam while the generator is implemented using multibody dynamics. The simulation is validated via a full scale wind tunnel experiment performed at the National Renewable Energy Laboratory Unsteady Aerodynamics Experiment Phase VI.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 3.2.3 Choice of macro steps

In this section, methods employing a dynamic choice of the macro step are presented. Schierz and Arnold (2011) point out challenges in macro step size control such as slow-down by small step sizes and error calculations, accuracy loss in case of large steps or, specific to co-simulation, the unknown influences between macro and micro steps. Most of the methods described below are adaptive algorithms where the macro step size, at which all subsystem simulators communicate, is chosen according to varying estimates. Busch (2012) and Schmoll (2015) realize automatic adaption of macro step sizes via a predictor-corrector method while Völker (2011) takes into account eigen frequencies of the overall and/or partial systems instead of local error estimates. Benedikt et al. (2010) include an iterative approach with *increasing* macro size which is reduced again if a maximum of iterations is reached.
Liang et al. (2011), on the other hand, present an algorithm without common macro steps, where the subsystems are solved sequentially with their individual step size, determining after every step the slowest und thus next system to be executed. A similar approach without synchronized time steps is applied by González et al. (2011), of which a more detailed description can be found in Section 3.2.5.
Details on other mentioned methods follow below.

Benedikt et al. (2010) present one iterative and one non-iterative coupling method with macro step size control. The regarded subsystems are assumed to be described by algebraic equations or a linear time invariant model. In the iterative approach, the step size is initially chosen small and grows with a user-defined factor. In case a defined maximum of iterations is reached, the step size is adaptively reduced.
The non-iterative algorithm compares the results after one macro step to the extrapolated solution and reduces the step size to a predefined minimum in case of rapid signal changes. Otherwise, it is weighted depending on the difference between estimated and simulated values.
The methods are tested on the model of a hybrid electric vehicle partitioned into two subsystems. In comparison to a conventional, non-adaptive approach, the main benefit of the non-iterative method is significant computational speedup (thereby leading to some loss in accuracy), whereas the iterative approach can reduce the error if advanced scheduling (i.e., selecting execution sequence of subsystems) is applied.

Schierz and Arnold (2011) first provide a general description of common loose coupling co-simulation methods and then point out the challenges with variable step sizes: if the steps

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

are chosen too big, the internal integrators might set very small steps to maintain tolerances, hence slowing down the whole process and increasing round-off errors. If the macro-steps are too small, however, the whole co-simulation can become quite inefficient. What is more is that error approximation requires additional calculations and many simulators do not offer possibilities to discard several time-steps and re-start from former ones without loss of information. Approaches for improvement of step size control are methods from control theory refined by filters, so local errors which are insignificant for the global discretization error are left out.
Schierz and Arnold also point out that step size control in co-simulation is not yet state of the art. Open questions regarding adaptive step sizes are for example influences between macro and micro steps.

Liang et al. (2011) propose an asynchronous co-simulation algorithm where the individual solvers use their own time step independently from all other partial systems, so they do not necessarily have any step in common. The sequence of computation is decided by the system with the smallest simulation time: this system calculates one step while using extrapolation for the values from other systems, afterwards again the system with smallest simulation time is chosen (which can then use interpolation for values from the first system and extrapolation for values from the other systems) and so on.
Regarding stability, they state that the "order of interpolation techniques can be within two to avoid instability", which is only tested by examples, not thoroughly proven in the paper. The improved stability comes with higher computational costs in comparison to Jacobi and Gauß-Seidl methods since more evaluations are required.

The work of Völker (2011) discusses the choice of the macro step size in co-simulation algorithms. In general, the choice of step sizes can be determined by local error estimates, but as these are often hard if not impossible (depending on the respective software) to obtain, other approaches are pursued. Eigen frequencies of the system itself are taken into account for the investigation on effects on average step sizes for several simulation runs. The approach presented for the synchronization interval of a coupled simulation suggests determination of the eigen frequency of the overall system by implementing the whole system – in less detail, if needed – in one simulation software. While this can also be helpful for comparison and validation, it is hard to accomplish for arbitrary systems, as many systems are co-simulated for the lack of a simulation environment which can sufficiently satisfy the individual needs of every partial system. Another possibility would be the approximation of the frequencies of the overall system via the eigen frequencies of the partial systems. The

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

communication interval is chosen as a tenth of the reciprocal value of the highest frequency occurring in the system.
The approach is tested on several test examples implemented in MATLAB/Simulink, SIMPACK, AMESim and DSHplus with MATLAB/Simulink acting as master.
In conclusion, an idea for automatic determination instead of particular investigation of the communication frequency by the master algorithm is mentioned.

Busch (2012) aims at the automatic adaption of macro step sizes by taking the accuracy of the master as well as minion systems into account. An explicit approach with only one execution of each macro step via predictor-corrector method is investigated. A comparison of extrapolation by Hermite and Lagrange polynomials shows that the behavior is similar for explicit methods but Hermite polynomials yield better results for implicit schemes, especially if higher order polynomials are used.
Busch also proposes an idea for adaptive methods to control the polynomial degree for future studies.

### 3.2.4 Decomposition and coupling of mechanical systems

In this section, coupling techniques for mechanical systems described as DAEs are assembled. Due to their specific structure regarding their description, several investigations on the decomposition (Featherstone 1999a,b; Jia and Leimkuhler 2003; Tseng and Hulbert 2001) and further, gluing of these systems in the form of different force-force, force-displacement, and displacement-displacement coupling approaches (cf. Section 2.9) are made. On the one hand, they differ by this distinction: Tseng et al. (2003) present an X-X (i.e. displacement-displacement) strategy, Wang et al. (2003) and Wang et al. (2005) a T-T (force-force) method and Rustin et al. (2009) a T-X (force-displacement) method, while the works of Schweizer et al. compare all three and consider systems coupled by applied forces/torques (Schweizer et al. 2015a; Schweizer and Lu 2014b) and also systems coupled by reaction forces/torques (Schweizer and Lu 2015; Schweizer et al. 2016; Schweizer and Lu 2014a). In addition, they apply different stabilization techniques: by additional Lagrange multipliers (Schweizer and Lu 2014b), consideration of derivatives or integrals of coupling conditions (Schweizer et al. 2015b), or Baumgarte stabilization (Schweizer et al. 2016). Iterative methods are found in (Tseng and Hulbert 2001; Wang et al. 2003, 2005), semi-implicit (i.e. predictor-corrector) approaches are considered f.i. by Schmoll (2015) and Schweizer and Lu (2014a). Furthermore, Rustin et al. (2009) apply automatic partitioning and parallel computing. The techniques are described below in more detail.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

In the two-part article of Featherstone (1999a,b), a so-called “divide-and-conquer algorithm” (DCA) for the parallel simulation of articulated-body dynamics is presented. Articulated-body dynamics consider a system of rigid bodies connected to each other by joints and to the outside world by handles, which can apply external forces. Unknown subjects of interest are the accelerations in response.
To apply the algorithm, the system is described by a binary assembly tree that represents the hierarchical decomposition of the system. The idea of the presented algorithm is to describe the coefficients in the equations for every tree node by terms from the equations of its children. Further, the algorithm consists of four passes through the assembly tree. Two preliminary passes are necessary to calculate the position and velocity of every body as well as coordinate transformation matrices where required. In the main pass, calculations of the equations (described according to above idea) are performed. As soon as the root node is reached in the main pass, a back-substitution pass follows. The root node has only one child and optionally connects the body to a fixed base. The interaction with this base has to be calculated and is passed through the tree top-down.
Featherstone (1999a) describes the basic algorithm by an example only consisting of two parts without deeper hierarchy. It states that the DCA can achieve an asymptotic time complexity of $\mathcal{O}(log(n))$ where $\mathcal{O}(n)$ is the number of processors on the used parallel computer and $n$ the number of bodies in the system.
In the second part of the article (Featherstone 1999b) the algorithm is extended to handle closed-loop systems, as for systems without a constant upper limit to joints participating in one assembly operation, time complexity for the general DCA would increase up to $\mathcal{O}(n^3)$. The modification presented in this article also increases accuracy especially for systems with large numbers of bodies, as stiffness increases typically with the number of participating parts. In the modified algorithm, one node can represent a body or only parts of a body, as some have to be artificially split to obtain a balanced assembly tree. When including kinematic loops, some connections can represent more than one joint (met by link splitting), loop constraints require stabilization (met by inclusion of complete constraint equations, modified with the Baumgarte method, in equations of motion) and some matrices may not be invertible anymore. Lastly, numerical conditioning of the back-substitution pass is improved to increase accuracy. Tests on systems up to 1024 body parts show that while the general DCA is significantly less accurate than a good serial algorithm, the improvement with the pivoted DCA presented in (Featherstone 1999b) compensates this shortcoming.

In (Jia and Leimkuhler 2003), a method for the parallelization of the simulation of mechanical dynamics is presented. It is based on the impulse method which splits the system into

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

slow and fast parts. This approach is mollified by reversible averaging (see also Leimkuhler and Reich (2001)) and further modified to meet certain deficiencies by so-called *slaved* reversible averaging. This approach assumes that simulation is dominated by the fast parts. Still, instabilities arise for certain problems which they intend to approach in further studies. The given method comprises not only the coupling algorithm but also the subsystem solvers, so a separation of simulations onto several simulation tools seems neither possible nor desired.

Tseng and Hulbert (2001) present an algorithm for decoupling mechanical systems in a manner that constraint equations are solved separately from the dynamic equations. Further, the algorithm uses Newton iteration and is enhanced by Gauß's principle of least constraint.

Tseng et al. (2003) present a gluing algorithm for multibody dynamics systems which allows subsystems to maintain their preferred integration method and step size. This means that in this case, the term "gluing algorithm" does not imply strong coupling but means that this algorithm imposes the joint constraint equations representing the interconnections between the subsystems, hence "gluing" them together.
Before presenting their own development, a general description of the DAEs behind multibody systems is given and transformed to obtain *Maggi's equation* (see Section 3.2.1 of the paper). These equations reduce the index of the DAE and eliminate algebraic variables from the dynamic equations without modifying the constraints, but also reduce the sparsity of the tangent matrix.
An overview of stabilization and index reduction techniques (for example Baumgarte and Gear Gupta Leimkuhler formulation) follows, which are the basis for several existing gluing algorithms also described, including Baumgarte's stabilization with Conjugate Gradient solver and the relaxed augmented Lagrangian method using waveform relaxation. The algorithm of Tseng et al. uses a coordinate-split technique with modification to the Newton iteration so sparsity is preserved and the dynamics are independent from the constraint equations. In the MEPI (Maggi's equations with perturbed iteration) algorithm, the unconstrained problem is solved iteratively and separately in the subsystems, followed by projections to satisfy the constraints. However, as this algorithm soon fails, it is improved by basis indifference and Gauß's principle of least constraint to not completely neglect the constraints in the subsystems, thus implicitly coupling the systems. "One distinguishing characteristic of MEPI is that it eliminates the involvement of the generalized constraint forces. Instead, the information from the constraint equations is conveyed by the projection operation on dy-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

namics residues."(Tseng et al. 2003)
The algorithms are applied to a test case of three arms linked by two rotational joints. The MEPI algorithm is classified by Wang et al. (2003) as an X-X strategy, see below.

Wang et al. (2003) present a gluing algorithm (called “T-T method”) - understood in the sense of “gluing” initially separated (separately developed) system parts together, i.e. coupling from an integrate-and-collaborate perspective. They have developed a concept platform for the simulation of distributed mechanical systems (FEM and/or multibody). Starting with a standardized model description for every subsystem in XML, integrated models are instantiated thereafter by assembling component models. Finally, the separate models are simulated along with the gluing algorithm. Model details are not modified by the algorithm.
Wang et al. use the term *leaf model* for a simulation code along with its input data set. Upper level models which contain leaf models and information on connecting them are called *integrated models*. Apart from leaf models, integrated models can also contain lower-level integrated models, which implies allowing hierarchical coupling, cf. Chapter 6.
At the interfaces, kinematic (X) and force (T) information can be available. Thus, T-T coupling refers to a strategy where kinematic vectors are used as inputs to the “coordinator” (i.e. orchestrator) which calculates the force (T-) vectors. Those are transferred to the subsystems to use in the next time step. Vice versa, in an X-X coupling strategy, the force vectors are used by the coordinator, which outputs the kinematic quantity vectors. An X-T gluing strategy, on the other hand, uses the X-vector of one and the T-vector of the other system as coordinator inputs, obtaining the respective other to pass to the subsystems.
Further on, the paper focuses on an iterative T-T strategy. In the iteration, the interface forces are updated using only the kinematic information until compatibility conditions are sufficiently satisfied respecting a given tolerance. For demonstration, they use a Newton-Raphson updating method in the algorithm which can, in practice, be replaced by more sophisticated methods.
The gluing algorithm is tested on a FEM truck frame model, a double pendulum model and a four-bar link multibody dynamics model. In the truck frame model, two gluing layers are nested: the first couples two subsystems, one of them comprising five subsystems itself. Since the truck frame model is linear, no iteration is required. All three examples show comparable accuracy for the gluing method and the simulation of the respective all-in-one models.

Wang et al. (2005) present a manner to describe models in XML format which is the base for the application of a “T-T” gluing method. The aim is to glue methods which can be imple-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

mented in different simulation tools and distributed on different computers while maintaining integrity of the individual models. The coupling takes place by the iteration of an interface force vector via a Newton-Rhapson algorithm, where the calculation of the gluing matrix for the iteration proves the most crucial task in the method. The co-simulation is achieved via a webserver which extracts all necessary information of the subsystems to create an integrated model and connections to the subsystem models. The global step size is determined by the topmost model and all children are synchronized at every iteration, which continues in every step until convergence is reached. The algorithm is tested on an example consisting of two gluing layers with a total of nine subsystems. Since the problem is linear, no iteration is needed.

Rustin et al. (2009) focus on the application of a T-T method on rigid multibody systems. The gluing algorithm is validated by comparison with the monolithic simulation for two test examples, a double pendulum and a six-legged robot consisting of 49 bodies divided into seven subsystems. Results show that the errors remain acceptable and simulation time is not accelerated in spite of parallelization due to the iterations, so the main advantage of the gluing algorithm is the possibility to independently model the subsystems. They state that the algorithm can be extended for other kinematic approaches and finite element methods.

Schweizer and Lu (2014b) present the loose coupling co-simulation of a stabilized index-2 formulation of mechanical systems, where stabilization is achieved by introduction of additional Lagrange multipliers (see also Gear et al. (1985)).
As they present a predictor-corrector co-simulation method, this necessitates two executions of the same macro time step, hence requiring the subsystem solvers to be able to step back and be re-initialized to the former synchronization reference. In the predictor step, extrapolated values are used in both subsystems for integration until the next synchronization reference. In a second step, the predicted coupling variables are corrected: renewed integration with perturbed predicted variables along with the non-perturbed variables allows an approximation of the partial derivatives of the state variables by finite differences, by which corrected coupling variables are derived. Finally, a corrector step follows to obtain the corrected *state* variables by using the corrected *coupling* variables.
The method is first explained for a simple test system of a linear oscillator with one degree of freedom and then generalized for the coupling of two arbitrary multibody systems.
Case studies show that the semi-implicit approach remains stable while an explicit one can deliver unstable results even when the given system is stable. As it takes about twice the calculation time in comparison to the explicit method, the semi-implicit approach is only re-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

ally advantageous for stiff systems where stability issues arise for the explicit method.
In conclusion, Schweizer and Lu state that the semi-implicit method could be further improved by macro step size control via error estimates or fully implicit coupling, which would require more repetitions of the macro step.

While Schweizer and Lu (2014b) consider subsystems coupled by physical force/torque laws (i.e. by applied forces/torques), Schweizer and Lu (2014a) propose a similar approach but for coupling by algebraic constraint equations (i.e. by reaction forces/torques) and including different types of subsystem decomposition (force-force coupling, force-constraint coupling, constraint-constraint coupling). Again, this semi-implicit method shows better stability properties than an explicit approach. Stabilization is not considered in this paper.

Schweizer et al. (2015b) extend the methods proposed in Schweizer and Lu (2014b) and Schweizer and Lu (2014a) by introducing two stabilization techniques: in addition to the coupling conditions, either their derivatives or their integrals are taken into account. Three kinds of extrapolation are used for the coupling variables: constant, linear and quadratic. Stability analysis shows that for force-force coupling and constant extrapolation, the original method (without derivatives or integrals of the constraints) yields the best results. For linear extrapolation, they are – depending on the system – quite similar but for quadratic extrapolation, the extension with derivatives proves the best method regarding stability. For force-displacement, the conclusions are almost the same as for force-force coupling. For displacement-displacement coupling, all methods show similar stability properties, so extension by derivatives or integrals is not recommended due to the increased computational effort. In case of a nonlinear model, the original and the method extended with derivatives become unstable for constant extrapolation when using force-displacement or displacement-displacement coupling. As soon as stiffness is increased, the method extended with derivatives of the coupling conditions proves the most stable. Convergence analysis reveals that the method stabilized by using derivatives behaves similarly to the original method, whereas the method using integrals shows faster convergence for force-force decomposition. Still, Schweizer et al. warn that these results might be influenced to a great extent by the given models and their parameters.

In Schweizer et al. (2015a), different loose coupling co-simulation methods of Jacobi type are applied to the linear two degree of freedom oscillator, interpreted as two linear one degree of freedom oscillators coupled by applied forces/torques (i.e. constitutive laws, not algebraic constraints). Macro-steps are held constant for all applications. The implicit ap-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

proach uses a predictor-corrector method consisting of three steps, see (Schweizer and Lu 2014a) and (Schweizer et al. 2015b), respectively. In the explicit approach, only the predictor step is carried out. Stability analysis shows that in general, instable regions become larger for higher extrapolation polynomials and displacement-displacement coupling renders a more stable system than force-displacement or force-force coupling. For all scenarios, the implicit method proves to have better stability properties than the explicit method. The latter also holds for convergence, especially when the order of extrapolation is increased.

Schweizer and Lu (2015) present the predictor-corrector method for systems coupled by *constraint equations* applied to a test case in index-1, index-2 and index-3 formulation, stabilized with the Baumgarte method on the one hand and introduction of a weighed multiplier on the other hand. This method shows better stability behavior than explicit methods but requires the repetition of every macro step in the subsystem solvers. The approach is illustrated by the decoupling and co-simulation of the linear one degree of freedom oscillator and further generalized for coupled systems of two arbitrary multibody subsystems. While the index-1 formulation yields stable results for constant, linear and quadratic extrapolation of coupling variables, higher index formulations remain stable only for constant extrapolation. For the given test cases, both the Baumgarte and the weighed multiplier approach show similar stability properties, although Schweizer and Lu state that these depend on the choice of parameters and further investigations will be needed for better comparison.

Schweizer et al. (2016) investigate stability properties of three different implicit coupling techniques for co-simulation with algebraic constraints by testing them on the two-mass oscillator which can be interpreted as extended Dahlquist test system. The first considered method is based on the Baumgarte method, the second on a weighted multiplier approach and the third is a classical projection method. Thus, it extends the investigations in (Schweizer and Lu 2015) with the projection method and the consideration of more parameters in the stability analysis. Furthermore, the test system is decomposed by the three different coupling approaches for mechanical systems, i.e. force-force, force-displacement and displacement-displacement (see also Section 2.9). For the force-force coupling approach, the crucial parameter for stability of the implicit method based on the Baumgarte stabilization technique proves to be the frequency ratio of subsystem 2. In addition, the stability region is significantly smaller for constant compared to linear or quadratic extrapolation. The weighted multiplier approach shows instabilities for quadratic extrapolation if the real and imaginary part of the eigenvalue of subsystem 1 converge to zero. The projection technique exhibits reduced stability regions for increased damping or frequency ratio in subsystem 2. For the

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

force-displacement coupling, the Baumgarte method and the weighted multiplier approach both prove to be more stable for constant extrapolation but less so for linear or quadratic extrapolation in comparison to the respective method with force-force coupling. For the projection technique, results are similar to the force-force decomposition. The projection technique shows its best results with displacement-displacement coupling. Specific tests are also carried out for higher index approaches of the Baumgarte and weighted multiplier methods. Furthermore, convergence and numerical error are considered to compare the three methods. The weighted multiplier approach shows slightly better results in comparison to the Baumgarte based approach for the index-1 case and similar ones for differential index 2. The global error of the projection technique is similar regarding its order in comparison to the weighted multiplier approach. Finally, the methods are applied to a nonlinear example where they yield mostly stable results with the exceptions of the weighted multiplier approach for force-force and force-displacement decomposition if quadratic extrapolation polynomials are used. One important conclusion that can be drawn is that no general statements on the best technique can be made, since even higher order extrapolation or higher macro step sizes can yield more stable results in some cases.

### 3.2.5 Comparisons

In this section, comparisons regarding performance, accuracy, stability or suitability among different co-simulation methods or versus a monolithic approach are presented. Some comparative works have already been considered in other sections, as f.i. those of Schweizer et al., which specialize in mechanical systems and are therefore found in section 3.2.4. Matthies et al. (2006) and Matthies and Steindorf (2002, 2003) present and compare different strong coupling schemes to simulate fluid-structure interaction. Comparisons of loose with strong coupling schemes for the application in building energy systems are performed by Trčka et al. (2009, 2007), concluding that selecting one of these methods comes down to a choice between performance and independent time steps or accuracy. Regarding the possibility of modularity in multiphysics system simulation, Schmoll (2015) comes to the conclusion that classical co-simulation is advantageous compared to coupling of dynamic with static subsystems. Pühringer (2017) even implements a framework with the aim of comparing protocols for data exchange and different coupling methods.
Rather than algorithms themselves, different implementations of co-simulation masters are compared by González et al. (2010). González et al. (2011) compare non-iterative slowest first and fastest first approaches with inter- and extrapolation polynomials of varying degrees with a – maybe for some researchers frustrating yet crucial – conclusion that the choice of the best coupling algorithm has to be exercised individually for every given problem.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Once again, further details and results are given below.

The works of Matthies et al. describe strong coupling algorithms applied to fluid-structure interaction models. By discretization and order reduction of the original PDEs, the fluid-structure interaction can be described as coupled DAEs, which are regularized to obtain an index-1 DAE. A fully implicit formulation is used, where every subsystem is solved implicitly. The algorithm presented in (Matthies and Steindorf 2002) considers not equilibrium equations but the fixed-point problems from the subsystem solver iterations wherein necessary derivatives are approximated. The method is applied to a one-dimensional test case for which the developed Block-Newton method is stable and accurate, which is shown by comparison to the monolithic reference solution, and more efficient than the block-Gauß-Seidel iterative method which is taken into account for comparison.
Matthies and Steindorf (2003) show that an approximative block-Newton method yields better convergence properties than block-Jacobi, block-Gauß-Seidel or relaxation methods, which are commonly used to solve large systems of nonlinear equations that occur when fully implicit formulations are used. The Navier-Stokes equation for the fluid part is formulated in an Arbitrary Lagrangean-Eulerian (ALE) framework, while for the equilibrium equation for the structure, a Lagrangean framework is used. The convergence of the full Newton-Rhapson method used as coupling algorithm is independent of the order in which the subsystems are solved.
In (Matthies et al. 2006), pure differential and also algebraic coupling of the participating systems is considered. It is assumed that all participating subsystems' solver algorithms use fixed-point iterations. The coupling equations are taken into consideration in one of the subsystems. The paper discusses convergence criteria for the iterative block-Gauß-Seidel method including the dependence on subsystem sequence, which is Matthies et al.'s motivation for the development of a new method which is independent of this sequence and further does not require pre-conditioning (as does for example Arnold and Günther (2001)).
An inexact block-Newton method using the Newton-Rhapson method and a Quasi-Newton method similar to the Newton-Rhapson algorithm (but approximating the Jacobians and thus not requiring the derivatives themselves), called Broyden-Fletcher-Goldfarb-Shanno algorithm, are presented. The formulation is given for two subsystems but can, according to Matthies et al., also be applied to more.
To implement the coupling, a middleware called the Component Template Library (CTL) developed at Institute of Scientific Computing of Technische Universität Braunschweig was used. The methods are compared for two examples, one structure-structure coupling and one structure-fluid interaction which both show that the block-Newton method requires sig-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

nificantly less iterations than the block-Gauß-Seidel method while maintaining the same robustness and convergence properties as a monolithic method. Matthies et al. state that the methods may also be applied to other problems such as thermo-mechanical or soil-pore fluid interaction.

Trčka et al. (2007) aim at the comparison of the following coupling strategies for specific building energy simulations:

- loose coupling with iteration within the subsystems in the following manner: TRNSYS receives the data from EnergyPlus at the last synchronization reference, then performs its integration steps with the use of this data and iterates until a certain accuracy is reached. Then, EnergyPlus receives the data from TRNSYS and performs its integration and iteration.
- strong coupling: iteration of coupled programs within one time step: EnergyPlus may request further iterations of TRNSYS until subsequent data from TRNSYS is sufficiently small.

To compare these two methods, a room model in EnergyPlus is coupled with an air system model in TRNSYS in a case study. The strongly coupled co-simulation leads to convergence problems for larger time steps, which can be faced with relaxation, whereas loose coupling leads to big oscillations. The introduction of a first order predictor for the loose coupling approach leads to further inaccuracies for non-smooth input changes.
Drawn conclusions are that loose coupling is faster and allows every subsystem solver to use individual time steps while strong coupling is more accurate even for larger synchronization time steps. They also state that the most sensible decomposition is achieved by separation into different domains.

In (Trčka et al. 2009), different kinds of loose and strong coupling methods as well as monosimulation for the simulation of a building and its HVAC system are compared with respect to performance and accuracy. Depending on the step size, all approaches lead to sufficient results. An interesting aspect is that for strong coupling an implicit algorithm is used, so time steps can be chosen larger than for the loose coupling simulation and still deliver stable results. Due to computation time and implementation complexity, loose coupling with small time steps is recommended.

González et al. (2010) describe the coupling of a C++ multibody simulation with MATLAB and Simulink. Integration by only one tool (called *function evaluation* and can be seen in

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

analogy to the FMI terminology *model-exchange*) is compared to Jacobi type co-simulation with respect to execution time. In particular, one coupling algorithm employs Simulink as master, calling a library of compiled MBS code; in another, the multibody simulation acts as master by calling .dlls from Simulink compiled with RTW and in the third one, the co-simulation is realized by network connection with communication via sockets between simultaneously running processes. Stability issues are not investigated, only referenced. They conclude that co-simulation is faster than mono-simulation in general; thereby they observe that speed depends further on the choice of the master: while all three solutions allow real-time simulation for models up to 300 variables, the network connection method is slowest due to overhead from socket communications, and the version with the MBS software as master performs best since both executable and library are coded in C++ and can communicate directly. However, this method also turns out to be the most complex regarding its implementation.

González et al. (2011) describe the development of a generic implementation of a multirate method for loose coupling of block diagrams and multibody simulations. The time grids are not necessarily synchronized (thus showing similarities to the method presented in (Liang et al. 2011)) and the multirate method is independent of the participating subsystem simulators. The technique is tested on an example with known analytical solution.
They compare slowest first and fastest first approaches without iteration (as no knowledge about and interference with subsystem simulators is presumed) but with inter- and extrapolation polynomials (depending on whether or not the current time step of the master is currently behind or ahead of the minion's simulation time) of orders zero to four. Algebraic loops are broken via memory blocks, implying that they have to be detected by the modeler. Damping is introduced for highly differing time steps allowing speed-up of the process while maintaining accuracy.
Although it would be desirable, they conclude from several tests on different examples that "it is not possible to find an optimal general purpose co-simulation method" but the most suitable choice has to be found out by comparison for every considered problem individually. In case of the presented engine-kart co-simulation, linear extrapolation (and interpolation respectively) yields the best results.

Schmoll (2015) considers two types of solver coupling for multiphysics systems in his thesis: "classical" co-simulation and the coupling of dynamic with static subsystems. The methods are applied to a high-pressure pump, where known results regarding stability are verified for classical Jacobi and Gauß-Seidel type methods with and without macro step size control

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

(via predictor-corrector). In comparison to the solver coupling of dynamic with static subsystems, co-simulation is advantageous considering the possibilities of modular modeling. Stability has not been an issue for the considered application.

Pühringer (2017) presents a comparison of different loose coupling methods and protocols for data exchange in co-simulation. A framework is implemented using the Simple Object Access Protocol (SOAP) and the OPC (Open Platform Communications) Unified Architecture binary protocol for data exchange. Supported subsystem simulation tools are Matlab/Simulink and OpenModelica. Methods are compared by an application in industrial energy efficiency. As coupling strategies, parallel non-iterative as well as parallel and sequential iterative coupling methods are considered. Results show that dynamic coupling strategies yield better results even for larger macro step sizes. Within dynamic coupling strategies, sequential methods require less iterations for the same accuracy, which is to be expected. Regarding the used protocols, SOAP performs significantly worse than OPC UA, to an amount that makes Pühringer doubt the usability in real world applications.

### 3.2.6 Stability and error estimates

To quantify the worth of coupling methods, these have to be investigated for the numerical effects they have on separately nicely working integration algorithms.
Already in 1984, Gear and Wells use error estimates for the truncation error to adapt the macro step size (cf. Section 3.1). In general, the order of the global error of the coupled method is bounded by the error of the subsystem solvers and the extrapolation method, as shown by Knorr (2002) (cf. Section 3.1), Arnold (2007) and Arnold et al. (2014). Further error estimates, based on Richardson extrapolation, can be found in (Zhang et al. 2011) and (Arnold et al. 2013); an investigation on relative consistency by calculating the defect in (Glumac and Kovacic 2019). Bartel et al. (2014) quantify the convergence rate of co-simulation with more than two participating subsystems.
While in the area of partitioned methods, investigations on stability have been published since the 1980s (see Section 3.5), they gain currency only since the year 2000 for classical loose coupling schemes. As the field of numerics of differential equations and differential algebraic equations itself comes as a vast area of research, the combination of different methods out of this area is even harder to investigate from a general point of view. That generalized stability analysis is difficult to accomplish is also pointed out by Arnold et al. (2011): “A detailed stability analysis for modular time-integration methods is technically very complicated since it has to take into account several types of stiff coupling terms and different extrapolation and interpolation methods”. Hence, many studies on stability of cou-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

pling methods are done on systems with certain limitations, such as constant extrapolation (Arnold 2010). To obtain higher accuracy, however, a higher extrapolation order may be preferred, which can increase stability issues. These can be met by methods for stabilization such as iteration (Arnold and Günther 2001; Kübler and Schiehlen 2000b), asynchronous algorithms (González et al. 2011; Liang et al. 2011, see Section 3.2.3) or weighting algorithms (Schierz and Arnold 2012).
Some promising stabilization techniques, such as the bilinear delay line by Larsson and Krus (2003) or the introduction of filters in (Kübler and Schiehlen 2000b), require alteration of the models themselves, which is often not possible with complex problems of the integrate-and-collaborate kind.
The approach found in (Sadjina and Pedersen 2016) stands out as they aim to increase stability by energy conservation between co-simulated systems, thereby using power bonds to calculate energy residuals. Stabilization of strongly coupled systems is addressed by Viel (2014).
Kübler and Schiehlen (2000b) deduce that zero-stability cannot be guaranteed for loose coupling co-simulation in case algebraic loops occur and Arnold (2010) shows that for sequential algorithms, the order in which the subsystems are executed is crucial for the stability properties of their co-simulation.
The reader interested in details on the mentioned investigations may consult the following summaries (or, of course, the sources themselves) for further information.

Kübler and Schiehlen (2000b) show that under the assumptions that

- one-step methods are used for integration (not necessary for iterative approaches)
- output equations are time-invariant
- output equations are linearly dependent on inputs (not necessary for iterative approaches)

and for the special case of two participating systems, zero-stability of the co-simulation of two coupled DAEs is guaranteed if there is no feed-through in one of the systems (i.e. one of the outputs is not explicitly dependent on the inputs), which means no algebraic loop can occur (see Chapter 6 for a detailed description of their investigation).
In addition, methods of simulator coupling allowing zero-stable integration even if they include algebraic dependencies are presented. On the one hand, two iterative schemes are described which always guarantee stability by iterating the variables in the loop over each global time step. The presented method is a quasi-Newton iteration with approximation of

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

the Jacobian. As the Jacobian is not always available, *Broyden's method* is used for approximation. On the other hand, a method introducing a filter causing the elimination of algebraic loops and hence again warranting zero-stability is shown. This last method, however, requires modification of the system itself.
For further information see also (Kübler and Schiehlen 2000a), where possible other problems are described for non-iterative algorithms such as the ones with filters, which can lead to wrong results even if they maintain zero-stability.

Larsson and Krus (2003) investigate stability of methods with minimal intrusion into subsystem solver algorithms and simulation tools respectively, as requiring the possibility of re-starts, for example, naturally limits the applicability of the co-simulation algorithms.
For testing, Larsson and Krus take a mass-damper co-simulation as benchmark example. Depending on the execution sequence, different stability properties can be expected for this application.
Considered approaches are an explicit method coupled with an implicit Euler method, bilinear delay line (described in detail on pages 2f of the paper) coupled with the trapezoidal rule or the implicit Euler method. The bilinear delay line yields a superior stability area but requires alteration of models which is often not possible with commercial software tools. In case of more than two participating subsystems, they are divided into two groups.

Trčka (2008) states that results on consistency in general show that a problem remains consistent if the local truncation error (not the round-off error!) converges to zero for $\Delta t \to 0$. When consistent methods are co-simulated, consistency is maintained, but maybe of lower order.
It is mentioned that possible ways to improve stability and accuracy would be decreasing the synchronization time step, strong coupling, or variable macro time steps. For the latter, step size control can depend on the rate of change per step or the estimation of the truncation error.

Arnold (2007) discusses the convergence and stability properties of multirate weak coupling methods. Mechanical systems with highly different time constants by the examples of a combustion engine with chain drive and the dynamical interaction between pantograph and catenary are considered. In comparison to many methods applied in electrical circuit simulation, where communication between the subsystems is rather frequent to compensate errors introduced by the differing time steps, Arnold reduces the communication between subsystems to a minimum sufficient for the application in large scale multibody systems. The order

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

of the global error of the multirate method is bounded by the error of the subsystem solvers and the extrapolation method. For the engine with chain drive example, significant speed-up can be achieved by the partitioning. Stability regions can be given for the macro step size in this particular example, but the only available general bounds are ofttimes too restrictive to be of practical use in many applications.

Arnold (2010) investigates the stability of loose, Gauß-Seidl-type coupling of multibody systems. He finds that instabilities can occur even for small step sizes if the coupling equations are algebraic constraints. A stability condition in case of constant extrapolation is given and for higher order extrapolation (used to achieve higher accuracy), a stabilization technique is introduced ("linearly implicit stabilization of coupling terms").

Arnold et al. (2011) focus on modeling and simulation methods for systems describing vehicle system dynamics. Among other approaches, co-simulation techniques for multidisciplinary problems are considered.
The paper starts with an overview of numerical algorithms for the solution of ODEs and DAEs frequently occurring in models for vehicle system dynamics. Co-simulation of one master and several minions via export of code and solver algorithms is described, where the subsystem solver algorithms are imported by the master via MATLAB S-functions and the Functional Mockup Interface.
The mentioned coupling algorithms are on the one hand parallel execution with constant extrapolation, where the macro step size is restricted for accuracy and/or stability reasons (for stiff systems), and on the other hand sequential algorithms where for certain examples, appropriate step sizes are given. As methods with step size control in both master and minion algorithms (called *nested* step size control) can lead to problems, these are not considered.
The methods are applied to a case study of servo-hydraulic steering, coupling mechanical and hydraulic components implemented with SIMPACK and Modelica.
It is explained that accuracy studies are similar to analysis of classical numerical methods. In general, constant extrapolation yields a co-simulation error of order 1 ($\mathcal{O}(H)$ for macro step size $H$) while for larger extrapolation order, the error might also increase, which is shown by an example in multibody mechanics.

Schierz and Arnold (2012) address loose coupling of DAEs with coupling constraints. Stabilization is achieved by optimal weighting algorithms which are obtained by focusing on the algebraic part and following some minor assumptions.
An overlapping Jacobi type algorithm is presented where coupling constraints are evaluated

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

in more than one or even all subsystems, thus yielding a local algebraic variable in every subsystem. Therefore, post-processing is used to determine the resulting algebraic variable for further global use by a linear combination of the local variables.
Numerical tests show that for the stabilized problems a higher approximation order leads to more accurate results (which it does not for the non-stabilized problems) and the overlapping Jacobi type method delivers significantly better results than the Gauß-Seidl type method and the non-overlapping Jacobi type method, which yield almost coincident curves.

Schierz et al. (2012) discuss co-simulation with step size control where the error in the co-simulation is estimated via Richardson extrapolation and a newly developed modification of Richardson extrapolation as well. Values from other systems are extrapolated using an interpolation polynomial from known values of former synchronization time steps. As co-simulation benchmark model, a quarter car consisting of two combined masses, connected to each other and the road respectively via spring-damper elements, is considered. Schierz et al. claim that their method improves computing time and accuracy in comparison to a fixed step size.

Detailed error estimates for the approach in (Schierz et al. 2012) regarding extrapolations of different order are given in (Arnold et al. 2013). An interesting outcome is that the reliability of estimates can depend on the kind of DAE coupling: in case of force-displacement coupling, direct feed through can occur, which leads to unreliable estimates when Richardson extrapolation is applied. In addition to Richardson extrapolation, they provide a modified estimate. The numerical results are tested by the benchmark of a quarter car where it is shown that their alternative estimate is as reliable as the classical one.

Schmoll and Schweizer (2012) investigate the influences of subsystem solvers on the stability and convergence of loose coupling co-simulation. Loose coupling co-simulation of Jacobi type is applied to a linear oscillator with two degrees of freedom, interpreted as two linear one degree of freedom oscillators coupled via force-displacement coupling. Coupling variables are extrapolated using Lagrange polynomials of varying degrees and the subsystems are solved with explicit Runge-Kutta methods without step size control. It is shown – as expected – that both the order of the subsystem methods and the micro step size influence the global error. Additionally, high degrees of extrapolation polynomials have significant impact on the global error, especially if some initial values have to be approximated.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Zhang et al. (2011) describe an approach to approximate the local truncation error for a variable-step co-simulation method based on Richardson extrapolation and taking interpolation errors by Hermite interpolation for values from other subsystems into account. Compared to coupling algorithms with fixed step size and constant extrapolation, the developed approach is more efficient and accurate.

Bartel et al. (2013) discuss stability issues of co-simulation of partial differential-algebraic equation (PDAE) systems (which, for their investigations, are spatially discretized to yield an DAE problem) via dynamic iteration. Introductorily, they explain that dynamic iteration on ODEs remains unconditionally stable (referring to Miekkala and Nevanlinna (1987) and Burrage (1995)) and for DAE systems, difficulties with windowing techniques are that "The contraction of the fixed point operator can be guaranteed only if a stability constraint is fulfilled." (see also Lelarasmee et al. (1982) and Jackiewicz and Kwapisz (1996)). It is shown that for index-1 DAEs with constant mass matrices, convergence is guaranteed if the system is stable and that the sequence of subsystem execution can influence the *order* of convergence (even if the system is convergent independently of the sequence). Assuming that the splitting error dominates the time discretization errors of the subsystems, the latter are disregarded. The numerical results are tested on two scenarios, a coupled semiconductor-circuit system and a coupled field-circuit system. The stated convergence conditions and the impact of the coupling sequence of PDE-DAE systems on convergence are thus verified. In conclusion, they state that systems of more than two subsystems and coupling structures converging with a rate higher than $\mathcal{O}(H)$ (where $H$ denotes the macro step size) are to be investigated in future studies.

Bartel et al. (2014) extend the work of Bartel et al. (2013), quantifying the convergence rate also for systems of more than two coupled subsystems. Starting with a system of partial differential-algebraic equations, the method of lines is applied to obtain a system of coupled DAEs. It is shown that for systems where coupling takes place only in differential variables, a higher convergence rate ($\mathcal{O}(H^2)$) can be achieved for the case of two subsystems. When more subsystems are coupled, the convergence rate is $\mathcal{O}(\sqrt{H})$. In case the $i$-th algebraic constraint depends only on the local variables of the $i$-th system, the convergence rate is $\mathcal{O}(H^{\frac{r}{r-1}})$ for $r \geq 2$ coupled systems. The results are applied to an extension to DAEs of the Prothero-Robinson test equation for stiff ODEs.The application proves that the estimations are sharp for $H \to 0$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Viel (2014) presents an algorithm to stabilize the co-simulation of strongly coupled systems. If the systems are strongly coupled, this means that the participating systems depend to a great extent on values from respective other systems. For stability reasons, this can force standard coupling methods to take macro steps in the order of magnitude of the smallest micro step, hence also inducing equally small steps in all other systems, which might have been unnecessary for the integration of the subsystem equations themselves. Consequently, this reduces the computational efficiency of the overall simulation. To face this problem, the authors apply a linearly implicit stabilization method. This method requires the subsystems to not only exchange state variables but also their Jacobi matrices to build linearized models. These are integrated numerically in all other subsystems, hence increasing accuracy and maintaining stability for larger step sizes (up to the micro step size of the reference simulation of a monolithic implementation). Although this method allows faster simulation while maintaining stability, the requirements for participating subsystem solvers such as the exchange of Jacobi matrices are hard to meet.

Arnold et al. (2014) describes numerical tests with the SNiMoWrapper, a FMI-compatible testbed for co-simulation. The results show that for constant, linear and quadratic extrapolation (extrapolation order $k \in \{0, 1, 2\}$), the global error is of size $O(H^{k+1})$, thus – as long as there are no algebraic loops in the coupled system – they do not observe the order reduction phenomenon that Busch (2012) describes for force-displacement coupling.

Sadjina and Pedersen (2016) aim at energy conservation in co-simulated systems to increase stability. They use power bonds to study the energy flow between co-simulated simulators with a method called ECCO (Energy-Conservation-based Co-Simulation method), which is described in detail in (Sadjina et al. 2017). In addition, ECCO gives suggestions for an adaptive step size according to energy residuals. These properties are used to improve NEPCE, the Nearly Energy Preserving Coupling Element (Benedikt et al. 2013). The resulting approach is non-iterative and thus computationally inexpensive.
As the residual energy scales quadratically with the macro step size, energy can approximately be conserved by controlling the macro step size. Since subsystem states are typically not directly accessible in a co-simulation, inputs are corrected until residual energies between subsystem simulators are approximately satisfied. This is further enhanced by directly modifying the coupling variables. In non-iterative approaches, corrections have to be made according to available coupling data, i.e. data from previous time steps instead of the current one. These considerations are combined with the ECCO method to allow adaptive step size control in non-iterative co-simulation. The method is tested on a benchmark ex-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

ample of a quarter car, which basically corresponds to a two-mass oscillator and thus two coupled Dahlquist test equations. The combined approach reduces errors by around 90 percent for the benchmark example with no extra computational costs in comparison to a fixed macro step size.

Glumac and Kovacic (2019) present a sequential co-simulation master algorithm which they apply to the test case of a two-mass oscillator with force-displacement coupling. They investigate the relative consistency by calculating the *defect* (see Enright (2000)), which they declare a valid measurement of consistency for non-iterative co-simulation. This also complies with the specific results from their test case. Further, they propose using the stability radius estimate of Hinrichsen and Son (1989) in order to formalize the comparison of co-simulation masters based on the size of stability regions and consider treating the problem of choosing the most suitable co-simulation master as a multi-objective optimization problem.

## 3.3 Standards for co-simulation

The variety of co-simulation methods and tools to be coupled with their origin from different fields of application has led to the desire of unification, which is aimed by the specification of standards. Still, these are constantly revised by the developers and also extended by other researchers to meet specific requirements. The two most popular standards which are also frequently found in the literature, the *High Level Architecture* and the *Functional Mockup Interface*, are presented here along with the DEV&DESS formalism. The latter – whether it may or may not be regarded as standard for co-simulation (cf. Section 2.12) – constitutes an important approach that therefore also occurs occasionally, be it directly utilized or adapted, in the literature presented in this chapter.

### 3.3.1 High Level Architecture

The High Level Architecture (HLA) has been specified by the US Department of Defense to address the need for reuse and interoperability of simulations within the department. It provides an architecture defining functional elements, interfaces and design rules for simulation applications and a common framework for the definition of specific system architectures (Dahmann et al. 1997). The HLA is software and programming language independent.
Its key functional components are *federates*, the *runtime infrastructure* (RTI) and the *run-*

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*time interface*. Federates can range from computer simulations to manned simulators and even interfaces to live players: the representation of a federate is not restricted as long as it allows the interaction with other objects through data exchanges via services from the RTI. The RTI is a distributed operating system offering these services for interaction and federation management. The runtime interface specification defines a standardized manner of interaction between the federates and the RTI independently from the implementation. Monitoring of simulation activities and interfaces to live participants such as control systems are also supported.
Formally, the HLA is defined by the following three components: *object model template*, *interface specification*, and the *HLA rules*. There are two kinds of object models: "the HLA Federation Object Model (FOM) and the HLA Simulation Object Model (SOM). The HLA FOM describes the set of objects, attributes, and interactions which are shared across a federation. The HLA SOM describes the simulation (federate) in terms of the types of objects, attributes and interactions it can offer to future federations. (...) The HLA interface specification describes the runtime services provided to the federates by the RTI, and by the federates to the RTI"(Dahmann et al. 1997). The six classes of services are: Federation management, declaration management, object management, ownership management, time management and data distribution management. "Time management is concerned with the mechanisms for controlling the advancement of each federate in simulation time. (...) Data management (DM) and data distribution management (DDM) in the HLA are used to specify which federates should receive messages for each state update and interaction"(Dahmann et al. 1997). More details on these services can be found in the description. It is also made clear that while the HLA provides the minimum essential tools for interoperability, it is itself insufficient to guarantee interoperability.

Different timing services by the HLA are described in (Awais 2015): When using the *Time Advance Request* timing service of the HLA, each federate is required to provide a *lookahead* – a time frame after the currently elapsed time in which no event is expected to happen. The *Time Advance Request Available* service is needed for zero lookahead simulations – which means information exchange can take place without advancing in simulation time. *Next Event Request (Available)* services specifically require the next scheduled event of each federate. With the *Flush Queue Request*, the times of exchange are only determined by the federate's events, which are scheduled in non-decreasing order (so the federate cannot send messages timed earlier than a received one).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 3.3.2 Functional Mockup Interface

The Functional Mockup Interface (FMI) is a standard for model exchange and co-simulation initiated by the project MODELISAR and now maintained and developed by the Modelica Association. In a nutshell, the FMI defines the manner in which Functional Mockup Units (FMUs) have to be built so they can be imported by tools serving as master orchestrator and the functionalities and interfaces for the latter. When an FMU *for model exchange* is exported, the tool where the respective model has been implemented translates it into a dynamic system model in C-code with inputs and outputs. The models can contain events as well as differential, algebraic or discrete equations. In the FMI *for co-simulation*, not only the model but also the solution algorithm is included in the exported code. Master algorithms can then define points in time where participating FMUs exchange data and control this data exchange. In addition to the C-code file, an FMU contains an XML file with the definition of input and output variables and other model information. Further C-functions for the setup of co-simulation minions or execution of model equations and optional data such as icons or documentation are also included in the zip-file (extension ".fmu") which finally constitutes a complete FMU. In the current version of the standard (FMI 2.0, see Blockwitz et al. (2012) and Modelica Association (2014)), the interfaces for model exchange and co-simulation are unified. Additional features such as getting and setting an FMU state (thus potentially enabling rollback) are introduced, but not mandatory for tools that support the FMI. Input and output dependencies of variables and their derivatives (important for algebraic loop detection) or Jacobian information (potentially needed for implicit integration methods or linearization) can also be included in an FMU.

The great potential and renown but also drawbacks and possibilities for improvements are assessed in the empirical survey of Schweiger et al. (2019b), see also Chapter 4. Some of the main difficulties are accounted for by the optional features of the FMI, many of which are not supported by most (in particular open source) tools that often do not even properly define which features they support and which they do not. This hampers the implementation of coupling methods requiring specific functionalities such as simulator rollback, information on derivatives or input-output dependencies. Another problem regarding discrete event or hybrid co-simulation is the requirement of time passing between two synchronization references, which means that simultaneous events cannot be handled by several exchanges of data at the same time step. This has led to extensions to the FMI standard f.i. by Broman et al. (2013), which of course are not supported by all tools currently supporting the FMI 2.0

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

itself[2], or Tripakis (2015), who aims to formalize models currently not supporting the FMI in a way to allow their combination nevertheless.

The formalized model of the FMI introduced by Broman et al. (2013) includes the model of the set of all input/output dependencies of an FMU instance (which can be expressed in the FMU XML file) as binary relation, so it is known which variables need to be calculated before others. The connections between FMU instances are also formalized in a model via a mapping function, assuming a closed model (i.e., "every input is connected to some output"). Furthermore, they give utilization constraints (called *FMU contract*) including what every FMU has to provide and what it can assume (i.e., conditions a caller must respect). In addition, two master algorithms are introduced. First, in a pre-processing step, the variables are ordered according to the graph of input-output dependencies, which has to be sorted. If it is cyclic (implying the existence of an algebraic loop), an error is returned.
In the actual master algorithm, inputs are set (in the order determined before), the states of all FMUs are saved, the communication step size is set to a default value, and then an acceptable step size for all FMUs is found: `doStep`, a function demanding the execution of an FMU for a given macro step and returning the actual elapsed simulation time (which can be smaller in case of terminal events or errors), is called for all FMUs and the step size set to the minimal returned. If thus the step size has been reduced, the step is repeated for all FMUs with the smaller step size. Note that this requires all participating FMUs to support rollback.
For the second algorithm, Broman et al. propose an extension of the FMI standard by the procedure `fmiGetMaxStepSize` which returns an upper bound for the FMU's acceptable step size. The algorithm requires all FMUs except for (at most) one to support either rollback or predictable step sizes (meaning the procedure described above). The algorithm works as follows: after getting and setting the inputs and corresponding outputs, the minimal step size from all FMUs supporting `fmiGetMaxStepSize` is determined. Then, the states are saved and `doStep` is called for all FMUs allowing rollback. Similar to the first algorithm, this is repeated in case one or more `doStep` calls reject this step size. Finally, `doStep` is called with the thereby determined step size for the one FMU which supports neither `fmiGetMaxStepSize` nor step revision. If this stops with a smaller step size, the FMUs supporting rollback are set to their previous states and `doStep` is called for them again with the – final – step size. Finally, `doStep` is called for all FMUs supporting `fmiGetMaxStepSize`. These will not end prematurely as their minimum step size has already been taken into account in the beginning and thus, according to the assumptions in

[2] A complete list of tools supporting the FMI 2.0 can be found on https://fmi-standard.org/tools/

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

the paper, have to be able to finish properly for a (possibly) even smaller step.
In the paper it is proven that both algorithms terminate and are determinate, meaning that repeated executions with the same settings and inputs return the same results.

Tripakis (2015) discusses "the principles of encoding different modeling formalisms (state machines, discrete event, and synchronous dataflow) as FMUs.", thus facing the challenge of "how to bridge the gap between the semantics of the original formalisms, and the FMI API." For untimed Mealy or Moore machines (see Definition A.22), they suggest two types of timed wrappers: On the one hand a periodic wrapper by assigning an equal amount of elapsed time between two transitions and on the other hand an aperiodic wrapper, where the environment (resp. the master algorithm) maps each `doStep` invocation to a transition. This, of course, can cause highly differing results for different time steps. To represent discrete event signals, the value "absent" is introduced so the FMU `get` function returns this value for all time instances where no event occurs. For synchronous data flow (SDF) models, they present a method where every actor is represented by an individual FMU, independently from the other ones, which makes it possible to interface them with other FMUs. In the case of discrete event FMUs, this can be achieved via converter FMUs which – in simplified terms – output the incoming events as list and vice versa. They also propose semantics for encoding timed automata and state machines – with some restrictions – as FMUs and explain the encoding for continuous-time models. A case of particular interest is the one where piecewise continuous signals occur, meaning discontinuous state changes (due to events). In this case, an additional variable is introduced which specifies if super-dense time is required in the considered time step.

Gomes et al. (2019) present a formalization for FMU execution which is similar to the one presented by Broman et al. (2013, 2015). They, like Broman et al. (2013) and Cremona et al. (2016a) argue that the validity of a master algorithm depends on the input-output dependencies inside FMUs, an information often not available as it is not required in the FMU description according to the standard alone. In addition, Gomes et al. present an algorithm based on topological ordering of a graph constructed according to input/output dependencies and further information such as feed-through and reactivity. However, this algorithm requires unique calls of `doStep` for each FMU and thus does not allow step rejections and rollback. Gomes et al. claim to consider this restriction in future investigations.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 3.3.3 DEVS-based formalisms

The Discrete Event System Specification (DEVS) is a formalism to describe hierarchically structured Discrete Event systems based on systems theory. Similarly, the Differential Equation System Specification (DESS) allows the description of ODE systems. Both have been introduced by Zeigler et al. (2000) and combined for the description of hybrid systems to the DEV&DESS (Discrete Event System & Differential Equation System Specification) formalism. On the deepest level of hierarchy, an *atomic DEVS* can be described as a set of inputs, outputs, states, internal and external transition functions, an output function and a time advance function. Instead of transition functions and the time advance function, an *atomic DESS* contains a *rate of change function* corresponding to the right side of an ODE. In contrast to DESS of Moore type, where the output function has only states in its argument, for Mealy type DESS the output function may depend directly on the inputs as well. In an *atomic DEV&DESS*, both are combined, resulting in discrete and continuous inputs, outputs, states, transition and output functions, a rate of change function and, in addition, a state event condition function. Two or more DEVS (or DESS, DEV&DESS respectively) can be combined into a *coupled DEVS* (or DESS or DEV&DESS), enhancing clarity and supporting modularity. The problem of concurrent events can be tackled by parallel DEVS (P-DEVS), where concurrency is resolved locally in every DEVS. Hybrid P-DEVS (introduced by Preyser (2015)) are designed to represent discrete and continuous systems as parallel DEV&DESS. Since the DEVS constitutes a formalism, it is software independent. Specific implementations are found in (Camus et al. 2016; Deatcu and Pawletta 2012; Heinzl et al. 2018; Preyser 2015).

## 3.4 Specific applications and developments

This section covers on the one hand specifically implemented frameworks for co-simulation (Section 3.4.1) and on the other hand developments for a particular model description (hybrid systems in Section 3.4.2, FEM in Section 3.4.3) or application (Section 3.4.4).

### 3.4.1 Frameworks

The introduction of frameworks has become more and more popular to allow easy “plug-and-play” co-simulation. However, many frameworks have again been designed motivated by a specific problem or area of application, such as building simulation (Wetter 2011), automotive systems (Zhang et al. 2014) or traffic (Ferreira et al. 2008) and are limited to the co-simulation of certain tools, leaving gaps aimed to be filled by further developments. What

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

is more is that these seemingly simple “enablers” of co-simulation bear the risk that systems are not properly checked for stability properties but rashly coupled, which can be amended by notwithstanding mindful consideration and inspection of every user. Many recent, independent developments (Ben Khaled et al. 2014; Galtier et al. 2015; Thule et al. 2019b; Wang et al. 2017) respect the FMI standard. Awais (2015) even utilizes the HLA as well as the FMI, for details see below. An implementation of a framework extending the FMI to allow hybrid co-simulation is found in (Cremona et al. 2019, 2016b), which is described in Section 3.4.2 along with further frameworks that are specifically tailored to support hybrid co-simulation.

Ferreira et al. (2008) present a framework enabling cooperative simulation in the field of traffic and transportation in urban areas. This is realized by interpreting the application domain as multi-agent system where experts as well as entities are represented as agents. Thus, agents steer agents. The three basic subsystems are *real world*, *virtual domain*, and *control strategies* and *management policies inductor*. The framework, of which the implementation is still in progress, can become a platform for collaboration and integrated analysis in urban planning.

Karsai and Sztipanovits (2008) describe the concept for a framework for the model-integrated design of cyber-physical systems. It is based on the interaction of physical and simulation models on different levels and varying manifestations, which are available for all parts (application, platform, plant, environment). Zhang et al. (2014) present the implementation of the framework based on virtual prototyping of automotive systems, for which it has been specialized.

In (Friedrich 2011), a framework for parallel co-simulation of multi-domain systems is presented. Loose coupling of Jacobi type using a master/minion concept is applied. For the required extrapolation, several methods are discussed: polynomial interpolation of earlier, known values; Hermite interpolation; and constant, linear, and quadratic extrapolation using a linear combination of previous values and thus enabling influences on the stability by the choice of coefficients. For a mechanical test problem of two coupled systems, optimal extrapolation parameters with regard to stability are investigated. Furthermore, other domains as well as inter-domain couplings are considered. The framework itself is implemented in C++.

Wetter (2011) describes the development of the Building Controls Virtual Test Bed (BCVTB),

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

a co-simulation tool based on on Ptolemy II and aimed for buildings simulation. The BCVTB acts as middleware for data exchange via BSD sockets (see (Stevens et al. 2004) for further information) allowing coupling of EnergyPlus, Modelica, Radiance, BACnet, and MATLAB including its toolboxes Simulink and Simscape among others. Data synchronization takes place at equidistant, predefined time steps. Between two synchronization references, all participating simulators are parallelly executed.

Galtier et al. (2015) present an open-source framework (DACCOSIM) for multi-threaded co-simulation using the FMI standard. Instead of one master managing all communication between FMUs, DACCOSIM distributes this functionality among three different components to avoid overload of one machine. Two algorithms with variable communication step size are implemented, using Euler's and Richardson's method respectively. These require the participating FMUs to allow rollback. If one FMU rejects a step, all participating FMUs must also roll back. Until an FMI-internal solution will be introduced, State Events are handled as follows: if changes in Boolean or integer values are detected (in comparison to the last communication step – which allows events to go undetected), the FMUs reject their step and continue with the smallest possible step size until the event is detected. A detailed description of the framework, which comes as an Eclipse plugin, and its functionalities can be found on pages 4f of the paper. As test example, heat transfer in a building with four rooms is modeled, partitioned into nine FMUs. Compared to a co-simulation with Dymola without multi-threading, a speedup of 3.5 is achieved with DACCOSIM.

Ben Khaled et al. (2014) also use multi-threading with FMUs, but on multiple-core machines (as opposed to deployment on a cluster as in (Galtier et al. 2015)), by which supra-linear speedup can be achieved. Synchronization intervals are extended while the possibly resulting error increase is met by the introduction of extrapolation polynomials. Instead of fixed polynomial prediction, which fails for the test example of a (hybrid) four-cylinder engine model due to discontinuities and sharp variations, a context-based polynomial predictor is used.

Awais (2015) has developed a framework for distributed hybrid co-simulation using the HLA as well as the FMI 1.0. For this purpose, FMUs are converted into processes which communicate by using the HLA. Awais implements several co-simulation algorithms – standalone and master/minion algorithms, implicit and explicit ones for DE and CT systems separately, and finally a version for hybrid systems enhancing a waveform iteration algorithm to allow discrete events and, in a further development step, macro step size control. These are also

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

included in the developed framework called “SAHISim”.

Wang et al. (2017) present a co-simulation framework for urban energy simulation based on the FMI standard. The framework is an extension of Mosaik (a co-simulation framework developed by OFFIS) supporting the FMI standard and providing a library of master algorithms. A database supporting the CityGML, a standard for information and data models of urban areas, is included in the framework. The application is demonstrated by the coupled simulation of EnergyPlus and No-MASS (Nottingham Multi Agent Stochastic Simulation) simulations. Wang et al. state that there is no limitation to the number of simulations that can be coupled.

Pühringer (2017) implements a framework using the Simple Object Access Protocol (SOAP) and the OPC (Open Platform Communications) Unified Architecture binary protocol for data exchange, aiming at the comparison of these protocols and different coupling methods. Pühringer claims that the developed framework mainly offers benefits regarding usability and extendibility compared to existing middleware and standards; according to him, models barely need to be adapted for the co-simulation and moreover, the used high-level data transfer protocols can be used to automate f.i. unit conversion between simulations.

Thule et al. (2019b) propose an architecture for the design of a modular co-simulation framework which is extendible and can integrate existing co-simulation approaches. They claim that a co-simulation framework should relieve users from having to establish communication with the minions or configuring the master algorithm and the minions for reliable results. As the first part (establishing communication) is mostly covered by using the FMI standard, they focus on an architecture for easy integration of novel master algorithms in a co-simulation framework. This integration can take place on three different levels with increasing flexibility. These levels are: *legacy integration* (integration of existing master algorithms by the master API developed in the INTO-CPS project (Thule et al. 2019a)), *master algorithm integration* (the algorithm has to implement the master API and use the provided runtime API for FMU management) and *approach integration* (integration of new co-simulation approaches in an extensible DSL (Domain Specific Language), the “Master Specification Language”). A specification of this DSL is given in the paper, including synchronization protocols, scenario and adaptions, and transformations (of scenarios into master algorithms). Further, they give possibilities how this DSL can be extended via plugins. Future goals are the formalization and validation of the language and the development of stable and usable language and extension interfaces.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 3.4.2 Hybrid (co-)simulation

Hybrid systems – in the sense of combined continuous time (CT) and discrete event (DE) systems – have been an ever-present challenge of special interest within modeling and simulation. Only recently, co-simulation has emerged as a possible solution approach that brings along advantages but also approach-specific complications. Although several investigations considered in this section are not focused on *co*-simulation, the peculiarities as well as methods for hybrid simulation frequently apply regardless whether the combination of DE with CT approaches is realized via co-simulation or integrated models.

This pertains for instance to Mosterman (1999), who presents an overview of phenomena in hybrid simulation reported in the literature: event handling, run-time equation processing, discontinuous state changes, event iteration, chattering, and comparing Dirac pulses. These, of course, are equally important issues in hybrid co-simulation and are therefore described in detail below. Solutions for event respectively zero crossing detection are addressed by Zhang et al. (2008) and Cremona et al. (2016a), event ordering by Thule et al. (2018), chattering avoidance by Barros (2017) and Zhang et al. (2008), zeno-behavior by Zhang et al. (2008), and debugging in hybrid simulations by Van Mierlo et al. (2017).

Gheorghe (2009) describes events update schemata and presents a generic methodology for developing hybrid co-simulation tools. For a similar purpose, formalisms have been introduced, f.i. by Barros (2008), who proposes the Heterogeneous Flow System Specification (HFSS), or Cremona et al. (2016a), who formalize the FMI, taking input-output dependencies and abstraction of functions into regard to create a non-cyclic graph of the overall system. Camus et al. (2016) present a DEVS wrapper for hybrid co-simulation of FMUs implemented in MECSYCO using the DEV&DESS standard.

The work of Broman et al. (2015) shall be emphasized at this point, as they define a range of requirements for hybrid co-simulation standards along with test components and acceptance criteria.

In many specific approaches, one part is controlled respectively set back by the other: methods with the DE simulation as master are found in (Fitzgerald et al. 2014), CT simulation is taken as master by Tudoret et al. (2000), and Gheorghe (2009) employs both of these options. Tong et al. (2014) present parallel approaches with potential rollback in both parts. Awais (2015) uses an iterative approach and Farkas et al. (2019) apply step size control. Comparisons of different hybrid simulation approaches can be found in (Heinzl 2016; Palensky et al. 2014; Quaglia et al. 2012) and (Li et al. 2014), who compare platforms rather than approaches per se.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Prominent applications are various kinds of controlled systems. These seem predestined as hybrid systems due to their common representation by a continuous time system with a discrete control (Camus et al. 2016; Tudoret et al. 2000; Widl et al. 2015). Specific applications range from power systems (Palensky et al. 2014; Tong et al. 2014), networked control systems (Li et al. 2014; Quaglia et al. 2012), voltage distribution control (Savicks et al. 2014), tanks with controller (Camus et al. 2016; Tudoret et al. 2000), room temperature control (Widl et al. 2015) and manufacturing systems (Thiede et al. 2016) to cyberphysical systems in general (Cremona et al. 2019).

Especially developed frameworks are FIDE by Cremona et al. (2016b), SAHISim by Awais (2015) (see Section 3.4.1), CODIS by Gheorghe (2009), an adaption of the Crescenco tool to combine Overture and 20-sim by Fitzgerald et al. (2014), and a systematic approach for multi-level simulation by Thiede et al. (2016). Tong et al. (2014) consider the EPOCHS and GECO framework in their review.

Recent developments in particular are utilizing the FMI in their solution approaches for hybrid systems (Awais 2015; Broman et al. 2015; Camus et al. 2016; Cremona et al. 2016b; Farkas et al. 2019; Savicks et al. 2014; Thule et al. 2018; Widl et al. 2015). As the FMI by itself proves insufficient to satisfy requirements for hybrid co-simulation (see f.i. Chapter 4, Widl et al. (2015) and cf. Broman et al. (2015)), Cremona et al. (2016a) and Cremona et al. (2019) propose extensions to the FMI standard.

Rather than purely chronologically, details are given below starting by hybrid simulation phenomena and how they are tackled, followed by requirements for hybrid co-simulation and formalisms aiming to fulfill them, specific master algorithms for hybrid (co-)simulation and comparisons of these or specific platforms, and concluded by several specific developments.

#### 3.4.2.1 Hybrid simulation phenomena

Mosterman (1999) summarizes the following hybrid simulation phenomena:

- time events: Events are generated at predetermined times (. . . )
- state events: If events occur because of system variables crossing threshold values, the time of their occurrence is not known a priori. In this situation,
  - the event needs to be detected, and
  - its time of occurrence needs to be located.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

- simulation model: The system of equations may change.
  - Blocks of sorted and solved equations may simply appear or disappear (...), and, therefore, can be dynamically added/removed.
  - In some cases equations can be replaced by others, changing computational causality, and the system of equations may have to be sorted again.
  - In other cases, algebraic constraints between state variables may become active and the system of equations needs to be solved again (...).
- reinitialization: There may be a discontinuous change in state variable values.
  - This change may be explicitly specified by the user by a new initial state equation (...).
  - The system of equations may have to be integrated to derive physically consistent initial values for a new mode. (...).
- event iteration: When an event occurs, new system variable values may immediately trigger a further event. Two types of event iteration exist,
  - the state vector is invariant across the entire iteration, and
  - the state vector is updated after each iteration step.
- chattering: If the system moves back and forth between modes the system starts to chatter. Root finding to locate the exact time of occurrence of the event causes continuous integration to become excessively slow. An equivalence relation eliminates the fast chattering motion, but preserve the dynamics of the slow motion along the chattering surface.
- Dirac pulses: Discontinuous changes in continuous variables may cause Dirac pulses to occur. If their magnitudes are numerically approximated, comparison may be affected by non-Dirac type variables (...). To ensure numerically precise treatment, Dirac pulse values should be distinguished from non-Dirac pulse values and evaluation of Dirac pulses can be based on their areas.

In addition, Mosterman describes common simulation tools for hybrid systems and features they offer to tackle (some of the) aforementioned phenomena. Note that the article is from the year 1999, so some of the mentioned tools are out of date, recent developments are – naturally – not included and neither is hybrid co-simulation, but the phenomena they address

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

have lost none of their topicality and are equally relevant to hybrid co-simulation.

Tudoret et al. (2000) present the co-simulation of discrete event systems in Signal with continuous time systems in Simulink by using automatically generated C-code of the models. Since the automatic export in Simulink is only possible for fixed-step solvers, only these are used. They introduce an algorithm with aperiodic synchronous selector activations, meaning the discrete part is waiting for a (state) event from the continuous part by which it is "woken up". Thereby, one Simulink model represents the master (which can contain further subsystems that can be enabled and disabled). The algorithm is applied to the model of a siphon pump consisting of interconnected tanks with a discrete controller. Among the problems occurring with their solution are overflow of tanks and chattering of the controller.

Zhang et al. (2008) introduce a zero-crossing detection and location algorithm that tackles some of the inefficiencies that come with existing methods. Among those, they name the prevention of detecting the same event twice at consecutive time steps (which can happen if an event takes place exactly on a taken time step), masked events (possible in the case of even root functions), chattering, and zeno behavior (discontinuities in a decreasing interval, "converging to a limit point in time"). The method is applied to a benchmark example of two balls bouncing down stairs.

Barros (2008) has developed a formalism (Heterogeneous Flow System Specification – HFSS) to describe hybrid systems, aiming at a unifying description rather than the combination of existing methods. To this end, he introduces hyperdense time and regards the discrete part as push system and the continuous flows as pull systems, described by generalized sampling. The applicability of the method is illustrated by the implementation of a PID controller.

Barros (2017) implements a discrete sliding mode control to enable chattering avoidance using the HyFlow formalism. In the HyFlow (Hybrid (formerly: "Heterogeneous") Flow System Specification, see Barros (2008)), continuous variables are represented using multi-sampling, while discrete event representation follows the DEVS formalism (cf. Zeigler et al. (2000) and Section 3.3.3). Performance is impeded due to necessary re-initializations of the continuous solvers at every discrete event. Depending on the considered system, chattering cannot always be avoided by sliding mode control. Another presented method is the use of dynamic topologies, where parts of the model are switched to be able to fulfill hard constraints.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Thule et al. (2018) identify the property *event ordering* and characterize it as model checking problem. First, they give examples for properties which are desired to be *preserved* by a co-simulation, meaning that if the coupled system satisfies a property, the co-simulation of this system does, too. These examples are stability, energy conservation and event synchrony, none of which are preserved per se. There are works dedicated to these characteristics and introducing methods to correct the automatically introduced errors in this respect. Thule et al., on the other hand, relax the requirement for preservation of event *synchrony* by preservation of event *ordering*: occurring events shall be processed in the same order in the co-simulation as they would in a monolithic approach. By a simple example of several FMUs propagating the delay of an event (since this cannot be processed without a step, as explained before and by Broman et al. (2015)), they show that this cannot be guaranteed (resp. not achieved at all in some cases) with a parallel (Gauß-Seidl type) master algorithm. With Jacobi type algorithms, on the other hand, event ordering can be maintained if minimal information on the subsystems is available. Detailed inner properties which might be restricted by their respective developers do not need to be revealed. The presented example is expected to be valuable as test case for valid hybrid co-simulation procedures.

#### 3.4.2.2 Requirements and formalisms for hybrid co-simulation

Broman et al. (2015) define requirements for hybrid co-simulation along with test components whose behavior can be compared to their mathematically correct behavior and acceptance criteria to determine whether a standard supports these components. They assume superdense time $\tau(t, n) \in T = \mathbb{R}_+ \times \mathbb{N}$. $n$ corresponds to a "microstep"[3] defining the sequence of values at time $t$. For two times $\tau_1, \tau_2 \in T$ with $\tau_1 = (t_1, n_1)$ and $\tau_2 = (t_2, n_2)$, $\tau_1 < \tau_2$ if $t_1 < t_2$ or $t_1 = t_2$ and $n_1 < n_2$, else $\tau_1 > \tau_2$. Based on this, they define continuous time (CT) and discrete event (DE) signals, therein continuous signals, piecewise continuous CT signals and piecewise continuous signals, which extend the latter to signals with absent values.

Regarding the representation of time, Broman et al. (2015) declare the following requirements to ensure a valid notion of simultaneity:

[3] not to be mistaken for co-simulation micro steps of subsystems, these so-called "microsteps" are introduced for ordering simultaneous events

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

1. The precision with which time is represented should be finite and should be the same for all observers in a model. Infinite precision (as provided by real numbers) is not practically realizable in computers, and if precisions differ to different observers, then the different observers will not agree on which events are simultaneous.

2. The precision with which time is represented should be independent of the absolute magnitude of the time. In other words, the time origin (the choice for the meaning of time zero) should not affect the precision.

3. Addition of time should be associative. That is, for any three time intervals $t_1$, $t_2$, and $t_3$,
$$(t_1 + t_2) + t_3 = t_1 + (t_2 + t_3). \tag{3.4}$$

*Remark* 3.2. Note that floating point numbers, which are usually employed by computers to represent real numbers, do not fulfil requirements 2 and 3.

Furthermore, a mathematical description of a number of test components to represent hybrid systems is given, including gain, adder, periodic piecewise constant signal generator, periodic discrete signal generator, modal model with discrete control, integrator, integrator with reset, zero-crossing detector, zero-order hold, sampler, and discrete time delay.
They recommend that the master should not invoke components on times when the input is absent (and would also deliver absent output), as this would unnecessarily consume computational time. Unfortunately, this is deemed impossible to implement following the FMI 2.0 standard due to the requirement of the call of `fmi2DoStep` (and with it, time advancing) between `fmi2SetXXX`, which sets the inputs, and `fmi2GetXXX`, which reads the outputs.
In addition to the test components mentioned above, Broman et al. present compositions that represent scenarios whose desired outcomes effectively pose acceptance criteria for master algorithms. These ensure, for example, the correct handling of simultaneous events or integration of discontinuous signals and zero-crossing detection in a feedback loop.

Cremona et al. (2016b) introduce “FIDE”, an environment based on Ptolemy II and designed to import, connect and co-simulate FMUs. It implements the master algorithm described in (Broman et al. (2013), see also Section 3.3.2). They also introduce superdense time and allow “zero" time steps (only differing in the “microstep” ($n$ in the tuple $(t, n)$)) as well as the absence of signals (necessary to include actual discrete event signals), by which they extend the FMI standard. They test the implementation on a model with FMUs in a feedback loop including zero-crossing detection for state events. Since the algorithm from Broman et al. (2013) does not support backtracking, the zero-crossing event is predicted by derivatives.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

At the time of the event, a step of duration zero is taken by all FMUs so the event can be processed immediately.

Cremona et al. (2016a) extend the FMI master algorithm presented in (Broman et al. 2013) to enable step revision. Zero-crossing event detection, which is necessary when dealing with hybrid systems, can on the one hand make use of derivatives to predict the times of zero-crossing (as it is done in (Broman et al. 2013)) – which would also mean that all subsystems have to provide information on their derivatives (which f.i. in the FMI 2.0 is only optional). On the other hand, rollbacks may be induced if zero-crossing is detected to have taken place between two synchronization references. This is the approach addressed by Cremona et al. The step determination algorithm described in (Broman et al. 2013) requires all FMUs either to report their maximum step size or allow rollbacks to the last synchronization reference. The overall step size is determined by the minimum of all maximum step sizes, either by the `fmi2GetMaxStepSize` function (introduced in (Broman et al. 2013), see Section 3.3.2) for those implementing it or by rejecting or accepting the proposed step size for those supporting rollback. The step revision algorithm presented by Cremona et al., however, expects all subsystems to allow rollback to the last synchronization step. “Pure” step revision can be summarized as follows: all participating FMUs try to move forward by a step size $h$. If one or more return an error flag, the step is repeated with a step size $h_1 < h$. If this fails again, an even smaller step size $h_2 < h_1$ is tried, and so on. Cremona et al. combine step determination and step revision, thus allowing step rejection and further repetition and iteration even after a maximum step size has been presumed. For the implementation of their master algorithm, they make use of FIDE (FMI Integrated Development Environment), which extends the FMI standard by allowing “absent" input or output values and zero-duration steps which are deemed necessary to support hybrid co-simulation. The latter can be realized by superdense time. Cremona et al. mention that algebraic loops should be rejected in hybrid systems, as most otherwise viable algebraic loop solvers require smooth functions and are thus not compatible with discontinuities as induced by discrete events. On the other hand, achieving this by disallowing strict (i.e. enabling feed-through by input-output dependencies) components as the FMI 2.0 for co-simulation does comes with disadvantages since this would also forbid components like adders to be implemented in Mealy-fashion (see Def. A.22). This is bypassed by allowing several calls of `fmi2GetXXX` before `fmi2DoStep`. Initially, Cremona et al. formalize the FMI so that input and output dependencies are included and some functions abstracted with the aim to form a graph which can be ordered and must not contain cycles. Then, the maximum step $h$ is initialized and further set to the minimum of this initial value and the maximum step sizes gained by the `getMaxStepSize` function from all FMUs which implement it. Afterwards, all FMUs which do not implement

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

this function (called “unpredictable" by Cremona et al.) are to try performing `doStep` with step size $h$. In case of prior finishing, `doStep` will return an error (and also the step size to which the FMU could be executed, rem.). Consequently, a new, smaller step size is chosen (see detailed algorithm for this in (Zhang et al. 2008)), to which all participating FMUs shall proceed after being restored to their previous states. This can be repeated with further refinement, if necessary (i.e. if another error occurs). If no error occurs, this means all FMUs can proceed by $h$, which they do. Then the next macro step can be initiated, starting with the default step size before adapting it again. Application of the presented algorithm is demonstrated on an example of a bouncing ball. The commonly known tunneling effect (the ball falling through the surface due to undetected events) can be postponed in comparison to a monolithic solution with a fixed-step solver until the refined step size exhausts the given numerical precision.

Cremona et al. (2019) are investigating hybrid co-simulation with the FMI-standard which is up to now not suitable for discrete event systems or event handling. They introduce a solution with superdense time and an extension to the FMI standard for supporting hybrid co-simulation. They list options for time resolution of coupled systems along with their (dis-)advantages (e.g. floating point numbers in double precision leading to rounding errors) and finally decide on decimal numbers with an integer exponent. The master has to have the smallest or greatest common divisor as resolution. In co-simulation with the FMI, the master tells subsystems how far to advance in time until the next synchronization takes place. Cremona et al. introduce the function `doStepHybrid`, a modification of `fmi2DoStep` from the FMI 2.0 so the subsystem reports whether it has accepted this step size or only advanced to a smaller time. In addition, the master may ask `getMaxStepSizeHybrid` (an adaption of `getMaxStepSize`), which returns an upper bound for the accepted step size of the respective FMU, before invoking `doStepHybrid`. According to Cremona et al., implementation involves “relatively small adaptations to existing master algorithms. In a nutshell, it requires adopting our integer-based representation time, using specific wrappers for FMUs based on their category, and letting the master negotiate a time resolution as part of its initialization procedure.” While their considerations are linked to the FMI for co-simulation, they state that the same principles can be used to develop independent solutions for simulating hybrid systems.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

#### 3.4.2.3 Algorithms for hybrid (co-)simulation

In (Gheorghe 2009), two sequential master algorithms (there called *synchronization models*) are described. In the so-called canonical synchronization model, the CT system is executed first and stops either at the next synchronization reference $t_{k+1}$ – which corresponds to the next timed event – or, if a state event occurs, at the time $t_{se}$ the event is triggered. The system's values and the time are then transferred to the DE system, which handles the next event in its queue (which takes place at $t_{k+1}$) or, in case of a state event, the processes triggered by the state event at $t_{se}$. In the rollback-based synchronization model, the DE system is advanced first until time $t_{k+1}$ (which is to say, the next event and its processes are handled) and the values are transferred back to the CT system. If a state event occurs, this is reported to the DE system, which goes back to the former synchronization reference $t_k$ and is advanced until $t_{se}$, executes the processes reacting to the event at that point and then proceeds to the next event, which can differ to the first iteration.
Furthermore, events update schemata are described in detail. Gheorghe then presents a generic methodology for developing hybrid co-simulation tools and applies it to co-simulate SystemC and Simulink simulations by a specifically developed framework called CODIS.

Fitzgerald et al. (2014) present a tool for model-based design of embedded systems which allows co-modeling and further co-simulation of discrete and continuous systems. It combines the tool *Overture* (for Vienna Development Method (VDM) models (of discrete event systems)) and 20-sim (for bond graph models (of continuous time systems)) via the Crescendo tool.
The model of the discrete event system and the continuous time model are combined in a *co-model* including a *contract* defining shared variables, parameters and events, i.e. information that is accessible for both models. In the book, the term *co-simulation* is defined as the simulation of the co-model. This is slightly misleading as it is further explained that this co-simulation takes place via an engine interacting with the simulators of the DE and CT models. So actually the co-simulator does not simulate the co-model directly but coordinates the simulations of the models, thus the approach qualifies as co-simulation using a middleware.
Synchronization time steps are determined by the discrete event simulation. Starting at a synchronized point in time, the discrete part tells the continuous part to advance to the time scheduled for its next event (which has not yet taken place). While the DE system does not go back in time, the continuous time system may step back if an event occurs between two internal time steps of the continuous system. Additionally, in case of state events (defined in the contract) occurring in the continuous simulation, it communicates them to the discrete

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

event part, hence setting the next synchronized time step.
The developed tool is tested on case-studies of varying complexity, including a line-following robot and a personal transportation device similar to a Segway.

Camus et al. (2016) present a DEVS wrapper for hybrid co-simulation of FMUs implemented in MECSYCO, using the DEV&DESS formalism. The presented method is applied to the model of a barrel-filter factory, consisting of a queue of barrels to be filled, a tank and two controllers: one for the valve between tank and barrels and one to set the barrel target water level and stop the current filling. The water level of tank and barrels as well as the flow from tank to barrels are represented by continuous models, whereas the controller models are event-based. The main challenge is posed by instantaneous reaction on (state) events from the respective other systems, such as reaching the desired level in barrels or abortion of the current filling. These can hardly be met with the usage of a purely FMU-based co-simulation, as events between two synchronizations are only transferred at the later communication point. MECSYCO is a platform for co-simulation of complex systems implementing a DEVS-wrapper. It is based on a paradigm regarding heterogenous co-simulation as multi-agent system, where every participating subsystem is represented by an agent which sees its model as atomic DEVS. Interactions between agents are considered indirectly by so-called artifacts (see Ricci et al. (2007) for further information). To be able to integrate FMU components, every communication point is seen as event (both internal events producing output and external input events). They make use of rollback to determine the next state events, which is required for the `getNextInternalEventTime` function of DEVS (this is not supported by all tools following the FMI 2.0 standard, as it is not mandatory, rem.). They mention the known drawback that state changes might not be detected if, for example, a Boolean changes twice in between two synchronization references. The application to the use case shows that in comparison to a purely FMI-based solution, events are processed without delay. In conclusion, Camus et al. mention possible improvements and extensions to the presented implementation.

Van Mierlo et al. (2017) present a hybrid simulation algorithm supporting debugging at the hybrid level combining Timed Finite State Automata and causal block diagrams (CBD). The SCCD (State Charts and Class Diagrams) formalism is used to model the simulator structure in a hierarchical canonical representation. This is enhanced to enable debugging by using Hybrid Automata. In this formalism, states and transitions are used, where states can also contain a CBD model.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Gomes et al. (2018a) have developed an algorithm to determine the communication step size for the co-simulation of bi-modal hybrid systems which ensures stability provided the original, monolithic simulation of the system is stable. In this approach, event detection is not necessary but the error due to event delay is taken into account. However, the method is limited to bi-modal hybrid systems consisting of a continuous part which can switch between two modes according to a discrete controller. The system is represented as hybrid automaton with at most one equilibrium in each mode, at the origin. Additionally, it needs to take the physics of the original system into account and access its equations. Still, this is a first approach to ensure maintaining stability. Gomes et al. point out the limitations of the method themselves and aim to address them in future work by extending the presented algorithm on multi-modal systems with multiple equilibria.

Farkas et al. (2019) propose an adaptive master algorithm designed to meet some challenges of hybrid CT simulation, which is understood as CT simulation with wrappers for DE components. The step size control algorithm requires a so-called sensitivity model as input which basically informs on intervals where the step size should be adjusted to detect discrete events with small latency. To get this information, a certain amount of insight on the participating FMUs is needed. If not yielded by the developers, Farkas et al. claim that the required details can be estimated after a few fixed-step simulation runs. Zeno behavior is avoided by the introduction of a minimum step size. While the approach shows promising results for some exemplary implementations, the authors also point out difficulties and leverage points for improvement, most importantly the challenge of obtaining enough data for a good sensitivity model.

#### 3.4.2.4 Comparisons of coupling methods for DAEs

Quaglia et al. (2012) present a real-time simulation of networked control systems by co-simulating MATLAB, which is well suited for modeling and simulating control systems, and SystemC for networking aspects and hardware-software embedding. One of the most interesting aspects of this work is the co-simulation of time-driven (MATLAB solvers) and event-based (SystemC simulation) approaches.
Communication is achieved via sockets and additional entities for MATLAB/Simulink and SystemC. The simulations can be synchronized by four different approaches. In the so-called “exact” one, as soon as one simulator has to communicate with the other, the respective faster one is set back to the smaller time and re-started. Two approaches are issuing warnings if events from the respective other system have been missed or simulation times are asynchronous but (partially) carrying on the simulation. The last method requires knowl-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

edge of future events and timing of synchronization steps accordingly, which necessitates the introduction of additional, virtual events for synchronization in the SystemC simulation. The approach is validated by simulating a teleoperated system communicating via a packet-based network.

Palensky et al. (2014) express the importance of modeling and simulating hybrid systems for the area of power grids. They give an overview of existing tools and further present a comparison of simulation approaches for hybrid systems: one of them based on events, the other on continuous time. As a model example, they consider prize-sensitive energy distribution among buildings of which each has a heating system and stochastic interventions such as window opening. First, they present the event-based approach where they use two types of schedules: one with a fixed time step, the other a dynamic schedule which uses the – in the considered test case available – analytical solution of the underlying system. Both show similarly good results if the fixed step is not chosen too big. They implement the model in Modelica and simulate it with Dymola with a continuous time approach, using several libraries and applying different solvers, fixed step as well as solvers with step size control. “Even though both approaches yield comparable simulation results, there are basic differences in performance, usability and flexibility. For the considered test model, the discrete event-based approach performs better with respect to runtime performance and memory usage” due to the large number of events which impede performance of Dymola’s – otherwise, for purely continuous problems, efficient – solution algorithms. As advantage of the continuous approach they name mostly the implementation of physical models, i.e. modeling and not simulation aspects.

Tong et al. (2014) review simulation methods of both communication and power systems, as the importance of their interdependencies has only recently come to broader awareness. First, monolithic approaches like Petri nets, realizations in MATPOWER (MATLAB Power System Simulation Package) or communication network simulators like NS2 or OPNET are mentioned, which are limited regarding the complexity of the considered scenario. Further, they refer to embedded simulation approaches like DEVS before describing co-simulation approaches: the framework EPOCHS links PSCAD/EMTDC, PSLF and NS2 via a formalism in one platform. Therein, the participating simulations are executed in parallel in between points of synchronization. Events occurring between two synchronization references are stalled, so errors are accumulated. In the GECO framework, which combines PSLF and NS2, the master algorithm queues the continuous time steps as well as the discrete events in a global event queue. To be able to establish such an event queue, iteration is required

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

to determine the order in which the events have to be scheduled, starting with independent, parallel executions up to the first synchronization reference and re-starting them afterwards, thus eliminating errors in comparison to the approach described before. Finally, a hardware in the loop approach, where the power system is simulated and the communication hardware included, is discussed.
Although the approaches presented are designed and implemented specifically for (co-)simulations of communication networks and power systems, Tong et al. provide a nice overview of approaches also applicable (with adaptions, possibly) for other hybrid systems. The methodology of the presented master algorithms, for instance, can also be applied to problems in other fields if all participating simulators and solution algorithms allow re-calculations and varying communication step sizes.

Li et al. (2014) give an overview of existing co-simulation platforms and emulators for networked control systems. These are systems of actuators, sensors and controllers which are coordinated via a communication network, thus network and physical simulations need to be combined. This poses amongst others the commonly known challenges of combining event and time driven systems. The presented platforms are compared regarding synchronization and underlying formalisms as well as performance for the test example of a networked converter system.

Heinzl (2016) and Heinzl et al. (2018) present a comparison between co-simulation and DEVS-based approaches for modeling and simulating hybrid production systems. The Hybrid PDEVS (hyPDEVS, see Preyser (2015)) approach, a formalism to describe hybrid systems based on DEVS (Zeigler et al. 2000) is compared to parallel loose coupling co-simulation via the BCVTB (cf. Wetter (2011)). While co-simulation offers the advantages of “convenient modeling, suited simulation algorithms and simultaneous model engineering”, the coupling and data exchange algorithms still remain challenging. In the applied approach, data exchange does only take place at pre-defined macro-steps in the considered co-simulation method, so numerical errors are introduced due to the delayed processing of events, which naturally consitutes a disadvantage in comparison to the hyPDEVS approach. Heinzl et al. point out that this could be met with step revision. However, not all simulators chosen for the respective subsystems can be expected to allow such interference in their algorithm by an external coordinator. In addition, they mention that integration of hybrid aspects on the semantic level is not possible in the co-simulation approach.
The DEVS-based approach demands high effort for initial model development but can be beneficiary for long-term application regarding maintainability and extendibility. Advantages

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

considering hybrid modeling in particular are the integration of continuous and discrete aspects on model level with the hyPDEVS approach.

#### 3.4.2.5 Specific developments

Savicks et al. (2014) present an approach to combine discrete event models implemented with Event-B and continuous models exported as FMU. Event-B is a method to formally model and analyze systems on different representation levels allowing mathematical proof of consistency between these levels and formal verification.
They extend the Rodin platform, which was designed as IDE for the Event-B method, to allow co-simulation of FMUs and Event-B models. Synchronization takes place at fixed time steps, where the `fmiDoStep` function is used for the continuous part and a "Wait" event is introduced to indicate synchronization reference. A Wait event is the only one allowed at these points in time, although naturally arbitrary events can take place in between two references within the discrete event system and unknown to the master. The concept is validated by the simulation of a voltage distribution control consisting of a continuous model of the control system implemented in Modelica and an Event-B state machine monitoring the voltage from the Modelica model and, depending on the voltage changes, controlling the switch of a transformer in the Modelica model. Savicks et al. point out that the tool is still under development and further research and experiments are required, but the present results are promising.

Widl et al. (2015) address "the FMI-based co-simulation of hybrid models representing closed-loop control systems, where a continuous time-based plant model is connected to a discrete event-driven controller model." As test case they consider a thermal room model with a heater which is switched on or off depending on the room temperature, thus combining a continuous system for the room temperature (implemented in Simulink) with the discrete controller (realized with TRNSYS). In one approach, the output of the FMU corresponding to the Simulink control model is processed directly in TRNSYS, where the respond is delayed. Another realization uses the co-simulation environment FUMOLA, where both FMUs from TRNSYS and Simulink are imported. This implementation shows the same results as those from a monolithic reference simulation in Simulink alone. This shows that even when applying the FMI standard, results may differ according to the manner or extent to which the standard is implemented by different tools (as not all features are mandatory, rem.).

Thiede et al. (2016) present a systematic approach to address multi-level simulation and methods for level selection and coupling. They start by breaking down manufacturing sys-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

tems into participating entities and defining their levels and corresponding hierarchy. Afterwards, interactions between these entities are defined and methods for coupling (offline coupling, model integration and co-simulation) are described. In addition, they give recommendations for the choice of a preferable approach depending on characteristics of the participating models. Further, they express the close linkage between water and energy in terms of, on the one hand, water for mining, fuel production, energy conversion but also cooling or heating purposes, cleaning etc., and on the other hand energy for water cleaning, conveyance, pumping, and energy contained in water (kinetic, potential, thermal). Due to these connections, joint considerations are necessary in order to save resources and solve problems integrally instead of shifting between energy and water flows. Thiede et al. consider a case study of a ball hub manufacturing in the automotive industry where water transports thermal energy and electrical energy is needed to transport and treat water. To realize the application, they develop five models: machine model (state chart), process chain model (DE), compressed air model (hybrid (DE&CT)), warm water model (CT) and cooling water model (CT). As implementations of the participating submodels were already available in the same software and parallel execution with frequent data exchange is required, they chose model integration (in Anylogic 7) over co-simulation. Results show that the amount of energy for water and vice versa account for over two thirds of the total energy demand and more than half of the costs and environmental impact, which confirms the significance of the water-energy nexus in the considered case. They also simulate the impact of different measures of improvement and their combination, which is higher than the sum of individual improvements, thus highlighting the importance of a holistic consideration of the system.

Nguyen et al. (2017) propose a conceptual structuration of co-simulation frameworks consisting of five generic layers:

i) conceptual (generic structure with models as black boxes)

ii) semantic (interaction between models, role of the framework for the specific problem)

iii) syntactic (formalization of the framework, “the manner in which semantic models and interactions are represented”)

iv) dynamic (execution, synchronization techniques)

v) technical (implementation details, simulation evaluation).

“As for synchronization techniques, two well established approaches are called conservative processing and optimistic processing. In general, the conservative approach requires all the

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

participating simulators to wait for each other to finish their step before advancing to the next step. (...) On the other hand, the optimistic approach allows the individual simulators to advance on their own events. When a conflict is detected, then the simulators must perform a leap backwards (...) and discard the all the results from the moment in question."(Nguyen et al. 2017)

### 3.4.3 Coupled simulation of FEM models

This section covers work on the co-simulation of FEM models, either amongst themselves (Esgandari and Olatunbosun 2015; Ibrahimbegović and Markovič 2003) or with multibody (Mousseau et al. 1999), BEM (Felippa et al. 2001) or circuit models (Wünsche et al. 1997b). The developments focus on specific kinds of applications such as fluid-structure interaction (Farhat and Lesoinne 2000; Felippa et al. 2001), electro-thermal systems (Petegem et al. 1994; Wünsche et al. 1997b) or vehicle dynamics (Mousseau et al. 1999).
Most approaches are either plainly sequential (Farhat and Lesoinne 2000; Felippa et al. 2001; Wünsche et al. 1997b) or iterative ones (Ibrahimbegović and Markovič 2003; Mousseau et al. 1999; Petegem et al. 1994). Details are given below.

Petegem et al. (1994) investigate a specific application in electrothermal simulation consisting of two subsystems. One execution is run without interaction to determine critical time intervals regarding temperature changes, then sequential runs follow until the overall system converges. The presented method is called *blockwise Gauß-Seidl waveform relaxation method with time windowing*.

Wünsche et al. (1997a,b) present an approach for adaptive macro step size control. System 1 is integrated for one macro step with extrapolated values for System 2, then System 2 is intended to simulate for the same interval but stops if the firstly approximated values of System 2 and currently calculated ones differ too much. If this is the case, the stop time is chosen as actual end time of the current macro step and System 1 is recalculated with the now known values from System 2. With that, one macro step is finished.
The presented method can also be used if one of the participating simulators cannot step back in time. However, the work does not explain whether the method can be extended to divisions into more than 2 systems. It is applied to electro-thermal systems, co-simulating ANSYS (FEM) and SABER (circuit models).

Mousseau et al. (1999) describe co-simulation in vehicle dynamics by combining multibody dynamics and finite element models. The vehicle dynamics model is approached by multi-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

body system simulation implemented in ADAMS using a BDF method for integration. The tires, due to their complex interaction with the ground, are modeled with a nonlinear Finite Element method using the software FEAP. In the corrector step of the BDF method, data from the FE model is transferred during every iteration step, requiring many calculations in the FE models which can lead to long computation times. Convergence can be accelerated by additionally taking Jacobians into account. Inter-process communication is enabled via the transport layer interface. The method is validated by comparison with measured test results, demonstrating that for certain forces, the simulation yields reliable results while for others, deviations from the measured values occur which are to be investigated in future work.

Farhat and Lesoinne (2000) present a staggered algorithm without subiterations applied to fluid-structure interaction for aeroelastic problems. The method shows superior accuracy to conventional serial staggered procedures due to extra value exchanges at half-step.

Felippa et al. (2001) review sequential coupling algorithms for fluid-structure interaction, f.i. the simulation of a submerged structure hit by a shock wave. Finite element methods (FEM) are used to discretize the structure while a boundary element method (BEM) is used for the fluid. To maintain stability, the model of the fluid has to be augmented by some properties of the structure. In a later approach, the fluid model is decomposed once more to be able to consider nonlinearities, so all in all three subsystems are co-simulated while maintaining stability. Some further applications of the sequential algorithms are aeroelasticity and control-structure interaction. In the paper's appendix, stability is investigated by the amplification method (spectral analysis) and accuracy by the modified equation method rather than standard truncation error analysis.

Ibrahimbegović and Markovič (2003) introduce a strong coupling method for multi-scale finite element models which is applied to models of inelastic behavior of engineering structures. The method is explained for the coupling of two scales, one macroscale and one microscale, thus resulting in a so-called micro-macro finite element model. The coupling approach is motivated by the method for classical plasticity models, where all local equations are iterated within each iteration step for the global equation so it is unconditionally stable according to Matthies and Steindorf (2003). The method is applied to test cases of tension and bending of porous material and two-phase material. Compared to simulation with the exact finite element representation, efficiency is improved while accuracy depends on the manner the micro-structure is represented. For the considered applications, the incompati-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

ble mode structured mesh representation is found to be the most suitable.

Esgandari and Olatunbosun (2015) describe a “hybrid” implicit/explicit Finite Element Analysis method combining frequency and time domain solution algorithms to simulate brake noise, both implemented in Abaqus. The implicit solver uses Newton-Raphson iteration, the explicit part is solved via an explicit Euler method. There are specified points in time for data exchange, so the coupling is loose. It is validated by a vehicle test but no general numerical investigations are presented.

### 3.4.4 Application-specific research

The following summaries describe very specific applications that do not necessarily offer potential to aid general developments but are kept in this work to further emphasize the vast field of areas profiting from the concept of co-simulation.

Steinebach et al. (2004) discuss methods of coupling for flow simulation, therein some model coupling methods and a Gauß-Seidl-type simulator coupling method where the difficulty of the choice of a good step size is expressed.

Brecher et al. (2009) present a survey on the state of the art in process-machine interactions focusing on metal-working processes. For different phenomena such as defined and undefined cutting edge and forming, they examine developments in offline coupling, model integration, and co-simulation, refer to related projects, and point out potential for future investigations.

Spiryagin et al. (2012) describe the simulation of rail traction vehicles by coupling MATLAB/Simulink with GENSYS using TCP/IP and making use of Simulink S-functions. The vehicle is modeled in GENSYS while the traction control is implemented in MATLAB/Simulink. The depicted model description implies the usage of the same time steps for every participating model part but without iteration as only open-loop coupling is considered.

Nouidui et al. (2014) present the development of a Functional Mockup Unit (FMU) in EnergyPlus for loose coupling co-simulation of Jacobi type using EnergyPlus as master. The applied algorithm is non-iterative and uses fixed time steps for data exchange (controlled by EnergyPlus). Values from other subsystems are approximated via constant extrapolation between two synchronization references. Since rollback and storage of previously calculated data are not supported in general, they only consider use cases where the EnergyPlus

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

subsystem contains a differential variable and hence no iterations are necessary. Two examples coupling a room model in EnergyPlus with an HVAC system and a finite state shading controller respectively implemented in Modelica and exported as FMU demonstrate the applicability of the development. Both systems do not contain algebraic loops.

Stettinger et al. (2015) describe co-simulation in real-time hardware applications, using model-based coupling to diminish unwanted latency and noise effects: when systems are coupled in real-time, latency effects occur due to sending and receiving time delays. By run-time system identification, missing data can partially be replaced by model-based prediction. To achieve valid predictions, latency times have to be approximated accurately. The system identification for the models presumes basic knowledge about the underlying, participating systems. The method is applied to the academic "air ball" test system, a table tennis ball maintained at a certain height in a pipe by controlled air flow from the bottom. The co-simulation platform ICOS is used to perform the model-based coupling. The approach has also been tested on further, industrial applications such as coupling existing offline simulation tools with a HIL test bench for internal combustion engines, where a thermal model is linked to a real engine control unit, and the control of the driven machine of an engine test bench.

## 3.5 Partitioned multirate schemes

The introduction of partitioned multirate schemes is motivated by dividing stiff systems of ordinary differential equations into an active and latent part (as in (3.5)) depending on the time constants of the respective subsystems: The active parts need to be integrated with a small step size, the latent parts with a comparatively large step size which is also used as macro time step. Stiffness is thus isolated in the latent parts which can therefore be integrated with an implicit algorithm while the active subsystems can be solved with an explicit solver (Günther et al. 2001; Günther and Rentrop 1994), incorporated in one partitioned solver algorithm. This way, computational effort can be reduced up to 90% (Rice 1960).

$$\begin{aligned} \dot{y}_L &= f_L(y_L, y_A, t), \quad y_L(t_0) = y_{L,0}, \quad y_L : \mathbb{R} \to \mathbb{R}^{n_L} \\ \dot{y}_A &= f_A(y_L, y_A, t), \quad y_A(t_0) = y_{A,0}, \quad y_A : \mathbb{R} \to \mathbb{R}^{n_A}, \quad n_L + n_A = n \end{aligned} \tag{3.5}$$

Explicit criteria to determine whether division into a system like (3.5) is sensible for a specific problem are given by Striebel (2006) (see below). While this division is mostly done "by hand" or even assumed to be given initially (Rice 1960; Verhoeven et al. 2006b), several

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

approaches include automatic partitioning of the system (depending on step size comparisons, asymptotic behavior, precision of extrapolated values or error estimates), which is sometimes renewed after every macro step (Engstler and Lubich 1997; Günther and Rentrop 1994; Kvaernø¸ and Rentrop 1999).

The presented multirate schemes range from one-step (f.i. Bartel and Günther (2002), who are co-simulating partitioned electrical networks with a w-multirate method (w-method: see Def. A.17) or Savcenco et al. (2007), who present an adaptive multirate strategy with a two-stage second-order Rosenbrock method (see Def. A.15)) to multi-step methods (Skelboe and Andersen 1989) and variants (Biesiadecki and Skeel 1993) including slowest first (Esposito and Kumar 2001; Günther and Rentrop 1994; Verhoeven et al. 2006b), fastest first (Kvaernø¸ and Rentrop 1999) and compound methods (Günther et al. 2001; Verhoeven et al. 2006b). Adaptive approaches have been developed by Savcenco et al. (2007) and Verhoeven et al. (2008), who control the step size of both micro and macro steps.

Detailed investigations on stability properties of different multirate schemes are found in (Gomm 1981; Skelboe and Andersen 1989; Verhoeven et al. 2006b) and (Verhoeven et al. 2007). Savcenco et al. (2007) conduct error estimates for Rosenbrock methods depending on integration and interpolation orders which are utilized in the adaption of the step size. Verhoeven et al. (2006a) present an error analysis for the BDF compound-fast multirate method presented in (Verhoeven et al. 2006b).

While hierarchical structures have up to now been mostly neglected in classical integrate-and-collaborate co-simulation (cf. Section 6.1), they have long been introduced for partitioned schemes: Skelboe and Andersen (1989) already consider three different step sizes. Esposito and Kumar (2001) do not restrict the number of levels as long as these show “triangular” dependencies – i.e., equations can be ordered so that System 1 does not depend on values from any other subsystems, System 2 may only depend on values from Systems 1 and 2 and so on. Striebel (2006) develops an approach suitable for an arbitrary number of activity levels that does not actually restrict dependencies but acknowledges that the partitioning only makes sense for weakly coupled systems, meaning relatively small magnitudes of dependencies (measured by the derivative of the right hand side by the respective state variables, see below).

An apt summary of limitations of multirate methods has been formulated by González et al. (2011):

> (. . . ) if the mechatronic system is modelled according to the weakly coupled strategy, these multirate integration methods cannot be applied directly due to their particular features:

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

a) They introduce modifications in the integration schemes, something that is not possible in commercial off-the-shelf modeling and simulation tools used for weakly coupled co-simulation. For example, the aforementioned block diagram simulators and multibody system simulation packages offer their own set of integration schemes that cannot be modified.

b) They assume that the coarse and refined time-grids are equidistant and synchronized, which means that the large stepsize $H$ is a multiple of the small stepsize $h$. This condition cannot be guaranteed in weakly coupled co-simulations if one or more subsystems are integrated with a variable time-step integrator, since the stepsize control algorithms of the different commercial simulation environments cannot be synchronized.

c) They mitigate the unstable behavior caused by the explicit extrapolation of some equation terms by introducing implicit schemes, which involve some kind of iterative process. Again, off-the-shelf simulation tools such as block diagram simulators do not allow this kind of iteration with other simulation tools.

Details on the mentioned methods and investigations are given below in chronological order.

Rice (1960) is the first to introduce multirate solution algorithms (according to Engstler and Lubich (1997) and Esposito and Kumar (2001)), in particular “split Runge-Kutta methods”. He considers a system of two interdependent ODEs, of which one, let us call it Equation $II$, is integrated with a fraction of the step size for the other ODE, Equation $I$. This requires extrapolation of one value for the smaller steps of Equation $II$, which is achieved by using the extrapolation in the Runge-Kutta stages taken for the solution of Equation $I$. The split method (in different variants) is applied to a six degree of freedom missile simulation, where it yields results of similar accuracy as the original method while reducing computation by up to 90%.

Gomm (1981) present an approach to perform stability analysis of explicit linear multirate methods. For this purpose, they construct a stability polynomial for the considered methods by using the multirate z-transform method.

Skelboe and Andersen (1989) point out that for multirate formulas, the analysis of stability properties requires at least one scalar test equation for every step size in use. In particular, they consider backward Euler multirate formulas (a definition can be found on p.2 of the paper) and give stability theorems for absolute and A-stability for the cases of two and three

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

different step lengths. The method is suitable if application of a multi-step formula to subsystems yields highly differing norms of the increment function (see A.2 for background information and cf. Striebel (2006)). In this case, if ordered by that property, increasingly larger step sizes can be applied for each subsystem without loss of accuracy (in the sense of no increase in the maximum local truncation error), where the smaller steps are always factors of the larger steps. The values from the slower reacting subsystems are obtained by zero order interpolation (since backward formulas are used, all considered values lie in the past, thus *inter*polation instead of *extra*polation, rem.). Skelboe and Andersen state that implicit formulas are not needed in their application of simulating MOS (metal-oxide-semiconductor) circuits, where waveform iteration is sufficient to solve the algebraic equations, although the convergence is neither guaranteed nor subject of the paper. They show that absolute stability can be proven depending on properties of the linearization matrices (but not only their eigenvalues, as would be the case for single rate methods) of the presented method for two and three different step lengths. It should be noted that these properties are sufficient, yet not necessarily required for stability. Although the same methods as used in the corresponding proofs cannot be applied to arbitrary numbers of step sizes, they could not invalidate the assumption by experiments either.

Biesiadecki and Skeel (1993) discuss three multirate algorithms for dynamic simulations called Verlet-I, Verlet-II (introduced by Grubmüller et al. (1991)) and Verlet-X. Verlet-I is a generalization of the Verlet algorithm (see also Grubmüller et al. (1991)) with the introduction of distance classes: Right-hand side function parts are classified according to the rapidity of changes in their force values and are solved with the same time step per class. In comparison to Verlet-I, Verlet-II takes the values of the previous into account in addition to the current macro step and Verlet-X includes linear extrapolation. The three algorithms are tested on systems including linear and nonlinear forces (e.g. a system of particles connected by springs) as well as artificial resonance. Depending on the test system, the different algorithms show instabilities, which implies that suitability of these methods is highly dependent on the test case.

Günther and Rentrop (1994) apply partitioned Runge-Kutta methods and multirate Rosenbrock-Wanner schemes to latent electric circuits (by the example of an inverter chain) to test the advantages and disadvantages of both strategies. In highly integrated electric circuits, most parts remain latent. To apply these partitioned methods, the system is divided along the “boundaries” of active and latent parts, where the active ones can be integrated by solvers for non-stiff problems and the latent parts require stiff solution algorithms.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

First, Günther and Rentrop use a partitioned Runge-Kutta (PRK) method where a semi-implicit Rosenbrock-Wanner method is used for the stiff part and an explicit Runge-Kutta method for the non-stiff part. Both are linked by coupling conditions, of which a detailed description can be found in (Rentrop 1985). The partitioning takes place according to step size comparisons for simultaneously applied methods, asymptotic behavior, the number of rejected steps and special conditions for matrices occurring during the integration (see pp. 38ff of the considered paper), and the precision of extrapolated values.
Secondly, they present a multirate Rosenbrock-Wanner method (called MROW2(3)). There again the circuits are divided into latent and active parts and extrapolation is used in both parts for the respective other values, so parallelization is possible. To maintain A-stability (see also Gear and Wells (1984)), linear extrapolation is used instead of higher order estimates. They apply a slowest first multirate strategy with step size control for the individual components. According to the proposed step size for the next step and the deviation of the approximated solution to its linear extrapolation, every component is categorized anew in each step as an active or latent one.
The methods are tested by the example of an inverter chain and compared to an A-stable one-step method. For the PRK-method, significant speed-up can only be achieved for strongly stiff problems with few stiff components, which is not the case for the inverter chain. The MROW method is very efficient due to the step size adaption and switching of components from and to active and latent, respectively. For a large number of inverters (several hundred) a speed-up of computing time of three to four times is reached compared to the reference solution, while accuracy is maintained. Concluding, Günther and Rentrop point out that “multirate strategies must be based both on numerical information and on circuit information, e.g. neighborhood of active elements”, which is available for the simulation of inverter chains.

Engstler and Lubich (1997) propose a multirate extrapolation method based on Richardson extrapolation which automatically partitions the system by inactivating components where the error estimator is below a given tolerance. Inactivation implies that the extrapolation tableau is not built up further for these components. While the method does not integrate the system parts with multiple rates, the concept can be utilized for automatic partitioning of the system in multirate schemes.

Kvaernø, and Rentrop (1999) investigate a multirate Runge-Kutta scheme for systems divided into active and latent parts according to two different strategies: *fastest first* and *slowest first*. They implement a third order explicit multirate Runge-Kutta method where new

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

step sizes are selected and subsystems are partitioned anew into latent and active after each macro step. Numerical stability is shown by applying the method to the mathematical model of an inverter chain.

The Multirate Partitioned Runge-Kutta method presented in Günther et al. (2001) generalizes the method from Kvaernø¸ and Rentrop (1999) to handle stiff systems. These are partitioned into an active and latent part depending on the time constants of the respective subsystems, so the stiffness is isolated in the latent parts. These are integrated with an implicit algorithm while the active subsystems are solved with an explicit solver. A compound multirate approach is applied so extrapolation is needed for all but the first micro steps. Based on the theory on Partitioned Runge-Kutta methods (see Hairer (1981)), order conditions for the Multirate Partitioned Runge-Kutta methods are derived. The method is tested on the model of an inverter chain implemented in MATLAB and compared to the MATLAB specific solver implementation ode23s. The results show that the method presented in the paper in general takes less steps and rejects less steps than ode23s and after a certain number of steps, ode23s terminates prematurely with an out of memory exception. The errors of both methods stay in the same order of magnitude (below $10^{-5}$). However, instabilities can occur in the Multirate Partitioned Runge-Kutta method if the active parts also show stiff behavior.

Esposito and Kumar (2001) state that for multirate numerical integration methods “Areas of application include simulating integrated circuits and molecular and stellar dynamics.” They, on the other hand, apply them to robotic systems which can be described by hierarchical differential equation systems, meaning that there are not interdependencies of all subsystem equation systems but they can be ordered so System 1 depends on no other subsystem, System 2 may depend on values from Systems 1 and 2, System 3 may depend on values from Systems 1, 2 and 3 and so on. For thus structured systems, they present a slowest first multirate predictor-corrector method. To analyze the integration error, they regard the additional error by interpolation, as this plus the error of the original method with which it shall be compared yields the total error of the multirate method. If interpolation polynomials of order $m$ (where $m$ is the number of steps of the multistep method) are chosen, the interpolation error is small compared to the original one and thus the multirate method performs similarly accurate to the original method. Regarding stability, the triangular form of the considered problem, meaning the slower components not depending on the faster ones, is essential as this means that extrapolation errors, which could become unbounded, do not occur. For the considered triangularly structured systems, stability is maintained compared to the original

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

method. For the simplified assumptions of fixed step sizes, a constant step size ratio of two to the respective next level and sufficiently high and equal right hand side function evaluation costs, estimates show that only about half (0.7-0.4) as many operations are required for multirate methods with three to five levels in comparison to the single-rate method, so efficiency is increased. They present an exemplary implementation of the method in MATLAB for the simulation of a standard differential drive cart. For this example, the costs of the multirate method come up to only 27 percent of a comparable singlerate method. The estimates are surpassed due to the higher ratios of step sizes between levels.

Maten et al. (2005) propose a multirate procedure to integrate hierarchical circuits. In this case, they use the property that the latent and active part of the considered circuit are submodels.

According to Striebel (2006), applying a multirate scheme is sensible if

- the systems are weakly coupled, meaning $\left\| \frac{\partial f_L}{\partial y_A} \right\| \ll \left\| \frac{\partial f_L}{\partial y_L} \right\|$ and $\left\| \frac{\partial f_A}{\partial y_L} \right\| \ll \left\| \frac{\partial f_A}{\partial y_A} \right\|$,
- "the activity levels are widely separated", meaning the micro steps are much smaller than the macro steps
- the activity is concentrated on a small part, meaning there are much less subcircuits in the active system

In the first part of his thesis, Striebel (2006) focuses on the division of a system into one active and one latent part. The time steps taken by the latent part are also the macro time steps where the systems are synchronized. In the second part, a multirate approach for two levels of activity is presented which is then modified to obtain a hierarchical multirate method for arbitrary numbers of activity levels.

In (Verhoeven et al. 2006b), several basic multirate algorithms for electric circuits partitioned into active and latent parts are described. Following the *slowest first* method, one step with size $H$ of the latent system is calculated first with extrapolated values of $y_A$; afterwards the active system is integrated with the corresponding smaller time step $h$ using interpolated values of $y_L$.
This can be modified to a *compound step* where both systems are integrated simultaneously for the large time step given by the latent part and afterwards the active system is integrated using interpolated values of $y_L$ which allows comparison of values and hence error estimation. In the *mixed compound* method, both systems are also integrated simultaneously for

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

one step, but each with its individual step size, before the remaining active ones follow. A compromise of the last two methods would be the *general compound method* where these are modified by calculating the active system for a step size of an arbitrarily choosable factor $\alpha > 0$ of the number $m$ of small step sizes during one large step size, which also means that the compound step and the mixed compound step can both be seen as special cases of the general compound step.

These methods are compared regarding stability by a linear, two-dimensional differential equation system. It is stated that the stability not only depends on the eigenvalues but also the eigenvectors of the coefficient matrix $A$ (for the specific equations see pp. 4ff of the paper).

The resulting conditions for asymptotic stability are more practical: For $H \to 0$, both the slowest first and the general compound method are stable if $A$ is a stable matrix (i.e., all eigenvalues have a negative real part).

A detailed stability analysis for the BDF slowest first method is given in (Verhoeven et al. 2007), an error analysis for the Compound-Fast algorithm can be found in (Verhoeven et al. 2006a).

Verhoeven et al. (2008) introduce an improvement of the Compound-Fast method by adaptive step size control of both mirco and macro steps to meet a certain error tolerance. Therefore, the local discretization errors of the latent and active parts as well as the interpolation errors are analyzed and controlled by adapting the step size via Error Per Unit Step control. The developed method is applied to several numerical test examples which show that instabilities occur in case of higher index DAE subsystems but else yield promising results, especially regarding the speed-up in comparison to single-rate methods.

Savcenco et al. (2007) present an adaptive multirate strategy in which, after one time step with a common step size, system parts exhibiting error estimates above a certain tolerance perform a rollback and repeat integration with smaller time steps. Values from systems which do not repeat the step are interpolated or obtained by dense output formulas. If necessary, the time grid is refined repeatedly until error tolerances are met. The method is designed for one-step methods and in the paper it is applied with the two-stage second-order Rosenbrock method. To maintain the method's convergence order, integration polynomials of the same order are chosen. This is reasoned as follows: "In general, the order of the interpolation should be related to the order of the time stepping method. With a basic integration method of order $p$, the error in one step will be $\Delta t_n^{p+1}$. Interpolation with a $q$-th order polynomial will introduce an interpolation error $\Delta t_n^{p+1}$ at the components in which we interpolate. Since we are interested in the errors in the maximum norm, the choice $q = p$ is natural. On the other

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

hand, it was observed, also for higher-order methods, that taking $q = p - 1$ often produces an order of accuracy equal to $p$ for the whole scheme, due to damping and cancellation effects."
In the first coupling scheme, the time step is bisected in every refinement step for all components with unsatisfyingly large error estimates. The next macro step is chosen adaptively, according to estimates depending on "the minimum time step over the components and an expected number of levels of refinement" in the last macro step. Rejection is also possible in case of rapid changes which would require refinement for all components anyway. While in this approach, every further refinement is applied to the whole macro time step, a second strategy is presented in which the need for further refinement is determined separately for every micro step.
The multirate schemes are tested on ODE systems originating from different problems: For the semi-discretization a reaction-diffusion problem, both multirate strategies show equal orders of magnitude in the maximum error with only one fourth of CPU time of the multi-rate methods in comparison to single-rate integration. Therein, the second strategy performs slightly better. The solution of the semi-discretized Allen-Cahn equation has two stable and one unstable equilibria. For this problem, the errors are at some points in time even smaller than in the single-rate scheme and again, significant computational speedup can be observed. Applied to an inverter chain problem, CPU cost is reduced to a sixth.

Striebel et al. (2009) employ a compound multirate method using ROW schemes and dense output formulas for the solution of index-1 DAEs describing electrical circuits. The overall system is divided into an active and a latent part of equidistant time steps respectively, the macro step being an integer multiple of the micro step. Via graphical differentiation (formulating the system as multirate DAE trees), stability conditions for the multirate method are deduced.

## 3.6 General information

In this section, general strategies for coupling methods (Tseng and Hulbert 2001), validation and verification of co-simulation (Trčka 2008) and results from a survey by Gomes et al. (2017) on the state of the art in co-simulation, including challenges in DE, CT and hybrid co-simulation, are summarized.

Tseng and Hulbert (2001) present guidelines for an effective gluing algorithm, aiming to "execute coupled system simulation without sacrificing the integrity of subsystem modeling

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

and solution and to maintain the efficacy of the overall results." They state that such an algorithm has to be

- *Sticky*: The inter-connection relations between subdomains should be well satisfied, i.e. coupling between subdomains should be resolved and captured.
- *Green*: It should not contaminate subdomain solution strategy. The integrity of the individual model and solution methods should be maintained. Minimum modification of the original solution scheme is desired.
- *Inexpensive*: The overhead should be minimized.
- *Pretty*: The results should be pretty; that is, the overall solution should be numerically correct within the bounds of the desired accuracy. (Tseng and Hulbert 2001)

While these guidelines can be adopted for coupling algorithms in general, the focus of Tseng and Hulbert lies on mechanical systems with certain requirements.

Chapter 6 of (Trčka 2008) is dedicated to validation and verification of co-simulation. In general, validation is about whether the conceptual model describes the regarded system accurately, verification about the correct implementation and simulation of the conceptual model. The co-simulation implemented by Trčka (coupling of EnergyPlus and TRNSYS as well as ESP-r and EARTH for integrative building systems simulation focusing on HVAC systems) has been verified by: static verification (structural properties of the code) and dynamical verification (exact synchronization and data transfer tested by varying of time constants). Validation for coupled simulation is tricky as for different simulation tools ofttimes only different validation approaches exist and comparison with mono-simulation might not be expedient as modeling and simulation of the same system in only one (and hence different for at least one subsystem) simulator could yield different results due to the differences in the simulation tools. Trčka uses a method based on inter-model comparison. Only one simulator is used for mono- and co-simulation and afterwards different implementations of the co-simulation are compared.

Gomes et al. (2017) provide a survey on state-of-the-art techniques for co-simulation. They start by defining their used terminology (some of which can be found in Section 2) as well as challenges for DE, CT and hybrid co-simulation. The DEVS (Zeigler et al. 2000) notation is adapted to describe discrete event co-simulation in general. As challenges specific to DE co-simulation, Gomes et al. names causality (especially for parallel execution with the possibility of rollback), determinism and confluence (the same results for all possible interleavings

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

of executions), dynamic structure (varying dependencies), and distribution. Co-simulation of CT systems is formalized with a notation similar to the one used for DE co-simulation. Fulfillment of algebraic constraints and algebraic loops (closed-loop feed-through in input-output dependencies), which are of special interest for coupled DAE systems, are named as typical challenges in CT co-simulation next to consistent initialization, compositional convergence (error control), compositional stability, compositional continuity (discontinuities in input trajectories due to extrapolation), and real-time constraints. Formalization of hybrid (CT/DE) co-simulation is considered a non-trivial task and thus not given specifically. However, the idea is explained and specific challenges are given, the latter being semantic adaptation (the choice of wrappers depends on the co-simulation scenario); predictive step sizes (fixed step sizes will miss events, adaptive approaches require detailed information on the subsystems); event location (related to step size prediction, requires information for prediction or rollback functionality); discontinuity identification; discontinuity handling (re-initializing might cause others and not terminate, energy conservation has to be respected); algebraic loops, legitimacy (infinite events at the same time step), and zeno behavior (infinite, consecutive events in ever decreasing intervals but in a bounded time frame, hard to detect in hybrid co-simulation); stability (issues of different origin; further analysis required); theory of DE approximated states (error bounds for the DE part) and establishing a standard for hybrid co-simulation. By starting with publications from the last five years and examining the references therein, a taxonomy has been made with the following distinction on the top level: Non-functional requirements, simulation unit requirements, framework requirements. Each of these is again grouped further, for details the interested reader may investigate Figures 13-15 of Gomes et al. (2017). For every considered publication, they have determined which of the requirements are addressed. This investigation has led to the results that the most observed non-functional requirements are accuracy, protection of intellectual property and performance whereas extensibility, a property deemed highly important by Gomes et al. themselves, is among the least observed. Within framework requirements, least observed are dynamic structure co-simulation, interactive visualization, multi-rate, algebraic coupling, and partial/full strong coupling support. In general, they find by their classification that there is a lack of research in methods which are both DE and CT based and in leveraging features from simulation units. The information in (Gomes et al. 2017) is summarized in (Gomes et al. 2018b).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 3.7 Concluding remarks on the state of the art in multirate and co-simulation

This chapter has given insights on various developments in the area of multirate and co-simulation, therein common methods, standards and tools. While there are broad areas of application and research, most investigations and developments are specialized on a certain kind of underlying equation system and may demand restrictions on the manner of coupling. This is not altogether surprising, as special problems come with specialized demands on their solution, which leads us to the most important conclusion to be drawn from these reviews: that the choice for the one or the other method cannot be made globally but depends on the underlying system, the status of model development, know-how and interdisciplinarity of the team of developers.

This holds true for selecting special coupling algorithms – see f.i. Schweizer et al. (2016), who show that depending on the system, even higher order extrapolation or higher macro step sizes can yield more stable results – as well as determining whether or not to approach a problem via co-simulation at all: For instance, the disadvantage mentioned by Heinzl et al. (2018) that integration of hybrid aspects on the semantic level is not possible with their chosen co-simulation approach (in comparison to a DEV&DESS-based solution) could for some use cases be seen as advantage, as co-simulation does not require detailed insight and understanding of the partial models' description but *allows* them to be developed independently by experts in the corresponding domain or field. With regard to the additional capabilities or intrusions into subsystem simulators which would be required for rollbacks in co-simulation, this is a minor requirement of insight in comparison to the renewed formalization of every participating model.

In addition, we can observe that, while sensible for the reasons given above, restriction of investigations to systems fulfilling certain requirements holds a few risks: There exist several software tools allowing the more or less easy coupling of certain simulators. Unfortunately, these are often used without further investigation on the consequences regarding numerical stability – such as, for example, testing the system and used algorithms for the requirements necessary to guarantee stability. This, among others, holds true for hierarchical or nested co-simulation, which is allowed by some tools and even, although scarcely, performed, but has not been investigated regarding consistency and stability up to now. This underlines the importance to fill this gap, which is aimed with the investigations presented in Chapter 6.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

The restriction to cases with special requirements also leaves a lot of unexploited methods for further investigations. Likewise does the pressing topic of hybrid co-simulation, for which promising developments are in progress in the research groups around the authors of (Broman et al. 2015; Cremona et al. 2019; Gomes et al. 2018a). We conclude with the observation in the words of González et al. (2011) that "it is not possible to find an optimal general purpose co-simulation method", which leaves co-simulation as ever present topic of interest with plenty of open research questions to be addressed in the future.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

CHAPTER 4

# Empirical Survey on Co-Simulation

To facilitate the possibility of consensus between different research groups and appliers of co-simulation, colleagues from national and international research groups and myself have collaboratively developed and conducted a two-stage Delphi study with more than 50 experts. The aim of this study was to help identifying current research needs, challenges and promising standards in co-simulation. Results of this study (published in (Schweiger et al. 2019a; Schweiger et al. 2018, 2019b)) are summarized in the following.

## 4.1 Method

The Delphi method (Dalkey and Helmer 1963) is an empirical method to explore problems characterized by an incomplete state of knowledge (Powell 2003) or a lack of agreement within the studied field (Okoli and Pawlowski 2004), which, as explained before in this thesis, clearly applies to co-simulation. In the first round of the study, a SWOT analysis was conducted. This is a technique to analyze strengths, weaknesses, opportunities, and threats (thus the abbreviation *SWOT*) in any item (project, product, person, etc.) (Kotler et al. 2016). In addition to this classical approach, an Analytic Hierarchy Process (AHP) was included in the second round of the study (resulting in a SWOT-AHP) to weight the factors in every category on a 9-point scale of importance. The first round was completed by 12 (out of 15 contacted) experts, while 53 experts answered the second questionnaire (out of 70 contacted ones). The participants were a mixture of experts from academia and industry, working in varying fields.

## 4.2 Results

The experts were asked for properties of the simulators they used within their co-simulation to determine their purpose of applying co-simulation. According to the results of the first round, the properties , “the simulator approximates the solution to sets of DAEs", “the simulator is a dedicated piece of hardware", “the simulator receives input from a human-machine interface”, “the simulator specializes in finite element modeling", “the simulator specializes in networks" and “the simulator specializes in software controllers" were pre-defined. Most of the experts (62%) use (among others) a simulator for numerically solving differential (algebraic) equation systems, but simulators specializing in networks or software controllers, or receiving input from a human-machine interface are also represented by equal to or more than 20%, see Figure 4.1. Only two experts answered with properties other than the predefined ones (solving PDES with finite volume methods and proving a theorem, respectively).

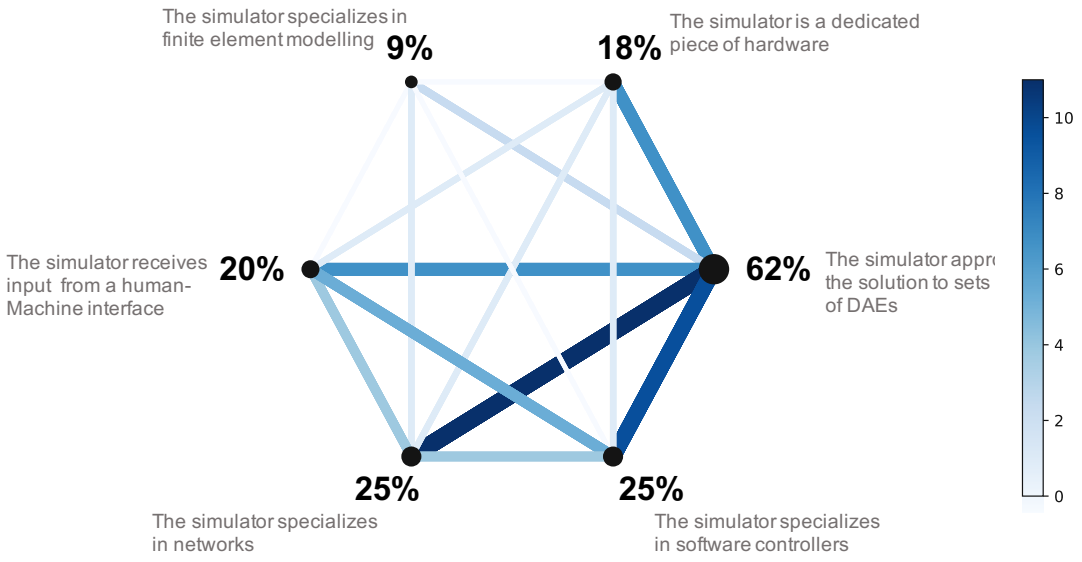


Figure 4.1: Answers to “which properties apply to the simulators that you have worked in co-simulation”. The amount of positive responses given in percent is indicated by the thickness of the corresponding node. Thickness and colors of the connections between the nodes correspond to the number of experts who responded positive to both connected nodes (which does not necessarily mean that both properties apply to simulators used in the same co-simulation) (Schweiger et al. 2019a).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

When asked about established and used standards for co-simulation, the functional mockup interface (FMI) turned out to be the most established as well as the most used for any kind of co-simulation (continuous time, discrete event or hybrid), as can be seen in Figures 4.2 and 4.3.

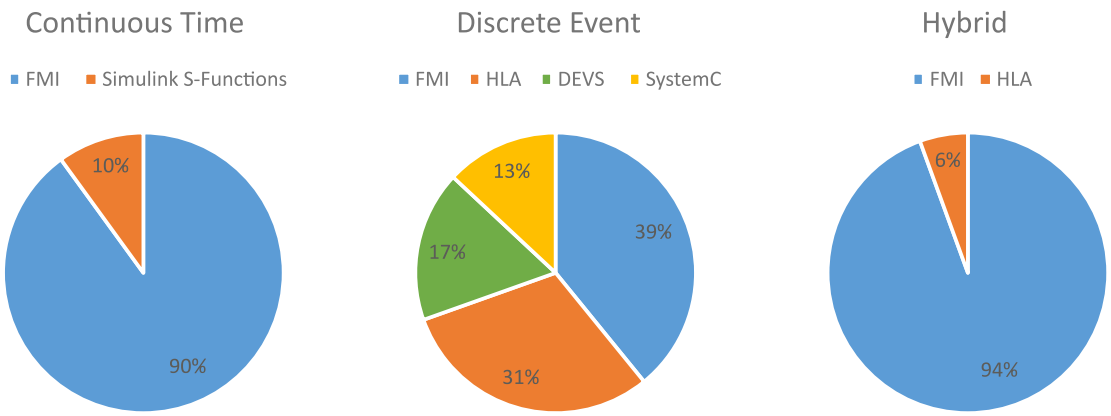


Figure 4.2: Experts' answers when asked for a widely accepted standard for continuous time/ discrete event/ hybrid co-simulation (Schweiger et al. 2019a).

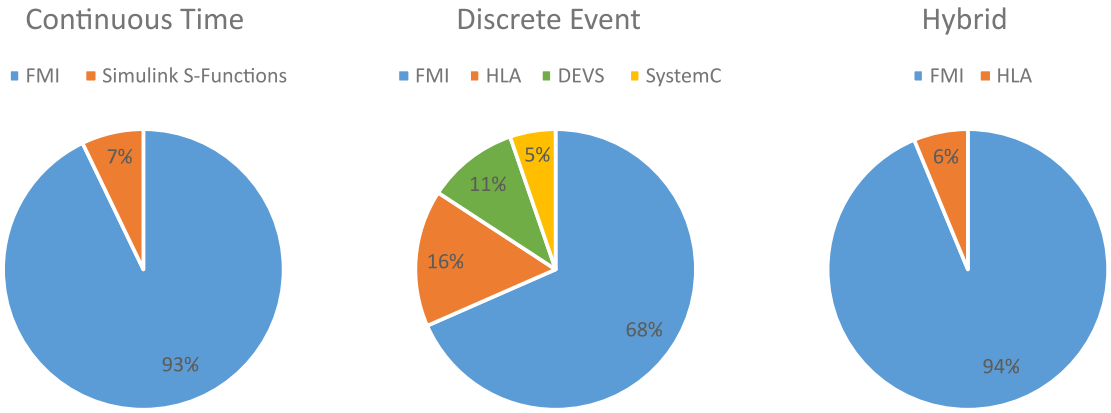


Figure 4.3: Answers to "What standard do you use for continuous time/ discrete event/ hybrid co-simulation?" (Schweiger et al. 2019a).

While for continuous time and hybrid co-simulation, the answers about used and established standards mostly concur, distinct differences can be seen between accepted and used standards for discrete event co-simulation, especially regarding the usage of the FMI.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

In addition to standards, the experts were asked which tools they use for co-simulation. Although many different tools were listed, Simulink turned out to be most used for hybrid (15%) as well as discrete event co-simulation (19%), Modelica-based tools are used for hybrid co-simulation by 40% of the questioned experts, followed by Matlab/Simulink (25%). The detailed segmentation is illustrated in Figures 4.4 to 4.6.

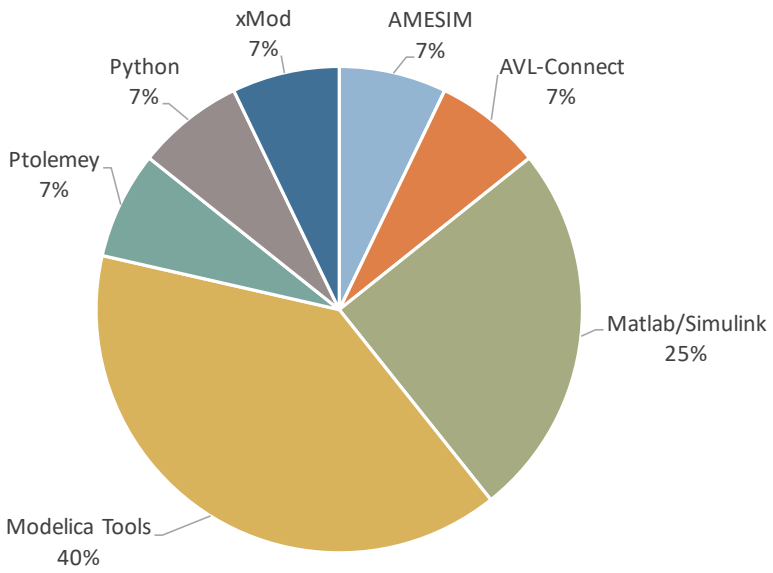


Figure 4.4: Experts' answers to "Which tools do you use for continuous time co-simulation?" (Schweiger et al. 2019a).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

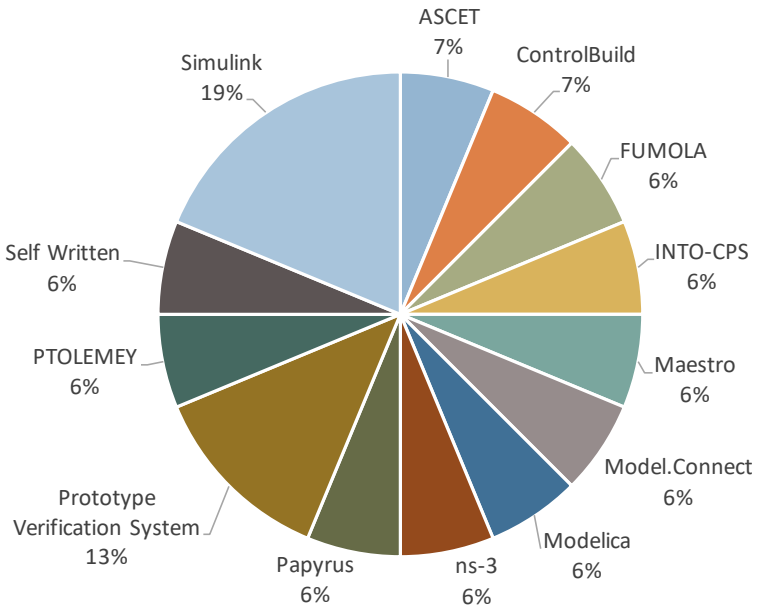


Figure 4.5: Experts' answers to "Which tools do you use for discrete event co-simulation?" (Schweiger et al. 2019a).

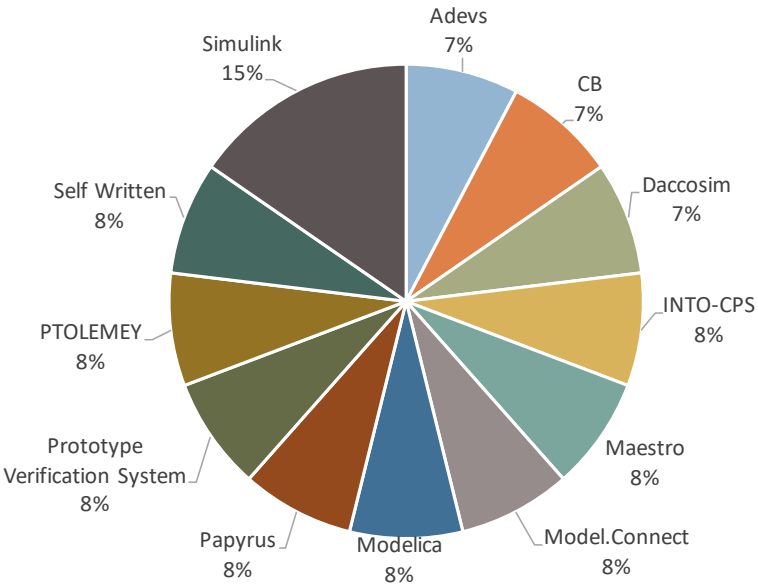


Figure 4.6: Experts' answers to "Which tools do you use for hybrid co-simulation?" (Schweiger et al. 2019a).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

As the prevalence of the FMI standard stood out even after the first round of the Delphi study, certain FMI-specific questions have been addressed in the second round. The experts were asked to assess current barriers for FMI in academia and research based on a seven-point Likert scale, ranging from 7 = “entirely agree” to 1 = “entirely disagree”. The results of this assessment are illustrated in Figure 4.7.

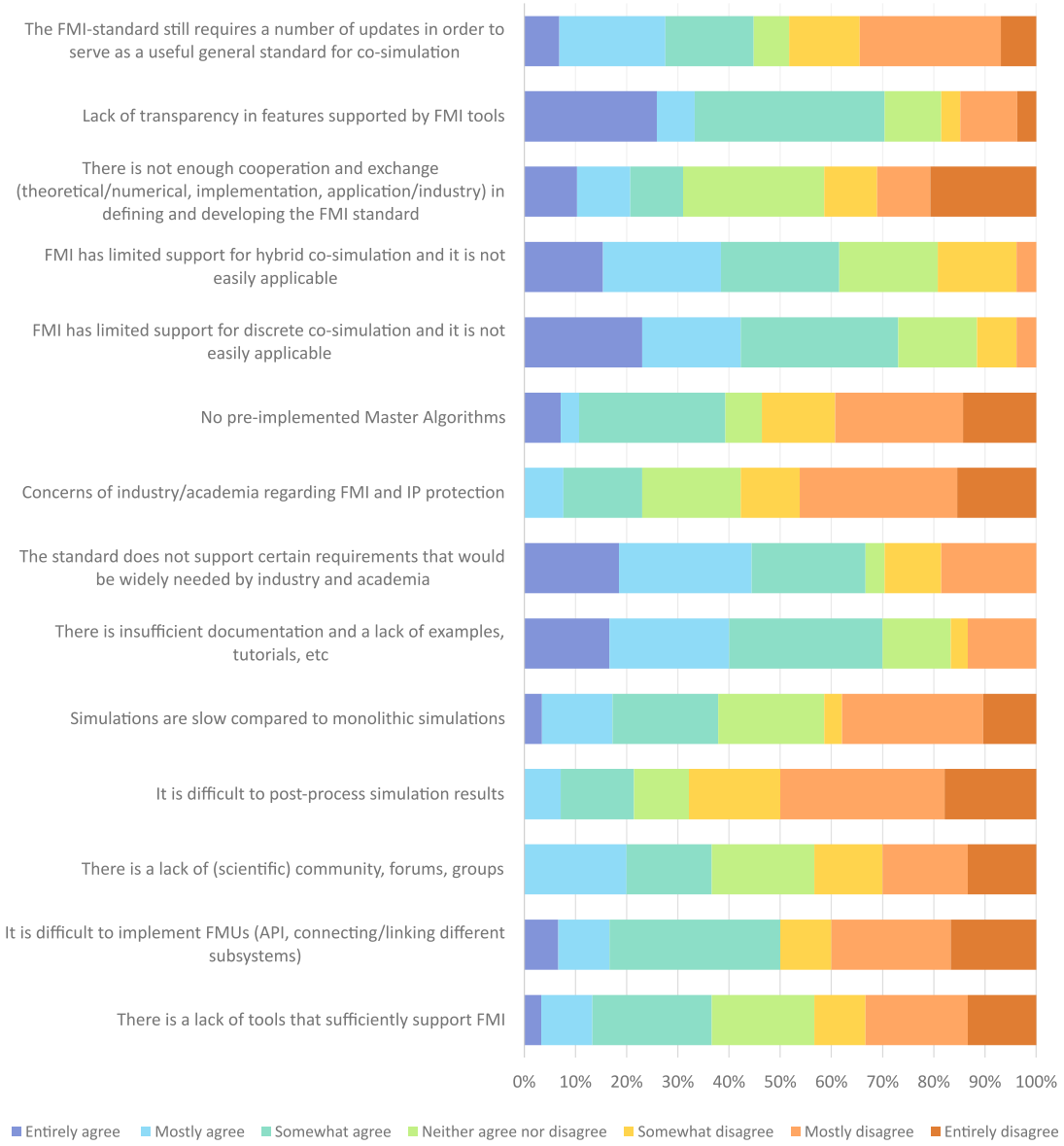


Figure 4.7: Experts’ assessment of current barriers for the FMI in industry and academia (Schweiger et al. 2019b).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

To evaluate these answers, the interpolated median is calculated for every potential barrier, since according to Sachs (2004), the interpolated median is more precise than the conventional median due to better consideration of the frequency of answers within one category in comparison to all answers. Only options with an interpolated median of 5 or more are classified as “barrier”, those between 3.5 and 5 as “somewhat of a barrier” and those below as “not a barrier”. The barriers and their medians are listed in Table 4.1.

Table 4.1: Barriers for the FMI according to the experts’ assessment. *Score: Entirely agree (7) Mostly agree (6) Somewhat agree (5) Neither agree nor disagree (4) Somewhat disagree (3) Mostly disagree (2) Entirely disagree (1)* (Schweiger et al. 2019b).

| | Mean | Median | **Interpolated Median** |
|---|---|---|---|
| The standard does not support certain requirements that would be widely needed by industry and academia | 5.42 | 5.00 | **5.25** |
| FMI has limited support for discrete co-simulation and it is not easily applicable | 5.67 | 5.00 | **5.25** |
| There is insufficient documentation and a lack of examples, tutorials, etc | 5.14 | 5.00 | **5.17** |
| Lack of transparency in features supported by FMI tools | 5.12 | 5.00 | **5.05** |
| FMI has limited support for hybrid co-simulation and it is not easily applicable | 5.82 | 5.00 | **5.00** |
| It is difficult to implement FMUs (API, connecting/linking different subsystems) | 4.07 | 4.00 | **4.00** |
| Simulations are slow compared to monolithic simulations | 3.82 | 4.00 | **3.92** |
| There is a lack of tools that sufficiently support FMI | 4.04 | 4.00 | **3.83** |
| There is a lack of (scientific) community, forums, groups | 4.27 | 4.00 | **3.83** |
| There is not enough cooperation and exchange (theoretical/numerical, implementation, application/industry) in defining and developing the FMI standard | 4.12 | 4.00 | **3.81** |
| The FMI-standard still requires a number of updates in order to serve as a useful general standard for co-simulation | 4.52 | 4.00 | **3.75** |
| No pre-implemented Master Algorithms | 4.08 | 3.00 | **3.25** |
| Concerns of industry/academia regarding FMI and IP protection | 3.52 | 3.00 | **2.83** |
| It is difficult to post-process simulation results | 3.57 | 2.50 | **2.50** |

This shows that up to now, FMI has only limited support for hybrid and discrete event co-simulation, even if it is already used for this purpose by many of the experts according to the results above. In addition, it is often unclear which of the FMI’s optional features are supported by FMI tools. What is more is that better tutorials and examples would facilitate a broadened use of the standard, f.i. for academic purposes. The barrier “The standard does not support certain requirements that would be widely needed by industry and academia” can encompass various aspects which will have to be investigated in further surveys.
Based on the experts’ qualitative answers on current challenges in the first round of the Delphi study and the authors’ personal experience, different statements were given in the second round, headed by “have you experienced. . . ” which could be answered on a 6-point Likert scale ranging from 1 = “very frequently” to 6 = “never”. A summary of the answers is illustrated in Figure 4.8.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

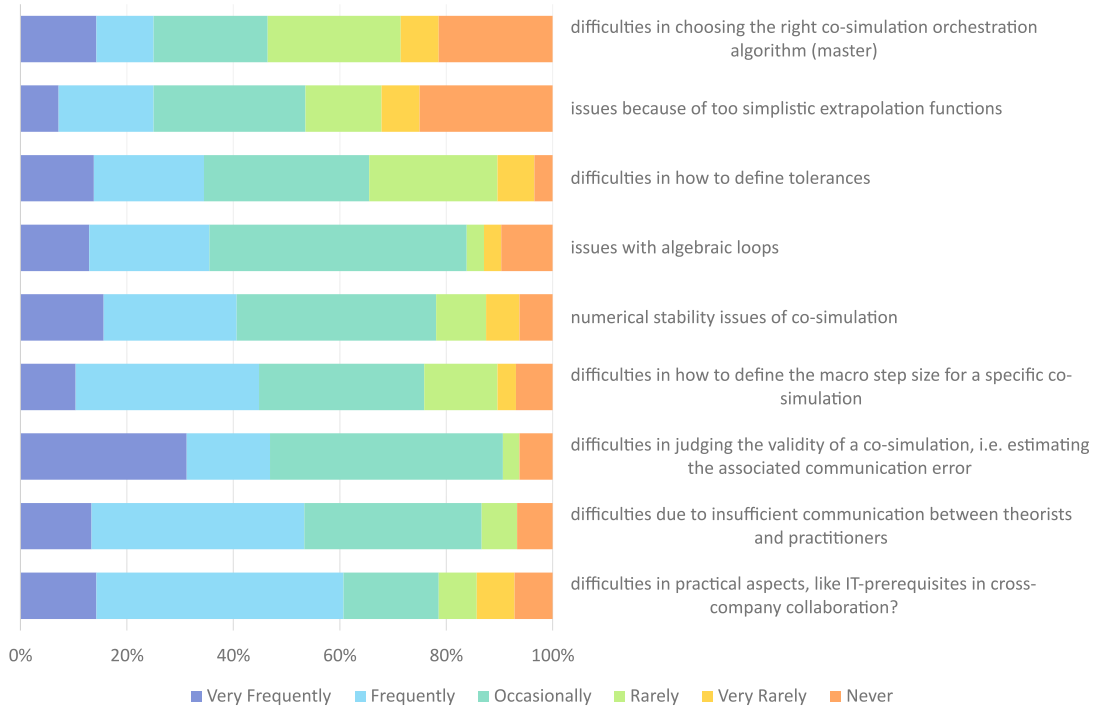


Figure 4.8: Experts' assessment of current challenges in co-simulation (Schweiger et al. 2019a).

Again, The interpolated mean has been used to determine the most present challenges, see Table 4.2. It can be seen that the experts mostly experience practical difficulties as opposed to scientific ones. The table also illustrates that most of the challenges chosen to be addressed in the second round are indeed experienced by most experts, at least occasionally.

Table 4.2: Experts' assessment of current challenges in co-simulation. *Score: Very Frequently (6) Frequently (5) Occasionally (4) Rarely (3) Very Rarely (2) Never (1)* (Schweiger et al. 2019a).

| | Mean | Median | **Interpolated Median** |
|---|---|---|---|
| difficulties in practical aspects, like IT-prerequisites in cross-company collaboration? | 4.7 | 5.0 | **4.7** |
| difficulties due to insufficient communication between theorists and practitioners | 4.4 | 5.0 | **4.6** |
| difficulties in judging the validity of a co-simulation, i.e. estimating the associated communication error | 4.6 | 4.0 | **4.4** |
| difficulties in how to define the macro step size for a specific co-simulation | 4.3 | 4.0 | **4.3** |
| numerical stability issues of co-simulation | 4.4 | 4.0 | **4.3** |
| issues with algebraic loops | 4.2 | 4.0 | **4.2** |
| difficulties in how to define tolerances | 4.3 | 4.0 | **4.0** |
| issues because of too simplistic extrapolation functions | 3.5 | 4.0 | **3.6** |
| difficulties in choosing the right co-simulation orchestration algorithm (master) | 3.6 | 3.0 | **3.4** |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Similar to current challenges, experts were asked to rate which research topics in the field of co-simulation have not received enough attention up to now. The scale ranges from 1 = “entirely disagree” to 7 = “entirely agree”.

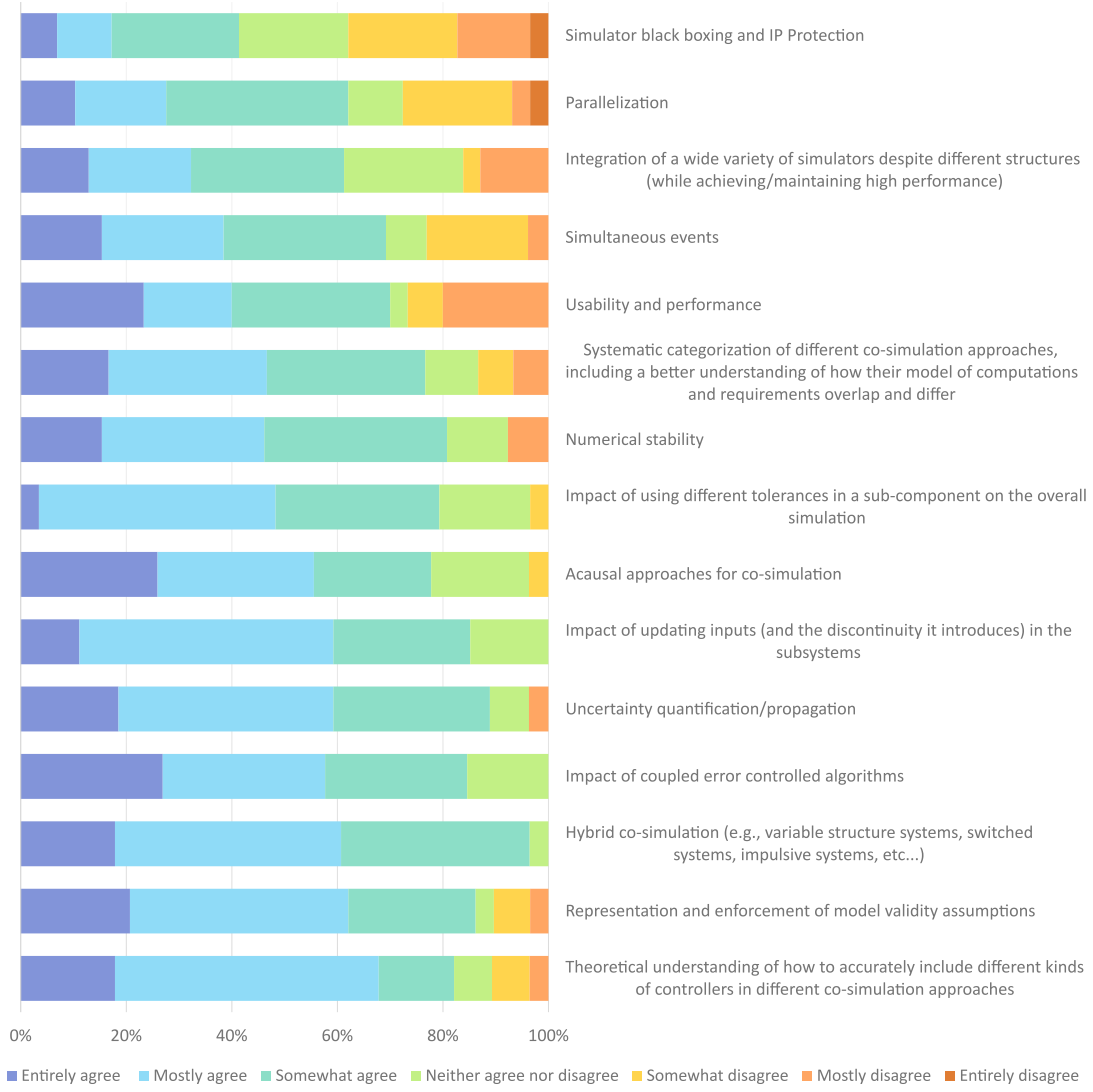


Figure 4.9: Experts’ assessment of research topics that have not received enough attention up to now (Schweiger et al. 2019a).

The results in Figure 4.9 and Table 4.3 show that the most pressing research needs are considered to be accuracy, validity, and uncertainty aspects as well as investigations on and possibilities for hybrid co-simulation and acausal approaches.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Table 4.3: Experts' assessment of research needs. *Score: Very Frequently (6) Frequently (5) Occasionally (4) Rarely (3) Very Rarely (2) Never (1)* (Schweiger et al. 2019a).

| | Mean | Median | **Interpolated Median** |
|---|---|---|---|
| Theoretical understanding of how to accurately include different kinds of controllers in different co-simulation approaches | 5.5 | 6.0 | **5.9** |
| Representation and enforcement of model validity assumptions | 5.6 | 6.0 | **5.8** |
| Hybrid co-simulation (e.g., variable structure systems, switched systems, impulsive systems, etc...) | 5.8 | 6.0 | **5.8** |
| Impact of coupled error controlled algorithms | 5.7 | 6.0 | **5.8** |
| Uncertainty quantification/propagation | 5.6 | 6.0 | **5.7** |
| Impact of updating inputs (and the discontinuity it introduces) in the subsystems | 5.6 | 6.0 | **5.7** |
| Acausal approaches for co-simulation | 5.6 | 6.0 | **5.7** |
| Impact of using different tolerances in a sub-component on the overall simulation | 5.3 | 6.0 | **5.5** |
| Numerical stability | 5.3 | 5.0 | **5.4** |
| Systematic categorization of different co-simulation approaches, including a better understanding of how their model of computations and requirements overlap and differ | 5.2 | 5.0 | **5.4** |
| Usability and performance | 4.9 | 5.0 | **5.2** |
| Simultaneous events | 5.0 | 5.0 | **5.1** |
| Integration of a wide variety of simulators despite different structures (while achieving/-maintaining high performance) | 4.8 | 5.0 | **4.9** |
| Parallelization | 4.6 | 5.0 | **4.9** |
| Simulator black boxing and IP Protection | 4.1 | 4.0 | **4.1** |

Figure 4.10 illustrates the results of the SWOT-AHP conducted in the survey. The length of the lines and distance of circles from the center indicate the priority assigned to the corresponding group or factor, respectively. It can be seen that factors in the categories "Strengths" and "Opportunities" are considered more important, amongst these the strength "Every sub-system can be implemented in a tool that meets particular requirements for the domain, the structure of the model and the simulation algorithm" and the opportunity "user-friendly tools including pre-defined master algorithms, integrated error estimation, etc.". "Incompatibility of different standards and co-simulation approaches" is rated as the threat with the highest priority and "robustness of co-simulation compared to monolithic simulation" as the most important weakness. In an overall comparison, the most prioritized factors are (in receding order) the opportunity of "user-friendly tools. . . " followed by the strengths "sub-systems can be implemented in a tool that meets the particular requirements. . . " and "co-simulation supports cross-discipline developments".

A more detailed exposition and discussion of results can be found in (Schweiger et al. 2019a) and (Schweiger et al. 2019b).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

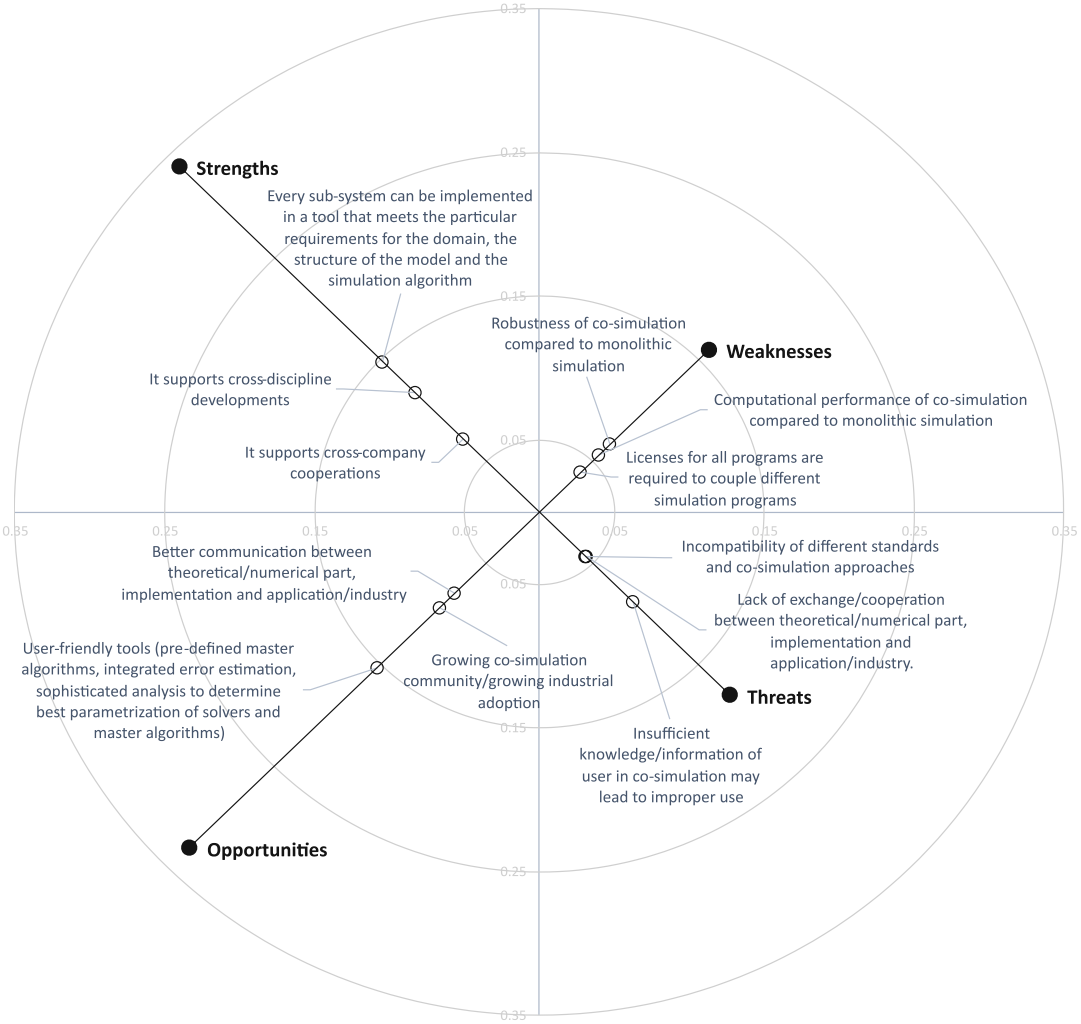


Figure 4.10: Graphical illustration of the results of the SWOT-AHP. Factors and groups are depicted as circles whose distance to the center corresponds to their assigned priorities (after (Schweiger et al. 2019a)).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 4.3 Concluding remarks on the empirical survey

All in all, the empirical survey has revealed important insights in the current developments and perceptions of co-simulation. It has shown that the FMI is considered the most promising standard in co-simulation, and even though it still entails various barriers, the survey has helped to identify them, which the Modelica Association can use as guidelines for future developments. Most difficulties in the experts' experience with co-simulation are practical ones such as cross-company cooperation or problems in the understanding between theorists and practitioners. The most important research needs were identified as accurate inclusion of different kinds of controllers in different co-simulation approaches, validity and accuracy aspects, hybrid co-simulation and acausal approaches. It is also interesting that in spite of the many challenges in co-simulation nowadays, the "strengths" and "opportunities" factors predominate in the SWOT-AHP. We hope that these results allow better targeted research and motivate better cooperation in the area of co-simulation; the variety in fields of origin and application notwithstanding.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

CHAPTER 5

# Structuring and Analysis

*Structuring of multirate and co-simulation methods and statistical analysis of selected literature*

Depending on the disciplines they originate from as well as the level and depth of development, co-simulation methods can be classified by various different means. Similarly, literature on co-simulation can be associated with attributes of this structure and further aspects such as theoretical focus or usage of standards. Consequently, this chapter consists of two interwoven parts, emerging in a not congruent, but overlapping *structuring of methods* and *analysis of publications*, complementing one another to a resulting list rounded off by analyzing illustrations.

## 5.1 Method and limitations

In the following, several possibilities of classifying and structuring methods of co-simulation by various aspects on different levels are presented.
Some of these listings have been introduced in preceding publications related to this area, others are presented as the result of an extensive literature research, outlining specifics of every publication found and thereupon re-structuring the aforementioned literature with respect to the considered viewpoints.

*Remark* 5.1. The words “publications”, “contributions”, “papers” and “works” are used synonymously throughout this chapter to allow for variety in the reader’s perception while covering articles, technical reports, books and theses as well.



Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 5.1.1 Literature selection.

The considered literature has been accumulated by strategic, but not all-encompassing search in different catalogues and search machines (ACM Digital Library[1], IEEE Xplore[2], Elsevier's ScienceDirect[3], Springer Link[4], SIAM[5], TU Wien library[6] and Google Scholar[7] by the keywords "co-simulation", "cooperative simulation", "coupled simulation", "hybrid simulation", "multi-level simulation", "hierarchical (co-)simulation"), contributions to conferences I have attended, recommendations by fellow researchers in the area of co-simulation and citations in papers found in the first iteration and again, in these, etc.

This implies that, for example, the high share of publications regarding mainly DAEs could have been caused by further investigation of references of found work, thus accidentally creating a "publication bubble" without intentionally channeling the research.
On the other hand, some publications, although found, have been omitted in the following analysis due to various reasons: some present only preliminary results of other, later included papers; some even basically share the same content (f.i., results of a dissertation or a technical report published in a journal), others have been discarded as the keywords by which they have been found have been interpreted with a different meaning than intended in this work (such as "multi-level simulation" for non-communicating different-scale simulation of one process to be compared post execution, not nested as in Chapter 6) and some have simply been deemed irrelevant for this work, such as basic applications in already frequently contemplated areas.

To sum up, the selection of literature may, at the very least, only reflect the statistics of the limited research. Therein, interesting correlations are revealed, even if these do not necessarily represent all existing literature on this topic – which, as Chapter 2 shows, is not easily delimited anyway.

[1] https://dl.acm.org/
[2] https://ieeexplore.ieee.org/
[3] https://www.sciencedirect.com/
[4] https://link.springer.com/
[5] https://epubs.siam.org/
[6] https://catalogplus.tuwien.at/
[7] https://scholar.google.at/

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 5.1.2 Classification

As indicated at the beginning, the selected work has been classified in two iterations. First, every publication has been carefully read, summed up and assigned certain properties in evidence, such as used model description, software and coupling algorithm. That completed, different manners of structuring in compliance with these properties, classifications of co-simulation methods found in the literature and further, own distinctions based on reflective considerations of the gathered information have been outlined (a preliminary version of which has been published in (Hafner and Popper 2017)). In compliance to the found structure, a list of properties has been defined. Thereby, it shall be noted that some aspects for classifying *methods* have not been incorporated in the list of viewpoints to categorize the selected *publications* (f.i. the distinction by participating subsystem solver algorithms, which apart from being too numerous are mentioned in barely any paper) and on the other hand, some further analysis has been made which only makes sense for literature, not methods, such as the number of publications using specific frameworks or a network of co-authorships.

In the next iteration of contemplating the selected literature, each work has been studied for these properties (or indisputable indications of them) and classified accordingly. A complete list of these assignments can be found in the Appendix, Section A.4.1. It is important to bear in mind that some properties could not be defined since the respective authors neither explicitly mention them (for example, whether a fixed or adaptive macro step is used) nor could the property be found out from the paper's context (which, however, could be accounted for by my subjective perception). Simply put, the presented numbers only declare that a certain share of papers *specifically* reveal that they use an adaptive macro step, for instance, not that the rest do not. Further, it shall be noted that while for a paper describing an application, the assignment of a specific model description, f.i. agent based models (ABM), would mean that ABM are used in the described application, while for a publication describing a framework, it might simply mean that partial simulations of ABM are supported.

Clear definitions in some publications and non-assignability in others – partitions which, moreover, change for every considered property – are a dilemma amplified by the heterogeneous nature of the various publications in consideration. To enable focusing attention on the relevant publications when viewing the statistics, publications for which a property has been defined are taken as new total for every considered aspect. However, this amount is declared at the beginning of each description so as not to neglect this information and risk a distorted perception.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 5.2 Publications over the years

Selected by the method described above, a total of 139 publications remains to be considered in the statistical analysis.

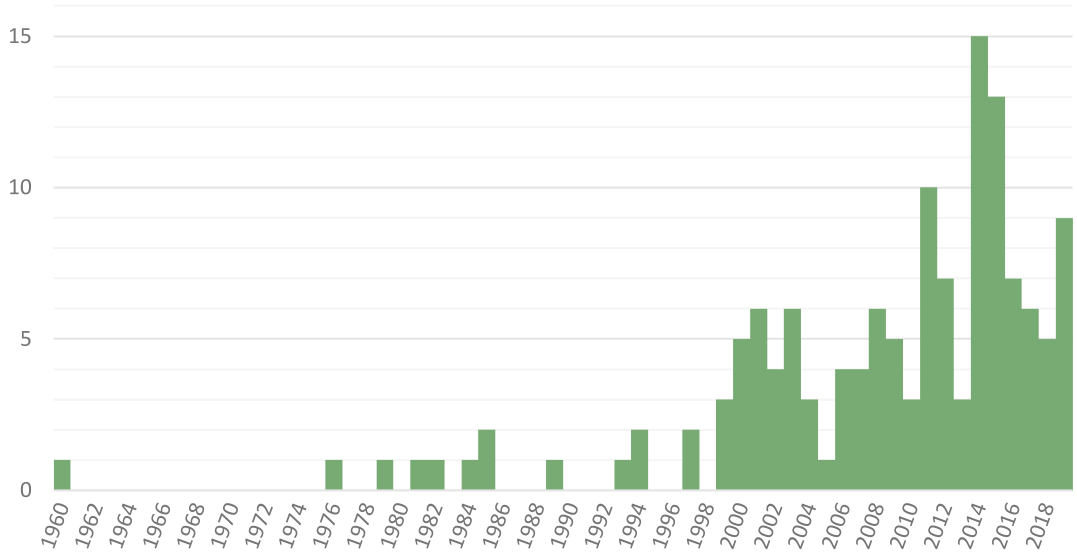


Figure 5.1: Number of publications on co-simulation considered in this analysis per year.

Observing the distribution of these among the years (Figure 5.1) reveals that although cooperative simulation has been touched incidentally before the millennium, research in this area has seen a significant upswing since. This trend is even more pronounced when the literature is cumulated in five-year time frames, as depicted in Figure 5.2. This also reflects that (de-)coupling of processes or methods has been investigated even in the eighties due to the small computational power of individual processors and consequent need for parallelization to utilize distributed processing power. Owed to the seemingly unbounded ascent of the latter – a perception fueled by Dennard scaling and Moore's law – parallelization then seemed to become almost obsolete. However, transistor scaling slowed down in this millennium in contrast to the prediction by Moore's and Dennard's scaling law (Bohr 2007; Esmaeilzadeh et al. 2011; Hennessy and Patterson 2018). In addition, the heterogeneity in computational solutions has increased with the gain in demands on simulation, developments in mathematical modeling and design of specific software according to modeling paradigms. Although this allows customized approaches with respect to individual requirements, new challenges arise with the ever-increasing complexity of comprehensive problems emerging in industry

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

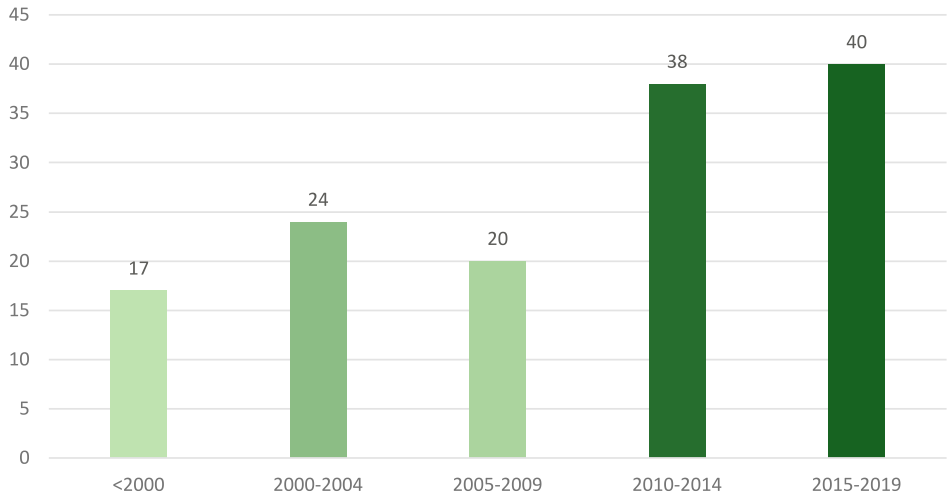


Figure 5.2: Number of publications on co-simulation considered in this analysis, partitioned into five-year time frames.

or urban energy systems: Specifically designed simulations have to be coupled for a holistic solution, thus the development of coupling methods, frameworks and standards has seen a new boost, confer also Chapter 1.

## 5.3 Main emphasis in the literature

Depending on the emphasis of the respective publication, each is assigned one main topic out of “theory”, “application”, “survey”, “standard” or “framework”. It shall be noted that this labelling describes whether the publication focuses *mainly* on theory, application, etc., not solely. Those where theory and application are quite balanced have, however, been classified “both theory and application”. The result of this classification of considered papers along their main content is shown in Figure 5.3. It can be seen that a majority (63%) of papers mainly covers theoretical aspects. 21 publications (15%) are applications of already known methods and for eight papers (6%), the theoretical and applied part are quite evenly matched. Eight of the 139 papers are pure surveys (some limited to an application area of interest) and two describe a standard (the HLA (Dahmann et al. 1997) and FMI (Blockwitz et al. 2012)). Thirteen present a framework, on which more information can be found in Section 5.10.2. These shares, however, have changed throughout the years, as Figure 5.4 illustrates.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

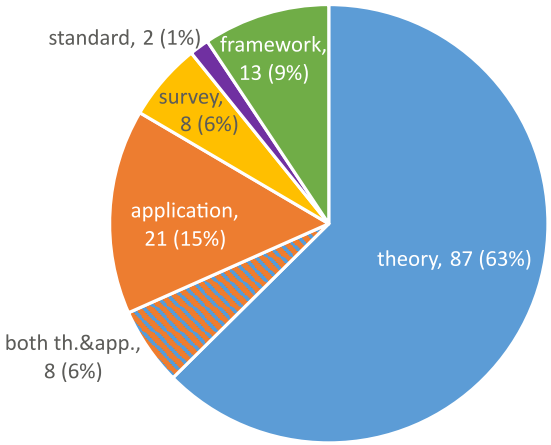


Figure 5.3: Main emphasis in the considered literature.

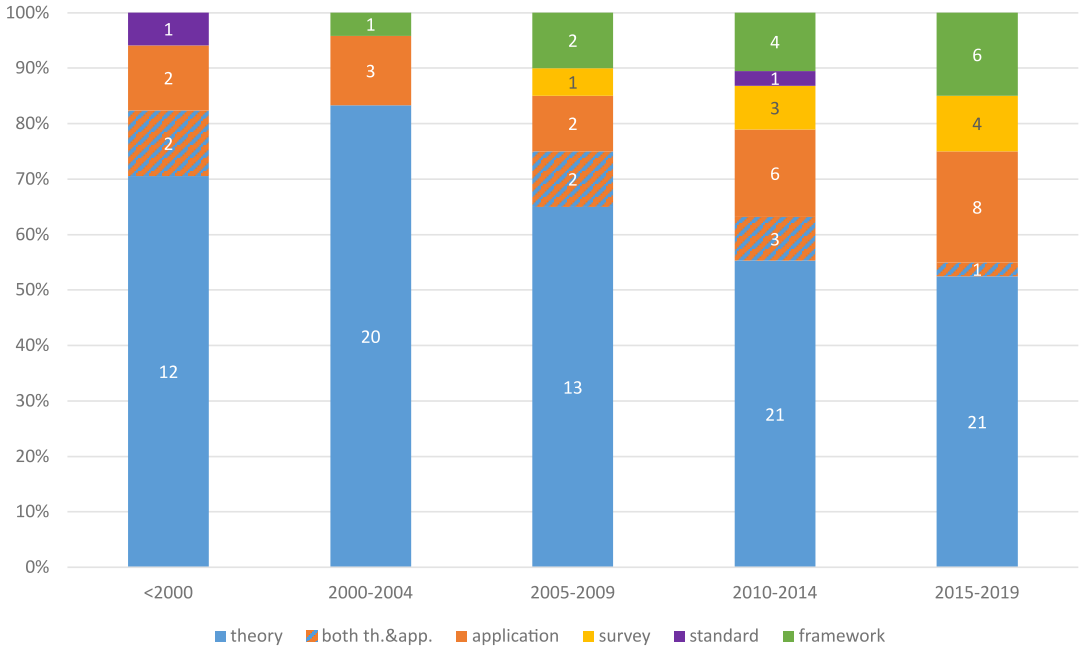


Figure 5.4: Variation of the main emphases' shares over time regarding five-year intervals. Numbers in the bars denote the quantity of publications per category in the respective time frame. Corresponding percentages can be read off the left-hand axis.

The first thing that catches the eye is the outstandingly high share of theoretical papers in the years 2000 to 2004, which amounts to more than eighty percent of regarded works. In later years, this percentage gradually declines, even if papers covering application as well as

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

theoretical aspects are included in the consideration. On the other hand, the development of frameworks seems – according to the considered selection – to have become more popular over the last two decades. Surveys start to occur in 2009 and have become more frequent ever since. This might simply be explained by the ever-increasing amount of research in this area, thus increasing the necessity of aggregating studies. The share of publications focusing on application does not vary much with regard to time. Although the percentage in the last five years is slightly higher compared to the time frames before, this observation cannot be upheld if papers with shared focus on theory and application are included (apart from the comparison to the already mentioned exceptional interval from 2000 to 2004).

In addition to this contentual distinction, it shall be noted that while most considered publications have been published in journals, books or in conference proceedings, ten publications are PhD theses and three diploma (i.e. master) theses. The distribution on the main thesis topics is depicted in Figure 5.5.

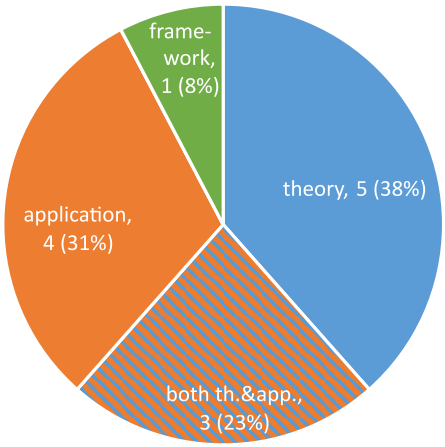


Figure 5.5: Share of different main topics in PhD and diploma theses on co-simulation.

Standards and surveys are not represented, one PhD thesis focuses on the development of a framework and the majority is almost evenly shared amongst theory and applications.

### 5.3.1 Theoretical subcharacterization of the literature

Although only 95 of the 139 publications exhibit a mainly theoretical or shared applied and theoretical focus, 122 (88%) consider at least one theoretical aspect. This share has not changed much over the years (see Table 5.1), although in the time frame from 2000 to 2004, which also exhibits an extraordinarily high share of purely theoretical papers (cf. Figure 5.4), all of the considered papers (24) comprise a theoretical part.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Table 5.1: Amount of publications covering at least one theoretical aspect over the years.

| | <2000 | 2000-2004 | 2005-2009 | 2010-2014 | 2015-2019 |
|---|---|---|---|---|---|
| theoretical aspect | 88% | 100% | 85% | 87% | 83% |

We further distinguish the following subcategories of theoretical aspects: error estimates (“error”), stability properties (“stability”), “coupling methods”, “performance”, “debugging”, “formalism” and “classification”. Every publication (of the above mentioned 122) can be assigned one or more of these categories. Figure 5.6 depicts the number of works in which the respective aspects are considered.

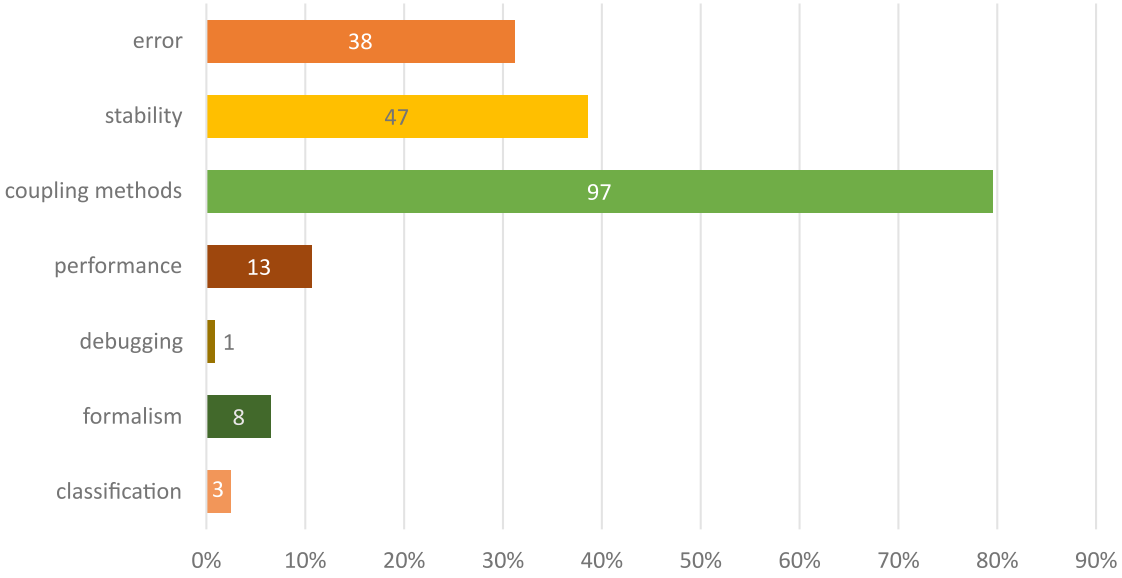


Figure 5.6: Share of publications considering specific theoretical subcategories. Absolute numbers are given in the bars, percentages are found on the horizontal axis.

Of all papers with theoretical aspect, a vast majority of eighty percent is investigating coupling methods. Of these, over one third (36%) is dealing with the stability of these methods and 28% with error estimation (cf. Table 5.2).

Table 5.2: Publications on coupling methods combined with further theoretical categories.

| | coupling methods and | | | |
|---|---|---|---|---|
| | error | stability | performance | no further category |
| number of publications | 35 | 27 | 11 | 40 |
| % of "coupling methods" | 36% | 28% | 11% | 41% |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Note that these are not exclusive: again about one third thereof (12% of all that study coupling methods) considers both investigations on stability and error estimates for the regarded coupling methods. About one tenth is analyzing performance properties of these methods. Forty papers introducing coupling methods (40% of the latter and approximately one third of total papers with theoretical aspect, respectively) are not explicitly covering any other theoretical subcategory, which is interesting as these seem to introduce or compare methods without explicitly considering potential threats to stability or accuracy. However, over half of these have not assigned “theory" as their main focus (cf. Section 5.3) but may simply apply methods or introduce a framework.

Taking a look at the overall parts of theoretical subcategories again in Figure 5.6, “stability” and “error” are the next most commonly investigated aspects with 39 and 31 percent, respectively. Within those, 19 consider both, which amounts to 50% of all papers investigating error estimates and 40% of those analyzing stability properties. 8 publications (11%) describe formalisms. These have mainly been published in the last five years, as can be seen in Figure 5.7. All three papers presenting classifications have also only come out in this time frame (more precisely: from 2017 to 2019). These circumstances again relate to the increase of research and the variety of methods in co-simulation over the last two decades. In contrast, the share of publications including investigations on stability properties and error estimates, which had its peak in the time frame from 2005 to 2009 (at the expense of coupling methods), has dwindled in the last years. Performance is represented with a quite steady share around ten percent (two to four papers) throughout all intervals.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

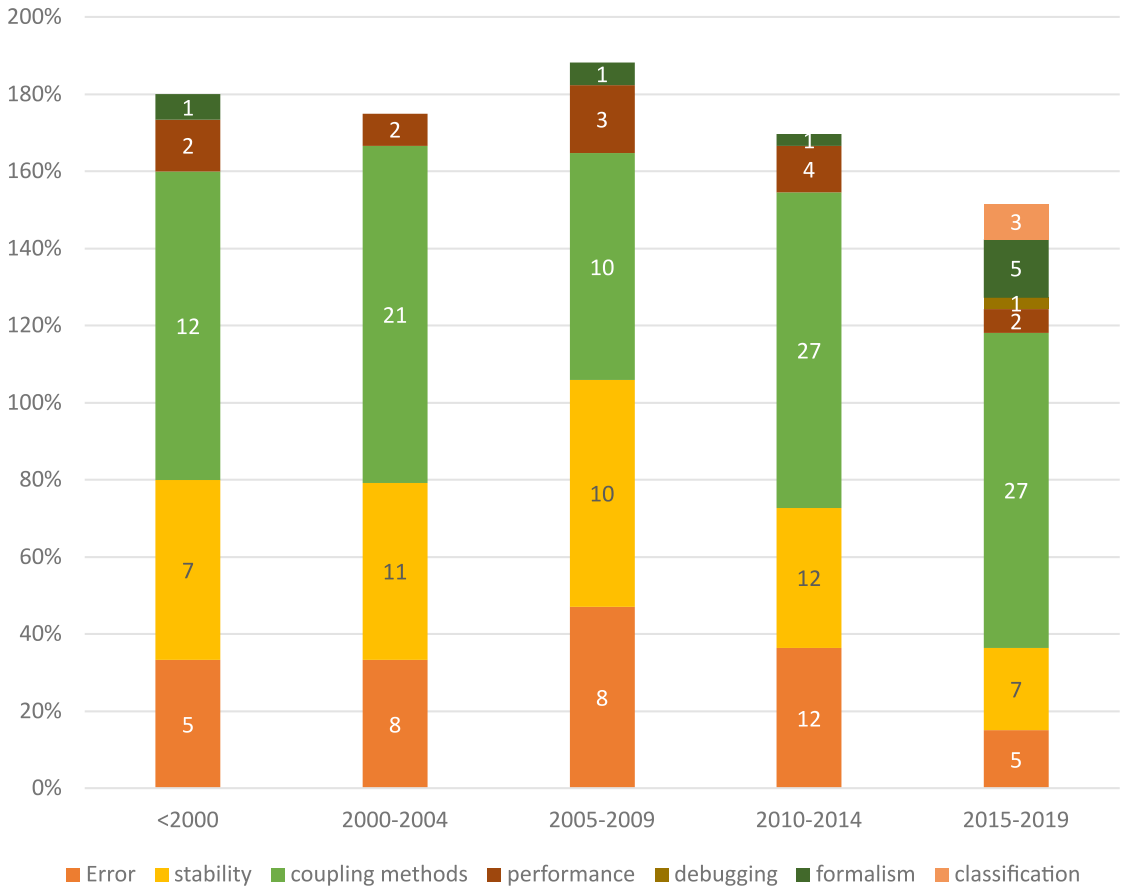


Figure 5.7: Appearance of theoretical subcategories over the years, given in numbers and percentages, respectively. As the categories are not exclusive, percentages may exceed 100%.

## 5.4 Distinction by the state of development

Research on co-simulation methods originates from needs at different states in the development of simulation models. On the one hand, facing a complex real system with partial systems differing to a great extent in their modeling requirements, these have to be approached with different techniques (or might already have been approached by experts in the respective fields). The resulting separate simulations, possibly implemented using individual tools, need to be coupled thereafter to sufficiently represent the whole system, which is illustrated in Figure 5.8.

On the other hand, complex systems within one physical domain (for example large mechanical or electrical systems) may be described by one mathematical model. However, this

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

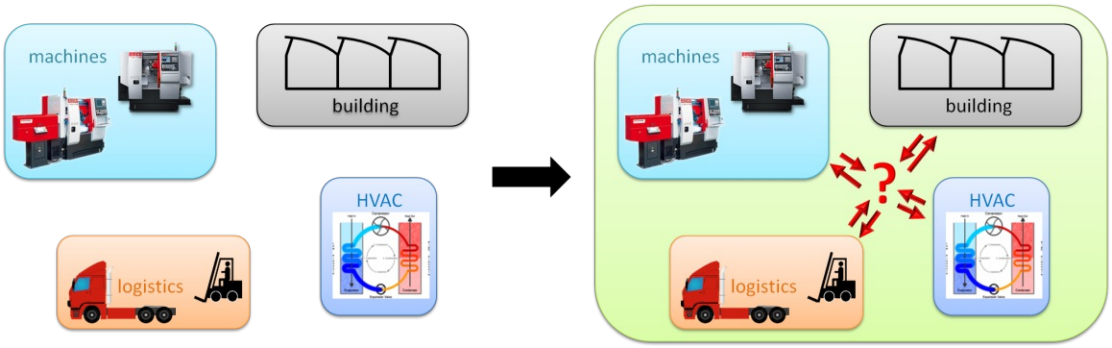


Figure 5.8: Illustration of an *integrate-and-collaborate* approach: coupling simulations of already existing model implementations (Hafner and Popper 2017).

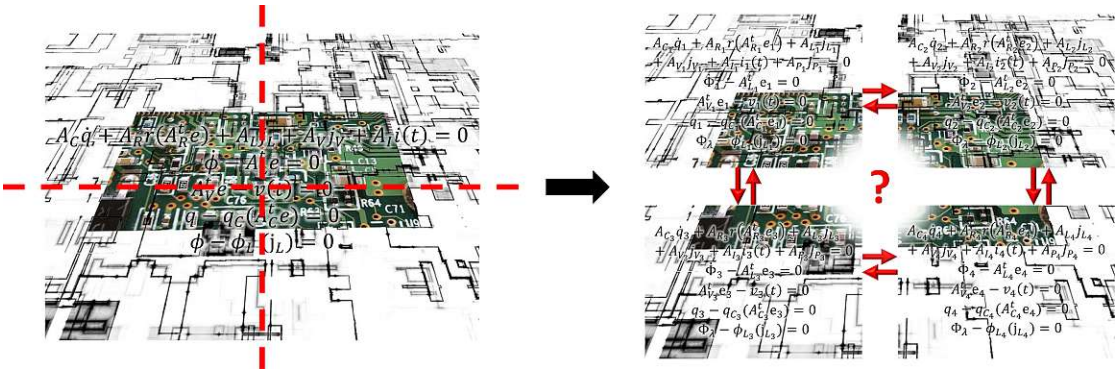

Figure 5.9: Illustration of the idea behind a *divide-and-conquer* method: separation of models due to overall complexity or differing time constants in system parts (after (Hafner and Popper 2017)).

model consists of many equations which may be solved more efficiently by parallelization. The equation system might also consist of more and less active parts which encourages separation depending on stiffness. Again, the separated partial models will be simulated individually and coupled again in an overall simulation, but in this case, the need for separation arises from consideration of the system on the (mathematical) model level instead of the real system, see Figure 5.9. Of course, there are overlaps – even with systems of one domain, predetermined breaking points of the system (for example joints in a mechanical systems) may also be those where the separation in the equations would commonly take place. However, this distinction is essential to understand two of the main sources of development in this area: the need to combine individually implemented models ready to be simulated by themselves versus the need to separate complex models in order to be able to simulate the whole system at all. A characterization of these two approaches can

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

be found in (Tseng 2000), where solutions to couple already distributed submodels are described as *integrate-and-collaborate* algorithms and for the partitioning of complex systems into subsystems, the term *divide-and-conquer algorithm* is introduced.

### 5.4.1 Analysis of literature by the state of development

Whether a decomposition or collaborative coupling approach is used can be defined for 121 of the regarded publications. 60% of these are approaching their task with an integrate-and-collaborate strategy, 38% use divide-and-conquer methods and 2% utilize or compare both (abbreviated “d&c and i&c”), see Figure 5.10.

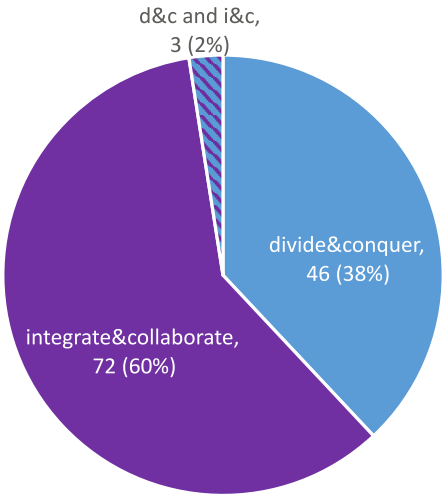


Figure 5.10: Publications categorized by the state of development in which the (de-)coupling is considered.

Over time, the share of works on integrate-and-collaborate approaches increases noticeably, from 25% to 80%, as the illustration by five-year time frames in Figure 5.11 demonstrates. This consorts with the increase of surveys, standard and framework descriptions in the literature (cf. Section 5.3). These, as Figure 5.12 shows, exclusively represent an integrate-and-collaborate point of view. The illustration of this cross-connection further reveals that publications mainly presenting an application mostly address an integrate-and-collaborate problem, while for theoretical papers, shares of divide-and-conquer and integrate-and-collaborate approaches are quite balanced.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

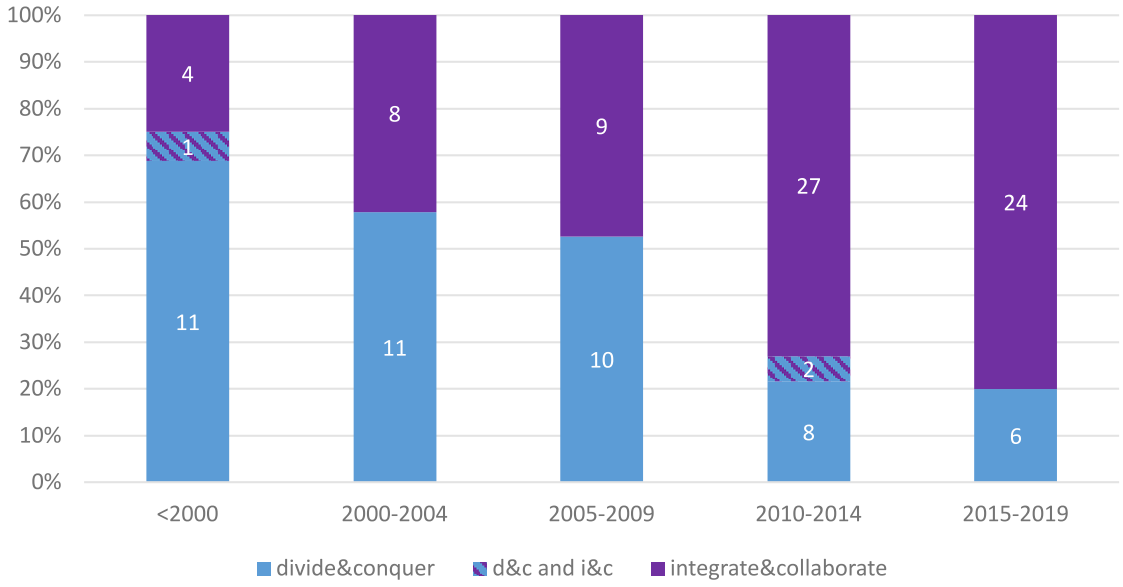


Figure 5.11: Share (resp. number) of integrate-and-collaborate and divide-and-conquer approaches in the literature per five-year interval.

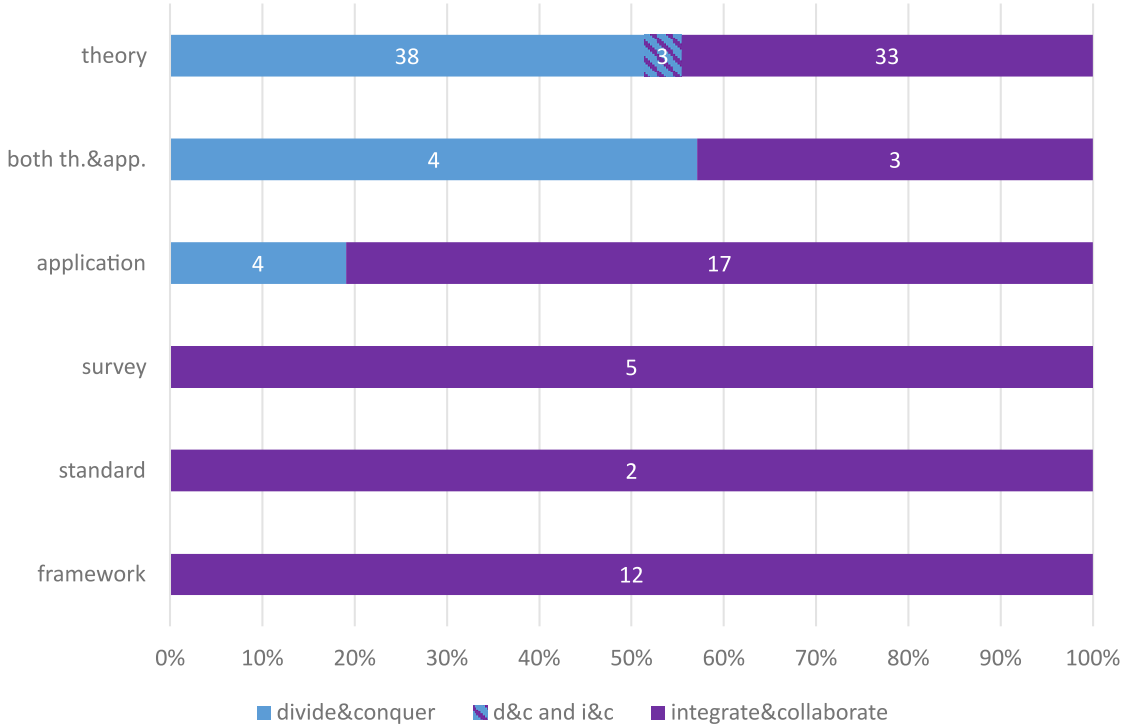


Figure 5.12: Decomposition of coupling point of view per main topic. The shares of divide-and-conquer and integrate-and-collaborative approaches are given separately for publications assigned to a specific main topic.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

## 5.5 Distinction by field of application

The need for co-simulation arises in various fields of application. These fields are closely related to the kind of coupling methods the systems are approached with and lead to varying developments, so it seems natural to classify co-simulation approaches by the areas they are applied to. Due to their variety and complexity, a complete list of applications can and will not be established in this work. However, a few examples of some of the most common applications are listed below:

- physical systems
  - mechanical systems
  - electrical systems
  - hydraulic systems
  - thermal systems
  - etc.
- systems which require coupling of different domains
  - HVAC systems
  - vehicle dynamics
  - fluid-structure interaction
  - electro-thermal circuits
  - mechanical-electrical systems
  - field-circuit models
  - etc.

### 5.5.1 Applications in literature

Let it be noted that many theory-based papers also apply their method or the like to at least one benchmark example. The share of these, as outlined in Table 5.3, has increased slightly up to 87% in the time frame from 2010 to 2015, only to drop again in the last five years. This reflects the concurrent upswing in surveys, frameworks, formalisms, and classifications.
Overall, 34 of the 139 papers considered in the classification do not include any application, the rest can be broken down as shown in Figure 5.13. We can observe that the considered applications are mostly physical and almost all cover at least a physical aspect (or include

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Table 5.3: Share of total publications with a defined application, progress over time.

| | <2000 | 2000-2004 | 2005-2009 | 2010-2014 | 2015-2019 |
|---|---|---|---|---|---|
| publ. with application | 65% | 79% | 80% | 87% | 65% |

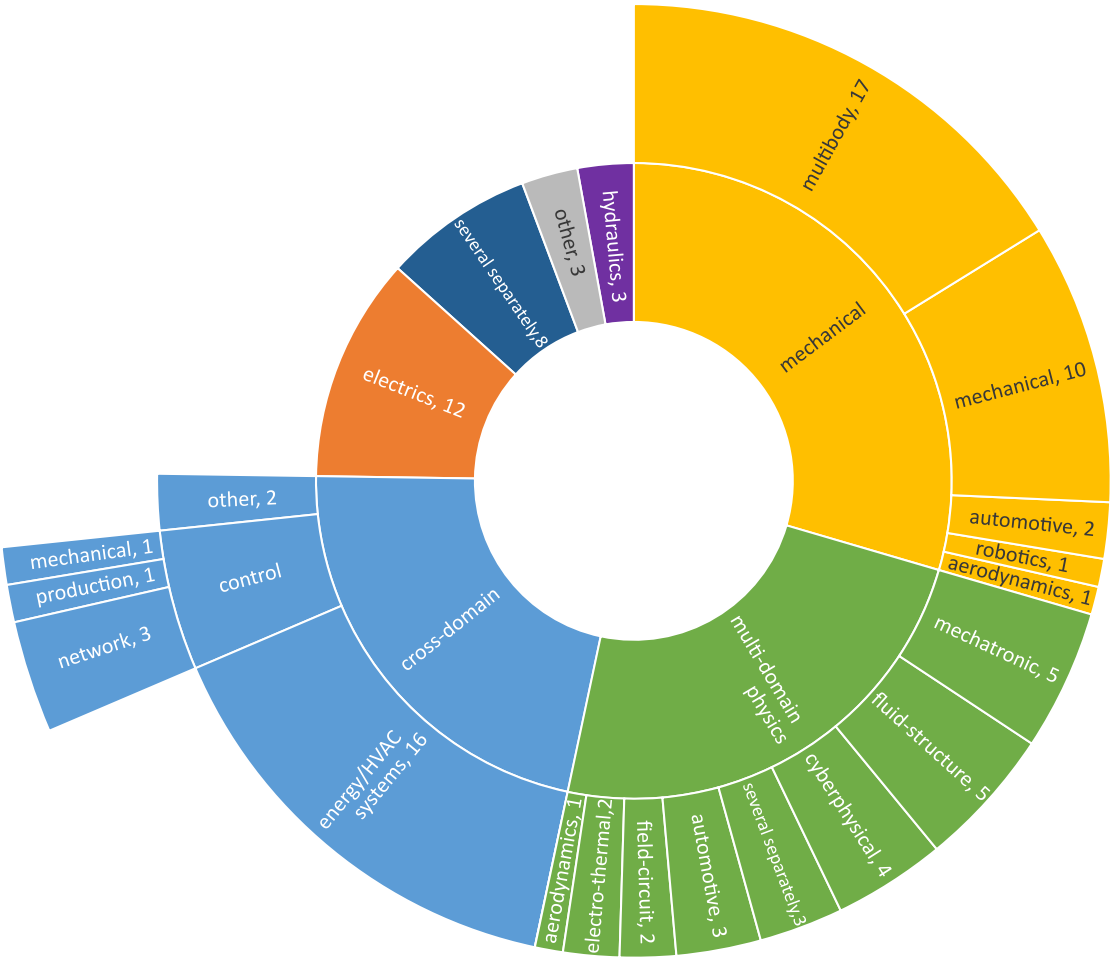


Figure 5.13: Breakdown of application areas found in the selected literature.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

a physical partial system). Applications in mechanics constitute the largest share with 30%. Within these, multibody simulations are most common, followed by general mechanical applications. 24% of all papers where an application is found combine two or more physical domains (“multi-domain physics”) in various manifestations.
While applications defined as “cross-domain” (amounting to 22% of the total of 105 with any application) often also involve multiple physical domains, some non-physical model part is also included – e.g. control strategies or logistics, which are often needed for networked controlled systems or in the simulation of production facilities.
Summarized as “other” are specific isolated applications such as molecular dynamics or a particular hardware in the loop (HIL) implementation. The lower-level classification “several separately” describes publications where a concept is applied to several examples in different areas, f.i. an electrical circuit and a mechanical system, but separately, not combining these domains. The sub-category of “multi-domain physics" that is also named “several separately” likewise comprises several applications in one work but in this case, every separate example couples different domains, such as field-circuit interaction as well as thermal-electromagnetic systems.

The progress of the basic shares over the years (Figure 5.14) reveals that while purely electrical applications have vanished in the last years, cross-domain applications have considerably increased. For mechanical and multi-domain physical problems, the shares go up and down without a distinguishable pattern.

### 5.5.2 Distinction of mechanical systems by manner of separation

As explained in Section 5.4, the need for co-simulation can on the one hand arise from requiring a holistic simulation of already existing solutions of partial models (“integrate-and-collaborate”) or the decomposition of complexly built systems (“divide and conquer"), f.i. due to differing stiffness properties in system parts. For mechanical systems, Schweizer and Lu (2014b) provide a basic classification of coupling approaches:

- coupling by physical force-/torque-laws
- coupling by algebraic constraint equations

Schweizer and Lu explain coupling by physical force-/torque-laws as coupling by constitutive laws or coupling by applied forces/torques while Busch (2012)) talks of *applied force coupling* or *coupling via applied cutting forces*. Coupling by algebraic constraint equations means coupling by reaction forces/torques according to Schweizer and Lu, called *constraint*

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

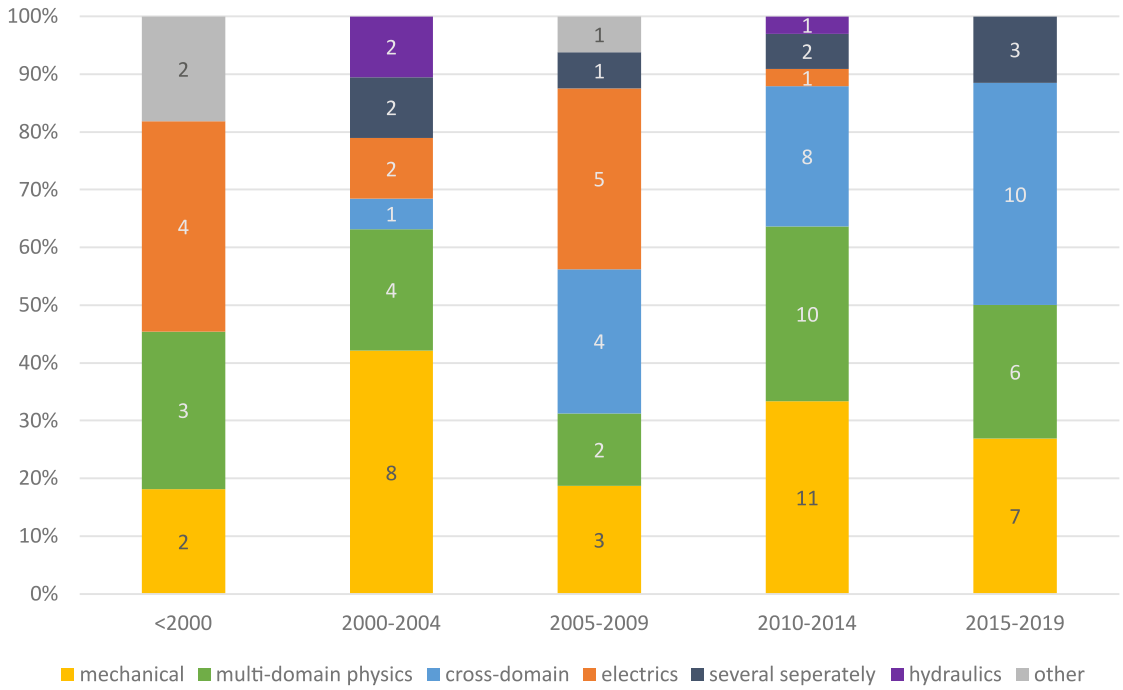


Figure 5.14: Share and numbers of application areas in the regarded literature per five-year time frame.

*coupling* or *coupling via constraint forces* (i.e. Lagrange multipliers) by Busch.
In case of applied force coupling (according to Busch), the overall system can be partitioned in the following ways (see also Schweizer et al. (2015b)):

- force-force coupling
- force-displacement coupling
- displacement-displacement coupling

The kind of coupling defines whether force or displacement variables are exchanged as coupling variables, consequently leading to different coupling equations respectively. For a detailed explanation see Section 2.9 or Schweizer and Lu (2014a), who also provide a comparison of simulation results for these approaches.

## 5.6 Distinction by model description

Different mathematical model descriptions also require individual solution algorithms and further coupling approaches, which leads to the following distinction:

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

Models described by

- agent based approaches
- system dynamics
- equation systems in continuous time domain
  - ordinary differential equation systems
  - differential-algebraic equation systems (DAEs)
    - index-1 DAEs
    - high-index DAEs
  - partial differential (algebraic) equation systems
- discrete event systems
- finite elements, boundary elements
- synchronous data flow
- etc.

Of course, co-simulation often combines more than one of these model descriptions and therefore requires approaches which are customized specifically for the exchange between these types of descriptions. Examples for those would for instance be certain differential-algebraic equation systems where the coupling can take place by the constraint equations (as one solution possibility, cf. Section 5.5.2) or hybrid systems where the continuous integrator may be interrupted every time the discrete part triggers an event or, as another or a combined approach, every state event occurring due to the continuous part is scheduled as an event and synchronization reference in the discrete part (Quaglia et al. 2012,Fitzgerald et al. 2014).

Undoubtedly, this list is not nearly complete, especially regarding the possibilities for the combination of several model descriptions. Nevertheless, this section is intended to emphasize the complexity of approaches as well as the impossibility of the establishment of a “holy grail”, a co-simulation method suitable to approach each and every composed simulation problem, which cannot even be found for problems with a common model description[8].

[8]See for example Arnold (2010) who demonstrates that by employing the same co-simulation method (Gauß-Seidl type loose coupling, cf. Section 5.7) on the same system, instabilities can occur if only the sequence of calculation of the subsystems is varied.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 5.6.1 Model descriptions in the considered literature

In the following analysis, the two publications describing standards have been excepted since these are not restricted to one or few types of model descriptions. In particular, the description of the HLA is so general that in theory, no restrictions on model description or even requirement of implementation is made (i.e., human interaction and HIL are also possible), while for the FMI, which focuses mostly on ODE and DAE models due to its origins, a restriction could be made to those model descriptions for which simulators supporting the FMI currently exist, which seems an unequal evaluation.

In 129 of the remaining papers, one or more considered model descriptions out of *ordinary differential equations* (“ODE”), *differential algebraic equations* (“DAE”), *partial differential equations* (“PDE”), *agent based models* (“ABM”), *system dynamics* (“SD”), *synchronous data flow* (“SDF”), *finite element models* (“FEM”), *boundary element models* (“BEM”) and *discrete event systems* (“DE”) can be identified. ODEs and DAEs are by far the most frequently considered descriptions, as depicted in Figure 5.15. Over half of these publications (70) cover more than one model description. Thereof, 23 are exclusively dealing with ODEs and DAEs.

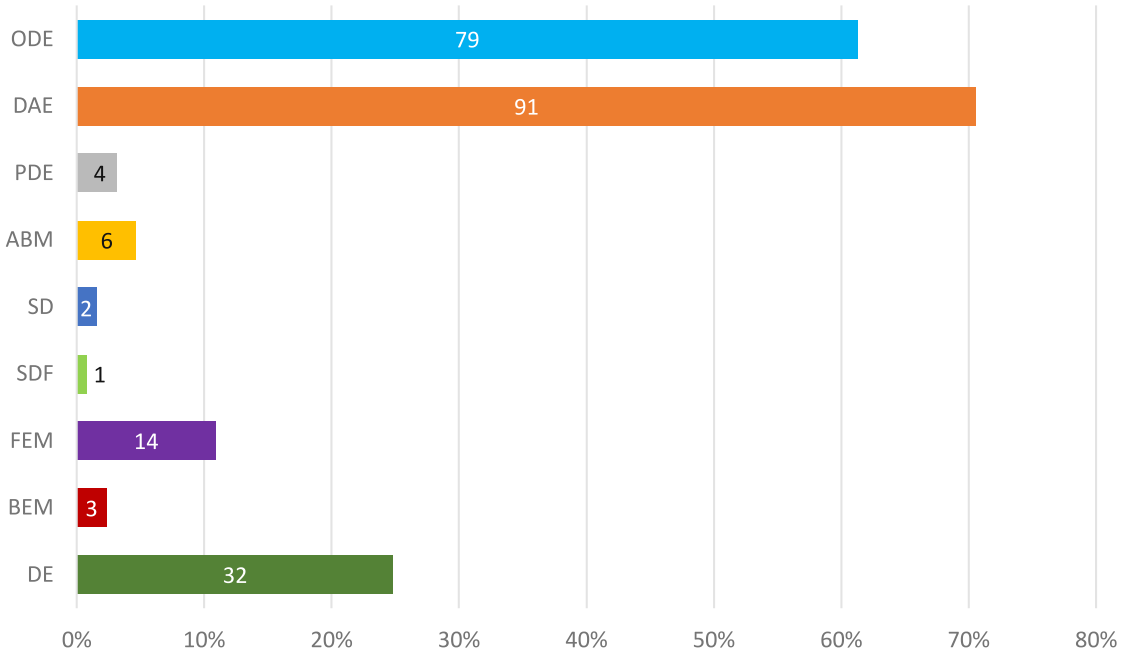


Figure 5.15: Amount of publications utilizing different kinds of model description.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

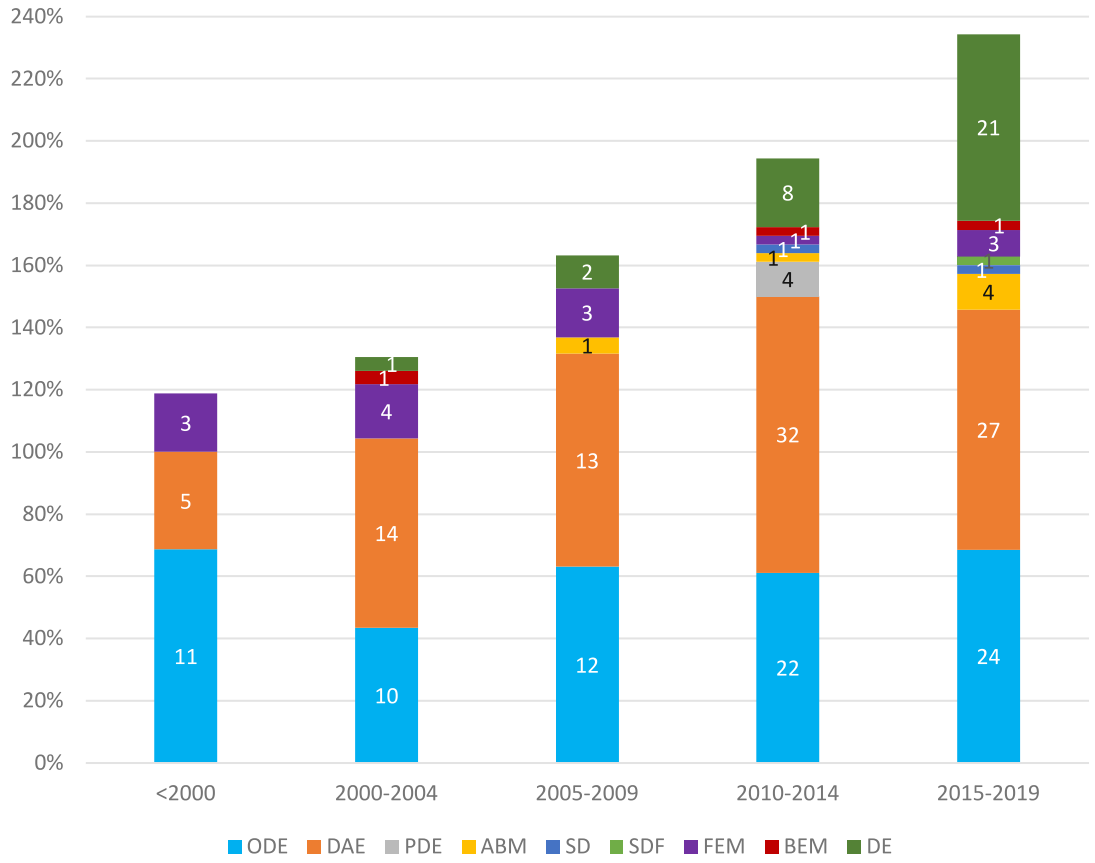


Figure 5.16: Change in percentages of model description categories in the selected literature on co-simulation over time.

While ODEs have always been considered in similar measure throughout the “history” told by this selection in research on co-simulation, DAEs have become a more popular topic of interest after the millennium, as illustrated in Figure 5.16.

33 papers explicitly cover hybrid co-simulation – in the sense of coupling continuous time models with discrete event systems[9]. Research in this area has increased drastically over time, as Figure 5.17 shows: 21 of these publications (64%) have been published after 2014. This – non-surprisingly – correlates with the consideration of discrete event systems, which has also become more frequent in recent years. Further, it has been noted that only five publications out of all regarded ones consider hardware in the loop (HIL) and two human interaction.

[9]To be precise, the coupling of CT with DE systems and also systems in which state events triggered by continuous state changes are handled. This allows for a slight discrepancy between the number of publications on hybrid systems and those explicitly including Discrete Event systems. See also Section 2.3.4.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

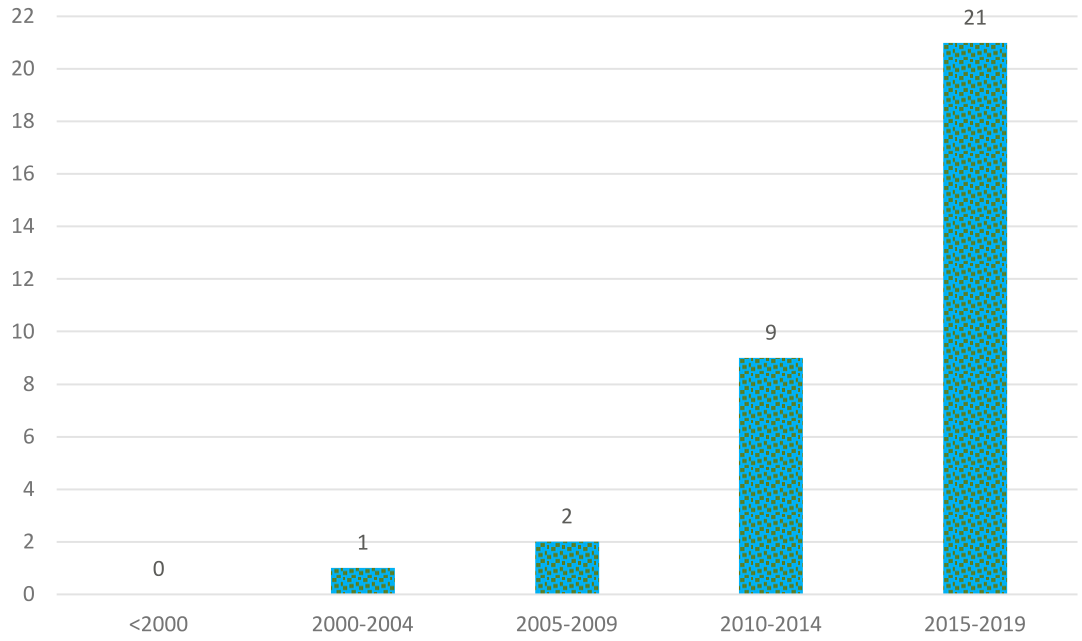


Figure 5.17: Publications on hybrid co-simulation per five-year time frame.

## 5.7 Distinction of algorithms

One major point to be considered when categorizing co-simulation methods are the numerical approaches that come with the nature of the topic. In that respect, the focus can lie on the solution algorithms used by the participating subsystems or the coupling algorithm itself. Prior to the specific algorithms, however, different concepts to approach the intended cooperation can be discerned.

### 5.7.1 Distinction by coordination concept

Regarding the general concept of coupling coordination, the first distinction we can make is whether or not an external orchestrator is used to organize the co-simulation.

In the present consideration, co-simulations where communication is orchestrated *outside* one of the participating subsystems are classified as using an orchestrator. Many co-simulation frameworks (the BCVTB (Wetter 2011), for instance) act as middleware – a software designed to orchestrate time synchronization and data exchange between participating sub-simulations and as such, their software. Thus, these naturally serve as orchestrator. However, an orchestrator does not necessarily require specific software but can even be

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

implemented in a co-simulation within the same simulator if, for instance, the focus lies on different time scales (see e.g. the benchmark example in Section 6.2.3).

Without an external orchestrator, data exchange can be handled in one of the subsystems themselves, thus acting as the so-called *master* of one or more other subsystems (*minions*). Note that there are overlaps: If a master or orchestrator coordinates several subsystems while solving an equation, it may be considered as both. In the FMI for co-simulation, the “master" is also the one orchestrating all other FMUs, without necessarily solving any system part itself, so in this case it depends on the respective implementation.

This brings us to the next distinction: Depending on whether or not equations are solved by the coupling algorithm, we differ between orchestrators that only define interfaces and manage input-output connections between the subsystems (as in (Hafner et al. 2014)) and those actively solving equations by employing numeric algorithms, f.i. to enforce fulfillment of constraint equations (Gu et al. 2000). A master may also be interfacing for the controlled minions. Nguyen et al. (2017) even describe the usage of a master algorithm as “simulation with an orchestrator” vs. “ad-hoc co-simulation” via interfaces without a coordinator.

#### 5.7.1.1 Orchestrator usage in the literature

Whether or not an external orchestrator is used is defined for 103 papers. Figure 5.18 illustrates that a majority (81%) of publications present approaches using an orchestrator. Only 18 publications (17%) do not use an orchestrator.

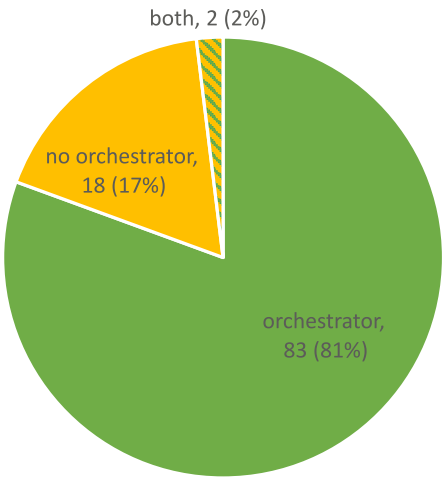


Figure 5.18: Share resp. number of publications using an external orchestrator.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

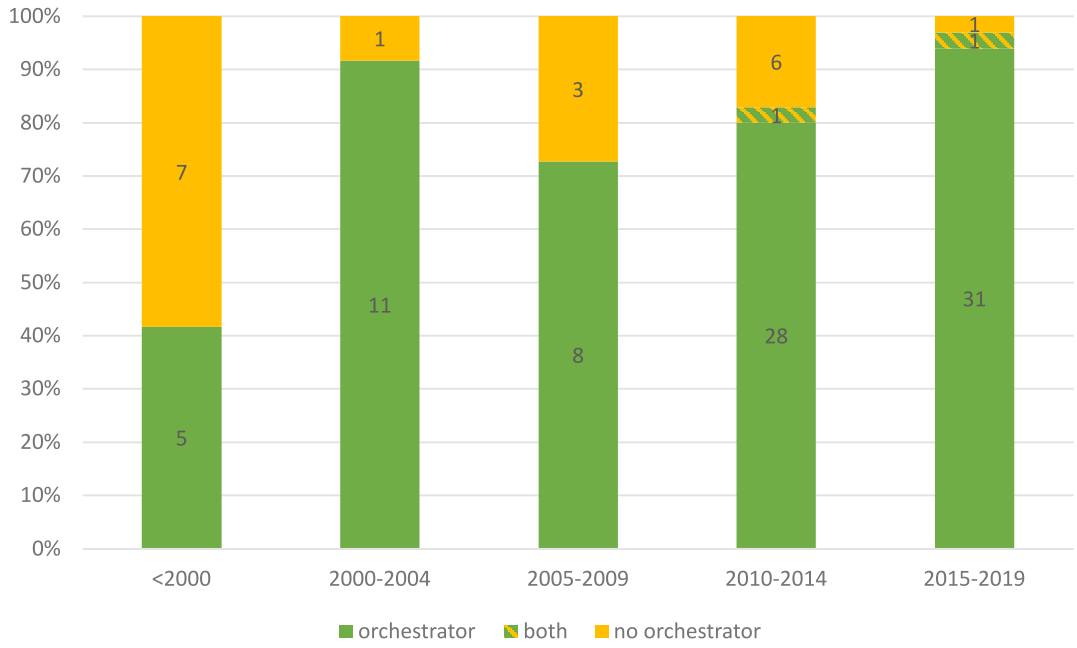


Figure 5.19: Histogram of the share and number of publications that describe the usage of an orchestrator, decidedly do not use an orchestrator, or compare both.

While this trend is also observable in most time frames (Figure 5.19), publications before the turn of the millennium stand out: within these, more than half do not use an external orchestrator.

### 5.7.2 Distinction by interfaces

Although slightly different interpretations for these terms are circulating the literature (cf. Section 2.5), we adopt the term *distinction on "interface level"* from Busch (2012) for the division of coupling approaches into

- *strong coupling* and
- *loose coupling*

methods. Following the definition in their introduction in Section 2.5, *strong coupling* allows different solvers but requires the same time steps in all subsystems, permanent exchange of coupling data and iteration in every time step while *weak* or *loose coupling* allows different, individual time steps in the partial systems. In general, strong coupling is applied when high accuracy results are desired but the partial simulations require different modeling and/or

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

simulation approaches. Loose coupling is suitable to speed up the otherwise computationally expensive simulation of complex systems, especially if the partial *systems* are loosely coupled (see (2.1)) so splitting errors are comparably negligible.

#### 5.7.2.1 Loose and strong coupling approaches in the literature

Whether loose or strong coupling methods are applied can be determined for 115 of the 139 publications. All but one publication can be classified in this respect in the time frame from 2000 to 2004, yet only 70% in the last five years (see Table 5.4), given the high share of publications which describe frameworks, surveys or formalisms.

Table 5.4: Share of publications revealing whether a loose or strong coupling approach is considered.

| | <2000 | 2000-2004 | 2005-2009 | 2010-2014 | 2015-2019 |
|---|---|---|---|---|---|
| loose/strong defined | 82% | 96% | 85% | 87% | 70% |

Considering the total of 115, these can be divided according to the coupling strength as depicted in Figure 5.20.

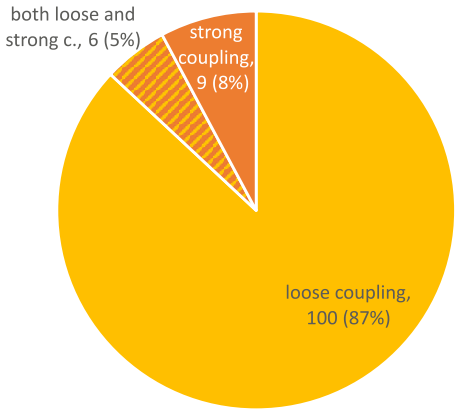


Figure 5.20: Quantity of publications on loose and strong coupling approaches in the considered literature.

An overwhelming majority of 87% focuses solely on loose coupling algorithms, nine papers (8%) consider only strong coupling methods and five percent cover both approaches. This general observation of a loose coupling dominance is also reflected in the histogram in Figure 5.21, although strong coupling slightly gains in popularity up to 2009, only to drop again afterwards.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

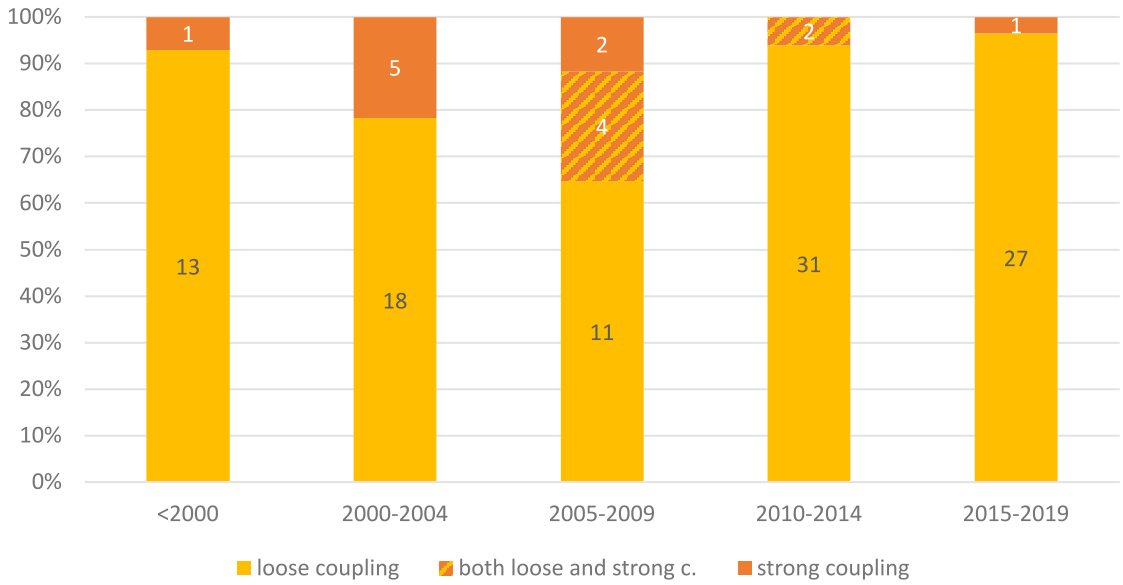


Figure 5.21: Share and number of publications on loose and strong coupling approaches over the years.

### 5.7.3 Distinction of coupling algorithms

Busch (2012) structures numerical coupling algorithms as follows:

- explicit coupling approaches
  - Jacobi type
  - Gauß-Seidl type
- semi-implicit coupling approaches
  - stabilized extrapolation
  - bdf-multirate
- implicit coupling approaches
  - waveform iteration
  - nonlinear projection

While this classification certainly has its justification and is therefore quoted here, I would nevertheless like to take a different turn with a distinction by several individual aspects, since iterative approaches also start with a sequential or parallel step which is iterated thereafter.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

Consequently, we distinguish coupling algorithms by

- sequence
- iterations
- macro step

in the following sections.

#### 5.7.3.1 Distinction by execution sequence

An important characterizing property of master algorithms is the sequence in which participating subsimulations are executed. Thereby, we differ between

- *parallel methods* and
- *sequential methods*.

Most commonly, parallel (also called *Jacobi* type, cf. Section 2.7) methods are applied. This does not necessarily require de facto computational parallelization on multiple cores, but means that every time data is exchanged, simulation time is the same in every subsystem, so no part can obtain future information on any other system. In sequential (*Gauß-Seidl* type) methods, values at a (in general, one) reference ahead can be used for more accurate approximations, f.i. by interpolation, in “slower” (in the sense of simulation time, i.e. later executed) subsystems. A detailed description of these approaches is given below and in Section 2.7.

**Gauß-Seidl type approach.** When systems are coupled with a Gauß-Seidl type approach, one of the systems is first to be simulated for one macro step, using its individual time step in between. Values required from other subsystems are extrapolated by using values at past synchronization references. After this system has finished its calculations for that macro step, a second subsystem is simulated for the same macro time step. For variables from the first system, interpolated values can now be used since the values for those variables have already been calculated at the end of the current macro step. Values from other subsystems have to be extrapolated. This procedure is repeated for all further partial systems. The stability of Gauß-Seidl type methods is highly dependent on the sequence of execution of the participating subsystems, see for example (Arnold 2010).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Note that apart from the classical Gauß-Seidl approach described above, where subsystems exchange data at a given macro time step, there exist sequential approaches that do not require the participating subsystem solvers to employ a common step at all, see Section 5.7.3.3.

**Jacobi type approach.** A Jacobi type algorithm allows all systems to be simulated in parallel for every macro time step using only extrapolated values from the respective other subsystems. This increases computational speed but also error accumulation and stability issues in comparison to the sequential Gauß-Seidl method.

**Sequence of computation in the literature**

The sequence of execution is defined in 103 of the considered publications. Looking at the progress of this share over time, it stands out that while in the time frame from 2010 to 2014 only three papers (8% of the total quantity of considered ones in this year) do not reveal the sequence of computation, this share amounts to 40% in the last five years – again, partly due to the high share of surveys as well as framework and formalism descriptions, which mostly do not limit themselves in this subject.

Over half (55%) of the 103 papers only investigate or apply parallel methods, almost a quarter exclusively focus on sequential ones and the rest discuss both parallel and sequential approaches, as depicted in Figure 5.22.

The share of (purely) parallel methods gradually increases over time, peaking at 70% in the last five years (see Figure 5.23). This might partly be explained by simultaneous progress in computational parallelization, which can be utilized to speed up coupled simulations, and partly by reasons of implementational simplicity.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

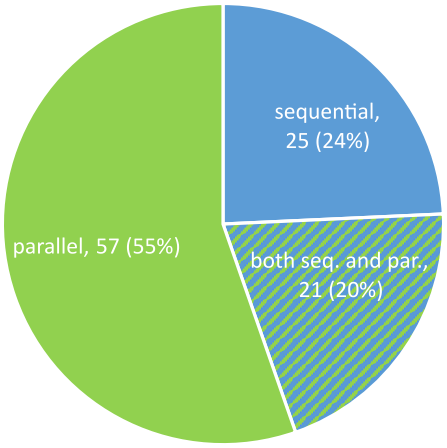


Figure 5.22: Division by consideration of parallel and sequential approaches in the selected literature.

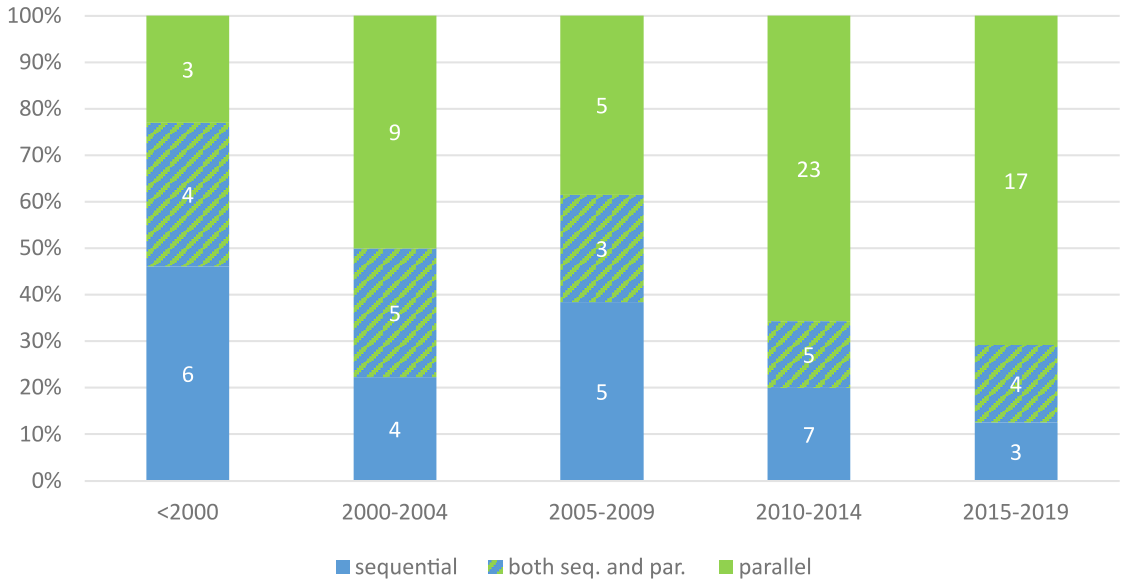


Figure 5.23: Share and number of parallel and sequential approaches in the selected literature per five-year time frame.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Figure 5.24 illustrates the correlation between the sequence of subsystem execution and the perspective concerning the state of development in which the system is partitioned and/or coupled. It is interesting to note that all three publications in which both an integrate-and-collaborate and a divide-and-conquer approach are considered also investigate both parallel and sequential methods. Publications on sequential algorithms show the highest share of those approaching their research question by a divide-and-conquer perspective.

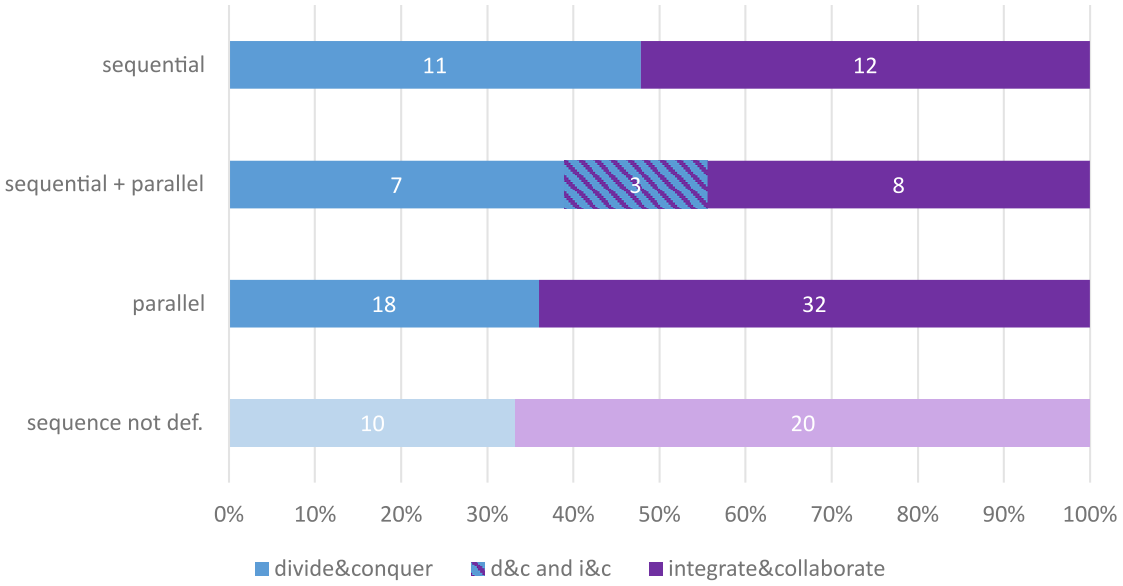


Figure 5.24: Nexus of sequence of execution and perspective of (de-)coupling.

Neither the sequence of computation nor whether the problem is approached from a integrate-and-collaborate or divide-and-conquer perspective can be made out for six papers. Twelve define the sequence but not the state of development in which the (de-)coupling takes place. The thirty papers with unknown sequence but defined perspective of (de-coupling) are included in the statistics, see the lighter colored bar in Figure 5.24.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

#### 5.7.3.2 Distinction by iterations

Depending on whether or not the coupling algorithm iterates over its macro steps, we differ between

- non-iterative approaches
  - including a predictor-corrector step
  - without any rollback
- iterative approaches
  - repetition of a Jacobi or Gauß-Seidl macro step
  - including a predictor-corrector step

Non-iterative coupling algorithms are also called *explicit* in the literature, while iterative ones are referred to as *implicit*. For methods which perform a predictor-corrector step (see A.2) or, a little more generally put, allow step rejection and thus require rollback even if no iteration as such takes place, the term *semi-implicit* has been introduced.

The waveform iteration method starts with the execution of one parallel or sequential macro step which is then repeated until a given tolerance is reached and the algorithm proceeds to the next macro step. In every repetition, interpolated values for variables from other subsystems can be used since all partial systems have already been calculated for this macro step.

By nature, non-iterative approaches are on the one hand faster than implicit ones but may also be unable to successfully handle instabilities (occurring for example due to feedback loops and subsequently possible algebraic loops when coupling differential-algebraic equation systems), as can be observed throughout the literature (see f.i. (Arnold and Günther 2001)).

Implicit methods are able to tackle these issues to some extent: waveform iteration allows simulations to remain stable which would fail with an explicit Jacobi or Gauß-Seidl based approach, albeit with the drawback of high calculation costs and particular requirements on participating simulators (such as rollback) which are not inherently fulfilled. Semi-implicit methods, while non-iterative, present better stability properties similar to implicit ones according to Busch (2012).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

**Publications divided by macro step iterations**

Whether the coupling algorithm uses iterations or not can be determined for 112 of the considered publications, whereat this rate has in general – with the exception of the years 2010 to 2014 – decreased over time, as Table 5.5 shows. The share of papers without clarification on iterations is particularly high in the time frame from 2005 to 2009 (30%) and from 2015 to 2019 (35%, which can partly be explained by the higher share of surveys and framework descriptions, see below), while it stays between six and eleven percent in all other intervals.

Table 5.5: Disclosure of implicit and explicit approaches in publications over time.

| | <2000 | 2000-2004 | 2005-2009 | 2010-2014 | 2015-2019 |
|---|---|---|---|---|---|
| iterations defined | 94% | 92% | 70% | 89% | 65% |

The distinction within all 112 papers where this property is defined is given in Figure 5.25, where the term “predictor-corrector” is abbreviated “p-c”. In two thirds (74), only non-iterative master algorithms are used. This emphasizes the importance of the development of explicit methods, as – even though implicit approaches surpass explicit ones in every comparison that I know with respect to accuracy and stability – a majority adheres to explicit methods, be it due to performance reasons or just for the sake of simplicity regarding implementation and software requirements. Eleven (15%) of the papers using non-iterative methods (amounting to 10% of all 112) apply a predictor-corrector method. A quarter (28) of the total are exclusively using iteration, of which one also applies a predictor-corrector method. Both iterative and non-iterative algorithms are considered in ten papers (8%), predictor-corrector steps in half of these.

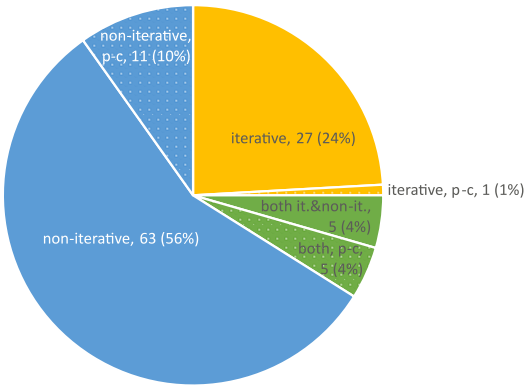


Figure 5.25: Overall partition of non-iterative and iterative co-simulation methods in the selected literature.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN


Figure 5.26: Variation of shares of iterative and non-iterative methods over the years.

Figure 5.26 illustrates that while the research on iterative methods has increased in the years before 2010 (up to around half, if work regarding both iterative and non-iterative research is included), it has diminished again in recent years. While this could be interpreted as preference of explicit methods in praxis (as 2000 to 2009 are also the years peaking in a focus on theory), this correlation is disproved by the cross-connection shown in Figure 5.27. One expectable observation to be made from there is that iterations are not defined in most surveys. Further, in most frameworks, non-iterative methods are applied or it is not defined whether or not iterations (can) take place.

Taking a look at the nexus of the sequence of execution and the iterations of master algorithm steps (Figure 5.28), it is interesting to note that the category of publications exclusively using parallel methods also shows the highest share of non-iterative master algorithms. This implies that those using methods that are more prone to stability issues also do not aim to increase accuracy by iteration. *Both* iterative and non-iterative methods are only considered in publications which also use sequential algorithms (exclusively or in a comparison to parallel ones).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

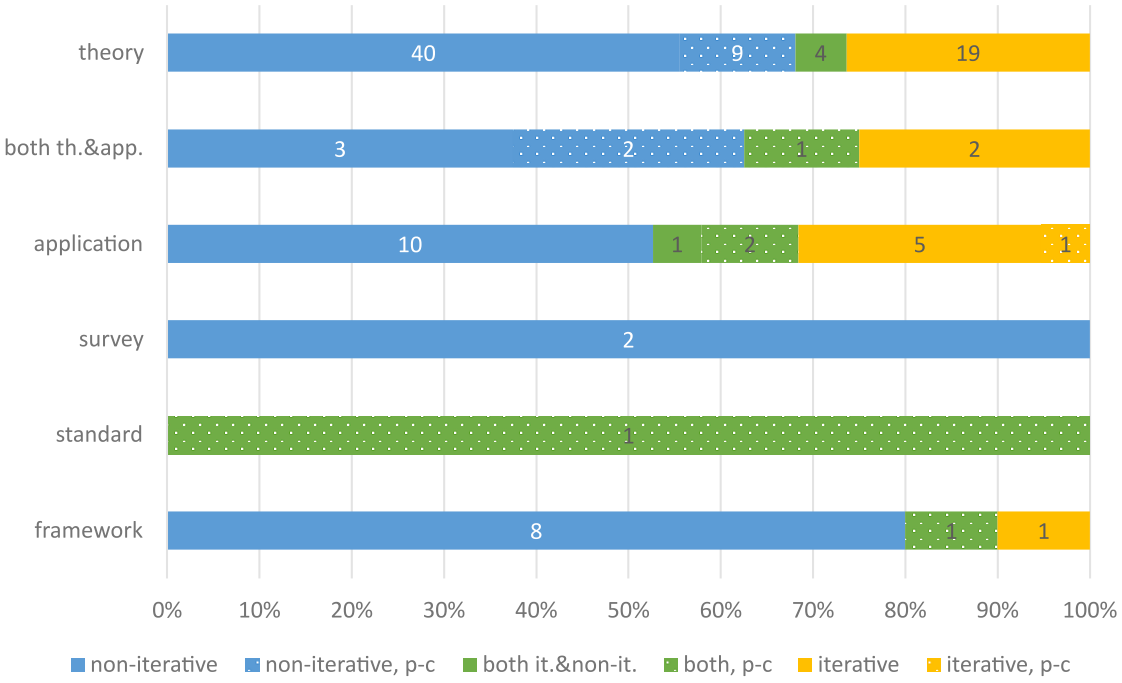


Figure 5.27: Connection between the main topic of publications and iterations in the master algorithm.

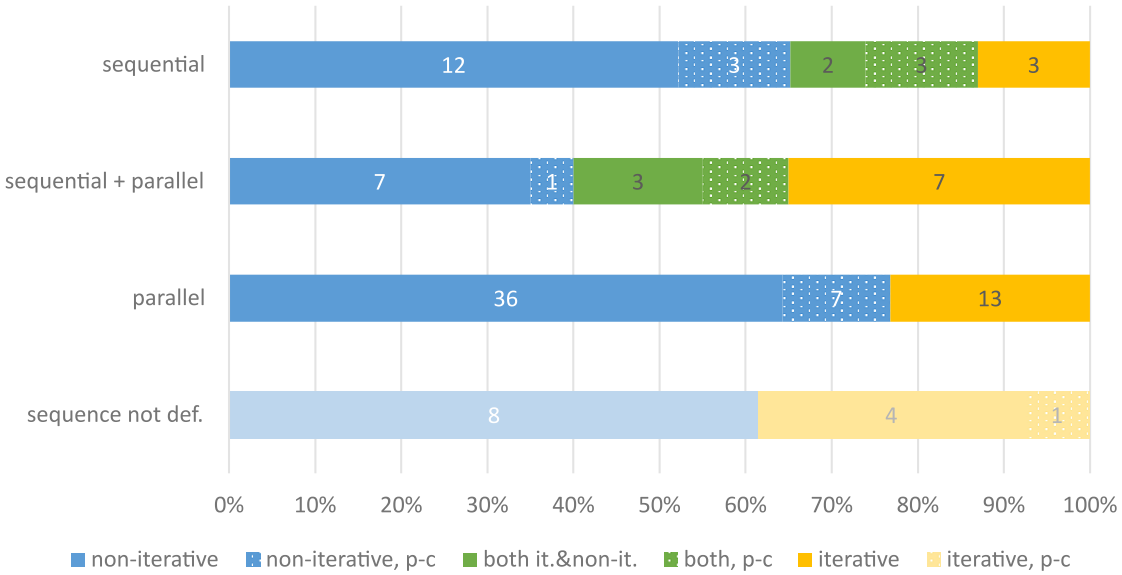


Figure 5.28: Nexus of sequence of execution and iterations of the master algorithm.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

There are 23 publications where neither sequence nor iterations are defined and four that define the sequence but do not reveal whether an iterative master algorithm is used.

#### 5.7.3.3 Distinction by macro steps

Regarding the macro step of the master algorithm, we can distinguish

- fixed macro step
- adaptive macro step and
- no common macro step.

Algorithms with a fixed macro step require all participating simulations to exchange data at previously defined synchronization references in – generally – equidistant intervals, no matter whether the subsystem solver algorithms would originally set a step at this point in time.

Other solutions adapt the macro step in the course of a simulation via step size control, either by step rejection and repetition with a smaller step size if certain tolerances are violated (see f.i. (Benedikt et al. 2010)) or by adaption before every macro step execution by predictive error estimation, f.i. via extrapolation of known values of state variables (Zhang et al. 2011) or calculation of energy residuals as in (Sadjina et al. 2017).

Some loose coupling methods do not necessarily require any synchronized time steps from the sub-simulations apart from the start time, see f.i. the approaches presented by González et al. (2011) and Liang et al. (2011). The latter is an explicit sequential one where after every micro step, the simulation time in all participating simulations is compared and the system with the smallest simulation time is allowed to execute its next step. Values from other systems are extra- or interpolated, depending on the current simulation time in the respective other systems. According to Liang et al. (2011), this approach yields better results than classical Jacobi or Gauß-Seidl type methods. An illustration of this approach can be found in Section 2.7, Figure 2.14.

**Fixed, adaptive and no common macro step in the literature**

Whether a fixed, adaptive, or individual step size without any simultaneous steps is used, is defined for 100 publications. While the share of these papers remains around 60-70% in most time frames, between 2010 and 2014, this property is defined in 97% (i.e. all but one) of the considered publications.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

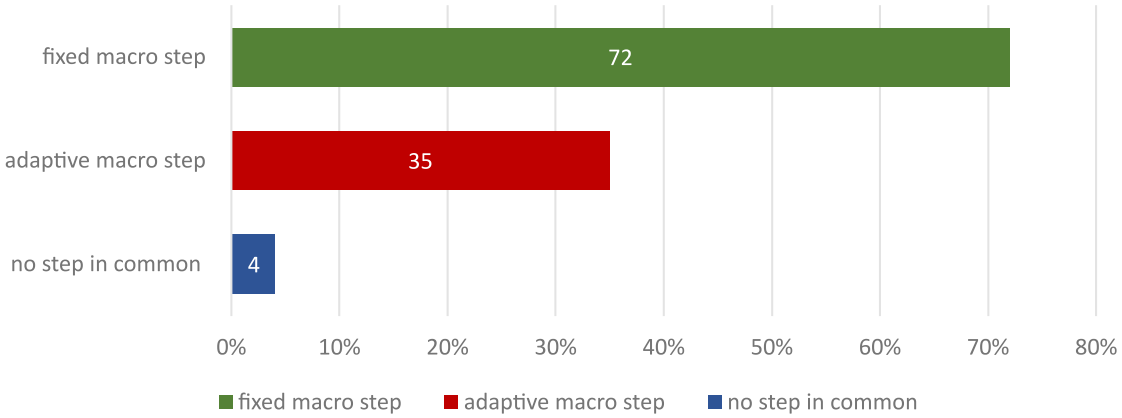


Figure 5.29: Shares and numbers of publications using a fixed, adaptive or no common macro step.

Figure 5.29 illustrates the amount of papers using different kinds of macro step sizes. A clear majority of 72% is applying or developing a method with a fixed communication time step. Nevertheless, 35% are using some kind of macro step size control and four present methods allowing no common synchronization reference. There are overlaps, which are shown in detail in the histogram in Figure 5.30. While from 2005 on, the amount of publications using fixed or adaptive macro steps does not vary substantially, an exceptional share of those in the category of fixed step sizes stands out in the time frame from 2000 to 2004. On the other hand, while in all other intervals, methods with fixed macro steps clearly dominate those with adaptive ones, shares are almost equal before 2000. All four works presenting master algorithms that do not require the subsystem solvers to have any step in common are found in the time frame from 2010 to 2014 (more precisely: 2011 and 2012).

Further, we want to take a closer look at the connection between the kind of master step and utilization of iterations. In 17 publications, neither if iterations take place nor whether fixed, adaptive, or completely independent step sizes are used has been revealed. 22 did not specify the macro steps, but did clarify whether iterative methods have been used. The partition of the rest is shown in Figure 5.31. Not altogether surprisingly, the highest share of adaptive algorithms is found in publications covering non-iterative methods. Apart from the one standard theoretically allowing all three kinds of master steps and iterations (note that the distinction on iterations is not exclusive!), methods that do not require a common synchronization step are always non-iterative.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

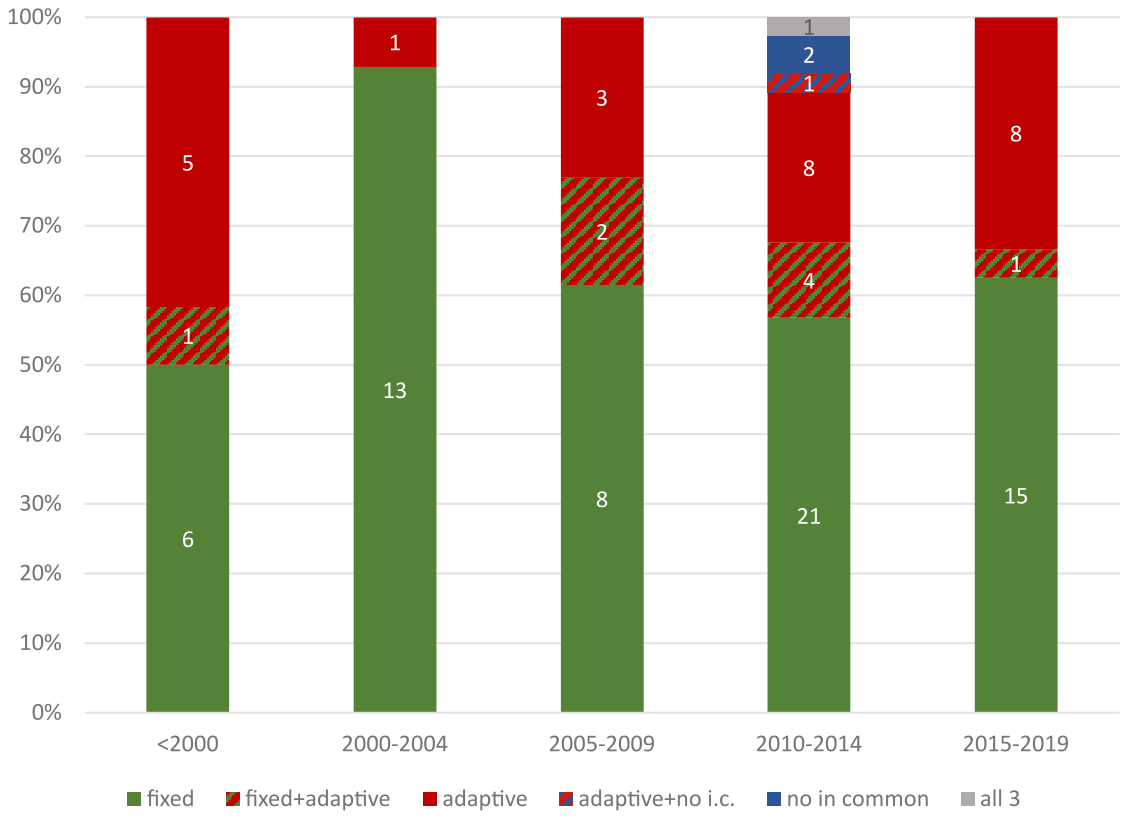


Figure 5.30: Change in percentages of fixed, adaptive or no common macro step usage in the selected literature over five-year time frames.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

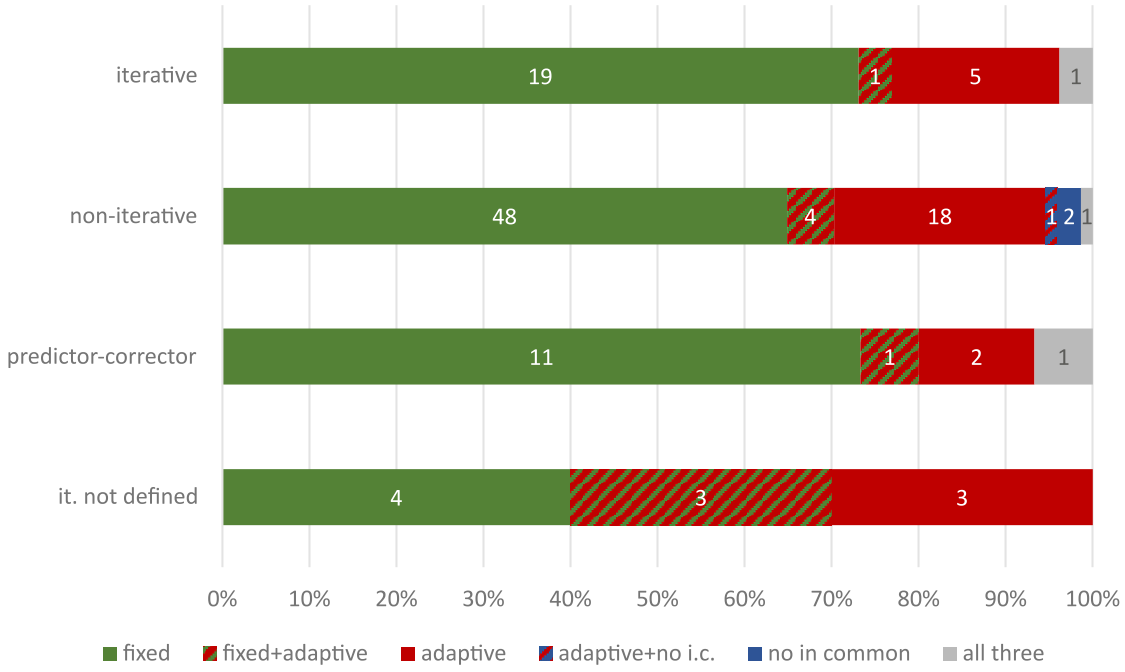


Figure 5.31: Connection between macro steps and iterations in the considered literature.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

### 5.7.4 Distinction by participating subsystem solver algorithms

Most co-simulation approaches (such as the Jacobi type or Gauß-Seidl type algorithms described in the last section) are not restricted to a specific kind of solver algorithm for the participating subsystems and thus open to explicit as well as implicit one-step methods (such as Runge-Kutta (RK) or Rosenbrock-Wanner (ROW) methods) or multi-step methods (such as backwards differentiation formulas (BDF)). Some co-simulation approaches, however, are developed regarding certain kinds of subsystem solution algorithms in particular. Approaches combining multi-step methods, for instance, can make use of this knowledge and will further require specific interfaces, additional information on former time steps from all the subsystems and the possibility to incorporate this information in the solution algorithms of the respective other systems. It is shown by Schmoll and Schweizer (2012) that the choice of subsystem solver algorithms and their order has a high influence on error accumulation and stability. Further, this distinction is of special interest in multirate methods that focus on the partition of the solution algorithm and are hence inevitably linked to the type of this algorithm. Thus, it is expedient to classify developments in co-simulation according to the kind of solution algorithms they can be applied to as well as the degree of suitability for these methods. Striebel (2006) provides the following *strategy distinction* which is motivated by, but not exclusive for, partitioned methods:

- multistep methods; for example:
  - employing slowest-first to BDF (allowing an arbitrary number of activity levels but restricting the quotient of successive step sizes to be a power of two)
  - local timestep control (BDF-based fastest-first approach)
  - general compound multirate method
- one-step methods: RK- or ROW-based methods
  - extrapolation/interpolation
  - generalised multirate
  - mixed multirate
  - hierarchical multirate
- sequence of computation (see Section 5.8)

While the last item in this listing does not necessarily relate to subsystems solvers, it brings us to the next aspect of structuring, where we focus particularly on multirate methods.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 5.8 Classification of partitioned multirate methods

Multirate methods can be further distinguished depending on the number of activity levels the participating subsystems are divided into or the sequence of subsystem execution in case of sequential methods.

### 5.8.1 Distinction by the number of activity levels

Depending on the number of activity levels into which a system is being separated – therefore determining the number of different step sizes used - Striebel (2006) distinguishes

- systems with two levels of activity and
- systems with arbitrary levels of activity.

In addition to the general distinction according to the number of participating subsystems following in Section 5.9, *partitioned* multirate methods stand out by the divide-and-conquer point of view – thus, decomposition of a complex system, in this case according to activity levels. Further, in contrast to master algorithms that allow cooperation of several subsystem solver algorithms, they utilize a partitioned solution algorithm (see Section 3.5 for further information).

Systems with two levels of activity are divided into an active and a latent part where the latent one defines the macro step size for communication as illustrated in Figure 5.32. The active parts are simulated with a smaller time step (micro step). For the separation into more than two levels of activity (illustrated in Figure 5.33), Striebel (2006) introduces a specifically developed hierarchical algorithm.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

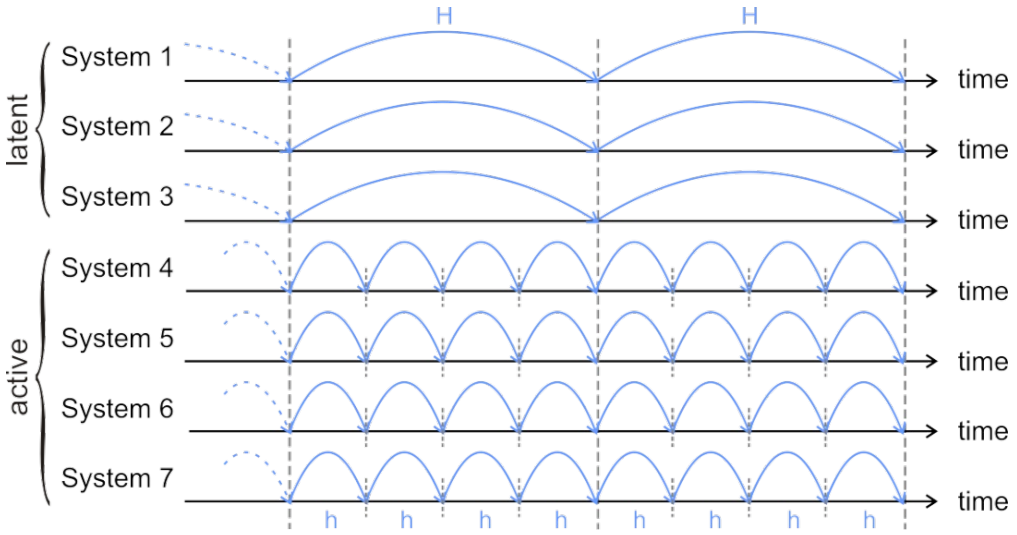


Figure 5.32: Illustration of multirate simulation with two levels of activity, macro step size $H$ and micro step size $h$ (Hafner and Popper 2017).

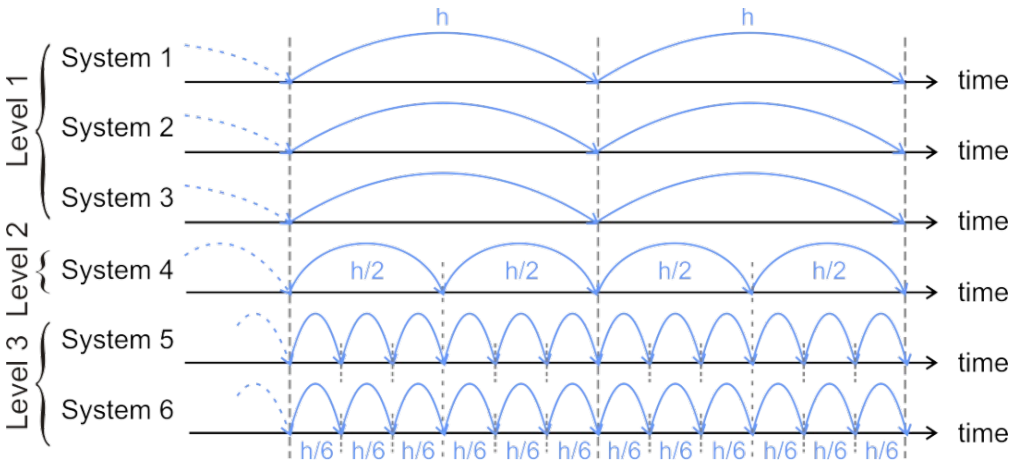


Figure 5.33: Illustration of a hierarchical multirate simulation for arbitrary levels of activity (Hafner and Popper 2017).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 5.8.2 Distinction by sequence of execution

Multirate methods can be further classified by the sequence of computation (see (Striebel 2006)) in the case of (at least partially) sequential execution of the systems for one macro step:

- slowest first
- fastest first
- mixed

Applying the *slowest first* method, the latent part (or, in the case of more than two levels of activity, the most latent part, i.e. the subsystem using the biggest time step which equals the communication time step) is executed first and uses extrapolated values from the active part. Then, the active part is calculated for the same macro interval using interpolated values for the variables required from the latent part at the micro steps.

The *fastest first* method calculates the active part at all micro steps within the next macro step using extrapolated values from the latent part before executing the macro step in the latent part.

Mixed approaches calculate one step in the active part in parallel to one step in the latent part (also called *compound step*) using extrapolated values in both systems before executing the remaining steps for the current macro step by the use of interpolated values. Illustrations of these variants can be found in Section 2.4.

#### 5.8.2.1 Publications by sequence in multirate methods

Of all publications, 19 cover multirate methods where the sequence of execution can be categorized as “slowest first”, “fastest first” or “compound step”. Of these, none have been published after 2011. Four of these publications address more than one method and of these, two cover both a slowest and fastest first method, one a compound and slowest first approach and one compares all three methods, see Table 5.6.

Table 5.6: Sequence of execution in multirate methods (number of publications).

| (only) compound step | slowest first | fastest first | comp.&s.f. | s.f.&f.f. | all three | total |
|---|---|---|---|---|---|---|
| 7 | 5 | 3 | 1 | 2 | 1 | 19 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Of those considering only one approach, most apply a compound step; overall (shown in Figure 5.34) compound and slowest first methods are equally represented, each 1.5 as often as fastest first approaches. This circumstance is not altogether surprising considering increased stability issues for fastest-first approaches (as explained f.i. by González et al. (2011)).

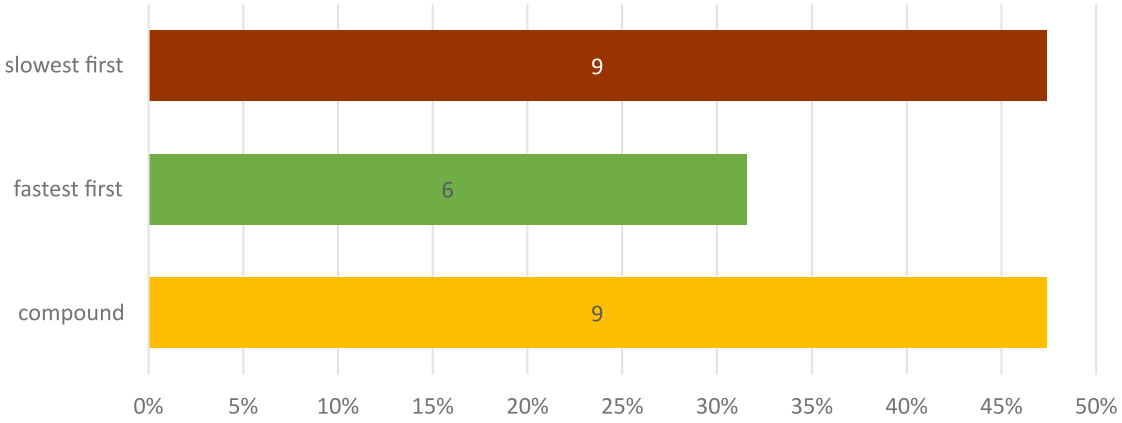


Figure 5.34: Share of publications employing a slowest first, fastest first or compound multi-rate method.

Since the number of publications in this category is too small to allow for a valid representation if partitioned any further, a histogram is omitted in this case.

## 5.9 Distinction by the number of coupled subsystems

Another interesting topic of consideration is the number of co-simulated subsystems. While partitions due to latency or activity are sometimes limited to dividing the overall system into two subsystems, further variations in time constants (cf. (Striebel 2006)) or problems of the integrate-and-collaborate kind (see f.i. (Galtier et al. 2015)) often require the cooperation of several or even an arbitrary number of subsystems. Many theoretical investigations start by considering an arbitrary number ($n$ subsystems), only to restrict more detailed investigations to only two systems. Regarding, for example, sequential approaches where the sequence – which, as we know, comes with $n!$ possible permutations – has major influence on the stability, this restriction is understandable, albeit not entirely satisfactory for certain issues, where the reader is basically left with gaps in this area of research: in practice, coupling of more systems is frequently sought.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 5.9.1 Number of coupled systems in the literature

The number of co-simulated systems has been defined in all but eight publications, all of which are surveys or formalism descriptions. The remaining ones are divided into those considering two ("2 systems"), more than two but an explicit, finite number ("$>$ 2 systems") and an arbitrary number of subsystems ("n systems"), see Figure 5.35.

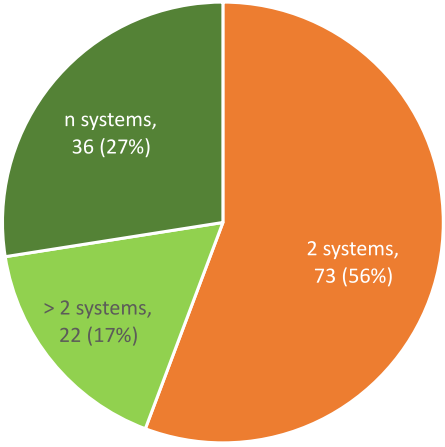


Figure 5.35: Publications divided by the number of coupled subsystems.

More than half (56%) of these publications consider exactly two coupled subsystems. Of the remaining 58 papers, 22 are coupling more than two, yet an explicit, integer number of systems. 27% describe methods that can be applied to an arbitrary number of subsystems. It shall be noted that there are six publications in which the number of coupled subsystems differs in theory and practice, most of which present a theoretical approach for $n$ coupled subsystems which have then been tested only on two, or pursued their investigations with only two systems for the sake of simplicity while claiming that these considerations can be extended to more systems without further ado. These have also been classified as allowing an arbitrary number of coupled systems.

While no major changes can be observed over time, Figure 5.36 shows that the share of publications allowing an arbitrary number of coupled systems diminishes slightly until 2009 only to rise again afterwards.

For additional information, Figure 5.37 shows how the different main orientations (cf. Section 5.3) are partitioned by the number of considered subsystems. It is not surprising that there is only one publication that focuses on an application and still considers an arbitrary number of coupled subsystems: Barros (2017) theoretically explains an approach (albeit tai-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN


Figure 5.36: Number of coupled subsystems, variation in shares over the years by five-year intervals.

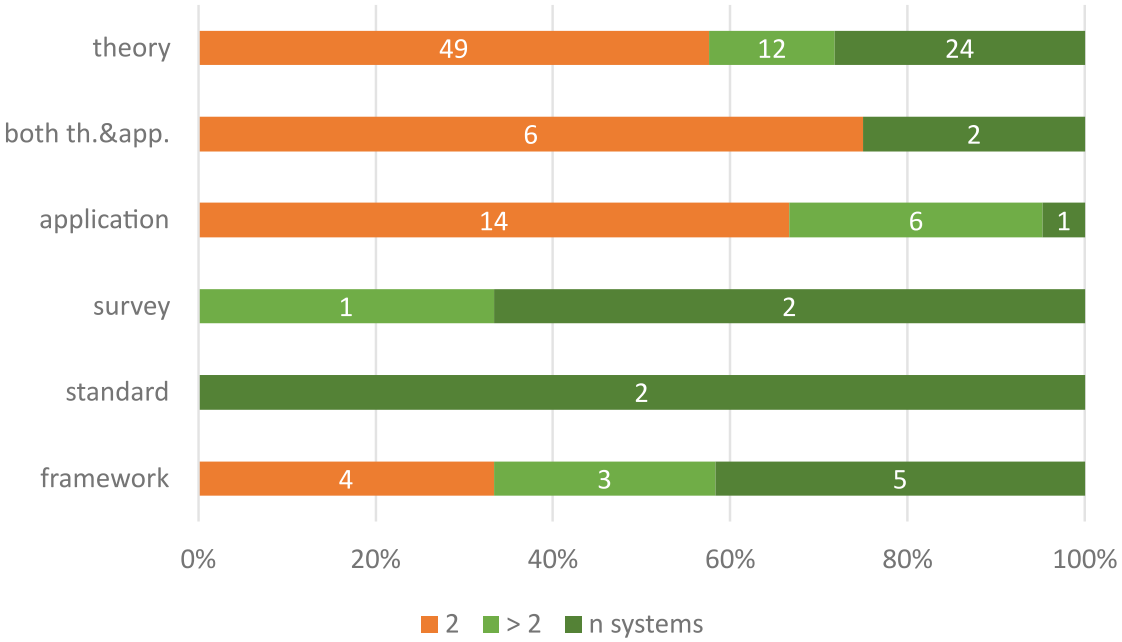


Figure 5.37: Papers' main emphases partitioned by different numbers of coupled subsystems.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

lored to a specific problem) suitable for $n$ systems and applies it on two, consequently being allotted to the first category, as explained above. However, all mainly theoretical and applied work is composed predominantly of publications coupling only two systems. Surveys and standards almost always allow an arbitrary number (except for (Thiede et al. 2016), where different methods – only one of them co-simulation – to approach a specific problem with a finite number of levels and thus subsystems (five) are presented).

Taking a look at the cross-connection of the number of coupled systems and their sequence of execution, eight publications that have been assigned neither can be discerned. For the remaining 131, the general observation that publications considering only two coupled systems constitute the majority is also reflected in all sub-categories, see Figure 5.38. Therein, the highest percentage of only two coupled subsystems is found in papers covering both sequential and parallel coupling methods. In this subcategory, no publication covers more than two but not an arbitrary number of subsimulations, which implies that either, parallel and sequential schemes are applied to two coupled test systems or theoretical studies are conducted on coupling $n$ systems, i.e. an arbitrary number of systems.

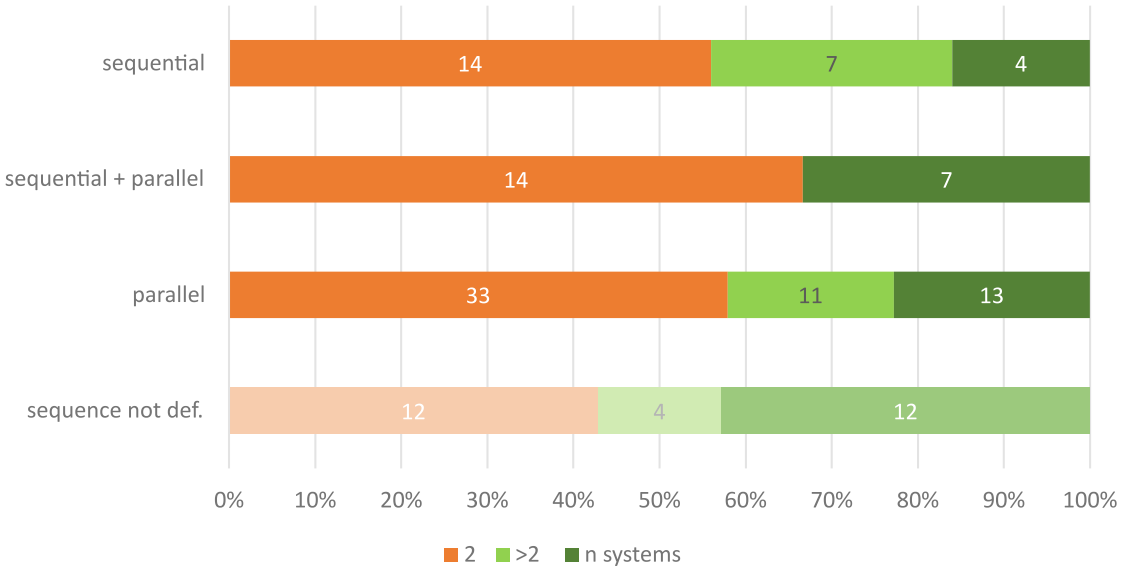


Figure 5.38: Nexus of the number of coupled subsystems and their execution sequence.

Another interesting aspect in this constellation is the fact that the considered literature barely covers investigations of sequential methods for an arbitrary number of systems. This may simply be explained by the impact of the order of execution in sequential approaches, which gains in possibilities and thus complexity with the number of systems, as explained before.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

It shall be noted that for all papers where the sequence is defined, the number of participating subsystems is also known. However, there are 28 papers with unknown sequence where the number of subsystems is given. It seems interesting that in this category alone, there are as many publications considering an arbitrary number of partial simulations as those taking only two participating systems into account.

### 5.9.2 Hierarchical approaches

Owing to specific relevance for this thesis, we furthermore characterize whether a hierarchical approach is considered in the literature. Thereby, we count only those publications that decidedly allow or enable a hierarchical structure.
Note that the construction of a non-trivial hierarchy is only possible for three or more systems (otherwise, no co-simulation of a co-simulation could occur), which already restricts the selection. Only nine publications allow hierarchy, four of which describe multirate methods where in addition to the equation systems the solution algorithm itself is partitioned. Apart from a general lack of existing studies and thus present potential in this area of research, Figure 5.39 illustrates that although publications that consider hierarchical structures have increased up to 2009, they have since dropped to one per five-year time frame. This chronological information, however, can hardly be ascribed too much importance given the small total number in this category.

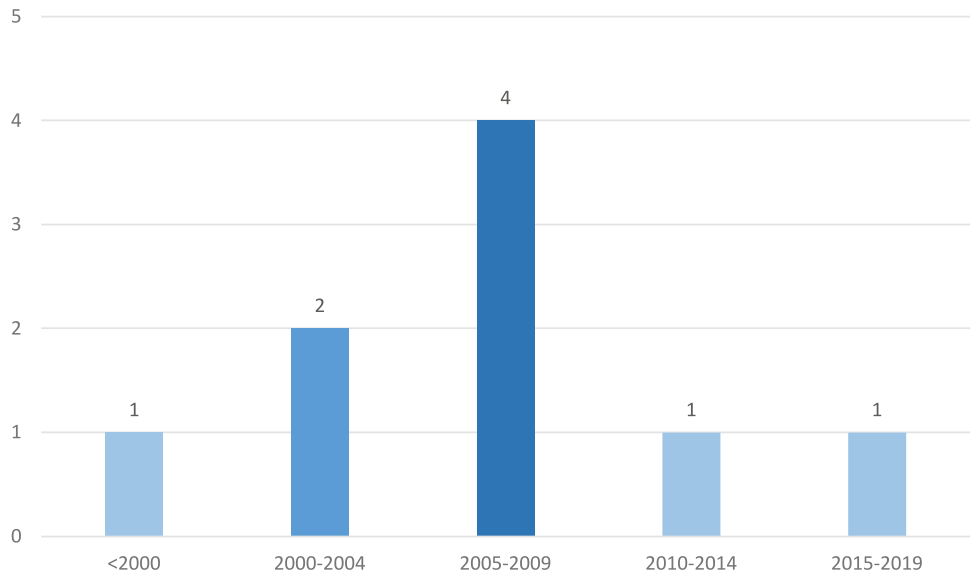


Figure 5.39: Publications on cooperative or multirate simulation that allow a hierarchical structure, illustrated per five-year time frames.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 5.10 Software, framework and standard usage in the literature

Through the years, frameworks and standards for co-simulation have been established with the aim to unify heterogeneous approaches and enable coupling of simulations without the need for renewed, effortful solutions for every individual problem. Even so, these developments have partly taken place in parallel and been motivated by particular applications, thus again leading to individually designed and therefore limited environments. This section gives an overview of all revealed software, standards, and frameworks developed and/or used in the regarded selection of literature.

### 5.10.1 Software

Sixty publications reveal one or more software programs or at least programming language they used. Most frequently named are MATLAB (plain or toolboxes, in 33 papers) and Modelica-based simulators (19 mentions). Next are EnergyPlus with six occurrences, ADAMS, FEAP and SIMPACK with four each and COMSOL and TRNSYS with three, followed by many different programs which have only been used once to twice. The reader interested in the detailed list is referred to Figure 5.40. Sixteen papers are using plain MATLAB, fifteen Simulink and two the Simscape toolbox. Within the Modelica-based software, Dymola is mentioned in five, OpenModelica by four, AMESim by two and SIMPLORER by one publication. Seven use the Modelica Language standard without explicitly naming the simulation program. The vast dominance of MATLAB and Modelica-based tools matches the high extent of ODE and DAE model descriptions, cf. Section 5.6.1.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

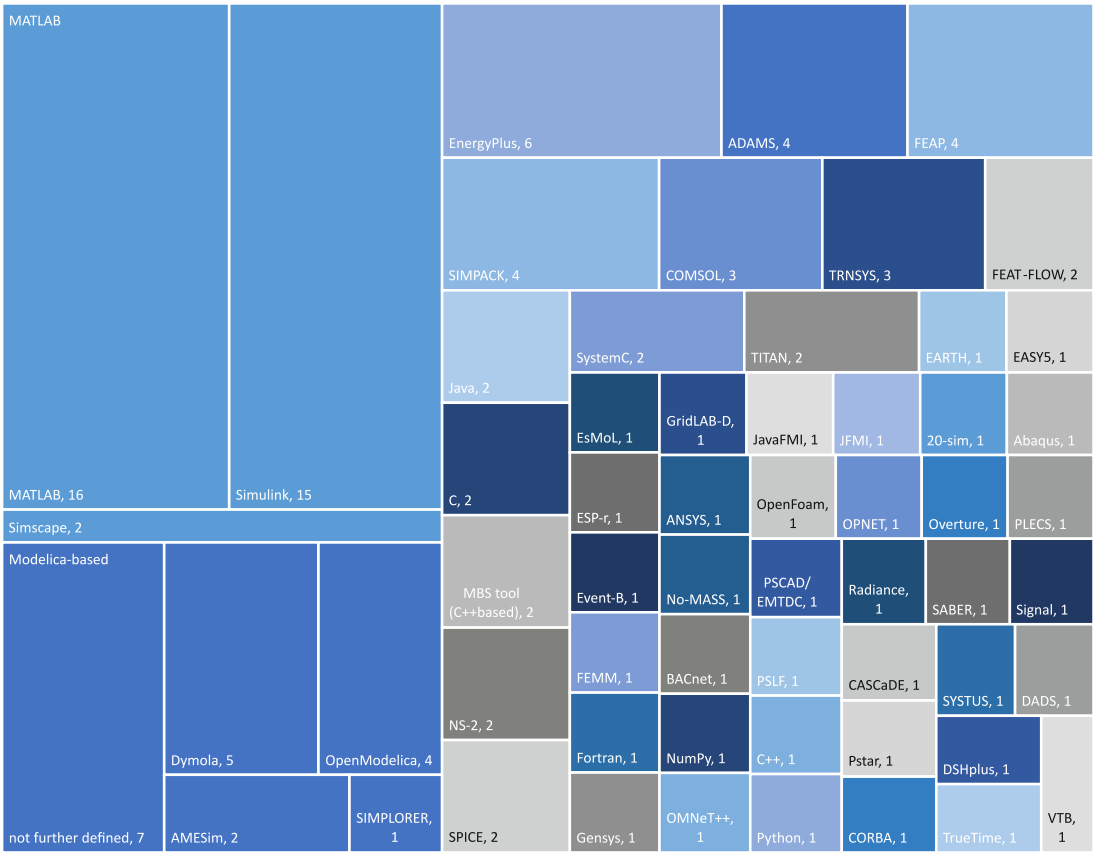


Figure 5.40: Software that has been used according to the considered literature.

### 5.10.2 Frameworks

All in all, 42 different frameworks are declared. Seventeen of these are nameless self-implemented ones and have, despite their independence, been combined in one group. Apart from these, most used is the BCVTB by four, followed by FIDE and Maestro with three publications. The rest are only mentioned in one publication each and can be looked up in Figure 5.41. A historical view on the individual frameworks would be futile due to the small number of respective publications. However, looking at the number of publications in which any framework is defined shows clearly that the usage of frameworks has increased remarkably in the last decade.

It is further interesting that although many frameworks are already available, individual, specific requirements still motivate the implementation of new ones instead of settling for an unsatisfactory compromise. This brings us to the development of standards and formalisms that aim to unify these different needs.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

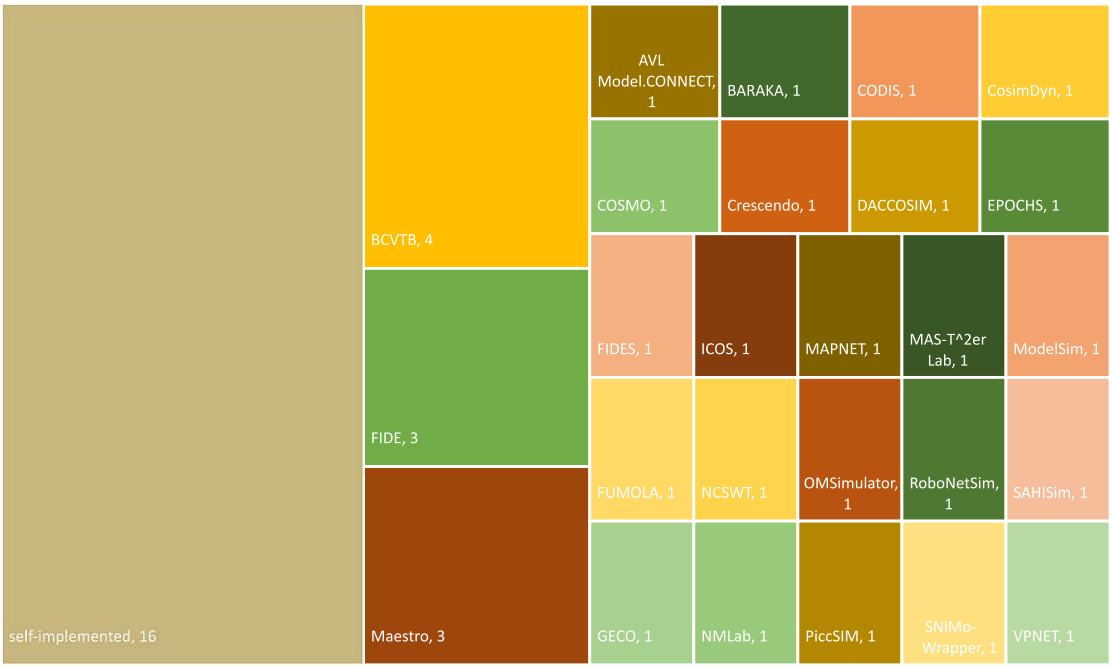


Figure 5.41: Frameworks that have been used according to the considered literature.

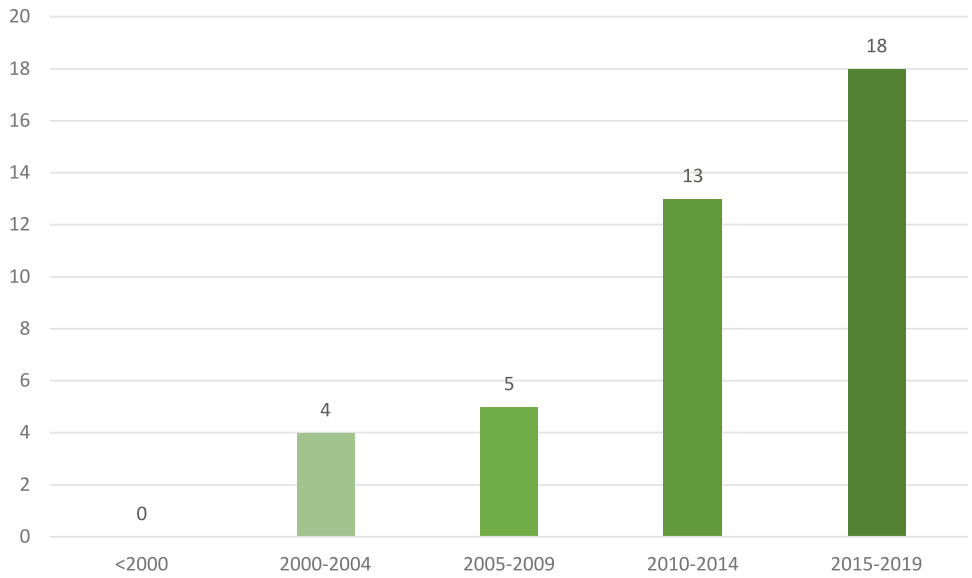


Figure 5.42: Framework usage according to the considered publications per five-year time frames.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 5.10.3 Standards and formalisms

Of those 35 publications naming a standard or formalism they use, develop or extend, an overwhelming majority of 29 works utilize the FMI (see Figure 5.43). Three consider DEVS-based approaches, within these plain DEVS, the extension on the combination with continuous parts called DEV&DESS and also hyPDEVS, which enables local resolving of concurrent events in hybrid systems. The HLA is represented in two publications, as well as Matlab S-functions and the HyFlow or HFSS formalism.

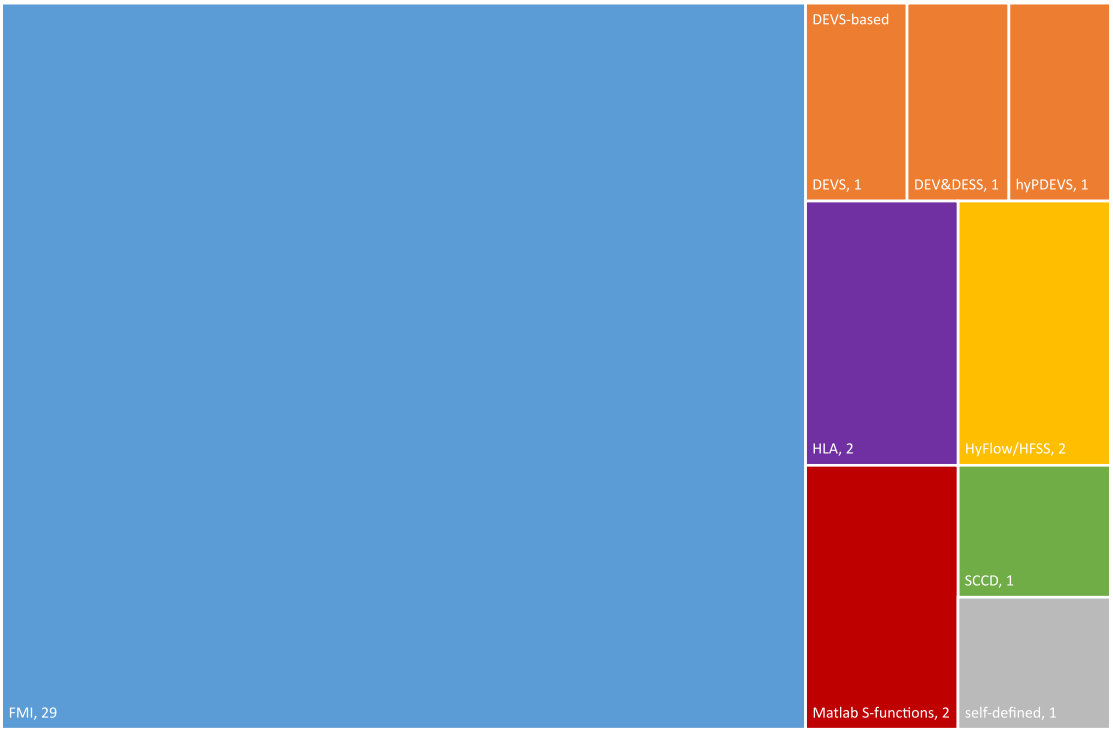


Figure 5.43: Standards and formalisms that have been used according to the considered literature.

Note that five works are using more than one standard or formalism, f.i. Awais (2015) who utilizes the FMI for model description as well as the HLA to develop a framework for hybrid co-simulation, or Heinzl (2016) who compares a DEVS-based approach with co-simulation via the BCVTB.
Again, a historical view of standard or formalism usage shows an increase from 2010 and particular accumulation in the last five years (Figure 5.44).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

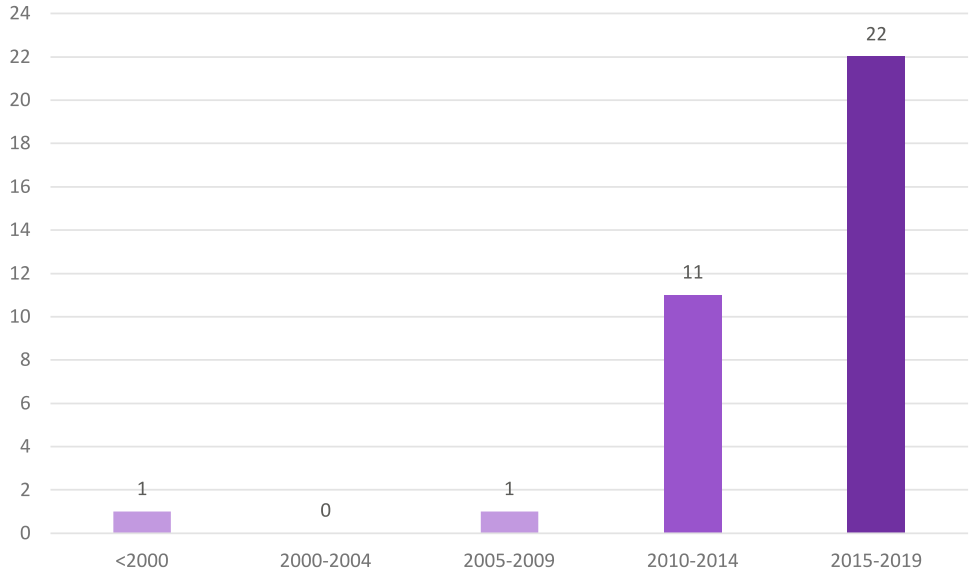


Figure 5.44: Chronological illustration of publications describing the usage or development of standards and formalisms.

## 5.11 Author and affiliation network

As mentioned before, research in co-simulation has manifold origins and often lacks even common terminology and knowledge on similar studies. An interesting illustration pointing out the interweaving of cooperations can be found in Figure 5.45, where all authors and co-authors of the 139 considered publications are depicted as colored dots. Their colors correspond to the countries of the authors' affiliation (in case of differing affiliations in different publications, the latest one). The circles are sized depending on the total number of publications the respective author has contributed to (see also Table 5.7). The connecting lines and their thickness correspond to (the number of) co-authored publications. All following network graphics have been created by utilizing `https://observablehq.com/@mbostock/hello-cola`. Labels are omitted in Figure 5.45 on purpose to clearly feature the emergence of clusters and solitary dots respectively.

There are a few groups that are conducting research in an exemplary manner via manifold cooperations with international partners. Others seem – judged by co-authorships – to work mostly alone (thirteen) or small clusters: thirteen are pairs, eleven groups consist of three and thirteen of four authors. Then, for the next few numbers, the groups seem to diminish: There are five groups of five, one of six and two of eight authors. Of these 45 groups of

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

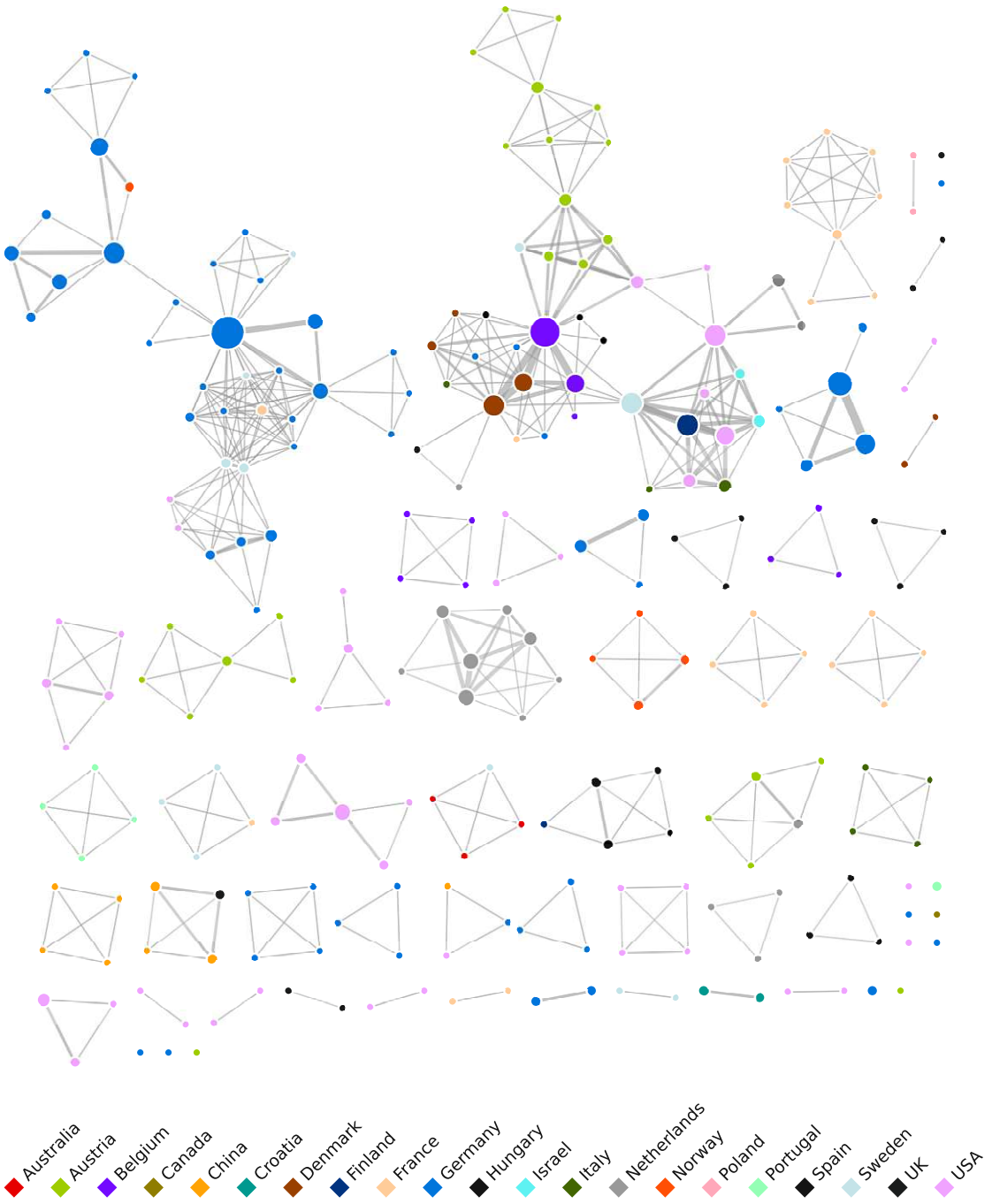


Figure 5.45: Illustration of the author network of the considered literature. Dots correspond to authors, lines indicate that connected authors have co-authored at least one publication. Colors refer to the country where the respective author's (latest) affiliation is located.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

two to eight authors, only six are cooperating internationally and 28 are even secluded to a single institution – but more on that below (Figures 5.47 and 5.48). Next are already the two largest clusters: The one to the top left around Martin Arnold with 38 and the one to its right with 44 authors, connected mainly through Cláudio Gomes and David Broman, cf. Figure 5.46, which shows the same network including the names of all researchers who have (co-)authored at least three of the considered publications. It is interesting to note that there are no group sizes between eight and 38 authors, implying that research is either done within a small, straightforwardly assessable team or, by only few international cooperations per author, the corresponding connection network inevitably attains a substantial dimension. We also see that the activity of a few authors suffices for the formation of these vast networks – without Cláudio Gomes, Michael Wetter and David Broman, the group of 44 would be split into three considerably smaller ones. Similar considerations can be made for the group of 38, which in comparison to the multicolored composition of the largest one is plainly dominated by authors from German institutions.

Regarding the ranking by contributions given in Table 5.7, we see that Martin Arnold leads with ten publications, closely followed by Cláudio Gomes with nine. Next is David Broman with seven and several authors with six contributions. In this table, at least, countries and affiliations occur variedly.

For a closer look at the latter, we regard the network of institutions, which can be depicted similarly to the one of authors. Again, every circle corresponds to one institution and its color to the establishment's location. Even more pronounced than in the author network, Figure 5.47 shows that many institutions (thirty of all 128) only issue publications without any external partners even if they publish a considerable number of total papers, as the Technische Universität Braunschweig with five or the Massachusetts Institute of Technology with three contributions (see also Figure 5.48 and Table 5.8). The two biggest clusters consist of 43 (found on the top left of Figure 5.47) and 22 (group on the bottom left) different institutions, respectively. Apart from these, there are four groups of two, seven of three and one of four. Six of these have international links. Similar to the corresponding observation in the author network, the lack of any clusters between four and 22 linked institutions implies that authors and their affiliations either work mostly within a very small research group or, if open to cooperation with many different, international researchers, their network expands immensely. All of these deliberations are, of course, limited by the selection of literature considered here and could potentially show different results if chosen by other criteria, as explained in Section 5.1.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

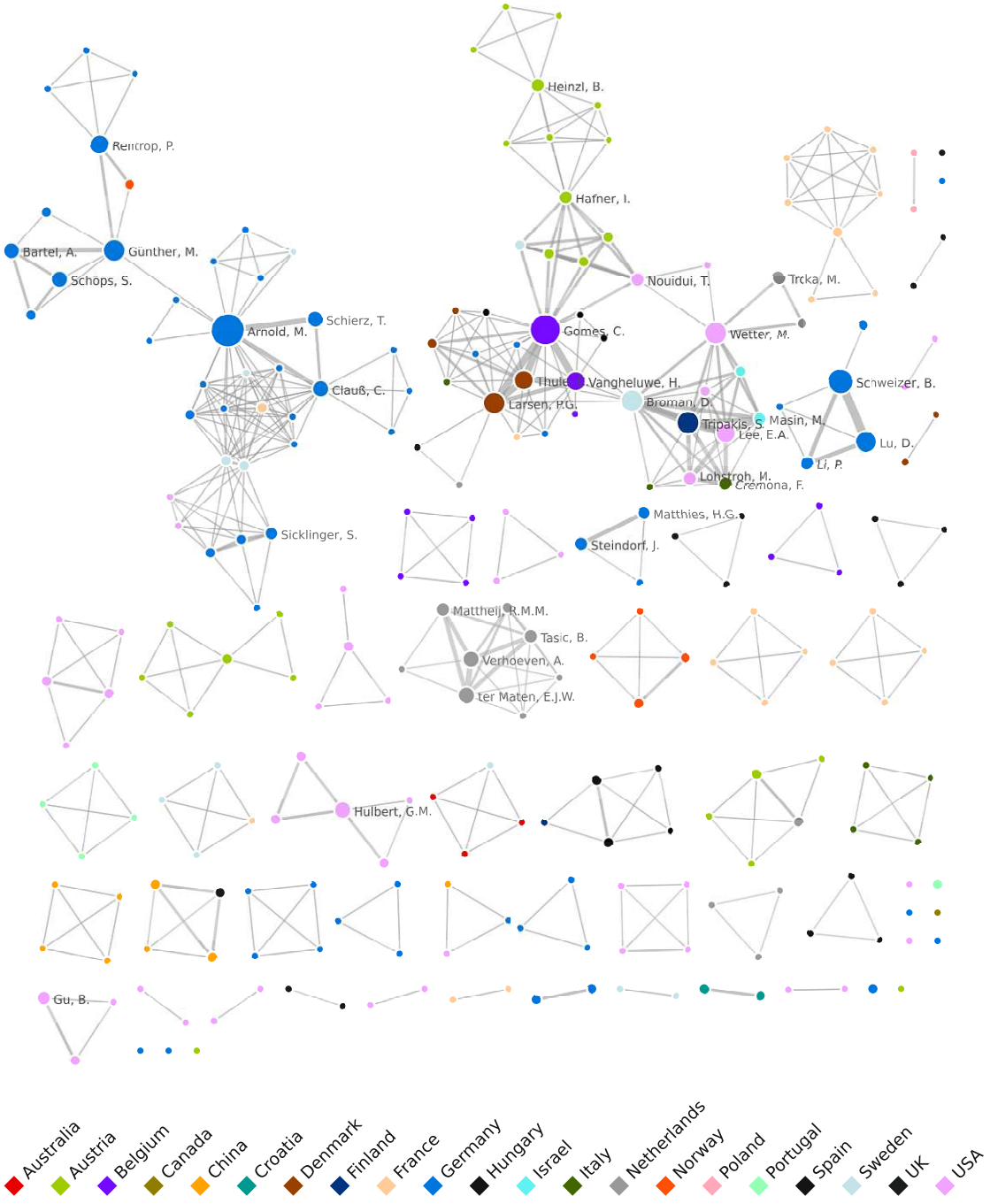


Figure 5.46: Illustration of the author network, colored by the affiliation's country. Authors with more than two publications in the selection are labeled.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Table 5.7: Publications per author (shortened to those with more than 2 contributions), in descending order.

| author | publications | (latest) affiliation |
|---|---|---|
| Arnold, M. | 10 | Martin Luther University Halle-Wittenberg (Germany) |
| Gomes, C. | 9 | Flanders Make (Belgium) |
| Schweizer, B. | 7 | Technische Universität Darmstadt (Germany) |
| Broman, D. | 6 | KTH Royal Institute of Technology (Sweden) |
| Günther, M. | 6 | Bergische Universität Wuppertal (Germany) |
| Larsen, P.G. | 6 | Aarhus University (Denmark) |
| Lu, D. | 6 | Technische Universität Darmstadt (Germany) |
| Tripakis, S. | 6 | Aalto University (Finland) |
| Wetter, M. | 6 | Lawrence Berkeley National Laboratory (USA) |
| Lee, E.A. | 5 | University of California, Berkeley (USA) |
| Rentrop, P. | 5 | University of Karlsruhe (Germany) |
| Thule, C. | 5 | Aarhus University (Denmark) |
| Vangheluwe, H. | 5 | Flanders Make (Belgium) |
| Bartel, A. | 4 | Bergische Universität Wuppertal (Germany) |
| Clauß, C. | 4 | Fraunhofer IIS (Germany) |
| Hulbert, G.M. | 4 | University of Michigan (USA) |
| Schierz, T. | 4 | Martin Luther University Halle-Wittenberg (Germany) |
| Schöps, S. | 4 | Technische Universität Darmstadt (Germany) |
| ter Maten, E.J.W. | 4 | NXP Semiconductors (Netherlands) |
| Verhoeven, A. | 4 | Technische Universiteit Eindhoven (Netherlands) |
| Cremona, F. | 3 | ALES (Italy) |
| Gu, B. | 3 | Massachusetts Institute of Technology (USA) |
| Hafner, I. | 3 | dwh GmbH (Austria) |
| Heinzl, B. | 3 | TU Wien (Austria) |
| Li, P. | 3 | Technische Universität Darmstadt (Germany) |
| Lohstroh, M. | 3 | University of California, Berkeley (USA) |
| Masin, M. | 3 | IBM IL (Israel) |
| Mattheij, R.M.M. | 3 | Technische Universiteit Eindhoven (Netherlands) |
| Matthies, H.G. | 3 | Technische Universität Braunschweig (Germany) |
| Nouidui, T. | 3 | Lawrence Berkeley National Laboratory (USA) |
| Sicklinger, S. | 3 | Technische Universität München (Germany) |
| Steindorf, J. | 3 | Technische Universität Braunschweig (Germany) |
| Tasic, B. | 3 | NXP Semiconductors (Netherlands) |
| Trcka, M. | 3 | Technische Universiteit Eindhoven (Netherlands) |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN


Figure 5.47: Illustration of the network of institutions with respect to cooperatively published literature. Dots correspond to institutions, lines indicate that researchers from connected institutions have co-authored at least one publication. Colors refer to the country where the affiliation is located.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

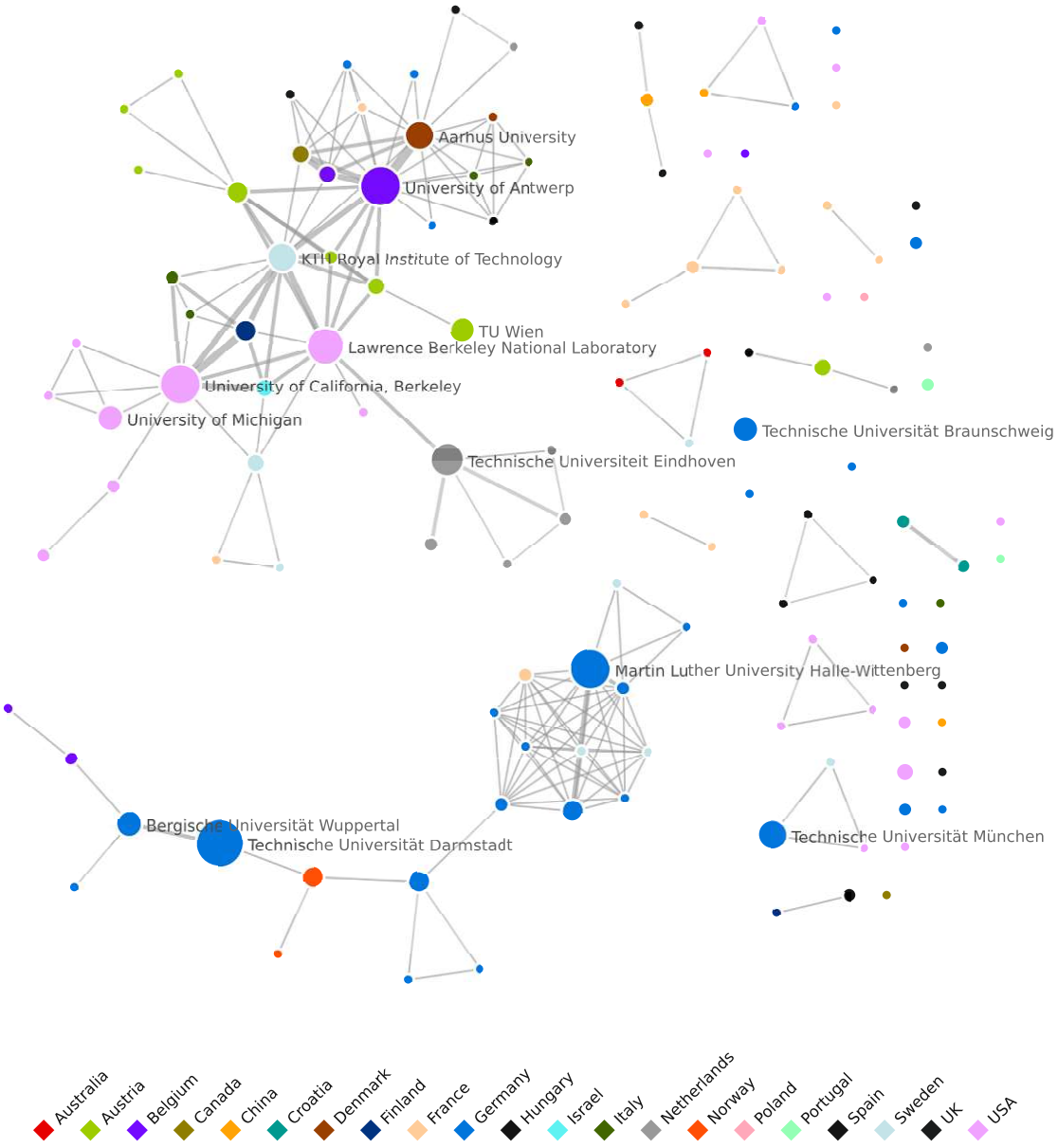


Figure 5.48: Illustration of the affiliation network, colored by the country. Institutions with more than four publications in the selection are labeled.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

Figure 5.48 shows the same network graphic as Figure 5.47 but with labeled institutions for those with five or more publications. Not altogether surprisingly, all except two of these – namely Technische Universität Braunschweig and Technische Universität München - are found within the two largest groups, which could put the above observation in perspective considering that this suggests that the more publications of one institution are regarded, the higher the chance of various co-authorships with other companies or universities. On the other hand, it may simply mean that most research on co-simulation is conducted within just a few, yet well-connected groups of broadly spread members.
A further interesting fact observable in this network is the different architecture of the two largest groups. While links between circles of the top-most (with regard to the number of participants) cluster are crisscrossing diversely, the second-largest group – apart from the web around Martin Luther University Halle-Wittenberg – rather resembles a chain with links of mostly one or at most two papers, the latter between Bergische Universität Wuppertal and Technische Universität Darmstadt. This means that although the total group consists of many different institutions, those are not multiply joined among each other but only linked to a few others. A prominent incidence of institutions based in Germany can be detected in the second largest group, in accordance with the observation in the author network graphic.

All institutions with at least three publications are given in Table 5.8 in descending order. Technische Universität Darmstadt is undisputedly first with 11 publications, followed by three different universities with nine (Halle-Wittenberg, Antwerp and Berkeley) and the Lawrence Berkeley National Laboratory with eight publications. While the latter is the first non-university establishment in this list, it is managed by the University of California. Looking at the first three-quarters of Table 5.8, we recognize almost exclusively universities or university related organizations. This might, on the one hand, imply that most scientific research is still conducted under the aegis of higher education institutions – at least for the specific topic of co-simulation – but may also reflect the effects of pressure for publications in universities.

All in all, the networks of authors and affiliations have revealed that while cooperations are already appreciated to advance research in co-simulation by some scientists, even in times of globalization with digital libraries and means of communication, several research groups seem to prefer restriction to their capabilities over cooperation with other institutions. This is affirmed by the depiction of the institutional network colored by the year of the latest publication (Figure 5.49), which shows that secluded research is not (only) a phenomenon of earlier work.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

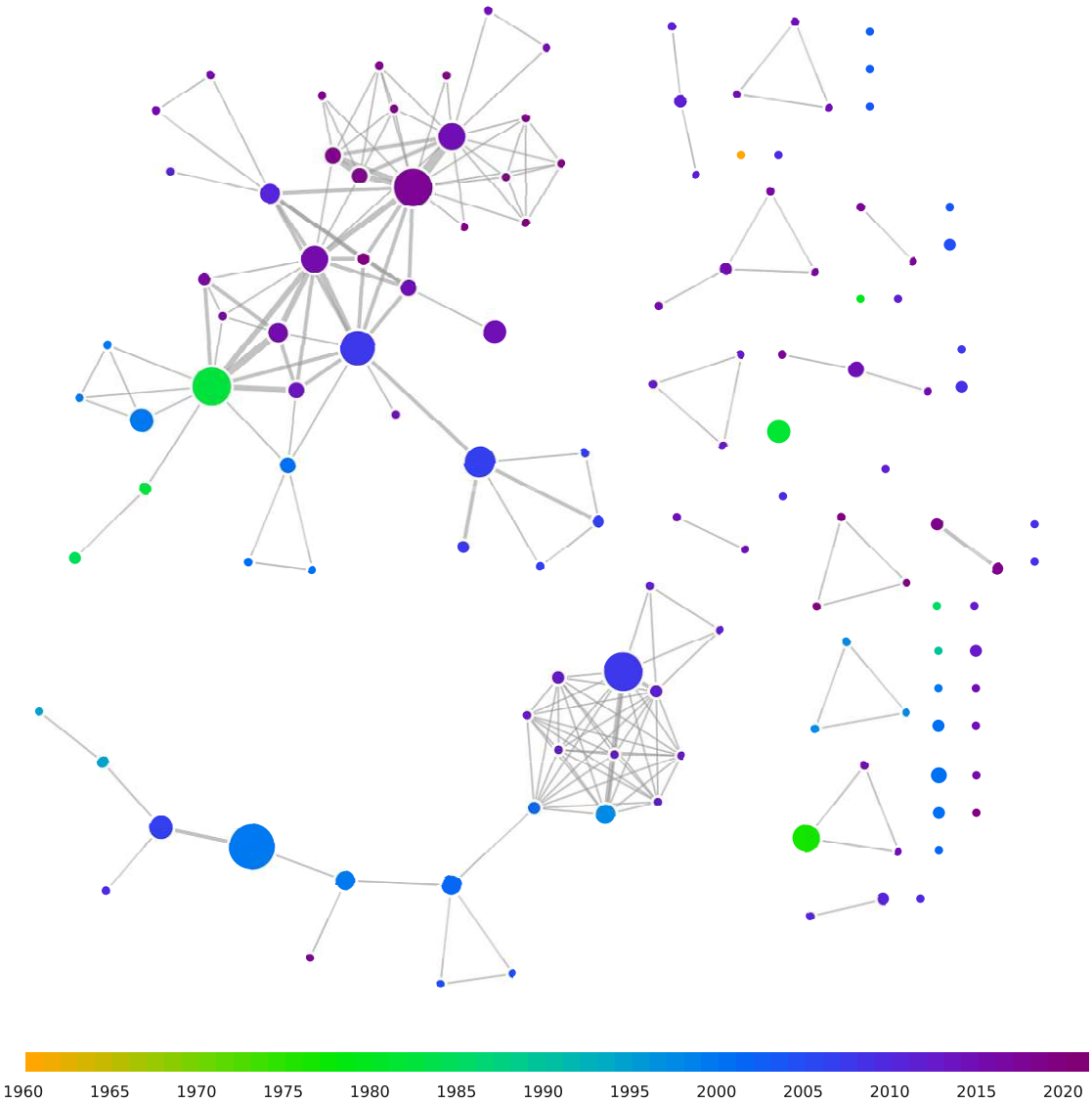


Figure 5.49: Network of institutions colored by the year of the latest considered publication.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Table 5.8: Publications per institution (shortened to those with more than 2 contributions), in descending order.

| institution | publications | country |
|---|---|---|
| Technische Universität Darmstadt | 11 | Germany |
| Martin Luther University Halle-Wittenberg | 9 | Germany |
| University of Antwerp | 9 | Belgium |
| University of California, Berkeley | 9 | USA |
| Lawrence Berkeley National Laboratory | 8 | USA |
| Technische Universiteit Eindhoven | 7 | Netherlands |
| Aarhus University | 6 | Denmark |
| KTH Royal Institute of Technology | 6 | Sweden |
| Technische Universität München | 6 | Germany |
| Bergische Universität Wuppertal | 5 | Germany |
| Technische Universität Braunschweig | 5 | Germany |
| TU Wien | 5 | Austria |
| University of Michigan | 5 | USA |
| Aalto University | 4 | Finland |
| Fraunhofer IIS | 4 | Germany |
| Graz University of Technology | 4 | Austria |
| Norwegian University of Science and Technology | 4 | Norway |
| University of Karlsruhe | 4 | Germany |
| Austrian Institute of Technology | 3 | Austria |
| dwh GmbH | 3 | Austria |
| Flanders Make | 3 | Belgium |
| IBM IL | 3 | Israel |
| Linköping University | 3 | Sweden |
| Massachusetts Institute of Technology | 3 | USA |
| McGill University | 3 | Canada |

## 5.12 Publications per country and continent

Publications per country are depicted in Figure5.50, where the respective country is colored in increasing intensity according to the number of contributions. The exact numbers are given in Table 5.9. We can observe a clear dominance of Germany with 54 publications, followed by the USA with 42 contributions and, placed a distant third, Sweden with thirteen publications. Even if the states of the USA (and the UK respectively) are listed separately, this barely affects the overall ranking since California itself has contributed to seventeen publications. However, this allows additional insight to the distribution among these states.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Table 5.9: Publications per country in descending order.

| country | number of publications |
|---|---|
| Germany | 54 |
| US (California) | 17 |
| Sweden | 13 |
| Austria | 12 |
| Belgium | 12 |
| Netherlands | 10 |
| France | 9 |
| UK (England) | 9 |
| Denmark | 7 |
| US (Michigan) | 5 |
| Finland | 5 |
| China | 4 |
| Italy | 4 |
| Norway | 4 |
| Canada | 4 |
| Portugal | 3 |
| US (Massachusetts) | 3 |
| US (New York) | 3 |
| Israel | 3 |
| Croatia | 2 |
| Spain | 2 |
| US (Colorado) | 2 |
| US (Florida) | 2 |
| US (Illinois) | 2 |
| Australia | 1 |
| Hungary | 1 |
| Poland | 1 |
| UK (Wales) | 1 |
| US (Georgia) | 1 |
| US (Louisiana) | 1 |
| US (North Carolina) | 1 |
| US (Pennsylvania) | 1 |
| US (South Carolina) | 1 |
| US (Tennessee) | 1 |
| US (Virginia) | 1 |
| USA | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

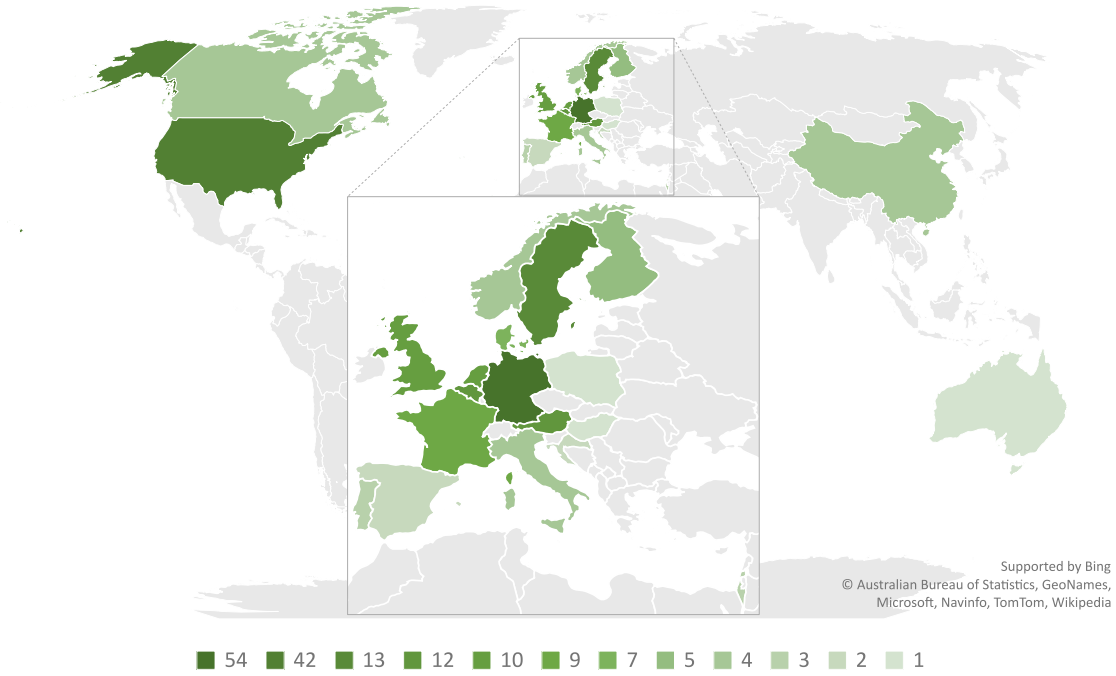


Figure 5.50: Map of countries colored with respect to the number of publications.

The ranking of continents by the number of contributions (given in Table 5.10) shows a clear prevalence in Europe, followed by not even one third as many in North America, occasional publications in Asia and only one contribution from Australia. South America, Africa and Antarctica are not represented at all. While this table may speak only for research in co-simulation, the influence of industrial development in individual countries and continents on possibilities for higher education and scientific research cannot be neglected. On the other hand, as the small representation of Australia can hardly be explained by this reflection, the dominance of European contributions may simply result from the location of my university with possible influences on accessibility to certain research and restrictions due to lack of specific intercontinental interchanges regarding communication of scientific resources.

Table 5.10: Contributions per continent, in descending order. Continents without publications in the selected literature are omitted.

| continent | number of publications |
|---|---|
| Europe | 114 |
| North America | 38 |
| Asia | 7 |
| Australia | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## 5.13 Summary of the classification

In this chapter, various ways of structuring co-simulation methods have been presented along with the characterization of a selection of literature on this wide-ranging topic. To reflectively enable a comprehensive perception of these extensive descriptions, Figure 5.51 gives an overview of different aspects by which co-simulation approaches can be distinguished, illustrating the complexity and multifacetedness of the given classification.

This diversity has also been reflected in the analysis of the regarded literature, which has furthermore unveiled tendencies towards more popular methods. While most publications cover mainly theoretical aspects, applications are nevertheless manifold and range from mostly physical systems in one or many domains to cross-domain applications including complex controlled systems up to urban scale. Model descriptions are dominated by Differential (Algebraic) Equations but also cover Agent Based or Finite Element models and even Discrete Event systems. Hybrid systems, albeit sparsely represented, remain a challenge if approached via coupled simulations as well as they do in a mono-simulation. Non-iterative, parallel loose coupling methods are applied predominantly, even though iterative and sequential approaches entail higher accuracy and better stability properties. Similarly, fixed macro steps are more frequently used than adaptive algorithms, which, as well as the preference to avoid rollback, may be explained by the implementational limitations of commonly known software tools that support co-simulation.

The visualization of co-authored publications by author and affiliation networks has unearthed huge differences in cluster sizes, which seem to be either very small and often even restricted to an inter-institutional research group, or eminently large and comprising many companies or universities from various countries.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

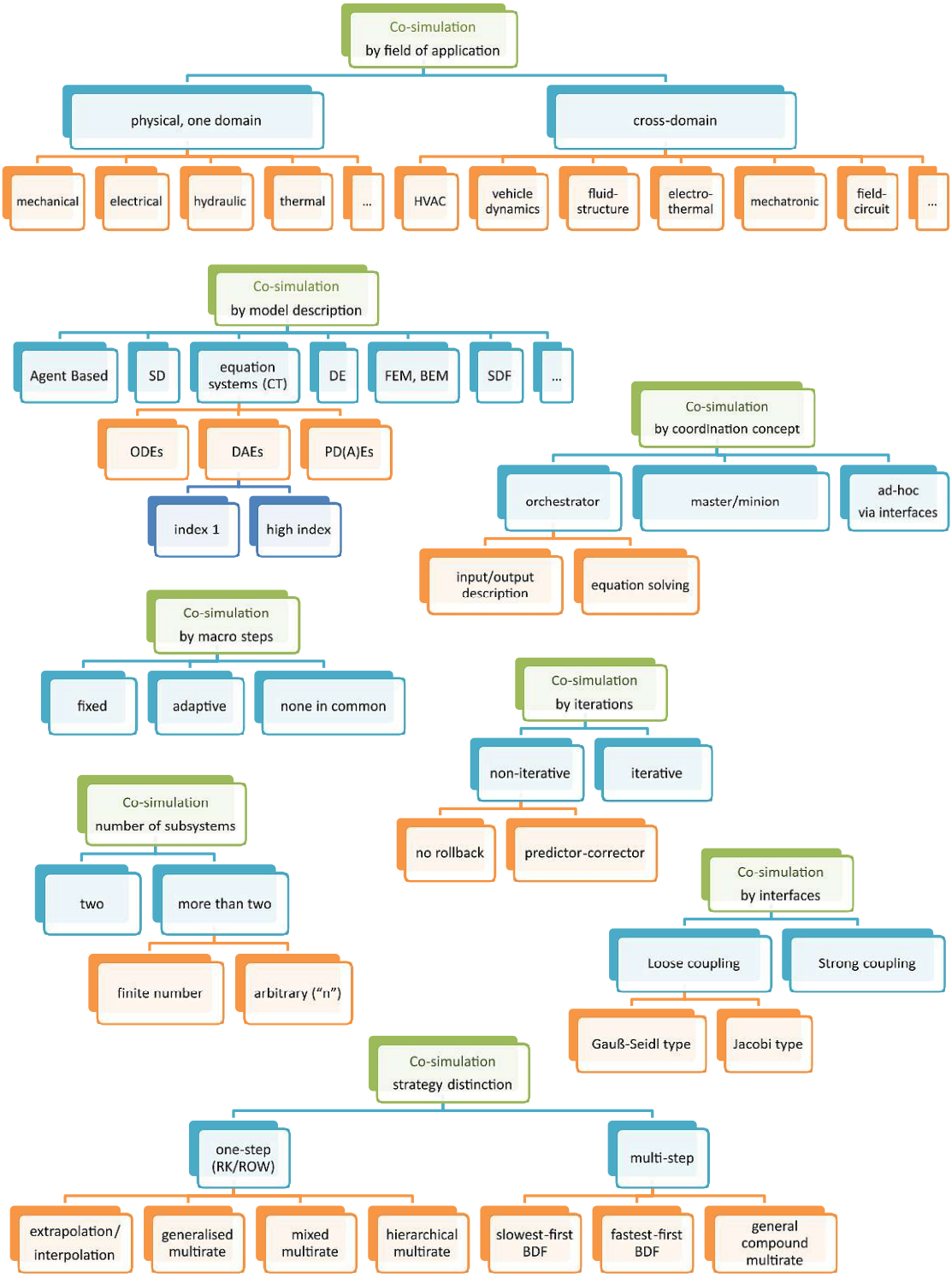


Figure 5.51: Summarizing illustration of the presented classification: different ways in which co-simulation methods can be structured.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

CHAPTER 6

# Hierarchical Co-Simulation

In this chapter, conventional co-simulation approaches, where one master algorithm orchestrates the synchronization of all participating subsystems simultaneously, are extended to allow hierarchical co-simulation, i.e. co-simulation on several levels, where certain subsystems are wrapped up in another, lower-level co-simulation. Parts of Sections 6.2.2 and 6.2.3 have been published in (Hafner and Popper 2020).

## 6.1 Motivation

While hierarchical structures occur in other areas of modeling and simulation, hierarchical co-simulation has not been properly investigated up to now. This section outlines the motivation and strategies for the introduction of further levels of hierarchy in co-simulation.

### 6.1.1 Hierarchical structures in modeling and simulation

Modularity and hierarchical approaches are no novelty in modeling and simulation in general, as the following references show.

Zeigler (2014) introduced the *Discrete Event System Specification* (DEVS), a formalism based on systems theory which allows the description of hierarchically structured discrete event systems, see also Section 3.3.3. The formalism enables the combination of several so-called *atomic DEVS* in a *coupled DEVS*, which can again be connected to other atomic or coupled DEVS, resulting in a hierarchical description of the considered system.



Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*Multi-agent systems* or *multi-level agent-based modeling* approaches introduce several levels of description in agent-based models. The individual agent-based models within a multi-level agent-based model (ML-ABM) can be based on different paradigms and represent different scales of the considered process (Morvan 2013; Servat et al. 1998). On the one hand, levels in ML-ABM can co-exist and interact if they describe hierarchically controlled parts. On the other hand, they can be activated and de-activated depending on the state of the system if different levels of detail (macroscopic and microscopic models) or heterogeneous modeling approaches (with respect to time representation or modeling paradigm) are required.

Similar to the latter case, dynamical systems requiring switches between different descriptions of the state space (due to behavioral changes, different levels of detail or removal or adding of components) are also known as *variable structure systems* or *structural-dynamic systems*. These can be dealt with in various ways, ranging from sequential execution in different simulators to parallel co-simulation approaches or preliminary simulation of auxiliary models for consistent initialization, see (Mehlhase 2015).

Alur et al. (2003) introduce a modeling language for the modular design and analysis of embedded hybrid systems. Encouraging the structuring of complex specifications by hierarchical design, the language supports both architectural hierarchy (parallel, communicating agents) and behavioral hierarchy (description of one agent by hierarchical sequential composition).

Hierarchical control architectures are common within *Model Predictive Control* (MPC). As centralized control is often difficult to apply to complex, large-scale systems with many interacting subsystems, various different control structures for decentralized, distributed and hierarchical MPC have been developed. A review and classification of these is given by Scattolini (2009).

Karnik et al. (1994) have developed a simulation tool for hierarchical VHDL (Very High Speed Integrated Circuit Hardware Description Language) descriptions, thereby extending the description to fulfil requirements for handling MOS circuits such as bi-directional flow. Every event in the circuit is evaluated hierarchically in contrast to other, single-level VHDL simulators where the system has to be flattened beforehand.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Mukherjee and Fedder (1999) present a hierarchical partitioning method for mixed-domain circuit simulation, where elements are represented on different levels to speed up the development process of microelectromechanical system components.

In the approach presented in (Yang and Becerik-Gerber 2015), calibration of energy consumption takes place on different levels simultaneously (building level, zone level etc.) to improve accuracy and robustness.

Most related to the topic at hand are *partitioned* or *split* methods as described in (Esposito and Kumar 2001; Günther and Rentrop 1994; Maten et al. 2005; Striebel 2006), also called *multirate schemes*. There, solution algorithms such as Runge-Kutta methods or Rosenbrock-Wanner schemes are partitioned to enable more active system parts to be executed with smaller time steps and latent parts with a multiple of these small steps, see also Section 3.5.

Structuring a complex system by creation of subclasses (as in object-oriented programming languages) or by wrapping parts of the system in subsystems (in graphical interfaces for block diagrams) on the modeling design level to allow a comprehensive view while no modification of the resulting simulation or the underlying solution algorithm takes place are of course common practice.

Nevertheless, hierarchical co-simulation as explained in the following has, to the best of my knowledge, not been investigated up to now, although several frameworks and standards do not prohibit the realization of further co-simulations within a co-simulation: Thule et al. (2019b) acknowledge the possibility of nested co-simulation in their specification of a domain specific language for master algorithms. They mention “Sub-co-simulations: (. . . ) co-simulation scenarios that have FMUs that may spawn a new co-simulation are constructed with a Hierarchical Cosim FMU.” This possibility, however, is just given as exemplary possibility for the adaption of master algorithms in their DSL and not investigated further with regards to stability or error accumulation. Neither so by Wang et al. (2003), who apply a gluing algorithm where coupled models contain coupled models.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### 6.1.2 Exposition and objective of hierarchical co-simulation

For the introduction of hierarchical co-simulation, we consider an overall system consisting of eight interacting partial systems. The reasons for the partition into these systems can vary from differing time constants or stiffness properties within the subsystems to the requirement for individual modeling approaches and consequently simulators (see also Chapter 1). Traditionally, synchronization between the subsystems is orchestrated by one co-simulation master algorithm as illustrated in Figure 6.1.

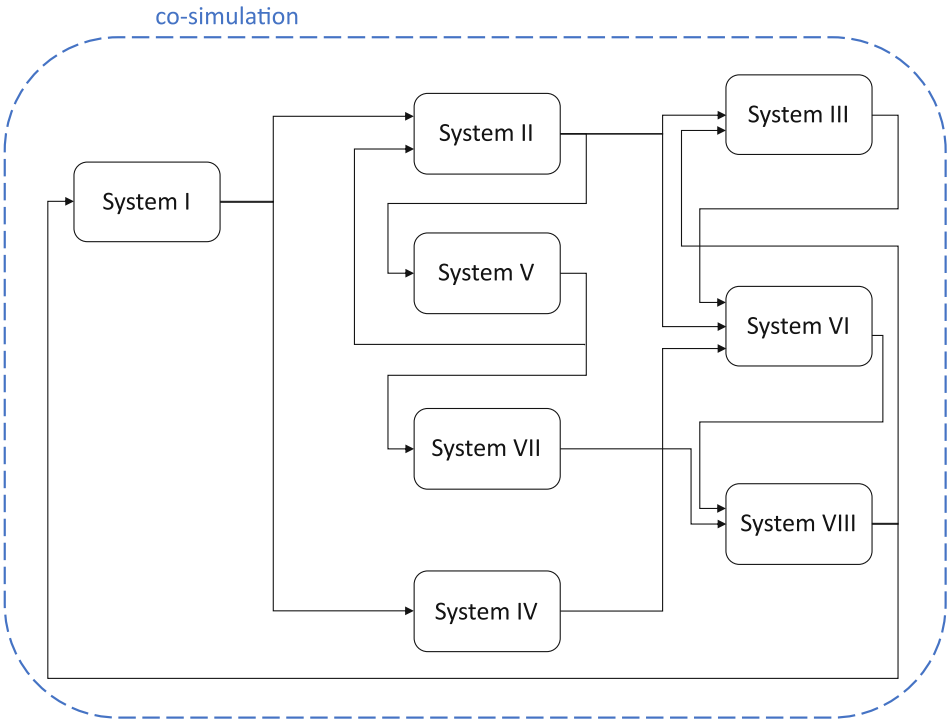


Figure 6.1: Schematic depiction of a traditional co-simulation example. Eight systems are coordinated by one master algorithm that manages the communication between all subsystems.

If the properties leading to the system partition mentioned above are highly diverse for the individual subsystems, some of these subsystems may require closer interaction than others. This motivates the introduction of one or more further co-simulations on a lower level, resulting in a hierarchical approach with several nested co-simulations, see Figure 6.2. Here, on the lowest level, Subsystems II and V are combined in a co-simulation (labelled Co-simulation 3 in the illustration). This co-simulated system exchanges values with Subsystem VII in another co-simulation (Co-simulation 2a). On the upmost level, Subsystems I and IV

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

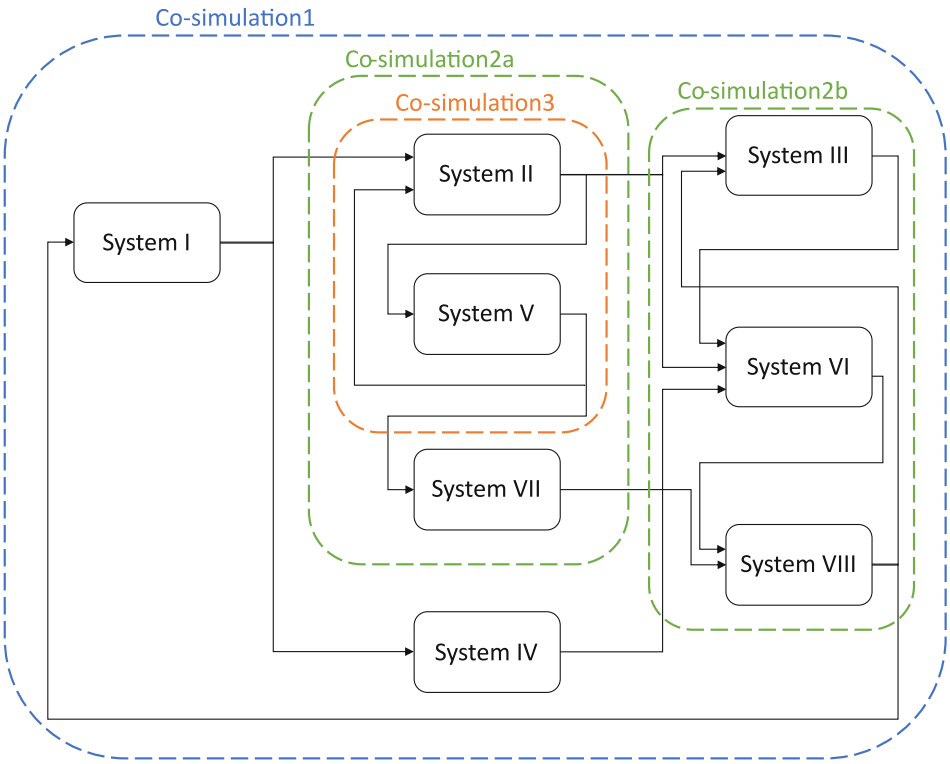


Figure 6.2: Schematic depiction of a hierarchical co-simulation approach. Coordination takes place on several levels by one top-level co-simulation that manages the communication between subsystems and further co-simulations. These may again coordinate subsystems and co-simulations on lower levels (Hafner and Popper 2020).

are synchronized with the systems resulting from Co-simulation 2a and Co-simulation 2b, in which Subsystems III, VI and VIII are coupled.

One of the most evident examples where it would be sensible to nest co-simulations this way is an application in which certain partial systems depend much more closely on values from one another than others (but have to be in separate simulations and therefore subsystems themselves due to highly differing modeling paradigms, for instance) and thus require more frequent data exchange. If this were handled by using a smaller time step for the original overall co-simulation, the whole simulation process would be slowed down unnecessarily. By the hierarchical approach, the more frequent synchronization between some subsystems can be achieved within the additional co-simulation.

In the example above, time steps could, for instance, be taken as illustrated in Figure 6.3 for the traditional, single-level co-simulation approach. The co-simulation synchronizes all

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

subsystems at the macro time steps depicted as blue, dotted lines; while in between, the individual subsystem solvers can take time individual steps (shown as small, grey ticks) depending on the dynamics of the respective subsystem. The additional depiction of subsystem steps taken at each synchronization reference, which is required by many master algorithms, is omitted for reasons of clearness.

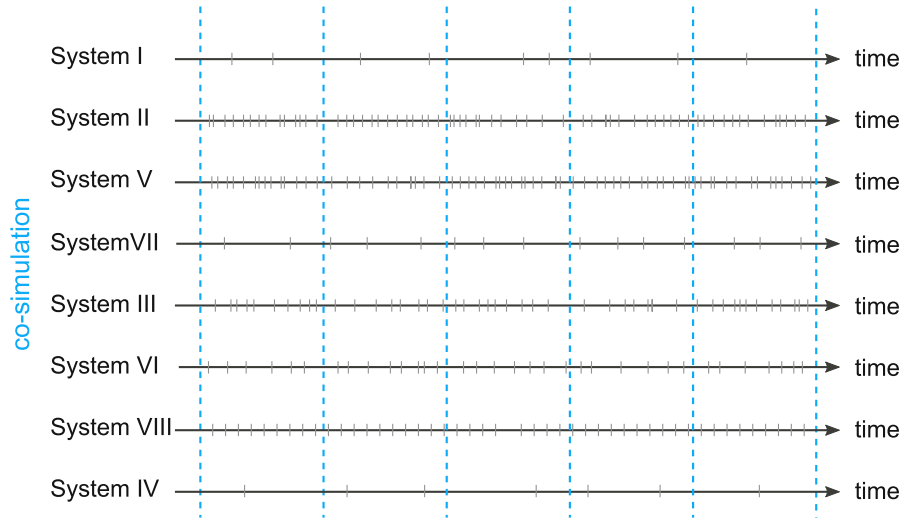


Figure 6.3: Illustration of steps taken in a traditional co-simulation approach. Micro steps are depicted in grey, macro steps (synchronization of all simulations) as dotted lines.

In Figure 6.4, possible time steps for the hierarchical approach are illustrated. Time steps taken by the subsystem solvers are again depicted as grey ticks. The most frequent synchronization takes place between Systems II and V in Co-simulation 3 (shown as orange, dotted lines). The latter is further on synchronized with System VII by Co-simulation 2a. Meanwhile, data exchange between Systems III, VI and VIII is scheduled independently by Co-simulation 2b (both green, dotted lines). All remaining subsystems and Co-simulations 2a and b synchronize at the time steps given by Co-simulation 1 (shown as blue, dotted lines). These can indeed be less frequent in comparison to the overall approach seen before and still yield more accurate results due to the additional exchange between more closely coupled subsystems, as will be shown in Section 6.2.3.

The motivation for hierarchical co-simulation by additional time steps alone may seem similar to the one for hierarchical multirate approaches. However, system parts may not only require different time steps but also different solution algorithms (f.i. implicit vs. explicit ODE solvers for stiff and non-stiff system parts) or modeling approaches, an issue which

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

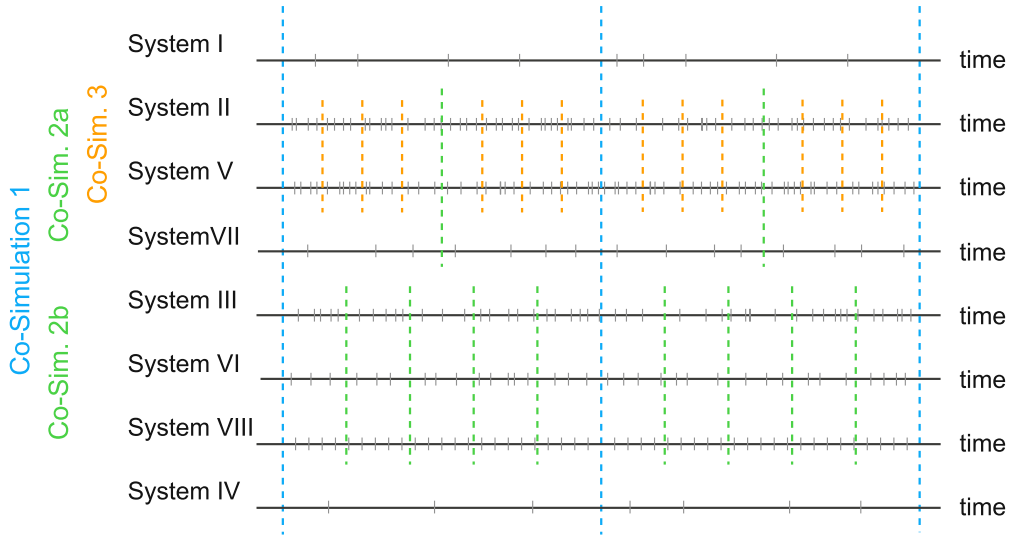


Figure 6.4: Illustration of steps taken in a hierarchical co-simulation approach: Individual communication step sizes are possible for every co-simulation. Micro steps are depicted in grey, macro steps on different levels as dotted lines.

cannot be met with partitioned methods. By applying the hierarchical co-simulation concept presented in this thesis, individual solution algorithms can be applied to the respective subsystems. In addition, these can be treated as black boxes as no inside information on the subsystems is required.

Application examples justifying a hierarchical approach of loose coupling co-simulation methods are large-scale physical systems, holistic simulation of industrial facilities, energy systems with varying time constants or even health systems, see Section 6.3.

## 6.2 Convergence theory

This section covers investigations on convergence of the proposed method, starting by considerations on mono-simulation and traditional, single-level co-simulation methods which are then extended on the hierarchical approach presented in the previous section.

### 6.2.1 Consistency

It has been shown in the literature that local error control is a valid method to bound the global co-simulation error (see f.i. (Arnold et al. 2014)). This justifies investigating the con-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

sistency error, i.e. the error of the method in one step, in a co-simulation. For this aim we need to start by calling to mind some background information on numerics of differential equations.

In the following, we consider a uniquely solvable ordinary differential equation IVP

$$\dot{\boldsymbol{x}} = f(t, \boldsymbol{x}), \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{6.1}$$

with Lipschitz continuous right side $f$ with respect to $\boldsymbol{x}$.

For a given approximation $\boldsymbol{x}_{t_n+h}$ of $\boldsymbol{x}$ at time $t_n + h$ by a numerical integration method with step size $h$, the consistency error is defined as the error of the method in one step and therefore, calculated by

$$\mathcal{E}(t_n, \boldsymbol{x}_n, h) = \boldsymbol{x}(t_n + h) - \boldsymbol{x}_{t_n+h}. \tag{6.2}$$

for given initial values $\boldsymbol{x}(t_n) = \boldsymbol{x}_n$. A method is called consistent if

$$\lim_{h\to 0} \left( \frac{\mathcal{E}(t_n, \boldsymbol{x}_n, h)}{h} \right) = \mathbf{0} \tag{6.3}$$

for every choice of $t_n, \boldsymbol{x}_n$. A method is called consistent of order $p$ if there exists a constant $C > 0$ with

$$\left\| \frac{\mathcal{E}(t_n, \boldsymbol{x}_n, h)}{h} \right\| \leq C \cdot h^p. \tag{6.4}$$

Since the consistency error is a measure for the *local* error of a method, state values are taken to be exact for all previous points in time.

*Remark* 6.1. In case they are not directly needed in the following calculations, the initial values $t_n, \boldsymbol{x}_n$ will be omitted in the notation of $\mathcal{E}$ to simplify the notation.

Important for the error estimates following below are Gronwall's Lemma (Theorem 6.2) and "the fundamental lemma" (Theorem 6.3).

*Theorem* 6.2 (Gronwall's Lemma (Thompson and Walter 2013)). *Let the real function* $m(t)$ *be continuous in* $J := [0, a]$*, and let*

$$m(t) \leq \alpha + \beta \int_0^t m(\tau) d\tau \quad \textit{in } J \textit{ with } \beta > 0$$

*then*

$$m(t) \leq \alpha e^{\beta t} \quad \textit{in } J.$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*Theorem* 6.3 (The "fundamental lemma" (Hairer et al. 1993)). *Supposing that* $\boldsymbol{x}(t)$ *is a solution of the system of differential equations 6.1 with* $f$ *Lipschitz continuous in the second argument with Lipschitz constant* $L$*, and* $\boldsymbol{v}(t)$ *an approximate solution fulfilling*

$$\|\dot{\boldsymbol{v}}(t) - f(t, \boldsymbol{v}(t))\| \leq \epsilon,$$

*then, for* $t \geq t_0$*, we have the error estimate*

$$\|\boldsymbol{x}(t) - \boldsymbol{v}(t)\| \leq \|\boldsymbol{x}(t_0) - \boldsymbol{v}(t_0)\| \, e^{L(t-t_0)} + \frac{\epsilon}{L}\left(e^{L(t-t_0)} - 1\right).$$

*Remark* 6.4. If $\boldsymbol{v}$ is also an exact solution of $\dot{\boldsymbol{x}} = f(t, \boldsymbol{x})$, from Theorem 6.3 follows

$$\|\boldsymbol{x}(t) - \boldsymbol{v}(t)\| \leq \|\boldsymbol{x}(t_0) - \boldsymbol{v}(t_0)\| \, e^{L(t-t_0)},$$

which directly implies that in case of the same initial values, $\boldsymbol{v}$ is identical to $\boldsymbol{x}$.

#### 6.2.1.1 Consistency in co-simulation

To investigate consistency in co-simulation, we consider the decomposition of (6.1) into a system of coupled ODEs (6.5):

$$\dot{\boldsymbol{x}}^i(t) = \boldsymbol{f}^i(\boldsymbol{x}^i, \boldsymbol{u}^i, t), \quad \boldsymbol{x}^i(t_0) = \boldsymbol{x}_0^i \tag{6.5a}$$

with $i = I, \ldots, N$, $\boldsymbol{x}^i \in \mathbb{R}^{n_x^i}$, $\boldsymbol{u}^i \in \mathbb{R}^{n_u^i}$, and

$$\boldsymbol{u}^i = \boldsymbol{L}^i \boldsymbol{x} = \begin{bmatrix} \boldsymbol{L}^{i,I} & \ldots & \boldsymbol{L}^{i,i-1} & 0 & \boldsymbol{L}^{i,i+1} & \ldots & \boldsymbol{L}^{i,N} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}^I \\ \vdots \\ \boldsymbol{x}^{i-1} \\ \boldsymbol{x}^i \\ \boldsymbol{x}^{i+1} \\ \vdots \\ \boldsymbol{x}^N \end{bmatrix} \tag{6.5b}$$

with $\boldsymbol{L}^{i,j} \in \mathbb{R}^{n_u^i \times n_x^j} \quad \forall i, j \in \{I, \ldots, N\}$ and the elements of $\boldsymbol{L}^{i,j}$ being equal to zero or one. Thereby, we assume again a unique solution and Lipschitz continuous right-side functions $\boldsymbol{f}^i$ in the first and second argument.

*Remark* 6.5. Notation with elements of $\mathbb{G} := \{I, II, \ldots\}$ is used to avoid confusion with exponents and allow easy identification of subsystems. In arithmetic operations where elements of $\mathbb{G}$ and $\mathbb{N}$ are mingled, these are to be understood as operations between elements of $\mathbb{N}$ by assigning every element of $\mathbb{G}$ its image under the bijection that uniquely assigns the $i$-th element of $\mathbb{G}$ the $i$-th element of $\mathbb{N}$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

In the following, investigations on convergence of traditional co-simulation analogously as given by Knorr (2002)[1] are presented and extended on hierarchical approaches in Section 6.2.1.2. We start by considering the $i$-th subsystem of (6.5). In case of a multirate co-simulation, values $\boldsymbol{u}^i$ have to be extrapolated in between two synchronization time steps and will be named $\tilde{\boldsymbol{u}}^i$. Further, $\boldsymbol{x}^i(t)$ will denote the exact solution of (6.5a) and $\tilde{\boldsymbol{x}}^i(t)$ the exact solution of

$$\dot{\boldsymbol{x}}^i(t) = \boldsymbol{f}^i(\boldsymbol{x}^i, \tilde{\boldsymbol{u}}^i, t), \quad \boldsymbol{x}^i(t_0) = \boldsymbol{x}_0^i. \tag{6.6}$$

The approximated solution of (6.6) at $t_{n,k}$ will be named $\tilde{\boldsymbol{x}}_{n,k}$.

To begin with, we regard the error $\mathcal{E}^i(t_{n,k}, \boldsymbol{x}_{n,k}, h_i)$ of the $i$-th subsystem in one micro step $h_i$ at $t_{n,k}$, where $n$ is the current macro step and $k$ the current micro step, counted anew for each macro interval. Thus $t_{n+1} := t_{n+1,0} = t_{n,m_i} = t_n + m_i \cdot h_i = t_n + H$ in case of $m_i$ micro steps per macro step, hence $m_i$ denoting the multirate factor of subsystem $i$ in case of fixed, equidistant micro steps which are integer divisors of the (also fixed) macro step size $H$, which we will assume w.l.o.g.[2] in the following calculations.

Starting with the consistency of the integration of every subsystem for one micro step, we will deduce consistency of the integration of every subsystem for one macro step and further of the co-simulation (cf. (Knorr 2002)[1]). Lemma 6.6 shows that the consistency order for one subsimulation depends on the original method as well as the extrapolation order.

*Lemma* 6.6 (Consistency error for one micro step). *Let $p_i$ denote the consistency order of the original method and $q_i$ the order of extrapolation for input values $\boldsymbol{u}^i$. Then*

$$\left\| \frac{\mathcal{E}^i(t_{n,k}, \boldsymbol{x}_{n,k}, h_i)}{h_i} \right\| = \mathcal{O}\left( h_i^{\min\{p_i, q_i+1\}} \right). \tag{6.7}$$

*Proof.* Considering exact values at $t_{n,k}$, per definition

$$\left\| \mathcal{E}^i(t_{n,k}, \boldsymbol{x}_{n,k}, h_i) \right\| = \left\| \boldsymbol{x}^i(t_{n,k} + h_i) - \tilde{\boldsymbol{x}}^i_{n,k+1} \right\| = \left\| \boldsymbol{x}^i(t_{n,k+1}) - \tilde{\boldsymbol{x}}^i_{n,k+1} \right\| \tag{6.8}$$

[1] The investigations of Knorr are restricted to two participating subsystems where the larger micro step size is also taken as macro step size. Following her strategy, we allow an arbitrary number of participating subsystems and macro step size $H$ with the possibility of $H > h_i$ for all subsystem solver step sizes $h_i$ in this work.

[2] All considerations can be performed analogously for unequally distanced grids with $h_i$ taken as upper bound of all $h_{i_j}$ with $i_j \in \{1, \ldots, m_{i_n}\}$ and $m_{i_n}$ the number of micro steps of subsystem $i$ in the $n$-th macro step. However, as this would only lead to more complex notation, we will restrict the step sizes as described above for reasons of clarity.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

with the notation described above. Adding and subtracting $\tilde{\boldsymbol{x}}(t_{n,k+1})$ gives

$$\left\|\mathcal{E}^i(t_{n,k}, \boldsymbol{x}_{n,k}, h_i)\right\| = \left\|\boldsymbol{x}^i(t_{n,k+1}) - \tilde{\boldsymbol{x}}^i(t_{n,k+1}) + \tilde{\boldsymbol{x}}^i(t_{n,k+1}) - \tilde{\boldsymbol{x}}^i_{n,k+1}\right\| \tag{6.9}$$

$$\overset{\text{triangle inequ.}}{\leq} \left\|\boldsymbol{x}^i(t_{n,k+1}) - \tilde{\boldsymbol{x}}^i(t_{n,k+1})\right\| + \underbrace{\left\|\tilde{\boldsymbol{x}}^i(t_{n,k+1}) - \tilde{\boldsymbol{x}}^i_{n,k+1}\right\|}_{\leq C_{i,1}\cdot h_i^{p_i+1}}. \tag{6.10}$$

The second term of (6.10) is the difference of the exact to the approximated solution of the modified system (6.6) and is therefore bounded by $C_{i,1} \cdot h_i^{p_i+1}$ for a constant $C_{i,1} > 0$ and with $p_i$ being the order of the numerical integration method given for system $i$.
To provide an estimate for the first term in (6.10), we use the assumption that $\boldsymbol{x}(t)$ and $\tilde{\boldsymbol{x}}(t)$ are the exact solutions of (6.5a) and (6.6), respectively, and can therefore be replaced by the integral over their derivatives (since they fulfill conditions like uniqueness, continuity, and differentiability by definition):

$$\begin{aligned}\left\|\boldsymbol{x}^i(t_{n,k+1}) - \tilde{\boldsymbol{x}}(t_{n,k+1})\right\| &= \left\|\int_{t_{n,k}}^{t_{n,k+1}} \left(f^i(\boldsymbol{x}^i, \boldsymbol{u}^i, \tau) - f(\tilde{\boldsymbol{x}}^i, \tilde{\boldsymbol{u}}^i, \tau)\right) d\tau\right\| \\ &\leq \int_{t_{n,k}}^{t_{n,k+1}} \left\|f^i(\boldsymbol{x}^i, \boldsymbol{u}^i, \tau) - f(\tilde{\boldsymbol{x}}^i, \tilde{\boldsymbol{u}}^i, \tau)\right\| d\tau\end{aligned} \tag{6.11}$$

Adding and subtracting $f(\tilde{\boldsymbol{x}}^i, \boldsymbol{u}^i, \tau)$ gives with the triangle inequality

$$(6.11) \leq \int_{t_{n,k}}^{t_{n,k+1}} \left\|f^i(\boldsymbol{x}^i, \boldsymbol{u}^i, \tau) - f(\tilde{\boldsymbol{x}}^i, \boldsymbol{u}^i, \tau)\right\| d\tau + \int_{t_{n,k}}^{t_{n,k+1}} \left\|f^i(\tilde{\boldsymbol{x}}^i, \boldsymbol{u}^i, \tau) - f(\tilde{\boldsymbol{x}}^i, \tilde{\boldsymbol{u}}^i, \tau)\right\| d\tau \tag{6.12}$$

$$\overset{\text{Lipschitz}}{\leq} \int_{t_{n,k}}^{t_{n,k+1}} L_{f^i,x} \left\|\boldsymbol{x}^i - \tilde{\boldsymbol{x}}^i\right\| d\tau + \underbrace{\int_{t_{n,k}}^{t_{n,k+1}} L_{f^i,u} \underbrace{\left\|\boldsymbol{u}^i - \tilde{\boldsymbol{u}}^i\right\|}_{\leq C_{i,2}\cdot h_i^{q_i+1}} d\tau}_{\leq L_{f^i,u}\cdot C_{i,2}\cdot h_i^{q_i+2}} \tag{6.13}$$

with Lipschitz constants $L_{f^i,x}$ and $L_{f^i,u}$ of $f^i$ with respect to $\boldsymbol{x}$ and $\boldsymbol{u}$, respectively, and $q_i$ denoting the order of the extrapolation method for the approximation of $\tilde{\boldsymbol{u}}^i$. Declaring $C_{i,3} := L_{f^i,u} \cdot C_{i,2}$ and $\boldsymbol{m}(t) := \left\|\boldsymbol{x}^i(t) - \tilde{\boldsymbol{x}}^i(t)\right\|$, above estimates can be summarized as

$$\boldsymbol{m}(t_{n,k+1}) \leq \int_{t_{n,k}}^{t_{n,k+1}} L_{f^i,x} \left\|\boldsymbol{x}^i - \tilde{\boldsymbol{x}}^i\right\| d\tau + C_{i,3} \cdot h_i^{q_i+2}. \tag{6.14}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Now we can apply the Lemma of Gronwall (Theorem 6.2) to $\boldsymbol{m}$ with $\alpha = C_{i,3} \cdot h_i^{q_i+2}$ and $\beta = L_{f^i,x}$ and obtain

$$\boldsymbol{m}(t_{n,k+1}) \leq C_{i,3} \cdot h_i^{q_i+2} \cdot \underbrace{e^{L_{f^i,x} \cdot \overbrace{(t_{n,k+1} - t_{n,k})}^{h_i}}}_{=\sum_{j=0}^{\infty} \frac{(L_{f^i,x} \cdot h_i)^j}{j!}} = \mathcal{O}\left(h_i^{q_i+2}\right) \tag{6.15}$$

and therefore

$$\left\| \frac{\mathcal{E}^i(t_{n,k}, \boldsymbol{x}_{n,k}, h_i)}{h_i} \right\| \overset{(6.10),(6.15)}{\leq} C_{i,1} \cdot h_i^{p_i} + \mathcal{O}\left(h_i^{q_i+1}\right) = \mathcal{O}\left(h_i^{\min\{p_i, q_i+1\}}\right). \tag{6.16}$$

□

This shows that while consistency is maintained in co-simulation, the order may be reduced if the extrapolation order is chosen too low. Constant extrapolation, for example, only maintains the order of integration methods of order one. For higher-order methods, the order is reduced but the method remains consistent (as $\left\| \frac{\mathcal{E}^i(t_{n,k}, \boldsymbol{x}_{n,k}, h_i)}{h_i} \right\|$ still converges to zero, but only linearly). However, higher order extrapolation can also lead to increased stability issues, which is shown for example in (Arnold 2010).

Next, we want to estimate the error for the integration of one subsystem per macro step.

*Lemma* 6.7 (Consistency error per subsystem for one macro step). *With the notations above*

$$\left\| \frac{\mathcal{E}^i(t_n, \boldsymbol{x}_n, H)}{H} \right\| = \mathcal{O}\left(H^{\min\{p_i, q_i+1\}}\right). \tag{6.17}$$

*Proof.* To extend the considerations for one micro step to one macro step, we will employ the method of "Lady Windermere's Fan", which, apart from (Knorr 2002), is also shown f.i. in (Hairer et al. 1993) and named after the eponymous play of Oscar Wilde. The main idea of this approach is to describe the error of the approximate solution after an interval – in our case, a macro step – by the analytical solutions at every point of a refined mesh – in our case, every micro step – assuming an exact value at the beginning of the considered interval. This is illustrated for a one-dimensional problem in Figure 6.5.
Let $\boldsymbol{w}_{n,k}^i(t), k = 0, \dots m_i$ denote the exact solution of system (6.6) but for the initial values $\boldsymbol{w}_{n,k}^i(t_{n,k}) = \tilde{\boldsymbol{x}}_{n,k}$, implying $\boldsymbol{w}_{n,0}^i(t) = \boldsymbol{x}^i(t)\ \forall t > t_n$ since we assume exact values at $t_{n,0}$. Then we can write

$$\left\| \mathcal{E}^i(t_n, \boldsymbol{x}_n, H) \right\| = \left\| \boldsymbol{x}^i(t_{n+1}) - \tilde{\boldsymbol{x}}_{n+1}^i \right\| = \left\| \boldsymbol{x}^i(t_{n,m_i}) - \tilde{\boldsymbol{x}}_{n,m_i}^i \right\| \tag{6.18}$$

$$\leq \sum_{k=0}^{m_i-1} \left\| \boldsymbol{w}_{n,k}^i(t_{n,m_i}) - \boldsymbol{w}_{n,k+1}^i(t_{n,m_i}) \right\|. \tag{6.19}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

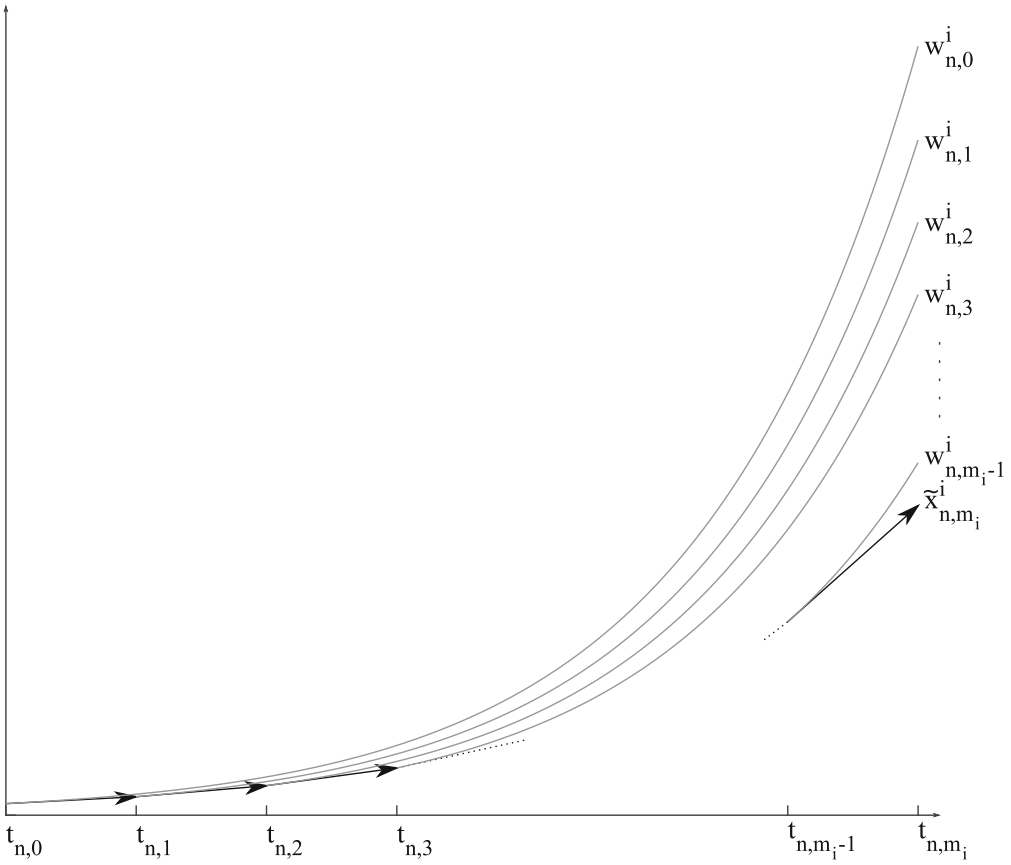


Figure 6.5: "Lady Windermere's Fan": exact solutions at every time step of the approximate solution are used to describe the error of the approximate solution in one macro step (after Knorr (2002)).

Since $\boldsymbol{w}^i_{n,k}$ are solutions to the same system with different initial values, we can apply Theorem 6.3 and obtain for every summand

$$\left\|\boldsymbol{w}^i_{n,k}(t_{n,m_i}) - \boldsymbol{w}^i_{n,k+1}(t_{n,m_i})\right\| \leq \left\|\boldsymbol{w}^i_{n,k}(t_{n,k+1}) - \boldsymbol{w}^i_{n,k+1}(t_{n,k+1})\right\| \cdot e^{L_{f^i,x}\cdot\overbrace{(t_{n,m_i}-t_{n,k+1})}^{(m_i-k-1)h_i}} \tag{6.20}$$

and thus

$$\left\|\mathcal{E}^i(t_n,\boldsymbol{x}_n,H)\right\| \leq \sum_{k=0}^{m_i-1} \left\|\boldsymbol{w}^i_{n,k}(t_{n,k+1}) - \boldsymbol{w}^i_{n,k+1}(t_{n,k+1})\right\| \cdot e^{L_{f^i,x}\cdot(m_i-k-1)h_i} \tag{6.21}$$

$$\overset{\boldsymbol{w}^i_{n,k}(t_{n,k})=\tilde{\boldsymbol{x}}_{n,k}}{=} \sum_{k=0}^{m_i-1} \left\|\boldsymbol{w}^i_{n,k}(t_{n,k+1}) - \tilde{\boldsymbol{x}}^i_{n,k+1}\right\| \cdot e^{L_{f^i,x}\cdot(m_i-k-1)h_i}. \tag{6.22}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

As $\left\| \boldsymbol{w}^i_{n,k}(t_{n,k+1}) - \boldsymbol{x}^i_{n,k+1} \right\|$ is the error in one micro step, according to Lemma 6.6 we can estimate this term with $\mathcal{O}(h_i^{\min\{p_i+1,q_i+2\}})$.
Therefore

$$\begin{aligned}
\left\| \mathcal{E}^i(t_n, \boldsymbol{x}_n, H) \right\| \leq & \mathcal{O}\left(h_i^{\min\{p_i+1,q_i+2\}}\right) \sum_{k=0}^{m_i-1} e^{L_{f^i,x}\cdot(m_i-k-1)h_i} && (6.23)\\
\leq & \mathcal{O}\left(h_i^{\min\{p_i+1,q_i+2\}}\right) \cdot m_i \cdot e^{L_{f^i,x}\cdot(m_i-1)h_i} && (6.24)\\
\overset{h_i=H/m_i}{=} & \mathcal{O}\left(\left(\frac{H}{m_i}\right)^{\min\{p_i+1,q_i+2\}}\right) \cdot m_i \cdot e^{L_{f^i,x}\cdot\frac{m_i-1}{m_i}H} && (6.25)\\
= & \mathcal{O}\left(H^{\min\{p_i+1,q_i+2\}}\right) && (6.26)\\
\Rightarrow \left\| \frac{\mathcal{E}^i(t_n, \boldsymbol{x}_n, H)}{H} \right\| = & \mathcal{O}\left(H^{\min\{p_i,q_i+1\}}\right). && (6.27)
\end{aligned}$$

□

*Corollary* 6.8 (Consistency error of co-simulation)*. With the notations above, consistency of the co-simulation in one macro step can be determined by*

$$\left\| \frac{\mathcal{E}(t_n, \boldsymbol{x}_n, H)}{H} \right\| = \mathcal{O}\left(H^{\min\limits_{i=I,\dots,N}\{p_i,q_i+1\}}\right). \tag{6.28}$$

*Proof.* Since we have

$$\boldsymbol{x}(t) = \begin{bmatrix} \boldsymbol{x}^I(t) \\ \vdots \\ \boldsymbol{x}^{i-1}(t) \\ \boldsymbol{x}^i(t) \\ \boldsymbol{x}^{i+1}(t) \\ \vdots \\ \boldsymbol{x}^N(t) \end{bmatrix} \tag{6.29}$$

(see (6.5b)), the approximation of the overall system at a synchronization point $t_{n+1}$ is given by the concatenation of the approximations of the states of the $N$ individual subsystems, i.e.

$$\tilde{\boldsymbol{x}}_{n+1} = \begin{bmatrix} \tilde{\boldsymbol{x}}^I_{n+1} \\ \vdots \\ \tilde{\boldsymbol{x}}^{i-1}_{n+1} \\ \tilde{\boldsymbol{x}}^i_{n+1} \\ \tilde{\boldsymbol{x}}^{i+1}_{n+1} \\ \vdots \\ \tilde{\boldsymbol{x}}^N_{n+1} \end{bmatrix}. \tag{6.30}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

With this, we can simply infer

$$\|\mathcal{E}(t_n, \boldsymbol{x}_n, H)\| = \|\boldsymbol{x}(t_{n+1}) - \tilde{\boldsymbol{x}}_{n+1}\| \leq \sum_{i=0}^{N} \left\|\boldsymbol{x}^i(t_{n+1}) - \tilde{\boldsymbol{x}}^i_{n+1}\right\| \tag{6.31}$$

$$\leq N \cdot \mathcal{O}\left(H^{\min\limits_{i=1\dots N}\{p_i+1, q_i+2\}}\right) = \mathcal{O}\left(H^{\min\limits_{i=1\dots N}\{p_i+1, q_i+2\}}\right) \tag{6.32}$$

with the estimates from Lemma 6.7.

$$\Rightarrow \left\|\frac{\mathcal{E}(t_n, \boldsymbol{x}_n, H)}{H}\right\| = \mathcal{O}\left(H^{\min\{p_i, q_i+1\}}\right). \tag{6.33}$$

□

These estimates show that while overall consistency is maintained in the co-simulation of ODE systems, the convergence order may be reduced in case of lower-order extrapolation of input values. Higher order extrapolation, while enhancing the order of consistency of the coupled method (bounded by the order of the original integration method), can also lead to increased stability issues, as shown f.i. in (Arnold 2007, 2010).
For DAEs that are only coupled via differential variables, the implicit function theorem (see f.i. (Zeidler 2013)) implies that locally, an equivalent ODE system can be found for which above considerations also apply. In case of coupling via algebraic variables, similar estimates are given f.i. in (Arnold and Günther 2001).

#### 6.2.1.2 Consistency in hierarchical co-simulation

Now we want to extend above deliberations to co-simulation on several levels of hierarchy. As already explained, consistency is defined locally (i.e. per step), and what is more is that it is a property regarded for the limit $h \to 0$. Section 6.2.1.1 has shown that apart from consistency of the original integration method, consistency of the co-simulation only depends on the error introduced by extrapolation of external input values – and this, again, per step. As this property is not affected by the method used in the respective other subsystems or the time steps and further synchronizations happening there in-between, this already suggests that consistency in hierarchical co-simulation is also maintained with its order depending on the applied extrapolation method. For detailed estimation, we will first consider the simplest case to which a hierarchical co-simulation can be applied: Three subsystems of which w.l.o.g. Systems $II$ and $III$ are co-simulated on the lowest level and this co-simulation communicates again on the topmost level with the simulation of System $I$, as illustrated in Figure 6.6.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

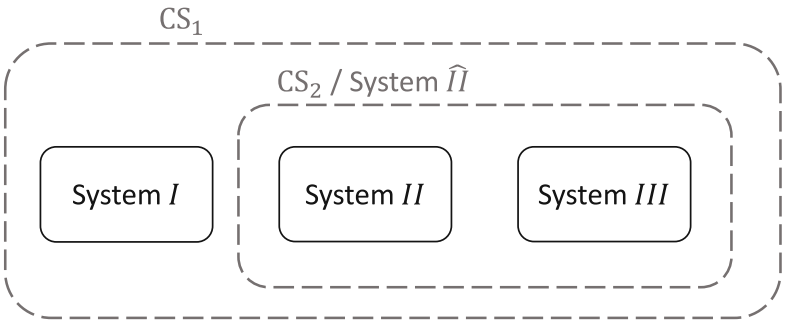


Figure 6.6: Illustration of hierarchical co-simulation of three systems on two levels. Co-simulation $CS_1$ coordinates System $I$ and System $\widehat{II}$, i.e. co-simulation $CS_2$, which manages the communication between systems $II$ and $III$.

The co-simulation between Systems $II$ and $III$ will be called $CS_2$ henceforth, and the corresponding system seen from the perspective of the upper level System $\widehat{II}$. The top-level co-simulation ($CS_1$) macro step will be denoted $H_1$ and the second-level co-simulation macro step $H_2$. For $CS_1$, we start with the error in one macro step $H_1$ of System $I$, for which we obtain from Lemma 6.7

$$\left\| \frac{\mathcal{E}^I(H_1)}{H_1} \right\| = \mathcal{O}\left(H_1^{\min\{p_I, q_I+1\}}\right). \tag{6.34}$$

For System $\widehat{II}$, we start by applying Corollary 6.8 to $CS_2$, which yields for one step of size $H_2$

$$\left\| \frac{\mathcal{E}^{\widehat{II}}(H_2)}{H_2} \right\| = \mathcal{O}\left(H_2^{\min\limits_{i=II,III}\{p_i, q_i+1\}}\right). \tag{6.35}$$

To estimate the error in one macro step $H_1$, we can repeat the strategy from the proof of Lemma 6.7 with $M_2$ describing the quotient of $H_1$ and $H_2$ and obtain

$$\left\| \frac{\mathcal{E}^{\widehat{II}}(H_1)}{H_1} \right\| = \mathcal{O}\left(H_1^{\min\limits_{i=II,III}\{p_i, q_i+1\}}\right) \tag{6.36}$$

and further for the top-level co-simulation $CS_1$ with (6.34), (6.36) and Corollary 6.8

$$\left\| \frac{\mathcal{E}(H_1)}{H_1} \right\| = \mathcal{O}\left(H_1^{\min\limits_{i=I,II,III}\{p_i, q_i+1\}}\right) \tag{6.37}$$

and therefore consistency. The order again depends on the extrapolation and consistency orders of all subsystems. This can also be concluded for arbitrary levels of hierarchy and participating subsystems, as Theorem 6.9 shows.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*Theorem* 6.9 (Consistency error of hierarchical co-simulation). *In a hierarchical co-simulation with a total of $N$ participating subsystems, consistency orders $p_i$, $i = I, \ldots, N$ of their corresponding integration algorithms and extrapolation orders $q_i$, $i = I, \ldots, N$, the consistency error of the overall co-simulation with macro step $H$ can be estimated as*

$$\left\| \frac{\mathcal{E}(H)}{H} \right\| = \mathcal{O}\left( H^{\min\limits_{i=I,\ldots,N}\{p_i, q_i+1\}} \right). \tag{6.38}$$

*Proof.* To begin with, we need to establish comprehensible notation of all considered systems, co-simulations, and step sizes. For this purpose, all participating simulations are depicted in a tree structure, see Figure 6.7.

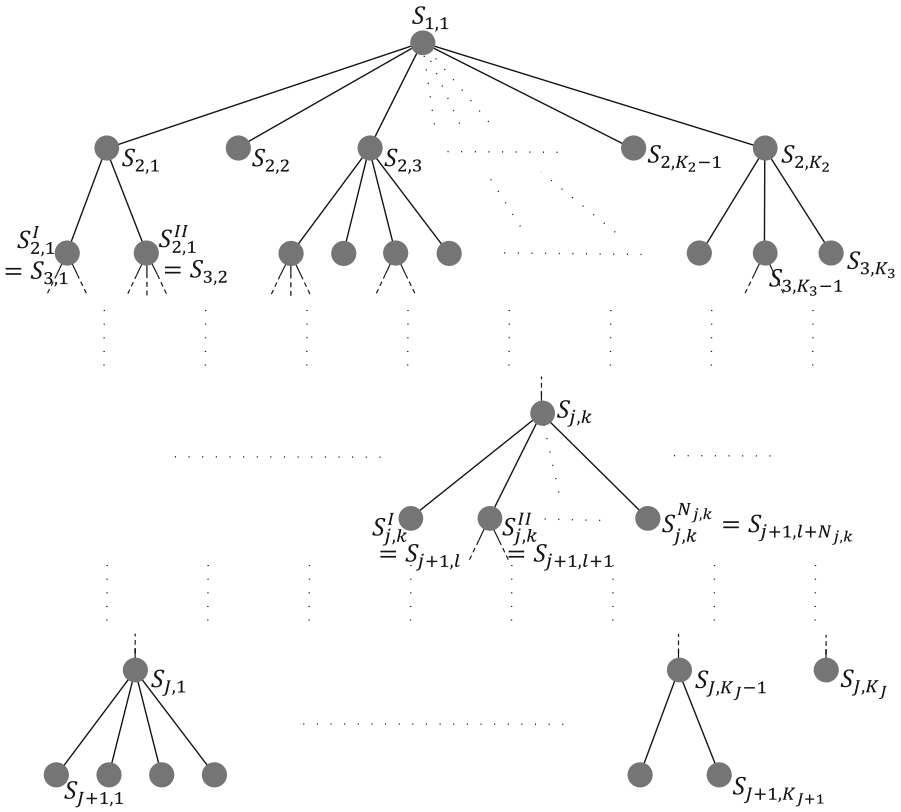


Figure 6.7: Illustration of the co-simulation hierarchy in a tree structure.

We will start from the topmost level, naming the overall co-simulation $S_{1,1}$. Beneath $S_{1,1}$, all further simulations enfold on $J$ levels in total. On every level $j \in 1, \ldots, J+1$ all simulations – be they co-simulations themselves or "leaf" nodes without further branching beneath – are numbered from $1$ to $K_j$. This means that on level $j$, we find simulations $S_{j,k}$ with $k = 1 \ldots K_j$. While the ordering of these may be arbitrary, this notation is necessary to

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

uniquely identify every co-simulation on every level in a fairly intelligible notation. Nevertheless, to clarify the belonging to the respective co-simulation, the sub-simulations of one node, i.e. all $N_{j,k}$ simulations coordinated by one co-simulation $S_{j,k}$ may be identified by $S^I_{j,k}, S^{II}_{j,k}, \ldots, S^{N_{j,k}}_{j,k}$ in addition. This means that the $i-$th subsimulation of $S_{j,k}$ may be called $S^i_{j,k}$ and equals, using the notation on the next level, $S_{j+1,l}$ for one $l \in \{1, \ldots, K_{j+1}\}$:

$$S^i_{j,k} = S_{j+1,l} \quad \text{for } l = i + \sum_{m=1}^{k-1} N_{j,m} \tag{6.39}$$

Note that naturally, the sum of all simulations that are co-simulated by simulations on level $j$ equals the number of simulations on level $j+1$:

$$\sum_{k=1}^{K_j} N_{j,k} = K_{j+1} \tag{6.40}$$

with the convention that for leaf nodes, $N_{j,k} := 0$.

In analogy to above example with three systems co-simulated on two levels, (6.38) follows from Lemmata 6.6, 6.7 and Corollary 6.8 when approached bottom-up with induction.

On the deepest level $J+1$, we only have leaf nodes. These systems $S_{J+1,l}, l = 1, \ldots, K_{J+1}$ are integrated with their individual time step $h_{J+1,l}$ and are coordinated by a co-simulation on level $J$. By considering one of these co-simulations $S_{J,k}$ with macro step size $H_{J,k}$ and its sub-simulations denoted as $S^i_{J,k}, i = I, \ldots, N_{J,k}$ , we know from Lemma 6.7 that for every $S^i_{J,k}$, the error per macro step can be estimated via

$$\left\| \frac{\mathcal{E}^i_{J,k}(H_{J,k})}{H_{J,k}} \right\| = \mathcal{O}\left( {H_{J,k}}^{\min\{p_{i_{J,k}}, q_{i_{J,k}}+1\}} \right) \tag{6.41}$$

with $p_{i_{J,k}}$ denoting the consistency order of the integration method of $S^i_{J,k}$ and $q_{i_{J,k}}$ the respective extrapolation order for external input values.
With Corollary 6.8 follows for the consistency order of $S_{J,k}$

$$\left\| \frac{\mathcal{E}_{J,k}(H_{J,k})}{H_{J,k}} \right\| = \mathcal{O}\left( H_{J,k}^{\min\limits_{i=I,\ldots,N_{J,k}} \{p_{i_{J,k}}, q_{i_{J,k}}+1\}} \right). \tag{6.42}$$

For every leaf simulation $S_{J,k}$ on level $J$ with micro step size $h_{J,k}$, we obtain an estimate for the error per micro step with Lemma 6.6:

$$\left\| \frac{\mathcal{E}_{J,k}(h_{J,k})}{h_{J,k}} \right\| = \mathcal{O}\left( h_{J,k}^{\min\{p_{J,k}, q_{J,k}+1\}} \right) \tag{6.43}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

for $p_{J,k}$ and $q_{J,k}$ again denoting the corresponding consistency and extrapolation order, respectively. As the indexing is unique, we can without confusion with some co-simulation declare $H_{J,k} := h_{J,k}$ and therefore in summary write the estimate for every simulation – cooperative as well as leaf simulation – on level $J$ as

$$\left\| \frac{\mathcal{E}_{J,k}(H_{J,k})}{H_{J,k}} \right\| = \mathcal{O}\left( H_{J,k}^{\min\{p_{J,k}, q_{J,k}+1\}} \right) \tag{6.44}$$

when for co-simulation nodes, we define $p_{J,k} := \min_{i=I,\dots,N_{J,k}} \{p_{i_{J,k}}\}$ and $q_{J,k} := \min_{i=I,\dots,N_{J,k}} \{q_{i_{J,k}}\}$.

In the next step, we will assume this estimate for every simulation on a level $j+1$, $j \in \{1,\dots,J\}$:

$$\left\| \frac{\mathcal{E}_{j+1,k}(H_{j+1,k})}{H_{j+1,k}} \right\| = \mathcal{O}\left( H_{j+1,k}^{\min\{p_{j+1,k}, q_{j+1,k}+1\}} \right) \tag{6.45}$$

again with $H_{j+1,k} := h_{j+1,k}$ in case $S_{j+1,k}$ is a leaf node and for co-simulation nodes $S_{j+1,k}$ defining $p_{j+1,k} := \min_{i=I,\dots,N_{j+1,k}} \{p_{i_{j+1,k}}\}$ and $q_{j+1,k} := \min_{i=I,\dots,N_{j+1,k}} \{q_{i_{j+1,k}}\}$ (using these definitions recursively in case for an $i$, the associated simulation $S_{j+1,k}^{i}$ ($= S_{j+2,l}$ for $l = i + \sum_{m=1}^{k-1} N_{j+1,m}$) is again a co-simulation). Based on that, we consider the simulations on level $j$. For every leaf node on level $j$, Lemma 6.6 can directly be applied:

$$\left\| \frac{\mathcal{E}_{j,k}(h_{j,k})}{h_{j,k}} \right\| = \mathcal{O}\left( h_{j,k}^{\min\{p_{j,k}, q_{j,k}+1\}} \right), \tag{6.46}$$

which with $H_{j,k} := h_{j,k}$ can be written

$$\left\| \frac{\mathcal{E}_{j,k}(H_{j,k})}{H_{i_{j,k}}} \right\| = \mathcal{O}\left( H_{j,k}^{\min\{p_{j,k}, q_{j,k}+1\}} \right). \tag{6.47}$$

For every co-simulation on level $j$, we can utilize (6.45) and Corollary 6.8 to obtain

$$\left\| \frac{\mathcal{E}_{j,k}(H_{j,k})}{H_{j,k}} \right\| = \mathcal{O}\left( H_{j,k}^{\min_{i=I,\dots,N_{j,k}} \{p_{i_{j,k}}, q_{i_{j,k}}+1\}} \right) = \mathcal{O}\left( H_{j,k}^{\min\{p_{j,k}, q_{j,k}+1\}} \right) \tag{6.48}$$

with $p_{j,k} := \min_{i=I,\dots,N_{j,k}} \{p_{i_{j,k}}\}$ and $q_{j,k} := \min_{i=I,\dots,N_{j,k}} \{q_{i_{j,k}}\}$ (recursively, if needed). Thus, with (6.47) we have

$$\left\| \frac{\mathcal{E}_{j,k}(H_{j,k})}{H_{j,k}} \right\| = \mathcal{O}\left( H_{j,k}^{\min\{p_{j,k}, q_{j,k}+1\}} \right) \tag{6.49}$$

for every cooperative and leaf simulation on level $j$.

This also holds for the topmost level $j = 1$, where only one co-simulation (and, naturally, no leaf node) remains. With $H := H_{1,1}$ and utilizing the fact that in this co-simulation, all

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

$N = \sum_{j=1}^{J} \sum_{k=1}^{K_j} N_{j,k}$ participating leaf simulations and therefore, the consistency and extrapolation orders of every solution algorithm are finally considered,

$$\left\| \frac{\mathcal{E}(H)}{H} \right\| = \mathcal{O}\left( H^{\min_{i=I,\dots,N}\{p_i, q_i+1\}} \right). \tag{6.50}$$

□

This means that consistency is also maintained in hierarchical co-simulation, although it may potentially converge with lower order in comparison to the corresponding mono-simulation, depending on the extrapolation of external inputs. Since this is also the case for traditional co-simulation, no further loss of the order of consistency is added by the introduction of further hierarchies. For the co-simulation of ODE systems with one-step integration methods, consistency already implies convergence. For DAE systems, however, instabilities may occur, which is discussed in the next section.

### 6.2.2 Zero-stability

While consistency is sufficient for convergence of one-step integration methods, multi-step methods have to be investigated for zero-stablility as well as consistency to ensure convergence. Zero-stability means convergence of the method if the step size converges to zero. Thus, it can be defined by using the root condition:

*Definition* 6.10 (Zero-stability of multi-step methods (Hairer et al. 1993)). A multistep method is called stable if the generating polynomial $\rho(\zeta)$ satisfies the root condition, i.e.,

i) the roots of $\rho(\zeta)$ lie on or within the unit circle and

ii) the roots on the unit circle are simple.

*Remark* 6.11. One-step methods always fulfill this property for Lipschitz continuous right-side function $f$, see also Section A.2.

#### 6.2.2.1 Zero-stability in co-simulation

For co-simulation methods, a definition of zero-stability can be found in (Busch 2012):

*Definition* 6.12 (Zero-stability of co-simulation (Busch 2012)). “A coupling approach is called *zero-stable* if the co-simulation solution converges for an infinitesimal macro step size, i.e. $H \to 0$."

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

For linear systems, zero-stability is independent of the initial values and thus an important quality. In this work, we will focus on zero-stability based on the considerations from Kübler and Schiehlen (2000b). They analyze zero-stability of loose-coupling co-simulation, which lays the groundwork for our further investigations regarding zero-stability of hierarchical co-simulation, and thus is explained in detail in the following. The mathematical description of coupled DAEs is given as follows:

$$\dot{\boldsymbol{x}}^i(t) = \boldsymbol{f}^i(\boldsymbol{x}^i, \boldsymbol{u}^i, t), \quad \boldsymbol{x}^i(t_0) = \boldsymbol{x}_0^i \tag{6.51a}$$

$$\boldsymbol{y}^i(t) = \boldsymbol{g}^i(\boldsymbol{x}^i, \boldsymbol{u}^i, t) \tag{6.51b}$$

with $i = I, \ldots, N$, $\boldsymbol{x}^i \in \mathbb{R}^{n_x^i}$, $\boldsymbol{u}^i \in \mathbb{R}^{n_u^i}$, $\boldsymbol{y}^i \in \mathbb{R}^{n_y^i}$ and

$$\boldsymbol{u}^i = \boldsymbol{L}^i \boldsymbol{y} = \begin{bmatrix} \boldsymbol{L}^{i,I} & \ldots & \boldsymbol{L}^{i,i-1} & 0 & \boldsymbol{L}^{i,i+1} & \ldots & \boldsymbol{L}^{i,N} \end{bmatrix} \begin{bmatrix} \boldsymbol{y}^I \\ \vdots \\ \boldsymbol{y}^{i-1} \\ \boldsymbol{y}^i \\ \boldsymbol{y}^{i+1} \\ \vdots \\ \boldsymbol{y}^N \end{bmatrix} \tag{6.51c}$$

with $\boldsymbol{L}^{i,j} \in \mathbb{R}^{n_u^i \times n_y^j} \quad \forall i, j \in \{I, \ldots, N\}$ and the elements of $\boldsymbol{L}^{i,j}$ being equal to zero or one.

*Definition* 6.13 (Zero-stability of coupled integration (Kübler and Schiehlen 2000b)). The coupled integration

$$\boldsymbol{x}_{k+1}^i = \boldsymbol{\Phi}^i(\boldsymbol{\phi}^i, m^i, \tilde{\boldsymbol{u}}^i) \tag{6.52a}$$

$$\boldsymbol{y}_{k+1}^i = \boldsymbol{g}^i(\boldsymbol{x}_{k+1}^i, \tilde{\boldsymbol{u}}_{k+1}^i, t_{k+1}) \tag{6.52b}$$

$$\boldsymbol{u}_k^i = \boldsymbol{L}^i \boldsymbol{y}_k \tag{6.52c}$$

of $N$ subsystems is zero-stable if the discrete coupled system

$$\boldsymbol{x}_{k+1}^i = \boldsymbol{\Phi}^i(\boldsymbol{\phi}^i(h^i \to 0), m^i) \tag{6.53a}$$

$$\boldsymbol{y}_{k+1}^i = \boldsymbol{g}^i(\boldsymbol{x}_{k+1}^i, \boldsymbol{u}_k^i, t_{k+1}) \tag{6.53b}$$

$$\boldsymbol{u}_k^i = \boldsymbol{L}^i \boldsymbol{y}_k, \quad i = I, \ldots N \tag{6.53c}$$

is stable.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Here, $\tilde{\boldsymbol{u}}^i$ denotes the extrapolation of unknown inputs, $\boldsymbol{\phi}^i$ the integration method including extrapolation and $m_i$ the multirate factor (constant per subsystem). Details for the latter can be found in (Kübler and Schiehlen 2000b).

Assumptions (given on page 100 of (Kübler and Schiehlen 2000b)) are:

- one-step integration methods are used
- output equations are time-invariant
- output equations are linearly dependent on inputs

Under these assumptions the outputs can be written as follows:

$$\boldsymbol{y}^i = \overline{\boldsymbol{g}}^i(\boldsymbol{x}^i) + \boldsymbol{D}^i(\boldsymbol{x}^i)\boldsymbol{u}^i \tag{6.54}$$

The discretized output equations yield

$$\boldsymbol{y}^i_{k+1} = \overline{\boldsymbol{g}}^i + \boldsymbol{D}^i\boldsymbol{u}^i_k \text{ with constant } \overline{\boldsymbol{g}}^i, \boldsymbol{D}^i. \tag{6.55}$$

Using this, it holds for the outputs of global system

$$\boldsymbol{y}_{k+1} = \overline{\boldsymbol{g}} + \underbrace{\begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}^I\boldsymbol{L}^{I,II} & \dots & \boldsymbol{D}^I\boldsymbol{L}^{I,N} \\ \boldsymbol{D}^{II}\boldsymbol{L}^{II,I} & \boldsymbol{0} & \dots & \boldsymbol{D}^{II}\boldsymbol{L}^{II,N} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{D}^N\boldsymbol{L}^{N,I} & \boldsymbol{D}^N\boldsymbol{L}^{N,II} & \dots & \boldsymbol{0} \end{bmatrix}}_{=:\boldsymbol{D}} \boldsymbol{y}_k \tag{6.56}$$

that stability is guaranteed if the spectral radius $\rho$ of $\boldsymbol{D}$ is less than or equal to $1$. In the special case of two participating systems, $\rho(\boldsymbol{D}) = 0$ if $\boldsymbol{D}^I = \boldsymbol{0} \vee \boldsymbol{D}^{II} = \boldsymbol{0}$, which means no feed-through in at least one of the systems (one of the outputs is not explicitly dependent on the inputs), thus no algebraic loop occurs.
This, however, is a very strict restriction (as the requirement would be $\rho(\boldsymbol{D}) \leq 1$ and no algebraic loop means $\rho(\boldsymbol{D}) = 0$), so specific investigations of the systems in consideration are preferable if enough information on the participating systems is available.

#### 6.2.2.2 Zero-stability in hierarchical co-simulation

In the following, we will show that zero-stability can, depending on the corresponding one-level co-simulation, only be guaranteed for hierarchical decomposition in case of not only $\rho(\boldsymbol{D}) \leq 1$ but also $\|\boldsymbol{D}\|_\infty \leq 1$. In other cases (or those where we do not presume to know

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

the stability properties of the corresponding single-level co-simulation), zero-stability has to be investigated separately for every co-simulation layer.

To begin our investigations on zero-stability of hierarchical co-simulation, we take the system given in 6.51, which is illustrated in Figure 6.8 and called $CS_0$ from now on. Therein, we introduce a second level of co-simulation: w.l.o.g., systems $M, \dots, N$ for an arbitrary, but fixed $M$ with $1 < M < N$ are wrapped up in a coupled system which serves as new $M^{th}$ system on the upper level co-simulation. This is illustrated in Figure 6.9.

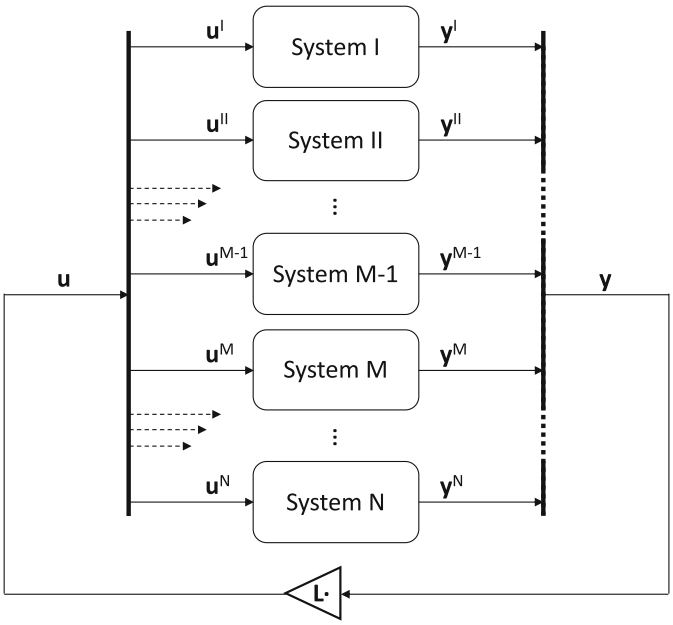


Figure 6.8: Illustration of the input-output relations in a traditional co-simulation approach for $N$ coupled systems.

The coupling equation for the original coupled system is given (cf. (6.51c)) as

$$\begin{bmatrix} \boldsymbol{u}^{I} \\ \boldsymbol{u}^{II} \\ \vdots \\ \boldsymbol{u}^{M-1} \\ \boldsymbol{u}^{M} \\ \vdots \\ \boldsymbol{u}^{N} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{0} & \boldsymbol{L}^{I,II} & \dots & \boldsymbol{L}^{I,M-1} & \boldsymbol{L}^{I,M} & \dots & \boldsymbol{L}^{I,N} \\ \boldsymbol{L}^{II,I} & \boldsymbol{0} & \dots & \boldsymbol{L}^{II,M-1} & \boldsymbol{L}^{II,M} & \dots & \boldsymbol{L}^{II,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{L}^{M-1,I} & \boldsymbol{L}^{M-1,II} & \dots & \boldsymbol{0} & \boldsymbol{L}^{M-1,M} & \dots & \boldsymbol{L}^{M-1,N} \\ \boldsymbol{L}^{M,I} & \boldsymbol{L}^{M,II} & \dots & \boldsymbol{L}^{M,M-1} & \boldsymbol{0} & \dots & \boldsymbol{L}^{M,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{L}^{N,I} & \boldsymbol{L}^{N,II} & \dots & \boldsymbol{L}^{N,M-1} & \boldsymbol{L}^{N,M} & \dots & \boldsymbol{0} \end{bmatrix}}_{=:\boldsymbol{L}} \cdot \begin{bmatrix} \boldsymbol{y}^{I} \\ \boldsymbol{y}^{II} \\ \vdots \\ \boldsymbol{y}^{M-1} \\ \boldsymbol{y}^{M} \\ \vdots \\ \boldsymbol{y}^{N} \end{bmatrix}.$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

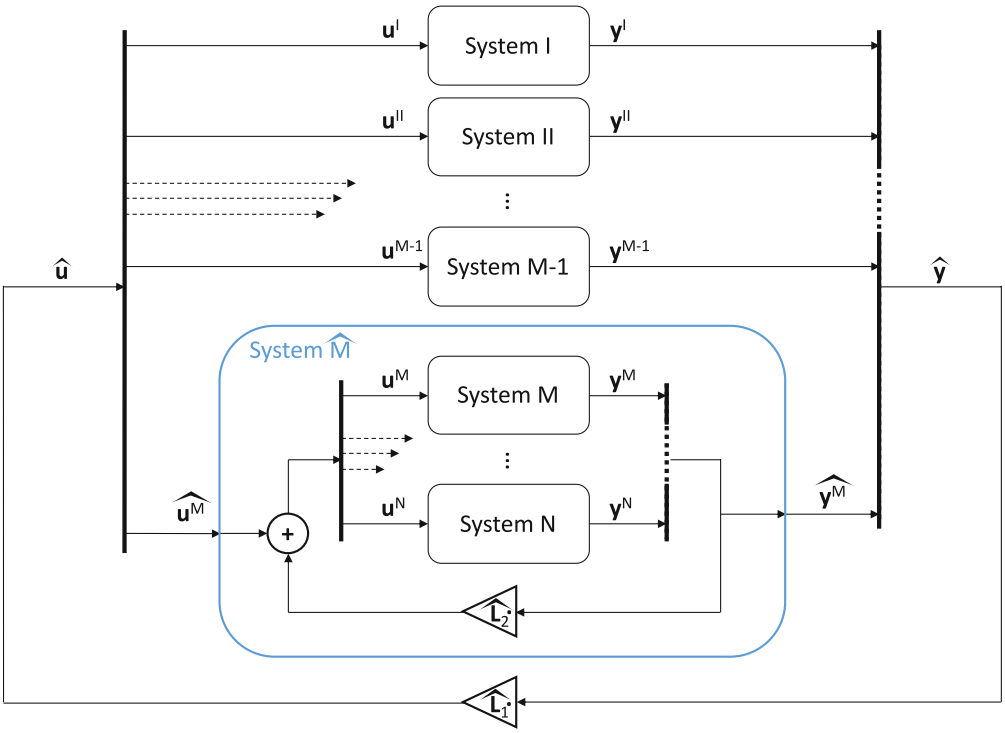


Figure 6.9: Illustration of the input-output relations in the hierarchical co-simulation of $N$ systems on two levels (Hafner and Popper 2020).

For the hierarchical co-simulation illustrated in Figure 6.9, we obtain the coupling equations (6.57) for the upper co-simulation level ($CS_1$).

$$\begin{bmatrix} \boldsymbol{u}^{I} \\ \boldsymbol{u}^{II} \\ \vdots \\ \boldsymbol{u}^{M-1} \\ \widehat{\boldsymbol{u}^{M}} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{0} & \boldsymbol{L}^{I,II} & \dots & \boldsymbol{L}^{I,M-1} & \widehat{\boldsymbol{L}^{I,M}} \\ \boldsymbol{L}^{II,I} & \mathbf{0} & \dots & \boldsymbol{L}^{II,M-1} & \widehat{\boldsymbol{L}^{II,M}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{L}^{M-1,I} & \boldsymbol{L}^{M-1,II} & \dots & \mathbf{0} & \widehat{\boldsymbol{L}^{M-1,M}} \\ \widehat{\boldsymbol{L}^{M,I}} & \widehat{\boldsymbol{L}^{M,II}} & \dots & \widehat{\boldsymbol{L}^{M,M-1}} & \mathbf{0} \end{bmatrix}}_{=:\widehat{\boldsymbol{L}_1}} \begin{bmatrix} \boldsymbol{y}^{I} \\ \boldsymbol{y}^{II} \\ \vdots \\ \boldsymbol{y}^{M-1} \\ \widehat{\boldsymbol{y}^{M}} \end{bmatrix} \tag{6.57}$$

with $\widehat{\boldsymbol{u}^{M}}$ as input to the new subsystem which replaces Systems $M$ to $N$ of $CS_0$, $\widehat{\boldsymbol{y}^{M}}$ as its

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

output and

$$\widehat{\boldsymbol{L}^{i,M}} = \begin{bmatrix} \boldsymbol{L}^{i,M} & \boldsymbol{L}^{i,M+1} & \dots & \boldsymbol{L}^{i,N} \end{bmatrix}, i = I, \dots, M-1 \text{ and} \tag{6.58a}$$

$$\widehat{\boldsymbol{L}^{M,i}} = \begin{bmatrix} \boldsymbol{L}^{M,i} \\ \boldsymbol{L}^{M+1,i} \\ \vdots \\ \boldsymbol{L}^{N,i} \end{bmatrix}, i = I, \dots, M-1. \tag{6.58b}$$

Given this, $\widehat{\boldsymbol{L_1}}$ can also be written as follows:

$$\widehat{\boldsymbol{L_1}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{L}^{I,II} & \dots & \boldsymbol{L}^{I,M-1} & \boldsymbol{L}^{I,M} & \dots & \boldsymbol{L}^{I,N} \\ \boldsymbol{L}^{II,I} & \boldsymbol{0} & \dots & \boldsymbol{L}^{II,M-1} & \boldsymbol{L}^{II,M} & \dots & \boldsymbol{L}^{II,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{L}^{M-1,I} & \boldsymbol{L}^{M-1,II} & \dots & \boldsymbol{0} & \boldsymbol{L}^{M-1,M} & \dots & \boldsymbol{L}^{M-1,N} \\ \boldsymbol{L}^{M,I} & \boldsymbol{L}^{M,II} & \dots & \boldsymbol{L}^{M,M-1} & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{L}^{N,I} & \boldsymbol{L}^{N,II} & \dots & \boldsymbol{L}^{N,M-1} & \boldsymbol{0} & \dots & \boldsymbol{0} \end{bmatrix} \tag{6.59}$$

Thus, the only difference between $\boldsymbol{L}$ and $\widehat{\boldsymbol{L}_1}$ is the increased number of zero matrices in the lower right corner.

The discretized output equations of $CS_1$ are

$$\boldsymbol{y}^I_{k+1} = \bar{\boldsymbol{g}}^I + \boldsymbol{D}^I \boldsymbol{u}^I_k \tag{6.60a}$$

$$\boldsymbol{y}^{II}_{k+1} = \bar{\boldsymbol{g}}^{II} + \boldsymbol{D}^{II} \boldsymbol{u}^{II}_k \tag{6.60b}$$

$$\vdots \tag{6.60c}$$

$$\widehat{\boldsymbol{y}^M_{k+1}} = \widehat{\bar{\boldsymbol{g}}^M} + \widehat{\boldsymbol{D}^M} \widehat{\boldsymbol{u}^M_k}. \tag{6.60d}$$

Note that while $\widehat{\boldsymbol{y}^M}$ in general corresponds to the stacked output vectors $\boldsymbol{y}^M, \dots, \boldsymbol{y}^N$ of $CS_0$, the input vectors do not as the coupling with the outputs of systems $M$ to $N$ is considered within the new system $\widehat{M}$, cf. Figure 6.9 and (6.62).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

The outputs of the global system can with (6.60a) be written as

$$\begin{bmatrix} \boldsymbol{y}_{k+1}^{I} \\ \vdots \\ \boldsymbol{y}_{k+1}^{M-1} \\ \widehat{\boldsymbol{y}_{k+1}^{M}} \end{bmatrix} = \begin{bmatrix} \overline{\boldsymbol{g}}^{I} \\ \vdots \\ \overline{\boldsymbol{g}}^{M-1} \\ \widehat{\overline{\boldsymbol{g}}^{M}} \end{bmatrix} + \underbrace{\begin{bmatrix} \boldsymbol{D}^{I} & & \boldsymbol{0} \\ & \ddots & \\ & & \boldsymbol{D}^{M-1} \\ \boldsymbol{0} & & & \widehat{\boldsymbol{D}^{M}} \end{bmatrix} \cdot \widehat{\boldsymbol{L}_1} \cdot \begin{bmatrix} \boldsymbol{y}_{k}^{I} \\ \vdots \\ \boldsymbol{y}_{k}^{M-1} \\ \widehat{\boldsymbol{y}_{k}^{M}} \end{bmatrix}}_{=:\boldsymbol{D}_{CS_1}}. \tag{6.61}$$

In analogy to the case of one co-simulation level, the co-simulation of the upper level is stable if $\rho(D_{CS_1}) \leq 1$. To find out whether this can be determined depending on the original coupled system, we have to find out the structure of $\boldsymbol{D}_{CS_1}$. The only unknown in comparison to $\boldsymbol{D}$ of $CS_0$ is $\widehat{\boldsymbol{D}^{M}}$, for which we have to take a look at System $\widehat{M}$, i.e. the second-level co-simulation $CS_2$. The coupling equations within this co-simulation can be written (cf. Figure 6.9) as follows:

$$\begin{bmatrix} \boldsymbol{u}_{k}^{M} \\ \boldsymbol{u}_{k}^{M+1} \\ \vdots \\ \boldsymbol{u}_{k}^{N-1} \\ \boldsymbol{u}_{k}^{N} \end{bmatrix} = \underbrace{\begin{bmatrix} \boldsymbol{0} & \boldsymbol{L}^{M,M+1} & \dots & \boldsymbol{L}^{M,N-1} & \boldsymbol{L}^{M,N} \\ \boldsymbol{L}^{M+1,M} & \boldsymbol{0} & \dots & \boldsymbol{L}^{M+1,N-1} & \boldsymbol{L}^{M+1,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{L}^{N-1,M} & \boldsymbol{L}^{N-1,M+1} & \dots & \boldsymbol{0} & \boldsymbol{L}^{N-1,N} \\ \boldsymbol{L}^{N,M} & \boldsymbol{L}^{N,M+1} & \dots & \boldsymbol{L}^{N,N-1} & \boldsymbol{0} \end{bmatrix}}_{=:\widehat{\boldsymbol{L_2}}} \cdot \begin{bmatrix} \boldsymbol{y}_{k}^{M} \\ \boldsymbol{y}_{k}^{M+1} \\ \vdots \\ \boldsymbol{y}_{k}^{N-1} \\ \boldsymbol{y}_{k}^{N} \end{bmatrix} + \widehat{\boldsymbol{u}_{k}^{M}} \tag{6.62}$$

The discretized output equations are

$$\boldsymbol{y}_{k+1}^{i} = \overline{\boldsymbol{g}}^{i} + \boldsymbol{D}^{i}\boldsymbol{u}_{k}^{i},\ i = M, \dots, N. \tag{6.63}$$

Thus follows for the global output of $CS_2$

$$\widehat{\boldsymbol{y}_{k+1}^{M}} = \begin{bmatrix} \boldsymbol{y}_{k+1}^{M} \\ \vdots \\ \boldsymbol{y}_{k+1}^{N} \end{bmatrix} = \underbrace{\begin{bmatrix} \overline{\boldsymbol{g}}^{M} \\ \vdots \\ \overline{\boldsymbol{g}}^{N} \end{bmatrix} + \begin{bmatrix} \boldsymbol{D}^{M} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D}^{N} \end{bmatrix} \widehat{\boldsymbol{L_2}} \begin{bmatrix} \boldsymbol{y}_{k}^{M} \\ \vdots \\ \boldsymbol{y}_{k}^{N} \end{bmatrix}}_{\widehat{\overline{\boldsymbol{g}}^{M}}} + \begin{bmatrix} \boldsymbol{D}^{M} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D}^{N} \end{bmatrix} \widehat{\boldsymbol{u}_{k}^{M}}. \tag{6.64}$$

The part with $\boldsymbol{y}_{k}^{i}$, $i = M, \dots, N$ can be included in $\widehat{\overline{\boldsymbol{g}}^{M}}$ as these are only internal states of $CS_2$ which are unknown in $CS_1$. Hence (6.64) can be written as

$$\widehat{\boldsymbol{y}_{k+1}^{M}} = \widehat{\overline{\boldsymbol{g}}^{M}} + \begin{bmatrix} \boldsymbol{D}^{M} & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D}^{N} \end{bmatrix} \widehat{\boldsymbol{u}_{k}^{M}} \tag{6.65}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

whence we obtain

$$\widehat{\boldsymbol{D}^M} = \begin{bmatrix} \boldsymbol{D}^M & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D}^N \end{bmatrix}, \tag{6.66}$$

which yields for $\boldsymbol{D}_{CS_1}$ from (6.61)

$$\begin{aligned} \boldsymbol{D}_{CS_1} &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}^I\boldsymbol{L}^{I,II} & \dots & \boldsymbol{D}^I\widehat{\boldsymbol{L}^{I,M}} \\ \boldsymbol{D}^{II}\boldsymbol{L}^{II,I} & \boldsymbol{0} & \dots & \boldsymbol{D}^{II}\widehat{\boldsymbol{L}^{II,M}} \\ \vdots & \ddots & \dots & \vdots \\ \boldsymbol{D}^{M-1}\boldsymbol{L}^{M-1,I} & \dots & \boldsymbol{0} & \boldsymbol{D}^{M-1}\widehat{\boldsymbol{L}^{M-1,M}} \\ \widehat{\boldsymbol{D}^M\boldsymbol{L}^{M,I}} & \dots & \widehat{\boldsymbol{D}^M\boldsymbol{L}^{M,M-1}} & \boldsymbol{0} \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}^I\boldsymbol{L}^{I,II} & \dots & \boldsymbol{D}^I\boldsymbol{L}^{I,M-1} & \boldsymbol{D}^I\boldsymbol{L}^{I,M} & \dots & \boldsymbol{D}^I\boldsymbol{L}^{I,N} \\ \boldsymbol{D}^{II}\boldsymbol{L}^{II,I} & \boldsymbol{0} & \dots & \boldsymbol{D}^{II}\boldsymbol{L}^{II,M-1} & \boldsymbol{D}^{II}\boldsymbol{L}^{II,M} & \dots & \boldsymbol{D}^{II}\boldsymbol{L}^{II,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{D}^{M-1}\boldsymbol{L}^{M-1,I} & \boldsymbol{D}^{M-1}\boldsymbol{L}^{M-1,II} & \dots & \boldsymbol{0} & \boldsymbol{D}^{M-1}\boldsymbol{L}^{M-1,M} & \dots & \boldsymbol{D}^{M-1}\boldsymbol{L}^{M-1,N} \\ \boldsymbol{D}^M\boldsymbol{L}^{M,I} & \boldsymbol{D}^M\boldsymbol{L}^{M,II} & \dots & \boldsymbol{D}^M\boldsymbol{L}^{M,M-1} & \boldsymbol{0} & \dots & \boldsymbol{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{D}^N\boldsymbol{L}^{N,I} & \boldsymbol{D}^N\boldsymbol{L}^{N,II} & \dots & \boldsymbol{D}^N\boldsymbol{L}^{N,M-1} & \boldsymbol{0} & \dots & \boldsymbol{0} \end{bmatrix} \end{aligned}$$

due to

$$\widehat{\boldsymbol{D}^M\boldsymbol{L}^{M,i}} = \begin{bmatrix} \boldsymbol{D}^M & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D}^N \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{L}^{M,i} \\ \boldsymbol{L}^{M+1,i} \\ \vdots \\ \boldsymbol{L}^{N,i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{D}^M\boldsymbol{L}^{M,i} \\ \boldsymbol{D}^M\boldsymbol{L}^{M+1,i} \\ \vdots \\ \boldsymbol{D}^N\boldsymbol{L}^{N,i} \end{bmatrix}, \qquad i = I, \dots, M-1 \tag{6.67}$$

and

$$\begin{aligned} \boldsymbol{D}^i\widehat{\boldsymbol{L}^{i,M}} &= \boldsymbol{D}^i \cdot \begin{bmatrix} \boldsymbol{L}^{i,M} & \boldsymbol{L}^{i,M+1} & \dots & \boldsymbol{L}^{i,N} \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{D}^i\boldsymbol{L}^{i,M} & \boldsymbol{D}^i\boldsymbol{L}^{i,M+1} & \dots & \boldsymbol{D}^i\boldsymbol{L}^{i,N} \end{bmatrix}, \qquad i = I, \dots, M-1. \end{aligned} \tag{6.68}$$

In comparison to matrix $\boldsymbol{D}$ of co-simulation $CS_0$, the only difference is the increased number of zero matrices in the lower right corner. In the following, we try to use this information to gain information on the properties of the spectral radius of $\boldsymbol{D}_{CS_1}$ using knowledge on $\rho(\boldsymbol{D})$.

We know that for every matrix norm $\|.\|$ and arbitrary matrix $\boldsymbol{A} = (a_{ij})$; $i = 1, \dots, m$; $j = 1, \dots, n$; $m, n \in \mathbb{N}$

$$\rho(\boldsymbol{A}) \leq \|\boldsymbol{A}\| \tag{6.69}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

holds (Horn and Johnson 2010, Thm. 5.6.9).
If we consider $\|.\|_\infty$ given as

$$\|\boldsymbol{A}\|_\infty = \max_{i=1,\dots,m} \sum_{j=1}^{n} |a_{ij}| \tag{6.70}$$

we immediately see that $\|\boldsymbol{D}_{CS_1}\|_\infty \leq \|\boldsymbol{D}\|_\infty$. Unfortunately, this does *not* imply $\rho(\boldsymbol{D}_{CS_1}) \leq \rho(\boldsymbol{D})$, see e.g. Example 6.14.

*Example* 6.14. Let matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ given as

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0.1 & 0.5 & 0 \\ 0.1 & 0 & 0 & 0.5 \\ 0.2 & 0 & 0 & -0.1 \\ 0 & 0.2 & -0.1 & 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0.1 & 0.5 & 0 \\ 0.1 & 0 & 0 & 0.5 \\ 0.2 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \end{bmatrix}. \tag{6.71}$$

Here $\|\boldsymbol{A}\|_\infty = \|\boldsymbol{B}\|_\infty = 0.6$ but $\rho(\boldsymbol{A}) \approx 0.3317 < \rho(\boldsymbol{B}) \approx 0.3702$.

This means that in general, stability for hierarchical co-simulation has to be determined anew, even if the starting point is a stable co-simulation on one level. An exception is the case where not only $\rho(\boldsymbol{D}) \leq 1$ but also $\|\boldsymbol{D}\|_\infty \leq 1$, as from this follows further

$$\rho(\boldsymbol{D}_{CS_1}) \leq \|\boldsymbol{D}_{CS_1}\|_\infty \leq \|\boldsymbol{D}\|_\infty \leq 1 \tag{6.72}$$

which ensures zero-stability of the co-simulation on the upper level $CS_1$.

For the stability properties of the coupling in $CS_2$, we are interested in the input-output dependencies within the system only, thus we need to look at the spectral radius of $\boldsymbol{D}_{CS_2}$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

$\boldsymbol{D}_{CS_2}$ is found in (6.64):

$$\begin{aligned}
\boldsymbol{D}_{CS_2} &= \begin{bmatrix} \boldsymbol{D}^M & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D}^N \end{bmatrix} \widehat{\boldsymbol{L_2}} \\
&= \begin{bmatrix} \boldsymbol{D}^M & \dots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \dots & \boldsymbol{D}^N \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{0} & \boldsymbol{L}^{M,M+1} & \dots & \boldsymbol{L}^{M,N-1} & \boldsymbol{L}^{M,N} \\ \boldsymbol{L}^{M+1,M} & \boldsymbol{0} & \dots & \boldsymbol{L}^{M+1,N-1} & \boldsymbol{L}^{M+1,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{L}^{N-1,M} & \boldsymbol{L}^{N-1,M+1} & \dots & \boldsymbol{0} & \boldsymbol{L}^{N-1,N} \\ \boldsymbol{L}^{N,M} & \boldsymbol{L}^{N,M+1} & \dots & \boldsymbol{L}^{N,N-1} & \boldsymbol{0} \end{bmatrix} \\
&= \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}^M\boldsymbol{L}^{M,M+1} & \dots & \boldsymbol{D}^M\boldsymbol{L}^{M,N-1} & \boldsymbol{D}^M\boldsymbol{L}^{M,N} \\ \boldsymbol{D}^{M+1}\boldsymbol{L}^{M+1,M} & \boldsymbol{0} & \dots & \boldsymbol{D}^{M+1}\boldsymbol{L}^{M+1,N-1} & \boldsymbol{D}^{M+1}\boldsymbol{L}^{M+1,N} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{D}^{N-1}\boldsymbol{L}^{N-1,M} & \boldsymbol{D}^{N-1}\boldsymbol{L}^{N-1,M+1} & \dots & \boldsymbol{0} & \boldsymbol{D}^{N-1}\boldsymbol{L}^{N-1,N} \\ \boldsymbol{D}^N\boldsymbol{L}^{N,M} & \boldsymbol{D}^N\boldsymbol{L}^{N,M+1} & \dots & \boldsymbol{D}^N\boldsymbol{L}^{N,N-1} & \boldsymbol{0} \end{bmatrix}
\end{aligned}$$

Since we see that $\boldsymbol{D}_{CS_2}$ is a submatrix of $\boldsymbol{D}$, here again $\|\boldsymbol{D}_{CS_2}\|_\infty \leq \|\boldsymbol{D}\|_\infty$ holds, and thus $\rho(\boldsymbol{D}_{CS_2})$ has to be determined separately only if $\|\boldsymbol{D}\|_\infty > 1$.

To sum up, we can conclude that zero-stability of hierarchical co-simulation can be determined analogously to customary co-simulation on one level. To this end, the matrices referring to the global system outputs on every co-simulation level have to be examined – except for the cases where the origin is a stable co-simulation with matrix $\boldsymbol{D}$ fulfilling $\|\boldsymbol{D}\|_\infty \leq 1$, which is satisfied in particular for couplings where no feed-through occurs in at least one system, so $\|\boldsymbol{D}\|_\infty = \rho(\boldsymbol{D}) = 0$. Simple induction shows that these considerations apply to more than two levels of co-simulation as well:

*Corollary* 6.15 (Zero-stability of hierarchical co-simulation). *Zero-stability of hierarchical co-simulation approaches can be determined by separately investigating zero-stability of the co-simulations on every level. If* $\boldsymbol{D} := \mathbb{D}_{1,1}$ *of the "flattened" overall co-simulation fulfills* $\|\boldsymbol{D}\|_\infty \leq 1$*, this implies* $\|\boldsymbol{D}_{j,k}\|_\infty \leq 1$ *for all* $k \in 1, \dots, K_j$*,* $j \in 1, \dots, J$*, when* $J$ *stands for the number of levels and* $K_j$ *for the number of co-simulations per level.*

*Proof.* Following the indexing in the proof of Theorem 6.9, we start from the top-most co-simulation $CS_{1,1}$, whose zero-stability is determined by the matrices $\boldsymbol{D}_{1,1}$ and $\boldsymbol{D}^i_{1,1}$, $i = 1, \dots, N_{1,1}$. The zero-stability of every $CS^i_{1,1}$, on the other hand, depends – according to the investigations above – apart from $\boldsymbol{D}^i_{1,1}$, on $\boldsymbol{D}^r_{2,i}$, $r = 1, \dots, N_{2,i}$. This can be

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

continued to the co-simulations on the next-to-last level. Finally, zero-stability of every co-simulation $CS_{J,k}$ on level $J$ is determined by the corresponding discretized output coefficient matrix denoted as $\boldsymbol{D}_{J,k}$. For the second part of Corollary 6.15, we need to climb back up the notation tree: co-simulation $CS_{J,k}$ is again the subsimulation of a co-simulation $CS_{J-1,l}$ on level $J-1$ for one $l \in 1,\ldots,K_{J-1}$. Following the considerations above, we further regard the corresponding "flattened" co-simulation $\mathbb{CS}_{J-1,l}$, meaning a co-simulation in which the subsimulations of $CS^i_{J-1,k}$ would be coordinated directly, with corresponding matrix $\mathbb{D}_{J-1,l}$. Similar to the example above follows that every $\boldsymbol{D}^i_{J-1,l}$ is a sub-matrix of $\mathbb{D}_{J-1,l}$, and $\mathbb{D}_{J-1,l}$ and $\boldsymbol{D}_{J-1,l}$ only differ by the increased number of zero matrices in $\boldsymbol{D}_{J-1,l}$, thus from $\|\mathbb{D}_{J-1,l}\|_\infty \leq 1$ follows $\|\boldsymbol{D}^i_{J-1,l}\|_\infty \leq 1$ for all $i \in 1,\ldots,N_{J-1,l}$. This can be continued for decreasing $j$ until the topmost co-simulation. □

### 6.2.3 Numerical stability

Depending on the coupling method, instabilities can still occur for zero-stable coupling methods due to the errors introduced by extra- or interpolation, which leads to another important stability measure:

*Definition* 6.16 (Numerical stability of co-simulation (Busch 2012)). A weak coupling approach is called *numerically stable* if it yields a stable solution for a finite macro-step size $H > 0$.

To investigate stability properties for finite communication step sizes (as opposed to $H \to 0$ with zero-stability), we consider a three-mass oscillator as benchmark example. The underlying equation system can be interpreted as coupled Dahlquist equations ($\dot{z} = \lambda z$ would be Dahlquist's linear test equation, which can mechanically be interpreted as linear 1-DOF oscillator, see f.i. Chapter 2.3 of (Busch 2012)). This equation poses a valid measure for numerical stability, see f.i. (Schweizer and Lu 2015): "assuming that the system is stable from the mechanical point of view, a numerical time integration method is called numerically stable, if the discretized Dahlquist equation yields a sequence of exponentially decaying values".
The oscillator with two masses has been taken into consideration in numerous investigations on stability of conventional, single-level co-simulation approaches (see for example (Busch 2012; Glumac and Kovacic 2019; Schweizer and Lu 2014a)). Stability properties for co-simulations of this example are highly sensitive to the choice of parameters and macro step size. Detailed parameter variations including stability regions for different (single-level) coupling approaches can be found in chapter 2.3.5 and Appendix 2B of (Busch 2012).

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

To test the hierarchical co-simulation approach, the example has to be extended to three masses, which is illustrated in Figure 6.10.

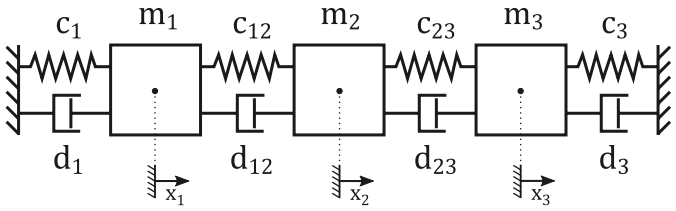


Figure 6.10: Illustration of a three-mass oscillator.

The underlying equations for this system are given as follows:

$$\dot{x}_1 = v_1 \tag{6.73a}$$
$$m_1\dot{v}_1 = -c_1x_1 - d_1v_1 + c_{12}(x_2 - x_1) + d_{12}(v_2 - v_1) \tag{6.73b}$$
$$\dot{x}_2 = v_2 \tag{6.73c}$$
$$m_2\dot{v}_2 = -c_{12}(x_2 - x_1) - d_{12}(v_2 - v_1) + c_{23}(x_3 - x_2) + d_{23}(v_3 - v_2) \tag{6.73d}$$
$$\dot{x}_3 = v_3 \tag{6.73e}$$
$$m_3\dot{v}_3 = -c_{23}(x_3 - x_2) - d_{23}(v_3 - v_2) + c_3(-x_3) + d_3(-v_3) \tag{6.73f}$$

which can be written in matrix form as

$$\dot{\boldsymbol{z}} = \boldsymbol{A} \cdot \boldsymbol{z} \tag{6.74}$$

with

$$\boldsymbol{z} = \begin{bmatrix} x_1 & v_1 & x_2 & v_2 & x_3 & v_3 \end{bmatrix}^T \tag{6.75}$$

and

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -\frac{c_1+c_{12}}{m_1} & -\frac{d_1+d_{12}}{m_1} & \frac{c_{12}}{m_1} & \frac{d_{12}}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{c_{12}}{m_2} & \frac{d_{12}}{m_2} & -\frac{c_{12}+c_{23}}{m_2} & -\frac{d_{12}+d_{23}}{m_2} & -\frac{c_{23}}{m_2} & \frac{d_{23}}{m_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{c_{23}}{m_3} & \frac{d_{23}}{m_3} & -\frac{c_{23}+c_3}{m_3} & -\frac{d_{23}+d_3}{m_3} \end{bmatrix}. \tag{6.76}$$

For the intended co-simulation, the system is split along the individual masses and coupled via force-displacement-coupling (cf. Section 3.2.4 or f.i. (Schweizer and Lu 2014a) for further information on the coupling approach), as illustrated in Figure 6.11.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

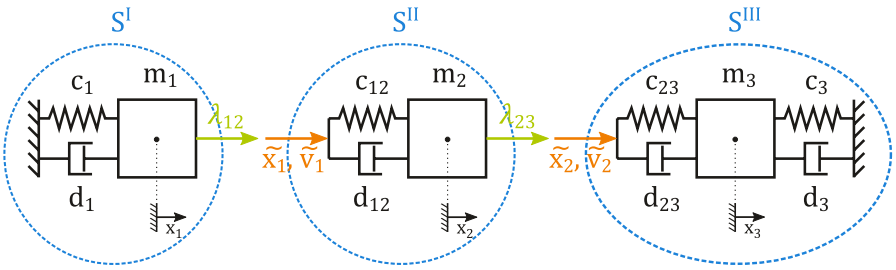


Figure 6.11: Force-displacement coupling of the three-mass oscillator.

By this coupling approach we obtain the subsystem equations for systems $S^I$, $S^{II}$ and $S^{III}$:

$$S^I: \quad \begin{aligned} \dot{x}_1 &= v_1 \\ m_1\dot{v}_1 &= -c_1 x_1 - d_1 v_1 + \lambda_{12} \end{aligned} \tag{6.77a}$$

$$S^{II}: \quad \begin{aligned} \dot{x}_2 &= v_2 \\ m_2\dot{v}_2 &= -c_{12}(x_2 - \widetilde{x_1}) - d_{12}(v_2 - \widetilde{v_1}) + \lambda_{23} \end{aligned} \tag{6.77b}$$

$$S^{III}: \quad \begin{aligned} \dot{x}_3 &= v_3 \\ m_3\dot{v}_3 &= -c_{23}(x_3 - \widetilde{x_2}) - d_{23}(v_3 - \widetilde{v_2}) + c_3(-x_3) + d_3(-v_3) \end{aligned} \tag{6.77c}$$

with the coupling conditions

$$\lambda_{12} - c_{12}(x_2 - x_1) - d_{12}(v_2 - v_1) = 0 \tag{6.78a}$$

$$\widetilde{x_1} - x_1 = 0 \tag{6.78b}$$

$$\widetilde{v_1} - v_1 = 0 \tag{6.78c}$$

$$\lambda_{23} - c_{23}(x_3 - x_2) - d_{23}(v_3 - v_2) = 0 \tag{6.78d}$$

$$\widetilde{x_2} - x_2 = 0 \tag{6.78e}$$

$$\widetilde{v_2} - v_2 = 0. \tag{6.78f}$$

Following the notation in (Glumac and Kovacic 2019; Kübler and Schiehlen 2000b), we obtain for the internal subsystem states $\boldsymbol{x^i}$, inputs $\boldsymbol{u}^i$ and outputs $\boldsymbol{y}^i$, $i = I, II, III$ for the traditional co-simulation:

$$\dot{\boldsymbol{x}}^I(t) = \begin{bmatrix} 0 & 1 \\ -\frac{c_1}{m_1} & -\frac{d_1}{m_1} \end{bmatrix} \boldsymbol{x}^I(t) + \begin{bmatrix} 0 \\ \frac{1}{m_1} \end{bmatrix} \boldsymbol{u}^I(t) \tag{6.79a}$$

$$\boldsymbol{y}^I(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{x}^I(t) + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \boldsymbol{u}^I(t) \tag{6.79b}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

$$\dot{\boldsymbol{x}}^{II}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{c_{12}}{m_2} & -\frac{d_{12}}{m_2} \end{bmatrix} \boldsymbol{x}^{II}(t) + \begin{bmatrix} 0 & 0 & 0 \\ \frac{c_{12}}{m_2} & \frac{d_{12}}{m_2} & \frac{1}{m_2} \end{bmatrix} \boldsymbol{u}^{II}(t) \tag{6.80a}$$

$$\boldsymbol{y}^{II}(t) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ c_{12} & d_{12} \end{bmatrix} \boldsymbol{x}^{II}(t) + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -c_{12} & -d_{12} & 0 \end{bmatrix} \boldsymbol{u}^{II}(t) \tag{6.80b}$$

$$\dot{\boldsymbol{x}}^{III}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{c_{23}+c_3}{m_3} & -\frac{d_{23}+d_3}{m_3} \end{bmatrix} \boldsymbol{x}^{III}(t) + \begin{bmatrix} 0 & 0 \\ \frac{c_{23}}{m_3} & \frac{d_{23}}{m_3} \end{bmatrix} \boldsymbol{u}^{III}(t) \tag{6.81a}$$

$$\boldsymbol{y}^{III}(t) = \begin{bmatrix} c_{23} & d_{23} \end{bmatrix} \boldsymbol{x}^{III}(t) + \begin{bmatrix} -c_{23} & -d_{23} \end{bmatrix} \boldsymbol{u}^{III}(t) \tag{6.81b}$$

The coupling equations can be written as

$$\begin{bmatrix} \boldsymbol{u}^{I} \\ \boldsymbol{u}^{II} \\ \boldsymbol{u}^{III} \end{bmatrix} = \boldsymbol{L} \cdot \begin{bmatrix} \boldsymbol{y}^{I} \\ \boldsymbol{y}^{II} \\ \boldsymbol{y}^{III} \end{bmatrix} \tag{6.82}$$

with

$$\boldsymbol{L} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{6.83}$$

This yields for the matrix $\boldsymbol{D}$ in (6.56):

$$\boldsymbol{D} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{D}^{I}\boldsymbol{L}^{I,II} & \boldsymbol{D}^{I}\boldsymbol{L}^{I,III} \\ \boldsymbol{D}^{II}\boldsymbol{L}^{II,I} & \boldsymbol{0} & \boldsymbol{D}^{II}\boldsymbol{L}^{II,III} \\ \boldsymbol{D}^{III}\boldsymbol{L}^{III,I} & \boldsymbol{D}^{III}\boldsymbol{L}^{III,II} & \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{12} & -d_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & -c_{23} & -d_{23} & 0 & 0 \end{bmatrix}$$

Whence follows $\rho(\boldsymbol{D}) = 0$, thus guaranteeing zero-stability in case of originally convergent integration algorithms.

For the hierarchical co-simulation approach, systems $S^{II}$ and $S^{III}$ are combined in a second-level co-simulation, as illustrated in Figure 6.12.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

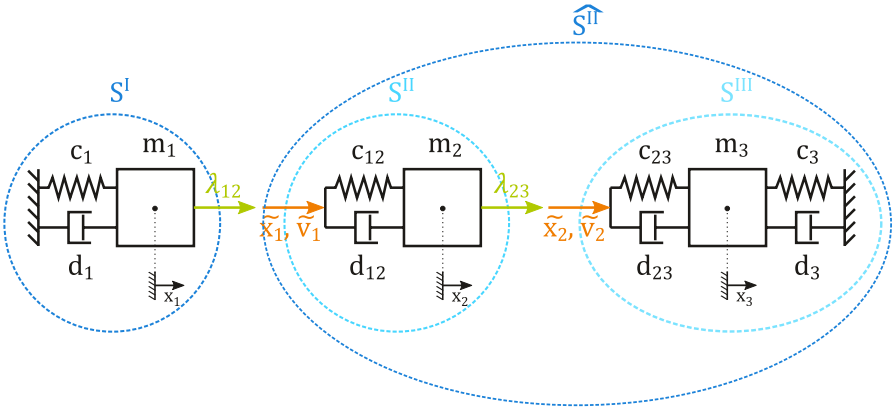


Figure 6.12: Illustration of the hierarchical coupling of a three-mass oscillator.

In this approach, the coupling equations on the upper-level co-simulation result in

$$\begin{bmatrix} \boldsymbol{u}^I \\ \widehat{\boldsymbol{u}^{II}} \end{bmatrix} = \widehat{\boldsymbol{L}_1} \cdot \begin{bmatrix} \boldsymbol{y}^I \\ \widehat{\boldsymbol{y}^{II}} \end{bmatrix} \tag{6.84}$$

with

$$\widehat{\boldsymbol{L}_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.85}$$

and for the second-level co-simulation

$$\begin{bmatrix} \boldsymbol{u}^{II} \\ \boldsymbol{u}^{III} \end{bmatrix} = \widehat{\boldsymbol{L}_2} \cdot \begin{bmatrix} \boldsymbol{y}^{II} \\ \boldsymbol{y}^{III} \end{bmatrix} + \widehat{\boldsymbol{u}^{II}} \tag{6.86}$$

with

$$\widehat{\boldsymbol{L}_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \tag{6.87}$$

As expected (cf. Section 6.2.2.2), we obtain $\rho(\boldsymbol{D}_{CS_1}) = \rho(\boldsymbol{D}_{CS_2}) = 0$ for

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

$$\boldsymbol{D}_{CS_1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -c_{12} & -d_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{6.88}$$

and

$$\boldsymbol{D}_{CS_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -c_{23} & -d_{23} & 0 & 0 \end{bmatrix}, \tag{6.89}$$

thus, the conditions for zero-stability are satisfied for both levels of co-simulation.

In the following, several scenarios are performed for both co-simulation approaches to compare numerical stability properties. All scenarios have been implemented in MATLAB 2018b. For all settings, explicit Euler methods are used to solve the individual subsystems. These simple methods have been chosen to enable the focus on the different methods of co-simulation without additional corrections: Adaptive methods in the subsystems would decrease their step size after each synchronization reference (due to the resulting leap in the progression of external variables), which, on the one hand, would improve accuracy but also blow up the taken micro time steps. As synchronization method, Jacobi-type coupling without iteration using zero-order extrapolation for external variables has been used. The initial conditions for all scenarios have been chosen as $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ and $v_1 = v_2 = v_3 = 0$.

**Scenario 1.** The parameters for the first scenario to be considered are given in Table 6.1.

Table 6.1: Parameter settings for Scenario 1.

| $c_1$ | $c_{12}$ | $c_{23}$ | $c_3$ | $d_1$ | $d_{12}$ | $d_{23}$ | $d_3$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1E-02 | 1E-01 | 1 | 10 | 0.1 | 0.4 | 1 | 2 | 10 | 10 | 10 |

As can be seen, the spring stiffnesses are chosen to increase from left to right (cf. Figure 6.10) to result in slower and faster varying subsystems. The step sizes for the individual subsystem solvers are chosen accordingly with $h_I = 0.005$, $h_{II} = 0.0025$ and $h_{III} = 0.00125$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

The monolithic reference system is of the form $\dot{\boldsymbol{y}} = \boldsymbol{A} \cdot \boldsymbol{y}$ and can thus be solved analytically. In addition to the analytical solution, the results of the hierarchical co-simulation are compared to a conventional single-level co-simulation. For the latter, a macro step size $H$ of $0.1$ seconds is chosen. For the hierarchical co-simulation, the communication step sizes are chosen as $H_1 = 0.1s$ for the upper level and $H_2 = 0.0125s$ for the lower level in the first experiment to emphasize the advantages of additional communication on the lower level, see Figure 6.13.

The results in Figure 6.14 show that even if both the overall communication step size and the communication step size on the lower level are doubled ($H_1 = 0.2s$, $H_2 = 0.05s$) – hence $H_1$ being also twice as large as the macro step size of the traditional co-simulation – the hierarchical approach yields significantly more accurate results for systems $S^{II}$ and $S^{III}$.

Figures 6.15 and 6.16 show the behavior over a longer period of time ($t_{end} = 25s$). We see that in spite of plainly distinct errors in specific phases, both approximations remain stable.

Table 6.2: Maximum error and elapsed time for the traditional and hierarchical co-simulation approach in Scenario 1.

| approach | $t_{end}$ | $H$ | $H_1$ | $H_2$ | $err_{x_1}$ | $err_{v_1}$ | $err_{x_2}$ | $err_{v_2}$ | $err_{x_3}$ | $err_{v_3}$ | elapsed time |
|---|---|---|---|---|---|---|---|---|---|---|---|
| traditional | 1 | 0.1 | | | 1.21E-04 | 5.30E-04 | 8.05E-03 | 1.53E-02 | 9.51E-04 | 6.32E-04 | 0.0066 |
| hierarchical | 1 | | 0.1 | 0.025 | 4.52E-05 | 2.89E-04 | 2.10E-03 | 3.67E-03 | 8.22E-04 | 1.38E-03 | 0.0161 |
| hierarchical | 1 | | 0.2 | 0.05 | 8.26E-05 | 5.34E-04 | 4.08E-03 | 7.44E-03 | 8.51E-04 | 1.06E-03 | 0.0099 |
| traditional | 25 | 0.1 | | | 3.96E-02 | 7.13E-03 | 9.35E-02 | 3.68E-02 | 1.78E-02 | 1.24E-02 | 0.1845 |
| hierarchical | 25 | | 0.1 | 0.025 | 1.76E-02 | 3.29E-03 | 2.14E-02 | 8.75E-03 | 6.63E-03 | 5.48E-03 | 0.4018 |
| hierarchical | 25 | | 0.2 | 0.05 | 3.53E-02 | 6.64E-03 | 4.28E-02 | 1.75E-02 | 9.67E-03 | 7.20E-03 | 0.2514 |

The elapsed time (averaged over 100 runs) and the maximum absolute errors for the different settings are given in Table 6.2, where we see that while the execution time is significantly higher in case of the same step size on the upper level and the traditional co-simulation – which has to be expected due to the additional synchronization on the lower level – the high difference can be overcome while still maintaining better accuracy by increasing both macro step sizes in the hierarchical approach.

Figure 6.17 shows the overall error – calculated by $\|.\|_2$ of the maximum errors of all states – depending on the macro step sizes $H = H_1$ and $H_2$ for a simulation over 25 seconds. $H_2$ ranges from the biggest micro-step size $0.025$ over all multiples that are divisors of $H_1$ up to

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

$H_1/2$ (for $H_2 = H_1$, the same results as for the traditional approach would be expected). For both methods, the error increases with the macro step size, but with clearly less curvature for the hierarchical co-simulation. By decreasing only $H_2$, the curve is flattened further. Note that the triangular shape of the error of the traditional approach in the frontal view of the $err$-$H_2$ plane is owed to the additional dimension of $H_2$ which apart from its irrelevance on a single-level simulation is bounded by the half of $H_1 = H$. Similar illustrations for each component can be found in the Appendix, Section A.5.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

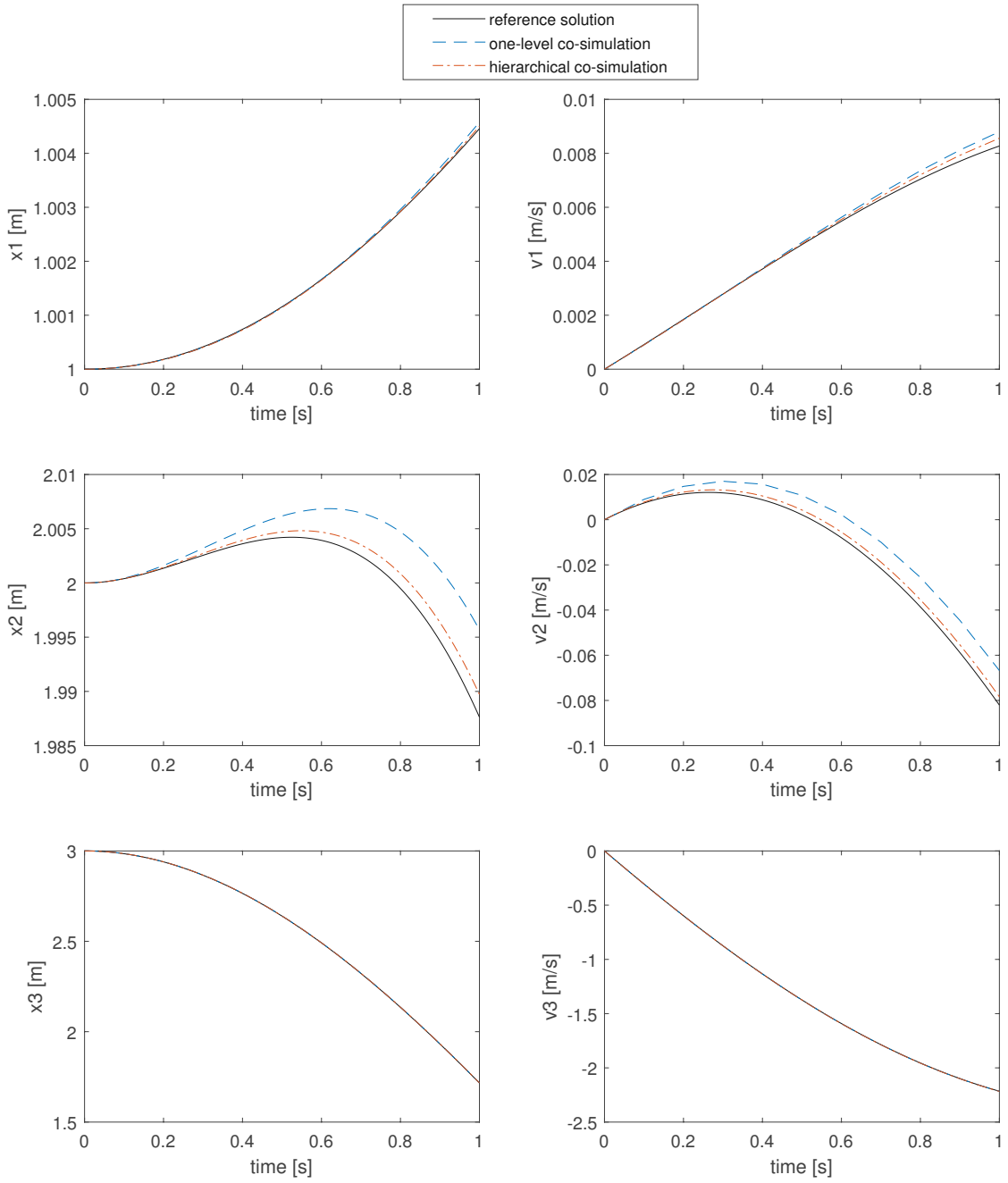


Figure 6.13: Trajectories of the system variables for Scenario 1 with $H = H_1 = 0.1s$ and $H_2 = 0.025s$ from $t_{start} = 0s$ to $t_{end} = 1s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

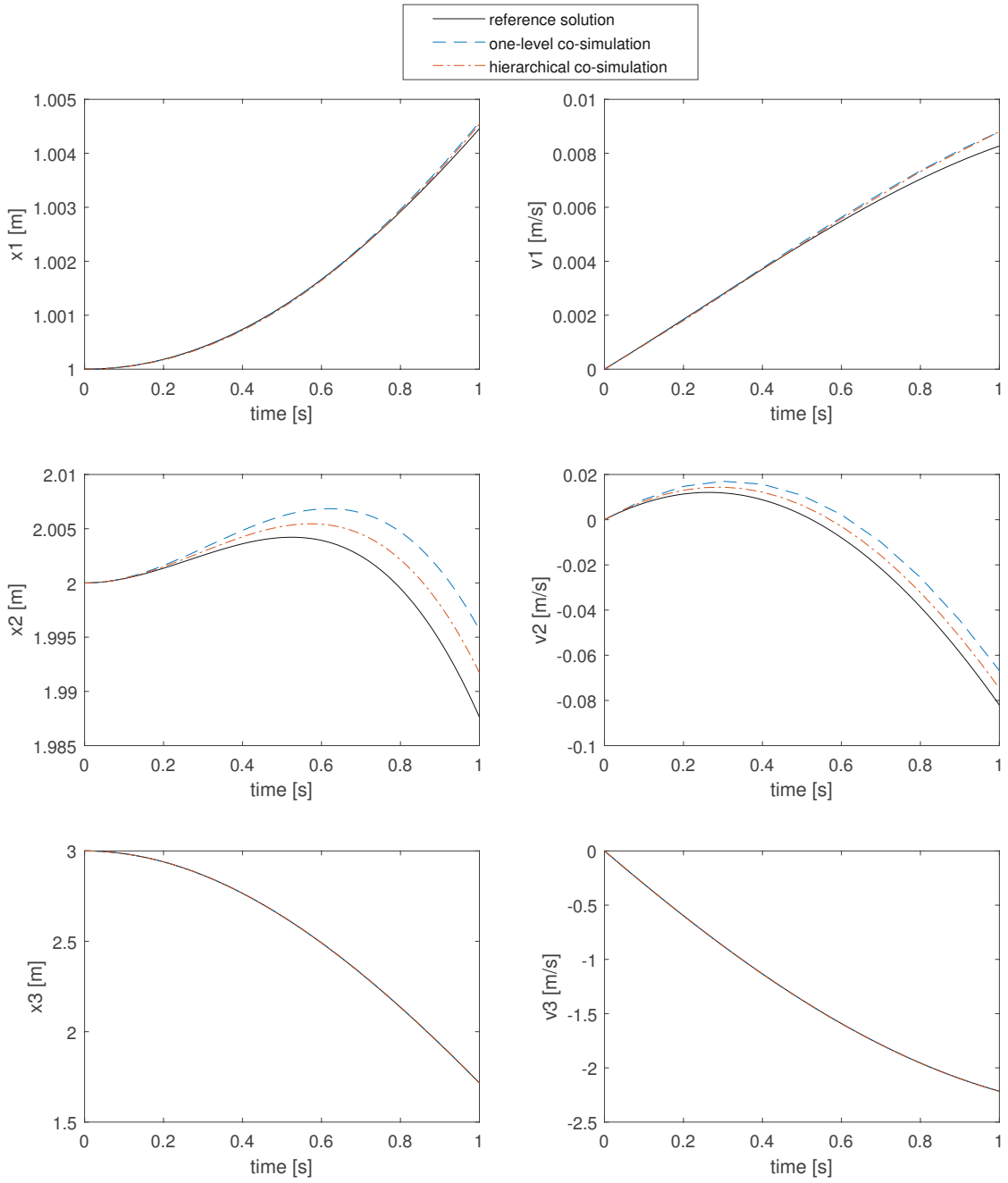


Figure 6.14: Trajectories of the system variables for Scenario 1 with $H = 0.1s$, $H_1 = 0.2s$ and $H_2 = 0.05s$ from $t_{start} = 0s$ to $t_{end} = 1s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

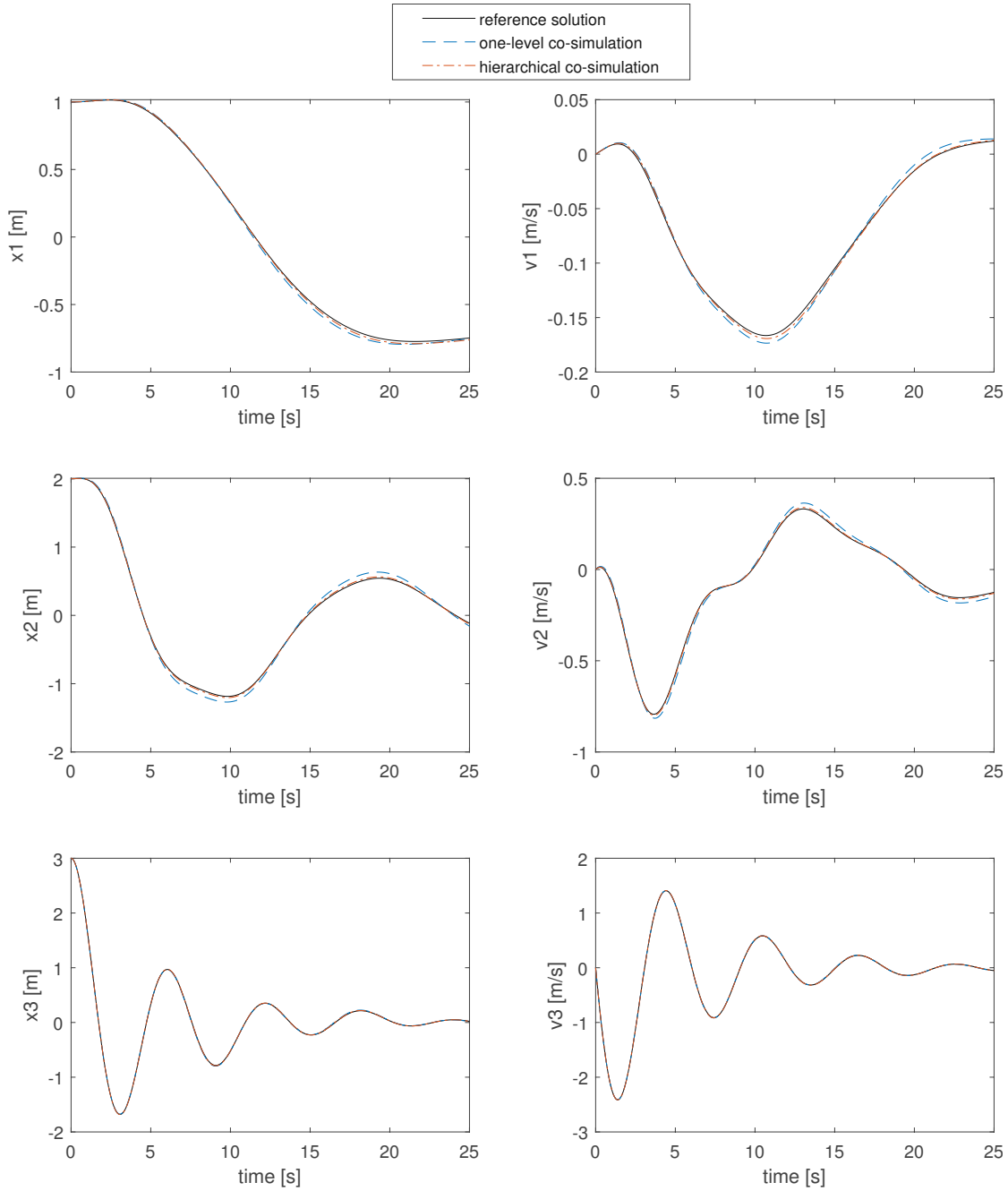


Figure 6.15: Trajectories of the system variables for Scenario 1 with $H = H_1 = 0.1s$ and $H_2 = 0.025s$ from $t_{start} = 0s$ to $t_{end} = 25s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

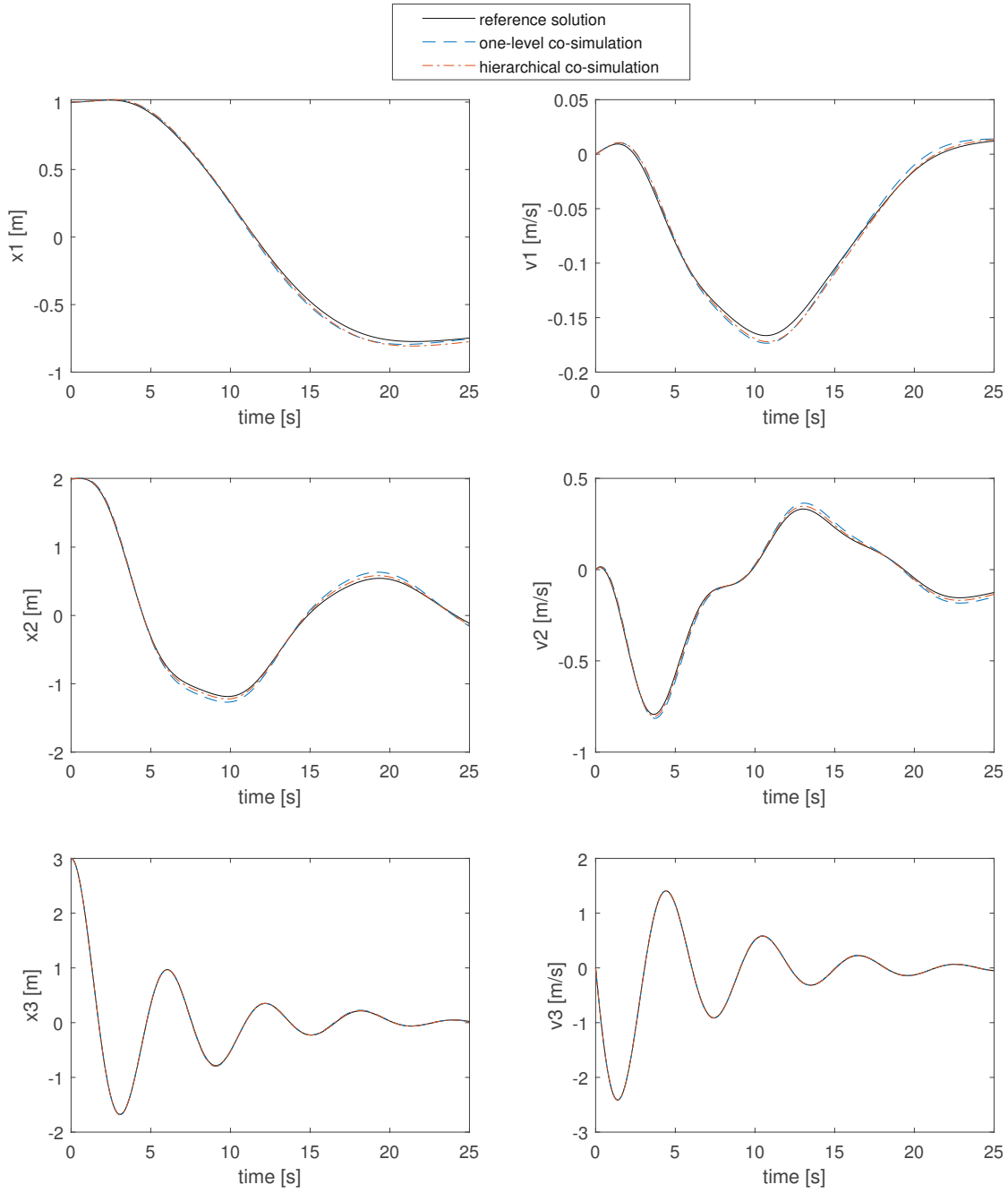


Figure 6.16: Trajectories of the system variables for Scenario 1 with $H = 0.1s$, $H_1 = 0.2s$ and $H_2 = 0.05s$ from $t_{start} = 0s$ to $t_{end} = 25s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

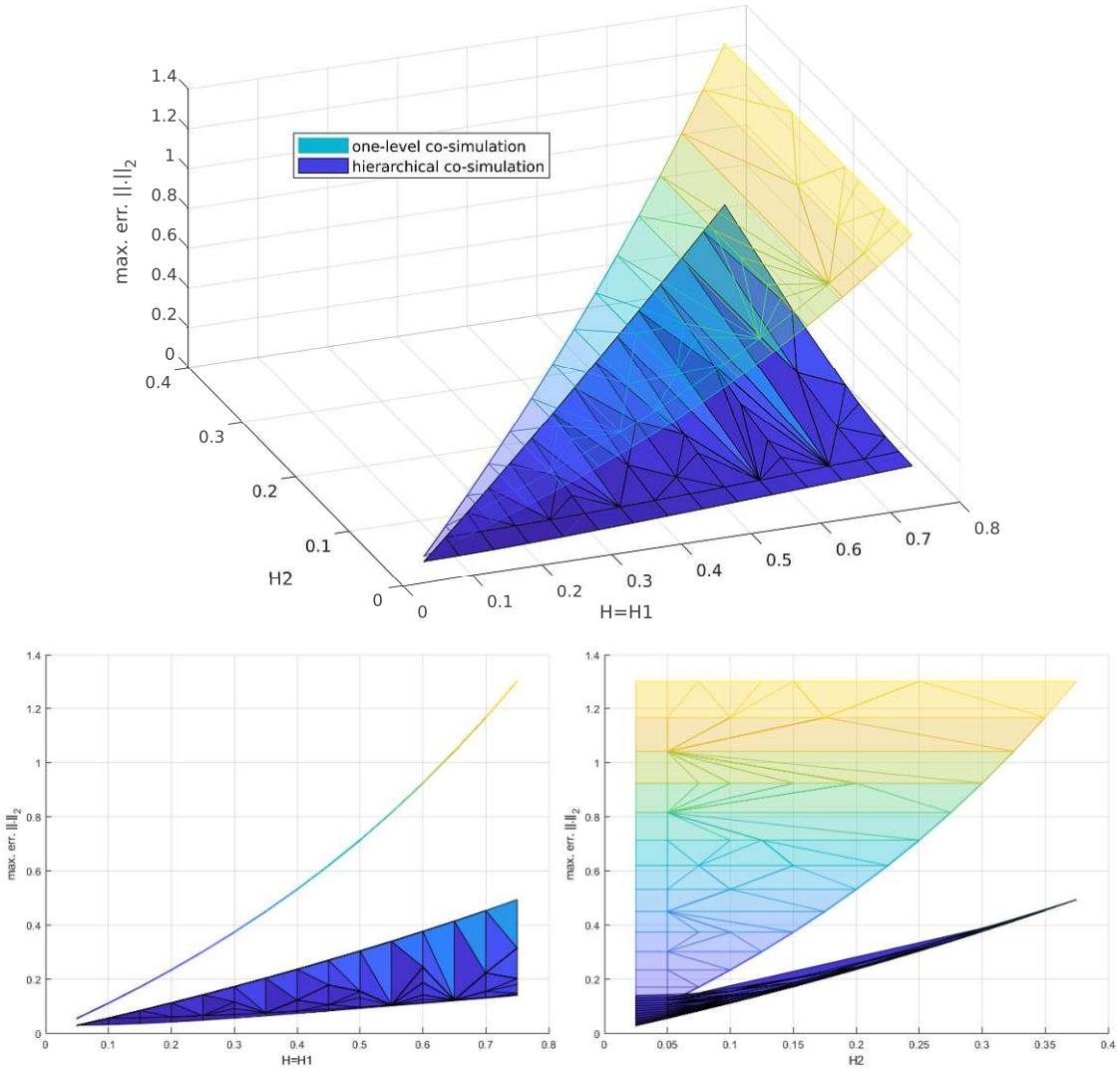


Figure 6.17: Error ($\|.\|_2$ of all component errors) for the simulation of Scenario 1 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

**Scenario 2.** In Scenario 2, the stiffnesses differ to a greater extent (see parameters in Table 6.3), which can lead to stability issues if communication step sizes are chosen too large. The conventional co-simulation already yields unstable results for the same step size as in Scenario 1 ($H = 0.1$). The solution obtained by the hierarchical approach with the same upper-level communication step size but, as in Scenario 2, additional synchronization between subsystems $S^{II}$ and $S^{III}$, remains stable. Figure 6.18 shows the progression from $t_{start} = 0$ to $t_{end} = 3$, where large errors in the conventional approach already indicate instabilities later on, which can be seen clearly in the trajectories until $t_{end} = 100$ (Figure 6.19).

Table 6.3: Parameter settings for Scenario 2.

| $c_1$ | $c_{12}$ | $c_{23}$ | $c_3$ | $d_1$ | $d_{12}$ | $d_{23}$ | $d_3$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1E-03 | 1E-01 | 10 | 100 | 0.1 | 0.4 | 1 | 2 | 10 | 10 | 10 |

Even for a larger communication step size on the upper level ($H_1 = 0.2$), stability is maintained with the hierarchical approach, as the coupling between systems $S^{II}$ and $S^{III}$ is the crucial one (cf. Figure 6.20). If the synchronization time on the second level is also increased (to $H_2 = 0.05$), qualitative behavior is still maintained but errors are too high to consider the solution still acceptable (see Figure 6.21 and Table 6.4).

Table 6.4: Maximum error and elapsed time for the traditional and hierarchical co-simulation approach in Scenario 2.

| approach | $t_{end}$ | $H$ | $H_1$ | $H_2$ | $err_{x_1}$ | $err_{v_1}$ | $err_{x_2}$ | $err_{v_2}$ | $err_{x_3}$ | $err_{v_3}$ | exec. time |
|---|---|---|---|---|---|---|---|---|---|---|---|
| traditional | 3 | 0.1 | | | 1.19E-02 | 1.56E-02 | 4.03E-01 | 4.61E-01 | 9.78E-02 | 2.37E-01 | 0.0202 |
| hierarchical | 3 | | 0.1 | 0.025 | 7.59E-03 | 7.01E-03 | 9.54E-02 | 1.10E-01 | 5.01E-02 | 1.33E-01 | 0.0376 |
| hierarchical | 3 | | 0.2 | 0.025 | 1.54E-02 | 1.44E-02 | 1.94E-01 | 2.24E-01 | 6.56E-02 | 1.66E-01 | 0.0372 |
| hierarchical | 3 | | 0.2 | 0.05 | 1.46E-02 | 1.29E-02 | 9.48E-02 | 1.10E-01 | 5.00E-02 | 1.33E-01 | 0.0258 |
| traditional | 100 | 0.1 | | | 2.37E-01 | 2.28E-01 | 5.38E+00 | 5.06E+00 | 5.35E-01 | 5.06E-01 | 2.2885 |
| hierarchical | 100 | | 0.1 | 0.025 | 2.57E-02 | 1.30E-02 | 2.90E-01 | 2.79E-01 | 6.86E-02 | 1.98E-01 | 6.4143 |
| hierarchical | 100 | | 0.2 | 0.025 | 5.81E-02 | 3.08E-02 | 6.96E-01 | 6.64E-01 | 1.06E-01 | 2.53E-01 | 5.3847 |
| hierarchical | 100 | | 0.2 | 0.05 | 3.90E-02 | 1.47E-02 | 2.88E-01 | 2.77E-01 | 6.85E-02 | 1.98E-01 | 2.5811 |

The overall error depending on the macro step sizes is illustrated in Figure 6.22 for 25 seconds simulation time. Again, we observe a much faster ascent for the traditional approach. In contrast to Scenario 1, where the difference in interdependencies for the individual subsimulations is less distinct, the impact of the choice of $H_2$ is even more pronounced in this scenario, which comes out clearly in the separate illustration of the hierarchical approach in Figure 6.23. Error plots for all components are found in the Appendix, Section A.5.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

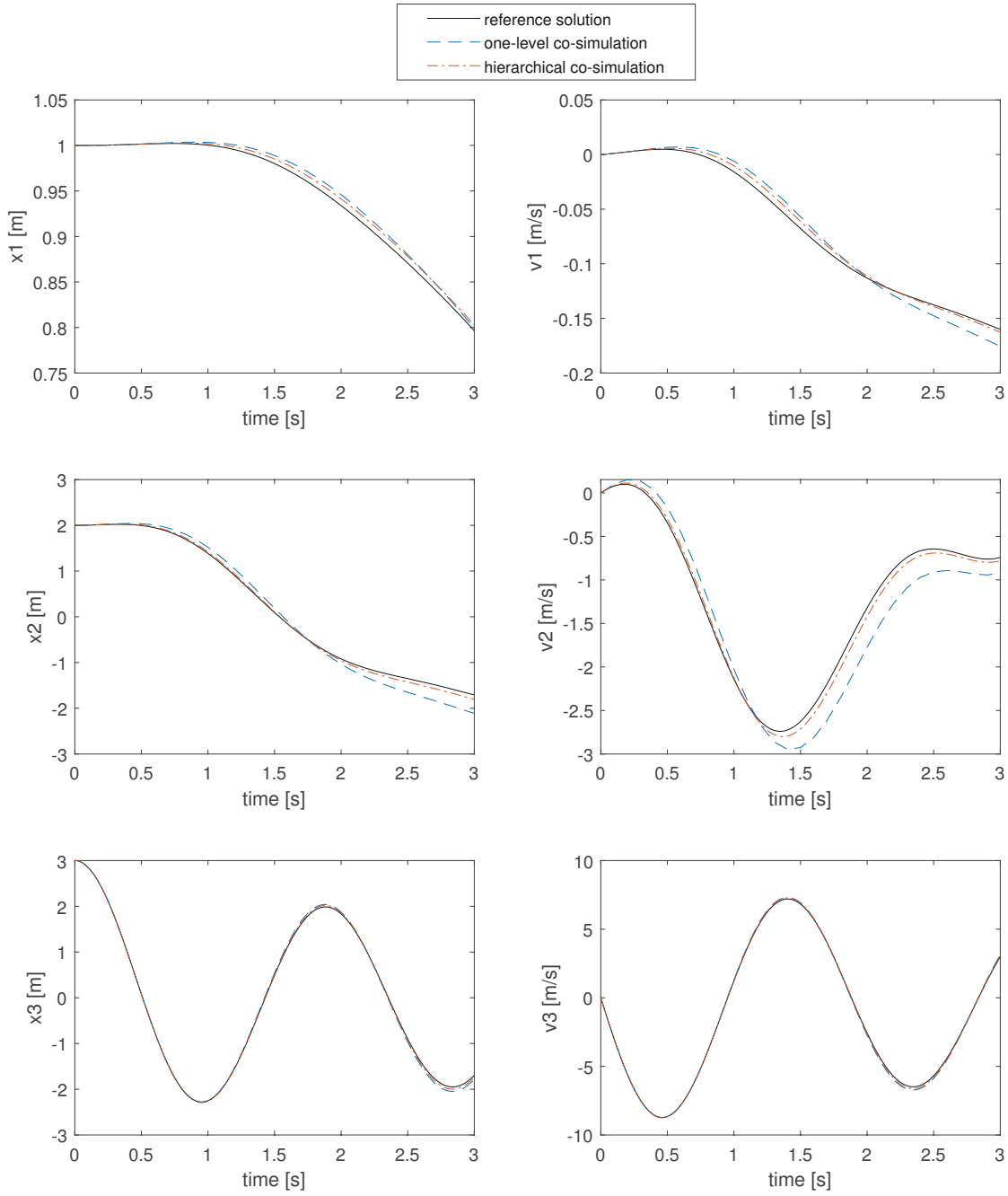


Figure 6.18: Trajectories of the system variables for Scenario 2 with $H = H_1 = 0.1s$ and $H_2 = 0.025s$ from $t_{start} = 0s$ to $t_{end} = 3s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

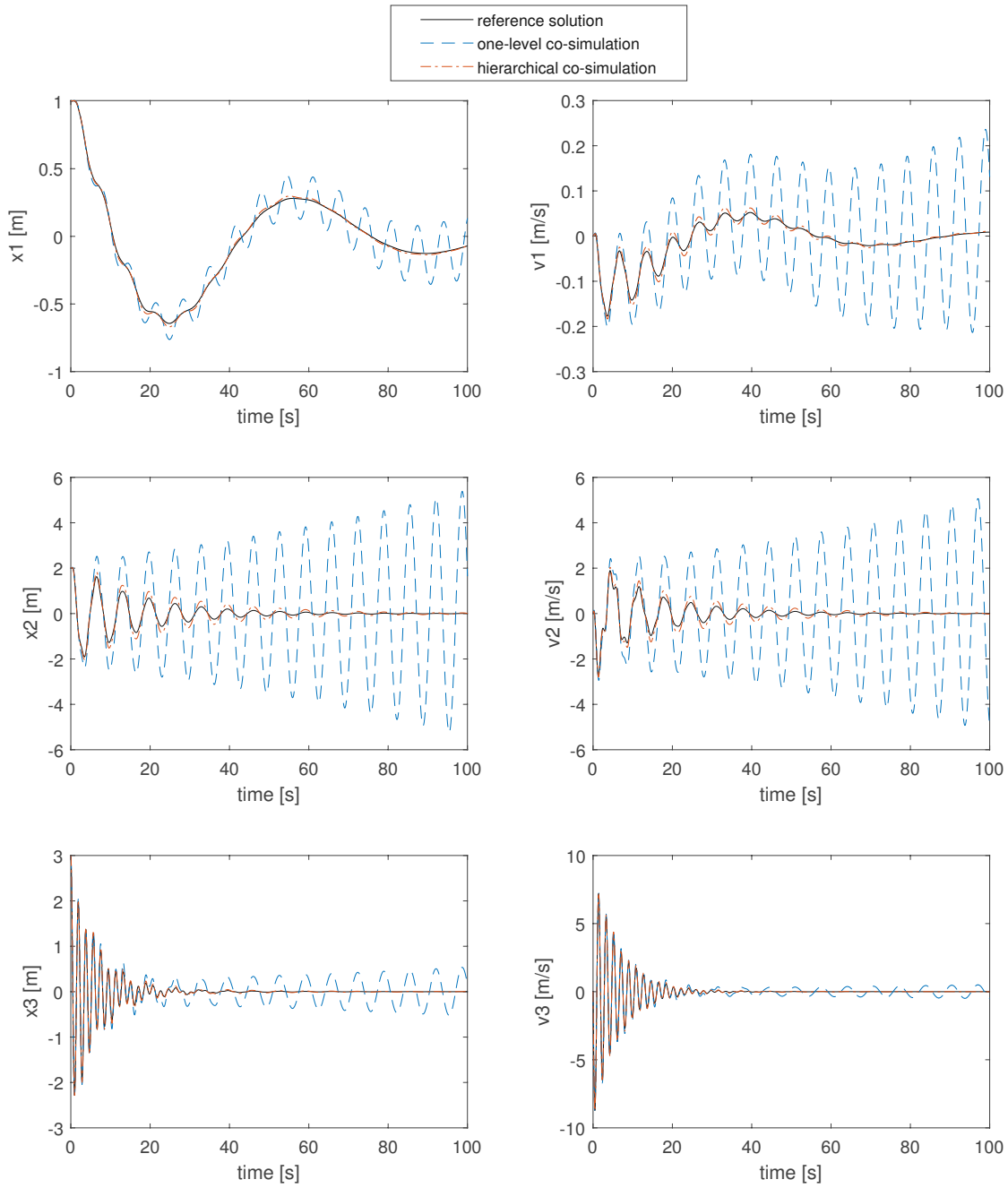


Figure 6.19: Trajectories of the system variables for Scenario 2 with $H = H_1 = 0.1s$ and $H_2 = 0.025s$ from $t_{start} = 0s$ to $t_{end} = 100s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

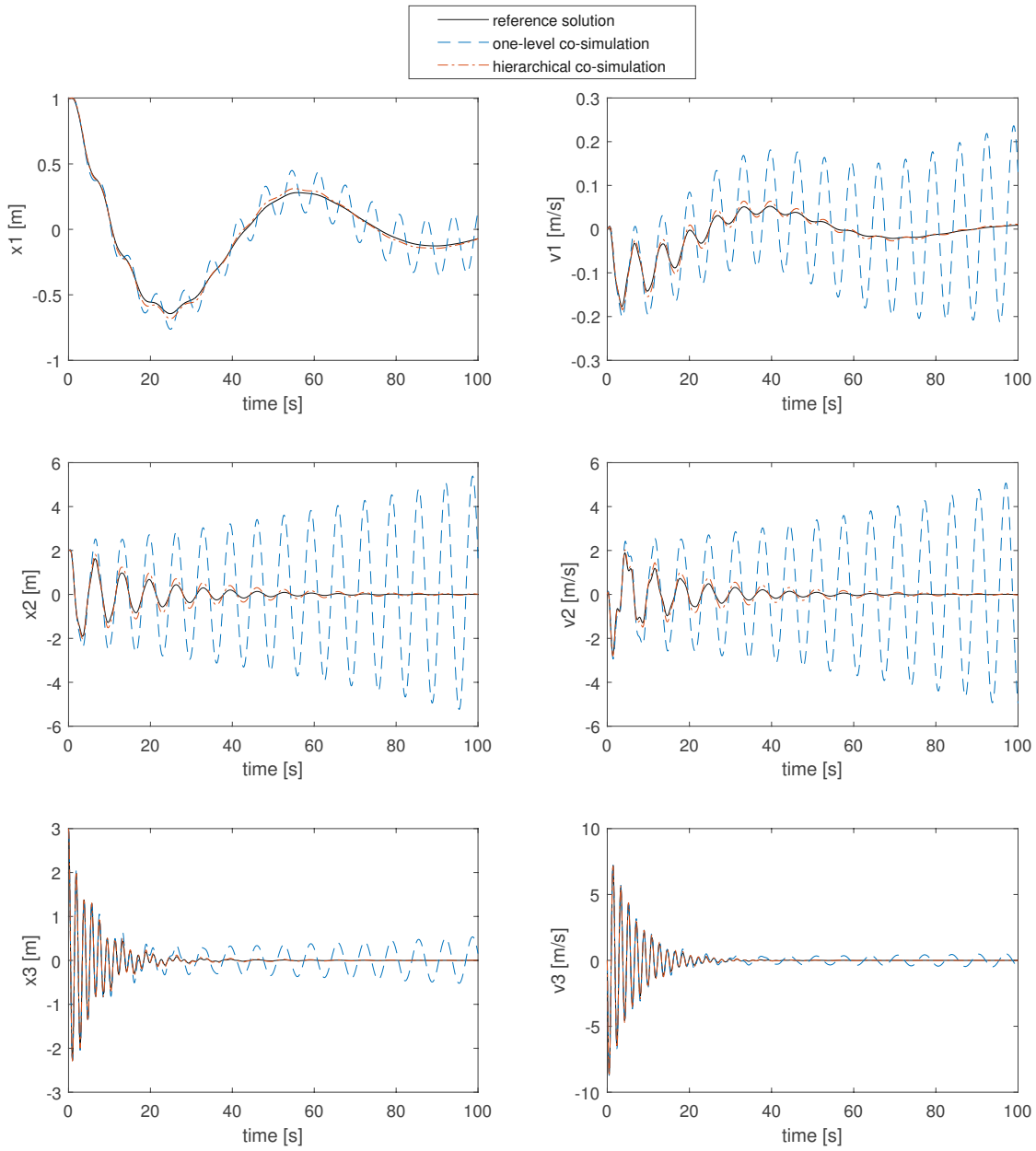


Figure 6.20: Trajectories of the system variables for Scenario 2 with $H = 0.1s$, $H_1 = 0.2s$ and $H_2 = 0.025s$ from $t_{start} = 0s$ to $t_{end} = 100s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

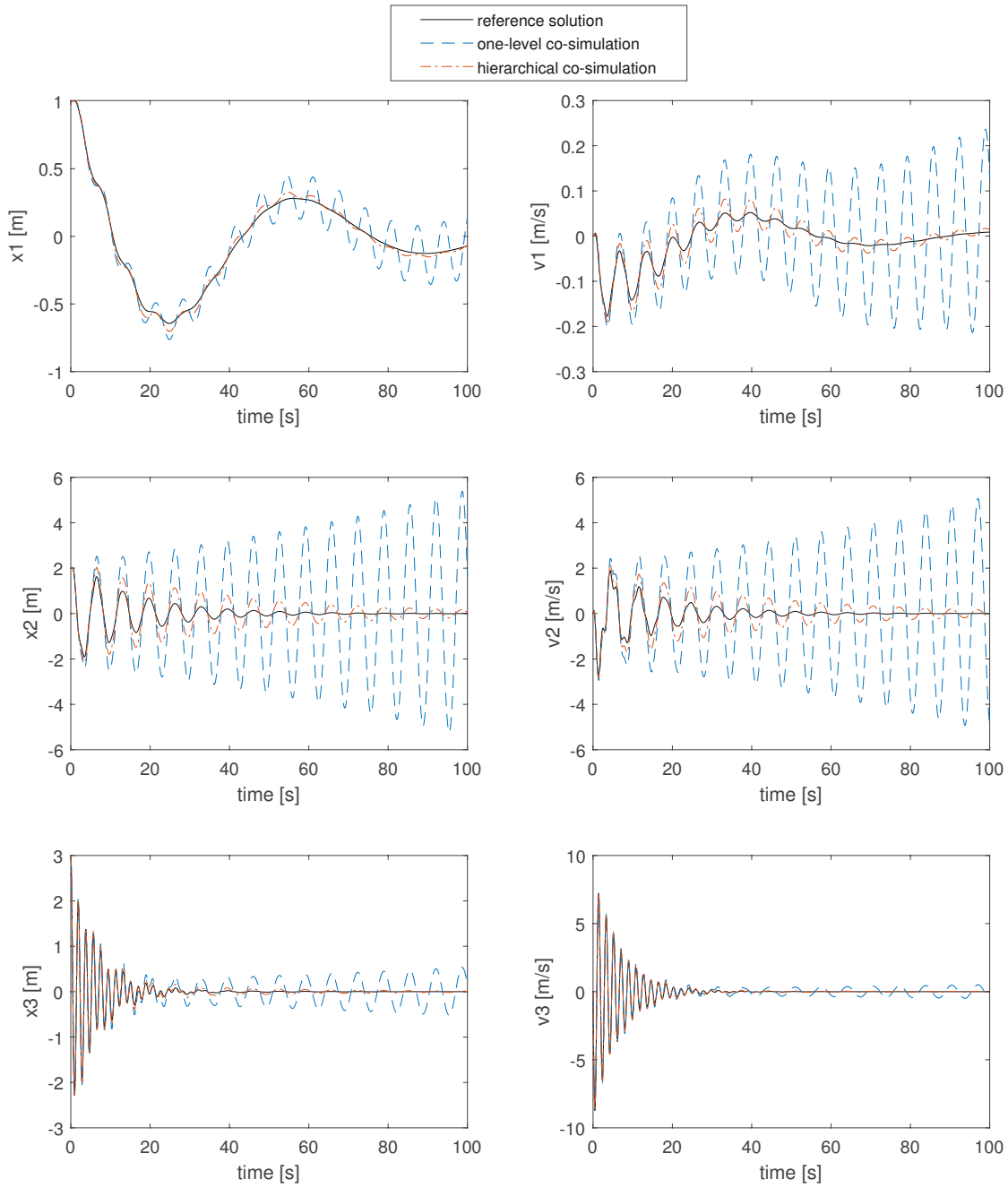


Figure 6.21: Trajectories of the system variables for Scenario 2 with $H = 0.1$, $H_1 = 0.2s$ and $H_2 = 0.05s$ from $t_{start} = 0s$ to $t_{end} = 100s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

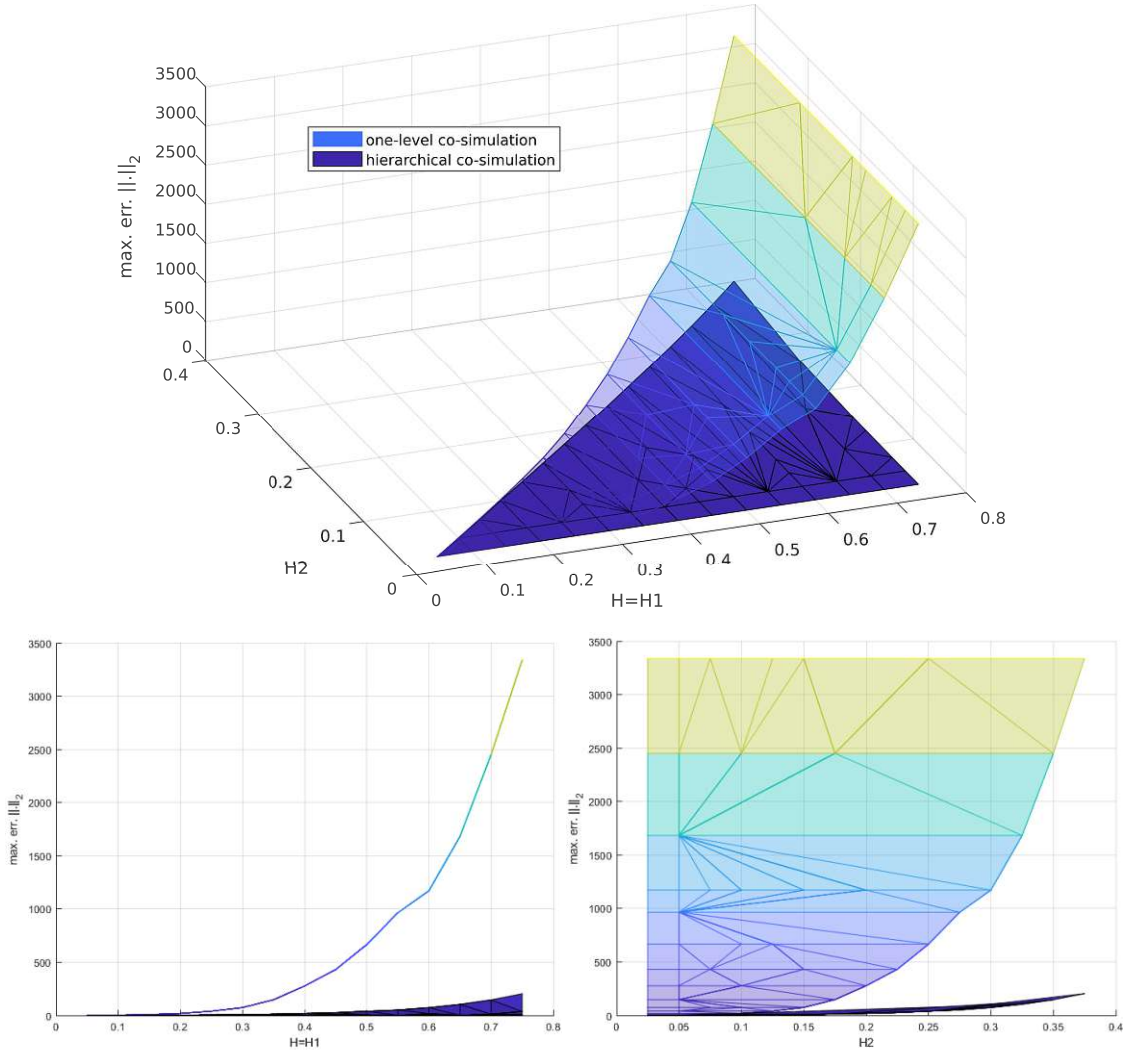


Figure 6.22: Error ($\|.\|_2$ of all component errors) for the simulation of Scenario 2 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

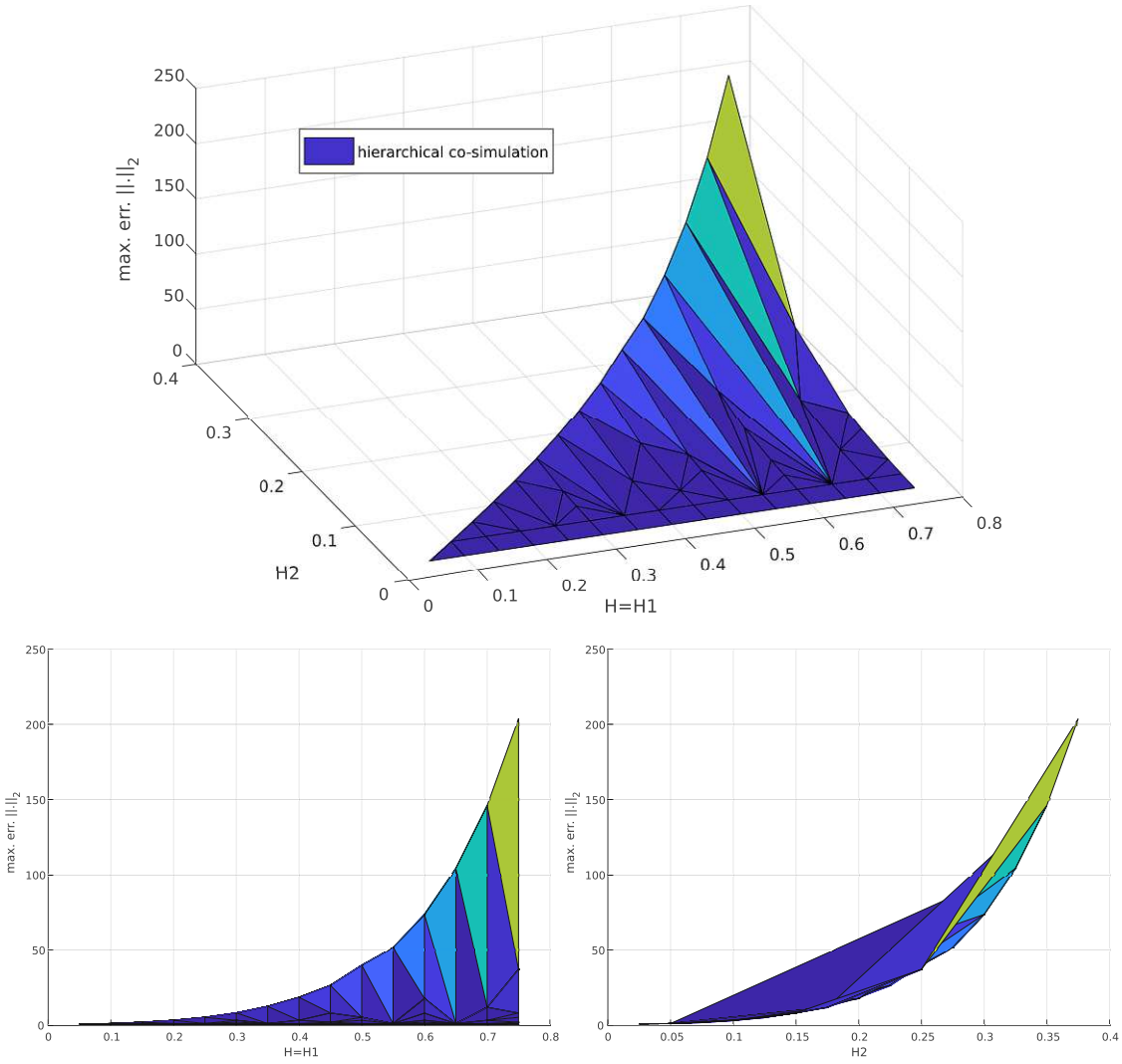


Figure 6.23: Error ($\|.\|_2$ of all component errors) for the hierarchical co-simulation of Scenario 2 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

**Scenario 3.** In Scenario 3, the stiffnesses for the springs attached to mass $m_1$ are increased, too (see Table 6.5), which leads to unstable results for the traditional as well as hierarchical approach with step sizes $H = 0.1s$, $H_1 = 0.1s$ and $H_2 = 0.025s$, see Figure 6.24. This makes sense as the increased stiffness for System $I$ cannot be compensated by closer communication of Systems $II$ and $III$.

Table 6.5: Parameter settings for Scenario 3.

| $c_1$ | $c_{12}$ | $c_{23}$ | $c_3$ | $d_1$ | $d_{12}$ | $d_{23}$ | $d_3$ | $m_1$ | $m_2$ | $m_3$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 10 | 10 | 100 | 0.1 | 0.4 | 1 | 2 | 10 | 10 | 10 |

This is emphasized by Figure 6.25, which shows $\|.\|_2$ of the maximum component errors for different values of $H = H_1$ and $H_2$ over 25 seconds simulation time. Compared to Scenario 2, we observe that although $H_2$ still impacts the magnitude of the error, it may not enforce maintenance of qualitative behavior. This is also reflected in the error plots of the individual components, which are again found in the Appendix, Section A.5.

The macro step sizes $H$ and $H_1$ would have to be chosen as low as $0.03$ to keep the error in bounds at all while still producing outputs too far from the reference solution to be of use. These results show that a hierarchical co-simulation approach can, in comparison to conventional, single-level coupling, enhance numerical stability as long as the additional composition is chosen with careful consideration of subsystem dependencies. This underlines once more that the optimal choice of coupling method depends on the given system.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

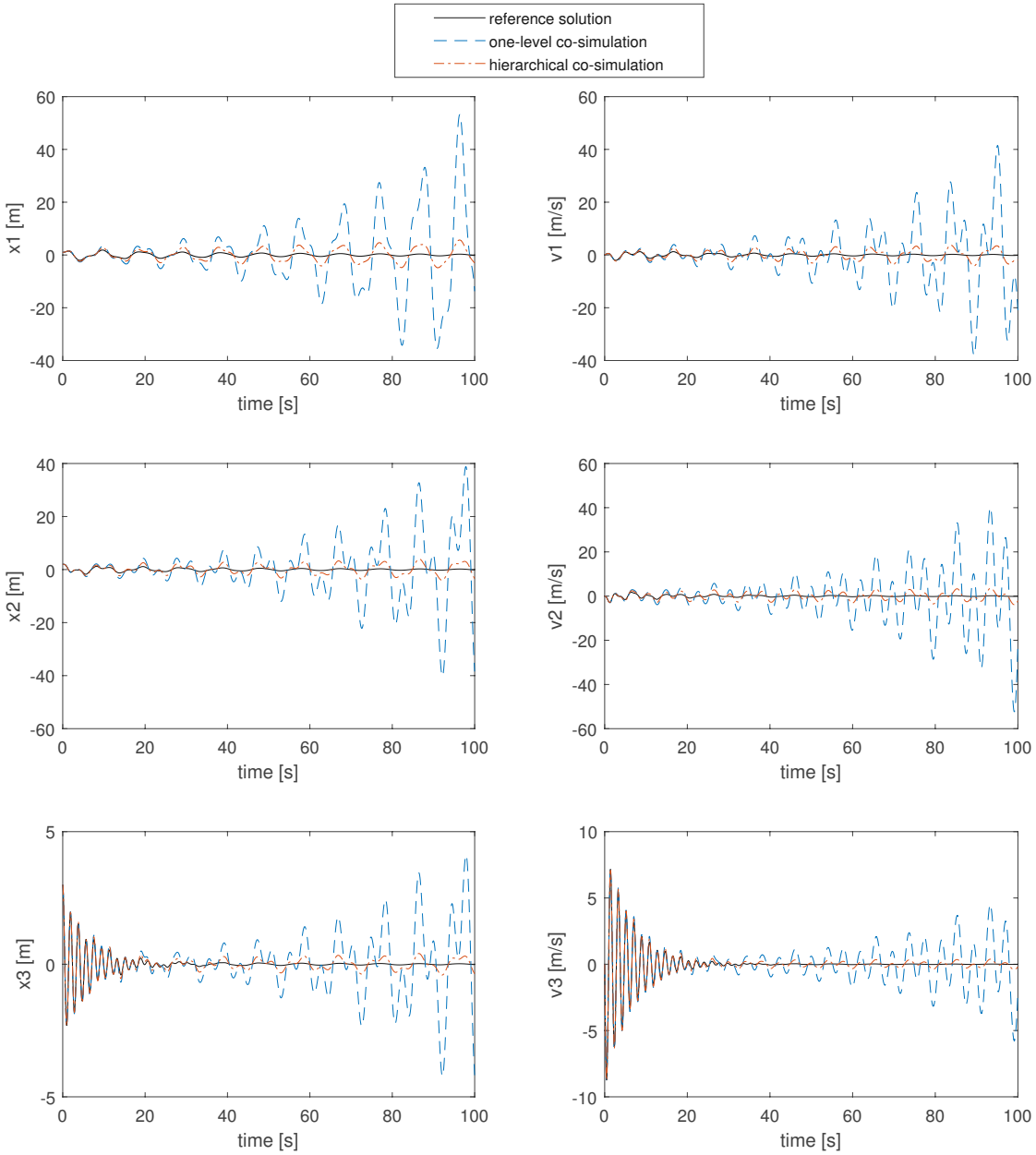


Figure 6.24: Trajectories of the system variables for Scenario 3 with $H = 0.1s$, $H_1 = 0.1s$ and $H_2 = 0.025s$ from $t_{start} = 0s$ to $t_{end} = 100s$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

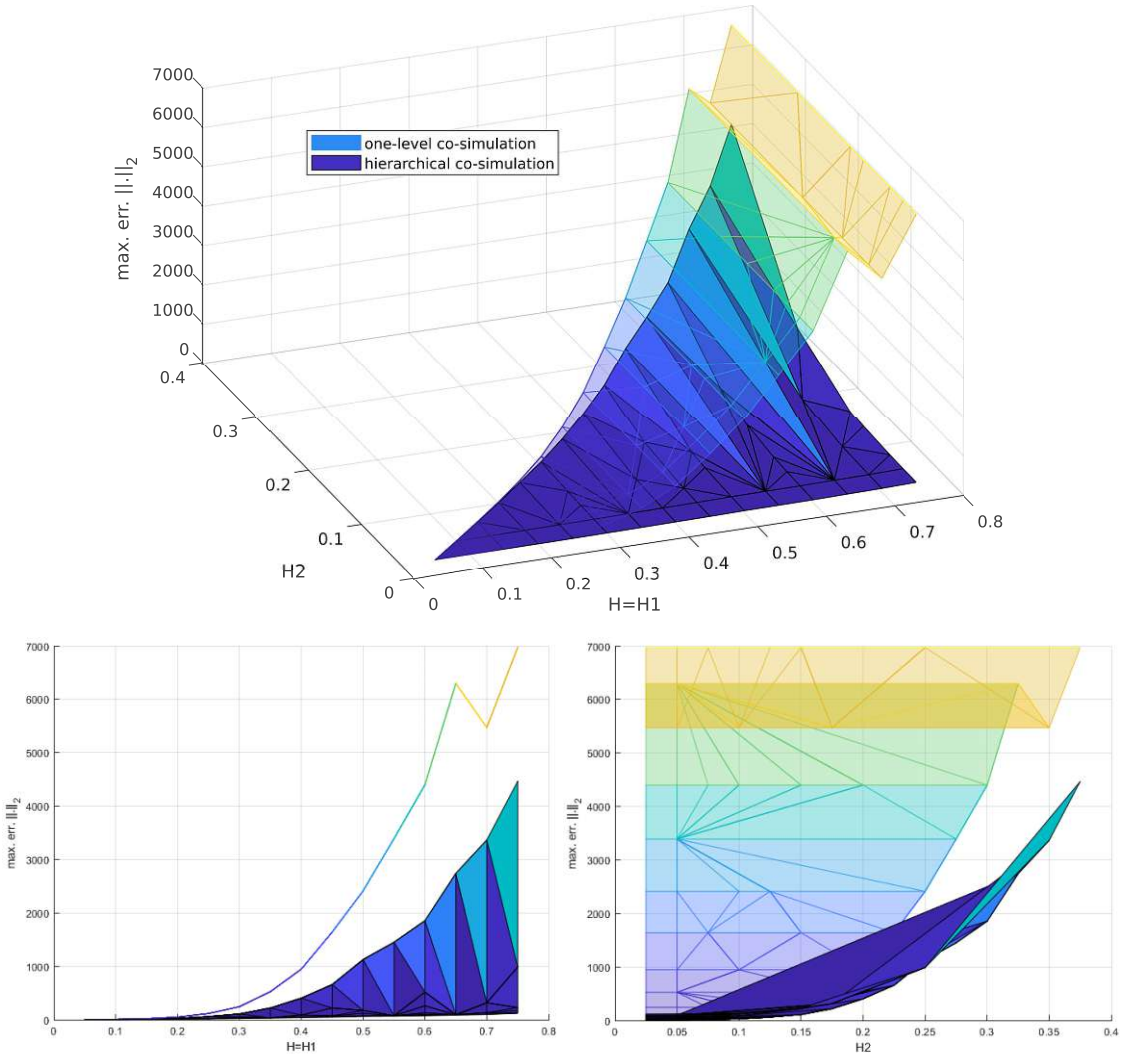


Figure 6.25: Error ($\|.\|_2$ of all component errors) for the simulation of Scenario 3 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

### 6.2.4 Concluding remarks on convergence

It should be noted that above considerations on consistency in hierarchical co-simulation are restricted to ODE systems. Estimates for more complex systems are difficult to give in general but require information on the participating systems and can be found in the literature for single-level co-simulation in specific applications, cf. Chapter 3.2.6.

Error estimates for higher order extrapolation or interpolation including the global error are given by Arnold et al. (2013) for the application in DAEs resulting from mechanical systems that are coupled in different ways. Considering the global error, it is shown that for DAEs with force-displacement coupling as well as displacement-displacement coupling, their order is not reduced, even if the local error is, as long there are no algebraic loops in the coupled system. In general, higher order extrapolation leads to higher accuracy (as the error between the correct and approximated solution are higher order terms of the step size), as is shown among others in (Arnold et al. 2013) by applying Richardson extrapolation for error estimation, which is reliable for systems without direct feed-through in at least one system. In case of instabilities (f.i. due to algebraic loops in the coupled system), however, this would also lead to faster divergence. Detailed error estimates in co-simulation using Richardson extrapolation are given in (Schierz et al. 2012; Zhang et al. 2011).

Zero-stability depends on invariable properties of the system, which means that a system can only be zero-stable or not and there is no “rate” of zero-stability in between. Therefore, other, relative stability measures for co-simulation methods have been investigated (Glumac and Kovacic 2019). While some authors (Busch 2012; Schweizer et al. 2015a) have compared co-simulation algorithms by plotted stability regions, Glumac and Kovacic (2019) present estimates for the stability radius. These properties are independent of internal states of the subsystems.

*Numerical* stability, on the other hand, depends on properties of the considered system. Improvement methods commonly used in single-level co-simulation approaches such as variations of extrapolation order, coupling methods (sequential or mixed algorithms and waveform iteration) and stabilization techniques can of course be utilized in hierarchical co-simulation as well. Detailed studies on the advantages of said techniques for traditional co-simulation are ample in the literature (see Chapter 3). In addition, the results from Section 6.2.3 show that stability issues can be tackled by introducing another layer of communication instead of having to decrease the overall communication step size, thus providing an innovative method for stabilization.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

In comparison to hierarchical partitioned multirate approaches as presented in (Günther and Rentrop 1994; Rice 1960; Striebel 2006), the application of the hierarchical co-simulation method presented in this thesis does not require any knowledge on the underlying system per se. The subsystems can, as in common co-simulation methods, be treated as black boxes with information on the input and output dependencies without interfering with the subsystem solvers. This can be beneficial when using co-simulation platforms such as the BCVTB or standards such as the FMI, and in particular for interdisciplinary collaborative projects where partial systems are developed independently and possibly protected by company-specific privacy agreements. On the other hand, due to the circumstance that the hierarchical co-simulation method does not use specific information on the subsystems and from their solution algorithms, it cannot provide the same convergence and accuracy properties as split multi-rate methods.

To sum up, depending on the given system, applying hierarchical instead of single-level co-simulation can exhibit significant improvements to accuracy and stability. The usage in preference to hierarchical multirate approaches has to be decided depending on the specifics of the given application.

## 6.3 Areas of application of hierarchical co-simulation

The presented method can be applied to various problems in industry and scientific decision support.

Applications naturally suggesting themselves are large-scale electrical circuits or mechanical systems. Striebel (2006) has already demonstrated the suitability of hierarchical partitioned multirate schemes to these kinds of problems. In case the system parts differ in detail to an extent they cannot be approached with partitioned methods but require separate solver algorithms or even simulation tools, hierarchical co-simulation presents an ideal strategy.

Another prominent use case would be the simulation of production facilities with the aim of predicting resource and energy flows along with costs in $CO_2$ as well as financial aspects. Regarding such a manufacturing process, machines have to exchange data rather frequently with logistics while only from time to time transferring their waste heat data to a slow varying, thermal room model. This, in turn, has to be synchronized with the simulation of an HVAC system controlling the room temperature which by itself does not require any

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

communication with manufacturing machines or logistic devices, let alone evaluation and data exchange at the same, considerably small, time steps.

Albeit focused beyond co-simulation, Duflou et al. (2012) present five levels of manufacturing activities: *device/unit process* (individual device or machine), *line/cell/multi-machine system* (organization of a series of devices necessary for a certain activity), *facility* (the physical entity where the device(s) are found, including HVAC systems and power generation), *multi-factory system* (facilities in close proximity allowing f.i. synergies with respect to energy), and *enterprise/global supply chain* (including all facilities, traffic, supporting infrastructure, etc.). These can also act as a guideline for the division of the system into hierarchical structures for co-simulation.

Holistic simulation of urban energy systems likewise intrinsically brings along several different levels of consideration: households, factories, traffic, network, and power plants represent different entities which, depending on the scale of the regarded system, can each per se prove complex enough to be addressed by individual co-simulations, which then have to communicate in order to portray the overall system. Intending communication of all partial systems to the utmost depth at one, inevitably small common time step would be excessive and redundant as well, yet inevitable with a traditional co-simulation approach. Aiming at the fulfillment of certain tolerances, however, may require very small macro steps and slow down the process to an impracticable extent. Larger macro steps or simplification of the system, on the other hand, may lead to insufficiently accurate results. With a hierarchical approach, the increase of accuracy comes at a comparably small cost if frequent exchanges are restricted to certain, deliberately selected system parts, thus sidestepping above mentioned dilemma.

These considerations can also be conferred upon other industrial systems of varying scales or infrastructure planning, where not only energy flows but people and resources have to be taken into account as well. This applies for instance to airports, hospitals, train stations and locomotive scheduling or even epidemiological simulations including health system infrastructure and logistics in addition to population dynamics. Due to their complexity along with differences in modeling approaches and time scales for system parts, all these examples invite cooperative simulation in hierarchical structures.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

CHAPTER 7

# Conclusion and Outlook

The present work has explained the basic principles of cooperative and multirate simulation with regard to terminology, related work along with present challenges, and ways of classifying existing coupling approaches. Based on these preliminary investigations, an innovative approach to enhance stability properties of co-simulated systems by the introduction of hierarchies has been presented.

## 7.1 Summary and conclusion

Due to the broad range of applications and methods in the area of multirate and co-simulation, a thorough study of related publications served as the natural starting point for the work on this thesis: Only thereby, current challenges and open research questions could be made out as basis for specific investigations. However, in the course of literature research it soon became clear that a comprehensive conception of existing methods is a challenge in itself due to major discrepancies in terminology. This circumstance as well as the multitude of variants to structure coupling and decomposition approaches respectively have motivated the elaboration of these topics as separate, minor research questions.

Chapter 2 (*Terminology: Present Perceptions and Unification*) has on the one hand highlighted different perceptions and, moreover, aimed at unifying certain terms. Although some discrepancies will remain since a few of these can hardly be standardized without inconsistencies due to already contradictory usage, the deliberations have certainly raised the readers' awareness to different interpretations. Apart from the clarification and initiation of



Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

discussion of certain terms, relations and overall nexus of methods have been elaborated at the end of Chapter 2, which together can prevent misunderstandings, allow the demarcation of one's own research and speed up early development or research stages of related projects.

Existing developments, current problems and, in particular, the ascertainment that the idea of hierarchical co-simulation has not already been investigated by some other research group, have been presented in the extensive overview of the state of the art in Chapter 3. Although the research on simulator coupling goes back more than two decades, a lot of questions still remain unanswered and some even need to be posed in the first place. Most of the presented developments are very specific, which along with particular investigations in search of a general-purpose method or criteria for the selection of an optimal one lead to the conclusion that the choice of an optimal method cannot be made without taking into account the underlying system, status of model development, and the know-how and interdisciplinarity of the team of developers. Depending on the considered use case, certain properties might even be seen as an advantage for some applications and a drawback for others, such as the necessity or possibility of intrusions to subsystem solvers.

To enable even broader and up-to-date insight in current developments and perceptions of co-simulation, an empirical survey has been developed and conducted in cooperation with colleagues from international and national research groups. Its results are presented in Chapter 4, therein pressing challenges, most commonly used applications, and standards. The Functional Mockup Interface stands out as the most established standard, yet also brings along the most specific challenges. These frequently include problems concerning hybrid simulation (which also proves to be one of the most pressing challenges in co-simulation in general, not only specific to the FMI). Among other specific problems, communication in cross-company collaborations and teams of both theorists and practitioners is mentioned in the experts' assessment of current challenges in co-simulation (see Table 4.2), where the terminology established in Chapter 2 may be of help. Further named are estimates of the associated communication error, difficulties in the choice of the macro step size for a specific co-simulation, and numerical stability issues, all of which are addressed in Chapter 6: While no particular guidelines for the choice of the macro step size are given, the introduction of further levels allows the selection of differing macro steps for the communication of different selections of subsystems, thus enhancing accuracy without necessarily slowing down the whole simulation process, as would be the case for only one adjustable common macro step. Numerical stability, on the other hand, is shown to clearly improve by

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

the careful introduction of further co-simulation levels (see Section 6.2.3).
Considering the overall results of the empirical survey, these enable better targeted research and hopefully motivate better cooperation in the area of co-simulation.

The research on the state of the art has brought out many different aspects by which co-simulation approaches can be distinguished. Novel ideas for classification have been brought forth by the combination of existing categorizations presented by other researchers and the widespread overview that was gained by the extensive study of related work. These have resulted in the complex and multi-faceted structure presented in Chapter 5. Similar to the elaborations on terminology, it has proved important to be able to classify one's own research and guarantee its originality: Only with the information of Chapters 3 and 5 was it possible to dissociate hierarchical co-simulation as presented here from partitioned hierarchical multirate integration.
Referring to this structuring, a selection of the literature presented in Chapter 3 has been classified and analyzed, thus highlighting more and less popular topics. In addition, the illustration of networks of (co-)authoring researchers and institutions has identified clusters and isolated authors, which further emphasizes the need for the unification of terminology and support in establishing cross-company and international cooperations.

The main focus of this work has been laid on the investigation of hierarchical co-simulation approaches. These, as evidenced by the results from Chapters 3 and 5, have up to now barely been mentioned in the literature (cf. the acknowledgement of co-simulations within a co-simulation by Thule et al. (2019b) and Wang et al. (2003)) and, even more importantly, not been investigated with regard to consistency or stability (confer Section 6.1). As explained at the beginning of Chapter 6, hierarchical approaches in general are no novelty in the area of modeling and simulation, see f.i. DEVS (Zeigler 2014), multi-level agent-based modeling (Morvan 2013) and, most similar to the co-simulation approach here, hierarchical partitioned multirate schemes (Maten et al. 2005; Striebel 2006). The latter, however, require the usage of the same instance of the same simulation tool for every subsystem and one partitioned solution algorithm. Applying hierarchical co-simulation, every sub-simulation may be integrated with an individual solution algorithm and even different simulators. Apart from the formal introduction of hierarchical structures in cooperative simulation, detailed investigations on consistency, zero-stability, and numerical stability properties of hierarchical co-simulation have been presented in this thesis. These have shown that accuracy and stability may be improved by the introduction of further co-simulation levels in comparison to common co-simulation while no additional errors are to be expected due to the extra

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

splitting. On the contrary: Even though the final estimate for the consistency error in the hierarchical approach is denoted only in dependence of the top-most and therefore largest macro-step (thus yielding an estimate corresponding to those for traditional co-simulation, cf. (Arnold and Günther 2001; Knorr 2002; Trčka 2008)), estimates on lower levels occurring in the proofs in Section 6.2.1.2 and the error plots in Sections 6.2.3 and A.5 show even faster convergence in case of carefully chosen application. Keeping in mind the information from Chapter 3 (cf. in particular (Kübler and Schiehlen 2000b)), we know from the investigations in Section 6.2.2 that zero-stability in hierarchical co-simulation can be determined similarly to zero-stability of single-level co-simulation and expect increased numerical stability, as shown in Section 6.2.3.

However, as with every modeling and simulation approach, hierarchical co-simulation cannot be expected to be a universally applicable solution with superior properties in general. Its usage and if so, details in its application such as level attribution, macro step sizes, and coupling methods per level have to be chosen with care and in accordance with the present use case.
Unfortunately, specific formal criteria for the selection of an optimal approach to a particular problem cannot be given for several reasons: apart from the considered real system aimed at being represented by a digital twin, the purpose of its simulation influences the choice of the particular approach. Upcoming questions during the decision process include but are not limited to the following ones: *Does the system need to be decomposed? If so, due to different time constants and/or the necessity for different modeling approaches for individual system parts? Does the system show more and less pronounced dependencies in certain parts? Which subsystem solvers are required? Does the application require real-time simulation? Do implementations of system parts already exist? If so, are these modifiable or to be treated as black boxes?* These and more need to be posed in varying order and repeatedly – sometimes even after several, possibly failing, tries for implementation, simulation runs, or comparisons. Even so, no determinate answers may be found, if like in Scenario 3 of 6.2.3, none of the chosen approaches delivers sufficiently accurate results, or in other cases, several methods perform satisfactorily, such as adaptive versus implicit approaches. Even if several differing magnitudes in time constants invite two or more levels of hierarchy, a homogeneous structure of the representing mathematical system may justify the preference of partitioned multirate approaches over hierarchical co-simulation and vice versa. Some criteria may present themselves straightforwardly (such as the optimal choice of an integration method for a certain ODE system) or can be determined by explicitly given conditions (such as boundaries for dependencies in form of partial derivatives with respect

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

to external variables to suggest loose or strong coupling approaches). However, this may not be the case for the combination of subsystem representation, solution method and coupling method with regard to macro steps, orchestration, extrapolation order and method, loose or strong coupling, implicit or adaptive algorithms and number of co-simulation levels. Due to the variety of options for each of these individual aspects, the possibilities for their combination amount to a barely graspable multitude.
Nevertheless, there are some informal guidelines that suggest advantages in the introduction of further co-simulation levels, such as manufacturing or urban energy systems, where hierarchical structures are already presenting themselves in the corresponding real system. For systems with pre-implemented parts that have to be treated as black boxes or simply require different modeling approaches, partitioned schemes would not be applicable as these would require interference with the subsystem solvers and homogeneous implementation. The same holds true for DEVS-based approaches, which even claim unrestricted knowledge of every subsystem as these need to be (re-)implemented in the formalism. While both these methods are advantageous in other respects, in case of facing above mentioned conditions, the application of hierarchical co-simulation as presented in this thesis proves the most suitable.
Summing up the results of Chapter 6, we can conclude that depending on the given system and objective, hierarchical co-simulation can exhibit significant improvements to accuracy and stability in comparison to hitherto known approaches.

## 7.2 Outlook

While this work has brought to light the variety co-simulation methods can cover, it has simultaneously shown that many of these specific methods and ways to combine different model descriptions and solution algorithms are not yet explored, which leaves a lot of open research questions to be faced in future investigations.

Establishing specific criteria for determining whether mono-simulation, traditional or hierarchical co-simulation – and which coupling method exactly therein – is the most suited method to approach a given problem is hardly realistic due to the variety in problems, their description and purpose as well as solution approaches and combinations thereof. Nevertheless, even informal guidelines, which up to now are lacking as well (Thiede et al. 2016), would be of help in the selection of a suitable method for a given problem. However, we need to acknowledge that it has been shown that specific choices (like for instance the in-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

terpolation order (González et al. 2011)) may not be determined beforehand but have to be found out by the much-despised, yet owing to a lack of alternatives still frequently applied strategy of trial-and-error. Any aid in this respect would be highly valuable throughout the area of modeling and simulation.
Further examples for open research questions are coupling methods for highly contrastive simulation approaches such as discrete and continuous model descriptions. Therein but also when discrete event systems are coupled among themselves, the processing of events presents a particular challenge.

The hierarchical co-simulation approach presented in this thesis provides several aspects for further enhancement. Instead of parallel, non-iterative coupling algorithms with fixed macro step size, zero-order extrapolation and Euler integration methods used in the benchmark example from Section 6.2.3, strategies which are known from related work (cf. Chapter 3) to improve stability, performance, or accuracy for traditional co-simulation may be utilized in hierarchical co-simulation as well. Among these, the utilization of sequential, iterative or adaptive orchestration algorithms, different extrapolation orders and higher order and/or multistep subsystem solvers remain a topic for future investigations. In addition, extensive case studies comparing hierarchical co-simulation to partitioned methods, mono-simulation and common co-simulation on complex application examples in the area of energy systems, electrical circuits or manufacturing processes will be essential to provide further guidelines for the selection of the most suited approach to a given problem.
For “divide-and-conquer” approaches, automatic partitioning not only into subsystems (like in traditional coupling methods) but also sub-co-simulations are of interest, on the one hand for initialization but on the other hand dynamically during runtime, too.

This shows that apart from substantial clarification and filling of certain gaps in the research on multirate and co-simulation, this thesis has opened possibilities and illuminated leverage points for continuing, targeted investigations.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

APPENDIX A

# Appendix

## A.1 Abbreviations

**AB** Agent Based

**ABM** Agent Based Models

**CA** Cellular Automata

**CBD** causal block diagram

**CT** continuous time

**DAE** differential-algebraic equation

**DCA** divide-and-conquer algorithm

**DE** discrete event

**DESS** Differential Equation System Specification

**DEV&DESS** Discrete Event System and Differential Equation System Specification

**DEVS** Discrete Event System Specification

**DOF** degree of freedom

**DSL** domain specific language

**FE** finite element

**FEM** finite element method, finite element model



Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

**FMI** functional mockup interface

**FMU** functional mockup unit

**HIL** hardware-in-the-loop

**HLA** High Level Architecture

**HVAC** Heating, Ventilation, Air Condition and Cooling

**hyPDEVS** Hybrid P-DEVS

**IJCSA** Interface Jacobian-based Co-Simulation Algorithm

**IVP** initial value problem

**MPC** Model Predictive Control

**ODE** ordinary differential equation

**PDAE** partial differential-algebraic equation

**PDE** partial differential equation

**P-DEVS** Parallel DEVS

**QSS** quantized state systems

**RTW** real-time workshop

**SD** System Dynamics

**SDF** synchronous data flow

**VHDL** Very High Speed Integrated Circuit Hardware Description Language

**WR** waveform relaxation

## A.2 Numerical basics and background

This chapter provides some essential information on methods and results for the numerical solution of differential equations, taken from Melenk 2008,Hairer et al. 1993,Hairer et al. 1996 and Zeidler 2013.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

### A.2.1 Ordinary differential equations

A typical ordinary differential equation (ODE) initial value problem given (IVP) is given in (A.1)

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(t, \boldsymbol{x}(t)), \quad t \in [t_0, T] \tag{A.1}$$

$$\boldsymbol{x}(t_0) = \boldsymbol{x}_0 \tag{A.2}$$

for $f$ continuous in $(t, \boldsymbol{x}(t))$ and Lipschitz-continuous in $\boldsymbol{x}(t)$. The aim of numerics of differential equations is to approximate a solution $\boldsymbol{x}(t)$ in the interval $[t_0, T]$ at least for specific points in time $t_0 \leq t_1 \leq t_2 \ldots t_{N-1} \leq t_N = T$. These values can be determined by

$$\boldsymbol{x}(t_{i+1}) = \boldsymbol{x}(t_i) + \int_{t_i}^{t_{i+1}} \boldsymbol{f}(s, \boldsymbol{x}(s))ds \tag{A.3}$$

Since $f$ is not necessarily integrable, numerical methods for this integration have to be found.

**Definition A.1** (One step method)**.** A numerical method which, given $x_0$, calculates $x_i, i = 1 \ldots, N$ by a recursive formula

$$x_{i+1} = x_i + h_i \Phi(t_i, x_i, x_{i+1}, h_i), \qquad i = 0, \ldots, N-1 \tag{A.4}$$

is called *one step method*. $\Phi$ is called *increment function*. If $\Phi$ is explicitly dependent on $x_{i+1}$, the method is called *explicit one step method*, otherwise *implicit one step method*.

*Remark* A.2. A one step method given by A.4 is called *convergent* if

$$\max_{i=0,\ldots N} |x(t_i) - x_i| \to 0 \quad for \quad h_\Delta = \max_{i=0,\ldots N} h_i \to 0. \tag{A.5}$$

The *consistency error* provides a measurement for the error made by the method in one step.

**Definition A.3** (Consistency error, Consistency)**.** Consider a region $G \subset \mathbb{R}^2$, $f \in C(G)$ locally Lipschitz continuous in its second argument and the increment function $\Phi$ defined on $\mathcal{G} := G \times [0, \underline{h}] \subset \mathbb{R}^3$ for an $\underline{h} > 0$. The *consistency error* $\tau(t_0, x_0, h)$ is defined for $(t_0, x_0, h) \in \mathcal{G}$ as

$$\tau(t_0, x_0, h) = x_{t_0,x_0}(t_0 + h) - (x_0 + h\Phi(t_0.x_0, h))$$

where the function $t \mapsto x_{t_0,x_0}(t)$ is defined as solution of

$$x'_{t_0,x_0}(t) = f(t, x_{t_0,x_0}(t)), \qquad x(t_0) = x_0.$$

If for every $(t_0, x_0) \in G$

$$\lim_{h \to 0^+} \frac{\tau(t_0, x_0, h)}{h} = 0$$

holds, the one-step method (A.4) is called *consistent* on G. The method is called *consistent of order* $p > 0$, if for every compact subset $K \subset G$ there exists a constant $C$ and $h' > 0$ so that

$$|\tau(t_0, x_0, h)| \leq Ch^{p+1} \qquad \forall (t_0, x_0) \in K \quad \forall h \in [0, h'].$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*Remark* A.4. A method of consistency order $p$ has a global error $O(h^p)$.

**Theorem A.5** (Consistency of one step methods)**.** *Let $J \subset \mathbb{R}$ be an open interval, $x_{ex} \in C^1(J)$, $[t_0, T] \subset J$. If for a $\delta > 0$ and $\underline{h} > 0$ the increment function $\Phi$ fulfils*

*(i) $\Phi$ is defined and continuous on $\mathcal{G} := S_\delta \times [0, \underline{h}]$ with*

$$S_\delta = \bigcup_{t \in [t_0, T]} \{t\} \times [x_{ex}(t) - \delta, x_{ex}(t) + \delta]$$

*(ii) $\Phi$ is Lipschitz continuous in its second argument, i.e. there exists an $L_\Phi > 0$ so that*

$$|\Phi(t, x, h) - \Phi(t, \hat{x}, h)| \leq L_\Phi |x - \hat{x}| \quad \forall (t, x, h), (t, \hat{x}, h) \in \mathcal{G}$$

*(iii) There exists $C_\tau > 0, p \in \mathbb{N}$ so that $\forall t \in [t_0, T]$ and $h > 0$ with $t + h \leq T$ holds*

$$|x_{ex}(t + h) - (x_{ex}(t) + h\Phi(t, x_{ex}(t), h))| \leq C_\tau h^{p+1}$$

*then it follows that there exists an $\overline{h} \in (0, \underline{h})$ so that for every discretisation $\Delta = \{t_i | i = 0 \ldots, N\}$ with $h_\Delta \leq \overline{h}$*

*(a) the approximations $x_i$, $i = 1, \ldots, N$ given by (A.4) exist and*

*(b) they fulfil $|x_{ex}(t_i) - x_i| \leq C_\tau (t_i - t_0) e^{L_\Phi (T - t_0)} h_\Delta^p$.*

*This assures in particular that*
$\max_{i=0,\ldots,N} |x_{ex}(t_i) - x_i| \leq C_\tau (T - t_0) e^{L_\Phi (T - t_0)} h_\Delta^p$.

**Definition A.6** (Runge-Kutta methods)**.** A Method is called *Runge-Kutta method* if there exist $s \in \mathbb{N}$, $a_{ij}, b_i \in \mathbb{R}$, $c_i \in [0, 1]$ so that its increment function $\Phi$ can be written as

$$\Phi(t, y, h) := \sum_{i=1}^{s} b_i k_i$$

where the $k_i$ are given as solution of

$$k_i := f(t + c_i h, y + h \sum_{j=1}^{s} a_{ij} k_j), \quad i = 1, \ldots, s$$

and the $b_i$ have to fulfill the consistency requirement

$$\sum_{i=1}^{s} b_i = 1. \tag{A.6}$$

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

**Definition A.7** (Linear multi-step methods)**.** The idea of multi-step methods is to approximate the integrand instead of the integral. To achieve this, information of calculations from earlier time steps is used for the calculation of an approximation polynomial of $\boldsymbol{f}(t, \boldsymbol{x}(t))$. In a *linear $k$-step method* each approximation $x_{i+1}$ is calculated via

$$\sum_{j=0}^{k} \alpha_{k-j} x_{i+1-j} = h \sum_{j=0}^{k} \beta_{k-j} f(t_{i+1-j}, x_{i+1-j}). \tag{A.7}$$

with $|\alpha_0| + |\beta_0| \neq 0$. Without loss of generality, $\alpha_k$ is set to $1$.

**Definition A.8** (Characteristic polynomial)**.** For a linear k-step method as in Definition A.7, the polynomial $\rho : \mathbb{C} \to \mathbb{C}$ given by

$$\rho(\zeta) = \sum_{j=0}^{k} \alpha_j \zeta^j \tag{A.8}$$

is called its *first characteristic polynomial* and $\sigma : \mathbb{C} \to \mathbb{C}$ given by

$$\sigma(\zeta) = \sum_{j=0}^{k} \beta_j \zeta^j \tag{A.9}$$

is called the *second characteristic polynomial* of the method.

In general, linear Multi-Step methods based on integration are called *Adams methods* and methods based on differentiation yield *Backwards Differentiation Formulas* (*BDF methods*) which are mainly used for dealing with stiff problems.

While for linear one-step methods consistency is sufficient for convergence, it is only a necessary but not sufficient condition for linear multi-step methods. To assure convergence, linear multi-step methods have to satisfy *zero-stability* as well [1]. Zero-stability stands for stability of the method for $h \to 0$ and can be defined as follows:

**Definition A.9** (Zero-Stability of Linear Multi-Step Methods)**.** A linear multi-step method is called *zero-stable* if every zero $\lambda$ of the first characteristic polynomial $\rho$ fulfils $|\lambda| \leq 1$ and every zero with $|\lambda| = 1$ is a single zero.

Zero-stability means stability for step size $h \to 0$.

**Definition A.10** (Convergence of linear multi-step methods)**.** A linear k-step method (A.7) applied to a problem $x' = f(t, x)$, $x(t_0) = x_0$ is called *convergent* on $[t_0, T]$ if

$$\underset{i=0,\ldots,N}{max} |x_i - x(t_i)| \to 0 \quad \text{for} \quad h \to 0$$

holds as long as the initial values fulfil

$$\underset{i=0,\ldots,k-1}{max} |x_i - x(t_i)| \to 0 \quad \text{for} \quad h \to 0$$

[1] Remark: Zero-stability is fulfilled for every one-step method.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

**Theorem A.11** (Convergence of linear multi-step methods)**.** *A consistent and zero-stable linear multi-step method is convergent: Consider an open region $G \subset \mathbb{R}$, $f \in C^p(G)$, the exact solution $x \in C^{p+1}(J)$, $[t_0, T] \subset J$ of the problem $x' = f(t, x)$, $x(t_0) = x_0$ and a zero-stable linear multi-step method as in Definition A.7 of consistency order $p$. Then follows that there exist $\underline{h} > 0$, $\overline{\varepsilon}$ and $C > 0$ so that for $0 < h \leq \underline{h}$ and initial error*

$$\max_{i=0,\dots,k-1} |x_i - x(t_i)| =: \varepsilon \leq \overline{\varepsilon}$$

*follows*

$$\max_{i=0,\dots,N} |x_i - x(t_i)| \leq C(h^p + \varepsilon). \tag{A.10}$$

For implicit methods, non-linear equation systems have to be solved. This can be achieved for non- or barely stiff problems by a fixed-point iteration, whose initial value is calculated by an explicit method called *predictor*. The implicit method is called *corrector* and the method on the whole is called *predictor-corrector method*.

**Definition A.12** (Stability Region of Linear Multi-Step Methods, A-Stability, A($\alpha$)-Stability)**.** Consider a linear multi-step method as in Definition (A.7). Its *stability region* $S \subset \mathbb{C}$ is defined as

$$S := \{z \in \mathbb{C} | \text{ every zero } \xi \text{ of } \rho(\xi) - z\sigma(\xi) \text{ fulfils } |\xi| \leq 1 \\ \text{and every zero with } |\xi| = 1 \text{ is a single zero}\}.$$

Linear multi-step methods with $\mathbb{C}^- \subset S$ are called *A-stable*, methods with $C(\alpha) := \{re^{i\varphi} \in \mathbb{C} | r > 0, \varphi \in (\pi - \alpha, \pi + \alpha)\} \in S$ are called *A($\alpha$)-stable*.

*Remark* A.13. $0 \in S$ for zero-stable methods.

*Remark* A.14. A RK method with stability function R is A-stable iff

$$|R(iy)| \leq 1 \quad \forall y \in \mathbb{R} \tag{A.11}$$

and

$$R(z) \text{ is analytic } \forall z : Re(z) < 0 \tag{A.12}$$

A RK method fulfilling (A.11) is called *I-stable* (meaning stability on the imaginary axis).

Rosenbrock-Type methods are basically obtained by linearization of a Rung-Kutta method.

**Definition A.15** (Rosenbrock method)**.** An s-stage *Rosenbrock method* is given by

$$k_i = hf\left(y_0 + \sum_{j=1}^{i-1} \alpha_{ij} k_j\right) + hJ \sum_{j=1}^{i} \gamma_{ij} k_j, \qquad i = 1, \dots, s \tag{A.13}$$

$$y_1 = y_0 + \sum_{j=1}^{s} b_j k_j \tag{A.14}$$

where $\alpha_{ij}$, $\gamma_{ij}$, $b_i$ are the determining coefficients and $J = f'(y_0)$.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*Remark* A.16. Rosenbrock methods are also called Rosenbrock-Wanner methods, ROW methods or (linearly implicit / semi-implicit / generalized / modified / adaptive / additive) Runge-Kutta methods.

*Remark* A.17 (W -methods). In so-called *W-Methods* the Jacobian is not calculated in every step but maintains an approximation to assure stability for several steps, which allows a reduction computational costs. However, the number of conditions for a certain convergeance order increases rapidly

### A.2.2 Differential-algebraic equations

**Definition A.18** (Differential-algebraic equation (system))**.** A differential-algebraic equation (DAE) is given by an implicit equation

$$F(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) = 0,$$

with a function $F\colon I \times D_x \times D_{\dot{x}} \to \mathbb{R}^n$, where $I \subseteq \mathbb{R}$ is a real interval and $D_x$, $D_{\dot{x}} \subseteq \mathbb{R}^n$ are open sets, $n \in \mathbb{N}$, $\boldsymbol{x}\colon I \to \mathbb{R}^n$ is a differentiable function and $\dot{\boldsymbol{x}}$ denotes the derivative of $\boldsymbol{x}$ with respect to $t$.

*Remark* A.19. According to the implicit function theorem, $F$ can be solved for $\dot{\boldsymbol{x}}$ if the matrix $\frac{\partial F}{\partial \dot{\boldsymbol{x}}}$ is regular.

**Definition A.20** (Constraint equations)**.** The algebraic equations of the DAE $F(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) = 0$ are of the form $g(\boldsymbol{x}) = \mathbf{0}$, where $g\colon \mathbb{R}^n \to \mathbb{R}^m$ is a function with $m < n$, and they are called constraints or constraint equations.

**Definition A.21** (Differential index)**.** The minimal number of derivatives $k \in \mathbb{N}_0$ so that an ODE can be extracted from the system

$$F(t, \boldsymbol{x}, \dot{\boldsymbol{x}}) = 0 \tag{A.15}$$

$$\frac{dF(t, \boldsymbol{x}, \dot{\boldsymbol{x}})}{dt} = 0 \tag{A.16}$$

$$\vdots \tag{A.17}$$

$$\frac{d^k F(t, \boldsymbol{x}, \dot{\boldsymbol{x}})}{dt^k} = 0 \tag{A.18}$$

is called the differential index of the DAE.

### A.2.3 Varia

**Definition A.22** (Moore machine, Mealy machine)**.** State machines whose output depends on both input and state variables are called Mealy machines. In case of no direct dependence on inputs, they are called Moore machines.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## A.3 References to mentioned software, programming languages, tools, and frameworks

*Remark* A.23. All of the following web pages have been last accessed on October 11, 2021 with the exception of the links to VTB (02/18/2013) and PiccSIM (02/17/2016).

| | |
|---|---|
| 20-sim | https://www.20sim.com/ |
| Abaqus | https://www.3ds.com/products-services/simulia/products/abaqus/ |
| ADAMS | https://www.mscsoftware.com/de/product/adams |
| AMESim | https://www.plm.automation.siemens.com/global/de/products/simcenter/simcenter-amesim.html |
| ANSYS | https://www.ansys.com/ |
| Anylogic | https://www.anylogic.com/ |
| AVL Model.CONNECT | https://www.avl.com/-/model-connect- |
| BACnet | http://www.bacnet.org/ |
| BARAKA | Labella et al. (2010) |
| BCVTB | https://simulationresearch.lbl.gov/bcvtb/ |
| C | Kernighan and Ritchie (1978) |
| C++ | https://www.stroustrup.com/C++.html |
| CASCaDE | *Computer Aided Simulation of Car, Driver, and Environment* (DaimlerChrysler) |
| CityGML | https://www.ogc.org/standards/citygml |
| CODIS | Nicolescu et al. (2006) |
| COMSOL | https://www.comsol.com/ |
| CORBA | https://www.corba.org/ |
| CosimDyn | Rustin et al. (2009) |
| COSMO | Zhang et al. (2010) |
| Crescendo | http://crescendotool.org/ |
| DACCOSIM | https://bitbucket.org/simulage/daccosim |
| DADS | Smith and Haug (1990) |
| DSHplus | http://dshplus.com/dshplus |
| Dymola | https://www.3ds.com/products-services/catia/products/dymola/ |
| EASY5 | https://www.mscsoftware.com/product/easy5 |
| Eclipse | https://www.eclipse.org/ |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| EnergyPlus | https://energyplus.net/ |
| EPOCHS | Hopkinson et al. (2006) |
| EsMoL | Porter et al. (2009) |
| ESP-r | http://www.esru.strath.ac.uk/Courseware/ESP-r/tour/ |
| Event-B | http://www.event-b.org/ |
| FEAP | http://projects.ce.berkeley.edu/feap/ |
| FEAT-FLOW | https://www.mathematik.tu-dortmund.de/~featflow/en/software/featflow.html |
| FEMM | https://www.femm.info/wiki/HomePage |
| FIDE | Cremona et al. (2016b) |
| FIDES | Schöps (2008) |
| Fortran | https://fortran-lang.org/ |
| FUMOLA | https://sourceforge.net/projects/fumola/ |
| GECO | Lin et al. (2012) |
| Gensys | http://www.gensys.se/ |
| GridLAB-D | https://www.gridlabd.org/ |
| ICOS | http://www.v2c2.at/icos |
| Java | https://www.java.com/en/ |
| JavaFMI | https://bitbucket.org/siani/javafmi/wiki/Home |
| JFMI | https://ptolemy.berkeley.edu/java/jfmi/ |
| Maestro | Thule et al. (2019a) |
| MAPNET | Li et al. (2011b) |
| MAS-T$^2$er Lab | Ferreira et al. (2008) |
| MATLAB | https://de.mathworks.com/products/matlab.html |
| MECSYCO | http://mecsyco.com/ |
| Modelica | https://modelica.org/modelicalanguage.html |
| ModelSim | https://eda.sw.siemens.com/en-US/ic/modelsim/ |
| Mosaik | https://mosaik.offis.de/ |
| NCSWT | Eyisi et al. (2012) |
| NMLab | Heimlich et al. (2010) |
| No-MASS | Chapman (2017) |
| NS-2 | https://www.isi.edu/nsnam/ns/ |
| NumPy | https://numpy.org/ |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| OMNeT++ | https://omnetpp.org/ |
| OMSimulator | https://openmodelica.org/openmodelicaworld/tools/2-uncategorised/191-omsimulator |
| OpenFoam | https://www.openfoam.com/ |
| OpenModelica | https://openmodelica.org/ |
| OPNET | https://opnetprojects.com/opnet-network-simulator/ |
| Overture | https://www.overturetool.org/ |
| PiccSIM | http://wsn.aalto.fi/en/tools/piccsim/ |
| PLECS | https://www.plexim.com/products/plecs/plecs_blockset |
| PSCAD/EMTDC | https://www.pscad.com/software/pscad/overview |
| PSLF | https://www.geenergyconsulting.com/practice-area/software-products/pslf |
| Pstar | analog circuit simulator by NXP Semiconductors |
| Ptolemy II | https://ptolemy.eecs.berkeley.edu/index.htm |
| Python | https://www.python.org/ |
| Radiance | http://radsite.lbl.gov/radiance/ |
| RoboNetSim | http://www.giannidicaro.com/robonetsim.html |
| SABER | https://www.synopsys.com/verification/virtual-prototyping/saber.html |
| SAHISim | Awais (2015) |
| Signal | Le Guernic et al. (1991) |
| SIMPACK | https://www.3ds.com/products-services/simulia/products/simpack/ |
| SIMPLORER | https://www.ansys.com/Products/Systems/ANSYS-Simplorer |
| Simscape | https://de.mathworks.com/products/simscape.html |
| Simulink | https://www.mathworks.com/products/simulink.html |
| SNiMoWrapper | Hante et al. (2019) |
| SPICE | http://bwrcs.eecs.berkeley.edu/Classes/IcBook/SPICE/ |
| SystemC | https://systemc.org/overview/ |
| TITAN | circuit simulator by Infineon Technologies |
| TRNSYS | http://www.trnsys.com/ |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| TrueTime | https://www.control.lth.se/research/tools-and-software/truetime/ |
| VPNET | Li et al. (2011a) |
| VTB | http://vtb.engr.sc.edu/vtbwebsite/#/Overview |

## A.4 Source for the literature analysis

This section contains aggregated information from the data that constitutes the source for the analysis in Chapter 5. This includes a list of the selected literature and its attributed properties and the complete tables of publications per author and institution.

### A.4.1 List of classified literature

Rice (1960), “Split Runge-Kutta method for simultaneous equations”
main: theory
application: missile simulations
theoretical aspect: error, coupling methods, performance
model description: ODE
number of coupled systems: 2
orchestrator: no
state of development: divide-and-conquer
coupling: non-iterative, fixed macro step, loose
multirate strategy: slowest first

Hofer (1976), “A Partially Implicit Method for Large Stiff Systems of ODEs with Only Few Equations Introducing Small Time-Constants”
main: theory
theoretical aspect: stability, coupling methods
model description: ODE
number of coupled systems: 2
orchestrator: no
state of development: divide-and-conquer
coupling: parallel, non-iterative, semi-implicit, fixed macro step, adaptive macro step

Andrus (1979), “Numerical Solution of Systems of Ordinary Differential Equations Separated into Subsystems”
main: theory
application: (mechanical (orbiting bodies), but odes)
theoretical aspect: error, coupling methods
model description: ODE
number of coupled systems: 2

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

orchestrator: no
state of development: divide-and-conquer
coupling: sequential, non-iterative, fixed macro step, loose
multirate strategy: fastest first

Gomm (1981), “Stability analysis of explicit multirate methods”
main: theory
theoretical aspect: stability
model description: ODE
number of coupled systems: 2
state of development: divide-and-conquer
coupling: sequential, parallel, non-iterative, fixed macro step, loose
multirate strategy: fastest first

Lelarasmee et al. (1982), “The Waveform Relaxation Method for Time-Domain Analysis of Large Scale Integrated Circuits”
main: theory
application: electrical circuits
theoretical aspect: error, coupling methods
model description: DAE
number of coupled systems: n
orchestrator: yes
state of development: divide-and-conquer
coupling: sequential, parallel, iterative, loose

Gear and Wells (1984), “Multirate linear multistep methods”
main: theory
theoretical aspect: error, stability, coupling methods
model description: ODE
number of coupled systems: 3
state of development: divide-and-conquer
coupling: sequential, non-iterative, adaptive macro step, loose
multirate strategy: slowest first

Rentrop (1985), “Partitioned Runge-Kutta methods with stiffness detection and stepsize control”
main: theory
theoretical aspect: stability, coupling methods
model description: ODE
number of coupled systems: 2
orchestrator: no
state of development: divide-and-conquer
coupling: sequential, non-iterative, adaptive macro step, loose

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

multirate strategy: fastest first

White et al. (1985), *Waveform Relaxation: Theory and Practice*
main: theory
theoretical aspect: error, coupling methods
model description: ODE
number of coupled systems: n
orchestrator: yes
state of development: divide-and-conquer, integrate-and-collaborate
coupling: sequential, parallel, iterative, loose

Skelboe and Andersen (1989), “Stability Properties of Backward Euler Multirate Formulas”
main: theory
application: electrical circuits
theoretical aspect: stability
model description: ODE
number of coupled systems: 2,3
orchestrator: no
state of development: divide-and-conquer
coupling: sequential, non-iterative, fixed macro step, loose
multirate strategy: compound
hierarchy possible

Biesiadecki and Skeel (1993), “Dangers of Multiple Time Step Methods”
main: theory
application: molecular dynamics
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
coupling: parallel, non-iterative, fixed macro step, loose

Günther and Rentrop (1994), “Partitioning and Multirate Strategies in Latent Electric Circuits”
main: both theory and application
application: electrical circuits
theoretical aspect: stability, coupling methods, performance
model description: ODE
number of coupled systems: 2
orchestrator: no
state of development: divide-and-conquer
coupling: sequential, parallel, non-iterative, adaptive macro step, loose
multirate strategy: slowest first

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Petegem et al. (1994), “Electrothermal simulation and design of integrated circuits”
main: application
application: electro-thermal systems
model description: ODE, FEM
software: SPICE, SYSTUS
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, iterative, fixed macro step, loose

Dahmann et al. (1997), “The Department of Defense High Level Architecture”
main: standard
theoretical aspect: formalism
model description: ODE, DAE, PDE, ABM, SD, SDF, FEM, BEM, DE, hybrid, HIL, human interaction
standard: HLA
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate

Wünsche et al. (1997a), “Microsystem Design Using Simulator Coupling”
main: theory
application: electro-thermal systems
theoretical aspect: coupling methods
model description: DAE, FEM
software: SABER, ANSYS
number of coupled systems: 2
orchestrator: no
state of development: integrate-and-collaborate
coupling: sequential, non-iterative, adaptive macro step, loose

Featherstone (1999a,b), “A Divide-and-Conquer Articulated-Body Algorithm for Parallel O(log(n)) Calculation of Rigid-Body Dynamics"
main: both theory and application
application: multibody (articulated-body dynamics)
theoretical aspect: coupling methods
model description: DAE, HIL, human interaction
number of coupled systems: n (1024)
state of development: divide-and-conquer
coupling: parallel, non-iterative

Kvaernø¸ and Rentrop (1999), *Low Order Multirate Runge-Kutta Methods in Electric Circuit Simulation*

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

main: theory
application: electrical circuits
theoretical aspect: coupling methods
model description: ODE
number of coupled systems: n (active/latent)
state of development: divide-and-conquer
coupling: non-iterative, adaptive macro step, loose
multirate strategy: slowest first, fastest first

Mousseau et al. (1999), “Vehicle dynamics simulations with coupled multibody and finite element models”
main: application
application: vehicle dynamics
model description: DAE, FEM
software: ADAMS, FEAP
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: iterative, semi-implicit, strong

Farhat and Lesoinne (2000), “Two efficient staggered algorithms for the serial and parallel solution of three-dimensional nonlinear transient aeroelastic problems”
main: theory
application: fluid-structure interaction
theoretical aspect: coupling methods
model description: FEM
number of coupled systems: 2
coupling: sequential, parallel, non-iterative, fixed macro step

Gu et al. (2000), “Co-simulation of coupled dynamic subsystems: a differential-algebraic approach using singularly perturbed sliding manifolds”
main: theory
application: multibody
theoretical aspect: coupling methods
model description: DAE
framework: self-implemented (Java, Maple)
number of coupled systems: 2
orchestrator: yes
coupling: parallel, non-iterative, fixed macro step, loose

Kübler and Schiehlen (2000a), “Modular Simulation in Multibody System Dynamics”
main: theory
theoretical aspect: stability, coupling methods

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

model description: DAE
number of coupled systems: 2
orchestrator: yes
coupling: sequential, parallel, iterative, non-iterative, fixed macro step, loose

Kübler and Schiehlen (2000b), “Two Methods of Simulator Coupling”
main: theory
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
coupling: sequential, parallel, iterative, non-iterative, fixed macro step, loose

Tudoret et al. (2000), “Co-Simulation of Hybrid Systems: Signal-Simulink”
main: framework
application: hydraulic system
theoretical aspect: coupling methods
model description: ODE, DE, hybrid
software: Signal, Simulink
framework: self
number of coupled systems: 2
state of development: integrate-and-collaborate
coupling: loose

Arnold and Günther (2001), “Preconditioned Dynamic Iteration for Coupled Differential-Algebraic Systems”
main: theory
application: electrical circuits, multibody dynamics
theoretical aspect: error, stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, parallel, iterative, adaptive macro step, loose

Esposito and Kumar (2001), “Efficient dynamic simulation of robotic systems with hierarchy”
main: theory
application: robotic systems
theoretical aspect: error, stability, performance
model description: ODE
software: Matlab
number of coupled systems: n
orchestrator: no

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

state of development: divide-and-conquer
coupling: sequential, non-iterative, semi-implicit, fixed macro step, loose
multirate strategy: slowest first
hierarchy possible

Felippa et al. (2001), “Partitioned analysis of coupled mechanical systems”
main: application
application: fluid-structure
theoretical aspect: error, stability
model description: FEM, BEM
number of coupled systems: 3
state of development: divide-and-conquer
coupling: sequential, non-iterative, fixed macro step, loose

Gu (2001), “Co-simulation of algebraically coupled dynamic subsystems”
main: theory
application: HVAC
theoretical aspect: coupling methods
model description: DAE
software: multiple
framework: self-implemented (Java)
number of coupled systems: 10
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Günther et al. (2001), “Multirate Partitioned Runge-Kutta Methods”
main: theory
application: electrical circuits
theoretical aspect: error, stability, coupling methods
model description: ODE
software: MATLAB
number of coupled systems: 2
state of development: divide-and-conquer
coupling: loose
multirate strategy: compound

Tseng and Hulbert (2001), “A Gluing Algorithm for Network-Distributed Multibody Dynamics Simulation”
main: theory
application: multibody
theoretical aspect: coupling methods
model description: DAE

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

number of coupled systems: 2
state of development: divide-and-conquer
coupling: iterative, strong

Bartel and Günther (2002), “A multirate W-method for electrical networks in state-space formulation”
main: theory
application: electrical circuits
theoretical aspect: coupling methods
model description: ODE
software: MATLAB
number of coupled systems: 2
state of development: divide-and-conquer
coupling: non-iterative, loose
multirate strategy: compound

Knorr (2002), “Multirate-Verfahren in der Co-Simulation gekoppelter dynamischer Systeme mit Anwendung in der Fahrzeugdynamik”
main: theory
application: multibody
theoretical aspect: error
model description: ODE
software: CASCaDE, DADS, MATLAB/Simulilnk
number of coupled systems: 2
state of development: divide-and-conquer
coupling: parallel, non-iterative, fixed macro step, loose

Matthies and Steindorf (2002), “Partitioned but strongly coupled iteration schemes for nonlinear fluid–structure interaction”
main: application
application: fluid-structure
theoretical aspect: coupling methods
number of coupled systems: 2
state of development: integrate-and-collaborate
coupling: parallel, iterative, strong

Rathinam and Petzold (2002), “Dynamic Iteration Using Reduced Order Models: A Method for Simulation of Large Scale Modular Systems”
main: theory
theoretical aspect: error, coupling methods, performance
model description: ODE
software: Matlab
number of coupled systems: 2 (n)

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

state of development: integrate-and-collaborate
coupling: parallel, iterative, fixed macro step, loose

Ibrahimbegović and Markovič (2003), “Strong coupling methods in multi-phase and multi-scale modeling of inelastic behavior of heterogeneous structures”
main: theory
theoretical aspect: stability, coupling methods
model description: FEM
software: FEAP
number of coupled systems: 2
state of development: divide-and-conquer
coupling: iterative, strong

Jia and Leimkuhler (2003), “A parallel multiple time-scale reversible integrator for dynamics simulation”
main: theory
application: multibody
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: n
state of development: divide-and-conquer
coupling: parallel, non-iterative, loose

Larsson and Krus (2003), “Stability Analysis of Coupled Simulation”
main: theory
application: mechanical
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 3
orchestrator: yes
state of development: divide-and-conquer
coupling: sequential, iterative, non-iterative, fixed macro step, loose

Matthies and Steindorf (2003), “Strong Coupling Methods”
main: theory
application: fluid-structure interaction
theoretical aspect: coupling methods
model description: ODE, DAE
software: FEAT-FLOW, FEAP
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: iterative, strong

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Tseng et al. (2003), “Efficient numerical solution of constrained multibody dynamics systems”
main: theory
application: multibody
theoretical aspect: coupling methods
model description: DAE
number of coupled systems: 3
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, iterative, fixed macro step, loose

Wang et al. (2003), “A Gluing Algorithm for Distributed Simulation of Multibody Systems”
main: theory
application: multibody dynamics, structural dynamics
theoretical aspect: coupling methods
model description: ODE, DAE, FEM
software: ADAMS, Matlab
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, iterative, strong
hierarchy possible

Ebert (2004), “Convergence of relaxation methods for coupled systems of ODEs and DAEs”
main: theory
theoretical aspect: error, stability, coupling methods
model description: ODE, DAE
number of coupled systems: 2
state of development: integrate-and-collaborate
coupling: sequential, parallel, iterative, loose

Gu and Asada (2004), “Co-Simulation of Algebraically Coupled Dynamic Subsystems Without Disclosure of Proprietary Subsystem Models”
main: theory
application: mechanical, electrical, . . .
theoretical aspect: stability, coupling methods
model description: DAE
framework: self-implemented (Java)
number of coupled systems: n
orchestrator: yes
coupling: parallel, non-iterative, fixed macro step, loose

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Steinebach et al. (2004), “Mechanisms of coupling in river flow simulation systems”
main: application
application: flow simulation
theoretical aspect: error, coupling methods
model description: ODE, DAE
number of coupled systems: 3
orchestrator: yes
state of development: divide-and-conquer
coupling: sequential, non-iterative, fixed macro step, loose

Wang et al. (2005), “A Distributed Mechanical System Simulation Platform Based on a “Gluing Algorithm””
main: theory
application: mechanical
theoretical aspect: coupling methods
model description: DAE, FEM
software: Fortran, CORBA, Java
framework: self (.net, xml)
number of coupled systems: 9
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, iterative, loose, strong
hierarchy possible

Matthies et al. (2006), “Algorithms for strong coupling procedures”
main: both theory and application
application: structure-structure, structure-fluid
theoretical aspect: stability, coupling methods
model description: DAE, FEM
software: FEAP, FEAT-FLOW
number of coupled systems: 2
coupling: parallel, iterative, strong

Striebel (2006), “Hierarchical Mixed Multirating for Distributed Integration of DAE Network Equations in Chip Design”
main: theory
application: electrical circuits
theoretical aspect: coupling methods
model description: ODE, DAE
software: TITAN
number of coupled systems: n
state of development: divide-and-conquer
coupling: non-iterative, fixed macro step, loose

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

multirate strategy: compound
hierarchy possible

Verhoeven et al. (2006a), “Error analysis of BDF Compound-fast multirate method for differential-algebraic equations”
main: theory
application: electrical circuits
theoretical aspect: error
model description: DAE
number of coupled systems: 2
state of development: divide-and-conquer
coupling: fixed macro step, adaptive macro step, loose
multirate strategy: compound

Verhoeven et al. (2006b), “A General Compound Multirate Method for Circuit Simulation Problems”
main: theory
theoretical aspect: stability, coupling methods
model description: ODE
number of coupled systems: 2
state of development: divide-and-conquer
coupling: sequential, parallel, fixed macro step, loose
multirate strategy: slowest first, fastest first, compound

Arnold (2007), “Multi-Rate Time Integration for Large Scale Multibody System Models”
main: theory
application: mechanical (multi-body) systems
theoretical aspect: error, stability
model description: ODE, DAE
number of coupled systems: 2
state of development: divide-and-conquer
coupling: sequential, parallel, non-iterative, fixed macro step, loose

Savcenco et al. (2007), “A multirate time stepping strategy for stiff ordinary differential equations”
main: theory
application: physics (circuits, reaction-diffusion,..)
theoretical aspect: error, coupling methods, performance
model description: ODE
software: C
number of coupled systems: n
state of development: divide-and-conquer
coupling: sequential, non-iterative, semi-implicit

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

multirate strategy: slowest first
hierarchy possible

Trčka et al. (2007), “Comparison of co-simulation approaches for building and HVAC/R system simulation.”
main: application
application: building energy (HVAC) sim
theoretical aspect: error, stability
model description: ODE, DAE
software: EnergyPlus, TRNSYS
number of coupled systems: 2
orchestrator: no
state of development: integrate-and-collaborate
coupling: sequential, iterative, non-iterative, semi-implicit, fixed macro step, loose, strong

Verhoeven et al. (2007), “Stability analysis of the BDF Slowest-first multirate methods”
main: theory
theoretical aspect: stability
model description: DAE
number of coupled systems: 2
state of development: divide-and-conquer
coupling: fixed macro step, loose
multirate strategy: slowest first, compound

Barros (2008), “Semantics of Dynamic Structure Event-based Systems”
main: theory
application: electrical circuits
theoretical aspect: formalism
model description: ODE, DE, hybrid
standard: HFSS
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: loose
hierarchy possible

Ebert (2008), “On Partitioned Simulation of Electrical Circuits using Dynamic Iteration Methods”
main: theory
application: electrical circuits
theoretical aspect: error, stability, coupling methods, performance
model description: ODE, DAE
software: SPICE, Matlab

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: sequential, parallel, iterative, fixed macro step, adaptive macro step, loose

Ferreira et al. (2008), “A Cooperative Simulation Framework for Traffic and Transportation Engineering”
main: framework
application: traffic and transportation
model description: ABM, human interaction
framework: MAS-T$^2$er Lab
number of coupled systems: 3
orchestrator: yes
state of development: integrate-and-collaborate
coupling: iterative

Karsai and Sztipanovits (2008), “Model-Integrated Development of Cyber-Physical Systems”
main: framework
application: cyberphysical systems
model description: ODE, HIL
software: Simulink/Stateflow, EsMoL, TrueTime
framework: pending
number of coupled systems: 3
orchestrator: (yes)
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, loose

Trčka (2008), “Cosimulation for Performance Prediction of Innovative Integrated Mechanical Energy Systems in Buildings”
main: application
application: building energy (HVAC) sim
theoretical aspect: error, stability
model description: ODE, DAE
software: EnergyPlus, TRNSYS, ESP-r, EARTH
number of coupled systems: 2
orchestrator: no
state of development: integrate-and-collaborate
coupling: sequential, iterative, non-iterative, semi-implicit, fixed macro step, loose, strong

Verhoeven et al. (2008), “BDF Compound-Fast Multirate Transient Analysis with Adaptive Stepsize Control”
main: theory
theoretical aspect: error, stability, coupling methods

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

model description: ODE, DAE
software: MATLAB, Pstar
number of coupled systems: 2
state of development: divide-and-conquer
coupling: non-iterative, adaptive macro step, loose
multirate strategy: compound

Brecher et al. (2009), “Interaction of manufacturing process and machine tool”
main: survey
application: process-machine interaction
orchestrator: yes
state of development: integrate-and-collaborate

Gheorghe (2009), “Continuous/Discrete Co-Simulation Interfaces from Formalization to Implementation”
main: theory
theoretical aspect: coupling methods
model description: ODE, DE, hybrid
framework: self ("CODIS")
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, adaptive macro step, loose

Rustin et al. (2009), “A Cosimulation T-T Procedure Gluing Subsystems in Multibody Dynamics Simulations”
main: theory
application: mechanical
theoretical aspect: coupling methods
model description: DAE, FEM
framework: self ("CosimDyn" – Library, C++)
number of coupled systems: 7
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, iterative, adaptive macro step, strong

Striebel et al. (2009), “A multirate ROW-scheme for index-1 network equations”
main: theory
application: electrical circuits
theoretical aspect: stability, coupling methods
model description: DAE
software: TITAN
number of coupled systems: 2

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

state of development: divide-and-conquer
coupling: parallel, non-iterative, fixed macro step, loose
multirate strategy: compound

Trčka et al. (2009), “Co-simulation of innovative integrated HVAC systems in buildings”
main: both theory and application
application: building energy (HVAC) sim
theoretical aspect: error, stability, performance
model description: ODE, DAE
number of coupled systems: 2
orchestrator: no
state of development: integrate-and-collaborate
coupling: sequential, iterative, non-iterative, semi-implicit, fixed macro step, loose, strong

Arnold (2010), “Stability of Sequential Modular Time Integration Methods for Coupled Multibody System Models”
main: theory
application: multibody
theoretical aspect: stability
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, fixed macro step, loose

Benedikt et al. (2010), “An Adaptive Coupling Methodology for Fast Time-Domain Distributed Heterogeneous Co-Simulation”
main: theory
application: automotive (multidomain physics)
theoretical aspect: error, coupling methods, performance
number of coupled systems: n (2)
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, iterative, non-iterative, adaptive macro step, loose

González et al. (2010), “Efficient coupling of multibody software with numerical computing environments and block diagram simulators”
main: both theory and application
application: mechatronic
theoretical aspect: coupling methods, performance
model description: ODE, DAE
software: MATLAB/Simulink, mbs tool C++based
framework: self

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

number of coupled systems: 2
orchestrator: no
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Arnold et al. (2011), “Numerical methods in vehicle system dynamics: state of the art and current developments”
main: survey
application: vehicle system dynamics
theoretical aspect: coupling methods
model description: ODE, DAE
software: SIMPACK, Modelica
standard: FMI, Matlab S-functions
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, parallel, non-iterative, fixed macro step, loose

Friedrich (2011), “Parallel Co-Simulation for Mechatronic Systems”
main: application
application: multi-domain systems (mechatronic)
theoretical aspect: stability
model description: ODE, DAE
framework: self-implemented C++
number of coupled systems: 7
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

González et al. (2011), “On the effect of multirate co-simulation techniques in the efficiency and accuracy of multibody system dynamics”
main: theory
application: mechatronic
theoretical aspect: coupling methods, performance
model description: ODE, DAE
software: MATLAB/Simulink, mbs tool C++based
framework: self (within MBS software)
number of coupled systems: 2
orchestrator: no
state of development: integrate-and-collaborate
coupling: sequential, non-iterative, no common macro step, loose
multirate strategy: slowest first, fastest first

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Liang et al. (2011), “Combinative Algorithms for the Multidisciplinary Collaborative Simulation of Complex Mechatronic Products Based on Major Step and Convergent Integration Step”
main: theory
application: physical benchmark expls
theoretical aspect: coupling methods
model description: ODE, DAE
software: ADAMS, MATLAB, EASY5
number of coupled systems: 3
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, non-iterative, no common macro step, loose

Schierz and Arnold (2011), “MODELISAR: Innovative numerische Methoden bei der Kopplung von multidisziplinären Simulationsprogrammen”
main: theory
theoretical aspect: error, coupling methods
model description: ODE, DAE
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, adaptive macro step, loose

Schöps (2011), “Multiscale Modeling and Multirate Time-Integration of Field/Circuit Coupled Problems”
main: both theory and application
application: field-circuit co-sim
theoretical aspect: error, stability, coupling methods
model description: ODE, DAE, PDE
software: MATLAB
framework: FIDES
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, iterative, fixed macro step, loose

Tomulik and Fraczek (2011), “Simulation of multibody systems with the use of coupling techniques: a case study”
main: application
application: multibody
theoretical aspect: coupling methods
model description: DAE
software: Matlab

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, iterative, fixed macro step, loose

Völker (2011), *Untersuchung des Kommunikationsintervalls bei der gekoppelten Simulation*
main: theory
application: mechanical-hydraulic
theoretical aspect: error
model description: ODE, DAE
software: MATLAB/Simulink, SIMPACK, AMESim, DSHplus
number of coupled systems: 3
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, adaptive macro step, loose

Wetter (2011), "Co-simulation of building energy and control systems with the Building Controls Virtual Test Bed"
main: framework
application: building simulation
model description: ODE, DAE
software: EnergyPlus, Modelica, Radiance, BACnet, MATLAB, Simulink, Simscape
framework: BCVTB
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Zhang et al. (2011), "Truncation error calculation based on Richardson extrapolation for variable-step collaborative simulation"
main: theory
theoretical aspect: error, coupling methods
model description: ODE, DAE
number of coupled systems: 2
orchestrator: no
coupling: sequential, non-iterative, adaptive macro step, loose

Blockwitz et al. (2012), "Functional Mockup Interface 2.0: The Standard for Tool independent Exchange of Simulation Models"
main: standard
theoretical aspect: formalism
model description: ODE, DAE, PDE, FEM, DE, hybrid, HIL
software: https://fmi-standard.org/tools/

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

standard: FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, parallel, iterative, non-iterative, semi-implicit, fixed macro step, adaptive macro step, no common macro step, loose, strong
hierarchy possible

Busch (2012), *Zur effizienten Kopplung von Simulationsprogrammen*
main: both theory and application
application: multibody, fluid-structure
theoretical aspect: error, stability, coupling methods
model description: DAE, PDE, FEM
software: SIMPACK, COMSOL
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: sequential, non-iterative, semi-implicit, adaptive macro step, loose

Quaglia et al. (2012), “A SystemC/Matlab co-simulation tool for networked control systems”
main: framework
application: networked control systems
theoretical aspect: coupling methods
model description: ODE, DE, hybrid, HIL
software: MATLAB, SystemC
framework: self-implemented
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, adaptive macro step, no common macro step, loose, strong

Schierz et al. (2012), “Co-simulation with communication step size control in an FMI compatible master algorithm”
main: theory
application: multibody
theoretical aspect: error, coupling methods
model description: DAE
software: Dymola
standard: FMI
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, adaptive macro step, loose

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Schierz and Arnold (2012), “Stabilized overlapping modular time integration of coupled differential-algebraic equations”
main: theory
application: multibody (benchmark ex.)
theoretical aspect: stability, coupling methods
model description: DAE
software: MATLAB
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, parallel, non-iterative, fixed macro step, loose

Schmoll and Schweizer (2012), “Convergence Study of Explicit Co-Simulation Approaches with Respect to Subsystem Solver Settings”
main: theory
application: mechanical
theoretical aspect: error, stability
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, fixed macro step, loose

Spiryagin et al. (2012), “Co-simulation of a mechatronic system using Gensys and Simulink”
main: application
application: mechatronic (rail traction vehicles)
model description: ODE
software: Matlab/Simulink, Gensys
standard: Matlab S-functions
number of coupled systems: 2
orchestrator: no
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step

Arnold et al. (2013), “Error Analysis and Error Estimates for Co-Simulation in FMI for Model Exchange and Co-Simulation V2.0”
main: theory
application: multibody
theoretical aspect: error
model description: DAE
standard: FMI
number of coupled systems: 2

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, adaptive macro step, loose

Bartel et al. (2013), “Dynamic Iteration for Coupled Problems of Electric Circuits and Distributed Devices”
main: theory
application: field-circuit
theoretical aspect: error, stability, coupling methods
model description: DAE, PDE
software: FEMM
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer, integrate-and-collaborate
coupling: sequential, parallel, iterative, fixed macro step, loose

Broman et al. (2013), “Determinate Composition of FMUs for Co-simulation”
main: theory
theoretical aspect: coupling methods
model description: ODE, DAE, BEM, DE, hybrid
standard: FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, adaptive macro step, loose

Arnold et al. (2014), “Error analysis for co-simulation with force-displacement coupling”
main: theory
application: multibody
theoretical aspect: error, coupling methods
model description: DAE
software: Simpack, Matlab/Simulink
framework: SNiMoWrapper
standard: FMI
number of coupled systems: 2
orchestrator: no
state of development: divide-and-conquer
coupling: parallel, non-iterative, fixed macro step, loose

Bartel et al. (2014), “On the convergence rate of dynamic iteration for coupled problems with multiple subsystems”
main: theory
theoretical aspect: error, coupling methods

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

model description: DAE, PDE
number of coupled systems: n
state of development: divide-and-conquer, integrate-and-collaborate
coupling: sequential, parallel, iterative, fixed macro step, loose

Ben Khaled et al. (2014), “Context-based polynomial extrapolation and slackened synchronization for fast multi-core simulation using FMI”
main: theory
application: automotive
theoretical aspect: coupling methods, performance
model description: ODE, DAE, hybrid
software: Modelica
framework: self
standard: FMI
number of coupled systems: 5
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Fitzgerald et al. (2014), *Collaborative Design for Embedded Systems*
main: framework
application: controlled mech sys (embedded systems)
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
software: 20-sim, Overture
framework: Crescendo
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, adaptive macro step, loose

Hafner et al. (2014), “Investigating communication and step-size behaviour for co-simulation of hybrid physical systems”
main: application
application: energy, production facilities
model description: ODE, DAE, DE, hybrid
software: Modelica, Simscape, Matlab
framework: BCVTB
number of coupled systems: 6
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Li et al. (2014), “Co-simulation platforms for co-design of networked control systems: An overview”
main: survey
application: networked control systems
model description: ODE, DAE, DE, hybrid
software: Matlab/Simulink, Modelica, VTB, SIMPLORER, COMSOL, PLECS, NS-2, OPNET, OMNeT++, SystemC
framework: PiccSIM, NMLab, NCSWT, ModelSim, MAPNET, VPNET, COSMO, RoboNet-Sim, BARAKA
state of development: integrate-and-collaborate

Nouidui et al. (2014), “Functional mock-up unit for co-simulation import in EnergyPlus”
main: application
application: building energy simulation
model description: ODE, SD
software: EnergyPlus, Modelica
standard: FMI
number of coupled systems: 2
orchestrator: (no)
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Palensky et al. (2014), “Simulating Cyber-Physical Energy Systems: Challenges, Tools and Methods”
main: application
application: power grids
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
software: GridLAB-D, Dymola, C++
number of coupled systems: 1000
orchestrator: yes and no
state of development: integrate-and-collaborate
coupling: fixed macro step, adaptive macro step

Savicks et al. (2014), “Co-simulating event-B and Continuous Models via FMI”
main: framework
application: power systems
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
software: Event-B, JFMI, (arb. FMUs)
framework: self (ext. of RODIN-platform)
standard: FMI (partly)
number of coupled systems: n

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

orchestrator: yes
state of development: integrate-and-collaborate
coupling: adaptive macro step, loose

Schweizer and Lu (2014a), “Semi-implicit co-simulation approach for solver coupling”
main: theory
application: mechanical
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, semi-implicit, fixed macro step, loose

Schweizer and Lu (2014b), “Stabilized index-2 co-simulation approach for solver coupling with algebraic constraints”
main: theory
application: mechanical
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, semi-implicit, fixed macro step, loose

Sicklinger et al. (2014), “Interface Jacobian-based Co-Simulation”
main: theory
application: multiphysics
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, iterative, fixed macro step

Sicklinger (2014), “Stabilized Co-Simulation of Coupled Problems Including Fields and Signals”
main: theory
application: multiphysics
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

state of development: integrate-and-collaborate
coupling: parallel, iterative, fixed macro step

Tong et al. (2014), “Reviews and perspectives of hybrid system simulation for power and communication”
main: survey
application: power systems and communication networks
theoretical aspect: coupling methods
model description: ODE, ABM, DE, hybrid, HIL
software: PSCAD/EMTDC, PSLF, NS-2
framework: EPOCHS, GECO
number of coupled systems: 2 (n)
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, adaptive macro step, loose

Viel (2014), “Implementing stabilized co-simulation of strongly coupled systems using the Functional Mock-up Interface 2.0”
main: theory
application: hydraulic system
theoretical aspect: stability, coupling methods
model description: ODE, DAE
software: AMESim, Python, C
standard: FMI
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, fixed macro step, loose

Awais (2015), “Distributed hybrid co-simulation”
main: framework
theoretical aspect: error, coupling methods, performance
model description: ODE, DAE, DE, hybrid
framework: self (SAHISim)
standard: HLA, FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, parallel, iterative, non-iterative, semi-implicit, fixed macro step, loose

Broman et al. (2015), “Requirements for Hybrid Cosimulation Standards”
main: theory
theoretical aspect: coupling methods, formalism
model description: ODE, DAE, DE, hybrid

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

standard: (FMI)
number of coupled systems: n
orchestrator: (yes)
state of development: integrate-and-collaborate

Esgandari and Olatunbosun (2015), “Implicit–explicit co-simulation of brake noise”
main: application
application: brake noise
theoretical aspect: coupling methods
model description: FEM
software: Abaqus
number of coupled systems: 2
state of development: divide-and-conquer
coupling: parallel, non-iterative, fixed macro step, loose

Galtier et al. (2015), “FMI-based distributed multi-simulation with DACCOSIM”
main: framework
application: smart grids (building HVAC)
theoretical aspect: coupling methods, performance
model description: ODE, DAE, hybrid
software: Modelica
framework: DACCOSIM
standard: FMI
number of coupled systems: 7
orchestrator: yes (master split on three components)
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, adaptive macro step, loose

Schmoll (2015), “Co-Simulation und Solverkopplung”
main: both theory and application
application: multiphysics
theoretical aspect: coupling methods
model description: DAE
software: ADAMS, COMSOL
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: sequential, parallel, non-iterative, semi-implicit, adaptive macro step, loose

Schöps (2015), “Iterative Schemes for Coupled Multiphysical Problems in Electrical Engineering”
main: application
application: field-circuit, mech.-electromagn., thermal-electromagn.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

model description: ODE, DAE
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: sequential, iterative, fixed macro step, loose

Schweizer et al. (2015a), “Explicit and Implicit Cosimulation Methods: Stability and Convergence Analysis for Different Solver Coupling Approaches”
main: theory
application: mechanical
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, semi-implicit, fixed macro step, loose

Schweizer and Lu (2015), “Predictor/corrector co-simulation approaches for solver coupling with algebraic constraints”
main: theory
application: mechanical
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, semi-implicit, fixed macro step, loose

Schweizer et al. (2015b), “Stabilized implicit co-simulation methods: solver coupling based on constitutive laws”
main: theory
application: mechanical
theoretical aspect: stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
state of development: divide-and-conquer
coupling: parallel, non-iterative, semi-implicit, fixed macro step, loose

Sicklinger et al. (2015), “Fully coupled co-simulation of a wind turbine emergency brake maneuver”
main: application
application: Aerodynamics

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

theoretical aspect: coupling methods
model description: DAE, FEM
software: OpenFoam
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, iterative, fixed macro step

Stettinger et al. (2015), “Modellbasierte Echtzeit-Co-Simulation: Überblick und praktische Anwendungsbeispiele”
main: application
application: real-time hardware
theoretical aspect: coupling methods
model description: ODE, HIL
software: Matlab/Simulink
framework: ICOS
number of coupled systems: 2
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, loose

Tripakis (2015), “Bridging the semantic gap between heterogeneous modeling formalisms and FMI”
main: theory
theoretical aspect: formalism
model description: ODE, SDF, DE, hybrid
standard: FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate

Widl et al. (2015), “FMI-based co-simulation of hybrid closed-loop control system models”
main: application
application: plant and controller (HVAC)
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
software: TRNSYS, Simulink
framework: FUMOLA
standard: FMI
number of coupled systems: 2
orchestrator: yes and no
state of development: integrate-and-collaborate
coupling: non-iterative

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Camus et al. (2016), “Hybrid Co-simulation of FMUs using DEV&DESS in MECSYCO”
main: framework
application: barrel-filter factory (queue, tank, controllers)
theoretical aspect: coupling methods
model description: ODE, DAE, ABM, DE, hybrid
software: Java, JavaFMI, Dymola
framework: self (building on MECSYCO)
standard: DEV&DESS, (FMI)
number of coupled systems: 4 (n)
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, adaptive macro step, loose

Cremona et al. (2016b), “FIDE: An FMI Integrated Development Environment”
main: framework
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
framework: FIDE (based on Ptolemy II)
standard: FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: non-iterative, loose

Cremona et al. (2016a), “Step revision in hybrid Co-simulation with FMI”
main: theory
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
framework: FIDE
standard: FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, iterative, adaptive macro step, loose

Heinzl (2016), “Hybrid Modeling of Production Systems: Co-simulation and DEVS-based Approach”
main: application
application: production facilities (energy)
model description: ODE, DAE, DE, hybrid
software: MATLAB, Dymola, EnergyPlus
framework: BCVTB
number of coupled systems: 3

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Sadjina and Pedersen (2016), “Energy Conservation and Coupling Error Reduction in Non-Iterative Co-Simulations”
main: theory
application: physical systems
theoretical aspect: error, stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
coupling: parallel, non-iterative, adaptive macro step, loose

Schweizer et al. (2016), “Implicit co-simulation methods: Stability and convergence analysis for solver coupling approaches with algebraic constraints”
main: theory
application: Dahlquist test, mechanical systems
theoretical aspect: error, stability, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
coupling: parallel, non-iterative, semi-implicit, fixed macro step, loose

Thiede et al. (2016), “Multi-level simulation in manufacturing companies: The water-energy nexus case”
main: survey
application: production facilities
theoretical aspect: coupling methods
model description: SD, DE, hybrid
software: (Anylogic (not co-sim))
framework: self
number of coupled systems: 5
orchestrator: no
state of development: integrate-and-collaborate
coupling: strong

Barros (2017), “Chattering Avoidance in Hybrid Simulation Models: A Modular Approach Based on the HyFlow Formalism”
main: application
application: mechanical, hydraulic
theoretical aspect: coupling methods
model description: ODE, DE, hybrid

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

standard: HyFlow (HFSS?), DEVS
number of coupled systems: 2 (n)
orchestrator: yes
state of development: integrate-and-collaborate
coupling: loose
hierarchy possible

Nguyen et al. (2017), “On Conceptual Structuration and Coupling Methods of Co-Simulation Frameworks in Cyber-Physical Energy System Validation”
main: theory
application: (energy systems (just field of interest))
theoretical aspect: classification
model description: ODE, DE, hybrid
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate

Pühringer (2017), “Analysis of Coupling Strategies and Protocols for Co-Simulation”
main: application
application: production facilities (industrial energy efficiency)
model description: ODE, DAE, DE, hybrid
software: MATLAB/Simulink, OpenModelica
framework: self-implemented (C++)
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, parallel, iterative, non-iterative, fixed macro step, loose

Sadjina et al. (2017), “Energy conservation and power bonds in co-simulations: non-iterative adaptive step size control and error estimation”
main: theory
application: physical systems
theoretical aspect: error, coupling methods
model description: DAE
number of coupled systems: 2
orchestrator: yes
coupling: parallel, non-iterative, adaptive macro step, loose

Van Mierlo et al. (2017), “Explicit Modelling and Synthesis of Debuggers for Hybrid Simulation Languages”
main: theory
theoretical aspect: debugging
model description: ODE, ABM, DE, hybrid

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

standard: SCCD
number of coupled systems: n
state of development: integrate-and-collaborate
coupling: parallel, fixed macro step, adaptive macro step

Wang et al. (2017), “Towards Generalized Co-simulation of Urban Energy Systems”
main: framework
application: urban energy systems
model description: ODE, ABM
software: EnergyPlus, No-MASS
framework: self-implemented (modified Mosaik)
standard: FMI
number of coupled systems: 2
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Glumac and Kovacic (2018), “Calling Sequence Calculation for Sequential Co-simulation Master”
main: theory
application: automotive (hybrid electric vehicle)
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
framework: AVL Model.CONNECT
standard: FMI
number of coupled systems: 10
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, non-iterative, loose

Gomes et al. (2018a), “Approximated Stability Analysis of Bi-modal Hybrid Co-simulation Scenarios”
main: theory
application: (cyberphysical systems)
theoretical aspect: stability, coupling methods
model description: ODE, DAE, DE, hybrid
coupling: adaptive macro step, loose

Gomes et al. (2018b), “Co-Simulation: A Survey”
main: survey
theoretical aspect: classification
model description: ODE, DAE, ABM, FEM, DE, hybrid

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Heinzl et al. (2018), “Simulation-based Assessment of Energy Efficiency in Industry: Comparison of Hybrid Simulation Approaches”
main: theory
application: energy
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
software: OpenModelica, MATLAB, Simulink
framework: BCVTB
standard: hyPDEVS
number of coupled systems: 2
orchestrator: (yes)
state of development: integrate-and-collaborate
coupling: parallel, non-iterative, fixed macro step, loose

Thule et al. (2018), “Towards the Verification of Hybrid Co-simulation Algorithms”
main: theory
application: (physics)
theoretical aspect: coupling methods
model description: ODE, DAE, DE, hybrid
software: OpenModelica
framework: Maestro
standard: FMI
number of coupled systems: 2 (n)
orchestrator: yes
state of development: integrate-and-collaborate
coupling: sequential, parallel, non-iterative, fixed macro step, loose

Cremona et al. (2019), “Hybrid co-simulation: it’s about time”
main: theory
application: (cyberphysical systems)
theoretical aspect: coupling methods, formalism
model description: ODE, DAE, DE, hybrid
framework: FIDE
standard: FMI
number of coupled systems: 2 (n)
orchestrator: yes
state of development: integrate-and-collaborate
coupling: parallel, iterative, adaptive macro step, loose

Farkas et al. (2019), “Adaptive Step Size Control for Hybrid CT Simulation without Rollback”
main: theory
application: cyberphysical systems
theoretical aspect: coupling methods

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

model description: ODE, DAE, DE, hybrid
software: OpenModelica, Dymola, Simulink
framework: OMSimulator
standard: FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: non-iterative, loose

Glumac and Kovacic (2019), “Relative Consistency and Robust Stability Measures for Sequential Co-simulation”
main: theory
application: mechanical
theoretical aspect: error, stability, coupling methods
model description: DAE
software: NumPy
standard: FMI
number of coupled systems: 2
orchestrator: yes
coupling: sequential, non-iterative, fixed macro step, loose

Gomes et al. (2019), “Generation of Co-simulation Algorithms Subject to Simulator Contracts”
main: theory
theoretical aspect: formalism
model description: ODE, DAE, BEM, DE
standard: FMI
number of coupled systems: n
orchestrator: yes
state of development: integrate-and-collaborate
coupling: fixed macro step

Schweiger et al. (2019a), “An empirical survey on co-simulation: Promising standards, challenges and research needs”
main: survey

Schweiger et al. (2019b), “Functional Mock-up Interface: An empirical survey identifies research challenges and current barriers”
main: survey
standard: FMI

Stecken et al. (2019), “Classification method for an automated linking of models in the co-simulation of production systems”

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

main: theory
application: production systems
theoretical aspect: coupling methods, classification
standard: own, FMI
number of coupled systems: 4
orchestrator: yes
state of development: integrate-and-collaborate
coupling: loose

Thule et al. (2019a), “Maestro: The INTO-CPS co-simulation framework”
main: framework
application: cyber-physical systems
framework: Maestro(INTO-CPS)
standard: FMI
orchestrator: yes

Thule et al. (2019b), “Towards Reuse of Synchronization Algorithms in Co-simulation Frameworks”
main: theory
theoretical aspect: formalism
framework: INTO-CPS
standard: FMI
orchestrator: yes

### A.4.2 Publications per author and institution

Table A.2: Complete list of publications per author in descending order.

| Author | Number of publications |
|---|---|
| Arnold, M. | 10 |
| Gomes, C. | 9 |
| Schweizer, B. | 7 |
| Broman, D. | 6 |
| Günther, M. | 6 |
| Larsen, P.G. | 6 |
| Lu, D. | 6 |
| Tripakis, S. | 6 |
| Wetter, M. | 6 |
| Lee, E.A. | 5 |
| Rentrop, P. | 5 |
| Thule, C. | 5 |
| Vangheluwe, H. | 5 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Bartel, A. | 4 |
| Clauß, C. | 4 |
| Hulbert, G.M. | 4 |
| Schierz, T. | 4 |
| Schöps, S. | 4 |
| ter Maten, E.J.W. | 4 |
| Verhoeven, A. | 4 |
| Cremona, F. | 3 |
| Gu, B. | 3 |
| Hafner, I. | 3 |
| Heinzl, B. | 3 |
| Li, P. | 3 |
| Lohstroh, M. | 3 |
| Masin, M. | 3 |
| Mattheij, R.M.M. | 3 |
| Matthies, H.G. | 3 |
| Nouidui, T. | 3 |
| Sicklinger, S. | 3 |
| Steindorf, J. | 3 |
| Tasic, B. | 3 |
| Trcka, M. | 3 |
| Asada, H.H. | 2 |
| Barros, F.J. | 2 |
| Beelen, T.G.J. | 2 |
| Benedikt, M. | 2 |
| Bletzinger, K. | 2 |
| Brooks, C. | 2 |
| Brunk, M. | 2 |
| Ebert, F. | 2 |
| Elmqvist, H. | 2 |
| Engel, G. | 2 |
| Farhat, C. | 2 |
| Friedrich, M. | 2 |
| Galtier, V. | 2 |
| Glumac, S. | 2 |
| Gonzalez, F. | 2 |
| Gonzalez, M. | 2 |
| Greenberg, L. | 2 |
| Hensen, J.L.M. | 2 |
| Kovacic, Z. | 2 |
| Kübler, R. | 2 |
| Kvaerno, A. | 2 |
| Lausdahl, K. | 2 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Liang, S. | 2 |
| Ma, Zheng-Dong | 2 |
| Olsson, H. | 2 |
| Palensky, P. | 2 |
| Pedersen, E. | 2 |
| Posch, A. | 2 |
| Ruehli, A.E. | 2 |
| Sadjina, S. | 2 |
| Sangiovanni-Vincentelli, A.L. | 2 |
| Schiehlen, W. | 2 |
| Schmoll, R. | 2 |
| Schoeggl, J. | 2 |
| Schweiger, G. | 2 |
| Striebel, M. | 2 |
| Tseng, F. | 2 |
| Viel, A. | 2 |
| Wang, H. | 2 |
| Wang, J. | 2 |
| Widl, E. | 2 |
| Wüchner, R. | 2 |
| Zhang, H. | 2 |
| Akesson, J. | 1 |
| Andersen, U. | 1 |
| Andrus, J.F. | 1 |
| Awais, M.U. | 1 |
| Battle, N. | 1 |
| Belsky, V. | 1 |
| Benveniste, A. | 1 |
| Bergmann, G. | 1 |
| Besanger, Y. | 1 |
| Biesiadecki, J.J. | 1 |
| Blochwitz, T. | 1 |
| Bombled, Q. | 1 |
| Bragantini, R. | 1 |
| Brauer, J. | 1 |
| Brecher, C. | 1 |
| Breitenecker, F. | 1 |
| Burgermeister, B. | 1 |
| Busch, M. | 1 |
| Butler, M. | 1 |
| Camus, B. | 1 |
| Caujolle, M. | 1 |
| Cole, C. | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Colley, J. | 1 |
| Dad, C. | 1 |
| Dahmann, J.S. | 1 |
| Deantoni, J. | 1 |
| Di Natale, M. | 1 |
| Duval, L. | 1 |
| Eder, K. | 1 |
| El Guennouni, A. | 1 |
| Elsheikh, A. | 1 |
| Engelmann, B. | 1 |
| Esgandari, M. | 1 |
| Esposito, J.M. | 1 |
| Esser, M. | 1 |
| Esteves, E.F. | 1 |
| Farkas, R. | 1 |
| Featherstone, R. | 1 |
| Felippa, C.A. | 1 |
| Ferreira, P.A.F. | 1 |
| Fiorini, P. | 1 |
| Fitzgerald, J. | 1 |
| Fraczek, J. | 1 |
| Führer, C. | 1 |
| Fujimoto, R.M. | 1 |
| Gaid, M.B. | 1 |
| Gear, C.W. | 1 |
| Geeraerts, B. | 1 |
| Gheorghe, L. | 1 |
| Gomm, W. | 1 |
| Gordon, B.W. | 1 |
| Graindourze, B. | 1 |
| Guennouni, A. El | 1 |
| Hante, S. | 1 |
| Herrmann, C. | 1 |
| Hippmann, G. | 1 |
| Hofer, E. | 1 |
| Horn, M. | 1 |
| Horvath, A. | 1 |
| Hundsdorfer, W. | 1 |
| Ibrahimbegovic, A. | 1 |
| Jia, Z. | 1 |
| Judex, F. | 1 |
| Junghanns, A. | 1 |
| Karalis, P. | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Karsai, G. | 1 |
| Kastner, W. | 1 |
| Khaled, A.B. | 1 |
| Knorr, S. | 1 |
| Köbis, M.A. | 1 |
| Körner, A. | 1 |
| Krus, P. | 1 |
| Kuhlenkötter, B. | 1 |
| Kumar, R.V. | 1 |
| Kurle, D. | 1 |
| Kyllingstad, L. T. | 1 |
| Lam-Yee-Mui, J. | 1 |
| Landsiedl, M. | 1 |
| Larsson, J. | 1 |
| Laursen T.A. | 1 |
| Leimkuhler, B. | 1 |
| Lelarasmee, E. | 1 |
| Lenkenhoff, K. | 1 |
| Lerch, C. | 1 |
| Lesoinne, M. | 1 |
| Li, H. | 1 |
| Li, W. | 1 |
| Li, Y. | 1 |
| Lidberg M. | 1 |
| Luaces, A. | 1 |
| Lucio, L. | 1 |
| Ma, Z. | 1 |
| Macedo, H.D. | 1 |
| Markovic, D. | 1 |
| Mattheij, R.M.M | 1 |
| Mauss, J. | 1 |
| Meisl, G. | 1 |
| Meyer, T. | 1 |
| Mikkola, A. | 1 |
| Mousseau, C.W. | 1 |
| Muradore, R. | 1 |
| Nadjm-Tehrani, S. | 1 |
| Navarro-Lopez, E.M. | 1 |
| Naya, M.A. | 1 |
| Neumerkel, D. | 1 |
| Nguyen, T.L. | 1 |
| Nguyen, V.H. | 1 |
| Ni, M. | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Niekamp, R. | 1 |
| Odeh, F. | 1 |
| Olatunbosun, O. | 1 |
| Olivieira, E.C | 1 |
| Otter, M. | 1 |
| Palmieri, M. | 1 |
| Park, K.C. | 1 |
| Persson, I. | 1 |
| Petzold, L.R. | 1 |
| Plessis, G. | 1 |
| Preyser, F. | 1 |
| Pühringer, C. | 1 |
| Quaglia, D. | 1 |
| Rademacher, S. | 1 |
| Raich, P. | 1 |
| Rathinam, M. | 1 |
| Rice, J.R. | 1 |
| Rill, G. | 1 |
| Robinson, D. | 1 |
| Rossetti, R.J.F. | 1 |
| Rößler, M. | 1 |
| Rustin, C. | 1 |
| Sansen, W. | 1 |
| Savcenco, V. | 1 |
| Savicks, V. | 1 |
| Schönemann, M. | 1 |
| Schulz, M. | 1 |
| Schwarz, P. | 1 |
| Siebers, P. | 1 |
| Simon, D. | 1 |
| Simson, S. | 1 |
| Skeel, R.D. | 1 |
| Skelboe, S. | 1 |
| Skjong, S. | 1 |
| Song, S. | 1 |
| Spiryagin, M. | 1 |
| Stecken, J. | 1 |
| Steinebach, G. | 1 |
| Stettinger, G. | 1 |
| Stippel, H. | 1 |
| Strömberg, J. | 1 |
| Sztipanovits, J. | 1 |
| Tavella, J. | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Taylor R.L. | 1 |
| Thiede, S. | 1 |
| Tomulik, P. | 1 |
| Tong, H. | 1 |
| Tran, Q.T. | 1 |
| Tudoret, S. | 1 |
| Van Mierlo, S. | 1 |
| van Petegem, W. | 1 |
| Verhoef, M. | 1 |
| Verlinden, O. | 1 |
| Verwer, J.G. | 1 |
| Vialle, S. | 1 |
| Völker, L. | 1 |
| Wang, K. | 1 |
| Wangda, Z. | 1 |
| Watzenig, D. | 1 |
| Weatherly, R.M. | 1 |
| Wells, D.R. | 1 |
| White, J. | 1 |
| Winkler, F. | 1 |
| Witt, S. | 1 |
| Wünsche, S. | 1 |
| Yu, W. | 1 |
| Zehetner, J. | 1 |
| Zhang, X. | 1 |

Table A.3: Complete list of publications per affiliation in descending order.

| Affiliation | Number of publications |
|---|---|
| Technische Universität Darmstadt | 11 |
| Martin Luther University Halle-Wittenberg | 9 |
| University of Antwerp | 9 |
| University of California, Berkeley | 9 |
| Lawrence Berkeley National Laboratory | 8 |
| Technische Universiteit Eindhoven | 7 |
| Aarhus University | 6 |
| KTH Royal Institute of Technology | 6 |
| Technische Universität München | 6 |
| Bergische Universität Wuppertal | 5 |
| Technische Universität Braunschweig | 5 |
| TU Wien | 5 |
| University of Michigan | 5 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Aalto University | 4 |
| Fraunhofer IIS | 4 |
| Graz University of Technology | 4 |
| Norwegian University of Science and Technology | 4 |
| University of Karlsruhe | 4 |
| Austrian Institute of Technology | 3 |
| dwh GmbH | 3 |
| Flanders Make | 3 |
| IBM IL | 3 |
| Linköping University | 3 |
| Massachusetts Institute of Technology | 3 |
| McGill University | 3 |
| AVL-AST d.o.o. | 2 |
| DLR German Aerospace Center | 2 |
| EDF R&D | 2 |
| IBM NY | 2 |
| Katholieke Universiteit Leuven | 2 |
| LMS Imagine | 2 |
| NXP Semiconductors | 2 |
| Philips Research Laboratories | 2 |
| Scuola Superiore Sant'Anna | 2 |
| SIMPACK AG | 2 |
| Technische Universität Berlin | 2 |
| Tsinghua University | 2 |
| Universidad de A Coruña | 2 |
| Universität Kassel | 2 |
| University of Coimbra | 2 |
| University of Colorado | 2 |
| University of Graz | 2 |
| University of Illinois | 2 |
| University of Stuttgart | 2 |
| University of Zagreb | 2 |
| AB DEsolver | 1 |
| ALES | 1 |
| Alternative Energies and Atomic Energy Commission (CEA) | 1 |
| Autonetics | 1 |
| AVL List GmbH | 1 |
| Bradken Resources Ltd | 1 |
| Budapest University of Technology and Economics | 1 |
| Bundesanstalt für Gewässerkunde | 1 |
| Cambridge University | 1 |
| Central Queensland University | 1 |
| CentraleSupelec | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| Chess WISE B.V. | 1 |
| CWI | 1 |
| Daimler AG | 1 |
| Dassault Systemes | 1 |
| Dassault Systemes CATIA | 1 |
| Dassault Systemes SIMULIA | 1 |
| Defense Modeling and Simulation Office | 1 |
| Delft University of Technology | 1 |
| DST Control AB | 1 |
| Duke University | 1 |
| Ecole Normale Superieure de Cachan | 1 |
| Faculte Polytechnique de Mons | 1 |
| fortiss | 1 |
| Georgia Institute of Technology | 1 |
| IFP Energies nouvelles | 1 |
| IncQuery Labs Ltd | 1 |
| independent | 1 |
| INRIA and LIRMM-DEMAR team | 1 |
| IRISA-INRIA, Campus de Beaulieu | 1 |
| ITI GmbH | 1 |
| Karlsruher Institut für Technologie | 1 |
| Lappeenranta University of Technology | 1 |
| Lund University | 1 |
| Magma Design Automation | 1 |
| Mechanical Dynamics Inc. | 1 |
| MIETEC-ALCATEL | 1 |
| Mjolner Informatics A/S | 1 |
| Modelon | 1 |
| MTA-BME Lendület Cyber-Physical Systems Research Group | 1 |
| NARI Group Corporation | 1 |
| Newcastle University | 1 |
| Northwestern Polytechnical University | 1 |
| Pisa University | 1 |
| Polytech Nice Sophia | 1 |
| Qtronic | 1 |
| Regensburg University of Applied Sciences | 1 |
| Ruhr-Universität Bochum | 1 |
| RWTH Aachen | 1 |
| RWTH Aachen University | 1 |
| SINTEF Fisheries and Aquaculture | 1 |
| Technische Universität Chemnitz | 1 |
| The MITRE Corporation | 1 |
| The University of Nottingham | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

| | |
|---|---|
| TWT GmbH Science & Innovation | 1 |
| Univ. Grenoble Alpes | 1 |
| Universität Kaiserslautern | 1 |
| Universität Ulm | 1 |
| Universite de Lorraine | 1 |
| Universite de Montreal | 1 |
| Universite Paris-Saclay | 1 |
| University of Applied Sciences | 1 |
| University of Birmingham | 1 |
| University of California, Santa Barbara | 1 |
| University of Cambridge | 1 |
| University of Copenhagen | 1 |
| University of Florence | 1 |
| University of Leicester | 1 |
| University of Manchester | 1 |
| University of Miami | 1 |
| University of New Orleans | 1 |
| University of Pennsylvania | 1 |
| University of Porto | 1 |
| University of South Carolina | 1 |
| University of Southampton | 1 |
| University of Verona | 1 |
| University of Wales | 1 |
| Vanderbilt University | 1 |
| Verified Systems International GmbH | 1 |
| Virtual Vehicle Competence Center Austria | 1 |
| Virtual Vehicle Research GmbH | 1 |
| Warsaw University of Technology | 1 |
| Yacht Technology | 1 |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## A.5 Error plots

In this section, componentwise error plots for Section 6.2.3 are given.

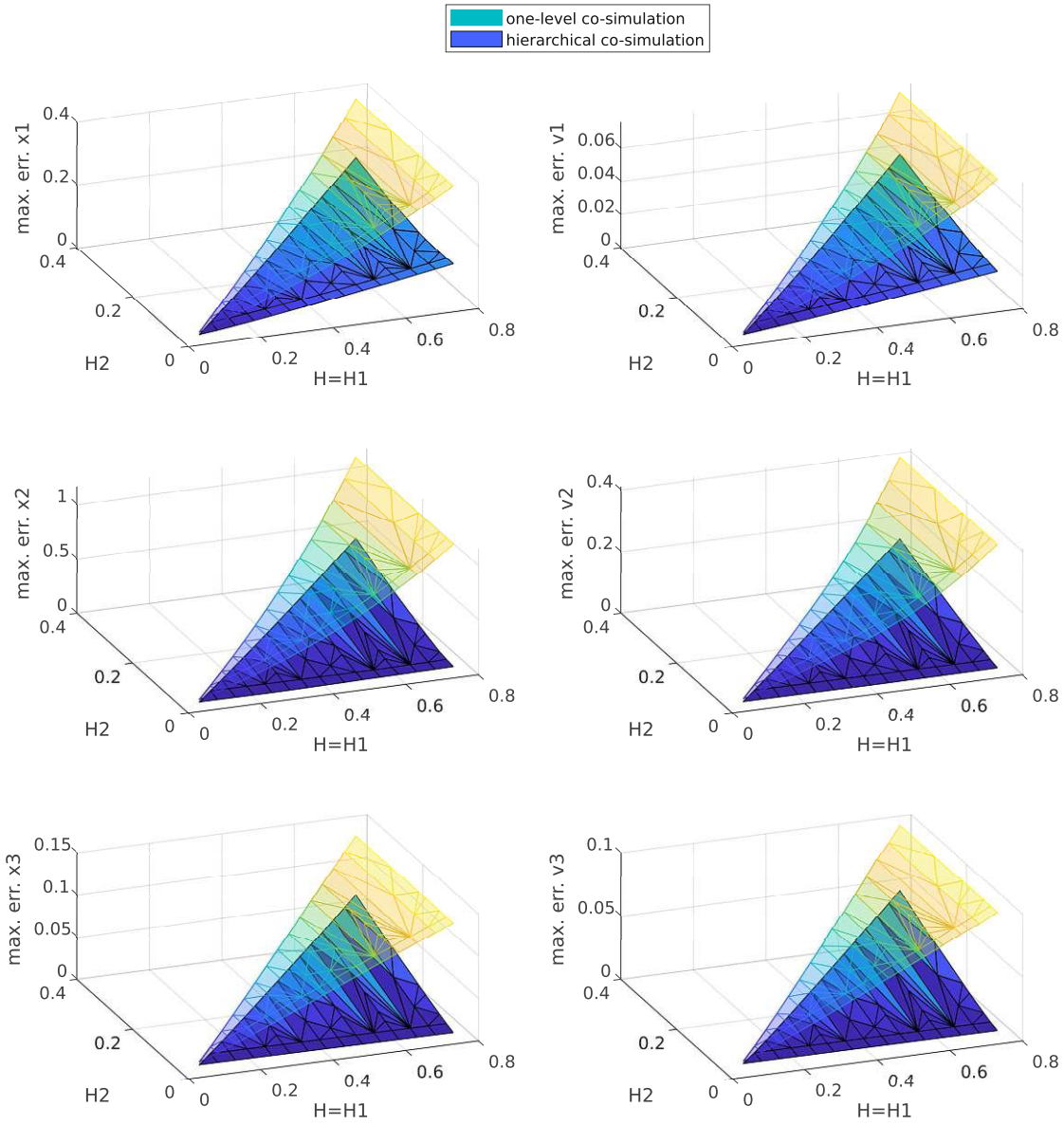


Figure A.1: Component errors for the simulation of Scenario 1 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

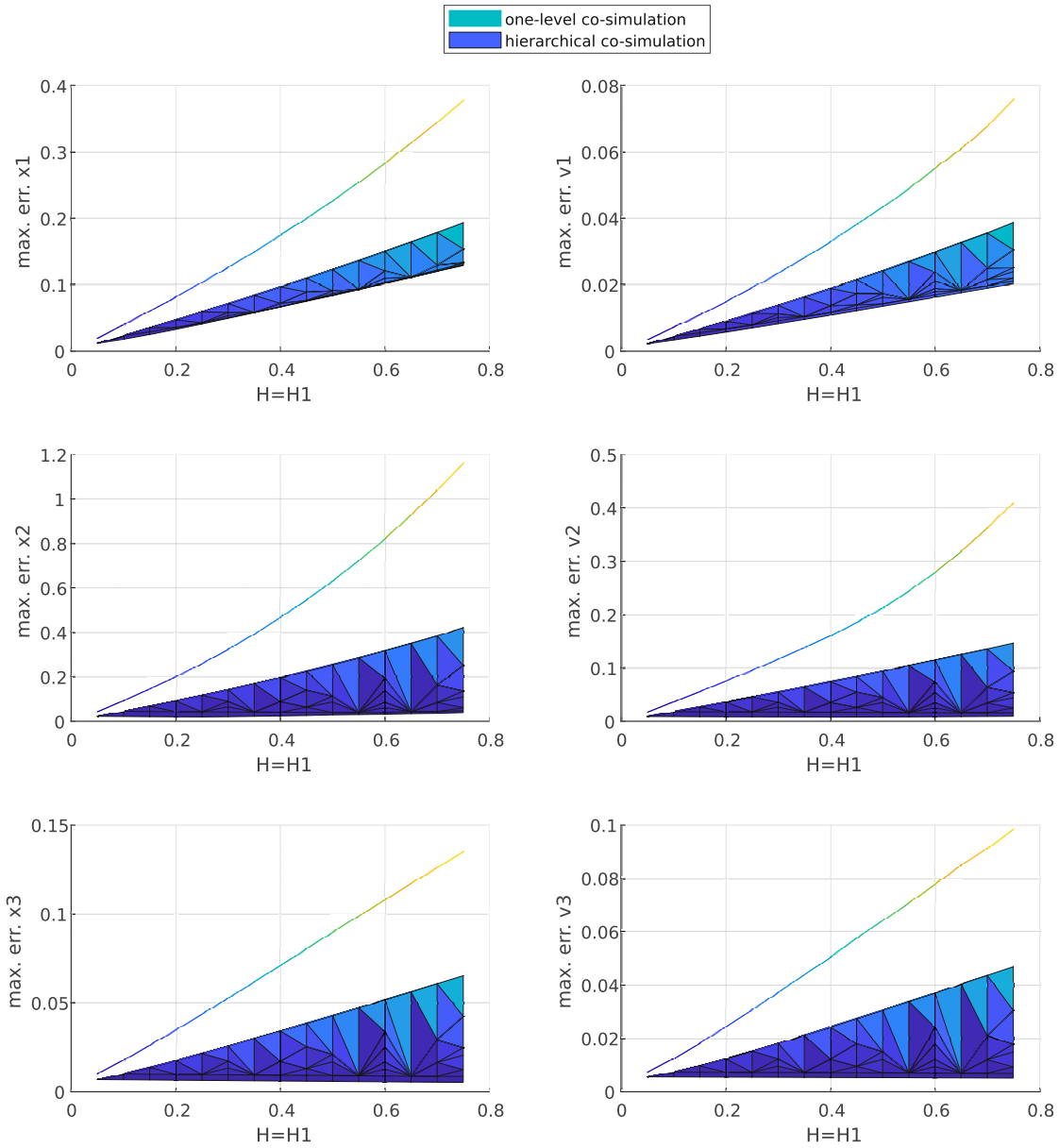


Figure A.2: Component errors for the simulation of Scenario 1 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes, $err$-$H_1$ plane view.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

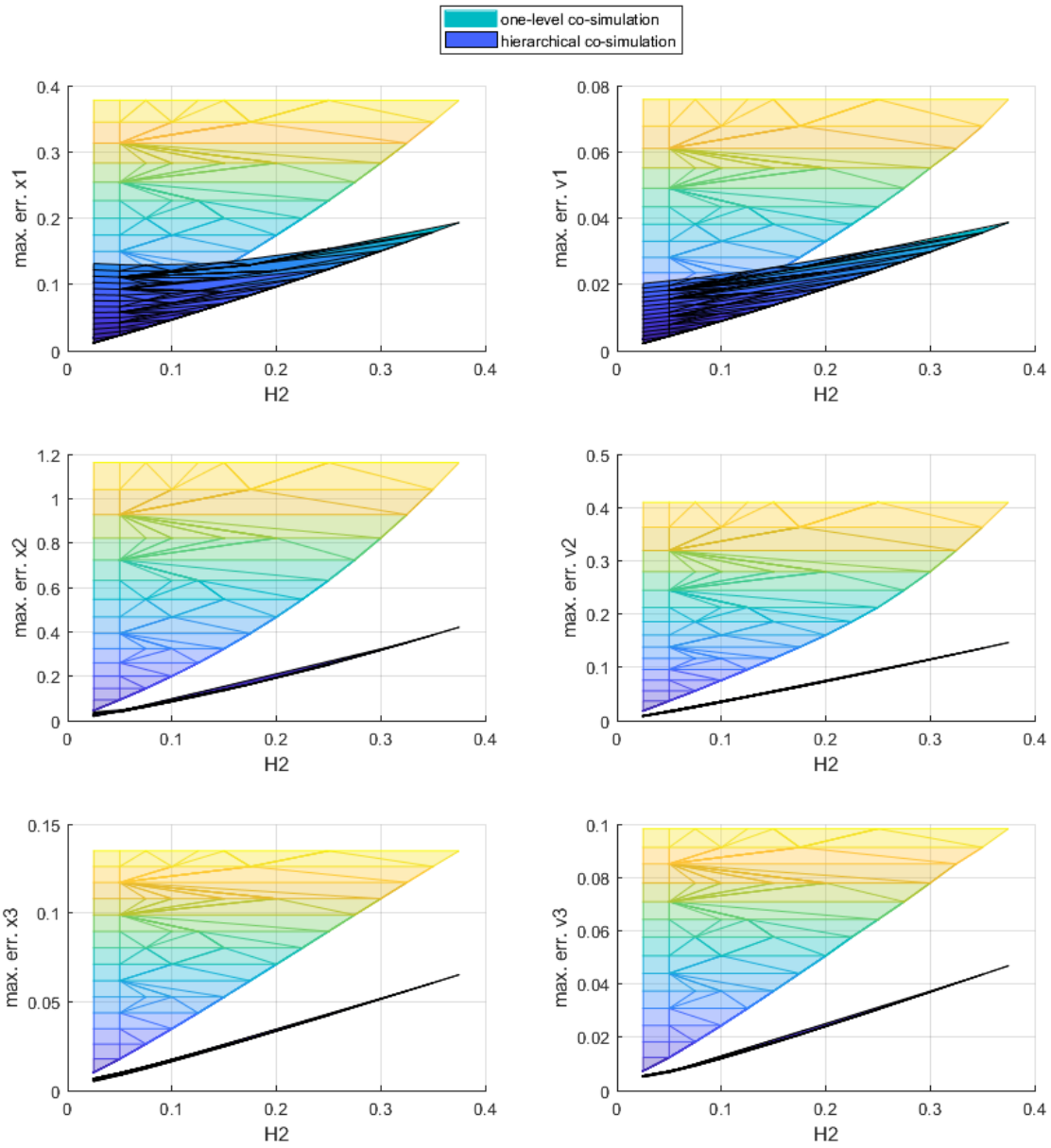


Figure A.3: Component errors for the simulation of Scenario 1 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes, $err$-$H_2$ plane view.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

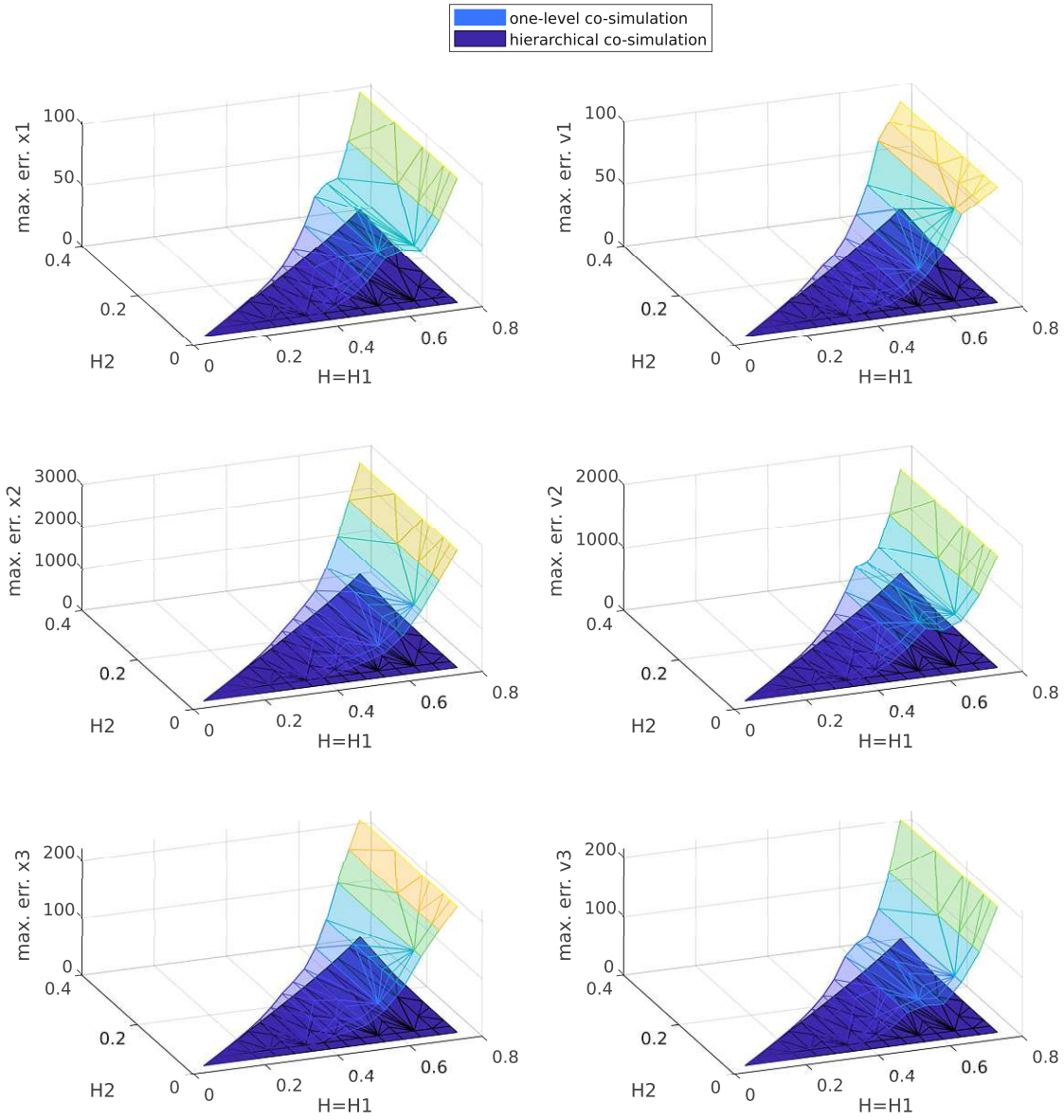


Figure A.4: Component errors for the simulation of Scenario 2 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

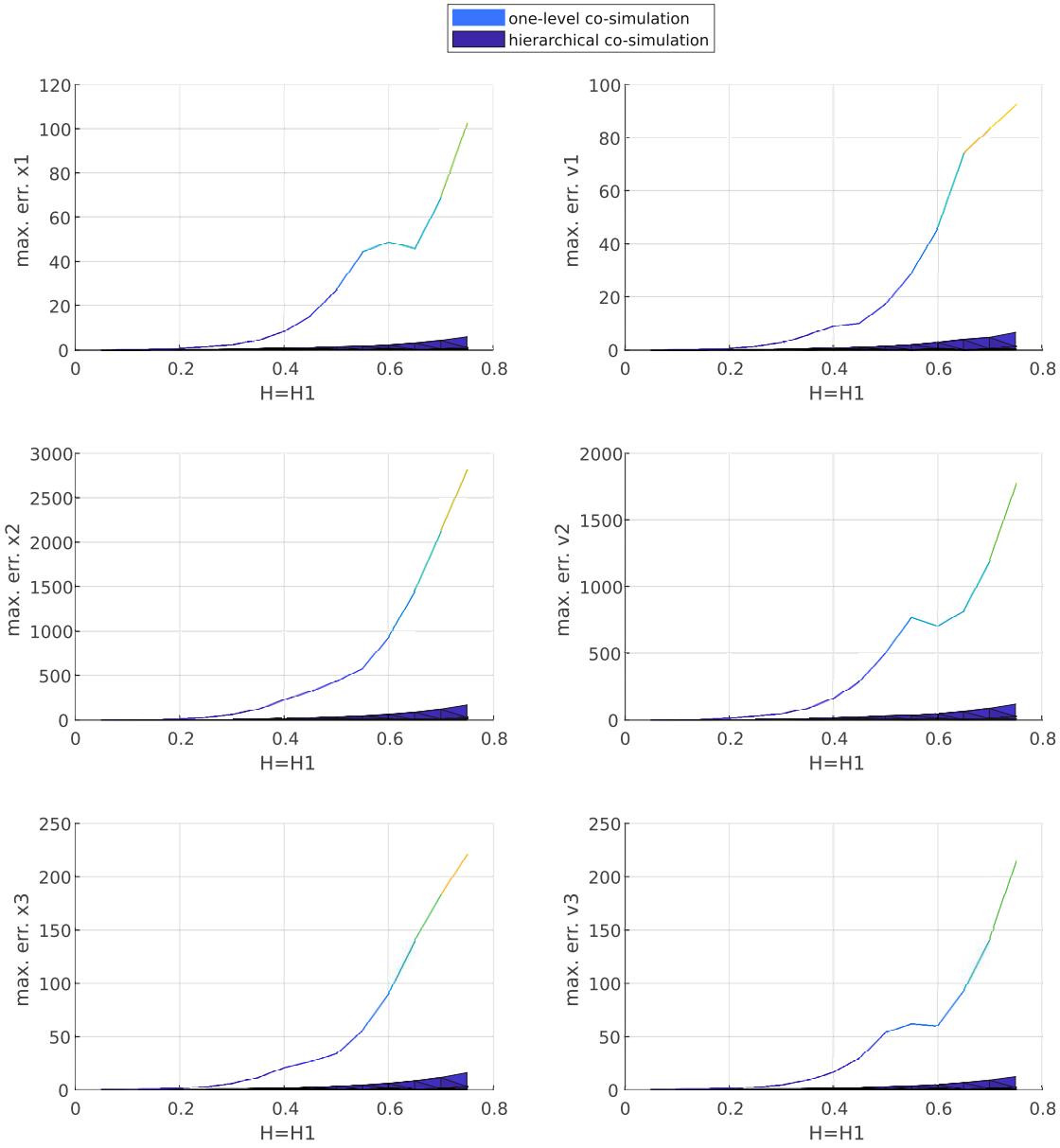


Figure A.5: Component errors for the simulation of Scenario 2 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes, $err$-$H_1$ plane view.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

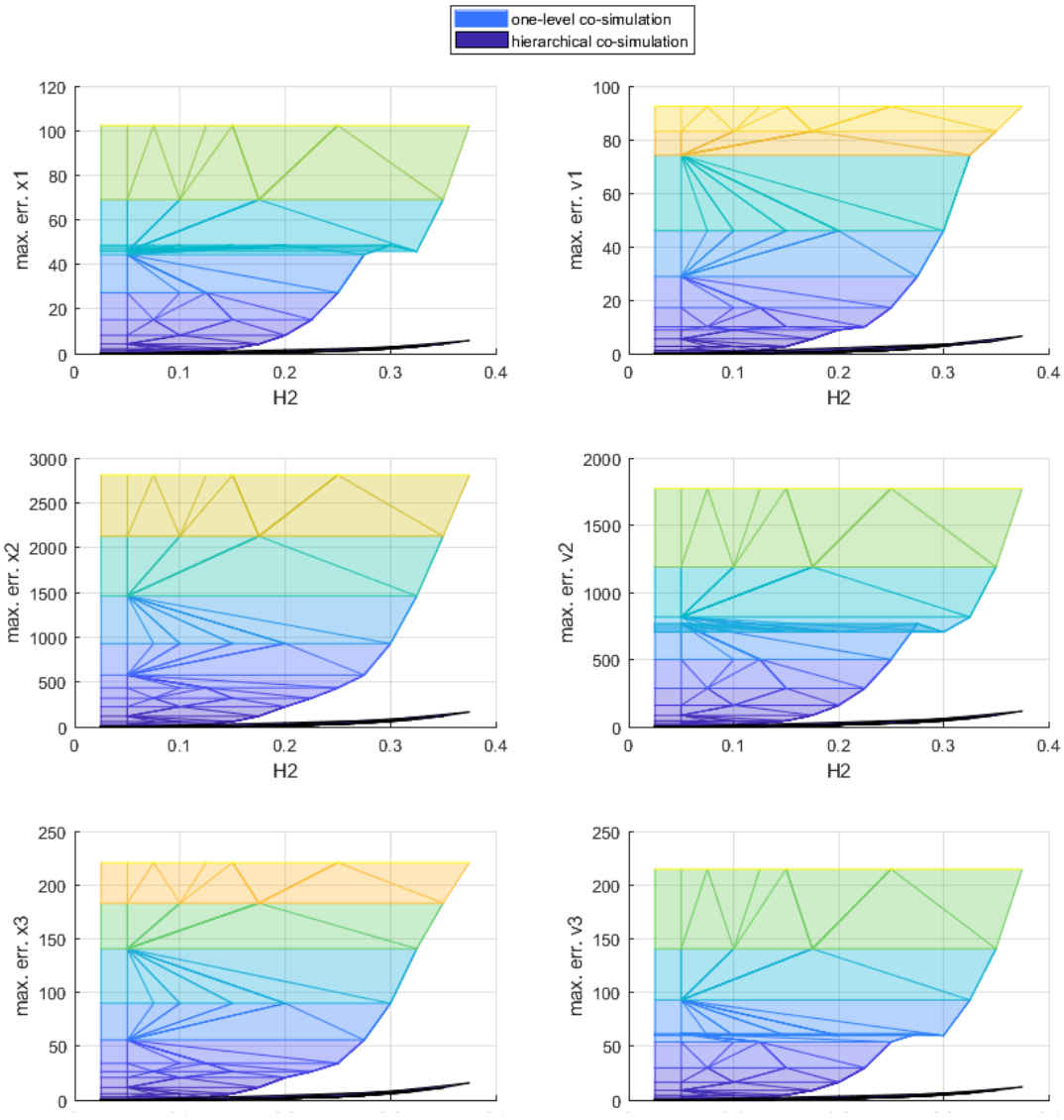


Figure A.6: Component errors for the simulation of Scenario 2 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes, $err$-$H_2$ plane view.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

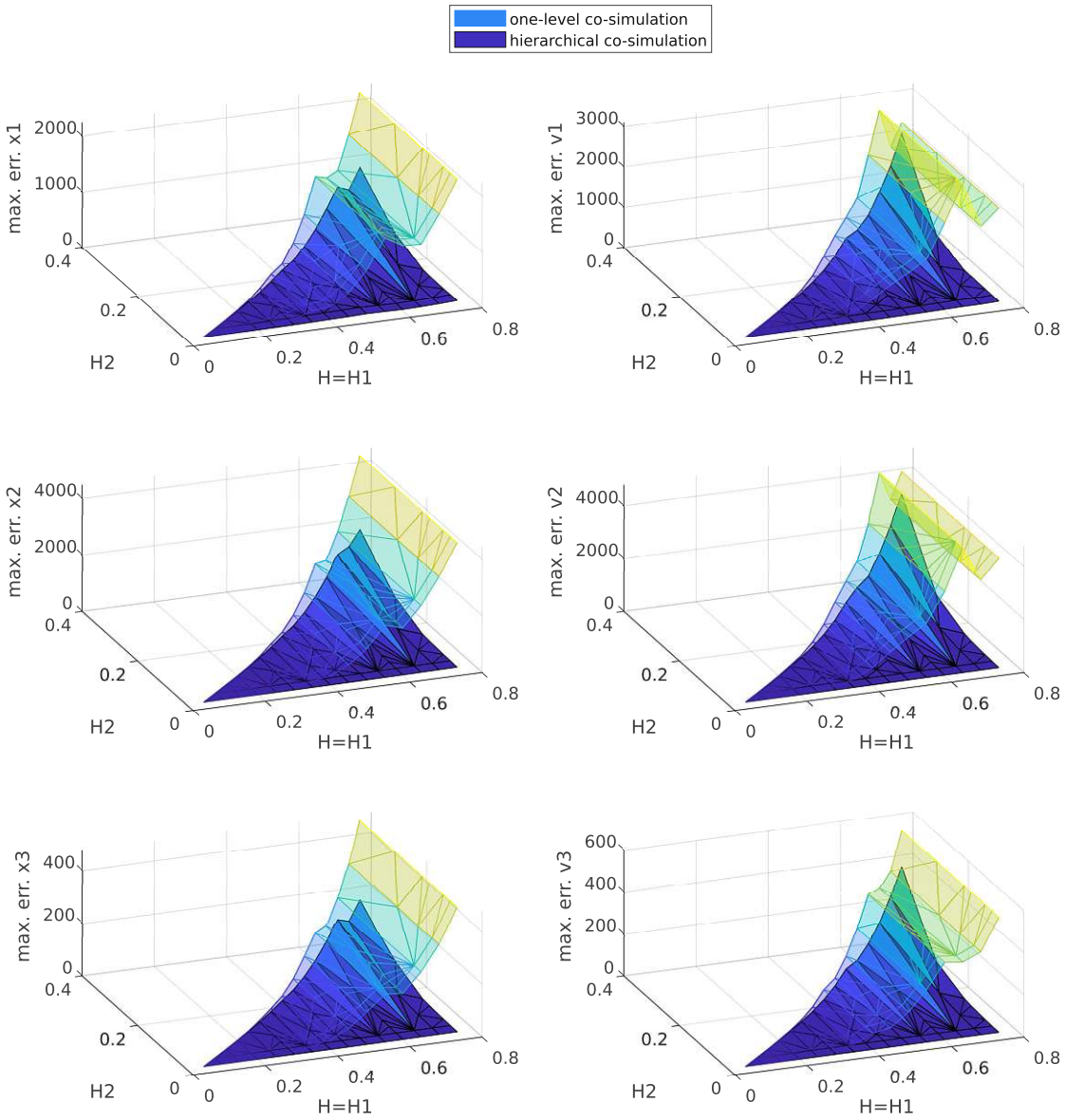


Figure A.7: Component errors for the simulation of Scenario 3 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

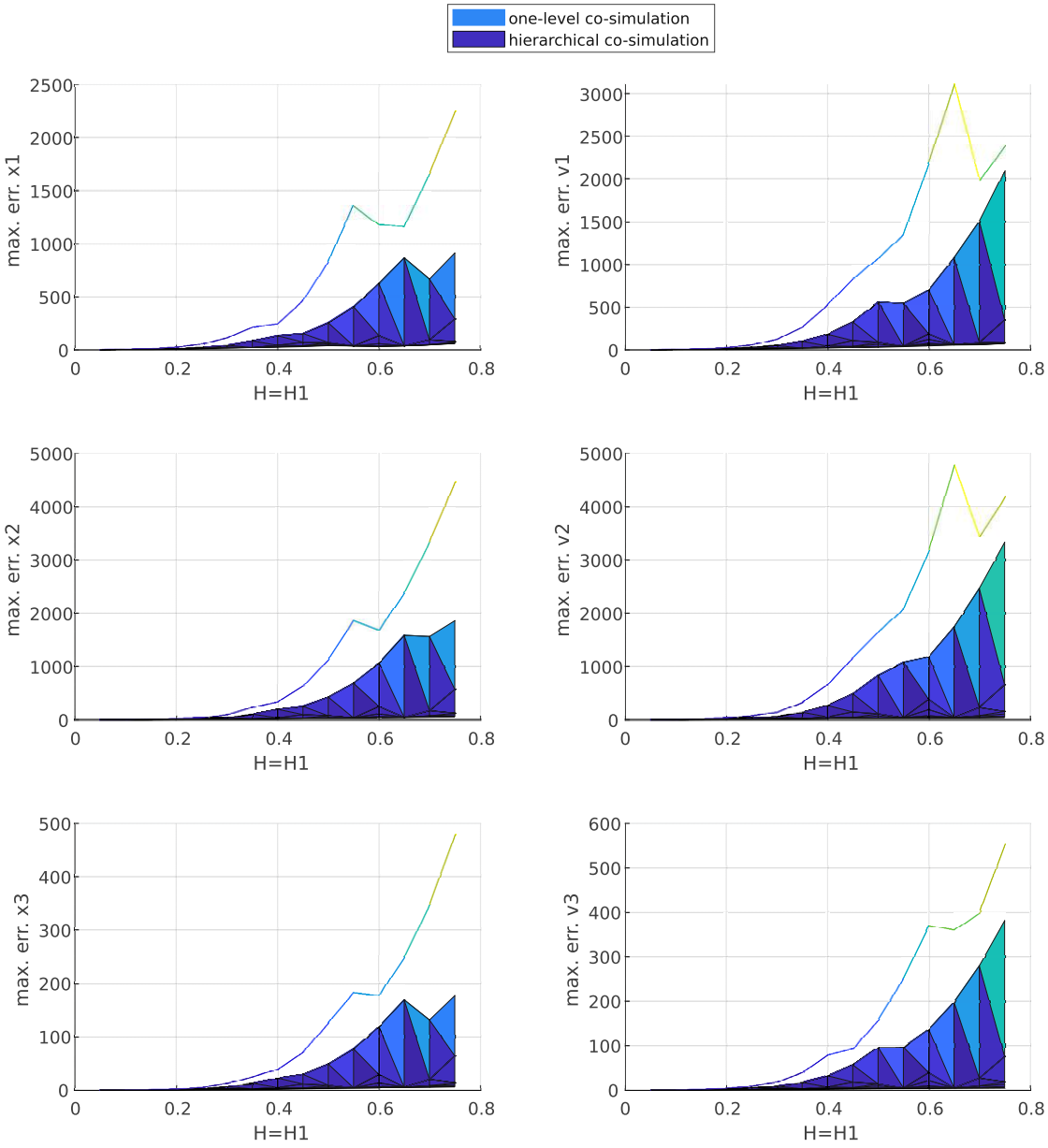


Figure A.8: Component errors for the simulation of Scenario 3 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes, $err$-$H_1$ plane view.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

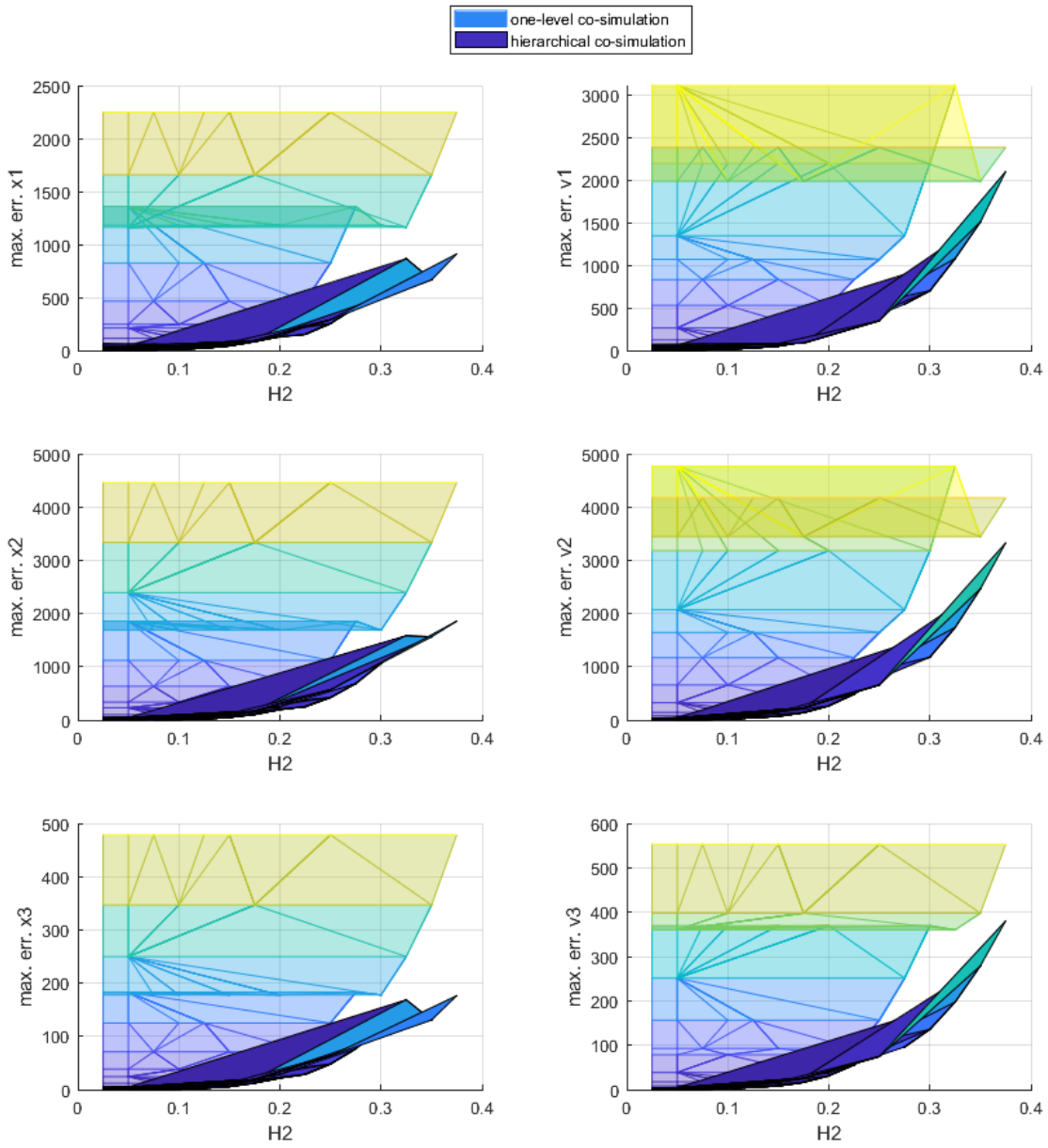


Figure A.9: Component errors for the simulation of Scenario 1 from $t_{start} = 0s$ to $t_{end} = 25s$ depending on macro step sizes, $err$-$H_2$ plane view.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.

# List of Figures





Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

# List of Tables





Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

# Bibliography


Alur, R., Thao Dang, J. Esposito, Yerang Hur, F. Ivancic, V. Kumar, P. Mishra, G.J. Pappas, and O. Sokolsky (Jan. 2003). "Hierarchical modeling and analysis of embedded systems". In: *Proceedings of the IEEE* 91.1, pp. 11–28. ISSN: 1558-2256. DOI: 10.1109/JPROC.2002.805817.

Andrus, J. (Aug. 1979). "Numerical Solution of Systems of Ordinary Differential Equations Separated into Subsystems". In: *SIAM Journal on Numerical Analysis* 16.4, pp. 605–611. ISSN: 0036-1429. DOI: 10.1137/0716045.

Arnold, M., B. Burgermeister, C. Führer, G. Hippmann, and G. Rill (July 2011). "Numerical methods in vehicle system dynamics: state of the art and current developments". en. In: *Vehicle System Dynamics* 49.7, pp. 1159–1207. ISSN: 0042-3114, 1744-5159. DOI: 10.1080/00423114.2011.582953.

Arnold, Martin (2007). "Multi-Rate Time Integration for Large Scale Multibody System Models". en. In: *IUTAM Symposium on Multiscale Problems in Multibody System Contacts*. Ed. by Peter Eberhard. IUTAM Bookseries 1. Springer Netherlands, pp. 1–10. ISBN: 978-1-4020-5980-3 978-1-4020-5981-0. DOI: 10.1007/978-1-4020-5981-0_1.

Arnold, Martin (2010). "Stability of Sequential Modular Time Integration Methods for Coupled Multibody System Models". en. In: *Journal of Computational and Nonlinear Dynamics* 5.3, p. 031003. ISSN: 15551423. DOI: 10.1115/1.4001389.

Arnold, Martin, Christoph Clauss, and Tom Schierz (Jan. 2013). "Error Analysis and Error Estimates for Co-Simulation in FMI for Model Exchange and Co-Simulation V2.0". In: *Archive of Mechanical Engineering* LX.1. ISSN: 0004-0738. DOI: 10.2478/meceng-2013-0005.

Arnold, Martin and Michael Günther (2001). "Preconditioned Dynamic Iteration for Coupled Differential-Algebraic Systems". English. In: *BIT Numerical Mathematics* 41.1, pp. 1–25. ISSN: 0006-3835. DOI: 10.1023/A:1021909032551.

Arnold, Martin, Stefan Hante, and Markus A. Köbis (Dec. 2014). "Error analysis for co-simulation with force-displacement coupling". en. In: *PAMM* 14.1, pp. 43–44. ISSN: 1617-7061. DOI: 10.1002/pamm.201410014.




Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Awais, Muhammad Usman (2015). “Distributed hybrid co-simulation”. eng. PhD thesis. Vienna, Austria: TU Wien. URL: https://resolver.obvsg.at/urn:nbn:at:at-ubtuw:1-78451 (visited on 10/01/2021).

Barros, Fernando J. (2008). “Semantics of Dynamic Structure Event-based Systems”. In: *Proceedings of the Second International Conference on Distributed Event-based Systems*. DEBS ’08. Rome, Italy: ACM, pp. 245–252. ISBN: 978-1-60558-090-6. DOI: 10.1145/1385989.1386020.

Barros, Fernando J. (2017). “Chattering Avoidance in Hybrid Simulation Models: A Modular Approach Based on the HyFlow Formalism”. In: *Proceedings of the Symposium on Theory of Modeling & Simulation*. TMS/DEVS ’17. Virginia Beach, Virginia: Society for Computer Simulation International, 15:1–15:12. URL: http://dl.acm.org/citation.cfm?id=3108905.3108920 (visited on 10/01/2021).

Bartel, A., M. Brunk, M. Günther, and S. Schöps (Jan. 2013). “Dynamic Iteration for Coupled Problems of Electric Circuits and Distributed Devices”. In: *SIAM Journal on Scientific Computing* 35.2, B315–B335. ISSN: 1064-8275. DOI: 10.1137/120867111.

Bartel, A. and M. Günther (2002). “A multirate W-method for electrical networks in state-space formulation”. In: *Journal of Computational and Applied Mathematics* 147.2, pp. 411–425. ISSN: 0377-0427. DOI: http://dx.doi.org/10.1016/S0377-0427(02)00476-4.

Bartel, Andreas, Markus Brunk, and Sebastian Schöps (May 2014). “On the convergence rate of dynamic iteration for coupled problems with multiple subsystems”. en. In: *Journal of Computational and Applied Mathematics* 262, pp. 14–24. ISSN: 03770427. DOI: 10.1016/j.cam.2013.07.031.

Ben Khaled, Abir, Laurent Duval, Mongi Ben Gaid, and Daniel Simon (Mar. 2014). “Context-based polynomial extrapolation and slackened synchronization for fast multi-core simulation using FMI”. In: Lund, Sweden: inköping University Electronic Press, pp. 225–234. DOI: 10.3384/ecp14096225.

Benedikt, M., D. Watzenig, J. Zehetner, and A. Hofer (2013). “NEPCE - a nearly energy-preserving coupling element for weak-coupled problems and co-simulations”. eng. In: *COUPLED V : proceedings of the V International Conference on Computational Methods for Coupled Problems in Science and Engineering*. CIMNE, pp. 1021–1032. ISBN: 978-84-941407-6-1. URL: https://upcommons.upc.edu/handle/2117/192675 (visited on 10/05/2021).

Benedikt, Martin, Hannes Stippel, and Daniel Watzenig (Apr. 2010). “An Adaptive Coupling Methodology for Fast Time-Domain Distributed Heterogeneous Co-Simulation”. In: *SAE 2010 World Congress & Exhibition*. DOI: 10.4271/2010-01-0649.

Biesiadecki, Jeffrey J. and Robert D. Skeel (Dec. 1993). “Dangers of Multiple Time Step Methods”. In: *Journal of Computational Physics* 109.2, pp. 318–328. ISSN: 0021-9991. DOI: 10.1006/jcph.1993.1220.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Blockwitz, Torsten, Martin Otter, Johan Akesson, Martin Arnold, Christoph Clauss, Hilding Elmqvist, Markus Friedrich, Andreas Junghanns, Jakob Mauss, Dietmar Neumerkel, Hans Olsson, and Antoine Viel (Nov. 2012). "Functional Mockup Interface 2.0: The Standard for Tool independent Exchange of Simulation Models". en. In: *Proceedings of the 9th International MODELICA Conference*, pp. 173–184. DOI: 10.3384/ecp12076173.

Bohr, Mark (2007). "A 30 Year Retrospective on Dennard's MOSFET Scaling Paper". In: *IEEE Solid-State Circuits Society Newsletter* 12.1, pp. 11–13. DOI: 10.1109/N-SSC.2007.4785534.

Brailsford, Sally C., Tillal Eldabi, Martin Kunc, Navonil Mustafee, and Andres F. Osorio (Nov. 2019). "Hybrid simulation modelling in operational research: A state-of-the-art review". In: *European Journal of Operational Research* 278.3, pp. 721–737. ISSN: 0377-2217. DOI: 10.1016/j.ejor.2018.10.025.

Brecher, C., M. Esser, and S. Witt (2009). "Interaction of manufacturing process and machine tool". en. In: *CIRP Annals* 58.2, pp. 588–607. ISSN: 00078506. DOI: 10.1016/j.cirp.2009.09.005.

Breitenecker, Felix (Sept. 1992). "Models, methods and experiments — A new structure for simulation systems". en. In: *Mathematics and Computers in Simulation* 34.3-4, pp. 231–260. ISSN: 03784754. DOI: 10.1016/0378-4754(92)90004-Z.

Breitenecker, Felix, Horst Ecker, and Ingrid Bausch-Gall (1993). *Simulation mit ACSL*. de. Wiesbaden: Vieweg+Teubner Verlag. ISBN: 978-3-528-06381-8 978-3-322-96175-4. DOI: 10.1007/978-3-322-96175-4.

Breitenecker, Felix and Nikolas Popper (Mar. 2007). "Structure of Simulation Systems for Structural-Dynamic Systems". In: *First Asia International Conference on Modelling Simulation (AMS'07)*, pp. 574–579. DOI: 10.1109/AMS.2007.97.

Broman, David, Christopher Brooks, Lev Greenberg, Edward A. Lee, Michael Masin, Stavros Tripakis, and Michael Wetter (2013). "Determinate Composition of FMUs for Co-simulation". In: *Proceedings of the Eleventh ACM International Conference on Embedded Software*. EMSOFT '13. event-place: Montreal, Quebec, Canada. Piscataway, NJ, USA: IEEE Press, 2:1–2:12. ISBN: 978-1-4799-1443-2. URL: http://dl.acm.org/citation.cfm?id=2555754.2555756 (visited on 07/31/2019).

Broman, David, Lev Greenberg, Edward A. Lee, Michael Masin, Stavros Tripakis, and Michael Wetter (2015). "Requirements for Hybrid Cosimulation Standards". In: *Proceedings of the 18th International Conference on Hybrid Systems: Computation and Control*. HSCC '15. Seattle, Washington: ACM, pp. 179–188. ISBN: 978-1-4503-3433-4. DOI: 10.1145/2728606.2728629.

Burrage, Kevin (Aug. 1995). *Parallel and Sequential Methods for Ordinary Differential Equations*. Numerical Mathematics and Scientific Computation. Oxford, New York: Oxford University Press. ISBN: 978-0-19-853432-7.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Busch, M. (2012). *Zur effizienten Kopplung von Simulationsprogrammen*. Kassel University Press. ISBN: 9783862192960. URL: https://www.uni-kassel.de/ub/?id=39129&h=9783862192960 (visited on 10/01/2021).

Busch, M, M. Arnold, A. Heckmann, and S. Dronka (2007). *Interfacing SIMPACK to Modelica/Dymola for multi-domain vehicle system simulations*. Tech. rep. 11(2). Wessling, Germany: INTEC GmbH, pp. 01–03.

Camus, B., V. Galtier, and M. Caujolle (Apr. 2016). "Hybrid Co-simulation of FMUs using DEV&DESS in MECSYCO". In: *2016 Symposium on Theory of Modeling and Simulation (TMS-DEVS)*. 2016 Symposium on Theory of Modeling and Simulation (TMS-DEVS), pp. 1–8. DOI: 10.23919/TMS.2016.7918814.

Chapman, Jacob (July 2017). "Multi-agent stochastic simulation of occupants in buildings". en. phd. Nottingham: University of Nottingham. URL: http://eprints.nottingham.ac.uk/39868/ (visited on 10/10/2021).

Cremona, F., M. Lohstroh, D. Broman, M. Di Natale, E. A. Lee, and S. Tripakis (Nov. 2016a). "Step revision in hybrid Co-simulation with FMI". In: *2016 ACM/IEEE International Conference on Formal Methods and Models for System Design (MEMOCODE)*, pp. 173–183. DOI: 10.1109/MEMCOD.2016.7797762.

Cremona, Fabio, Marten Lohstroh, David Broman, Edward A. Lee, Michael Masin, and Stavros Tripakis (June 1, 2019). "Hybrid co-simulation: it's about time". In: *Software & Systems Modeling* 18.3, pp. 1655–1679. ISSN: 1619-1374. DOI: 10.1007/s10270-017-0633-6.

Cremona, Fabio, Marten Lohstroh, Stavros Tripakis, Christopher Brooks, and Edward A. Lee (2016b). "FIDE: An FMI Integrated Development Environment". In: *Proceedings of the 31st Annual ACM Symposium on Applied Computing*. SAC '16. event-place: Pisa, Italy. New York, NY, USA: ACM, pp. 1759–1766. ISBN: 978-1-4503-3739-7. DOI: 10.1145/2851613.2851677.

Dahmann, Judith S., Richard M. Fujimoto, and Richard M. Weatherly (1997). "The Department of Defense High Level Architecture". en. In: *Proceedings of the 29th conference on Winter simulation - WSC '97*. Atlanta, Georgia, United States: ACM Press, pp. 142–149. ISBN: 978-0-7803-4278-1. DOI: 10.1145/268437.268465.

Dalkey, Norman and Olaf Helmer (Apr. 1963). "An Experimental Application of the DELPHI Method to the Use of Experts". en. In: *Management Science* 9.3, pp. 458–467. ISSN: 0025-1909, 1526-5501. DOI: 10.1287/mnsc.9.3.458.

Deatcu, Christina and Thorsten Pawletta (Apr. 2012). "A Qualitative Comparison of Two Hybrid DEVS Approaches". en. In: *SNE Simulation Notes Europe* 22.1, pp. 15–24. ISSN: 23059974, 23060271. DOI: 10.11128/sne.22.tn.10107.

Duflou, Joost R., John W. Sutherland, David Dornfeld, Christoph Herrmann, Jack Jeswiet, Sami Kara, Michael Hauschild, and Karel Kellens (Jan. 2012). "Towards energy and

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

resource efficient manufacturing: A processes and systems approach". en. In: *CIRP Annals* 61.2, pp. 587–609. ISSN: 0007-8506. DOI: 10.1016/j.cirp.2012.05.002.

Ebert, Falk (Nov. 2004). "Convergence of relaxation methods for coupled systems of ODEs and DAEs". In: URL: https://opus4.kobv.de/opus4-matheon/frontdoor/index/index/docId/177 (visited on 07/18/2016).

Ebert, Falk (Oct. 2008). "On Partitioned Simulation of Electrical Circuits using Dynamic Iteration Methods". en. In: DOI: http://dx.doi.org/10.14279/depositonce-1997.

Engine Yard (2017). *Set Up Database Replication*. https://support.cloud.engineyard.com/hc/en-us/articles/205408108-Set-Up-Database-Replication. (Visited on 09/03/2021).

Engstler, Ch. and Ch. Lubich (June 1997). "Multirate extrapolation methods for differential equations with different time scales". en. In: *Computing* 58.2, pp. 173–185. ISSN: 1436-5057. DOI: 10.1007/BF02684438.

Enright, W.H. (Dec. 2000). "Continuous numerical methods for ODEs with defect control". en. In: *Journal of Computational and Applied Mathematics* 125.1-2, pp. 159–170. ISSN: 03770427. DOI: 10.1016/S0377-0427(00)00466-0.

Esgandari, Mohammad and Oluremi Olatunbosun (July 2015). "Implicit–explicit co-simulation of brake noise". In: *Finite Elements in Analysis and Design* 99, pp. 16–23. ISSN: 0168-874X. DOI: 10.1016/j.finel.2015.01.011.

Esmaeilzadeh, Hadi, Emily Blem, Renée St. Amant, Karthikeyan Sankaralingam, and Doug Burger (2011). "Dark silicon and the end of multicore scaling". In: *2011 38th Annual International Symposium on Computer Architecture (ISCA)*, pp. 365–376.

Esposito, J.M. and V. Kumar (2001). "Efficient dynamic simulation of robotic systems with hierarchy". en. In: *Proceedings 2001 ICRA. IEEE International Conference on Robotics and Automation (Cat. No.01CH37164)*. Vol. 3. Seoul, South Korea: IEEE, pp. 2818–2823. ISBN: 978-0-7803-6576-6. DOI: 10.1109/ROBOT.2001.933049.

Eyisi, Emeka, Jia Bai, Derek Riley, Jiannian Weng, Wei Yan, Yuan Xue, Xenofon Koutsoukos, and Janos Sztipanovits (2012). "NCSWT: An integrated modeling and simulation tool for networked control systems". In: *Simulation Modelling Practice and Theory* 27, pp. 90–111. ISSN: 1569-190X. DOI: https://doi.org/10.1016/j.simpat.2012.05.004.

Farhat, C. and M. Lesoinne (Feb. 2000). "Two efficient staggered algorithms for the serial and parallel solution of three-dimensional nonlinear transient aeroelastic problems". en. In: *Computer Methods in Applied Mechanics and Engineering* 182.3-4, pp. 499–515. ISSN: 00457825. DOI: 10.1016/S0045-7825(99)00206-6.

Farkas, Rebeka, Gábor Bergmann, and Ákos Horváth (2019). "Adaptive Step Size Control for Hybrid CT Simulation without Rollback". en. In: *Proceedings of the 13th International Modelica Conference*. Vol. 157. Linköping Electronic Conference Proceedings. Regens-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

burg, Germany: Linköping University Electronic Press, p. 10. ISBN: 978-91-7685-122-7. DOI: 10.3384/ecp19157503.

Farsi, Maryam, John Ahmet Erkoyuncu, Daniel Steenstra, and Rajkumar Roy (July 2019). "A modular hybrid simulation framework for complex manufacturing system design". In: *Simulation Modelling Practice and Theory* 94, pp. 14–30. ISSN: 1569-190X. DOI: 10.1016/j.simpat.2019.02.002.

Featherstone, Roy (Jan. 1999a). "A Divide-and-Conquer Articulated-Body Algorithm for Parallel O(log(n)) Calculation of Rigid-Body Dynamics. Part 1: Basic Algorithm". en. In: *The International Journal of Robotics Research* 18.9, pp. 867–875. ISSN: 0278-3649, 1741-3176. DOI: 10.1177/02783649922066619.

Featherstone, Roy (Jan. 1999b). "A Divide-and-Conquer Articulated-Body Algorithm for Parallel O(log(n)) Calculation of Rigid-Body Dynamics. Part 2: Trees, Loops, and Accuracy". en. In: *The International Journal of Robotics Research* 18.9, pp. 876–892. ISSN: 0278-3649, 1741-3176. DOI: 10.1177/02783649922066628.

Felippa, Carlos A., K.C. Park, and Charbel Farhat (Mar. 2001). "Partitioned analysis of coupled mechanical systems". en. In: *Computer Methods in Applied Mechanics and Engineering* 190.24-25, pp. 3247–3270. ISSN: 00457825. DOI: 10.1016/S0045-7825(00)00391-1.

Ferreira, Paulo A. F., Edgar F. Esteves, Rosaldo J. F. Rossetti, and Eugénio C. Oliveira (Sept. 2008). "A Cooperative Simulation Framework for Traffic and Transportation Engineering". en. In: *Cooperative Design, Visualization, and Engineering*. Lecture Notes in Computer Science. Springer, Berlin, Heidelberg, pp. 89–97. ISBN: 978-3-540-88010-3 978-3-540-88011-0. DOI: 10.1007/978-3-540-88011-0_12.

Fitzgerald, John, Peter Gorm Larsen, and Marcel Verhoef, eds. (2014). *Collaborative Design for Embedded Systems*. en. Berlin, Heidelberg: Springer Berlin Heidelberg. ISBN: 978-3-642-54117-9 978-3-642-54118-6. DOI: 10.1007/978-3-642-54118-6.

Friedrich, Markus (2011). "Parallel Co-Simulation for Mechatronic Systems". PhD thesis. Technische Universität München. URL: http://mediatum.ub.tum.de/node?id=1063436 (visited on 03/29/2016).

Fritzson, Peter A. (2004). *Principles of object-oriented modeling and simulation with Modelica 2.1*. OCLC: ocm52920516. Piscataway, N.J. : [New York]: IEEE Press ; Wiley-Interscience. ISBN: 978-0-471-47163-9.

Galtier, Virginie, Stephane Vialle, Cherifa Dad, Jean-Philippe Tavella, Jean-Philippe Lam-Yee-Mui, and Gilles Plessis (Apr. 2015). "FMI-based distributed multi-simulation with DACCOSIM". In: *Proceedings of the Symposium on Theory of Modeling & Simulation: DEVS Integrative M&S Symposium*. DEVS '15. San Diego, CA, USA: Society for Computer Simulation International, pp. 39–46. ISBN: 978-1-5108-0105-9.

Ge, Xiaolong, Botong Liu, Botan Liu, Xigang Yuan, and Hongxing Wang (Apr. 2019). "Numerical simulation of evaporation with Rayleigh convection using LBM-FDM hybrid method

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

and proposal of the interfacial mass transfer coefficient model". In: *International Journal of Heat and Mass Transfer* 133, pp. 107–118. ISSN: 0017-9310. DOI: 10.1016/j.ijheatmasstransfer.2018.12.100.

Gear, C. W. (Mar. 1988). "Parallel methods for ordinary differential equations". en. In: *CALCOLO* 25.1-2, pp. 1–20. ISSN: 0008-0624, 1126-5434. DOI: 10.1007/BF02575744.

Gear, C. W. and D. R. Wells (Dec. 1984). "Multirate linear multistep methods". en. In: *BIT Numerical Mathematics* 24.4, pp. 484–502. ISSN: 0006-3835, 1572-9125. DOI: 10.1007/BF01934907.

Gear, C.W., B. Leimkuhler, and G.K. Gupta (1985). "Automatic integration of Euler-Lagrange equations with constraints". In: *Journal of Computational and Applied Mathematics* 12–13, pp. 77 –90. ISSN: 0377-0427. DOI: http://dx.doi.org/10.1016/0377-0427(85)90008-1.

Geimer, M., T. Krüger, and Linsel P (2006). "Co-Simulation, gekoppelte Simulation oder Simulatorkopplung? Ein Versuch der Begriffsvereinheitlichung". In: *O+P Ölhydraulik und Pneumatik* 50.11-12. ISSN: 0341-2660. URL: https://publikationen.bibliothek.kit.edu/1000011318 (visited on 02/16/2017).

Gheorghe, Luiza (Aug. 2009). "Continuous/Discrete Co-Simulation Interfaces from Formalization to Implementation". en. phd. École Polytechnique de Montréal. URL: https://publications.polymtl.ca/137/ (visited on 09/06/2019).

Glumac, Slaven and Zdenko Kovacic (2018). "Calling Sequence Calculation for Sequential Co-simulation Master". In: *Proceedings of the 2018 ACM SIGSIM Conference on Principles of Advanced Discrete Simulation*. SIGSIM-PADS '18. event-place: Rome, Italy. New York, NY, USA: ACM, pp. 157–160. ISBN: 978-1-4503-5092-1. DOI: 10.1145/3200921.3200924.

Glumac, Slaven and Zdenko Kovacic (Feb. 2019). "Relative Consistency and Robust Stability Measures for Sequential Co-simulation". In: pp. 197–206. DOI: 10.3384/ecp19157197.

Günther, M., A. Kværnø, and P. Rentrop (June 2001). "Multirate Partitioned Runge-Kutta Methods". en. In: *BIT Numerical Mathematics* 41.3, pp. 504–514. ISSN: 0006-3835, 1572-9125. DOI: 10.1023/A:1021967112503.

Günther, Michael and Peter Rentrop (1994). "Partitioning and Multirate Strategies in Latent Electric Circuits". en. In: *Mathematical Modelling and Simulation of Electrical Circuits and Semiconductor Devices*. Ed. by R. E. Bank, H. Gajewski, R. Bulirsch, and K. Merten. ISNM International Series of Numerical Mathematics. Basel: Birkhäuser, pp. 33–60. ISBN: 978-3-0348-8528-7. DOI: 10.1007/978-3-0348-8528-7_3.

Gomes, Cláudio, Paschalis Karalis, Eva M. Navarro-López, and Hans Vangheluwe (2018a). "Approximated Stability Analysis of Bi-modal Hybrid Co-simulation Scenarios". In: *Software Engineering and Formal Methods*. Ed. by Antonio Cerone and Marco Roveri. Vol. 10729.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Cham: Springer International Publishing, pp. 345–360. ISBN: 978-3-319-74780-4 978-3-319-74781-1. DOI: 10.1007/978-3-319-74781-1_24.

Gomes, Cláudio, Casper Thule, David Broman, Peter Gorm Larsen, and Hans Vangheluwe (Feb. 2017). "Co-simulation: State of the art". In: *arXiv:1702.00686 [cs]*. arXiv: 1702.00686. URL: http://arxiv.org/abs/1702.00686 (visited on 09/04/2019).

Gomes, Cláudio, Casper Thule, David Broman, Peter Gorm Larsen, and Hans Vangheluwe (May 2018b). "Co-Simulation: A Survey". en. In: *ACM Computing Surveys* 51.3, pp. 1–33. ISSN: 03600300. DOI: 10.1145/3179993.

Gomes, Cláudio, Casper Thule, Levi Lúcio, Hans Vangheluwe, and Peter Gorm Larsen (Sept. 2019). "Generation of Co-simulation Algorithms Subject to Simulator Contracts". In: *CoSim-CPS-19*. Oslo, Norway, p. 15.

Gomm, W. (Mar. 1981). "Stability analysis of explicit multirate methods". In: *Mathematics and Computers in Simulation* 23.1, pp. 34–50. ISSN: 0378-4754. DOI: 10.1016/0378-4754(81)90005-7.

González, Francisco, Manuel González, and Aki Mikkola (Oct. 2010). "Efficient coupling of multibody software with numerical computing environments and block diagram simulators". en. In: *Multibody System Dynamics* 24.3, pp. 237–253. ISSN: 1384-5640, 1573-272X. DOI: 10.1007/s11044-010-9199-6.

González, Francisco, Miguel Ángel Naya, Alberto Luaces, and Manuel González (Apr. 2011). "On the effect of multirate co-simulation techniques in the efficiency and accuracy of multibody system dynamics". en. In: *Multibody System Dynamics* 25.4, pp. 461–483. ISSN: 1384-5640, 1573-272X. DOI: 10.1007/s11044-010-9234-7.

Grubmüller, H., H. Heller, A. Windemuth, and K. Schulten (Mar. 1991). "Generalized Verlet Algorithm for Efficient Molecular Dynamics Simulations with Long-range Interactions". en. In: *Molecular Simulation* 6.1-3, pp. 121–142. ISSN: 0892-7022, 1029-0435. DOI: 10.1080/08927029108022142.

Gu, Bei (2001). "Co-simulation of algebraically coupled dynamic subsystems". eng. Thesis. Massachusetts Institute of Technology. URL: http://dspace.mit.edu/handle/1721.1/8695 (visited on 10/06/2016).

Gu, Bei and H. Harry Asada (Apr. 2004). "Co-Simulation of Algebraically Coupled Dynamic Subsystems Without Disclosure of Proprietary Subsystem Models". In: *Journal of Dynamic Systems, Measurement, and Control* 126.1, pp. 1–13. ISSN: 0022-0434. DOI: 10.1115/1.1648307.

Gu, Bei, B. W. Gordon, and H. H. Asada (2000). "Co-simulation of coupled dynamic subsystems: a differential-algebraic approach using singularly perturbed sliding manifolds". In: *American Control Conference, 2000. Proceedings of the 2000*. Vol. 2, 757–761 vol.2. DOI: 10.1109/ACC.2000.876599.

Hafner, Irene and Niki Popper (2017). "On the Terminology and Structuring of Co-simulation Methods". In: *Proceedings of the 8th International Workshop on Equation-Based Object-*

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*Oriented Modeling Languages and Tools*. EOOLT '17. event-place: Weßling, Germany. New York, NY, USA: ACM, pp. 67–76. ISBN: 978-1-4503-6373-0. DOI: 10.1145/3158191.3158203.

Hafner, Irene and Niki Popper (Oct. 2020). "Investigation on Stability Properties of Hierarchical Co-Simulation". en. In: *Proceedings ASIM SST 2020, 25. Symposium Simulationstechnik*. Vol. 59. ARGESIM Report. Online-Tagung: ARGESIM Verlag, pp. 41–48. ISBN: 978-3-901608-93-3. DOI: 10.11128/arep.59.a59007.

Hafner, Irene, Niki Popper, and Felix Breitencker (2016). "On the Methodology of Cooperative and Multirate Simulation". In: *Tagungsband ASIM 2016 23. Symposium Simulationstechnik*. 23. Symposium Simulationstechnik. Dresden, Germany: ASIM, pp. 127–130. ISBN: 978-3-901608-49-0.

Hafner, Irene, Matthias Rößler, Bernhard Heinzl, Andreas Körner, Michael Landsiedl, and Felix Breitenecker (2014). "Investigating communication and step-size behaviour for co-simulation of hybrid physical systems". In: *Journal of Computational Science* 5.3, pp. 427–438. ISSN: 1877-7503. DOI: http://dx.doi.org/10.1016/j.jocs.2013.08.007.

Hairer, E. (1981). "Order conditions for numerical methods for partitioned ordinary differential equations". In: *Numerische Mathematik* 36.4, pp. 431–445. ISSN: 0945-3245. DOI: 10.1007/BF01395956.

Hairer, E., S.P. Nørsett, and G. Wanner (1993). *Solving Ordinary Differential Equations I: Nonstiff Problems*. Vol. 8. Springer Series in Computational Mathematics. Springer Berlin Heidelberg. ISBN: 9783540788621. DOI: 10.1007/978-3-540-78862-1.

Hairer, E., S.P. Nørsett, and G. Wanner (1996). *Solving Ordinary Differential Equations II: Stiff and Differential-Algebraic Problems*. Vol. 14. Springer Series in Computational Mathematics. Springer Berlin Heidelberg. ISBN: 9783540604525. DOI: 10.1007/978-3-642-05221-7.

Hante, Stefan, Martin Arnold, and Markus Köbis (2019). "The SNiMoWrapper: An FMI-Compatible Testbed for Numerical Algorithms in Co-simulation". en. In: *IUTAM Symposium on Solver-Coupling and Co-Simulation*. Ed. by Bernhard Schweizer. IUTAM Bookseries. Cham: Springer International Publishing, pp. 99–116. ISBN: 978-3-030-14883-6. DOI: 10.1007/978-3-030-14883-6_6.

Heimlich, Oliver, Rudolf Sailer, and Lukasz Budzisz (Aug. 2010). "NMLab: A Co-simulation Framework for Matlab and NS-2". en. In: *2010 Second International Conference on Advances in System Simulation*. Nice, France: IEEE, pp. 152–157. ISBN: 978-1-4244-7783-8. DOI: 10.1109/SIMUL.2010.24.

Heinzl, Bernhard (2016). "Hybrid Modeling of Production Systems: Co-simulation and DEVS-based Approach". en. Diplomarbeit. Wien: TU Wien.

Heinzl, Bernhard, Philipp Raich, Franz Preyser, and Wolfgang Kastner (Jan. 2018). "Simulation-based Assessment of Energy Efficiency in Industry: Comparison of Hybrid Simulation

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Approaches". In: *IFAC-PapersOnLine*. 9th Vienna International Conference on Mathematical Modelling 51.2, pp. 689–694. ISSN: 2405-8963. DOI: 10.1016/j.ifacol.2018.03.117.

Hennessy, John L. and David A. Patterson (2018). *A New Golden Age for Computer Architecture: Domain-Specific Hardware/Software Co-Design, Enhanced Security, Open Instruction Sets, and Agile Chip Development*. https://iscaconf.org/isca2018/turing_lecture.html. (Visited on 07/30/2021).

Hinrichsen, D. and N.K. Son (1989). "The complex stability radius of discrete-time systems and symplectic pencils". In: *Proceedings of the 28th IEEE Conference on Decision and Control*. Tampa, FL, USA: IEEE, pp. 2265–2270. DOI: 10.1109/CDC.1989.70573.

Hofer, E. (Oct. 1976). "A Partially Implicit Method for Large Stiff Systems of ODEs with Only Few Equations Introducing Small Time-Constants". en. In: *SIAM Journal on Numerical Analysis* 13.5, pp. 645–663. ISSN: 0036-1429, 1095-7170. DOI: 10.1137/0713054.

Holmes, Philip and Philip Holmes, eds. (2012). *Turbulence, coherent structures, dynamical systems and symmetry*. 2nd ed. Cambridge monographs on mechanics. Cambridge, UK ; New York: Cambridge University Press. ISBN: 978-1-107-00825-0.

Hopkinson, K., X. Wang, R. Giovanini, J. Thorp, K. Birman, and D. Coury (May 2006). "EPOCHS: A Platform for Agent-Based Electric Power and Communication Simulation Built From Commercial Off-the-Shelf Components". en. In: *IEEE Transactions on Power Systems* 21.2, pp. 548–558. ISSN: 0885-8950. DOI: 10.1109/TPWRS.2006.873129.

Horn, Roger A. and Charles R. Johnson (2010). *Matrix analysis*. eng. 23. print. OCLC: 711847070. Cambridge: Cambridge Univ. Press. ISBN: 978-0-521-38632-6.

Ibrahimbegović, Adnan and Damijan Markovič (July 2003). "Strong coupling methods in multi-phase and multi-scale modeling of inelastic behavior of heterogeneous structures". In: *Computer Methods in Applied Mechanics and Engineering*. Multiscale Computational Mechanics for Materials and Structures 192.28–30, pp. 3089–3107l. ISSN: 0045-7825. DOI: 10.1016/S0045-7825(03)00342-6.

Jackiewicz, Zdzislaw and Marian Kwapisz (1996). "Convergence of waveform relaxation methods for differential-algebraic systems". In: *SIAM Journal on Numerical Analysis* 33.6, pp. 2303–2317.

Jackson, Kenneth R. (Sept. 1991). "A survey of Parallel Numerical Methods for Initial Value Problems for Ordinary Differential Equations". eng. In: *IEEE Transactions on Magnetics* 27.5, pp. 3792–3797. ISSN: 0018-9464.

Jia, Zhidong and Ben Leimkuhler (Apr. 2003). "A parallel multiple time-scale reversible integrator for dynamics simulation". In: *Future Generation Computer Systems*. Special Issue on Geometric Numerical Algorithms 19.3, pp. 415–424. ISSN: 0167-739X. DOI: 10.1016/S0167-739X(02)00168-1.

Karnik, T., D.G. Saab, S.M. Kang, Y.K. Lee, and K.H. Kim (Sept. 1994). "Hierarchical mixed-level simulation of VHDL descriptions". In: *Proceedings Seventh Annual IEEE Inter-*

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*national ASIC Conference and Exhibit*, pp. 170–173. DOI: 10.1109/ASIC.1994.404583.

Karsai, Gabor and Janos Sztipanovits (2008). “Model-Integrated Development of Cyber-Physical Systems”. en. In: *Software Technologies for Embedded and Ubiquitous Systems*. Ed. by Uwe Brinkschulte, Tony Givargis, and Stefano Russo. Lecture Notes in Computer Science. Springer Berlin Heidelberg, pp. 46–54. ISBN: 978-3-540-87785-1.

Kübler, R. and W. Schiehlen (Aug. 2000a). “Modular Simulation in Multibody System Dynamics”. en. In: *Multibody System Dynamics* 4.2-3, pp. 107–127. ISSN: 1384-5640, 1573-272X. DOI: 10.1023/A:1009810318420.

Kübler, R. and W. Schiehlen (2000b). “Two Methods of Simulator Coupling”. In: *Mathematical and Computer Modelling of Dynamical Systems* 6.2, pp. 93–113. DOI: 10.1076/1387-3954(200006)6:2;1-M;FT093.

Kernighan, Brian W. and Dennis M. Ritchie (1978). *The C programming language*. Prentice-Hall software series. Englewood Cliffs, N.J: Prentice-Hall. ISBN: 978-0-13-110163-0.

Knorr, Stephanie (Oct. 2002). “Multirate-Verfahren in der Co-Simulation gekoppelter dynamischer Systeme mit Anwendung in der Fahrzeugdynamik”. de. Diploma Thesis. Ulm, Germany: Universität Ulm. URL: http://www-num.math.uni-wuppertal.de/fileadmin/mathe/www-num/theses/da_knorr.pdf (visited on 10/03/2021).

Kofman, Ernesto and Sergio Junco (2000). “Quantized-State Systems: A DEVS Approach for Continuous System Simulation”. en. In: *TRANSACTIONS of The Society for Modeling and Simulation International* 18.1, p. 10. ISSN: 0740-6797/01. URL: https://www.fceia.unr.edu.ar/~kofman/files/qss.pdf (visited on 10/05/2021).

Kotler, Philip, Roland Berger, and Nils Bickhoff (2016). *The Quintessence of Strategic Management*. Quintessence Series. Berlin, Heidelberg: Springer Berlin Heidelberg. ISBN: 978-3-662-48489-0 978-3-662-48490-6. DOI: 10.1007/978-3-662-48490-6.

Kvaernø, Anne and Peter Rentrop (1999). *Low Order Multirate Runge-Kutta Methods in Electric Circuit Simulation*. en. URL: https://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.9.3084 (visited on 10/05/2021).

Labella, Thomas Halva, Isabel Dietrich, and Falko Dressler (Aug. 2010). “Hybrid simulation of Sensor and Actor Networks with BARAKA”. en. In: *Wireless Networks* 16.6, pp. 1525–1539. ISSN: 1022-0038, 1572-8196. DOI: 10.1007/s11276-008-0134-1.

Larsson, Jonas and Petter Krus (Nov. 2003). “Stability Analysis of Coupled Simulation”. In: ASME International Mechanical Engineering Congress and Exposition Dynamic Systems and Control, Volumes 1 and 2, pp. 861–868. DOI: 10.1115/IMECE2003-41192.

Lättilä, Lauri, Per Hilletofth, and Bishan Lin" (2010). “Hybrid simulation models – When, Why, How?” In: *Expert Systems with Applications* 37.12, pp. 7969 –7975. ISSN: 0957-4174. DOI: http://dx.doi.org/10.1016/j.eswa.2010.04.039.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Le Guernic, Paul, Thierry Gautier, Michel Le Borgne, and Claude Le Maire (1991). “Programming Real-Time Applications with Signal”. In: *Proceedings of the IEEE*. Another look at Real-time programming 79.9, pp. 1321–1336. DOI: 10.1109/5.97301.

Leimkuhler, Ben and Sebastian Reich (2001). “A Reversible Averaging Integrator for Multiple Time-Scale Dynamics”. In: *Journal of Computational Physics* 171.1, pp. 95 –114. ISSN: 0021-9991. DOI: http://dx.doi.org/10.1006/jcph.2001.6774.

Lelarasmee, E., A.E. Ruehli, and A.L. Sangiovanni-Vincentelli (July 1982). “The Waveform Relaxation Method for Time-Domain Analysis of Large Scale Integrated Circuits”. en. In: *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems* 1.3, pp. 131–145. ISSN: 0278-0070. DOI: 10.1109/TCAD.1982.1270004.

Lelarasmee, Ekachai (1982). “The Waveform Relaxation Method for Time Domain Analysis of Large Scale Integrated Circuits: Theory and Applications”. PhD thesis. EECS Department, University of California, Berkeley. URL: http://www.eecs.berkeley.edu/Pubs/TechRpts/1982/9614.html (visited on 10/03/2021).

Li, W., A. Monti, M. Luo, and Roger A. Dougal (2011a). “VPNET: A co-simulation framework for analyzing communication channel effects on power systems”. In: *2011 IEEE Electric Ship Technologies Symposium*, pp. 143–149. DOI: 10.1109/ESTS.2011.5770857.

Li, Weilin, Huimin Li, and Antonello Monti (2011b). “Using Co-simulation Method to Analyze the Communication Delay Impact in Agent-Based Wide Area Power System Stabilizing Control”. In: *ISMC 2011, International Simulation Multi-conference, The Hague, Netherlands 27-29 June*. URL: https://publications.rwth-aachen.de/record/127882 (visited on 10/10/2021).

Li, Weilin, Xiaobin Zhang, and Huimin Li (Feb. 2014). “Co-simulation platforms for co-design of networked control systems: An overview”. In: *Control Engineering Practice* 23, pp. 44–56. ISSN: 0967-0661. DOI: 10.1016/j.conengprac.2013.10.010.

Liang, Silü, HeMing Zhang, and HongWei Wang (2011). “Combinative Algorithms for the Multidisciplinary Collaborative Simulation of Complex Mechatronic Products Based on Major Step and Convergent Integration Step”. en. In: *Chinese Journal of Mechanical Engineering* 24.03, p. 355. ISSN: 1000-9345. DOI: 10.3901/CJME.2011.03.355.

Lin, Hua, Santhosh S. Veda, Sandeep S. Shukla, Lamine Mili, and James Thorp (2012). “GECO: Global Event-Driven Co-Simulation Framework for Interconnected Power System and Communication Network”. In: *IEEE Transactions on Smart Grid* 3.3, pp. 1444–1456. DOI: 10.1109/TSG.2012.2191805.

Linietsky, Juan, Ariel Manzur, and the Godot community (n.d.). *MultiplayerAPI*. https://docs.godotengine.org/en/stable/classes/class_multiplayerapi.html. (Visited on 09/03/2021).

Maten, J. ter, A. Verhoeven, A. El Guennouni, and Th Beelen (2005). “Multirate hierarchical time integration for electronic circuits”. en. In: *PAMM* 5.1, pp. 819–820. ISSN: 1617-7061. DOI: 10.1002/pamm.200510381.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Matthies, Hermann G., Rainer Niekamp, and Jan Steindorf (2006). “Algorithms for strong coupling procedures”. In: *Computer Methods in Applied Mechanics and Engineering*. Fluid-Structure Interaction 195.17–18, pp. 2028–2049. ISSN: 0045-7825. DOI: 10.1016/j.cma.2004.11.032.

Matthies, Hermann G. and Jan Steindorf (Nov. 2002). “Partitioned but strongly coupled iteration schemes for nonlinear fluid–structure interaction”. In: *Computers & Structures* 80.27–30, pp. 1991–1999. ISSN: 0045-7949. DOI: 10.1016/S0045-7949(02)00259-6.

Matthies, Hermann G. and Jan Steindorf (2003). “Strong Coupling Methods”. en. In: *Analysis and Simulation of Multifield Problems*. Ed. by Professor Wolfgang Wendland and Professor Messoud Efendiev. Lecture Notes in Applied and Computational Mechanics 12. Springer Berlin Heidelberg, pp. 13–36. ISBN: 978-3-642-05633-8 978-3-540-36527-3. DOI: 10.1007/978-3-540-36527-3_2.

Mehlhase, Alexandra (2015). “Konzepte für die Modellierung und Simulation strukturvariabler Modelle”. Doctoral Thesis. Berlin: Technische Universität Berlin, Fakultät IV - Elektrotechnik und Informatik. DOI: 10.14279/depositonce-4514.

Melenk, Markus (2008). *Numerik von Differentialgleichungen*. Vorlesungsskriptum zur LVA Numerik von Differentialgleichungen.

Miekkala, U. and O. Nevanlinna (July 1987). “Convergence of Dynamic Iteration Methods for Initial Value Problems”. In: *SIAM Journal on Scientific and Statistical Computing* 8.4, pp. 459–482. ISSN: 0196-5204. DOI: 10.1137/0908046.

Modelica Association (2014). *Functional Mock-up Interface for Model Exchange and Co-Simulation*. https://svn.modelica.org/fmi/branches/public/specifications/v2.0/FMI_for_ModelExchange_and_CoSimulation_v2.0.pdf. (Visited on 10/19/2017).

Modelica Association (2021). *Functional Mockup Interface*. https://fmi-standard.org/. (Visited on 09/14/2021).

Morvan, Gildas (Nov. 2013). “Multi-level agent-based modeling - A literature survey”. en. In: *arXiv:1205.0561 [cs]*. arXiv: 1205.0561. URL: http://arxiv.org/abs/1205.0561 (visited on 02/29/2020).

Mosterman, Pieter J. (1999). “An Overview of Hybrid Simulation Phenomena and Their Support by Simulation Packages”. en. In: *Hybrid Systems: Computation and Control*. Ed. by Gerhard Goos, Juris Hartmanis, Jan van Leeuwen, Frits W. Vaandrager, and Jan H. van Schuppen. Vol. 1569. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 165–177. ISBN: 978-3-540-65734-7 978-3-540-48983-2. DOI: 10.1007/3-540-48983-5_17.

Mousseau, C. W., T. A. Laursen, M. Lidberg, and R. L. Taylor (Feb. 1999). “Vehicle dynamics simulations with coupled multibody and finite element models”. In: *Finite Elements in Analysis and Design* 31.4, pp. 295–315. ISSN: 0168-874X. DOI: 10.1016/S0168-874X(98)00070-5.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

MTS Systems (2017). *MTS Hybrid Simulation*. https://www.mts.com/en/products/application/civil-engineering/structures/hybrid-simulation/index.htm. (Visited on 09/12/2017).

Mukherjee, Tamal and Gary K. Fedder (July 1999). "Hierarchical Mixed-Domain Circuit Simulation, Synthesis and Extraction Methodology for MEMS". en. In: *Journal of VLSI signal processing systems for signal, image and video technology* 21.3, pp. 233–249. ISSN: 0922-5773. DOI: 10.1023/A:1008122921631.

Murray, Justin Adam, Mehrdad Sasani, and Xiaoyun Shao (Nov. 2015). "Hybrid simulation for system-level structural response". In: *Engineering Structures* 103, pp. 228–238. ISSN: 0141-0296. DOI: 10.1016/j.engstruct.2015.09.018.

Mustafee, N., S. Brailsford, A. Djanatliev, T. Eldabi, M. Kunc, and A. Tolk (2017). "Purpose and benefits of hybrid simulation: Contributing to the convergence of its definition". In: *2017 Winter Simulation Conference (WSC)*, pp. 1631–1645. DOI: 10.1109/WSC.2017.8247903.

Nguyen, Van, Yvon Besanger, Quoc Tran, and Tung Nguyen (Nov. 2017). "On Conceptual Structuration and Coupling Methods of Co-Simulation Frameworks in Cyber-Physical Energy System Validation". en. In: *Energies* 10.12, p. 1977. ISSN: 1996-1073. DOI: 10.3390/en10121977.

Ni, Yunyun and Johannes F. Broenink (Oct. 2012). "Hybrid systems modelling and simulation in DESTECS: a co-simulation approach". In: *26th European Simulation and Modelling Conference, ESM 2012*. Ed. by M. Klumpp. EUROSIS-ETI, pp. 32–36. ISBN: 978-90-77381-73-1.

Nicolescu, Gabriela, Faouzi Bouchhima, and Luiza Gheorghe (2006). "CODIS – A FRAMEWORK FOR CONTINUOUS/DISCRETE SYSTEMS CO-SIMULATION". In: *IFAC Proceedings Volumes* 39.5. 2nd IFAC Conference on Analysis and Design of Hybrid Systems, pp. 274–275. ISSN: 1474-6670. DOI: https://doi.org/10.3182/20060607-3-IT-3902.00052.

Nouidui, Thierry, Michael Wetter, and Wangda Zuo (May 2014). "Functional mock-up unit for co-simulation import in EnergyPlus". In: *Journal of Building Performance Simulation* 7.3, pp. 192–202. ISSN: 1940-1493. DOI: 10.1080/19401493.2013.808265.

Okoli, Chitu and Suzanne D. Pawlowski (Dec. 2004). "The Delphi method as a research tool: an example, design considerations and applications". en. In: *Information & Management* 42.1, pp. 15–29. ISSN: 0378-7206. DOI: 10.1016/j.im.2003.11.002.

Open Compute Project (2020). *OCP Terminology Guidelinesfor Inclusion and Openness*. https://www.opencompute.org/documents/ocp-terminology-guidelines-for-inclusion-and-openness. (Visited on 07/14/2021).

Palensky, Peter, Edmund Widl, and Atiyah Elsheikh (Mar. 2014). "Simulating Cyber-Physical Energy Systems: Challenges, Tools and Methods". In: *IEEE Transactions on Systems*

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

*Man and Cybernetics Part C (Applications and Reviews)* 44, pp. 318 –326. DOI: 10.1109/TSMCC.2013.2265739.

Petegem, W. van, B. Geeraerts, Willy Sansen, and B. Graindourze (Feb. 1994). "Electrothermal simulation and design of integrated circuits". In: *IEEE Journal of Solid-State Circuits* 29.2, pp. 143–146. ISSN: 0018-9200. DOI: 10.1109/4.272120.

Pühringer, Clemens (2017). "Analysis of Coupling Strategies and Protocols for Co-Simulation". en. Diplomarbeit. Wien: TU Wien.

Porter, Joseph, Zsolt Lattmann, Graham Hemingway, Nagabhushan Mahadevan, Sandeep Neema, Harmon Nine, Nicholas Kottenstette, Peter Volgyesi, Gabor Karsai, and Janos Sztipanovits (2009). "The ESMoL Modeling Language and Tools for Synthesizing and Simulating Real-Time Embedded Systems". In: *15th IEEE Real-Time and Embedded Technology and Applications Symposium*. San Francisco, CA.

Powell, Catherine (2003). "The Delphi technique: myths and realities". en. In: *Journal of Advanced Nursing* 41.4, pp. 376–382. ISSN: 1365-2648. DOI: 10.1046/j.1365-2648.2003.02537.x.

Preyser, Franz Josef (2015). "An approach to develop a user friendly way of implementing DEV&DESS models in powerDEVS". en. Thesis. URL: https://repositum.tuwien.at/handle/20.500.12708/9211 (visited on 07/19/2021).

PTV AG (2017). *PTV Vissim*. http://vision-traffic.ptvgroup.com/de/produkte/ptv-vissim/anwendungsfaelle/mesoskopische-und-hybrid-simulation/. (Visited on 09/12/2017).

Quaglia, Davide, Riccardo Muradore, Roberto Bragantini, and Paolo Fiorini (Apr. 2012). "A SystemC/Matlab co-simulation tool for networked control systems". In: *Simulation Modelling Practice and Theory* 23, pp. 71–86. ISSN: 1569-190X. DOI: 10.1016/j.simpat.2012.01.003.

Rathinam, M. and L. Petzold (Jan. 2002). "Dynamic Iteration Using Reduced Order Models: A Method for Simulation of Large Scale Modular Systems". In: *SIAM Journal on Numerical Analysis* 40.4, pp. 1446–1474. ISSN: 0036-1429. DOI: 10.1137/S0036142901390494.

Redis Ltd (n.d.). *Replication*. https://redis.io/topics/replication. (Visited on 09/03/2021).

Rentrop, Peter (Dec. 1985). "Partitioned Runge-Kutta methods with stiffness detection and stepsize control". en. In: *Numerische Mathematik* 47.4, pp. 545–564. ISSN: 0945-3245. DOI: 10.1007/BF01389456.

Ricci, Alessandro, Mirko Viroli, and Andrea Omicini (2007). "Give Agents Their Artifacts: The A&A Approach for Engineering Working Environments in MAS". In: *Proceedings of the 6th International Joint Conference on Autonomous Agents and Multiagent Systems*. AAMAS '07. Honolulu, Hawaii: ACM, 150:1–150:3. ISBN: 978-81-904262-7-5. DOI: 10.1145/1329125.1329308.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Rice, John R. (July 1960). “Split Runge-Kutta method for simultaneous equations”. en. In: *Journal of Research of the National Bureau of Standards Section B Mathematics and Mathematical Physics* 64B.3, p. 151. ISSN: 0022-4340. DOI: 10.6028/jres.064B.018.

Rumsey, F. and J. Watkinson (2004). *Digital Interface Handbook*. ITPro collection. Elsevier/-Focal Press. ISBN: 9780240519098.

Rustin, Cédric, Olivier Verlinden, and Quentin Bombled (2009). “A Cosimulation T-T Procedure Gluing Subsystems in Multibody Dynamics Simulations”. en. In: ASME, pp. 83–92. ISBN: 978-0-7918-4901-9. DOI: 10.1115/DETC2009-86617.

Sachs, Lothar (2004). *Angewandte Statistik*. de. Berlin, Heidelberg: Springer Berlin Heidelberg. ISBN: 978-3-540-40555-9 978-3-662-05744-5. DOI: 10.1007/978-3-662-05744-5.

Sadjina, Severin, Lars T. Kyllingstad, Stian Skjong, and Eilif Pedersen (July 2017). “Energy conservation and power bonds in co-simulations: non-iterative adaptive step size control and error estimation”. en. In: *Engineering with Computers* 33.3, pp. 607–620. ISSN: 0177-0667, 1435-5663. DOI: 10.1007/s00366-016-0492-8.

Sadjina, Severin and Eilif Pedersen (June 2016). “Energy Conservation and Coupling Error Reduction in Non-Iterative Co-Simulations”. In: *arXiv:1606.05168 [cs]*. arXiv: 1606.05168. URL: http://arxiv.org/abs/1606.05168 (visited on 09/10/2019).

SaltStack (2021). *Configuring the Salt Minion*. https://docs.saltproject.io/en/latest/ref/configuration/minion.html. (Visited on 09/03/2021).

Samin, J. C., O. Brüls, J. F. Collard, L. Sass, and P. Fisette (Oct. 2007). “Multiphysics modeling and optimization of mechatronic multibody systems”. en. In: *Multibody System Dynamics* 18.3, pp. 345–373. ISSN: 1573-272X. DOI: 10.1007/s11044-007-9076-0.

Savcenco, V., W. Hundsdorfer, and J. G. Verwer (Mar. 2007). “A multirate time stepping strategy for stiff ordinary differential equations”. en. In: *BIT Numerical Mathematics* 47.1, pp. 137–155. ISSN: 1572-9125. DOI: 10.1007/s10543-006-0095-7.

Savicks, Vitaly, Michael Butler, and John Colley (2014). “Co-simulating event-B and Continuous Models via FMI”. In: *Proceedings of the 2014 Summer Simulation Multiconference*. SummerSim ’14. event-place: Monterey, California. San Diego, CA, USA: Society for Computer Simulation International, 37:1–37:8. URL: http://dl.acm.org/citation.cfm?id=2685617.2685654 (visited on 08/05/2019).

Scattolini, Riccardo (May 2009). “Architectures for distributed and hierarchical Model Predictive Control – A review”. en. In: *Journal of Process Control* 19.5, pp. 723–731. ISSN: 09591524. DOI: 10.1016/j.jprocont.2009.02.003.

Schierz, Tom and Martin Arnold (Sept. 2011). “MODELISAR: Innovative numerische Methoden bei der Kopplung von multidisziplinären Simulationsprogrammen”. en. In: *Tagungsband ASIM-Konferenz STS/GMMS 2011*. ZHAW Winterthur, Schweiz: Shaker. ISBN:

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

978-3-905745-44-3. URL: http://sim.mathematik.uni-halle.de/~arnold/pdf/papers/2011/SchierzArnold11b.pdf (visited on 10/27/2015).

Schierz, Tom and Martin Arnold (Oct. 2012). "Stabilized overlapping modular time integration of coupled differential-algebraic equations". en. In: *Applied Numerical Mathematics* 62.10, pp. 1491–1502. ISSN: 01689274. DOI: 10.1016/j.apnum.2012.06.020.

Schierz, Tom, Martin Arnold, and Christoph Clauß (Nov. 2012). "Co-simulation with communication step size control in an FMI compatible master algorithm". In: *Proceedings of the 9 th International Modelica Conference*. Munich, Germany, pp. 205–214. DOI: 10.3384/ecp12076205.

Schmoll, Robert (2015). "Co-Simulation und Solverkopplung". de. Buch. Kassel Univ. Press. URL: https://kobra.bibliothek.uni-kassel.de/handle/urn:nbn:de:hebis:34-2015110449285 (visited on 06/03/2016).

Schmoll, Robert and Bernhard Schweizer (Dec. 2012). "Convergence Study of Explicit Co-Simulation Approaches with Respect to Subsystem Solver Settings". en. In: *PAMM* 12.1, pp. 81–82. ISSN: 16177061. DOI: 10.1002/pamm.201210032.

Schöps, Sebastian (2008). "Coupling and Simulation of Lumped Electric Circuits Refined by 3-D Magnetoquasistatic Conductor Models Using MNA and FIT". eng. PhD thesis. Wuppertal, Germany: Bergische Universität Wuppertal. URL: https://www-num.math.uni-wuppertal.de/fileadmin/mathe/www-num/theses/ma_schoeps.pdf (visited on 10/10/2021).

Schöps, Sebastian (2011). "Multiscale Modeling and Multirate Time-Integration of Field/Circuit Coupled Problems". en. PhD thesis. Wuppertal: Universität Wuppertal. URL: http://elpub.bib.uni-wuppertal.de/edocs/dokumente/fbc/mathematik/diss2011/schoeps/dc1107.pdf (visited on 10/27/2015).

Schöps, Sebastian (Jan. 2015). "Iterative Schemes for Coupled Multiphysical Problems in Electrical Engineering". In: *IFAC-PapersOnLine*. 8th Vienna International Conferenceon Mathematical Modelling 48.1, pp. 165–167. ISSN: 2405-8963. DOI: 10.1016/j.ifacol.2015.05.174.

Schweiger, G., C. Gomes, G. Engel, I. Hafner, J. Schoeggl, A. Posch, and T. Nouidui (Sept. 2019a). "An empirical survey on co-simulation: Promising standards, challenges and research needs". In: *Simulation Modelling Practice and Theory* 95, pp. 148–163. ISSN: 1569-190X. DOI: 10.1016/j.simpat.2019.05.001.

Schweiger, Gerald, Georg Engel, Josef Schoeggl, Irene Hafner, Cláudio Gomes, and Thierry Nouidui (2018). "Co-Simulation – an Empirical Survey: Applications, Recent Developments and Future Challenges". In: ARGESIM Publisher Vienna, pp. 125–126. ISBN: 978-3-901608-91-9. DOI: 10.11128/arep.55.a55286.

Schweiger, Gerald, Cláudio Gomes, Georg Engel, Irene Hafner, Josef-Peter Schoegg, Alfred Posch, and Thierry Nouidui (Feb. 2019b). "Functional Mock-up Interface: An em-

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

pirical survey identifies research challenges and current barriers". In: pp. 138–146. DOI: 10.3384/ecp18154138.

Schweizer, B. and D. Lu (Sept. 2015). "Predictor/corrector co-simulation approaches for solver coupling with algebraic constraints". en. In: *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik* 95.9, pp. 911–938. ISSN: 1521-4001. DOI: 10.1002/zamm.201300191.

Schweizer, Bernhard, Pu Li, and Daixing Lu (Sept. 2015a). "Explicit and Implicit Cosimulation Methods: Stability and Convergence Analysis for Different Solver Coupling Approaches". en. In: *Journal of Computational and Nonlinear Dynamics* 10.5, p. 051007. ISSN: 1555-1415. DOI: 10.1115/1.4028503.

Schweizer, Bernhard, Pu Li, and Daixing Lu (Aug. 2016). "Implicit co-simulation methods: Stability and convergence analysis for solver coupling approaches with algebraic constraints". en. In: *ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift für Angewandte Mathematik und Mechanik* 96.8, pp. 986–1012. ISSN: 00442267. DOI: 10.1002/zamm.201400087.

Schweizer, Bernhard, Pu Li, Daixing Lu, and Tobias Meyer (June 2015b). "Stabilized implicit co-simulation methods: solver coupling based on constitutive laws". en. In: *Archive of Applied Mechanics* 85.11, pp. 1559–1594. ISSN: 0939-1533, 1432-0681. DOI: 10.1007/s00419-015-0999-2.

Schweizer, Bernhard and Daixing Lu (Aug. 2014a). "Semi-implicit co-simulation approach for solver coupling". en. In: *Archive of Applied Mechanics* 84.12, pp. 1739–1769. ISSN: 0939-1533, 1432-0681. DOI: 10.1007/s00419-014-0883-5.

Schweizer, Bernhard and Daixing Lu (June 2014b). "Stabilized index-2 co-simulation approach for solver coupling with algebraic constraints". en. In: *Multibody System Dynamics* 34.2, pp. 129–161. ISSN: 1384-5640, 1573-272X. DOI: 10.1007/s11044-014-9422-y.

Servat, David, Edith Perrier, Jean-Pierre Treuil, and Alexis Drogoul (1998). "When Agents Emerge from Agents: Introducing Multi-scale Viewpoints in Multi-agent Simulations". en. In: *Multi-Agent Systems and Agent-Based Simulation*. Ed. by Jaime Simão Sichman, Rosaria Conte, and Nigel Gilbert. Lecture Notes in Computer Science. Berlin, Heidelberg: Springer, pp. 183–198. ISBN: 978-3-540-49246-7. DOI: 10.1007/10692956_13.

Sicklinger, S., V. Belsky, B. Engelmann, H. Elmqvist, H. Olsson, R. Wüchner, and K.-U. Bletzinger (May 2014). "Interface Jacobian-based Co-Simulation". en. In: *International Journal for Numerical Methods in Engineering* 98.6, pp. 418–444. ISSN: 1097-0207. DOI: 10.1002/nme.4637.

Sicklinger, Stefan (2014). "Stabilized Co-Simulation of Coupled Problems Including Fields and Signals". PhD thesis. Technische Universität München. DOI: 10.13140/2.1.1103.7762.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Sicklinger, Stefan, Christopher Lerch, Roland Wüchner, and Kai-Uwe Bletzinger (Sept. 2015). "Fully coupled co-simulation of a wind turbine emergency brake maneuver". In: *Journal of Wind Engineering and Industrial Aerodynamics*. Selected papers from the 6th International Symposium on Computational Wind Engineering CWE 2014 144, pp. 134–145. ISSN: 0167-6105. DOI: 10.1016/j.jweia.2015.03.021.

Skelboe, Stig. and Per Ulfkjaer. Andersen (Sept. 1989). "Stability Properties of Backward Euler Multirate Formulas". In: *SIAM Journal on Scientific and Statistical Computing* 10.5, pp. 1000–1009. ISSN: 0196-5204. DOI: 10.1137/0910059.

Smith, R. C. and E. J. Haug (1990). "DADS — Dynamic Analysis and Design System". In: *Multibody Systems Handbook*. Ed. by Werner Schiehlen. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 161–179. ISBN: 978-3-642-50995-7. DOI: 10.1007/978-3-642-50995-7_11.

Spiryagin, Maksym, Scott Simson, Colin Cole, and Ingemar Persson (Mar. 2012). "Co-simulation of a mechatronic system using Gensys and Simulink". In: *Vehicle System Dynamics* 50.3, pp. 495–507. ISSN: 0042-3114. DOI: 10.1080/00423114.2011.598940.

Stecken, Jannis, Kay Lenkenhoff, and Bernd Kuhlenkötter (Jan. 2019). "Classification method for an automated linking of models in the co-simulation of production systems". In: *Procedia CIRP*. 52nd CIRP Conference on Manufacturing Systems (CMS), Ljubljana, Slovenia, June 12-14, 2019 81, pp. 104–109. ISSN: 2212-8271. DOI: 10.1016/j.procir.2019.03.019.

Steinebach, G., S. Rademacher, P. Rentrop, and M. Schulz (July 2004). "Mechanisms of coupling in river flow simulation systems". en. In: *Journal of Computational and Applied Mathematics* 168.1-2, pp. 459–470. ISSN: 03770427. DOI: 10.1016/j.cam.2003.12.008.

Stettinger, Georg, Martin Benedikt, Martin Horn, and Josef Zehetner (July 2015). "Modellbasierte Echtzeit-Co-Simulation: Überblick und praktische Anwendungsbeispiele". de. In: *e & i Elektrotechnik und Informationstechnik* 132.4-5, pp. 207–213. ISSN: 0932-383X, 1613-7620. DOI: 10.1007/s00502-015-0305-6.

Stevens, W.R., B. Fenner, and A.M. Rudoff (2004). *Unix Network Programming: The Sockets Networking Api*. Addison-Wesley Professional Computing Series Bd. 1. Addison-Wesley. ISBN: 9780131411555.

Stevenson, A. (2010). *Oxford Dictionary of English*. Oxford Dictionary of English. OUP Oxford. ISBN: 9780199571123.

Striebel, Michael (Apr. 2006). "Hierarchical Mixed Multirating for Distributed Integration of DAE Network Equations in Chip Design". en. PhD thesis. Bergische Universität Wuppertal. ISBN: 9783183404209.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Striebel, Michael, Andreas Bartel, and Michael Günther (Mar. 2009). “A multirate ROW-scheme for index-1 network equations”. en. In: *Applied Numerical Mathematics* 59.3-4, pp. 800–814. ISSN: 01689274. DOI: 10.1016/j.apnum.2008.03.014.

Swinerd, Chris and Ken R. McNaught (2012). “Desgin Classes for Hybrid Simulations Involving Agent-Based and System Dynamics Models”. Engl. In: *Elsevier* Simulation Modelling Practice and Theory.25, pp. 118–133. DOI: 10.1016/j.simpat.2011.09.002.

Thiede, Sebastian, Malte Schönemann, Denis Kurle, and Christoph Herrmann (Dec. 2016). “Multi-level simulation in manufacturing companies: The water-energy nexus case”. en. In: *Journal of Cleaner Production* 139, pp. 1118–1127. ISSN: 0959-6526. DOI: 10.1016/j.jclepro.2016.08.144.

Thompson, R. and W. Walter (2013). *Ordinary Differential Equations*. Graduate Texts in Mathematics. Springer New York. ISBN: 978-1-4612-0601-9.

Thule, Casper, Cláudio Gomes, Julien Deantoni, Peter Gorm Larsen, Jörg Brauer, and Hans Vangheluwe (2018). “Towards the Verification of Hybrid Co-simulation Algorithms”. In: *Software Technologies: Applications and Foundations*. Ed. by Manuel Mazzara, Iulian Ober, and Gwen Salaün. Vol. 11176. Cham: Springer International Publishing, pp. 5–20. ISBN: 978-3-030-04770-2 978-3-030-04771-9. DOI: 10.1007/978-3-030-04771-9_1.

Thule, Casper, Kenneth Lausdahl, Cláudio Gomes, Gerd Meisl, and Peter Gorm Larsen (Apr. 2019a). “Maestro: The INTO-CPS co-simulation framework”. en. In: *Simulation Modelling Practice and Theory* 92, pp. 45–61. ISSN: 1569190X. DOI: 10.1016/j.simpat.2018.12.005.

Thule, Casper, Maurizio Palmieri, Cláudio Gomes, Kenneth Lausdahl, Hugo Daniel Macedo, Nick Battle, and Peter Gorm Larsen (Sept. 2019b). “Towards Reuse of Synchronization Algorithms in Co-simulation Frameworks”. en. In: *CoSim-CPS-19*. Oslo, Norway, p. 16.

Tomulik, Pawel and Janusz Fraczek (Feb. 2011). “Simulation of multibody systems with the use of coupling techniques: a case study”. en. In: *Multibody System Dynamics* 25.2, pp. 145–165. ISSN: 1384-5640, 1573-272X. DOI: 10.1007/s11044-010-9206-y.

Tong, H., M. Ni, W. Yu, and Y. Li (June 2014). “Reviews and perspectives of hybrid system simulation for power and communication”. In: *The 4th Annual IEEE International Conference on Cyber Technology in Automation, Control and Intelligent*. The 4th Annual IEEE International Conference on Cyber Technology in Automation, Control and Intelligent, pp. 302–306. DOI: 10.1109/CYBER.2014.6917479.

Tripakis, Stavros (July 2015). “Bridging the semantic gap between heterogeneous modeling formalisms and FMI”. en. In: *2015 International Conference on Embedded Computer Systems: Architectures, Modeling, and Simulation (SAMOS)*. Samos, Greece: IEEE, pp. 60–69. ISBN: 978-1-4673-7311-1. DOI: 10.1109/SAMOS.2015.7363660.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Trčka, Marija (Oct. 2008). “Cosimulation for Performance Prediction of Innovative Integrated Mechanical Energy Systems in Buildings”. en. PhD thesis. Eindhoven: Technische Universiteit Eindhoven.

Trčka, Marija, Jan L.M. Hensen, and Michael Wetter (Sept. 2009). “Co-simulation of innovative integrated HVAC systems in buildings”. en. In: *Journal of Building Performance Simulation* 2.3, pp. 209–230. ISSN: 1940-1493, 1940-1507. DOI: 10.1080/19401490903051959.

Trčka, Marija, Michael Wetter, and Jan Hensen (2007). “Comparison of co-simulation approaches for building and HVAC/R system simulation.” In: *Proc. of the 10th IBPSA Conference*. Beijing, China, pp. 1418–1425. URL: http://www.ibpsa.org/proceedings/BS2007/p503_final.pdf (visited on 10/03/2021).

Troch, I. and F. Breitenecker (1990). “Hybride Simulation”. In: *Simulation in der Regelungstechnik*. Ed. by K. H. Fasol and K. Diekmann. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 296–323. ISBN: 978-3-642-84261-0. DOI: 10.1007/978-3-642-84261-0_14.

Tseng, Fan-Chung, Zheng-Dong Ma, and Gregory M. Hulbert (Jan. 2003). “Efficient numerical solution of constrained multibody dynamics systems”. In: *Computer Methods in Applied Mechanics and Engineering* 192.3–4, pp. 439–472. ISSN: 0045-7825. DOI: 10.1016/S0045-7825(02)00521-2.

Tseng, Fang-Chung and GregoryM. Hulbert (2001). “A Gluing Algorithm for Network-Distributed Multibody Dynamics Simulation”. English. In: *Multibody System Dynamics* 6.4, pp. 377–396. ISSN: 1384-5640. DOI: 10.1023/A:1012279120194.

Tseng, F.C. (2000). *Multibody Dynamics Simulation in Network-distributed Environments*. University of Michigan. ISBN: 9780599680395.

Tudoret, Stéphane, Simin Nadjm-Tehrani, Albert Benveniste, and Jan-Erik Strömberg (2000). “Co-Simulation of Hybrid Systems: Signal-Simulink”. In: *Formal Techniques in Real-Time and Fault-Tolerant Systems*. Ed. by Mathai Joseph. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 134–151. ISBN: 978-3-540-45352-9.

Van Mierlo, Simon, Cláudio Gomes, and Hans Vangheluwe (2017). “Explicit Modelling and Synthesis of Debuggers for Hybrid Simulation Languages”. In: *Proceedings of the Symposium on Theory of Modeling & Simulation*. TMS/DEVS ’17. Virginia Beach, Virginia: Society for Computer Simulation International, 4:1–4:12. URL: http://dl.acm.org/citation.cfm?id=3108905.3108909 (visited on 10/03/2021).

Varga, Richard S. (Nov. 1999). *Matrix Iterative Analysis*. en. Springer Science & Business Media. ISBN: 978-3-540-66321-8.

Verhoeven, A., T. G. J. Beelen, A. El Guennouni, E. J. W. ter Maten, R. M. M. Mattheij, and B. Tasić (2006a). “Error analysis of BDF Compound-fast multirate method for differential-algebraic equations”. In: *CASA-Report*. CASA-Report 06.10. ISSN: 0926-4507. URL: https://www.researchgate.net/publication/254810206_Error_analy

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

sis_of_BDF_Compound-fast_multirate_method_for_differential-algebraic_equations (visited on 02/16/2017).

Verhoeven, A., A. El Guennouni, E. J. W. ter Maten, and R. M. M. Mattheij (2006b). "A General Compound Multirate Method for Circuit Simulation Problems". In: *Scientific Computing in Electrical Engineering*. Ed. by Angelo Marcello Anile, Giuseppe Alì, and Giovanni Mascali. Berlin, Heidelberg: Springer Berlin Heidelberg, pp. 143–149. ISBN: 978-3-540-32862-9. DOI: 10.1007/978-3-540-32862-9_21.

Verhoeven, A., E. J. W. Ter Maten, R. M. M. Mattheij, and B. Tasić (June 2007). "Stability analysis of the BDF Slowest-first multirate methods". In: *International Journal of Computer Mathematics* 84.6, pp. 895–923. ISSN: 0020-7160. DOI: 10.1080/00207160701458641.

Verhoeven, A., B. Tasić, T. G. J. Beelen, E. J. W. ter Maten, and R. M. M. Mattheij (2008). "BDF Compound-Fast Multirate Transient Analysis with Adaptive Stepsize Control". In: *Journal of Numerical Analysis, Industrial and Applied Mathematics* 3.3-4, pp. 275–297. ISSN: 1790-8140. URL: https://www.researchgate.net/publication/228627395_BDF_Compound-Fast_Multirate_Transient_Analysis_with_Adaptive_Stepsize_Control1 (visited on 02/08/2017).

Viel, Antoine (Mar. 2014). "Implementing stabilized co-simulation of strongly coupled systems using the Functional Mock-up Interface 2.0". In: pp. 213–223. DOI: 10.3384/ecp14096213.

Völker, Lars (2011). *Untersuchung des Kommunikationsintervalls bei der gekoppelten Simulation*. ger. Karlsruher Schriftenreihe Fahrzeugsystemtechnik Bd. 6. Karlsruhe: KIT Scientific Publ. ISBN: 978-3-86644-611-3.

Wang, Jidong, Jiahui Wu, and Yanbo Che (Aug. 2019). "Agent and system dynamics-based hybrid modeling and simulation for multilateral bidding in electricity market". In: *Energy* 180, pp. 444–456. ISSN: 0360-5442. DOI: 10.1016/j.energy.2019.04.180.

Wang, Jinzhong, Zheng-Dong Ma, and Gregory M. Hulbert (Oct. 2003). "A Gluing Algorithm for Distributed Simulation of Multibody Systems". en. In: *Nonlinear Dynamics* 34.1, pp. 159–188. ISSN: 1573-269X. DOI: 10.1023/B:NODY.0000014558.70434.b0.

Wang, Jinzhong, Zheng-Dong Ma, and Gregory M. Hulbert (2005). "A Distributed Mechanical System Simulation Platform Based on a "Gluing Algorithm"". en. In: *Journal of Computing and Information Science in Engineering* 5.1, p. 71. ISSN: 15309827. DOI: 10.1115/1.1846056.

Wang, Kunpeng, Peer-Olaf Siebers, and Darren Robinson (Jan. 2017). "Towards Generalized Co-simulation of Urban Energy Systems". In: *Procedia Engineering*. Urban Transitions Conference, Shanghai, September 2016 198, pp. 366–374. ISSN: 1877-7058. DOI: 10.1016/j.proeng.2017.07.092.

Wetter, Michael (Sept. 2011). "Co-simulation of building energy and control systems with the Building Controls Virtual Test Bed". en. In: *Journal of Building Performance Simulation*

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

4.3, pp. 185–203. ISSN: 1940-1493, 1940-1507. DOI: 10.1080/19401493.2010.518631.

White, J., F. Odeh, A.L. Sangiovanni-Vincentelli, and A.E. Ruehli (1985). *Waveform Relaxation: Theory and Practice*. Technical Report M85/65. Berkeley: EECS Department, University of California. URL: http://www.eecs.berkeley.edu/Pubs/TechRpts/1985/ERL-85-65.pdf (visited on 07/18/2016).

Widl, E., F. Judex, K. Eder, and P. Palensky (Nov. 2015). "FMI-based co-simulation of hybrid closed-loop control system models". In: *2015 International Conference on Complex Systems Engineering (ICCSE)*. 2015 International Conference on Complex Systems Engineering (ICCSE), pp. 1–6. DOI: 10.1109/ComplexSys.2015.7385981.

Wünsche, S., C. Clauß, P. Schwarz, and F. Winkler (1997a). "Microsystem Design Using Simulator Coupling". In: *Proceedings of the 1997 European Conference on Design and Test*. EDTC '97. Washington, DC, USA: IEEE Computer Society, pp. 113–. ISBN: 978-0-8186-7786-1. URL: http://dl.acm.org/citation.cfm?id=787260.787646 (visited on 12/03/2015).

Wünsche, S., C. Clauss, P. Schwarz, and F. Winkler (Sept. 1997b). "Electro-thermal circuit simulation using simulator coupling". In: *IEEE Transactions on Very Large Scale Integration (VLSI) Systems* 5.3, pp. 277–282. ISSN: 1063-8210. DOI: 10.1109/92.609870.

Yang, Zheng and Burcin Becerik-Gerber (July 2015). "A model calibration framework for simultaneous multi-level building energy simulation". en. In: *Applied Energy* 149, pp. 415–431. ISSN: 0306-2619. DOI: 10.1016/j.apenergy.2015.03.048.

Zeidler, Eberhard, ed. (2013). *Springer-Taschenbuch der Mathematik*. de. Wiesbaden: Springer Fachmedien Wiesbaden. ISBN: 978-3-8351-0123-4 978-3-8348-2359-5. URL: http://link.springer.com/10.1007/978-3-8348-2359-5 (visited on 02/16/2016).

Zeigler, Bernard P (2014). *Object-Oriented Simulation with Hierarchical, Modular Models: Intelligent Agents and Endomorphic Systems.* English. OCLC: 1041189428. Saint Louis: Elsevier Science. ISBN: 978-1-4832-6491-2. URL: http://qut.eblib.com.au/patron/FullRecord.aspx?p=1876958 (visited on 12/02/2019).

Zeigler, Bernard P., Herbert Praehofer, and Tag Gon Kim (2000). *Theory of modeling and simulation: integrating discrete event and continuous complex dynamic systems*. 2nd ed. San Diego: Academic Press. ISBN: 978-0-12-778455-7.

Zhang, Fu, Murali Yeddanapudi, and Pieter J. Mosterman (2008). "Zero-Crossing Location and Detection Algorithms For Hybrid System Simulation". en. In: *IFAC Proceedings Volumes* 41.2, pp. 7967–7972. ISSN: 14746670. DOI: 10.3182/20080706-5-KR-1001.01346.

Zhang, HeMing, SiLü Liang, ShiJi Song, and HongWei Wang (June 2011). "Truncation error calculation based on Richardson extrapolation for variable-step collaborative simulation". en. In: *Science China Information Sciences* 54.6, pp. 1238–1250. ISSN: 1674-733X, 1869-1919. DOI: 10.1007/s11432-011-4274-z.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

Zhang, Zhenkai, Emeka Eyisi, Xenofon Koutsoukos, Joseph Porter, Gabor Karsai, and Janos Sztipanovits (Apr. 2014). “Co-simulation framework for design of time-triggered cyber physical systems”. In: *Simulation Modelling Practice and Theory* 43, 16–33. DOI: 10.1109/ICCPS.2013.6604006.

Zhang, Zhi, Zhonghai Lu, Qiang Chen, Xiaolang Yan, and Li-Rong Zheng (2010). “COSMO: CO-Simulation with MATLAB and OMNeT++ for Indoor Wireless Networks”. In: *2010 IEEE Global Telecommunications Conference GLOBECOM 2010*, pp. 1–6. DOI: 10.1109/GLOCOM.2010.5683583.

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

# Curriculum Vitae

## Personal Information


| | |
|---|---|
| Name | Irene Hafner |
| Address | Sonnseitn 23 |
| | 7432 Oberschützen |
| | Austria |
| email | irene.hafner@tuwien.ac.at |
| Nationality | Austrian |
| Date of Birth | 13 June 1988 |


## Work Experience

| | |
|---|---|
| since 2013 | Employee at dwh GmbH |
| 2010 – 2013 | Tutor, Project Assistant at TU Wien |
| 2009 – 2012 | Tutor in Mathematics at Schülerhilfe Oberwart |
| 2008 and 2009 | Internship at PROFACTOR GmbH |

## Academic Experience and Education

| | |
|---|---|
| since 2013 | Doctoral programme in Engineering Sciences<br>Dissertation field: Technical Mathematics at TU Wien |
| 2012 – 2013 | Project Assistant at TU Wien |
| 2010 – 2013 | Tutor at TU Wien |
| 2010 – 2013 | Master programme Mathematics in Science and Technology at TU Wien |
| 2006 – 2010 | Bachelor programme Mathematics in Science and Technology at TU Wien |
| 1998 – 2006 | Bundesgymnasium Oberschützen |
| 1994 – 1998 | Volksschule Oberschützen |



Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN

## PERSONAL SKILLS AND COMPETENCES

| | |
|---|---|
| Languages | German (native) |
| | English (fluent) |
| | French (basic knowledge) |
| | Latin (basic knowledge) |
| Others | AMSDM Masterschool |
| | Cambridge First Certificate in English |
| | Voluntary youth worker in programs of the local Lutheran parish |

Die approbierte gedruckte Originalversion dieser Dissertation ist an der TU Wien Bibliothek verfügbar.
The approved original version of this doctoral thesis is available in print at TU Wien Bibliothek.
TU Bibliothek Your knowledge hub WIEN