Characterisation of multipartite entanglement beyond the single-copy paradigm

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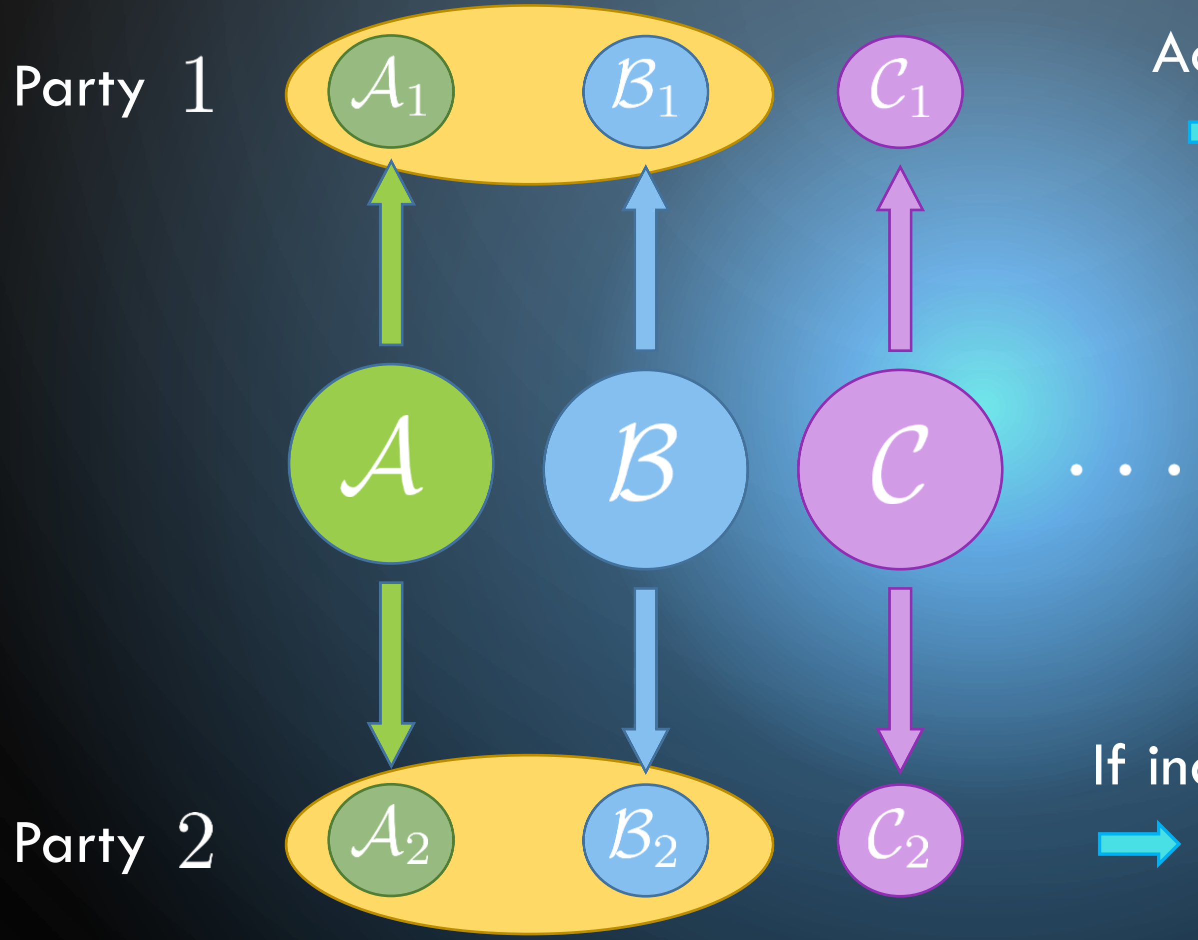
Quantum information processing with multiple copies

Typical scenario in QIP: distribution of entangled pairs $\mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$



Access to Quantum Memories

→ Copies $\mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$ can be processed jointly

If individual copies are entangled*

→ Entanglement distillation

→ Fewer copies of more strongly entangled states

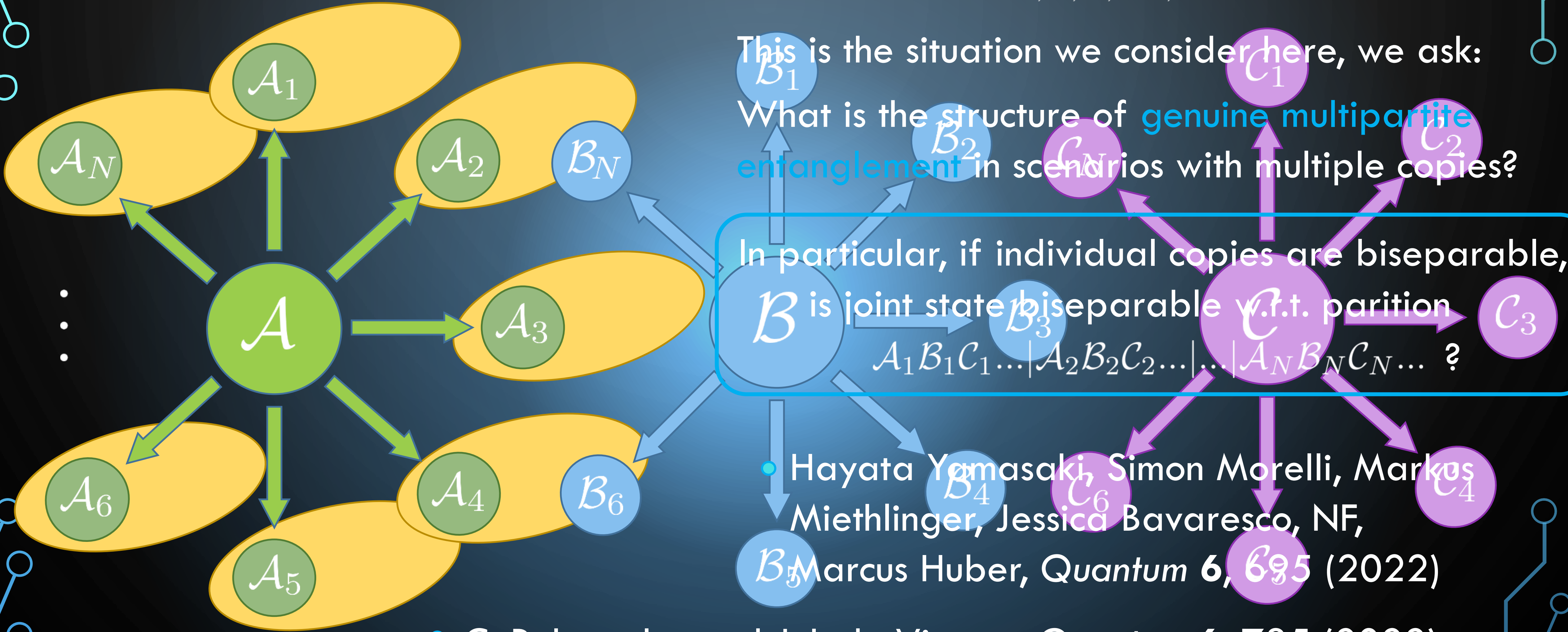
If individual copies are separable

→ joint state is separable w.r.t. biparition $\mathcal{A}_1\mathcal{B}_1\mathcal{C}_1 \ldots | \mathcal{A}_2\mathcal{B}_2\mathcal{C}_2 \ldots$

* But not always possible, $\exists$ undistillable entanglement, in particular, all PPT-entangled states are undistillable

Quantum information processing with multiple copies

But typical scenario in QIP involves multiple parties $1, 2, 3, \ldots, N$



and multiple copies $\mathcal{A}, \mathcal{B}, \mathcal{C}, \ldots$

This is the situation we consider here, we ask:

What is the structure of genuine multipartite entanglement in scenarios with multiple copies?

In particular, if individual copies are biseparable, is joint state biseparable w.r.t. parition $\mathcal{A}_1\mathcal{B}_1\mathcal{C}_1\ldots|\mathcal{A}_2\mathcal{B}_2\mathcal{C}_2\ldots|\ldots|\mathcal{A}_N\mathcal{B}_N\mathcal{C}_N\ldots$?

- Hayata Yamasaki, Simon Morelli, Markus Miethlinger, Jessica Bavaresco, NF, Marcus Huber, *Quantum* **6**, 695 (2022)
- C. Palazuelos and J. I. de Vicente, *Quantum* **6**, 735 (2022)

Genuine Multipartite Entanglement

Pure state $|\Phi^{(k)}\rangle$ separable w.r.t. to k-partition $\mathcal{A}_1|\mathcal{A}_2|\ldots|\mathcal{A}_k$ if

$$|\Phi^{(k)}\rangle = \bigotimes_{i=1}^{k} |\phi_{\mathcal{A}_i}\rangle$$

Mixed state $\rho^{(k)}$ is called k-separable if it can be written as a convex combination of pure states that are separable w.r.t. to some k-partition

$$\rho^{(k)} = \sum_i p_i |\Phi_i^{(k)}\rangle\langle\Phi_i^{(k)}|$$

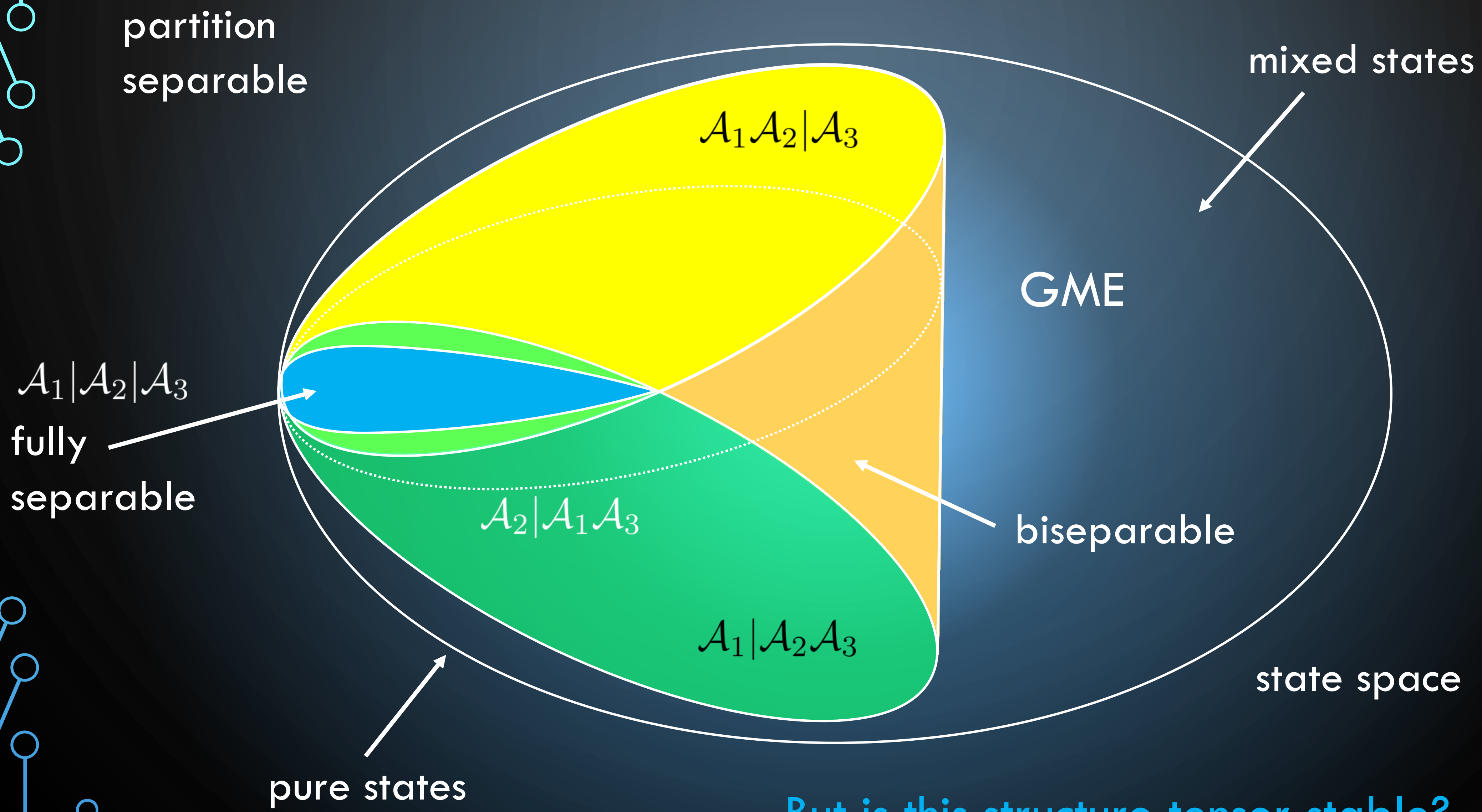
➡ k-separability does not imply separabilty w.r.t. any specific partition except when $\rho^{(k)}$ is pure or when $k = N$ (fully separable)

Here: States that are separable w.r.t. any partition: partition separable

For $k = 2$: biseparable

For $k = 1$: genuinely k-partite entangled (here, just „GME“)

Structure of Genuine Multipartite Entanglement $(N = 3)$



But is this structure tensor-stable?

Activation of Genuine Multipartite Entanglement

Why would it NOT be? Consider tripartite biseparable state

$$\rho_{\text{bisep}} = p\,\rho_{\mathcal{A}_1} \otimes \rho_{\mathcal{A}_2\mathcal{A}_3} + (1-p)\,\rho_{\mathcal{A}_1\mathcal{A}_2} \otimes \rho_{\mathcal{A}_3}$$

If we take two copies: $\mathcal{A}$, $\mathcal{B}$

$$\rho_{\text{bisep}}^{\otimes 2} = p^2\,\rho_{\mathcal{A}_1} \otimes \rho_{\mathcal{A}_2\mathcal{A}_3} \otimes \rho_{\mathcal{B}_1} \otimes \rho_{\mathcal{B}_2\mathcal{B}_3} + (1-p)^2\,\rho_{\mathcal{A}_1\mathcal{A}_2} \otimes \rho_{\mathcal{A}_3} \otimes \rho_{\mathcal{B}_1\mathcal{B}_2} \otimes \rho_{\mathcal{B}_3}$$

$$+\,p(1-p)\left[\rho_{\mathcal{A}_1} \otimes \rho_{\mathcal{A}_2\mathcal{A}_3} \otimes \rho_{\mathcal{B}_1\mathcal{B}_2} \otimes \rho_{\mathcal{B}_3} + \rho_{\mathcal{A}_1\mathcal{A}_2} \otimes \rho_{\mathcal{A}_3} \otimes \rho_{\mathcal{B}_1} \otimes \rho_{\mathcal{B}_2\mathcal{B}_3}\right]$$

→ Some terms not necessarily separable w.r.t. $\mathcal{A}_1|\mathcal{A}_2\mathcal{A}_3$, $\mathcal{A}_2|\mathcal{A}_1\mathcal{A}_3$ or $\mathcal{A}_1\mathcal{A}_2|\mathcal{A}_3$

→ Two copies might be GME

Indeed, example for such 2-copy activatable states have been found before [1]

But is this structure tensor-stable?

[1] Marcus Huber and Martin Plesch, *Phys. Rev. A* **83**, 062321 (2011).

The Tools

Here, we examine GME activation in more detail

Consider one-parameter family of isotropic N-qubit GHZ states

$$\rho(p) = p\,|\mathrm{GHZ}_N\rangle\langle\mathrm{GHZ}_N| + (1-p)\,\frac{1}{2^N}\mathbb{1}_{2^N} \quad \text{with} \quad |\mathrm{GHZ}_N\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right)$$

We can then leverage two results:

(i) For any N-qubit state ρ_X in X-form, a nonzero value of the genuinely multipartite concurrence $C_{\mathrm{GM}}(\rho_X)$ [2] provides a necessary and sufficient GME criterion [3].

(ii) For any two states ρ and σ in $\mathcal{H}$, the Hadamard map $\mathcal{E}_\circ[\rho\otimes\sigma] = \frac{\rho\circ\sigma}{\mathrm{Tr}(\rho\circ\sigma)} \in \mathcal{H}$, can be implemented via SLOCC [4].

[2] Rafsanjani, Huber, Broadbent, and Eberly, *Phys. Rev. A* **86**, 062303 (2012), arXiv:1208.2706.

[3] Ma, Chen, Chen, Spengler, Gabriel, and Huber, *Phys. Rev. A* **83**, 062325 (2011), arXiv:1101.2001.

[4] Lami and Huber, *J. Math. Phys.* **57**, 092201 (2016), arXiv:1603.02158.

The Works

We observe: (1) isotropic N-qubit GHZ states are in X-form

(2) Hadamard (Schur) product preserves this X-form

(3) Hadamard (Schur) product can be implemented via SLOCC

(4) SLOCC cannot create GME from biseparable states

(5) Nonzero GM concurrence detects GME for states in X-form

For one copy: If $C_{\mathrm{GM}}\big(\rho(p)\big) > 0$ ➡ $\rho(p)^{\otimes 2}$ GME ➡ Condition: $p > p^{(1)}_{\mathrm{GME}}(N) := \frac{2^{N-1}-1}{2^N-1}$

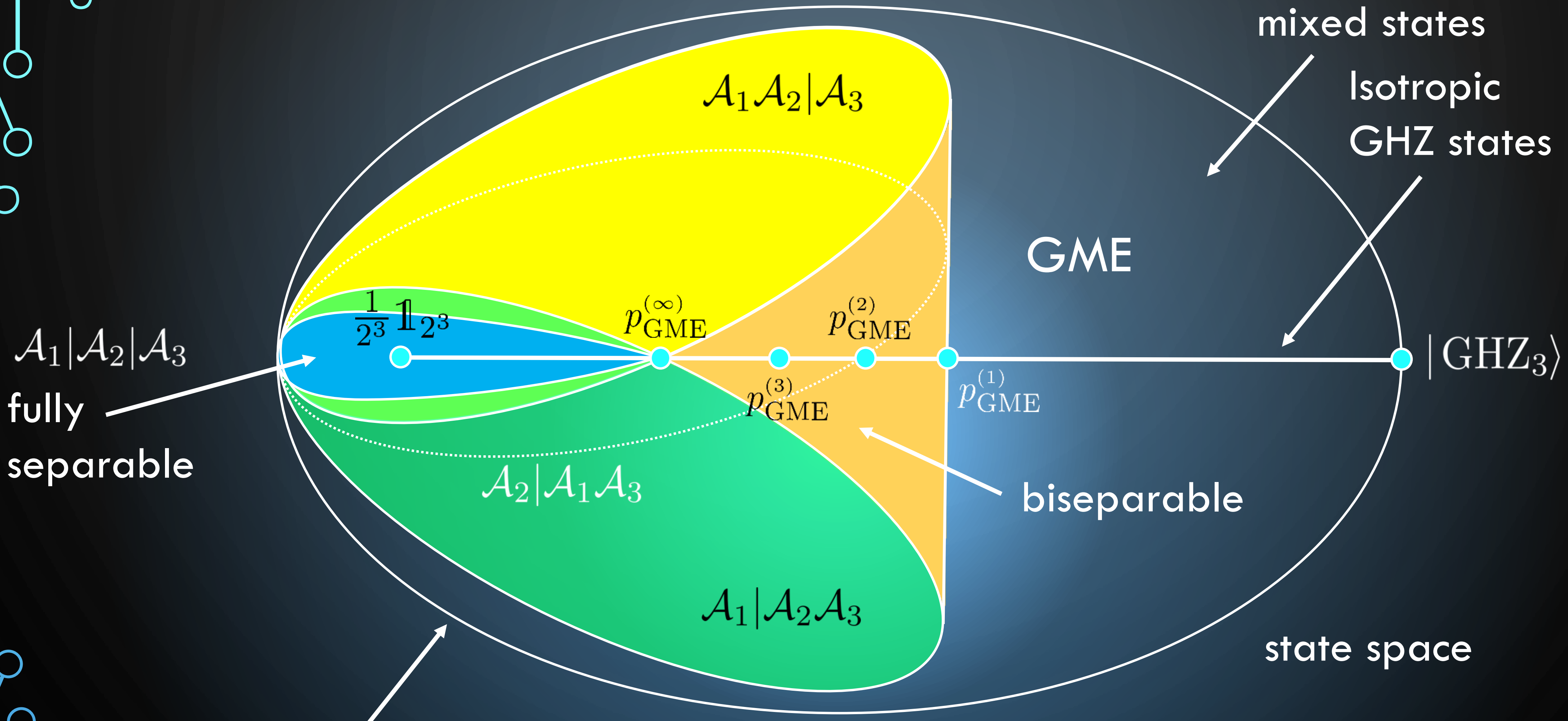
For two copies: $\rho(p)^{\otimes 2} \longmapsto \mathcal{E}_\circ[\rho(p) \otimes \rho(p)]$ If $C_{\mathrm{GM}}\big(\mathcal{E}_\circ[\rho(p)^{\otimes 2}]\big) > 0$ ➡ $\rho(p)^{\otimes 2}$ GME

For k copies: If $C_{\mathrm{GM}}\big(\mathcal{E}_\circ^{\circ(k-1)}[\rho(p)^{\otimes k}]\big) > 0$ ➡ $\rho(p)^{\otimes k}$ GME

➡ Family of k-copy GME thresholds:

$$p > p^{(k)}_{\mathrm{GME}}(N) := \frac{\sqrt[k]{2^{N-1}-1}}{2^{N-1} + \sqrt[k]{2^{N-1}-1}}$$

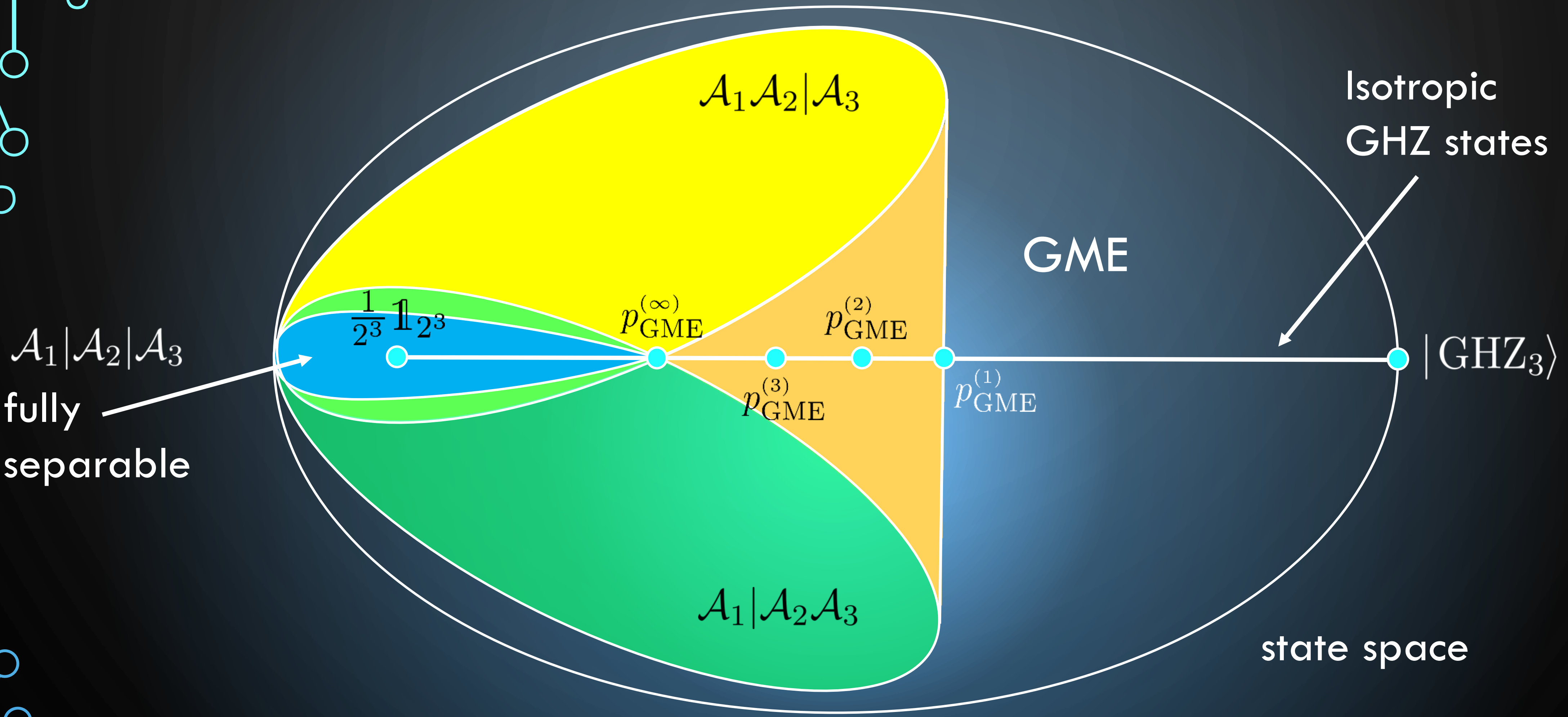
The Results (illustrated for $N = 3$)



We show: (i) All biseparable isotropic N-qubit GHZ states are activatable

(ii) All isotropic 3-qubit GHZ states between $p_{\mathrm{GME}}^{(3)}$ and $p_{\mathrm{GME}}^{(2)}$ require 3 copies for GME activation (2 copies biseparable)

The Results (illustrated for $N = 3$)



Moreover: (iii) We consider a family of biseparable three-qutrit states with no distillable bipartite entanglement across any cut and show that three copies can become GME

The Conjectures

Our results lead us to two main conjectures

(I) There exists a hierarchy of states with k-copy activatable GME, i.e., for all $k \geq 2$ there exists a biseparable but not partition-separable state ρ such that $\rho^{\otimes k-1}$ is biseparable, but $\rho^{\otimes k}$ is GME.

(II) GME may be activated for any biseparable but not partition-separable state of any number of parties as $k \to \infty$.

Confirmation of Conjectures

While we have been battling reviewers and the pandemic…

… our conjectures have been proven by our talented colleagues in Madrid

Carlos Palazuelos

Julio I. de Vicente

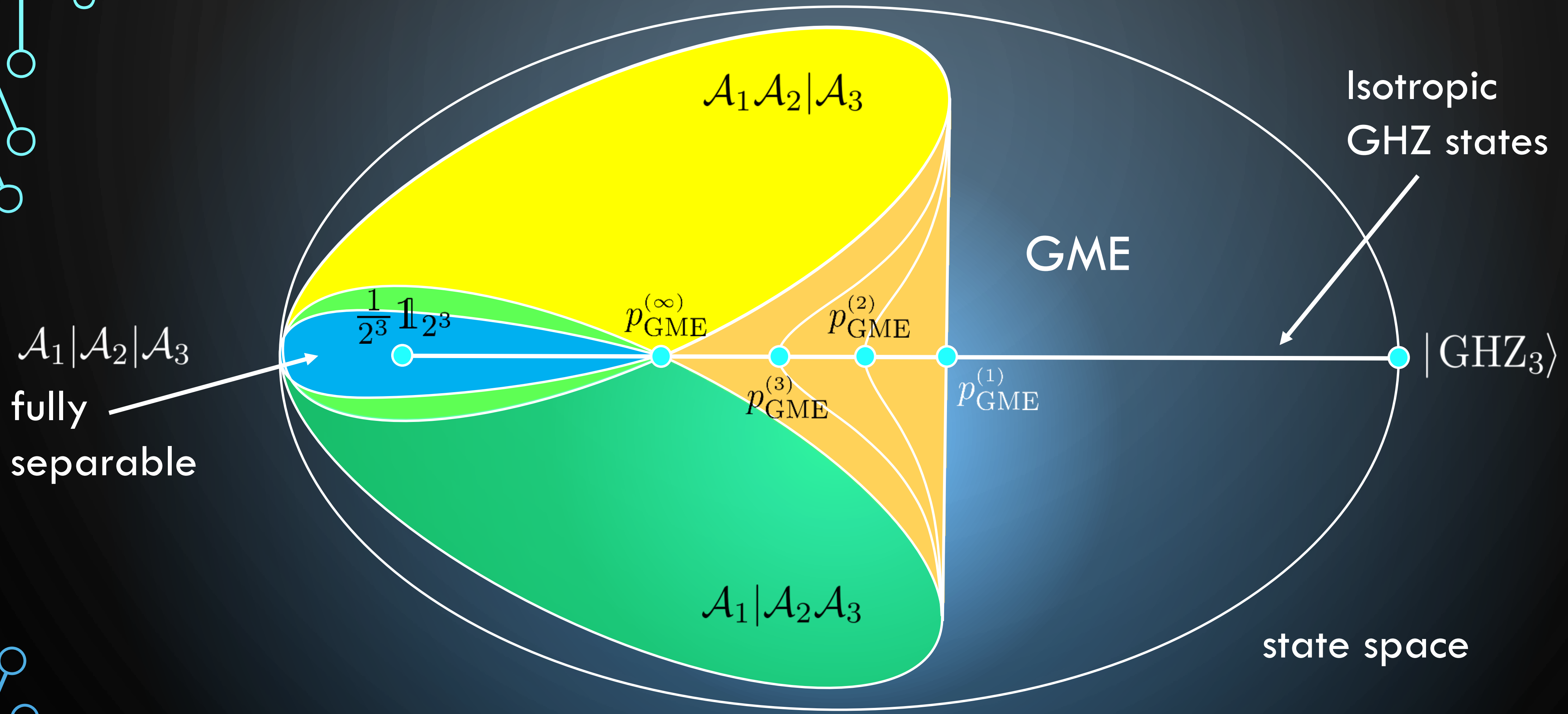
C. Palazuelos and J. I. de Vicente,
Quantum **6**, 735 (2022)

They show that:

(1) An n-partite state is GME-activatable if and only if it is not partition separable.

(2) For any $k \in \mathbb{N}$ and n (with $n > 2$), there exists an n-partite GME-activatable state such that k copies of it are not GME.

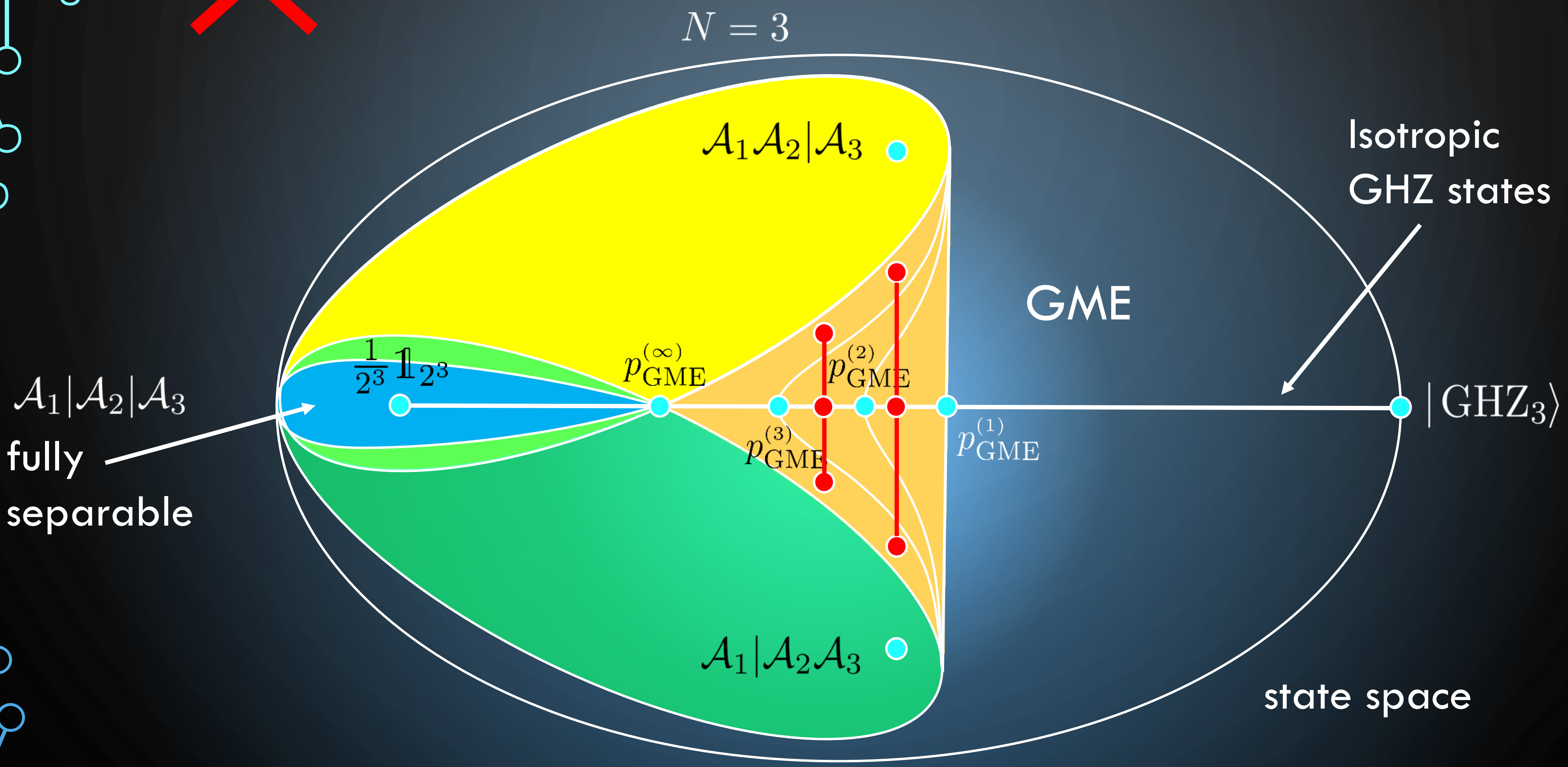
The Results (illustrated for $N = 3$)



This means… a hierarchy of k-copy activation

… but asymptotically (as $k \to \infty$) a breakdown of hierarchy of genuinely n-partite entangled states (everything either partition-separable or GME)

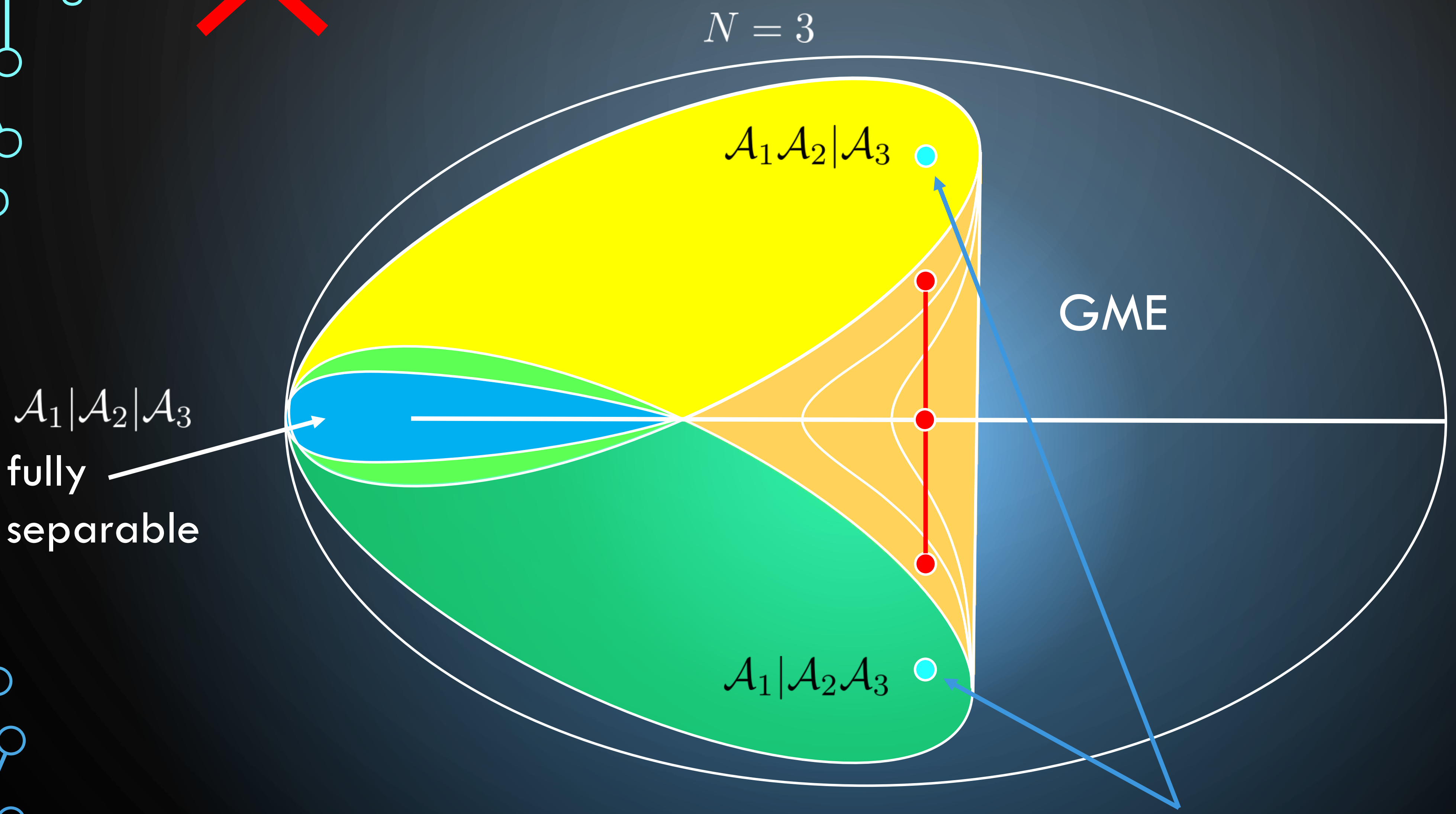
Convexity & Shared Randomness in Multi-Copy Scenarios



Shared randomness: incoherent mixture of k-copy activatable states

➡ $(k-k')$-copy activatable state for $k' < k$

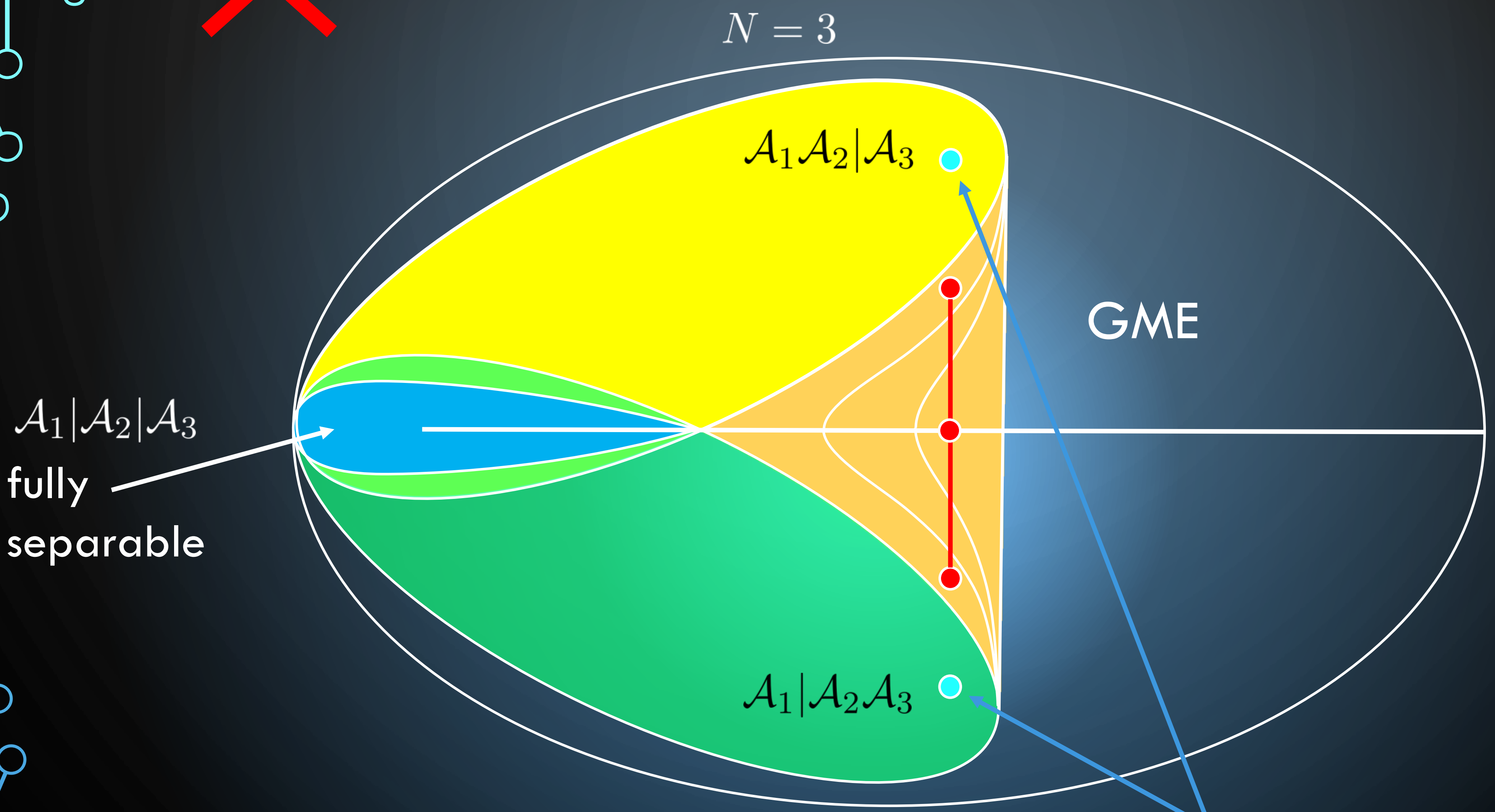
Convexity & Shared Randomness in Multi-Copy Scenarios



Perhaps not so shocking: If given two systems in partition-separable states

$\rho_{\mathcal{A}_1\mathcal{A}_2|\mathcal{A}_3}$ and $\rho_{\mathcal{A}_1|\mathcal{A}_2\mathcal{A}_3}$ ➡ $\rho_{\mathcal{A}_1\mathcal{A}_2|\mathcal{A}_3} \otimes \rho_{\mathcal{A}_1|\mathcal{A}_2\mathcal{A}_3}$ is GME

Convexity & Shared Randomness in Multi-Copy Scenarios



"Forgetting" which system is in which state ➡ 2 copies of incoherent mixture which might now be 2-copy activatable ➡ no net gain in GME via randomness

Collaborators on this project

now Ass. Prof. in Tokyo

Hayata Yamasaki

now Postdoc in Bilbao

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Jessica Bavaresco

Nicolai Friis

Marcus Huber

Activation of genuine multipartite entanglement:
beyond the single-copy paradigm of entanglement characterisation
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