

# Using Stochastic Geometry and Sequential Spatial Point Process Model for Estimation of Stand Density Based on ALS-ITD.

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## 1. Introduction

In individual tree detection (ITD), trees are delineated from a pre-processed ALS point cloud. A problem in ITD is that all trees are not visible when forest is seen from above. Here we focus on estimation of the hidden trees. Mehtätalo (2006) proposed a Horvitz-Thompson-like estimator for stand density ( $N$ , trees per ha) in ITD. The estimator was based on a sequential construction of the detected crown discs. The crown discs are ordered with respect to their diameter from the largest to the smallest. The largest tree is observed for sure. The other trees are observed from above only if they are not hidden below the crowns of the larger trees. Therefore, the probability to detect the  $i$ :th largest detected tree with radius  $r_i$  was computed by using the union of larger crown segments. Especially, it was assumed that the tree is not detected if the center point of the crown is within the union of larger tree crowns.

Kansanen et al. (2016, 2019) generalized the detection condition by defining a set  $A_i$ , which was obtained by removing a buffer of width  $\alpha r_i$  from the union of larger crown segments. Here  $\alpha$  is a tuning parameter that can be estimated either based on the properties of the applied ITD algorithm or empirically by using error of stand density as a criterion. If tree locations follow complete spatial randomness, the detection probability for tree  $i$  can be approximated as the proportion of the whole plot area that was not covered by  $A_i$ ,

$$\hat{\pi}_i = 1 - \frac{|A_i \cap W|}{|W|}, \quad (1)$$

where  $W$  is the sample plot of interest. After computing the detection probabilities, stand density (trees per plot) was estimated using a Horvitz-Thompson-like estimator

$$\hat{N} = \sum \frac{1}{\hat{\pi}_i} = N_{\text{detected}} + \sum \left( \frac{1}{\hat{\pi}_i} - 1 \right), \quad (2)$$

where the summation runs from 1 to the detected tree count. The latter form explicitly shows that the estimator is a sum of detected tree count  $N_{\text{detected}}$  and an estimated hidden tree count  $\sum \left( \frac{1}{\hat{\pi}_i} - 1 \right)$ .

An implicit assumption in the above constructions is that the density of trees of a given size in the parts that are not visible to the scanner is similar to the density in the visible parts. This assumption may not be realistic because the parts that are hidden provide worse growing conditions for small trees than the visible parts. This problem is implicitly taken into account if the tuning parameter  $\alpha$  is estimated empirically so that the estimated stand densities match with the field measurements. However, a better solution might be to model the variability in stand density explicitly.

A finite sequential spatial point process model (SSPP) was recently proposed for forest data in Yazigi et al (2021). A similar model has been previously applied with eye movement data (Penttinen and Ylitalo 2016). In SSPP, the points are ordered according to the time and the locations of the latter points are affected by the earlier points. In the forestry context, it is realistic to assume that the order in terms of

tree age is well approximated by the order of trees in terms of size. The model is based on an accept-reject construction as follows. The first tree location is generated uniformly at random. For each of the latter trees, a uniform location is proposed. The proposal is accepted with probability  $\theta$ , ( $0 < \theta < 1$ ) if the proposed location is within a  $r$ -radius neighbourhood of at least one of the previous locations, and with probability  $1 - \theta$  if the proposed location is not within any neighbourhood. The acceptance probability  $\theta$  and interaction radius  $r$  are parameters of the model, which were estimated using maximum likelihood. The model allows a wide range of spatial point patterns. Especially  $\theta = 1/2$ , leads to complete spatial randomness,  $\theta < 1/2$  a regular tree pattern and  $\theta > 1/2$  a clustered pattern.

If the parameter  $r$  is selected so that it corresponds to the applied detection condition, the ratio  $\frac{\theta}{1-\theta}$  describes the ratio of stand densities in the hidden and visible parts of the forest in the estimation method of Kansanen et al (2016, 2019). However, application of the model requires an estimate of  $\theta$  be available. It might be possible to model parameter  $\theta$  empirically using such predictors based on the ALS data which include information of the spatial pattern of tree locations, see Häbel et al. (2021).

In this paper, we apply the model of Yazigi et al. (2021) in the context of the method of Kansanen et al. (2016) to allow different stand densities in the hidden and visible parts of the plot. The developed model is evaluated with empirical data and compared with the previous methods.

## 2. Data and methods

A data set of 111 square fixed-area plots of size 30 by 30 meters from Liperi, North Carelia, Finland is used. The number of stems, quadratic mean diameter and basal area of the plots varied within 211-3900 trees per ha, 8.2-37.2 cm, and 7.84-46.19 m<sup>2</sup>/ha, respectively. 39% of the plots were dominated by Scots pine, 43% by Norway spruce, and 18% by birch species. All trees were measured for location, diameter (DBH) and height. ALS data were acquired using an Optech Titan instrument on July 2-10, 2016. Although this scanner operates in three wavelengths, we used only the 1064 nm channel in this study. The scanning altitude was 850 meters, the scanning half angle 20 degrees, pulse repetition frequency 250 kHz and sampling density 4.8 pulses per m<sup>2</sup>.

We assume that a tree is detected if the center point of the crown is not within the union of larger tree crowns so that the detectability is given by Equation (1) where  $A = \bigcup_{j=1}^{i-1} C_j$  and  $C_j$  is the crown disc of tree  $j$ . However, we assume that for each tree  $i$ , the density in the hidden part relative to the density in the visible part is  $\frac{\theta}{1-\theta}$ . Therefore, we propose here the following new estimator for the stand density:

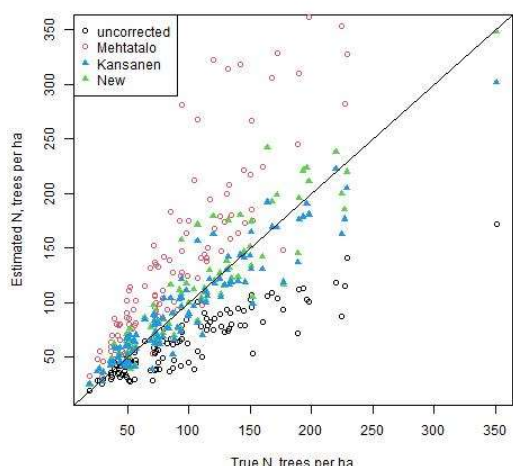
$$\hat{N} = N_{\text{detected}} + \frac{\theta}{1-\theta} \sum \left( \frac{1}{\hat{\pi}_i} - 1 \right). \quad (3)$$

Notice that the estimator provides weights for every tree, so extension to estimation of other population characteristics (such as basal area) is straightforward.

The estimator was evaluated by using an analysis with the following steps:

1. Individual tree detection was conducted using a method that provided estimates of tree locations and (maximum) tree crown radius (Lähivaara et al. 2014).
2. The detected trees were matched with field-measured trees with a manual procedure. A linear mixed-effect model (see e.g. Mehtätalo and Lappi 2020) was fitted to model the crown width (based on ALS) on the field-measured tree DBH. The model was used to predict crown radius for all field-measured trees.
3. Using the predicted crown radii from step (2) as a known interaction radii, the “true” value of  $\theta$  of in the SSPP model (Yazigi et al. 2021) was estimated separately for each plot.
4. The estimate of  $\theta$  was modeled using different plot-level characteristics proposed by Häbel et al. (2021). Prediction based on that model was used in step 5.
5. Stand density was estimated for each plot using the estimator (3).

For comparison, the estimator of Kansanen et al. (2016) was also implemented. The tuning parameter  $\alpha$  was estimated by finding for each sample plot such a value of  $\alpha$  that provided minimum difference between the field-measured  $N$  and ALS-estimated  $N$ . The estimates were modeled using the same predictors as used in step 4 above and predicted for all plots for a procedure comparable with steps 4 and 5 above. Estimation using  $\alpha = 0$  based on Mehtätalo (2006) was also implemented.



**Figure 1.** The estimated stand density using the four applied methods against field measurement

new method. The new method was comparable with that of Kansanen (2016) with respect to bias, but slightly worse in terms of RMSE. Figure 1 shows the plot-specific estimates of different methods against the true stand density.

The presented results are only tentative. For example, a leave-one-out cross-validation should be implemented to confirm the findings. In addition, the applied correction method uses either  $\theta$  or  $\alpha$  in estimation, but also a method that utilizes both of them is possible and of interest. Furthermore, instead of the MLE of  $\theta$  in step 4, an estimate based on similar  $N$ -matching could be used as we used for  $\alpha$ .

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**Table 1.** The relative bias and RMSE of the four applied methods.

Method	bias, %	RMSE, %
uncorrected	-35	48.8
Mehtatalo	57	80.9
Kansanen	-3.5	20.0
New	3.5	22.5

## 3. Results and discussion

From among the variables proposed by Häbel et al (2021) the total number of distinct patches in the CHM at 80% height of the total height and integrated deviation of the F-function from the theoretical reference were the best predictors of both  $\theta$  and  $\alpha$ . The R-square was 16% in the model of  $\theta$  and 10% in the model of  $\alpha$ .

Table 1 shows the relative bias and RMSE of the