Controllability of PDEs with hysteresis

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We focus on the problem of controllability of the equation

$$u_t(x,t) - \Delta u(x,t) + \mathcal{F}[u](x,t) = v(x,t), \quad x \in \Omega \subset \mathbb{R}^N,\ t \in (0,T), \tag{1}$$

with a hysteresis operator $\mathcal{F}$, a right-hand side v called the *control*, and given initial and boundary conditions. The controllability problem for equation (1) consists in proving that for an arbitrary initial condition, an arbitrary final time T, and an arbitrary admissible final state $\bar{u}(x)$, it is possible to choose the control v in such a way that the solution satisfies $u(x,T) = \bar{u}(x)$.

In [1] we propose a constructive method based on a two-parameter penalty argument. One small parameter penalizes the distance of the solution at final time from the expected value, whereas the second one is used to approximate the underlying rate independent variational inequalities in the hysteresis term by smooth viscous constitutive relations. Our main result is the construction of a control algorithm based on passing to the limit in the two small parameters.

The motivation for considering equations with hysteresis of the form (1) comes from models for phase transition in a two-phase system. In particular our result can be applied to prove that the configuration of the so-called *relaxed Stefan problem*, which models the phase transition in solid-liquid systems, can be made constant in finite time.

References

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