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Lensing with Generalized Symmetrons

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Abstract: Generalized symmetrons are models that have qualitatively similar features to the archetypal symmetron, but have barely been studied. In this article, we investigate for what parameter values the fifth forces induced by disformally coupling generalized symmetrons can provide an explanation for the difference between baryonic and lens masses of galaxies. While it is known that the standard symmetron struggles to provide an alternative source for the lensing otherwise attributed to particle dark matter, we show that some generalized symmetron models are more suitable for complying with existing constraints on disformal couplings. This motivates future studies of these only little-explored models.

Keywords: screened scalar fields; symmetrons; dark matter; lensing

1. Introduction

Some of modern physics' most prominent open problems can be found in cosmology, i.e., the questions about the natures of dark energy (DE) and dark matter (DM). In order to tackle these problems, a range of modifications of general relativity have been considered. Amongst those, scalar-tensor theories [1] are some of the most studied. An overview of models that address the problems of DE and DM can be found in Refs. [2,3].

Some of the scalar fields appearing in scalar-tensor theories are expected to cause a fifth force of Nature, which, however, is in tension with Solar System-based experiments [4–6]. This led to some of these models being already ruled out [7]. Despite this, so-called screened scalar fields, see Refs. [8,9] for reviews, have ways of circumventing Solar System constraints by screening their fifth forces in environments of higher mass densities, such that the forces are effectively rendered feeble. The most well-known screened scalar field models include the chameleon [10,11], the symmetron [12–19], the environment dependent dilaton [14,17,20–24], and the galileon [25–27]. In recent years these models have been (proposed to be) tested in various experiments, see, e.g., Refs. [8,28–44], studied as quantum fields [45–48], and proposed for investigations in analog gravity simulations [49].

Screened scalar field models have a rich phenomenology, which allows them to serve as promising candidate theories for DM or DE. For example, in Ref. [50], it was shown that the symmetron fifth force could explain the stability and rotation curves of disk galaxies, while Ref. [51] demonstrated how the same force could lead to the observed motion of stars perpendicular to the plane of the Milky Way disk. This made the symmetron a promising alternative to particle DM. However, in Refs. [48,52] it was shown that using the same parameter values as in Ref. [50], the difference between the baryonic masses of galaxies and the masses required for causing the observed (gravitational) lensing cannot be explained by a symmetron fifth force alone, but potentially by adding another scalar field or considering a hybrid model between modified gravity and particle DM.

In the present article, we revisit the idea of a symmetron fifth force being an alternative to particle DM. However, instead of only considering the archetypal symmetron, we discuss so-called generalized symmetrons, which were discovered using tomographic methods [53,54]. This class of models comprises the standard symmetron, but also an, in principle, infinite number of qualitatively similar generalizations. Generalized symmetrons

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have barely been studied in the literature [8], and their parameter spaces are still unconstrained. As we show in the present article, some generalized symmetron models can actually explain the observed lensing otherwise attributed to particle DM while complying with existing constraints [55,56] on the necessary disformal coupling [57]. Since the parameter spaces of those models have not yet been constrained by experiments, they offer significantly more freedom to discuss them as alternatives to particle DM than many other screened scalar field models, including the standard symmetron. This motivates future studies of these yet only little explored models.

The article is structured as follows: In Section 2 we review the generalized symmetron models, while in Section 3, based on the discussion in Ref. [48], we derive the deviation of a galaxy's lensing from general relativity's predictions due the presence of a generalized symmetron fifth force. Subsequently, in Section 4, we study which parts of the generalized symmetron parameter spaces are suitable for explaining the difference between baryonic and lens masses of galaxies. Finally, we draw our conclusions in Section 5.

2. Generalized Symmetrons

The standard symmetron model is a scalar-tensor modification of gravity and introduces a new type of scalar field—the symmetron. It was first mentioned in Refs. [12–17] and then introduced with its current name in Refs. [18,19]. Besides being a potential DM candidate, the symmetron also motivated a new inflationary scenario [58] and was proposed as a solution to the H_0-tension [59].

The symmetron φ is described by the action

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R - \frac{1}{2}(\partial\varphi)^2 - V(\varphi)\right) , \quad (1)$$

where M_P is the reduced Planck mass, and the effective symmetron potential is given by

$$V(\varphi) = \frac{1}{2}\left(\frac{\rho}{\mathcal{M}^2} - \mu^2\right)\varphi^2 + \frac{\lambda}{4}\varphi^4 . \quad (2)$$

Here, $\rho = -T^\mu{}_\mu$ is the density for dust-like matter, $\mathcal{M}$ is a constant with a dimension of a mass that parametrizes the coupling to matter, μ is a tachyonic mass, and λ is the dimensionless self-coupling constant of the symmetron. The universal coupling to the trace of the matter energy-momentum tensor $T^\mu{}_\mu$ arises since the symmetron couples conformally to the metric tensor via a factor

$$A(\varphi) = 1 + \frac{\varphi^2}{\mathcal{M}^2} + \mathcal{O}\left(\frac{\varphi^4}{\mathcal{M}^4}\right) , \ \varphi \ll \mathcal{M} . \quad (3)$$

This factor is used in order to translate from one conformal frame [1] to another, e.g., from the Einstein frame given in terms of the metric g to the Jordan frame given in terms of $\tilde{g}$ via $\tilde{g}_{\mu\nu} = A(\varphi)g_{\mu\nu}$.

Due to its coupling to matter, an additional force of Nature, a so-called fifth force, is expected to be mediated by the symmetron. However, in order to avoid Solar System constraints on fifth forces [4–6], the symmetron is equipped with a screening mechanism, which renders the fifth force feeble in regions where the trace of the energy-momentum tensor, or, in case of dust, the density ρ, is sufficiently large. This screening is achieved by the behavior of the symmetron's non-linear effective potential given in Equation (2). As long as $\mu^2\mathcal{M}^2 > \rho$, this potential has minima at

$$\varphi_0 = \pm\sqrt{\frac{1}{\lambda}\left(\mu^2 - \frac{\rho}{\mathcal{M}^2}\right)} . \quad (4)$$

However, if this condition is not fulfilled, the vacuum expectation value (vev) of the symmetron can only be $\varphi_0 = 0$. Both possible scenarios are depicted in Figure 1. Separating the symmetron into its vev and a small fluctuation $\delta\varphi$, such that $\varphi = \varphi_0 + \delta\varphi$, it can be shown that the symmetron fifth force at leading order follows $F_\varphi \sim \varphi_0 \nabla \delta\varphi$. Consequently, in situations where $\mu^2 \mathcal{M}^2 < \rho$, the symmetron-induced fifth force is screened since its leading term vanishes, leaving only small corrections of higher order in $\delta\varphi$.

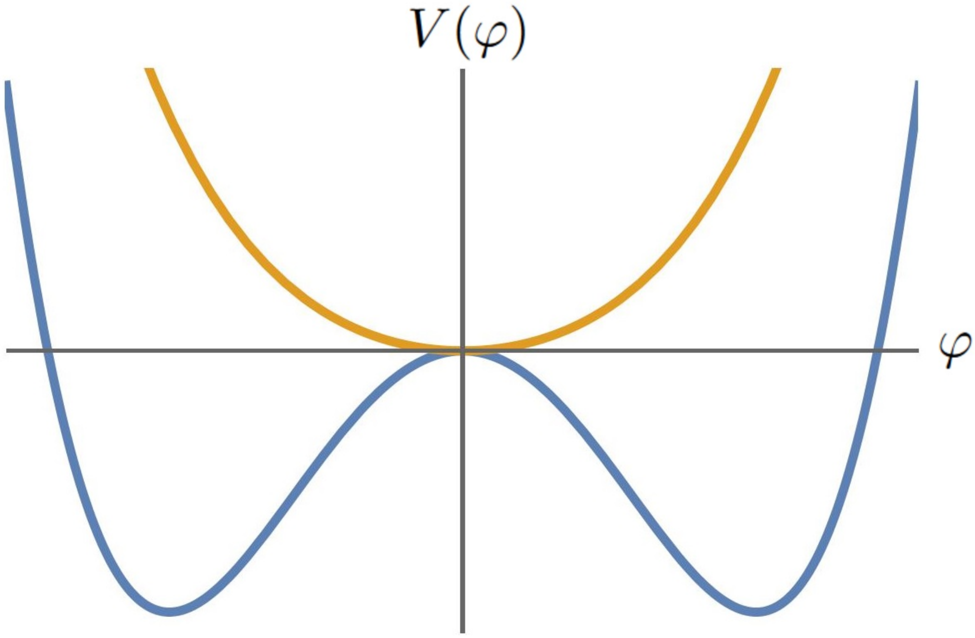

Figure 1. Potential $V(\varphi)$ of the (generalized) symmetron: if the density ρ is sufficiently small, then the potential takes on the shape of the blue curve, and the field has a non-vanishing vev. However, if $\rho > \mu^2 \mathcal{M}^2$ (orange curve), then the potential can only have a minimum at $\varphi = 0$, which results in the fifth force being screened.

Besides this standard symmetron model, a radiatively stable symmetron has been developed [60], and generalizations were discovered by using tomographic methods [53,54]. The latter is the main subject of this article and is discussed in what follows. Generalized symmetrons couple to the metric tensor via

$$A(\varphi) \quad = \quad 1 + \frac{\varphi^{2\alpha}}{\mathcal{M}^{2\alpha}} + \mathcal{O}\left(\frac{\varphi^{4\alpha}}{\mathcal{M}^{4\alpha}}\right) \ , \ \ \varphi \ll \mathcal{M} \ , \tag{5}$$

and have potentials [8]

$$V(\varphi) \quad = \quad \left(\frac{\rho}{\mathcal{M}^{2\alpha}} - \mu^{4-2\alpha}\right)\varphi^{2\alpha} + \frac{\varphi^{2\beta}}{\Lambda^{2\beta-4}} \ , \tag{6}$$

where Λ describes a symmetron self-coupling constant which generally has the dimension of a mass, but is replaced $1/\Lambda^{2\beta-4} \to \lambda/4$ for $\beta = 2$. Note that, for $\alpha = 2$, μ must also be replaced by a dimensionless constant. The numbers $\alpha, \beta \in \mathbb{Z}^+$ label each model and are, in principle, arbitrary, but must adhere to $\beta > \alpha$. Choosing the smallest possible pair $(\alpha, \beta) = (1, 2)$ recovers the standard symmetron model as given in Equation (2). Each choice for (α, β) leads to a model that has similar qualitative features to the standard symmetron. This means the potential in Equation (6) is also represented by Figure 1, a generalized symmetron fifth force is screened in regimes where $\rho > \mu^{4-2\alpha}\mathcal{M}^{2\alpha}$ since there $\varphi_0 = 0$, and in unscreened regimes the field's vev takes on the form

$$\varphi_0 = \pm \sqrt[2(\beta-\alpha)]{\frac{\alpha}{\beta}\Lambda^{2\beta-4}\left(\mu^{4-2\alpha} - \frac{\rho}{\mathcal{M}^{2\alpha}}\right)} \; . \tag{7}$$

In an unscreened situation, a generalized symmetron has a mass

$$m^2 = 4\beta(\beta-\alpha)\frac{\varphi_0^{2(\beta-1)}}{\Lambda^{2\beta-4}} \; , \tag{8}$$

while in case of total screening, this becomes

$$m^2 = \begin{cases} \frac{\rho}{\mathcal{M}^2} - \mu^2 & , \; \alpha = 1 \\ 0 & , \; \alpha > 1 \end{cases} \; . \tag{9}$$

3. Lensing

Gravitational lensing became the first experimentally confirmed novel effect predicted by general relativity when, in 1919, the bending of light in the Sun's gravitational field was observed [61]. Today gravitational lensing serves as a valuable tool for indirect observations of DM [62]. If a symmetron fifth force is supposed to act as an alternative to particle DM, as suggested by the findings in Refs. [50,51], then it must be able to explain the observed roughly 1:5 ratio between baryonic mass and DM in galaxies [63], i.e., contribute to the lensing about 5 times as much as the Newtonian potential Φ of a galaxy. In Refs. [48,52] it was shown that the standard or $(1,2)$-symmetron cannot provide such an explanation for the parameter values required by Ref. [50]. Although other models of the class of generalized symmetrons might still be able to serve as sensible alternatives to particle DM because they show the same qualitative features as the $(1,2)$-symmetron, they are as yet much less constrained.

In order to study the effect of a generalized symmetron on lensing, we now derive a measure that enables us to compare the contribution from the fifth force with the one from Newtonian gravity. For this, we follow the discussion in Ref. [48], which in turn is based on elaborations made in Ref. [64]:

We consider a perturbed Friedmann–Lemaître–Robertson–Walker (FLRW) background

$$ds^2 = a^2(\tau)[-(1+2\Psi)d\tau^2 + (1+2\Phi)\delta_{ij}dx^i dx^j] \tag{10}$$

with conformal time $d\tau = dt/a(t)$ and Newtonian potentials $\Phi, \Psi \ll 1$, which fulfill the no-slip condition $\Phi = -\Psi$. Furthermore, we assume that the thickness of the lens (e.g., a galaxy) is much smaller than the distance d_L between the lens and an observer (on Earth), and the deflection angles are very small. This assumption justifies a thin-lens approximation, which effectively considers the lens to have no thickness and lie within a two-dimensional plane. This so-called lens plane is perpendicular to the line between the light source and the observer. Using this approximation, we introduce the coordinate system $\{\tau, r, x^1 \approx r\theta^1, x^2 \approx r\theta^2\}$ with radial coordinate r and deflection angles $\theta^{1,2}$. See Figure 2 for a depiction of the setup.

We introduce the photon momentum $k^\mu := dx^\mu/d\kappa$ with some affine parameter κ, which gives the geodesic equation

$$\frac{dk^\mu}{d\kappa} + \Gamma^\mu_{\nu\sigma}k^\nu k^\sigma = 0 \; . \tag{11}$$

The photon momentum can be separated into a background vector $\hat{k}^\mu$ and a small perturbation δk^μ due to the presence of a lens mass, such that $k^\mu = \hat{k}^\mu + \delta k^\mu$. Without the lens mass, the photon trajectory is not subject to deflection, which means, for the background momentum $\hat{k}^{x^{1,2}} = 0$ must hold. Consequently, from the on-shell condition $k^{\mu}k_{u} = 0$ follows $\hat{k}^{r} = \hat{k}^{0}$.

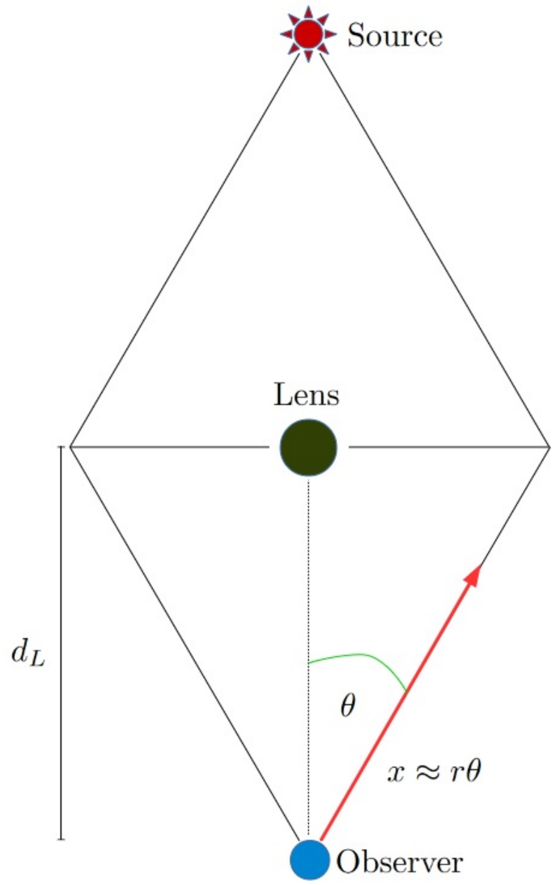

Figure 2. Schematics of gravitational lensing in two dimensions with thin-lens approximation: The source sends out light which is distorted by the lens and then received by an observer. d_L denotes the distance between the observer and the lens plane in which the lens is situated. The radial coordinate r equals 0 at the position of the observer and is orthogonal to the lens plane. θ denotes the angle between r and the light ray reaching the observer (here, it represents either θ^1 or θ^2). The coordinate x is approximated by $r\theta$ since θ is assumed to be very small [48].

Taking all this into account, and considering only terms up to first order in the Newtonian potentials and the momentum perturbations, for the purely Newtonian case without any fifth forces, the geodesic Equation (11) can be used to derive the lensing force law

$$\frac{d^2x^i}{dr^2} = (\Phi - \Psi)_{,x^i} \quad , \quad i \in \{1,2\} \ . \tag{12}$$

A similar force law can be derived when also including generalized symmetrons. However, it must be stressed that a solely conformal coupling of the scalar field φ to the metric tensor is not sufficient to have any effect on lensing. This was, for example, explicitly shown in Ref. [48], but can also simply be concluded from the fact that the energy–momentum tensor of a massless particle, such as the photon, has a vanishing trace. Therefore, the scalar φ must not only couple conformally but also disformally via

$$\bar{g}_{\mu\nu} = A(\varphi)g_{\mu\nu} + B\varphi_{,\mu}\,\varphi_{,\nu} \quad , \tag{13}$$

$$\bar{g}^{\mu\nu} = A^{-1}(\varphi)\left(g^{\mu\nu} - \frac{B}{C(\varphi)}\varphi_{,}{}^{\mu}\,\varphi_{,}{}^{\nu}\right) \quad , \tag{14}$$

where $A(\varphi)$ is the conformal factor from Equation (5), B is the disformal coupling parameter, which for simplicity is assumed to be a constant, and $C(\varphi) := A(\varphi) + B(\partial\varphi)^2$. $\bar{g}$ denotes the metric in the Jordan frame and g the metric in the Einstein frame [1].
For the Jordan frame Christoffel symbols, we find

$$\begin{aligned}\bar{\Gamma}^{\mu}_{\nu\sigma} &= \Gamma^{\mu}_{\nu\sigma} + A^{-1}\left[A,_{(\nu}\, g^{\mu}_{\sigma)} - \frac{1}{2}A,^{\mu}\, g_{\nu\sigma}\right] \\ &\quad + \frac{B}{C}\varphi,^{\mu}\left[\varphi,_{\nu\sigma} - \varphi,_{\rho}\,\Gamma^{\rho}_{\nu\sigma} + \frac{A^{-1}}{2}\varphi,^{\rho}\, A,_{\rho}\, g_{\nu\sigma} - A^{-1}A,_{(\nu}\,\varphi,_{\sigma)}\right] , \end{aligned} \tag{15}$$

where the second term in the first line represents a contribution that arises purely from the conformal coupling and therefore does not affect the lensing. Substituting Equation (5) into Equation (15), following the same procedure as for the derivation of Equation (12), and assuming the lens object to be static, then the force law

$$\begin{aligned}\frac{d^2x^i}{dr^2} &= (\Phi - \Psi),_{x^i} - \frac{B}{C}\varphi,^{x^i}\left[\left(\varphi,_{yz} - \frac{2\alpha}{\mathcal{M}^{2\alpha}}\varphi^{2\alpha-1}\varphi,_{y}\,\varphi,_{z}\right)\frac{dx^y}{dr}\frac{dx^z}{dr}\right. \\ &\quad \left. -2\mathcal{H}\varphi,_{y}\,\frac{dx^y}{dr} + a^2\varphi,_{y}\,(\Phi - \Psi),^{y} - 2\varphi,_{r}\,\Phi,_{r}\right] , \end{aligned} \tag{16}$$

where $y, z \in \{r, x^1, x^2\}$, and $\mathcal{H}$ denotes the conformal Hubble parameter, can be obtained. Comparing this force law with the one in Equation (12), we find that the term is proportional to B in Equation (16) corresponds to the contribution of the generalized symmetron fifth force to the lensing effect.

Next, we assume $a(\text{today}) = 1$, use the no-slip condition for the Newtonian potentials, and consider the force law only in the lens plane $r = d_L$, where we expect the lensing to be maximal and dx/dr to vanish:

$$\left.\frac{d^2x^i}{dr^2}\right|_{d_L} = \left[2\Phi,_{x^i} - \frac{B}{C}\varphi,^{x^i}\left(\varphi,_{rr} - \frac{2\alpha}{\mathcal{M}^{2\alpha}}\varphi^{2\alpha-1}(\varphi,_{r})^2 - 2\mathcal{H}\varphi,_{r} + 2\varphi,_{x^j}\,\Phi,^{x^j}\right)\right]\Bigg|_{d_L} . \tag{17}$$

In order to obtain a rough numerical estimate of the generalized symmetron contribution, we assume its source, i.e., a galaxy acting as a lens, to be a disk lying in the lens plane with homogeneous, constant mass density and radius R, leading to a total galaxy mass M. Since this approximated lens has a spherical symmetry, we can restrict our investigation to the case $\theta^2 = 0$, and consequently only work with $\theta^1 =: \theta$. Applying Newton's shell theorem and considering that the radial coordinate within the lens plane originating from the disk's center can be expressed as $r\theta$ in the observer's coordinates, the Newtonian potential becomes

$$\Phi = -\frac{GM}{r\theta} . \tag{18}$$

Some references, including Refs. [48,54], suggest an approximation for the symmetron field profile outside a homogeneous, spherically symmetric source of radius R, which is valid for a symmetron mass m_{out} outside the source fulfilling $r\theta < m_{\text{out}}^{-1}$. However, in what follows, this approximation will not always be good. Therefore, we use the symmetron profile [37]

$$\varphi = v - \frac{D}{r\theta}e^{-m_{\text{out}}r\theta} \tag{19}$$

with

$$D := (v - w)Re^{m_{\text{out}}R}\frac{Rm_{\text{in}} - \mathcal{T}}{Rm_{\text{in}} + Rm_{\text{out}}\mathcal{T}} , \tag{20}$$

where m_{in} is the field's mass within the source, $v := \varphi_{0,\text{out}}$ and $w := \varphi_{0,\text{in}}$ are the vev in the environment surrounding the source and within the source, respectively, and

$\mathcal{T} := \tanh(m_{\rm in}R)$. The solution in Equation (19) requires that we only consider terms up to first order in $\delta\varphi$, and, at this order, is even valid for any generalized symmetron model. Substituting Equations (18) and (19) into Equation (17) leads us to

$$\left.\frac{d^2x}{dr^2}\right|_{d_L} = \frac{2GM}{d_L^2\theta^2}[1+F] \tag{21}$$

with

$$\begin{aligned} F := & \frac{BD^2}{Cd_L^4\theta^2}e^{-2m_{\rm out}d_L\theta}(1+m_{\rm out}d_L\theta)\Bigg\{\left(1+\frac{d_L\theta}{2GM}\right)\Bigg[1+(1+m_{\rm out}d_L\theta)^2 \\ & +\frac{2\alpha D}{d_L\theta\mathcal{M}^{2\alpha}}\left(v-\frac{D}{d_L\theta}e^{-m_{\rm out}d_L\theta}\right)^{2\alpha-1}e^{-m_{\rm out}d_L\theta}(1+m_{\rm out}d_L\theta)^2+2\mathcal{H}d_L(1+m_{\rm out}d_L\theta)\Bigg] \\ & -\frac{1+m_{\rm out}d_L\theta}{\theta^2}\Bigg\} \end{aligned} \tag{22}$$

being a term that describes the contribution of the generalized symmetron fifth force to lensing in comparison to the one originating from Newtonian gravity. Furthermore, in the lens plane, we have:

$$\mathrm{C}|_{d_L} = 1+\frac{1}{\mathcal{M}^{2\alpha}}\left(v-\frac{D}{d_L\theta}e^{-m_{\rm out}d_L\theta}\right)^{2\alpha}+(1-2\Phi|_{d_L})\frac{BD^2}{d_L^4\theta^4}e^{-2m_{\rm out}d_L\theta}(1+m_{\rm out}d_L\theta)^2(1+\theta^2)\ . \tag{23}$$

In Equation (22) we see that $F\sim D^2$, such that from Equation (20) we can find $F\sim(v-w)^2$. This result implies that if v and w are very similar, and have the same sign, for example, in some situations where the field is unscreened both in and outside the source, the contribution of the generalized fifth force to lensing can be very small. However, it is not necessarily true that both vev need to have the same sign. Since we are interested in checking whether it is at all possible to explain the observed lensing by a generalized symmetron fifth force, we consider the best-case scenario, in which $v = +|v|$ and $w = -|w|$.

4. Model Parameters

We now want to consider different generalized symmetron models and see for what points in their parameter spaces the expression in Equation (22) gives $F \approx 5$, such that the contribution of a fifth force can be interpreted as an alternative to particle DM, at least when it comes to lensing.

For this, we consider the same galaxy parameters as in Ref. [48], i.e., we look at a Milky Way-like galaxy with $M = 6\times10^{11}M_\odot \approx 6.67\times10^{77}\,$eV and a scale length $R = 5\,$kpc $\approx 2.69\times10^{26}\,$eV^{-1} [65]. Ref. [66] reports lensing by galaxies at redshift $z = 1$ under angles of about 1 arcmin. Therefore, we choose $\theta = \frac{\pi}{10{,}800}$ and $d_L \approx 6.60\times10^{32}\,$eV^{-1}, where we obtained the latter from $d_L \approx zd_H$ [67] using the Hubble length d_H. For the density around the galaxy we assume $\rho_{\rm out} \approx 2.59\times10^{-11}\,$eV^4 [68]. In addition, we use $G \approx 6.71\times10^{-57}\,$eV^{-2} and $\mathcal{H} \approx 1.51\times10^{-33}\,$eV.

We start the discussion by again looking at the $(1,2)$-symmetron before we show that moving to larger values of (α,β) is beneficial for complying with the existing constraint $B < 5.6\times10^{-48}\,$eV^{-4} [55,56] on the disformal coupling parameter.

4.1. $(1,2)$-*Symmetron*

Ref. [50] suggested that the $(1,2)$-symmetron could explain the stability and rotation curves of disk galaxies, and therefore be a possible candidate for an alternative to particle DM. For this to work, the symmetron parameters were chosen to be $\mathcal{M} = M_P/10$, $v = \mathcal{M}/150$, and $\mu = 3\times10^{-30}\,$eV. In Refs. [48,52], it was shown that those parameter choices require a disformal coupling parameter B much larger than permitted by the constraints given in Refs. [55,56]. This result also held for realistic variations of the galaxy parameters.

We now study for what values of $\mathcal{M}$ and the self-coupling constant λ the $(1,2)$-symmetron can actually comply with the constraints on disformal couplings. For the disformal coupling parameter, we choose $B = 10^{-49}\,\text{eV}^{-4}$ in order to consider a value that is not yet excluded by experiments, and for μ we initially take the same value as in Ref. [50]. We find that $\lambda \approx 10^{-174.6}$ is the largest and $\mathcal{M} \approx 10^{58.7}\,\text{eV}$ the smallest possible value in order to find $F \approx 5$, while simultaneously fulfilling the perturbative condition $\varphi \ll \mathcal{M}$. Even though there are not necessarily any restrictions on the permitted values for the coupling constants, it is certainly peculiar to have a theory with such a small self-coupling and, more strikingly, a mass scale that is more than 30 orders of magnitude above the Planck mass.

Note that for these parameter values, the field is not screened within the galaxy. Furthermore, since in this parameter regime $1/m_{\text{in}} > R$, i.e., the Compton wavelength of the field is larger than the galaxy scale, the field adapts to the size of the galaxy and consequently rather takes on the mass $m_{\text{in}} \approx q/R$ and the vev

$$w \approx \pm \sqrt[\beta-1]{\frac{q\Lambda^{\beta-2}}{2\sqrt{\beta(\beta-\alpha)}R}}\,, \tag{24}$$

where q is a fudge factor that would have to be computed numerically taking into account more detailed properties of a galaxy, but is assumed to be of order 1 (compare with the fudge factor, e.g., in Ref. [69]).

Considering smaller values of the disformal coupling parameter only worsens the situation, i.e., requires smaller values of λ and larger $\mathcal{M}$. As was also observed in Ref. [48], changing the galaxy parameters to other realistic values does not significantly improve the situation.

What remains to be checked is how the result is affected by a change in the tachyonic mass μ. As it turns out and can be seen in Figure 3, the value for μ used in Ref. [50] is sitting within a small part of the parameter space that allows for $F \approx 5$.

Looking at Equations (19) and (20), and remembering that $F \sim D^2$, we see that significantly increasing or decreasing μ beyond this small section of the parameter space, while keeping all other parameters at the same values, leads to F becoming rapidly smaller since the combined exponential function in Equations (19) and (20) becomes smaller due to the enlarged symmetron mass or the vev reduces, respectively. In order to counteract the declining F, such that $F \approx 5$ can be recovered, we have to consider even smaller values of λ and consequently larger values of $\mathcal{M}$. This is the exact opposite of what we had hoped to achieve, which is why varying μ can be excluded as a possibility for improving the case for the $(1,2)$-symmetron.

4.2. $(1,\beta)$-Symmetrons with $\beta \geq 3$

Now we move to the first set of non-standard symmetrons and consider models with $(1,\beta)$ for $\beta \geq 3$. For those, Equation (22) has still the same form as for the $(1,2)$-symmetron, but the vev and the unscreened masses are different according to Equations (7) and (8). Furthermore, instead of using a dimensionless constant λ, we now have to work with Λ as another mass scale besides $\mathcal{M}$. Again choosing $\mu = 3 \times 10^{-30}\,\text{eV}$, we find the results presented in Figure 4 for $3 \leq \beta \leq 10$.

We observe that going from $\beta = 2$ to $\beta = 3$ requires a slightly lower $\mathcal{M}$ for $F \approx 5$ and $\varphi/\mathcal{M} \ll 1$, but the Λ of the $(1,3)$-symmetron must be more than 100 orders of magnitude above the Planck mass. This renders this theory to be unrealistic as an alternative to particle DM.

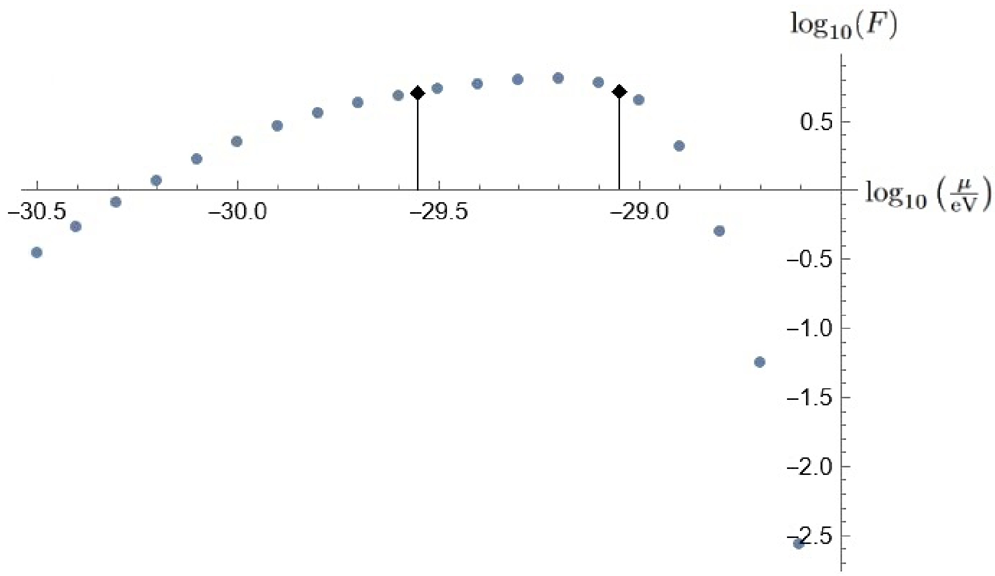

Figure 3. Behavior of the function $F(\mu)$ for the $(1,2)$-symmetron with parameters $\lambda = 10^{-174.6}$, $\mathcal{M} = 10^{58.7}$ eV, and $B = 10^{-49}$ eV^{-4}; the black rhombuses with vertical lines depict the two points where $F = 5$.

At $\beta = 4$ there is another slight decrease in $\mathcal{M}$, while from $\beta = 5$ on this mass scale increases again until it settles around $\mathcal{M} \approx 10^{58.3}$ eV even for very large β. This means, compared to the standard symmetron, only changing β barely improves the value of the matter coupling mass scale.

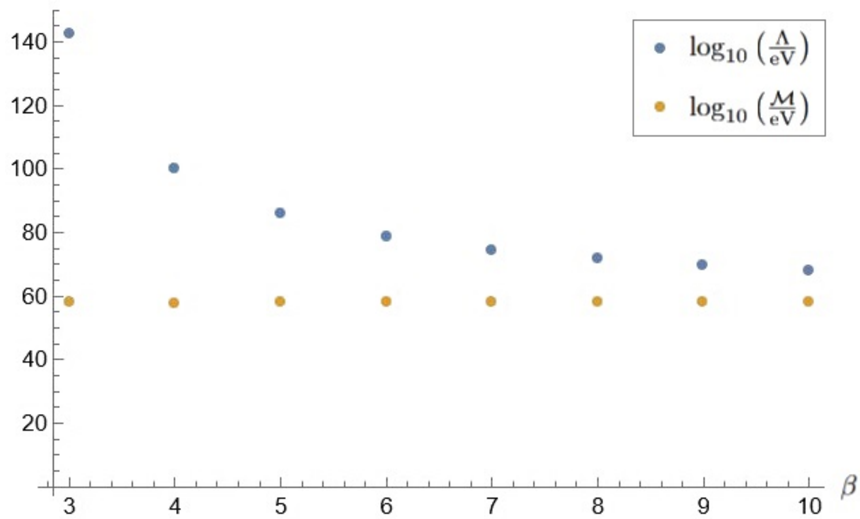

Figure 4. Values of Λ and $\mathcal{M}$ that give $F = 5$ for $(1, \beta \geq 3)$-symmetrons with $\mu = 3 \times 10^{-30}$ eV up to $\beta = 10$.

The self-coupling constant Λ is strictly monotonically decreasing and approaches $\Lambda \approx 10^{57.2}$ eV for large β. This means, the two mass scales in $(1, \beta \geq 3)$-symmetron theories are getting closer to each other with increasing β until Λ becomes smaller than $\mathcal{M}$ and both finally differ by approximately 1 order of magnitude.

Even though there are parts of their parameter spaces for which generalized symmetrons with $\alpha = 1$ can explain the lensing otherwise attributed to particle DM, they require coupling constants that are at least 30 order of magnitude above the Planck mass.

4.3. (α, β)-Symmetrons with $\alpha \geq 2$

Finally, we also consider generalized symmetron models with $\alpha \geq 2$. Now we encounter a couple of differences to the previously considered models: Equation (22) changes with every possible value of α, and μ is either dimensionless (for $\alpha = 2$) or appears in Equation (6) with negative order (for $\alpha \geq 3$).

Generally, it can be said that studying these models is numerically more intricate than it was for the ones we considered before. This is due to the fact that within the considered astrophysical setup, $\alpha \geq 2$-symmetrons can lead to extremely small terms that have to be compensated by extremely large terms in order to find any results of order 1. For example, Equation (22) can be separated into two terms, one outside (F_o) and one inside (F_i) the curly brackets. F_o tends to become very small since it is suppressed by at least $B/d_L^3\theta \sim 10^{-144}\,\text{eV}^{-1}$ (for $B = 10^{-49}\,\text{eV}^{-4}$). While generalized symmetrons with only $\alpha = 1$ seem to not struggle with compensating such smallness by leading to a sufficiently large F_i, the models discussed here are more problematic since they lead to even more extreme values for F_o and F_i.

Beginning with the $(2,3)$-symmetron model, we can find no point in the parameter space that allows for $F \approx 5$. The closest value we find is around $F \sim 10^{-139}$. This is caused by F_i not reaching sufficiently large values and whenever it increases for parameter values beyond the maximum of F, F_o decreases even faster. Increasing β does not lead to greatly improved results.

Moving to $\alpha = 3$, from where on μ is a mass scale, we again do not find any point in the parameter space that allows for $F \approx 5$, but can at least reach $F \sim 10^{-117}$.

The first model we find that allows for $F \approx 5$ is the $(5,7)$-symmetron around the parameter space point $(\Lambda, \mathcal{M}, \mu) = (10^{37.4}, 10^{58.7}, 10^{24.0})\,\text{eV}$. Interestingly, from $\alpha = 5$ on, changing β can actually have a significant impact on the results. However, while for $\alpha = 6$ we still need at least $\beta = \alpha + 2$, from $\alpha = 7$ on we can obtain $F \approx 5$ even for $\beta = \alpha + 1$.

With increasing α we have to use smaller Λ in order to reach $F \approx 5$, such that we are getting closer to the Planck scale. For example, in the $(11,12)$-symmetron model we find $F \approx 5$ at the point $(\Lambda, \mathcal{M}, \mu) = (10^{27.3}, 10^{58.7}, 10^{24.0})\,\text{eV}$, where Λ is close to M_P.

As can be seen in the examples presented above, increasing α is useful for reducing Λ, but $\mathcal{M}$ remains more than 30 order of magnitude above the Planck scale independent of the choice of (α, β).

5. Conclusions

In this article, we discussed generalizations of the symmetron model, characterized by a pair of positive integers (α, β), and investigated for what parameter values their fifth forces can explain the difference between baryonic and lens masses of galaxies, which is otherwise attributed to particle DM. For this, we first reviewed generalized symmetrons and then derived a measure F that allowed us to compare the lensing contribution from Newtonian gravity with one from a gravity-like fifth force induced by a disformally coupling generalized symmetron.

We looked at a Milky Way-like galaxy and checked for a selection of generalized symmetron models for what parameter space values $F \approx 5$, which corresponds to the expected ratio between DM and baryonic matter in galaxies, is fulfilled. With $B = 10^{-49}\,\text{eV}^{-4}$ we chose a disformal coupling parameter close to the maximal value allowed by current experimental constraints. For the standard symmetron, corresponding to $(\alpha, \beta) = (1,2)$, we found that a tiny self-coupling constant λ and a matter coupling mass scale $\mathcal{M}$ more than 30 orders of magnitude larger than the Planck mass M_P are required in order to find $F \approx 5$. Increasing β, which required us to work with a mass scale Λ instead of a dimensionless λ, led to both Λ and $\mathcal{M}$ being at least 30 orders of magnitude above the Planck scale. Finally, also varying α was most promising since it allowed us to use generalized symmetron fifth forces as explanations for the observed lensing excess at mass scales Λ and μ around or even below the Planck scale. However, in no model was it possible to significantly reduce the value for $\mathcal{M}$, which means it must always be much larger than M_P.

From Refs. [48,52], we know that the $(1,2)$-symmetron is not able to successfully act as an alternative to particle DM since it cannot explain lensing. This is also reflected in the fact that, as we showed in the present article, the standard symmetron fifth force would require an extremely small λ for $F \approx 5$. In contrast, some generalized symmetron models with larger α are so far better at explaining the observed lensing because they instead require the self-interaction mass parameter Λ to be around the Planck scale, which is a typical value expected for many screened scalar field models. Despite this, every model, even for very large α and β, required $\mathcal{M}$ to be at least 30 orders of magnitude above M_P in order to reach $F \approx 5$. There are several possible ways around this conundrum: either we accept that we have such a large fundamental mass scale in Nature, we relax the requirement on F and want a generalized symmetron fifth force to only partially explain DM, we find a way to relax the requirement $\varphi \ll \mathcal{M}$, or we consider a hybrid model between modified gravity and particle DM, as was suggested for the standard symmetron in Ref. [52].

In any case, generalized symmetrons beyond $(\alpha, \beta) = (1,2)$ are interesting theories to study because, to date, there are no experimental constraints on their parameter spaces. In addition, redoing the analyses made in Refs. [50,51] for some of the more promising models, for example, theories such as $(\alpha, \beta) = (5,7)$ and beyond, might in the future demonstrate that generalized symmetrons are actually good alternatives to particle DM.

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Abbreviations

The following abbreviations are used in this manuscript:

DE	Dark energy
DM	Dark matter
vev	Vacuum expectation value
FLRW	Friedmann–Lemaître–Robertson–Walker

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