# A CONTRIBUTION TO THE THEORY OF SLIP FACTOR FOR RADIAL FLOW FANS

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## ABSTRACT

**The preliminary design of radial flow (centrifugal) fans is usually performed by the theory of fluid-flow machinery, based on the change of angular momentum between impeller inlet and exit. As a consequence of the rotation of the impeller, the relative flow at impeller exit does not follow the blade exit angle. This effect is modeled by a so-called slip velocity, induced by a relative eddy in the blade channel. The nondimensional slip velocity characterizes the so-called slip factor. A number of empirical correlations, semi-empirical and theoretical equations are available to calculate the slip factor for impellers of radial flow fans: Stodola, Busemann, Wiesner, Eck, and others. The present paper extends the slip factor model of Stodola for parallel hub and shroud walls (b = const.) to impellers with contoured shroud wall (b · r = const.). The model is based on Helmholtz' vorticity theorem on the convective transport of the relative eddy through the impeller blade channel. Using simplified velocity triangles, a transformation of the equations of Stodola is performed to make them sensitive to the impeller inlet to exit diameter ratio. Finally, a linearization of the original slip factor equations of Eck for high blade solidities is performed using a Taylor series expansion.**

**KEYWORDS: RADIAL FLOW FAN, RELATIVE EDDY, SLIP FACTOR**

## NOMENCLATURE

| | |
|---|---|
| a | diameter of relative eddy at blade channel exit (m) |
| A | cross section area of vortex tube ($m^2$) |
| b | blade channel width in meridional plane (m) |
| c | absolute velocity (m/s) |
| d | diameter (m) |
| l | blade length (m) |
| r | radius (m) |
| S | blade solidity (-) |
| t | blade spacing (m) |
| u | circumferential velocity (m/s) |
| V | Volume of vortex tube ($m^3$) |
| w | relative velocity (m/s) |
| z | number of impeller blades (-) |
| $\alpha$ | absolute flow angle, measured from the circumferential direction (°) |
| $\beta$ | relative flow angle, measured from the circumferential direction (°) |
| $\mu$ | slip factor (-) |
| $\sigma$ | blade (metal) angle (°) |

$\omega$ angular velocity of the relative eddy (rad/s)
$\Omega$ angular velocity of the impeller (rad/s)

## SUBSCRIPTS

m meridional component
u circumferential component, measured in positive circumferential direction
E English definition of slip factor
G German definition of slip factor
1 impeller inlet
2 impeller exit
$\infty$ infinite number of impeller blades

## INTRODUCTION

The preliminary design of radial flow fans (centrifugal fans) is usually performed by the theory of fluid-flow machinery, based on the change of angular momentum between impeller inlet and impeller exit. Such a preliminary design method, based on the Euler-equation is given for example in Eck (1972). A basic information for the calculation of the change of angular momentum is the impeller exit flow angle. Usually, the flow angle deviates from the blade angle (metal angle) at impeller exit. Various physical mechanisms are responsible for this deviation: vorticity of the relative flow in the channel between adjacent blades, displacement effect of blade boundary layers, boundary layer separation, endwall and secondary flow, tip-leakage flow in compressor impellers with unshrouded blades. The present paper deals with the first effect, since it is normally the one considered in the preliminary design process. During the preliminary design process, the deviation of the flow from the blade exit angle is quantified by the so-called slip factor. In the next section, the physical background of the slip effect is briefly discussed and a definition of slip factor is provided.

## SLIP EFFECT AND DEFINITION OF SLIP FACTOR

Figure 1 shows velocity triangles at the exit of a radial impeller with backswept blades. The impeller rotates in the anti-clockwise direction with $\Omega$ as angular velocity. $\sigma_2$ is the blade (metal) angle at impeller outer radius and $\beta_2$ is the corresponding relative flow angle.

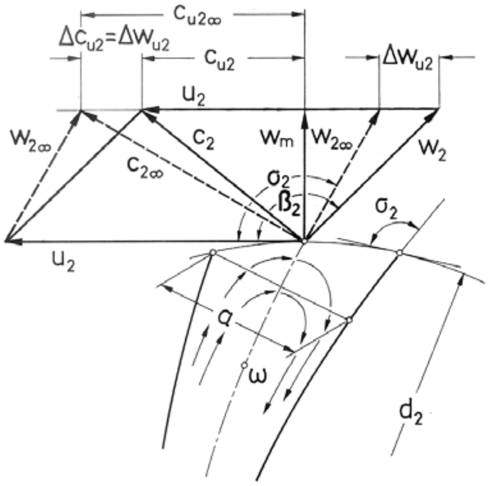



**Figure 1: Impeller blade channel and velocity triangles at exit**

For an infinite number of blades with zero thickness, the relative flow follows exactly the blade shape and the flow exit angle would be identical to the blade exit angle. This situation is represented in Fig. 1 by the velocities with subscript $\infty$, indicating an infinite number of blades.

For a finite number of blades z, a relative eddy establishes in the channel between two adjacent blades. This relative eddy with its angular velocity ω rotates in the opposite direction compared to the impeller rotation. As a result, the relative flow at impeller exit does not follow the blade exit angle and a so-called slip velocity $\Delta c_{u2} = \Delta w_{u2}$ appears. This slip effect turns the impeller exit flow in the direction against the circumferential speed. The slip velocity is induced by the relative eddy in the blade channel and, therefore, the effect depends at least on the number of impeller blades.

The relative effect of the slip velocity is described by the so-called slip factor $\mu$. When slip factors from different sources are compared (Dixon and Hall, 2014), one has to consider that there exists a German definition

$$\mu_G = \frac{c_{u2}}{c_{u2\infty}} = \frac{c_{u2\infty} - \Delta c_{u2}}{c_{u2\infty}} = 1 - \frac{\Delta c_{u2}}{c_{u2\infty}}, \tag{1}$$

as well as an English definition

$$\mu_E = \frac{u_2 - \Delta c_{u2}}{u_2} = 1 - \frac{\Delta c_{u2}}{u_2}. \tag{2}$$

Both definitions differ by the velocity which is used to normalize the slip velocity. The derivations in the present paper are based on the German definition of the slip factor (Eq. (1)) and $\mu = \mu_G$ is set throughout. For radial ending blades ($\sigma_2 = 90°$), both definitions give identical results.

## LITERATURE OVERVIEW

A number of empirical correlations, semi-empirical and theoretical equations are available to calculate the slip factor for impellers of radial flow fans, compressors and pumps.

The first slip factor model which can be found in the literature is due to Stodola (1924). It is based on the concept of a relative eddy. This relative eddy rotates against the direction of impeller rotation and induces the slip velocity at impeller exit.

Busemann (1928) published a paper on the slip effect in radial flow pump impellers. He applied the method of conformal mapping on the two-dimensional, inviscid flow in impellers with logarithmic spiral blades. These are blades with constant blade angle. The slip factors, depending on blade angle and impeller inlet to exit diameter ratio are presented in diagrams. One outcome was that the slip factor is independent of the inlet to exit diameter ratio below a limiting value. The results of Busemann (1928) are treated as an “exact” solution of the problem. Therefore, they are often used as a reference to check the quality of new or improved slip factor models.

Wiesner (1967) summarizes results from slip factor models published in the previous literature. He found, that the model of Busemann (1928) gives still generally applicable results for the prediction of slip factor. Furthermore, he presented an empirical equation which fits the results of Busemann up to a limiting impeller inlet to outlet radius ratio. The equation of Wiesner (1967) correlates the slip factor to the following two parameters: blade number, blade exit angle.

In his textbook on axial and radial flow fans, Eck (1972) presented a model for the prediction of slip factor. The model is also based on the concept of the relative eddy in the blade channel. However, the strength of the relative eddy is related to the mean blade pressure difference using arguments of blade circulation. It is interesting to note that Eck (1972) differentiates between impellers with parallel hub and shroud walls (b = const.) and impellers with contoured shroud walls according to b · r = const. To the author's knowledge, this differentiation is not present in the remaining slip factor models.

Apart from these historical papers, publications on the slip factor for radial flow fan or compressor impellers can still be found in the literature. Recently, von Backström (2006a) presented a model which is based on the relative eddy concept, too. However, a single relative eddy (SRE), located in the impeller is postulated instead of multiple relative eddies in the blade channels. It is stated that the SRE-model is a feasible replacement of the well established model of Wiesner. Furthermore, the author points out that solidity is an important parameter, since many factors that contribute to slip depend on this quantity. The slip factor depends on solidity, weighted by an auxiliary coefficient that is a function of blade number and blade exit angle. Recently, an improvement of the SRE-method, mainly of the accuracy of the auxiliary coefficient has been presented (von Backström, 2006b).

Qiu et al. (2007, 2011) presented a unified slip factor model for radial flow, mixed flow and also axial flow impellers. For radial flow impellers, the model is similar to the relative eddy based model of Stodola. In contrast to many other slip factor models, the model of Qiu et al. gives also accurate results for off-design conditions.

Therefore, it can be summarized that an improvement of the understanding of the physical effect and the derivation of simple and accurate slip factor models is still of interest. The present paper discusses, compares and extends the slip factor models of Stodola (1924) and Eck (1972).

## ORIGINAL SLIP FACTOR MODELS OF STODOLA AND ECK

A short description of the slip factor models of Stodola (1924) and Eck (1972) will be provided in this chapter. The description ends up with the slip factor equations, given in the original papers. For a detailed description of the models, the reader is referred to the original literature.

### Model of Stodola

According to the kinematics of velocity triangles it is $\vec{c} = \vec{u} + \vec{w}$. It is assumed that the flow field upstream and downstream of the impeller is a potential flow. Therefore, the absolute flow field inside the impeller can also be treated as a potential flow with zero vorticity. However, the relative flow in the impeller is a vortical flow, since $\mathrm{rot}\vec{w} = -\mathrm{rot}\vec{u} = -2\vec{\Omega}$. This can be interpreted as a relative eddy, which is located in the channel between adjacent blades (Fig. 1). The relative eddy rotates as a forced vortex in the opposite direction of the impeller. Its' magnitude of angular velocity is $\omega = \Omega$. The diameter of the relative eddy is identical to the length a, which is the distance between the trailing edge of the suction side and the pressure side of the adjacent blade. Under these assumptions, the slip velocity induced by the relative eddy is $\Delta c_{u2} = \Delta w_{u2} = \omega a / 2$. The final result for the slip factor according to the model of Stodola is given in Tab. 1. It is valid for an impeller with parallel walls at hub and shroud (b = const.).

### Model of Eck

The model of Eck is also based on the concept of relative eddy. However, it differs from the model of Stodola by the calculation of the strength of the relative eddy. The basic idea of the model is the fact that the relative eddy is responsible for the non-uniform velocity distribution at impeller channel exit. A linear velocity distribution is assumed. The velocity difference between blade suction side and blade pressure side is $\Delta w_2$. This velocity difference is related to the angular velocity of the relative eddy according to $\Delta w_2 = 2\omega a$. On the other hand, the velocity difference is related to the mean pressure difference between blade pressure und suction side according to Bernoulli's equation. The final result for the slip factor according to the model of Eck is given in Tab. 1. It provides individual equations for impellers with parallel walls at hub and shroud (b = const.) and impellers with contoured shroud wall according to b · r = const. The reason is that the static moment $\int_{r_1}^{r_2} b \cdot r dr$ is required for the calculation of the impeller torque which is balancing the mean blade pressure difference. This static moment is different for impeller channels with b = const. or b · r = const., respectively.

As can be seen from Tab. 1, the impeller blade number z as well as the impeller blade exit angle $\sigma_2$ appear in each equation. Furthermore, the impeller inlet to exit diameter ratio $d_1/d_2$ can be found in the original slip factor equations of Eck. In contrast, the original slip factor equation of Stodola contains velocities $u_2$ and $c_{u2\infty}$. Finally, the inconsistency is further increased by the fact that the theory of Stodola provides no equation for the case b · r = const. In the next step, the original slip factor model of Stodola will be extended to the case b · r = const.

| | Eck (1972) | Stodola (1924) |
|---|---|---|
| b = const. | $\mu = \dfrac{1}{1+\dfrac{\pi \sin\sigma_2}{z\left[1-\left(\dfrac{d_1}{d_2}\right)^2\right]}}$ | $\mu = 1-\dfrac{u_2 \pi \sin\sigma_2}{c_{u2\infty} z}$ |
| b · r = const. | $\mu = \dfrac{1}{1+\dfrac{\pi \sin\sigma_2}{2z\left[1-\left(\dfrac{d_1}{d_2}\right)\right]}}$ | ? |

**Table 1: Original slip factor equations according to Eck (1972) and Stodola (1924)**

## EXTENDED SLIP FACTOR MODEL OF STODOLA

The original slip factor model of Stodola is limited to the case of parallel walls at hub and shroud, that is b = const. In this case, the angular speed of the single relative eddy in the blade channel $\omega$ is equal to the angular speed of the impeller $\Omega$. To keep the meridional velocity component $c_m$ constant, the shroud wall can be contoured according to b · r = const. Now it is postulated that there is not one single relative eddy fixed in the blade channel but relative eddies are convected continuously through the blade channel from impeller inlet to impeller exit. Figure 2 shows one impeller blade channel with the relative eddies $\omega_1$ at the inlet, $\omega_2$ at the exit and $\omega(r)$ at an arbitrary radius r. Each relative eddy is related to a vortex tube which undergoes a squeezing process from impeller inlet to impeller exit. This is due to the fact that the blade channel width b decreases from impeller inlet to impeller exit. Using the cross section areas of the vortex tubes, their volume is

$$V = A_1 \cdot b_1 = A_2 \cdot b_2 = A \cdot b = \text{const.} \quad (3)$$

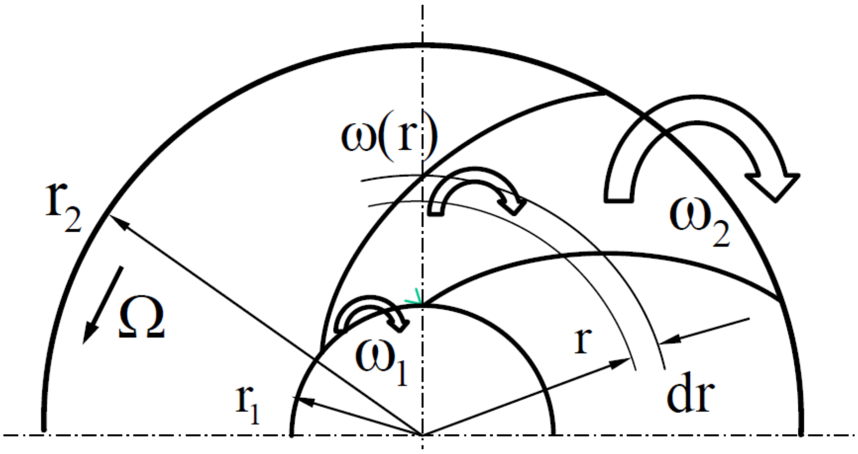



**Figure 2: Impeller blade channel and relative eddies for b · r = const.**

According to Helmholtz' vorticity theorem on the convective transport of a vortex tube (Greitzer et al., 2004) it is

$$A_1 \cdot \omega_1 = A_2 \cdot \omega_2 = A \cdot \omega = \text{const.} \tag{4}$$

This means that the strength of the relative eddy decreases from impeller inlet to impeller exit since its' cross section area increases. The angular speed of the relative eddy convected through the blade channel is no longer constant but it depends on the radius according to

$$\omega_1 \cdot r_1 = \omega_2 \cdot r_2 = \omega \cdot r = \text{const.} \tag{5}$$

The integration of the area-weighted vorticity from impeller inlet to impeller exit results in the circulation of the relative velocity field in the blade channel. This circulation is set equal to the angular speed of the impeller times the area of one blade channel. Therefore, it is

$$\int_{r_1}^{r_2} \frac{\omega 2\pi r}{z} dr = \frac{2\pi\omega_2 r_2}{z} \int_{r_1}^{r_2} dr = \Omega \frac{\pi\left(r_2^{\,2} - r_1^{\,2}\right)}{z}. \tag{6}$$

Since the slip velocity is induced by the relative eddy at impeller exit, its' angular velocity $\omega_2$ is of interest. Performing the integration in Eq. (6), the ratio of this angular velocity to the impeller angular velocity gets

$$\frac{\omega_2}{\Omega} = \frac{1}{2}\left(1 + \frac{d_1}{d_2}\right). \tag{7}$$

As can be seen from Eq. (7), the ratio of angular velocity of the relative eddy at impeller exit $\omega_2$ to the impeller angular velocity $\Omega$ depends on the diameter ratio $d_1/d_2$. It varies between $\omega_2/\Omega = 0.5$ at $d_1/d_2 = 0$ and $\omega_2/\Omega = 1.0$ at $d_1/d_2 = 1.0$. Table 2 shows the final result of the extended equation of Stodola's model for b · r = const. using Eq. (7). Both equations of Stodola's model differ by the number of blades: z, respectively 2z. This influence can be found in the same manner in the equations of Eck for b = const. and b · r = const., respectively. Furthermore, the diameter ratio $d_1/d_2$ appears in the extended equation of Stodola for b · r = const. Unfortunately, a direct comparison of the equations of Stodola and Eck is not possible, since the ratio $u_2 / c_{u2\infty}$ appears in the equations of Stodola but not in the equations of Eck. Using Eqs. (1) and (2) it can be shown that

$$\frac{u_2}{c_{u2\infty}} = \frac{1-\mu_G}{1-\mu_E}. \tag{8}$$

Switching from the German to the English definition of slip factor means that the ratio $u_2 / c_{u2\infty}$ vanishes in the equations of Stodola but appears in the equations of Eck and a direct comparison is still not possible. Therefore, the equations of Stodola will be transformed in such a manner that the slip factors depend on a common set of impeller geometry parameters.

| | Eck (1972) | Stodola (extended) |
|---|---|---|
| b = const. | $\mu = \frac{1}{1+\frac{\pi \sin\sigma_2}{z\left[1-\left(\frac{d_1}{d_2}\right)^2\right]}}$ | $\mu = 1-\frac{u_2 \pi \sin\sigma_2}{c_{u2\infty} z}$ |
| b · r = const. | $\mu = \frac{1}{1+\frac{\pi \sin\sigma_2}{2z\left[1-\left(\frac{d_1}{d_2}\right)\right]}}$ | $\mu = 1-\frac{u_2 \pi \sin\sigma_2 \left(1+\frac{d_1}{d_2}\right)}{c_{u2\infty} 2z}$ |

**Table 2: Original slip factor equations of Eck (1972) and extended equations of Stodola**

## TRANSFORMED SLIP FACTOR MODEL OF STODOLA

In the next step, the equations of Stodola are transformed to eliminate the velocity ratio $u_2 / c_{u2\infty}$. The objective is that both slip factor models should depend on the same set of quantities. This transformation can be performed based on simplified velocity triangles. It is assumed that the absolute inlet flow field is free of swirl ($\alpha_1$ = 90°). The basic assumption for the simplified velocity triangles is $\sigma_2 = \beta_1$. Under zero incidence conditions, this means that the exit blade angle is equal to the inlet blade angle. If the blade angle is constant throughout, the blade shape is a logarithmic spiral. One the one hand, this assumption restricts the study to fans with low flow coefficient and two-dimensional blades. On the other hand, impellers with logarithmic spiral blades are often assumed for the investigation of slip factor, like Busemann (1928) or von Backström (2006a). The zero incidence condition restricts the process of transformation to the design point. Figure 3 shows the simplified velocity triangles for the case of parallel end walls (b = const.).

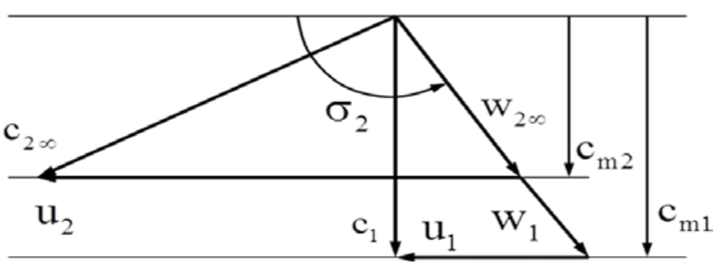



**Figure 3: Simplified velocity triangles for b = const.**

Due to the increasing flow area, the meridional velocity component $c_m$ decreases from inlet to exit. For constant density, it is

$$\frac{c_{m2}}{c_{m1}} = \frac{d_1}{d_2} = \frac{u_1}{u_2}. \tag{9}$$

Using the similarity of the velocity triangles at impeller inlet and exit, it can be shown that

$$\frac{c_{u2\infty}}{u_2} = \frac{u_2 - |w_{u2\infty}|}{u_2} = 1 - \frac{|w_{u2\infty}|}{u_2} = 1 - \frac{|w_{u2\infty}|}{u_1}\frac{u_1}{u_2} = 1 - \frac{c_{m2}}{c_{m1}}\frac{u_1}{u_2} = 1 - \left(\frac{u_1}{u_2}\right)^2 = 1 - \left(\frac{d_1}{d_2}\right)^2. \tag{10}$$

This means that the velocity ratio is related directly to the impeller inlet to exit diameter ratio. The same procedure can be applied to the simplified velocity triangles for b · r = const., which are plotted in Fig. 4. Under constant density conditions, b · r = const. means that the meridional velocity component $c_m$ is constant.

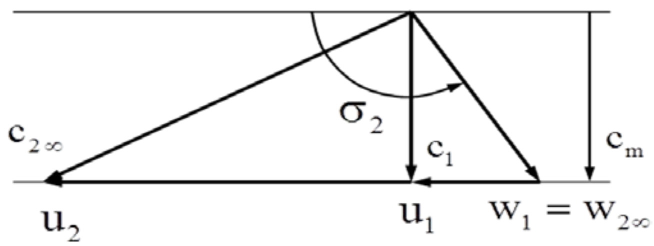



**Figure 4: Simplified velocity triangles for b·r = const.**

Now, the velocity ratio is related to the diameter ratio according to

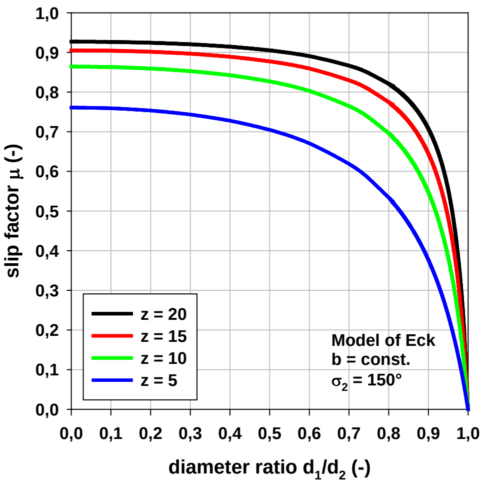
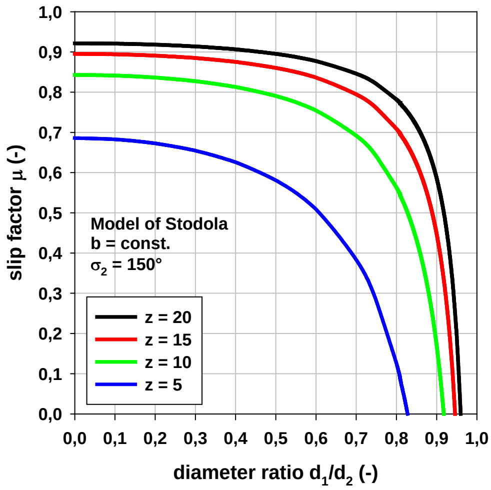
$$\frac{c_{u2\infty}}{u_2} = \frac{u_2 - |w_{u2\infty}|}{u_2} = 1 - \frac{|w_{u2\infty}|}{u_2} = 1 - \frac{u_1}{u_2} = 1 - \frac{d_1}{d_2}. \tag{11}$$

Table 3 shows the transformed equations of Stodola's model, which are now sensitive to the impeller inlet to exit diameter ratio. It can be seen that both slip factor models now depend on the same set of parameters: blade number z, blade exit angle $\sigma_2$, diameter ratio $d_1/d_2$.

| | Eck (1972) | Stodola (transformed) |
|---|---|---|
| b = const. | $\mu = \frac{1}{1 + \frac{\pi \sin\sigma_2}{z\left[1 - \left(\frac{d_1}{d_2}\right)^2\right]}}$ | $\mu = 1 - \frac{\pi \sin\sigma_2}{z\left[1 - \left(\frac{d_1}{d_2}\right)^2\right]}$ |
| b · r = const. | $\mu = \frac{1}{1 + \frac{\pi \sin\sigma_2}{2z\left[1 - \left(\frac{d_1}{d_2}\right)\right]}}$ | $\mu = 1 - \frac{\pi \sin\sigma_2 \left(1 + \frac{d_1}{d_2}\right)}{2z\left[1 - \left(\frac{d_1}{d_2}\right)\right]}$ |

**Table 3: Original slip factor equations of Eck (1972) and transformed equations of Stodola**

At this point, a direct comparison of the results of Stodola and Eck is possible. Table 4 shows a graphical representation of the equations from Tab. 3. A typical impeller blade exit angle for radial flow fans with backswept blades ($\sigma_2$ = 150°) is set. The blade number has been varied from z = 5 to 20. The lower range is typical for centrifugal pumps whereas the upper range is representative for centrifugal compressors. Typical blade numbers for radial flow fans are in the range z = 10 to 15. The slip factors in Tab. 4 are plotted versus the impeller inlet to exit diameter ratio $d_1/d_2$.

<table>
<tr><th></th><th>Eck (1972)</th><th>Stodola (transformed)</th></tr>
<tr><td>b = const.</td><td>
</td><td>
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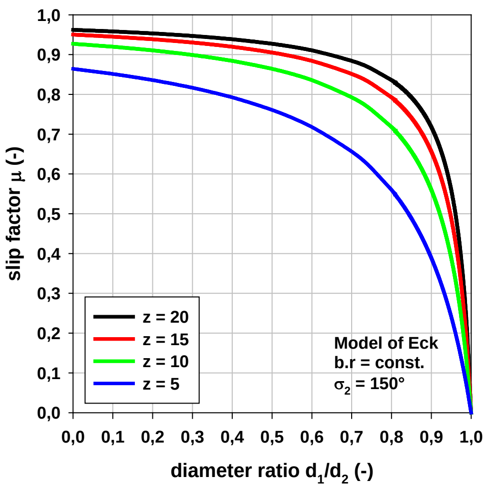
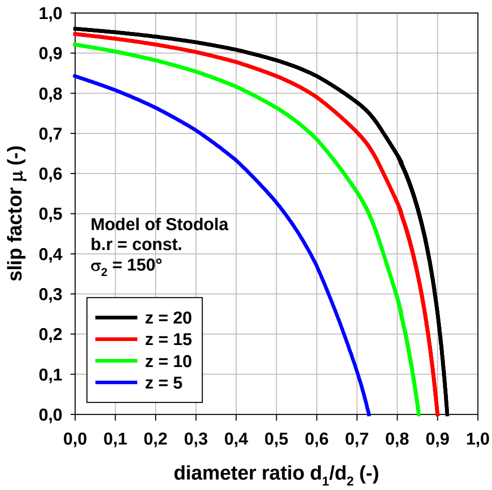
<tr><td>b · r = const.</td><td>
</td><td>
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**Table 4: Results of original slip factor equations of Eck (1972) and transformed equations of Stodola**

Some general trends can be observed. The slip factor increases with increasing blade number. The slip factor decreases with increasing impeller inlet to exit diameter ratio. At low impeller inlet to exit diameter ratios, the influence of this parameter is rather weak. Some slip factor models (Busemann, Wiesner, von Backström) propose a constant slip factor for impeller inlet to exit diameter ratios below a limiting value.

For the model of Eck, slip factors for $b \cdot r$ = const. are higher than their corresponding values for b = const. The reason is that for impellers with the same exit width $b_2$ and the same diameter ratio $d_1/d_2$, the static moment for $b \cdot r$ = const. is higher than for b = const. At the same torque, the mean blade pressure difference and, therefore, the velocity difference between suction side and pressure side at impeller exit for $b \cdot r$ = const is smaller than for b = const. This results in a lower slip velocity and a higher slip factor for $b \cdot r$ = const. compared to b = const.

For the extended model of Stodola for $b \cdot r$ = const., the angular velocity of the relative eddy at impeller exit is lower than the angular velocity of the impeller (Eq. (7)). As can be seen from Tab. 4, this results in a higher slip factor for $b \cdot r$ = const. compared to b = const. up to a limiting diameter ratio $d_1 / d_2 = \sqrt{2} - 1 \approx 0.41$. At higher diameter ratios, the slip factor is lower for $b \cdot r$ = const. compared to b = const.

As pointed out by von Backström (2006a), the slip factor should depend on blade solidity. The common definition of blade solidity is the ratio of chord length to blade spacing. In the present case, chord length is replaced by the blade length. For logarithmic spiral blades, the blade length is

$$l = \frac{d_2 - d_1}{2 \sin \sigma_2}. \tag{12}$$

The blade spacing is set at the impeller outer diameter

$$t_2 = \frac{d_2 \pi}{z}. \tag{13}$$

Under these definitions, blade solidity is

$$S = \frac{l}{t_2} = \frac{z\left(1 - \frac{d_1}{d_2}\right)}{2\pi \sin \sigma_2}. \tag{14}$$

As can be seen from Tab. 3, the slip factor according to Eck for $b \cdot r$ = const. depends only on the solidity. The other slip factors depend on solidity and also on the impeller diameter ratio $d_1/d_2$. For $d_1/d_2 \rightarrow 1$, solidity goes to zero and the model of Eck gives a zero slip factor. This is not the case for the model of Stodola, since the slip factors approach minus infinity.

## LINEARIZED SLIP FACTOR MODEL OF ECK

Finally, a mathematical manipulation is applied to the equations of Eck. It is based on the assumptions that

$$\frac{\pi \sin \sigma_2}{z\left[1 - \left(\frac{d_1}{d_2}\right)^2\right]} \ll 1 \tag{15}$$

for b = const. and

$$\frac{\pi \sin \sigma_2}{2z\left[1 - \left(\frac{d_1}{d_2}\right)\right]} \ll 1 \tag{16}$$

for $b \cdot r$ = const., respectively. This assumption is valid for high blade numbers z, large blade exit angles $\sigma_2$ (backswept blades) and low diameter ratios $d_1/d_2$. According to Eq. (14), blade solidity depends on these three quantities and Eqs. (15) and (16) are fulfilled if the blade solidity is high. In the textbook of Eck (1972) an equation is given for the optimum number of impeller blades. This equation is based on the condition that the blade solidity has to be larger than a certain value. Therefore, Eqs. (15) and (16) can be justified and a Taylor series expansion is applied to the equations of Eck from Tab. 3 with respect to the arguments in Eqs. (15) and (16). Table 5 shows the linearized equations of Eck as well as the (transformed) equations of Stodola. In spite of the differences in their underlying theories, the results are identical for b = const. and at least similar for b · r = const.

| | Eck (linearized) | Stodola (transformed) |
|---|---|---|
| b = const. | $\mu = 1 - \frac{\pi \sin\sigma_2}{z\left[1-\left(\frac{d_1}{d_2}\right)^2\right]}$ | $\mu = 1 - \frac{\pi \sin\sigma_2}{z\left[1-\left(\frac{d_1}{d_2}\right)^2\right]}$ |
| b · r = const. | $\mu = 1 - \frac{\pi \sin\sigma_2}{2z\left[1-\left(\frac{d_1}{d_2}\right)\right]}$ | $\mu = 1 - \frac{\pi \sin\sigma_2 \left(1+\frac{d_1}{d_2}\right)}{2z\left[1-\left(\frac{d_1}{d_2}\right)\right]}$ |

**Table 5: Linearized slip factor equations of Eck and transformed equations of Stodola**

## CONCLUSIONS

The main contributions of this work to the theory of slip factor are as follows: The slip factor equation of Stodola, which has been originally derived for impellers with parallel hub and shroud walls (b = const.) is extended to impellers with contoured shroud wall (b · r = const.). Since the majority of the slip factor models do not differentiate between different meridional contours, this may be one reason for their scatter in relation to test data. Using velocity triangle arguments, the equations of Stodola have been transformed to make them sensible to the impeller inlet to exit diameter ratio. This transformation is valid for impellers with logarithmic spiral blades. As a consequence, the slip factor equations of Eck and Stodola can be compared directly, since they rely now on the same set of parameters: blade number z, blade exit angle $\sigma_2$, impeller inlet to exit radius ratio $d_1/d_2$. For high blade solidity, a linearization of Eck's equations has been performed and it has been shown that they are similar, respectively identical, to the equations of Stodola.

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