Dissertation

Investment in renewable energy technologies under uncertainty

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Abstract

This thesis investigates the investment problem in renewable energy technologies (primarily wind and solar technology) under uncertain power output generation. Throughout the thesis we consider the energy manager of a firm who aims at minimizing the expected power procurement costs to cover the firm's electricity demand. In the first part of the thesis we propose a "reliability-based planning" approach and determine the optimal (investment costs minimizing) renewable energy portfolio subject to a probabilistic constraint that forces the firm to cover its demand with an ex-ante specified level of reliability. In the second part we compare this "reliability-based planning paradigm" to the "balancing-costs-based planning mechanism" in which we additionally include the expected shortfall costs in scenarios where the demand cannot be covered. In a use case we demonstrate that the optimal portfolio choice in the balancing-cost-based planning mechanism refers to a higher degree of technological diversification at lower levels of reliability compared to the optimal solution obtained in the reliability-based planning mechanism.

Up to this point all conclusions are drawn based on a static optimization framework where the energy manager faces a "now-or-never" investment decision in a setting where the level of the feed-in-tariff and the prices of the investment goods are deterministic. We loosen this assumption in the third part of the thesis and analyze the optimal investment decision in a dynamic framework under policy and technology uncertainty, where besides the optimal renewable energy choice also the optimal timing of the investment has to be determined. We apply real options theory and demonstrate in a use case that whenever the expected technological innovations are low there is little incentive to postpone the investment. That means, under these circumstances we recover the case of the previously discussed "now-or-never" investment decision. However, in case that major technological innovations are expected in solar technology, the energy manager makes use of the flexibility options available and either defers the investment decision or adopts a staged investment strategy where

he or she immediately invests a fraction of the budget available in wind technology. With increasing energy price for purchasing power in case of a shortfall in the power supply the staged investment strategy becomes increasingly important.

Zusammenfassung

In dieser Dissertation untersuchen wir das Investitionsproblem in erneuerbare Energietechnologien (primär in Wind- und Solartechnologie). Erneuerbare Energietechnologien werden zur nachhaltigen Energieversorgung eingesetzt, haben aber auch den Nachteil, dass deren Stromerzeugung unsicher ist. In dieser Arbeit betrachten wir ein Unternehmen, welches das Ziel hat, die erwarteten Strombeschaffungskosten, die zur Deckung des Strombedarfs notwendig sind, durch Investition in erneuerbare Energietechnologien zu minimieren. Im ersten Teil der Arbeit bestimmen wir das optimale Energieportfolio, welches die Investitionskosten minimiert und die zusätzliche Nebenbedingung, dass der Energiebedarf des Unternehmens durch das Energieportfolio mit einem ex-ante spezifizierten Zuverlässigkeitsniveau gedeckt werden kann, erfüllen soll. In diesem "zuverlässigkeitsbasiertem Planungsansatz" ist das Sicherheitsniveau welches verlangt wird, ein exogener Parameter. Im zweiten Teil der Arbeit vergleichen wir diesen zuverlässigkeitsbasierten Planungsansatz mit einem "ausgleichskostenbasierten Planungsansatz", in dem zusätzlich die erwarteten Kosten, die durch eine Unterdeckung entstehen, berücksichtigt werden. Durch Anwendung der Planungsmodelle in einem Use Case zeigen wir, dass beim optimalen Energieportfolio im ausgleichskostenbasierten Planungsmechanismus höhere Diversifizierungsgrade bei niedrigeren Zuverlässigkeitsniveaus als im zuverlässigkeitsbasierten Planungsmechanismus realisiert werden.

Diese Resultate basieren auf einem statischen Optimierungssetting, bei dem eine "now-or-never" Investitionsentscheidung getroffen werden soll und die Höhe des Einspeistarifes, sowie die Preise der Investitionsgüter als deterministisch angenommen werden. Diese Annahmen werden im letzten Teil der Dissertation gelockert, indem die optimale Investitionsentscheidung unter regulatorischer und technologischer Unsicherheit in einem dynamischen Optimierungsrahmen analysiert wird. Durch die Anwendung der Realoptionsanalyse bestimmen wir neben dem optimalen Energieportfolio auch den optimalen Zeitpunkt der Investition. Wir demonstrieren in einem Use Case,

dass immer dann, wenn die erwarteten technologischen Innovationen gering sind, der zuvor diskutierte Fall einer “now-or-never” Investitionsentscheidung reproduziert wird. Werden jedoch große technologische Innovationen in der Solartechnologie erwartet, ist es optimal die vorhandenen Flexibilitätsoptionen zu nutzen. Das bedeutet, dass die Investitionsentscheidung entweder verschoben wird oder eine gestaffelte Investitionsstrategie, bei der ein Bruchteil des verfügbaren Budgets früh in Windtechnologie investiert wird, optimal ist.

Papers

- Ondra, M., Dangl, T., and Hilscher, C. (2021). A probabilistically constained extension to the generation expansion problem. *Available at SSRN 3771789.* Presented at the 25th Annual Conference of the European Association of Environmental and Resource Economists (EAERE) 2020.
- Ondra, M., and Dangl, T. (2021). Optimal investment strategy in renewable energy technologies. *Available at SSRN 3742080.* Presented at the 7th Annual Conference of the French Association of Environmental and Resource Economists (FAERE) 2020.
- Ondra, M. and Dangl, T. (2021). Strategic capacity choice in renewable energy technologies under uncertainty. *Available at SSRN 3916999.* Presented at the 12. Internationale Energiewirtschaftstagung (IEWT) 2021, organized by the Austrian Association for Energy Economics (AAEE).

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1 Introduction

Electricity is an essential commodity contributing to society's welfare which posses several peculiarities that make accurate planning of electricity production and consumption – in both, the long and the short run – a difficult task. From an environmental point of view electricity generation and energy consumption are considered to have a major impact on climate change. One approach to mitigating climate change induced by anthropogenic greenhouse gases is to increase the share in renewable energy sources (RES). Therefore, renewable generation expansion planning is a key factor in policy making to cope with climate change. The worldwide cumulated renewable power capacity[1] approximately doubled from 1.2 TW in 2010 to 2.5 TW in 2019 (IRENA, 2020). Taking into account all different renewable energy technologies today, wind and solar power plants are among the most popular alternatives, because these resources are easily available.[2]

At a microeconomic level each firm participating in the liberalized electricity market has to face the power procurement problem which is considered as the problem to determine the optimal amount of power from different energy sources to cover the electricity demand at minimum possible costs (Shafieezadeh et al., 2019). Besides directly purchasing power, e.g., by negotiating contracts with energy retailers, each firm also has the opportunity to invest in renewable self-generation facilities and thereby act as a *prosumer* (Espe et al., 2018; Zafar et al., 2018) by covering its own demand, at least to some extent. Generally, a prosumer can be defined as *"an energy user who generates renewable energy in his or her domestic environment and either stores the surplus energy for future use or trades to interested energy customers"* (Rathnayaka et al., 2015).

From a private investor's perspective a major concern when investing in RES is the fact that renewable energy technologies are capital intensive with high fixed costs.

[1] The maximum net generating capacity of power plants.

[2] In 2015 wind and solar technology accounted for approximately 77% of new capacities installed, with hydropower plants covering most the rest (REN21, 2016).

Ubiquitous risks when investing in power generation facilities arise due to the fact that future cash flows are uncertain (Tietjen et al., 2016). More specifically, in this paper the authors determine various different sources of investment risks: (i) revenue risk (due to risky electricity prices), (ii) variable cost risk (due to risky fuel and carbon prices) and (iii) RES availability risk (due to uncertain production volumes).[3]

The scope of this thesis is to analyze the energy manager's investment decision in RES under uncertainty where optimally installed capacities of the renewable energy technologies have to be determined. Since each renewable energy technology exhibits specific power output distributions, each of the technologies contributes differently to the cumulated risk of demand coverage violations. Therefore, by choosing the optimal renewable energy portfolio the energy manager is able to shape the risk distribution of a shortfall in the power supply.

1.1 Methodology

1.1.1 Optimization under uncertainty

In this thesis we analyze direct investment problems in renewable energy technologies under various sources of uncertainty. Typically, the decision maker is interested in the optimal decision under uncertainty and therefore, we model the underlying decision problem using an optimization framework under uncertainty. In the first part of this thesis we consider a particular static stochastic modeling approach, also known as the probabilistically constrained optimization paradigm. The probabilistically constrained optimization paradigm was first introduced in Charnes and Cooper (1959) and is getting increasing attention also in energy-economic related problems (Geng and Xie, 2019).

The generic form of the probabilistically constrained optimization problem is as follows: We denote by $\mathbf{x} \in \mathbb{R}^n$ the decision variable, Ω denotes the set of deterministic constraints, $\mathbf{c} \in \mathbb{R}^n$ specifies the coefficients of the linear objective and $\boldsymbol{\xi} \in \mathbb{R}^d$ denotes the random vector representing the sources of uncertainties (in the frame-

[3] Soroudi and Amraee (2013) classify uncertain parameters in power system studies into two categories: (i) technical parameters and (ii) economical parameters. Technical parameters are associated with topological properties of the network and operational parameters associated with the operational decisions of the energy park. Economical parameters can be further classified into microeconomic and macroeconomic parameters.

work of the thesis, the uncertain parameters are the power output from wind and solar technology). The generic form of the probabilistically constrained optimization problem, where ϵ denotes the probability of violation, is given by

$$\begin{aligned} &\min_{\mathbf{x}\in\Omega} \mathbf{c}'\mathbf{x} \qquad \text{s.t.} \\ &\Pr\{f(\mathbf{x},\boldsymbol{\xi}) \leq 0\} \geq 1-\epsilon. \end{aligned} \tag{1.1}$$

Generally, the inner constraint $f(\mathbf{x},\boldsymbol{\xi}) : \mathbb{R}^n \times \mathbb{R}^d \to \mathbb{R}^m$ consists of m individual constraints $f(\mathbf{x},\boldsymbol{\xi}) = (f_1(\mathbf{x},\boldsymbol{\xi}),\ldots,f_m(\mathbf{x},\boldsymbol{\xi}))$. In this thesis we consider the case $m=1$, i.e., a single probabilistic constraint.[4] Hence, in this framework the stochastic inequality $f(\mathbf{x},\boldsymbol{\xi}) \leq 0$ has to hold true with probability larger than $\chi = 1-\epsilon$, where χ denotes the level of reliability. The application of probabilistic constraints is closely related to commonly used risk measures in the theory of risk management. A single probabilistic constraint can be re-written in terms of the Value-at-Risk (VaR), which is a commonly used risk-metric. Let X be a random variable with distribution function $F_X(u) = \Pr\{X \leq u\}$, where we denote by $F_X^{-1}(u)$ its inverse.[5] For a fixed confidence parameter α, the Value-at-Risk VaR_α is defined as the α-quantile

$$\mathrm{VaR}_\alpha(X) = F^{-1}(\alpha) \tag{1.2}$$

of the distribution. An analytic expression of the probabilistic constraint exists in some cases when specific parametric distribution, e.g., a normal distribution is imposed. In case that this is not possible, data-driven solution methodologies to solve the probabilistically constrained optimization problem have to be used. Two solution methodologies which provide distribution-free results are: (i) the sample approach (Calafiore and Campi, 2005; Calafiore, 2010; Campi and Garatti, 2011) and (ii) the sample average approximation (Sen, 1992; Ruszczyński, 2002; Luedtke and Ahmed, 2008; Pagnoncelli et al., 2009).

Some properties of the VaR risk-measure are, that VaR_α is translation-invariant: $\mathrm{VaR}_\alpha(X+c) = \mathrm{VaR}_\alpha(X) + c$, $\forall c \in \mathbb{R}$ and positively homogeneous: $\mathrm{VaR}_\alpha(cX) = c\mathrm{VaR}_\alpha(X)$, $\forall c > 0$. An important property of a general risk measure ρ is the sub-additivity property $\rho(X_1+X_2) \leq \rho(X_1)+\rho(X_2)$. However, the VaR generally lacks

[4]In the case $m=1$ the probabilistic constraint is referred to as a single probabilistic constraint, whereas for $m>1$ the probabilistic constraint is referred to as a joint probabilistic constraint.

[5]More specifically, the right continuous inverse $F_X^{-1}(v) = \inf\{u : F_X(u) \geq v\}$.

the subadditivity property and therefore might misinterpret diversification effects (Artzner et al., 1999). A related downside of the Value-at-Risk is, that the VaR risk metric only determines the frequency of scenarios where the constraint is not met and not the extent of constraint violation. A risk measure which fulfills this property and exhibits subadditivity – and therefore is a coherent risk measure – is the conditional Value-at-Risk CVaR_α (or the expected shortfall), which is defined for continuous distributions via the conditional tail expectation

$$\mathrm{CVaR}_\alpha(X) = \mathbb{E}[X|X \geq \mathrm{VaR}_\alpha(X)]. \tag{1.3}$$

Therefore, the CVaR includes the extent of constraint violation and furthermore can be easily implemented in optimization problems (Rockafellar et al., 2000).

1.1.2 Real options

In a static optimization framework the decision maker faces a "now-or-never" investment opportunity under uncertainty and irreversibility, i.e., the investment decision has to be made at a fixed time and cannot be postponed. However, in various applications the timing of the investment is not exogenously fixed but can be chosen by the decision maker. Such a dynamic setting that allows for deferring the investment introduces managerial flexibility which is not reflected in traditional, e.g., Net-Present-Value based approaches. Relying on this standard methodology can lead to misleading conclusions because the flexibility of waiting (i.e., deferring the investment opportunity) can have a positive value. In a financial context this flexibility can be treated as a collection of call and put options that account for the managerial flexibility to adapt later decisions (after uncertainty is revealed) and more information is available to the decision maker, see also Dixit and Pindyck (1994) and Trigeorgis et al. (1996).

In order to formalize this approach we consider a simple one-period example. Consider an energy manager who values the opportunity to invest an amount I_0 in a power plant with infinite lifetime that generates expected cash flows at t_1 of V^+, when the economy is in the good state or V^-, if the economy is in the bad state. Furthermore, we assume that $V^- < I_0 < V^+$ holds true.[6] The probability that the economy is in the good state is given by p and consequently, the probability that the economy is in

[6]This condition ensures that only in the good state of the economy investing is the optimal decision.

the bad state is $q = 1 - p$. Furthermore, we denote the interest rate by r. When the decision to invest in the project is only available at t_0, the project's NPV is given by

$$\begin{aligned} \text{NPV} &= \frac{\mathbb{E}[V]}{1+r} - I_0 = \frac{pV^+ + (1-p)V^-}{1+r} - I_0 \\ &= V_0 - I_0, \end{aligned} \tag{1.4}$$

where V_0 denotes the (gross) present value of the investment. The traditional (static) NPV investment-rule, where the option to defer the investment is not included, accepts the project whenever $\text{NPV} > 0$ and otherwise the project is rejected.

Now consider the additional flexibility to defer the investment and denote the value of the investment by $\tilde{V}$. In the good state of the economy at t_1 the value of the investment at t_1 is $\tilde{V}^+ = \max\{V^+ - I_0; 0\} = V^+ - I_0$ and in the bad state $\tilde{V}^- = \max\{V^- - I_0; 0\} = 0$. Therefore, the expected value of the investment is given by

$$\mathbb{E}[\tilde{V}] = p\tilde{V}^+ + q\tilde{V}^- = p(V^+ - I_0). \tag{1.5}$$

In this illustrative example we assume that deferring the investment decision generates additional costs c. Since the energy manager can decide whether to invest immediately at t_0 or to postpone the investment to t_1, the value of this strategy at t_0 is given by

$$\tilde{V}_0 = \max\left\{V_0 - I_0; \frac{1}{1+r}\mathbb{E}[\tilde{V}] - c\right\}. \tag{1.6}$$

This equation captures the idea of Dynamic Programming, where Bellman's optimality principle advises how to make the current optimal decision under full consideration of the conditional solution of the continuation problem, which contains the full sequence of future decisions.[7] Therefore, waiting becomes valuable (i.e., $\mathbb{E}[\tilde{V}]/(1+r) - c > V_0 - I_0$) whenever the value of the investment in the bad state of the economy is lower than the threshold value

$$V^- \leq I_0 \frac{q+r}{q} - c\frac{1+r}{q}, \tag{1.7}$$

which indicates that deferring the investment is the optimal decision whenever the potential losses in the bad state of the economy are too high. The value of the option

[7]For further information, see Dixit and Pindyck (1994).

to defer the investment – given that (1.7) holds true and waiting is indeed profitable – can be re-written as

$$\tilde{V}_0 = \text{NPV} + \underbrace{\max\left\{0; \frac{I_0(q+r) - qV^-}{1+r} - c\right\}}_{\text{option premium } \geq 0}, \tag{1.8}$$

which splits up the value of the option into the "classical" NPV plus an additional option premium that increases the value due to the addition flexibility, see also Trigeorgis et al. (1996). This simple example therefore demonstrates that under the presence of uncertainty and irreversibility, postponing the investment decision can have a positive value.

1.2 Structure of the thesis

First paper: A probabilistically constrained extension to the generation expansion problem

In the first paper (Chapter 2) we propose a probabilistic modeling approach to the generation expansion problem including renewable energy technologies, where the energy manager aims at minimizing the investment costs of an energy park subject to the stochastic supply-demand constraint. In this reliability-based framework, robust generation expansion plans are obtained by implementing the demand coverage constraint as a probabilistic constraint, specifying required system reliability. The probabilistic constraint can be equivalently written as a Value-at-Risk (VaR) constraint which characterizes the admissible set of renewable energy portfolios. Among all feasible renewable energy portfolios the energy manager prefers the one referring to the minimum investment costs. We demonstrate that an analytic solution exists when the uncertain parameter instances are assumed to be jointly normally distributed. Therefore, we recover the energy manager's efficient frontier characterizing the optimal level of investment as a function of the imposed level of reliability. Within the probabilistically constrained framework we analyze the potential of reducing the investment costs via Demand Side Management (DSM). With increasing participation in DSM the energy manager is able to reduce the necessary level of investment in RES to obtain the required level of reliability, thereby mitigating investment risks. However, in real life problems the assumption of normally distributed random variables can be too restrictive. We propose a data-driven solution methodology to the probabilistically constrained optimization problem which gives distribution free results and increases the applicability of the model. The application to a use case shows, that the optimal renewable energy portfolio associated with (i) normally distributed random variables and (ii) empirical real-world data are different. More specifically, the Gaussian assumption underestimates the tail risk associated with an investment in wind technology. This tail risk emerges due to the fact that below and above a threshold wind speed no power output from wind turbines can be generated. In the Gaussian case the optimal portfolio decision is to choose an equally diversified portfolio when required reliability is high, whereas in the data-driven solution methodology the share in solar technology is considerably higher, which serves as a hedge against this tail risk from wind-turbine characteristics.

Second paper: Optimal investment strategy in renewable energy technologies

In the second paper (Chapter 3) we extend the probabilistic view on the generation expansion problem under uncertain production volumes and compare the optimal planning problem introduced in the first paper, referred to as the "reliability-based planning" paradigm to the alternative "balancing-cost-based planning" paradigm. In the reliability-based planning paradigm, the VaR constraint specifies the admissible level of risk via an exogenous threshold on the demand coverage probability. The VaR risk measure lacks the subadditivity property and therefore might not correctly reflect upon diversification effects (Artzner et al., 1999). Moreover, the VaR only measures the frequency of scenarios violating the demand coverage requirement, but not the extent of constraint violation. We overcome these conceptual shortcomings by proposing the balancing-cost-based planning paradigm, where the energy manager also includes the expected costs of a shortfall in the power supply. In case that the energy park alone cannot cover the demand, the energy manager has to make use of an outside option and purchase additional power to cover the demand at the market. Following this modeling approach, the underlying objective associated with the investment problem can be written in terms of the conditional Vale-at-Risk (CVaR).[8] The exogenous planning parameter in this planning approach is the price of the balancing energy, where we assume that the firm is a price taker. Therefore, in the cost-based planning approach the probability of demand coverage associated with the energy park is an endogenous parameter which depends on the price of the balancing energy. We demonstrate, that even in case that the level of reliability is the same in both planning approaches, the underlying portfolio selection might differ considerably. To demonstrate this we apply the model in a use case, where we consider different scenarios concerning the price of the balancing energy. We compare the scenario of a deterministic price of the balancing energy (corresponding to the situation, where the energy manager purchases pre-contracted energy at a fixed price) to a stochastic energy price (corresponding to the situation where the energy manager purchases energy at the spot market). We show that there exists a threshold energy price below which the energy manager is reluctant to invest in renewable energy technologies. This is due to the fact, that the opportunity costs of purchasing external power are lower than the capital expenditures associated with the investment opportunity. The optimal level of investment in RES depends on the price of contracted energy and increases with increasing energy price. The scenario of purchasing external power at the spot market introduces another source of uncertainty, i.e., uncertain electricity prices. In this case the energy manager increases investment in renewable self generation facilities with increasing spot price volatility to hedge against spot price risk. These results are obtained given the assumption of an uncorrelated energy price with RES power output. In reality however, we expect that RES power output and energy price are negatively correlated, i.e., whenever RES power output is low, the energy price at the spot market is high. Incorporating the assumption of a correlated energy price we find that the energy manager's optimal decision is to increase (decrease) the optimal level of investment depending on weather the level of investment in the benchmark scenario of an uncorrelated energy price scenario is high (low).

[8] Using the CVaR in the investment problem also has the advantage, that the stochastic optimization problem can be formulated as a linear program, which can be solved efficiently (Rockafellar et al., 2000).

Third paper: Strategic capacity choice in renewable energy technologies under uncertainty

The third paper (Chapter 4) extends the investment model to a dynamic optimization framework, where besides determining the optimal renewable energy portfolio also the timing of the investment is investigated. We propose a real options approach to the energy manager's investment problem in renewable energy technologies under multiple sources of uncertainty. Therefore, we consider the combined impact of uncertain renewable energy output, policy uncertainty and technology uncertainty. Policy uncertainty is modeled by assuming that the level of the feed-in tariff is subject to multiplicative geometric Brownian shocks and technology uncertainty is introduced by assuming that the investment price of solar technology is subject to random exogenous innovation shocks. In this framework it is due to the increased managerial flexibility that the optimal investment decision can be to defer the decision and invest after the uncertainty is revealed and more information is available to the decision maker. Moreover, the investment model also allows for a staged investment strategy, where the energy manager exercises the option to invest a fraction of the budget in wind technology and keep the option to invest in solar technology alive. However, by following this investment strategy, the energy manager sacrifices a part of the flexibility options, since an early investment in wind technology excludes the opportunity to end up with a renewable energy portfolio consisting only of solar technology. The application to a use case demonstrates, that the energy manager follows a staged investment strategy whenever the expected technological innovations in solar technology are sufficiently high and the penalty for purchasing external power in case of a shortfall in the power supply from the renewable energy park is also high. Based on the energy manager's optimal investment decision in this partial equilibrium model we also infer the optimal subsidy retraction rate that is set by the regulator such that the energy manager is indifferent between investing now and to defer the investment decision.

2 A probabilistically constrained extension to the generation expansion problem [1]

Abstract. This paper presents a probabilistic modeling approach to the generation expansion problem, where the energy manager of a firm aims at minimizing the investment costs of an energy park subject to the stochastic supply-demand balance constraint. We consider a reliability-based framework, where the energy manager determines robust generation expansion plans by imposing a probabilistic guarantee on the demand coverage distribution, which specifies the required system reliability. We compare two solution methodologies to solve the probabilistically constrained generation expansion problem, i.e., (i) the sample approach and (ii) the sample average approximation. Applicability of the model is demonstrated in the use case, where the two renewable energy sources wind and solar technology are considered. We recover the energy manager's efficient frontier characterizing the optimal level of investment in renewable energy technologies as a function of the ex-ante imposed level of reliability which quantifies the substitution rate of investment and reliability. In a use case we find, that the portfolio selection depends on the required reliability and is shifted towards a higher share in solar power when required reliability increases. Within the probabilistically constrained framework we demonstrate the potential of Demand Side Management (DSM) in reducing capital expenditures. With increasing participation in DSM the energy manager is able to reduce the necessary level of investment to obtain required reliability, thereby mitigating investment risks.
Keywords: Generation expansion planning, Probabilistically constrained optimization, Risk management, Demand Side Management

[1] Joint work together with Thomas Dangl and Christoph Hilscher, Vienna University of Technology, Institute of Management Science, Theresianumgasse 27, 1040 Vienna. The full paper (Ondra et al., 2021) was presented at the EAERE 2020.

2.1 Introduction

In the course of the generation expansion problem (GEP), the energy manager of a firm has to answer several questions. Besides determining the optimal level of investment in power generation facilities, one of the most pressing questions to be answered in this context is to decide upon the optimal generation mix of the technologies (Koltsaklis and Dagoumas, 2018).[2] In this paper we consider an energy manager who faces the "here-and-now" decision of investment in self-generation facilities to cover the firm's electricity demand.[3] Therefore, an energy manager who aims at minimizing the capital expenditures associated with the energy park in a green-field approach has to constitute the optimal energy portfolio by determining the capacities to be installed in the different technologies available.

In view of a firm which adopts the transition to a green management (Shu et al., 2019), we consider an energy manager who faces the investment decision in renewable energy technologies. Investment in renewables is considerably risky due to various sources of economical, technological and policy risks. However, prices of renewable energy technologies dropped in recent years and are forecast to continue this trend (Carlsson et al., 2014). Therefore, investment in renewable energy sources (RES)[4] becomes more and more interesting for large electricity consumers or producers. One type of an energy manager participating in the energy market and whose behavior differs from a classical investor is the so-called "prosumer" (Espe et al., 2018). From the prosumer's point of view, the primary goal is to cover the firm's electricity demand at minimum possible costs via self-generation facilities like wind or solar power plants.[5]

One aspect when considering investment in RES from the prosumer's perspective is, that renewable power output is uncertain. The immanent uncertain availability of wind and solar power (RES availability risk) (Hemmati et al., 2017) affects energy planning problems. Stochastic production volumes associated with renewable energy technologies introduces uncertainty – and thus also risk – in the energy manager's investment problem (Tietjen et al., 2016). Therefore, the energy manager has to

[2] In Koltsaklis and Dagoumas (2018), the authors also consider the optimal time to build but we leave this aspect for future research and consider a static framework.
[3] Or at least a fraction of the demand that has to be supplied by self-generation facilities.
[4] We consider primarily wind and solar power.
[5] We neglect that surplus power can be sold to the grid and focus entirely on the security aspect to cover the demand.

determine expansion plans which are robust enough to hedge against unfavorable events in the future, like a shortfall in the power supply.[6] Consequently, due to the intermittent character of RES the firm's electricity demand can not be supplied with certainty, but is exposed to a certain amount of risk. Each technology included in the renewable energy portfolio exhibits a different exposure to risk, which contributes to the power shortfall probability. From the aspect of energy security, the question of determining optimally installed capacities and identifying the optimal generation mix therefore plays a significant role. Hence, the portfolio selection affects the shortfall probability.

Due to RES availability risk, power generation robustness can only be achieved to a certain level of reliability. Recently, various authors considered energy planning problems within the conceptual framework of probabilistically constrained problems (PCP) (see Geng and Xie (2019) for a review on the applications of probability constrained optimization in power systems). This probabilistically constrained paradigm was first introduced in Charnes and Cooper (1959) and later studied on a theoretical level (Prékopa, 1971; Pinter, 1989; Prekopa et al., 1998). Most applications of probabilistically constrained optimization in energy planning problems can be found in short-term economic dispatch (Vrakopoulou et al., 2013; Bienstock et al., 2014) and medium-term unit commitment problems (Bertsimas et al., 2012; Zheng et al., 2014). In generation expansion planning, the energy manager determines the capacities to be installed and it therefore is considered as a long-term energy planning problem. We consider an energy manager who makes the investment decision subject to the stochastic supply-demand constraint, which has to hold true with an ex-ante specified level of reliability. In the regime of probabilistically constrained optimization problems, the energy manager imposes a threshold confidence parameter on the demand coverage distribution. This level of reliability therefore acts as a tuning parameter in the investment decision and represents the probability of demand coverage via self-generation facilities. The majority of papers addressing the probabilistically constrained long-term generation expansion problem however, assume Gaussian distributions (Sanghvi et al., 1982; López et al., 2007; Manickavasagam et al., 2015) and use the second order cone equivalent form in the formulation of stochastic optimiza-

[6]From a macroeconomical point of view, investment in RES might also come along with some unfavorable properties. Liebensteiner and Wrienz (2020) give empirical evidence, that increased shares in intermittent RES has a negative impact on investment in flexible peak-load capacities.

tion problems to obtain computationally tractable results. Jabr (2013) presents a solution algorithm which does not require knowledge of the probability distribution associated with the uncertain parameters and considers the transmission network expansion planning problem to incorporate uncertainties of renewable power output and load via uncertainty sets.

Another research stream addressing the problem of uncertainty in energy economical problems focuses on portfolio theoretic applications in energy economics. Awerbuch and Berger (2003) consider fossil fuel price risk and show that adding a renewable energy technology in the energy portfolio consisting of conventional plants can decrease the costs at the same level of risk, or decrease the risk at the same level of the costs, see Odeh et al. (2018) for a review of portfolio applications in the electricity market. Such a diversification effect between fossil-fuel based and renewable energy technologies, which was introduced in the context of portfolio theoretic approaches in energy management in (Awerbuch and Berger, 2003; Awerbuch and Yang, 2007) via fossil fuel price risk, can also be observed in generation expansion planning of renewable energy technologies when RES availability risk is considered as the major source of uncertainty.

This paper intends to analyze the energy manager's investment decision in renewable energy technologies, when a threshold probability on demand coverage is imposed. Therefore, we consider a probabilistic modeling approach to the long-term generation expansion problem. Besides proposing an analytic solution in case of normally distributed power sources, we use a purely data-driven approach and apply non-parametric techniques which provide distribution free results and thereby allow the use of real-world output data of renewable energy technologies. First, we construct the model by implementing the probabilistic supply-demand constraint with the objective to minimize capital expenditures (CAPEX). We then set up the model in a use case using real-world output data of wind speed and solar irradiance for a typical location in Central Europe. The solution quality (its reliability) is validated ex-post based on resampled scenarios, where we compare two solution methodologies to solve the probabilistically constrained investment problem (PCIP), (i) the sample average approximation and (ii) the sample approach of Calafiore and Campi (2005). Based on the optimal solution of the probabilistically constrained optimization problem, we recover the energy manager's efficient frontier of the optimal portfolio of installed capacities as a function of the required level of reliability. The efficient portfolio frontier implicitly contains the information associated with the costs of an additional unit of reliability. Within the scope of the probabilistic modeling approach, we therefore determine the marginal rate of substitution between reliability and investment costs.

The optimal solution defines a generation plan that is robust enough to meet the reliability requirement. This efficient portfolio frontier is strictly convex, i.e., high levels of reliability require high levels of investment in order to obtain robust expansion strategies. In view of the volatile power output from RES, Demand Side Management (DSM) is considered as a valuable strategy to mitigating risks resulting in demand coverage violations. One particular strategy within DSM is Demand Response (DR), where the demand is considered to be responsive and deferrable, i.e., the demand can be reduced temporarily and increased at another time. This corresponds to a shift of critical peak demand scenarios in order to reduce the stress on the self-generation facilities. Since the demand is traditionally considered as relatively inelastic (Paterakis et al., 2017) this is possible only to some extent. Generally, DSM introduces flexibility by including the option to shifting weights in the distribution of the power shortfall, i.e., reducing the weight in the tail of the shortfall distribution. This increase in the flexibility decreases the necessary level of investment in order to obtain robust generation expansion plans.

Behboodi et al. (2016) considers the problem of determining the optimally installed capacity in wind power technology, when demand response is considered simultaneously and shows, that demand response can reduce uncertainty costs of wind technology, which is defined in terms of the producer surplus. Strbac (2008) shows that DSM increases the utility of existing plants, since the demand can be used to balance fluctuations from volatile RES power output. Pinson et al. (2014) suggests, that the flexibility of demand shifting could potentially reduce installed capacities of power utilities and therefore the necessary level of investment. Similarly, Zhang and Li (2012) argues that DSM serves as an alternative to the investment in new power plants or, that investment arising due to growing demand can be postponed (Paterakis et al., 2017).

Within the probabilistically constrained optimization approach we study the effect of Demand Side Management on the optimal investment decision in renewable energy technologies. In a risk neutral evaluation of the investment costs, the difference of the optimal level of investment in RES with and without DSM relates to the value of implementing DSM. Moreover, we show that introducing DSM ex-post, i.e., after the optimal level of investment is determined without the flexibility option to shift the peak demand, leads to an increase in the obtained level of reliability, i.e., reduces the probability of a power shortfall.

The rest of the paper is organized as follows: Section 2.2 introduces the probabilistic modeling approach using the probabilistically constrained optimization paradigm. Section 2.3 introduces and compares two approaches to solve the model, i.e., the sample average approximation and the sample approach. Section 2.4 presents the use case and Section 2.5 reflects upon the computational simulations carried out. Section 2.6 concludes the paper.

2.2 From deterministic to probabilistic generation expansion planning

In the deterministic GEP, we consider an energy manager who aims at minimizing the investment costs by choosing optimally installed capacities $\mathbf{x} \in \mathbb{R}^n$, where n denotes the number of different technologies considered in the investment scenario. The prices of the investment goods are denoted by p_i and represent the costs per one unit of capacity installed of the i-th technology. To account for different power output profiles, we denote the power available per unit of installed capacity of the i-th technology at time t by P_{it}. In the deterministic regime, the future values of the power available are assumed to be perfectly known. Therefore, the energy manager disregards uncertainties in the power output which arise in real world problems (Oree et al., 2017).

We consider a power output model which exhibits constant economies of scale. Therefore, the total power output associated with the i-th technology is given by $x_i P_{it}$ and the firm's hourly demand that has to be supplied at time t is denoted by d_t. The mathematical formulation of the deterministic GEP is given by the optimization problem of minimizing the capital expenditures (CAPEX) associated with the renewable energy portfolio, subject to the deterministic supply-demand constraint over the planning horizon which is specified by elementary hourly time intervals $t = 1, \ldots, T$. Hence, the mathematical formulation of the energy manager's deterministic investment problem is given by

$$
\begin{aligned}
&\min_{x_1,\dots,x_n} p_1x_1+\ldots+p_nx_n \qquad \text{s.t.}\\
&x_1P_{1t}+\ldots+x_nP_{nt}\geq d_t, \quad t=1,\dots,T \\
&\mathbf{x}\in\Omega.
\end{aligned}
\tag{2.1}
$$

In this formulation, the set $\Omega = \{\mathbf{x} \in \mathbb{R}^n : x_1 \geq 0, \dots, x_n \geq 0\}$ restricts the installed capacity to positive values.[7] Oree et al. (2017) discuss different sources of uncertainty that traditionally arise in the context of the GEP and potentially affect the energy manager's investment decision. Disregarding uncertain elements present in the decision process can lead to investment decisions which are infeasible or overly expensive (Beraldi et al., 2017). Therefore, stochastic modeling techniques to address the generation expansion problem are needed.

In the probabilistic modeling approach, the major source of uncertainty affecting the risk of demand coverage violations is due to uncertain production volumes and uncertain demand. Therefore, we consider the hourly RES power output per unit of installed capacity in the i-th technology P_i as random variables in the supply-demand constraint, thereby converting the deterministic into a stochastic supply-demand constraint. In this way, the intermittent and stochastic characteristics of non-dispatchable renewable energy technologies are represented in the investment decision.

The energy manager implements risk-awareness associated with RES power output by requiring that the solution has to be robust against demand coverage violations. We introduce the loss function

$$f(\mathbf{x},\mathbf{P}) = d - x_1P_1 - \ldots - x_nP_n, \tag{2.2}$$

where a positive value $f(\mathbf{x},\mathbf{P}) > 0$ denotes a shortfall of the power supply. In this case, the power available from renewable energy technologies is not sufficient to cover the demand. Conversely, a negative value of the loss function $f(\mathbf{x},\mathbf{P}) < 0$ denotes the scenario of surplus power being available. The loss itself is a random variable and its distribution is induced by the joint distribution of the demand and the power

[7] Generally, in the context of the probabilistically constrained optimization approach, the domain Ω can be used to model (convex) deterministic constraints which do not involve uncertain parameters, e.g. capacity limitations.

available from the installed capacities.

Demand coverage robustness associated with the renewable energy portfolio $\mathbf{x}$ is achieved, by requiring an ex-ante chosen level of reliability $\chi \in [0, 1)$ on the loss function, which acts as a threshold of the energy manager's requirement on the system's reliability. The level of reliability can thus be considered as a tuning parameter associated with the robustness of the solution. Therefore, the probability of demand coverage[8], has to exceed the imposed level of reliability. The mathematical formulation of the probabilistic GEP is given by

$$\begin{aligned} &\min_{x_1,\ldots,x_n} p_1 x_1 + \ldots + p_n x_n \qquad \text{s.t.} \\ &\Pr\{f(\mathbf{x}, \mathbf{P}) \leq 0\} \geq \chi, \\ &\mathbf{x} \in \Omega. \end{aligned} \tag{2.3}$$

In financial applications of probabilistically constrained optimization problems, the probabilistic constraint is referred to the Value-at-Risk (VaR) which is defined as the quantile of the underlying risk distribution. The VaR has become a popular risk measure of investment practitioners and is accepted in various financial institutes. However, a fundamental issue arising in optimization problems including the VaR risk metric is, that it is generally hard to compute unless the risk distribution is of a special parametric form, e.g., normal or log-normal (Duffie and Pan, 1997; Jorion, 2000). To allow for a distribution free result, i.e., where no parametric distribution of the random variables has to be imposed, approaches based on empirical samples are introduced e.g. in Sen (1992); Calafiore and Campi (2005); Gaivoronski and Pflug (2005).

2.3 Methodologies to solve the probabilistic GEP

First of all, we consider the approximation of Gaussian variables and derive an analytic solution to the PCIP problem (2.3). To generalize this concept to arbitrary distributions, we impose a data-driven approach that allows to use general empirical distributions. Two of the most frequently used data-driven methodologies to solve probabilistically constrained optimization problems are: (i) the sample average approximation (SAA) and (ii) the scenario approach (SA) (Geng and Xie, 2019).

[8]Measured in terms of the loss function (2.2).

2.3.1 Analytic solution of normally distributed uncertain parameters

Fist we consider the case, where the uncertain parameters associated with the power output from RES, i.e., the hourly power per one unit of installed capacity of the different technologies is normally distributed $\mathbf{P} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$. Therefore, the total power output of the energy park is again normally distributed $\mathbf{x}'\boldsymbol{P} \sim \mathcal{N}(\mathbf{x}'\boldsymbol{\mu}, \mathbf{x}'\boldsymbol{\Sigma}\mathbf{x})$. Furthermore, let the demand be normally distributed $d \sim \mathcal{N}(\mu_d, \sigma_d^2)$ and uncorrelated with the total power output of the energy park. Under these assumptions, the power shortfall (2.2) is again normally distributed $f(\mathbf{x}, \boldsymbol{P}) \sim \mathcal{N}(\mu_d - \mathbf{x}'\boldsymbol{\mu}, \mathbf{x}'\boldsymbol{\Sigma}\mathbf{x} + \sigma_d^2)$. Therefore, the probabilistic constraint can be equivalently written as[9]

$$\frac{\mathbf{x}'\boldsymbol{\mu} - \mu_d}{\sqrt{\mathbf{x}'\boldsymbol{\Sigma}\mathbf{x} + \sigma_d^2}} = \Phi^{-1}(\chi), \tag{2.4}$$

where Φ^{-1} denotes the quantile of the standard normal distribution. By taking the square of both sides we obtain that the probabilistic constraint is given in terms of a quadratic form which – in the case of two energy assets, i.e., $n = 2$ – represents a conic section. However, due to the fact that we square (2.4), we obtain an artefact solution that has to be discarded later. For this artefact solution, the probabilistic constraint (2.4) does not hold true. We find

$$\begin{gathered}
(\mathbf{x}'\boldsymbol{\mu} - \mu_d)(\mathbf{x}'\boldsymbol{\mu} - \mu_d) = \Phi^{-1}(\chi)^2(\mathbf{x}'\boldsymbol{\Sigma}\mathbf{x} + \sigma_d^2) \\
\mathbf{x}'(\Phi^{-1}(\chi)^2\boldsymbol{\Sigma} - \boldsymbol{\mu}\boldsymbol{\mu}')\mathbf{x} + 2\mu_d\boldsymbol{\mu}'\mathbf{x} + \Phi^{-1}(\chi)^2\sigma_d^2 - \mu_d^2 = 0
\end{gathered} \tag{2.5}$$

In order to rewrite this expression in terms of a quadratic constraint $\mathbf{x}'\boldsymbol{Q}\mathbf{x} + \mathbf{x}'\mathbf{b} + c = 0$, we define $\boldsymbol{Q} = \Phi^{-1}(\chi)^2\boldsymbol{\Sigma} - \boldsymbol{\mu}\boldsymbol{\mu}'$, $\boldsymbol{b} = 2\mu_d\boldsymbol{\mu}$, $c = \Phi^{-1}(\chi)^2\sigma_d^2 - \mu_d^2$. In terms of this

[9] Solving the integral in the probabilistic constraint $\Pr\{d - \mathbf{x}'\mathbf{P} \leq 0\} \geq \chi$ gives the constraint

$$\frac{\mathbf{x}'\boldsymbol{\mu} - \mu_d}{\sqrt{\mathbf{x}'\boldsymbol{\Sigma}\mathbf{x} + \sigma_d^2}} \geq \Phi^{-1}(\chi).$$

We replace the inequality by an equality sign due to the fact that the cost minimizing solution is on the boundary of the associated feasible set.

quadratic form, the energy manager's investment problem is given by

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{p}'\mathbf{x} \qquad \text{s.t.} \\ &\mathbf{x}'\boldsymbol{Q}\mathbf{x} + \mathbf{x}'\mathbf{b} + c = 0. \end{aligned} \tag{2.6}$$

Note, that the parameters of the quadratic constraint depend on the exogenously given level of reliability χ. In case of $\chi = 0.5$, the original probabilistic constraint (2.3) gives $\mathbf{x}'\mu = \mu_d$. Therefore, in this special case the investment problem reduces to a linear program

$$\begin{aligned} &\min_{\mathbf{x}} \mathbf{p}'\mathbf{x} \qquad \text{s.t.} \\ &\mathbf{x}'\mu = \mu_d. \end{aligned} \tag{2.7}$$

In this special case, the optimal solution is the single energy investment $x^* = \mu_d/\mu^*$ in the most profitable technology characterized by $p^*/\mu^* = \min\{p_1/\mu_1, \ldots, p_n/\mu_n\}$.

Proposition 2.1. *Consider the investment problem* (2.6) *based on the probabilistically constrained optimization problem* (2.3) *for $n \in \mathbb{N}$ energy assets and let the RES power output $\boldsymbol{P} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ be normally distributed with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. Furthermore, let the demand d be normally distributed $d \sim \mathcal{N}(\mu_d, \sigma_d^2)$ and uncorrelated with the power output of the renewable energy portfolio $\mathbf{x}$. Assume that the matrix of the quadratic form $\boldsymbol{Q} = \Phi^{-1}(\chi)^2\boldsymbol{\Sigma} - \boldsymbol{\mu}\boldsymbol{\mu}'$ is nonsingular and furthermore, let $\boldsymbol{b}'\boldsymbol{Q}^{-1}\boldsymbol{b} \geq 4c$[10] Then the critical points (i.e., candidates for extreme values)*

[10]For the case of one energy asset, this condition gives:

$$\phi^{-1}(\chi)^2 < \frac{\mu^2}{\sigma^2} + \frac{\mu_d^2}{\sigma_d^2}$$

and relates to the fact that the uncertainty within the system has to be sufficiently small to guarantee the existence of real valued solution candidates that meet the required reliability.

are given by

$$\mathbf{x}(\chi) = \begin{cases} \frac{\boldsymbol{Q}^{-1}}{2}\left(-\sqrt{\frac{\boldsymbol{b}'\boldsymbol{Q}^{-1}\boldsymbol{b} - 4c}{\boldsymbol{p}'\boldsymbol{Q}^{-1}\boldsymbol{p}}}\boldsymbol{p} - \boldsymbol{b}\right), & \textit{for} \quad \chi > 0.5, \\ \frac{\boldsymbol{Q}^{-1}}{2}\left(\sqrt{\frac{\boldsymbol{b}'\boldsymbol{Q}^{-1}\boldsymbol{b} - 4c}{\boldsymbol{p}'\boldsymbol{Q}^{-1}\boldsymbol{p}}}\boldsymbol{p} - \boldsymbol{b}\right), & \textit{for} \quad \chi < 0.5, \\ x^*, & \textit{for} \quad \chi = 0.5, \end{cases}$$

where $x^* = \mu_d/\mu^*$ *denotes the capacity of the most profitable energy investment, characterized by* $p^*/\mu^* = \min\{p_1/\mu_1, \ldots, p_n/\mu_n\}$.

Moreover, for the single energy investment problem $n = 1$, *the optimally installed capacity is given by*

$$x(\chi) = \frac{\mu_d \mu}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)} \times \begin{cases} \left(1 + \sqrt{1 - \frac{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}{\mu_d^2\mu^2}}\right), & \textit{for} \quad \chi > 0.5 \\ \left(1 - \sqrt{1 - \frac{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}{\mu_d^2\mu^2}}\right), & \textit{for} \quad \chi < 0.5, \\ \frac{\mu_d}{\mu}, & \textit{for} \quad \chi = 0.5. \end{cases}$$

Proof. The proof is given in the Appendix 2.7.1. $\square$

In case the matrix $\boldsymbol{Q}$ is positive semidefinite the optimization problem is a convex program. To find the global cost optimal renewable energy portfolio in the general case of n energy assets, the types of the extreme values have to be analyzed and these candidate solutions have to be compared to the corner point solutions corresponding to feasible single energy investment solutions, in order to account for the non-negativity constraint of the installed capacities. Therefore, the optimization problem can be solved by applying the Karush-Kuhn-Tucker conditions, see e.g., Boyd and Vandenberghe (2004).

Demand response programs

One strategy to mitigating risks of a shortfall in the power supply is given by the application of demand response programs (DRP). Participating in DRP shifts the demand curve to better match the power supply of intermittent power generation facilities. However, total power consumption is not reduced but the peak demand is shifted and consumed later. Therefore, demand and RES power supply become correlated with correlation parameter $\rho \geq 0$, where the value of the correlation indicates the level of participation in the DRP. A high level of correlation implies, that whenever the RES power output is low, the demand is also low, which reduces the risk of a shortfall in the power supply. Conversely, when RES power output is high, the demand is also high, which reduces surplus capacities. Therefore, a high level of correlation refers to an effective DRP since the power output can be used more efficiently. An energy manager who invests in RES and additionally adopts DRP is able to mitigate investment risks. To see this, consider the single energy investment scenario $n = 1$ and denote by $f_0 \sim \mathcal{N}(\mu_d - x\mu, x^2\sigma^2 + \sigma_d^2)$ the stochastic power shortfall without DRP, i.e., where demand and RES power output are uncorrelated and denote by $f_1 \sim \mathcal{N}(\mu_d - x\mu, x^2\sigma^2 + \sigma_d^2 - 2\rho x\sigma\sigma_d)$ the stochastic power shortfall associated with a DRP. For every choice of the installed capacity $x > 0$ and every level of participation in DRP $\rho > 0$, we observe that DRP reduces the variance of the risk distribution $\sigma^2(f_1) < \sigma^2(f_0)$, i.e., f_0 has higher uncertainty. Due to the assumption of normally distributed risk distributions with equal expected power shortfall $\mathbb{E}[f_0] = \mathbb{E}[f_1]$, this implies that f_1 second order stochastically dominates f_0 (Levy, 2015). Therefore, the energy manager prefers to adopt DRP in order to decrease the risk associated with demand coverage violations imposed by the reliability constraint. Introducing DRP in the investment decision therefore potentially reduces the risk of a shortfall in the power supply associated with the investment decision, as the optimal level of investment is decreasing with increasing level of DRP participation, i.e., increasing level of correlation. The next proposition quantifies the necessary capacities in the presence of an DRP:

Proposition 2.2. *Consider the investment problem* (2.6) *based on the probabilistically constrained optimization problem* (2.3) *for a single energy asset $n = 1$ and let the RES power output and the demand $(P, d)^T$ be jointly normally distributed, with mean $(\mu, \mu_d)^T$ and covariance matrix*

$$\Sigma = \begin{pmatrix} \sigma^2 & \rho\sigma\sigma_d \\ \rho\sigma\sigma_d & \sigma_d^2 \end{pmatrix},$$

i.e., the demand is assumed to be correlated with the RES power output with correla-

tion parameter $\rho \geq 0$. The candidates for extreme values are given by

$$x(\chi,\rho) = \frac{1}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)}$$
$$\times \begin{cases} \left((\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d) + \sqrt{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)^2 - (\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}\right) \\ \textit{for}\, \chi \geq 0.5, \\ \left((\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d) - \sqrt{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)^2 - (\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}\right) \\ \textit{for}\, \chi < 0.5. \end{cases}$$

In case of $\chi = 0.5$, the optimal solution is given by $x(0.5,\rho) = \mu_d/\mu$.

Proof. The proof is given in the Appendix 2.7.2. □

On the one hand, imposing normally distributed uncertain variables allows for an analytic solution of the investment problem. On the other hand, in real life applications the assumption of Gaussian distributions can be too restrictive, which oversimplifies the system or does not correctly reflect upon shortfall risk characteristics which depend on the shape of the risk distribution. In order to generalize this approach towards a data-driven framework where distribution free results are obtained, i.e., no assumption on the underlying distribution has to be made, more advanced numerical methodologies have to be used.

2.3.2 The sample average approximation

Geng and Xie (2019) note, that the underlying idea of using the sample average approximation to handle probabilistic constraints first appeared in Sen (1992). In this approach, the probabilistic constraint is replaced by a set of N realizations of the uncertain parameter instances $\{\mathbf{P}^{(1)}, \ldots, \mathbf{P}^{(N)}\}$, where $\mathbf{P}^{(i)}$ denotes the i-th sample of the output power of the different technologies. According to the imposed probabilistic constraint with confidence parameter χ, a renewable energy portfolio is considered as feasible when for at least $\chi \cdot 100\%$ of the sampled constraints demand coverage $f(\mathbf{x}, \mathbf{P}^{(i)}) \leq 0$ holds true. These are the scenarios where the energy manager does not observe a loss. The associated empirical probability of demand coverage for a renewable energy portfolio $\mathbf{x}$ based on the observations is given by

$$\hat{\chi}_N(\mathbf{x}) = \frac{1}{N}\sum_{i=1}^{N} \mathbf{1}_{\{f(\mathbf{x},\mathbf{P}^{(i)})\leq 0\}}, \tag{2.8}$$

where $\mathbf{1}$ denotes the characteristic function, which is defined by

$$\mathbf{1}_{\{f(\mathbf{x},\mathbf{P}^{(i)})\leq 0\}} = \begin{cases} 1 & \text{if} \quad f(\mathbf{x},\mathbf{P}^{(i)}) \leq 0 \\ 0 & \text{else.} \end{cases} \tag{2.9}$$

The required probabilistic guarantee can be achieved, by imposing that the empirical probability has to be greater than the ex-ante specified level of reliability $\hat{\chi}_N(\mathbf{x}) \geq \chi$. That is, in the sample average approximation, the probabilistic constraint is modeled via the proportion of sampled scenarios in which the energy manager observes full demand coverage (Pagnoncelli et al., 2009).

In the sample average approximation, we reformulate the estimation of the empirical demand coverage probability (2.8) via binary variables $z_m \in \{0,1\}$, $m = 1,\ldots,N$, which select the responsive and non-responsive scenarios. This corresponds to the reformulation of the problem as a mixed integer problem (Ruszczyński, 2002; Luedtke and Ahmed, 2008)

$$\begin{aligned} &\min_{\substack{x_1,\ldots,x_n \\ z_1,\ldots,z_N}} \{p_1x_1 + \ldots + p_nx_n\} \qquad \text{s.t.} \\ &x_1P_1^{(1)} + \ldots + x_nP_n^{(1)} + Mz_1 \geq d^{(1)}, \\ &\vdots \\ &x_1P_1^{(N)} + \ldots + x_nP_n^{(N)} + Mz_N \geq d^{(N)}, \\ &z_m \in \{0,1\}, \qquad m = 1,\ldots,N, \\ &\sum_{m=1}^{N} z_m \leq (1-\chi)\cdot N \\ &\mathbf{x} \in \Omega, \end{aligned} \tag{2.10}$$

where M is a large enough constant in the big-M approach. Thus, for $z_m = 1$ the constraint associated with the m-th scenario is always fulfilled, irrespective of the capacity choice in the renewable energy portfolio and corresponds to a non-responsive scenario.

Therefore, such a constraint can be ignored within the optimization problem. The cardinality constraint imposes the condition that only a fraction of $(1-\chi)\cdot 100\%$ of the scenarios are allowed to be discarded and therefore imposes a threshold on the empirical probability of constraint violation. Under some regularity conditions, the optimal value of the SAA approach and the associated optimal solution converges to its true counterpart with probability one as N approaches infinity (Ahmed and Shapiro, 2008; Pagnoncelli et al., 2009).

2.3.3 The scenario approach

In the SA based on Calafiore and Campi (2005) we use the same empirical dataset consisting of N sampled realizations of the uncertain parameters. The original proababilistically constrained optimization problem (2.3) is approximated by the associated sampled program

$$
\begin{aligned}
&\min_{x_1,\ldots,x_n} \{p_1 x_1 + \ldots + p_n x_n\} \qquad \text{s.t.}\\
&x_1 P_1^{(1)} + \ldots + x_n P_n^{(1)} \geq d^{(1)},\\
&\vdots\\
&x_1 P_1^{(N')} + \ldots + x_n P_n^{(N')} \geq d^{(N')},\\
&\mathbf{x} \in \Omega,
\end{aligned}
\tag{2.11}
$$

of size N' (we can also use the full empirical sample, i.e., $N' = N$ constraints). In the SA the solution has to be valid for all constraints entering the optimization problem. Calafiore and Campi (2005) show, that the solution of the sampled program (2.11) is feasible with probability of at least $1-\beta$, whenever the sample size is chosen such that

$$
N' \geq \frac{n}{\beta(1-\chi)} - 1 \tag{2.12}
$$

holds true.[11] This equation relates the a-priori sample size N' to the a-posteriori solution validity χ, i.e., its reliability. This a-priori bound on the sample size has been refined in Campi et al. (2009). The advantage of this approach in the context of the energy manager's investment problem is, that we can utilize the underlying linear

[11] I.e., $\Pr\{\hat{\chi} \geq \chi\} \geq 1-\beta$ holds true (Calafiore and Campi, 2005) .

structure of the supply-demand constraint and therefore end up with a simple linear program that can be solved efficiently.

In order to obtain a less conservative solution to the probabilistcally constrained optimization problem, we apply the sample and discard algorithm introduced in Campi and Garatti (2011). Therefore, we initialize the problem with the full empirical sample of N observations. The sample and discard algorithm allows for discarding a number of r scenarios according to any algorithm A, where r is given by

$$r \leq (1-\chi)N - n + 1 - \sqrt{2(1-\chi)N \ln\left(\frac{((1-\chi)N)^{n-1}}{\beta}\right)}. \tag{2.13}$$

To identify the constraints to be removed from the set of empirical observations $I = \{1, \ldots, N\}$, the algorithm A is applied to the set of initial constraints and returns an index set of the r constraints to be excluded, i.e., $A(I) = \{i_1, \ldots, i_r\}$. The feasibility result of Campi and Garatti (2011) shows, that the associated solution of the reduced problem is robustly feasible with probability of at least $1-\beta$. We apply a discard algorithm which is also suggested in Calafiore (2010) and corresponds to the iterative solution of the updated optimization problem according to the marginal costs, where in each iteration one constraint is discarded. In this procedure, we sequentially discard the constraint which refer to the highest shadow prices.[12] Since discarding binding constraints relaxes the problem, the associated objective value of the optimal solution decreases with every iteration.

2.3.4 Comparison of the data-driven approaches

The SA is generally considered as an approximation of the SAA approach in the following sense: When both procedures SA and SAA are initialized with the same empirical sample of size N, the SAA approach globally discards $(1-\chi)\cdot 100\%$ of the scenarios in an optimal way. On the other hand, the sample and discard algorithm in the SA approach allows to remove a fraction of $r/N \leq (1-\chi)\cdot 100\%$ constraints. As it can be obtained in (2.13), only in the limit of large sample sizes the number of constraints to be discarded converges to the same fraction of $(1-\chi)\cdot 100\%$ as in the SAA. However, in the SA approach the constraints are discarded according to an

[12]In order to account for feasibility in the sampled program the algorithm is equipped with a pre-solving procedure which discards constraints, where $P_{1t}^{(i)} + \ldots + P_{nt}^{(i)} = 0$ holds true.

not necessarily optimal algorithm, whereas in the SAA the constraints are discarded in a globally optimal way. Hence, the SA is considered as a suboptimal routine compared to the SAA whenever the sample size N is sufficiently large. However, this might not hold true when the sample size is small, s.t. the characteristics of the underlying distribution are not properly reflected by the empirical sample. The sample and discard algorithm in the SA approach accounts for a sampling error (which can be large in small sample sizes), where the number of constraints to be discarded is adjusted via the sample size N, see (2.13). In contrast to that, the SAA approach discards a fraction of $(1-\chi) \cdot 100\%$ of the scenarios, regardless of the sample size N. However, the data availability of wind speed and solar irradiance is generally high, s.t. a lack of data is not a problem in the use case. Therefore, as we later demonstrate in the use case, the sample size N can be chosen, s.t. sampling error is small. Another advantage of the SA approach is, that the underlying linear structure of the problem can be utilized efficiently. A small simulation study that illustrates this is given in Appendix 2.7.3.

2.4 Computational experiments

2.4.1 The use case

To demonstrate the applicability of the model as well as the data-driven solution methodology, we analyze the energy manager's decision in minimizing the investment costs of an energy park when the probability of hourly demand coverage is exogeneously fixed. We consider investment in the renewable energy technologies wind ($i = 1$) and solar power ($i = 2$) in a daytime model.[13] The demand $d = 0.1MW$ is assumed to be deterministic and constant. The associated prices of the investment goods per one unit of installed capacity are given by $p_1 = 1400€/kW$ and $p_2 = 1000€/kW$[14] (Carlsson et al., 2014).

Uncertainty in the power output is modeled by translating empirical hourly data on solar irradiance I and on wind speed v[15] via the physical energy model into supply

[13] The data are sampled in the time from 10:00-18:00 for the time span of one year, i.e., we consider seasonal variations of the power output.

[14] Approximated Values.

[15] The wind speed is measured at the ground level and extrapolated to the hub height of the wind turbine.

of power.[16] Solar power output for one unit of installed capacity is given by

$$P_{\text{solar}}(I) = \frac{I}{I_{\text{ref}}}, \tag{2.14}$$

where $I_{\text{ref}} = 1[\text{kW/m}^2]$ is the reference irradiance. The associated power output of wind power is given by[17]

$$P_{\text{wind}}(v) = \begin{cases} 0, & \text{for} \quad v \leq v_{\text{CI}} \quad \text{and} \quad v > v_{\text{CO}} \\ \frac{1}{v_{\text{RO}} - v_{\text{CI}}}(v - v_{\text{CI}}), & \text{for} \quad v_{\text{CI}} \leq v \leq v_{\text{RO}} \\ 1, & \text{for} \quad v_{\text{RO}} \leq v \leq v_{\text{CO}}. \end{cases} \tag{2.15}$$

A plot of the histogram of the power available per one unit of installed capacity of both power sources considered in the use case is given in Fig. 2.1. Due to the threshold wind speeds below and above of which no power output from wind technology is obtained, the wind power distribution exhibits the properties of a heavy tailed distribution.

2.5 Computational results

First, we analyze the manager's investment problem by assuming that the underlying distribution of the uncertain parameters is Gaussian. In order to investigate the error introduced by this approximation, we compare the optimal investment decision in the Gaussian case with the optimal investment decision based on a data-driven approach where empirical data are used. Concerning the data-driven approach using real world output data, we average $\tilde{n} = 100$ runs of the optimization problem to obtain robust results. Each run of the optimization problem is carried out with a block-bootstrapped sample of hourly values of the power available per one unit of installed capacity for one year, i.e., $N = 2880$ samples.[18] Therefore, we incorporate auto-correlation structures that are present in the empirical data in the investment problem. Moreover, to include short-term weather trend, we choose a block size of 3 days.

[16] Sources:www.soda-pro.com (solar irradiance), www.mesonet.agron.iastate.edu (wind speed), location: Schwechat, Austria, hourly data available from 2012 to 2018.

[17] The wind turbine is specified via the cut-in speed $v_{\text{CI}} = 3m/s$, the rated-output speed $v_{\text{RO}} = 11m/s$ and the cut-out speed $v_{\text{CO}} = 25m/s$.

[18] We consider $8h$ in the daytime for 12 months, where each month is assumed to have 30 days.

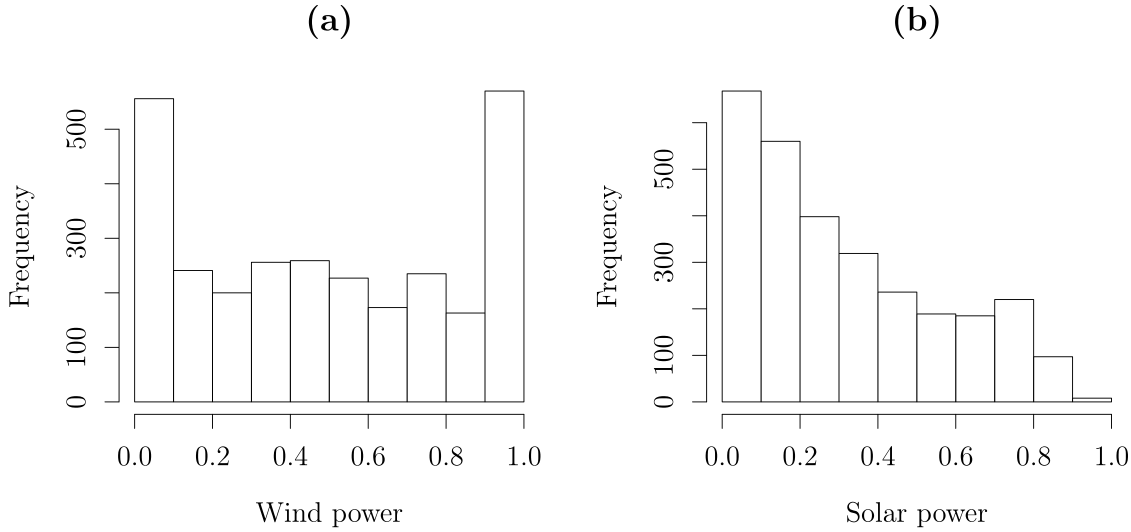

Figure 2.1: Histograms of the hourly values of the power per installed capacity in the daytime for (a) wind and (b) solar power for one sampled year. The values are given in MW for one MW of installed capacity.

2.5.1 Investment costs

Based on the solution of the energy manager's investment problem according to the different approaches proposed to solve the probabilistically constrained optimization problem, we recover the energy manager's frontier of the optimal level of investment as a function of the level of reliability. The optimal capacity choice of the renewable energy portfolio in the $i-$th optimization run $\mathbf{x}^{*(i)}$ induces the minimum expected capital expenditures in the $i-$th optimization run as a function of the ex-ante chosen level of reliability $\text{CAPEX}^{(i)} = \mathbf{p}'\mathbf{x}^{*(i)}$ for both data driven approaches. We determine the overall capital expenditures as a function of the imposed level of reliability by averaging over the $\tilde{n}$ optimization runs

$$\text{CAPEX}(\chi) = \frac{1}{\tilde{n}} \sum_i \text{CAPEX}^{(i)}(\chi), \tag{2.16}$$

which is illustrated in Fig. 2.2(a). In this plot, also the analytic solution based on the assumption of Gaussian variables is illustrated. Obviously, the energy manager increases optimally installed capacities with increasing levels of reliability. In the regime of lower levels of reliability, the analytic solution reproduces the optimal level of investment of the data-driven solution. However, when required reliability increases, the tail risks associated with the empirical distribution are not well approximated in the Gaussian case. Therefore, the underlying assumption of normally distributed uncertain parameters does not hold true and the optimal level of investment under the Gaussian assumption differs from the optimal level of investment using real-world output data.

We demonstrate that over a large range of reliability levels the efficient expansion frontier is linear, i.e., additional reliability comes at a constant price. Only at very high levels of reliability the frontier is expected to be strictly convex. The frontier implicitly contains the rate of substitution (SR) between reliability and investment costs

$$\mathrm{SR} = \frac{\mathrm{dCAPEX}}{\mathrm{d}\chi}, \tag{2.17}$$

which quantifies the expected additional investment costs for an additional unit of reliability. Considering the energy manager's conflict in interests, i.e., maximizing reliability vs. minimizing invested capital, the expected rate of substitution economically evaluates the additional required investment costs for one additional unit of reliability. The convexity of the investment frontier indicates that the marginal rate of substitution is increasing. However, for lower levels of required reliability we observe that the efficient frontier is approximately linear.

The strong increase in the optimal level of investment in response to increased required reliability can be explained by the portfolio choice of the optimal renewable energy portfolio, which is illustrated in Fig. 2.2(c) for the SAA approach. With increasing required reliability, the energy manager avoids the tail risk introduced by wind technology and consequently increases the share in solar technology. However, covering the demand with solar power requires a higher level of investment, due to less power available from solar panels in the morning and in the afternoon.

As it can also be observed in Fig. 2.2(a) the investment costs associated with the solution of the SA are higher than the investment costs associated with the SAA approach. The conservatism in the SA approach manifests itself by introducing additional contingency capacities which overall increase the invested capacity and therefore refers to higher capital expenditures.

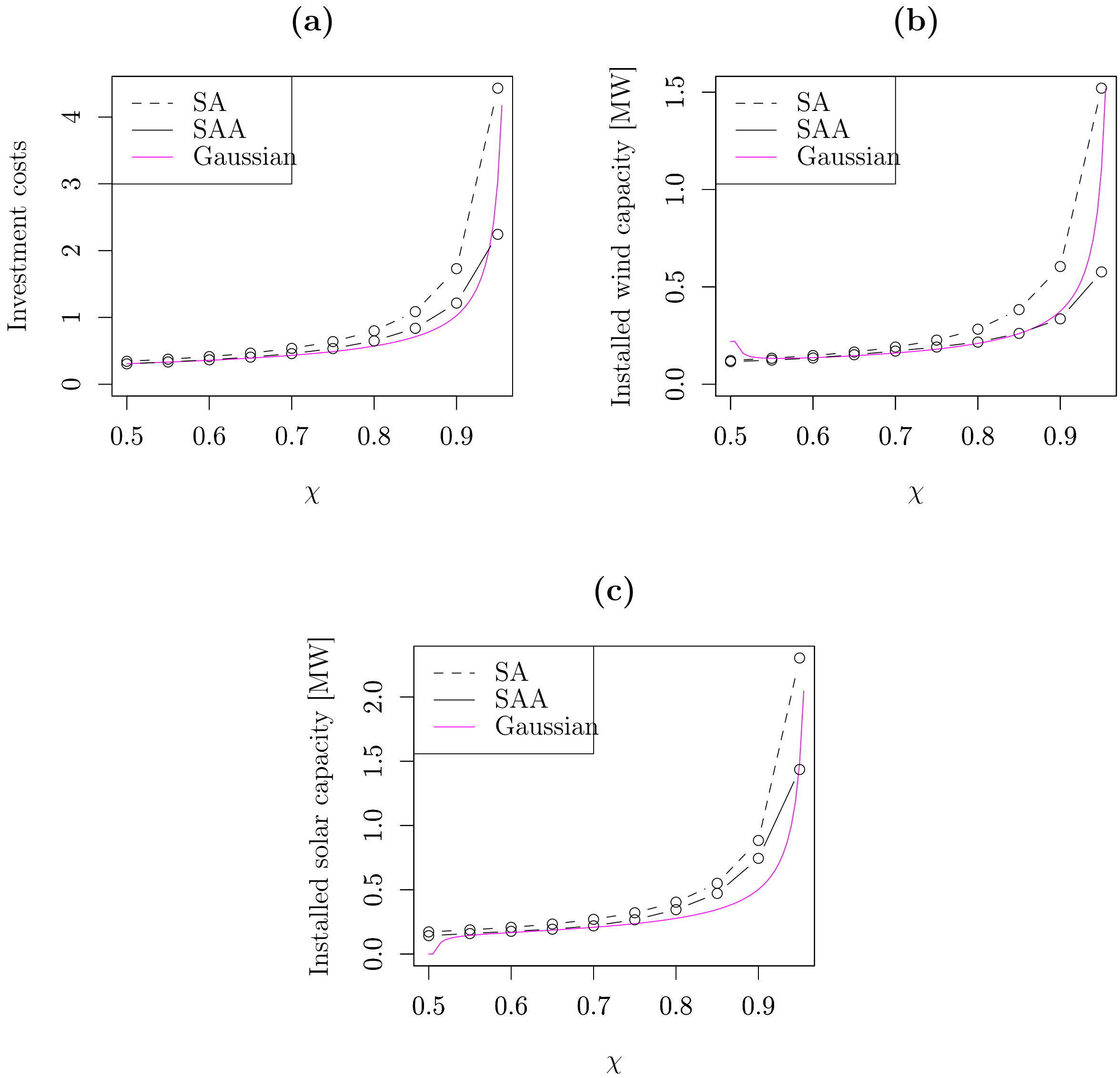

Figure 2.2: Fig. (a) shows the expected invested capital in units of 10^6€ for different levels of reliability. Fig. (b) and (c) show the installed capacities for wind and solar technology.

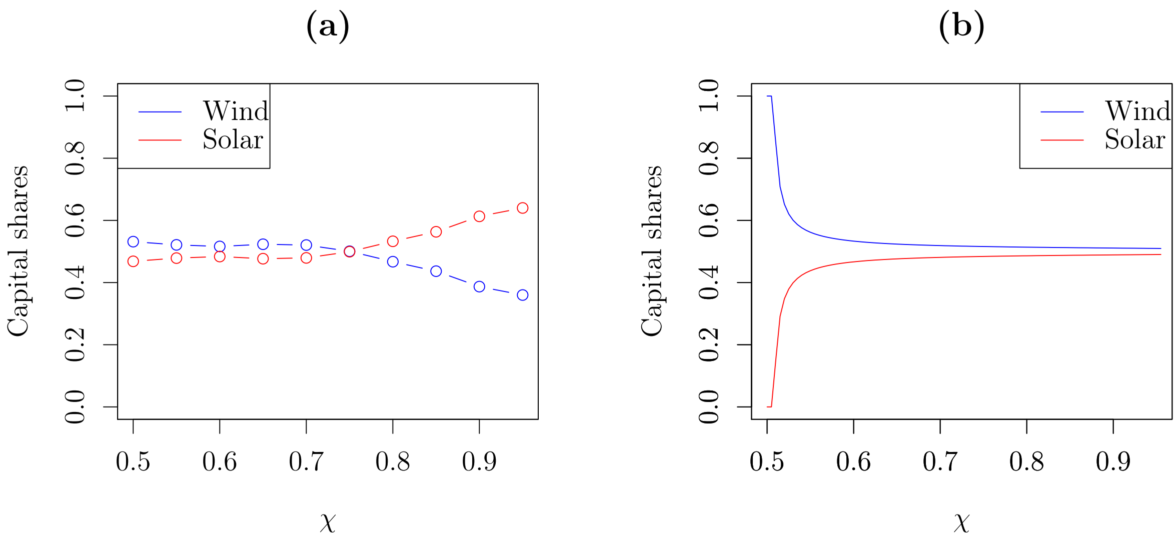

Figure 2.3: Fig. (a) shows the invested shares in the different technologies for the SAA approach and (b) for the Gaussian approximation.

2.5.2 Portfolio shares

A plot of the capacities installed in the different technologies is given in Fig. 2.2(b) for wind technology and Fig. 2.2(c) for solar technology. We observe, that the optimal renewable energy portfolio is a proper mix in the different technologies. Including the probabilistic supply-demand constraint therefore introduces a diversification effect in the portfolio selection. This diversification effect in the optimal capacities installed translates to the ex-post portfolio shares $\alpha_i(\chi) = x_i^* p_i/(x_1^* p_1 + x_2^* p_2)$, i.e., the share of invested capital[19] in the i-th technology for a given level of reliability. In Fig. 2.3 the portfolio shares are illustrated for the approximation of Gaussian variables (Fig. 2.3(b)) and for the data-driven approach of the SAA (Fig. 2.3(a)). Due to the fact, that the optimal renewable energy portfolio depends on the underlying distribution of the power shortfall the portfolio selection differs considerably when using Gaussian approximations or using real-world output data.

First, consider the case of empirical distributions. Due to differences in the empirical distribution of the power available (see Fig. 2.1), technology weights in the optimal portfolio change when requested reliability increases. We observe, that for

[19]Since the associated solutions of the optimization problem are random variables itself, the capital shares are also realizations of the underlying random variable.

low reliability levels $\chi \approx 0.5$ the energy manager's optimal decision is to opt for a diversified technology portfolio and to invest in similar shares of wind and solar technology, with slightly higher weight in favor of wind technology. Due to the low levels of reliability imposed, the tail risk emerging from the integration of wind technology is within the acceptable region.[20] However, for higher levels of reliability this tail risk becomes increasingly important. Therefore, the energy manager's optimal decision is to decrease the share in wind technology and increase the share in solar technology to reduce exposure to the tail risk associated with a power outage of wind technology.

Under the assumption of normally distributed random variables (Fig. 2.3(b)), these properties emerging from the tail characteristics of the distribution of the power shortfall are not well represented. At an imposed level of reliability of $\chi \approx 0.5$, the optimal renewable energy portfolio under the assumption of normally distributed variables is the single energy investment in wind technology. With increasing required reliability the optimal portfolio selection converges to a diversified portfolio with approximately equal shares. Therefore, although the Gaussian scenario approximates the optimal level of investment in renewable energy technologies – at least in the low-reliability regime – the tail characteristic of the shortfall distribution is not well captured and therefore refers to a different portfolio selection, especially for higher required levels of reliability.

2.5.3 Ex-post validation and runtime

The lack of the Gaussian approximation in adequately capturing the tail characteristics can also be observed in an ex-post validation of the probabilistic constraint based on resampled scenarios of the empirical data, which is illustrated in Fig. 2.4(a). Over a wide range of reliability levels, the Gaussian model underestimates the risks of a shortfall in the power supply. The SAA approach, however, reproduces on average the exact required levels of reliability. In this case the sample size is sufficiently large, s.t. the sampling error becomes small and a correction term for small samples is not needed. The fact that the SA approach is generally a suboptimal procedure compared to the SAA approach (whenever the sample size is sufficiently large) can be observed by comparing the optimal level of investment in both approaches, which is higher in the SA approach. The increased investment costs arising due to additional

[20]This risk arises from the physical energy model of wind power in (2.15), which introduces a threshold wind speed below and above of which no power can be produced from wind turbines.

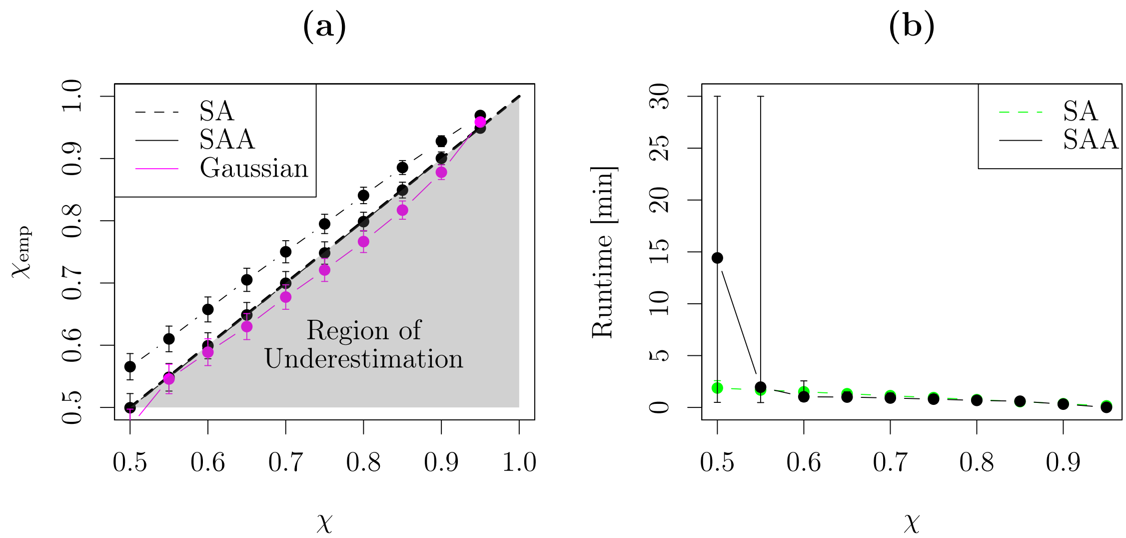

Figure 2.4: Fig. (a) shows the result of the ex-post validation. The simulations are carried out for a sample size of $N = 2880$ constraints. Fig. (b) shows a comparison of the runtime of the algorithm, where in the SAA approach, the gap tolerance was set to 6% and an upper bound on the runtime for one optimization run of 30 min is imposed. The errorbars show the min and the max value of the 100 runs of the optimization problem. The computations have been carried out on Windows 7 with a Intel Core i7 @ 2.50 GHz.

Table 2.1: Summary of the t-test to test the hypothesis that the average reliability obtained in the SA approach μ_{SA} is lower compared to the SAA approach μ_{SAA}. The model is validated for 100 resampled years.

Methodology		Level of reliability χ									
		0.5	0.55	0.6	0.65	0.7	0.75	0.8	0.85	0.9	0.95
SA:	μ_{SA}	0.57	0.61	0.66	0.71	0.75	0.79	0.84	0.89	0.93	0.97
	σ_{SA}	0.021	0.021	0.020	0.019	0.018	0.016	0.013	0.011	0.009	0.004
SAA:	μ_{SAA}	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95
	σ_{SAA}	0.023	0.022	0.021	0.020	0.019	0.018	0.015	0.013	0.010	0.006
p-value		$p < 10^{-12}$									

contingency capacities can also be observed by higher ex-post levels of reliability.

In order to quantify this results of the ex-post reliability, which can also be seen in Fig. 2.4(a), we test for the mean value of the ex-post reliability level in the SA and the SAA approach. More specifically, we compare the average level of reliability obtained in the SA approach μ_{SA} with the average reliability obtained in the SAA approach μ_{SAA} for different levels of reliability based on 100 optimization runs. For each reliability level we perform a t-test $H_0: \mu_{\mathrm{SA}} \leq \mu_{\mathrm{SAA}}$ vs. $H_1: \mu_{\mathrm{SA}} > \mu_{\mathrm{SAA}}$. The results of the test are reported in Tab. 2.1. We find the mean value of the probability of demand coverage is significantly higher in the SA approach, for a significance level of $\alpha = 0.05$.

2.5.4 Demand fluctuations and DSM

To this point, we have considered uncertainty only in the RES power output and assumed the demand d to be a fixed, i.e., deterministic value. However, generally also the demand that has to be supplied by the energy park is subject to fluctuations and therefore subject to uncertain. Hence, demand volatility introduces another source of uncertainty in the energy manager's investment decision. To illustrate the consequence of demand uncertainty in the energy manager's investment problem, assume that the global solution to the investment problem is given by the interior solution stated in Proposition 2.1 for the single energy asset case and for $\chi \geq 0.5$ (which is true in this use case). Then the optimal level of investment as a function of the required reliability $I(\chi) = px(\chi)$, for $\chi \geq 0.5$ is given by

$$I(\chi) = \frac{p\mu_d\mu}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)}\left(1 + \sqrt{1 - \frac{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}{\mu_d^2\mu^2}}\right). \tag{2.18}$$

Now, fix a level of reliability $\chi \geq 0.5$ and consider the optimal level of investment to obtain the required reliability as a function of the volatility of the demand σ_d^2. We find that

$$\frac{\mathrm{d}I}{\mathrm{d}\sigma_d^2} = \frac{p\Phi^{-1}(\chi)^2}{\mu_d\mu}\sqrt{\frac{1}{1 - \frac{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)}{\mu^2}\frac{(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}{\mu_d^2}}}. \tag{2.19}$$

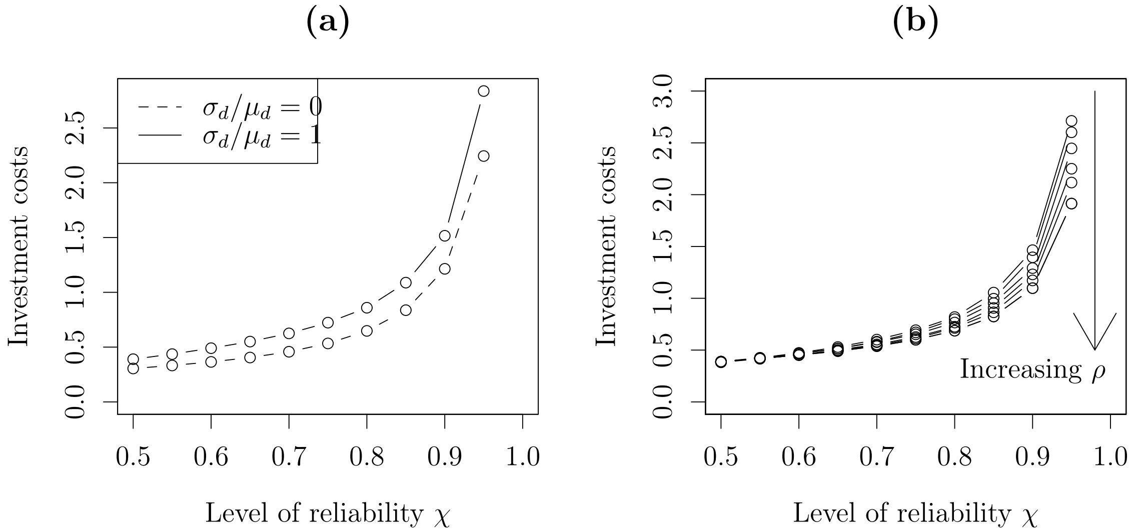

Figure 2.5: Fig. (a) shows the optimal level of investment, when the demand is assumed to be deterministic $\sigma_d = 0$ and in the case where the demand is assumed to be volatile $\sigma_d = 0.1[MW]$. Fig. (b) shows the optimal level of investment in case the demand is uncertain $\sigma_d/\mu_d = 1$ and correlated, with different correlation parameters $\rho \in \{0, 0.1, 0.2, 0.3, 0.4, 0.5\}$ between RES power output and the demand.

Therefore, the optimal level of investment is increasing with increasing uncertainty in the demand $\mathrm{d}I/\mathrm{d}\sigma_d^2 > 0$. However, this theoretical result holds true under the assumption of normally distributed uncertain parameters. The application to the use case shows, that in the data driven-approach using real-world RES output data, where the demand is assumed to be truncated normally distributed,[21] this effect also occurs, see Fig. 2.5(a). Therefore, with increasing demand uncertainty the energy manager increases optimally installed capacities to meet the required reliability and thus obtains generation expansion plans which are also robust against demand uncertainty.

To analyze the effect of DSM on the optimal level of investment, assume that the global solution to the investment problem is given by the interior solution stated in Proposition 2.2 for the single energy asset case and for $\chi \geq 0.5$. Therefore, Proposition 2.2 gives the optimally installed capacities in the one energy asset scenario when the demand is shifted according to DSM. In this case, the demand is correlated with

[21]With left truncation parameter $a = 0$ to obtain positive demand and no right truncation $b = \infty$.

the RES power output, with correlation parameter $\rho \geq 0$. Therefore, the optimal level of investment in RES technology as a function of the level of reliability and the correlation parameter for $\chi \geq 0.5$ is given by

$$\begin{aligned}\tilde{I}(\chi,\rho) = \frac{p}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)}\Big(&(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d) \\ &+ \sqrt{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)^2 - (\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}\Big).\end{aligned} \tag{2.20}$$

Thus, the marginal decrease of the optimal level of investment in response to increasing DSM effectiveness for a fixed level of reliability is given by

$$\frac{\partial\tilde{I}}{\partial\rho} = \frac{-p\sigma\sigma_d}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)}\left(1 + \frac{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)}{\sqrt{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)^2 - (\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \Phi^{-1}(\chi)^2\sigma_d^2)}}\right) \tag{2.21}$$

Therefore, we observe that whenever the ratio of the mean demand and the standard deviation of the demand as well as the ratio of the mean power available and the standard deviation of the power available is not too low,[22] i.e., for $\mu_d/\sigma_d > |\Phi^{-1}(\chi)|$ and $\mu/\sigma > |\Phi^{-1}(\chi)|$, that $\partial\tilde{I}/\partial\rho < 0$ holds true. Hence, with increasing participation in DSM the energy manager reduces the optimal level of investment and thereby is able to mitigate investment risks.

In the empirical approach we model DSM participation by introducing a correlation factor between the demand and the cumulated RES power output per unit of installed capacity. Therefore, when RES availability is low, active participation in DSM shifts the peak demand to a lower level. Using the data-driven solution methodology which incorporates real-world output data of the RES power output, we observe that the optimal level of investment is decreasing with increasing level of the correlation ρ, i.e., with increasing efficiency of DSM, see Fig. 2.5(b). Moreover, cost savings increase with increasing required reliability. Due to the risk-neutral evaluation of the investment problem the difference in the efficient frontiers with ($\rho > 0$) and without ($\rho = 0$) DSM represents the value of implementing DSM mechanism.

However, in case the energy manager does not consider DSM ex-ante and plans optimally installed capacities without the flexibility option to shift the peak demand, implementing DSM ex-post (i.e., after the optimal renewable energy portfolio is de-

[22] This conditions refer to the existence of the solution, i.e., positive installed capacities.

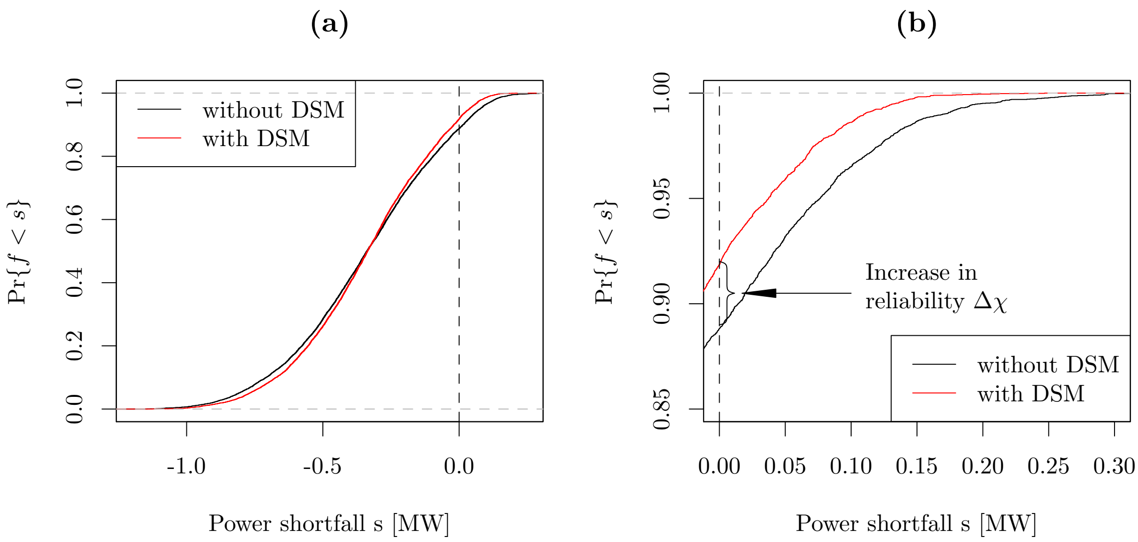

Figure 2.6: Fig. (a) shows the empirical cdf for the ex-ante optimal investment strategy associated without DSM in two cases: (i) without DSM (black line) and (ii) with ex-post DSM adjustment, Fig. (b) shows a section of plot (a) to illustrate the increase in reliability, when DSM is included. The plots are given for $\mu_d = 0.1MW, \sigma_d = 0.1MW$, $\rho = 0.5$ and $\chi = 0.9$.

termined without DSM) leads to an increase in the obtained level of reliability, i.e., an increase in the system's reliability. This is illustrated in a plot of the cumulative distribution function of the stochastic power shortfall without DSM f_0 and with implementing DSM ex-post f_1. The plot Fig. 2.6(a) illustrates the probability $\Pr\{f_i < s\}$ to obtain a power shortfall smaller than s, without DSM ($i = 0$) and with DSM ($i = 1$), based on resampled scenarios. We observe that in the regime of a power shortfall $s > 0$, the level of reliability including DSM $\chi_1 = \Pr\{f_1 < 0\}$ is increased compared to the level of reliability associated with not adopting DSM $\chi_0 = \Pr\{f_0 < 0\}$, see Fig. 2.6(b). Moreover, in the regime of surplus power $s < 0$ the probability to observe unused capacities is reduced, due to demand shifting.

2.6 Conclusion

In this paper we analyze the energy manager's investment decision in renewable energy technologies when a threshold reliability on demand coverage is required. We thereby extend the application of probability constrained programming to the long-term generation expansion problem, where we use a data-driven decision model, which allows for using empirical data. Two solution methodologies, the sample approach and the sample average approximation, are compared. Whenever the sample size is sufficiently large, s.t. the sampling error associated with small sample sizes vanishes, we demonstrate in the use case that the sample average approximation refers to solutions that are closer to the required level of reliability compared to the sample approximation. Furthermore, we derive a closed-form solution for the underlying investment model when the uncertain parameters are assumed to be Gaussian. Within the probabilistic modeling approach the energy manager's attitude towards risk can be directly incorporated in the model formulation via the ex-ante chosen level of reliability. The model supports the energy manager's decision to find the optimal investment decision such that the hourly supply-demand balance holds true with the imposed probability. The application of this model to the use case, where we consider wind and solar power plants points out, that the optimal (cost minimizing) generation mix shifts to increased shares of solar technology in response to increasing required reliability. This is due to the fact, that the optimal portfolio avoids the tail risk introduced by the wind power distribution. We recover the efficient frontier of investment in renewable energy technologies and demonstrate that over a wide range of reliability levels additional reliability comes at a constant price, whereas at higher levels of required reliability it is strictly convex. Based on this efficient expansion frontier we determine the substitution rate of the level of investment and the level of reliability and thereby evaluate the additional costs for an extra unit of reliability. Furthermore, we study the effect of demand uncertainty in the optimal investment decision. With increasing demand uncertainty, the energy manager increases investment in RES technology in order to provide robust generation expansion plans which meet the required reliability. This effect is shown in the case of a single energy investment and normally distributed random variables but we demonstrate that it is also present in the use case, i.e., using real-world RES output data. Within the probabilistically constrained framework we analyze the effect of DSM on the optimal level of investment in RES. With increasing effectiveness of DSM, i.e., with increasing correlation parameter of RES power output and demand the energy manager is able to reduce investment in RES, while still providing the required system reliability.

2.7 Appendix

2.7.1 A: Proof of Proposition 2.1

The Lagrangian associated with the optimization problem is given by

$$\mathcal{L}(\mathbf{x}, \lambda) = \mathbf{p}'\mathbf{x} - \lambda(\mathbf{x}'\boldsymbol{Q}\mathbf{x} + \boldsymbol{b}'\mathbf{x} + c). \tag{A1}$$

The first-order condition is therefore given by

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}} = \mathbf{p} - 2\lambda\boldsymbol{Q}\mathbf{x} - \lambda\mathbf{b} = \mathbf{0} \tag{A2}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \mathbf{x}'\boldsymbol{Q}\mathbf{x} + \boldsymbol{b}'\mathbf{x} + c = 0. \tag{A3}$$

Equation (A2) gives the candidates for the solution

$$\begin{aligned} \mathbf{x} &= \frac{1}{2\lambda}\boldsymbol{Q}^{-1}(\mathbf{p} - \lambda\mathbf{b}) \\ &= \frac{\boldsymbol{Q}^{-1}}{2}(\frac{\mathbf{p}}{\lambda} - \mathbf{b}). \end{aligned} \tag{A4}$$

The Lagrange multiplier can be determined by inserting into (A3) and we obtain two candidates for the solutions according to the different branches of the conic section

$$\begin{aligned} \frac{1}{4\lambda^2}\left(\boldsymbol{Q}^{-1}(\mathbf{p} - \lambda\mathbf{b})\right)'\boldsymbol{Q}\left(\boldsymbol{Q}^{-1}(\mathbf{p} - \lambda\mathbf{b})\right) + \frac{1}{2\lambda}\boldsymbol{b}'\boldsymbol{Q}^{-1}(\mathbf{p} - \lambda\mathbf{b}) + c = 0 \\ (\mathbf{p}' - \lambda\mathbf{b}')\boldsymbol{Q}^{-1}(\mathbf{p} - \lambda\mathbf{b}) + 2\lambda\boldsymbol{b}'\boldsymbol{Q}^{-1}\mathbf{p} - 2\lambda^2\boldsymbol{b}'\boldsymbol{Q}^{-1}\mathbf{b} + 4\lambda^2 c = 0 \\ \mathbf{p}'\boldsymbol{Q}^{-1}\mathbf{p} - \lambda\mathbf{p}'\boldsymbol{Q}^{-1}\mathbf{b} - \lambda\mathbf{b}'\boldsymbol{Q}^{-1}\mathbf{p} + \lambda^2\mathbf{b}'\boldsymbol{Q}^{-1}\mathbf{b} + 2\lambda\boldsymbol{b}'\boldsymbol{Q}^{-1}\mathbf{p} - 2\lambda^2\boldsymbol{b}'\boldsymbol{Q}^{-1}\mathbf{b} + 4\lambda^2 c = 0 \\ \lambda^2(4c - \boldsymbol{b}'\boldsymbol{Q}^{-1}\mathbf{b}) + \boldsymbol{p}'\boldsymbol{Q}^{-1}\mathbf{p} = 0. \end{aligned} \tag{A5}$$

Therefore, the solutions of the Lagrange multiplier are given by

$$\lambda_{1,2} = \pm\sqrt{\frac{\boldsymbol{p}'\boldsymbol{Q}^{-1}\boldsymbol{p}}{\boldsymbol{b}'\boldsymbol{Q}^{-1}\boldsymbol{b} - 4c}}. \tag{A6}$$

Since we are interested in the cost-minimal solution the Karush-Kuhn-Tucker conditions require the Lagrange multiplier to be negative (for the minimization problem). Hence, the candidate for a minimum corresponds to the Lagrange multiplier $\lambda^* < 0$ (and the candidate for a maximum is associated with $\lambda^* > 0$). The candidate for the cost-minimizing energy portfolio is

$$\mathbf{x}^* = \frac{\boldsymbol{Q}^{-1}}{2}\left(-\sqrt{\frac{\boldsymbol{b}'\boldsymbol{Q}^{-1}\boldsymbol{b}-4c}{\boldsymbol{p}'\boldsymbol{Q}^{-1}\boldsymbol{p}}}\boldsymbol{p}-\boldsymbol{b}\right). \tag{A7}$$

To check for the sufficient condition to obtain a local minimum, we have to analyze the bordered Hessian. For the case of one constraint and n variables we have to check if the last $n-1$ leading principal minors have alternating signs, beginning with $+1$. Let us call the quadratic constraint $g(\mathbf{x}) = \mathbf{x}'\mathbf{Q}\mathbf{x}+\mathbf{x}'\mathbf{b}+c$. Then the bordered Hessian is given by

$$\begin{aligned} B &= \begin{pmatrix} 0 & \nabla g(\mathbf{x}^*)^T \\ \nabla g(\mathbf{x}^*) & \lambda^* H_g(\mathbf{x}^*) \end{pmatrix} && \text{(A8)} \\ &= \begin{pmatrix} 0 & 2\mathbf{x}^{*T}\mathbf{Q}+\mathbf{b}^T \\ 2\mathbf{Q}\mathbf{x}^*+\mathbf{b} & \lambda^*\mathbf{Q}, \end{pmatrix} && \text{(A9)} \end{aligned}$$

where $H_g = \mathbf{Q}$ denotes the Hessian of the constraint g. These conditions depend on the parameters of the distribution and the imposed level of reliability have to be evaluated for the particular application. For the case of $n = 2$ energy assets, we obtain

$$B = \begin{pmatrix} 0 & 2(Q_{11}x_1^*+Q_{21}x_2^*)+b_1 & 2(Q_{12}x_1^*+Q_{22}x_2^*)+b_2 \\ 2(Q_{11}x_1^*+Q_{21}x_2^*)+b_1 & \lambda^*Q_{11} & \lambda^*Q_{21} \\ 2(Q_{12}x_1^*+Q_{22}x_2^*)+b_2 & \lambda^*Q_{21} & \lambda^*Q_{22} \end{pmatrix} \tag{A10}$$

and the condition for a minimum is given by $\det(B) > 0$.

Alternatively, we can compare this solution to a special case to find the correct sign of the solution. Hence, the candidates for solutions of the optimal renewable energy portfolio are given by

$$\mathbf{x}_{1,2} = \frac{\boldsymbol{Q}^{-1}}{2}\left(\pm\sqrt{\frac{\boldsymbol{b}'\boldsymbol{Q}^{-1}\boldsymbol{b}-4c}{\boldsymbol{p}'\boldsymbol{Q}^{-1}\boldsymbol{p}}}\boldsymbol{p}-\boldsymbol{b}\right). \tag{A11}$$

To obtain optimally installed capacities in the one energy asset scenario, consider the case $n = 1$ and use the formula from the general case:

$$x_{1,2} = \frac{\mu_d\mu}{(\mu^2-\Phi^{-1}(\chi)^2\sigma^2)}\left(1 \mp \sqrt{1-\frac{(\mu^2-\Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2-\sigma_d^2\Phi^{-1}(\chi)^2)}{\mu_d^2\mu^2}}\right). \tag{A12}$$

In order to discard the artifact solution, we note that this equation has to hold true for all possible values of σ_d^2. Therefore, it has to hold true in the special case $\sigma_d = 0$.

In this special case (A12) gives

$$x_{1,2}(\chi) = \mu_d \frac{\mu \mp |\Phi^{-1}(\chi)|\sigma}{(\mu - \Phi^{-1}(\chi)\sigma)(\mu + \Phi^{-1}(\chi)\sigma)}. \tag{A13}$$

Therefore, we have

$$x_{1,2}(\chi) = \begin{cases} \mu_d \dfrac{\mu \mp |\Phi^{-1}(\chi)|\sigma}{(\mu - |\Phi^{-1}(\chi)|\sigma)(\mu + |\Phi^{-1}(\chi)|\sigma)} & \text{for} \quad \chi \geq 0.5 \\ \mu_d \dfrac{\mu \mp |\Phi^{-1}(\chi)|\sigma}{(\mu + |\Phi^{-1}(\chi)|\sigma)(\mu - |\Phi^{-1}(\chi)|\sigma)} & \text{for} \quad \chi < 0.5. \end{cases} \tag{A14}$$

In this special case, the probabilistic constraint gives

$$\begin{aligned} \mu_d - x\mu &= -\Phi^{-1}(\chi)x\sigma \\ x(\chi) &= \frac{\mu_d}{\mu - \Phi^{-1}(\chi)\sigma}, \end{aligned} \tag{A15}$$

which can be rewritten by a case distinction

$$x(\chi) = \begin{cases} \dfrac{\mu_d}{\mu - |\Phi^{-1}(\chi)|\sigma} & \text{for} \quad \chi \geq 0.5 \\ \dfrac{\mu_d}{\mu + |\Phi^{-1}(\chi)|\sigma} & \text{for} \quad \chi < 0.5 \end{cases} \tag{A16}$$

Therefore, to obtain the correct sign the solution has to be given by

$$x(\chi) = \frac{\mu_d \mu}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)} \times \begin{cases} \left(1 + \sqrt{1 - \dfrac{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}{\mu_d^2\mu^2}}\right), & \text{for} \quad \chi \geq 0.5 \\ \left(1 - \sqrt{1 - \dfrac{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)}{\mu_d^2\mu^2}}\right), & \text{for} \quad \chi < 0.5. \end{cases} . \tag{A17}$$

2.7.2 B: Proof of Proposition 2.2

By the reproduction property of the normal distribution, the power shortfall $f(x, P) = d - xP \sim \mathcal{N}(\mu_d - x\mu, x^2\sigma^2 - 2\rho x\sigma\sigma_d + \sigma_d^2)$ is normally distributed. Therefore, the boundary of the probabilistic constraint is given by

$$x^2(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2) + x(2\rho\sigma\sigma_d\Phi^{-1}(\chi)^2 - 2\mu_d\mu) + \mu_d^2 - \Phi^{-1}(\chi)^2\sigma_d^2 = 0. \tag{B1}$$

Hence, we find that by applying the general formula (A11) for $n = 1$ and adapted coefficients

$$x_{1,2}(\chi,\rho) = \frac{1}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)} \times \left((\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d) \pm \sqrt{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)^2 - (\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)} \right), \tag{B2}$$

where again for $\rho \to 0$, we recover the uncorrelated case. A case distinction gives, similar to Proposition 2.1

$$x(\chi,\rho) = \frac{1}{(\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)} \times \begin{cases} \left((\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d) + \sqrt{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)^2 - (\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)} \right) \\ \text{for } \chi \geq 0.5, \\ \left((\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d) - \sqrt{(\mu_d\mu - \rho\Phi^{-1}(\chi)^2\sigma\sigma_d)^2 - (\mu^2 - \Phi^{-1}(\chi)^2\sigma^2)(\mu_d^2 - \sigma_d^2\Phi^{-1}(\chi)^2)} \right) \\ \text{for } \chi < 0.5. \end{cases} \tag{B3}$$

2.7.3 C: Simulation study

In order to demonstrate the effect of the sampling error for smaller sample sizes in the SA and the SAA approach we perform a small simulation study. We assume that the power available from the power facilities is normally distributed[23]. In a situation where the decision maker has access to only a small data sample[24] he or she can only weakly rely on the assumption that the observed data sample sufficiently represents the population distribution, on which the SAA approach is built. However, by using the sample and discard algorithm accounts for the sampling error by discarding a fraction of the constraints, where the number of constraints that can be removed depends on a user-specified confidence parameter β, see (2.13).[25] We validate the solution of the SA and the SAA approach, respectively, based on resampled scenarios. The results are given in Fig. 2.7. This figure demonstrates, that the SAA approach on average underestimates the true solution due to the lack of data. The SA approach, however, on average exceeds the reliability requirement. One reason for this is that the discard algorithm does not discard the constraints in a globally optimal way and thus, introduces conservatism in the solution. Another reason is that the number of constraints that can be removed accounts for a correction of the sampling error.

[23]For the two power sources, we choose: $\mu_1 = 0.4$, $\sigma_1 = 0.2$ and $\mu_2 = 0.3$, $\sigma_2 = 0.1$. Negative sampled values are replaced by 0.

[24]In this simulation we choose $N = 50$.

[25]In the simulation study $\beta = 0.4$ was chosen.

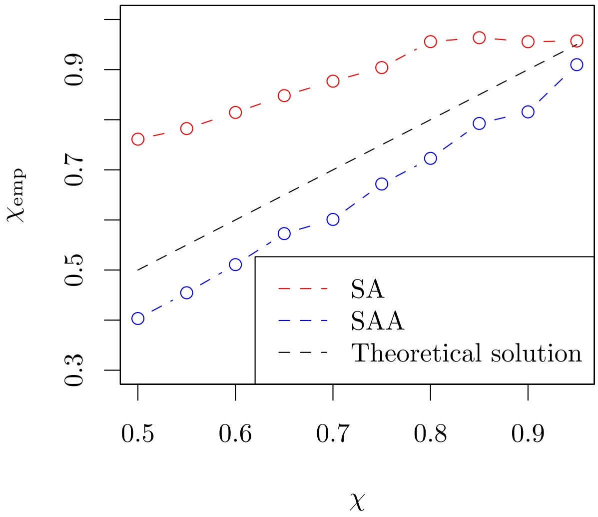

Figure 2.7: This figure shows the average level of reliability associated with the solution of the SA and the SAA approach, where in each of the 100 optimization runs the sample size (i.e., the number of constraints in the optimization problem) is $N = 50$.

Therefore, for small sample sizes, the SA approach can lead to more accurate results (in terms of the ex-post level of reliability, see also Fig. 2.4 where the ex-post level of reliability for the empirical data and a large data sample is plotted).

3 Optimal investment strategy in renewable energy technologies [1]

Abstract. This paper analyzes the energy manager's investment decision in renewable energy technologies in the presence of uncertain production volumes under different planning methodologies. We assume the energy manager to be a price taker who aims at minimizing the cost to cover the firm's electricity demand by deciding upon the optimal level of investment in renewable self-generation facilities. The "reliability-based planning paradigm", where a constraint on the probability of demand coverage is imposed, leads to different renewable energy portfolios compared to the "balancing-cost-based planning paradigm", where the price of demand coverage violations is exogeneously fixed. We analyze the energy manager's optimal investment decision in renewables in the balancing-cost-based approach for two different types of the outside option, i.e., purchasing residual power to cover the demand (i) via pre-contracted energy at a fixed price or (ii) at the balancing market with a stochastic energy price. We find that the energy manager is reluctant to invest in renewable energy technologies when the price of pre-contracted energy is below a critical threshold price, which is decreasing with decreasing prices of the investment goods. Moreover, the energy manager increases investment in renewables with increasing spot price volatility in order to hedge against spot price risk. In the presence of a negative correlation between the power output and spot price, i.e., whenever there is a shortfall in the power supply energy prices tend to be higher, the energy manager's optimal decision is to increase (decrease) the optimal level of investment depending on whether the level of investment in the uncorrelated scenario is high (low).
Keywords: Generation expansion problem; Risk management in Energy Economics;

[1] Joint work together with Thomas Dangl, Vienna University of Technology, Institute of Management Science, Theresianumgasse 27, 1040 Vienna. The full paper (Ondra and Dangl, 2021a) was presented at the FAERE 2020 and parts of it at the AIEE 2020.

3.1 Introduction

One approach to mitigating climate change induced by anthropogenic greenhouse gases is increasing the share in renewable energy sources (RES). Taking into account all available power generation facilities today, wind and solar technologies are among the most popular alternatives because these resources are available throughout the globe. A decarbonization policy which requires high shares of renewables, however, can simultaneously induce a transition to an autark energy system, at least to some degree (Tröndle et al., 2019). In this context, energy autarky is considered an idea of using global resources locally rather than a concept of isolation (Pieńkowski and Zbaraszewski, 2019). A necessary condition in this autarkic energy policy scenario is, that the potential for renewable electricity is sufficiently high to cover the demand on all national and sub-national levels. In the trend of decreasing prices of renewable energy technologies (Carlsson et al., 2014), the opportunity to invest in RES becomes increasingly valuable, not only from an ecological but also from an economical point of view. Therefore, each enterprise participating in the liberalized energy market has the option to invest in RES and thereby act as a prosumer by covering a part of its demand via self-generation facilities. The optimal level of investment in renewable energy technologies is determined in the course of the generation expansion problem.[2] The major question arising in this context is "how much to invest in which kind of technology?" such that the supply meets the demand.

From a private investor's perspective a major concern when investing in RES is given by the fact that renewable energy technologies are capital intensive with high investment costs. Portfolio theory has been applied in several studies (Awerbuch, 2000; Awerbuch and Berger, 2003; Awerbuch and Yang, 2007) which show, that adding RES in a portfolio of conventional plants with volatile fuel costs lowers portfolio risk for a given level of costs (Tietjen et al., 2016). However, approaches following this research stream have focused on fossil fuel price uncertainty solely (Odeh et al., 2018).

In a more general setting, optimal dynamic capacity expansion models for a firm have been addressed early in Manne (1961). This problem can be considered in a real options framework, in which optimal investment timing is determined (Dixit and Pindyck, 1994). Dangl (1999) discusses an investment problem, where a firm has to determine optimal investment timing and optimal capacity choice simultaneously

[2] For a review of the generation expansion problem, see Koltsaklis and Dagoumas (2018).

under demand uncertainty. A general result which highlights the effect of uncertainty is, that with higher levels of uncertainty, the firm finds it optimal to invest later in larger quantity. In this paper, however, we use a static optimization framework and focus on the optimal portfolio decision under uncertainty in a "now-or-never" setting.

When it comes to integrating RES in the generation expansion problem, the energy manager has to take into account that the associated power output of the renewable energy park is volatile, see Fig. 3.1 where the histograms of the power available per installed capacity for wind (Fig. 3.1(a)) and solar (Fig. 3.1(b)) technology are shown. The empirical distributions of the power output are generated via the physical energy model, which translates wind speed into power output of wind technology, see (3.1), and solar irradiance into the power output of solar power, see (3.2). Each renewable energy technology exhibits a specific exposure to shortfall risk. Especially the distribution of wind power per installed capacity, see Fig. 3.1(a), exploits the characteristics of a heavy-tailed distribution. This is due to the fact, that below the cut-in and above the cut-out wind speed no power can be generated due to technical limitations. Therefore, uncertain production volumes associated with RES introduces risk in the energy manager's investment problem. The shape of the distribution of the power output induces the exposure to shortfall risk, since the energy park has to supply a certain demand. Therefore, an energy manager who chooses optimally installed capacities is concerned with avoiding the tail risk introduced by investment in wind technology. In the course of the energy manager's investment decision in RES, he or she has to shape the risk distribution (induced by the joint density of the total power output), by determining the optimal renewable energy portfolio, which is defined by the installed capacities of the different technologies. This effect of risk shaping is illustrated in Fig. 3.1(c), where the power output of a renewable energy portfolio with $x_w = 1MW$ and $x_s = 2MW$ installed capacity is plotted. As it can be observed in this example, a diversified portfolio shifts the weight in the probability distribution such that the tail risk is reduced.

In the liberalized energy market where energy is traded like any other commodity, a higher penetration of renewable energy technology has an impact on electricity price variability. However, the literature is inconclusive about the overall effect. Green and Vasilakos (2010) show, that the increased use of wind technology can increase price volatility in the British electricity market. Rintamäki et al. (2017) find, that wind power decreases the daily volatility of prices by flattening the hourly price profile in

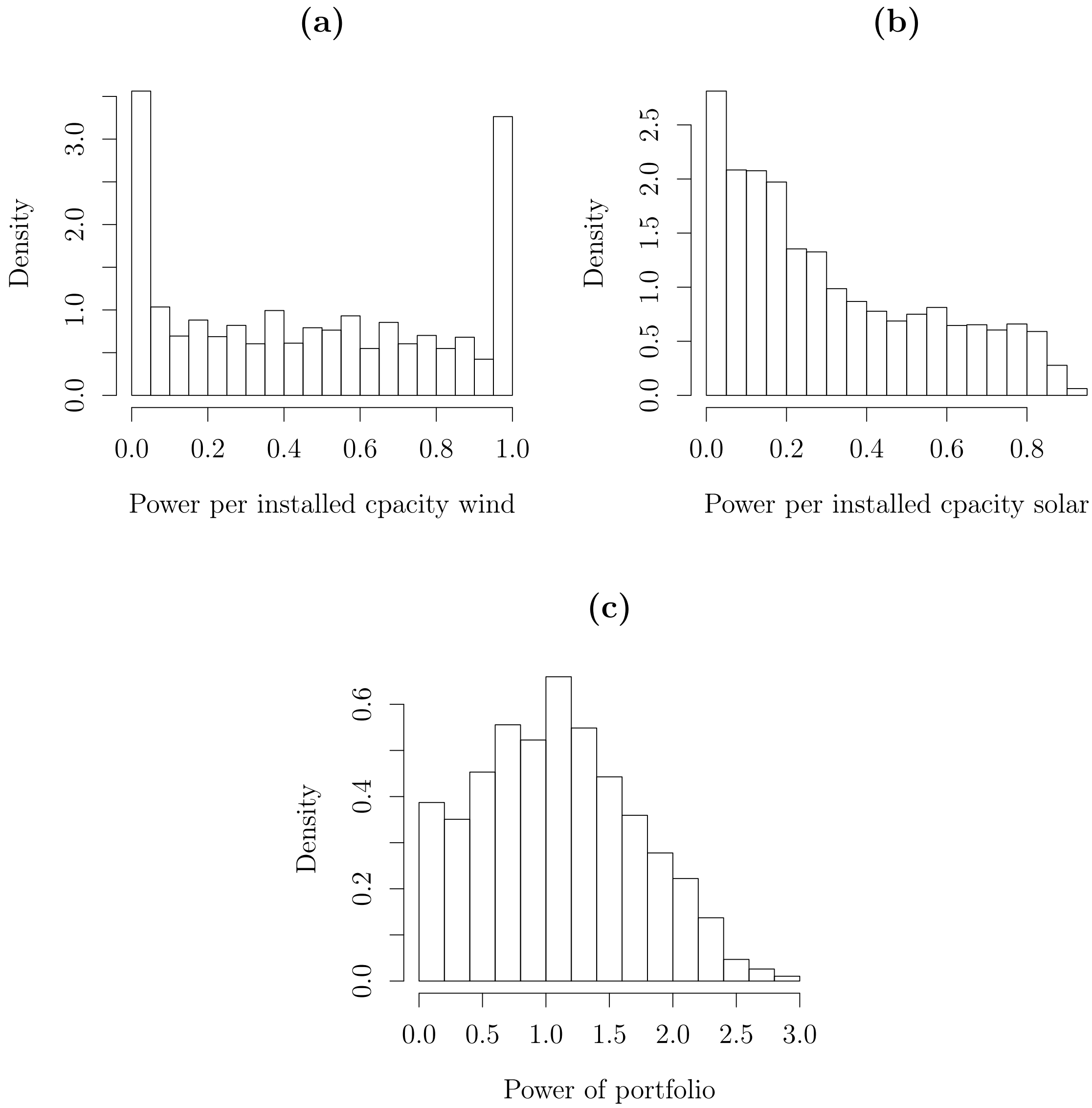

Figure 3.1: Fig. (a) shows a histogram of the wind power per installed capacity. Fig. (b) shows a histogram of the solar power per installed capacity, both for typical locations in Central Europe. Fig. (c) illustrates the distribution of the power available for an energy portfolio of 1 MW installed wind capacity and 2 MW installed solar capacity.

Denmark.

In this paper we analyze different renewable generation expansion planning mechanisms imposed by a system planner who is responsible for electricity supply of an entire electricity system. These two approaches are (i) the "reliability-based planning mechanism", where the planner imposes a threshold level of reliability on the supply-demand constraint and (ii) the "balancing-cost-based planning approach", where the planner introduces the price of the balancing energy for a constraint violation as the planning parameter. The underlying foundations of the two planning mechanisms are also used in a similar way in (Saez-Gallego et al., 2014), who compare the efficiency of transmission system planning mechanisms in order to determine optimal reserve capacities.[3] In our approach, we compare these planning mechanisms in the light of renewable generation expansion planning strategies and consider RES availability risk as a key factor on which the investment decision is based. In our approach the design variables, i.e., the installed capacities of the different technologies directly affect the risk distribution of power-shortfall and therefore introduces the need of "risk-shaping" by determining the optimal renewable energy portfolio, see Fig. 3.1.

3.1.1 The reliability-based and balancing-cost-based planning approach

We consider a planner controlling security of electricity supply by planning or regulating generation expansion of the electricity utility of the individual subsidiary system. Due to uncertain production volumes associated with renewable energy technologies, the planner incorporates risk management into strategic planning of renewable generation expansion.

In the "reliability-based planning mechanism", the planner is considered to be a decentral planner who prescribes a necessary level of reliability χ on the demand coverage distribution of the electric utility. In the reliability-based planning mechanism, the central planner establishes a minimum constraint on the reliability of the subsidiary system. The manager of the subsidiary electric utility will then determine the optimal technology portfolio which minimizes investment costs of the self-generation facilities subject to the reliability requirement.

One approach to incorporate the reliability requirement in optimization based plan-

[3]The availability of the reserve capacities is assumed to be deterministic.

ning problems is via the implementation of probabilistic constraints. The use of probabilistic constraints in stochastic optimization problems dates back to the work of Charnes and Cooper (1959). Recently, this approach also gained increasing attention in the generation expansion problem, although mostly restricted by the assumption of normally distributed random variables (Geng and Xie, 2019). Ondra et al. (2021) propose a data-driven approach to the generation expansion problem including uncertainty in the power supply. The probability of supply-demand coverage, i.e., the level of reliability $\chi \in [0, 1)$ with which the supply-demand constraint holds true, is incorporated within the energy manager's generation expansion problem via a probabilistic constraint. Generally, a probabilistic constraint can be equivalently stated as a Value-at-Risk (VaR) constraint on the risk distribution, with confidence level χ. The solution of the probabilistic generation expansion problem recovers the energy manager's efficient frontier of investment costs as a function of the imposed level of reliability. In the reliability-based planning scenario based on the VaR risk measure, the planning parameter is the exogeneously given level of reliability, which is imposed on the demand coverage distribution. In this context, the question arises whether the level of reliability is a sufficient planning parameter that contains all relevant information such that the energy manager is able to determine optimal renewable energy portfolios regarding cost efficiency? Imposing the level of reliability as the planning parameter within the model comes along with some conceptual shortcomings. First, only the frequency of scenarios violating the demand coverage constraint is considered, but not the extent of constraint violation. This is also indicated in (Saez-Gallego et al., 2014) where the authors remark, that load shedding costs are not considered in the probabilistically constrained planning approach. The VaR risk measure is therefore indifferent to extreme tails, i.e., in the scenarios where the power output of the energy park is low and thus, the power shortfall is large. Moreover, using the VaR as a risk measure comes along with the unfavorable property of the lack of subadditivity (Artzner et al., 1999). The lack of subadditivity introduces the possibility of the VaR of a portfolio to be higher than the sum of VaRs of the assets in the portfolio. Therefore, VaR might misinterpret diversification effects and investment decisions based on the VaR risk measure can lead to suboptimal solutions (Embrechts et al., 2014).

In the "balancing-cost-based planning" approach we consider the planner to be a regulator who follows a decentralized planning mechanism. In this decentralized planning approach, the regulator requires that the energy manager realizes an integrated view of the total costs associated with the electric utility in the investment decision. Therefore, he or she has to account for the expected expenses in case of a shortfall in the power supply, which are not in the scope of the reliability-based planning approach. However, in the decentralized approach the planner does not impose the reliability requirement, such that the energy manager can autonomously determine the optimal level of reliability. Hence, in the balancing-cost-based planning approach reliability is not an explicit objective imposed on the subsidiary but an implicit choice of decentralized planning. And even if – by coincidence – both planning approaches result in the same sub-system reliability, the underlying portfolio selection might differ considerably. The expected shortfall costs in the power supply are given by the price of the balancing energy, ξ. Therefore, the planning parameter in the balancing-cost-based planning framework is the price of the balancing energy. This corresponds to introducing a penalty on the violation of demand coverage, when the energy manager has to make use of an outside option and purchase residual power to cover the demand at the balancing market. On the one hand, with increasing investment in renewable self-generation facilities the energy manager can cover the demand via self-generation facilities with a higher probability. Hence, additional payments which come as the expected costs of a power shortfall are reduced. On the other hand, imposing high levels of reliability requires the installation of high capacities which are used on rare occasions and therefore come as idle costs. An optimal decision therefore must balance costs and economic benefits associated with a certain level of system reliability, χ. Evaluating the capitalized costs of making use of an outside option in case of a shortfall in the power supply consequently endogenizes[4] the choice of the optimal level of reliability of supply-demand coverage of the energy park, which is illustrated in Fig. 3.2. A low level of reliability $\chi = 0$, which is obtained whenever the price of balancing energy is also low, corresponds to the energy manager's procurement policy of purchasing total power at the market to cover the demand and to refuse investment in self-generation facilities. With increasing prices of the balancing energy, the energy manager increases the level of investment in RES to supply the demand via self-generation facilities and therefore chooses a higher level

[4]Ovaere et al. (2019) shows, that the value of lost load which corresponds to the price of the balancing energy affects the level of reliability when total costs of the energy system consist of reliability costs and interruption costs.

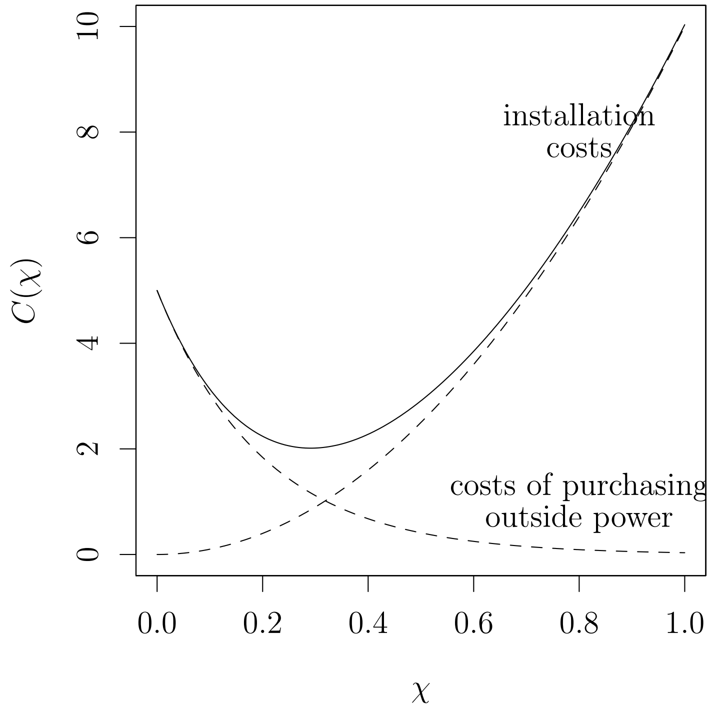

Figure 3.2: Illustration of the total costs associated with the energy park as a function of the confidence parameter χ of the supply demand constraint.

of reliability. An energy manager who aims at minimizing the total costs of the energy park determines the optimal level of reliability, when the costs for an additional unit of reliability equals the expected reduction of costs for purchasing additional power at the balancing market in case of a shortfall in the power supply. Hence, based on the price of the balancing energy the energy manager determines the optimal level of investment and therefore also the optimal level of reliability. Therefore, prescribing a necessary level of reliability associated with the power output, as it is the case in the reliability-based planning approach, affects the energy manager's decision and he or she chooses a suboptimal renewable energy portfolio. Thus, incorporating the price of balancing energy instead of the level of reliability itself yield efficient renewable energy portfolios and therefore the price of the balancing energy is a sufficient planning parameter.

Moreover, another motivation to discuss optimal portfolio selection within the balancing costs based planning approach is, that not every firm is willing to invest in capital intensive renewable self-generation facilities to cover the demand, but to make use of an outside option and, e.g., purchase power at the balancing market or negotiate contracts with retailers (Gómez-Villalva and Ramos, 2004). However, besides uncertain production volumes of RES, volatile spot prices introduce another source of uncertainty in the investment decision and forward contracts can be used to hedge against spot price volatility. Vehviläinen and Keppo (2003) discuss the importance of using risk management techniques to manage electricity market price risk. An early application of using forward contracts as risk sharing instruments for spot price risks in the electricity market is conducted in Kaye et al. (1990). Woo et al. (2004b) consider an electricity distribution company and approach the problem of determining the optimal amount of forward electricity to reduce the exposure to inherent risks of spot price volatility. Based on this model, an efficient frontier of tradeoff between expected cost and cost risk measured in terms of cost variance is constructed in Woo et al. (2004a). In this constrained least cost setting however, the authors do not include the option to invest in self-generation facilities. Bjorgan et al. (1999) discuss hedging using future contracts and also investigate how production scheduling of non-intermittent technologies can be used to reduce overall risk, where stochastic input variables are assumed to be normally distributed. Conejo et al. (2008) consider an existing energy park with thermal power plants and addresses the problem of optimal investment in the electricity futures market, where price uncertainty is described by a set of scenarios.

This paper contributes to the existing literature by analyzing the energy manager's investment decision in renewable energy technologies, in the presence of uncertain production volumes, when the outside option to purchase power at the market also exists. We analyze the optimal renewable portfolio selection in the balancing-cost-based planning approach, where the price of balancing energy is exogenously fixed. Therefore, investment in RES comes as opportunity costs of a shortfall in the power supply. We determine the energy manager's optimal level of self coverage of the demand, characterized by the energy park's level of reliability which acts as a threshold probability on the supply-demand constraint. We compare the reliability-based planning approach with the cost-based balancing approach in a use case and show that the energy manager chooses different renewable energy portfolios, depending on the type of the regulatory mechanism. Moreover, we analyze the optimal renewable energy portfolio in the balancing-cost-based approach, when the energy price is assumed to be uncertain and show that the energy manager increases investment in RES in response to an increase in the energy price volatility, when the spot price is assumed to be uncorrelated with RES power output. We analyze the effect of a correlated energy price on the level of investment, by simulating different levels of correlation of the energy price and RES power output. Thereby, we observe two different scenarios:

(i) The energy manager increases the optimal level of reliability (i.e., the probability of covering the demand via RES) in response to increasing levels of the correlation of the energy price and the power shortfall, which corresponds to an insurance effect. This effect occurs whenever the optimal level of investment in the benchmark scenario of an uncorrelated energy price is high.

(ii) The energy manager decreases the optimal level of reliability in response to increasing correlation in order to avoid scenarios, where power is procured power from capital intensive RES although the energy price is low. This effect occurs, whenever the optimal level of investment in the benchmark scenario of a uncorrelated energy price is low.

The rest of the paper is organized as follows. In Section 3.2 we introduce the formal model. Section 3.3 presents the use case and illustrates the results. Section 3.4 concludes the paper.

3.2 The model

We divide the construction of the model into two parts. In the first part, we consider the reliabiltiy based planning approach, where an exogenous probability of demand coverage χ is imposed. We obtain the energy manager's investment decision in renewable energy technologies as a function of the exogenously given level of reliability. As a result we recover the efficient generation-portfolio frontier, that allows to characterize (i) the optimal generation mix and (ii) the marginal cost for an additional unit of system reliability. In the second part of the model construction we extend this approach to the balancing-cost-based planning approach and additionally include the expected capitalized costs of making use of an outside option in case of a shortfall in the power supply of the energy park within the evaluation of the total costs. In

Table 3.1: Notation.

χ	Level of reliability
$\mathbf{x}^-$	Optimal technology portfolio in the reliability-based planning approach [$MW\ inst.$]
$\mathbf{x}^+$	Optimal technology portfolio in the cost-based-planning approach [$MW\ inst.$]
P_i	Power per installed capacity of the i-th technology [$MW/MW\ inst.$]
p_i	Price per installed capacity of the i-th technology [€$/MW\ inst.$]
d	Demand [MW]
ξ	Price of balancing energy [€$/MWh$]
X	Power shortfall [MW]
ΔT ...	Expected useful life time of the energy park [y]
$\delta(\mathbf{x})$ ...	Expected excess payments for purchasing residual power in case of a power shortfall of the renewable energy portfolio $\mathbf{x}$ at the market [€]
$I(\mathbf{x})$ ...	Capital expenditures to install the renewable energy portfolio $\mathbf{x}$ [€]
$C(\chi)$ ..	Total costs of the energy park at the level of reliability χ [€]

the cost-based-planning approach the penalty for constraint violation ξ, i.e., the price for making use of an outside option is exogenous. Table 3.1 shows the notation used throughout the paper.

3.2.1 General framework

A central aspect in this paper is to analyze the energy manager's optimal investment decision in renewable energy technologies under uncertain production volumes. The power output of RES is uncertain due to the dependence on the environmental conditions, i.e., the wind speed v in case of wind power technology and the solar irradiance I in case of solar power. Each technology exhibits a specific exposure to shortfall risk.[5] For two of the most popular renewable energy technologies, the power per

[5]Which is introduced by the physical power model that translates the environmental conditions into the power output.

installed capacity is given by

$$P_{\mathrm{w}}(v) = \begin{cases} 0, & \text{for} \quad v \le v_{\mathrm{CI}} \quad \text{and} \quad v > v_{\mathrm{CO}} \\ \frac{v - v_{\mathrm{CI}}}{v_{\mathrm{RO}} - v_{\mathrm{CI}}}, & \text{for} \quad v_{\mathrm{CI}} \le v \le v_{\mathrm{RO}} \\ 1, & \text{for} \quad v_{\mathrm{RO}} \le v \le v_{\mathrm{CO}} \end{cases} \tag{3.1}$$

for wind technology[6] and

$$P_{\mathrm{s}}(I) = \frac{I}{I_{\mathrm{ref}}} \tag{3.2}$$

for solar technology[7]. The economies of scale of such an investment in renewable energy are (approximately) constant over a the range of capacity that we want to consider[8] The power output associated with an installed capacity of x_w in wind technology and x_s in solar technology is given by $P'_w = x_w P_w$ for wind technology and $P'_s = x_s P_s$ for solar technology, respectively. The joint density of wind and solar power output of one unit of capacity installed in each technology is denoted by $f(P_w, P_s)$. Hence, the joint distribution of wind and solar output when x_w units of wind and x_s units of solar power is installed is given by

$$f'(P'_w, P'_s) = \frac{1}{x_w x_s} f\left(\frac{P'_s}{x_s}, \frac{P'_w}{x_w}\right). \tag{3.3}$$

The total power output of the energy park is given by the cumulated power output of the power sources, $P' = P'_w + P'_s$. Therefore, the probability to obtain a total power output smaller than z can be derived via the convolution

$$\Pr\{P' \le z\} = \int_{-\infty}^{\infty} \int_{-\infty}^{z-P'_s} \frac{1}{x_s x_w} f\left(\frac{P'_s}{x_s}, \frac{P'_w}{x_w}\right) \mathrm{d}P'_w \mathrm{d}P'_s. \tag{3.4}$$

[6]In this model the wind turbine is specified via the cut-in speed $v_{\mathrm{CI}} = 3m/s$, the rated-output speed $v_{\mathrm{RO}} = 11m/s$ and the cut-out speed $v_{\mathrm{CO}} = 25m/s$.

[7]$I_{\mathrm{ref}} = 1[\mathrm{kW/m}^2]$ denotes the reference irradiance.

[8]Constant economies of scale are only violated if the area of the energy park is large such that wind speed and solar irradiance vary considerably between different locations in the park.

Consequently the joint density of the total power output is given by

$$f(z) = \frac{\mathrm{d}\,\mathrm{Pr}\{P' \leq z\}}{\mathrm{d}z} = \int_{-\infty}^{\infty} \frac{1}{x_s x_w} f\left(\frac{P'_s}{x_s}, \frac{z - P'_s}{x_w}\right) \mathrm{d}P'_s$$
$$= \int_{-\infty}^{\infty} \frac{1}{x_s x_w} f\left(\frac{z - P'_w}{x_s}, \frac{P'_w}{x_w}\right) \mathrm{d}P'_w, \tag{3.5}$$

which can be obtained either by integrating over the solar or the wind power distribution. The general framework of the model is, that the energy manager can shape the risk distribution of the total power output of the energy park by choosing the installed capacities of the different technologies and thereby control the distribution of the total power output. Therefore, the energy manager chooses optimally installed capacities of renewable energy sources based on the joint density of the solar and wind power output distribution f.

3.2.2 Reliability-based planning approach

In the reliability-based planning approach to the generation expansion problem we consider a scenario, where the energy manager is instructed to determine the minimum costs of investment in an energy park, such that the firm's electricity demand can be covered with an ex-ante specified level of reliability χ. We denote by p_i the price per installed capacity and by x_i the capacity installed of the i-th technology. The vector of installed capacities represents the renewable energy portfolio and is denoted by $\mathbf{x} \in \mathbb{R}^n$, where n denotes the number of different technologies considered. The energy manager aims at minimizing the investment costs associated with the renewable energy portfolio choice. The capital expenditures associated with a renewable energy portfolio $\mathbf{x}$ are given by $I(\mathbf{x}) = \sum_{i=1}^{n} p_i x_i$. Moreover, in the probabilistic approach to the generation expansion problem, the energy manager requires that the stochastic hourly supply-demand imbalance, or power shortfall, $X(\mathbf{x}) = d - \mathbf{x}'\mathbf{P} \leq 0$, where d denotes the demand and P_i denotes the stochastic power available per installed capacity of the i-th technology, has to hold true with an ex-ante specified level of reliability $\chi \in [0, 1)$, i.e.,

$$\mathrm{Pr}\{X(\mathbf{x}) \leq 0\} \geq \chi. \tag{3.6}$$

This constraint can be equivalently formulated via the Value-at-Risk (VaR) of the probability distribution of power shortfall.[9] Since the power output of the energy park is subject to uncertainty, the hourly supply-demand imbalance is also stochastic. Therefore, the power shortage X is a random variable with a distribution induced by that of the demand and the supply associated with the renewable energy portfolio $\mathbf{x}$. The level of reliability χ acts as the threshold confidence parameter of the energy manager's requirement on the system's reliability and consequently, $1-\chi$ is the tolerance to constraint violation. We denote by $\theta(\mathbf{x}) = \Pr\{X(\mathbf{x}) \leq 0\}$ the demand coverage probability of the renewable energy portfolio $\mathbf{x}$. The mathematical formulation of the probabilistically constrained generation expansion problem as a constrained least cost problem is given by

$$\begin{aligned} &\min_{x_i} \sum_{i=1}^{n} x_i p_i \\ &\Pr\{X(\mathbf{x}) \leq 0\} = \chi \\ &x_i \geq 0, \qquad i = 1, \ldots, n. \end{aligned} \tag{3.7}$$

Let us assume that we can exclude singularities[10]in the distribution of the power shortfall and impose further regularity conditions on the quantile function of the power shortfall (i.e., that $\theta(\mathbf{x}) = \Pr\{X(\mathbf{x}) \leq 0\}$ is quasi-concave s.t. for each level of the required reliability χ the upper contour set $\{\mathbf{x} : \theta(\mathbf{x}) \geq \chi\}$ is a convex set (Mas-Colell et al., 1995)). For a discussion of this assumption, see Appendix 3.5.1. Then we obtain the following proposition.

Proposition 3.1. *Let the power shortfall $X(\mathbf{x})$ be a continuous random variable with a differentiable and strictly quasi-concave distribution function. The energy manager*

[9] The probabilistic constraint can be equivalently written in terms of the Value-at-Risk $\mathrm{VaR}_\chi(X(\mathbf{x})) \geq 0$, with the loss function induced by the supply-demand imbalance, i.e., a shortfall in the power supply $X(\mathbf{x}) > 0$ denotes a loss. Due to monotonicity of the VaR, a portfolio where this inequality is strict is optimal. An energy manager who chooses a renewable energy portfolio which has the property of a strictly positive VaR contradicts the cost efficiency conjecture.

[10] Due to the threshold wind speeds above and below of which no power output from wind technology is available, the wind distribution shows atomic features which leads to discontinuities in the distribution function. This, however, is not the case for the distribution function of solar technology. Typically, wind power plants available are not build at a single, but geographically diverse locations. Whenever one wind power plant does not produce any power output it is unlikely that all other power plants, which are located sufficiently far away, do also produce no power output. Therefore, geographical diversification helps to smoothen this discontinuity. The slope of distribution function can be very steep, however, singularities are ruled out.

chooses optimally installed capacities, such that for all technologies

$$p_i \frac{\partial \mathrm{VaR}_\chi}{\partial x_i} = p_j \frac{\partial \mathrm{VaR}_\chi}{\partial x_j} \tag{3.8}$$

holds true. Moreover, the shadow price is given by

$$\lambda = \left(\frac{1}{p_i} \frac{\partial \theta}{\partial x_i} \right)^{-1}, \tag{3.9}$$

which is a global constant, independent of the technology i.

Proof. See Appendix 3.5.2. □

The optimal renewable energy portfolio for a given level of reliability is denoted by $\mathbf{x}^-(\chi)$ and induces the energy manager's optimal investment frontier as a function of the level of reliability $I(\chi) = \sum_i p_i x_i^-(\chi)$. Therefore, the marginal costs for an additional unit of reliability are given by

$$\frac{\mathrm{d}I}{\mathrm{d}\chi} = \sum_{i=1}^{n} p_i \frac{\mathrm{d}x_i^-}{\mathrm{d}\chi}, \tag{3.10}$$

where $\mathrm{d}x_i^-/\mathrm{d}\chi$ are the total derivatives of portfolio investments along the efficient frontier, i.e., for the optimal portfolio choice $\mathbf{x} = \mathbf{x}^-(\chi)$. However, we can also apply the Envelope theorem $\mathrm{d}I/\mathrm{d}\chi = -\lambda \partial h/\partial \chi$, which gives ($h(\mathbf{x}, \chi) = \theta(\mathbf{x}) - \chi$ denotes the constraint in the Lagrange function)

$$\frac{\mathrm{d}I}{\mathrm{d}\chi} = \lambda = p_i \left(\frac{\partial \theta}{\partial x_i} \right)^{-1}, \tag{3.11}$$

which is the same for each technology i. Moreover, for the optimal renewable energy portfolio $\theta(\mathbf{x}) = \mathrm{VaR}_\chi^{-1}(X(\mathbf{x}))$ holds true. Therefore,

$$\frac{\partial \theta}{\partial x_i} = \frac{1}{\frac{\partial \mathrm{VaR}_\chi}{\partial x_i}} = \left(\frac{\partial \mathrm{VaR}_\chi}{\partial x_i} \right)^{-1}. \tag{3.12}$$

The marginal contribution to the investment costs for an additional unit of reliability is determined by the marginal contribution of the $i-$the technology to the VaR

$$\frac{\mathrm{d}I}{\mathrm{d}\chi} = p_i \frac{\partial \mathrm{VaR}_\chi}{\partial x_i}. \tag{3.13}$$

Note, however, that in this setting the level of reliability is an exogenous parameter chosen by the energy manager.

3.2.3 Balancing-cost-based planning approach

To account for an integrated view on the costs of the energy park, the energy manager who follows the balancing-cost-based planning approach aims at minimizing the total costs associated with covering the demand. This includes (i) investment costs in RES and (ii) expected excess payments in case of a shortfall in the power supply. Additional expenses of making use of an outside option and purchasing power at the balancing market therefore might come as the opportunity costs of a shortfall in the power supply. However, this expenses only incur, whenever $X(\mathbf{x}) > 0$. Therefore, we introduce the loss function[11] which penalizes a shortfall in the power supply

$$l(\mathbf{x}) = \Delta T \xi \max\{X(\mathbf{x}), 0\}, \tag{3.14}$$

where the energy price is denoted by ξ and ΔT denotes the expected useful life time of the energy park. The loss $l(\mathbf{x})$ itself is therefore a random variable. Its distribution is induced by the joint distribution of energy supply, demand and the energy price. Whenever the demand is higher than the power supply of the energy park, the loss is positive and requires additional excess payments to cover the demand. By the law of total probability, the expected excess payments $\delta(\mathbf{x})$ can be split into two parts

$$\begin{aligned} \delta(\mathbf{x}) &= E[l(\mathbf{x})|X(\mathbf{x}) > 0] \cdot \Pr\{X(\mathbf{x}) > 0\} \\ &\quad + E[l(\mathbf{x})|X(\mathbf{x}) \leq 0] \cdot \Pr\{X(\mathbf{x}) \leq 0\} \\ &= \Delta T(1-\chi) E[\xi X(\mathbf{x})|X(\mathbf{x}) > 0], \end{aligned} \tag{3.15}$$

[11]We consider a scenario without a feed-in tariff. In this case, excess power can neither be sold at the market, nor stored in the absence of a storage device and therefore has no economic value. However, in case of a power shortfall, there is an outside option of last resort to purchase energy at the electricity market.

where $\chi = \Pr\{X(\mathbf{x}) \leq 0\}$ denotes the level of reliability. Due to the fact that we consider positive energy prices $\xi \geq 0$ and by definition of the level of reliability χ, we have that $\mathrm{VaR}_\chi(\xi X(\mathbf{x})) = \mathrm{VaR}_\chi(X(\mathbf{x})) = 0$. Therefore, we find that the additional expected expenses in case of a shortfall in the power supply are given by

$$\begin{aligned} \delta(\mathbf{x}) &= \Delta T(1-\chi)E[\xi X(\mathbf{x})|X(\mathbf{x}) > \mathrm{VaR}_\chi(X(\mathbf{x}))] \\ &= \Delta T(1-\chi)\mathrm{CVaR}_\chi(\xi X(\mathbf{x}(\chi))), \end{aligned} \tag{3.16}$$

where CVaR_χ denotes the conditional Value-at-Risk with the confidence parameter χ. Thus, the total costs associated with demand coverage are given by

$$\begin{aligned} \tilde{C}(\mathbf{x}) &= I(\mathbf{x}) + \delta(\mathbf{x}) \\ &= \sum_{i=1}^{n} p_i x_i + \Delta T(1-\chi)\mathrm{CVaR}_\chi(\xi X(\mathbf{x})). \end{aligned} \tag{3.17}$$

In the balancing costs based planning approach, the energy manager measures riskiness of electricity supply associated with an investment in RES using the concept of the CVaR. In the course of the investment decision, the energy manager has to shape the risk distribution by choosing the amount of installed capacity in each technology in order to find the minimum of the total costs. The optimal renewable energy portfolio in the balancing-cost-based planning approach, denoted by $\mathbf{x}^+$, is given by solving

$$\begin{aligned} &\min_{\mathbf{x}} \sum_{i=1}^{n} p_i x_i + \Delta T(1-\chi)\mathrm{CVaR}_\chi(\xi X(\mathbf{x})) \\ &x_i \geq 0, \qquad i = 1, \ldots, n. \end{aligned} \tag{3.18}$$

Proposition 3.2. *Let the stochastic energy price at the balancing market ξ and the power shortfall X be uncorrelated. The energy manager chooses optimally installed capacities, such that*

$$p_i = \Delta T E[\xi] E[P_i \mathbf{1}_{X(\mathbf{x})>0}] \tag{3.19}$$

holds true also for all renewable energy technologies. Moreover, the marginal costs of *an additional unit of reliability is given by*

$$\frac{\mathrm{d}C}{\mathrm{d}\chi} = -\Delta T E[\xi]\mathrm{VaR}_{\chi}(X(\mathbf{x}^{+})) \tag{3.20}$$

and the endogenized optimal level of reliability χ^* *minimizing the total costs is given by* $\mathrm{VaR}_{\chi^*}(X(\mathbf{x}^{+})) = 0$

Proof. See Appendix 3.5.3. □

Whenever the outside option of buying power at the market exists, the level of reliability is endogenized and the energy manager chooses the renewable energy portfolio which corresponds to the optimal level of reliability χ^*. We observe that the energy manager chooses a different portfolio when he or she includes the expected expenses of a shortfall in the power supply, due to the different first-order condition in Proposition 3.1 and 3.2, respectively. In the reliability-based planning approach, where the energy park has to supply the demand with an ex-ante chosen level of reliability, the energy manager chooses optimally installed capacities according to the price of the investment goods and the contribution to the risk shape of a power shortage, see Proposition 3.1. In the scenario of an integrated evaluation of the total costs, where the energy manager also considers the expected costs in case of a shortfall of the supply, he or she chooses optimally installed capacities by evaluating the costs associated with the technology and the expected costs of making use of an outside option, see Proposition 3.2.

3.2.4 The single technology case

In the balancing-cost-based planning approach, the energy manager can also refuse to invest in RES and purchase total power to cover the demand via pre-contracted energy. In order to evaluate if the investment decision in the balancing-cost-based planning approach agrees with economic intuition in a simplified scenario, we consider the scenario of a single energy asset, i.e., $n = 1$. More specifically, we formulate a participation constraint which denotes the regime, where the energy manager invests in RES, i.e., such that for the optimal level of reliability $\chi > 0$ holds true. However, this single energy asset scenario ignores diversification effects in the renewable energy portfolio. By using the definition of the covariance $\mathrm{Cov}(X, Y) = E[XY] - E[X]E[Y]$, Proposition 3.2 gives

$$\begin{aligned} p &= \Delta T E[\xi] \left(\mathrm{Cov}[P, \mathbf{1}_{X>0}] + E[P]E[\mathbf{1}_{X>0}]\right) \\ &= \Delta T E[\xi] \left(\mathrm{Cov}[P, \mathbf{1}_{X>0}] + E[P](1-\chi)\right) \\ &= \Delta T E[\xi] \left(\sigma \varrho(\chi)\sqrt{\chi(1-\chi)} + \mu(1-\chi)\right), \end{aligned} \tag{3.21}$$

where μ and σ denotes the expected power output of the renewable energy technology and the volatility for the power output of one unit of capacity installed, respectively. The correlation of the power output with the power shortfall is denoted by $\rho(\chi)$ and depends on the optimal level of reliability. Furthermore, we denote by

$$\alpha = \frac{p/\mu}{\Delta T E[\xi]} \tag{3.22}$$

the cost ratio of the average price of one unit of power produced by the self-generation facility and the average price associated with purchasing one unit of power at the market. Therefore, (3.21) gives

$$\alpha = \frac{\sigma \varrho(\chi)}{\mu}\sqrt{\chi(1-\chi)} + (1-\chi). \tag{3.23}$$

In the absence of uncertain production volumes, i.e., $\sigma = 0$ or whenever the coefficient of variation associated with the technology is small $\sigma/\mu \ll 1$, the optimal level of reliability is determined by the cost ratio $\chi = 1 - \alpha$. Therefore, in a deterministic scenario of the power output and a stochastic scenario, where power output is weakly volatile such that $\sigma/\mu \ll 1$ holds, lower values of the cost ratio α implies higher optimal levels of reliability. Hence, a low price of the investment good incentivizes the energy manager to increase the optimal level of investment in renewable energy technologies. The energy manager decides to invest in RES, i.e., $\chi > 0$, whenever the participation constraint

$$\alpha < 1 \quad \Leftrightarrow \quad \frac{p}{\mu} < \Delta T E[\xi] \tag{3.24}$$

holds true. Hence, in order to obtain investment in RES, the average costs of one unit of power output associated with the RES has to be smaller than the expected costs for purchasing one unit of power at the market over the expected useful life time of the technology.

The presence of a volatile power output ($\sigma > 0$), were the volatility cannot be considered negligible, generally introduces a nonlinear effect in the energy manager's optimal decision. However, the participation condition obtained in the deterministic and weakly volatile power scenarios holds true for general levels of the volatility $\sigma > 0$. To see this, note that the correlation is bounded from above by $|\rho(\chi)| \leq 1$. Therefore, in the limit $\chi \to 0$, we have that $\alpha = 1$. Since we are interested in the participation constraint, where the energy manager just starts to invest in RES, we consider the level of reliability to be small. To obtain the asymptotic behavior for small values of χ, we assume differentiability and expand the right hand side of (3.23) in a Taylor series. Therefore, (3.23) gives

$$\begin{aligned}\alpha &= \frac{\sigma}{\mu}\left(\rho(0) + \rho'(0)\chi + o(\chi^2)\right)\left(\chi^{1/2} + o(\chi^{3/2})\right) + (1-\chi) \\ &= \frac{\sigma}{\mu}\left(\rho(0)\chi^{1/2} + [\rho(0) + \rho'(0)]o(\chi^{3/2})\right) + (1-\chi).\end{aligned} \tag{3.25}$$

In the limit $\chi \to 0$, the asymptotic behavior for parameter values $\mu, \sigma, \rho(0) \neq 0$ is determined by

$$\alpha = 1 + \frac{\sigma}{\mu}\rho(0)\chi^{1/2}. \tag{3.26}$$

Since $\rho(\chi)$ denotes the correlation of the power shortfall and the power output for a given level of reliability, we expect that the correlation to be negative $\rho(\chi) \leq 0$. Therefore, we have that

$$\chi^{1/2} = \left(\frac{\mu}{\sigma|\rho(0)|}\right)(1-\alpha). \tag{3.27}$$

Therefore, also in the general case $\sigma > 0$, the energy manager invests in RES whenever the right hand side of this equation is larger than zero, which is the case when

$$\alpha < 1, \tag{3.28}$$

which reproduces the participation constraint (3.24) obtained in the deterministic and weakly volatile power scenario. Moreover, (3.27) shows how the optimal level of reliability depends on the price of the investment good for small levels of investment,

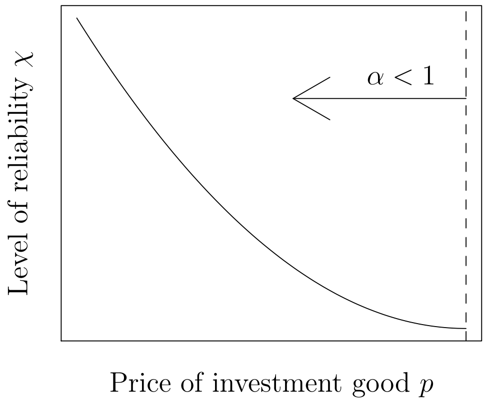

Figure 3.3: This figure shows the optimal level of reliability as a function of the price of the investment good p for a low level of investment, i.e., when χ is small.

i.e., whenever the level of reliability is small

$$\chi(p) = \left(\frac{\mu}{\sigma|\rho(0)|}\right)^2 \left(1 - \frac{2p}{\mu\Delta TE[\xi]} + \frac{p^2}{(\mu\Delta TE[\xi])^2}\right), \tag{3.29}$$

see Fig. 3.3, which illustrates the quadratic dependence of the level of reliability on the price of the investment good for low levels of the investment in the participation regime $\alpha < 1$. Therefore, in the participation regime $\alpha < 1$ the energy manager increases the optimal level of reliability with decreasing price of the investment good.

3.3 Computational experiments

3.3.1 The use case

We demonstrate the applicability of the model in a use case, where the energy manager considers to invest in wind ($i = 1$) and solar ($i = 2$) technology in the absence of a feed-in tariff. The energy manager decides upon the optimal level of investment in renewable energy technologies and the optimal generation mix of technologies.

Uncertainty in the power available from both technologies is modeled by translating empirical data on solar irradiance and on wind speed via the physical energy model into supply of power. We sample from real world output data of the solar irradiance and the wind speed in Schwechat, Austria.[12] A sample to initialize the data-driven optimization problem is generated via blockbootstrapping with a block size of three days to incorporate short-term weather trends and contains hourly values of wind and solar output power for one year to incorporate also long-term (seasonal) weather characteristics. The demand that has to be supplied by the energy park is assumed to be deterministic and constant $d = 1MW$. The costs of investment are specified by the prices of the investment goods per installed kW, where we consider two price scenarios based on Carlsson et al. (2014): (i) the high price scenario of 2013, given by $p_1 = 1400€/kW$ for wind technology and $p_2 = 1000€/kW$ for solar technology and (ii) the low price scenario of 2050, where the price for wind technology is $p_1 = 800€/kW$ and the price for solar technology is $p_2 = 640€/kW$.[13]

Moreover, we investigate the energy manager's investment decision in two alternative frameworks concerning the type of the outside option to purchase residual power at the market. First, we consider the case where the energy manager includes a fixed price contract with pre-contracted power price ξ per purchased unit of power and inspect how his or her optimal investment strategy changes with increasing price ξ. We compare this deterministic price scenario with the stochastic price scenario, where ξ is assumed to be random. This corresponds to purchasing power in case of a shortfall of the power supply at the spot market. In an economy characterized by a low share of renewable energy technologies, RES availability risk is only a minor contribution to spot price volatility. In this case, spot price and RES power output are assumed to be uncorrelated, $\rho \approx 0$. Conversely, in an economy characterized by higher shares of renewable energy technologies the latter does not hold true any more. Cumulated RES power output is then a major driver of the spot market price. Hence, whenever there is a shortfall in the power supply of the energy park, we expect that spot prices are high. In this case, the correlation of the RES power output and spot market price $\rho < 0$ affects the energy manager's optimal investment policy.

[12]source: www.soda-pro.com (solar irradiance), www.mesonet.agron.iastate.edu (wind speed), location: Schwechat, Austria, hourly data available from 2012 to 2018 in the daytime 10:00-18:00.
[13]Approximated values.

3.3.2 Numerical solution of the reliability-based planning scenario

The energy manager's probabilistically constrained optimization problem[14] in (3.7) is solved numerically by the sample average approximation, introduced in Sen (1992). In this data-driven stochastic optimization approach, the probabilistic constraint is replaced by an empirical sample of the power output of size $\{\mathbf{P}^{(1)}, \dots, \mathbf{P}^{(N)}\}$. The confidence parameter χ of the probabilistic constraint is included in this approach via the fraction of scenarios violating demand coverage. To indicate the responsive and non-responsive scenarios, we introduce a binary variables $z_i \in \{0,1\}$, $i = 1, \dots, N$, one for each constraint in the optimization problem. A constraint is discarded as a non-responsive scenario when $z_i = 1$ and is included in the optimization problem for $z_i = 0$, respectively. The cardinality constraint ensures, that the proposed level of reliability is below the imposed limit. The mathematical formulation as a constrained mixed-integer problem is given by

$$
\begin{aligned}
&\min_{\substack{x_1,x_2\\ z_1,\dots,z_N}} \; p_1x_1 + p_2x_2 \qquad \text{s.t.}\\
&x_1P_1^{(i)} + x_2P_2^{(i)} \geq d(1 - z_i)\\
&z_i \in \{0,1\}, \qquad i = 1, \dots, N\\
&\sum_{i=1}^{N} z_i \leq (1-\chi)\cdot N\\
&x_1 \geq 0, \quad x_2 \geq 0.
\end{aligned}
\tag{3.30}
$$

A plot of the optimal investment frontier as a function of the level of reliability $I(\chi) = \sum p_i x^-(\chi)$, where $\mathbf{x}^-(\chi)$ denotes the optimal solution of (3.30), is given in Fig. 3.4(a). Based on the efficient frontier, the technical rate of transformation between investment costs and the level of reliability can be derived. Hence, we also quantify the additional costs of one extra unit of reliability in the energy park.

[14]For more information on the probabilistically constrained generation expansion problem, see Ondra et al. (2021)

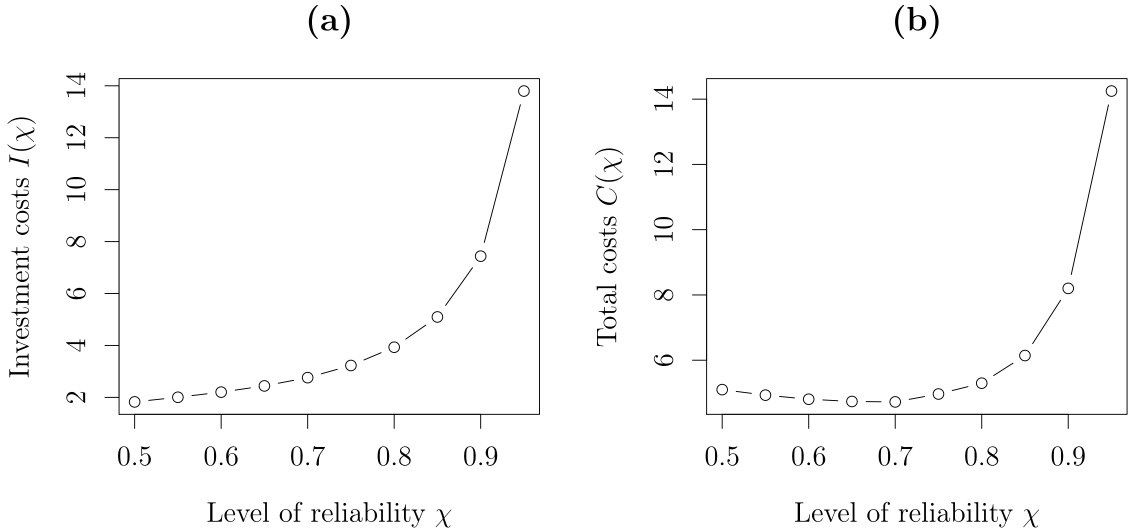

Figure 3.4: Fig. (a) shows the energy manager's optimal investment frontier for different values of the reliability parameter. The costs of investment is given in units of 10^6€. Fig. (b) shows a simulation of the total costs when the expected costs for a shortfall of the power supply are integrated and a shortfall in the power supply is penalized with 200€$/MWh$.

3.3.3 Numerical solution of the balancing-cost-based planning approach

In an integrated evaluation of the total costs associated with supplying the demand, the expected costs of a power shortfall have to be included. We simulate the total costs $C(\chi)$ for different levels of the level of reliability based on the solution obtained in the reliability-based planning scenario ex-post. The total costs as a function of the level of reliability are given in Fig. 3.4(b). Introducing a penalty on scenarios of a shortfall in the power supply increases the total costs, especially when the level of reliability is low. For higher levels of reliability, there are only few scenarios of a shortfall in the power supply and hence, the penalty due to purchasing outside power is also low, however, in this case the investment costs are high, see also Fig. 3.2. Overall, at a specific level of reliability the total costs attain a minimum value. Therefore, the price of balancing energy endogenizes the optimal choice of the level of reliability. Fig. 3.4(b) illustrates the existence of this optimal level of reliability and demonstrates the insufficiency of the level of reliability as a planning parameter.

An energy manager who additionally includes the expected costs of a shortfall in the power supply determines the optimal renewable energy portfolio by evaluating riskiness of the power supply via the CVaR and therefore implicitly chooses the optimal level of reliability. Rockafellar et al. (2000) show, that optimization problems including the CVaR can be reformulated in terms of a linear program via

$$\begin{aligned} &\min_{\substack{x_1,x_2,\\ z_1,\dots,z_N}} \; p_1x_1 + p_2x_2 + \frac{\Delta T}{N}\sum_{i=1}^{N} z_i \\ &z_i \geq \xi^{(i)}(d - x_1P_1^{(i)} - x_2P_2^{(i)}) \\ &z_i \geq 0, \quad \forall i = 1,\dots,N, \\ &x_1 \geq 0, \quad x_2 \geq 0. \end{aligned} \tag{3.31}$$

The optimal solution to the optimization problem (3.31) is denoted by $\mathbf{x}^+$ and constitutes a portfolio which exhibits the property, that the probability to cover the demand is χ^*, i.e., $\mathrm{VaR}_{\chi^*}(X(\mathbf{x}^+)) = 0$ holds true.

In the optimization approach corresponding to the empirical scenario approach, the participation constraint introduced in the one asset scenario can be made explicit for an arbitrary number of technologies considered, where in the use case $n = 2$ is considered. Therefore, we define the function

$$g(x_1, x_2) = p_1x_1 + p_2x_2 + \frac{\Delta T}{N}\sum_{i=1}^{N} \xi^{(i)}(d - x_1P_1^{(i)} - x_2P_2^{(i)}). \tag{3.32}$$

The energy manager invests in RES, whenever $\nabla g(0,0) < 0$, i.e., whenever

$$\begin{aligned} p_i - \frac{\Delta T}{N}\sum_{j=1}^{N} \xi^{(j)}P_i^{(j)} &< 0 \\ p_i - \Delta T E[\xi P_i] &< 0 \\ \frac{p_i/\mu_i}{\Delta T E[\xi]} &< 1, \qquad \forall i, \end{aligned} \tag{3.33}$$

where in the last line we consider the case where ξ is uncorrelated with RES power output. Note, that in this case each technology i introduces a technology specific participation constraint $\alpha_i < 1$ referring to an investment in the i-th technology, where $\alpha_i = p_i\mu_i/(\Delta T E[\xi])$. In the one-asset scenario, this reproduces the participation constraint (3.24). In the following, we discuss two different types of the outside options in case of a shortfall in the power supply, i.e., (i) purchasing power via pre-contracted energy at a deterministic energy price and (ii) purchasing power at the balancing market at a stochastic energy price.

Investment and pre-contracted energy

The fixed-price scenario corresponds to a deterministic penalty in the stochastic loss function (3.14). The only source of uncertainty in this investment scenario is introduced by the uncertain production volumes associated with the power output from renewable energy technologies.

In the absence of investment in RES, the loss function is deterministic due to the constant demand. This corresponds to a scenario, where the energy manager decides not to invest in RES, i.e., $\chi = 0$, but purchases total power to cover the demand via pre-contracted energy. This constitutes total costs of $C(\xi) = \Delta T \xi d$. In this case, total costs increase linearly with the energy price and the option to consume total power to cover the demand at the market introduces an upper bound of the total costs, i.e., investing in RES comes as opportunity costs.

The total costs in an integrated evaluation of the energy park are given in Fig. 3.5(a) for both price scenarios of the investment goods. We observe that the energy manager does not invest in capital intensive renewable self-generation facilities until the pre-contracted energy price ξ exceeds a threshold price ξ^*. This threshold price illustrates an investment barrier, which reflects upon the energy manager's willingness to invest in RES and is lower, the lower the associated prices of the investment goods. More specifically, the participation constraint can be used to illustrate the existence of the threshold price below which the energy manager refuses to invest in RES. To see this, note that (3.33) gives

$$\xi^* = \min \frac{1}{\Delta T} \left\{ \frac{p_1}{\mu_1}, \frac{p_2}{\mu_2} \right\}, \tag{3.34}$$

which is $\xi^*_{\text{high}} \approx 41€/MWh$ in the high price scenario and $\xi^*_{\text{low}} \approx 23€/MWh$ in the low price scenario of the investment goods. Therefore, lower prices of the investment goods create an incentive for the energy manager to invest in RES. The optimally installed capacities in wind and solar technology induce the ex-post portfolio shares

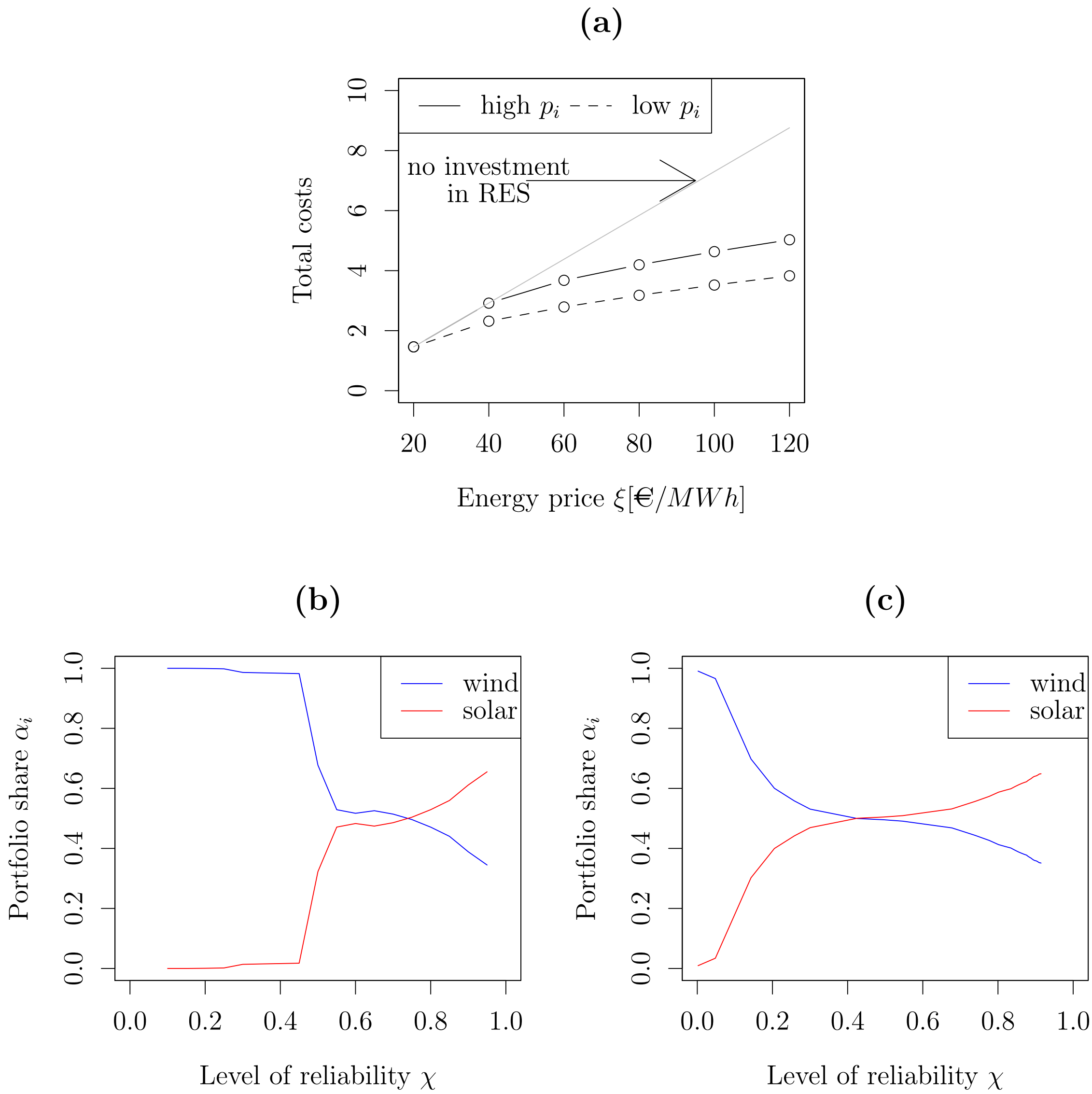

Figure 3.5: Fig. (a) shows the total costs of the energy park for the two price scenarios of the renewable technology investment goods (solid line: low-price reference case 2013, dashed line: high-price price scenario of 2050). The values are given in units of 10^6€. Fig. (b) shows the portfolio shares in the reliability-based planning approach and Fig. (c) shows the portfolio shares in the cost-based planning approach. Both figures illustrate the optimal portfolio shares in the scenario of the low prices of the investment goods.

α_i, which are defined by the share of the investment in RES in the i-th technology

$$\alpha_i = \frac{x_i p_i}{x_1 p_1 + x_2 p_2}. \tag{3.35}$$

The energy manager chooses optimally installed capacities according to the different planning mechanisms, i.e., the reliability-based planning approach, where an exogenous level of reliability is imposed and the balancing-cost-based approach, where the level of reliability is endogenized via an exogenous energy price. Therefore, the portfolio shares associated with the planning approaches are different. The portfolio shares obtained in the reliability-based planning approach are given in Fig. 3.5(b) and Fig. 3.5(c) shows the portfolio shares obtained in the balancing-cost-based approach. Both plots show the optimal renewable energy portfolio as a function of the level of reliability. In the reliability-based planning approach, the energy manager opts for a technology portfolio, where investment in wind technology is dominant. However, the optimal share in wind technology decreases as the required level of reliability increases. This is due to the fact that investment in wind technology comes along with increased tail risk in the distribution of the power shortfall. Therefore, with higher levels of reliability, the energy manager opts for a technology portfolio which prefers solar technology over wind technology in order to avoid the tail risk. In the cost-based-planning approach Fig. 3.5(c) the level of reliability is endogenized by the energy price ξ, where the optimal level of reliability $\chi \to 1$, whenever the energy price $\xi \to \infty$. Compared to the reliability-based planning approach, the energy manager increases the share in solar technology at a lower level of the reliability. Therefore, imposing a required level of reliability in the reliability-based planning approach overestimates the technology share in wind technology and underestimates the technology share in solar technology, especially for lower levels of reliability. In the limit of high levels of the reliability $\chi \to 1$, both approaches avoid the tail risk introduced by wind technology such that there is no shortfall in the power supply and the total costs in the balancing-cost-based planning approach are given by the capital expenditures. Therefore, in the limit of high levels of reliability, the different planning approaches yield the same generation mix. In the reliability-based planning approach (Fig. 3.5(b)), the tail risk introduced by wind technology is not in the scope of the probabilistic constraint and therefore investment is only obtained in the technology which is more profitable in terms of the unit costs per average power output, which is wind technology. Similarly, in the balancing-cost-based planning approach (Fig. 3.5(c)) the participation constraint (3.33) shows that below a critical threshold price the less profitable technology, i.e., the technology with higher costs per power output is excluded from the optimal renewable energy portfolio. However, in the transition region where $\chi \to 0$ the energy manager obtains noticeable different energy portfolios for the different planning mechanisms. Whereas in the reliability-based planning approach (Fig. 3.5(b)), the energy manager obtains single energy investment in wind technology for lower levels of the reliability, an energy manager following the balancing-cost-based approach (Fig. 3.5(c)) considers expected payments in the scenario of a shortfall of the power supply and opts for a more diversified energy portfolio.

The renewable energy portfolio defines the energy park's optimal level of reliability χ, i.e., the probability that the energy park supplies the demand. We evaluate the optimal level of reliability ex-post, by estimating the energy park's capability to supply the demand based on resampled scenarios. The empirical level of reliability is then given by

$$\hat{\chi}(\mathbf{x}) = \frac{1}{N'} \sum_{i=1}^{N'} \mathbf{1}_{\{\mathbf{x}'\mathbf{P}^{(i)} \geq d\}} \tag{3.36}$$

and is illustrated in Fig. 3.6 as a function of the exogenous energy price ξ for the solution obtained in the balancing-cost-based planning approach. Obviously, with increasing prices of the balancing energy the energy manager increases installed capacities in the renewable energy technologies and therefore increases the optimal level of reliability.

Investment and volatile energy balancing prices

Next, we consider the case where the energy manager faces the decision to invest in RES and to purchase power at the spot market. In this setting, the energy price ξ in the loss function (3.14) is assumed to be stochastic. Therefore, the energy manager considers two potential sources of uncertainty in the investment decision which enables the possibility of exceptionally high losses. First, investment in renewable technologies introduces RES availability risk and therefore also the risk of a shortfall in the power supply. Second, spot price at the balancing market is also volatile. The energy

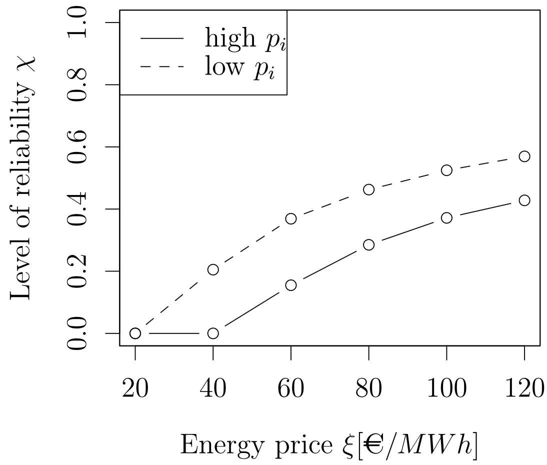

Figure 3.6: This figure shows the ex-post reliability level for the two different scenarios associated with the prices of the investment goods.

manager's problem is therefore to simultaneously balance these risks and to find the optimal investment policy in renewable energy technologies. We investigate two correlation scenarios in which the energy manager has to determine the optimal level of investment. In the first case, spot price and RES power output are assumed to be independent and thus also uncorrelated, i.e., $\rho = 0$. To analyze the energy manager's investment decision, spot market price is simulated via a truncated normal distribution[15] with mean μ' and volatility σ'.[16] In the second case, spot price and cumulated RES power output per installed capacity is assumed to be correlated with correlation parameter $\rho < 0$, i.e., whenever the cumulated RES power output is low, spot prices tend to be higher.[17] Since the demand is assumed to be constant in the use case, the negative correlation parameter $-\rho$ agrees with the correlation of the power shortfall and the energy price, i.e., whenever there is a shortfall in the power supply, the price of balancing energy is high.

[15]We do not consider the possibility of negative prices and left-truncate the distribution at zero.
[16]In this framework, the volatility measures the uncertainty associated with the energy price at the spot market.
[17]The construction of the correlated spot market price is illustrated in the appendix 3.5.4.

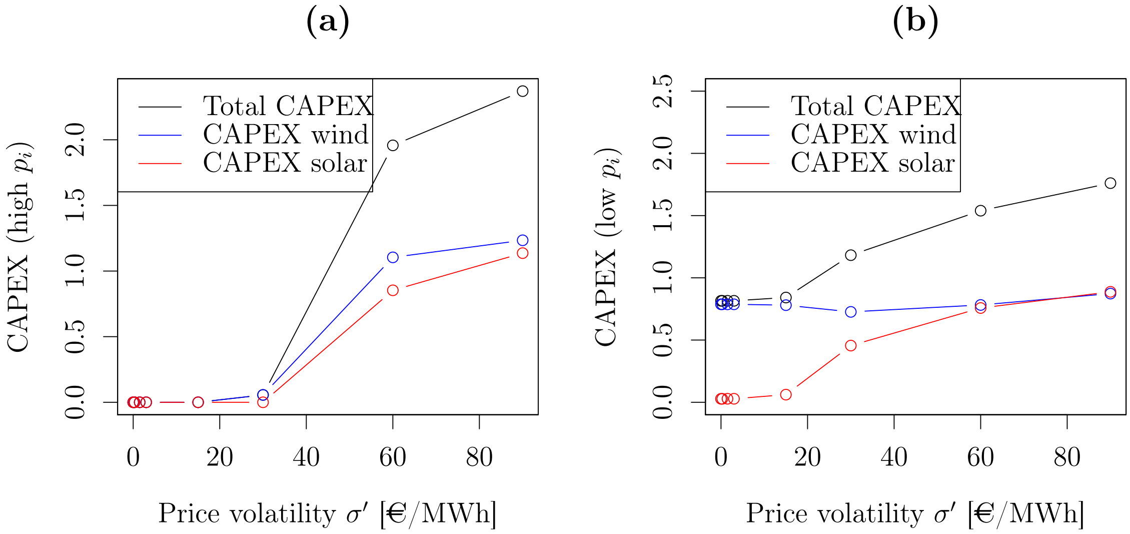

Figure 3.7: The investment costs as a function of spot price volatility in the low investment price case for $\mu' = 30€/MWh$. Fig. (a) shows the results in the high price scenario of the investment goods and Fig. (b) shows the scenarios in the low price scenario of the investment goods. The computational experiments are carried out for values $\sigma'/\mu' = \{0, 0.01, 0.05, 0.1, 0.5, 1, 2, 3\}$. The black line denotes the total investment costs, the red line corresponds to the partial investment costs in solar technology and the blue line corresponds to the partial investment costs in wind technology. All values are the mean values of the 100 optimization runs.

Case 1: Uncorrelated spot market price. The plot of the energy manager's optimal investment decision as a function of spot price volatility is given in Fig. 3.7(a) and (b), for both price scenarios of the investment goods. The case $\sigma = 0$ corresponds to the deterministic energy price scenario with the energy price $\xi = \mu$ and has been discussed in the previous case. We find, that with increasing spot price volatility $\sigma > 0$, the energy manager increases investment in RES to hedge against the spot price risk at the electricity market. Due to the occurrence of multiple sources of uncertainty, the energy manager is sensitive to an increase in the spot price volatility and increases investment in RES with increasing price risk.

In the high price scenario of the investment goods (see Fig. 3.7(a)) we obtain a threshold value for the spot price volatility below which no investment in RES is obtained, similar to the threshold energy price ξ^*. Note that in this scenario, the mean value of the energy price is $\mu' = 30€/MWh$. Compared to the situation of a

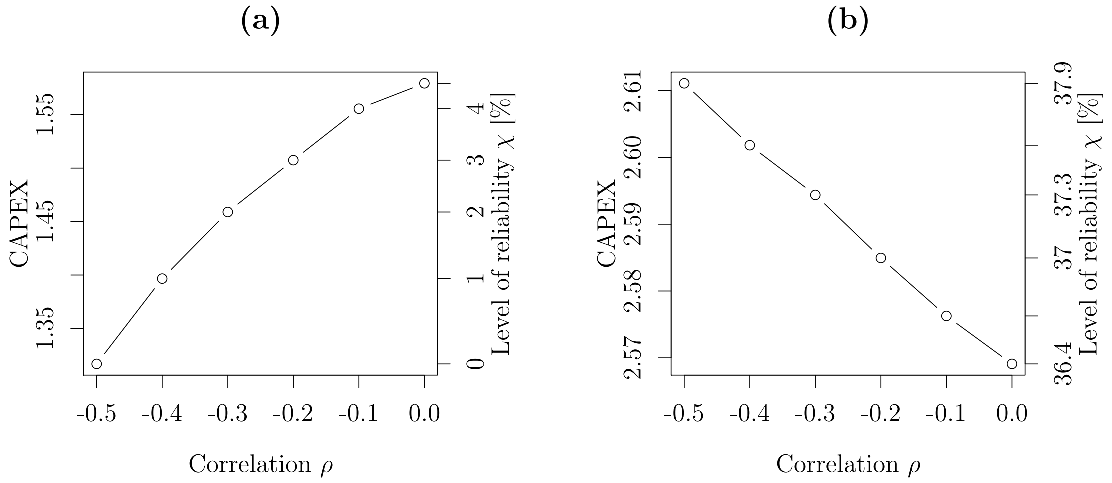

Figure 3.8: This Figure shows the energy manager's optimal investment decision in RES, when the energy price at the balancing market is correlated with the cumulated RES power output. Both plots represent the situation of the high price scenario of the investment goods. Fig. (a) shows the optimal level of investment, when the mean energy price is $\mu' = 40€/MWh$ and spot price risk is $\sigma' = 40€/MWh$. Fig. (b) shows the optimal level of investment for $\mu' = 100€/MWh$ and a spot price risk of $\sigma' = 40€/MWh$.

deterministic energy price of $\xi = 30€/MWh$, where the energy manager does not invest in RES (see Fig. 3.5(a)), the energy manager now has to additionally consider the aspect of a risky energy price. We obtain that whenever spot price risk exceeds a certain threshold, also in this scenario the energy manager invests in renewable energy technologies. Fig. 3.7(b) shows the capital expenditures in the scenario of low prices of the investment goods.

Scenario 2: Correlated spot market price. Let us now consider a scenario, where the energy price is negatively correlated with RES power output. In this case one intuitively expects increased investment in RES in order to avoid scenarios of a power shortfall where balancing energy has to be purchased in case of a power shortfall compared to the benchmark case of an uncorrelated energy price. This corresponds to using RES investment as an insurance against shortfall scenarios of high balancing prices. However, analyzing the optimal investment decision with respect to different levels of the correlation, we find that the optimal investment decision in response to increasing magnitude of the correlation depends on the price of balancing energy (and therefore on the optimal level of investment in the benchmark scenario $\rho = 0$) and can be either to increase or to decrease the level of investment in RES.

In case the initial level of investment at $\rho = 0$ is low, i.e., the expected price of balancing energy is low, increasing the level of investment with decreasing levels of correlation turns out to be not the optimal investment decision. This is due to the fact, that the energy manager prevents himself or herself from purchasing balancing power at low costs. Instead of purchasing cheap balancing power, the energy manager covers the demand via additionally installed capacities. Overall, this corresponds to an increase in the total costs and therefore it is not optimal to increase investment in renewable energy technologies with decreasing correlation. This can be seen in Fig. 3.8(a), where the optimal level of investment as a function of the correlation for the mean price of balancing energy $\mu' = 40€/MWh$, referring to an initially low level of investment, is plotted. Hence, in order to minimize the total costs, the energy manager decreases the optimal level of investment in RES with decreasing levels of the correlation.

At a higher initial level of investment we observe the opposite investment policy. In Fig. 3.8(b) the optimal level of investment as a function of the correlation for the mean price of balancing energy $\mu' = 100€/MWh$ is plotted. In this case, the energy manager increases investment in RES with decreasing level of the correlation to reduce scenarios where the energy park does not cover the demand and expensive power has to be purchased at the market. Therefore, in this scenario we observe the insurance effect of RES investment in order to avoid scenarios of expensive balancing power.

3.4 Conclusion

In this paper we analyze the energy manager's investment decision in renewable energy technologies, characterized by uncertain production volumes. The energy manager aims at minimizing the costs to cover the firm's electricity demand. In this framework, the stochastic production volumes associated with renewable energy technologies introduces uncertainty and thus also risk in the investment decision. In the reliability-based planning approach to the investment problem, the energy manager imposes a threshold reliability level with which the supply-demand constraint has to hold true. However, an energy manager who imposes the reliability-based planning approach constitutes suboptimal renewable energy portfolios, since the expected costs of shortfall in the power supply are not included. Therefore, prescribing a level of reliability is an insufficient planing parameter. We extend this approach by penalizing the power shortfall in the worst case scenarios, which are not in the scope of the probabilistic constraint. Increasing investment in volatile renewable self-generation facilities reduces expected additional expenses of purchasing power at the balancing market and therefore increases the energy park's level of reliability. However, costs that emerge from a high level of reliability can be economically infeasible and come as idle costs. By considering the expected costs of a shortfall in the power supply over the expected lifetime of the energy park, the energy manager evaluates both (i) investment costs and (ii) costs for making use of an outside option in case of a shortfall in the power supply. Hence, in this balancing-cost-based approach, where the expected costs of constraint violation are included, the level of reliability is endogenized. Therefore, the price of the balancing energy, which acts as a penalty per unit of demand coverage violation is an adequate planning parameter. Penalizing the power shortfall from the energy park corresponds to the situation, where the energy manager a-priori includes the possibility to purchase power at the electricity wholesale market. We compare two different scenarios concerning the type of the outside option in case of a shortfall in the power supply. Within the scope of the model, the energy manager has the option to purchase residual power to cover the demand either via (i) pre-contracted power at a fixed price or (ii) at the spot market with a volatile spot market price.

The application of the model to a use case without a feed-in tariff shows, that in the case of a fixed price contract the energy manager is reluctant to invest in RES whenever the pre-contracted energy price is below a critical threshold. This critical threshold price depends on the prices of the RES investment goods and is decreasing with decreasing prices of the investment goods. Moreover, we also compare the technology portfolio in the reliability-based planning approach to the balancing-cost-based planning approach. The application to the use case shows, that the energy manager overestimates the technology share in wind technology for lower levels of reliability in the reliability-based planning approach compared to the balancing-cost-based planning approach.

In the second case, where the energy manager has the option to purchase power at the balancing market, volatile spot market prices introduces another source of uncertainty in the investment decision. Whenever the spot price is assumed to be uncorrelated with the power output from renewable energy technologies, we find that the energy manager is sensitive to spot price volatility and increases investment in renewable energy technologies to hedge against spot price uncertainty. In real life applications, we expect the spot market price to be negatively correlated (correlation parameter ρ) with RES power output, i.e., in scenarios where the RES power output is low, energy prices tend to be high or equivalently in scenarios of a shortfall in the power supply, spot prices tend to be high. In the presence of such a correlation structure, $\rho < 0$, we observe that the energy manager's investment decision in increasing or decreasing the optimal level of investment with respect to the benchmark case of an uncorrelated energy price $\rho = 0$ in response to a stronger correlation, depends on the initial level of investment in the benchmark scenario. Whenever the initial investment at $\rho = 0$ is low, the energy manager decreases the optimal level of investment. Conversely, for higher levels of investment with respect to the benchmark scenario $\rho = 0$, the energy manager increases installed capacities to reduce scenarios of a power shortfall, which corresponds to an insurance effect.

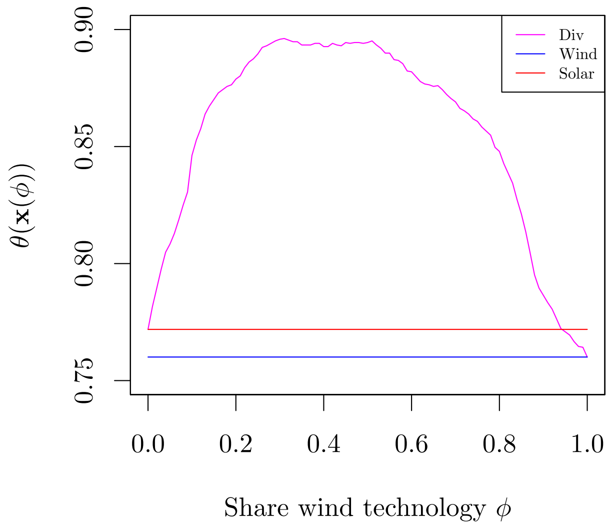

Figure 3.9: This figure demonstrates the quasi-concavity of the distribution function in an example.

3.5 Appendix

3.5.1 A: Quasi-concavity of the empirical distribution function

Let us now discuss the economic intuition behind the assumption of the distribution function of the power shortfall as a function of the renewable energy portfolio $\theta(\mathbf{x})$ to be quasi-concave. Formally this condition can be written for two renewable energy portfolios $\mathbf{x}_1$ and $\mathbf{x}_2$, as

$$\theta(\phi\mathbf{x}_1 + (1-\phi)\mathbf{x}_2) \geq \min\{\theta(\mathbf{x}_1); \theta(\mathbf{x}_2)\}, \tag{A1}$$

with $\phi \in [0, 1]$. Since $\theta(\mathbf{x})$ is the level of reliability that can be obtained with the renewable energy portfolio $\mathbf{x}$, imposing this condition implies that diversifying the portfolio (with weights ϕ and $1-\phi$) increases the reliability compared to the minimum level of reliability of the two portfolios. This also implies that there is a diversification effect between wind and solar technology at work. Consider for example the case where $\mathbf{x}_1$ is the single energy portfolio consisting only of wind technology and $\mathbf{x}_2$ is the single energy portfolio consisting only of solar technology. Then, according to (A1) mixing these technologies refers to higher levels of reliability than the inferior single energy investment. In order to check if this assumption is compatible with the empirical wind and solar data we demonstrate this condition graphically for a fixed amount of total installed capacities. Fig. 3.9 shows a plot that demonstrates this property in an example. To ensure convexity of the optimization problem, one has to check this condition for each level of totally installed capacities along the efficient frontier.

3.5.2 B: Proof of Proposition 3.1

Proof. We denote by $\theta(\mathbf{x}) = \Pr\{X(\mathbf{x}) \leq 0\}$ the demand coverage probability of the renewable energy portfolio $\mathbf{x}$. Therefore, for the optimal portfolio $\theta(\mathbf{x}) = \mathrm{VaR}_{\chi}^{-1}(X(\mathbf{x}))$ holds true. The Lagrangian associated with the probabilistically constrained least cost generation expansion problem is given by

$$\mathcal{L}(\mathbf{x}, \lambda) = \sum_{i=1}^{n} p_i x_i - \lambda(\theta(\mathbf{x}) - \chi). \tag{B1}$$

The stationary condition for each of the n commodities to be optimal is given by $\partial\mathcal{L}/\partial x_i = \partial\mathcal{L}/\partial\lambda = 0$, for all technologies $i = 1, \ldots, n$. Hence the energy manager chooses the technology portfolio for which

$$p_i = \lambda \frac{\partial\theta}{\partial x_i} \tag{B2}$$

holds true. This equation also gives an interpretation of the Lagrange multiplier

$$\lambda = \left(\frac{\partial\theta}{\partial(p_i x_i)}\right)^{-1} \tag{B3}$$

as the inverse ratio of the marginal contribution to the demand coverage probability per monetary unit invested in the i-th technology. However, from (B2) we obtain, that for all technologies $i = 1, \ldots, n$

$$p_i \left(\frac{\partial\theta}{\partial x_i}\right)^{-1} = \mathrm{const} \tag{B4}$$

has to hold true. By using

$$\frac{\partial\theta}{\partial x_i} = \frac{1}{\frac{\partial \mathrm{VaR}_{\chi}}{\partial x_i}} = \left(\frac{\partial \mathrm{VaR}_{\chi}}{\partial x_i}\right)^{-1}, \tag{B5}$$

we have that

$$p_i \frac{\partial \mathrm{VaR}_{\chi}}{\partial x_i} = \mathrm{const} \tag{B6}$$

and that the shadow price is given by

$$\lambda = p_i \frac{\partial \mathrm{VaR}_\chi}{\partial x_i}. \tag{B7}$$

□

3.5.3 C: Proof of Proposition 3.2

Proof. The energy manager determines the optimal portfolio by solving the optimization problem. The FOCs are given by

$$\begin{aligned} 0 &= p_i + \Delta T E[\xi] \frac{\partial}{\partial x_i}(1-\chi)\mathrm{CVaR}_\chi(X(\mathbf{x})) \\ &= p_i - \Delta T E[\xi](1-\chi)E[P_i | X(\mathbf{x}) > 0], \end{aligned} \tag{C1}$$

where the last equation follows from Tasche (2001). Since the conditional expectation can be rewritten in terms of the probability of a power shortfall

$$E[P_i | X(\mathbf{x}) > 0] = \frac{E[P_i \mathbf{1}_{X>0}]}{1-\chi}, \tag{C2}$$

the FOCs are given by

$$p_i = \Delta T E[\xi] E[P_i \mathbf{1}_{X>0}]. \tag{C3}$$

The optimal level of reliability, such that the total costs associated with the technology portfolio $\mathbf{x}^+(\chi)$ are minimized is given by $\frac{\mathrm{d}C}{\mathrm{d}\chi} = 0$. The Envelope theorem characterizes the costs of an additional unit of reliability

$$\begin{aligned} \frac{\mathrm{d}C}{\mathrm{d}\chi} &= \left.\frac{\partial \tilde{C}}{\partial \chi}\right|_{\mathbf{x}=\mathbf{x}^+} \\ &= \Delta T E[\xi] \frac{\partial}{\partial \chi}(1-\chi)\mathrm{CVaR}_\chi(X(\mathbf{x}^+)), \end{aligned} \tag{C4}$$

where $\tilde{C}$ denotes the objective function. By the integral representation of the CVaR

$$\mathrm{CVaR}_\chi = \frac{1}{1-\chi}\int_\chi^1 \mathrm{VaR}_\beta \mathrm{d}\beta, \tag{C5}$$

we have that

$$\frac{\partial}{\partial \chi}(1-\chi)\mathrm{CVaR}_\chi(X(\mathbf{x}^+)) = -\mathrm{VaR}_\chi \tag{C6}$$

holds true. Hence, the marginal cost of reliability is

$$\frac{\mathrm{d}C}{\mathrm{d}\chi} = -\Delta T E[\xi] \mathrm{VaR}_\chi(X(\mathbf{x}^+)). \tag{C7}$$

A necessary condition for the optimal reliability level is that the marginal costs for an addition unit of reliability is zero, which is equivalent to

$$\mathrm{VaR}_{\chi^*}(X(\mathbf{x}^+)) = 0. \tag{C8}$$

□

3.5.4 D: Simulation of correlated spot prices

We simulate spot market price via a left truncated normal distribution with $\mu = 30€/MWh$ and spot price volatility is indicated by σ. The left-truncation parameter is set to $a = 0$, i.e. we consider only positive spot market prices. We generate correlated samples (with correlation ρ) of spot market price and RES power output[18] by introducing the systematic risk component in the energy market $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$.

In order to generate a sequence of uniformly distributed observations of RES power output we transform the empirical data of the cumulated RES power output $P_t^{(i)}$ using the standard Gaussian cdf via $\hat{P}_t^{(i)} = \Phi(P_t^{(i)})$. This sample is an observation of a uniformly distributed random variable on the set $[0, 1]$ and thus has mean $\mu_P = 1/2$ and standard deviation $\sigma_P = 1/\sqrt{12}$. We construct a corresponding, i.e. correlated sequence of uniformly distributed observations of spot prices via the transformation

$$\hat{\xi}_t = \alpha + \rho \hat{P}_t + \epsilon_t. \tag{D1}$$

The parameter ρ induces correlation of spot market price and RES power output

$$\begin{aligned}
\mathrm{Corr}(\hat{\xi}_t, \hat{P}_t) &= \frac{\mathrm{Cov}(\hat{\xi}_t, \hat{P}_t)}{\sigma_\xi \sigma_P} \\
&= \frac{\mathrm{Cov}(\alpha + \rho \hat{P}_t + \epsilon_t, \hat{P}_t)}{\sigma_\xi \sigma_P} \\
&= \frac{\rho^2 \sigma_P^2}{\rho \sigma_P^2} \\
&= \rho.
\end{aligned} \tag{D2}$$

Furthermore, we impose that the mean and the variance of both samples are the same, i.e. that the price characteristics follows the characteristics from the cumulated power

[18]we consider the random variable of the cumulated RES power output.

distribution.

$$E[\hat{\xi}_t] = E[\hat{P}_t] \tag{D3}$$

$$\mathrm{Var}[\hat{\xi}_t] = \mathrm{Var}[\hat{P}_t]. \tag{D4}$$

The first equation (D3) imposes a condition on the parameter α via

$$\begin{aligned} \alpha &= \mu_P(1-\rho) \\ &= \frac{1}{2}(1-\rho) \end{aligned} \tag{D5}$$

and the second equation (D4) reflects the influence of the correlation on the systematic risk

$$\begin{aligned} \sigma_\epsilon &= \sigma_P\sqrt{1-\rho^2} \\ \sigma_\epsilon &= \frac{1}{\sqrt{12}}\sqrt{1-\rho^2}. \end{aligned} \tag{D6}$$

Given these parameter choices, the re-transformed values $\xi_t^{(i)} = F^{-1}(\hat{\xi}_t^{(i)})$, where F^{-1} is the quantile of the truncated normal distribution, constitutes a sample of spot prices which are correlated with the observed empirical data of RES power output with correlation ρ. In case of $\hat{\xi}_t^{(i)} \geq 1$ or $\hat{\xi}_t^{(i)} < 0$, we repeat the procedure.

4 Strategic capacity choice in renewable energy technologies under uncertainty [1]

Abstract. In this paper we discuss optimal renewable energy investment (in wind and solar technology) under uncertainty in a real options approach framework. We consider the combined impact of uncertain production volumes associated with renewable energy power output, policy uncertainty via uncertain remuneration of surplus power and stochastic technological learning, which – in expectation – decreases future costs of solar technology. An energy manager who determines the optimal dynamic investment strategy aims at minimizing expected power procurement costs, which consist of investment costs in renewable energy technologies, expected shortfall costs and expected benefits from selling surplus power to the grid. This results in nonlinear costs of power procurement and introduces – similar to classical portfolio theory – a diversification effect between wind and solar technology. Concerning the optimal timing of the investment, we show that a staged investment strategy can reduce expected power procurement costs compared to a lumpy investment strategy. Therefore, if technological innovations in solar technology are expected, an early investment in wind technology and keeping the option to expand the energy park can be the optimal strategic renewable portfolio choice.
Keywords: Feed-in tariff; Renewable energy policy; Renewable energy investment under uncertainty

[1] Joint work together with Thomas Dangl, Vienna University of Technology, Institute of Management Science, Theresianumgasse 27, 1040 Vienna. The full paper (Ondra and Dangl, 2021b) was presented at the IEWT 2021.

4.1 Introduction

Nowadays energy managers of industrial firms are facing investment decision in power generation facilities in a risky environment, where multiple potential sources of uncertainty arise. On the one hand renewable energy sources (RES) are more sustainable investment choices from an environmental point of view. On the other hand they are capital intensive and exposed to uncertain production volumes, a fact that increases the shortfall risk in the power supply. Therefore, in order to overcome the investment burden in RES, remuneration policies that promote environmentally friendly power technologies are put into place. However, the level of the remuneration is uncertain and is expected to decrease in the future. Therefore, besides facing uncertain production volumes, the energy manager is also exposed to policy uncertainty. In a competitive environment technology manufacturers of RES decrease the prices of the investment goods by active research and development. These technological innovation shocks occur randomly over time and thus the prices of the investment goods are also considered as uncertain. This illustrates that the energy manager is exposed to various sources of uncertainties which increases the complexity of the investment decision.

The scope of the paper is to consider an investment project in RES (wind and solar technology), where the timing of the investment decision is not exogenously fixed but can be chosen by the energy manager. In such a dynamic optimization framework the opportunity to postpone the investment decision to acquire knowledge over time and perform better-informed investment decisions at some time in the future is explicitly included in the model. We analyze the optimal investment decision in RES in a real options framework, where uncertainty associated with the investment opportunity in RES not only arises due to stochastic production volumes of RES, but also due to policy risk (uncertain remuneration of surplus energy that can be delivered to the grid) and investment price risk (uncertain prices of the investment goods). Since all of these uncertain parameters potentially affect the optimal investment decision, we analyze their combined impact.[2]

[2]Policy risk arises due to the uncertain remuneration policy of surplus power, where the level of the FIT is assumed to be subject to multiplicative geometric Brownian shocks and is expected to decrease over time. Investment price risk is due to stochastic technological learning and diffusion which decreases the prices of the investment goods. Therefore, the prices of the investment goods are assumed to be subject to exogenous technological innovation shocks.

The bulk of the real options literature focuses on optimal timing of the investment (Dixit and Pindyck, 1994; Trigeorgis et al., 1996). A general result of applications of real options theory to investment models is that the option to defer the investment decision to later periods introduces managerial flexibility, which constitutes potentially significant economic value – the value of the real option. Investment, i.e., the exercise of the real option, is inevitably associated with a loss in flexibility and hence the value of the real option has to be considered in the investment decision. On the one hand, by investing a large amount in RES the firm takes the risk that if ex-post a significant innovation occurs, deferring the investment would have yielded higher profits or lower costs. On the other hand, postponing the investment decision to later periods not only waives potential cashflows but waiting for technological innovations bears the risk of a decreased remuneration policy which reduces expected benefits from selling surplus power. Therefore, in this setup the two sources of uncertainty drive the timing of the investment towards opposite directions, i.e., a high subsidy retraction rate implies that immediate investment is beneficial, whereas uncertain investment prices imply that the investment should be postponed to later periods.

Balcer and Lippman (1984) analyze the optimal timing problem associated with adopting a new technology when innovations are uncertain and show that the current best practice technology will be adopted if the technological lag exceeds a certain threshold. Grenadier and Weiss (1997) consider an option pricing model to evaluate technological innovations, which are assumed to be stochastic in their arrival times as well as their profitability and show that depending on the structure of the innovations the firm might adopt the initial technology, even if potentially more valuable innovations might occur in the future.[3] (Sendstad and Chronopoulos, 2020) emphasizes that many studies ignore technological uncertainty. In their paper the authors compare different investment strategies under policy and technological uncertainty. The authors demonstrate that *"[...] the option to invest sequentially in improved technology raises the value of the investment opportunity"* (Sendstad and Chronopoulos, 2020). Boomsma et al. (2012) analyze different support schemes associated with renewable energy output and demonstrate in a use case, that feed-in tariffs encourage earlier investment. Ritzenhofen and Spinler (2016) show that under market-independent, fixed and sufficiently attractive FIT schemes investment projects in RES can be con-

[3]This is also due to benefits from learning and the resulting easy adaption to technologies arising in the future, which makes them better able to benefit from future innovations.

sidered as “now-or-never” decisions. Nagy et al. (2021) analyze the effect of subsidy withdrawal on the optimal investment decision under demand uncertainty and show that increasing probability of subsidy withdrawal accelerates the investment, however, at a smaller size. Dalby et al. (2018) propose a real options model that incorporates Bayesian learning, through which the investor updates his or her subjective beliefs on subsidy retraction. The authors demonstrate that *“[...] investors are less likely to invest when the arrival rate of a policy change increases”* (Dalby et al., 2018).

In our investment model, the optimal renewable energy portfolio choice, as well as the optimal timing of the investment have to be determined simultaneously. In several applications of standard real options theory the investment opportunity is assumed to be of a given size. Dangl (1999) was among the first to consider optimal timing and optimal capacity choice in a monopolistic setup simultaneously and shows that with increasing uncertainty the investment decision occurs later in a higher capacity, which highlights the effect of uncertainty in the investment decision. Huisman and Kort (2015) extend this approach by considering a duopoly setting and found that under an entry deterrence policy the first investor overinvests in capacity and that the entrant invests in less capacity.

Generally, the energy manager of a firm has available a bundle of different investment opportunities in renewable energy technologies (we focus on wind and solar technology). Dixit (1993) evaluates investment opportunities in a general setting under output price uncertainty, when a menu of different projects exist. He argues, that each project should be evaluated separately, and that the optimal solution is the one with the highest option value, see also Décamps et al. (2006). Therefore, the analysis in Dixit (1993) can be considered as the multi-project extension of the single project case discussed in McDonald and Siegel (1986). In our paper – where the energy manager faces the opportunity to invest in wind and solar technology – we adopt a different view and do not consider the investment opportunity in different renewable energy technologies as mutually exclusive, but highlight the diversification effect arising from investment in a mix of different generation technologies.

This portfolio diversification effect is due to the nonlinear pricing relation of the expected power procurement costs which have to be minimized. In the investment decision in renewable energy technologies the energy manager evaluates the total costs of the energy park by including investment costs, expected shortfall costs as well as expected remunerations from surplus power. Each technology included in the renewable energy portfolio exhibits different characteristics of the power output. Therefore, each technology contributes differently to the shortfall risk. By choosing the optimal technology portfolio, the energy manager can shape the risk distribution associated with a shortfall in the power supply (Ondra and Dangl, 2021a). Due to the existence of a portfolio diversification effect we do not consider investment opportunities in different RES technologies as mutually exclusive but as interrelated projects, where the synergy gains result from risk shaping associated with the renewable energy portfolio selection.[4]

Due to the fact, that the timing of the investment is not exogenously fixed, the investment model basically allows for different investment strategies: (i) a lumpy investment strategy and (ii) a staged investment strategy. In the lumpy investment strategy the budget available for building the energy park is spent at one specific point in time. In contrast to that, it might be valuable to adopt a staged investment strategy and partially invest in a single technology at an early stage of the investment period and invest later in the lagged technology. This investment strategy corresponds to investing a fraction of the budget and to keep the option to expand the energy park alive. Sequential investment is investigated for example in Dixit and Pindyck (1998) and Bar-Ilan and Strange (1998). Applications of sequential investment models in the power sector can be found in Gollier et al. (2005), who discuss an investment model of nuclear power plants and evaluate the flexibility of investing in a sequence of small power plants in contrast to investing in a large scale power plant. The authors demonstrate that despite the presence of economies of scale, the option to invest in a modular project can have a higher value and therefore is able to outperform a lumpy investment strategy. Sendstad and Chronopoulos (2020) consider policy risk and technological uncertainty together and show that a greater likelihood of subsidy retraction lowers the incentive to invest. Moreover, the authors demonstrate how sequential investment facilitates earlier technology adoption compared to lumpy investment.

This paper aims at investigating the energy manager's investment decision in RES (specifically wind and solar technology) associated with uncertain production volumes, which are subsidized by a remuneration policy that is uncertain over time.

[4] Childs et al. (1998) discuss the effect of project interrelationships on investment decisions, where there is a development and implementation stage and projects are considered as complements in the sense that implementing projects together yields synergy gains.

Moreover, we consider the prices of the investment goods to be subject to random exogenous innovation shocks and therefore also consider technological uncertainty. The energy manager consequently faces the power procurement problem under multiple sources of uncertainty and has to determine whether an investment in RES is beneficial, or if power to cover the firm's demand should be purchased via pre-contracted energy at a fixed exogenous energy price. Therefore, we extend the real options literature in the field of energy economics by highlighting the optimal dynamic investment behavior in renewable energy technologies in the presence of a renewable portfolio diversification effect under policy and investment price uncertainty. The rest of the paper is organized as follows. Section 4.2 introduces the investment model. Section 4.3 values the investment decision and Section 4.4 derives the Bellman equation. Section 4.5 reports on the numerical results of the use case and Section 4.6 concludes the paper.

4.2 The investment model

We consider an energy manager who aims at minimizing the firm's costs of power supply by investing in renewable self generation facilities (wind and solar technology), where the firm is considered to be a price taker. Furthermore, we assume a regulatory framework promoting renewable energy such that surplus power from renewable self generation facilities can be sold to the grid at the level of the feed-in tariff (FIT). In case of a shortfall in the power supply of the energy park (or in the absence of an investment in renewable energy sources (RES)) there exists an outside option, where pre-contracted power can be purchased at a fixed exogenous energy price.[5] Therefore, the expected costs of power supply of the firm are given by: (i) the investment costs in self generation facilities, where the budget that can be used to build the energy park is constrained by I_0, (ii) plus expected costs in case of a shortfall associated with the self generation facilities and (iii) minus expected remunerations for selling surplus power to the grid.

We consider a dynamic investment framework, where the timing of the investment opportunity is not exogeneously fixed but can be chosen by the energy manager. Therefore, the energy manager has to determine simultaneously: (i) optimally installed capacities in wind and solar technology subject to a budget constraint and (ii)

[5]We assume, that the exogenous energy price is fixed and uncorrelated with power supply.

the optimal timing of the investment. Moreover, the energy manager faces the decision in an uncertain environment, i.e., under multiple sources of uncertainties which potentially affect the optimal investment decision. Generally, the major sources of uncertainty associated with an investment in RES are: (i) uncertainty in the renewable energy output (uncertain production volumes) (ii) policy risk (uncertain levels of the remuneration of surplus power from renewable energy technologies) and (iii) technology risk (uncertain prices of the investment goods).

One of the most important aspects discussed in this paper arises from the fact, that power output from renewable energy technologies is uncertain. Wind and solar technology can be associated with different distributions of the power output per unit of installed capacity. A special characteristic of the power output distribution associated with wind technology is that due to the existence of a threshold wind speed below and above which no power output can be generated, the wind distribution exhibits the characteristics of a heavy-tailed distribution. Therefore, investment in wind technology comes along with a higher tail-risk of a power shortfall compared to an investment in solar technology. However, by choosing optimally installed capacities the energy manager is able to shape the underlying risk distribution of a shortfall in the power supply. Hence, by diversifying the energy portfolio the energy manager can lower the power shortfall risk which introduces the renewable energy portfolio effect. The histograms associated with the distribution of the power output are illustrated for a numerical example in Fig. 4.1 for the case of (a) a single energy investment in wind technology, (b) a single energy investment in solar technology and (c) a diversified energy portfolio with equal capital shares invested in wind and solar technology.

In classical portfolio theory, the risk diversification effect is due to maximizing expected utility of a risk-averse investor. In our approach, we don't maximize expected utility of wealth but minimize total expected power procurement costs, i.e., we consider a risk-neutral energy manager. In this scenario, risk diversification is formally introduced via the underlying non-linear pricing relation of expected surplus and expected shortfall costs.[6] For the sake of tractability we consider 3 different types of

[6]The optimal decision of a risk neutral investor is to invest in a portfolio consisting of a single asset (the asset with the highest return), since the costs as well as the return of the portfolio is linear in its portfolio weights. It is only the degree of risk-aversion that leads to non-linear effects and thus introduces diversification. An energy manager who aims at minimizing the expected power procurement costs, however, will also invest in a diversified renewable energy portfolio since the expected power procurement costs is a non-linear function of the portfolio weights.

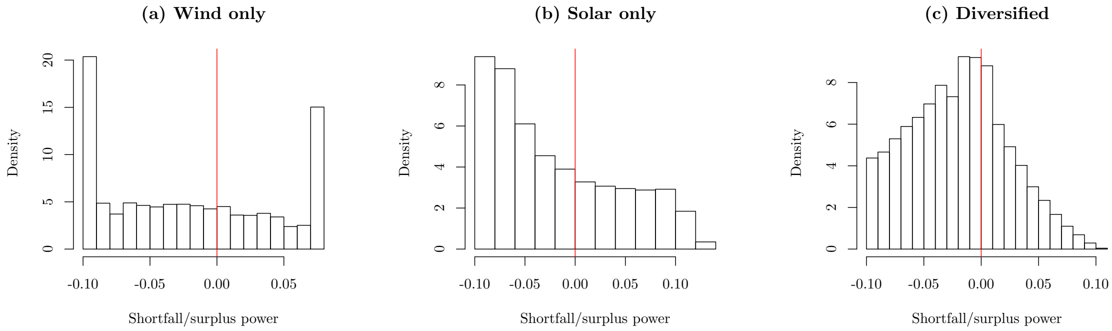

Figure 4.1: These figures show the empirical distribution of the shortfall/surplus power in case of: (a) single energy investment in wind technology, (b) single energy investment in solar technology and (c) a diversified energy portfolio with equal capital shares in wind and solar technology. The red line indicates demand and supply equality and separates the regions where a shortfall in the power supply occurs (left from the red line) from the region of surplus power (right from red line).

renewable energy portfolios that reflect the characteristic features of the underlying shortfall risk distribution: (i) the single energy investment in wind technology, (ii) the single energy investment in solar technology and (iii) a diversified portfolio consisting of equal capital shares in wind and solar technology. Of course, the portfolio consisting of equal shares of both technologies might not be the optimally diversified energy portfolio whenever the full range of possible capacity choices is considered, however, it demonstrates the characteristic feature of portfolio diversification and allows us to study conditions under which the diversified portfolio dominates the pure choices (i) and (ii).

To highlight the benefits of the portfolio diversification effect we consider an illustrative example. More specifically, we investigate the static problem (i.e., the "now-or-never" decision problem) where the exogenous parameters of the pre-contracted energy price and the prices of the investment goods are assumed to be deterministic, i.e., perfectly known. To do so, we determine the optimal portfolio choice as a function of the level of the feed-in tariff ξ_+ and the price for solar technology p_s in two scenarios. First, where the opportunity to invest either in wind or in solar exists (i.e., a diversified portfolio is not allowed). And second, where the opportunity to invest in wind, solar or a diversified portfolio exists. The optimal portfolio decision associated with the static problem is illustrated in Fig. 4.2, where the type of renew-

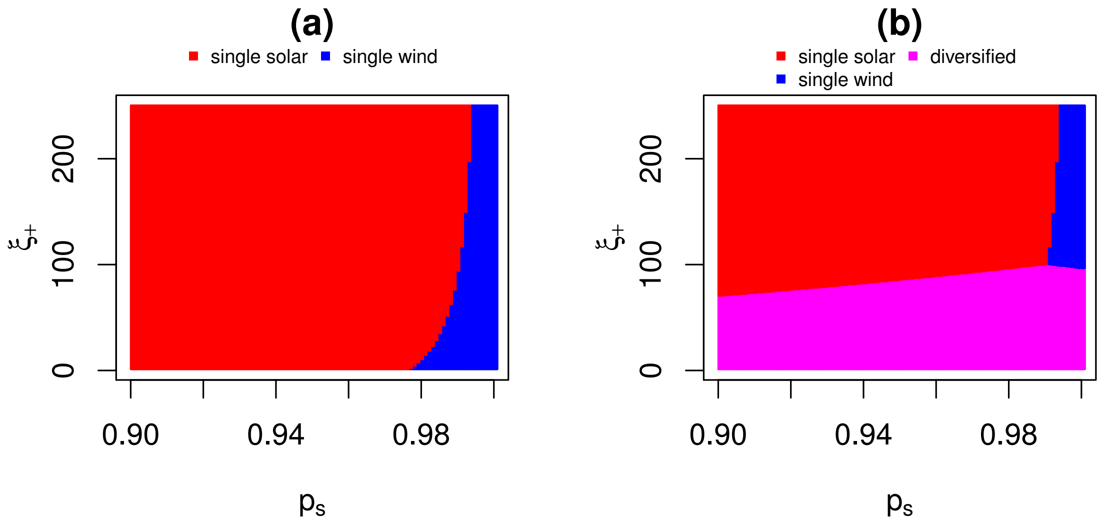

Figure 4.2: This figure illustrates the optimal renewable energy portfolio choice in a static framework (a) in case a diversified portfolio is not included and (b) in case a diversified portfolio is included. The price of wind technology is assumed to be $p_w = 1.4M€/MW$, the pre-contracted energy price is $\xi_- = 100€/MWh$ and the budget $I_0 = 0.25M€$.

able energy portfolio for different levels of the FIT and prices for solar technology (i.e., different levels of technological innovations) is plotted. Fig. 4.2(a) shows the optimal portfolio choice, whenever only the pure investment choices are considered, i.e., a diversified portfolio is not in the scope of the decision maker. Fig. 4.2(b) illustrates the situation when the diversified renewable energy portfolio is considered as a feasible investment opportunity. Despite the fact that the average power output per unit of installed capacity in wind technology is higher than the average power output per unit of installed capacity in solar technology,[7] the optimal investment is not necessarily to invest in wind technology, but depends on the level of the exogenous parameters. The optimal strategy might even be to reject investment in RES and purchase total power to cover the demand from outside. This occurs e.g., in the absence of a remuneration policy (or whenever the level of the FIT is exceptionally low) and when the costs of purchasing total power to cover the demand are lower than the capital expenditures associated with the RES investment. However, we con-

[7] The average hourly power output per monetary unit of the investment are $0.317MW/M€$ for wind technology and $0.314MW/M€$ for solar technology, when daytime data are used.

sider a situation where the investment in renewable energy technologies is affordable. Fig. 4.2 generally demonstrates that for lower levels of the FIT the cost-minimal choice is to invest in the diversified energy portfolio. This portfolio choice can be explained by taking into account the different shortfall/surplus power distributions of the underlying energy assets. A low level of FIT weakens the disadvantage of solar energy in terms of lower average energy output per unit of invested capital and puts more emphasis on avoiding large power shortages (the advantage of solar power as discussed earlier). In the case of very high levels of FIT, the optimal decision is in favor of a technology that maximizes energy output, i.e., wind energy. Higher risk of shortfalls associated with wind technology is less critical in this case. A diversified energy portfolio can balance out the expected costs in case of a shortfall in the power supply and the expected remunerations for selling surplus power to the grid.

This example illustrates that due to the stochastic production volumes of wind and solar technology, the investment decision in the optimal generation mix (or investment in RES at all) highly depends on the level of the exogenous parameters, i.e., the level of the FIT and the energy price, even in case they are assumed to be deterministic. Of course, the complexity of the investment problem increases when the exogenous parameters are assumed to be subject to uncertainty that also impact the investment decision, which is the scope of this paper.

Concerning policy risk, we expect the level of the FIT to undergo multiplicative geometric Brownian shocks. Since support schemes for RES are gradually withdrawn, the drift of the geometric Brownian motion is taken to be negative. At the time the investment in wind and solar technology is made, the current level of the FIT is locked in and used to price surplus power that is sold to the grid over the expected useful lifetime of the energy park. Therefore, the current level of the FIT has also an impact on the optimal generation mix, since the current level of the FIT enters as a parameter in the non-linear pricing relation affecting the optimal renewable energy portfolio. A detailed description of the stochastic process associated with the remuneration policy can be found in Appendix 4.7.1.

Due to technology diffusion and technological learning, the prices of the investment goods for renewable energies are subject to random exogenous innovation shocks. Therefore, the prices of the investment goods are uncertain and are expected to decrease. Since renewable energy investments are characterized by high capital costs, uncertainty over the capital expenditures is a major driver of investment risk. Generally, the price of both technologies can be considered as subject to uncertainty. However, we assume that major technological process and product innovations occur only for solar technology. In contrast to that, only minor technological innovations are expected in wind technology and are considered as negligible.[8] Therefore, the exogenous price for wind technology is assumed to be fixed. A detailed description of the stochastic process associated with the stochastic innovations in solar technology can be found in Appendix 4.7.2.

4.2.1 Timing of the investment

Let us illustrate the effects of investment timing in a simplified one step-model before turning to the fully dynamic model. In the one-step model only at two points in time (today t_0 and the future state t_1) an investment in RES can be made. Since the timing of the investment in wind and solar technology is not exogenously fixed, the energy manager has the option to postpone the investment decision today at t_0 to the future t_1 and to receive new information about the evolution of the uncertain parameters, in order to re-evaluate the investment opportunity. During this time period $[t_0, t_1]$ the energy manager has to secure the electricity supply of the firm and purchases power to cover the demand. However, including the possibility of deferring the investment decision to the future state t_1 generally introduces managerial flexibility and therefore creates a value which has to be considered in the investment decision.

Since we consider the combined impact of the multiple sources of uncertainty, we solve for the optimal investment decision on the joint grid representing uncertainty of the states of nature, i.e., the level of the remuneration policy and the stochastic price per unit of solar technology installed, which is illustrated in Fig. 4.3. These two dynamic sources of uncertainty, i.e., policy uncertainty and uncertainty over the investment price of solar technology, drive the optimal timing of the investment towards different directions. Due to the expected decrease of the level of the FIT, the energy manager tends to invest in RES earlier, since the likelihood of the remuneration policy to offer a higher compensation for selling surplus power to the grid is also higher. In contrast to policy uncertainty, due to the expected decreasing price of solar technology the energy manager tends to invest later in order to reduce the expected capital expenditures of the energy investment. The optimal investment

[8] This represents a model limitation which, however, can methodologically be treated in the same way as uncertainty associated with solar technology.

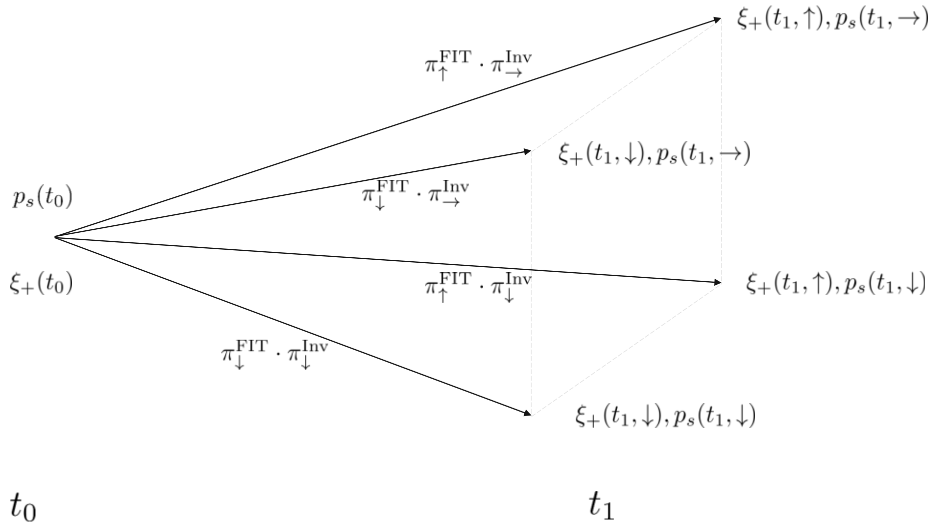

Figure 4.3: This figure represents the joint grid of the states of nature in the one-step problem with the 4 possible states arising in the future.

decision therefore has to balance the expected trade-off associated with investing in RES at t_0 or investing in RES at t_1.

The energy manager not only has the opportunity to adopt a lumpy investment strategy or to postpone the investment decision as such to the future state t_1 (which includes the opportunities to invest in wind technology/solar technology/a diversified energy portfolio at t_0 or t_1, or to not invest at all), but also to follow a staged investment strategy. In the staged investment strategy, the energy manager introduces additional flexibility in terms of including the possibility to invest partially (we assume – as a simplifying assumption – that in staged investment the investment budget is split in two equally sized portions) in wind technology at the first stage decision at t_0 and to keep the option to expand in solar/wind technology in the second stage decision at t_1 alive.[9] Therefore, following this strategy, the energy manager has the option to choose the timing of the partial investments in wind and / or solar technology independent of each other. For the energy manager who follows a staged investment strategy, the current level of the FIT is locked in at time t_0 the early investment in wind technology is made. However, if he or she decides to expand this energy portfolio

[9]This also includes to reject in expanding the energy park at t_1.

in the future t_1, the new level of the FIT at t_1 is assigned from t_1 onwards to price surplus power and therefore overwrites the old level of the FIT at t_0.[10] Hence, the benefit associated with a staged investment strategy is that the energy manager can immediately alter cash-flows that arise from purchasing outside power to cover the demand. The trade-off is that the energy manager sacrifices a part of the flexibility options, since the single solar energy portfolio – which is valuable in case of a low price of solar technology – is not attainable due to the early investment in wind technology. For completeness we remark that also the investment strategy to partially invest in solar technology at t_0 and to keep the option to invest in wind/solar technology at t_1 alive, exists. However, technological innovations are expected only in solar technology and due to the characteristic features of the wind distribution, an investment in wind technology is more valuable in this case. Therefore, an early partial investment in solar technology and keeping the option to expand the energy park is not in the scope of the model.[11]

In order to simplify the problem, we assume an effective infinite lifetime of the energy park. This can be made plausible by assuming that the energy manager re-invests in the same energy portfolio after the expected useful lifetime of the energy park.[12] The underlying assumptions in the investment model are summarized in Tab. 4.1.

Table 4.1: This table summarizes the model assumptions.

Variable	Assumptions
FIT ξ_+ [€$/MWh$]	(i) GBM with negative drift $\mu < 0$ (ii) At time of investment the current level of FIT is locked in
Price per unit of solar technology p_s [€$/MWh\,inst.$]	(i) Number of innovations per year Poisson distributed (ii) Fixed size of innovation α
Energy price ξ_- [€$/MWh$]	Fixed price per MWh of shortfall in the power supply
Demand d $[MW]$	Deterministic demand over efficient lifetime of the energy park
Budget I_0 [€]	Max. amount that can be invested in RES. We impose $I_0 \leq 2p_w d$, ensuring that with a staged investment no surplus power can be sold.
Timing of the investment	Only at two points in time t_0, t_1 an investment in RES can be made
Renewable energy portfolio $\mathbf{x} = (x_w, x_s)$ $[MW]$	We consider 3 different energy portfolios: (a) single energy investment wind $x_w = I_0/p_w$, $x_s = 0$ (b) single energy investment solar $x_s = I_0/p_s$, $x_w = 0$ (c) diversified energy portfolio $x_w = I_0/(2p_w)$, $x_w = I_0/(2p_w)$
Investment strategy	(i) *Lumpy investment*: invest in portfolios (a)-(c) either at t_0 or at t_1 (ii) *Staged investment:* invest in wind capacity $x_w = I_0/(2p_w)$ at t_0 and keep the option to expand $x_s = I_0/(2p_s)$ in solar technology at t_1 alive

4.3 Valuing the renewable energy investment

In order to determine the value of the option (in terms of the total costs of the firm's power supply) of investing in a renewable energy park (with different renewable energy portfolio options), we have to determine expected costs of every energy portfolio in every possible state of the future (see Fig. 4.3). Generally, in this dynamic investment problem at two points in time a decision has to be made: The first stage decision at t_0 and the second stage decision at t_1. We assume, that the firm's power demand d is deterministic and that the budget that can be used for the energy investment is given by I_0. The costs of purchasing one unit of pre-contracted power in case of a power shortfall is exogenously fixed and denoted by ξ_-. Since the power output per unit of installed capacity in wind technology P_w and the power output per unit of installed capacity in solar technology P_s are stochastic, the energy manager takes into account the expected shortfall costs (where balancing energy has to be purchased) and the expected remunerations for selling surplus power to the grid. For a given level of the FIT ξ_+, the current price of solar technology p_s (which is different in the future states since they are subject to uncertainty) and the price of wind technology p_w (which is fixed), the expected costs of power procurement are given by: (i) the investment costs I to build the energy park, minus (ii) expected remunerations from selling surplus power to the grid and plus (iii) purchasing pre-contracted power in case of a shortfall in the power supply. The investment costs (i) have to be paid instantaneously and are considered as sunk costs, whereas the cash flows associated with the expected remunerations and expected shortfall costs (ii) and (iii) arise continuously during the effective lifetime of the energy park and therefore have to be discounted.

[10] This assumption is done for reasons of tractability. If the level of the FIT for the early investment and the level of the FIT for later investment is fixed independently, the FIT fixed for the early invested capacity serves as a state variable for the second stage investment decision, increasing the dimensionality of the investment problem. Since the insight from our analysis is not driven by these subtleties, we avoid the overly complex model structure. It would also be possible to fix the level of the FIT with the early investment in wind technology for all times. In this case, however, we do not allow for small first stage investments.

[11] Moreover, a staged investment strategy where the first and the second stage decision is to invest in solar technology excludes the benefits from the renewable diversification effect.

[12] Formally, we introduce the re-investment in the energy assets by introducing an effective interest rate r', s.t. the present value associated with the finite investment opportunity at the interest rate r is the same as the present value of the infinite investment at the effective rate r'. Therefore, generally $r' \geq r$ holds true. This represents a model limitation since we don't consider the flexibility to re-balance the energy portfolio after the finite lifetime but continue to re-invest in the same energy portfolio.

We describe the evolution of the states of nature, i.e., the level of the FIT ξ_+ and the technological innovation α via a set of trees. Each tree characterizes the state of the investment, i.e., investment decisions which are already fixed. All possible energy portfolios considered in this investment model are shown in Fig. 4.4, i.e.: no investment in RES (A), staged investment in wind technology (B), the diversified energy portfolio (C), the single energy investment in wind technology (D) and the single energy investment in solar technology (E). In case of a RES investment that exhausts the budget (i.e., the trees C,D and E), the corresponding investment opportunities can be immediately valued since there are no further flexibility options left and nodes in these trees represent stopping at absorbing nodes. Since we assume irreversibility of the investment, we do not consider the opportunity of selling the power generation facilities. Therefore, undoing the investment and returning to the tree A representing no investment in RES is not in the scope of this model. In contrast to trees C, D, and E where the investment decision is already fixed, trees A and B represent states with investment flexibility.

Given that currently (at a given time t) no investment (tree A) or a staged investment (tree B) was made, the budget left (i.e., I_0 or $I_0/2$) can be used to expand the energy park in the future. Fig. 4.4 indicates these flexibility options associated with the investment strategy via arrows. Investing in a renewable energy technology and thereby expanding the current renewable energy portfolio corresponds to a jump between the trees A-E. Investment at $t+1$ can be done in the same way as at t, leading to a jump to the corresponding node in the tree that represents the investment decision (the state of nature is preserved). The costs associated with the jump between the trees corresponds to the investment $I \in \{I_0/2, I_0\}$ necessary to obtain the target renewable energy portfolio. In case the energy manager has not invested in RES (black arrows starting at tree A) he or she can decide to invest half the budget $I_0/2$ in wind technology (tree B) and keep the option to expand the energy park alive (stay in tree A) or invest the full budget I_0 in: a diversified portfolio (tree C), an undiversified portfolio in wind technology (tree D) or an undiversified portfolio in solar technology (tree E). The energy manager can always choose to postpone the investment decision, continue the current energy portfolio and therefore also to remain within the current tree, at least for the coming time step (thereby keeping the flexibility alive and reconsidering an investment after observing the shocks to the stochastic state variables p_s and ξ_+). In case of an early investment in wind technology (tree B), the energy manager has the option to expand in wind or solar technology (tree C or tree D). However, due to irreversibility of the investment the portfolio representing a single energy investment in solar technology cannot be obtained in this case.

4.3.1 Costs and cash flows of the investment

Let us now discuss the costs and expected cash flows associated with an investment in RES in more detail. The decision to invest in RES corresponds to a jump between the trees in Fig. 4.4 and generates sunk costs of either $I_0/2$ or I_0, depending on the type of the investment strategy.[13] However, the actual capacity installed in solar technology depends on the current level of the investment price of solar technology $x_s = I/p_s$ and therefore varies according to the possible states of nature in the future. In contrast to that, the price of wind technology is constant and therefore the installed capacity in wind technology is either $x_w = I_0/p_w$ or $x_w = I_0/(2p_w)$, for all possible states of nature that occur in the future. This is of special importance, since the value of the investment depends on the installed capacities in wind and solar technology, respectively. We assume that the hourly stochastic power output per capacity installed in wind P_w and solar technology P_s to be iid distributed. Therefore, if the capacity installed in wind technology is x_w, the capacity installed in solar technology is x_s and the level of the FIT at the time of the last investment in RES is ξ_+,[14] the value of the investment is given by

$$V_I(x_w, x_s, \xi_+) = \delta\big(\underbrace{-\xi_+\mathbb{E}[\max\{x_wP_w + x_sP_s - d; 0\}]}_{\substack{\text{expected remunerations from selling} \\ \text{surplus power to the grid}}} + \underbrace{\xi_-\mathbb{E}[\max\{d - x_wP_w - x_sP_s; 0\}]}_{\text{expected shortfall costs}}\big), \tag{4.1}$$

where δ denotes the present value factor. Therefore, the value associated to the definite decision of rejecting an investment in RES once and for all and, thus, purchasing total power to cover demand in the future is given by $V_I(0, 0, \xi_+) = \xi_- d\delta$ and is independent of the level of the FIT. However, if the energy manager decides to postpone the investment by Δt and previously an early investment in RES with x_w capacity

[13] Investing half of the budget available $I_0/2$ corresponds to a staged investment strategy (black arrows in Fig. 4.4) and investing the total budget available I_0 corresponds to a lumpy investment strategy (red arrows in Fig. 4.4).

[14] The level of the FIT at the time a consecutive investment in RES overwrites the previously locked-in FIT.

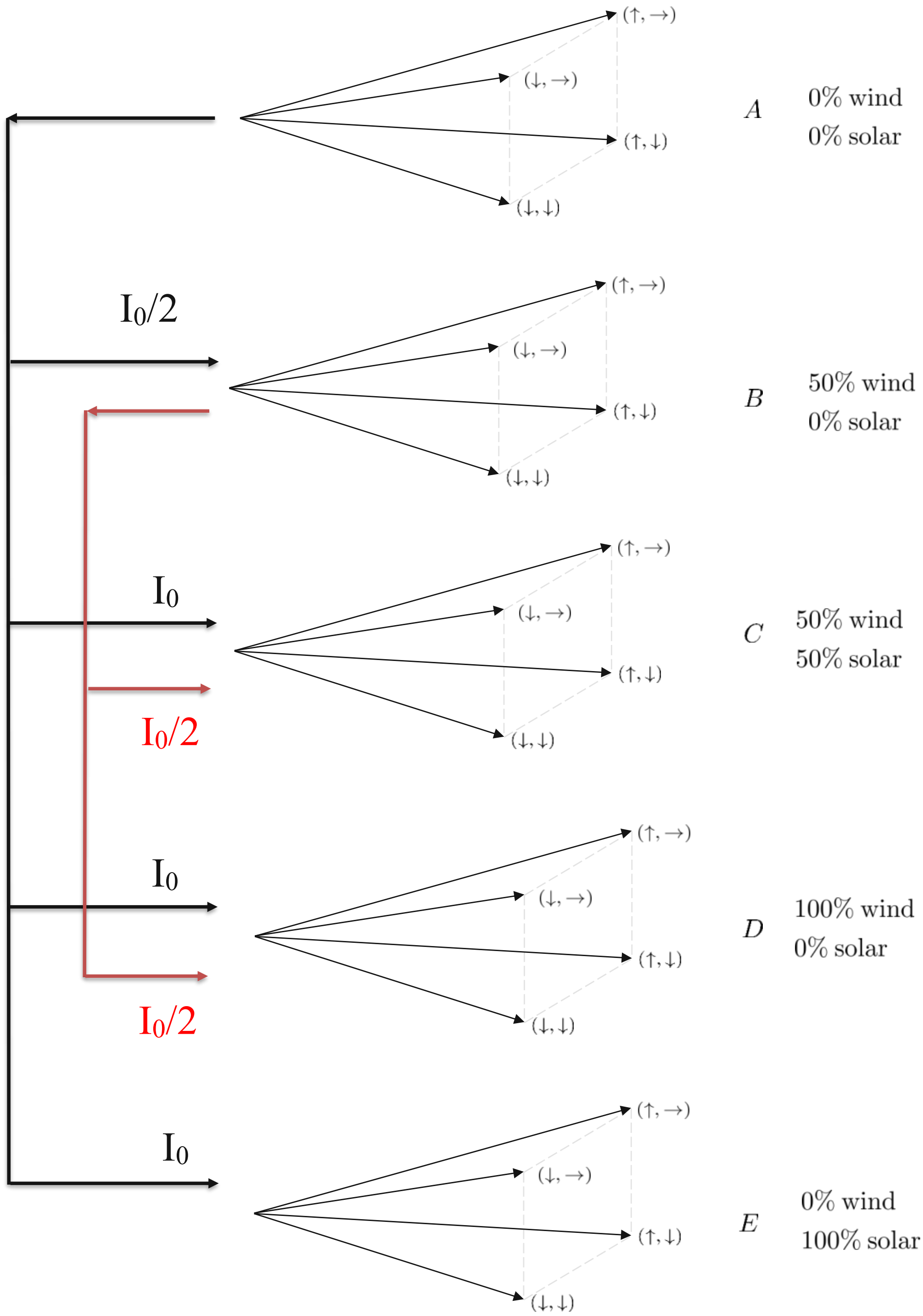

Figure 4.4: This figure illustrates all possible energy portfolios of wind and solar technology considered. The arrows illustrate the flexibility options (black: flexibility options when no investment has occured, red: flexibility options when a partial investment in wind technology has occured).

installed in wind technology (tree B) occurred, where the level of the FIT at the time of the investment in wind technology was ξ_+, the cash flow arising during the period Δt due to deferring the investment is simply given by

$$c(x_w, \xi_+) = \Delta t(\underbrace{-\xi_+ \mathbb{E}[\max\{x_w P_w - d; 0\}]}_{\substack{\text{expected remunerations from selling} \\ \text{surplus power to the grid}}} + \underbrace{\xi_- \mathbb{E}[\max\{d - x_w P_w; 0\}]}_{\text{expected shortfall costs}}). \tag{4.2}$$

If no investment was made in RES the cash flow arising due to deferring the investment is given by $c(0, \xi_+) = \Delta t \xi_- d$ and corresponds to purchasing total power to cover the firm's electricity demand during the time span Δt via pre-contracted energy.

Note that in (4.2) the cash-flow depends also on the current level of the FIT that is locked-in. However, we impose the constraint $I_0 \leq 2p_w d$ on the investment budget. This guarantees that in case of a staged investment the first stage investment in wind technology (with investment costs $I_0/2$) installs a capacity which is sufficiently low such that no surplus power can be generated, i.e., $\Pr\{x_w P_w - d \leq 0\} = 1$ holds true. In this case the cash-flow is independent of the level of the FIT but depends only on installed capacities in wind technology x_w. This is due to the fact that in this case $\mathbb{E}[\max\{x_w P_w - d; 0\}] = 0$ and (4.2) becomes

$$c(x_w) = \Delta t \xi_- \mathbb{E}[\max\{d - x_w P_w; 0\}]. \tag{4.3}$$

We impose this constraint to avoid the level of the FIT of the first stage investment as a state variable in the investment problem of the second stage. Since the power output per installed capacity of wind technology is bounded from above $P_w \leq 1$ and the capacity in case of a partial investment in wind technology is $x_w = I_0/(2p_w)$, we have that $x_w P_w \leq d$ holds true with certainty, given the budget is constrained by $I_0 \leq 2p_w d$.

4.4 Value of the option to invest

We determine the value of the investment opportunity in RES by using dynamic programming methods based on Bellman's Principle of Optimality. $V_t(\xi_+, p_s)$ denotes the value of the investment opportunity in terms of the minimum attainable present value of the power procurement costs at time t, given that the current state of nature is (ξ_+, p_s). The terminal value at the end of the decision horizon T is determined by investing in the energy portfolio that refers to the minimum expected power procurement costs under given flexibility, or refrain from investment at all. Due to different flexibility options, the value of the terminal nodes of the trees A and B are different. Let us denote the value of the investment at the final decision nodes of: (i) the diversified portfolio (tree C), (ii) the single wind technology portfolio (tree D) and (iii) the single solar technology portfolio (tree E) by:

$$
\begin{aligned}
V^C(p_s, \xi_+) &= V_I\left(\frac{I_0}{2p_w}, \frac{I_0}{2p_s}, \xi_+\right) \\
V^D(p_s, \xi_+) &= V_I\left(\frac{I_0}{p_w}, 0, \xi_+\right) \\
V^E(p_s, \xi_+) &= V_I\left(0, \frac{I_0}{p_s}, \xi_+\right).
\end{aligned}
\tag{4.4}
$$

Furthermore, the value of rejecting to invest in RES and purchasing total power to cover the demand is given by $V^{Ni}(p_s, \xi_+) = V_I(0, 0, \xi_+)$ and the value to abandon the option to expand, given that an early investment in wind technology has occurred is given by $V^{Ne}(p_s, \xi_+) = V_I(I_0/(2p_w), 0, \xi_+)$.

In case of a staged investment strategy, represented by tree B, the only investment opportunities at the final nodes are to: (i) invest $I_0/2$ to obtain the single wind technology portfolio (tree D) (ii) invest $I_0/2$ in solar technology to obtain the diversified technology portfolio (tree C) or (iii) abandon the option to expand the energy park (stay within tree B). Therefore, at the final nodes we have

$$
V_T^B(p_s, \xi_+) = \min\left\{\frac{I_0}{2} + V^D(p_s, \xi_+); \frac{I_0}{2} + V^C(p_s, \xi_+); V^{Ne}(p_s, \xi_+)\right\}. \tag{4.5}
$$

In case of no previous investment in renewable energy technologies, represented by tree A, the investment opportunities are to: (i) invest I_0 in the single wind technology portfolio (tree D) (ii) invest I_0 in the diversified technology portfolio (tree C), (iii) invest I_0 in the single solar technology portfolio (tree E), (iv) to abandon the option to invest in RES or (v) invest $I_0/2$ in wind technology (tree B). Therefore, at the final nodes we have

$$V_T^A(p_s,\xi_+) = \min\Big\{ I_0 + V^D(p_s,\xi_+); I_0 + V^C(p_s,\xi_+); I_0 + V^E(p_s,\xi_+); \\ V^{Ni}(p_s,\xi_+); \frac{I_0}{2} + V_T^B(p_s,\xi_+)\Big\}. \tag{4.6}$$

Having determined the value of the investment opportunity at the final nodes, we iterate backwards in time to determine the value of the investment opportunity at each preceding node. Therefore, assume that we have determined the value of the trees in each possible state of nature at time t. Since the value of the investment in RES depends on the value of expanding the energy park, given that an early investment in wind technology has already occurred (i.e., it is possible to jump from tree A to tree B), we first have to solve for the value of tree B.

Concerning the tree B, at each point in time $t-1$ the energy manager has the opportunity to: (i) invest $I_0/2$ to obtain the single wind technology portfolio (tree D) (ii) invest $I_0/2$ in solar technology to obtain the diversified technology portfolio (tree C) or (iii) defer the investment decision, obtain the cash flow and stay within tree B. Therefore, the value of the option to expand the energy park, given by the Bellman equation is

$$V_{t-1}^B(p_s,\xi_+) = \min\left\{ \frac{I_0}{2} + V^D(p_s,\xi_+); \frac{I_0}{2} + V^C(p_s,\xi_+); c\left(\frac{I_0}{2p_w}\right) + e^{-r\Delta t}\mathbb{E}_{t-1}[V_t^B(p_s,\xi_+)]\right\}, \tag{4.7}$$

where

$$\mathbb{E}_{t-1}[V_t^B(p_s,\xi_+)] = \sum_{p'_s,\xi'_+} p(p'_s,\xi'_+|p_s,\xi_+)V_t^B(p'_s,\xi'_+) \tag{4.8}$$

and $p(p'_s,\xi'_+|p_s,\xi_+)$ denotes the conditional probability to obtain the state of nature (p'_s,ξ'_+) in the next time step, given the current state of nature is (p_s,ξ_+).

Concerning the tree A representing the full flexibility, in every preceding node at time $t-1$ the energy manager has the opportunity to: (i) invest I_0 in the single wind technology portfolio (tree D) (ii) invest I_0 in the diversified technology portfolio (tree C), (iii) invest I_0 in the single solar technology portfolio (tree E), (iv) invest $I_0/2$ in wind technology and keep the option to expand the energy park alive (tree B) or

(v) defer the investment decision (stay in tree A). Therefore, the Bellman equation derives to

$$V_{t-1}^A(p_s,\xi_+) = \min\Big\{ I_0 + V^D(p_s,\xi_+); I_0 + V^C(p_s,\xi_+); I_0 + V^E(p_s,\xi_+); \frac{I_0}{2} + V_{t-1}^B(p_s,\xi_+); c(0) + e^{-r\Delta t}\mathbb{E}_{t-1}[V_t^A(p_s,\xi_+)]\Big\}, \tag{4.9}$$

where the expected value of the investment in the next period is

$$\mathbb{E}_{t-1}[V_t^A(p_s,\xi_+)] = \sum_{p'_s,\xi'_+} p(p'_s,\xi'_+|p_s,\xi_+)V_t^A(p'_s,\xi'_+). \tag{4.10}$$

This procedure can be followed iteratively to determine the current value of the investment opportunity in RES at $t = 0$, which is denoted by $V = V_0^A$.

4.5 Numerical results

We demonstrate the model in a use case, where we sample from real-world wind speed and solar irradiance data for a typical location in Central Europe where hourly data of the solar irradiance and the wind speed are available in the daytime.[15] The prices of the investment goods are given by $p_w = 1.4M€/MW$ for wind technology and at the starting time $t = 0$ $p_s = 1M€/MW$ for solar technology. The market rate is assumed to be $r = 1\%$[16]. All the results presented in this section are obtained for the one-step problem.

In order to analyze the sensitivity of the value of the option to invest in RES with respect to a change in the policy of the FIT and the innovations in solar technology, we perform "what-if" analysis by simulating different parameters of the underlying stochastic processes, which is illustrated in Fig. 4.5. Fig. 4.5(a) shows the value of the RES investment as a function of the parameters μ and σ of the stochastic process associated with the remuneration policy, for fixed values of the parameters describing stochastic innovations in solar technology. Since the drift is assumed to be negative $\mu < 0$, the absolute value $|\mu|$ indicates the long-term subsidy retraction

[15] From 10:00-18:00.

[16] Which gives an effective interest rate which considers re-investment of $r \approx 5\%$.

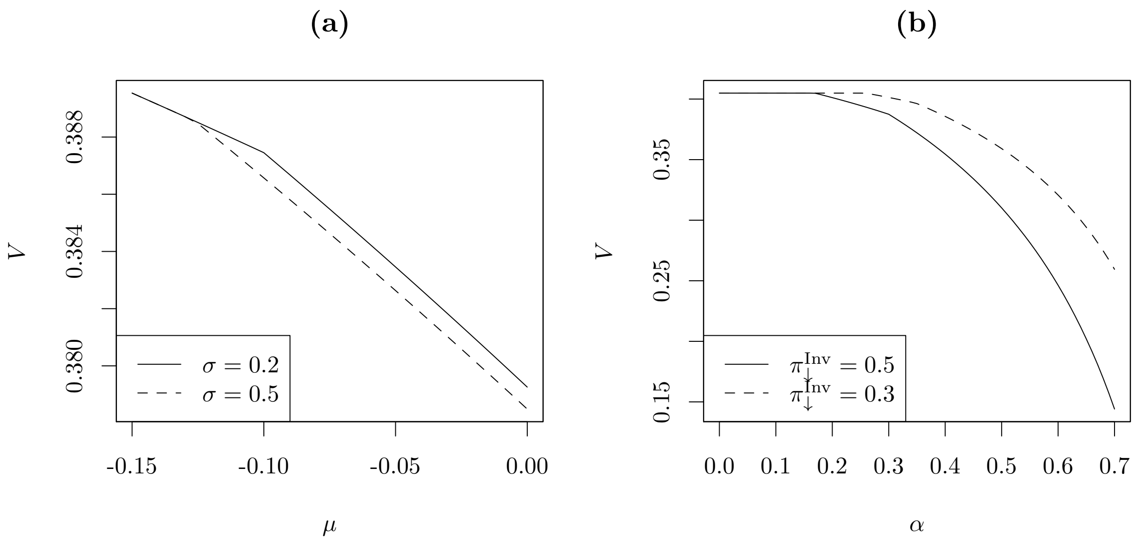

Figure 4.5: Figure (a) shows the value of the option to invest in RES as a function of the drift rate μ for two levels of σ ($\alpha = 0.3$, $\pi_{\downarrow}^{\mathrm{Inv}} = 0.5$). Figure (b) shows the value of the option as a function of the level of the innovation in solar technology for two levels of the probability to obtain an innovation in the next period ($\mu = -0.1$, $\sigma = 0.2$).

rate.[17] We observe expected power purchasing costs to be decreasing with lower values of the subsidy retraction rates. To highlight the effect of the uncertainty associated with the withdrawal of the remuneration policy, Fig. 4.5(a) shows the value of the investment opportunity in RES for two scenarios of the uncertainty σ associated with the remuneration policy. In this context, a higher uncertainty leads to lower expected power procurement costs as increasing uncertainty in the retraction of the FIT refers to a higher probability that the FIT will increase in the future.

Fig. 4.5(b) illustrates the value of the investment option in RES as a function of the parameters of the stochastic process associated with the stochastic innovations in solar technology α and $\pi_{\downarrow}^{\mathrm{Inv}}$, for fixed values of the parameters describing the stochastic level of the FIT. Obviously, the expected total power procurement costs are decreasing with increasing size of the expected innovation in solar technology α. Whenever the expected technological innovations in the future are below a threshold value, the

[17] Due to the fact that the energy manager expects an exponential decrease of the level of the FIT $\mathbb{E}[\xi_{+t}] = \xi_{+0}e^{-|\mu|t}$, higher values of $|\mu|$ refer to a scenario where the remuneration is withdrawn more quickly.

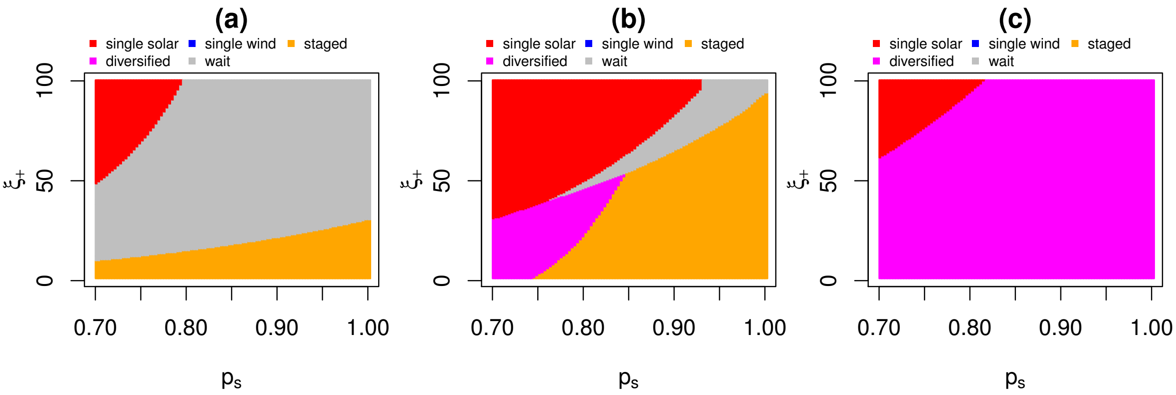

Figure 4.6: This plot shows the optimal investment strategy in RES for the scenario where minor innovations in solar technology are expected $\alpha = 0.1$ and: the (a) low $\xi_- = 50€/MWh$, (b) mid $\xi_- = 100€/MWh$ and (c) high energy price regime $\xi_- = 200€/MWh$.

optimal investment strategy is to invest immediately and obtain the benefits that arise from a potentially higher remuneration of excess power (therefore, for small values of α, the value of the option is constant, i.e., independent of α). Furthermore, we observe that the value of the option is more sensitive to an increase in the level of exogenous innovations in solar technology α compared to a decrease in the subsidy retraction rate. This highlights the impact of technological learning on the optimal investment strategy. To analyze the impact of uncertainty associated with the technological jumps, Fig. 4.5(b) shows the value of the investment opportunity in RES for two scenarios of the probability $\pi_\downarrow^{\text{Inv}}$ to obtain a technological innovation in the future. When the probability to obtain a technological innovation is higher, the expected power procurement costs are decreasing.

4.5.1 Strategic investment choice

We now discuss the optimal investment strategy in more detail. To do so, we illustrate the optimal investment strategy at $t = 0$ as a function of the current level of the FIT ξ_+ and the current price of solar technology p_s, for different scenarios of the exogenous energy price $\xi_- \in \{50€/MWh, 100€/MWh, 200€/MWh\}$ and the parameters of the stochastic processes $\pi_\downarrow^{\text{Inv}} = 0.5$, $\alpha \in \{0.1, 0.3\}$ (minor or major technological innovations in solar technology), $\mu = -0.1$ and $\sigma = 0.2$.

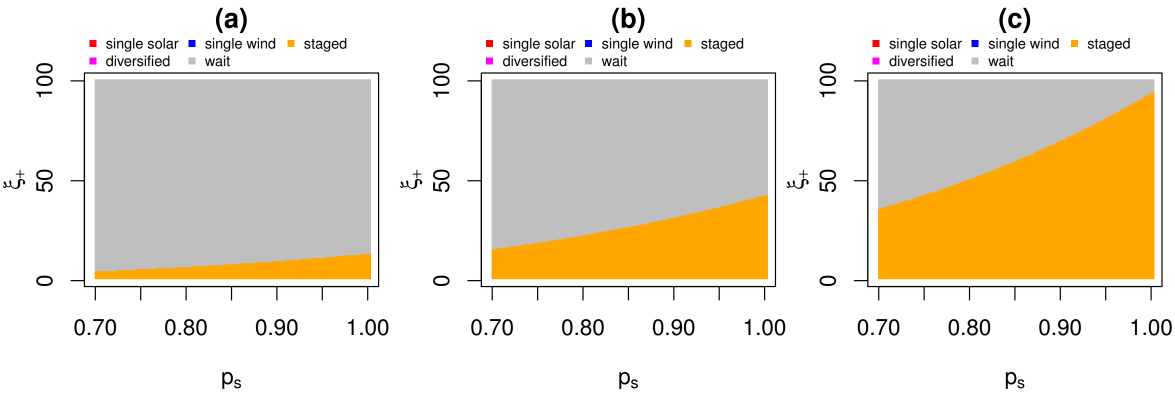

Figure 4.7: This plot shows the optimal investment strategy in RES for the scenario where major innovations in solar technology are expected $\alpha = 0.3$ and: the (a) low $\xi_- = 50€/MWh$, (b) mid $\xi_- = 100€/MWh$ and (c) high energy price regime $\xi_- = 200€/MWh$.

Fig. 4.6 illustrates the optimal investment strategy for the case of low technological innovations in solar technology $\alpha = 0.1$ and for different scenarios of the price of pre-contracted power: the low-range (Fig. 4.6(a)), mid-range (Fig. 4.6(b)) and high-range (Fig. 4.6(c)) energy price regime. In the case of a low energy price (Fig. 4.6(a)), we observe that both a lumpy and a staged investment strategy can be the optimal investment choice, depending on the current level of the FIT and the current price for solar technology. Generally we observe for the low energy price regime, that whenever the level of the FIT is sufficiently high the optimal strategy is to invest in RES immediately and obtain the benefits from selling surplus power to the grid due to the high level of the remuneration policy. However, for the majority of scenarios considered, deferring the investment decision is the dominant strategy. Therefore, low energy prices trigger early investment only on rare occasions and cause the energy manager to adopt a "wait-and-see" attitude. With increasing price of pre-contracted energy (Fig. 4.6(b) and (c)), the energy manager tries to avoid purchasing expensive power to cover the demand and speeds up investment in self-generation facilities. More specifically, higher energy prices emphasize the importance of avoiding power shortfall and thus, the optimal decision is investing early in a solar dominated production.

This situation is quite different, when there are major innovations in solar tech-

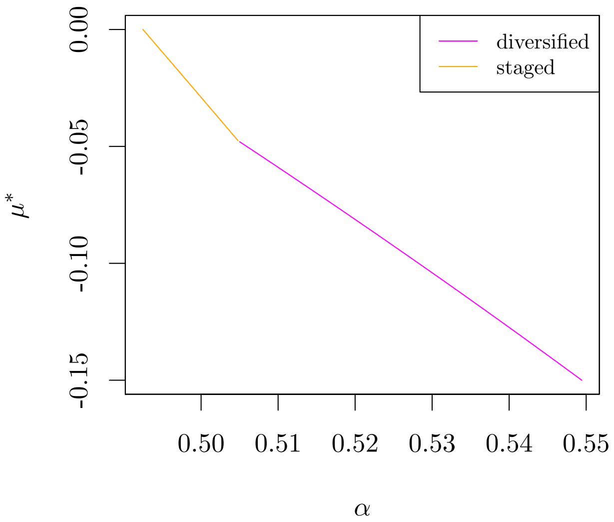

Figure 4.8: This figure shows the relationship between the level of innovation in solar technology and the endogenized drift rate μ^* s.t. the energy manager is indifferent in investing now or to defer the investment decision. The impact of the probability to obtain an innovation in solar technology is also demonstrated. In this numerical example $\sigma = 0.2$, $\pi_{\downarrow}^{\mathrm{Inv}} = 0.1$ and $\xi_{-} = 100€/MWh$ are chosen.

nology expected $\alpha = 0.3$ (Fig. 4.7). In this scenario, keeping the flexibility to invest in shares of solar power in the future when the price for solar technology is low becomes a valuable strategy. Fig. 4.7(a)-(c) illustrates the optimal investment choice in the low-, mid-, and high-energy price regime. With increasing energy price we observe, that adopting a staged investment strategy becomes increasingly important. With the early investment in wind technology, the energy manager sacrifices a part of the flexibility to invest in solar technology. However, in this scenario the staged investment strategy optimally balances the benefits of an expected decrease in the investment price of solar technology and the cash-flow due to purchasing power and deferring a part of the investment.

4.5.2 Policy implications

Based on the energy manager's optimal decision as a price taker, we now discuss policy implications associated with the optimal design of the remuneration policy. To do so, consider the regulator's point of view who is in charge of determining the level of the FIT that is used for pricing surplus power that is sold to the grid by power generation facilities. We assume, that the policy maker regulates the long term trend of the remuneration policy by setting the subsidy retraction rate. Given the exogenous level of the innovations in solar technology α and the probability $\pi_{\downarrow}^{\mathrm{Inv}}$ with which this innovation occurs in the next period, the regulator is interested in finding the critical subsidy retraction rate (i.e., the subsidy retraction rate $\mu^*(\alpha, \pi_{\downarrow}^{\mathrm{Inv}})$), where the energy manager is indifferent in investing immediately in RES or to postpone the investment decision to the future. In this setting, the parameter μ^* is therefore an endogenous parameter that depends on the innovations of solar technology of the market. This boundary region is of particular importance, since choosing slightly higher values of the subsidy retraction rate facilitates early investment in RES. In contrast to that, slightly lower values of the subsidy retraction rate incentives the energy manager to defer investment in renewable energy technologies.

The condition of how to obtain the endogenized subsidy retraction rate is indifference in the investment decision. The energy manager is indifferent in investing in RES at t_0 or deferring the investment decision to t_1, whenever the continuation value of the option to invest in RES is equal to the minimum power procurement costs associated with an investment at t_0. The subsidy retraction rate μ^* that fulfills this condition implicitly defines the boundary region denoting indifference of investing now or to postpone the investment decision, which is illustrated in Fig. 4.8.[18] Obviously, remunerations for surplus power must be withdrawn more quickly, whenever expected technological innovations in solar technology are higher. Fig. 4.8 also illustrates that the optimal portfolio choice is changing along the boundary region.

[18]This boundary region can be determined by applying a bi-sectioning algorithm to iteratively find the value of the drift s.t. equality of the continuation value and the optimal portfolio choice associated with the static problem holds true.

4.6 Conclusion

This paper extends the real options literature in the field of renewable energy investment. We analyze the optimal investment decision in renewable energy technologies (primarily wind and solar technology),[19] which is characterized by uncertain production volumes under policy uncertainty and stochastically decreasing prices for solar technology. The expected total power procurement costs to cover the firm's demand consists of the investment costs, minus expected remunerations for selling surplus power to the grid plus expected costs of a shortfall in the power supply. This nonlinear pricing relation introduces diversification benefits even for the risk neutral decision maker. Generally, the optimal investment decision in renewable energy technologies is not the investment in the energy technology with the highest expected power output per amount of invested capital, but to opt for a properly diversified energy portfolio that balances shortfall risks and benefits obtained from selling surplus power to the grid. Following the real options approach, we not only determine the optimal portfolio decision in renewable energy technologies but also the optimal timing of the investment. More specifically, the dynamic investment model also allows a staged investment strategy, i.e., an early partial investment in wind technology and keeping the option to expand in solar technology alive. An early investment in wind technology might be beneficial since it allows to immediately alter the cash-flow. In the use case we find that this staged investment strategy is of special importance, whenever the price of the energy that has to be purchased in case of a shortfall in the power supply is high, but major innovations in solar technology are expected. In this scenario, the optimal investment decision is to sacrifice a part of the flexibility for an early investment in wind technology. This demonstrates, that the option of a staged investment strategy in RES facilitates early investment in wind technology. Furthermore, with increasing price of pre-contracted energy (i.e., shifting more weight to the shortfall-tail of the cost distribution), the likelihood of the energy manager to adopt a staged investment strategy increases. Our investment model also provides valuable insights from the regulator's point of view, who sets the optimal subsidy retraction rate (i.e., creating a stimulus for early investment that counterbalances the incentive to delay investment which is usually present in investment decisions under uncertainty). Based on the partial equilibrium model referring to the energy

[19] I.e., we do not take conventional (fossil fuel based) power plants into account.

manger's optimal portfolio choice, we infer the optimal subsidy retraction to be set by the regulator.

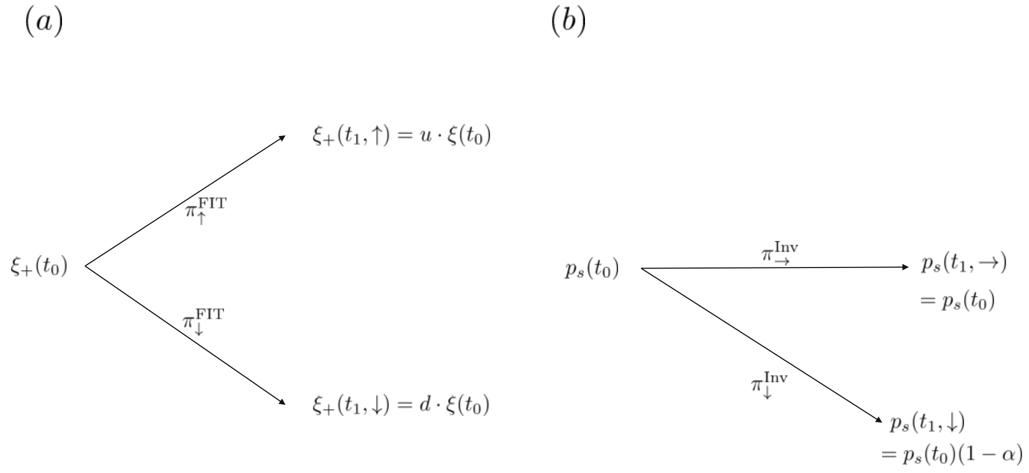

Figure 4.9: (a) shows the grid associated with the GBM of the level of the FIT and (b) shows the grid for the evolution of the investment price for solar technology.

4.7 Appendix

4.7.1 A: Remuneration policy

We assume the level of the FIT follows a geometrical Brownian motion (GBM) $\mathrm{d}\xi_{+t} = \mu\xi_{+t}\mathrm{d}t + \sigma\xi_{+t}\mathrm{d}z_t$, with drift μ and volatility σ^2, where $\mathrm{d}z_t$ is increment to a Wiener process. Therefore, future values of the level of the FIT are log-normally distributed with mean $\mathbb{E}[\xi_{+t}] = \xi_{+0}\exp(\mu t)$ and variance $\mathbb{V}[\xi_{+t}] = \xi_{+0}^2 \exp(2\mu t)(\exp(\sigma^2 t) - 1)$. Following Cox et al. (1979), we approximate the GBM via a binomial lattice, where the decision horizon is subdivided in elementary time intervals of length Δt. The up and down factors specifying the level of the FIT in the proceeding time step are given by

$$\begin{aligned} u &= e^{\sigma\sqrt{\Delta t}}, \\ d &= e^{-\sigma\sqrt{\Delta t}}. \end{aligned} \tag{A1}$$

The probability $\pi_{\uparrow}^{\mathrm{FIT}}$ to obtain an up movement of the level of the FIT in the proceeding time step is given by

$$\pi_{\uparrow}^{\mathrm{FIT}} = \frac{e^{\mu\Delta t} - e^{-\sigma\sqrt{\Delta t}}}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}. \tag{A2}$$

To obtain a valid probability $\pi_{\uparrow}^{\mathrm{FIT}} \in [0, 1]$ has to hold true. Since the level of the remuneration policy is expected to decrease over time, the drift is negative $\mu \leq 0$. The requirement to obtain a probability measure therefore imposes a condition on the size of the time step, which has to be sufficiently small $\sqrt{\Delta t} \leq \sigma/|\mu|$. For $\Delta t \to 0$ this time-discrete process converges to a GMB. The process associated with the one step problem is illustrated in Fig. 4.9(a).

4.7.2 B: Prices of the investment goods

Due to technological learning and diffusion, the price per one unit of installed solar capacity p_s can decrease over time. We consider a stochastic model of technological learning and diffusion and assume that stochastic exogenous technological innovations occur over time. Whenever an innovation shock occurs, the price of solar technology decreases instantaneously by a fraction of α% and when no innovation shock occurs, the price remains the same. We assume, that the number of innovations associated with solar technology ν follows a Poisson process with a rate of λ innovations per year. Therefore, the expected number of innovations in y years is given by $\mathbb{E}[\nu] = \lambda y$ and the probability to obtain k innovations over a time period of y years is given by

$$\Pr\{\nu = k\} = \frac{(\lambda y)^k}{k!} e^{-\lambda y}. \tag{A1}$$

Therefore, the price of solar technology in the future is

$$p_s(t_1) = \begin{cases} p_s(t_1, \downarrow) = p_s(t_0)(1-\alpha), & \text{if an innovation occurs} \\ p_s(t_1, \rightarrow) = p_s(t_0), & \text{if no innovation occurs.} \end{cases} \tag{A2}$$

Similar to the construction of the GBM, we divide the time horizon into time intervals of length Δt. Hence, the probability of obtaining a technological innovation in solar technology is approximated (linearly) with $\pi_{\downarrow}^{\mathrm{Inv}} = \lambda\Delta t$. The probability of multiple innovations within one step Δt is of order $(\Delta t)^2$ and can safely be ignored

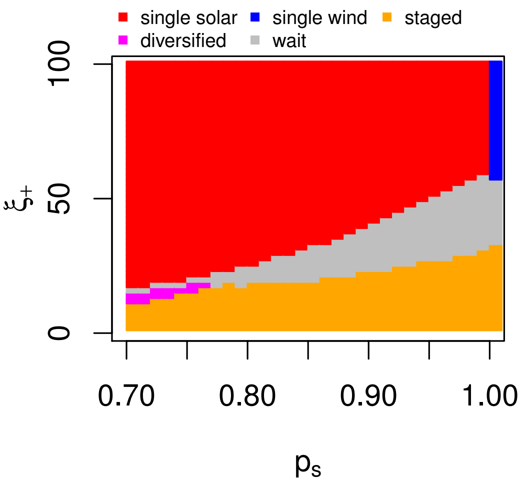

Figure 4.10: This figure illustrates the optimal investment strategy in the fully dynamic model as a function of the current price for solar technology and the current level of the FIT at $t = 0$.

for small Δt. Consequently, the probability that no innovation occurs is given by $\pi_{\rightarrow}^{\text{Inv}} = 1 - \pi_{\downarrow}^{\text{Inv}}$. To obtain a probability $\pi_{\downarrow}^{\text{Inv}} \in [0, 1]$, the condition $\Delta t \leq 1/\lambda$ has to hold true. The number of inventions in the decision period is Binomially distributed $\nu \sim \mathcal{B}(n, \pi_{\downarrow}^{\text{Inv}})$, where for the number of intervals within the decision horizon $n \rightarrow \infty$, the probability mass function of the Binomial distribution converges to the probability mass function of a Poisson distribution with rate λ. The process associated with the one step problem is illustrated in Fig. 4.9(b).

4.7.3 C: Dynamic N-period Problem

Up to this point we have illustrated the investment model in the one-step problem. Let us now discuss the fully dynamic model and analyze the solution which is obtained for an arbitrary but finite time horizon T.[20] Therefore, we split the time horizon into N equally spaced sub-intervals of length Δt.[21] In the fully dynamic model the same logic as in the one-step problem applies. At each point in time the energy manager faces the flexibility options to invest in RES, invest partially in wind technology or defer the investment decision, see Fig. 4.4. We solve the Bellman equations (4.7) and (4.9) backwards in time, starting at the terminal nodes at time T. We follow this procedure recursively and determine the value function iteratively up to time $t = 0$. In the one-step problem we have applied this iteration one time, whereas in the fully dynamic model we have to apply this step N times.

In the use case we assume that the decision to invest in RES can be made on a semi-annual basis, i.e., $\Delta t = 0.5$ with a time horizon $T = 10y$. Furthermore, we impose for the underlying process of the FIT $\mu = -0.1$, $\sigma = 0.2$ and for the underlying process of technological innovations in solar technology $\alpha = 0.025$ and $\pi_{\downarrow}^{\text{Inv}} = 0.25$ (per time step Δt, i.e., $\lambda = 0.5$). The energy price is $\xi_{-} = 50€/MWh$, the budget available is $I_0 = 0.25M€$ and the effective interest rate $r \approx 5\%$.

The fully dynamic model (Fig. 4.10) basically recovers the model effects obtained one-step problem. With increasing level of the remuneration policy, the optimal decision is to invest immediately in RES and with decreasing price of solar technology, the investment decision is in favor of solar technology. When the current level of the FIT is not sufficiently high, the optimal decision is either to postpone the investment decision or to follow a staged investment strategy and invest a fraction of the budget available in wind technology.

[20] The existence of a stationary solution requires some restrictions on the discount rate and on the expected rate of price reduction for solar production technology. The discount rate must be sufficiently large to outweigh the growth effect coming from expected price reduction. If expected price reduction is high, the area of solar panels that can be installed with fixed investment costs I_0 (or $I_0/2$) exhibits a large positive growth rate which must be more than offset by the discount factor in order to obtain stationarity. In real-life, however, also limited area available for solar panels and further limiting effects impose an upper bound to the installed capacity even when prices decline steeply. Hence, simple and realistic adaptations of the model will provide a stationary solution even with low interest rates and large expected price reductions. Therefore, for T sufficiently large, the solution approximates the stationary solution.

[21] With $N \to \infty$, i.e., $\Delta t \to 0$ the price process of the level of the FIT converges to a Geometric brownian motions.

5 Conclusion

This thesis aims at determining the optimal investment choice in renewable energy technologies under uncertain production volumes. One of the key features of the model presented in this thesis is the renewable energy portfolio effect. More specifically, this means that due to the different characteristics of the power output per installed capacity of wind and solar technology the optimal generation mix is properly diversified energy portfolio rather than the pure investment choices. In this way the risk of a shortfall in the power supply of the energy park can be balanced.

The first two papers (Chapter 2 and 3) discuss different planning mechanisms where the energy manager aims at minimizing the expected power procurement costs in the absence of a remuneration scheme of renewable energy technologies. The first paper (Chapter 2) introduces the reliability-based planning approach, where the energy manager's planning paradigm is to choose the renewable energy portfolio that refers to the minimum capital expenditures such that the probability that the energy park is able to cover the demand is larger than a pre-specified threshold. Obviously, when a very high level of reliability is imposed, the investment costs of the energy park become exceptionally high. Although the reliability-based planning approach statistically guarantees a threshold probability of supply-demand coverage, it does not take into account the extend of constraint violation in the scenarios where the supply does not cover the demand. The second paper (Chapter 3) introduces the cost-based planning approach, where the energy manager determines the renewable energy portfolio that minimizes the investment costs and the expected shortfall costs. In this planning approach the price of energy in case of a shortfall in the power supply endogenizes the level of reliability associated with the optimal portfolio choice. Comparing these two planning mechanisms, we find that the underlying optimal portfolio decision is different, even if – by coincidence – the level of reliability is the same. More specifically, including the expected shortfall costs implies that the energy manager opts for a properly diversified portfolio at lower levels of reliability compared to the reliability-based planning approach where the optimal portfolio choice is an undiversified portfolio. These two papers discuss the investment problem in a static context, i.e., where the investment opportunity is considered as a "now-or-never" problem. The third paper (Chapter 4) discusses the optimal portfolio selection problem under multiple sources of uncertainty in a dynamic framework, where the possibility to defer the investment decision to the future exists. In this paper, the optimal investment choice in renewable energy technologies is analyzed when the level of the feed-in tariff and the price of solar technology is subject to uncertainty in the future. This paper proposes a real options approach, where also the optimal timing of the investment is discussed. It is demonstrated in a use case that whenever only minor technological innovations are expected, a high level of the energy price facilitates early investment in renewable energy technologies. High expected innovations in solar technology on the contrary induces the energy manager to defer the investment decision. However in between theses two scenarios, the optimal decision is to follow a staged investment strategy and to invest partially in wind technology and to keep the option to expand the energy park alive. The results of this work can be used to determine the optimal subsidy retraction rate to avoid overcompensation and underinvestment.

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