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A MECHANIC-ACOUSTIC COUPLING CONDITION, INCLUDING VISCOUS AND THERMAL BOUNDARY LAYER EFFECTS

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ABSTRACT

Acoustic-structural interaction can be modeled by coupling the linearized compressible flow equations and the balance of momentum, governing fluid and solid mechanics respectively. The flow equations can accurately model the boundary layer effects but require the boundary layer to be resolved sufficiently fine, which increases the computational cost. In this work, we extend the so-called boundary condition approach which accounts for the boundary losses to include boundary motion, thereby arriving at a coupling condition between fluid and solid. So we describe the acoustics by the Helmholtz partial differential equation, the mechanics by the balance of momentum, and the coupling condition establishes the coupling and also accounts for the boundary losses. This strategy reduces the number of unknowns and the coupling condition does not require the boundary layer to be resolved explicitly, reducing the computational cost significantly. The formulation is validated using several test cases, and the results agree well with the analytical and fully resolved compressible flow equations.

Keywords: *vibro-acoustics, viscosity, boundary layers, coupling condition*

1. INTRODUCTION

The interaction between viscous fluid and an elastic structure under boundary layer effects are studied in various fields due to their applications such as Micro Electro Mechanical Systems (MEMS). The boundary layer effects are due to viscous dissipation caused by the shear motion due to the no-slip condition at the boundary and also due to heat exchange between the fluid and the structure. When the characteristic size of the domain is comparable to the boundary layer thickness, it is important to model the damping caused by the boundary layer effects. The above-mentioned boundary layer effects can be modeled by coupling the linearized compressible flow equations and balance of momentum, governing acoustics and mechanics respectively. But using this approach increases the computational cost due to the introduction of extra variables such as velocity (three variables), temperature, and pressure. And the computational mesh has to be very fine in the vicinity of the solid boundaries to resolve the large gradients in boundary layers.

A lot of approaches have been proposed to reduce the computational cost by approximating the effects due to the boundary layer effects. One such approach is proposed by Berggren et al [1], where they derive a boundary condition that accounts for boundary losses that can be used with isentropic models such as the Helmholtz equation for the acoustic pressure. The aim of this work is to extend this boundary condition approach by including the boundary motion thereby arriving at a coupling condition between the acoustic and mechanics. Once derived, the coupled problems can be treated using the standard Helmholtz equation for acoustic pressure and the balance of momentum for the mechanical displacement field along with boundary coupling conditions which account for the viscous and thermal boundary layer effects. The formulation is validated against a fully resolved linearised Navier-Stokes formulation for the fluid coupled to the solid via non-conforming interfaces [2].

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2. GOVERNING EQUATIONS

Let us consider a compressible, viscous fluid ($\boldsymbol{\Omega}_{\mathrm{a}}$) coupled to an elastic solid ($\boldsymbol{\Omega}_{\mathrm{m}}$) along a common interface Γ_{ma} as shown in fig. 1.

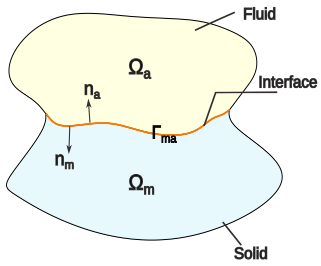


Figure 1: Simple sketch showing fluid-structure interaction along a common interface

2.1 Mechanics

The mechanical domain ($\boldsymbol{\Omega}_{\mathrm{m}}$) is governed by the conservation of momentum which is given by,

$$-\rho_{\mathrm{m}}\omega^2\boldsymbol{u} - \nabla\cdot\boldsymbol{\sigma}_{\mathrm{m}} = \boldsymbol{g}_{\mathrm{m}} \quad \text{in} \quad \boldsymbol{\Omega}_{\mathrm{m}} \tag{1}$$

where ρ_{m} is the density, $\boldsymbol{u}$ the displacement vector, $\boldsymbol{\sigma}_{\mathrm{m}}$ the stress tensor, and $\boldsymbol{g}_{\mathrm{m}}$ is the external force acting per unit volume in the mechanical domain. We assume linear elastic material behavior and a linearized strain displacement relationship

$$\boldsymbol{\sigma}_{\mathrm{m}} = \boldsymbol{C} : \boldsymbol{\epsilon}\,, \quad \boldsymbol{\epsilon} = \frac{1}{2}\left(\boldsymbol{\nabla}\boldsymbol{u} + (\boldsymbol{\nabla}\boldsymbol{u})^T\right) = \boldsymbol{\mathcal{B}}(\boldsymbol{u})\,, \tag{2}$$

where $\boldsymbol{C}$ denote the stiffness tensor and $\boldsymbol{\epsilon}$ the strain tensor. Equation (1) has been written in the frequency domain with angular frequency ω.

2.2 Acoustics

The viscous and compressible fluid is modeled by the acoustic wave equation, typically only used for inviscid fluids

$$-\frac{\omega^2}{c_0^2}p - \boldsymbol{\nabla}\cdot\boldsymbol{\nabla}p = 0 \quad \text{in} \quad \boldsymbol{\Omega}_{\mathrm{a}}\,, \tag{3}$$

where c_0 denotes the isentropic speed of sound in the medium and p is the acoustic pressure (perturbation). Viscous and thermal effects are confined to small regions at the boundaries of the fluid domain, i.e. to boundary layers, and should be considered via suitable boundary conditions [1] for maximum computational efficiency.

2.3 Coupling conditions

At the interface between flexible solid and viscous compressible fluid the dynamic and kinematic coupling conditions must be enforced. The kinematic coupling condition describes the equality of the time derivative of mechanical displacement and acoustic particle velocity. Boundary layer effects can be accounted for following Berggren [1] by splitting velocities into interface-normal and tangential components. In the tangential direction, the known boundary layer solution (Stokes 2. problem) can be used. After integration of the balance of mass, and accounting for mechanical displacements one obtains a suitable coupling condition, connecting the acoustic far-field pressure (outside the boundary layer) and the interface displacement. Similarly, by splitting the surface traction into tangential and normal components and considering the analytically known relation for the viscous boundary traction one arrives at a dynamic coupling condition accounting for viscous and thermal boundary layer effects.

3. FINITE ELEMENT FORMULATION

The final weak form of the governing equations is obtained by introducing appropriate test functions (denoted by ’) and integrating over the computational domain, taking advantage of integration by parts to incorporate boundary and coupling conditions. For the mechanical domain, i.e. from eq. (1), we obtain

$$\begin{aligned} &-\omega^2\int\limits_{\Omega_{\mathrm{m}}}\rho_{\mathrm{m}}\boldsymbol{u}'\cdot\boldsymbol{u}\,\mathrm{d}\Omega + \int\limits_{\Omega_{\mathrm{m}}}\boldsymbol{\mathcal{B}}(\boldsymbol{u}') : \boldsymbol{C} : \boldsymbol{\mathcal{B}}(\boldsymbol{u})\,\mathrm{d}\Omega \\ &+\int\limits_{\Gamma_{\mathrm{ma}}}\boldsymbol{u}'\cdot\boldsymbol{n}\,p\,\mathrm{d}\Gamma - \frac{\mu(i-1)}{\delta_{\mathrm{v}}\rho_{\mathrm{f}}\omega}\int\limits_{\Gamma_{\mathrm{ma}}}\boldsymbol{u}'\cdot\boldsymbol{\nabla}_{\mathrm{T}}p\,\mathrm{d}\Gamma \\ &+\frac{\mu\omega(i-1)}{\delta_v}\int\limits_{\Gamma_{\mathrm{ma}}}\boldsymbol{u}'\cdot\boldsymbol{u}_t\,\mathrm{d}\Gamma \\ &-\mu i\omega\int\limits_{\Gamma_{\mathrm{ma}}}\boldsymbol{u}'\cdot\boldsymbol{\nabla}\boldsymbol{u}\cdot\boldsymbol{n}\,\mathrm{d}\Gamma = \int\limits_{\Omega_{\mathrm{m}}}\boldsymbol{u}'\cdot\boldsymbol{g}\,\mathrm{d}\Omega\,, \end{aligned} \tag{4}$$

where $\delta_{\mathrm{v}} = \sqrt{2\mu/(\rho_{\mathrm{f}}\omega)}$ is the viscous boundary layer thickness related to the shear viscosity μ and density ρ of the fluid.

From the acoustics PDE (3) we obtain

$$
\begin{aligned}
&- k_0^2 \int_{\Omega_\mathrm{a}} p' p \, \mathrm{d}\Omega + \int_{\Omega_\mathrm{a}} \boldsymbol{\nabla} p' \cdot \boldsymbol{\nabla} p \, \mathrm{d}\Omega \\
&\quad + \delta_\mathrm{T} k_0^2 \frac{(i-1)(\gamma-1)}{2} \int_{\Gamma_\mathrm{ma}} p' p \, \mathrm{d}\Gamma \\
&\quad + \delta_\mathrm{v} \frac{i-1}{2} \int_{\Gamma_\mathrm{ma}} \nabla_t p' \cdot \nabla_t p \, \mathrm{d}\Gamma \\
&\quad + \omega^2 \int_{\Gamma_\mathrm{ma}} \rho_\mathrm{f} p' (\boldsymbol{u} \cdot \boldsymbol{n}) \, \mathrm{d}\Gamma \\
&\quad + \delta_\mathrm{v} \frac{\omega^2 (1+i)}{2} \int_{\Gamma_{ma}} \rho_0 p' (\nabla_t \cdot \boldsymbol{u}_T) \, \mathrm{d}\Gamma = 0 \, , \qquad (5)
\end{aligned}
$$

where the index $(\,)_t$ denotes the tangential direction. The thermal boundary layer thickness $\delta_\mathrm{T} = \sqrt{2k/(\omega \rho_\mathrm{f} c_p)}$ depends on the angular frequency and fluid properties including thermal conductivity k and specific heat capacity at constant pressure c_p.

4. NUMERICAL VALIDATION AND RESULTS COMPARISON

To verify the formulation, we test it using the 1D plane wave propagation in a flexible acoustic channel and compare the results with the reference solution obtained from the coupling between the flow equations and the balance of momentum (Linflow-Mech coupling). The formulation is implemented and also tested using the open-source FEM program openCFS [3].

The example illustrates 1D wave propagation in a flexible acoustic channel where the interface between the fluid and the solid can deform due to the excitation applied in the fluid. Figure (2) provides information about the model and the boundary conditions used. The chosen fluid is water and the channel is made up of rubber. The properties of the material utilized are provided in the Tables (1) and (2). Since the boundary layer coupling

Density in Kg/m^3	1000
Compression modulus in Pa	2.5$\cdot$10^9
Shear viscosity in Pa	1.002$\cdot$10^{-3}

Table 1: Assumed material properties of water.

Density in kg/m^3	920
Young's modulus in Pa	1$\cdot$10^5
Poisson's ratio	0.49

Table 2: Assumed material properties for rubber.

condition includes the boundary layer effects in terms of the boundary layer thicknesses δ_v and δ_T there is no need to resolve near the interface. This is contrary to the full linearised Navier-Stokes formulation [2] where boundary layers must be appropriately resolved by the FE mesh and pressure, velocity, and temperature degrees of freedom exist. The fluid is excited with a harmonic pressure

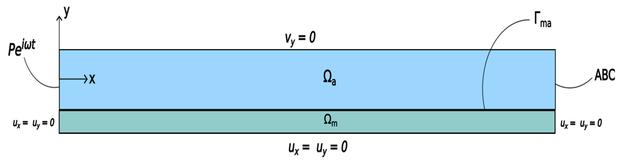


Figure 2: Geometry and BCs: Flexible acoustic channel

$pe^{i\omega t}$ for a range of frequencies in the excitation boundary. Symmetry boundary conditions are applied and an impedance-type absorbing boundary condition (ABC) is used to absorb the sound waves which ensures zero reflection. Figure 3 shows the field results of acoustic pressure p (in the background), acoustic velocity $\boldsymbol{v}$ (indicated using arrows) of the fluid, and the mechanical displacement $\boldsymbol{u}$ of the solid (in green contour). The particle velocity is high near the interface corresponding to the deformation of the channel. The tangential velocity at a cross-section of the channel is compared for the two formulations in fig. 4. One can recognize the typical Stokes boundary layer in the reference solution for the full linearised Navier-Stokes equation model in the fluid. For the boundary layer coupling model, we only consider the far-field solution in terms of acoustic pressure. The corresponding acoustic far-field particle velocity is calculated using the gradient of pressure. The calculated tangential velocity using pressure corresponds to the limit velocity of the fluid, which can be can be concluded by comparing the limit velocities of both formulations in Figure (4).

The amplitude decay of the pressure and velocity in

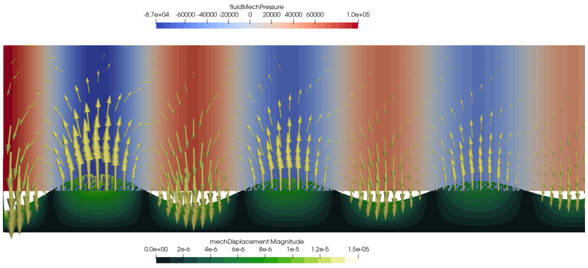


Figure 3: Field results plotted at a section of the channel (Linflow-Mech coupling)

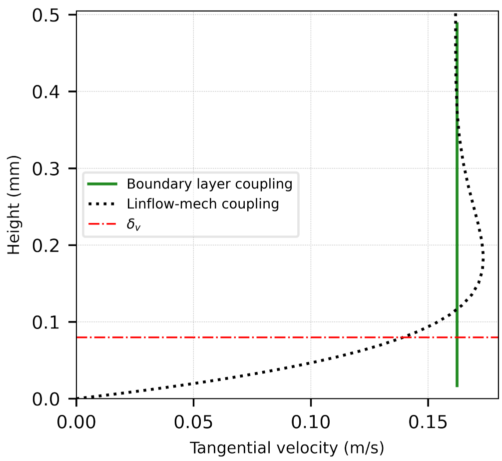
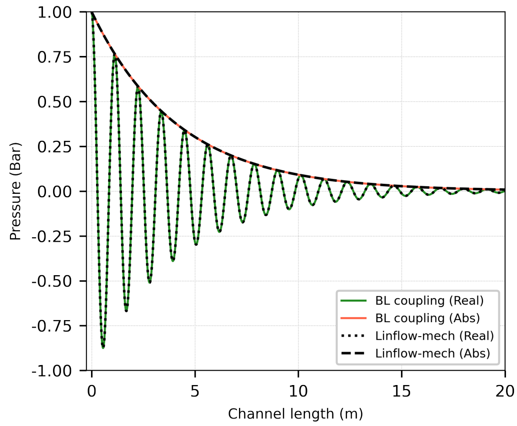


Figure 4: Comparison of tangential velocity at a cross-section showing boundary layer development.

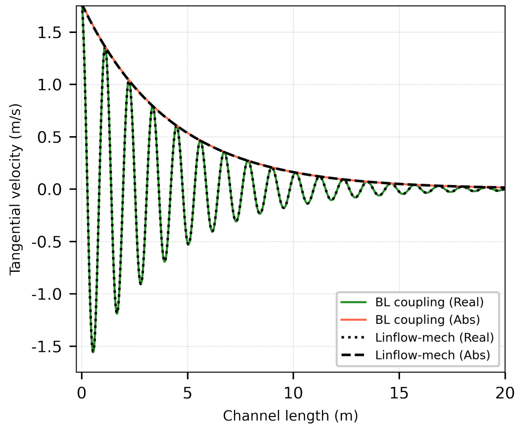
the fluid is compared in the figures (5a) and (5b). The amplitude decays from max value to zero at the end of the channel due to visco-thermal boundary layer effects. It is evident from the figure that the solutions from the boundary layer coupling formulation agree well with the reference solution.

5. CONCLUSIONS

In this work, we present a new boundary layer coupling formulation that allows us to model the acoustic-structural interaction with viscous and thermal boundary layer effects at the interface. The coupling condition is derived by extending the boundary condition approach by including the boundary motion. The coupling condition is implemented and tested using the finite element method by



(a) Pressure



(b) Velocity

Figure 5: Acoustic quantities along the center line of the channel.

comparing the results with the full linearized compressible flow equations. One of the test cases is presented and it is evident that the model works as expected and the results agree well with the reference solution. The proposed formulation reduces the number of unknowns and also does not require resolution of the boundary layers by the FE mesh which dramatically reduces the computational cost.

6. REFERENCES

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