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The Impact of COVID-19 on MacroeconomicVariables

A Cross-Country comparison using a Heterogeneous Agent Economy Model incorporating Idiosyncratic Risk and SIR Components

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Abstract

This thesis tries to better understand the dynamics of how macroeconomic models, combined with epidemiological models, describe the impact of COVID-19 on economies. Therefore, an epidemiological model (discretized baseline SIR) is integrated in an incomplete markets model, using uncertain labour efficiency, endogenous labour and non-linear taxation of capital and working income. The goal of this setup is to calibrate two model economies. One representing hard, immediate and multiple lockdowns as well as a social distancing policy embedded in a progressive tax system with minimal wages (Austria). The other representing mild, yet longer lasting multiple lockdowns, as well as a mild social distancing policy embedded in a less progressive tax system using minimal wages (United States of America). Based on the model derived and calibrated *value of a statistical life* as well as the *consumption equivalent variation*, a comparison of the two model economies *transitional dynamics* is conducted. These comparisons are based on exchanging lockdown policies of both economies. Consequences on **allocation** (aggregation of cross-sectional distributions) of the impact of COVID-19 of output, consumption, capital, labour, and investment during the transitional dynamics between the pre- and post-pandemic steady state are displayed. With respect to **welfare** households in the US model economy are willing to give up to 3.9% of consumption on average, (3.7% minimum, 4.1% maximum) to be allowed to adopt the *AUT policy*, therefore preferring the AUT pandemic policies. This is measured by the *consumption equivalent variation* [Lucas, 1992]. In comparison to a *no lockdown* policy, both the AUT policy (multiple, temporary, strict lock-downs with social distancing) and the USA policy (multiple, longer lasting, less-strict lock-downs and social distancing) are preferred by the households. Therefore, are willing to give up 7.7% in the AUT. vs no-policy case, and 4.1% in the USA. vs no-policy case, of household consumption to keep the nation states policies. Results for AUT model economies are similar, yet inverted, with respect to nation state policies. Therefore preferring its own policy.

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CHAPTER 1

Introduction

During the time of a pandemic, politicians company officers and in general decision makers around the globe are urged to make the right choices. Especially in case, they might end up being hard and unpopular ones, expert opinions are supposed to support the process in finding adequate solutions to the tasks ahead. Therefore, this thesis tries to better understand the dynamics of how macroeconomic models, combined with epidemiological models, describe the impact of COVID-19 on the model economies.

This document consists of five chapters ranging from *Introduction*, *Literature Review*, *The Model*, *Experiments, and Results* until *Concluding Remarks* Chapter 1 explains the motivation behind the thesis as well as the used methodology the on an abstract level. Chapter 2 consists of an overview of scientific articles which provide insights on how to tackle the tasks at hand. Chapter 3 presents the inner workings of the model used in this thesis to answer the research questions, and chapter 4 uses the defined model to conduct experiments in an artificial model environment. Finally, chapter 5 outlines general remarks about the thesis.

1.1 Problem Statement

Two countries, the United States of America (USA) and Austria (AUT), with contrary initial approaches in dealing with the COVID-19 crisis are modelled with respect to their impact on the cross-sectional distribution of consumption, wealth, and labour. Austria chose to respond with a hard and early lockdown as opposed to the USA which, in general, responded in a loosely and lockdown opposing manner. In addition to the COVID-19 policy responses, structural as well as fiscal differences apart from the pandemic are present in the chosen countries.

This thesis chose to focus on differences in asset and working income taxation, consumption taxation, as well as minimum wages as a proxy for the vast amount of social benefit programs in the chosen countries. AUT overall offers higher monetary support with respect to minimum income, temporary assistance and welfare programmes in comparison to the USA as seen in [OECD, 2019]. In addition, AUT, and in general all member states of the EU use a more progressive tax system compared to the USA who's functional form is estimated by [García-Miralles et al., 2019] and [Guner et al., 2013]. Note that [García-Miralles et al., 2019] estimates the functional form of Spain, which serves as a proxy for the AUT tax estimations.

How these differences impact the decision-making process of the individuals with respect to consumption, wealth, and labour is not obvious upfront. To be able to reason about possible scenarios, mathematical models are used to analyse the impact of COVID-19 upon the USA and AUT. In the case of models generated wealth distributions it seems hard to draw quantitative conclusions using the representative agent assumption, meaning all agents share the same state, therefore derive the same optimality conditions. The same reasoning applies to the modelling of the idiosyncratic nature of a pandemic, furthermore increasing the need to allow for some sort of *heterogeneous agents* within the model.

To deal with the vast complexity of truly heterogeneous agents, meaning all agents (households, nature, government, ...) share different models, different states within the model and different beliefs about the other agents models, as well as states within them. Because of the tractability of solutions, simplifications are needed to reduce the state variable space to a reasonable amount, without discarding too much of its possible explanatory power. A term coined by Thomas Sargent "communism of models" [Sargent, 2017], which makes the case for sharing the *same model* among all agents only differing in certain states within the model, as well as share the same beliefs about the other agents states. This seems to be useful to reduce the complexity at hand. Therefore, the main workhorse model to quantitatively evaluate policies in question ("lockdown", "social distancing") is therefore an adapted version (taxes, labour efficiency units, endogenous labour supply, state of infection or its recovery, minimum wages) of the *standard incomplete markets model* [Aiyagari, 1994] with elements from [Krueger et al., 2020] and [Hur, 2021].

1.2 Research Questions

The goal of this thesis is to provide quantitative insight of the impact of the COVID-19 Pandemic with respect to allocation of aggregated variables, namely of output Y, capital K, labour N, investment I and consumption C. The underlying cross-sectional distributions of assets, labour, and consumption are a basis to derive the mentioned allocation variables. With respect to welfare of households within the economies, the consumption equivalent variation [Lucas, 1992] g is used. To be more precise about the expected results, the research questions this thesis tries to answer are stated in the following enumeration.

1. To what extent does COVID-19 influence the aggregation of economic variables (Y, K, N, I, C) and welfare (g) of Austrian households as well as households of the United States of America?

2. To what extent do policies like "social distancing" and "lockdown" influence the aggregation and welfare in question?

1.3 Results

This section provides a coarse overview of the results displayed in more detail in chapter 4. The impact of the pandemic as well as pandemic policies of **aggregate** output Y, aggregate consumption C. aggregate capital K, aggregate labour N and aggregate net investments I are summarized as follows.

Regarding **high income households** and the **absence of a pandemic policy**, output is reduced below steady state during the period of the pandemic. In addition, labour supply drops abruptly and rises again during the course of the pandemic to again reach steady state levels weeks before the end of the pandemic, due to vaccination. A preference to work less and consume more can be observed for high income households. The resulting increase in consumption is financed using the household's savings and therefore reducing the model economies capital stock. Weeks before the end of the pandemic, capital stock increases again to reach pre-pandemic steady state levels. During the **presence of pandemic policies**, a similar behaviour of households is present, yet steady state deviations of labour supply are dampened, possibly due to a reduction of rates of infection. Output shows sharp declines before lockdowns due to the lagged behaviour of savings and consumption. Consumption during lockdowns is reduced due to mandatory reduction in consumption and therefore increase in savings. This leads to above steady state levels of consumption after lockdown periods and below levels during lockdowns. In general, the AUT policies lead to a higher increase in volatility than the USA policies in both model economies with respect to high income households.

Considering **low income households** and the **absence of a pandemic policy**, output is reduced below steady state during the period of the pandemic. In addition, labour supply increases slightly and drops weeks before the pandemic ends in the USA model economy. In the AUT model economy, no trend is present, possibly due to higher minimum income levels. Consumption of low income households is slightly below steady state and capital supply is increased slightly, to cope with the increase in uncertainty due to the pandemic shock. An inverse behaviour to high income households is visible, possibly due to the ability of high income households to make use of the accumulated assets to smooth utility governing factors as consumption and leisure. During the **presence of pandemic policies**, a similar behaviour of households is present, yet the level of capital supply of low income households is increased, labour supply during lockdowns is increased with respect to the steady state and consumption is in general above the steady state level, even during lockdown periods.

With respect to **welfare**, in general, households of both model economies (USA and AUT) prefer lockdown policies over no-lockdown policies. Households of the AUT model economy prefer policies compared to no-lockdown policies in a greater extent than households of the US model economy. Comparing *temporary* and *strict lockdown* policies, abbreviated in this thesis as the *AUT policy*, to *less temporary* and *less strict* policies, abbreviated in this thesis as the *US policy*, with each other, agents of both model economies prefer the AUT policy approach. These welfare consequences are measured by the *consumption equivalent variation* as defined in [Lucas, 1992] and are displayed in the following paragraph in a quantified manner.

Households in the **US model economy** are willing to give up to 3.8% of consumption on average, 3.9% of consumption on average, (3.7% minimum, 4.1% maximum) to be allowed to adopt the *AUT policy*, therefore preferring the AUT pandemic policies. In comparison to a *no lockdown* policy, both the AUT policy and the USA policy are preferred by the households. Therefore, are willing to give up 7.7% in the AUT. vs no-policy case, and 4.1% in the USA. vs no-policy case, of household consumption to keep the nation states policies. Results for **AUT model economies** are similar, yet inverted as well as higher, with respect to nation state policies. Therefore, AUT households are willing to give up to 19.1% of consumption on average, (18.5% minimum, 19.6% maximum) to be allowed to adapt the *AUT policy*, therefore preferring the AUT pandemic policies, in the AUT vs. USA policy case. In comparison with no policy and the AUT policy 35.8% of consumption will be traded for staying at the AUT pandemic policy. In case of the USA policy vs. no policy 20.63% of consumption may be forfeit, again asserting a strong preference of keeping the AUT pandemic policy. Further elaborations on results are displayed in chapter 4.

The methodological setup of the thesis will be explained in the next section. It explains what research method guides this thesis, as well as what kind of mathematical models and solution methods are incorporated to answer the stated research questions above.

1.4 Methodology

To answer the previously stated research questions, mathematical models are incorporated to reason about the policy choices in question. To better understand on an *abstract level* what activities and objects are needed to solve the task at hand, [Ortlieb, 2001] describes a control loop like structure which provides guidance in how to conduct research in the area of applied mathematics. It is worth noting that the author does **not** intend this process to represent an "algorithm" which provides a sufficient condition for successful studies, as can be seen in [Ortlieb, 2001]. It is rather intended as a guidance to think about mathematics and its relation to reality. Nevertheless, this modelling process consists of the following activities:

1. Modelling of Phenomenon in Question
2. Model Development (solving/simulating)
3. Interpretation of Results
4. Validation against Real Word Phenomenon

To describe this method in a more concise manner, an *UML Activity Diagram* is used in an attempt to capture the essence of [Ortlieb, 2001]. The *object flows* (grey) as well as *control flows* (white) are displayed in figure 1.1.

These activities result in objects which are passed along to the next activities to eventually create a model which represents the observed phenomena in question. With respect to the field of mathematical macroeconomics, a *model* may be characterized as of a mathematical (solving systems of equations), statistical (estimation of characteristics and testing hypothesis) or simulative (simple behavioural rules leading to complex interactions) nature. The main focus of the models in questions will be of *mathematical* nature, since purely *statistical models* tend to be looking backward in time. Meaning, a lack of data to derive or train estimators on for possible policy experiments is most likely present (how does someone know if a Black Swan will ever exist?). *(Agent Based) Simulation* on the other hand may be incorporated to solve the research questions at hand, yet the formalism of "stochastic control theory" like, utility based "intertemporal optimization" problems, formulated as *bellman equations* and solved using *dynamic programming* provides a more solid and guided approach of mathematical modelling of the research questions at hand. Of course, a clear separation of such models is not possible and ideas and formalism's of are used across the given characterization.

At first, the object **"Real World Phenomenon"** needs to be described by **"Modelling"** activity and transformed into an *"Mathematical Problem"*. To successfully do, so the resulting model should try to describe an economy with *incomplete markets* (agents cannot completely insure against loss of labour income) over a period of pandemic shocks (idiosyncratic) as well as capture some sort of heterogeneity of the model agent's

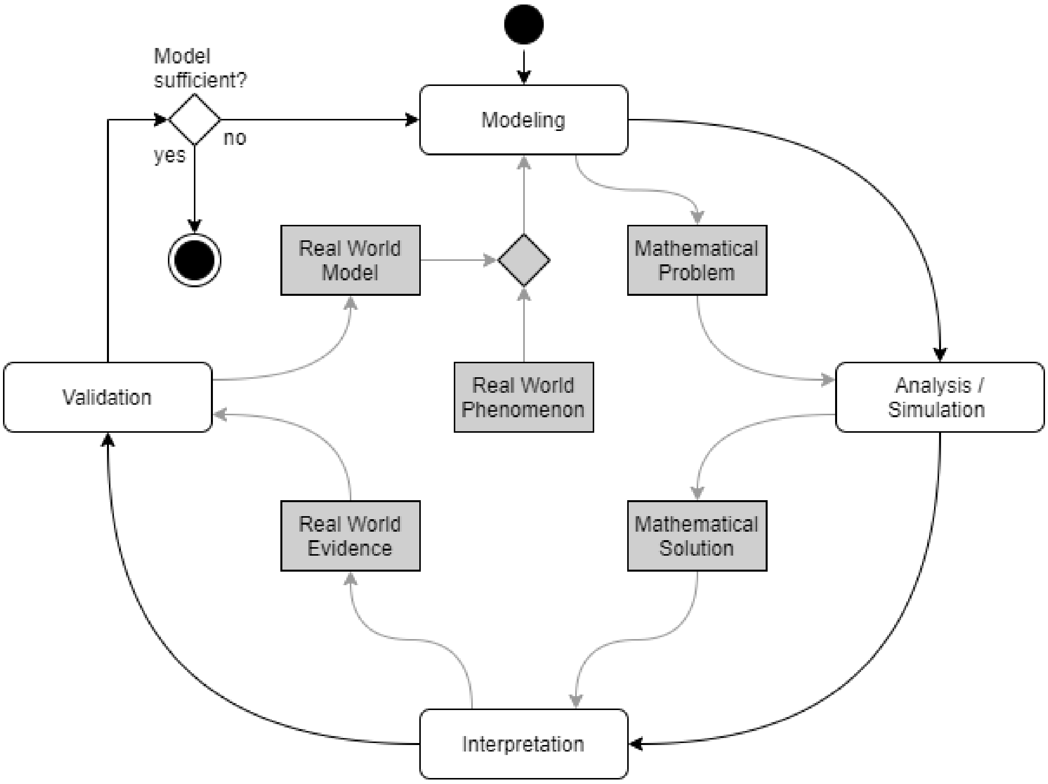

Figure 1.1: Activity diagram inspired from [Ortlieb, 2001] describing the states of mathematical modelling

decision-making, resulting in a *cross-sectional distribution* of *macroeconomic variables* (assets, labour, consumption, output). In addition to allow for a comparison of different Nations estimates of *structural parameters* (Consumption-Leisure Preferences, capital depreciation, discount factors, . . .), varying degrees of a *tax scheme's progressiveness*, as well as the incorporation of *minimal wages* needs to be accounted for. The reasons why these characteristics might be help-full will be explained in section 3. To allow for these model characteristics, a *standard incomplete markets model* [Aiyagari, 1994] is combined with an *epidemiological model (SIR)* using elements from [Krueger et al., 2020] and [Hur, 2021]. Furthermore, the model is adapted to analyse the transitional dynamics between pre- and post-pandemic steady states. The complete mathematical description of the model can be seen in section 3 in a more verbose manner.

Given the object **"Mathematical Problem"** one has to solve it to provide possible insights about the task at hand. How such a model is solved during the activity **"Analysis / Simulation"**. The problem at hand may be divided into coarse sub-problems and further solved in an incremental manner. First, a baseline model is solved for where reliable *testing data* is present (closed form solutions). After core algorithms as *Value Function Iteration* and *Monte-Carlo Simulation*, seem to present reasonable results for the defined *intertemporal, optimization problem* embedded in an *recursive, stationary equilibrium*, further model enhancements are added (labour efficiency units, taxation, ...) and afterwards *calibrated.* This results in the solution of the *stationary problem.* Afterwards the *transition dynamics* need to be solved for using a *Shooting Algorithm* (effects of the pandemic, lock-downs, social distancing, ...). All present algorithms and solution methods are defined and explained in chapter 3

The resulting object **"Mathematical Solution"** now stands as an alternative reality to be explored. During the activity *"Interpretation"* the solution needs to be understood and tested based on qualitative as well as quantitative criteria. In chapter 3 the resulting policy functions (optimal allocation of resources given the current state — what is my expected income? how probable is an infection? what are the optimal policies of other agents?) as well as resulting empirical cross-sectional distributions are checked for qualitative inconsistencies. Quantitative "sanity checks" are applied to the model output to increase the confidence in the model during normal times or past, observed shocks. These include matching targeted and non-targeted moments of real world distribution or indices (Gini coefficient of income and wealth, wealth to economic output ratio, rate of infection, value of a statistical life) against the model generated output, as well as model-specific "stability" results (the choice of certain hyperparameters are not supposed to impact results — e.g., grid size of variables).

The insights generated by the activity **"Interpretation"** results in the object **"Real World Evidence"** which is supposed to encapsulate consequences of the derived model. These consequences should be reflected against real world policies, the applicability of the model consequences given its practicality. As seen in chapter 2 and 4 economic intuition as well as comparison to other literature is conducted and differences presented.

CHAPTER 2

Literature Review

The chosen papers represent macroeconomic papers who already use epidemiological models as well as computational strategies to solve the occurring modelling. To do so, *macroeconomic models* need to include the notion of an *epidemiological model* as well as its impact on the modelled agents. The authors of the following papers chose to use a canonical epidemiological model, similar to [Kermack et al., 1927], to describe the dynamics of an epidemic spreading across individuals. This chapter will provide insight in chosen papers and its provided ideas and results used in this thesis.

By combining a SIR model with a macroeconomic model [Eichenbaum et al., 2020] established a model to compare policies with respect to vaccination, containment, and treatment of households and the resulting consumption and leisure decisions. Worth noting is that no capital, therefore no savings decisions and net investments, are present in [Eichenbaum et al., 2020]. These results seem insightful in the case the pandemic is studied on an aggregate level, described by representative households. In case one wishes to study the impact on different groups of people, one needs to advance the notions established in [Eichenbaum et al., 2020]. Therefore, [Krueger et al., 2020] and [Hur, 2021] provide an increasing complexity in modelling of the interaction between the pandemic and economic agents. A first increase in complexity is the addition of multiple production sectors offering different kinds of goods (depending on the goods consumption an in- or decrease in the endogenous probability of infection) for the households to consume [Krueger et al., 2020]. Again, as in [Eichenbaum et al., 2020] no capital is present, as the setup of the model in [Eichenbaum et al., 2020] is a basis for [Krueger et al., 2020]. The introduction of multiple production sections lead to a reduction of economic as well as pandemic suffering in comparison to [Eichenbaum et al., 2020]. Both papers use the notion of a *competitive general equilibrium* without production and social planners (social planner results are not discussed in this review since they do not affect the current thesis). A difference in modelling of the considered papers, is its approach to model the ability to respond to pandemic shocks. This includes consumption, labour, and savings decision as seen in [Hur, 2021] in a general equilibrium (partial during the pandemic shock), as opposed to [Eichenbaum et al., 2020] and [Krueger et al., 2020] which result in a model where households determine their labour supply and level of consumption, yet are not able to save assets for future consumption smoothing.

Another approach can be observed in [Hur, 2021] which tries to model heterogeneous income and savings decisions of agents, as well as endogenous labour in an incomplete market's scenario similar to [Aiyagari, 1994], yet enriches the model using an overlapping generation (OLG) structure. The benchmark model in [Eichenbaum et al., 2020] builds on a representative household not including idiosyncratic factors as well as an OLG structure. The absence of these factors might not change *qualitative* insight as seen in the main findings of [Hur, 2021], yet have impacts on the *quantitative* estimates of the models. In general, the introduced heterogeneity of [Hur, 2021] seems necessary to answer effects which are built on distributional consequences of the pandemic. Therefore, the approach of this thesis, as in [Hur, 2021], utilizes the notion of incomplete markets as well as heterogeneous income and savings decision as in [Aiyagari, 1994] with the absence of an OLG structure, as well as an explicit modelling of a health care system. The concept of consumption equivalent variations are considered (between different policies AUT vs. USA) by this thesis, and displayed in chapter 4.

This increase in model complexity may be contributed to the given problems the papers wish to take on. In [Eichenbaum et al., 2020] the construction of a model combining macroeconomic models with epidemiological ones to compare it to an only epidemiological one. So is it possible that households behave in a way the pandemic is controlled merely by their decision-making. [Eichenbaum et al., 2020] further expands on topics as *vaccination, medical preparedness, containment, and treatment* within the present macroeconomic model. These represent adjustments of the model, introducing a healthcare system and its involvement in the macroeconomic model, as well as uncertainty about when vaccines may be produced and distributed. Considering the paper of [Krueger et al., 2020], it tries to answer if a "Swedish" solution of the COVID-19 pandemic is feasible with respect to economic reasoning. "Swedish" is to be interpreted as a "no-lockdown" strategy. The resulting endogenous shift of consumption to different, more safe consumption possibilities will absorb the shock if such goods are available in the economy. The model used by [Krueger et al., 2020] is based and extended on the model of [Eichenbaum et al., 2020] and extended using the heterogeneous consumption and production sectors. It allows for a significant disruption in the dynamics of the pandemic, and an almost "business as usual" behaviour with respect to allocation. This however requires flexible markets providing alternatives to existing services and goods. In the paper of [Hur, 2021] the main findings represent the existence of externalities, which give rise to welfare-improving government intervention. Young and low income households do possess different optimally conditions with respect to consumption and leisure as opposed to older and rich households, affecting the dynamics of the pandemic. Since young people seem to be more resilient towards the pandemic than older people, as well as low income and wealth households are required to work to survive in comparison to rich and wealthy households (especially in an incomplete

Comparison - Results				
Result	[Eichenbaum]	[Krueger]	[Hur]	Present Thesis
aggregation of output	(same as consumption)	(same as consumption)	reduction of 17%	reduction of 3.09 % - using a "no policy" policy of the USA model economy
aggregation of capital	not present	not present	not recorded	reduction of 15.92 % - no policy experiment-USA
aggregation of consumption	reduction of 10%	reduction of 83%	reduced by at most 60% (82%) for old and low (high) wealth, 35% (62%) for middle and low (high) wealth and 10% (20%) for young and low (high) wealth households	Reduction of 7.0% - no policy experiment
aggregation of labour	reduction of 10%	reduction of 1.6%	reduction of 40 %	reduction of 5.61% - no policy—USA
welfare consequences	not present	not present	no lockdown -8.0 %,	reduction of no lockdown—USA -4.10 %

Table 2.1: A comparison results in literature and the present thesis with respect to allocation and welfare.

market setting), economic activities tend to be not socially optimal with respect to the spread of the pandemic. This finding is of most interest for the present thesis since it is directly related to the research questions presented in 1.2. In addition, an optimal policy frontier with respect to weekly subsidies for households is explored, resulting in a simultaneous improvement of health and economic outcomes.

A summary of used formalisms of the presented papers is present in the appendix at section 5. For a more detailed listing of the results, see 5 at the end of each subsection. Furthermore, in table 2.1 comparable results of this thesis and the chosen papers are presented. These range from allocation (C, K, N) and for chosen papers welfare consequences (g as seen in equation 4.6).

Worth noting is that the high reduction of labour in the model of [Hur, 2021] is based on the enforcement of limit on outside labour. A reason for this may be that [Hur, 2021] allows for a consumption savings decision of households, [Eichenbaum et al., 2020] and [Krueger et al., 2020] do not allow for savings and investments. This is due to the different problems they intend to tackle, therefore possible enhancements are not deemed necessary by the authors, as a slim model is often more desirable. To conclude the chapter "Literature Review" 2, in table 2.2 a general overview of models is presented as well as a general comparison of the model type, solution strategies and questions answered by the selected papers.

Comparison - Overview				
Topic	[Eichenbaum]	[Krueger]	[Hur]	Present Thesis
model components	representative household, SIR, endogen. labour	multiple industry sectors, SIR, endogen. labour	incomplete markets, heterogeneous income and asset allocation, OLG, SIR, endogen. labour	incomplete markets, heterogeneous income and asset allocation, SIR, endogen. labour, non-linear taxation
equilibrium	general, competitive	general, competitive	general, competitive, recursive, stationary (during transition path — partial instead of general)	general, competitive, recursive, stationary (also general during transition path)
welfare consequences	no	no	yes	yes
aggregation of Econ. vars.	yes	yes	yes	yes
goal of paper/thesis	Construct a model combining economic models with epidemiological models	shift towards flexible (indoor) consumption and multiple sectors of substitutes (outdoor) opposed to lockdowns	find optimal policy frontier with respect to subsidies paid to households instead of lockdowns	cross-country comparison to evaluate lockdown policies of nation states including social distancing
solution strategies	closed form solution	numeric solution (Dynare: integral equations)	stat. simulation, numeric solution (VFI, backward induction, shooting algorithm)	stat. simulation, numeric solution (VFI, backward induction, shooting algorithm)
consumption affecting pandemic	aggregate	aggregate with sectors	aggregate and individual, inside and outside	aggregate
labour affecting pandemic	aggregate	no effect	aggregate and individual, inside and outside	aggregate
lockdown policies	tax on consumption	not present	restriction on outside labour	restriction on consumption

Table 2.2: A general comparison of model components, equilibrium, aggregation, welfare, solution strategies and questions answered by the selected papers [Hur, 2021], [Krueger et al., 2020] and [Eichenbaum et al., 2020].

CHAPTER 3

The Model

Given the modelling approaches presented in chapter 2 and research questions to be answered presented in chapter 1 the built model to achieve this is presented in this chapter. The same structure as in chapter 2 is used, therefore the following sections include the Macroeconomic Environment, Pandemic, Equilibrium Characterization, Calibration, etc. The results of the thesis are presented in a separate chapter 4.

To get a coarse overview of the ideas behind the modelling of the economies, the following enumeration tries to summarize some key attributes of the used model:

1. **heterogeneous agents in an incomplete market setting [Aiyagari, 1994]**
   - using labour efficiency units influencing the amount of working income as a stochastic shock and
   - state of infection $\{S, I, R\}$ of a household — both as part of its state space.
2. **rational expectations — communism of models [Sargent, 2017]**
   - Households share the same model with every other possible agent within the model, differing in its state (poor or rich, infected or not, ...)
   - therefore each household knows all policy functions of all other households (same state implies same decision-making),
   - and furthermore allows forecasting next period variables using the derived "laws of motions".
3. **recursive stationary equilibria and transitional dynamics**
   - First the non-pandemic economy is described by a stationary distribution allowing for a notion of rest,

- after the outbreak of the pandemic the distribution is no longer stationary and is adapting with respect to the dynamics of the pandemic,
- at the end, after herd immunity or vaccination programs are considered, the economy returns to its original stationary distribution.

Furthermore, the approach of this thesis draws as in [Hur, 2021] on incomplete markets as well as heterogeneous income and savings decision as in [Aiyagari, 1994] with the absence of an OLG structure. The model includes the utility of leisure as well as consumption and allows for endogenous labour. This basis is enhanced using differences in progressiveness of taxation to represent a more accurate difference between the model representing USA and the model representing Austria. To achieve this, non-linear average taxation functions with respect to working- and asset income are defined, based on estimates by [Guner et al., 2013] and [García-Miralles et al., 2019]. Since Austrian estimates could not be found in the desired form in the literature, estimates originating from Spain are used, as seen in [García-Miralles et al., 2019]. The connection of the epidemiological model and the macroeconomic one is inspired by [Eichenbaum et al., 2020] and adapted in a similar fashion as seen in [Krueger et al., 2020]. Differing in losing the value of a statistical life (VSL) in case an infection results in death. Computational strategies achieving optimality with respect to the defined household problem are Value Function Iteration with Howard's policy improvement [Christiano, 1990] and linear interpolation. Parts of the solution are derived by closed form solutions (optimal trade-off between l and c derived from FOCs) and due to the non-linear taxation root finding (bisection) is used to solve for the resulting system of equations. These concepts will be elaborated in the following sections.

3.1 The Macroeconomic Environment

This section defines the macroeconomic model used within this thesis. A continuum of households is present in an incomplete market setting, therefore resulting in "fortune" and "less fortune" households with respect to its working income. Therefore, the overall savings and consumption decisions are differing based on each household state.

3.1.1 Households

Such a state is an element of the state space and defined as $x \in X \subseteq A \times E \times R \times W$ in the non-pandemic setting, where A describe all possible never binding amounts of assets a household may possess $a \in A := [\underline{a}, \overline{a}]$ $\underline{a} = 0, \underline{a} < \overline{a} \leq \infty$ and E all possible outcomes of labour efficiency $e \in E := [0, \infty)$ (E is based on an AR(1) process in discretized form as seen in equation (3.5) — therefore $[0, \infty)$ is just a theoretical set of values, $E = \{e_0, ..., e_m\}$, $m = 7$ being its discretized counterpart), $R := (0, \infty)$ representing all possible values of the real interest rate, and $W := (0, \infty)$ representing all possible values of the real wage rate. The overall state $x \in A \times E \times R \times W$ is therefore defined by the households *state variables* a, e, r and w. State variables are influenced by a household's choices, offered at each point in time t. These choices are represented as *controls* or *control variables* (consumption c, leisure l, net-savings $a' - a$) and allow households to forge a plan (a policy function) on how much to adjust its controls based on its state, resulting in states across time, which are optimal with respect to its stochastic value functions $v(x) = u(c, l) + \mathbb{E}[v(x')]$ (uncertainty is present). Variables with a prime attached represent next period values.

During the absence of the pandemic, the households tries to solve the following problem:

$$\textbf{Given} \quad \text{HH} := v(a, e, r, w) = max_{a' \geq \underline{a}, c \geq 0, l \geq 0} \left[\; u(c, l) + \bar{u} + \beta \, \mathbb{E}[v(a', e', r', w')] \; \right] \tag{3.1}$$

$$\textbf{such that} \quad \text{BC} := (1 - \tau^c)\, c + a' = [1 - \tau^{a,y}(y)]\; y - a \tag{3.2}$$

$$\textbf{where} \quad y := max(we, we_{min})(1 - l) + ra \tag{3.3}$$

$$we_{min} = \hat{w}e^{\text{usa,aut}}_{min} \cdot 1_{\{we < w_{min}\}} \tag{3.4}$$

$$e := ln(\tilde{e}') = \rho\, ln(\tilde{e}) + \varepsilon, \quad \varepsilon \sim \mathcal{N}(0, \sigma^2) \tag{3.5}$$

$$u(c, l) = \frac{(c^{\phi} l^{1-\phi})^{1-\gamma} - 1}{1 - \gamma} \tag{3.6}$$

$$\tau^{a,y}(y) = n + o\, y^p \; n, o, p \in \mathbb{R} \tag{3.7}$$

In addition, a stationary recursive equilibrium defined in section 3.2 is applied in equation (3.1). There $v(x)$ represents the value function, $u(c, l)$ the period utility function, β the discount factor, x being an element of the state space X. Because of the present stationarity, in short, it allows dropping the factor prices r and w from the household problem since $r := r'$ and $w := w'$ at steady state, allowing one to guess r and w respectively such that the equilibrium conditions hold. It furthermore implies that R and W are not present in the state space, leading to $X = A \times E$ in equilibrium.

To further expand on equation (3.1) one can use the budget constraint presented in equation (3.2) and derive the FOCs using the infinite series notation of the household's problem conditions using the Lagrangian multiplier approach. These conditions give insight into the relation of c and l and allow a procedure to be defined, determining FOCs $\rightsquigarrow l(x)$ and afterwards FOCs $\rightsquigarrow c(x, l)$ where $x \in X \subseteq A \times E$ defines an element of the state space. This procedure will be explained in more detail in section 3.4.

In equation (3.2) the symbol $\tau^{a,y}$ represents a non-linear taxation function of working as well as capital income and its partial derivatives with respect to l and c as $\tau^{a,y}_l(y)$ and $\tau^{a,y}_c(y)$. Whereas $\tau^c \in [0, 1]$ represents a simple tax on consumption. A minimum wage is defined as $we_{min} = \hat{w}e^{\text{usa,aut}}_{min} \cdot 1_{\{we < w_{min}\}}$ in case a household meet the minimum requirements based on current working income. The constant $\hat{w}e^{\text{usa,aut}}_{min}$ represents a rough estimate for the minimum wages of the respective nation state.

In equation (3.3) y represents the household's period income. The chosen approach of modelling working income $w_t\, e_t\, (1 - l_t)$ is the usage of *labour efficiency units* e. In equation

(3.5) the modelling of labour efficiency units as a log transformed AR(1) process is present, with persistence ρ and ε being a standard normal variable. The log transformation is used as a trick to impose non-negative income, as well as to arrive at a right skewed income distribution.

In equation (3.6) ϕ represent the tradeoff between consumption and leisure as a consumption labour share, and γ being the degree of relative risk aversion. The functional form is chosen to represent a utility function with constant relative risk aversion.

In equation (3.7) $\tau^{a,y}$ represents the functional form of the working as well as asset income tax, where estimated parameters n, o, p are estimated by [Glover et al., 2020] and [García-Miralles et al., 2019].

3.1.2 Firms

The modelling of **firms** is based on perfectly competitive markets, resulting in a price-taking behaviour of firms, and therefore a static optimization problem as seen in equation (3.8). The optimality conditions for the factor prices r and w with respect to the firm's profit Π are present in the same set of equations (3.8). The firm's factor inputs are defined as aggregated labour N and aggregated capital K, combined in a Cobb-Douglas production function F, where δ represents the capital's rate of depreciation and α the capital income share.

$$\begin{aligned} F(K,N) &= K^{\alpha}N^{1-\alpha} \\ \Pi(K,N,r,w) &= F(K,N) - (wN + (r+\delta)K) \\ \Pi_K(K,N,r,w) &= F_K(K,N) - r + \delta \stackrel{!}{=} 0 \implies r = F_K(K,N) + \delta \\ \Pi_N(K,N,r,w) &= F_N(K,N) - w \stackrel{!}{=} 0 \implies w = F_N(K,N) \end{aligned} \tag{3.8}$$

Therefore, by using a Cobb-Douglas production function $F(K,N)$ and the firm's profit function Π allows one to derive a relationship between w and r. This will be useful later in section 3.4 during a procedure where guessing r and w is needed to arrive at equilibrium conditions. Therefore, by using equation (3.9) guessing only r and using $w(r)$ is sufficient.

$$\begin{aligned} \left[\frac{\delta+r}{\alpha}\right]^{\frac{1}{\alpha-1}} = \frac{K}{N} \equiv \frac{K}{N} &= \left[\frac{w}{1-\alpha}\right]^{\frac{1}{\alpha}} \\ w = \left[\frac{\delta+r}{\alpha}\right]^{\frac{\alpha}{\alpha-1}} &(1-\alpha) \end{aligned} \tag{3.9}$$

3.2 Equilibrium Characterization

In this model we wish to aggregate the demand and supply for capital $K^d \stackrel{!}{=} K^s$ and labour $N^d \stackrel{!}{=} N^d$ as well as use the notion of a recursive stationary competitive equilibrium.

Therefore, this thesis' **equilibrium conditions** are defined as follows:

Given prices r, w and tax rates τ^c, $\tau^{a,y}$, aggregate capital K, aggregate labour N, a probability space $(X, \mathcal{B}(X), \lambda)$, where λ is a measure of households, X a state space and $\mathcal{B}(X)$ the Borel-sigma algebra, a value function $v(a, e)$, where $a'(a, e)$, $l(a, e)$ and $c(a, e)$ are policy functions (optimal decision rules), a stationary recursive competitive equilibrium is defined, such that:

1. For given prices and tax rates, the household's problem is solved by $v(a, e)$ and therefore defined by its policy functions $a'(a, e)$, $l(a, e)$, $y(a, e)$ and $c(a, e)$.
2. For given w, r, K and N satisfy the firm's first-order conditions (3.8);
3. Aggregate factor inputs are consistent with the stationary distribution.
$$\begin{aligned} N &= \int_{A\times E} (1-l)\, e\, d\lambda \\ K &= \int_{A\times E} a'(a,e)\, d\lambda \end{aligned} \tag{3.10}$$
4. The time-invariant measure λ defined on the probability space $(X, \mathcal{B}(X), \lambda)$, is determined by the transition function P as
$$\begin{aligned} \lambda(B) &= \int_{A\times E} P(a,e,B)\, d\lambda \ \text{ for all } B \in \mathcal{B} \\ P(B) &= \sum_{e\in B_e} \pi(e'|e) 1_{\{a'(a,e)\in B_a\}}. \end{aligned} \tag{3.11}$$
5. The government budget is balanced
$$G = \int_{A\times E} \tau^{a,y}(y(a,e)) \cdot y(a,e)\, d\lambda + \tau^c \int_{A\times E} c(a,e)\, d\lambda \tag{3.12}$$

Worth mentioning is that $\pi(e'|e)$ represents the conditional distribution of the labour efficiency units, modelled as discretized AR(1) process using Tauchen's method [Tauchen, 1986], and therefore resulting in a Markov Chain. The above definition allows now to have a notion of rest with respect to the resulting distributions. Single households are in constant change yet as a whole the distribution is not affected by a notion of change. As a minor note, the government income G is not utilized within the model as transfers. Nevertheless, the government is represented in this model as a pure spender, as public spendings in healthcare or infrastructure is not considered within this model.

3.3 The Epidemic

The **epidemic** in this thesis is modelled using the same dynamics as in [Krueger et al., 2020] as seen in equation (3.13). A more exhaustive explanation of the system can be seen in equation 5 at the appendix in chapter 5. The difference being, the rate of infection $\tau_t^{SIR}(\bar{c_{ss}}, \bar{l_{ss}})$ is dependent on aggregate steady state leisure $\bar{l_{ss}}$ as well as aggregate steady state consumption $\bar{c_{ss}}$. A difference in modelling to [Hur, 2021] is that no individual component of l and c is present, as well as no distinction between outside or inside consumption and leisure. Due to simplicity, no additional breakdown of c and l are considered within this thesis. Time t passes in the transitional dynamics in a discrete biweekly manner, over the time-span of 2 years, resulting in 52 points in time.

$$
\begin{aligned}
\tau_t^{sir} &= (\pi_a + \pi_s(\overline{c_{ss}}^{(1-\psi)}\overline{l_{ss}}^{\psi}))I_t \\
S_{t+1} &= S_t - T_t \\
I_{t+1} &= I_t + T_t - (\pi_r + \pi_d)I_t \\
R_{t+1} &= R_t + \pi_r I_t \\
D_{t+1} &= D_t + \pi_d I_t \\
Pop_{t+1} &= Pop_t - D_t
\end{aligned}
\tag{3.13}
$$

As seen in equation (3.13) a discretized SIR model where S represents the susceptible, I the infected, R the recovered, D the deceased part of households as well as π_r and π_d represent the exogenous probability to recover, and to fall victim to the disease. The rate of infection $\tau_t^{SIR}(\bar{c_{ss}}, \bar{l_{ss}})$ is a linear function of the number of infected households I_t. The interaction is modelled in a non-linear manner and is inspired by a Cobb-Douglas production function with increasing returns to scale. The idea is, given an increase (decrease) in consumption c and leisure l above (below) its steady state levels, the resulting impact on the rate of infection $\tau_t^{SIR}(\bar{c_{ss}}, \bar{l_{ss}})$ is higher (lower) than in the case of a linear combination of consumption c and leisure l. To compare l and, c the first moment of its distribution is used as an aggregate and afterwards related to its steady state aggregate value $\bar{l_{ss}} = \frac{l}{l_{ss}}$ and expressed as a fraction. The same rules apply to consumption $\bar{c_{ss}} = \frac{c}{c_{ss}}$.

The value function in equation (3.1) describes the household's decisions without the impact of the pandemic. Since the notion of a stationary recursive equilibrium as seen in section 3.2 is not sufficient to track the behaviour of households during the **transitional dynamics**. During the stationary recursive equilibrium, factor prices r and w are assumed to be constant. Therefore, allowing one to solve for a steady state with respect to the time-invariant measure $\lambda(B)$. As $r_t(K_t, N_t)$ and $w_t(K_t, N_t)$ are functions during the transitional dynamics, this complicates the problem significantly as households need to predict next period prices w' and r', including the prediction of all other households predictions of w' and r'. A procedure defined in section 3.4 allows one to solve for the needed law of motion with respect to the distribution of $\lambda(a', e, sir)$. The following equations define the value functions with respect to the pandemic. The household problem during the transitional dynamics is split into three sub problems to be solved. In case of the household's recovered state $sir = r$, the following problem applies. During this state, the pandemic is not influencing the household's decision-making in a direct manner.

The equilibrium conditions during the transitional dynamics are similar as in the steady-state definition, as seen in equation (3.2). An exception is the time invariance of the measure λ_t is no longer present, instead a law of motion with respect to λ_t is present, allowing households to transition from λ_t to λ_{t+1}. During the transition periods, the state space is increased to include the information about the state of infection, as well as the law of motion (3.16). Therefore, a the households assets, e the labour efficiency units, sir the state of infection, $\lambda_t(B)$ where $B \in \mathcal{B}$ the underlying probability measure defined on $(X, \mathcal{B}(X), \lambda_t)$, the state space $X = A \times E \times SIR$ ($A = [\underline{a}, \overline{a}], E = \{e_0, ..., e_m\}$, $m = 7$, $\underline{a} = 0, \underline{a} < \overline{a} \leq \infty$ being never binding amount of assets, SIR $= \{s, i, r\}$). The probability measure λ_t is not included in the state space directly. However, it is part of the state space since $r_t(K_t, N_t)$ and $w_t(K_t, N_t)$ are defined by $\lambda_t(x)$, r and w being considered state variables during the transition period. Therefore, information about the time-dependent measure $\lambda_t(x)$ according to its law of motion (3.16), enables agents to change their decision based on it.

$$\textbf{Given} \quad \text{HH} := v(a, e, \lambda, sir = r) = max_{a' \geq \underline{a}, c \geq 0, l \geq 0} \; [\; u(c, l) + \bar{u} + \beta \, \mathbb{E}[v(a', e', \lambda', sir' = r)] \;] \tag{3.14}$$

$$\textbf{such that} \quad (3.2),\ (3.13), \tag{3.15}$$

$$\text{LOM} := \lambda'(a', e', sir') = \sum_{E \times SIR} \pi(e'|e) \lambda(a^{-1}(a', e, sir), e) \tag{3.16}$$

$$\textbf{where} \quad (3.3),\ (3.4),\ (3.5),\ (3.6),\ (3.7) \tag{3.17}$$

Worth mentioning is the change of $y(sir) := max(we, we_{min})\,(1 - l)\,1_{\{sir \neq i\}} + ra$ in comparison to the stationary problem (3.1). In case of an infection, $sir = i$ the agent's working income is set to zero. This tries to model possible negative consequences of the infection as further financial burden. Furthermore, in case an infection arises, the household's next period may result in three possible outcomes. Either the household stays infected with probability $1 - \pi_r - \pi_d$ and uses the next period value function with respect to $v(a', e', \lambda, sir' = i)$, the household recovers with probability π_r and uses the next period value function with respect to $v(a', e', \lambda, sir' = r)$, or the household dies with probability π_d and uses the next period value function with respect to $v(a', e', \lambda, sir' = d)$. In the latter case, $v(a', e', \lambda, sir' = d)$ represents the negative value of the statistical life, resulting in a loss of possible future consumption in unity of utility.

$$\begin{aligned} \textbf{Given} \quad \text{HH} := v(a, e, \lambda, sir = i) = max_{a' \geq \underline{a}, c \geq 0, l \geq 0} \; [\; & u(c, l) + \bar{u} + \\ & \beta \mathbb{E}[v(a', e', \lambda', sir' = i)]\,(1 - \pi_r - \pi_d) + \\ & \beta \mathbb{E}[v(a', e', \lambda', sir' = r)]\,\pi_r \; - \beta \pi_d \cdot VSL \;] \end{aligned} \tag{3.18}$$

$$\textbf{such that} \quad (3.2),\ (3.13),\ (3.16) \tag{3.19}$$
$$\textbf{where} \quad y := ra \tag{3.20}$$
$$(3.4),\ (3.5),\ (3.6),\ (3.7) \tag{3.21}$$

The choice to set the household's income to $y = ar$ reflects the negative short term effects of the pandemic on households. Such a drastic loss is not based on any empirical evidence, yet is motivated as a modelling choice to introduce negative consequences for the households. In comparison, [Hur, 2021] considered a limitation on labour $1 - l$ during the duration of the pandemic resulting in reduced income.

In case the agent is in state susceptible, $sir = s$ the household's next period may result in two possible outcomes. Either the household stays susceptible with probability $1 - \tau^{sir}(c, l)$ and uses the next period value function with respect to, $v(a', e', \lambda, sir' = s)$ or the household becomes infected with probability $\tau^{sir}(c, l)$.

$$\begin{aligned} \text{HH} := v(a, e, \lambda, sir = s) = max_{a' \geq \underline{a}, c \geq 0, l \geq 0} \Big[\; & u(c, l) + \bar{u} + \beta \, \mathbb{E}[v(a', e', \lambda', sir' = s)] \, (1 - \tau^{sir}) \\ & + \beta \, \mathbb{E}[v(a', e', \lambda', sir' = i)] \, \tau^{sir} \; \Big] \end{aligned} \tag{3.22}$$

$$\textbf{such that} \quad (3.2),\ (3.13),\ (3.16) \tag{3.23}$$
$$\textbf{where} \quad (3.3),\ (3.4),\ (3.5),\ (3.6),\ (3.7) \tag{3.24}$$

This approach allows a step-wise solution of the transitional dynamics, meaning solving for $sir = r$ first allows one to solve for $sir = i$ and afterwards $sir = s$. Further details are present in the appendix 5 at algorithm .3.

The usage of the value of a statistical life (VSL) as discussed in the appendix in section 5 equation (17) and section 5 equation (29) is used in this thesis. It is defined as the marginal rate of substitution between an introduced mortality and consumption, yet without leisure. Worth noting is mortality is not modelled within the present model, apart from the pandemic, therefore to derive the VSL mortality is introduced (and removed shortly afterwards), as seen in equation (3.25).

$$\begin{aligned} \hat{v}(a, e) &= u(c^* + \Delta_c, \, l^*) + \bar{u} + (1 + \Delta_s)\beta \; v(a', e') \\ VSL(k, e) := MRS_j &= \left. \frac{\frac{\partial \hat{v}}{\partial \Delta_s}}{\frac{\partial \hat{v}}{\partial \Delta_c}} \right|_{\Delta_c = 0, \Delta_s = 0} = \frac{\beta v(a', e')}{c^{1 - \phi\gamma} l^{(1-\phi)(1-\gamma)}} \\ \int_X VSL(k, e) d\lambda(k, e, h) &\stackrel{!}{=} k \int_X c(k, e) d\lambda(k, e, h) \end{aligned} \tag{3.25}$$

In equation (3.25) $\hat{v}(a, e)$ is the steady state value function and $(c^*, \, l^*)$ steady state policy functions, adapted using Δ_c and Δ_s. Here, Δ_c describes a marginal increase (decrease) in consumption and Δ_s an increase (decrease) in appreciation of future utility, and is used as a proxy for an increase (decrease) of the probability of survival. The idea now is to find an expression of the marginal rate of substitution between introduced mortality and consumption, and remove Δ_s, Δ_r afterwards. This expression is defined as the value of a statistical life. The next step is to aggregate the VSL with respect to the first moment and equate it to the aggregated consumption in the economy. To balance biweekly consumption with yearly estimates of the literature, k is introduced. The parameter $\bar{u}$ can now be varied, such that the equations holds.

The described equations in this section need now to be solved to investigate the task at hand. Therefore, in the next section 3.4 used algorithms and computational strategies of this thesis are described and related to state-of-the-art procedures.

3.4 Computational Strategy and Closed Forms

To solve the system described in previous sections of chapter 3 algorithms are required to solve the household's problem, without the effects of the pandemic (3.1) and including it in the household's problem (3.14), (3.18), (3.22), since no closed form solutions exist in a meaningful way.

3.4.1 Value Function Iteration: Finding Fixed Points

The baseline algorithm to solve for a fixed point of equation (3.1) is the well established *value function iteration*. Using this approach allow for a relatively simple and versatile algorithm, yet lacking performance in comparison to a more state-of-the-art method, the endogenous grid-point method [Carroll, 2006]. Speeding up the value function iteration will be achieved using linear interpolation of grid points of, $v(a, e)$ as well as using Howard's policy adaptation [Christiano, 1990].

After a fixed-point for $v(a, e)$ is found the required policy functions are derived from the value function using the maximum value of the value function and map this to the current values of the relevant variables assets, leisure and consumption $a, c, l = arg\,max_{\,a,c,l}\, v(a, e)$ resulting in its policy functions.

3.4.2 Monte Carlo Simulation: Arriving at Equilibrium Conditions

Now to arrive at the distributions of a, c and l a Monte-Carlo simulation is used. It allows to approximate lebesgue-like integrals, $\mathbb{E}[g(Y)] = \int_X g(y)\, d\mu_Y(y)$, where Y is a random variable defined on a probability space $(\, X, \mathcal{B}(X), \mu_Y(Y \in B)\,)$, $g(x)$ being a *Borel-measurable* function form $\mathbb{R}$ to $\mathbb{R}$, $X \subseteq \mathbb{R}^n$ the state space (sample space), $\mathcal{B}$ the generated Borel sigma algebra and μ_Y being the distribution measure (λ being the Lebesgue measure and the completion of μ_Y), as seen in section 3.1, where the supply side of the equilibrium conditions are defined as such. Although the decisions of households are path dependent, the underlying discretized AR(1) process of e is known (as well as the probabilities of infection τ alter on) and allows for a path-wise evaluation of the household's decisions given its value function. Aggregating all simulated paths solves $\mathbb{E}[g] = \int_X g\, d\mu$. Therefore, given uniformly distributed random numbers and the discretized AR(1) process for labour efficiency units, e a procedure can be constructed such that different paths of the labour efficiency units e are present. Using a path of $(e_t)t \geq 0$ as a basis and an initial distribution of assets a_t, one can deduce a_{t+1} using the corresponding policy function $a'(a, e)$. The same applies to $c'(a, e)$ and $l'(a, e)$. The resulting values for each path at each time can be checked using the budged constraint as seen in equation (3.1) which has to hold for every household at each point in time. By aggregation with respect to the simulated households of the paths of $(a_t)t = t^*$, $(c)t = t^*$ and $(l)t = t^*$ at a point in time, t^* an "empirical" distribution is formed. In this case, the distribution of assets of households. Here t^* defines the point in time when distributions are at rest, as defined in equation (3.2). A Pseudo-Code like description of the Monte Carlo Simulation is present in the appendix at algorithm .2.

After the fixed point is found using the value function iteration including Howard's policy improvement and linear interpolation [Christiano, 1990] and the Monte Carlo Simulation, the equilibrium conditions as defined in equation (3.2) need to be considered. Specifically, the aggregation of supply and demand of $K^d \stackrel{!}{=} K^s$ and $N^d \stackrel{!}{=} N^s$ requires further clarification. As N is defined as $N := (1 - \bar{l}) \sum_{i=0}^{n_e} \pi^*(e_i) e_i$ no discrepancy between supply and demand is present. This is based on the modelling decision of using a discretized AR(1) process using Tauchen's method [Tauchen, 1986]. The usage of the stationary distribution π^* of the Markov Chain allows one to calculate the expected labour within the economy as defined above. Therefore, the expected labour supply $N^S := \int_{A \times E \times \{s,i,r\}} (1 - l_t)\, e\, d\lambda_t$ must equal N at all points in time and is therefore not to be simulated. This is noted in the following statement in a more concise manner. This holds for time-independent measure λ as well as during the transitional dynamics λ_t and the state space, including the pandemic $A \times E \times \{s, i, r\}$ or not $A \times E$.

$$N = (1 - \bar{l}) \sum_{i=0}^{n_e} \pi^*(e_i) e_i \implies N \equiv N^D \equiv N^S := \int_{A \times E \times \{s,i,r\}} (1 - l_t) e\, d\lambda_t \tag{3.26}$$

Since the absence of an exogenous term as in the case of labour N (AR(1) process) in the determination of capital, supply and demand need to be balanced using its factor price r. This in turn influences the policy function of leisure l and influences the level of, N leading to endogenous labour supply. The idea to solve for $K^d \stackrel{!}{=} K^s$ is to use $K^d = \left[\frac{w}{1-\alpha}\right]^{\frac{1}{\alpha}} N$ derived from the static firm problem (3.8) and including the first moment of the savings distribution as $K^s = \int_{A \times E \times \{s,i,r\}} a'(a, e, sir)\, d\lambda_t$. Again, this holds for time-independent measure λ as well as during the transitional dynamics λ_t and the state space, including the pandemic $A \times E \times \{s, i, r\}$ or not $A \times E$.

$$\left[\frac{w}{1-\alpha}\right]^{\frac{1}{\alpha}} N =: K^D \stackrel{!}{=} K^S := \int_{A \times E \times \{s,i,r\}} a'(a, e, sir) d\lambda_t \tag{3.27}$$

Overall, the procedure to solve for a fixed point in the household's problem (3.1) such that the equilibrium conditions (3.2) hold, as well as derive "empirical" distributions originating from a Monte Carlo method is described in the pseudocode (.1) present in the appendix 5. Worth mentioning is the missing variable r is used as a guess to equate supply and demand of capital. Therefore, a search procedure is required restricting r to values which further narrow down $K^s = K^d$. Therefore, a golden section search is used to eliminate nonsensical values of r. This is possible because r influences K^s in an increasing and K^d in a decreasing manner, allowing one to increase (decrease) r given $K^s < K^d$ ($K^s > K^d$) based on of the previous guess of r.

Now let us consider the relation between c and l needed in the household's problem. Since this relationship needs to be explicit to reflect the household's optimal choice of c and l with respect to the given optimization problem (3.1) the following equation

$$
\begin{gathered}
c(l, y) = \frac{\phi}{1-\phi}\, l \ \left[we\,[1 + \tau^{a,y}(y)] - \Delta y\ \tau_l^{a,y}(y)\right] \cdot (1 - \tau_c^{a,y}(y)) \\
BC := c(l, y) + a' - [1 - \tau^{a,y}(y)]\ y - a = 0 \\
\text{where} \\
y := we(1 - l) + ra \\
\Delta y := ra - we(1 + l) \\
\tau^{a,y}(y) = a + b\, y^p \ a, b, p \in \mathbb{R}
\end{gathered}
\tag{3.28}
$$

is the result of the series notation of the households problem and the budget constraints. Equation (3.28) is derived using Lagrangian Multipliers and afterwards using a bisection method (because of the non-linear tax-function) of the presented budged constraint (BC) such that it results in $BC \overset{!}{=} 0$, determining $FOC \rightsquigarrow l$ and afterwards $c(l, y)$.

3.4.3 Transitional Dynamics

The last piece of the puzzle is to solve for the transitional dynamics during the pandemic induced shock. Again, as the probability of infection τ^{sir} can be guessed in a shooting style algorithm, the Monte Carlo simulation defined in subsection 3.4.2 can be used to solve for the equilibrium conditions. Given such a guess of the factor price r (wages $w(r)$ are a function of r) and therefore implicitly τ^{sir} ($r_t \implies \lambda_t \implies v_t(x) \implies (c_t, l_t) \implies \tau_t^{sir}$) one can repeat such a procedure by changing r^i at iteration i, until changes in r^{i+1} are neglectable, given all conditions defined in the household's problem apply (3.14), (3.18), (3.22).

As one can write the law of motion for the distribution, $\lambda'(a', e', sir') = \sum_{E \times SIR} \pi(e'|e)\lambda(a^{-1}(a', e, sir), e)$ the approach of this thesis results in a shooting method style guess and update until convergence algorithm. Since the start and endpoints of the transitional dynamics, as well as its states, value functions and policy functions at these points in time, are known (stationary recursive equilibria), first a backward in time moving procedure, trying to figure out the best response to the boundary conditions $v^*_{t-1,sir} = u(c_t, l_t) + \beta\, v^{\text{known!}}_{t,sir}$, $\forall t$ is used. The missing path for the factor prices r and w may be guessed at the beginning since after each iteration, hopefully, a closer to the solution lying path of factor prices is present. After the backward optimization of $(v_{t,sir})_{0\leq t\leq T}$ a forward shooting procedure is responsible to figure out using a SIR-adapted Monte Carlo Simulation allows generating the distributions with respect to a, c, and l using the updated $(v_{t,sir})_{0\leq t\leq T}$. Afterwards, a third procedure guesses factor prices r to allow for the equilibrium conditions to hold, except for the stationary part. This is repeated until no more changes of the system are present with respect to $(r^i_t)_{0\leq t\leq T} - (r^{i+1}_t)_{0\leq t\leq T} \approx 0$ where i are the repeated iterations of the procedure. A pseudocode of the procedure is defined in algorithm .3 present in the appendix.

It is worth mentioning that $\tau^{sir}_t(c,l)$ is a function of c and l, yet at the same time, consumption $c(\tau^{sir})$ itself is a function of τ^{sir}. This cross dependency was initially dealt with in a separate approach, iterating until τ^{sir} and c do not change any more and only then to proceed with the above described forward shooting. This procedure is not used in the current state of this thesis, since the error resulting from it was in the $1e-5$ end therefore discarded for time efficiency reasons. This is because each τ^{sir}-iteration requires a recalculation of the value function, propagating the effect of a changed, τ^{sir} which is expensive. The resulting error will be reflected in the factor prices r and therefore dealt with in the next iteration of the main algorithm, since $(r^i_t)_{0\leq t\leq T} - (r^{i+1}_t)_{0\leq t\leq T} \approx 0$ is required to be around zero. An update rule of $(r^i_t)_{0\leq t\leq T}$ is present in the form of $(r^{i+1}_t)_{1\leq t\leq T} = (1-\xi) * (r^i_t)_{1\leq t\leq T} + \xi(r^{implied}_t)_{1\leq t\leq T}$, where $r^{implied}$ represents the supply and demand equating factor price of the current solution. This is considered due to convergence stability reasons of the present solution, as such an update rule intends to update in a conservative manner. To further make sure the value function converges to the old steady state value function, a vaccination is conducted after one year model time.

3.5 Matched Moments, Calibration, and Parameterization

In this section, the used model parameters used for calibration are presented in table 3.1, as well as hyperparameters as seen in subsection 3.5.2, representing parameters of used solution methods. Note that two separate models are calibrated. One representing the progressiveness of the income tax system as well as the intensity of consumption taxes and labour efficiency units of Austria and the other one of the United States of America. Structural parameters as α, β and δ differ slightly yet in a non-significant manner.

3.5.1 Model Parameters

Worth mentioning is that the functional form of [García-Miralles et al., 2019] $\tau^{a,y}(y) = n + o\, y^p \;\; n, o, p \in \mathbb{R}$ is different from [Guner et al., 2013] $\tau^{a,y}(y) = 1 - o\, y^{-p} \;\; o, p \in \mathbb{R}$ therefore resulting in a different number of parameters as seen in table 3.1. The US estimates result in a less progressive functional form as, e.g. the three times household

Model Parameters	AUT	USA
number of biweekly time periods within a yea	=26	=26
rate of depreciation δ	$1.075^{\frac{1}{T}} - 1$	$1.05^{\frac{1}{T}} - 1$
capital income share α [International Labour Office, 2019]	0.42	0.40
discount factor β	0.94	0.935
consumption labour share ϕ	0.5	0.5
degree of relative risk aversion [Gandelman and Hernandez-Murillo, 2015] γ	1.05	1.39
share of consumption affecting π_s, ψ	1.1	1.1
share of leisure affecting π_s, η	1.1	1.1
consumption tax τ_c	0.2	0.1
minimum wage (labour efficiency units e, $m = 7$ number of grid points of Tauchen's Method)	e_{m-2}	e_{m-1}
coefficient for income tax [Guner et al., 2013] n^{usa}	—	-0.089
coefficient for income tax [Guner et al., 2013] o^{usa}	—	0.186
coefficient for income tax [Guner et al., 2013] p	—	0.236
coefficient for income tax [García-Miralles et al., 2019] o^{aut}	0.8985	-
coefficient for income tax[García-Miralles et al., 2019] p^{aut}	0.1483	-
probability of recovery π_r	0.55	0.55
probability of death π_d	0.02	0.02
rate of autonomous infection spread π_a	0.01	0.01
rate of c and l based infection spread π_s	1.2	1.2
dampening factor of social distancing with respect to $k_{\text{social.dist}}$	0.65	0.70
fraction of consumption ex-change into net-savings during lockdown k_{lockdown}	0.5	0.75
beginning of multiple lock-downs $t_{\text{lockdown}+}$	$[5, 14, 20]$	$[6, 17]$
end of multiple lock-downs $t_{\text{lockdown}-}$	$[7, 16, 22]$	$[12, 23]$
beginning of social distancing measures $t_{\text{SD}+}$	6	5
end of the pandemic (instantaneous vaccination)	$\frac{T}{2} = 26$	$\frac{T}{2} = 26$
persistence of AR(1) process — labour efficiency units ρ	0.955	0.94
standard deviation of AR(1) process — labour efficiency units σ	0.13	0.19

Table 3.1: Used model parameters for the Austrian as well as the US model.

median income of the US model has an average tax rate of around, 13% opposed to the Spain model 25%.

Another not obvious fact is since the persistence ρ and standard deviation of the AR(1) process of labour efficiency units are estimated at a yearly basis in [Hur, 2021], they require a mapping to a biweekly basis. Since the present object is not an AR(1) process, yet instead a discretized form of it, using Tauchen's procedure [Tauchen, 1986] a Markov Chain is present. Therefore, one can use an eigenvalue decomposition of the stochastic matrix and taking the $\frac{1}{T}^{\text{th}}$ root of the diagonal matrix resulting in $\tilde{P} = P^{\frac{1}{T}} = V \cdot D^{\frac{1}{T}} \cdot V^{-1}$

as seen in [Chhatwal J, 2016]. The resulting $\tilde{P}$ is used as a basis to govern the transitions of labour efficiency during the transitional dynamics. The now missing $\tilde{s}$ which governs the standard deviation of the AR(1) process is chosen such that $T \cdot \tilde{s} = s$ where s is estimated on a yearly basis.

Epidemiological variables are chosen to represent the real life rate of growth of infected people, as well as aggregate infected people until the present day (21.11.2021). For the AUT model, an overall infection of 9% and for the USA 16.5% is targeted in the respective model economies. During the unrestricted spread of the virus, an approximate rate of growth of around $[1.8, 2.2]$ at point $0 \leq t \leq T$ is considered. More elaborate measures as reproduction numbers are not considered as a tool of calibration.

With respect to the pandemic, the duration of the considered transitional dynamics $T_{\text{transition}} = 52$ as well as the point in time the population is vaccinated $T^{\text{vacc}} = \frac{T_{\text{transition}}}{2}$. This implies an immediate elimination of the spread of the virus at $t = T^{\text{vacc}}$, resulting in $\tau^{\text{sir}} = 0.0$. This implies the value function $v(a, e, \lambda, sir)$ of any household now equal the value function of recovered households $v(a, e, \lambda, sir = r)$, allowing households to include the vaccination and therefore the end of the pandemic into the decision problem (3.18). Epidemiological parameters π_i, $i \in \{a, s, d, r\}$ are considered to be equal in both models.

The minimum wages $we_{\min}$ are calibrated roughly to be a half of the median income of the USA's model income distribution. The minimum wage with respect to AUT is calibrated as two thirds of the AUT's model income. These estimates are based on data published by the U.S. Department of labour as well as the federal ministry of Austria. As in Austria a minimal wage per se does not exist, the lowest wage based on collective agreements is used as a proxy. Therefore, jobs without collective agreements are not included.

With respect to the VSL defined in (3.25), this expression is balanced in such a way that the average yearly consumption equal to the VSL using the same value for $k = 6226$ as seen in [Hur, 2021] and in the chapter 3.5. The difference being leisure l as well as consumption c are present in the derivation of the VSL. This value is used to calibrate the system towards comparable value functions with respect to the US and AUT model. A further usage of the value of a statistical life is, the lost VSL as seen in equation (3.18) in case of a fatal course of the pandemic.

3.5.2 Hyperparameters

Present hyperparameters of the model are grid size of assets a, $n_a = n_a^{interp} \cdot n_a^{coarse} = 100 \cdot 50 = 5000$ as well as the quality of the stationary distribution. This is reflected in grid size n_a, length of paths with respect to the Monte Carlo Simulation $T_{\text{stationary}} = 3000$, convergence criteria for the difference of value functions $\Delta^{min}_{abs(v^i - v^{i+1})} = 1\text{E-}10$ and number of simulated households $n^{HH} = 1\text{E}4$. Furthermore, $n^{\text{Tauchen}} = 7$ as suggested in [Christiano, 1990] and [Hur, 2021], the lower bound on assets for a household to hold $\underline{a} = 0$ (no borrowing), as well as the non-binding upper bound $\overline{a} = 4$ and the number of Howard's policy iterations $n^{\text{Howard}} = 500$, are considered.

3.5.3 Matched and Unmatched Moments

Since not all parameters are chosen to match direct empirical estimates, these are chosen in such a way that the following characteristics of an economy match. Such characteristics are the $\frac{K}{Y}$, the VSL as well as leisure as seen in table 3.2. Non-targeted moments are displayed to give confidence the model displays. Similar as in [Hur, 2021] an underestimation of the Gini coefficient with respect to wealth and consumption is present.

Targeted / Non-Targeted Moments	Data AUT	DATA USA	Model AUT	Model USA
consumption Gini [Pirmin Fessler, 2017] [Hur, 2021]	0.33	0.29	0.27	0.36
wealth Gini [Pirmin Fessler, 2017] [Hur, 2021]	0.74	0.84	0.65	0.63
Total number of infected people	0.12	0.15	0.11	0.16
K/Y [Hur, 2021]	3.8	4.8	5.08	5.40
$\mathbb{E}[VSL]$ [Hur, 2021][Pirmin Fessler, 2017]	< 238.8	238.8	318.1	344.2
$\mathbb{E}[l]$ [Hur, 2021][Pirmin Fessler, 2017]	34.4	34.4	33.2	35.8

Table 3.2: Targeted as well as Non-targeted moments, as seen in [Hur, 2021]

CHAPTER 4

Experiments and Results

The present chapter intends to show the results and conclusions of the comparisons of swapping US with AUT policies and vice versa. The resulting model output, namely two categories of economic variables, are considered. Variables with respect to *allocation* of aggregated variables (output Y, labour N, capital K, consumption C, investment I) and variables with respect to *welfare* (consumption equivalent variation g). These are further divided into groups of low-, middle- as well as high-income households.

The experiment, therefore, is how the different pandemic policies affect the two model economies of AUT and USA during the pandemic shock. These policies are as follows:

1. no policy applied to model AUT,
2. no policy applied to model USA,
3. lockdown and social distancing policy of AUT applied to model AUT,
4. lockdown and social distancing policy of AUT applied to model USA,
5. lockdown and social distancing policy of the USA applied to model AUT,
6. lockdown and social distancing policy of the USA applied to model USA,

The exact differences in policies are described in the upcoming section 4.1. The goal of these experiments is to observe differences in allocation of economics variables (Y, N, K, C, I) of households with different levels of income. Another, and considered to be the main result of this thesis, is the policies effect on welfare consequences, measured by the consumption equivalent variation given two types of polices within the USA or AUT model. The result is a quantitative expression of how much consumption households are willing to give up to switch policies.

The mentioned allocation variables are defined in equation (4.1 - 4.5)

$$C := \int_X c(a,e,sir)\; d\lambda_t(a,e,sir) \tag{4.1}$$
$$N := \int_X (1-l)\cdot e\; d\lambda_t(a,e,sir) \tag{4.2}$$
$$K := \int_X a(a,e,sir)\; d\lambda_t(a,e,sir) \tag{4.3}$$
$$I = K' - (1-\delta)K \;\Longleftarrow\; I - \delta K = K' - K \tag{4.4}$$
$$Y := C + I \equiv F(K,N) = K^{\alpha}N^{1-\alpha} \tag{4.5}$$

where Y represents aggregated output, N aggregated labour, K aggregated capital, C aggregated consumption and I aggregated net investment. Here l represents the households leisure, a the households assets, e the labour efficiency units, sir the state of infection, $\lambda_t(B)$ where $B \in X$ the underlying probability measure defined on $(B, \mathcal{B}, \lambda_t)$, the state space $X = A \times E \times SIR$ ($A = [\underline{a}, \overline{a}], E = \{e_0, ..., e_m\}$, $m = 7$, $\underline{a} = 0, \underline{a} < \overline{a} \leq \infty$ being never binding amount of assets, SIR $= \{s, i, r\}$), δ the rate of depreciation and $F(K,N)$ the production function with its respective factor inputs. Variables with an added "prime" symbol indicate the next period value of the underlying variable. Further, the consumption equivalent variation [Lucas, 1992] g is defined in equation (4.6) as well as the value of a statistical life as seen in equation (3.25) is used as a measure of welfare.

$$\begin{aligned} v(a,e,\lambda,sir;\,g)^{\mathbf{0}} &= \mathbb{E}_0 \sum_{t\geq 0} \beta^t \frac{([c\cdot(1+g)]^{\phi} l^{1-\phi})^{1-\gamma} - 1}{1-\gamma} \\ \Longrightarrow\; v(a,e,\lambda,sir;\,g)^{\mathbf{0}} &= (1+g)^{(1-\gamma)\phi}\mathbb{E}\sum_{t\geq 0}\beta^t \frac{(c^{\phi}l^{1-\phi})^{1-\gamma} - 1}{1-\gamma} \\ = (1+g)^{(1-\gamma)\phi} v(a,e,\lambda,sir)^{\mathbf{0}} &\;\Longrightarrow\; \frac{v(a,e,\lambda,sir)^{\mathbf{1}}}{v(a,e,\lambda,sir)^{\mathbf{0}}}^{\frac{1}{(1-\gamma)\phi}} - 1 = g \end{aligned} \tag{4.6}$$

Since all goods produced are consumed in the model economy, equation (4.5) represents the model economies output including net investments I and consumption C (no import or export is present, as well as government spending). In equation 4.4 net investment is defined as the difference of aggregated capital K at time t and $t+1$ including the rate of depreciation δ. In equation (4.6) the consumption equivalent variation is defined. It represents the amount of consumption in percent a household requests to be indifferent to the solutions of the value functions $v(a,e,\lambda,sir;\,g)^0 = v(a,e,\lambda,sir;\,g)^1$. This coefficient can now be evaluated for each household using the value function at $t = 0$, evaluating the period during transitional dynamics. This is possible due to the construction of value functions, which encapsulate all future information about the household's optimal welfare gains. For, e.g., one value function is based on the USA pandemic policy and the other one is based on the pandemic policy of AUT. Therefore, it is a measure how much of consumption is needed to make up for the lost welfare with respect to the other pandemic policy.

4.1 Stay at Home and Social Distancing Policies

This section's results consider stay at home and social distancing policies. Until now, no such policies are considered within the model as seen in chapter 3. Stay at home orders are implemented as saving a *predetermined* fraction of the chosen consumption $k^{usa}_{\text{lockdown}} = 0.75$ and $k^{aut}_{\text{lockdown}} = 0.5$. As the idea is to have a harsh and temporary lockdown in the AUT model, a more drastic amount of lost consumption is considered as the effects of a more harsh and temporary lockdown seem to be more severe for the economy as a whole during the lockdown. The social distancing is represented by dampening the effect of π_s on τ^{sir} in equation (3.13). This reduces the rate or probability of infection by a linear amount, starting at $t^{\text{start}}_{\text{lockdown}} \leq t \leq T$ and lasting until the end of the transitional dynamics. Therefore, the social distancing measure affects the periods outside the lockdown as well. For $k^{usa}_{\text{social.dist}}$ a value of 0.7 is chosen in comparison to $k^{aut}_{\text{social.dist}} = 0.65$. This is based on the assumption that the AUT social distancing policy is enforced by the government as well as followed by the households in a more "cooperating" manner. The exact timings of the start of the social distancing policies is $t^{aut}_{\text{SD+}} = 5$ and $t^{usa}_{\text{SD+}} = 6$ and last until the end of the transitional dynamics.

Opposing this idea is a more moderate and less temporary "lockdown" in the USA model economy. A lockdown in the USA economy is considered as a "there is something wrong, yet households are allowed to proceed with their normal life in most sectors of their life", leading to a less severe restriction on $k^{usa}_{\text{lockdown}}$ in comparison to $k^{aut}_{\text{lockdown}}$. The timing of the lock-downs is present as vertical, shaded bars in the upcoming figures. The exact timings of the start of its three lock-downs of the AUT economy are $t^{\text{AUT}}_{\text{lockdown+}} = (5, 14, 20)$ as well as its endings, $t^{\text{AUT}}_{\text{lockdown-}} = (7, 16, 22)$ resulting in a total duration of 9 biweekly time units (18weeks) until the vaccination ends the pandemic at $t_{\text{vaccinate}}$. As a point of reference $T = 52$ and $t_{\text{vaccinate}} = 26$. The timings of the start of its two lock-downs of the USA economy are $t^{\text{AUT}}_{\text{lockdown+}} = (6, 17)$ as well as its endings, $t^{\text{AUT}}_{\text{lockdown-}} = (12, 23)$ resulting in a total duration of 14 biweekly time units (28weeks) until the vaccination ends the pandemic at $t_{\text{vaccinate}}$. As a point of reference $T = 52$ and $t_{\text{vaccinate}} = 26$.

Households within the model do not know about the exact lockdown policies, as they only observe distributions and factor prices. However, since the present macroeconomy is modelled in a complete information setting, agents observe the changes in factor prices, influenced in addition by the pandemic policies, as well as asset-, leisure- and consumption distributions during the backwards induction step and adapt accordingly to the scenario. Therefore, the value function encapsulates the presence of a future lockdown if visited at time $t = 0$. As these lock-downs result in a suboptimal allocation of economic variables during the next iteration i of the backwards induction step, an optimal response towards this suboptimal allocation is calculated. However, again to be interrupted by a lockdown during the next iteration. Leisure is not responding towards the change in $c(1-k) \not\Rightarrow l(c(1-k)$, directly, where k is the fraction of consumption which is not performed due to the lockdown policy, as the considered, biweekly time periods are considered too short and labour markets too rigid. The changed $c(1-k)$ however has an effect on the savings decision and therefore the optimal choice on $u(c, l)$, therefore influencing (c, l) pairs.

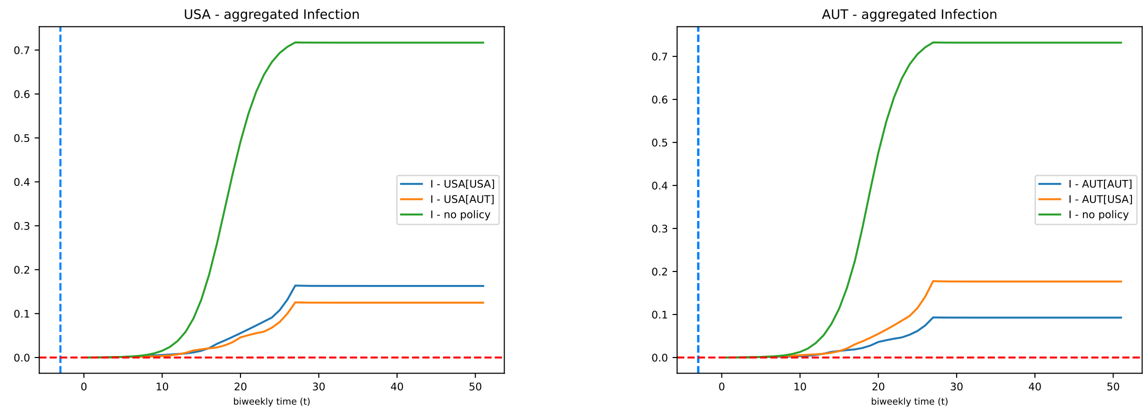

Figure 4.1: Aggregated, infected households using pandemic policies during transitional dynamics in the USA and AUT model economy

The resulting probability of infection of both economies, and therefore aggregated infected households, are present in figure 4.1. Here, a certain sensitivity towards the present pandemic policies is displayed. The US economy, using the AUT policy, seems to reduce overall infections by 30.52% and related to the total population by 3.83% (US:16.38%, AUT:12.55%), as in the AUT economy the USA policy increases aggregate infections by 90.76% and related to the total population by 8.45% (US:17.76%, AUT:9.31%). No lockdown policies applied in both models result in herd immunity (70% of households are infected). Note in [Hur, 2021], [Eichenbaum et al., 2020] and [Krueger et al., 2020] the macro SIR model including no policies achieve a significant reduction of infections, prohibiting herd immunity. The model in this thesis does not provide such a result, as the absence of a policy results in herd immunity and therefore a significant contamination of the population. Worth noting is the AUT[AUT] – and USA[USA] policies are calibrated such that the real world aggregated infected households are similar to the model output. The AUT[USA]- and USA[AUT] policies however can be used to see a course effect of the interchanged policies on the infection dynamics. Here, AUT[USA] represents the AUT model economy using the USA pandemic policy.

Based on the aggregated infections as seen in figure 4.1, one can derive the aggregated deceased households of both model economies. These are displayed in figure 4.2. Since the probability of mortality of infected households is exogenous, similar behaviour as in the

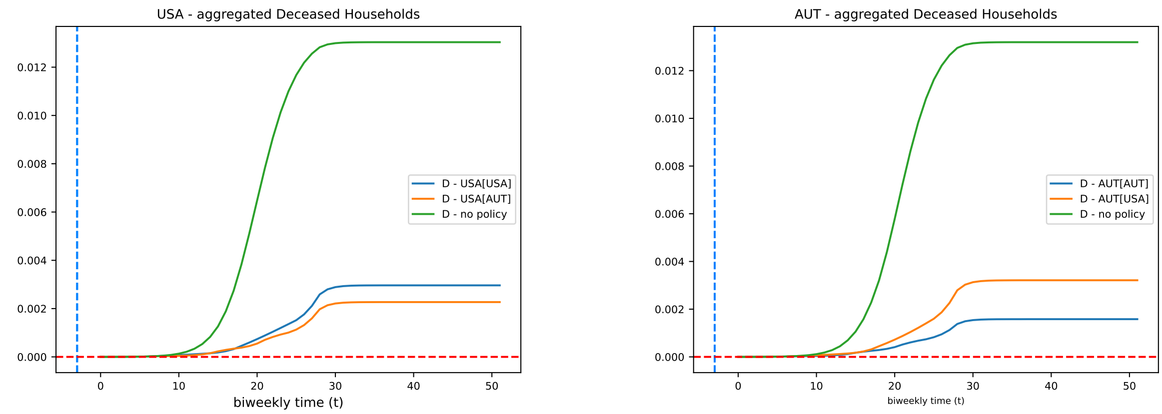

Figure 4.2: Aggregated, deceased households using pandemic policies during transitional dynamics in the USA and AUT model economy

aggregation of infected households is expected. Therefore, the AUT model economy using the AUT policy results in the least mortal cases with 0.16% of deceased households at the end of the pandemic. No policy and USA policy result in 1.32% and 0.32% considering the AUT model. With respect to the US model economy, the AUT policy results in 0.23% of deceased households. No policy and USA policy result in 1.30% and 0.30% considering the USA model. Again, the USA model economy seems to be less impacted by a change of policy than the AUT economy.

As a distribution of consumption, leisure, and households assets is present with differences in income and assets, a natural choice to calculate is a Lorenz Curve or a Gini index. The present Gini index of AUT and US model economies and its policies are present in figure 1 and figure 2 in the appendix. Each row represents the same variable of a nation state, as well as each column represents one pandemic policy. Worth noting here is a reduction of the Gini index across nation states and policies, during lockdown periods, implying a more evenly distributed variables across its distribution. Worth noting is here that due to the efforts of low income households increased savings, as well as forced savings during lock down periods, a tendency towards a fairer allocation of resources based on household's optimal decisions is present. This effect however is not permanent, as seen in between the lockdown periods, which in the end equals its starting state. Also, worth noting is the case that during the "no policy" case the Gini index of assets increases, as opposed to the lockdown policy cases.

4.1.1 Allocation of Aggregated Variables

With respect to the *allocation of aggregated variables*, the following results are present in the model economies. In figure 4.3, figure 4.7, figure 4.11, as well as figure 4.13 and

4.12, the USA model economy using USA-, AUT and no policies, as well as the AUT model economy using USA-, AUT and no policies, display output Y, consumption C, net investment I, labour N as well as capital K, during the transitional dynamics of the USA economy. Since the present model in this thesis uses a general equilibrium approach during the transition period, in figure 3 the wage rate and in figure 4 the real interest rate according to aggregate labour N, as well as aggregate capital K during the transitional period is displayed in the appendix. In general, in almost all figures the currently infected households are present in a secondary y-axis, to provide an overview of the state of the pandemic. There the vertical, dashes, light-blue lines represent start and end of the lockdown phases.

In figure 4.3, 4.4 and 4.5 the effect of the lagged net-investment are present. Due to the reduction in consumption at time t, the savings of households affect next period investments. This lag is responsible for the drop of output Y. In the case of no pandemic policies, aggregate output Y drops on average by $USA[no] = 3.09\%$, $AUT[no] = 2.1\%$ in comparison to its steady state during the two-year transition period. No present policy, compared to the USA and AUT policies, yield a lower drop in output Y on average based on its steady state ($AUT[AUT] = 5.52\%$, $AUT[USA] = 3.71\%$, $USA[AUT] = 5.57\%$, $USA[USA] = 3.39\%$) favouring no pandemic policies.

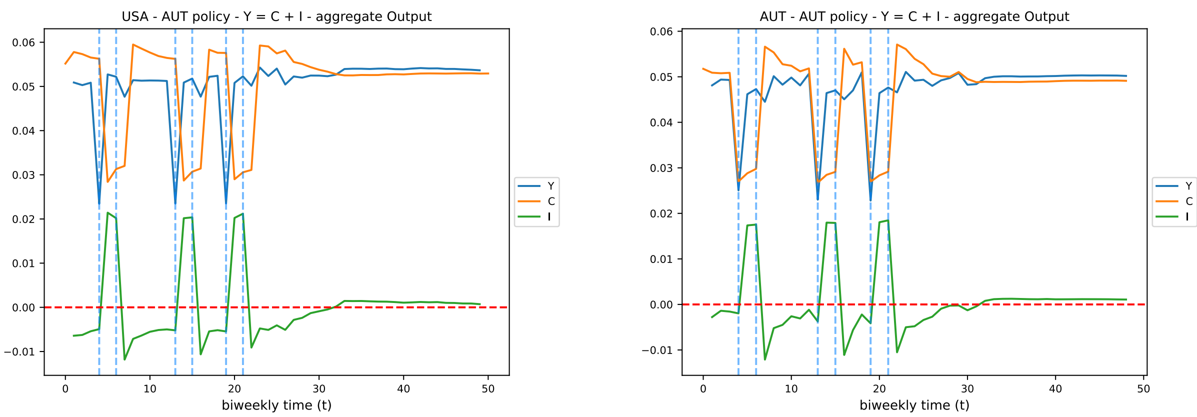

Figure 4.3: Present first moment of output Y, consumption C and investment I during transitional dynamics in the USA and AUT model economy using the AUT policy.

In figure 4.3 displayed on the left-hand side, is the AUT policy of the USA economic model. The largest magnitude of the output shocks are similar in principle and result in a drop of 56.3%, 56.2% and 56.1% in terms of the steady state level. On the right-hand side, is the AUT policy of the AUT economic model. The highest magnitude of the output shocks are similar in principle and result in a drop of 50.1%, 53.9% and 53.1% in terms of the steady state level.

In figure 4.4 displayed on the left-hand side, is the USA policy of the USA economic model. The largest magnitude of the output shocks are similar in principle and result in

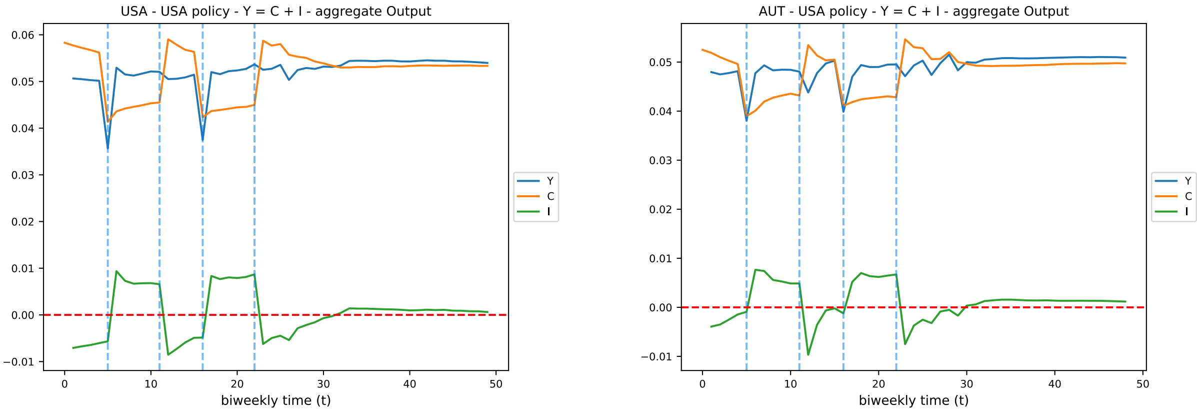

Figure 4.4: Present first moment of output Y, consumption C and investment I during transitional dynamics in the USA and AUT model economy using the USA policy.

a drop of 33.9% and 30.5% in terms of the steady state level. On the right-hand side, is the USA policy of the AUT economic model. The magnitude of the output shocks are similar in principle and result in a drop of 25.5% to 24.6%.

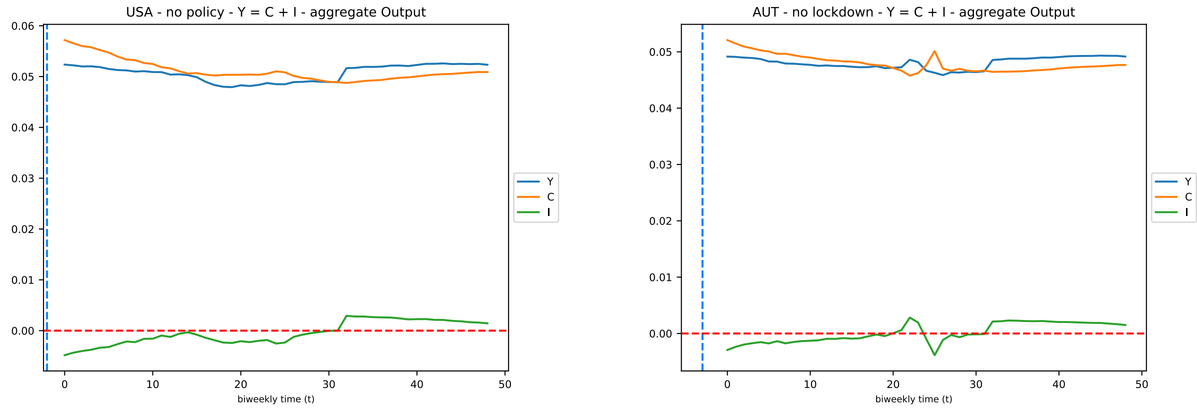

Figure 4.5: Present first moment of output Y, consumption C and investment I during transitional dynamics in the USA and AUT model economy using no policy.

In figure 4.6, 4.7 and 4.8, aggregate labour N (split in low N_p, middle N_n, and high income N_r households) during transitional dynamics of both model economies, is displayed. During all experiments, a reduction of labour supply of low-income households is visible as a downward trend, anti-correlating with an increase of the share of infected households. Overall, the high-income households seem to have smoother transition with respect to labour N as compared to low-income households. As high-income households tend not to be impacted with respect to labour supply, low-income households supply varies in the periods during and after the lockdown periods. In the case of no pandemic policies, aggregate labour N drops on average by $USA[no] = 5.61\%$, $AUT[no] = 5.39\%$ in comparison to its steady state during the two-year transition period. No present policy, compared to the USA and AUT policies, yield a lower drop in labour N on average based on its steady state ($AUT[AUT] = 4.92\%$, $AUT[USA] = 2.43\%$, $USA[AUT] = 4.23\%$, $USA[USA] = 3.17\%$) favouring the USA pandemic policies.

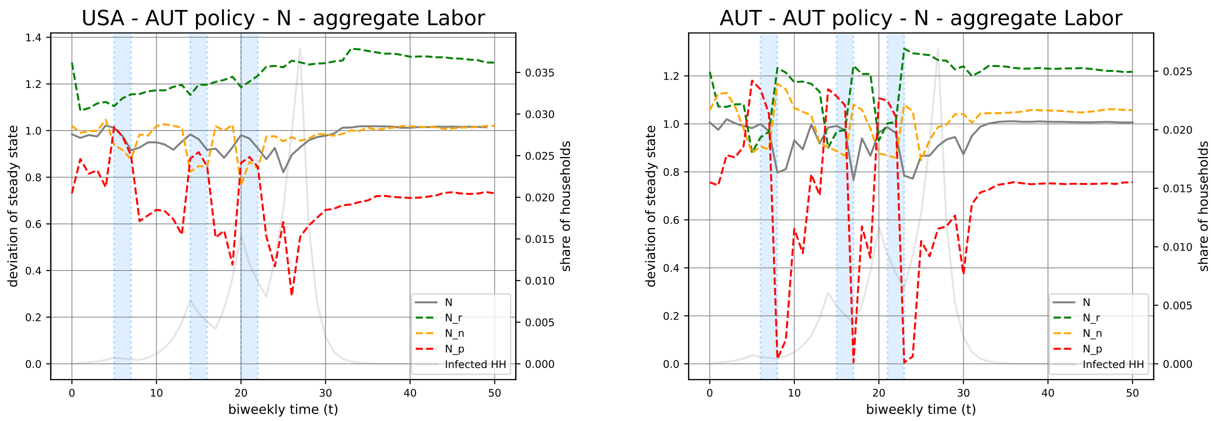

Figure 4.6: Present first moment of labour N (split in low N_p, middle N_n, and high income N_r households) during transitional dynamics of AUT and USA model economies using the AUT policy.

In figure 4.6 the impact on low-income households is very prominent in the AUT model using AUT policies. Including the behaviour of asset accumulation as seen in figure 4.9, it seems that the low-income household's successful acquisition of assets before and during lockdown and the resulting reduction of assets afterwards allows them to stop working after the lockdown periods. In figure 4.6 displayed on the left-hand side, is the AUT policy of the USA economic model. The largest magnitude of the labour shocks are similar in principle and result in a drop of 19.03%, 12.89% and 9.45% in terms of the steady state level. On the right-hand side, is the AUT policy of the AUT economic model. The highest magnitude of the output shocks are similar in principle and result in a drop of 21.35%, 21.56% and 22.65% in terms of the steady state level.

In figure 4.7 displayed on the left-hand side, is the USA policy of the USA economic model. The largest magnitude of the output shocks are similar in principle and result in a drop of 11.0% and 30.5% in terms of the steady state level. On the right-hand side, is the USA policy of the AUT economic model. The magnitude of the output shocks are similar in principle and result in a drop of 25.5% to 24.6% in terms of the steady state level.

In figure 4.8 on the right-hand side, the AUT model economy using no lockdown policy is present. Low-income households seem not related to the degree of infections, as opposed to the left-hand side of the USA model economy, where a steady decline is visible during

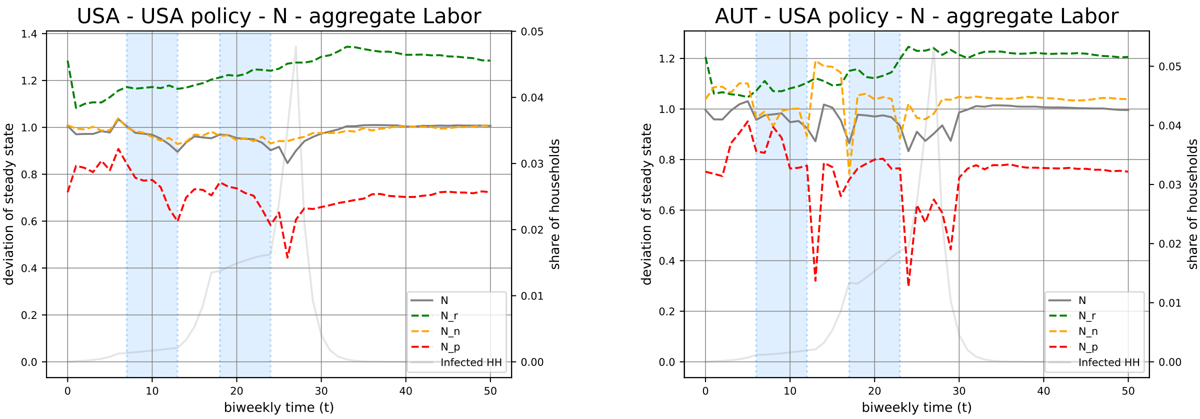

Figure 4.7: Present first moment of labour N (split in low N_p, middle N_n, and high income N_r households) during transitional dynamics of AUT and USA model economies using the USA policy.

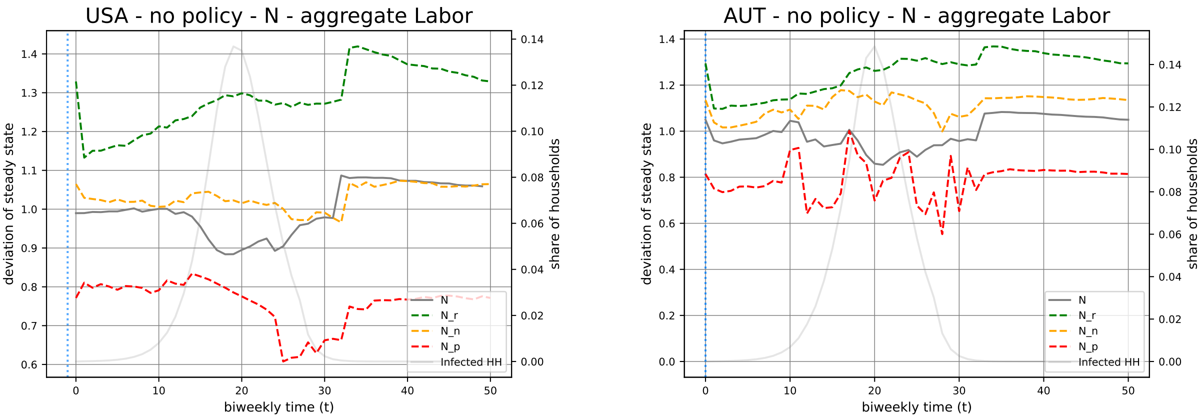

Figure 4.8: Present first moment of labour N (split in low N_p, middle N_n, and high income N_r households) during transitional dynamics of AUT and USA model economies using no policy.

the spread of the pandemic. This AUT behaviour can be explained by the presence of higher minimum wages in AUT economy, splitting the low income group into multiple subgroups with contrary decisions, not visible in the first moment of the distribution.

Worth mentioning is that during lockdown, households leisure and therefore labour, respond to the forced savings and reduced consumption only in an indirect manner. Households choose a different combination of consumption and leisure bundle c, l in order to compensate for the lost utility in the reduced consumption $u(c, l) + \beta v(a', e', \lambda', sir')$. Households do not adapt leisure immediately after the drop in consumption is taking effect. Computational complexity as well as the assumption that the lockdown introduces a sort of rigidity, that agents can not respond immediately.

In figure 4.9, 4.10 and 4.11, aggregate capital K (split in low K_p, middle K_n, and high income K_r households) during transitional dynamics of both model economies, is displayed. During all experiments, a reduction of capital supply of high-income households is visible as a downward trend, anti-correlating with an increase of the share of infected households. Implying, spending already allocated resources during a crisis, compensating for a reduction in labour supply as seen in, e.g., 4.7. Overall, the low-income households seem to have smoother transition with respect to capital K as compared to high-income households. As low-income households do not have the amount of assets (which serve as an insurance for idiosyncratic risks as infection and labour efficiency shocks), dealing with unforeseen shocks as a pandemic. In general, it is harder for low-income households to accumulate assets, as compared to high-income households. Nevertheless, it is deemed welfare increasing to accumulate assets by increasing its labour supply before the first lockdown as seen in e.g., 4.7, as increasing labour supply is not as impactful as the pandemic has not picked up momentum yet. This implies a preparing behaviour with respect to the upcoming pandemic and lockdowns. In the case of no pandemic policies, aggregate capital K drops on average by $USA[no] = 15.92\%$, $AUT[no] = 12.38\%$ in comparison to its steady state during the two-year transition period. No present policy, compared to the USA and AUT policies, yield a lower drop in capital K on average based on its steady state ($AUT[AUT] = 1.32\%$, $USA[AUT] = 5.46\%$, $USA[USA] = 3.38\%$) favouring the USA pandemic policies. Worth mentioning is the case of $AUT[USA] = 0.76\%$, where aggregate capital increases during the transition period on average.

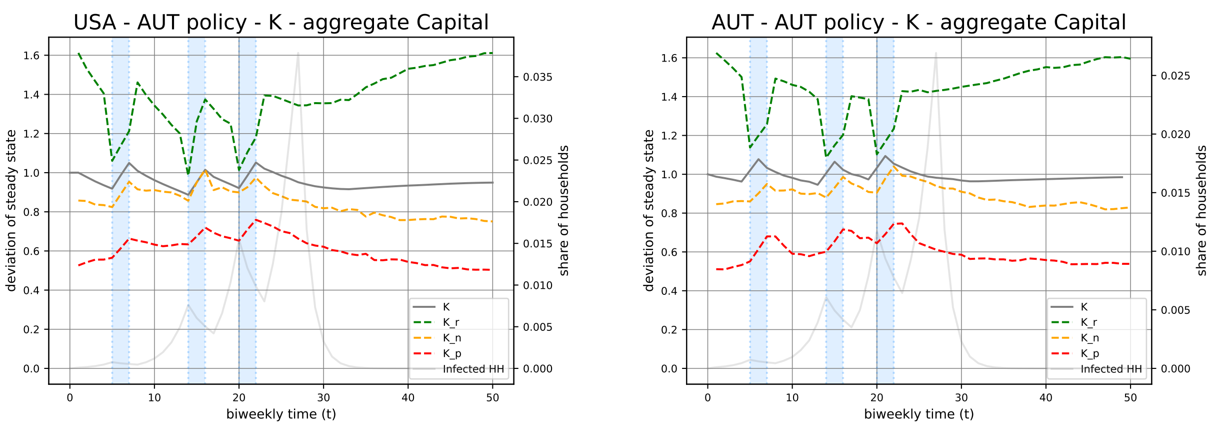

Figure 4.9: Present first moment of capital K (split in low K_p, middle K_n, and high income K_r households) during transitional dynamics in the USA and AUT model economy using the AUT policy.

In figure 4.12, 4.13 and 4.14, aggregate consumption C (split in low C_p, middle C_n, and high income C_r households) during transitional dynamics of both model economies,

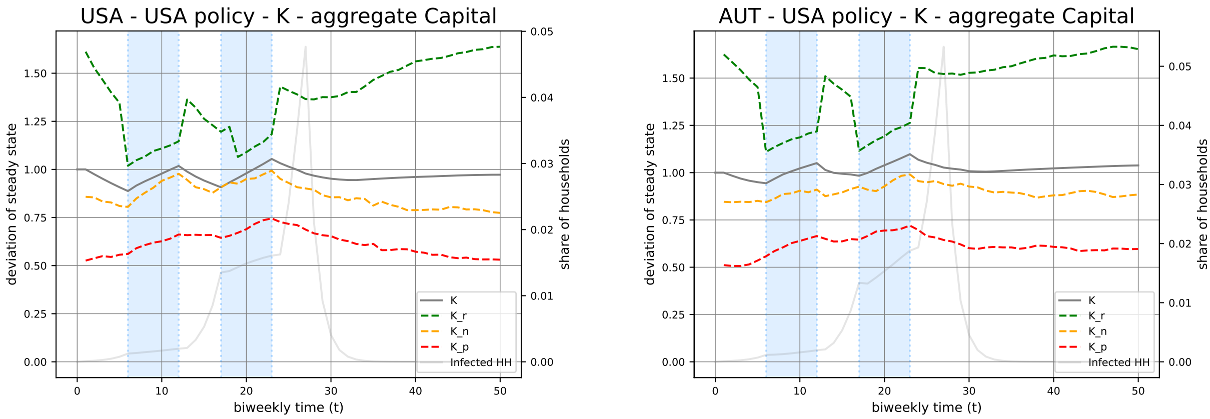

Figure 4.10: Present first moment of capital K (split in low K_p, middle K_n, and high income K_r households) during transitional dynamics in the USA and AUT model economy using the USA policy.

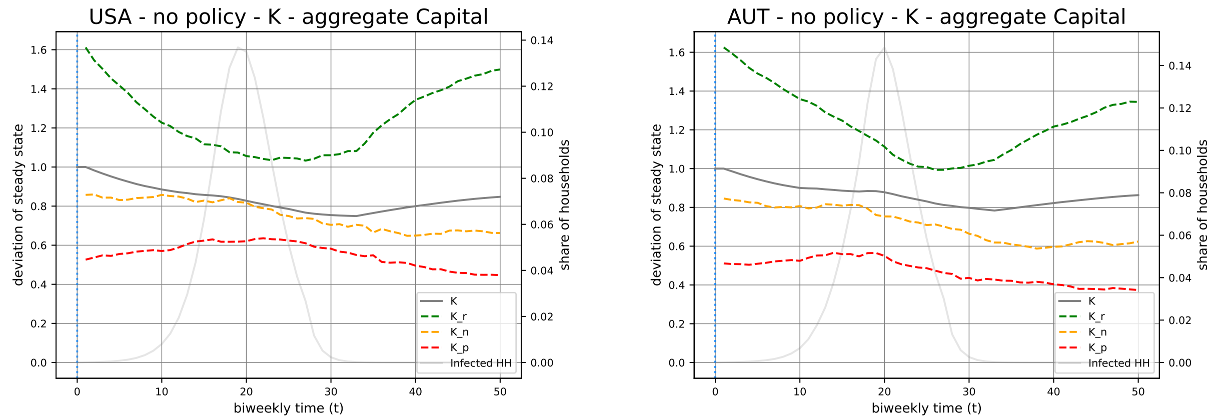

Figure 4.11: Present first moment of capital K (split in low K_p, middle K_n, and high income K_r households) during transitional dynamics in the USA and AUT model economy using no policy.

are displayed. During all experiments, a reduction of consumption during lockdown of high-income households is visible, as well as using no lockdown policy an initial increase and afterwards a steady reduction of consumption takes place. Indicating a “consume while you can” behaviour of agents before the pandemic picks up momentum, as well as before the lockdowns. Overall, the low-income households seem to have smoother transition with respect to consumption C as compared to high-income households. As low-income households do not have the amount of assets (which serve as an insurance for idiosyncratic risks as infection and labour efficiency shocks) before the pandemic starts, an initial increase in consumption is not deemed optimal, to prepare for the mandatory decrease in consumption during the lockdown periods. In e.g., 4.13 at first a strange behaviour is observed. A mandatory decrease in consumption results in an increase in consumption during lockdown for middle- as well as low-income households. Yet, due to the accumulated wealth during the pre lockdown periods it, as the amount of labour increased as well, the overall consumption is outpacing the mandatory reduction in consumption. Overall, on average, the consumption decreases with respect to the aggregation of all types of income households. In the case of no pandemic policies, aggregate consumption C drops on average by $USA[no] = 6.97\%$, $AUT[no] = 5.79\%$ in comparison to its steady state during the two-year transition period. No present policy, compared to the USA and AUT policies, yield a lower drop in consumption C on average based on its steady state ($AUT[AUT] = 7.56\%$, $AUT[USA] = 5.30\%$, $USA[AUT] = 4.27\%$, $USA[USA] = 5.72\%$) resulting in a not definite pandemic policy preference, with respect to variability.

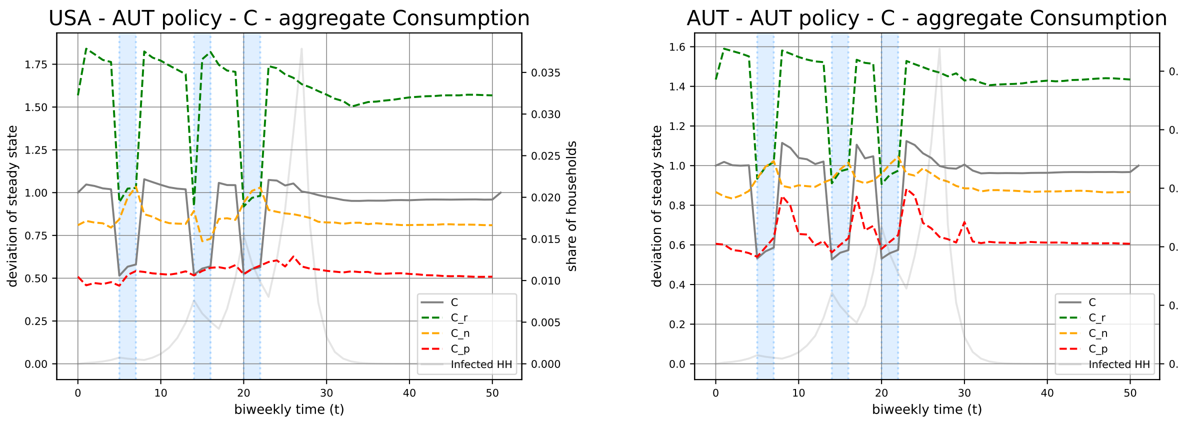

Figure 4.12: Present first moment of capital C (split in low C_p, middle C_n, and high income C_r households) during transitional dynamics in the USA and AUT model economy using the AUT policy.

In figure 4.12 the impact on low-income households is very prominent in the AUT model using AUT policies. Worth mentioning is the increase in consumption during and especially after lockdown, resulting in a decrease in capital as seen in figure 4.9, as well as a labour supply near zero, as seen in 4.6. Both effect are implied by the mandatory increase in savings due to the mandatory cut in consumption.

As the research questions in section 1.2 requires an answer on the effect on the allocation of aggregate economic variables, a qualitative summary of the findings with respect to the aggregation of variables during the transitional dynamics using different types of pandemic policies, based on groups of different income levels in the AUT and USA model economy is present.

Regarding **high income households** and the **absence of a pandemic policy**, labour

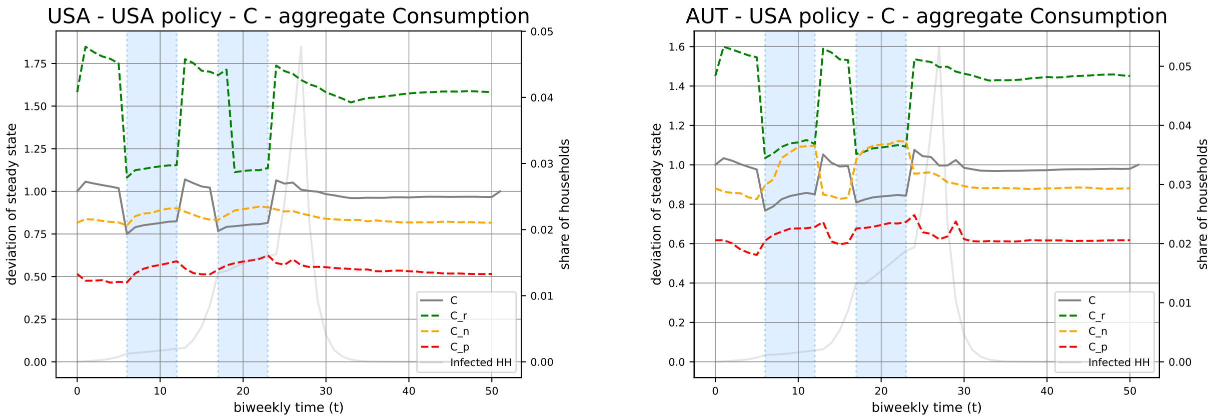

Figure 4.13: Present first moment of capital C (split in low C_p, middle C_n, and high income C_r households) during transitional dynamics in the USA and AUT model economy using the USA policy.

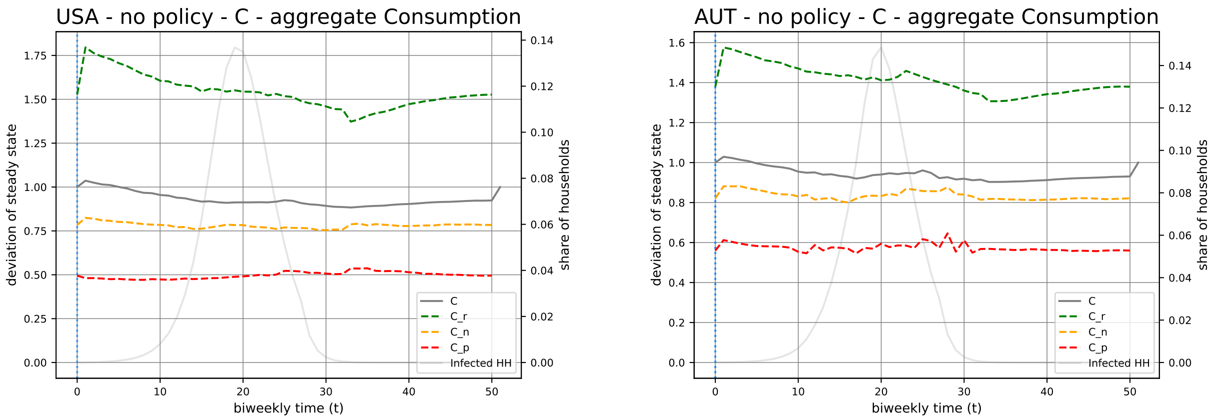

Figure 4.14: Present first moment of capital C (split in low C_p, middle C_n, and high income C_r households) during transitional dynamics in the USA and AUT model economy using no policy.

supply drops abruptly and rises again during the course of the pandemic to again reach steady state levels weeks before the end of the pandemic, due to vaccination. A preference to work less and consume more can be observed for high income households. The resulting increase in consumption is financed using the household's savings and therefore reducing the model economies capital stock. Weeks before the end of the pandemic, capital stock increases again to reach pre-pandemic steady state levels. During the **presence of pandemic policies**, a similar behaviour of households is present, yet steady state deviations of labour supply are dampened, possibly due to a reduction of rates of infection.

Consumption during lockdowns is reduced due to mandatory reduction in consumption and therefore increase in savings. This leads to above steady state levels of consumption after lockdown periods and below levels during lockdowns. In general, the AUT policies lead to a higher increase in volatility than the USA policies in both model economies with respect to high income households.

Considering **low income households** and the **absence of a pandemic policy**, labour supply increases slightly and drops weeks before the pandemic ends in the USA model economy. In the AUT model economy, no trend is present, possibly due to higher minimum income levels. Consumption of low income households is slightly below steady state and capital supply is increased slightly, to cope with the increase in uncertainty due to the pandemic shock. An inverse behaviour to high income households is visible, possibly due to the ability of high income households to make use of the accumulated assets to smooth utility governing factors as consumption and leisure. During the **presence of pandemic policies**, a similar behaviour of households is present, yet the level of capital supply of low income households is increased, labour supply during lockdowns is increased with respect to the steady state and consumption is in general above the steady state level, even during lockdown periods.

The following enumeration gives a course overview of the resulting and is supposed to give a broad statement about the resulting model economies aggregate variables.

- **Output** Y: The AUT lockdown policy led to a larger slump than the USA lockdown policy. The "no policy" policy provides the lowest volatility and lowest deviation from steady state on average.

- **Labour** N: The AUT lockdown policy led to higher volatility than the USA lockdown policy. The USA policy provides the lowest volatility and lowest deviation from steady state on average for high income households, as in contrast to low income households the "no-policy" policy provides the lower deviation from steady state.

- **Capital** K: The AUT and USA lockdown policy led to higher savings than in the "no-policy" policy case. The "no policy" policy provides the lowest variation and lowest deviation from steady state on average.

- **Consumption** C: The AUT lockdown policy led to a larger slump than the USA lockdown policy. The "no policy" policy provides the lowest variation and lowest deviation from steady state on average.

- **Investment** I: The AUT lockdown policy led to a larger spike than the USA lockdown policy. The "no policy" policy provides the lowest variation and lowest deviation from steady state on average.

4.1.2 Welfare

A comparison (based on welfare) of the different aggregations seem not obvious at first glance. Therefore, the usage of a measure of welfare, as seen in equation (4.6). The following figures, 4.15, 4.16 and 4.17 capture the distributions of simulated households with respect to the used *welfare* measure, the consumption equivalent variation g as defined in equation (4.6). Overall, a grouping of low-, middle- as well as, high-income households is present, showing the high income households to be less impacted by the chosen policies since their given up percentage of consumption is overall lower than other income groups. Aggregated, this results in an overall positive average in the USA model using the USA pandemic policy in comparison to the AUT policy, as well as vice versa in a negative form.

The biweekly VSL expresses a percentage change in weekly consumption to balance for a percent change in mortality. In [Hur, 2021] a recommendation of the U.S. Environmental Protection Agency, a VSL of 7.4 million USD in 2006 is used. By assuming a biweekly consumption of $C_{\text{biweekly}} = 1.2\text{E}3$, it implies k to equal 6.2 thousand USD, based on $\hat{\text{VSL}} = k \cdot C_{\text{biweekly}}$. This is the basis of equation (3.25) to define the VSL of the model economy. The VSL is calculated in a steady state setting and is therefore constant during the transitional dynamics. Yet, the AUT model results in an VSL of 318.1 USD ($318.1 = 6.2\text{E}3 \times 5.1\text{E-}2$) and the US model in an VSL of 344.2 USD ($344.2 = 6.2\text{E}3 \times 5.5\text{E-}2$). Therefore, households in the USA model require a higher compensation in case of their lost future consumption as in the AUT model on average, implying higher consumption over the lifespan of an agent in the USA economy, as well as a higher reduction in utility, in case of a fatal course of the pandemic. Swapping pandemic policies is not affecting the VSL, since it is only incorporated at the steady state.

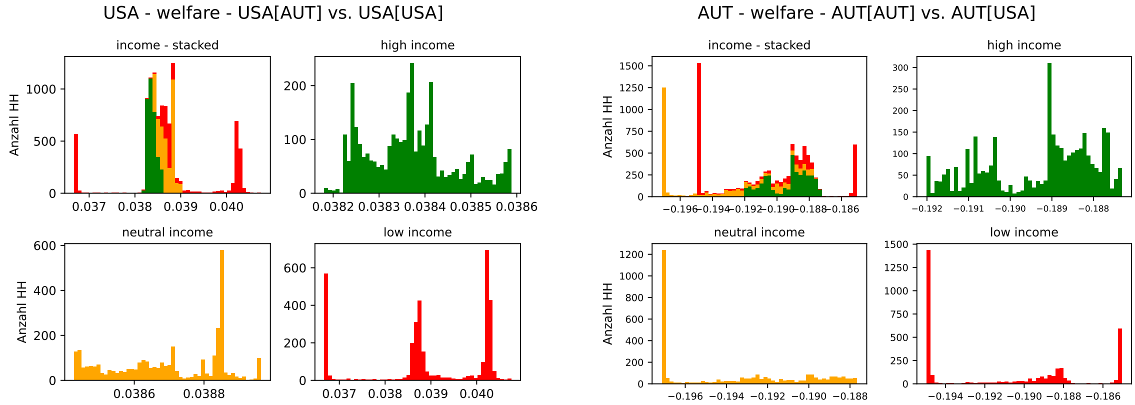

Figure 4.15: Consumption equivalent variation of the USA and AUT model economy using the USA pandemic policy as well as the AUT one.

In figure 4.15 the AUT model comparing AUT and USA policies on the right-hand side, as well as the USA model comparing AUT and USA policies on the left-hand side of consumption equivalent variation g. This allows one to conclude that households in the USA economy using the AUT policy are demanding an $g = 3.87\%$ increase in consumption on average in case they are forced to stay at the USA policy. Therefore, preferring the pandemic policy of the AUT model. Worth noting is no simulated household prefers the USA policy in contrast to the AUT model, allowing for a *Pareto improvement* since no household is worse off after a change in policy, and at least one household benefits from the policy change. Given the AUT model, one can conclude that households in the AUT policy are demanding an $g = 19.13\%$ increase in consumption on average in case they are forced to switch to the USA policy, therefore preferring the AUT policy. However, an unexpected result with respect to the low-income households is present in during all experiments. A subgroup of low-income households exist which prefers the AUT solution less than all the other groups of households. In this case, it is the first group who is above the minimal wages in the USA yet, receive minimal wages in AUT.

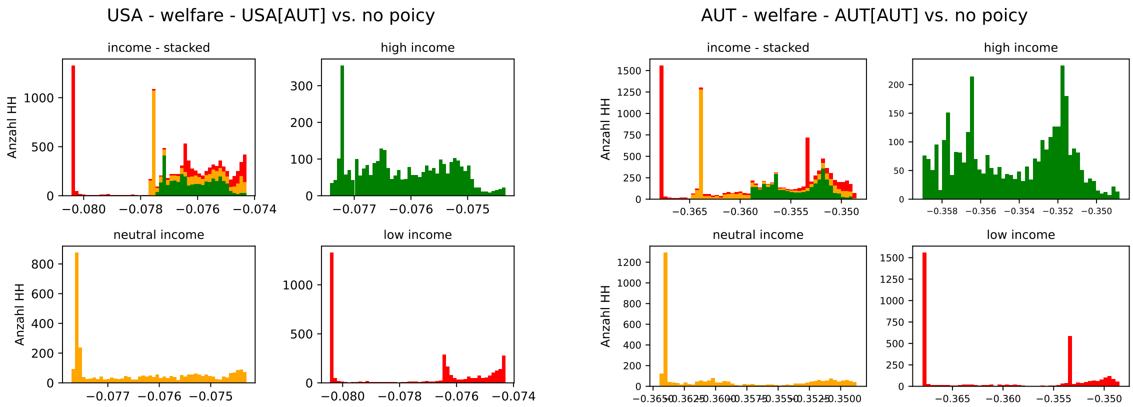

Figure 4.16: Consumption equivalent variation of the USA and AUT model economy using the AUT pandemic policy as well as no policy.

In figure 4.16 the AUT model comparing AUT and no policies on the right-hand side, as well as the USA model comparing no policies on the left-hand side of consumption equivalent variation g. This allows one to conclude that households in the USA economy are demanding an $g = 7.67\%$ increase in consumption on average in case they are forced to stay at the no policy. Therefore, preferring the pandemic policy of the AUT model. Worth noting is no simulated household prefers the “no policy” policy in contrast to the AUT policy, allowing for a *Pareto improvement* since no household is worse off after a change in policy, and at least one household benefits from the policy change. Given the USA model using the AUT policy, one can conclude that households in the USA economy are demanding an $g = 35.80\%$ increase in consumption on average in case they are forced to switch to the “no policy” policy, therefore preferring the AUT policy. Here, in both model economies, the outlier of the low-income subgroup represents the first group that receives neither minimal wages in AUT and the USA.

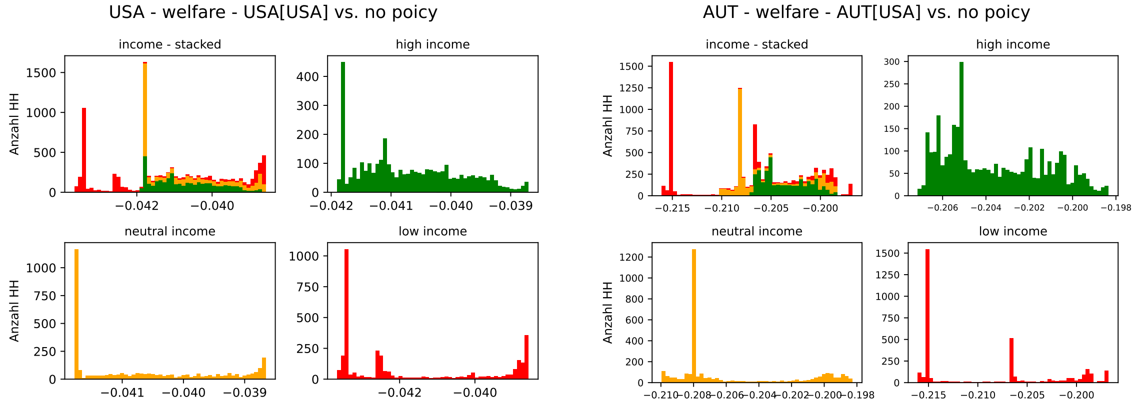

Figure 4.17: Consumption equivalent variation of the USA and AUT model economy using the USA pandemic policy as well as no policy.

In figure 4.17 the AUT model comparing USA and no policies on the right-hand side, as well as the USA model comparing no policies on the left-hand side of consumption equivalent variation g. This allows one to conclude that households in the USA economy are demanding an $g = 4.10\%$ increase in consumption on average in case they are forced to stay at the no policy. Therefore, preferring the pandemic policy of the USA model. Worth noting is no simulated household prefers the “no policy” policy in contrast to the USA policy, allowing for a *Pareto improvement* since no household is worse off after a change in policy, and at least one household benefits from the policy change. Given the USA model using the USA policy, one can conclude that households in the USA economy are demanding an $g = 20.62\%$ increase in consumption on average in case they are forced to switch to the “no policy” policy, therefore preferring the USA policy. Here, in both model economies, the outlier of the low-income subgroup represents the first group that receives neither minimal wages in AUT and the USA. Furthermore, are these decisions influenced by the progressiveness of the present income taxes.

As the research questions in section 1.2 requires an answer on the welfare effects of households, a qualitative summary of the findings with respect to the welfare during the transitional dynamics using different types of pandemic policies, based on groups of different income levels in the AUT and USA model economy is present.

Considering **AUT/USA** pandemic policies in comparison to the absence of one, an overall preference of a lockdown policy is present. With respect to its levels, a quite dense distribution of the consumption equivalent variation, overlapping all types of income, is present. One exception are parts of middle and highest levels of low income households, preferring lockdown policy even more than the quite lumped distribution of diverse income levels. In general, the distribution of the consumption equivalent variation is spread more apart in the AUT model economy as compared to the US model economy. Its levels are increased as well, by one order of magnitude (4% vs. 20%). The considered figures are 4.17 and 4.16.

policy	all income	low income	middle income	high income
AUT[AUT] vs. AUT[USA]	−0.191	−0.191	−0.194	−0.189
AUT[AUT] vs. no policy	−0.358	−0.360	−0.359	−0.354
AUT[USA] vs. no policy	−0.206	−0.210	−0.206	−0.203
USA[USA] vs. USA[AUT]	+0.039	+0.039	+0.039	+0.038
USA[USA] vs. no policy	−0.041	−0.042	−0.041	−0.041
USA[AUT] vs. no policy	−0.077	−0.078	−0.076	−0.076

Table 4.1: Mean values of consumption equivalent variation g based on grouped distributions of working income

With respect to the **USA pandemic policy** applied in the AUT and USA model economy, an overall preference of the AUT policy is present. With respect to its levels, a dense yet ordered distribution of the consumption equivalent variation. Ordered with respect to, high income households are willing to give up less consumption than middle income households and parts of low income households. An exception are the lowest levels of low income households, which seem to be the least indifferent to the policy change, possibly due to the presence of minimal wages in both economies and no desire to engage in working activities, with leisure close to 100%, and little desire to consume. A very similar behaviour is present applying the **AUT pandemic policy** to both model economies. Apart from no present ordering, where parts of middle income households prefer the AUT policy the most. The considered figures are 4.15.

To conclude the chapter 4 the present *mean*, *min* and *max* values of the distribution of g with respect to all policies (no, AUT, USA) are present in table 4.1.

CHAPTER 5

Conclusion and Remarks

One of the insight this thesis hopes to convey is, that different income groups of households behave in different manners during the pandemic shock, due to different levels of savings, involvement in the labour market, and desire to consume goods. Regarding **high income households** and the **absence of a pandemic policy**, labour supply drops abruptly and rises again during the course of the pandemic to again reach steady state levels weeks before the end of the pandemic, due to vaccination. A preference to work less and consume more can be observed for high income households. The resulting increase in consumption is financed using the household's savings and therefore reducing the model economies capital stock. Weeks before the end of the pandemic, capital stock increases again to reach pre-pandemic steady state levels. During the **presence of pandemic policies**, a similar behaviour of households is present, yet steady state deviations of labour supply are dampened, possibly due to a reduction of rates of infection. Consumption during lockdowns is reduced due to mandatory reduction in consumption and therefore increase in savings. This leads to above steady state levels of consumption after lockdown periods and below levels during lockdowns. In general, the AUT policies lead to a higher increase in volatility than the USA policies in both model economies with respect to high income households.

Considering **low income households** and the **absence of a pandemic policy**, labour supply increases slightly and drops weeks before the pandemic ends in the USA model economy. In the AUT model economy, no trend is present, possibly due to higher minimum income levels. Consumption of low income households is slightly below steady state and capital supply is increased slightly, to cope with the increase in uncertainty due to the pandemic shock. An inverse behaviour to high income households is visible, possibly due to the ability of high income households to make use of the accumulated assets to smooth utility governing factors as consumption and leisure. During the **presence of pandemic policies**, a similar behaviour of households is present, yet the level of capital supply of low income households is increased, labour supply during lockdowns is increased with respect to the steady state and consumption is in general above the steady state level, even during lockdown periods.

Another insight this thesis hopes to convey is, that even rational households suffer from similar conditions as real households do. A main result of this thesis can be described, as the consumption equivalent variation measured welfare consequences g, and the resulting preference of temporary, strict lockdowns in comparison to less temporary and strict lockdown policies. With respect to **welfare**, in general, households of both model economies (USA and AUT) prefer lockdown policies over no-lockdown policies. Households of the AUT model economy prefer policies compared to no-lockdown policies in a greater extent than households of the US model economy. Comparing *temporary* and *strict lockdown* policies, abbreviated in this thesis as the *AUT policy*, to *less temporary* and *less strict* policies, abbreviated in this thesis as the *US policy*, with each other, agents of both model economies prefer the AUT policy approach.

During this thesis, lockdown policies are clearly separated by the distribution of g, meaning all values of g are above or below zero. This allows one to state that households as an aggregate prefer the AUT policy over the US policy, during all conducted policy experiments, independent of the nation state economic model. Yet, in case of subgroups (e.g., low income households) being in favour and the rest of the distribution of households being not in favour of a certain policy, the generated insight can be used to target e.g., specific fiscal policies to make such a distribution clearly separable again.

Yet, it seems necessary, to state that these are model results and applicability, robustness and a more rigorous model validation is necessary and of utmost importance, as all modelling exercises do. Model results should always be interpreted as such. Therefore, a deep a thorough understanding is necessary before jumping to conclusions.

With respect to further improvements or model enhancements, several improvements are available. In case of more detailed analysis of household subgroups, an OLG structure may be included, as well as different working and consuming sectors. The latter enables the identification of areas with a particular need for policy action. In addition, modelling an individual impact of the leisure and consumption decisions on the risk of becoming infected as a household. In the current version, only the aggregate consumption and leisure determines the spread of the infection. This model does not include the health sector as an agent and its dependence tax payments, as well as health consequences of the agents. It's a life and death, black and white distinction, as no information about aftermaths of an infection are included in the state space of households. Yet this seems to be a very detailed and hard to calibrated endeavour.

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Appendix

Additional Information about used Literature

The Macroeconomics of Epidemics

This paper [Eichenbaum et al., 2020] is one of the first papers to link macroeconomic models with epidemiological models. The core of this paper seems to be a basis for many other papers, including [Krueger et al., 2020] as well as [Hur, 2021]. The paper in question tries to elaborate on how economic agents decision-making is influenced by the pandemic as well as the reverse relationship, how is the decision-making of agents affecting the pandemic.

The starting point of [Eichenbaum et al., 2020] is a three sector economy using households, firms, and governments as agents. **Households** solve an intertemporal decision problem with respect to its consumption decisions. Therefore let

$$U = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \tag{1}$$

be the lifetime utility of a household, with period utility defined by

$$u(c, n) = ln(c) - \theta \frac{n^2}{2} \tag{2}$$

the previous equation. The budget constraint of the **government** is balanced and used as a lump sump transfers to all agents. Below, the budget constraint for households is present with $\phi^{\{s,i,r\}}, \phi^{\{s,r\}} = 1, \phi^i < 1$ as being a productivity factor with respect to the states of infection during the pandemic. They equal to one in its pre-pandemic scenarios, as well as in states of recovery and susceptibility.

$$\begin{aligned} \mu_t c_t &= \Gamma_t \\ (1+\mu_t) c_t &= w_t n_t \phi^{\{s,i,r\}} + \Gamma_t \end{aligned} \tag{3}$$

The idea of μ_t is to be a proxy for containment measures defined exogenously, which will be termed *containment rate* from now on.

Using the lifetime utility in equation (1) as well as the budget constraint (3) one can derive the first order condition of the representative household using a constrained optimization approach (Lagrangian multipliers).

$$(1 + \mu_t)\theta_t = c_t^{-1} w_t \phi^{\{s,i,r\}} \tag{4}$$

Firms in this model do possess a notion of time invariant productivity A as well as the factor input labour N. They produce according to the needs of the households as price-takers. The chosen production function is assumed to be linear in labour $F(N) = AN$ as well as wages being constant in time $w = 1$ leading to time independent prices $\Pi(N) = pF(N) - wN \implies \frac{\partial \Pi(N)}{\partial N} = pA - w \stackrel{!}{=} 0 \implies p = \frac{1}{A}$ as no supply side effects of wages are modelled. This results in the price of the product being solely determined by the firm's productivity.

Since the research question of [Eichenbaum et al., 2020] focuses on how people change their consumption preferences based on its infection probability, the pandemic needs to be incorporated within the described model above. Given an **epidemiological model**, a discretized SIR model where S represents the susceptible, I the infected, R the recovered, D the deceased part of households as well as π_r and π_d represent the exogenous probability to recover, and to fall victim to the disease.

$$\begin{aligned} T_t &= \tau_t S_t \\ S_{t+1} &= S_t - T_t \\ I_{t+1} &= I_t + T_t - (\pi_r + \pi_d) I_t \\ R_{t+1} &= R_t + \pi_r I_t \\ D_{t+1} &= D_t + \pi_d I_t \\ Pop_{t+1} &= Pop_t - D_t \end{aligned} \tag{5}$$

The linkage between the economic model and the epidemiological model is the endogenous infection probability affected by consumption and labour decisions of the households.

$$\tau_t(c, n) = \pi_1 c_t^s (I_t C_t^I) + \pi_2 n_t^s (I_t N_t^I) + \pi_3 I_t \tag{6}$$

The exogenous probabilities $\pi_i, i \in \{1, 2, 3\}$ refer to the possibilities of how a transmission of the disease may transpire. The first term of equation (6) refers to contamination at consumption activities (Shopping, Bars, etc.), the second one refers to working related infections and the last one as a non-consumption or working related infections.

In **equilibrium,** each agent solves their maximization problem given their budget constraints. Labour and goods markets are assumed to clear. Since consumption and working hours are now indexed by the household's state of infection, one can rewrite the above equations using the following market clearing conditions.

$$An_t = c_t = S_t c_t^s + I_t c_t^i + R_t c_t^r$$
$$n = S_t n_t^s + I_t n_t^i + R_t n_t^r \tag{7}$$

To link the decision problem with the epidemiological dynamics, [Eichenbaum et al., 2020] defines the lifetime utility of different states of the in the following way

$$U_t^R = u(c_t^r, n_t^r) + \beta U_t^R \tag{8}$$
$$U_t^I = u(c_t^i, n_t^i) + \beta[(1 - \pi_r \pi_d) \cdot U_{t+1}^I + \pi_r \cdot U_{t+1}^R + \pi_d \times \mathbf{0}] \tag{9}$$
$$U_t^S = u(c_t^s, n_t^s) + \beta[(1 - \tau_t) U_{t+1}^S + \tau_t U_{t+1}^I] \tag{10}$$

allowing one to derive its first order condition in the usual manner (Lagrangian Multipliers). It is worth noting that a "sequential" definition of equation (8 - 10) does not allow the same households to be affected by multiple infections. "Sequential" in the sense of one can solve for the recurrence relations of U_t^R and afterwards use this result to proceed solving U_t^I. Allowing for multiple infections to occur seems to complicate the solution strategy of the present model in a significant way (finding fixed points of the interactions between $(U_t^i)_{t \leq T}, i \in \{R, I, S\}$) and is expected not to yield substantial insights in the decision-making of households.

Since the system is not completely described by the derived equations 4, 5, 6, 8 and 7 one can use a "guess and check" strategy to solve for the needed dynamics of the system. This is possible since initial conditions of the dynamics are defined in the sense that the state of start and end of the pandemic is considered to be the same. Therefore, by guessing the sequence $\{n_t^s, n_t^i, n_t^r\}_{t=0}^{T-1}$ for a defined period T one can solve for all states of the system, obtaining a solution of the macroeconomic SIR model (equations which define a sequence of $\{c_t, n_t, \mu_t, S_t, I_t, R_t\}$ such that at all t the equilibrium condition hold). Since such an approach is used by this thesis and explained in more detail in chapter 3 the precise algorithm is omitted in this chapter (detailed description can be found in the appendix of [Eichenbaum et al., 2020]).

[Eichenbaum et al., 2020] further expands on topics as *vaccination, medical preparedness, containment, and treatment* are not further explored within this literature review. These represent slight adjustments of the model described above, as well as an introduction of costs with respect to the healthcare system.

The **Robustness** of the model in [Eichenbaum et al., 2020] is explored by variation of parameters of interest. These include the discount factor of households $\beta \in \{0.94^{\frac{1}{52}}, \mathbf{0.96^{\frac{1}{52}}}\}$, mortality rate $\pi_d \in \{\mathbf{0.005 \times \frac{7}{18}}, 0.01 \times \frac{7}{18}\}$, productivity of infected households $\phi^i \in \{0.7, \mathbf{0.8}\}$, as well as initial amount of infected households. These parameter combinations yield different model results with respect to the severity of the pandemic and are judged on a qualitative basis. The bold parameters are considered as the chosen ones for the baseline macro SIR model. Based on these parameter scenarios and adjustments with respect to vaccination, medical preparedness, containment, and treatment of households [Eichenbaum et al., 2020] shows that

1. the basic SIR model ($\tau_t(I_t)$ only dependent on number of infected people, re-calibrated such that 40% of households are infected) is strictly more severe than the macro SIR model ($\tau_t(c_t, n_t, I_t)$, dependent on consumption and hours worked decision),

2. the deviation of consumption as well as aggregate hours worked form pre-pandemic steady state of the macro SIR model is *reduced* by 10% in contrast to the basic SIR model 2.8%, yet also *reducing* mortality by 5%.

A general comparison of the model type, solution strategies and questions answered by the paper is displayed in table 2.2.

Macroeconomic Dynamics and Reallocation in an Epidemic

This paper [Krueger et al., 2020] tries to answer if a "Swedish" solution of the COVID-19 pandemic is feasible with respect to economic reasoning. "Swedish" is to be interpreted as a "no-lockdown" strategy. The resulting endogenous shift of consumption to different, more safe consumption possibilities will absorb the shock if such goods are available in the economy. The model used by [Krueger et al., 2020] is based and extended on the model of [Eichenbaum et al., 2020] which is described in section 5 of chapter 5 in the appendix.

The starting point of [Krueger et al., 2020] is a two sector economy using households and firms as agents. Removing the balanced government constraint from the model in comparison to [Eichenbaum et al., 2020]. As in [Eichenbaum et al., 2020] households solve an intertemporal decision problem with respect to its consumption decisions. These decisions require taking into account the form of a continuum of consumption goods with different degrees of infection ϕ. Therefore let

$$U = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^j, n_t^j) \tag{11}$$

be the lifetime utility of a continuum of **household** $j \in [0, 1]$ with period utility defined as seen in equation 2 and choosing from a bundle across a continuum $k \in [0, 1]$ of sectors to consume from

$$c_t^j = \left[\int (c_{tk}^j)^{1-\frac{1}{\eta}} dk \right]^{\frac{\eta}{\eta-1}} \tag{12}$$

at time t. The elasticity of substitution across consumption sectors is denoted $\eta \geq 0$.

Since no income shocks or savings decision are modelled, the resulting budget constraint of a household j

$$BC \equiv \int c^j_{tk} dk = An^j_t \tag{13}$$

is different to [Eichenbaum et al., 2020] with respect to the missing lump sump transfers γ_t as well as consumption taxation/containment rate μ_t as seen in equation (3). As [Eichenbaum et al., 2020] is not focusing on containment strategies or policy interventions, these parts of the model of [Eichenbaum et al., 2020] seem only to complicate the model and are therefore removed.

Firms in [Krueger et al., 2020] do not differ in any aspect from [Eichenbaum et al., 2020]. In general, firms are of minor concern for the paper discussed, since they do not offer significant insight in the household's decision-making.

The **dynamics of the pandemic** are similar to equation (5) of ([Eichenbaum et al., 2020]) apart from the functional form of the endogenous transition probability of the disease τ_t.

$$\tau_t(c) = \pi_s I_t \int \phi(k) c^s_{tk} c^i_{tk} dk + \pi_a I_t \tag{14}$$

Note that in comparison to [Eichenbaum et al., 2020] leisure/hours worked does not affect the endogenous infection probability $\tau(c)$ is only a function depending on aggregate consumption across different sectors as well as individuals $c^j_t k, j \in [0,1], 0 \leq s < i \leq 1$. As seen in later sections of [Krueger et al., 2020] an exchange of sectorial consumption c_{tk} with sectorial hours worked n_{tk} is implemented and discussed. In this literature review, only the sectorial consumption as a basis for infection is considered.

In **equilibrium,** each agent solves their maximization problem given their budget constraints. Labour and goods markets are assumed to clear. The market clearing conditions are defined as follows and are further simplified using the equations derived from the first order conditions with respect to equation 8 and 9 representing recovered and infected optimality conditions.

$$\begin{aligned} S_t c^s_{tk} + (I_t + R_t) c^i_{tk} = An_{nk} &\implies S_t c^s_{tk} + (I_t + R_t)\frac{A}{\sqrt{\theta}} = An_{nk} \\ \int n_{tk} dk = S_t c^s_t + (I_t + R_t) n^i_t &\implies \int n_{tk} dk = S_t c^s_t + (I_t + R_t)\frac{1}{\sqrt{\theta}} \end{aligned} \tag{15}$$

The authors of [Krueger et al., 2020] argue, that given their assumptions the optimal decision for infected as well as recovered households are equal. So recovered and infected household's decision-making is not affected by the infection probability τ_t (infection has no effect on income or hours worked). Therefore, it is reasonable to assume $c^{*r}_t = c^{*i}_t$, $n^{*r}_t = n^{*i}_t$, $\lambda^r_{bt} = \lambda^i_{bt}$. As $u_{c_t}(c^i_t, n^i_t)\left(\frac{c^i_t}{c^i_{tk}}\right)^{\frac{1}{\eta}} = \lambda^i_{bt}$ represents the first order condition of infected households with respect to consumption as well as labour, the functional form is the usual Dixit-Stiglitz constant elasticity of substitution (CES), first order condition at constant prices with solution to the optimal decision problem $c^i_{tk} \equiv c^i_t$. The specific form of the utility function in equation (2) yields $\frac{1}{c^i_t} = \lambda^i_{bt}$, $\theta n^i_t = A\lambda^i_{bt} = \frac{A}{c^i_i}$ (λ^i_{bt} being the Lagrange multiplier with respect to of infected households of the budged constraint) and using the budget constraint (13) yields $n^i_t = \frac{1}{\sqrt{\theta}}$, $c^i_t = \frac{A}{\sqrt{\theta}}$. The derived policy functions n^{ri}_t, c^{ri}_t represent now a time independent solution to the decision problem of the infected and recovered state.

Combining equations (14), (15) as well as deriving its first order conditions with respect to the susceptible population (10) and further apply the equilibrium conditions (15) one can derive the following the policy rules with respect to n^s_t, c^s_t. Using a backward induction based approach to solve for U^s_t given a fixed value for U^s_{t+1}, allows the final set of equations to determine all necessary variables $(U^s_t, \tau_t, c^s_t, n^s_t, \lambda^s_{bt})_{t\leq T}$ as defined in equation (16). Combined with the time independent solutions of the recovered and infected state, this yields the "almost" solution of the macro SIR model using a competitive equilibrium, as shown in [Krueger et al., 2020]. "Almost" in a sense such that the present integral equations $\tau_t(U^s_{t+1})$ and $c^s_t(U^s_{t+1})$ requires solving. To do so the authors of [Krueger et al., 2020] make use of numerical solution strategies using the software platform *Dynare* which is a common tool for handling a wide class of economic models.

$$
\begin{aligned}
\left[n^s_t = \frac{\int c^s_{tk} dk}{A};\ \theta n^s_t = -A\lambda^s_{bt}\right] &\implies \lambda^s_{bt} = \frac{-\theta \int c^s_{tk} dk}{A^2} \\
c^s_t &= \left[\int \left(\lambda^s_{bt} + \pi_s I_t \beta(U^s_{t+1} - U^i_{t+1})\phi(k)\frac{A}{\sqrt{\phi}}\right)^{1-\eta} dk\right]^{\frac{1}{1-\eta}} \\
\tau_t = \pi_s I_t \int \phi(k) &\left(\lambda^s_{bt} + \pi_s I_t \beta(U^s_{t+1} - U^i_{t+1})\phi(k)\frac{A}{\sqrt{\phi}}\right)^{-\eta} (c^s_t)^{1-\eta}\frac{A}{\sqrt{\phi}} dk \\
U^S_t &= u(c^s_t, n^s_t) + \beta[(1-\tau_t)U^S_{t+1} + \tau_t U^I_{t+1}]
\end{aligned}
\tag{16}
$$

The authors of [Krueger et al., 2020] furthermore use the notion of a **social planner** who directs the consumption decisions such that infection is minimized given the complete epidemiological knowledge at any point in time of the system. Therefore, yielding an optimal consumption plan as a comparison for the competitive equilibrium defined above. As the focus of this thesis is not the notion of an optimal response to the pandemic, the notion of the social planner is not in the scope of this literature review.

However, what is of interest is the notion of a **value of a statistical life (VSL)** with respect to robustness and calibration. VSL defines a premium an agent accepts in case of an increase in an overall mortality rate. Its usage in economic models is to derive an equation encapsulating the idea of the VSL (e.g., marginal rate of substitution between survival and consumption) and furthermore to assert its non negativity with respect to its parameters or variables. Otherwise, an argument for households to become suicidal can be made. Households in [Eichenbaum et al., 2020] as well as [Krueger et al., 2020] are defined as immortal except of the pandemic. Yet, by introducing a small exposure of not surviving the next period, one can derive equation (/17). Using the recovered consumption policy $c^r = \frac{A}{\sqrt{\theta}}$ as well as the recovered hours worked policy $n_t^r = \frac{1}{\sqrt{\theta}}$ one can show the period utility equals $u_t^r = log(\frac{A}{\sqrt{\theta}}) - \frac{1}{2}$ and using the limit $t \longrightarrow \infty$ of this geometric series the lifetime utility equals $U^r = \frac{u^r}{1-\beta} u^r$. Combining the previous equations, the lifetime utility of a recovered agent is described by $U^r = u^r + \beta U^r$. In the risky scenario, one allows for an increase in consumption (compensation γ) to balance a present rate of mortality $\delta > 0$. This allows to define the lifetime utility in a risky scenario $U^r_{\delta,\gamma} = \gamma log(\frac{A}{\sqrt{\theta}}) - \frac{1}{2} + (1-\delta)\beta U^r_{\delta,\gamma}$. As seen in [Krueger et al., 2020] by equating the lifetime utility of both scenarios and defining the VSL to be the ratio $:= \frac{\gamma}{\delta}$, we can solve for the two unknowns γ and $delta$ and therefore define the value of a statistical life to be equation (17)

$$VSL := \frac{\beta}{1-\beta} \left[log(\frac{A}{\sqrt{\theta}} - \frac{1}{2}) \right] \tag{17}$$

Equation (17) allows for a **robustness** exercise with respect to estimated VSL of real life economies, as well as explore parameter settings where the VSL turns out to be negative. As [Krueger et al., 2020] define this to be the case if $\frac{A}{\sqrt{\theta}} < e^{0.5}$, therefore, avoiding such a parameter setting. In addition, the VSL equation (17) may be used to calculate the prevention value of a statistical life, being equal to $VSL_{unitc} = 8298$, which expresses a percentage change in weekly consumption to balance for a percent change in mortality. As [Krueger et al., 2020] point out, this value for about 500 USD of weekly consumption of Sweden yields an VSL_{real} of $500 \times 8298 \approx 4E6$ USD. In [Hur, 2021] a recommendation of the U.S. Environmental Protection Agency of $7E6$ USD is used. This yields the double of the VSL as compared to [Krueger et al., 2020], despite them stating that the VSL seem to be on the high end. In general, further estimates as seen in [Hur, 2021] and [Krueger et al., 2020] seem to indicate the estimation of the idea of an VSL seem to be very dependent on the referred study.

To showcase a glimpse of the **results** presented in [Krueger et al., 2020] a baseline “two sector model” and a “no sector model” $\phi = 1$ using the notion of a competitive equilibrium is shown in the following enumeration. The Model parameters in use are similar to [Eichenbaum et al., 2020] as seen in section 5. A comparison of the model type, solution strategies and questions answered by the paper is displayed in table 2.2 in chapter 2.

1. The $\phi = 1$ model yields a stock of infected households of 25% at peak. In comparison, the baseline model yields at peak a stock of infected households of 9% at peak, yet implies a more right skewed stock of infected households.

2. The $\phi = 1$ model yields the worst case drop in aggregate consumption to 73% of the steady state stock. In comparison, the baseline model yields the worst case

drop in aggregate consumption to 85%, yet prolongs the shock due to the right skewed stock of infected households.

3. At peak, the low-infection sector consumption is increased by 60% of the steady state stock, as compared to a drop of 78% in the high-infection sector of the steady state stock.

The Distributional Effects of COVID-19 and Optimal Mitigation Policies

The paper [Hur, 2021] discussed in this section differs in style and modelling techniques from [Eichenbaum et al., 2020] and [Krueger et al., 2020]. Its core is a general equilibrium model using the notion of incomplete markets such as in [Aiyagari, 1994] as well as a OLG structure, although during the pandemic a partial equilibrium is present, since factor prices are not adapted accordingly. Incomplete, meaning the present agents within the model do not have access to an "insurance policy" allowing them to completely avoid future undesirable states (individual income $\mathbb{P}[y = 0] > 0$ suffers from uncertainty). In addition, it allows for a more expressive (yet complicated) economic model since expressions as *heterogeneous income*, *stochastic ageing of households*, ***no*** *closed-form solutions of policy functions*, *aggregate capital/labour* derived from its *emerging distribution*, *market clearing factor prices r and w* as well as a notion of *working-* as well as *capital income* all combined within a *recursive stochastic general equilibrium model*.

The main findings discussed in [Hur, 2021] are that externalities give rise to welfare-improving government intervention. Young and low income households do possess different optimally conditions with respect to consumption and leisure as opposed to older and rich households, affecting the dynamics of the pandemic. Since young people seem to be more resilient towards the pandemic than older people, as well as low income and wealth households are required to work to survive in comparison to rich and wealthy households (especially in an incomplete market setting), economic activities tend to be not socially optimal with respect to the spread of the pandemic. This finding is of most interest for the present thesis since it is directly related to the research questions presented in 1.2. In addition, an optimal policy frontier with respect to weekly subsidies for households is explored, resulting in a simultaneous improvement of health and economic outcomes.

The model defined in [Hur, 2021] uses heterogeneous **households** within an incomplete market setting, as well as the notion of a stochastically ageing overlapping generations. Given the period utility with respect to consumption

$$u(c_i, c_o) = \frac{(c_i^{\gamma} c_o^{1-\gamma})^{1-\sigma}}{1-\sigma} \tag{18}$$

and dis-utility of labour

$$g(l) = \phi \frac{l^{1-\nu}}{1+\nu} + \mathbf{1}_{l=0}\tilde{u} \tag{19}$$

where γ represents the assumed to be constant elasticity of inside consumption c_i as well as outside consumption c_o, and σ (ν) represents the coefficient of relative risk aversion with respect to consumption (labour). In general, the usage of *constant relative risk aversion* using a *Cobb-Douglas aggregation* is present throughout all presented papers. A special case of equation (19) is its usage of an indicator function in case $l = 0$, resulting in a less than zero utility (based on the dis-utility of not working $\tilde{u}$). This allows for more flexibility during calibration of the model.

The chosen approach of **modelling working income** $w_t \varepsilon_t \eta_{jk} l_t$ is the usage of *labour efficiency units* η_{jk} based on age j and state of infection $k \in \{S, I, R\}$, as well as *idiosyncratic labour efficiency shocks* ε_t modelled as an AR(1) process $ln(\varepsilon_t) = \rho\, ln(\varepsilon_{t-1}) + \zeta_t,\ \zeta_t \sim N(0, \sigma^2))$ based on panel surveys of income data. Since an AR(1) process allows for an infinite range of values (and therefore an infinite state space) a discretization of the estimated AR(1) process using Tauchen's Method [Tauchen, 1986] is used to define a Markov Chain Γ which is used as a transition matrix in equation 20 and 20. These *labour efficiency units* themselves are used to impact households during the period of *infection,* reducing the household's working income $w_t \varepsilon_t \eta_{jk} l_t$.

Given the **OLG** structure, households age according to, $j \in J \equiv \{1, 2, .., \bar{J}\}$ including a probability of ageing $\{\psi_j\}$ with mandatory retirement age $j = J_R$. Using stochastic ageing as described in [Blanchard, 1985] allows reducing the age component of the state space of the model, since two states (working, retired) are now present instead of $\bar{J}$.

This is combined with an **epidemiological model** inspired by [Eichenbaum et al., 2020]. The households face as usual an intertemporal optimization problem given a budget constraint. Since value functions as in [Bellman, 1952] and its related solution strategies are used in this thesis as well as in [Hur, 2021], the household's optimization problem is presented in such a form. In the following, a classification of *control* as well as *state* variables is suitable since the problem to be solved is similar to optimal control problems. Therefore, let the recursive optimization problem of the households at *retiring* age be defined as

$$
\begin{aligned}
v^R_{jt}(k,h) = \max_{c_i,c_o,k'\geq 0} & u(c_i,c_o) + \bar{u} + \hat{u}^h \\
& + \beta\psi_j \sum_{h'\in\{S,I,R\}} \Pi_{jhh't}(c_o,0) v^R_{j+1,t+1}(k',h') \\
& + \beta(1-\psi_j) \sum_{h'\in\{S,I,R\}} \Pi_{jhh't}(c_o,0) v^R_{j,t+1}(k',h') \\
& \text{s.t. } (1+\tau_{ct})c + k' \leq s + k(1+r_t)
\end{aligned} \tag{20}
$$

as well as the recursive optimization problem of the households at *working* age be defined as,

$$
\begin{aligned}
v_{jt}^{W}(k,\varepsilon,h) = \max_{c_i,c_o,l_i,l_o,k'\geq 0} & u(c_i,c_o) - g(l) + \bar{u} + \hat{u}^h \\
& + \beta\psi_j \sum_{\varepsilon'\in E} \sum_{h'\in\{S,I,R\}} \Gamma_{\varepsilon\varepsilon'} \Pi_{jhh't}(c_o,l_o) v_{j+1,t+1}(k',\varepsilon',h') \\
& + \beta(1-\psi_j) \sum_{\varepsilon'\in E} \sum_{h'\in\{S,I,R\}} \Gamma_{\varepsilon\varepsilon'} \Pi_{jhh't}(c_o,l_o) v_{j,t+1}(k',h') \\
& \text{s.t. } (1+\tau_{ct})c + k' \leq w_t \eta_j^h (1-\tau_{lt})\varepsilon l + k(1+r_t) + T_t(l) \\
& l_i \leq \bar{\theta}_j(\varepsilon), l_0 \leq \bar{l}_{0t}
\end{aligned} \tag{21}
$$

given the dynamics of the pandemic,

$$
\pi_{It}(c_o,l_0;Z_t) = \beta_0 c_o C_{It}^0 + \beta_l l_0 L_{It}^0 + (\beta_e + \eta_t)\mu_{It}
$$
$$
Z_t \equiv \{\mu_{It}, C_{It}^0, L_{It}^0, \eta_t\}
$$
$$
\Pi_{jhh't}(c_o,l_o) = \begin{bmatrix} 1-\pi_{It}(c_o,l_o,;Z_t)-\pi_{jRt} & \pi_{It}(c_o,l_o,;Z_t) & \pi_{jRt} & 0 \\ 0 & 1-\pi_{Xt} & \pi_{Xt}(1-\delta_{jt}(Z_t)) & \pi_{X_t}\delta_{jt}(Z_t) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}
$$

Since equation (21), (20) and (22) contain numerous symbols for variables and constants, the following table 2 tries to elaborate on the used notation. In general, equation 20 defines the recursive problem of the households of a certain age $j \geq J_R$ and equation 21 of working people $j < J_R$. Equation 22 defines the dynamics of the pandemic as a stochastic matrix (Markov Chain), defining transitions between $\Pi \subseteq \{S,I,R,D\} \times \{S,I,R,D\}$. Note that a symbol, e.g., k with an attached prime k' at time t, defines its successor state at time $t+1$. Given the symbol k_{tij}^h given indices for age j, time t, health h and individual, i it represents simultaneously a value of a certain stock (e.g., assets) as well as its policy function $k'(k,\varepsilon,h)$. The latter describes a functional relationship between state variables ("where am I given my environment?" e.g., assets k, efficiency shocks ε, health h) and control variables ("how can I influence my state in the environment?" e.g., net-savings $k'-k$, hours worked l, consumption c). In the context of the equations given in section 5 the "prime" variables reflect policy functions in the context of dynamic programming/optimal control theory. Note that given the properties (functional, stochastic, recursive) of the optimization problem, closed form solutions of policy (value) functions are most likely not achievable. Therefore, the equations (20), (21), and definition (5) are to be interpreted as *imperative* statements defining properties of the model. Solutions methods of such models are discussed in chapter 3 in more detail.

Firms in [Hur, 2021] are defined as in [Eichenbaum et al., 2020] in a static, representative, price taking manner using a Cobb-Douglas production function resulting in,

Model Parameters	
Parameters	[Hur, 2021]
$\bar{u}$	estimated flow value of life based on equation 29
$\hat{u}^h$	estimated utility based on health consequences of an COVID-19 infection
$\Gamma\Pi v_j$	expected value of utility of households at age j given fixed point $V = \Gamma\Pi V$
C	aggregate consumption
L	aggregate labour
c_i	inside individual consumption based on stay at home order
c_o	outside individual consumption
l_i	inside labour based on stay at home order
l_o	outside labour
v^R	value function based on retired state
v	value function based on working state
Π	transition probabilities of SIR states (Markov Chain)
ψ	probability of ageing in stochastic ageing setting
β	discount factor as in 20 and 21 or factor controlling infection as in 22
τ_{ct}	tax or subsidies on consumption
τ_{lt}	tax or subsidies on labour
r_t	real interest rate
k	savings policy function
s	pension income
T	subsidies received based on stay at home order

Table 1: Symbols linked to its variables, providing a shortened explanation of [Hur, 2021]

$$\begin{aligned} F(K,N) &= K^\alpha N^{1-\alpha} \\ \underset{K_t,N_t}{max}&[F(K_t,N_t) - wN_t - (r+\delta)K_t] \end{aligned} \tag{23}$$

where δ represents the rate of depreciation of capital. Deriving its optimality condition yield the following expressions

$$\begin{aligned} w_t &= (1-\alpha)K^\alpha N^{-\alpha} \\ r_t &= \alpha K^{\alpha-1} N^{1-\alpha} - \delta \end{aligned} \tag{24}$$

Although the main object of study are the *transitional dynamics* between the *steady states,* one needs to define how to arrive at such *steady state.* Therefore, the notion of a *stationary recursive competitive equilibrium* is defined as follows [Hur, 2021].

> A stationary recursive competitive equilibrium, given fiscal policies $\{\tau_l, s\}$, is a set of value functions *val*, policy functions *pol*, prices *price* producer plans *plan*, the distribution of newborns ω, and invariant measures $\{\mu_j\}_j$ such that:

1. Given prices and fiscal policies, retirees, and workers solve 20 and 21, respectively.
2. Given prices, firms solve (24).
3. Markets clear:
    a) $Y = \int_X \sum_{j \in J} [c_{ij}(k, \varepsilon, h) + c_{oj}(k, \varepsilon, h) + \delta k] \, d\mu_j(k, \varepsilon, h)$
    b) $L = \int_X \sum_{j<J_r} \eta_{jh} \varepsilon [l_{ij}(k, \varepsilon, h) + l_{oj}(k, \varepsilon, h)] \, d\mu_j(k, \varepsilon, h)$
    c) $K = \int_X \sum_{j \in J} k \, d\mu_j(k, \varepsilon, h)$
4. The government budget constraint holds:

$$\tau_l w \int_X \sum_{j<J_R} \eta_{jh} \varepsilon [l_{ij}(k, \varepsilon, h) + l_{oj}(k, \varepsilon, h)] \, d\mu_j(k, \varepsilon, h) = s \int_X \sum_{j \geq J_R} d\mu_j(k, \varepsilon, h) \quad (25)$$

5. for any subset $(\mathcal{K}, \mathcal{E}, \mathcal{H}) \in \mathcal{B}$, the invariant measure μ_j satisfies, for $j > 1$,

$$\begin{aligned} \mu_j(\mathcal{K}, \mathcal{E}, \mathcal{H}) = & \int_X \psi_{j-1} 1_{\{k'_{j-1}(k\varepsilon, h) \in \mathcal{K}\}} \sum_{\varepsilon' \in \mathcal{E}} \sum_{' \in \mathcal{H}} \Gamma_{\varepsilon\varepsilon'} \Pi_{hh'} \, d\mu_{j-1}(k, \varepsilon, h) \\ & + \int_X (1 - \psi_j) 1_{\{k'_j(k\varepsilon, h) \in \mathcal{K}\}} \sum_{\varepsilon' \in \mathcal{E}} \sum_{' \in \mathcal{H}} \Gamma_{\varepsilon\varepsilon'} \Pi_{hh'} \, d\mu_j(k, \varepsilon, h) \end{aligned} \quad (26)$$

and

$$\mu_1(\mathcal{K}, \mathcal{E}, \mathcal{H}) = \int_X (1 - \psi_1) 1_{\{k'_1(k\varepsilon, h) \in \mathcal{K}\}} \sum_{\varepsilon' \in \mathcal{E}} \sum_{' \in \mathcal{H}} \Gamma_{\varepsilon\varepsilon'} \Pi_{hh'} \, d\mu_1(k, \varepsilon, h) + \omega(K, \mathcal{E}, \mathcal{H}) \quad (27)$$

6. The newborn distribution satisfies:

$$\int_X k \, d\omega(k, \varepsilon, h) = \int_X \psi_{\bar{J}} k'_{\bar{J}}(k, \varepsilon, h) \, d\mu_{\bar{J}}(k, \varepsilon, h) \quad (28)$$

In the defined recursive equilibrium as seen in definition (5) a notion of necessary condition to reach a steady state (invariant measure), and therefore time independence is defined. Since aggregation is based on idiosyncratic elements (Γ, Π), the stochastic components result in a distribution of individuals in the state space, therefore different applied policy function, and therefore a distribution of control as well as state variables. The use of an invariant measure μ based on a probability space $(\mathcal{K}, \mathcal{E}, \mathcal{H})$ is in a Lebesgue style integral results in first moments of the underlying distribution. An approximation of the resulting integrals and possible methods to do so are discussed in chapter (3). This is a key difference to the previous equilibrium conditions as seen in [Krueger et al., 2020] and [Eichenbaum et al., 2020].

The household's maximization problem as defined in (21) given the recursive equilibrium conditions in definition (5) seem to require an adaptation to the **solution strategies** employed. Since a fluctuation in income alone allows for models which seem hard to solve in an analytical way, now numeric and exact heuristic methods are employed to yield more promising results. These do not completely replace first order conditions but include them (closed form link between c and l) to find solutions faster. The definition in a recursive equilibrium as well as in value functions hints at the usage of dynamic programming as defined in [Bellman, 1952]. The exact methods used are not all discussed in [Hur, 2021]. In principle, after finding a value function for the household's problem for each state in the state space, the general equilibrium assumption need to be satisfied (prices w, r such that markets clear). Since in most cases one has more variables than equations defining them, guessing methods are used such that criteria in definition 5 are fulfilled. In short, given initial distributions $\mu(k', \varepsilon, h)$ guess r, update demand of Firms and calculate w, find fixed point of value functions v, v^R, use definition (5) to update $\mu(k', \varepsilon, h)$, evaluate market clearing and repeat until it is sufficiently close to supply equals demand. This represents an "iteration on the cumulative distribution function" style algorithm. An exhaustive search may be used to find the utility maximizing control variables (c, l). A detailed version of this statement is present in the appendix of [Hur, 2021]. Similar procedures are described in 3 employed in the current thesis in detail.

Worth to mention is in case of the stationary equilibrium the effects of the pandemic Γ are not present, yet present in the equations. These are just present for completeness's sake, since their effect is only present during the transitional dynamics. To solve for these, the general equilibrium assumption in [Hur, 2021] is discarded, and a partial equilibrium assumption is used. All factor prices (r, w), retirement income (s) and labour income tax (τ_l) stay at its steady state values. This is because of the computational burden associated with a general equilibrium setting during the transition path. A shooting algorithm combined with backward induction is used to solve for the transitional dynamics, as seen in the appendix of [Hur, 2021]. The equilibrium conditions during the transitional dynamics are similar to the steady-state ones as seen in definition (5), yet factor prices, retirement income and income tax are considered constant, as well as the measure μ_t is no longer invariant with respect to time.

With respect to **validation,** the following is conducted by [Hur, 2021]. During calibration of the pre-pandemic steady state, the goal is to reach a certain quality of model validity. Matching of moments of targeted variables (wealth/GDP, average weekly hours worked, average VSL) and reasoning about non targeted variables (disposable earnings Gini index, consumption Gini index, wealth Gini index) is used to achieve such an endeavour. First let us define the value of a statistical life (VSL) the marginal rate of substitution between survival and conceived utility of consumption of the pre-pandemic steady state. Using equation 21 (without the dynamics of Π due to the absence of the pandemic) one can include small changes of survival $\beta := \beta(1 + \Delta_s)$ as well as consumption $c := c + \Delta_c)$ to equation 21. This results in the value of statistical life as seen in [Hur, 2021]

$$()VSL_j(k,\varepsilon) := MRS_j = \left.\frac{\frac{\partial \hat{v}}{\partial \Delta_s}}{\frac{\partial \hat{v}}{\partial \Delta_c}}\right|_{\Delta_c=0,\Delta_s=0} = \frac{\beta \sum_{\varepsilon' \in E} \Gamma_{\varepsilon'\varepsilon}[\psi_j v_{j+1}(k',\varepsilon') + (1-\psi_j)v_j(k',\varepsilon')]}{(c_o + c_i)^{-\sigma}(\gamma^{\gamma}(1-\gamma)^{1-\gamma})^{1-\sigma}} \tag{29}$$

Model Parameters	Data	Model
disposable earnings Gini	0.37	0.36
consumption Gini	0.33	0.25
wealth Gini	0.74	0.59
wealth $p75/p25$	11.9	13.2
K/Y	4.8	4.8
$\mathbb{E}[VSL]$	238.8	238.8
$\mathbb{E}[l]$	34.4	34.4

Table 2: Targeted as well as Non-targeted moments, as seen in [Hur, 2021]

To calibrate the present model such that empirical evidence is included $\bar{u}$ is set such that $\int_X \sum_{j\in J} VSL(k,\varepsilon)d\mu_j(k,\varepsilon,h) \stackrel{!}{=} k \int_X \sum_{j\in J} c_{ij}(k,\varepsilon) + c_{oj}(k,\varepsilon)d\mu_j(k,\varepsilon,h)$ holds, where $k = 6226 := \frac{VSL_{year}}{c_{\frac{T}{26}}} = \frac{7.5E6\$}{1.204\$}$ represents used estimated of biweekly consumption per capita. This results in $\bar{u}$ being equal to 25.91 and $\mathbb{E}[VSL] = 238.8$ which matches the data used by [Hur, 2021]. Further, calibrated model parameters are similar to [Krueger et al., 2020] and can be seen in table 2.2 in an overall comparison of the models discussed in chapter 2. Targeted moments of $\frac{K}{Y} = 4.8$ as well as $\mathbb{E}[l] = 34.4$ are equal to the used real life estimates. Estimated Gini coefficients can be seen in the table below, which are not targeted and differ from real life estimates. Worth mentioning is the general case that $AR(1)$ processes seem no to capture the extreme cases of income and therefore the asset distribution. Introducing a handcrafted super rich state might help to introduce further inequality in the asset distribution.

An excerpt **results** of the [Hur, 2021] are mentioned below. These include the optimal policy frontier with respect to lockdown policies, SIR vs. macro SIR and welfare implication of the chosen policies (lockdown vs. no lockdown). Model lock-downs are implemented as a cap on outside hours (labour supply $l \leq \bar{l}_o = 0.13$, equivalent to 15 hours per week), without a corresponding cap on outside consumption. Yet, due to the presence of a flow value of infection $\hat{u}_I$ being a 50% reduction of flow utility value of an average agent, agents do consider the pandemic within the household's optimization problem (21) and (20) without the presence of $\bar{l}_o$. Results with respect to "subsidy vs. lockdown" discussed in [Hur, 2021] are left out on purpose due to not being connected to the topic of the thesis at hand.

1. the basic SIR model only dependent on number of infected people as seen in [Krueger et al., 2020], is strictly more severe than the macro SIR model, dependent on consumption and hours worked,

2. the deviation of outside consumption as well as aggregate hours worked from pre-pandemic steady state of the macro SIR model is *reduced* by at most 60% (82%) for old and low (high) wealth, 35% (62%) for middle and low (high) wealth and

10% (20%) for young and low (high) wealth households in contrast to the basic SIR model. In addition, *reducing* overall mortality by 3.5% to a level of 0.5%.

3. Infections differ around 0.2% with respect to young, middle and old households, with old households consuming the least.

4. Welfare consequences (consumption equivalent variation of SS and pandemic[Lucas, 1992]) for households are −8.0% for no mitigation, −6.4% for USA policy, −6.4% for a subsidy only policy, as well as −7.9% for a lockdown only policy. These values are grouped by age, wealth, and income respectively as seen in [Hur, 2021].

Additional Figures and Pseudocode

In this section additional figures as gini indices, value functions, policy functions as well as distributions of economic variables are present.

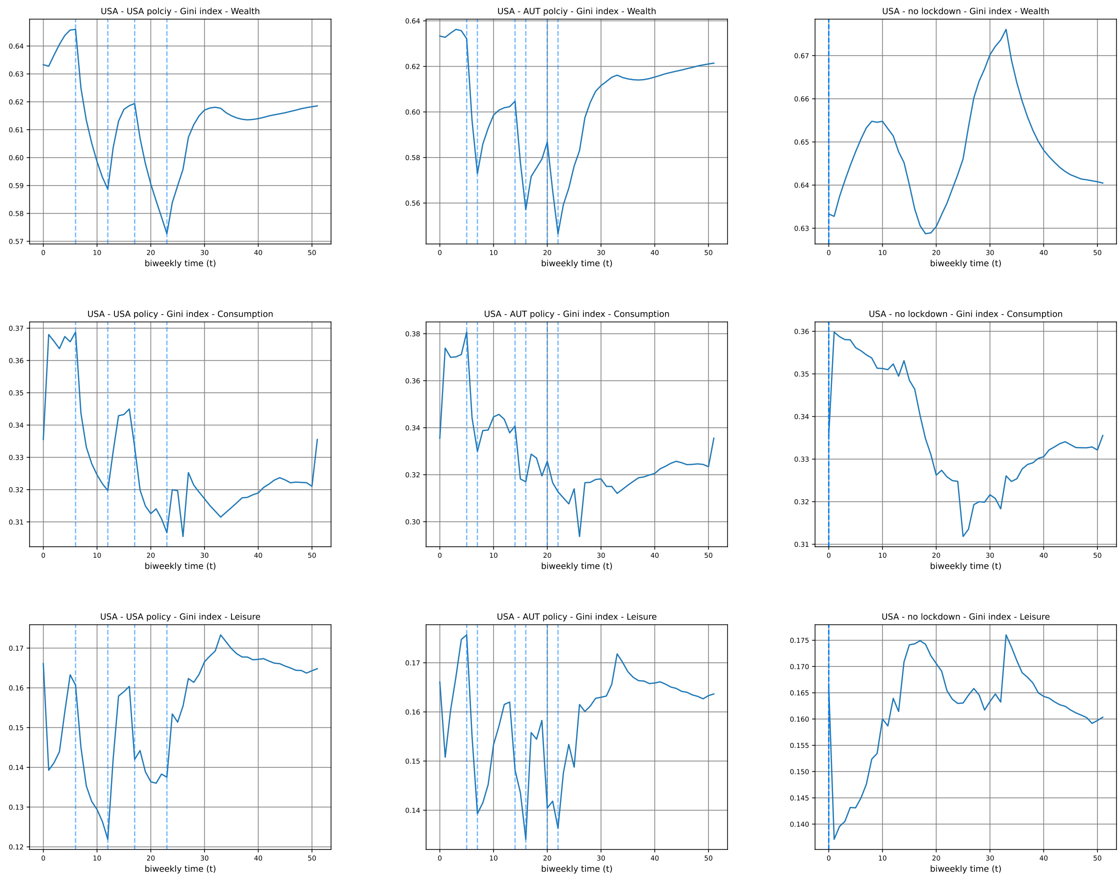

Figure 1: Present Gini index of aggregated households assets $A = K$, consumption C and leisure L during transitional dynamics in the USA model economy

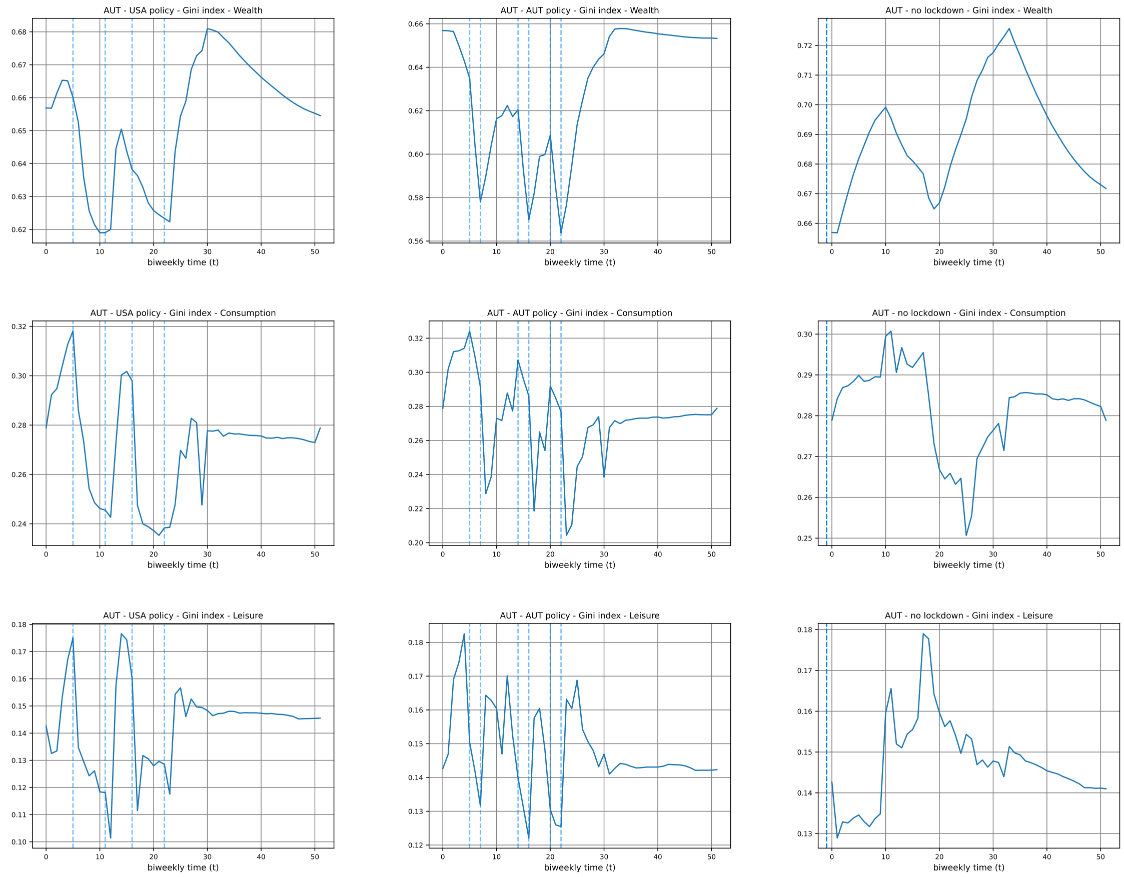

Figure 2: Present Gini index of aggregated households assets $A = K$, consumption C and leisure L during transitional dynamics in the AUT model economy

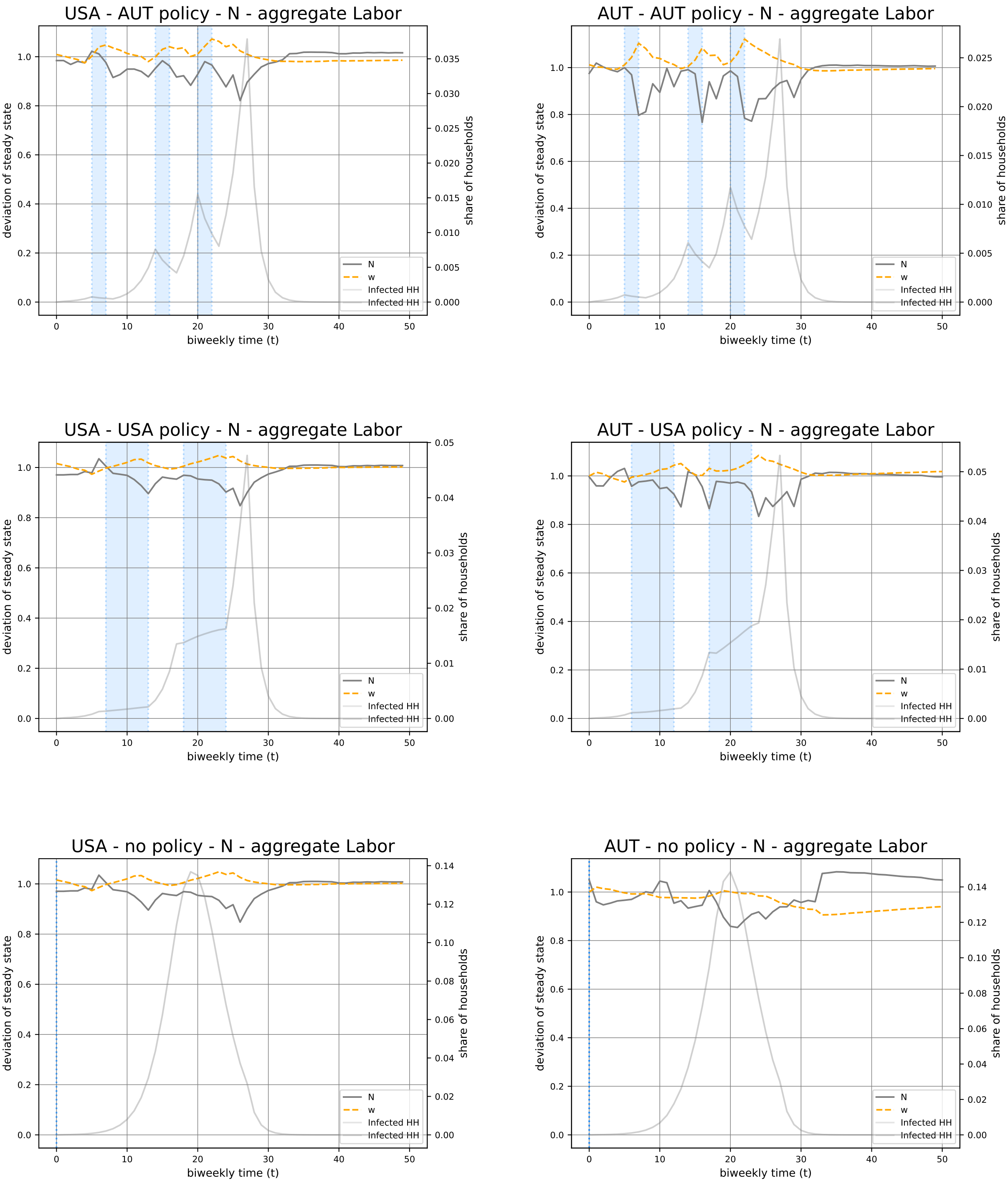

Figure 3: Present first moment of labour N and rate of wages w during transitional dynamics in the USA model economy

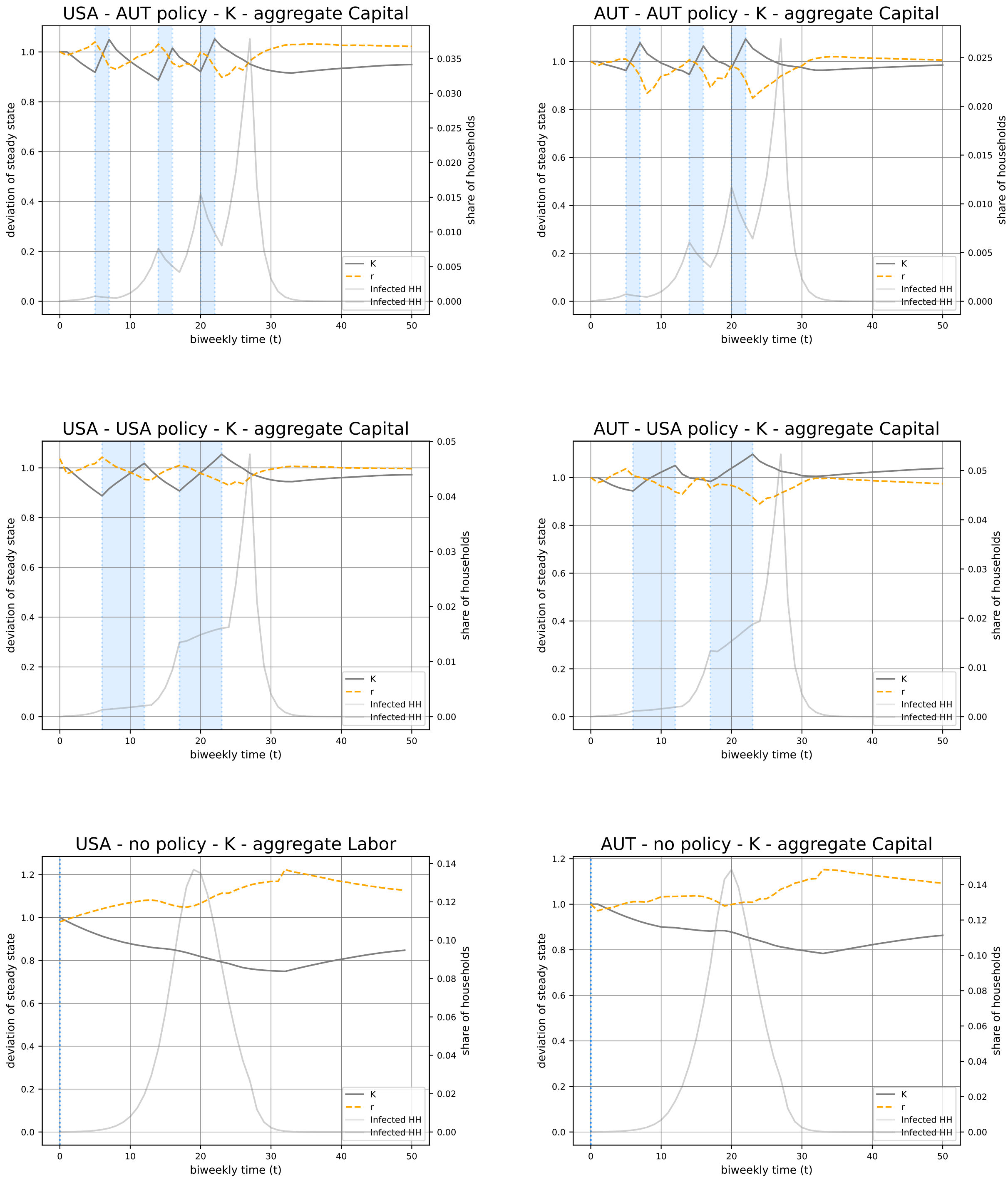

Figure 4: Present first moment of capital K and real interest rate r during transitional dynamics in the USA model economy

The following algorithms are supposed to serve as a rough overview of the main algorithms .1, .2, .3 used within the thesis.

Algorithm .1: Find fixed point of v(x)

Input: Initial economic model as defined in chapter 3 not in stationary equilibrium

Output: Economic model as defined in chapter 3 in stationary equilibrium

1. Start with an initial guess for the interest rate $r^{(0)}$

while $abs(K^s - K^d) > 0$ **do**

  2. Compute the wage rate $w^{(i)}$ implied by the firm's first-order condition

  while $abs(v(x)^{(i)} - v(x)^{(i-1)}) > 0$ **do**

    3. Given prices $r^{(0)}$ and $w^{(0)}$ solve dynamic problem of the households $v(x)^{(i)} = u(c,l) + \beta v(x)^{(i-1)}$ based on 1. and 2.

    4. Based on $v(x)^{(i)}$ derive policy functions $a'(a,e)$, $c(a,e)$ and $l(a,e)$

  end

  5. Use Monte Carlo Simulation .2 to derive invariant measure λ and calulate aggregate supply K^s given guessed $r^{(i)}$.

  6. Use Firms production function (3.8) to calculate K^d

  7. Adapt $r^{(i)}$ according to r^{i+1} = Golden Section Search$(\underline{r}, \overline{r})$

  8. Based on $l(a,e)$ calculate N

end

return $[v(x),\ a'(a,e),\ c(a,e),\ l(a,e),\ r,\ w]$;

Algorithm .2: Monte Carlo Simulation — Solving for Equilibrium conditions

Input: value function $v(x)$, policy functions $a'(a, e)$, $c(a, e)$, $l(a, e)$ and factor prices r, w

Output: simulated distribution of net-savings $a' - a$, consumption c, leisure l given factor prices r and w

1. Initialize: Household $n \in 1, \ldots, N$ is assigned wealth a_n^0 and efficiency $e_n^{(0)}$

for $n \in 1, \ldots, N$ **do**

    for $t \in 1, \ldots, T$ **do**

        2. Compute next-period wealth $a'(a_n^i; e_n^i)$

        3. Use a random number generator to obtain e_n^{i+1}

    end

end

while $abs(\mu^{(i)} - \mu^{(i-1)}) > 0$ **do**

    4. Join all N paths to come up with simulated distributions $[\lambda_a,\ \lambda_c,\ \lambda_l]$

    5. Compute statistics from the sample (e.g. mean $\mu^{(i)}$ and standard dev. sigma$^{(i)}$)

end

return $[\lambda_a,\ \lambda_c,\ \lambda_l]$;

Algorithm .3: Solve for Transitional Dynamics

Input: Economic model as defined in chapter 3 in stationary equilibrium
Output: Economic model as defined in chapter 3 in along transition path in equilibirum

1. Guess a time path for factor prices, $(r_t^0)_{2 \leq t \leq T-1}$, $(w_t^0)_{2 \leq t \leq T-1}$
while $abs(r^{(i)} - r^{(i-1)}) > 0$ **do**
  for $t \in (T-1, ..., 1)$ **do**
    if t *is lockdown* **then**
      2.1. Using guess $r_t^{(i)}$, compute the optimal decision rules iterating backwards in time $v(x)_t^{(i)} = u(c,l) + \beta v(x)_{t+1}^{(i-1)}$, where $v(x)_T^{(i-1)}$ is the value function in stationary equilibirum, based on forced savings $a_t + \Delta$ and forced reduction in consumption $c_t - \Delta$, where $\Delta = c_t \cdot (1 - k_{\text{lockdown}})$
    end
    else
      2.2. Using guess $r_t^{(i)}$, compute the optimal decision rules iterating backwards in time $v(x)_t^{(i)} = u(c,l) + \beta v(x)_{t+1}^{(i-1)}$, where $v(x)_T^{(i-1)}$ is the value function in stationary equilibirum,
    end
  end
  for $t \in (1, ..., T-1)$ **do**
    if t *is lockdown* **then**
      3.1. Using the Monte Carlo algorithm .2 simulate the dynamics of the distribution forward based on optimal decision rules derived during backward induction, including forced savings $a_t + \Delta$ and forced reduction in consumption $c_t - \Delta$, where $\Delta = c_t \cdot (1 - k_{\text{lockdown}})$
    end
    else
      3.2. Using the Monte Carlo algorithm .2 simulate the dynamics of the distribution forward based on optimal decision rules derived during backward induction
    end
  end
  for $t \in (1, ..., T-1)$ **do**
    while $abs(K_t^s - K_t^d) > 0$ **do**
      4. Adapt $r_t^{(i)}$ according to $r^{i+1} =$ Golden Section Search$(\underline{r}, \overline{r})$ such that $K^s = K^d$ where K^s is fixed (derived in 3. using old guess of factor prices r, w) and K^d is derived using the Firms production function.
      5. Use $r^{(i)} := (1-) * r^{(i-1)} + * r^{(i)}$ as an upate rule for adjustable convergance using ξ
      6. Based on $l(a,e)$ calculate N
    end
  end
end
return $[(v_t)_{1 \leq t \leq T}, \ (a_t)_{1 \leq t \leq T}, \ (c_t)_{1 \leq t \leq T}, \ (l_t)_{1 \leq t \leq T}, (\lambda_{a,t})_{1 \leq t \leq T}, (\lambda_{c,t})_{1 \leq t \leq T}, (\lambda_{l,t})_{1 \leq t \leq T}]$;