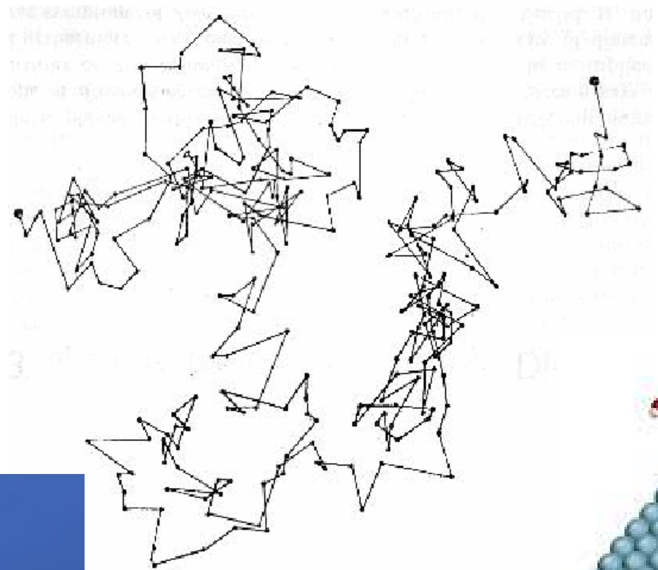
KPZ fluctuations and bosonic skin effect in the ASIP model

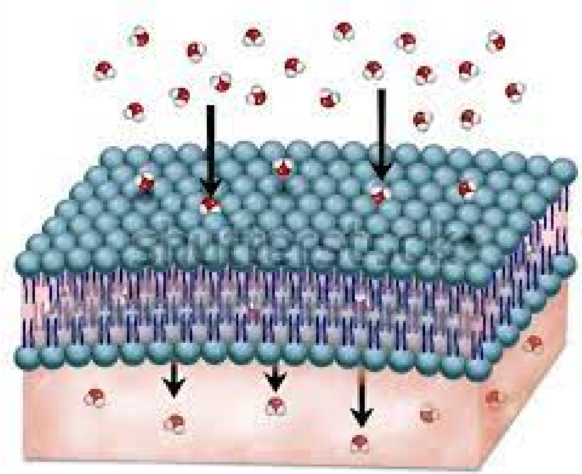
Louis Garbe

Les Houches, 08/03/2023

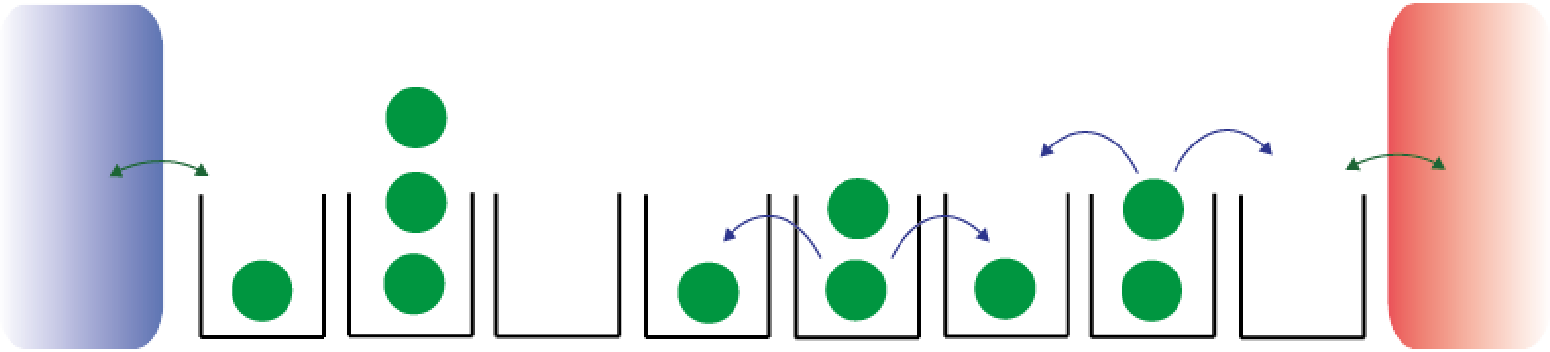
Der Wissenschaftsfonds.

Diffusive transport



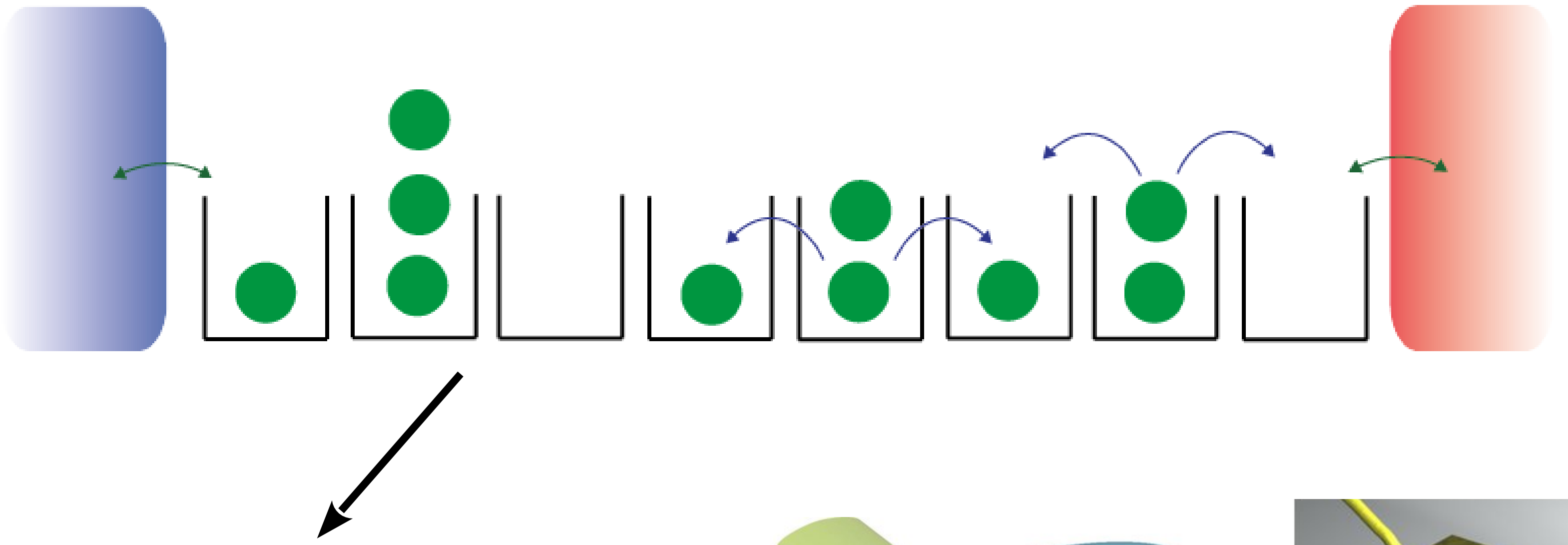


Minimal model: particles hopping incoherently, independently and symmetrically on a 1D lattice

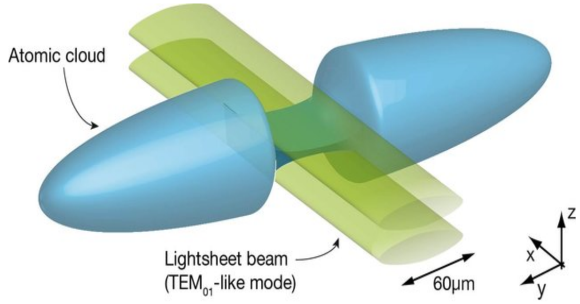


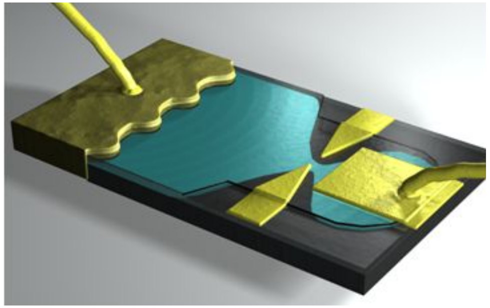
Fourier/Ohm/Fick’s law, Gaussian Fluctuations...

Minimal model: particles hopping ~~incoherently~~, independently and symmetrically on a 1D lattice

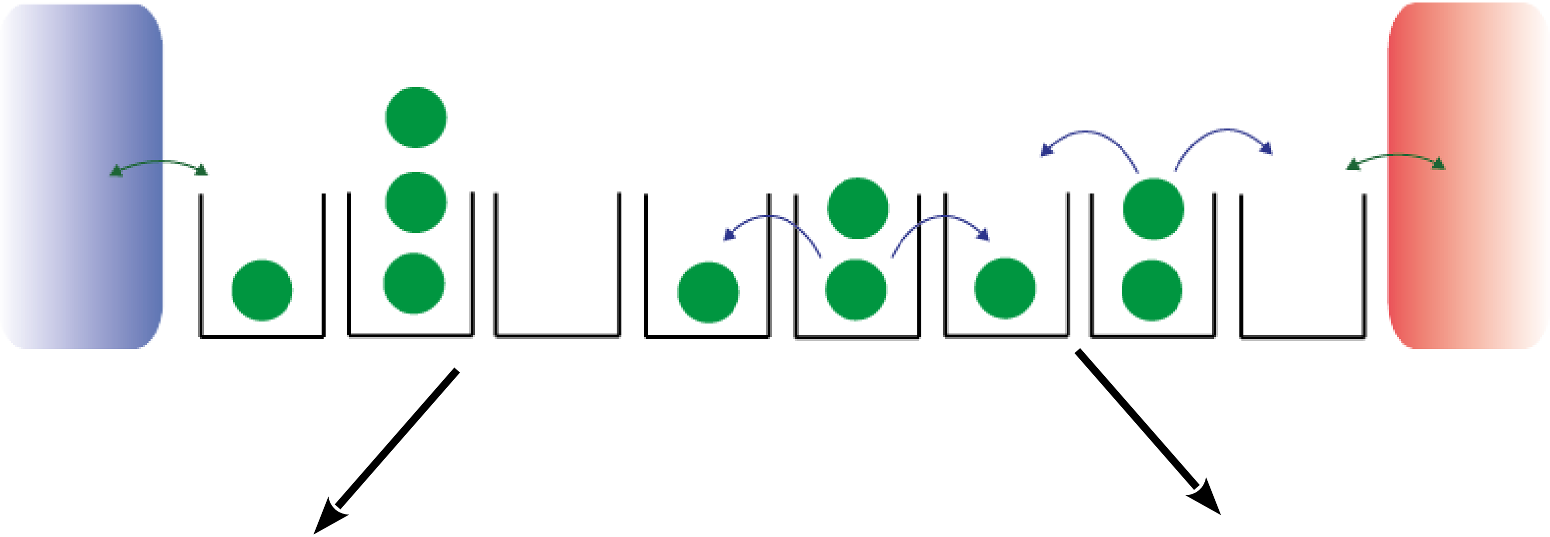


Coherent transport (See J.-P. Brantut's talk)





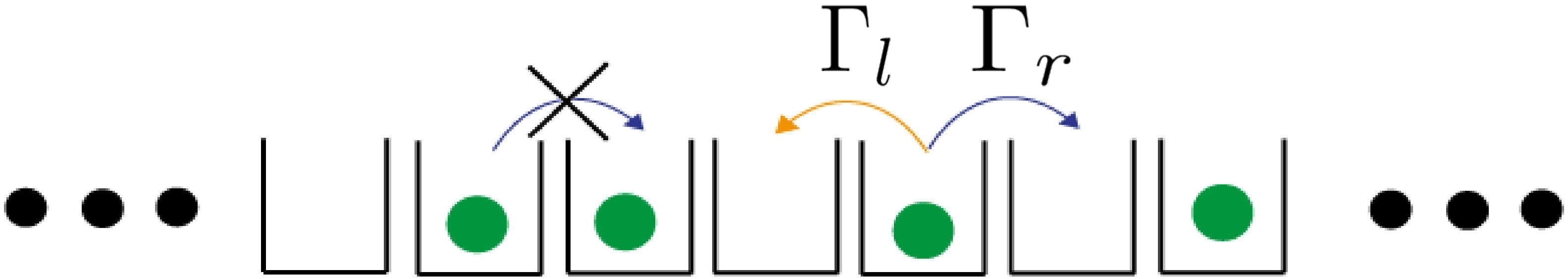
Minimal model: particles hopping incoherently, ~~independently~~ and ~~symmetrically~~ on a 1D lattice



Coherent transport (See J.-P. Brantut's talk)

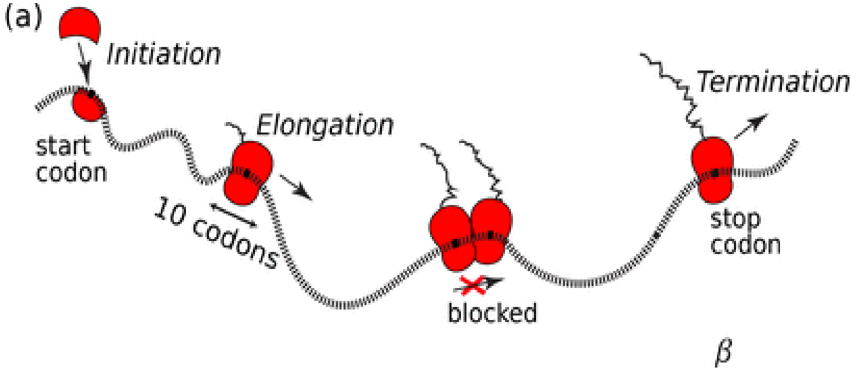
ASEP

Asymmetric Simple Exclusion Process

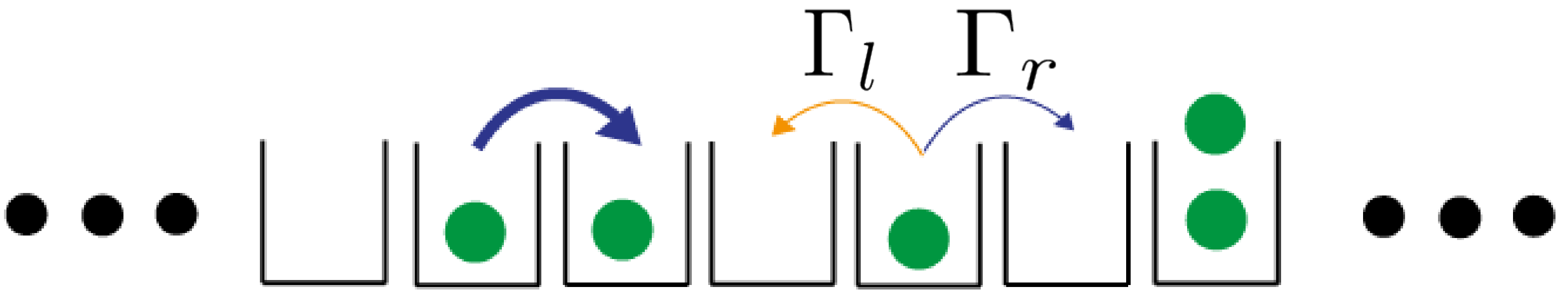


$$\Gamma_r \neq \Gamma_l$$

$$P[p \to p+1] \propto \Gamma_r(1 - n_{p+1})$$



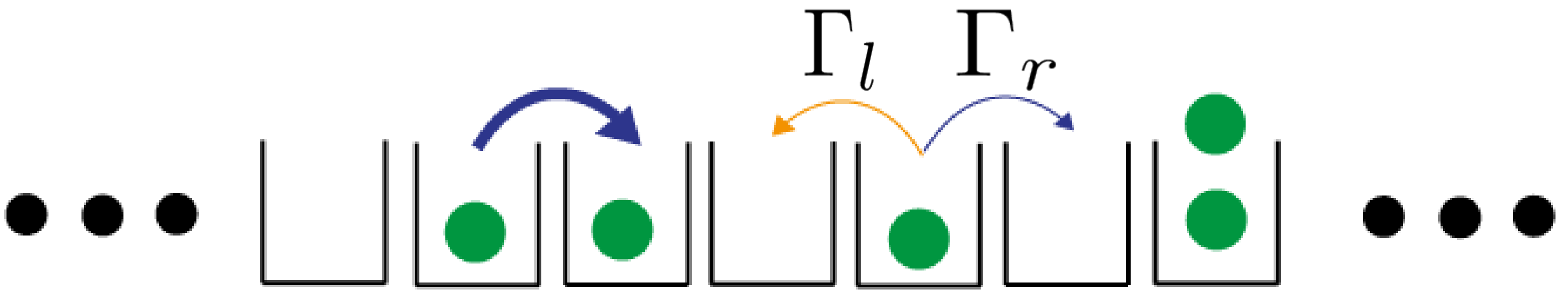
Asymmetric Simple Inclusion Process



$$\Gamma_r \neq \Gamma_l$$

$$P[p \to p+1] \propto \Gamma_r(1 + n_{p+1})$$

Asymmetric Simple Inclusion Process



$$\Gamma_r \neq \Gamma_l$$

$$P[p \to p+1] \propto \Gamma_r(1 + n_{p+1})$$

Grosskinsky et al., J.stat.phys 142, 952-974 (2011)
Reuveni et al., PRE 84, 041101 (2011)
D. Bernard et al., Europhy. Lett. 121, 60006 (2018)
Haga et al., PRL 127, 070402 (2021)

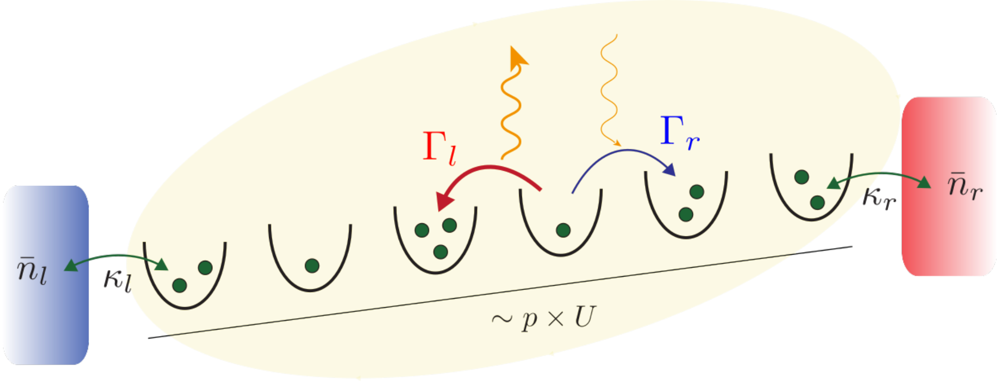
(Non-interacting) bosonic systems

$$\mathcal{L}_{\mathrm{hop}}\rho = \sum_{p=1}^{L-1} \Gamma_l \mathcal{D}[a_p^\dagger a_{p+1}]\rho + \Gamma_r \mathcal{D}[a_{p+1}^\dagger a_p]\rho$$

$$\mathcal{L}_{\mathrm{hop}}\rho = \sum_{p=1}^{L-1} \Gamma_l \mathcal{D}[a_p^\dagger a_{p+1}]\rho + \Gamma_r \mathcal{D}[a_{p+1}^\dagger a_p]\rho$$

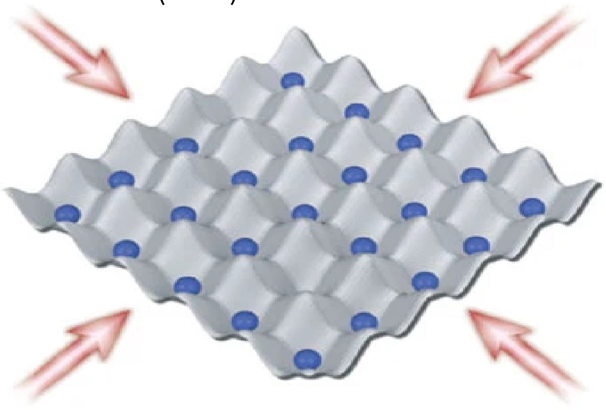
Bosons hop by emitting (absorbing) energy in an environment

Asymmetry set by the temperature of the environment



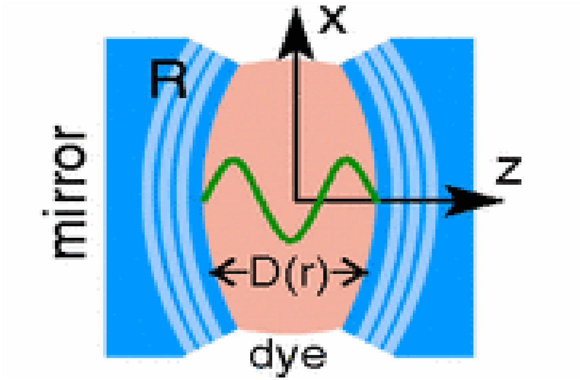
Some possible platforms:

Bloch, Nature 453, 1016-1022 (2008)



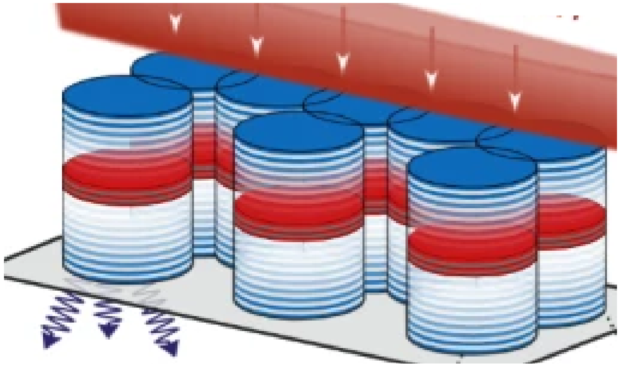
Cold atoms

Klaers and al., PRL 108, 160403 (2012)



Photons condensates

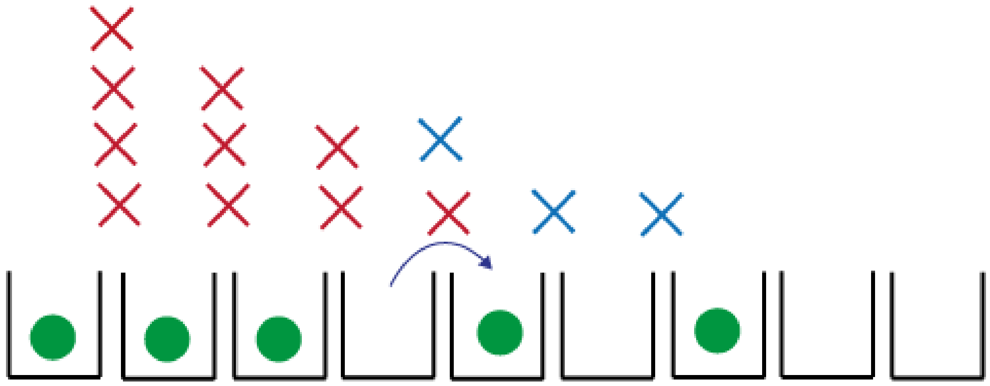
Fontaine and al., Nature 608, 687-691 (2022)



Polaritons condensates

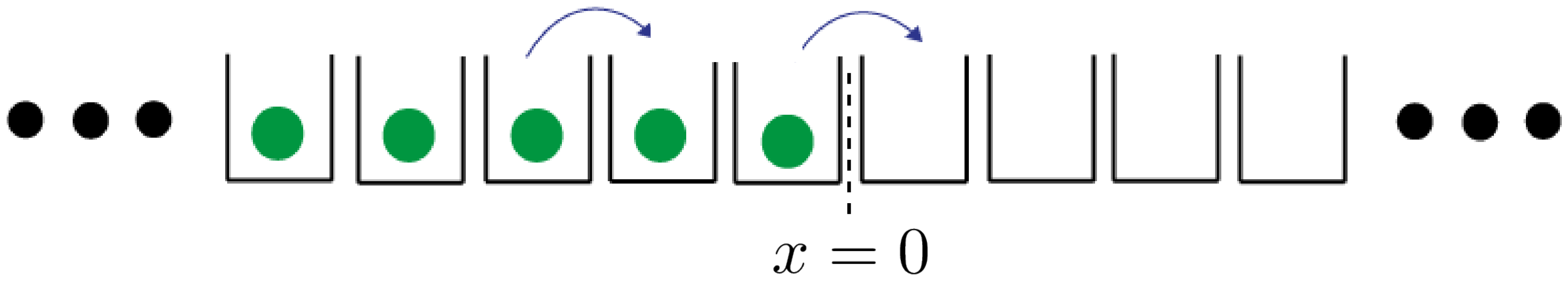
This talk

1) Current statistics: why ASIP=ASEP



2) Driven transport: why ASIP≠ASEP

Classical problem



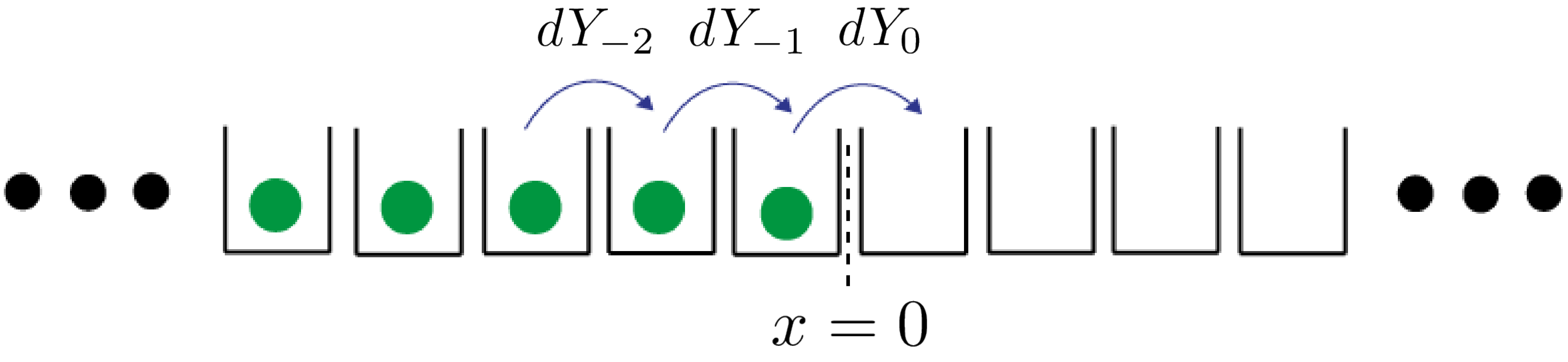
Configuration $\vec{n} = (n_1, n_2, \ldots)$

$$\Gamma_l = 0$$

$$P[\vec{n}, t] = \langle \vec{n} | \rho(t) | \vec{n} \rangle$$

Domain wall initial state

Monte-Carlo sampling



$$dY_i(t) = (0, 1)$$

$$P[dY_i(t) = 1] \sim \Gamma_r n_i (1 + \sigma n_{i+1}) dt$$

$$\sigma = +1 \ \text{(Bosons)}, \ \ -1 \ \text{(Fermions)}, \ \ 0 \ \text{(ZRP)}$$

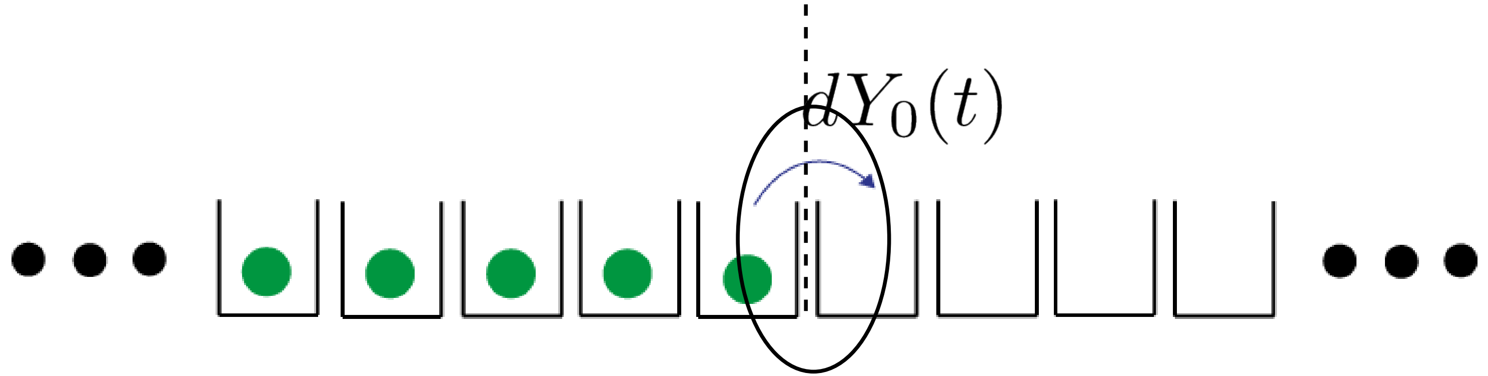
Integrated current

$$dn_i = dY_{i-1}(t) - dY_i(t)$$

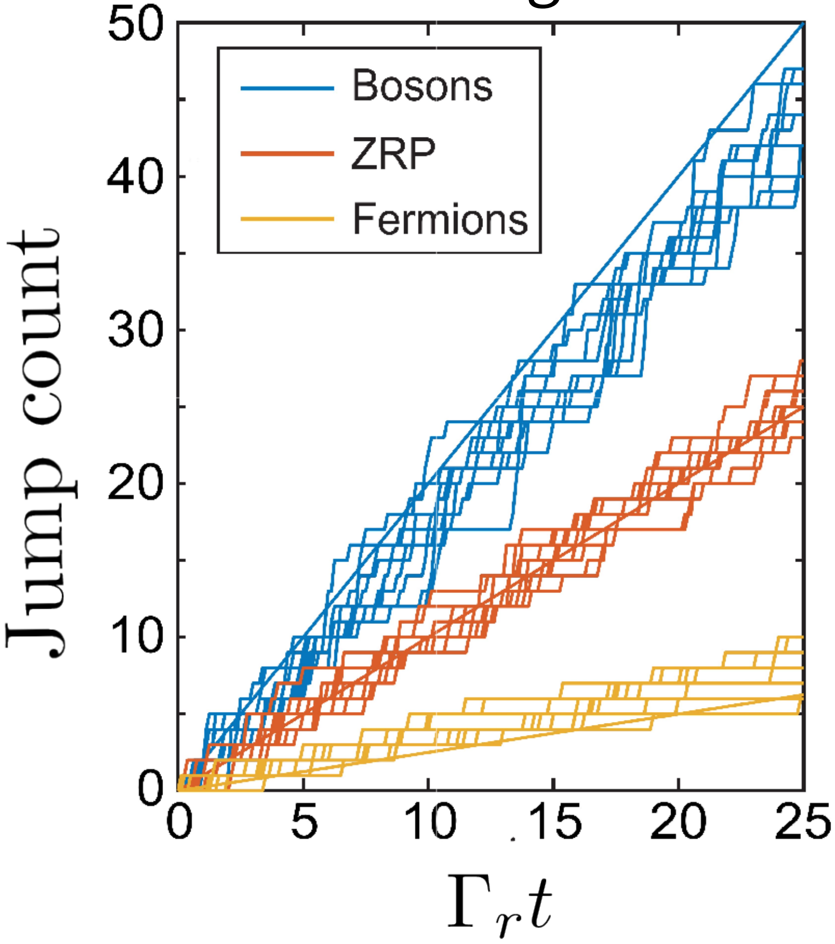
Conservation relation

$$n_i \leftrightarrow h_i(t) = h_i(0) + \int_0^t dY_i(\tau) d\tau$$

Integrated current

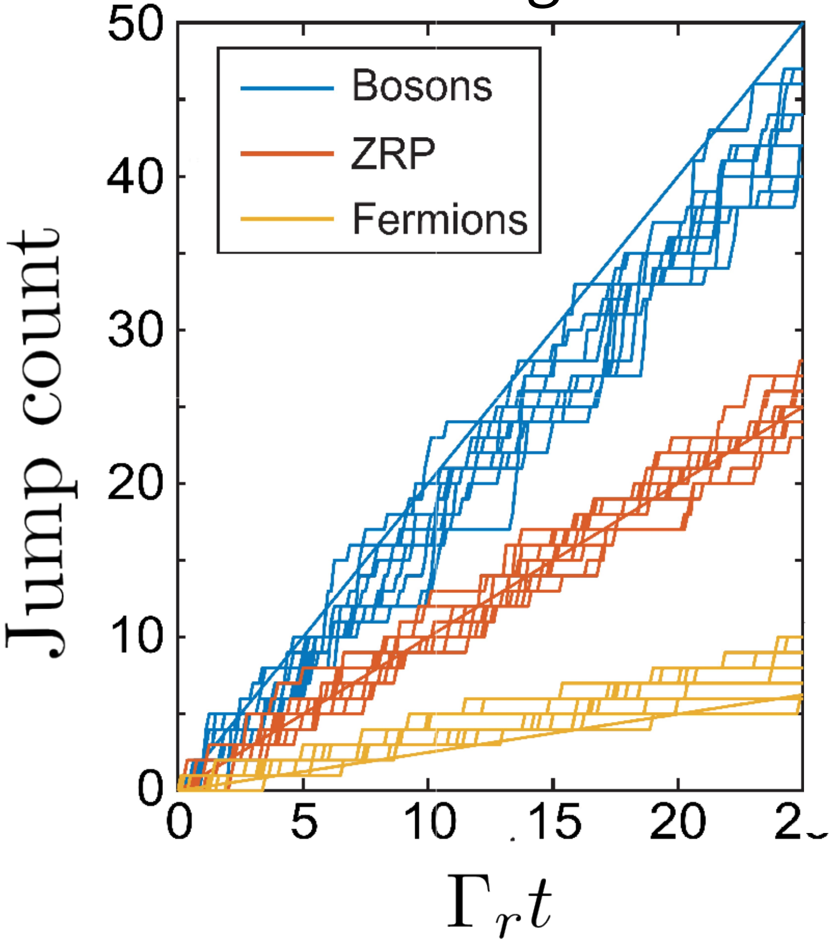


Integrated current across the origin $h_0(t)$

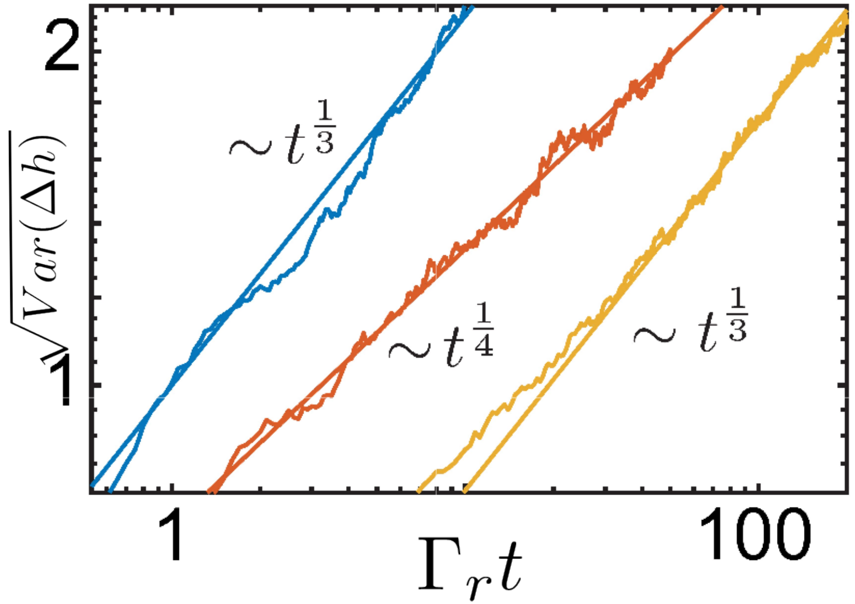


$$h_0 = j_\sigma t + \Delta h(t)$$

Integrated current across the origin $h_0(t)$



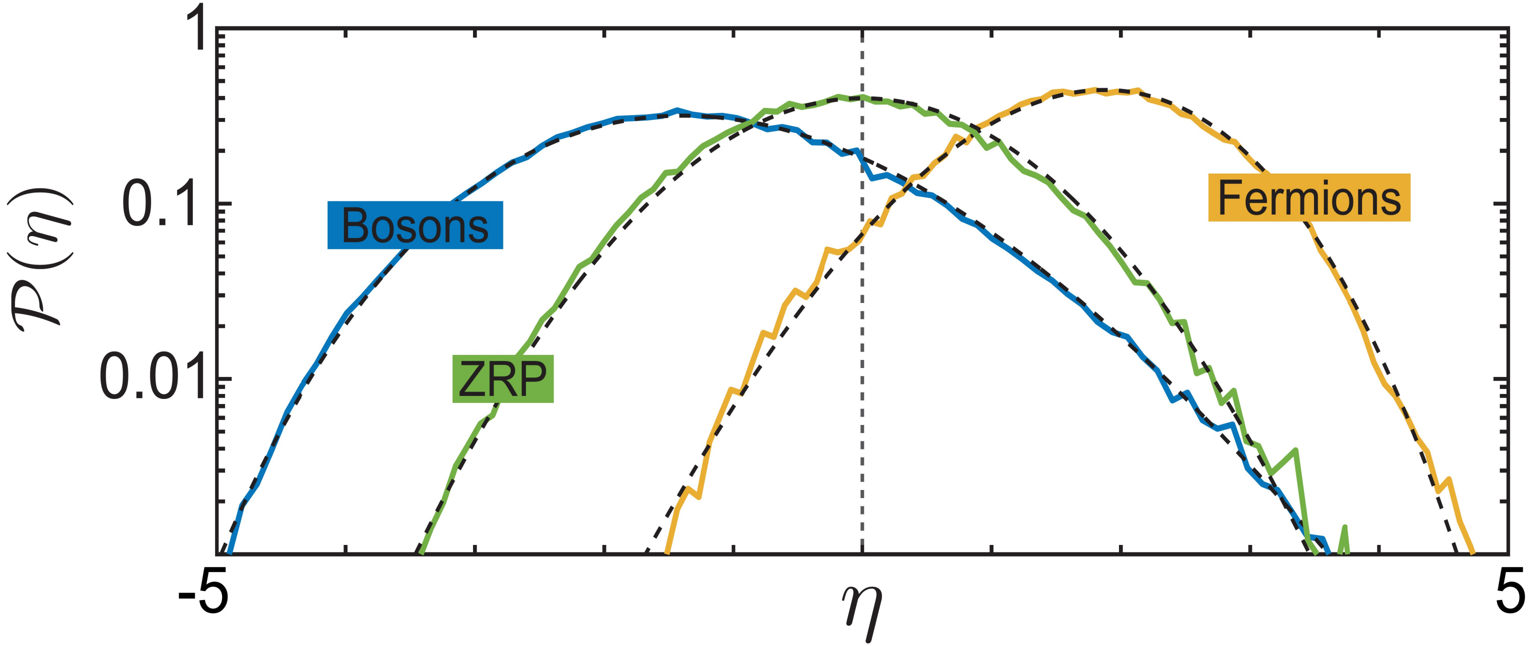
$$h_0 = j_\sigma t + \Delta h(t)$$



KPZ universality class

$$h_0 = j_\sigma t + (\Gamma_r t)^{1/3}\eta$$

Time-independent random variable



$$\mathcal{P}_{\mathrm{boson}}(\eta) = \mathrm{TW}_{\mathrm{GOE}}(\eta)$$

$$\mathcal{P}_{\mathrm{fermions}}(\eta) = \mathrm{TW}_{\mathrm{GUE}}(-\eta)$$

Connection to growth dynamics

Long-wavelength limit: $n_i(t) \to n(x,t)$

(Stochastic) Burgers $\boxed{\partial_t n = -\Gamma_r(1 \pm 2n)\partial_x n + \nu\partial_x^2 n - \partial_x \xi(x,t)}$

Connection to growth dynamics

Long-wavelength limit: $n_i(t) \to n(x,t)$

(Stochastic) Burgers $\boxed{\partial_t n = -\Gamma_r(1 \pm 2n)\partial_x n + \nu \partial_x^2 n - \partial_x \xi(x,t)}$

$$\partial_t n + \partial_x J = 0$$

$$h(x,t) = h_0(x) + \int_0^t J$$

$$\boxed{n(x,t) = -\partial_x h(x,t)}$$

Connection to growth dynamics

Long-wavelength limit: $n_i(t) \to n(x,t)$

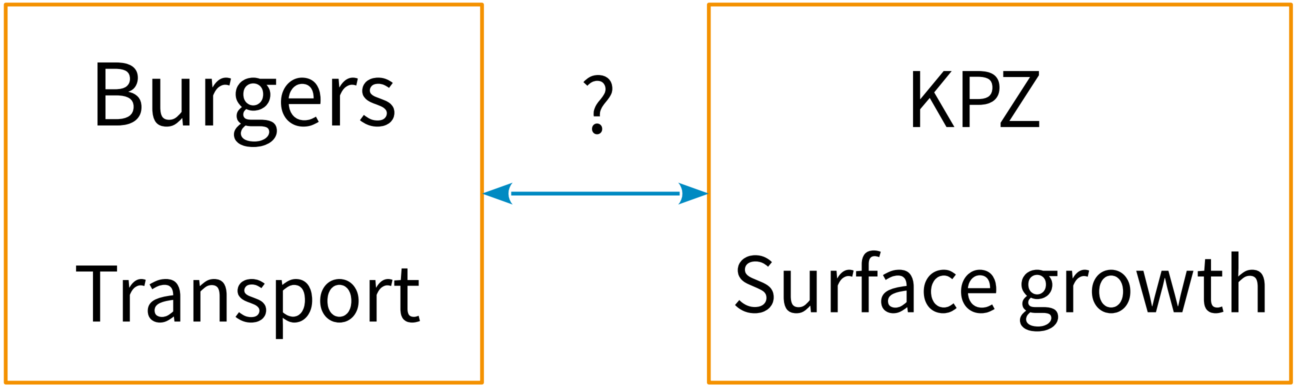
(Stochastic) Burgers $$\partial_t n = -\Gamma_r(1 \pm 2n)\partial_x n + \nu \partial_x^2 n - \partial_x \xi(x,t)$$

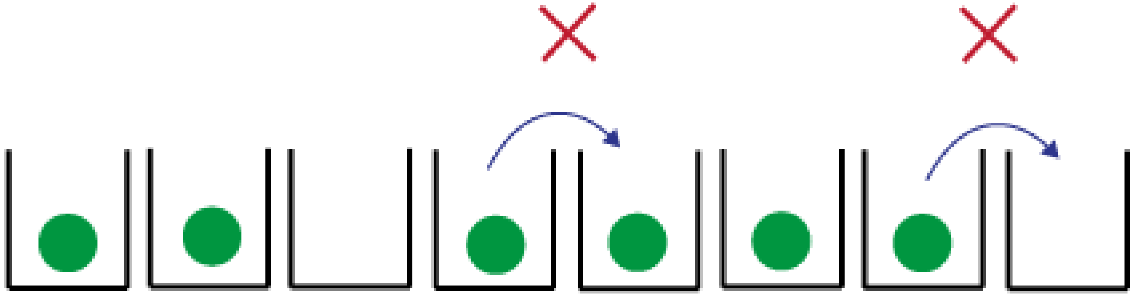
$$\partial_t n + \partial_x J = 0$$

$$h(x,t) = h_0(x) + \int_0^t J$$

$$n(x,t) = -\partial_x h(x,t)$$

KPZ $$\partial_t h = \Gamma_r \left[-\partial_x h \pm (\partial_x h)^2 + \partial_x^2 h \right] + \xi(x,t)$$


Burgers
Transport
?
KPZ
Surface growth

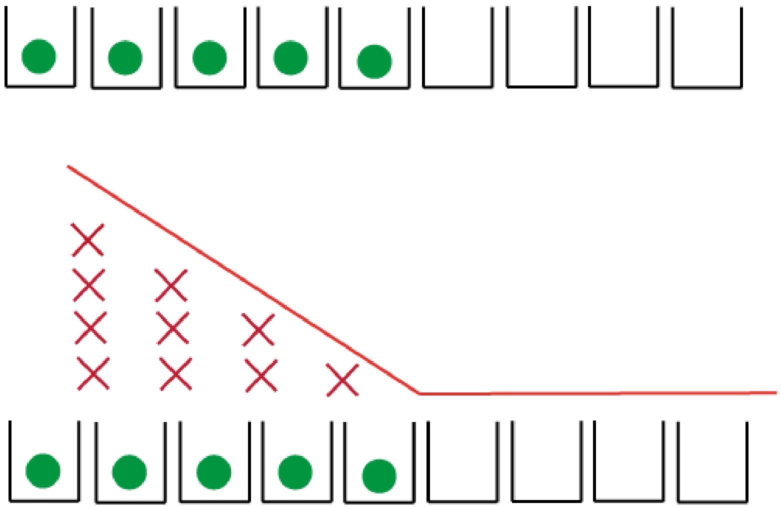


$$h(x,t) = h(x,0) + \int_0^t dY(x,\tau)d\tau \qquad \partial_x h(x,0) = -n(x,0)$$

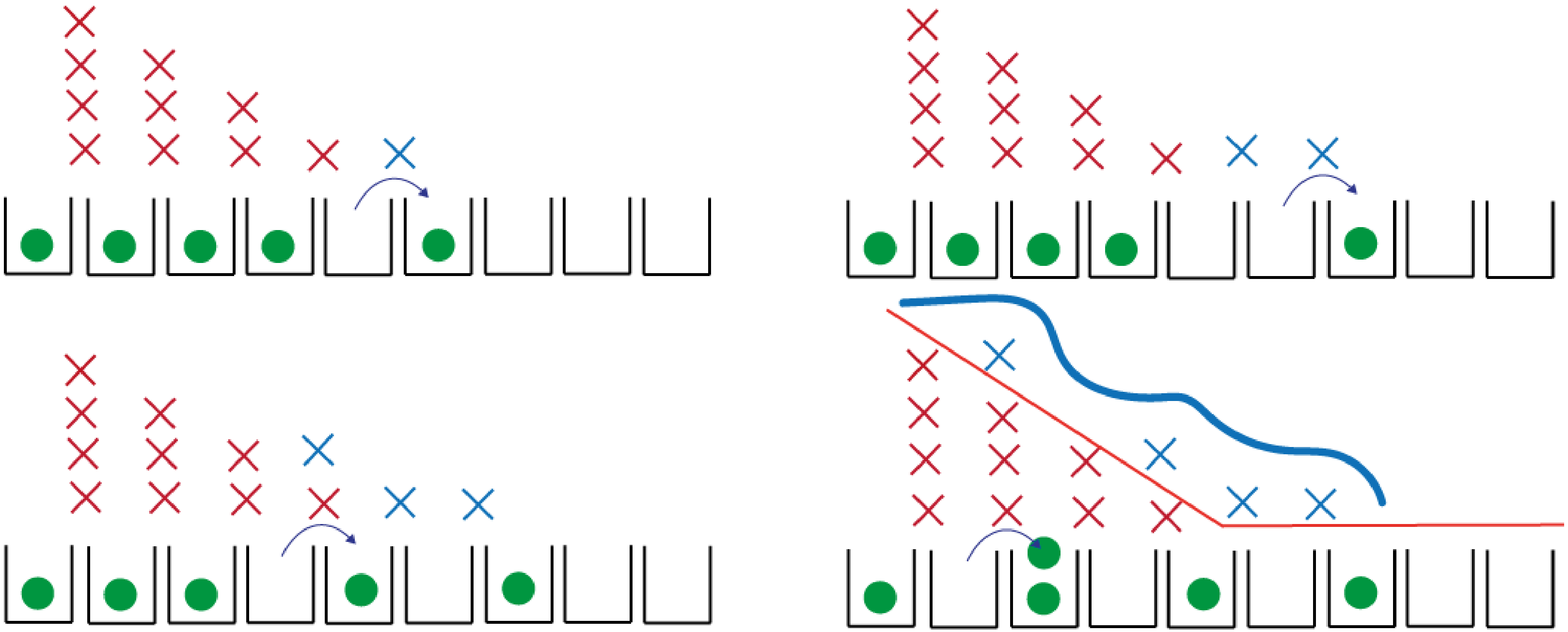
Domain wall:

$$n(x,0) = \theta(-x)$$

$$h(x,0) = -x\theta(-x)$$



$$h(x,t) = h(x,0) + \int_0^t dY(x,\tau)d\tau$$



The surface growth records the jumps events,
on a substrate given by the initial conditions

$$\partial_t h = \Gamma_r \left[-\partial_x h \pm (\partial_x h)^2 + \partial_x^2 h\right] + \xi(x,t)$$

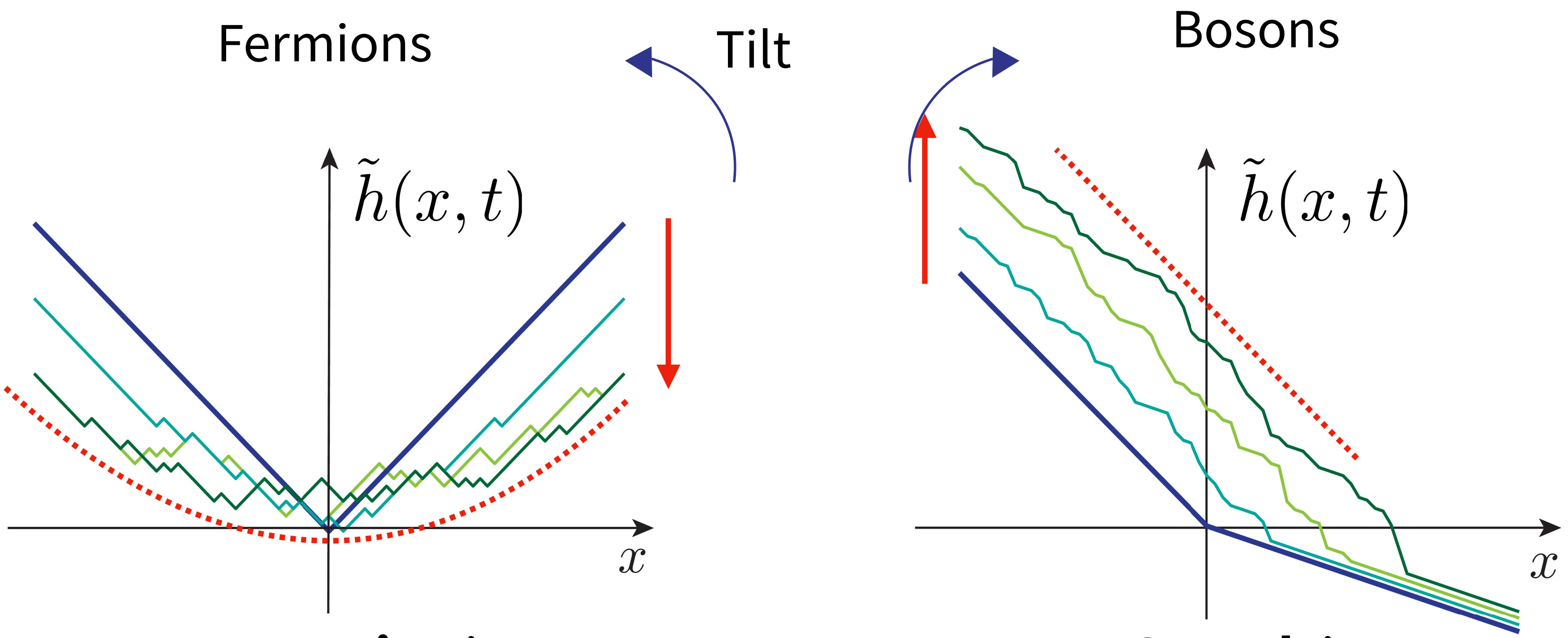
KPZ + lateral motion

Tilt transformation: $$\tilde{h}(x,t) = h(x,t) \pm \left(\frac{\Gamma_r t}{4} - \frac{x}{2}\right)$$

$$\partial_t \tilde{h} = \Gamma_r \left[\pm(\partial_x \tilde{h})^2 + \partial_x^2 \tilde{h}\right] + \xi(x,t)$$

Canonical KPZ equation

Statistics affects both the tilt and the final equation



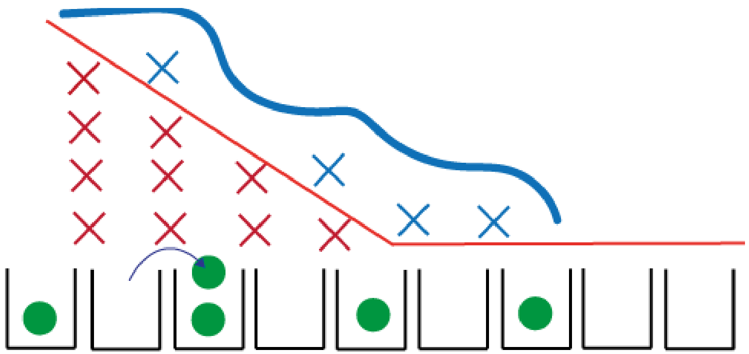
Evaporation in a
curved geometry

Growth in a
flat geometry

GUE, sign-flipped

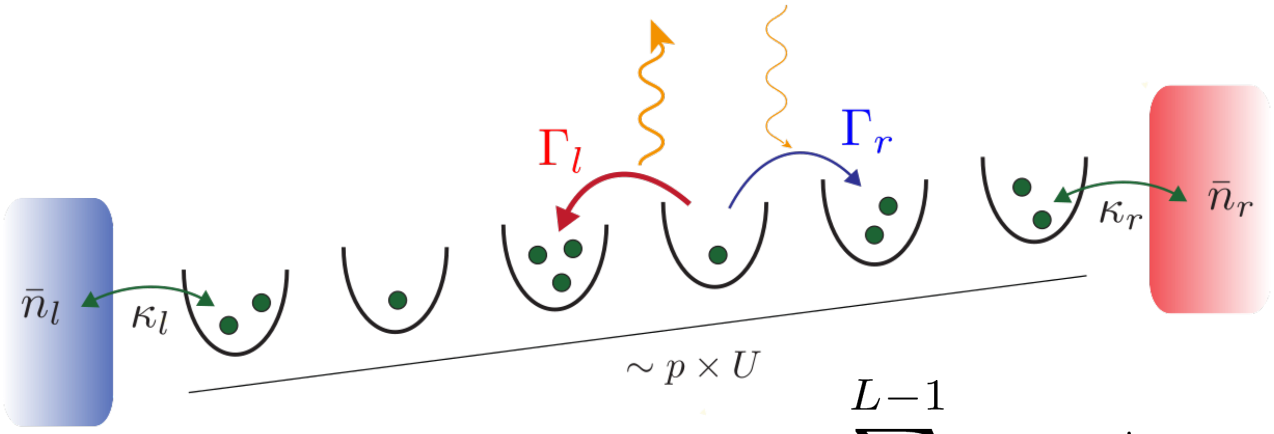
GOE, no sign-flip

Summary



- Connection between correlated transport and surface growth
- Both ASEP and ASIP belong to KPZ universality class
- Unified description of bosons and fermions: statistics reflected in substrate and growth properties

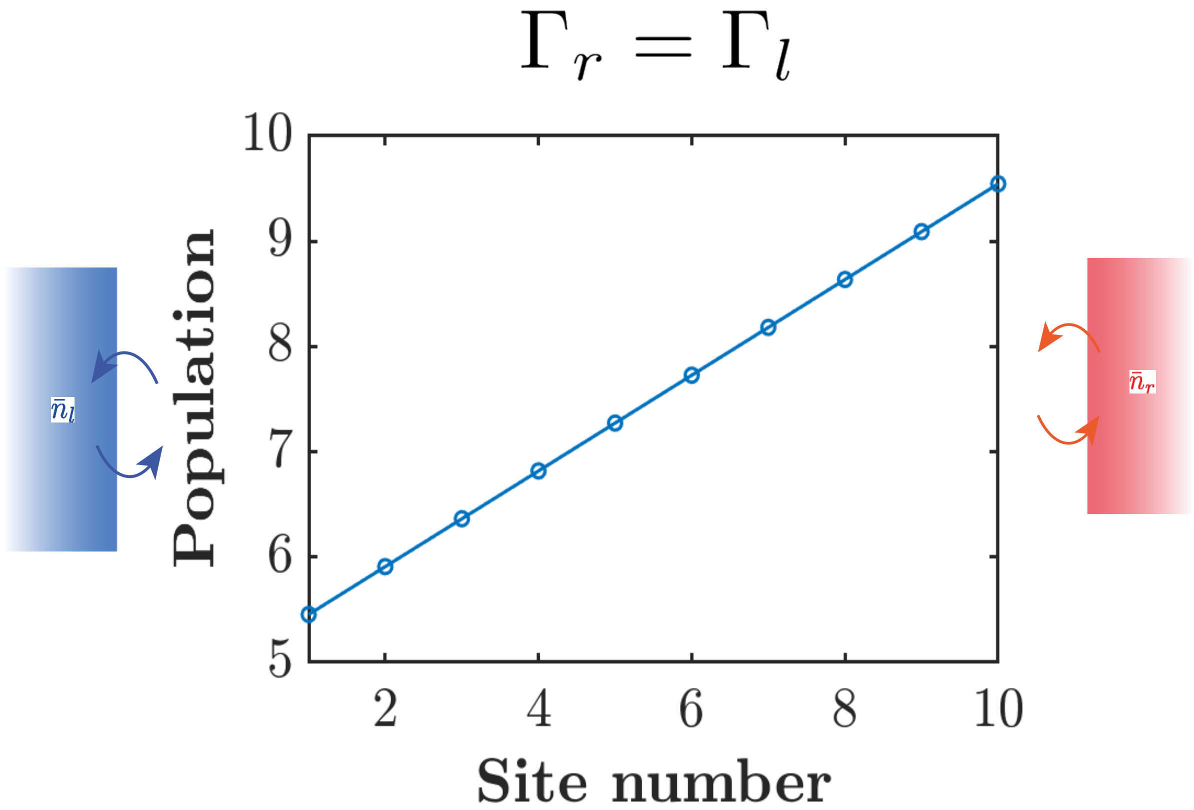
Driven transport



$$\mathcal{L}_{\text{hop}}\rho = \sum_{p=1}^{L-1} \Gamma_l \mathcal{D}[a_p^\dagger a_{p+1}]\rho + \Gamma_r \mathcal{D}[a_{p+1}^\dagger a_p]\rho$$

$$\dot{\rho} = (\underbrace{\mathcal{L}_{\text{hop}}}_{\text{Chain}} + \underbrace{\mathcal{L}_{B,r} + \mathcal{L}_{B,l}}_{\text{Reservoirs}})\rho$$

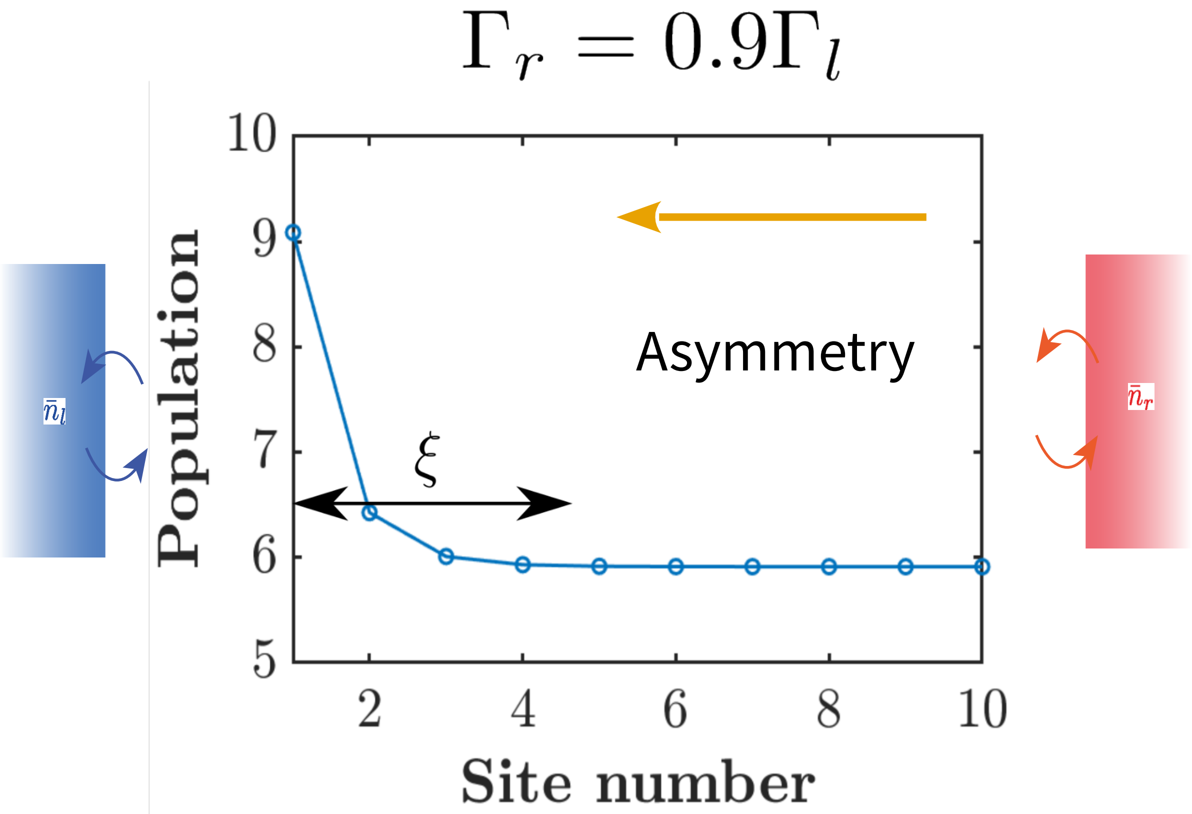
Density profile



Diffusive transport

Linear population profile

$$J \propto \frac{\bar{n}_r - \bar{n}_l}{L}$$

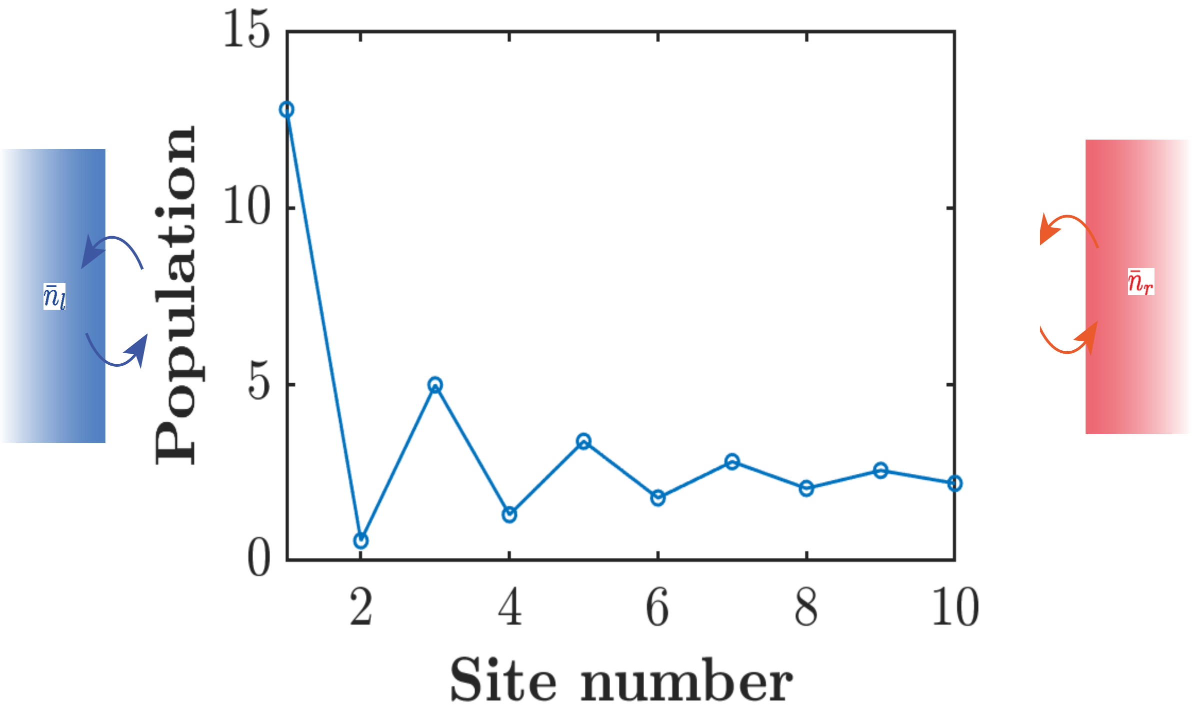


Accumulation on the edge
→ bosonic skin effect

Ballistic transport

$$J \propto \bar{n}_r$$

$$\Gamma_r = 0$$



Zig-zag structure → failure of hydrodynamic treatment

Pile-up and clustering of particles → bosonic behavior

Population fluctuations

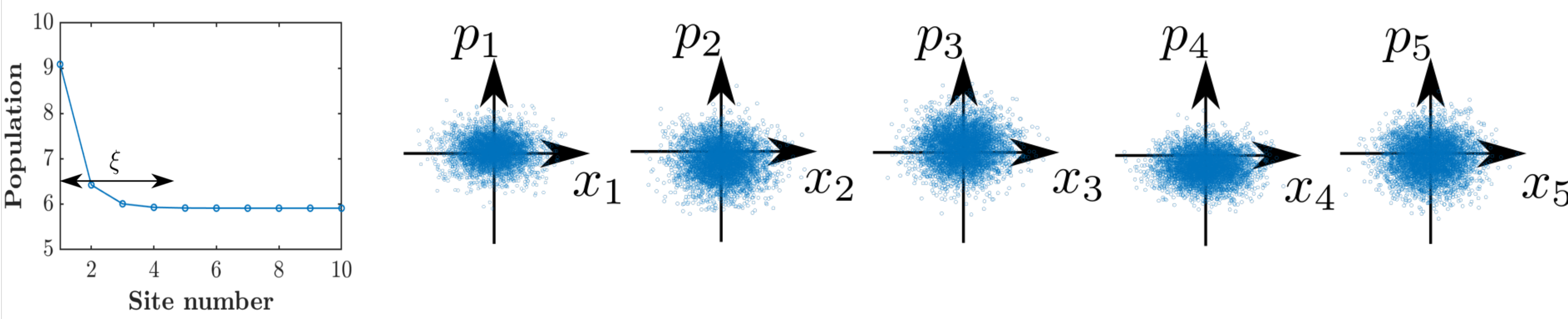
So far: we only considered the average population $\langle \hat{n}_p \rangle$. What about the fluctuations?

Wigner distribution in quadrature space:

$$x_q = a_q^\dagger + a_q$$

$$p_q = i(a_q^\dagger - a_q)$$

Weak asymmetry:



Population fluctuations

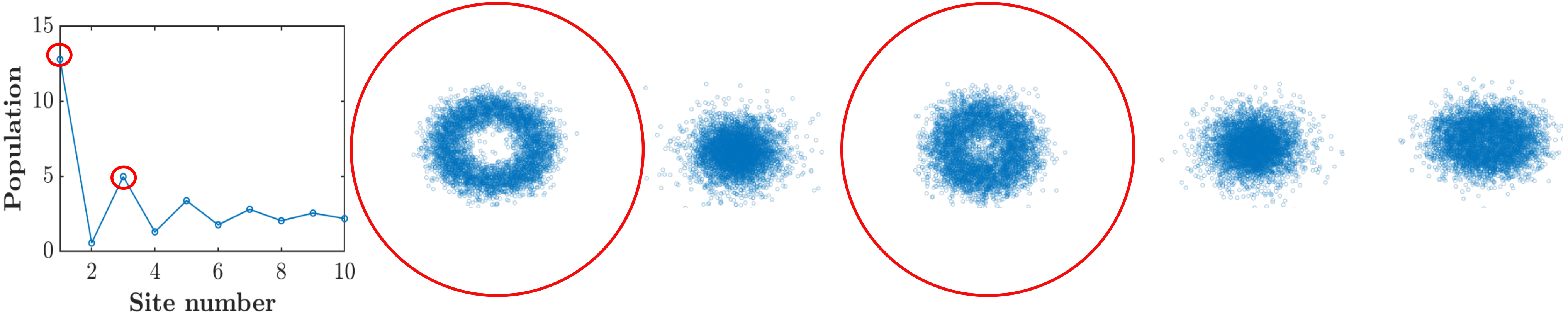
So far: we only considered the average population $\langle \hat{n}_p \rangle$. What about the fluctuations?

Wigner distribution in quadrature space:

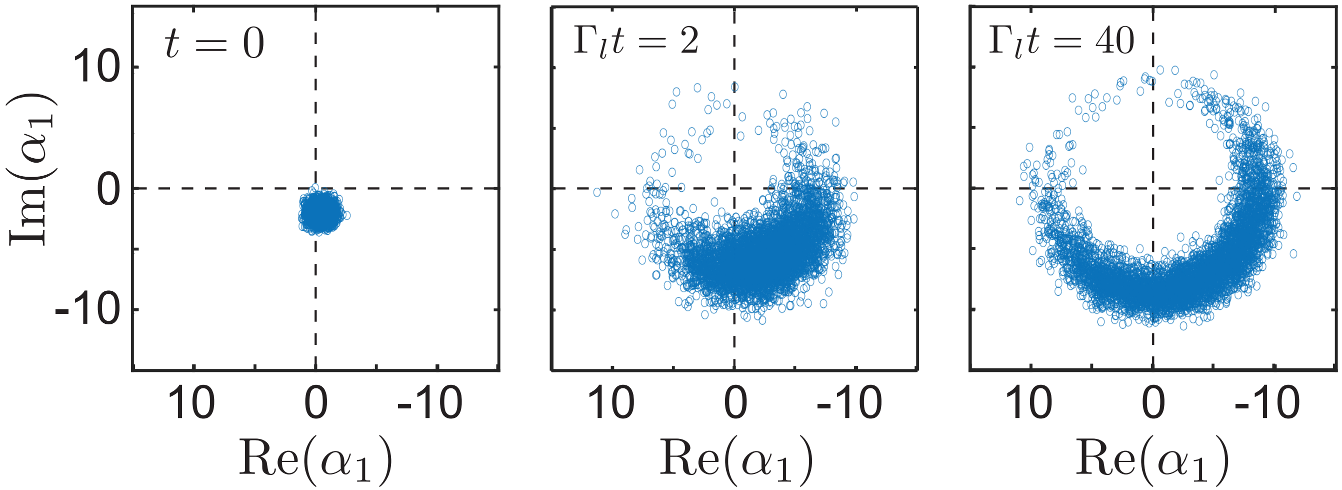
$$x_q = a_q^\dagger + a_q$$

$$p_q = i(a_q^\dagger - a_q)$$

Strong asymmetry:



Lasing transition

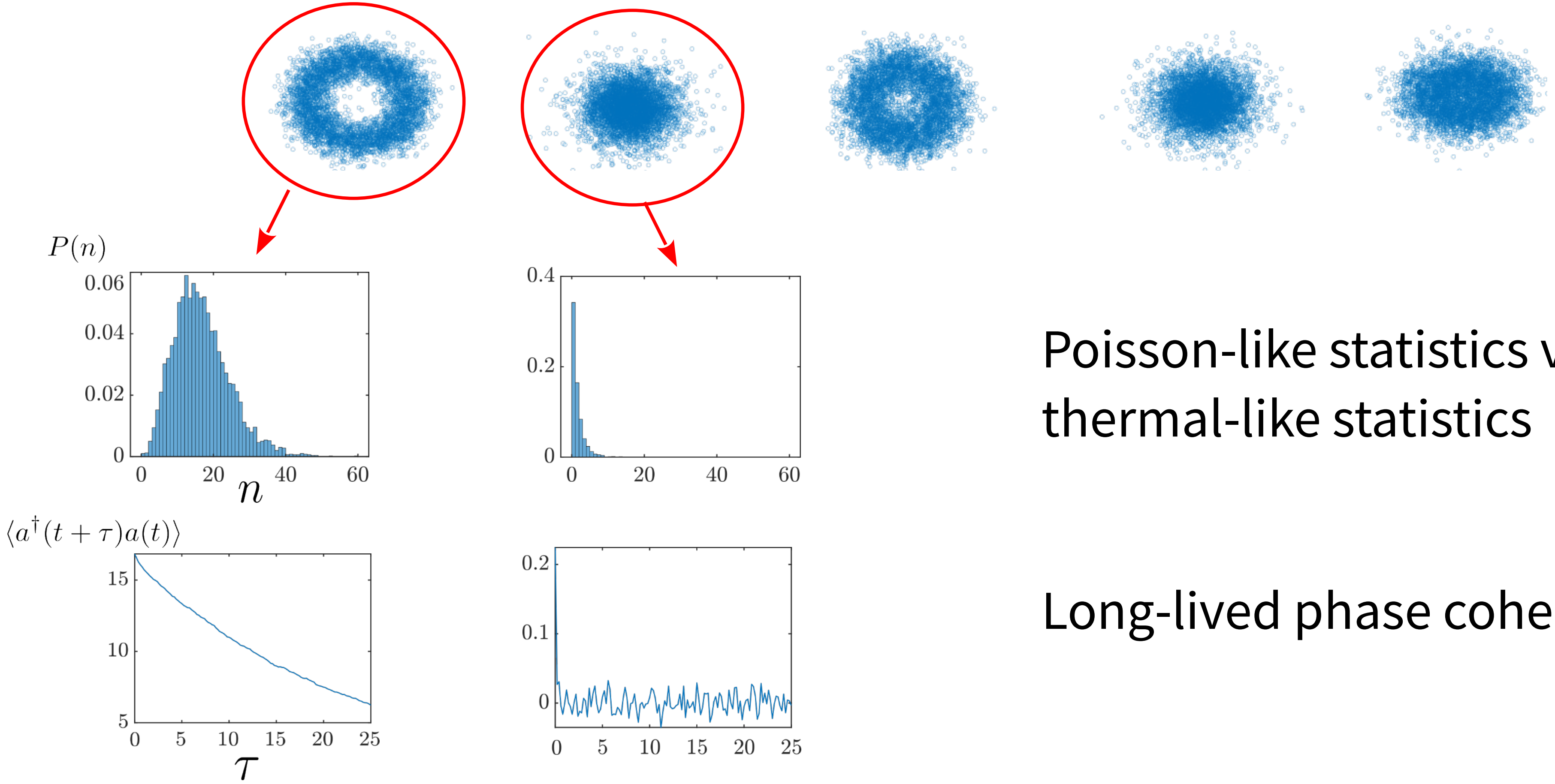


U(1) symmetry breaking→ lasing/condensation effect

on every other site

Signature of a properly bosonic behavior

Lasing transition



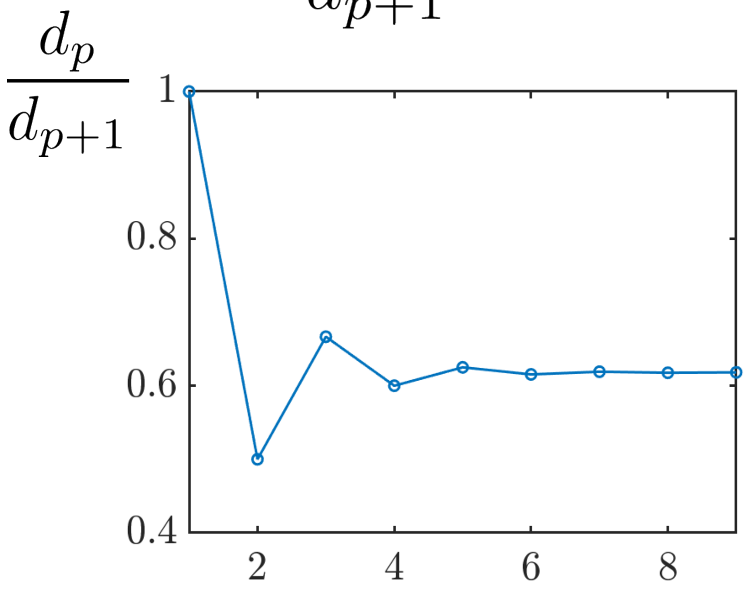
Poisson-like statistics vs thermal-like statistics

Long-lived phase coherence

Outro: of rabbits and sunflower

Fibonacci sequence: $d_p = 1, 1, 2, 3, \ldots$

$$\frac{d_p}{d_{p+1}} \to \phi$$

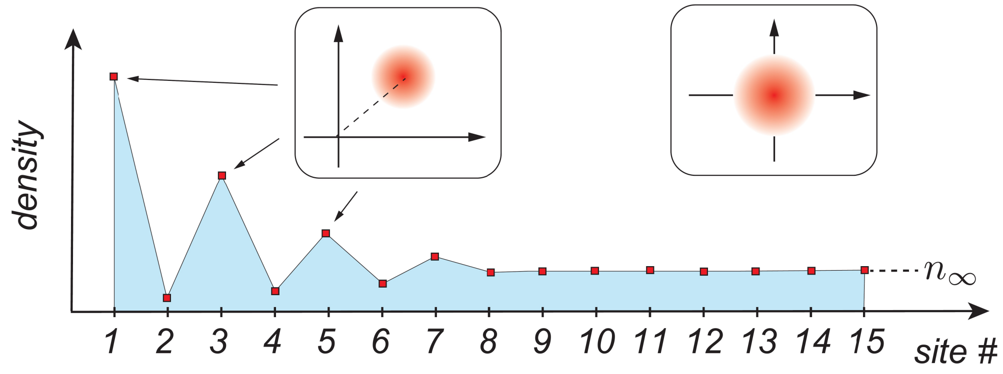


Here: $n_p \propto \frac{d_p}{d_{p+1}}$

$$d_{p+2} = d_{p+1} + a d_p$$

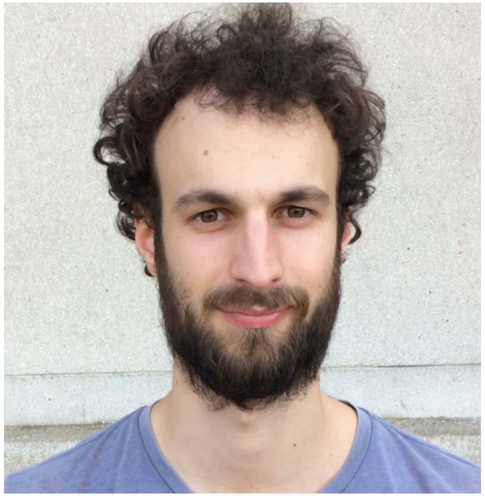
Lucas sequence

Summary



- Accumulation on the edge: bosonic skin effect
- Zig-zag phase with alternating thermal and 'condensed' state
- U(1) symmetry breaking and coherence in a purely dissipative transport scenario

Yuri Minoguchi



Julian Huber

Peter Rabl

Andrea Gambassi

L. Garbe, Y. Minoguchi, J. Huber, P. Rabl, arXiv:2301.11339

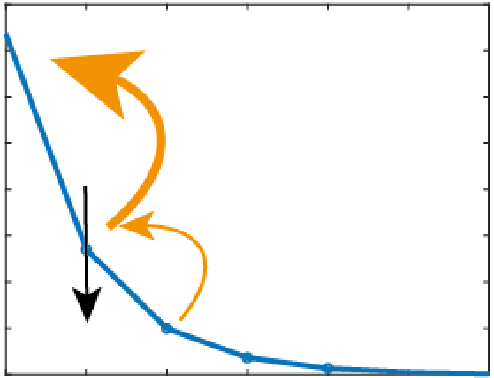
Y. Minoguchi, J. Huber, L. Garbe, A. Gambassi and P. Rabl, in preparation

Semi-intuitive explanation for the zigzag

$$J_{p,p+1} \simeq \Gamma_l n_{p+1} - \Gamma_r n_p + (\Gamma_l - \Gamma_r) n_p n_{p+1}$$

Extreme case: non-linear term only

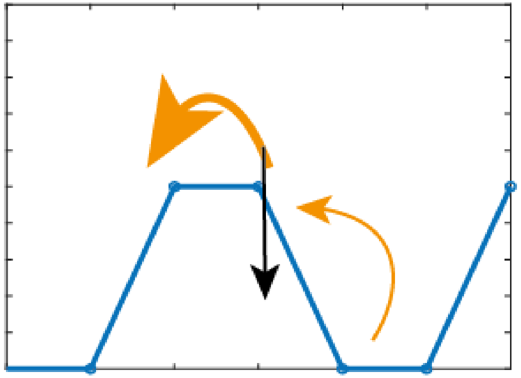
$$J_{p,p+1} \sim n_p n_{p+1}$$



$n_1 > n_2 > n_3$

$n_1 n_2 > n_2 n_3$

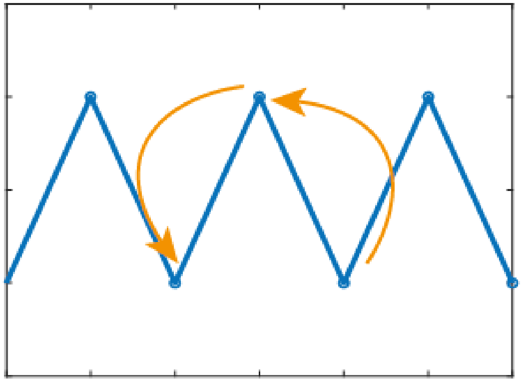
Unstable



$n_1 = n_2 > n_3$

$n_1 n_2 > n_2 n_3$

Unstable



$n_1 = n_3$

$n_1 n_2 = n_2 n_3$

Stable

Connection with non-Hermitian physics

Linearization: $n_p(t) = n_\infty + \epsilon_p(t)$ $\qquad$ $\frac{d\vec{\epsilon}}{dt} \sim -iH_{\text{eff}}\vec{\epsilon}$

Dynamical matrix gives a *non-Hermitian* Hamiltonian

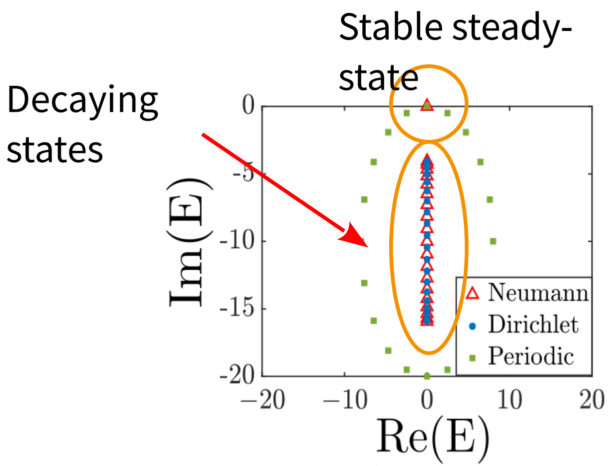
$$c = (\Gamma_r - \Gamma_l)n_\infty$$

$$\nu = \Gamma_r + \Gamma_l$$

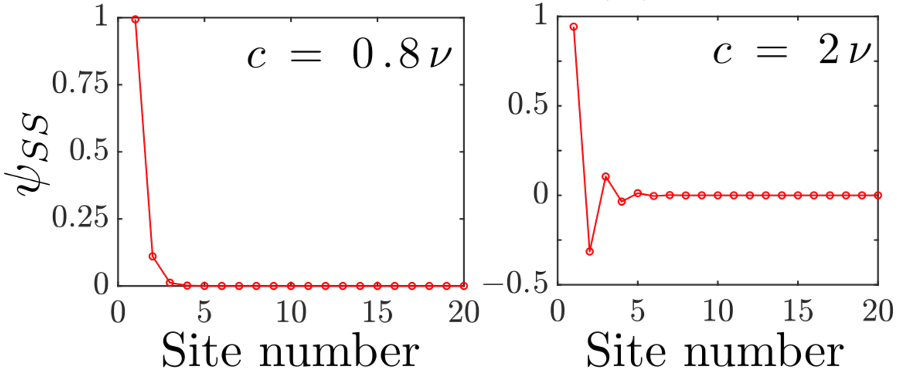
$$H_{\text{eff}} = i\begin{bmatrix} c+\frac{\nu}{2} & \frac{\nu}{2}+c & 0 & 0 & 0 & \dots \\ \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & 0 & 0 & \dots \\ 0 & \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & 0 & \dots \\ 0 & 0 & \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix} - i\nu\mathbf{1}$$

Connection with non-Hermitian physics

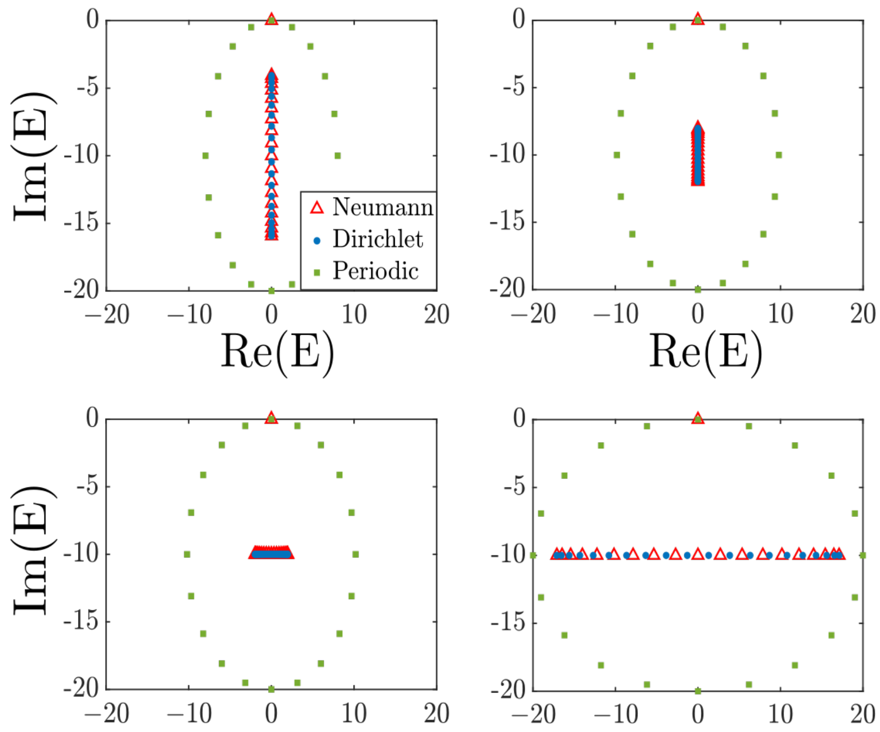
$$H_{\text{eff}} = i \begin{bmatrix} c+\frac{\nu}{2} & \frac{\nu}{2}+c & 0 & 0 & 0 & \dots \\ \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & 0 & 0 & \dots \\ 0 & \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & 0 & \dots \\ 0 & 0 & \frac{\nu}{2}-c & 0 & \frac{\nu}{2}+c & \dots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix} - i\nu\mathbf{1}$$



The steady-state shows a zig-zag transition for $c = \frac{\nu}{2}$



Connection with non-Hermitian physics



At the transition: excited states *coalesce*

$$H_{\text{eff}} = 2ci \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots \\ 0 & -1 & 1 & 0 & 0 & \cdots \\ 0 & 0 & -1 & 1 & 0 & \cdots \\ 0 & 0 & 0 & -1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots \end{bmatrix}$$

Non-diagonalizable Jordan form

→ *Exceptional point* in the excited states associated with the transition in the steady-state.