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Progression for Monitoring in Temporal ASP

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Abstract. In recent years, there has been growing interest in the application of temporal reasoning approaches and non-monotonic logics from artificial intelligence in dynamic systems that generate data. A well-known approach to temporal reasoning is the use of a progression technique, which allows for the online computation of logical consequences of a logical knowledge base over time. We consider a progression technique for Temporal Here and There and Temporal Equilibrium Logic, which is the logic underlying answer programming over linear-temporal logic (LTL). Compared to usual LTL online computation, where the goal is to check whether a trace is compliant with a temporal specification, our approach provides also the means to compute non-monotonic temporal reasoning over a trace of observations. Besides formal notions and results, we also present an algorithm for performing progression to monitor a dynamic system, which has been implemented as a proof of concept and allows for handling expressive application scenarios.

1 Introduction

With the increase of data-driven applications, reasoning about their behavior as they evolve over time has become crucial. Temporal logics provide a formal framework to specify the desired temporal behavior in an unambiguous way. Over the last decade, there has been a great effort in artificial intelligence to develop temporal reasoning approaches based on non-monotonic logics and answer set programming [10, 22] (ASP). A popular example is *Temporal Equilibrium Logic* [1, 2] (TEL), which combines Linear Temporal Logic [25] (LTL) and *Equilibrium Logic* [24] in an orthogonal way. TEL is a non-monotonic version of *Temporal Here-and-There Logic* [2] (THT), which is an intuitionistic version of LTL based on Heytings's Logic of Here-and-There [19] amounting to 3-valued Gödel Logic.

Tools for computing stable traces of temporal logic programs have been presented already in the literature, among them TELINGO [14], STELP [13], and the algorithm in [12]. The latter two are automata-based; a Büchi-based approach as in [13] may have to handle an exponential number of loop formulas to ensure the stability of traces. TELINGO instead uses *incremental ASP* and is limited to finite traces.

How to monitor TEL specifications over infinite traces is an open problem. In this paper, we take inspiration from rewriting-based techniques proposed in runtime verification [27], and from situation calculus [21] to reason about actions and in planning with goals specified using Metric Interval Temporal Logic [5]. However, all these works are based on classical semantics. The key idea of progression is to rewrite, as new observations of the system become available, an answer-set program expressing the obligations that need to be fulfilled in the future in order to have a true or a false verdict.

The main difference between TEL/THT and LTL is the semantics for the implication. TEL/THT is more expressive than LTL [2]: LTL formulas can be encoded into THT, but the converse is not always possible. Furthermore, LTL is monotonic while TEL is non-monotonic and thus more suited for handling exceptions and dealing with incomplete information. In contrast with rule-based languages such as temporal Datalog [29, 26], TEL/THT uses explicit temporal modal operators to reason about time instead of rules. In [28] there has been proposed a stream reasoning framework based on Metric Temporal Logic, under stable semantics.

Computing stable models (aka answer sets or equilibrium models) of logic programs where time is involved can be a challenging task, especially when dealing with infinite traces, which may be necessary when observing a reactive system. The behavior of such a system may depend on the environment; hence the possibility to monitor complex properties that require reasoning is desirable. In non-monotonic reasoning and logic programming, *skeptical* (*cautious*) reasoning is a common approach to inference, in which in a Tarskian manner the intersection of all answer sets of a program is considered. Our approach thus aims to compute the intersection of all prefixes of all stable traces of a temporal program, by taking also possible future observations on the system under scrutiny into account.

In order to compute prefixes of stable traces of the program, we introduce a novel online computation based on the idea of progression already used in runtime verification [7] and in planning where the goal is specified using Metric Temporal Logic [5]. To the best of our knowledge, this is the first time that this approach has been used in temporal answer set programming. In comparison to incremental ASP, this approach incorporates the observations into the program, which does not lead to an increase in the program size. Furthermore, it maintains a program at the symbolic level, making also visible which information is missing. It provides a basis for reasoning about future evolution in a flexible way, e.g., to find possible sequences of observations that will ensure a specification will be satisfied.

To validate our approach, we propose a temporal version of the well-known Σ_2^p strategic companies problem [15, 20], where we assume to have an incoming trace of observations about the evolution of ownership of companies and production of goods.

Our contribution We define a novel approach for monitoring by progression TEL/THT formulas over an infinite sequence of observations. In particular, we develop an algorithm to reason online on a stream of incoming observations. We provide a prototype and validate our approach with the temporal strategic companies problem.

Paper organization In Section 2 we provide the necessary background on THT and TEL. Section 3 and 4 introduce our approach of progression for monitoring TEL and THT, respectively. In Section 5

we present an algorithm for computing online the intersection of all the temporal traces of a given temporal program with an incoming trace of observations. In Section 6, we demonstrate an application of our approach to a case study on monitoring strategic companies and provide some experimental results. We discuss related work in Section 7 and conclude in Section 8 with an outlook on future work.

2 Preliminaries

As Equilibrium Logic (EL) [24] can be seen as the logic underlying Answer Set Programming (ASP) [24], TEL over infinite traces [2] can be interpreted as a pure logical temporal extension of ASP. Note that the definition of EL is based on Here-and-There Logic (HT), as TEL is defined over THT. The logics have the same formulas; we are interested in a fragment with the following syntax:

$$\begin{aligned} F &::= \bot \mid p \mid F \circ F \mid \mathbf{X}F \mid \mathbf{G}\ F \mid \mathbf{F}\ F \mid \mathbf{Y}\ F' \\ F' &::= \bot \mid p \mid \mathbf{Y}\ F' \end{aligned} \tag{1}$$

Where $p \in \mathcal{P}$ for a finite set $\mathcal{P}$ of propositional atoms and $\circ \in \{\wedge, \vee, \rightarrow\}$. Negation is defined as $\neg\phi \equiv \phi \rightarrow \bot$, and $\top \equiv \neg\bot$. The temporal operators $\mathbf{G}$ (globally) and $\mathbf{F}$ (finally) are defined as usual, viz. $\mathbf{G}\phi \equiv \bot\mathbf{R}\phi$ and $\mathbf{F}\phi \equiv \top\mathbf{U}\phi$ respectively. We denote by $\mathcal{L}$ the set of all the THT resp. TEL formulas.

The semantics of THT is defined over sequences of pairs of sets of atoms. A THT interpretation $\langle H, T\rangle$ is an infinite sequence of pairs $\langle H_i, T_i\rangle$ for $i \geq 0$, where $H_i \subseteq T_i$ for each i. In contrast, a TEL trace T can be viewed as a THT trace $\langle T, T\rangle$, and we may identify $\langle T, T\rangle$ by T if there is no confusion.

Definition 1 (THT-Satisfaction)**.** *Satisfaction of a THT formula by a THT-trace $I = \langle H, T\rangle$, at time k, where $k \geq 0$ is integer, is inductively defined as follows:*

- $I, k \vDash p$ *iff* $p \in H_k$, *for any atom* $p \in \mathcal{P}$
- $I, k \vDash \mathbf{Y}\ \phi$ *iff* $I, k-1 \vDash \phi$ *and* $k > 0$
- $I, k \vDash \phi \vee \psi$ *iff* $I, k \vDash \phi$ *or* $I, k \vDash \psi$
- $I, k \vDash \phi \wedge \psi$ *iff* $I, k \vDash \phi$ *and* $I, k \vDash \psi$
- $I, k \vDash \phi \rightarrow \psi$ *iff* $\begin{cases} \langle T, T\rangle, k \nvDash \phi \text{ or } \langle T, T\rangle, k \vDash \psi, \text{ and} \\ I, k \nvDash \phi \text{ or } I, k \vDash \psi \end{cases}$
- $I, k \vDash \mathbf{X}\ \phi$ *iff* $I, k+1 \vDash \phi$
- $I, k \vDash \phi\ \mathbf{U}\ \psi$ *iff there is* $j \geq k$ *s.t.* $I, j \vDash \psi$, *and for all* $j' \in [k, j-1]$, $I, j \vDash \phi$
- $I, k \vDash \phi\ \mathbf{R}\ \psi$ *iff for all* $j \geq k$ *s.t.* $I, j \nvDash \psi$, *there exists* $j' \in [k, j-1]$, $I, j \vDash \phi$
- $I, k \nvDash \bot$

A trace I is a model for a formula ϕ if $I, 0 \vDash \phi$.

We recall that $\langle T, T\rangle \vDash \phi$ if and only if $T \vDash_{LTL} \phi$ with $\phi \in \mathcal{L}$ [4]. An interpretation I is *total* if $H = T$. Furthermore, given two interpretations I and $\langle H', T\rangle$, and a trace of observations O such that $O_i \subseteq H_i$ and $O_i \subseteq H'_i$, for each $0 \leq i$, we say that $\langle H', T\rangle \leq_O \langle H, T\rangle$ if $H'_i \subseteq H_i$ for each $i \geq 0$. Intuitively, $\leq_O$ allows for H-minimality modulo observations; in our approach, observations are added online as facts and thus they do not need to be proven. To simplify the notation in the following sections if we do not have to make explicit the observation trace, we write $\langle H', T\rangle \leq \langle H, T\rangle$, instead of $\langle H', T\rangle \leq_O \langle H, T\rangle$. We are now ready to introduce the semantics of TEL.

Definition 2 (TEL-Satisfaction Modulo Observations)**.** *Given a trace of observations O, a trace T is an temporal equilibrium model of a formula $\phi \in \mathcal{L}$ modulo O if the following two conditions hold: (i) $\langle T, T\rangle \vDash \phi$, i.e., T is a total THT model of ϕ, and (ii) no $\langle H, T\rangle \leq_O \langle T, T\rangle$ s.t. $\langle H, T\rangle \vDash \phi$ exists, i.e., $\langle T, T\rangle$ has to be minimal modulo observations O.*

We note that if the trace of observations is the empty trace, Definition 2 collapses to classical TEL satisfaction [2]. Given two traces T and O, and a formula $\phi \in \mathcal{L}$, we denote by $T \vDash^O_{TEL} \phi$ that T is a equilibrium trace of ϕ modulo observations O. In case O is clear from the context, we may just write $T \vDash_{TEL} \phi$.

In the next sections, we will use interchangeably temporal equilibrium model and equilibrium/stable traces, furthermore, we will use a normal form for a generic THT resp. TEL formula $\phi \in \mathcal{L}$, called *temporal program*. The translation into normal form uses a Tseitin-style reduction and preserves equivalence under THT semantics and thus strong equivalence [3].

Definition 3 (Temporal program)**.** *Given a set $\mathcal{P}$ of propositional atoms, we define the set of* temporal literals *as $\{p, \neg p, \mathbf{X}p, \neg\mathbf{X}p, \mathbf{G}p, \mathbf{F}p\}$, where $p \in \mathcal{P}$. Atoms with the negation as failure in front of the atom are called* negative*, otherwise, they are called* positive*. A temporal rule is either:*

- *an* initial *rule of the form*

$$r : b_1 \wedge \ldots b_k \wedge \neg b_{k+1} \wedge \ldots \neg b_n \rightarrow c_1 \vee \cdots \vee c_l \tag{2}$$

  where all b_i, $c_j \in \{p, \mathbf{X}p\}$ and $\neg$ is negation as failure;
- *a* dynamic *rule of the form $\mathbf{G}r$, where r is an initial rule;*
- *a* fulfillment *rule of form either $\mathbf{G}(\mathbf{G}p \rightarrow q)$ or $\mathbf{G}(p \rightarrow \mathbf{F}q)$, where p, q are atoms.*

An initial or dynamic rule r is a constraint*, if its head is $\bot$, and is a* fact *if its body is empty (n=0) and its head is a single positive literal. A* temporal program *is any set of temporal rules.*

In the original definition of temporal program [11], negated literals were admitted in the head of a rule, while in Definition 3 we do not. We restrict the syntax to simplify our exposition in the upcoming sections. Note that by using a fresh auxiliary atom it is always possible to rewrite a rule with negation in the head into one without.

Temporal programs may be seen as a temporal extension of logic programs, which consist only of rules like (2), where $b_i, c_j \in \mathcal{P}$. We will use interchangeably the terms answer sets and stable models.

We introduce also some notations. If r is a temporal rule, we denote by $lits(r)$ the set of temporal literals appearing in r. Furthermore, let $B(r)$ and $H(r)$ be the set of temporal literals occurring, respectively, in the body and in the head of the rule r. Moreover, let $B^+(r)$ be the set of positive literals in $B(r)$, and $B^-(r)$ be the set of negative literals in $B(r)$. We also use the shortcuts $lits^+(r) = B^+(r) \cup H(r)$, and $lits^-(r) = B^-(r)$.

3 Progression for THT

In online computation, we usually do not have the full trace, but only a prefix of it. We thus propose the following THT_3 semantics. A prefix of a THT-trace I is any sequence $I^f = \langle H^f, T^f\rangle = \langle H_0, T_0\rangle, \ldots, \langle H_k, T_k\rangle$ (the prefix of length $k+1$) while a suffix of I is any sequence $I^{k,\cdots} = \langle H_k, T_k\rangle, \langle H_{k+1}, T_{k+1}\rangle, \ldots$ (the suffix at k or k-suffix), where $k \geq 0$. A THT-prefix is a prefix of any THT-trace I; by Pre_{THT} we denote the set of all THT-prefixes. For any prefix I^f and trace O, a THT-trace I is an extension if I^f is a prefix of I and $O \leq H$; by $ext(I^f, O)$ we denote the set of all such extensions.

Definition 4 (THT_3 semantics)**.** *The truth value of $\phi \in \mathcal{L}$ with respect to a THT-prefix I^f and a trace O of observations is as follows:*

$$I^f \models^O_{THT_3} \phi = \begin{cases} \top & \text{if } I \models \phi \text{ for every } I \in ext(I^f, O), \\ \bot & \text{if } I \not\models \phi \text{ for every } I \in ext(I^f, O), \\ ? & \text{otherwise.} \end{cases}$$

Note that in case O is the empty trace, Definition 4 is the Temporal Here and There version of the LTL_3 logic proposed in [7]. We add the observation trace O as a parameter since we use Definition 4 for a 3-valued logic for TEL, where minimality on the trace matters.

In order to process one state at a time, we resort to the concept of progression and introduce it for THT. In that, we omit the temporal operators $\mathbf{U}$ and $\mathbf{R}$, which do not appear in the normal form, and focus on $\mathbf{F}$ and $\mathbf{G}$.

In the progressive evaluation of a THT formula, we may be able to evaluate an implication $p \rightarrow \mathbf{X}q$ only partially in the current state, and we must delegate the remaining part of the evaluation to the future. To this end, we introduce $\rightarrow_c$ as a new type of implication for evaluation in the *There* part of the trace, in order to ensure that the remaining evaluation is compliant with the THT semantics.

We denote by $\mathcal{L}^P$ the set of formulas generated by the grammar in (1), where in place of $\rightarrow$ also $\rightarrow_c$ may occur. Note that we exclude nesting of $\mathbf{G}, \mathbf{F}, \mathbf{X}$ into $\mathbf{Y}$ operators. We can now introduce the definition of THT progression.

Definition 5 (THT progression on a state of a prefix)**.** *Progression $P_{THT} : \mathcal{L}^P \times Pre_{THT} \times \mathbb{N} \rightarrow \mathcal{L}^P$ is the partial function that maps a formula ψ, a THT-prefix $I^f = \langle H^f, T^f \rangle$ of length k, and an integer i such that $0 \leq i < k$ to an $\mathcal{L}^P$ formula as follows:*

- $P_{THT}(\bot, I^f, i) = \bot$
- $P_{THT}(p, I^f, i) = \top$ *if* $p \in H^f_i$, *and* $p \in \mathcal{P}$
- $P_{THT}(p, I^f, i) = \bot$ *if* $p \notin H^f_i$, *and* $p \in \mathcal{P}$
- $P_{THT}(\mathbf{Y}\phi, I^f, i) = P_{THT}(\phi, I^f_{i-1})$ *if* $i > 0$, *otherwise* $\bot$
- $P_{THT}(\phi_1 \vee \phi_2, I^f, i) = P_{THT}(\phi_1, I^f, i) \vee P_{THT}(\phi_2, I^f, i)$
- $P_{THT}(\phi_1 \wedge \phi_2, I^f, i) = P_{THT}(\phi_1, I^f, i) \wedge P_{THT}(\phi_2, I^f, i)$
- $P_{THT}(\mathbf{X}\,\phi, I^f, i) = \phi$
- $P_{THT}(\phi_1 \rightarrow \phi_2, I^f, i) =$
  $\begin{cases} P_{THT}(\phi_1, I^f, i) \rightarrow P_{THT}(\phi_2, I^f, i) \wedge \\ P_{THT}(\phi_1, \langle T^f, T^f \rangle, i) \rightarrow_c P_{THT}(\phi_2, \langle T^f, T^f \rangle, i) \end{cases}$
- $P_{THT}(\phi_1 \rightarrow_c \phi_2, I^f, i) =$
  $P_{THT}(\phi_1, \langle T^f, T^f \rangle, i) \rightarrow_c P_{THT}(\phi_2, \langle T^f, T^f \rangle, i)$
- $P_{THT}(\mathbf{G}\phi, I^f, i) = P_{THT}(\phi, I^f, i) \wedge \mathbf{G}\phi$
- $P_{THT}(\mathbf{F}\phi, I^f, i) = P_{THT}(\phi, I^f, i) \vee \mathbf{F}\phi$

In addition, $\top \rightarrow^* \bot$ is replaced by $\bot$; $\bot \rightarrow^* \phi$ by $\top$; and $\phi \rightarrow^* \top$ by $\top$, for each formula ϕ and $\rightarrow^* \in \{\rightarrow, \rightarrow_c\}$. Furthermore, $\top \vee \phi$ is replaced by $\top$; $\bot \vee \phi$ by $\bot$; $\bot \wedge \phi$ by $\bot$; and $\top \wedge \top$ by $\top$.

Note that we do not apply the progression recursively on the future states, but we indeed apply it recursively on the sub-prefix of the trace, as we assume to have access to the current and past states.

Let us denote the recursive application of the progression over a finite trace of length k in the following way.

Definition 6 (THT progression over prefixes)**.** *For any THT-prefix of length k and formula $\phi \in \mathcal{L}$, the application of the progression to ϕ over I^f is defined as*

$$P_{THT}(\phi, I^f) \equiv P_{THT}(\dots P_{THT}(\phi, I^f, 0) \dots, I^f, k-1) \quad (3)$$

We have now all the definitions needed to state the main result of this section.

Theorem 1 (THT verdict on prefixes)**.** *For every THT-prefix I^f, trace O of observations, and formula $\phi \in \mathcal{L}$, progression leads to the same verdict of the $\models^O_{THT_3}$ semantics, i.e.,*

$$P_{THT}(\phi, I^f) = v \implies I^f \models^O_{THT_3} \phi = v, \text{ for } v \in \{\top, \bot\}.$$

4 Progression for TEL

For progression of TEL formulas, we start with a 3-valued semantics.

Definition 7 (TEL_3 semantics)**.** *Let T^f be a TEL-prefix of length k, $\phi \in \mathcal{L}$, and O be a trace of observations. Then*

$$T^f \models^O_{TEL_3} \phi = \begin{cases} \top & \text{if } T^f O^{k,\cdots} \models^O_{TEL} \phi, \\ \bot & \text{if } T \not\models^O_{TEL} \phi \,\forall\, \langle H, T \rangle \in ext(\langle T^f, T^f \rangle, O), \\ ? & \text{otherwise.} \end{cases}$$

The following example explains why we require minimal LTL models in Definition 7.

Example 1. *Let us consider first $\phi = \mathbf{G}(\neg\neg p \rightarrow p)$, i.e., a TEL tautology. In this case, we have that each trace T is a temporal equilibrium trace, and we conclude that $T^f \models^O_{TEL_3} \phi = \top$ for any possible O.*

Let us focus on an LTL tautology that is not a TEL tautology. Let $\phi = \top$, and $T^f = \emptyset$ is the prefix of lengh 1. Then, because of minimality, the only possible extension that is a TEL trace is the $T^f O^{1,\cdots}$, which is indeed the minimal LTL trace modulo observation O. Therefore, $T^f \models^O_{TEL_3} \phi = \top$.

Given $T = \emptyset$, let us analyze now a more interesting LTL tautology, $\phi = \mathbf{GF}p \vee \mathbf{GF}\neg p$. Thanks to minimality in the Here, the only extension of T that is an equilibrium trace is the one $p \in T_i$ if and only if $p \in O_i$. We conclude also in this case that $T^f \models^O_{TEL_3} \phi = \top$.

Definition 8 (TEL progression on a prefix of a trace)**.** *Let T^f be a prefix of a TEL trace and $\phi \in \mathcal{L}$ a TEL formula. Then the TEL progression of ϕ on T^f is defined as*

$$P_{TEL}(\phi, T^f) = \begin{cases} \top & \text{if } \phi' = \top, \text{ and } \psi(H^f) = \bot \,\forall H^f \subset T^f \\ \bot & \text{if } \phi' = \bot, \text{ or } \psi(H^f) = \top \,\exists H^f \subset T^f \\ ? & \text{otherwise,} \end{cases}$$

where $\phi' = P_{THT}(\phi, \langle T^f, T^f \rangle)$, $\psi(H^f) = P_{THT}(\phi, \langle H^f, T^f \rangle)$.

Theorem 2 (TEL verdict on prefixes)**.** *Let T^f be a TEL-prefix of length k, O be a trace of observations, and ϕ be a formula. Then progression leads to the same verdict of the $\models_{TEL_3}$ semantics, i.e.,*

$$P_{TEL}(\phi, T^f) = v \implies T^f \models^O_{TEL_3} \phi = v, \text{ for } v \in \{\top, \bot\}.$$

Example 2 ($\mathbf{GF}\ \neg p$)**.** *Let $\phi = \mathbf{GF}\ \neg p$, $O = \emptyset^\omega$ and $T^f = \emptyset$, then $P_{TEL}(\phi, T^f) = ?$, and $T^f \models_{TEL_3} \phi = \top$. Therefore,*

$$T^f \models^O_{TEL_3} \phi = \top \not\Longrightarrow P_{TEL}(\phi, T^f) = \top$$
$$P_{TEL}(\phi, T^f) = ? \not\Longrightarrow T^f \models^O_{TEL_3} \phi = ?$$

Example 3 ($\mathbf{GF}\ p$)**.** *Let $\phi = \mathbf{GF}\ p$, $O = \emptyset^\omega$ and $T^f = \emptyset$, then $P_{TEL}(\phi, T^f) = ?$, and $T^f \models_{TEL_3} \phi = \bot$. Therefore,*

$$T^f \models^O_{TEL_3} \phi = \bot \not\Longrightarrow P_{TEL}(\phi, T^f) = \bot,$$
$$P_{TEL}(\phi, T^f) = ? \not\Longrightarrow T^f \models^O_{TEL_3} \phi = ?$$

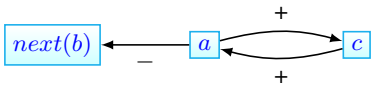

Figure 1: Dependency graph of the program of Example 4

We also notice that both the three-valued THT and TEL logic introduced in Definitions 4 and 7 take the observation trace into account, but the respective definitions of progression 5, 8 do not. The reason is that we are progressing a prefix of a trace and we assume to have the observations already encoded in the trace. Given a finite trace $I^f = \langle H^f, T^f \rangle$, we extend the definition of $<_O$ over finite prefixes by $H^f <_O T^f$ if for each $i = 0, \ldots, k-1$, $O_i \subseteq H_i$ and exists $i \in \{0, \ldots, k-1\}$ such that $H_i \subset T_i$. If we replace $H^f \subset T^f$ with $H^f <_O T^f$ in Definition 8, Theorem 2 still holds.

5 Computing the Intersection of TEL Traces

In this section, we describe an algorithm to compute online the intersection of all the equilibrium traces of a given temporal program π, and an incoming trace of observations O. During the computation, it may happen that the truth value of one atom is determined not only by other non-future atoms, but also by future atoms, e.g., `p:- X(q).`. In this case, if we are not able to determine the truth value of the atom, we may delay the computation, by pastifying an atom, i.e. adding a previous operator, obtaining, for instance, `Y(p) :- q.`. Furthermore, we may instantiate some dynamic or fulfillment rules, adding them to the initial part of the temporal program. Therefore, during the computation, we may deviate from the Definition 3.

In order to clearly define the set of atoms whose truth value we can compute, we proceed by introducing the dependency graph for programs with possibly disjunctive rules.

Definition 9 (Dependency graph)**.** *The* dependency graph *of a logic program π is the directed graph $DG_\pi = \langle N, E \times \{+, -\} \rangle$ where (i) each atom of π is a node in N, (ii) there is a positive (resp. negative) arc in E from a node a to a node b if $a \in H(r)$ and $b \in B^+(r)$ (resp. $b \in B^-(r)$) for some rule r in π, and (iii) for every rule r in π and $a \neq b \in H(r)$, there is a positive arc from a to b in E.*

Given the definition of the dependency graph, we can now introduce the following concept.

Definition 10 (Negative dependency)**.** *Given a logic program π, and its corresponding dependency graph DG_π, we say that an atom p* depends negatively *on q if there exists a path from p to q passing through a negative arc.*

Furthermore, we say that a set U of atoms *is closed under negative dependencies* if $q \in U$ holds for every atom q such that some atom $p \in U$ negatively depends on q.

Example 4. *If we apply Definition 9 to the following logic program, we obtain the dependency graph in Figure 1.*

$$\neg\, next(b) \rightarrow a, \quad a \vee b, \quad next(b)$$

Furthermore, by applying Definition 10, we see that $U = \emptyset$ is the only set closed under negative dependencies not containing $next(b)$, as both a and b depend negatively on $next(b)$.

In order to compute the intersection of all the equilibrium traces with a stream of incoming observations, we resort to some splitting techniques to decompose the program into a *lower* and an *upper* part. Intuitively, the lower part refers to the part of the program related to the current and the past states, while the upper part is related to future states. As we are interested in deriving as many facts as soon as possible, we extend the Splitting Theorem. We define two new functions that resemble the e_U and b_U functions of the Splitting Theorem [22], respectively $filter$ and $progress$. Instead of requiring U to be a splitting set, we have the more relaxed requirement of being a set closed under negative dependency. We first define $filter$.

Definition 11 (Filter)**.** *Given a logic program π and $U \subseteq \mathcal{P}$, we let $filter(\pi, U)$ consist of all rules $r \in \pi$ that contain only atoms from U and where each negative literal $\neg p$ occurring in r depends only on atoms in U.*

We are now ready to introduce the reduction $skep\_prog(\pi, C, B, U)$. Besides the program π, it takes as input a set C of atoms that are considered to be proved, a set B of atoms that may possibly be proved, and a set U of atoms closed under negative dependencies.

Definition 12 (Skeptical Progress)**.** *Let us consider a logic program π, and $C, B, U, U^s \subseteq \mathcal{P}$, where U^s is the maximal splitting set of π contained in U. Let us define $skep\_prog(\pi, C, B, U) = \pi'$ to be obtained by removing all the rules $r \in \pi$, such that either*

- *there is some $p \in H(r)$ such that $p \in C$,*
- *there is some $p \in B^+(r)$ such that $p \in U^s \setminus B$, or*
- *there is some $p \in B^-(r)$ such that $p \in C$.*

From each remaining rule r, all positive literals $p \in$ are deleted, and all negated literals $\neg p$ such that $p \in U^s \setminus B$ are deleted.

The following theorem says that by a proper application of the filter function on the progress function, we are able to compute an answer set of the program π. The intuition behind Theorem 3 is that given U, you can filter rules that are defeasible with respect to atoms in U only, where a rule is defeasible if its body depends on some negated by default literal.

Theorem 3 (Non-Defeasible Splitting Theorem)**.** *Let U be a set of atoms closed under negative dependencies w.r.t. a logic program π. Then π has an answer set Z only if $Z = X \cup Y$ such that*

- *X is an answer set of $filter(\pi, U)$ and*
- *Y is an answer set of $skep\_prog(\pi, X, X, U)$.*

The converse holds if each rule $r \in filter(\pi, U)$ with disjunctive head satisfies $H(r) \cap (U \setminus U^s) = \emptyset$, where U^s is the maximal splitting set of π contained in U.

If we compare Theorem 3 with the well-known Splitting Theorem [22], it is interesting to stress that Theorem 3 uses a notion of being closed *under defeasiblity*, while the Splitting Theorem requires the splitting set to be closed *under definition*, i.e., for each atom p in splitting set S, all atoms occurring in a rule r that defines p should be contained in S as well. Notably, for normal (disjunction-free) logic programs Theorem 3 extends the Splitting Theorem, providing an if and only if characterizations of answer sets.

As we are interested in the computation of the intersection of equilibrium traces, we exploit Theorem 3 to prove the following.

Theorem 4 (Skeptical Non-Defeasible Splitting Theorem)**.** *Suppose $U \subseteq \mathcal{P}$ is closed under negative dependencies w.r.t. a logic program π. Let C and C_X (B and B_X) be the skeptical (brave) consequences of π and $filter(\pi, U)$, respectively, and let C_Y (B_Y) be the skeptical (brave) consequences of $skep\_prog(\pi, C_X, B_X, U)$. Then $C_X \cup C_Y \subseteq C$ and $B \subseteq B_X \cup B_Y$.*

We remark that for normal logic programs π, we in fact can show that in Theorem 4 $C_X \cup C_Y = C$ and $B = B_X \cup B_Y$ holds, i.e. the cautious and brave conclusions remain invariant under progression.

Even if the results just stated are related to logic programs in general, in our framework we are interested in selecting U as the largest set of atoms closed under negative dependency containing only past atoms or present atoms, currently appearing in the temporal program. I.e., atoms preceded only by a non-negative number of previous operators, that is $Y^i p$ with $i \geq 0$. For convenience, let us introduce the notation $history\_atoms(\pi_{init}) = \{\mathbf{Y}^i p \mid \exists r \in \pi_{init}$ and $\mathbf{Y}^i p \in lits^+(r) \cup lits^-(r), i \geq 0\}$ for the past time literals in π_{init}.

Now we have all terminology to present our reasoning algorithm (Algorithm 1). At the very beginning, it copies the initial segment of the temporal program in π_i (line 2), then it instantiates rules from the dynamic part and from the fulfillment part via the function $inst$, adding them to the initial part of the program (line 4).

Next, it computes U_i, the set of current and past atoms such that they are closed under negation (line 5). It filters out the program π_i using U as the filtering parameter, obtaining a new filtered program π_f (line 6). If there is no *local answer set*, instability is detected and it is signaled to the user (line 8). Otherwise, if π_f admits a stable model, its skeptical and brave consequences (respectively C_i, and B_i) are computed, and the skeptical consequences C_i are fed into the output trace (lines 10-12). Once C_i and B_i are available, they can be used to simplify the program via the application of *skep_prog* (line 13). The intuition of (line 14) is to add a previous operator before atoms appear in the initial segment (see Defn. 13 for details).

Algorithm 1 Main algorithm. Input: π, O. Output: Skeptical trace

1: $i := 0$
2: $\pi_c^i := \pi_{init}$
3: **while** $\top$ **do**
4: $\quad \pi_c^i := \pi_c^i \cup inst(\pi_{dyn}) \cup inst(\pi_{ful}) \cup O_i$
5: $\quad U_c^i := history\_atoms(\pi_c^i) \setminus get\_neg\_deps(\pi_c^i)$
6: $\quad \pi_f := filter(\pi_c^i, U_c^i)$
7: $\quad$ **if** π_f does not admit any answer set **then**
8: $\quad\quad noStableTraceError$
9: $\quad$ **end if**
10: $\quad C_i := skeptical\_conseq(\pi_f)$
11: $\quad B_i := brave\_conseq(\pi_f)$
12: $\quad feed\_skeptical\_trace(C_i)$
13: $\quad \pi_c^i := skep\_prog(\pi_c^i, C_i, B_i, U_i)$
14: $\quad \pi_c^{i+1} := pastify(\pi_c^i)$
15: $\quad i := i + 1$
16: **end while**

Definition 13 (Pastify). *Given a temporal program of form $\pi = \pi_{init}$, $pastify(\pi)$ results by rewriting its rules as follows: rewrite*

- $\bigvee_{k=1}^{m} \mathbf{Y}^{i_k} b_k \bigvee_{k=m+1}^{n} \neg \mathbf{Y}^{i_k} b_k \rightarrow \bigvee_{k=1}^{l} \mathbf{Y}^{j_k} c_k$ *to* $\bigvee_{k=1}^{m} \mathbf{Y}^{i_k+1} b_k \bigvee_{k=m+1}^{n} \neg \mathbf{Y}^{i_k+1} b_k \rightarrow \bigvee_{k=1}^{l} \mathbf{Y}^{j_k+1} c_k$;
- $\mathbf{Y}^i q \rightarrow \mathbf{Y}^i p \vee \ldots p \vee \mathbf{F} p$ *to* $\mathbf{Y}^{i+1} q \rightarrow \mathbf{Y}^{i+1} p \vee \ldots p \vee \mathbf{F} p$;
- $\mathbf{G} q \wedge q \wedge \ldots \mathbf{Y}^i q \rightarrow \mathbf{Y}^i p$ *to* $\mathbf{G} q \wedge q \wedge \ldots \mathbf{Y}^{i+1} q \rightarrow \mathbf{Y}^{i+1} p$.

Note that each while-iteration in Alg.1 is feasible in polynomial time with a Σ_2^p oracle, but the program size may grow linearly.

In what follows, we present different results with the aim of showing that the algorithm computes an approximation of the intersection of all equilibrium traces of the input program π modulo observations O. For simplicity, we assume that the observations added in Algorithm 1 as facts at run-time are already encoded in the input program. Set operations such as intersection, union, and set minus over traces must be considered state-wise.

Definition 14 (Unfolding). *Given a temporal program π and $k \geq 0$, the temporal program $unfold(\pi, k)$ contains (i) all rules in π_{init} and (ii) for each rule $r = \bigwedge_{j=1}^{k} b_j \wedge \bigwedge_{j'=k+1}^{n} \neg b_{j'} \rightarrow \bigvee_{h=1}^{l} c_h$, where all b_js, $b_{j'}$ and c_h are positive temporal literals, in $\pi_{dyn} \cup \pi_{ful}$ the rules $r_i = r[\mathbf{X}^i]$, for each $0 \leq i \leq k$, where*

$$r[\mathbf{X}^i] = \bigwedge_{j=1}^{k} \mathbf{X}^i b_j \wedge \bigwedge_{j'=k+1}^{n} \neg \mathbf{X}^i b_{j'} \rightarrow \bigvee_{h=1}^{l} \mathbf{X}^i c_h$$

Notice that $unfold(\pi, k)$ can be viewed as a non-temporal ASP program that contains a set of atoms from $\mathcal{P}^k = \{\mathbf{X}^i p : p \in \mathcal{P}$ for $i = 0, \ldots, k\}$. Given a set $T^k \subseteq \mathcal{P}^k$, we define $trace(T^k)$, as the prefix of a trace starting from T^k, as follows. If $p \in T^k$, then $p \in trace(T^k)_0$. And, if $\mathbf{X}^i p \in T^k$, then $p \in trace(T^k)_i$ for each $0 < i \leq k$. Using this notation, we can introduce the limit version for $k \rightarrow \infty$, obtaining $\mathcal{P}^\omega$, $\pi^\omega = unfold(\pi)^\omega$, where $unfold(\pi) = unfold(\pi, 1)$. Furthermore, if T^ω is a set of atoms in π^ω, then $trace(T^\omega)$ is the corresponding infinite trace. In order to simplify notation, we will use $T \equiv trace(T^\omega)$.

Theorem 5 (Trace Equivalence). *Let π be a temporal program without fulfillment rules. Then T^ω is a stable model for π^ω iff $trace(T^\omega)$ is an equilibrium trace for π. Furthermore, if in the latter case $trace(T^\omega) \vDash \pi_{ful}$ for a set π_{ful} of fulfillment rules, then $trace(T^\omega)$ is an equilibrium trace for $\pi \cup \pi_{ful}$.*

Let $\pi^\omega = unfold(\pi)^\omega$ be an unfolded temporal program π, $\pi_p^0 = \pi^0$ and U^i maximal subset of $\cup_{j=0,\ldots,i} \mathcal{P}^j$ closed under negated dependencies in π_p^i. Let us denote by T^i a generic answer set of $filter(\pi_p^i, U^i)$. We define $\pi_p^{i+1} = skep\_prog(\pi^i, T^i, T^i, U^i)$ for some T^i non-deterministically chosen. Then,

Theorem 6 (Sequence Non-Defeasible Splitting). *Let π be a temporal program without fulfillment rules. If π admits a temporal equilibrium model T, then $T = trace(\cup_{i \geq 0} T^i)$, for some sequence $T^0, T^1, \ldots$ is an equilibrium trace of π. Furthermore, if $T \vDash \pi_{ful}$ for some set π_{ful} of fulfillment rules, then $T \vDash \pi \cup \pi_{ful}$.*

Theorem 5 and 6 just presented pertain to properties that establish a relationship between models of the unfolded program π^ω and those of the original one π, as well as a property about splitting a temporal program using the newly introduced concept of Non-Defeasible Splitting. However, in certain cases, it may be possible to observe a finite number of equilibrium traces and a large number of local answer sets at each step of the computation. Consequently, determining a single trace would require an accurate guess of the T^i stable model at time step i to be utilized during the program's evolution. Since a non-deterministic choice is involved, some backtracking procedures would also be required. To address this challenge, our proposed algorithm aims to compute an approximation of the unique intersection of all the equilibrium traces instead. Theorem 5 and 6 can be used to prove their skeptical counterpart.
Denote by $AS(\pi)$ the set of all stable models of a logic program π.

Theorem 7 (Skeptical Equivalence). *Let π be a temporal program without fulfillment rules. If π admits an equilibrium trace, then $trace(\bigcap AS(\pi^\omega)) = \bigcap TEL(\pi)$.*

Let π^ω be the unfolded temporal program π. $\pi_\cap^0 = \pi^0$ and U^i maximal subset of $\cup_{j=0,\ldots,i} \mathcal{P}^j$ closed under negative dependencies in $\pi_\cap^i$. Let us denote by C^i, and B^i, respectively, the skeptical

and the brave consequences of $filter(\pi_p^i, U^i)$. We define $\pi_\cap^{i+1} = skep\_prog(\pi_\cap^i, C^i, B^i, U^i)$. Then,

Theorem 8 (Sequence Skeptical Non-Defeasible Splitting)**.** *Let π be a temporal program where $\pi_{ful} = \emptyset$. If π has an equilibrium trace,*

$$trace(\bigcup_{i\geq 0}\bigcap AS(\pi_\cap^i)) \subseteq \bigcap TEL(\pi) \text{ and}$$
$$trace(\bigcup_{i\geq 0}\bigcup AS(\pi_\cap^i)) \supseteq \bigcup TEL(\pi).$$

We can extend Theorems 7 and 8, which are already applicable to a temporal program $\pi = \pi_{init} \cup \pi_{dyn}$, to include also a set of fulfillment rules π_{ful} under some assumption. Syntactically, (i) for each rule $r \in \pi_{ful}$ such that $p \in H(r)$ or $\mathbf{F}p \in H(r)$, p can occur only in heads of π, or with r added, would not feed back to the component in which q occurs in a modular program decomposition such as program splitting. Otherwise, (ii) for $\mathbf{F}p \in H(r)$, we may instead require the observation trace O to be *fair* with respect to p, i.e., infinitely many observations of p must occur. Under such constraints, if $trace(T) \vDash_{TEL} \pi_{ful}$ holds for each $T \in AS(\pi^\omega)$, then we can replace $TEL(\pi)$ with $TEL(\pi \cup \pi_{ful})$ in both Theorems 7 and 8. Intuitively, case (i) holds because p can be proved at the very last step and its truth value does not affect the other atoms in the answer set. Case (ii) holds because thanks to *fairness* on the observation, if we prove $\mathbf{F}p$, we do not have to do any guess where to add p in the trace, but just wait for the next occurrence in O thanks to minimality.

In Theorems 5–8 we deal with the π_{ful} part of a program differently from the π_{init} and π_{dyn} parts. Let us consider the following simple program $\pi = \pi_{init} \cup \pi_{dyn} \cup \pi_{ful}$, where $\pi_{init} = \{p\}$, $\pi_{dyn} = \{\neg p, q \rightarrow \bot\}$ and $\pi_{ful} = \{p \rightarrow \mathbf{F}q\}$. There is only one equilibrium trace for π, viz. the trace $\{p, q\}, \emptyset^\omega$, which trivially coincides with the intersection of all equilibrium traces. However, if we proceed as in Algorithm 1, we only compute the local intersection and union of all the stable models, but we never make a guess where to fulfill $\mathbf{F}q$. Therefore, we can fulfill in our approach a promise $\mathbf{F}q$ only if we derive q by some other rule in the initial or the dynamic part. On the other hand, by admitting some fulfillment rules more expressive possibilities are offered.

We point out that we cannot apply Theorem 8 directly to Algorithm 1 because it refers to the unfolded version of the input program π^ω, while in Algorithm 1 rules are added at runtime and the state-counter i is incremented at each step. However, we can still exploit Theorem 8 as it is not hard to see that for each rule r, we have $I, i \vDash r \iff I, i+1 \vDash pastify(r) \iff I, 0 \vDash \mathbf{X}^i r$.

6 Case study: Temporal Strategic Companies

In this section, we take the Strategic Companies [18] (Σ_p^2-complete) problem to show that we can reason online about it. We will conclude the section by analyzing how the program size grows over time taking the Strategic Companies problem as a running example.

In the original problem, a collection $C = c_1, ..., c_m$ of companies is given. Each company c_i produces some goods from a set G of goods and is possibly controlled by a set $W_i \subseteq C$ of owner companies, for each $i = 1, \ldots, m$. A set $C' \subseteq C$ is a "strategic set" of companies if it is $\subseteq$-minimal among all sets such that (1) the companies in C' produce all goods in G, and (2) if $W_i \subseteq C'$, all companies c_i owned by W_i must belong to C'.

For the presentation of this problem, we will assume that (i) each product is produced by at most two companies, and (ii) each company is controlled by at most two companies.

That product $g \in G$ is produced by c_0 and c_1 is represented by $prBy(p, c_0, c_1)$, that c_0 and c_1 control c by $ctrBy(c, c_0, c_1)$, and that c_i belongs to the strategic set by $str(c_i)$.

With this notation, we can now encode the previous conditions:

$$prBy(P, C_0, C_1) \rightarrow str(C_0) \vee str(C_1),$$
$$ctrBy(C, C_0, C_1) \wedge str(C_0) \wedge str(C_1) \rightarrow str(C).$$

A company c_i is *unnecessary*, $unn(c_i)$, if it does not belong to the strategic set; this is expressed by

$$\neg str(C) \rightarrow unn(C).$$

Since the set of ownerships and the set of companies producing each good can change over time, in this example, we want to reason whether two given companies can be unnecessary in two consecutive time steps. For this purpose, we will include the corresponding property for each company consisting of the following two rules:

$$unn(c_i) \wedge \mathbf{X}(unn(c_i)) \rightarrow prop(c_i),$$
$$\neg prop(c_i) \rightarrow negProp(c_i).$$

By introducing $prop(c_i)$, we are able to identify which property has been violated. This is a difference from usual LTL monitoring, in which one single automaton combining all the properties is built, which does not allow distinguishing between violations.

Unless new information is given, ownership and productions remain unchanged. This *inertia* principle can be encoded as follows:

$$prBy(P, C_0, C_1) \wedge \neg\mathbf{X}(chg(pr(P))) \rightarrow \mathbf{X}(prBy(P, C_0, C_1)),$$
$$prBy(P, C_0, C_1), \mathbf{X}(prBy(P, C_2, C_3)) \rightarrow \mathbf{X}(chg(pr(P))),$$

where $C_0 \neq C_2$ or $C_1 \neq C_3$; similarly for ownerships.

We may monitor temporal properties on the observations, such as:

$$prBy(p, c_0, c_1) \rightarrow \mathbf{F}(prBy(p, c_0, c_0) \vee prBy(p, c_1, c_1)), \quad \text{(P1)}$$
$$prBy(p, c_0, c_1) \rightarrow$$
$$prBy(p, c_0, c_1)\ \mathbf{U}\ (prBy(p, c_0, c_0) \vee prBy(p, c_1, c_1)) \quad \text{(P2)}$$

Property P1 requires that if the product p is produced by c_0 and c_1 at a certain point, that product will eventually only be produced by one of the two companies. Property P2 indicates a convergence in the companies producing a good. That is, if product p is produced by companies c_0 and c_1, they must keep producing it until it is only produced by one of them. While we omitted the until operator $\mathbf{U}$ here, it can be expressed in the normal form using the $\mathbf{X}$ and $\mathbf{F}$ operators and auxiliary predicates. As already mentioned after Theorem 8, fairness in the observations is assumed to ensure the approach remains valid.

We encoded this problem with the presented properties in an open-source prototype[1]. that uses the Clingo [17, 23] API for Python. A user can input a temporal logic program and enter new observations at each time step, as well as translate a TEL formula into normal form. The number of companies (3) and products (2) was not changed for the different tests as it did not affect the patterns we observed. The property related to a company being unnecessary in two consecutive steps was also successfully included but it did not affect the number of rules. The results of the tests are shown in Figure 2.

The number of total rules for the initial step is 288 (3 initial, $2 \cdot 1$ fulfillment, $2 \cdot 142$ dynamic). The number of non-initial rules is doubled since they are instantiated at each step.

In the case of property P1, the initial state fires the rule of the property, making the program wait for the atom inside of the $\mathbf{F}$. As an optimization, fulfillment rules of type $\mathbf{G}(p \rightarrow \mathbf{F}q)$ are not instantiated

[1] The tool is available in https://gitlab.tuwien.ac.at/ignacio.lopez/prolingo.

when the rule has been fired in the past and q does not depend on the future. Without optimization, as the rule by inertia fired in every step the number of rules keeps growing until an observation arrives that makes the head of the rule true (step 40). If no observation would have arrived, the number of rules would have kept growing. In step 42, a new observation arrives making the body of the rule true, which makes the number of rules start to grow again.

The traces of observations to test property P2 consisted in (T1) no observations, (T2) one observation at step 2 that makes the head of the rule true, and (T3) like (T2) but for every step. Since some of the rules in the program are simplified when the property is satisfied, we can see in Figure 2 that with the trace (T3), the number of rules is lower than with the other two traces. Since (T1) and (T2) are very similar, their respective program evolutions are almost overlapped.

The number of rules keeps growing at each step due to the instantiation of the dynamic and fulfillment rules. This is because the normal form from [2] results in future dependencies, which does not allow us to simplify the program at each step as it happened with property P1.

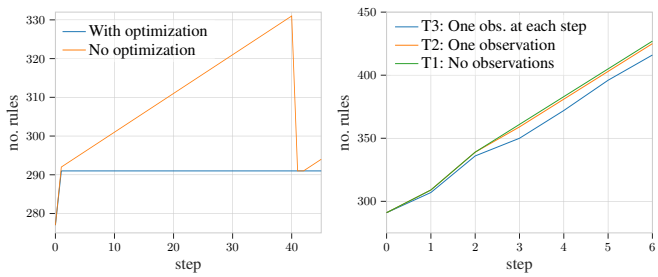

Figure 2: Evolution of the case study with property P1 containing the **F** operator (left) and property P2 containing the **U** operator (right).

7 Related Work

In the field of AI, there are other related approaches to reason about temporal properties, particularly in the context of stream reasoning. Streamlog [29] is a temporal Datalog language where the specification is expressed as a set of rules with time-stamped predicates, similar to the work in [26]. It is also interesting to point out some syntactic restrictions introduced in [29]. Requiring time stamps of the head to be greater than the time stamps of the body gives the possibility to easily compute a local stratification and obtain then a unique trace. We could investigate further these restrictions to use our algorithm for trace generation purposes, too. In particular, given a locally stratified program π_{init}, our algorithm eventually returns a prefix T^f such that $P_{TEL}(\pi_{init}, T^f) = \top$. Therefore, by application of Theorem 2, $T^f \models_{TEL_3} \pi_{init} = \top$.

Other works such as LARS [9, 8] provide streaming reasoning capabilities with rules using temporal operators such as `always` and `eventually` (but not `until` or `release`) that are evaluated within a finite window of time points. In contrast, our approach is geared to all temporal operators of LTL [25].

The solver TELINGO [14] also deals with TEL/THT, but is different from our approach in several respects. First, only it handles finite equilibrium traces, while our algorithm computes the intersection of infinite TEL traces online. TELINGO is more geared to tasks like planning with a finite horizon, while our algorithm addresses the monitoring problem. Furthermore, TELINGO has some syntactic constraints such as disallowing future operators in rule bodies and past operators in rule heads. In our approach, instead, future operators can occur both in heads and bodies. Another difference is that TELINGO uses incremental reasoning in ASP, while we use progression-based monitoring by rewriting the formula to be monitored at each step.

STELP [13] is another ASP solver for TEL that addresses a restricted class of temporal logic programs called "*Splittable temporal logic programs*" [4]. It handles temporal operators like `always` and `until` as constraints (the head of their rule is empty): this is helpful when one wants to discard particular TEL models. Instead, under some restrictions we can use `always` and `until` also for generating models (the head of the rule can also be non-empty).

In [12], the authors provide an approach to construct a Büchi automaton accepting TEL models. However, it is particularly computationally expensive (due to EXPSPACE-completeness of TEL satisfiability) and to the best of our knowledge has not yet been implemented. Another important difference with the automata-based approach is that in our approach it is easier to monitor multiple formulas in parallel and we can provide an explanation of the violation, by identifying the subformula responsible for it.

Our approach takes inspiration from the rewriting-based approach for runtime verification proposed first in [27], where the authors employed the Maude system [16] as a rewriting logic engine to implement LTL rewriting rules. Later, the authors of [6] leveraged a progression-based monitoring approach for LTL formulas using rewriting in the context of decentralized monitoring. A problem in this setting is that to satisfy the LTL specification, each node may need to know at a certain moment whether an event/proposition has occurred in another node before the next synchronization step. This problem is solved by rewriting the formula using a past operator in front of the formula when it contains propositions controlled by other nodes: in this case, the verdict is delayed until the synchronization with the other nodes occurs. We also use this trick but in a completely different context. In the ASP setting implication is interpreted differently than in LTL. For example, if we have a formula $\mathbf{X}p \rightarrow q$ in the classical interpretation corresponds to $\neg\mathbf{X}p \vee q$ and will result in the verdict true if q occurs at the current moment or will be rewritten in the obligation $\neg p$ to be held in the next step. However, in the ASP setting, it is not enough to have q true in the current moment, as the truth of q must be justified, which fails if $\mathbf{X}p$ is false in the future; this is why the body is rewritten as $\mathbf{Y}q$ using the past operator.

8 Conclusion and Future Work

In this paper, we presented a novel approach to temporal reasoning using a progression technique for Temporal Here and There and Temporal Equilibrium Logic. Our approach allows for non-monotonic reasoning over a trace of observations, providing the means to compute logical consequences of a temporal knowledge base over time. We presented the theoretical foundations to apply progression to these logics and proposed an algorithm to monitor dynamic systems that has been implemented as a proof-of-concept.

Our work contributes to the growing interest in the application of temporal reasoning approaches and non-monotonic logics. By using a progression technique, we have shown that it is possible to go beyond the usual LTL online computation and provide a more expressive approach to temporal reasoning. This can be useful in various domains, such as robotics, control systems, and autonomous vehicles, where real-time monitoring and decision-making are crucial.

In addition, we identified some future lines of work to improve and extend our approach. These cover the use of explicit negation and variables, and the optimization of the algorithm. Overall, our findings show the potential for our approach to be used in a variety of dynamic systems, paving the way for future research in this area.

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