Multi microphone extension of impedance tube systems

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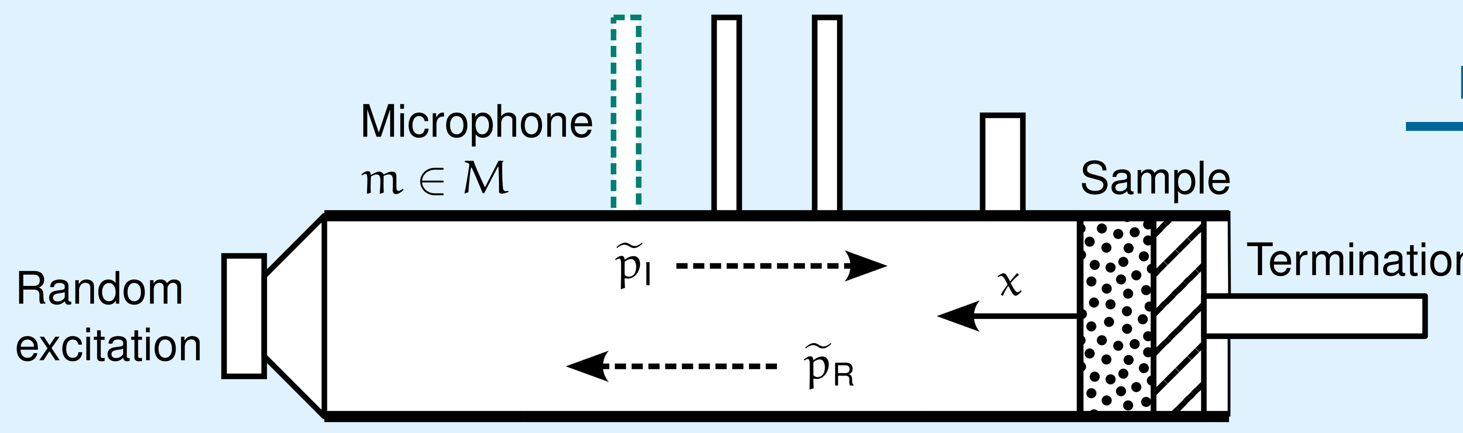
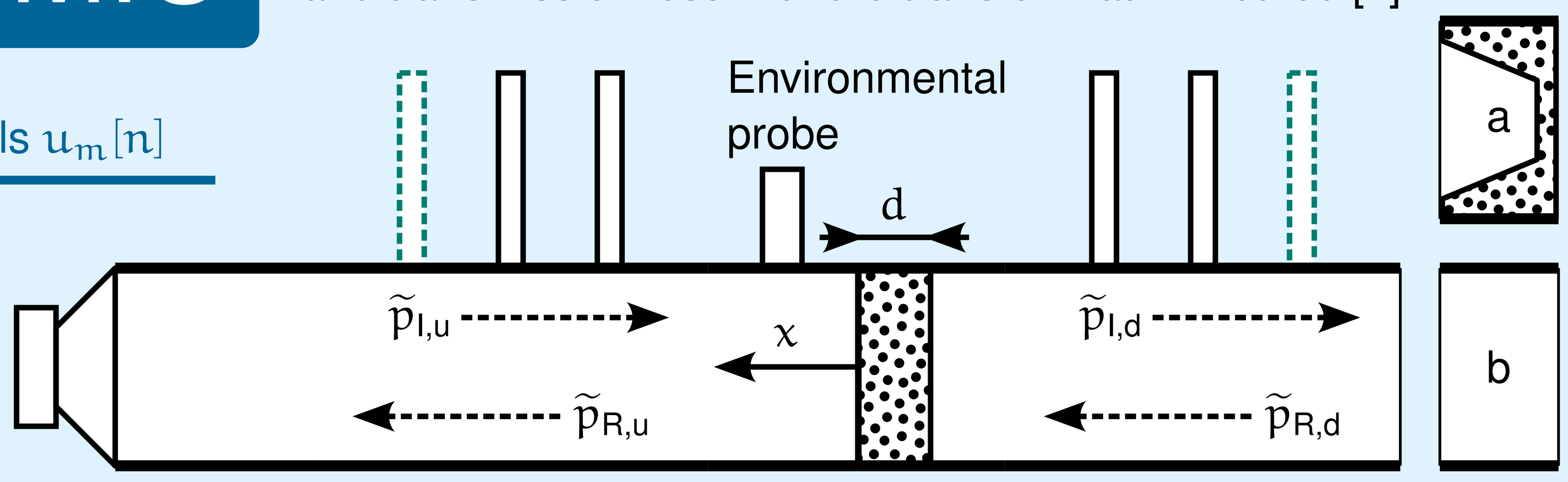
2MIC

Two microphone measurement of the reflection coefficient and derived quantities with the transfer function method [1].



4MIC

Four microphone measurement of the transmission coefficient and transmission loss with the transfer matrix method [2].



Microphone signals $u_m[n]$

DFT & Calibration (a & b)

$$\tilde{p}_m[k] = \frac{2 c_{SPL} \tilde{c}_{FR}[k]}{\sum_{n=0}^{N-1} w[n]} \sum_{n=0}^{N-1} u_m[n] w[n] e^{-j(2\pi/N)kn}, \quad k = 0, \dots, N/2$$

$m = A, B$

Wave decomposition

$$\tilde{p}_A = \tilde{p}_I e^{jk_0 x_A} + \tilde{p}_R e^{-jk_0 x_A}$$
$$\tilde{p}_B = \tilde{p}_I e^{jk_0 x_B} + \tilde{p}_R e^{-jk_0 x_B}$$

$m = A, \dots, D$

Wave decomposition (a & b)

$$\tilde{p}_A = \tilde{p}_{I,u} e^{jk_0 x_A} + \tilde{p}_{R,u} e^{-jk_0 x_A}$$
$$\tilde{p}_B = \tilde{p}_{I,u} e^{jk_0 x_B} + \tilde{p}_{R,u} e^{-jk_0 x_B}$$
$$\tilde{p}_C = \tilde{p}_{I,d} e^{jk_0 x_C} + \tilde{p}_{R,d} e^{-jk_0 x_C}$$
$$\tilde{p}_D = \tilde{p}_{I,d} e^{jk_0 x_D} + \tilde{p}_{R,d} e^{-jk_0 x_D}$$

$\tilde{p}_I, \tilde{p}_R$

Conventional methods

Material parameters

$\tilde{r} = \frac{\tilde{p}_R}{\tilde{p}_I}$ ... Complex reflection coefficient

$\alpha = 1 - |\tilde{r}|^2$ ... Absorption coefficient

$\tilde{Z} = Z_0 \frac{1+\tilde{r}}{1-\tilde{r}}$ ... Complex acoustic impedance

$\tilde{p}(x=0), \tilde{v}(x=0), \tilde{p}(x=d), \tilde{v}(x=d)$

$\tilde{p}_{I,u}, \tilde{p}_{R,u}$
$\tilde{p}_{I,d}, \tilde{p}_{R,d}$

Transmission Matrix

$$\begin{bmatrix} \tilde{p}_a & \tilde{p}_b \\ \tilde{v}_a & \tilde{v}_b \end{bmatrix}_{x=0} = \tilde{T} \begin{bmatrix} \tilde{p}_a & \tilde{p}_b \\ \tilde{v}_a & \tilde{v}_b \end{bmatrix}_{x=d}$$

Interface quantities (a & b)

$$\tilde{p}(x=0) = \tilde{p}_{I,u} + \tilde{p}_{R,u}$$
$$\tilde{v}(x=0) = (\tilde{p}_{I,u} - \tilde{p}_{R,u})/Z_0$$
$$\tilde{p}(x=d) = \tilde{p}_{I,d} e^{-jk_0 d} + \tilde{p}_{R,d} e^{jk_0 d}$$
$$\tilde{v}(x=d) = (\tilde{p}_{I,d} e^{-jk_0 d} - \tilde{p}_{R,d} e^{jk_0 d})/Z_0$$

$\tilde{T}$

Material parameters

$\tilde{\tau} = \frac{2e^{jkd}}{T_{11} + T_{12}/Z_0 + Z_0 T_{21} + T_{22}}$ ... Complex transmission coefficient

$\widetilde{TL} = 10 \log_{10} \left(\frac{1}{\tilde{\tau}} \right)$ ... Complex transmission loss

Wave decomposition for m microphones

MPAIR

Results of individual pairs/quads combined with pre-windowed weighted average [3].

Results of each pair

Pre-windowed average

$$\tilde{r} = \frac{\sum_{i,j}^{M} \tilde{r}(\tilde{p}_i, \tilde{p}_j, x_i, x_j) w(x_i, x_j)}{\sum_{i,j}^{M} w(x_i, x_j)}$$

$\sum_{i,j}^{M}$ ... Unique microphone pair sum

Rectangular window

$$w[k] = \begin{cases} 1 & k_l < k < k_u \\ 0 & \text{otherwise} \end{cases}$$

Pre-windowed average

$$\tilde{\tau} = \frac{\sum_{i,j,k,l}^{M} \tilde{\tau}(\tilde{p}_i, \tilde{p}_j, \tilde{p}_k, \tilde{p}_l, x_i, x_j, x_k, x_l) w(x_i, x_j, x_k, x_l)}{\sum_{i,j,k,l}^{M} w(x_i, x_j, x_k, x_l)}$$

$\sum_{i,j,k,l}^{M}$ ... Unique microphone quad sum

Results of each quad

LSTSQ

Least squares fit at the wave decomposition step to solve the overdetermined system for additional microphone pressures [3].

Wave decomposition

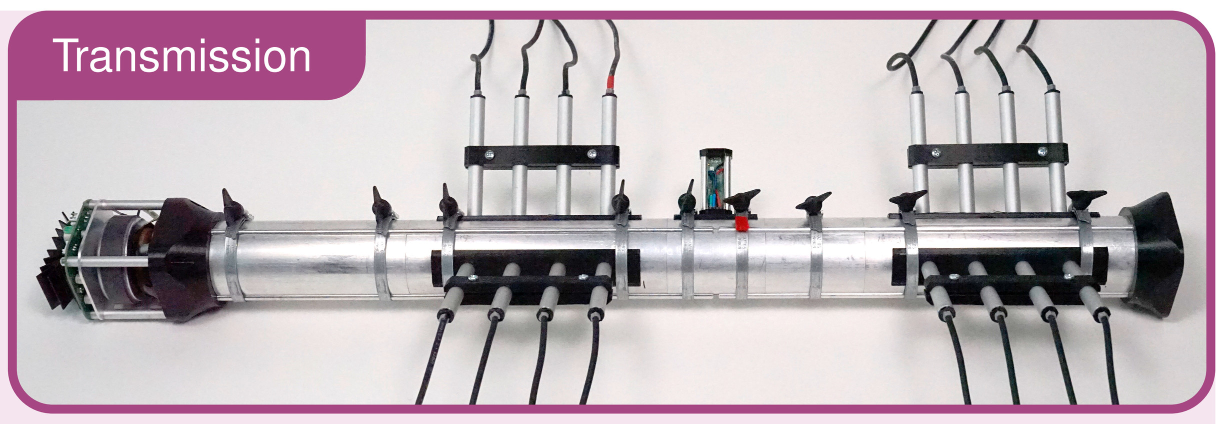
$$\begin{bmatrix} e^{jk_0 x_1} & e^{-jk_0 x_1} \\ \vdots & \vdots \\ e^{jk_0 x_{M_{u/d}}} & e^{-jk_0 x_{M_{u/d}}} \end{bmatrix} \begin{bmatrix} \tilde{p}_{I,u/d} \\ \tilde{p}_{R,u/d} \end{bmatrix} = \begin{bmatrix} \tilde{p}_1 \\ \vdots \\ \tilde{p}_{M_{u/d}} \end{bmatrix}$$

Multi microphone extensions

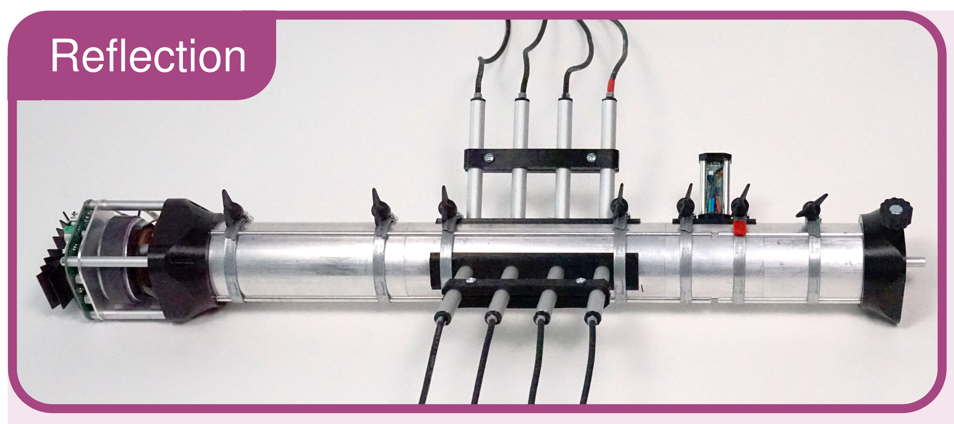
G50

Five foams from **BASF** tested. Brüel & Kjær Type 4206 large (**LT**) and small tube (**ST**) as reference [3].

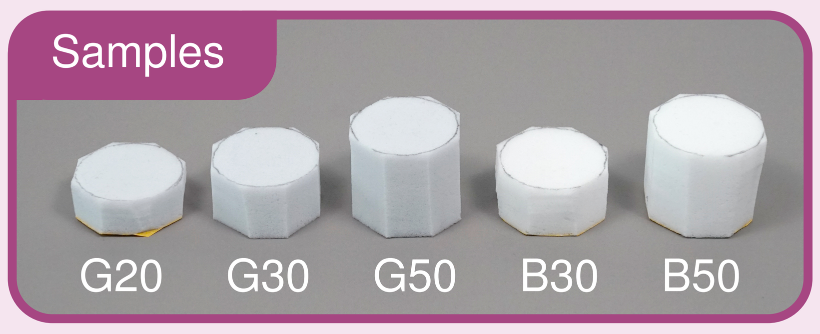
Transmission

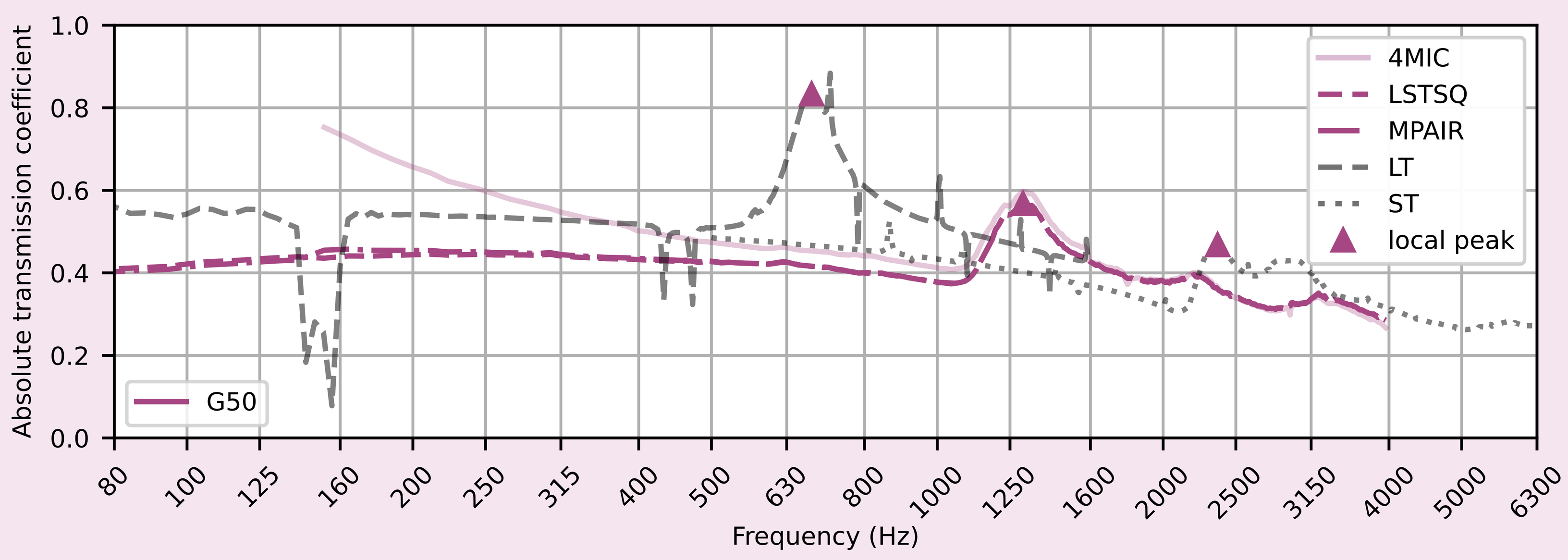


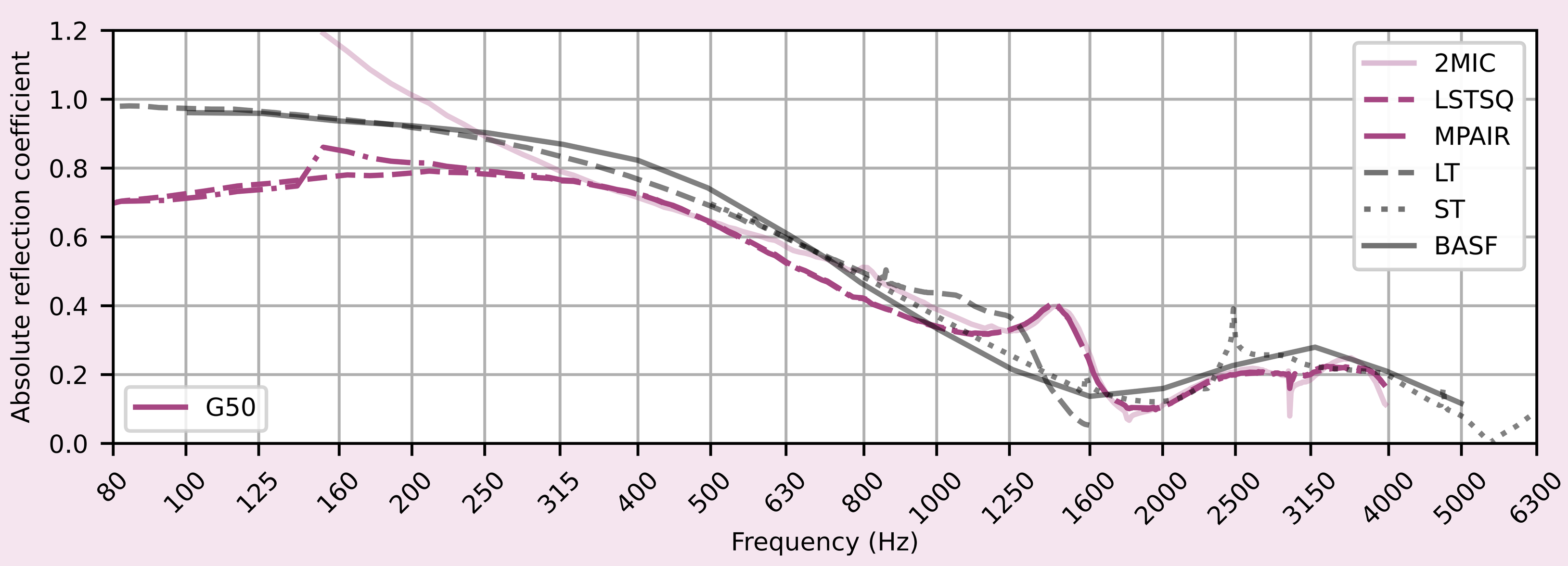
Reflection



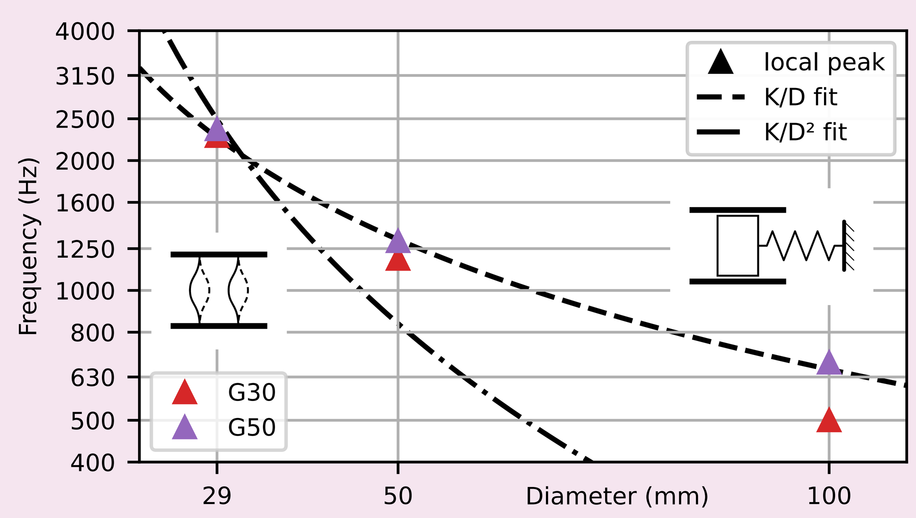
Samples







Experiments



- ⊕ Great error rejection and low frequency extension.
- ⊕ Electret microphones and consumer grade converters allow for large cost reduction.
- ⊘ Resonance effects for transmission (local peaks).
- ⊖ Wide spread of individual results due to cheap microphones.

[1] ISO 10534-2. Acoustics - Determination of sound absorption coefficient and impedance in impedance tubes Part 2: Transfer-function method. 2023.
[2] ASTM E2611. Standard Test Method for Normal Incidence Determination of Porous Material Acoustical Properties Based on the Transfer Matrix Method. 2019.
[3] Huber, Felix. Development of a multi microphone impedance tube for acoustic material characterization. reposiTUm. 2024.