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# A performance calculation approach for a robotic compact storage and retrieval system (RCS/RS) serving one picking station

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**ABSTRACT**

The number of robotic compact storage and retrieval systems (RCS/RS) is one of the fastest growing compared to other storage systems. Over a thousand systems with a large spectrum of design and order speed are already in operation. Nonetheless, there is hardly any information on throughput or optimal system design. This paper presents an analytical approach for the performance calculation of an RCS/RS, which operates with several robots serving one I/O shaft. One robot's cycle time is calculated by assuming a uniform distribution of container stacks and a probabilistic storage height. Based on this, the interaction of the robots at the I/O shaft is considered using an open queueing model with limited capacity. After validating the analytical approach using a discrete event simulation model of an RCS/RS, an extensive parameter variation is done. The easy and fast solvability with standard calculation programs and applicability are just two benefits.

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Automated warehouses; RCS/RS; cycle time model; multiple-deep storage stacks; grid-based storage system

## 1. Introduction

Continuous accessibility, seven days a week online open stores, and comparable prices are just three reasons for the still increasing trend towards E-commerce, complemented by the closing of the stationary trade due to the COVID-19 pandemic. Based on those facts, companies are forced to update their trading strategy. Besides the conventional shops in shopping streets or malls, they have online shops offering their entire product range or even more. After the customer orders something with a few clicks, the supply chain has to work perfectly to ensure prompt delivery. One part of an effective and functional supply chain are the warehouses, also called 'customer-fulfilment-centres'. Modern storage systems support the employees working in the warehouse by the pick-to-person principle and enable e-commerce accompanying standards like same-day-delivery, return-option or multi-article-orders. Such storage systems must be efficient, profitable, reliable, and sustainable. Robotic compact storage and retrieval systems (RCS/RS) possess most of the abovementioned characteristics. Moreover, those systems are scalable

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and can be expanded demand-based. In said system, goods are stored in to a block arranged plastic containers stacked above each other resulting in high storage densities. The warehouse is operated by robots from above, as can be seen in Figure 1.

Nowadays, material handling providers have to simulate every storage system to know the possible throughput, which requires, in most cases, much time and intensive computation. Furthermore, there are no generally valid statements about what throughput can be expected for neither one robot nor the total system. Based on this, potential customers can not receive any other information, e.g. the impact of the robot's velocity rate on the performance, besides the provider's fact sheets and the individual simulation results. Also, there are no commercial standards for RCS/RS (e.g. VDI 4480 for AS/RS or VDI 2692 for SBS/RS). While SBS/RS usually have storage depths up to 5, RCS/RS consist of container stacks up to 25 or even more. This impact also must be considered. Additionally, science has not yet been able to provide easy and quick analytical approaches to determine the throughput solvable with standard calculation programs. The fact that there are hardly any general statements, neither scientific nor concerning sales, about throughput or system design reveals the need for scientific consideration. It specifies the research gap this paper intends to address.

Based on the above-described research gap, this paper aims to develop a performance calculation approach for RCS/R systems with one picking station, considering an inhomogeneous article distribution and many variable system parameters. Every movement of the robot will be described by only one closed expression. A discrete event simulation was developed for this paper to validate the analytical approach. The results will be compared in a numerical simulation study.

Regarding the objectives, this investigation will give answers to the following research questions:

- How can the throughput of an RCS/RS with several robots serving one picking station be approximated analytically?
- How can the results of the analytical approach be validated using a discrete event simulation?

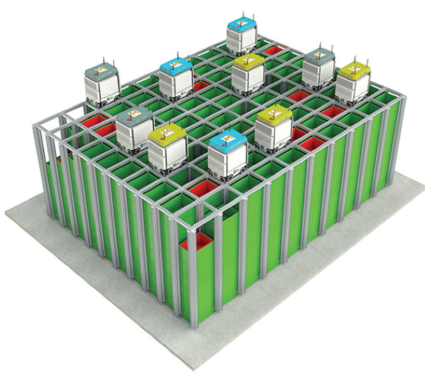


**Figure 1.** Robots on the grid. Source: Ocado (2022).

- How can an RCS/RS with one picking station be designed for a given parameter set?

This analytical approach's main advantage and novelty is its fast and straightforward applicability for customers, consultants, material handling suppliers, and providers. The possible throughput of an RCS/RS for a given set of input parameters and system configurations can be predicted easily. Moreover, the equations can uncomplicatedly be implemented in a table calculation program or a parametric computer algebra system and generate results immediately.

The approach determines one robot's cycle time and, based on that, the whole system's performance using an open queueing model with limited capacity. The model represents a seminal basis for further consideration of several picking stations along the grid.

Based on the abbreviations in Table 1, Section 2 gives an overview of the existing literature. Section 3 describes this paper's system and assumptions in more detail. Concerning Section 3, Section 4 presents the analytical approach. A numerical study is done in Section 5 to validate the analytical approach. The used parameters originate from a European RCS/RS supplier. To close this paper, Section 6 gives a summary and an outlook for further research.

## 2. Literature review

This section is intended to provide an overview of the existing literature in the context of RCS/R systems and the methods for performance calculation of such automated storage systems. Generally, the performance of these systems can be approximated with numerical simulation (e.g. DES) or analytical methods using a CTM and the queueing theory. The latter again provides several ways; some have already been used in storage systems, e.g. single queueing models using Markov chains, open-, semi-open- or closed queueing networks. An open queueing model enables in- and output from outside the system boundary compared to the closed queueing model, where neither in- nor output is

**Table 1.** Abbreviations.

| | |
|---|---|
| 3D-AS/RS | 3-dimensional automatic storage and retrieval systems |
| AS/RS | Automatic storage and retrieval systems |
| AVS/RS | Automatic vehicle storage and retrieval systems |
| CQN | Closed queueing network |
| CTM | Cycle time model |
| DCC | Dual command cycle |
| DES | Discrete event simulation |
| FCFS | First come, first served |
| I/O point | In- and output point |
| LIFO | Last in, first out |
| MSE | Mean squared error |
| OQN | Open queueing network |
| PBSS | Puzzle-based storage systems |
| RCS/RS | Robotic compact storage and retrieval systems |
| RMFS | Robotic mobile fulfilment systems |
| SBS/RS | Shuttle-based storage and retrieval systems |
| SCC | Single command cycle |
| SOQN | Semi-open queueing network |
| SQLC | Single queue with limited capacity |
| S/R machine | Storage and retrieval machine |
| VRCS/RS | Vertical robotic compact storage and retrieval systems |

**Table 2.** Literature overview.

| Author | System | Method | SCC/DCC | Policy | Validation with DES |
|---|---|---|---|---|---|
| Gue and Kim (2007) | PBSS | CTM | SCC | random | ✓ |
| de Koster et al. (2008) | 3D AS/RS | CTM | SCC | random | ✓ |
| Yu and de Koster (2008) | 3D AS/RS | CTM | SCC | 2-class based | |
| Yu and de Koster (2009) | 3D AS/RS | CTM | SCC | full-turnover | |
| Yang et al. (2014) | 3D AS/RS | CTM | DCC | random | |
| Nigam et al. (2014) | RMFS | CQN | DCC | class based | |
| Hao et al. (2015) | 3D AS/RS | CTM | SCC | random | |
| Kota et al. (2015) | PBSS | CTM | SCC | random | |
| Xu et al. (2017) | 3D AS/RS | CTM | DCC | random | ✓ |
| Zaerpour et al. (2017) | PBSS | CTM | SCC | random | ✓ |
| Lamballais et al. (2017) | RMFS | SOQN | DCC | class based | ✓ |
| Yuan and Gong (2017) | RMFS | OQN | SCC | random | ✓ |
| Zou et al. (2017) | RMFS | SOQN | SCC | random | ✓ |
| Beckschaefer et al. (2017) | RCS/RS | DES | DCC | random | |
| Tappia et al. (2017) | SBS/RS | OQN | SCC | random | |
| Zou et al. (2018) | RCS/RS | SOQN | DCC | random & dedicated | ✓ |
| Zou et al. (2018) | RMFS | DES | DCC | random | |
| Wang et al. (2019) | RMFS | CTM | DCC | random | ✓ |
| Tjeerdsma (2019) | RCS/RS | DES | DCC | random | |
| Azadeh et al. (2019) | RCS/RS | CQN | DCC | random | |
| Jin et al. (2020) | RMFS | SOQN | SCC | random | ✓ |
| Eder (2020b) | SBS/RS | SQLC | DCC | random | ✓ |
| Chi et al. (2021) | RMFS | SOQN | DCC | random | ✓ |
| Duan et al. (2021) | RMFS | SOQN | DCC | random | ✓ |
| Yang et al. (2021) | RMFS | SOQN | DCC | random | ✓ |
| Galka and Scherbarth (2021) | RCS/RS | DES | DCC | random | |
| Trost et al. (2022) | RCS/RS | DES | DCC | random | |
| Lamballais et al. (2022) | RMFS | SOQN | DCC | random | ✓ |
| Wang et al. (2022) | RMFS | CTM | DCC | random | ✓ |
| Luo and Zhao (2022) | RMFS | DES | DCC | random | |
| Trost et al. (2023) | RCS/RS | DES | DCC | random | |
| This paper | RCS/RS | SQLC | DCC | random | ✓ |

possible. Mixed networks with different types of users allow, for example, new input from outside into the system for one user category and departure from the system for another user category Baum (2013). Using one of the methods mentioned above, valid and precise approximations may be found for the performance calculation of storage systems.

Beckschaefer et al. (2017), Galka and Scherbarth (2021), Tjeerdsma (2019), Trost et al. (2022), Chen et al. (2022), and Trost et al. (2023) have all developed a discrete event simulation with specific system characteristics to gain statements about the system. While Beckschaefer et al. (2017) focused on warehousing strategies and whether a new product should be stored in an empty container or an already partially filled with the same product container should be removed from storage to store the new stock item, Tjeerdsma (2019) developed a multi-scenario discrete event simulation to redesign an order-processing line. Galka and Scherbarth (2021) developed a numerical simulation to determine the influence of the robots on the system performance using different access probabilities along the stack height for one specific warehouse scenario. Trost et al. (2022) investigated the marginal productivity of RCS/R systems. Chen et al. (2022) investigated overhead RCS/RS with overhead cranes (”bridge cranes”) by using dedicated and shared storage policies within the stacks and zoning within the warehouse by numerical discrete event simulation. Trost et al. (2023) analysed RCS/RS, listed several influencing factors and their interaction, and carried out a simulation study with a broad parameter variation.

Ko and Han (2022) chose another procedure to investigate RCS/RS and thereby proposed a roll-out heuristic algorithm to find the optimal order sequencing within an RCS/RS. Hameed et al. (2020) developed a numerical performance calculation approach using an optimal path algorithm for robot routing and compared the impact of a collision avoidance system within the robots. For one specific testing scenario, the total throughput decreased by around 10 percent with the consideration of obstacles compared to neglecting them.

Zou et al. (2018) presented an analytical approach for the performance evaluation of RCS/RS using an SOQN. This was done under the assumption of numerous simplifications and introducing a”wall parameter”. The central statement of the investigation was that the costs for the sorted warehousing – which is atypical for RCS/RS – could be twice as high as with the chaotic strategy, especially since sorting would reduce the great advantage of the high degree of space utilisation. The sorted system has a considerably higher throughput since relocations are eliminated. The presented approach is neither easily nor quickly analytically solvable with standard calculation programs.

Since RCS/RS are comparatively new, the number of scientific works about them is still small. However, AS/RS and their scientific research have a long history. The developed approaches for AS/RS are the basis for countless further considerations of resembling storage systems.

For example, when SBS/RS were introduced, the analytical approaches using cycle time models for throughput consideration of AS/RS were adapted and expanded to determine the performance of SBS/RS. Something similar can also be done for RCS/RS. Therefore, the existing literature for similar storage systems using vehicles serving multiple tiers, aisles and/or depths, such as SBS/RS, 3D-AS/RS, PBSS, or RMFS, is also part of this section.

3D-AS/RS enabled higher order speed and higher space utilisation rates by using a block layout and a gravity-supported or powered conveying mechanism for the third direction. de Koster et al. (2008) investigated the performance and the optimal design of 3D-AS/RS with a random storage strategy for SCC by developing a CTM. Yu and de Koster (2008, 2009) conducted further studies using different storage strategies and Yang et al. (2014) considered the acceleration/deceleration of the S/R machine. Hao et al. (2015) varied the location of the I/O point, and Xu et al. (2017) developed cycle time models for DCC and lower-mid dwell points.

Azadeh et al. (2019) developed an analytical model to predict the optimal layout of a vertical RCS/RS, which has a layout similar to SBS/RS, and to analyse the performance considering two different robot blocking protocols with a closed queueing network.

Many shuttles are also applied within a robotic mobile fulfilment system (RMFS). In this system, the robots transport the containers or the storage shelfs to an assigned picking station. Nigam et al. (2014) developed a throughput calculation approach using a CQN and a class-based storage strategy. In contrast, Lamballais et al. (2017) built an SOQN to determine robot utilisation. They asserted that the influence of the location of the working station on the throughput is way higher than the length-to-wide ratio of the storage area. Several further papers, such as Zou et al. (2017), Jin et al. (2020), Chi et al. (2021), Duan et al. (2021), Yang et al. (2021) and Lamballais et al. (2022), investigated RMFS by using SOQN with robots operating in a dual command cycle and a random storage policy. Jin et al. (2020) and Yang et al. (2021) extended to a multiple-deep layout while Duan et al. (2021) discussed the throughput of RMFS with time-varying arrivals of new orders. Yuan and Gong (2017) used an OQN to calculate the performance of an RMFS to determine the best ratio of robots to picking stations. Wang et al. (2022) and Wang et al. (2019) developed a CTM and discussed the system's throughput for different numbers of picking stations under the view of a zoning policy.

Lienert et al. (2018), Li et al. (2021), and Luo and Zhao (2022) all present a simulation-based performance analysis for different storage layouts, single or multiple-deep storage and high-density warehouses. The latter can also be found in literature under the term puzzle-based storage systems. Those are also arranged in a block but use load-captive shuttles for horizontal transport. Most notably, Gue (2006), Gue and Kim (2007), Gue et al. (2014), and Kota et al. (2015) studied PBSS. Gue et al. (2014) expanded the first approaches of Gue (2006) and Gue and Kim (2007) and developed the *GridStore*, which represents a modular, scalable and decentralised high-density storage system. The authors demonstrate that the performance underlies various operation configurations and that the system can operate deadlock-free. The system has generally just a few similarities to RCS/RS. Still, the deadlock-free logic and the number of shuttles within a small grid section can bring insights for the investigation of RCS/RS since a congestion-free routing and many robots operating on the grid are part of a well-working RCS/RS. Zaerpour et al. (2017) calculated the optimal system dimensions regarding a minimum retrieval time with multiple tiers.

SBS/RS also resembles RCS/RS in some respects since they can also work with vehicles serving multiple tiers and/or multiple deep storage slots and, in some cases, multiple aisles. The number of papers investigating SBS/RS is immense. To further limit it, the focus will be SBS/RS with shuttle vehicles serving multiple tiers of multiple deep storage racks. Tappia et al. (2017) developed an analytical approximation for such SBS/RS using a single queueing model with limited capacity, and Eder (2020b) presented a method for determining the performance using an open-queueing system with limited capacity. He delivered an easy and quick formula set, which was adapted and expanded steadily based on the cycle time calculations of an AS/RS. It will be further developed since this paper aims to present a straightforward and fast method for the throughput approximation of an RCS/RS.

As can be seen in Table 2, only a few scientific considerations exist on RCS/RS and its possible throughput. Only one scientific paper has presented an analytical calculation method, which is neither easily nor quickly analytically solvable with standard calculation programs. Therefore, this paper aims to showcase a straightforward and quickly solvable analytical approach to determine the throughput of an RCS/RS with several robots serving one picking station. The formulas are based on the existing CTM for AS/RS and SBS/RS, expanded and adapted for RCS/RS. Additionally, an open queueing model with a single queue with limited capacity is applied to consider the robots' interaction and calculate the total system's performance.

## 3. System description

Based on the literature in the second section, this chapter provides an explanation of the system investigated in this paper. RCS/R systems are fully automatic, by robots from above-operated warehouses, that store small goods in plastic containers stacked onto each other using the LIFO storage strategy within each stack. The storage and retrieval are carried out from the top, leading to high volume-density rates because of the loss of aisles (Trost et al., 2023). Figure 2 exhibits a small section of an RCS/RS.

The essential components of an RCS/RS are the storage grid, the containers, the robots, and the I/O shaft with the picking station. The grid serves as an orthogonal railway network for the robots and as a divisional grid for the stacked storage containers. The goods to be stored inside the warehouse are put into plastic containers stacked onto each other. The battery-operated robots carry out the storage and retrieval process by picking up the containers at the I/O shaft and transporting them along the railway grid to the assigned stack. The I/O shaft with

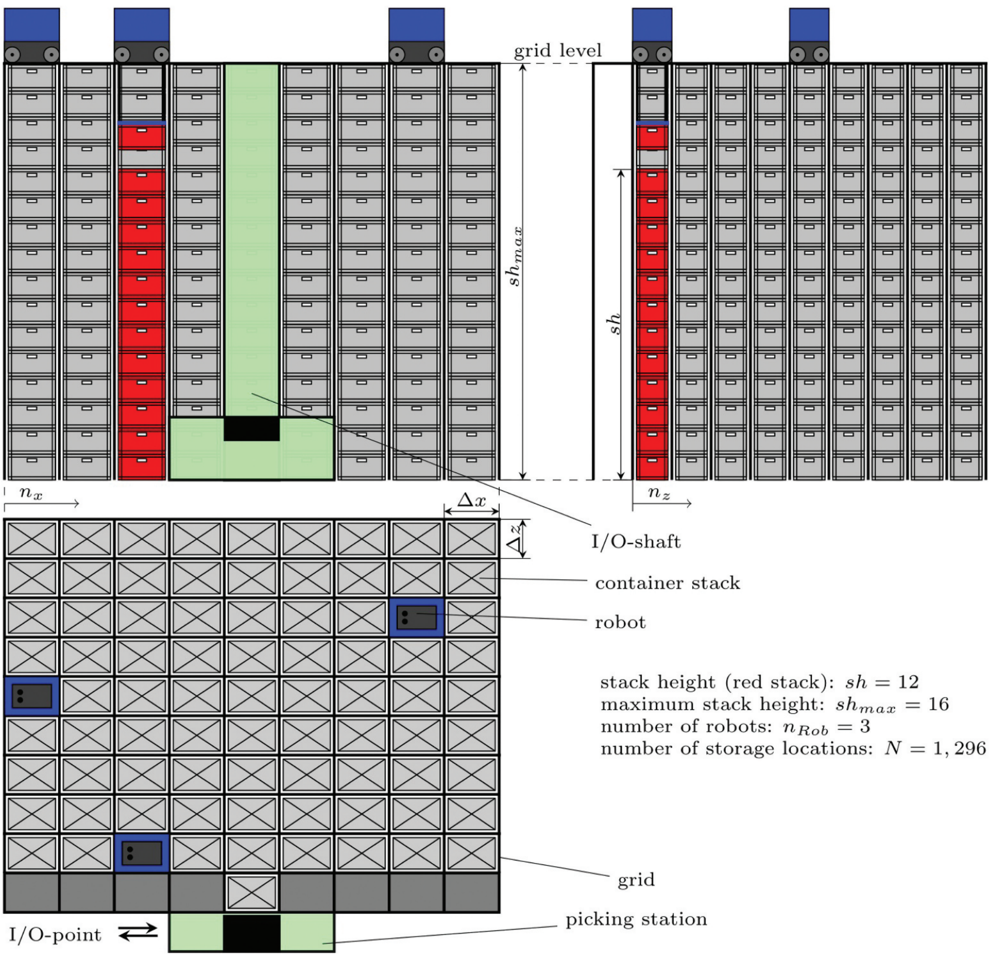



**Figure 2.** RCS/RS.

the picking station represents the I/O point of the storage system, where the containers are filled with new storage goods and emptied by picking the required goods. The I/O shaft connects the grid level and the picking station in front of the storage system. Although lift systems are conceivable at the I/O shaft, the system investigated in this paper consists of an I/O shaft where the robots lift and lower the containers (Trost et al., 2023).

If a new order to retrieve without having direct access to the required container arrives, it is necessary to relocate all the other containers stacked on top of the required one. In some storage systems, after the required container is retrieved, the relocated containers are return relocated in the sequence of their removal.

The main assumptions are listed below:

- The robots work in a dual command cycle under the FCFS rule.
- The systems dwell point is in front of the I/O shaft.
- The I/O shaft is located in the middle of one of the wide edges of the grid.
- There are always containers waiting at the dwell point in front of the I/O shaft.
- The robots pick up a new storage container after dropping off an order container.
- The robots routing along the grid is without collisions.
- The containers are stored and ordered evenly distributed.
- The container to be relocated is relocated to the next available storage location.
- If return relocations are done, the return sequence is identical to the relocations.
- The filling degree is limited to a specific value ensuring relocations.

## 4. Analytical approach

To predict the possible throughput of an RCS/RS, knowing the cycle time of one robot is mandatory. Since the RCS/RS investigated in this paper works with several robots, the interaction of the robots at the I/O shaft has to be considered. Collision, congestion, or deadlock situations of the robots on the grid can be ruled out of the analytical approach since the routing logic of the system ensures this. Moreover, the primary purpose of the analytical approach is a performance approximation tool to gain statements on the maximum throughput.

The analytical approach is based on Eder (2020a, 2020b) and his SBS/RS performance determination using an open-queueing model with limited capacity. Fundamentally, the analytical approach can be split up into the steps below:

- Cycle time of one robot (arrival rate):
  - Ride time from/to the I/O shaft to/from the grid element
  - Ride time from the grid element to another grid element
  - Lifting and lowering time of the container up from/down onto the stack
  - Probability of relocations
  - Ride time in the relocation cycle

- Time on the I/O shaft (service rate)
- Throughput with $M|M|1|K$

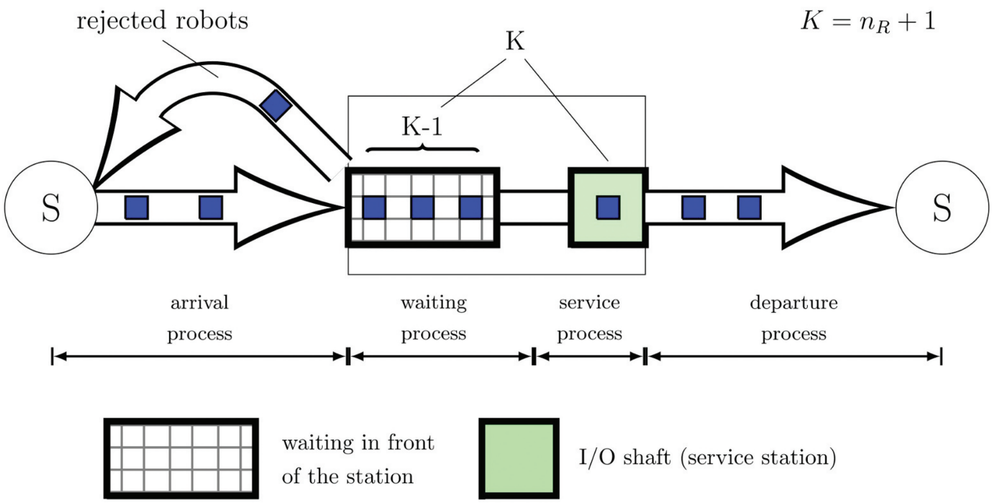



**Figure 3.** Open queueing model with limited capacity.

Based on one CTM, the throughput of the whole system is calculated using the queueing theory. The process is depicted in Figure 3. In the figure, K stands for the maximum number of robots waiting in front of the I/O shaft in addition to the one standing on the I/O shaft.

The Kendall notation with the four tuple (A|B|C|K) is common within the queueing theory and describes the queueing system. Here, A stands for the arrival process, B for the service process, C for the number of parallel service stations and K for the queue length. This approach assumes an M|M|1|K model with Markov characteristics both for arrival and service process (Baum, 2013). The complete notation used in this approach can be found in Table 3.

### *4.1. Cycle time calculations*

For the time function, a distinction must be made whether the maximum velocity is reached and thus the trapezoidal drive mode takes place or, in the case of short distances, the triangular drive.

$$\mathbf{t}(y) = \begin{cases} 2 \cdot \sqrt{\frac{y}{a_R}} & \text{for} \quad y < \frac{v_R^2}{a_R} \\ \frac{y}{v_R} + \frac{v_R}{a_R} & \text{for} \quad y \geq \frac{v_R^2}{a_R} \end{cases} \tag{1}$$

Equation 1 depicts the time function $\mathbf{t}(y)$ with the variable $y$, which describes the ride distance. When the robot reaches the maximum velocity and $y = \frac{v_R^2}{a_R}$, the trapezoidal drive mode starts and the ride time calculates by the below equation. Otherwise, the above one has to be applied.

RCS/R systems generally operate in a dual command cycle. Some systems prefer return relocations, while others do not. Thus, several system operation modes are conceivable. For this paper, the relevant robot arrival time Equations 2 (without return

**Table 3.** Notation.

| | |
|---|---|
| $\Delta x$ | Distance between two grid elements along the x-axis |
| $\Delta z$ | Distance between two grid elements along the z-axis |
| $\vartheta$ | Throughput of the RCS/RS |
| $\lambda$ | Arrival rate |
| $\mu$ | Service rate |
| $\rho$ | Utilisation rate of the picking station |
| $a_R$ | Acceleration rate of a robot in horizontal direction |
| $f$ | Filling degree |
| $h_C$ | Height of a storage container |
| $K$ | Capacity of the queueing system |
| $k_0$ | Position of the picking station along the x-axis |
| $n_R$ | Number of robots operating on the grid |
| $n_x$ | Number of grid elements along the x-axis |
| $n_z$ | Number of grid elements along the z-axis |
| $n_{St}$ | Number of stacks |
| $sh$ | Storage height of a container stack |
| $p_0$ | Probability for an empty system |
| $p_k$ | Blocking probability |
| $t_A$ | Arrival rate of the robot without return relocations |
| $t_{A\_RR}$ | Arrival rate of the robot with return relocations |
| $t_{CX}$ | Time for the container exchange at the picking station |
| $t_L$ | Time required for locking/unlocking the locking claws |
| $t_{R\_rel}$ | Time of a robot required to travel at the relocation cycle |
| $t_{R\_DCC}$ | Additional time of a robot to travel in a dual command cycle |
| $t_{R\_SCC}$ | Time of a robot required to travel in a single command cycle |
| $t_S$ | Service time |
| $t_T$ | Time required to lift or lower a container up from or down onto the stack |
| $t_{WX}$ | Time of a robot to change the wheels from one direction to another |
| $v_R$ | Velocity rate of a robot in horizontal direction |
| $v_T$ | Velocity rate of a robot for lifting and lowering |
| $w_{rel}$ | Probability of a relocation cycle |
| $y$ | Ride distance variable |

relocations) and 3 (with return relocations) for a combined storage and retrieval process performing in a dual command cycle are depicted below:

$$t_A = 2 \cdot t_{R\_SCC} + t_{R\_DCC} + 2 \cdot t_T + w_{rel} \cdot (t_{R\_rel} + 2 \cdot t_T) + t_S \quad (2)$$

$$t_{A\_RR} = 2 \cdot (t_{R\_SCC} + t_{R\_DCC} + t_T + w_{rel} \cdot (t_{R\_rel} + 2 \cdot t_T)) + t_S \quad (3)$$

### 4.1.1. Mean ride time

The ride time for a single command cycle can be calculated with Equation 4. Depending on the distance to ride, the time function (Equation 1) provides the correct formula. The first term describes the ride from the picking station along the z-direction. If the assigned stack is along the z-axis, a wheel change is unnecessary, and the term $|k - k_0|$ results in zero. The variable $k_0$ describes the position of the I/O shaft along the x-axis, and the variables $k$ and $l$ are the summation indices. An I/O shaft in the middle of the edge means $k_0 = \frac{n_x}{2}$. If the direction has to be changed once, i.e. the robot also moves along the x-axis, the wheels have to be changed. This is considered with an additional time component $t_{WX}$. Thus, for $|k - k_0| > 0$, the last term calculates to $sign(|k - k_0|) = 1$ since the *sign* function results in one for any number greater than zero.

$$t_{R\_SCC} = \frac{1}{n_x} \cdot \frac{1}{n_z} \cdot \sum_{k=1}^{n_x} \sum_{l=1}^{n_z} \mathbf{t}(l \cdot \Delta z) + \mathbf{t}((|k - k_0|) \cdot \Delta x) + t_{WX} \cdot \mathrm{sign}(|k - k_0|) \tag{4}$$

Analogous to Equation 4 for the single command cycle, the following expression (Equation 5) represents the ride time from the storage stack to the next retrieval stack. Without the change of direction, i.e. for $|k - l| = 0$ or $|m - n| = 0$, the last term results in zero because a wheel change is unnecessary.

$$t_{R\_DCC} = \frac{1}{n_x^2} \cdot \frac{1}{n_z^2} \cdot \sum_{k=1}^{n_x} \sum_{l=1}^{n_x} \sum_{m=1}^{n_z} \sum_{n=1}^{n_z} \mathbf{t}((|k - l|) \cdot \Delta x) + \mathbf{t}((|m - n|) \cdot \Delta z) + t_{WX} \cdot \mathrm{sign}((|k - l|) \cdot (|m - n|)) \tag{5}$$

### 4.1.2. Mean time for container lifting and lowering

Based on Eder (2020b), the average time for the lifting and lowering of the containers can be calculated with the following slightly modified expression 6. In this equation, $t_L$ is the time required to pick up or drop down the container with the locking claws. The binomial coefficient is necessary for having only one formula for $n$ stack heights. $f$ is the filling degree of the storage system. A precise and more detailed description of all terms can be found in Eder (2020a).

$$t_T = t_L + \sum_{n=1}^{sh} \sum_{i=0}^{n-1} \frac{1}{sh + 4 \cdot i} \cdot \binom{sh - 1}{i} \cdot f^{sh-1-i} \cdot (1 - f)^i \cdot 2 \cdot \frac{h_C}{v_T} \cdot n \tag{6}$$

The last fraction can be interpreted as the lifting and lowering of a container over the height of one container height and means the required time for lifting and lowering at the maximum storage height of $sh = 1$. It is multiplied with the run variable $n$ to gain the time required for each case.

### 4.1.3. Relocation cycle

If the stack height exceeds one, relocations could be necessary to retrieve the required container. The probability of a relocation cycle is based on Eder (2020a). For $sh = 2$, the probability of a possible relocation results to $w_{rel} = \frac{1}{2} \cdot f^2$. Eder (2020a) describes the factors as the following: $f^2$ represents the probability that both grid elements are occupied by containers. For $sh > 1$, the relocation probability results in:

$$w_{rel} = \sum_{n=0}^{sh-2} \sum_{i=1}^{sh-1-n} \cdot \frac{i}{sh - n} \cdot \binom{sh}{n} \cdot f^{sh-n} \cdot (1 - f)^n \tag{7}$$

Based on the relocation probability, the ride time for the relocations must be calculated. That, again, depends on the number of accessible storage locations next to the stack where the retrieval has to be carried out. Eder (2020a) assumed in his paper with a multiple-deep SBS/RS that the ordered container is located in the middle of an aisle. This approach was adjusted by Eder (2020b) to determine the throughput of a multiple-deep SBS with vehicles serving multiple tiers. He assumed that the order is stored at the end of an aisle, which results in a significantly smaller number of neighbouring storage slots. This approach for SBS is appropriate since an SBS commonly consists of several aisles.

RCS/RS, instead, are typically built quadratically without sub-tiers or aisles. Thus, they only consist of four edges and four corners. Compared to SBS, their higher storage heights and the fact that, for most systems, the number of stacks along the two horizontal axes is much higher than the number of edges and corners allow the assumption of an ordered container located in the middle (Figure 4). Nevertheless, the approaches for relocations at one edge of the grid and in one corner are also listed below for completeness. Equation 8 depicts the relocation ride time.

$$t_{R\_rel} = \sum_{i=1}^{n_x}\sum_{j=1}^{n_z} \frac{(f^{sh})^X \cdot (1-(f^{sh})^Y)}{\max(i,j)} \cdot 2 \cdot (\mathbf{t}(i \cdot \Delta x) + \mathbf{t}(j \cdot \Delta z) + t_{WX} \cdot \text{sign}(|i-j|)) \tag{8}$$

The first term $(f^{sh})^X$ describes the probability that the eight neighbouring stacks are fully occupied. Therein, $f^{sh}$ represents the probability that one stack has reached its maximum height, and $X$ describes the number of occupied stacks Eder (2020b). Correspondingly, the second expression $(1-f^{sh})^Y$ returns the probability of a free storage location on a stack within those eight stacks. The exponent $Y$ represents the number of storage slots within the same relocation circle/region. On the one hand, the term $\max(i,j)$ in the denominator guarantees that the denominator will not become zero. On the other hand, $n$ ensures that all stacks where the relocation containers could be relocated are considered.

Figure 4 depicts the relocation distance for a relocation cycle anywhere on the grid.

Referring to Figure 4, the stacks to which the relocation containers are transported are arranged around the retrieval stack. This assumption had to be made to gain only one expression for the relocation rides. In Figure 4, the number of stacks within the first relocation circle/region ($Y$) describes all the blue stacks (first free stacks).

In case of retrieval with relocations along one edge of the grid, Figure 5 presents the relocation options.

The worst case for the relocation cycle occurs in the four corners of the grid, as shown in Figure 6.

The probability of free stacks is indirectly proportional to the stack height and the filling degrees. Low-stacked systems with high filling degrees have a low probability of accessible relocation stacks. RCS/RS usually have stack heights up to $sh = 25$ or even

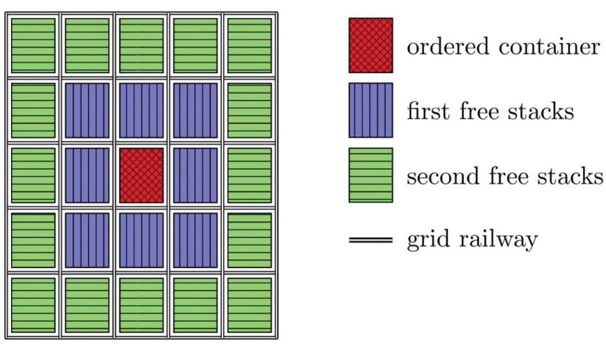



**Figure 4.** Relocation distance and next free stacks in the middle of the grid ($p = 8$).

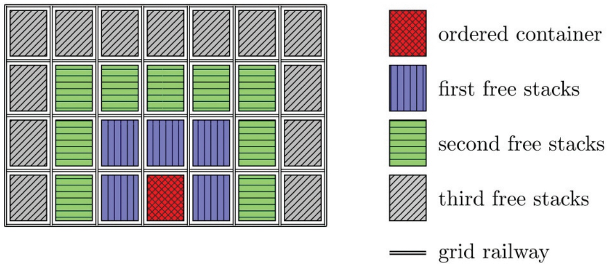



**Figure 5.** Relocation distance and next free stacks at one edge of the grid ($p = 4$).

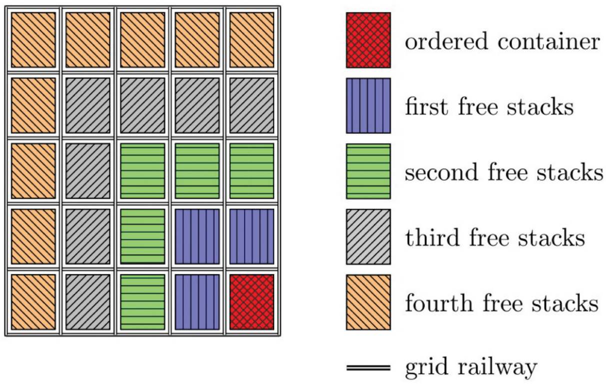



**Figure 6.** Relocation distance and next free stacks within the four corners of the grid ($p = 2$).

higher; thus, there is a very high probability of finding a free storage location on the first neighbouring stacks.

For all three cases, $X$ and $Y$ is calculated differently:

$$X(\max(i,j)) = \begin{cases} (1 + 2 \cdot (\max(i,j) - 1))^2 - 1 & p = 8 \\ \max(i,j) \cdot (1 + 2 \cdot (\max(i,j) - 1)) - 1 & p = 4 \\ 2 \cdot \max(i,j)^2 - 1 & p = 2 \end{cases} \tag{9}$$

$$Y(\max(i,j)) = \begin{cases} p \cdot \max(i,j) & p = 8 \\ p \cdot \max(i,j) + 1 & p = 4 \\ p \cdot \max(i,j) + 1 & p = 2 \end{cases} \tag{10}$$

Equation 9 and 10 consist of a factor $p$, which is necessary for the summations. Following Eder (2020b), the factor $X$ describes the number of occupied stacks. The first case assumes a relocation anywhere on the grid, hence all eight circular neighboured stacks can theoretically be relocation stacks ($p = 8$).

The total time for the relocation cycle is the sum of the ride time $t_{R\_rel}$ and the lifting and lowering time $t_T$.

### 4.2. Service time calculation

RCS/R systems generally operate in a dual command cycle. Equation 11 represents the time of a robot on the I/O shaft:

$$t_S = t_{CX} + 2 \cdot t_L + 2 \cdot \frac{h_C}{v_T} \cdot sh \tag{11}$$

In this equation, $t_{CX}$ stands for the container exchange time within the picking station and $t_L$ describes the unlocking of the retrieval container and the locking of the next storage container. The third term represents the time for lifting and lowering the old and the new container through the I/O shaft.

### 4.3. Open queueing model MM1K

The open-queueing model is based on the two processes interacting within the storage system:

- The interarrival time of the robots at the I/O shaft.
- The service time required on the I/O shaft.

The arrival rate at the I/O shaft is the reciprocal value of a cycle time multiplied by the number of robots operating on the grid:

$$\lambda = n_R \cdot \frac{1}{t_A} \tag{12}$$

Analogously, the service rate, which represents the possible number of pickings per time unit, is calculated like this:

$$\mu = \frac{1}{t_S} \tag{13}$$

The ratio of the arrival to the service rate defines the utilisation rate of the I/O shaft (service station):

$$\rho = \frac{\lambda}{\mu} = \frac{t_S}{t_A} \tag{14}$$

The throughput $\vartheta$ using an open queueing model with limited capacity assuming a queue providing space for a maximum of $K = n_R + 1$ robots can be calculated with the following expression Baum (2013):

$$\vartheta = \lambda \cdot (1 - p_K) = \mu \cdot (1 - p_0) = \lambda \cdot \left(1 - \frac{1 - \rho^K}{1 - \rho^{K+1}}\right) = \mu \cdot \left(1 - \frac{1 - \rho}{1 - \rho^{K+1}}\right) \tag{15}$$

As it can be seen in Equation 15, there are two options to determine the throughput: Either with the arrival rate $\lambda$ and the probability of blocking $p_K$ or with the service rate $\mu$ and the probability of emptiness $p_0$.

The presented approach for RCS/RS can be used to calculate the possible throughput serving multiple stack heights and one picking station at one edge of the grid. This approach can be used for several systems available on the market. The easy and fast solvability is one of the main advantages of this approach since it is neither computationally nor time-intensive. This means that results can be gained immediately.

## 5. Numerical study

On the one hand, the fifth section aims to validate the analytical approach from Section 4 with a discrete event simulation of an RCS/RS, which will be carried out in subsection5.1. On the other hand, it sets out to test different parameters and configurations (subsection 5.2 and 5.3). The main focus will be the throughput within the considered system. The system investigated operates in a DCC, and the cycle time is calculated with equation 2, assuming a storage system performing without return relocations. Table 4 presents the input parameters for the numerical study.

To validate the analytical approach presented in Section 4, the results will be compared with those of 30 independent scenarios of the DES. The simulation model, which rebuilds an RCS/RS with the processes controlling the system in the background, was created in the DES simulation software SIMIO (version 15.240). The containers were evenly distributed over all stacks and storage heights. All the parameters from Table 4, such as the number of stacks along both horizontal axes, the stack height, the filling degree, etc., can be varied in the simulation. The running time of the different simulation experiments varies from a few minutes to several hours.

Since this investigation considers more than one robot operating, the collision-avoiding system implemented in the DES can be summarised by the following two assumptions:

- If a robot occupies a node, this node and its four paths are blocked for all the other robots as long as the robot occupies this node.
- If a robot assigns a path, this path and the next node are blocked for all the other robots as long as the robot is going along this path.

Real RSC/R systems work with longer path-planning forecasts, i.e. the robots pre-reserve the whole path towards their destination and permanently block the following five grid elements. Nevertheless, as mentioned earlier, those easily applicable and practicable methods provided valuable and significant results.

**Table 4.** Parameters for the RCS/RS.

| | |
|---|---|
| Number of grid elements along the x-axis | $n_x \in \{10, 20, 25, 30, 40, 50\}$ |
| Number of grid elements along the z-axis | $n_z \in \{5, 10, 20, 25, 30, 40, 50\}$ |
| Storage height of a container stack | $sh \in \{1 \dots 25)\}$ |
| Filling degree | $f \in \{10\%, \dots, 75\%, 90\%, 95\%, 98\%\}$ |
| Container height | $h_C = 330mm$ |
| Number of robots | $n_R \in \{1, 2, \dots, 20\}$ |
| Robot horizontal velocity rate | $v_R = 2\frac{m}{s}$ |
| Robot lifting and lowering velocity rate | $v_T = 1.6\frac{m}{s}$ |
| Robot horizontal acceleration rate | $a_R = 0.8\frac{m}{s^2}$ |
| Robot time to lock/unlock the container | $t_L = 1s$ |
| Robot wheel change time | $t_{WX} = 1s$ |

### 5.1. Validation of the analytical approach

This section shall validate the analytical approach from Section 4. The complexity of the system will be limited in the beginning to ensure the following check steps can be confirmed:

- The ride time ($sh = 1$)
- The probability for a relocation ($w_{rel}$)
- The robot's lifting and lowering time ($n_R = 1$)
- The number of robots

First, the time required for one robot's ride will be under investigation. Thus, the system's stack height is limited to one. Secondly, the relocation probability calculated with equation 7 shall be validated. Based on that, the third step is to calculate the throughput of an RCS/RS with one operating robot. The number of vehicles is only varied fourthly.

Figure 7(a) displays one robot's throughput for different quadratic grid sizes and a stacking height limited to one, i.e. no relocations. The numerical results regarding the throughput are compared to the analytical, and, as shown in Figure 7(a), the discrepancy converges towards zero. Similar results for rectangular grids can be seen in Figure 7(b). The throughput decreases for larger grid sizes, regardless of whether the arrangements are quadratic or rectangular.

Based on one robot's ride time for an RCS/RS with stack height $sh = 1$, the next step will be having multiple containers stacked onto each other. The simulation was performed with a 25 by 25 grid to ensure that every relocation container gets a new stack assigned. The relocation probability for a filling degree of 90% is plotted in Figure 8(a), and the corresponding throughput with one operating robot can be seen in the curve in Figure 8(b).

The error rate both for the relocation probability and for the throughput is minimal, always remaining below 1%.

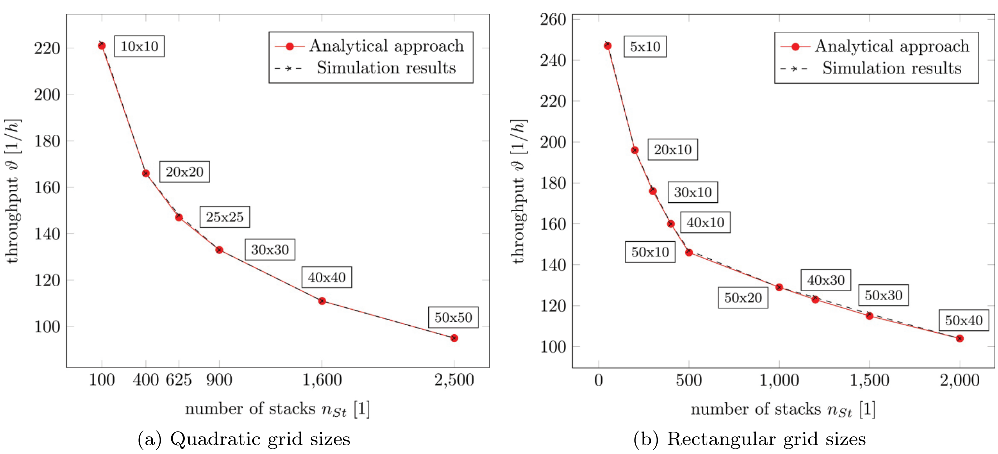
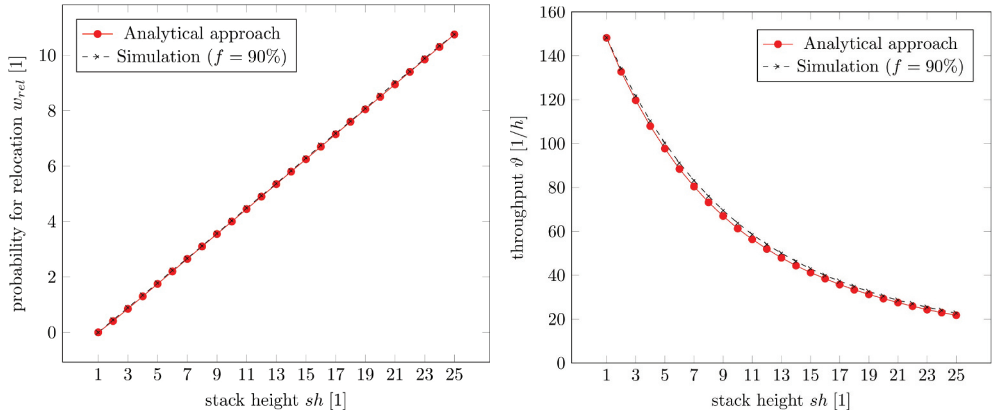



**Figure 7.** Throughput of an RCS/RS for a stack height of 1 for different grid sizes and a filling degree of 100%.




(a) Number of necessary relocations for one retrieval depending on the stack height for different filling degrees

(b) Throughput of a 25 by 25 RCS/RS depending on the stack height for a 90% filling degree

**Figure 8.** Relocations and throughput for a stack height of $sh = 25$.

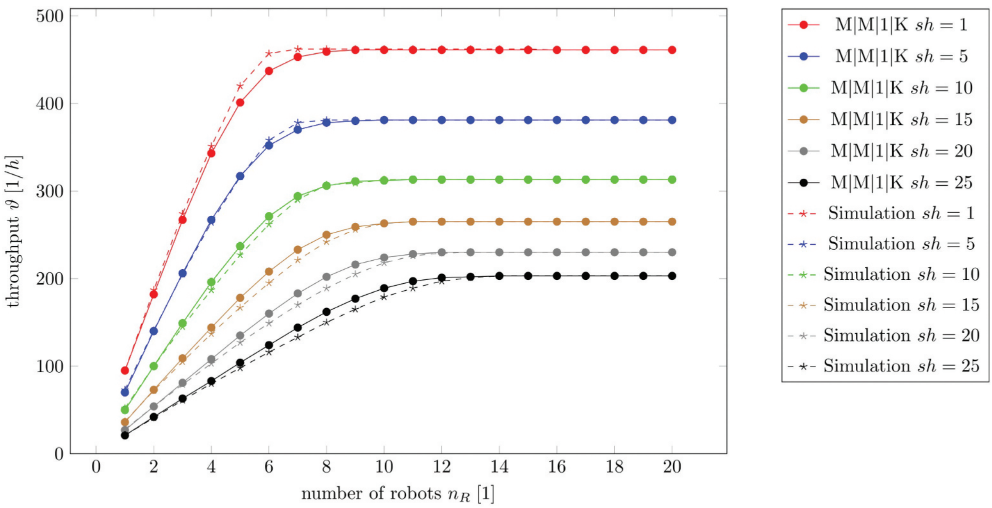



**Figure 9.** Throughput of a 50 by 50 RCS/RS depending on the number of robots for different stack heights comparing the results from DES with those from the analytical approach.

Based on the results for the use of one robot, the performance of several robots shall now be investigated. Therefore, the throughput of a 50 by 50 RCS/RS (2,500 stacks), depending on the number of robots operating, is plotted for different stack heights. Figure 9 compares the M|M|1|K model with the results from DES:

As shown in Figure 9, the system's throughput increases nearly linearly for the first few operating robots. Depending on the stack height, the curve converges towards the limit value of the picking station, which is different for each stack height. The M|M|1|K model provides a good approximation for several stack heights with a varying number of robots. The most significant estimation error is smaller than ten percent.

The deviation between the results of the DES and those of the analytical approach for a specific range of robots operating on the grid (e.g. 4 to 6 robots for $sh = 10$) can be explained by the traffic on the grid. The robots probably have to use a slightly longer track to their assigned destination or wait until another robot leaves a specific grid element. In most cases, a delay for the robots occurs on their track near the I/O shaft.

### 5.2. Parameter variations

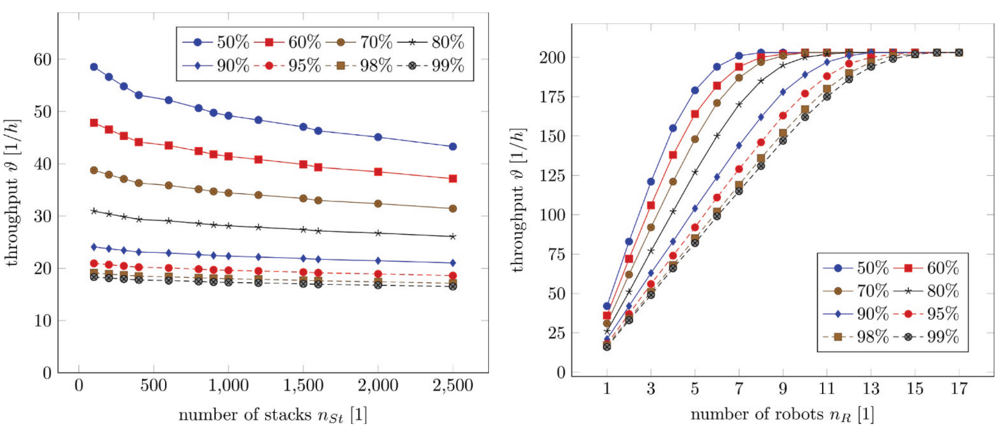
Since neither providers nor science still could not provide general statements on the performance of RCS/R systems, subsection 5.2 sets out to present results of a wide parameter variation. All results within this subsection have been gained from the analytical approach.

The key parameters are the filling degree, the stack height, the grid size, and the number of robots.

Starting with the filling degree, the left Figure 10(a) presents one robot's throughput depending on the number of stacks with varying filling degrees and a stack height of 25.

The number of stacks was successively increased to 2,500, corresponding to a 50 by 50 grid. A high filling degree, e.g. 98%, results in a smaller throughput nearly independent of the grid size. A low number of stacks combined with a small filling degree enables high performance. As shown in Figure 10(a), the influence of the grid size decreases more and more the higher the filling degree gets. An explanation for this is the broader grid and, thus, the longer ride times from and to the I/O shaft. This also reduces the impact of the high temporal number of relocations.

Using more than one operating robot on a 50 by 50 grid increases the throughput. Depending on the number of robots, the throughput is plotted for different filling degrees in the figure on the right. 9. The fuller the warehouse, the flatter the curve is. Assuming a nearly full storage system, e.g. filling degree $f = 99\%$, the number of free stacks for a new or relocation container is low. The robots have longer ride distances; thus, the whole system's throughput is




(a) the number of stacks ($n_R = 1$)

(b) the number of robots ($n_{St} = 2,500$)

**Figure 10.** Throughput of an RCS/RS with a stack height of 25 for different filling degrees depending on.

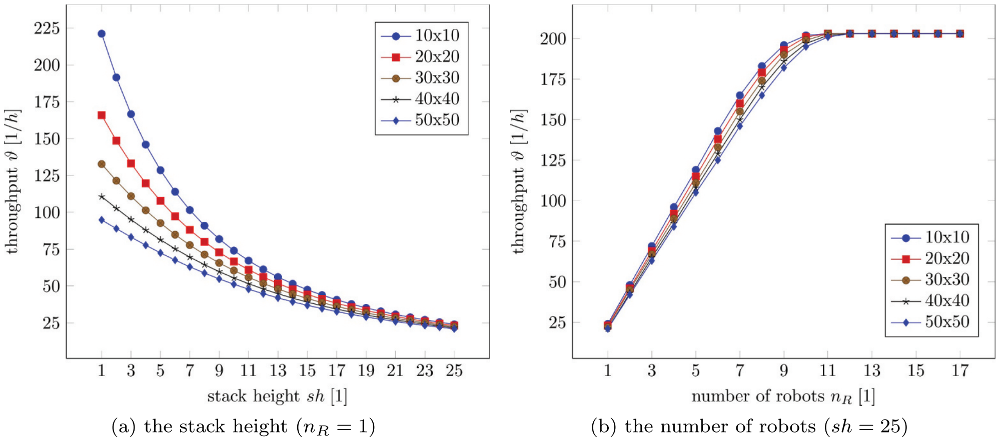



**Figure 11.** Throughput of an RCS/RS (filling degree 90%) for different grid sizes depending on.

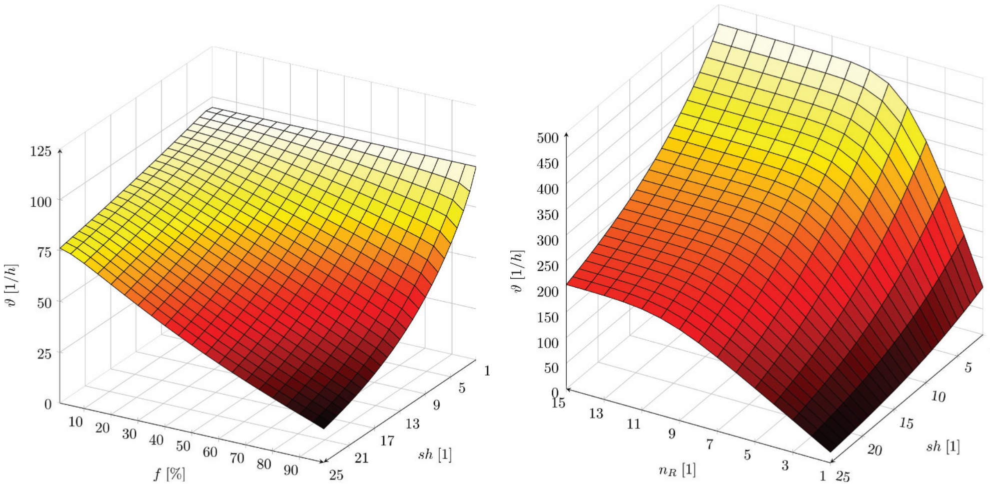
lower until the maximum utilisation of the picking station is reached. While for a low filling degree of, e.g. $f = 50\%$, only seven robots are necessary to retrieve around 200 containers per hour, between 14 to 15 robots must operate on the grid to achieve the same output for a filling degree of $f = 99\%$.

Continuing with a fixed filling degree ($f = 90\%$), one robot's throughput is displayed over the stack height for different quadratic grid sizes Figure 11(a).

The figure shows that the throughput decreases for increasing stack heights. The largest grid size results in the flattest curves. All curves converge, which means that e.g. for a stack height of 25, one robot's possible throughput is nearly identical for every grid size. This can be explained by the long relocation times due to the high stacks. At this point, the consideration of several robots shall again be discussed. Figure 10(b) depicts the throughput for different grid sizes depending on the number of robots. As can be seen, the most significant deviation between the curves arises for a number of 7 to 8 robots. Using fewer robots serving one picking station means having an underused station. In contrast, more operating robots would lead to longer waiting times at the I/O shaft since the utilisation rate of the service station converges towards $\rho = 1$. In summary, the impact of the grid size on the throughput is insignificant.

The last part of Section 5.1 shall provide some parameter variations to discuss the system's characteristics. Therefore, one robot's possible throughput is plotted over the stack height and the filling degree for a grid size of 50 by 50 Figure 12 (a). In addition, the right Figure 12(b) presents the throughput of a 50 by 50 RCS/RS with a filling degree of $f = 90\%$ depending on the stack height and the number of robots.

While the variation of the filling degree leads to a nearly linear progression, a rising stack height causes a parabolic throughput decrease, as can be seen in the left Figure 12(a). The right plot Figure 12(b) exhibits that the optimal number of operating robots varies between 7 for a stack height of $sh = 1$ and about 11 for container stacks up to 25.




(a) with one robot depending on the filling degree $f$ and the stack height $sh$.

(b) with a filling degree of 90% depending on the stack height $sh$ and the number of robots $n_R$.

**Figure 12.** Throughput of an RCS/RS of a 50 by 50 grid size.

**Table 5.** Required parameter setting.

| | |
|---|---|
| Storage capacity | $N = 20,000$ |
| Stack height | $sh \in \{8, \ldots, 25\}$ |
| Filling degree | $f = 90\%$ |
| Number of robots | $n_R \in \{1, \ldots, 15\}$ |

## *5.3. Optimisation example*

After the validation of the analytical approach in subsection 5.1 and the parameter variation in subsection 5.2, a throughput optimisation will be carried out. Therefore, the system's storage capacity shall be 20,000 storage locations. The stack height *sh* can be varied in the range of the realistic values for RCS/R systems, i.e. not less than $sh = 5$ and a maximum of $sh = 25$. The filling degree will be set to $f = 90\%$. Table 5 summarises the assumptions for the optimisation example with one picking station.

The results of the optimisation example sorted by the throughput can be found in Table 6. Besides the throughput, the required space area and volume are also evaluated for a given set of input parameters. The throughput $\vartheta$ is calculated with the maximum number of robots. The space and volume demand only considers the storage locations and the grid.

**Table 6.** Optimisation example.

| No. | $n_{St}$ | $sh$ | $N$ | $\vartheta$ $[\frac{1}{h}]$ | $n_{R\_opt}$ | Area $[m^2]$ | Volume $[m^3]$ |
|---|---|---|---|---|---|---|---|
| 1 | 800 | 25 | 20,000 | 203 | 14 | 290 | 2,971 |
| 2 | 1,000 | 20 | 20,000 | 230 | 13 | 361 | 3,105 |
| 3 | 1,000 | 20 | 20,000 | 230 | 13 | 361 | 3,105 |
| 4 | 1,250 | 16 | 20,000 | 257 | 11 | 450 | 3,275 |
| 5 | 2,000 | 10 | 20,000 | 313 | 10 | 716 | 3,793 |
| 6 | 2,500 | 8 | 20,000 | 337 | 10 | 893 | 4,141 |

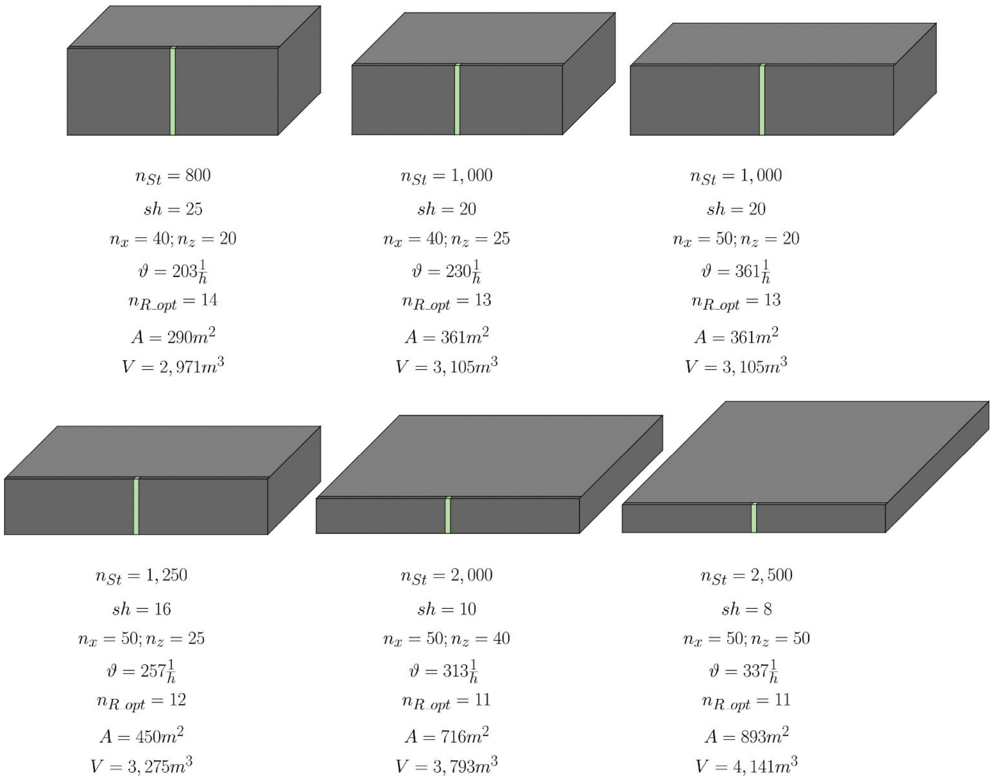



**Figure 13.** Optimisation example.

As can be seen in Table 6, the maximum throughput can be achieved with a 50 by 50 grid and a stack height of $sh = 8$ (case 6). This results in the desired number of $N = 20,000$ storage locations, a space demand of $893m^2$ and a volume demand of $4,141m^3$. The optimal number of robots serving one picking station is about 10. In contrast, a 40 by 20 grid with a stack height of $sh = 25$ (case 1), on the one hand, has a nearly three times smaller space demand but, on the other hand, a throughput reduced by more than a third. Case number 4 shows a scenario containing $n_{St} = 1,250$ stacks with a stack height of $sh = 16$, which leads to a possible throughput of 257 containers per hour with a space demand of $450m^2$ and a volume demand of $3,275m^3$. This could be a good compromise to solve the discrepancy between the required space and the throughput. Figure 13 visualises the geometry of the five cases beginning with the 40 by 20 grid, leading to the smallest throughput and footprint.

## 6. Conclusion

The E-commerce trend is one of the most significant incitements for material handling providers to install more robotic compact storage and retrieval systems. Especially, fully automated food distribution instead of conventional supermarkets is on the rise.

For the last 25 years, the RCS/RS market competition has been tiny. This is one reason for the absence of performance statements. Moreover, not only do the providers keep their data a secret, but just a small number of scientific papers deal with this topic. Most papers had specific targets for default system settings. However, some other storage systems have similar characteristics regarding the storage and retrieval process, the movement of the containers along the three axes, and the system's logic.

This paper's target was to present a fairly accurate analytical approach for the performance approximation of RCS/RS with one picking station. To answer the questions mentioned in the introduction, this paper's analytical performance approach used a cycle time model and an open queueing model with a single queue with limited capacity to predict the system's throughput. The study showed that many system parameters, such as the cubature, how many containers get stacked onto each other, the filling degree, or the number of robots, greatly influence the throughput. The accuracy of the analytical model was validated by comparing the results with those from a discrete event simulation. Based on the validation, a parameter variation was conducted. To sum up, this approach to calculate the cycle time of one robot and the queueing system for the whole system's performance can be used as a first step in designing an alike system. The approach provides easy-to-solve calculation formulas, constituting the basis for expanding the system to multiple picking stations.

To provide an outlook on further work, the next step is to consider more I/O shafts with picking stations along the edges of the grid to be able to calculate the performance of a more extensive system with more robots operating. Another topic of interest could be considering an article distribution (e.g. class-based article distributions) to improve the system performance due to fewer relocations. Moreover, comparing different RCS/RS types and choosing the best with regard to the resulting costs, the space demand, the performance, the storage capacity, or the energy demand could be interesting. All those targets will be part of future scientific discourses, which shall support the design process of RCS/RS.

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## Disclosure statement

No potential conflict of interest was reported by the author(s).

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## Data availability statement

The datasets generated during and/or analysed during the current study are available from the corresponding author upon reasonable request.

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