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Scalar potentials from string theory and (anti-) de Sitter critical points

Skalarpotentiale in der Stringtheorie und (Anti-)de Sitter-Lösungen als kritische Punkte

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DR David Andriot
Laboratoire d'Annecy-le-Vieux de Physique Théorique, CNRS

und

Univ.Prof. Dr. Anton Rebhan
E136 - Institut für Theoretische Physik, TU Wien

eingereicht an der Technischen Universität Wien
E130 - Fakultät für Physik

von

Ludwig Sebastian Horer
01525130

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Ludwig Sebastian HORER

Thèse dirigée par **David ANDRIOT** et
codirigée par **Anton REBHAN**

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Potentiels scalaires issus de la théorie des cordes et points critiques (anti-)de Sitter

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devant le jury composé de :

David ANDRIOT
Directeur de thèse
Joseph Patrick CONLON
Examinateur
Luc Jean Robert FRAPPAT
Examinateur
Susha Louise PARAMESWARAN
Rapporteuse
Anton REBHAN
Co-Directeur de thèse
Gary SHIU
Rapporteur
Washington TAYLOR
Examinateur

Abstract

In this thesis we study the classical de Sitter scenario in type II string theories: in a regime where string corrections to the effective theory are negligible, we analyze de Sitter solutions in flux compactifications with orientifolds – a crucial area for linking string theory to cosmological models. In particular, we explore classical scalar potentials derived from string theory and study in detail the constraints on the existence of their critical points. These extrema correspond to solutions with maximally symmetric spacetimes, including (anti-)de Sitter and Minkowski. Our results reveal that classical flux compactifications leading to d-dimensional (quasi-)de Sitter solutions face strong constraints on their existence. These no-go theorems categorically exclude classical de Sitter in dimensions $d \geq 7$, revealing a preference for four-dimensional spacetimes. This is rigorously checked by computing the parameter c of the de Sitter Swampland Conjecture for each no-go theorem, showing consistency with the bounds given by the Trans-Planckian Censorship Conjecture, with numerous saturation cases in dimensions $d > 3$. This analysis demonstrates a profound agreement between the low-energy limits of string theory and cosmology, while strengthening the validity of the proposed bounds.

In addition, the newly proposed Anti-Trans-Planckian Censorship Conjecture introduces a framework for characterizing negative scalar potentials in the quantum gravity effective theory. This conjecture states that in a contracting spacetime, modes squeezing to sub-Planckian lengths challenge the validity of the effective theory. As a consequence, it imposes bounds on the potential and its derivatives in the asymptotics of field space, which have been tested in various string compactifications. By extending these bounds to anti-de Sitter solutions characterized by radius l, we predict the presence of a scalar field with mass m satisfying $m^2 l^2 \lesssim -2$. This result has significant implications for the corresponding dual conformal field theory.

Recent constructions propose the existence of de Sitter solutions in models with O8-planes/D8-branes that circumvent a classical no-go theorem via unusual sources or, equivalently, corresponding boundary conditions on the bulk fields. Motivated by the ongoing debates on whether these sources arise in classical supergravity, we explore a minimal extension of the classical de Sitter scenario by including 4-derivative corrections in the α' expansion of the O-plane/D-brane action. While higher-order terms and bulk corrections are of minor importance, our analysis shows that even this extended model fails to yield the desired solutions; a conclusion that extends to models with additional O6-planes/D6-branes.

Keywords: string theory, supergravity, de Sitter, D-brane, orientifold

Kurzfassung

In dieser Arbeit untersuchen wir das klassische de Sitter-Szenario in Typ II Stringtheorien. In einem Regime, in dem die Stringkorrekturen gegenüber der effektiven Theorie vernachlässigbar sind, analysieren wir de Sitter-Lösungen in Flusskompaktifizierungen mit Orientifolds – ein wichtiger Ansatz, um die Stringtheorie mit kosmologischen Modellen zu verbinden. Insbesondere untersuchen wir klassische skalare Potentiale, die aus der Stringtheorie abgeleitet sind, und studieren im Detail die Bedingungen für die Existenz ihrer kritischen Punkte. Diese Extremstellen entsprechen Lösungen mit maximal symmetrischen Raumzeiten, einschließlich (Anti-)de Sitter und Minkowski. Unsere Ergebnisse zeigen, dass klassische Flusskompaktifizierungen, die zu d-dimensionalen (quasi-)de Sitter-Lösungen führen, strikten Einschränkungen bezüglich ihrer Existenz unterliegen. No-Go-Theoreme schließen klassische de Sitter-Lösungen in den Dimensionen $d \geq 7$ kategorisch aus und zeigen eine Präferenz für vierdimensionale Raumzeiten. Diese Einschränkungen werden rigoros überprüft, indem der Parameter c der „de Sitter Sumpfland Vermutung“ für jedes No-Go-Theorem berechnet und die Übereinstimmung mit dem Grenzwert der „Trans-Planckian Censorship Conjecture“ in Dimensionen $d > 3$ gezeigt wird. Dieses Ergebnis bestätigt die gute Übereinstimmung zwischen den Niedrigenergiegrenzen der Stringtheorie und den kosmologischen Modellen.

Darüber hinaus bietet die postulierte „Anti-Trans-Planckian Censorship Conjecture“ einen Rahmen zur Charakterisierung negativer Skalarpotentiale. Diese Hypothese besagt, dass auf Sub-Planck-Längen komprimierte Moden in einer kontrahierten Raumzeit die Gültigkeit der effektiven Theorie in Frage stellen. Daher werden Grenzen für das Potential und seine Ableitungen in den Asymptoten des Feldraums aufgestellt, die in verschiedenen Kompaktifizierungen getestet werden. Durch Anwendung dieser Grenzen auf Anti-de Sitter-Lösungen sagen wir die Existenz eines Skalarfeldes der Masse m voraus, das die Bedingung $m^2 l^2 \lesssim -2$ erfüllt. Dieses Ergebnis hat wichtige Implikationen für die duale konforme Feldtheorie.

Weiterhin deuten neuere Konstruktionen auf die Existenz von de Sitter-Lösungen in O8-plane/D8-brane-Modellen hin, die ein klassisches No-Go-Theorem durch ungewöhnliche Quellen oder entsprechende Randbedingungen für die Bulk-Felder umgehen. Angesichts der Debatten darüber, ob diese Quellen in der klassischen Supergravitation auftreten können, untersuchen wir eine minimale Erweiterung des klassischen de Sitter-Szenarios durch die Einführung von Korrekturen nächsthöherer Ordnung in der α'-Entwicklung der O-plane/D-brane-Wirkung. Während Terme höherer Ordnung und Bulk-Korrekturen von geringer Bedeutung sind, zeigt unsere Analyse, dass selbst dieses erweiterte Modell nicht die gewünschten Lösungen liefert. Diese Schlussfolgerung gilt auch für Modelle mit zusätzlichen O6-planes/D6-branes.

Schlüsselwörter: Stringtheorie, Supergravitation, de Sitter, D-Brane, Orientifold

Résumé

Dans cette thèse, nous étudions le scénario de Sitter classique dans les théories de cordes de type II : dans le régime où les corrections de cordes à la théorie effective sont négligeables, nous analysons les solutions de Sitter dans les compactifications avec flux avec des orientifolds – un domaine crucial pour lier la théorie des cordes aux modèles cosmologiques. En particulier, nous explorons les potentiels scalaires classiques dérivés de la théorie des cordes et étudions en détail les contraintes sur l'existence de leurs points critiques. Ces extrêma correspondent à des solutions avec des espaces-temps maximalement symétriques, incluant (anti-)de Sitter et Minkowski. Nos résultats révèlent que les compactifications avec flux classiques menant à des solutions (quasi-)de Sitter de dimension d font face à de fortes contraintes sur leur existence. Ces théorèmes no-go excluent catégoriquement de Sitter classique dans les dimensions $d \geq 7$, révélant une préférence pour les espaces-temps quadri-dimensionnel. Cela est rigoureusement vérifié en calculant le paramètre c de la Conjecture de Sitter du Swampland pour chaque théorème no-go, montrant la cohérence avec les limites fixées par la « Conjecture de Censure Trans-Planckienne », avec de nombreux cas de saturation dans des dimensions $d > 3$. Cette analyse démontre un accord profond entre les limites à basse énergie de la théorie des cordes et la cosmologie, tout en renforçant la validité des limites proposées.

En outre, la nouvelle « Conjecture de Censure Anti-Trans-Planckienne » proposée introduit un cadre pour caractériser les potentiels scalaires négatifs dans la théorie effective de la gravité quantique. Cette conjecture stipule que dans un univers en contraction, les modes atteignant des longueurs sub-Planckiennes remettent en question la validité de la théorie effective. En conséquence, elle impose des limites sur le potentiel et ses dérivées dans les asymptotiques de l'espace des champs, qui ont été testées dans diverses compactifications de cordes. En étendant ces bornes aux solutions anti-de Sitter caractérisées par le rayon l, nous prédisons la présence d'un champ scalaire de masse m satisfaisant $m^2 l^2 \lesssim -2$. Ce résultat a des implications significatives pour la théorie des champs conforme duale correspondante.

Des constructions récentes proposent l'existence de solutions de Sitter dans des modèles avec des O8-plans/D8-branes qui contournent un théorème no-go classique via des sources inhabituelles ou, de manière équivalente, des conditions aux limites correspondantes sur les champs du bulk. Motivés par les débats en cours sur l'origine de ces sources dans la supergravité classique, nous explorons une extension minimale du scénario de Sitter classique en incluant des corrections à 4 dérivées dans l'expansion en α' de l'action des O-planes/D-branes. Bien que les termes d'ordre supérieur et les corrections soient de moindre importance, notre analyse montre que même ce modèle étendu ne parvient pas à produire les vides désirés ; une conclusion qui s'étend aux modèles avec des O6-planes/D6-branes supplémentaires.

Mots clés : théorie des cordes, supergravité, de Sitter, D-brane, orientifold

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1 Introduction and outline

Towards a unified theory

In the landscape of 20th century theoretical physics, two fundamental theories stood out for their rigorous experimental verification and profound theoretical implications: General Relativity and the Standard Model of particle physics. Notably, no known experiment has yet contradicted the predictions of either theory. General Relativity has transformed our understanding of gravity as a geometric property of space and time, providing a sophisticated explanation of macroscopic cosmological phenomena. At the quantum level, the Standard Model provides a comprehensive quantum field theoretical framework that explains the electromagnetic, weak and strong interactions among fundamental particles. This model, when extended to include neutrino masses, is consistent with the observed particle spectrum up to currently available energy scales of several thousand GeV and competently explains physical phenomena well below the Planck scale.

Despite their remarkable successes, these theories leave important questions unresolved and face serious challenges when extended beyond their respective domains of validity at the cosmic and quantum frontiers. General Relativity and the Standard Model, as separate entities, provide incompatible descriptions under conditions where both quantum and gravitational effects dominate, such as close to the singularities of black holes and in the early stages of the universe. Here, the Standard Model falls short since it fails to include gravity or to explain the cosmological observations of dark matter, dark energy and the accelerated expansion of the universe.

Although the Standard Model effectively describes the interactions of the three fundamental forces at lower energies, it suffers from theoretical shortcomings that undermine its status as a fully comprehensive framework. Central to these issues is the dependence of the model on empirically derived parameters, such as the coupling constants of elementary particles. This introduces a problem of naturalness, as these parameters exhibit large variations in magnitude that are not predicted by the theory, but rather are adjusted to fit experimental results. Such inconsistencies have led to theories suggesting an underlying structure more fundamental than the Standard Model itself, perhaps indicated by the symmetry properties that the current framework does not satisfactorily explain.

One avenue that extends beyond the Standard Model and improves our understanding of fundamental interactions are Grand Unified Theories (GUTs), which propose that at energy levels above the GUT scale the electromagnetic, weak and strong forces may converge into a single force governed by a simple Lie group. The precise energy level at which unification might occur, assuming it is realized in nature, depends on the physics at scales that are currently unexplored by experimental capabilities. Supposing the existence of a vast range of energies without new physics – often referred to as the "desert" – and involving a theory of supersymmetry, the unification energy is estimated to be around 10^{16} GeV[1]. This potential unification would label the Standard Model as a low-energy effective

[1] The most advanced particle accelerator, the Large Hadron Collider (LHC), reaches energies up to 10^5 GeV in proton-proton collisions. This puts the hypothetical GUT scale only a few orders of magnitude below the Planck scale of 10^{19} GeV, and far beyond the operational capability of any current collider.

theory of a more fundamental one, which differentiates into the various forces observed in nature as the energy level decreases.

A "theory of everything" that aims to unify the electroweak and strong forces with gravity requires a significantly higher energy level, recognizing the conspicuous absence of gravitational forces in the Standard Model. At the energy scales typically explored in particle physics, gravity remains negligible until one reaches the Planck scale, $M_p \approx 10^{19}$ GeV, where gravitational effects become comparable to those of quantum physics. Consequently, a model that unifies all fundamental interactions, including gravity, is not considered essential at the energy levels commonly studied. However, this view changes dramatically in the context of extremely high energy densities or spacetime curvatures, such as those found in black holes or the early universe, where quantum theory interacts with strong gravitational fields. The main challenge in incorporating gravity into a unified model lies in its (perturbatively) non-renormalizable nature, due to the Einstein-Hilbert action. Unless there is some unexpected cancellation, its naive quantization results in divergent amplitudes. This limitation not only highlights the exclusion of gravity from the Standard Model due to differences in scales, but also emphasizes the need for a comprehensive new theory that effectively integrates gravity with quantum interactions to provide a unified description encompassing all fundamental forces – essentially, a theory of quantum gravity (QG).

A major conundrum in modern cosmology, known as the cosmological constant problem, involves the large discrepancy between the theoretically predicted value of the cosmological constant and its empirically observed value, which is several orders of magnitude smaller. Precise measurements that support the standard cosmological model (ΛCDM) indicate that the universe is undergoing an accelerated expansion, consistent with the characteristics of a de Sitter (dS) spacetime. This expansion is driven by a cosmological constant, Λ, with a corresponding mass scale, $M_\Lambda = \sqrt{\Lambda} \approx 10^{-12}$ GeV, orders of magnitude smaller than the Planck mass. The origin of this surprisingly small scale and properties of the associated "dark energy" remain elusive and pose significant conceptual challenges. These may go beyond current theoretical frameworks, possibly requiring a theory of QG or extra spatial dimensions.

In discussions about the underlying structure of the Standard Model, another theoretical issue is the Higgs boson, whose discovery was crucial for confirming the mechanism of electroweak symmetry breaking. Prior to its experimental verification in 2012, the predicted mass of the Higgs boson required fine-tuning to extremely precise values to avoid large quantum corrections that could destabilize the electroweak scale; an issue commonly referred to as the Higgs hierarchy problem. Such precise level of fine-tuning is considered unnatural. Supersymmetry proposes a theoretical framework that could solve this problem by introducing superpartners for each particle, thereby naturally balancing the quantum corrections and stabilizing the Higgs mass.

Supersymmetry (SUSY) is a robust extension of the Standard Model of particle physics. This global symmetry extends the Poincaré group by adding fermionic generators in the corresponding Lie superalgebra, thus avoiding the Coleman-Mandula no-go theorem. In a four-dimensional (4D) spacetime, the number of supersymmetries, denoted by $\mathcal{N}$, varies from one to four for fields with spin one, and up to eight for spin-two particles such as the graviton. In supersymmetric models, particles are grouped into multiplets containing both fermions and bosons, typically in equal numbers and with identical masses, ensuring that each particle is paired with a superpartner whose spin differs by one half. However, this exact symmetry is not observed at energies typical for the Standard Model, suggesting that SUSY must be broken at higher energy scales.

Despite the complexities introduced by SUSY, including multiple breaking schemes and unobserved superpartners, such extensions have significant advantages. The additional symmetry in supersymmetric gauge theories imposes strict constraints, so that quantum corrections are typically more manageable due to the cancellation between bosonic and fermionic contributions. Another asset is the refinement of the GUTs: the three coupling constants of the Standard Model converge exactly to a single point when projected to higher energies. These attractive properties have strongly driven the adoption of SUSY in the Standard Model.

Considering SUSY as a local symmetry inevitably leads to supergravity (SUGRA) theories. These are theories of gravity that are supersymmetric and more constrained than conventional General Relativity. Notably, in an 11D spacetime, the theory is uniquely defined by its symmetries. In reduced dimensions, different supergravity theories can be formulated, with possible connections between them. Although these theories provide a way to extend SUSY into a framework that includes gravity, achieving renormalizability remains a significant challenge. Despite these obstacles, supergravity theories, especially those in higher dimensions, provide a distinctive and promising path towards developing a potential theory of QG.

Physics beyond the Standard Model is not inherently related to a theory of QG, especially when considering the scales involved. As noted above, QG effects become significant at the Planck scale, whereas expansions of the Standard Model typically focus on phenomena at the electroweak scale[2]. Nonetheless, from a unification perspective, these two areas should eventually overlap. Moreover, if one can develop a theory of QG that also includes certain gauge groups, this raises the possibility that at lower energies some extensions of the Standard Model might be observable. The theory involved is string theory [2–6].

String theory

String theory proves to be a profound framework for a unified theory of QG, wherein the fundamental constituents are not point-like particles, but 1D "strings" embedded in a higher-dimensional (target) spacetime. The mass and charge of the particles are defined by the oscillations of these strings through different vibration modes. They also provide a solution to the UV divergences observed in point-particle theories, which are inherently resolved by the extended nature of the strings. Moreover, string theory inherently includes a

[2]Note that the concept of species scale can significantly alter the relationship between these scales [1].

rank-two symmetric tensor, the graviton, a quantum particle associated with gravity. This property implies a natural integration of gravity via the massless modes when string theory is quantized. Nonetheless, string theory is characterized by finite amplitudes, confirming the renormalizable nature of gravity within this model.

In the perturbative approach to string theory, a unique characteristic is the inherent spacetime SUSY, which includes both bosonic and fermionic degrees of freedom. This property is important because it ensures that all consistent string theories are anomaly-free, but it requires a 10D spacetime. Among these models are five superstring theories, including type IIA and IIB, which are characterized by $\mathcal{N} = 2$ supersymmetry. Type I and two heterotic string theories, distinguished by their gauge groups, either $E_8 \times E_8$ or $SO(32)$, are $\mathcal{N} = 1$.

In regimes of low-energy and weak coupling[3], these theories approximate classical SUGRA, albeit corrected by higher-order derivatives and both perturbative and non-perturbative quantum effects. This requires that the parent theory's massive modes are sufficiently large to be negligible, typically realized by a mass scale given by the string length, l_s. Remarkably, this is the only universal constant in string theory that is left undefined. Despite the lack of a specific reason, it is commonly associated with the Planck scale. Moreover, these theories, along with 11D M-theory[4], are connected by dualities and thus are understood as different representations of a unified theory under different constraints.

The connection to the observable universe, in particular the ability of string theory to reproduce known physical properties in its low-energy limit, is one of the major challenges. For consistency, superstring theories require a 10D spacetime, but only four of these dimensions are observed at the macroscopic scale. The remaining six dimensions are assumed to be compact and closed on themselves. These dimensions form a smooth compact manifold, typically called $\mathcal{M}_6$, which would only become observable at energy levels beyond those currently achieved experimentally. Therefore, $\mathcal{M}_6$ remains undetectable, which is why we call it "internal space". The 10D space is then a topological product of $\mathcal{M}_4 \times \mathcal{M}_6$. The product of six circles T^6 is a simple example where each geometric parameter, such as the mean radius, introduces an additional compactification scale, M_c. More complicated configurations include Calabi-Yau manifolds, which are favored for preserving $\mathcal{N} = 1$ supersymmetry in the 4D effective theory.

Starting from string theory, we develop its low-energy limit by a so-called dimensional reduction to obtain an effective 4D description. An important step in the development of this effective theory is the identification of light (or massless) modes of the full theory, which typically manifest as small fluctuations around a stable background or vacuum state of the potential. Only these modes are then retained in the theory, provided that this selective approach maintains consistency. The result is a robust low-energy effective theory, here 4D SUGRA. It is critical to proceed with caution. First identify the light modes and then

[3]For the SUGRA approximation to hold, the string coupling constant, g_s, must be small to ensure that the theoretical description remains perturbative.

[4]Its fundamental objects are higher-dimensional extended objects, known as M-branes. The low-energy effective theory is then considered to be 11D SUGRA.

determine the possibility of eliminating interactions with massive modes before truncating the spectrum. To illustrate the process of dimensional reduction to an effective 4D theory, it is instructive to consider a simple example in which light modes play a central role.

The concept of dimensional reduction originated in the 1920s. Consider a field theory defined on $\mathcal{M}_4 \times S^1$. In this model, the fields can be expanded into a Fourier series along the periodically compactified fifth dimension, with each term in the series $\sim \mathrm{e}^{-\mathrm{i}(n/R)x^5}$ representing quantized momenta, n/R, along the circle of radius R. Any value of the momentum contributes to the mass of the corresponding 4D modes, ${m_n}^2 \sim (n^2/R^2)$, supposing the fields are massless in 5D. The action can then be integrated over the compact dimension to deduce an effective 4D description. Higher Kaluza-Klein modes (corresponding to higher values of n) contribute progressively less to the action. Additionally, a smaller radius reduces the contribution of modes with $n \neq 0$. This process, known as Kaluza-Klein reduction, involves integrating out the heavy modes and retaining only the 4D massless modes to reformulate the full action. This is equivalent to the fields being independent of the internal coordinates.

Returning to 4D SUGRA, to be consistent with (extensions of) the Standard Model, this effective theory is typically given by $\mathcal{N} = 1$ supergravity and SUSY should be broken on a scale far below the compactification scale. Therefore, string compactifications are central to aligning the theoretical 10D spacetime with the observable world. However, this process poses several conceptual challenges, particularly in the stabilization of moduli fields φ^i. These massless scalars in the 4D effective (bosonic) action

$$S^{(4)} = \int \mathrm{d}^4x\sqrt{-g_4}\left(\frac{M_p^2}{2}\mathcal{R}_4 - \frac{1}{2}G_{ij}\partial_\mu\varphi^i\partial^\mu\varphi^j - V(\varphi)\right), \tag{1.1}$$

are critical for defining properties such as the volume of the internal space. Although, their effective potential, $V(\varphi)$, remains flat due to their inherent lack of mass. Therefore, their vacuum expectation values and the physical parameters they control remain undefined, which is a significant hurdle in the development of realistic models. The existence of moduli fields contradicts empirical observations, emphasizing the urgent need for effective moduli stabilization strategies across different string compactifications.

A promising approach to the moduli problem is flux compactifications [7–9], where background fluxes generate a potential for the moduli. While this method often successfully stabilizes many moduli, it does not universally address all moduli comprehensively, necessitating the incorporation of additional measures, such as quantum corrections to the effective potential. These corrections are difficult to calculate accurately. In addition, the potential generated by these fluxes, along with effects from local structures, may unintentionally destabilize some moduli, leading to the possible appearance of tachyons in the 4D theory.

Such complexities, together with the intricacies of the flux quantization conditions, make the explicit construction of realistic models technically challenging, especially in scenarios without inherent protective properties such as SUSY that could prevent the presence of tachyons. The quest for fully stable and realistic moduli stabilization is riddled with obstacles, emphasizing the ongoing challenge of mapping 10D string theories into observable phenomena in our universe.

Heterotic strings have been important due to their integration of non-abelian gauge groups, which are suitable for Grand Unified Theories and may potentially match the gauge interactions of the Standard Model. However, the full realization of this ambitious program remains incomplete. Meanwhile, type II string theory, which typically features abelian gauge fields in its perturbative spectrum, has gained prominence following advances in non-perturbative physics, in particular the AdS/CFT correspondence. The potential to introduce non-abelian gauge groups through non-perturbative objects known as D-branes has made type II a more adaptable and promising framework for effectively modeling field content, gauge groups and couplings. For this reason, type II SUGRA will be the main focus of this thesis.

Within type II string theory, initially defined in a perturbative sense, D-branes are indispensable extended objects, conceived as solitonic membranes or hypersurfaces of different dimensions that evolve dynamically in time. These branes, characterized by their ability to anchor the endpoints of open strings under Dirichlet boundary conditions, exhibit dynamics driven by the perturbative oscillations of these open strings. In classical SUGRA, the dynamics of D-branes are governed by scalar and vector fields arising from the massless modes of the open strings on the worldvolume of the branes. In particular, the vector fields can lead to non-abelian gauge theories, positioning D-branes as essential for exploring complex physical phenomena beyond the limits of perturbative string theory. Meanwhile, the scalar fields act as moduli that determine the position of the brane in spacetime.

In the landscape of string theory, alongside D-branes and fundamental strings, orientifold planes (O-planes) play a crucial role. O-planes are hypersurfaces defined by specific symmetries of the underlying spacetime. They are identified as fixed points within a manifold, resulting from the imposition of a finite symmetry on the fields coupled with an inversion of the worldsheet orientation. This symmetry operation involves not only spatial reflections, but also a reversal in the orientation of the strings, which profoundly affects the physical properties and types of allowed interactions in string models.

Both D-branes and O-planes act as sources of Ramond-Ramond (RR) charges, generating electric and magnetic fields, and they influence the gravitational landscape by adding tension, which alters the overall energy density of the vacuum. Together, they facilitate non-abelian gauge interactions through vector fields located on the worldvolumes and modify the theoretical construction of the universe by altering spacetime properties at their localized positions. These unique properties establish D-branes and O-planes as essential components within the broader framework of string theory. They interact intricately with other extended objects, such as NS5-branes, KK monopoles or anti-D-branes ($\overline{\text{D}}$-branes), all of which are manifestations of the deeper symmetries and dualities inherent in the theory. However, the latter are not relevant to our current studies.

D-branes/O-planes are localized on submanifolds within the 10D spacetime, which plays a critical role in several aspects of theoretical physics. Their localization is essential to ensure consistency during the compactification process, especially in scenarios involving background fluxes. From a phenomenological point of view, these localized sources offer several advantageous properties; they facilitate the breaking of SUSY and induce a significant warping of spacetime[5], highlighting the profound impact these structures have on the fabric of spacetime. In the broader context of string cosmology, which will be extensively explored in this thesis, local sources are essential for developing a coherent understanding of the fundamental properties of the universe. However, their presence significantly complicates the equations of motion. Achieving a complete solution that fully accounts for their effects is an ambitious challenge, often beyond current analytical capabilities in classical SUGRA. This complexity is explored in more detail in this thesis.

A practical approach to the challenges posed by localized sources is to replace them with a homogeneous charge distribution that is "smeared" across the internal manifold, a technique known as the smeared approximation [12–15]. In this method, the delta-functions, which localize the sources in the supergravity equations, are replaced by regular functions. The main advantage of this approach is the significant simplification of the calculations, since it allows the equations of motion to be treated on an integrated rather than a localized one. In addition, smearing facilitates well-established methods such as consistent truncations [16–18]. On the other hand, the treatment of localized sources would require a more sophisticated framework such as warped compactifications [19, 20], which remain ambiguous.

However, this simplification comes at the cost of neglecting source backreaction effects on both the internal fields and the geometry of the compact space. This omission can have significant effects on 4D observables, such as the values of the moduli or the cosmological constant. There is an ongoing debate as to whether these backreaction effects are non-negligible, since the fully backreacted 10D equations are not satisfied by the smeared solutions. Therefore, the validity of smeared solutions as accurate representations of localized scenarios is still under debate, underscoring the critical need to understand the backreaction of localized sources. This issue is particularly relevant since the majority of solutions currently in the literature rely on the smeared approximation, with only a limited number exploring fully localized configurations. We will explore this topic further in the course of this thesis.

The string landscape

A profound challenge in deriving low-energy physics from string theory is the vast number of compactification choices, resulting in a metaphorical landscape of around 10^{500} vacua[6], each with unique physical properties [22]. This is often referred to as the string landscape. How does string theory select a 4D effective theory that matches our observable universe from this extensive set? In other words, the chosen 4D spacetime must be phenomenologically viable.

[5] This warping is crucial and typically cannot be achieved by SUGRA alone. For more details, see the discussion of the Maldacena-Nuñez no-go theorem [10, 11].

[6] But it could be as high as $10^{272\,000}$ or more [21].

Maximally symmetric spacetimes are important in this context. Minkowski spacetimes are relevant for mimicking phenomena of the Standard Model. Anti-de Sitter (AdS) spacetimes are considered in holographic contexts [23, 24], certain dS constructions [25, 26] and discussions of scale separation [27, 28]. However, string theory must also account for the accelerated expansion of the universe, which can be achieved by a positive cosmological constant or by rolling scalar fields. This work focuses on the former, specifically on compactifications to a dS spacetime.

Despite extensive research over the past two decades, the feasibility of dS compactifications within string theory remains uncertain. While several promising scenarios have been proposed, including [25, 26], fully explicit models of dS space have yet to be realized. Moreover, several theoretical arguments have cast suspicion on the compatibility of dS spacetime with QG [29–31].

One major difficulty is that dS solutions are inherently non-supersymmetric, which complicates the task of satisfying all the equations of motion compared to supersymmetric solutions. Furthermore, without SUSY, there is no intrinsic mechanism to ensure the stability of the solutions, making moduli stabilization problematic. Additionally, dS solutions are ruled out in the simplest classical compactifications [10, 11], thus requiring corrections to the classical SUGRA approximation or localized sources of negative tension.

In string theory, two main strategies for obtaining dS vacua are prominent in the literature. One way is to include higher derivative corrections and non-perturbative effects in the classical SUGRA approximation. Another notable approach is the classical dS scenario [32, 33], which explores compactifications with O-planes and background fluxes in the classical regime of type II string theory [16, 34–36]. The low-dimensional effective theory is obtained by dimensional reduction of 10D type II SUGRA at the 2-derivative level, including the action of localized O-planes/D-branes at the leading-order in α'. This approach ensures that both perturbative and non-perturbative string corrections remain minimal in the effective theory, although not necessarily everywhere in 10D. This is the case for small string coupling and sufficiently large volume.

The classical dS scenario offers several advantages in type IIA, such as providing explicit models that avoid the need for complex moduli stabilization and the difficult computations of the full moduli dependence of instanton corrections. However, this approach has serious limitations. Many classical models are disqualified by no-go theorems that exclude dS extrema [32, 37–39]. The few identified dS extrema are often perturbatively unstable, or they encounter issues with strong curvature and large string coupling[7]. These instabilities are often associated with a tachyon [43, 44].

Despite these obstacles, the classical dS scenario has not been explicitly ruled out [45], leaving room for ongoing research and possible counterexamples in the field.

Outline and summary

This thesis builds upon the research published in [35, 36, 39, 46, 47].

[7] See also the arguments suggesting that classical dS vacua with small curvature and coupling are not possible in many classes [40, 41], as well as possible counterexamples [42].

In Section 2.2, we clarify the conventions adopted and provide an overview of basic aspects of type II flux compactifications with localized sources. In Section 2.3, we explore the various boundary conditions employed throughout this thesis, and in Section 2.4, we revisit and rationalize the use of the smeared approximation, a pivotal element of our analytical framework.

In Section 3, we illustrate the process of dimensional reduction using two distinct sets of scalar fields. Our objective is to establish the transformation laws for these fields into canonical forms and to derive the corresponding potential of a d-dimensional effective theory as specified in (1.1) for each set of scalars.

Constraints on (quasi-)de Sitter solutions in 10D SUGRA

String theory, traditionally formulated in 10D spacetime, presents a stark contrast to the 4D universe we observe, raising fundamental questions about the dimensional nature of our universe and the theoretical landscape of string theory [48]: why are only four dimensions observable? Given that string theory does not inherently favor any specific dimensionality, it becomes essential to explore the viability of higher-dimensional scenarios within this framework.

The study of classical dS solutions in dimensions $d \geq 3$ is therefore not just a theoretical curiosity, but is motivated by various factors [49]. To date, all explicit and robust solutions within string theory feature a non-positive cosmological constant. This observation supports the widespread view that the landscape of flux vacua is devoid of metastable dS solutions [30]. To challenge this prevailing belief, we need to find a single counterexample that is metastable, de Sitter and reliable, without any constraints on its dimension or SUSY breaking scale.

Exploring higher-dimensional solutions may simplify complexities associated with moduli stabilization, fluxes and branes, potentially easing the search for dS solutions. Additionally, these searches might also yield simpler models for testing theoretical propositions, such as the dS/CFT correspondence [50], and even facilitate the development of 4D quintessence models through dimensional reduction [51]. By extending our search to arbitrary dimensions, we aim to address both theoretical and phenomenological questions, thereby deepening our understanding of dS spaces in string theory and their implications for our universe.

In Section 4 we explore the possibility of identifying classical (quasi-)dS solutions across various dimensions. In an attempt to both identify and constrain such configurations, numerous studies have concentrated on classical dS solutions in four dimensions. For a comprehensive list of these studies, see [52]. Despite these efforts, the successful identification of these solutions remains limited, leading to the formulation of several no-go theorems that constrain fluxes, sources and manifold properties necessary for viable supergravity configurations [38, 45, 53].

This thesis contributes to this area by deriving no-go theorems within type II SUGRA, effectively ruling out dS solutions in a classical context. While proving that a dS solution is forbidden in SUGRA is sufficient to rule out its existence in a classical regime, the reverse is not necessarily true; discovering a dS solution in 10D SUGRA does not guarantee its presence in the classical string background. This validation even fails in known 4D cases [41, 54], and generic arguments for such failures have been given in [40, 45, 55].

Our discussion extends well-established no-go theorems [38] in $d = 4$ to arbitrary dimensions, $3 \leq d \leq 10$, introducing some novel aspects related to this dimensional expansion. This effort also builds upon the foundational work in [49], where such dimensional extensions were first explored; our work reproduces and expands on these results.

In Section 4.2 we establish no-go theorems using 10D equations which, under certain assumptions, yield an inequality $\mathcal{R}_d \leq 0$ that effectively forbids dS solutions. We deliberately ignore complications arising from smeared orientifolds and the backreaction of localized sources, focusing instead on scenarios that allow classical dS solutions with smeared sources. Further studies of non-classical approaches in higher dimensions may also face similar difficulties. This remains a fertile area for future research.

In Section 4.3 we systematically apply these no-go theorems in dimensions $d \geq 4$. As the dimension increases, the constraints on flux and source content become more stringent, often automatically satisfying some of the theoretical requirements of the no-go theorems. This is particularly evident in Section 4.3.2, where we focus on configurations that preserve SUSY.

Our conclusion, summarized in Section 4.3.3, explicitly rules out the existence of classical dS solutions in dimensions $d \geq 7$, while the possibilities in dimensions $d = 5, 6$ are limited and further dismissed by additional theoretical conjectures, as outlined in [35, 45]. These results suggest a theoretical bias towards $d \leq 4$, potentially indicating a preference for $d = 4$. We extend these observations to quasi-dS solutions in Section 4.4.

Asymptotic behavior of scalar flux potentials in lower-dimensional effective theories

Our earlier discussion revealed significant challenges in the search for classical (quasi-)dS solutions in arbitrary dimensions. Meanwhile, the swampland program [56, 57] seeks to establish criteria that distinguish whether a consistent effective theory can emerge as a low-energy limit of string theory. Theories that do not meet these criteria are considered to lie in the "swampland". From this perspective, all dimensions should be equally considered, implying that the scope of the swampland conjectures applies universally across string compactifications in arbitrary external dimensions, unless QG provides a compelling argument for a specific dimensional preference.

Within this context, the de Sitter Conjecture [30] proposes a systematic obstacle to dS solutions, formulated as an inequality

$$M_p |\nabla V(\varphi)| \geq c\, V(\varphi)\,, \qquad c \sim \mathcal{O}(1)\,. \tag{1.2}$$

Although primarily studied in 4D [38], this conjecture is valid in $d \geq 3$. More recently, this inequality is thought to hold only in the asymptotic limits of moduli space, as shown by the Trans-Planckian Censorship Conjecture (TCC) [58], introducing a theoretical minimum for the constant c,

$$c \geq c_0 \,, \qquad c_0 = \frac{2}{\sqrt{(d-1)(d-2)}} \,. \tag{1.3}$$

In Section 5, we rigorously test this bound through no-go theorems. If it were to be validated in our observable universe, it would necessitate more complex cosmological models [59, 60]. Furthermore, our motivation to study the classical regime of string theory lies in its agreement with the asymptotics where these inequalities are assumed to be valid, which is ideal for probing the TCC.

We recapitulate the SUGRA no-go theorems using a d-dimensional effective theory, where $V(\varphi) > 0$, similar to (1.1). Under certain assumptions, we derive an inequality akin to (1.2), which reveals a specific value for c and effectively rules out dS critical points. This derivation is consistent with the inequalities obtained in 10D, confirming that $|\nabla V(\varphi)|$ cannot vanish or even remain small. We elaborate on this in Section 5.3, where we detail how this derivation also precludes quasi-dS solutions, characterized by a positive potential with minimal gradient and slow field dynamics. While extensive analyses have previously focused on $d = 4$ [38], our work extends this to determine a d-dependent value of c, facilitating direct comparison with the bound in (1.3).

Our results, summarized in Section 5.4 and illustrated in Table 6 and Figure 4, confirm the TCC bound in dimensions $d \geq 4$, showing multiple instances of saturation consistent with those observed in $d = 4$. This consistency across dimensions serves as a substantial validation of the TCC bound, supporting the universal nature of swampland conjectures. Furthermore, in Section 5.4.1, we explore an intriguing anomaly in $d = 3$, where a newly established no-go theorem indicates a c value below the TCC threshold, related to the peculiarities of gravity in this dimension.

In Sections 5.2 and 5.4.2, we compare the TCC with other theoretical proposals that have appeared in the literature, including the Swampland Distance Conjecture. In Section 5.4.3, we also discuss an asymptotic upper bound on $|\nabla V(\varphi)|$ necessary for cosmic accelerated expansion. When violated, it suggests alternative cosmological scenarios and opens pathways for further research, especially if the TCC bound holds true.

We then shift our focus to negative scalar potentials. Although these might seem less relevant to cosmological models, despite ekpyrosis and bouncing cosmologies [61–66], AdS vacua, which are subject to extensive studies due to advances in holography [23, 24, 67], also fall under this category. The structural similarities between negative and positive scalar potentials in string theory, which primarily differ by variations in the values and signs of the coefficients, suggest a unified approach could be applied to both types. These similarities indicate that methods developed for analyzing positive potentials might be effectively adapted for their negative counterparts.

Our analysis employs models that describe spacetimes characterized by either expansion or contraction, as indicated by the scale factor $a(t)$ in the metric. Detailed discussions of this subject are provided in Sections 5.1 and 6.1.1. The concept of contracting spacetimes is particularly important in the context of the Anti-Trans-Planckian Censorship Conjecture (ATCC):

Anti-Trans-Planckian Censorship Conjecture. In any (lower-dimensional) effective theory of QG (1.1) with $V < 0$ admitting a contracting cosmology, modes with a wavelength close to the typical length scale of the universe at t_i should not shrink to sub-Planckian levels at some later time $t > ti$ without losing the validity of the effective theory.

This statement can be analytically expressed as

$$\frac{a(t)}{a(t_i)} \geq \frac{\sqrt{|V(\varphi(t_i))|}}{M_p^2}\,, \qquad \forall\, t > t_i\,. \tag{1.4}$$

The ATCC reinterprets our understanding of contracting universes with $V < 0$, which lack the horizon mechanisms utilized in dS spaces as proposed in the original TCC [58]. Instead, it emphasizes the importance of maintaining the validity of the effective theory by ensuring that the typical energy scale, i.e. the scalar potential, remains sub-Planckian. We delve deeper into this topic in Section 6.1.2, highlighting that solutions violating 1.4 fall outside the regime of validity. This refined approach is not merely speculative; it is grounded in the physical constraints dictated by the fundamental scales of the theory.

In Section 6.1.3, we derive an explicit bound on the lifetime, interpreted in the context of spacetime contraction. From 1.4, we develop a framework for characterizing negative potentials in dimensions $3 \leq d \leq 10$ using Planckian units, focusing on a single scalar field. This framework necessitates an assumption about V and $a(t)$ that, while naturally satisfied for $V > 0$, requires careful scrutiny for $V < 0$. We derive an exponential lower bound, $V(\varphi) \geq -\mathrm{e}^{-c_0|\varphi-\varphi_i|}$, applicable across the entire field space. This leads to the following condition in the asymptotic limit of field space,

$$\left\langle -\frac{V'}{V} \right\rangle_{\varphi\to\pm\infty} \geq c_0\,, \tag{1.5}$$

mirroring the one of the TCC despite subtle differences in their derivations. This condition does not categorically rule out AdS critical points in the asymptotic limits, similarly to how the TCC does not explicitly exclude dS spaces. Instead, it imposes constraints on the asymptotic behavior of the potential and sets a lower bound on the exponential rates. In Section 6.1.4, we verify the validity of these bounds as well as the previous assumption in various cosmological models, including AdS and dynamical solutions with rolling fields.

In Section 6.2, we introduce a novel condition on the second derivative of the potential,

$$\left\langle \frac{V''}{V} \right\rangle_{\varphi\to\infty} \geq \frac{4}{(d-1)(d-2)}\,. \tag{1.6}$$

This asymptotic condition suggests that in every d-dimensional AdS solution of typical length l, there exists a scalar field with mass m satisfying

$$m^2 l^2 \lesssim -2\,. \tag{1.7}$$

This flexible bound naturally holds for perturbatively unstable solutions [68, 69].

We rigorously test this condition against a variety of perturbatively stable configurations. Most supersymmetric setups obey this bound, despite some debated exceptions [25, 26, 70]. In contrast, non-supersymmetric models often require more flexibility and may exhibit some deviations from this established rule. However, many of these are prone to non-perturbative instabilities [71]. A comprehensive summary of these examples is provided in Tables 7 and 8. Furthermore, the holographic implications of this bound for a dual CFT are discussed in Section 6.3.3.

Finally, Sections 6.4 and 6.5 explore multi-field models and apply the ATCC to specific string compactifications. We validate the ATCC bounds for a semi-universal potential, $V(\rho, \tau, \sigma)$, initially derived in Section 3, and in the context of flux compactifications leading to so-called DGKT solutions [70].

Almost classical de Sitter

Recently, the authors of [72] claimed to have identified classical dS vacua in a remarkably simple model referred to as CDT1. This model, employing type IIA string theory with parallel O8-planes and the Romans mass as the sole flux, simplifies the equations of motion to solvable ordinary differential equations (ODEs) that include the nonlinear backreaction of the O-planes. Subsequent analyses, however, have shown that these localized sources are not consistent with the conventional string theory interpretation of O8-planes at the leading order in α' [73]. More generally, it has been found that the classical type IIA bulk action, together with the leading-order O8/D8 action within the α' expansion, excludes classical dS in all flux compactification lacking higher codimension sources [73, 74].

In a later paper [75], a potential loophole due to ambiguities in the SUGRA equations was suggested, and "permissive" boundary conditions were proposed that could allow source terms violating the assumptions made by [73]. In Sections 2.3 and 7.1.2, we will argue that these ambiguities are not evident at the level of classical SUGRA, and that the sources must conform to those specified in the no-go theorem.

As indicated by [73], this conclusion might be modified by considering the effect of leading α' corrections in the O-plane/D-brane action, which are known to appear at the 4-derivative level [76–84]. To explore further, Section 7.1.3 introduces the "almost classical" dS scenario, a minimal extension of the classical dS framework that incorporates leading α' corrections while neglecting higher-order derivatives. Despite potential concerns about regions where the α' expansion may not hold, it is claimed that the classical terms and their 4-derivative corrections dominate the contribution of the O-planes to the vacuum energy, with the effects of the non-perturbative hole region playing a role only in short-range physics. Nonetheless, this approach still fails to produce metastable dS solutions, thereby affirming the validity of the no-go theorem.

The CDT2 model, which incorporates both O6/O8-planes, is also discussed in Section 7.2.1. This model, though more complex, retains solvable equations that account for nonlinear O-plane backreaction. However, similar to the CDT1 model, the CDT2 model adheres to a classical dS no-go theorem outlined in Section 7.2.3, leading exclusively to AdS solutions. The introduction of 4-derivative couplings in Section 7.2.4 does not resolve these challenges, as their effects remain secondary in low-curvature regimes.

This section underscores the persistent challenges in achieving stable dS vacua within the framework of string theory and illustrates the limitations inherent in both classical and near-classical dS scenarios. Despite offering valuable insights, these models have not yet provided a viable path to stable dS solutions, highlighting the ongoing need for further research and exploration of alternative strategies in the field.

2 Preliminaries

In this section we review the basic principles and recent developments in flux compactifications from 10D type II supergravity to a d-dimensional effective theory characterized by a maximally symmetric spacetime. Given that both orientifolds and D-branes are intrinsically localized objects, it is essential to carefully analyze the boundary conditions they impose on the bulk fields. A thorough discussion of this topic is presented in Section 2.3, with a focus on models containing O8-planes, as discussed in [72]. Furthermore, Section 2.4 will provide a comprehensive analysis of backreaction effects and elucidate their connection to the smeared approximation.

2.1 Differential geometry

We follow [85–87] in our conventions for tensors and differential forms. The vector space of (smooth) r-forms on a m-dimensional manifold $\mathcal{M}$ is denoted by $\Omega^r(\mathcal{M})$. The set of r-forms,

$$\mathrm{d}x^{a_1} \wedge \ldots \wedge \mathrm{d}x^{a_r} = \sum_P \mathrm{sgn}(P)\, \mathrm{d}x^{a_{P(1)}} \otimes \mathrm{d}x^{a_{P(2)}} \otimes \ldots \otimes \mathrm{d}x^{a_{P(r)}}\,, \tag{2.1}$$

serves as a coordinate basis, where $\mathrm{sgn}(P) = +1$ (-1) for even (odd) permutations of $1, \ldots, r$. A differential form $\omega \in \Omega^r(\mathcal{M})$ can be expressed as a linear combination

$$\omega = \frac{1}{r!}\, \omega_{[a_1 \ldots a_r]}\, \mathrm{d}x^{a_1} \wedge \mathrm{d}x^{a_2} \wedge \ldots \wedge \mathrm{d}x^{a_r} \tag{2.2}$$

of the basis vectors. Indices in brackets indicate a unit-weight (anti-)symmetrization, $\omega_{(ab)} = \frac{1}{2}(\omega_{ab} + \omega_{ba})$ and $\omega_{[ab]} = \frac{1}{2}(\omega_{ab} - \omega_{ba})$, reflecting the antisymmetry of the basis (2.1). For differential forms $\xi \in \Omega^p(\mathcal{M})$ and $\eta \in \Omega^q(\mathcal{M})$, the wedge product is defined as

$$\xi \wedge \eta = \frac{1}{p!q!}\, \xi_{[a_1 \ldots a_p} \eta_{b_1 \ldots b_q]}\, \mathrm{d}x^{a_1} \wedge \ldots \wedge \mathrm{d}x^{a_p} \wedge \mathrm{d}x^{b_1} \wedge \ldots \wedge \mathrm{d}x^{b_q}\,, \tag{2.3}$$

with the following properties

$$\begin{gathered}(\xi \wedge \eta) \wedge \omega = \xi \wedge (\eta \wedge \omega)\,, \qquad \xi \wedge \eta = (-1)^{pq}\, \eta \wedge \xi\,, \\ \xi \wedge \xi = 0 \quad \text{if } q \text{ is odd}\,.\end{gathered} \tag{2.4}$$

The exterior derivative $\mathrm{d} : \Omega^r(\mathcal{M}) \to \Omega^{r+1}(\mathcal{M})$ of an r-form is defined by

$$\mathrm{d}\omega = \frac{1}{r!}\, \partial_{[b} \omega_{a_1 \ldots a_r]}\, \mathrm{d}x^b \wedge \mathrm{d}x^{a_1} \wedge \ldots \wedge \mathrm{d}x^{a_r}\,, \tag{2.5}$$

which satisfies the conditions $\mathrm{d}^2 = 0$ and

$$\mathrm{d}(\omega \wedge \xi) = (\mathrm{d}\omega) \wedge \xi + (-1)^r\, \omega \wedge (\mathrm{d}\xi)\,. \tag{2.6}$$

The Levi-Civita symbol, denoted by $\varepsilon^{a_1 \cdots a_m}$, and the corresponding tensor are defined such that $\varepsilon^{P(1) \ldots P(m)} = \mathrm{sgn}(P)$. If the manifold $\mathcal{M}$ is endowed with a metric g_{ab}, the m-dimensional Levi-Civita tensor is given by $\epsilon_{a_1 \ldots a_m} = \sqrt{|\mathrm{det} g_{ab}|}\, \varepsilon_{a_1 \ldots a_m}$, where the determinant of the m-dimensional metric is involved. Note that

$$\epsilon^{a_1a_2\dots a_m} = g^{a_1b_1}g^{a_2b_2}\dots g^{a_mb_m}\epsilon_{b_1b_2\dots b_m} = g^{-1}\epsilon_{b_1b_2\dots b_m}\,. \tag{2.7}$$

The Hodge star operator $* : \Omega^r(\mathcal{M}) \to \Omega^{m-r}(\mathcal{M})$ is defined by the action on the basis vectors,

$$*\left(\mathrm{d}x^{a_1}\wedge\dots\wedge\mathrm{d}x^{a_r}\right) = \frac{1}{(m-r)!}\,\epsilon^{a_1\dots a_r}{}_{b_{r+1}\dots b_m}\,\mathrm{d}x^{b_{r+1}}\wedge\dots\wedge\mathrm{d}x^{b_m}\,, \tag{2.8}$$

implying that

$$*1 = \frac{1}{m!}\,\epsilon_{a_1\dots a_m}\,\mathrm{d}x^{a_1}\wedge\dots\wedge\mathrm{d}x^{a_m}\,, \tag{2.9}$$

where the right-hand side is the invariant volume element vol_d. For any element $\omega \in \Omega^r(\mathcal{M})$,

$$*\omega = \frac{1}{r!(m-r)!}\,\omega_{a_1\dots a_r}\epsilon^{a_1\dots a_r}{}_{b_{r+1}\dots b_m}\,\mathrm{d}x^{b_{r+1}}\wedge\dots\wedge\mathrm{d}x^{b_m}\,, \tag{2.10}$$

and

$$**\omega = (-1)^{r(m-r)+t}\,\omega\,, \tag{2.11}$$

with the number t of timelike dimensions in the m-dimensional space[8]. For a 10D manifold decomposed into a product space $\mathcal{M}_d\times\mathcal{M}_{10-d}$, we introduce another relation for $\xi\in\Omega^p(\mathcal{M}_d)$ and $\eta\in\Omega^q(\mathcal{M}_{10-d})$,

$$*_{10}\left(\xi\wedge\eta\right) = (-1)^{p(10-d-q)}\,*_d\xi\wedge *_{10-d}\eta\,, \tag{2.12}$$

where we define the Hodge operators $*_d$ and $*_{10-d}$ of the lower-dimensional spaces with respect to the corresponding metrics. Finally, for all $\omega,\sigma\in\Omega^r(\mathcal{M})$, the relations

$$|\omega\cdot\sigma| = \frac{1}{r!}\,\omega_{a_1\dots a_r}\sigma^{a_1\dots a_r}\,,\qquad |\omega\cdot\sigma|_{ab} = \frac{1}{(r-1)!}\,\omega_{aa_2\dots a_r}\sigma_b{}^{a_2\dots a_r} \tag{2.13}$$

and the properties

$$\begin{gathered} *\omega\wedge\sigma = *\sigma\wedge\omega = |\omega\cdot\sigma|*1\,,\qquad |*\omega\cdot *\sigma| = (-1)^t\,|\omega\cdot\sigma|\,,\\ |*\omega\cdot *\sigma|_{ab} = (-1)^t\left(g_{ab}|\omega\cdot\sigma| - |\omega\cdot\sigma|_{ab}\right)\,, \end{gathered} \tag{2.14}$$

will be useful in later discussions.

2.2 Type II supergravity

In the following section, we introduce the framework, notation and equations necessary for the rest of this work [2, 3, 88]. Given the simplifying assumptions that we adopt, we will specify the compactification setting and refine our conventions at the beginning of each section.

Let us begin by defining our compactification ansatz in a more general way. We are working in (massive) 10D type II SUGRA, where the 10D spacetime is considered as a

[8]More specifically, $t = 0$ if $(\mathcal{M}, g)$ is Riemannian and $t = 1$ if it is Lorentzian.

warped product of the form $\mathcal{M}_d \times \mathcal{M}_{10-d}$. In its most general form, the metric is given by

$$\mathrm{d}s_{10}^2 = g_{MN}(X)\mathrm{d}X^M\mathrm{d}X^M = \mathrm{e}^{2A(y)}g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + g_{mn}(x,y)\mathrm{d}y^m\mathrm{d}y^n\,, \tag{2.15}$$

which preserves the d-dimensional Lorentz invariance (maximal symmetry). In this context, we use capital Latin indices M, N to label all 10 directions, while Greek indices, $\mu, \nu = 0, \ldots, d-1$, are employed to denote external directions. Furthermore, Latin indices, $m, n = d, \ldots, 9$, are used to label internal dimensions. The external spacetime is either (anti-)de Sitter or Minkowski, characterized by a constant scalar curvature $\mathcal{R}_d$. The Ricci scalar of the internal compact manifold, $\mathcal{R}_{10-d}$, is defined with respect to the metric g_{mn}. We assume that the signature of both the 10D and the d-dimensional spacetime is $(-,+,\ldots,+)$. The localized sources are accounted for by the warp factor e^{2A}.

We turn to the field content; the bulk fields include the dilaton ϕ, the metric g_{MN}, which is characterized by 10D curved indices, and the Kalb-Ramond 2-form B_2, which defines the NSNS field strength $H_3 = \mathrm{d}B_2$. The dilaton must be a function of the internal coordinates only in order to preserve d-dimensional Lorentz invariance. We now proceed to discuss the RR sector. In the democratic formulation of type II string theory [89, 90], odd-degree forms appear in type IIA and even-degree forms in type IIB. These differential forms are given in terms of gauge potentials C_q, where $q = 1, \ldots, 9$. Note, however, that these potentials are not completely independent. This point will be discussed in more detail below. The RR field strengths,

$$F_q = \mathrm{d}C_{q-1} - H_3 \wedge C_{q-3} + F_0 \mathrm{e}^{B_2}|_q\,, \tag{2.16}$$

with $q \geq 1$, are given in terms of B_2 and C_{q-1}, which are defined on local gauge patches away from the localized sources. Moreover, F_0 is the mass parameter (Roman mass) in massive type IIA SUGRA, which lacks propagating degrees of freedom.

In the classical SUGRA approximation of type II string theory, the bosonic part of the effective action is given by

$$S = S_{\text{bulk}} + S_{\text{loc}}\,. \tag{2.17}$$

This equation includes the (pseudo-)action

$$S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}\left(\mathrm{e}^{-2\phi}\left(\mathcal{R} + 4(\partial\phi)^2 - \frac{1}{2}|H_3|^2\right) - \frac{1}{4}\sum_{q=0}^{10}|F_q|^2\right)\,, \tag{2.18}$$

with $2\kappa_{10}^2 = (2\pi)^7\alpha'^4$, in the string frame and the democratic formalism. In the following, we set $2\pi\sqrt{\alpha'} = 1$. For $F_0 = 0$, this results in the standard massless type IIA theory, as presented in [3]. The term "pseudo" refers to the redundant degrees of freedom in C_q, which are convenient for deriving the equations of motion. To avoid the degenerate degrees of freedom compared to the standard type II string theory formulation in Appendix A, which contains only lower-degree forms, any redundancy must be removed on-shell, i.e. at the level of the equations of motion. This is done by imposing the self-duality constraint,

$$*_{10}F_q = (-1)^{\frac{q(q-1)}{2}} F_{10-q}\,, \tag{2.19}$$

on the RR field strengths. By breaking the democracy among the RR forms, a genuine action can be obtained that contains only the independent degrees of freedom. It is necessary to remove half of the F_q using the self-duality constraint. For the time being, we will continue with the democratic formalism, but in Appendix A we will discuss the standard action of type II SUGRA in more detail [3, 74].

We turn to the source content; we study spacetime-filling (anti-)O$p_i^\pm$-planes and Dp_i-branes with $d-1 \leq p_i \leq 9$, wrapped on (p_i+1)-dimensional sub-manifolds Σ_i. Thus, p_i is the space dimension of the i-th source. In particular, we exclude NS5-branes, Kaluza-Klein monopoles and fundamental strings due to tadpole cancellation problems. Moreover, the backreaction of KK monopoles raises concerns and the identification of stable cycles for these branes remains a challenge [33]. The sources are classified according to their dimensionality p_i, and each class contains sets i of parallel sources placed along identical directions. The superscripts $\pm$ indicate different signs of the tension, while "anti" refers to a reversed sign of the RR charge. The action of these sources contains two contributions,

$$S_{\text{loc}} = \sum_i S_{\text{loc}}^{(p_i)} = \sum_i \left(S_{\text{DBI}}^{(p_i)} + S_{\text{CS}}^{(p_i)} \right)\,, \tag{2.20}$$

the Dirac-Born-Infeld action and the Chern-Simons part[9]. We define

$$S_{\text{loc}}^{(p_i)} = -T_i \int_{\Sigma_i} \mathrm{d}^{p_i+1}x \sqrt{-g_{p_i+1}}\, \mathrm{e}^{-\phi} \left(1 + \mathcal{L}_{\alpha'^2,i}\right) + T_i \int_{\Sigma_i} C_{p_i+1}\,, \tag{2.21}$$

where g_{p_i+1} denotes the pullback of the metric g_{MN} to Σ_i and $T_i \in \{T_{\text{O}p_i^\pm}, T_{\text{D}p_i}\}$ is the tension (charge) of the sources, set to[10]

$$T_{\text{O}p_i^\pm} = \pm 2^{p_i-5} T_{\text{D}p_i}\,, \qquad T_{\text{D}p_i} = \frac{2\pi}{(2\pi\sqrt{\alpha'})^{p_i+1}}\,. \tag{2.23}$$

The couplings to B_2 and the worldvolume gauge field are not relevant to this study and the action for an anti-O$p_i^\pm$-plane/anti-Dp_i-brane is given by reversing the sign of the CS term. We also introduce 4-derivative corrections ($\mathcal{L}_{\alpha'^2,i}$) in the α' expansion of the classical source action. The location of an O-plane is defined by fixed points, i.e., the surface being invariant under its target space involution σ_{p_i}. In addition, the RR-form field strengths also transform under the involution function. To analyze the parity of the fluxes, we use

[9] The CS term is topological and, as such, does not contribute to the Einstein equations.

[10] On the covering space of an orientifold, equation (2.23) must be modified to reflect

$$T_{\text{O}p_i^\pm} = \pm 2^{p_i-4} T_{\text{D}p_i}\,. \tag{2.22}$$

the conventions outlined in [49], where $\sigma_{p_i}(H_3) = -H_3$ and

$$\begin{aligned} \sigma_{p_i}(F_q) &= +\chi(F_q)\,, \quad \text{for } p_i = 2,3,6,7\,, \\ \sigma_{p_i}(F_q) &= -\chi(F_q)\,, \quad \text{for } p_i = 0,1,4,5,8,9\,. \end{aligned} \tag{2.24}$$

This definition introduces the operator χ, which is used to invert the indices of a q-form according to

$$\begin{aligned} \chi(F_q) &= +F_q\,, \quad \text{for } q = 0,1,4,5,8,9\,, \\ \chi(F_q) &= -F_q\,, \quad \text{for } q = 2,3,6,7\,. \end{aligned} \tag{2.25}$$

In Section 7, where we explore an "almost classical" theory of type IIA SUGRA, we study 4-derivative corrections in the α' expansion of the source action, given by

$$\begin{aligned} \mathcal{L}_{\alpha'^2,i} = (2\pi)^4\alpha'^2\Big(& c_{1i}e^{4\phi}F_0^4 + c_{2i}e^{2\phi}F_0^2\mathcal{R} + c_{3i}\mathcal{R}^2 + c_{4i}H_3^4 + c_{5i}H_3^2\mathcal{R} \\ &+ c_{6i}e^{2\phi}F_0^2H_3^2 + c_{7i}e^{4\phi}F_2^4 + c_{8i}e^{4\phi}F_0^2F_2^2 + c_{9i}e^{2\phi}F_2^2\mathcal{R} + c_{10i}e^{2\phi}F_2^2H_3^2 + \dots\Big)\,. \end{aligned} \tag{2.26}$$

The terms $\mathcal{R}^2$, $H_3^2\mathcal{R}, \dots$ represent all possible scalars derived from the Riemann tensor or components of the field strengths pulled back to the source. Note that the α' expansion of the RR sector is characterized as an expansion in powers of $e^{\phi}F_q$. However, this does not cover all corrections in next-to-leading order. Both the bulk and the Chern-Simons actions also receive α' corrections. Furthermore, the equation (2.26) contains other terms, including F_4 and derivatives of the dilaton. These corrections, as well as the exact numerical values of the coefficients c_{ai}, are not critical for the discussion in this work, although the latter are easily obtainable [76–84].

At last, we will address the equations of motion and Bianchi identities for the classical action, which is defined by $\mathcal{L}_{\alpha'^2,i} = 0$. The dilaton equation of motion reads

$$\begin{aligned} e^{-2A}\mathcal{R}_d + \mathcal{R}_{10-d} = {}& 4e^{\phi}\nabla^2 e^{-\phi} + 2de^{-A}\nabla^2 e^{A} \\ &+ d(d-1)(\partial A)^2 - 4d(\partial A\cdot\partial\phi) + \frac{1}{2}|H_3|^2 + \frac{1}{2}\sum_i \frac{T_i}{2\pi}e^{\phi}\delta(\Sigma_i)\,, \end{aligned} \tag{2.27}$$

while the (trace-reversed) external and internal Einstein equations are

$$\begin{aligned} e^{-2A}\mathcal{R}_d = {}& \frac{d}{4}e^{\phi}\nabla^2 e^{-\phi} + de^{-A}\nabla^2 e^{A} + \frac{d}{4}(\partial\phi)^2 - \frac{d(8+d)}{4}(\partial A\cdot\partial\phi) + d(d-1)(\partial A)^2 \\ &- \frac{d}{8}|H_3|^2 - \sum_q \frac{(q-1)d}{16}e^{2\phi}|F_q|^2 - \sum_i \frac{T_i}{2\pi}\frac{(7-p_i)d}{16}e^{\phi}\delta(\Sigma_i)\,, \end{aligned} \tag{2.28}$$

$$\begin{aligned} \mathcal{R}_{mn} = {}& 2e^{\phi}\nabla_m\partial_n e^{-\phi} + de^{-A}\nabla_m\partial_n e^{A} + \frac{g_{mn}}{4}e^{\phi}\nabla^2 e^{-\phi} + \frac{g_{mn}}{4}(\partial\phi)^2 - \frac{d}{4}g_{mn}(\partial A\cdot\partial\phi) \\ &- 2(\partial_m\phi)(\partial_n\phi) + \frac{1}{2}|H_3|^2_{mn} - \frac{g_{mn}}{8}|H_3|^2 \\ &+ \frac{1}{2}e^{2\phi}\sum_{q=0}^{10-d}\left(|F_q|^2_{mn} - \frac{q-1}{8}g_{mn}|F_q|^2\right) + \frac{1}{2}e^{\phi}\left(T^{\text{loc}}_{mn} - \frac{g_{mn}}{8}T^{\text{loc}}_{10}\right)\,, \end{aligned} \tag{2.29}$$

where both the Laplacian and the covariant derivative are defined with respect to the warped metric g_{mn} and all spacetime-filling fluxes have been dualized to internal ones according to (2.19)[11]. We also define the energy-momentum tensor of the localized sources,

$$2\pi \mathrm{e}^{-\phi}\, T^{\mathrm{loc}}_{MN} = -\frac{2}{\sqrt{-g}}\frac{\delta S_{\mathrm{loc}}}{\delta g^{MN}} = -\mathrm{e}^{-\phi}\sum_i T_i \Pi^{(i)}_{MN}\delta(\Sigma_i) \tag{2.30}$$

with $T^{\mathrm{loc}}_{10} = \sum_i T^{(p_i)}_{10} = g^{MN}T^{\mathrm{loc}}_{MN}$. The projector $\Pi^{(i)}_{MN}$ to Σ_i, wrapped by the source p_i, is given by g_{MN} for directions parallel to the source and zero otherwise. A complete list of the Einstein equations can be found in Appendix A. The delta-distributions are most easily explained by considering their effect on a test function f defined over the internal coordinates,

$$\int \mathrm{d}^{10}x\sqrt{-g_{10}}\, f\delta(\Sigma_i) = \int_{\Sigma_i}\mathrm{d}^{p_i+1}x\sqrt{-g_{p_i+1}}\, f|_{\Sigma_i}\,, \tag{2.31}$$

or $\delta(\Sigma_i) = \delta(\vec{x})/\sqrt{g_{9-p_i}}$ in local coordinates. Moreover, the equations of motion and the Bianchi identities for the field strengths are given together by

$$\begin{gathered}\mathrm{d}\left(*_{10}F_q\right) = -H_3\wedge *_{10}F_{q+2}\,, \qquad \mathrm{d}F_q = H_3\wedge F_{q-2} - (-1)^{\frac{q(q+1)}{2}}\sum_{p_i=8-q}\frac{Q_i}{2\pi}\delta_{9-p_i}\,,\\ \mathrm{d}\left(\mathrm{e}^{-2\phi}*_{10}H_3\right) = -\sum_q *_{10}F_q\wedge F_{q-2}, \qquad \mathrm{d}H_3 = 0\,,\end{gathered} \tag{2.32}$$

where $\delta_{9-p_i} = \delta(\Sigma_i)\,\mathrm{vol}_{\perp_i}$ and $Q_i = \pm T_i$, with a negative sign for anti-Op_i/anti-Dp_i. When integrating the Bianchi identity over the internal space transverse to the sources, the combination of the integrated fluxes $H_3\wedge F_{q-2}$ must cancel the charge presented by the integrated source form. This requirement, which ensures that the net flux and source terms effectively cancel each other, is known as the tadpole cancellation condition.

2.3 Boundary conditions of localized sources

In anticipation of discussions on localized sources in string compactifications [72, 73, 75], we aim to clarify some ambiguities in the equations of motion. This complex topic will be discussed in more detail in Section 7 and [47], but an initial discussion here serves as a useful introduction to the effects of localized sources.

By varying the action shown in (2.21), the O-planes/D-branes manifest themselves as delta-functions in the equations of motion. This description is typically understood as the backreaction of these delta-function sources on the bulk fields. However, there are alternative interpretations, which we will briefly explore.

$$\begin{array}{ccccccc} V_{\mathrm{Op/Dp}}\text{ in} & & \text{Couplings} & & \delta(\Sigma_i)\text{ in 10D} & & \text{Boundary conditions} \\ \text{scalar potential} & \Longleftarrow & \text{in } S_{\mathrm{loc}} & \Longrightarrow & \text{equations} & \Longrightarrow & \varphi|_{\Sigma_i}\text{ at source positions}\end{array}$$

O-planes are often conceptualized not as extended physical objects but as expressions of the

[11]In $d = 3$, the H_3 flux is allowed to fill the entire 3D spacetime, thus introducing an additional term in the equations of motion, as detailed in Appendix A.

background structure inherent to orientifolds. The fields defined within this background are subject to specific boundary conditions that affect how the fields couple to the O-planes, as detailed in equation (2.21). Furthermore, these sources contribute to the scalar potentials in the effective theory derived from the 10D action by dimensional reduction, as discussed in Section 3. While these perspectives are alternative but equivalent views of the effects of localized O-planes/D-branes, ambiguities in this formalism remain [75].

To further illustrate these points, particularly in the context of the source configuration explored in Section 7, we study an O8$^-$-plane. The properties of this model, which will be discussed in detail later, are described by the metric

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2A(z)}\mathrm{d}s_4^2 + \mathrm{e}^{-2A(z)}\left(\mathrm{e}^{2\lambda(z)}\mathrm{d}s_{\kappa_5}^2 + \mathrm{d}z^2\right) , \tag{2.33}$$

with a warp factor e^{2A}, the dilaton ϕ and a conformal factor $\mathrm{e}^{2\lambda}$. The internal space consists of a 5D Einstein space κ_5 wrapped by the O8$^-$ and a circle S^1 with the orientifold localized at $z = 0$. Assuming that the delta-functions primarily describe the local behavior of the fields near an O8$^-$, the classical equations in Section 2.2 can be written as

$$\begin{aligned}
(\mathrm{e}^{A-\phi}\phi')' &= -20\,\delta(z) + \dots\,, \\
(\mathrm{e}^{A-\phi}A')' &= -4\,\delta(z) + \dots\,, \\
(\mathrm{e}^{A-\phi}\lambda')' &= -8\,\delta(z) + \dots\,,
\end{aligned} \tag{2.34}$$

with $'$ denoting ∂_z. The remaining terms, including contributions from curvature, fluxes and first derivatives, result only in higher-order corrections in the field expansion around $z = 0$, while the leading terms in the bulk fields are sourced by the delta-functions[12]. Combining the equations (2.34), we find

$$\left(\mathrm{e}^{A-\phi}\right)'' = 16\,\delta(z) + \dots\,. \tag{2.35}$$

Since $\left(\mathrm{e}^{A-\phi}\right)'$ remains finite, this local equation integrates well in a distributional sense, thus addressing the issue of ill-defined distributions previously claimed in [75]. The solution around $z = 0$ is then given by

$$\mathrm{e}^{A-\phi} = c_0 + 8|z| + \dots\,, \tag{2.36}$$

where c_0 is the integration constant. Therefore, we detail the dynamics of the bulk fields as z approaches zero is obtained from

$$\mathrm{e}^{A} = a_0 - \frac{2a_0}{c_0}|z| + \dots\,, \quad \mathrm{e}^{\phi} = \frac{a_0}{c_0} - \frac{10a_0}{c_0^2}|z| + \dots\,, \quad \mathrm{e}^{\lambda} = l_0 - \frac{4l_0}{c_0}|z| + \dots\,, \tag{2.37}$$

for $c_0 \neq 0$ with $a_0, l_0 \neq 0$. For $c_0 = 0$, the expansion becomes

$$\mathrm{e}^{A} = a_0|z|^{-\frac{1}{4}} + \dots\,, \quad \mathrm{e}^{\phi} = \frac{a_0}{8}|z|^{-\frac{5}{4}} + \dots\,, \quad \mathrm{e}^{\lambda} = l_0|z|^{-\frac{1}{2}} + \dots\,. \tag{2.38}$$

[12] We implicitly assume that only second-order derivative terms contribute to the delta-functions, a stance that proves to be self-consistent in [72, 75].

To analyze next-to-leading-order coefficients, we combine the equations detailed in (2.34) to eliminate the delta-function sources. For the case $c_0 \neq 0$, dividing by $\mathrm{e}^{A-\phi}$ simplifies the results, leading to

$$\left(A - \frac{\phi}{5}\right)'' = 0 + \dots, \qquad (2A - \lambda)'' = 0 + \dots . \tag{2.39}$$

With the delta-functions now excluded, the system becomes more tractable, although the case $c_0 = 0$ introduces more complexity. As indicated by the expansion (2.36), we may need to consider terms such as $\mathrm{e}^{A-\phi}\delta(z) \sim |z|\delta(z)$ within the equations of motion (2.34). This inclusion leads to ambiguities in the equations (2.39),

$$\left(A - \frac{\phi}{5}\right)'' = C_1\,\delta(z) + \dots, \qquad (2A - \lambda)'' = C_2\,\delta(z) + \dots , \tag{2.40}$$

where C_I are free parameters. Despite these changes, both modified equations (2.40) remain well-defined and integrable, yielding the following expressions

$$\begin{aligned} \mathrm{e}^{A} &= a_0|z|^{-\frac{1}{4}} + a_1|z|^{\frac{3}{4}} + \dots , \\ \mathrm{e}^{\phi} &= \frac{a_0}{8}|z|^{-\frac{5}{4}} + \frac{5}{16}\left(2a_1 - C_1 a_0\right)|z|^{-\frac{1}{4}} + \dots , \\ \mathrm{e}^{\lambda} &= l_0|z|^{-\frac{1}{2}} + \frac{l_0}{2a_0}(4a_1 - C_2 a_0)|z|^{\frac{1}{2}} + \dots . \end{aligned} \tag{2.41}$$

Setting $C_I = 0$ provides a solution to the equations (2.39). Further details and implications of these sub-leading coefficients will be explored in due course. For now, we provide a brief overview of the corresponding boundary conditions [47, 75], illustrated in the diagram below;

classical	permissive	restrictive
$C_I = 0$	C_I, a_1 indefinite	$C_I = a_1 = 0$

In the context of the equations of motion, we refer to delta-function sources according to the boundary conditions they impose. However, we will temporarily set aside the discussion of restrictive boundary conditions, as they are not the main focus of the current dialogue, despite their presence in simple solutions [91]. Instead, our interest shifts to exploring whether we are dealing with classical or permissive sources, a crucial distinction since only the latter are known to potentially facilitate dS solutions [72, 73, 75, 92].

We take a critical stance on the validity of permissive sources within the framework of classical SUGRA. This skepticism arises from the fact that c_0, which represents the zero mode of $\mathrm{e}^{A-\phi}$ and acts as a dynamical field, must vanish for permissive sources. As a degree of freedom, the modulus c_0 may experience fluctuations whether it is stabilized or not. However, such fluctuations should not affect the fundamental nature of the sources. Consequently, the derivation of the equations (2.39) and (2.40), where c_0 is then set to zero in the latter, is expected to yield identical results. This consistency justifies setting C_I to zero.

The possibility that c_0 lacks dynamical properties is considered negligible, given its role as a universal modulus in string compactifications, as illustrated in (2.37) and discussed in [20, 93, 94]. Moreover, the orientifold involution of $O8^-$ does not project out c_0 [90]. For a comprehensive and structured analysis in a consistent mathematical context, the reader is referred to [95]. In line with the results in [73], we conclude that permissive sources are implausible in classical SUGRA, justifying the claim that $C_I = 0$.

Considering the potential for string corrections to challenge this conclusion [73, 75], we initially study next-to-leading-order (4-derivative) corrections (2.26), with coefficients c_{ai}, in the α' expansion of the classical source action. These corrections introduce "almost classical" boundary conditions, which naturally reproduce the classical conditions when $c_{ai} = 0$;

classical	permissive	restrictive	almost classical
$C_I = c_{ai} = 0$	$c_{ai} = 0$	$C_I = a_1 = c_{ai} = 0$	$C_I = 0$

However, our initial optimism is tempered upon closer examination of the field dependence of the corrections in (2.26), which reveals that almost classical sources differ significantly from permissive ones, even when considering general values of c_{ai}. Furthermore, the 4-derivative terms affect all equations of motion and not just specific combinations such as (2.40). The subtle effect of these 4-derivative corrections casts doubt on their ability to generate non-classical source terms, as proposed in [72], which could potentially facilitate dS solutions. In Section 7, we will explore why achieving dS solutions with (almost) classical sources in the CDT1 model [72] or similar setups with O6/D6 cannot be realized.

2.4 Smeared versus localized

We revisit some arguments previously discussed in [14, 15, 96] regarding the effects of localized sources. These sources induce non-trivial profiles for the internal metric, the warp factor and the dilaton by their backreaction on spacetime. In the equations of motion, the action of O-planes/D-branes introduces these sources as delta-functions, which can lead to regions of intense curvature (or energy densities) near the sources, where the classical SUGRA description may become inadequate, necessitating the inclusion of string corrections. To circumvent these complexities, we explore a regime where the backreaction of the sources is minimized over the entire spacetime, thus reducing the regions prone to string corrections and preserving the integrity of the SUGRA framework.

A practical solution to the challenges posed by localized sources is to replace them with a continuous distribution of the tension "smeared" over the compact space. This approach, known as the smeared approximation, is discussed in detail in references such as [8, 12, 13, 97]. In this refined framework, the delta-functions $\delta(\Sigma_i)$, which represent the i-th source with support on the worldvolume Σ_i, are replaced in the equations by smooth functions. For example,

$$\delta(\Sigma_i) \to \frac{1}{V_i}\,, \tag{2.42}$$

where V_i is the volume of the $(9 - p_i)$-dimensional transverse space. These functions are designed to integrate to the same total value as the original delta-functions. Despite such approximations, where fields like the warp factor and dilaton may show negligible spatial variation and remain almost constant, the source tension (or charge) still plays a significant role in the dynamics described by the equations of motion. An alternative perspective involves the expansion of the fully backreacted 10D solution into Fourier modes. In this perturbative framework, the smeared solution represents the leading-order term, which is particularly dominant in the limit of large internal volumes, weak string coupling or small cosmological constants. Each of these serves as an expansion parameter, the nuances of which will be discussed in due course.

In essence, the smeared limit allows us to solve the equations in a controlled manner in regimes characterized by small g_s or large internal volumes, as highlighted in [15, 98]. The use of smeared sources allows consistent truncations in string compactifications on group manifolds, as discussed in [16, 99, 100], apart from simplifying solution finding. However, by their very definition, a D-brane is characterized by its boundary conditions and an O-plane by its involution, which implies that the smeared approximation does not adequately reflect the localized nature of these sources. This inherent discrepancy has led to criticism concerning the validity of such approximations in the literature. Nevertheless, recent developments suggest that the smeared approach remains a justifiable method [15,70,98,101].

In the following discussion, we will study in more detail the conditions under which the backreaction of sources is negligible, thus supporting the use of the smeared approach. For sources of dimensionality $p_i \leq 6$, the backreaction of an Op_i^--plane is quantified by

$$\frac{g_s T_i}{r^{7-p_i}}\,, \tag{2.43}$$

where r is the radial coordinate in the $(9-p_i)$-dimensional transverse space, and the source is localized at $r = 0$ [91, 102]. Notably, the backreaction becomes significantly pronounced in the vicinity of the sources. More precisely, backreaction effects are $\gtrsim \mathcal{O}(1)$ within a spherical region, defined by

$$r \lesssim r_{\text{crit}} \equiv (g_s|T_i|)^{\frac{1}{7-p_i}} \tag{2.44}$$

around the orientifold. Within this critical sphere, an interesting behavior of the warp factor is observed: e^{-4A} becomes negative, leading to an imaginary metric according to (2.15). Beyond this radius, the metric loses its physical interpretation, indicating that the classical SUGRA description becomes unreliable and string corrections gain importance. In this so-called "hole" region, the curvature diverges as one approaches r_{crit}, rendering the solution unreliable even some distance before reaching the boundary where the curvature becomes $\gtrsim \mathcal{O}(1)$, as illustrated in Figure 1. For additional discussions of similar phenomena in flat space, especially regarding brane solutions with negative tension, we refer to [91]. Building on the previous arguments, it can be deduced that as

$$g_s \to 0\,, \tag{2.45}$$

the unphysical hole disappears. Considering a characteristic length scale R of our space,

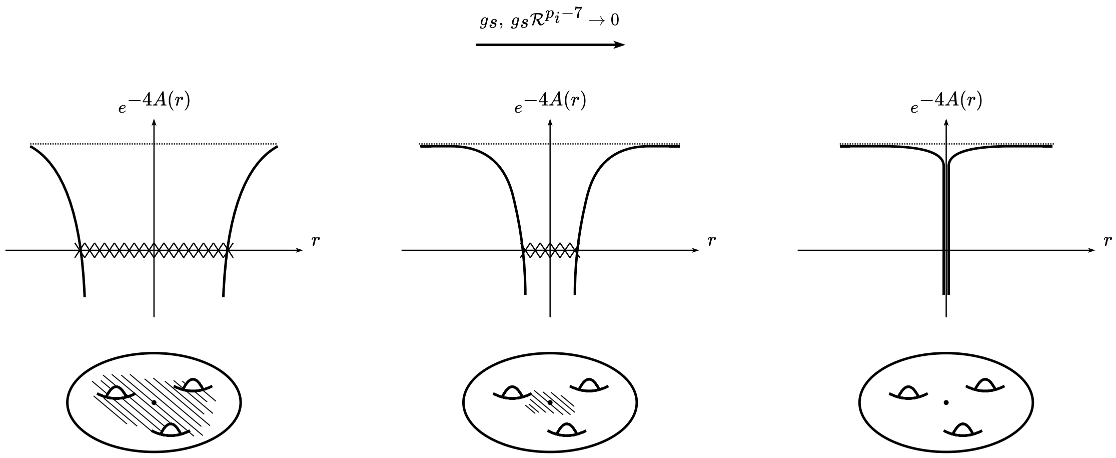


Figure 1: This figure illustrates the smeared limit within the transverse space of Op_i^--planes for $p_i \leq 6$ [96]. In regimes where both the volume and g_s are of order one, a pronounced backreaction of the sources is observed across large spatial regions (indicated as shaded areas). In these regions, where neither the warp factor nor the dilaton are well-defined, the classical SUGRA framework proves inadequate. However, the backreaction at generic points typically vanishes as $g_s,\ g_s R^{p_i-7} \to 0$, aligning the profiles of the warp factor and the dilaton with those anticipated by the smeared approach throughout space (represented as dashed lines).

the continuous limit[13]

$$\frac{g_s}{R^{7-p_i}} \to 0 \tag{2.46}$$

implies that backreaction effects are negligible at generic points in the 10D spacetime. Nevertheless, maintaining a finite and large R is essential to suppress α' corrections in the bulk, leading to $g_s/R^{7-p_i} \to 0$ iff $g_s \to 0$. In summary, under conditions where backreaction effects are effectively absent throughout the 10D spacetime, type II supergravity, or the dimensionally reduced effective theory and its scalar potential, can be properly described by the smeared approximation, as shown in Figure 1.

This analysis also leads to another insightful conclusion, identified in [96] and previously discussed in [15, 73, 103], known as the "Small-Hole Condition": as $g_s \to \epsilon$ with a small but finite ϵ – indicating a small hole – and with sufficiently large volume, the smeared approximation remains valid and the classical SUGRA framework remains reliable for deriving the potential, albeit with some leading higher-derivative corrections. It is important to clarify that while the smeared SUGRA background accurately represents the physics of the low-energy theory, it does not fully capture the local properties of the 10D parent theory near the O-planes. This discrepancy arises because negligible corrections at the level of the effective theory do not necessarily translate into negligible local corrections in ten dimensions. In particular, within any small but finite stringy region surrounding an O-plane, string or backreaction corrections persistently affect the local physics in 10D.

Recent studies [15, 96, 98] have developed a more precise mathematical framework to

[13] While it seems physically unreasonable, the possibility of a discontinuous limit, where backreaction effects remain significant in the effective theory even as $g_s \to 0$, cannot be entirely dismissed.

support what was previously a rather speculative discussion. These works have derived the perturbative expansion of the 10D solutions in regions characterized by small backreaction. The leading term is consistent with the smeared SUGRA assumption, while the subsequent higher-order terms, which contain backreaction corrections, behave as

$$g_s \sum_i T_i G_i \,, \tag{2.47}$$

where G_i, the Green's function of the transverse space to the i-th source normalized by a constant shift, satisfies $\nabla^2 G_i = 1/V_i - \delta(\Sigma_i)$ [96]. At generic points in an isotropic space, the Green's function scales as $1/R^{7-p_i}$, and at $r \ll R$ as $G \sim 1/r^{7-p_i}$, mirroring the behavior of the Green's function in flat space. Thus, the backreaction described in (2.47) is consistent with the heuristic result (2.43), i.e., of the order $g_s T_i / R^{7-p_i}$ at generic points. It is crucial to note, however, that isotropy is not a prerequisite for the validity of the equation (2.47); further details can be found in [96, 104].

In addition, the critical radius defined in (2.44), beyond which backreaction effects begin to diverge, can also be determined. Since the equation (2.47) is primarily concerned with the behavior of next-to-leading-order corrections, there is no a priori guarantee that a fully backreacted solution, including the complete backreaction of an Op_i^--plane, would necessarily exhibit a singular hole for $r \lesssim (g_s|T_i|)^{\frac{1}{7-p_i}}$. However, the local structure near an Op_i^- in any compactification is expected to resemble the one observed in flat space (which has a singularity), albeit with slight modifications. This link between strong backreaction and the appearance of singular regions underscores the importance of staying within regions where the smeared approximation is valid over most of 10D spacetime.

Building on our discussion of O8$^-$-planes in Section 7, we now focus on this specific case. The Green's function for S^1 is expressed as

$$G_8 = R\left(\frac{z^2}{4\pi} - \frac{|z|}{2} + \frac{\pi}{4}\right) \lesssim \mathcal{O}(R)\,, \tag{2.48}$$

where $z \in [0, 2\pi)$ and the radius is R. With the source located at $z = 0$, the backreaction from any O8$^-$ (as well as O8$^+$, D8) is approximately $\lesssim g_s|T_i|R$, vanishing in cases of large volume or small g_s, provided that $g_s R \to 0$. Unlike $p_i \leq 6$, G_8 remains bounded as $z \to 0$. This finite behavior extends to the warp factor and other functions, even within the fully backreacted solution [91], ensuring that the SUGRA description remains robust and reliable throughout space, without the complications typically associated with large curvature or singular holes, as shown in Figure 2. It is important to recognize that our analysis is carried out within a compact framework. From this discussion, it is clear that there are regimes where both the backreactions of O6$^-$-/O8$^-$-planes and the string corrections are negligibly small, preserving the integrity of the theoretical model.

As for Op_i^+-planes/Dp_i-branes with positive tension, they do not generate holes where the warp factor becomes negative for $3 < p_i \leq 6$. Nevertheless, there remains a stringy

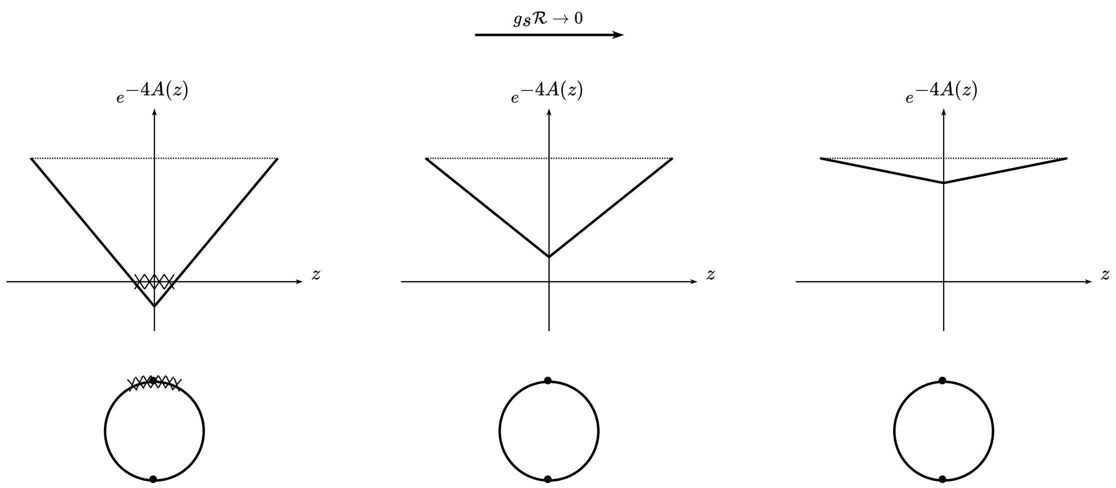


Figure 2: This figure illustrates the smeared limit in the compact transverse space S^1 of an O8$^-$ [47]. For $g_s R \gtrsim \mathcal{O}(1)$, backreaction effects are pronounced (left, center), although singular holes do not necessarily occur (center). To ensure tadpole cancellation, an O8$^+$-plane is strategically placed at the opposite end of S^1.

region surrounding the source where the curvature diverges and classical SUGRA ceases to be reliable [75, 91]. This issue underscores the necessity to include higher-order string corrections into our theoretical framework, although such problematic regions disappear in the smeared limit. But there are other regimes to be considered, as the dynamics are notably different for D-branes; in systems involving a stack of Dp_i, the curvature remains small at distances beyond the string length, indicating that classical SUGRA provides a consistent description for D-brane solutions even without the smeared approximation.

To summarize this complex discussion, in regions of spacetime severely affected by the backreaction from an Op_i^--plane with $p_i \leq 6$, singular holes or regions of large curvature manifest, leading to the failure of classical SUGRA. As $g_s R^{7-p_i} \to 0$, these singularities and the backreaction effects disappear, rendering the smeared solution valid over the entire 10D spacetime. On the other hand, for $O8^{\pm}$-planes and D-branes, despite pronounced backreaction, no singularities develop, and the SUGRA solution remains robust and reliable without resorting to the smeared approach.

Throughout this thesis, we will frequently use the smeared approximation because it provides control over string corrections and singular regions. However, it is crucial to confirm that our discussions in the smeared limit are supported by fully backreacted equivalents. This careful approach ensures the reliability and completeness of our theoretical models.

3 Dimensional reduction and effective action

We proceed to outline the mathematical framework for the dimensional reduction of a theory from D spacetime dimensions to an effective theory in d dimensions. First, we establish the relation between the actions of the D- and d-dimensional theories [53]. The D-dimensional action

$$S = \frac{1}{2\kappa_D^2} \int \mathrm{d}^D x \sqrt{-g_D}\, \mathrm{e}^{-2\phi} \left(\mathcal{R}_D + 4\partial_M \phi \partial^M \phi \right) + \dots \,, \tag{3.1}$$

with the gravitational constant κ_D^2, includes the standard Einstein-Hilbert term along with the kinetic term for the dilaton ϕ. The other terms, denoted by ellipses, include contributions from the H_3 flux, the RR field strengths and the O-planes/D-branes, hereafter collectively referred to as sources, details of which will be explained later. In contrast, the d-dimensional action in the Einstein frame,

$$S^{(d)} = \int \mathrm{d}^d x \sqrt{-g_d} \left(\frac{M_p^2}{2} \mathcal{R}_d - \frac{1}{2} G_{ij} \partial_\mu \varphi^i \partial^\mu \varphi^j - V(\varphi) \right) , \tag{3.2}$$

features the potential $V(\varphi)$ of the minimally coupled scalar fields φ^i, which arise from dimensional reduction, and the field space metric G_{ij}. The Planck mass is given by

$$M_p^2 = \frac{1}{\kappa_D^2} \int \mathrm{d}^{D-d} y \sqrt{g_{D-d}}\, g_s^{-2} \,, \tag{3.3}$$

where g_{D-d} is the metric of the $(D-d)$-dimensional compactified space. The Ricci scalars are defined by the metric tensors g_{MN} of the higher-dimensional theory and $g_{\mu\nu}$ of the reduced d-dimensional effective theory, where $\mu, \nu = 0, \dots, d-1$.

Within this lower-dimensional framework, a critical point of the scalar potential, characterized by the absence of the scalar kinetic energy, corresponds to a stable vacuum solution. This scenario implies that the dynamics of the d-dimensional theory is stationary with respect to variations in the scalar fields, thus providing a solution to the equations of motion derived from the effective potential. These critical points are crucial for identifying stable configurations in compactified theories and play an important role in the landscape of string theory vacua. They allow the calculation of physical properties such as field masses and interaction couplings. Furthermore, evaluating the trace of the d-dimensional Einstein equation on-shell, i.e. at the critical point, so that

$$\frac{d-2}{d} \mathcal{R}_d = \frac{2}{M_p^2} V|_0 \,, \tag{3.4}$$

shows that a dS solution corresponds to an extremum of a positive potential, $V(\varphi) > 0$.

In the following, we will explore additional terms in the scalar potential arising from contributions of the H_3 flux, the RR field strengths and the sources to the D-dimensional action. We will also discuss the kinetic terms for various scalar fields in type II SUGRA, whose definitions and corresponding scalar potentials are detailed in the text. For clarity and parametric control, we adopt the smeared approximation, which involves keeping a constant background dilaton, $\mathrm{e}^{\phi_0(x)} = g_s$, and neglecting the warp factor in the metric (2.15). As mentioned above, this approximation is justified by the expectation that an approximately smeared regime is sufficient to ensure the validity of classical SUGRA.

3.1 Semi-universal scalar fields (ρ, τ, σ)

To derive the effective action of any quantum field theory, we can use the background field method. This technique involves expanding the quantum fields around a classical background value. Specifically, the scalars in (3.2) are treated as perturbations around the 10D background fields, which include the metric and the dilaton. Starting with the initial set of scalar fields $\{\rho, \tau, \sigma\}$, we adopt the following formal expression for the D-dimensional metric [32, 43]

$$\mathrm{d}s_D^2 = \tau^{-2}(x) g_{\mu\nu}(x) \mathrm{d}x^\mu \mathrm{d}x^\nu + \rho(x) g_{mn}(x, y) \mathrm{d}y^m \mathrm{d}y^n \,, \tag{3.5}$$

which allows a decomposition of the Ricci scalar as follows,

$$\mathcal{R}_D = \tau^2 \mathcal{R}_d + \rho^{-1} \mathcal{R}_{D-d} + \dots \,, \tag{3.6}$$

where $\mathcal{R}_{D-d} = g^{mn}\mathcal{R}_{mn}$ and $\mathcal{R}_d = g^{\mu\nu}\mathcal{R}_{\mu\nu}$. The ellipses represent additional terms, which will be explained in more detail. The fluctuations of the dilaton field are expressed by the equation $\mathrm{e}^{\phi(x)} = g_s \mathrm{e}^{\delta\phi(x)}$. We will omit any explicit mention of the coordinate dependence in the following discussions. To eliminate the scalar field prefactors in the Einstein-Hilbert term of the d-dimensional action, we perform the following transformation,

$$\mathrm{e}^{\delta\phi} = \tau^{-\frac{d-2}{2}} \rho^{\frac{D-d}{4}} \,. \tag{3.7}$$

This definition leads to the first term in the scalar potential,

$$V = -\frac{1}{2\kappa_D^2} \int \mathrm{d}^{D-d} y \sqrt{g_{D-d}}\, g_s^{-2} \tau^{-2} \rho^{-1} \mathcal{R}_{D-d} \,. \tag{3.8}$$

In the following sections, additional contributions from the kinetic term of the dilaton and other terms of the D-dimensional action will be explored.

3.1.1 Kinetic terms

To derive the energy density of the scalar fields, we start with the kinetic term of the 10D dilaton,

$$4(\partial\phi)_D^2 = \tau^2 \Bigg(\frac{(D-d)^2}{4} \rho^{-2} (\partial\rho)^2 + \frac{(d-2)^2}{4} \tau^4 (\partial\tau^{-2})^2 + \frac{(d-2)(D-d)}{2} \tau^2 \rho^{-1} \partial_\mu \tau^{-2} \partial^\mu \rho \Bigg) \,, \tag{3.9}$$

expanding upon the results presented in [34]. We distinguish between D- and d-dimensional derivatives. Further evaluation of the Ricci scalar and an exact derivation of (3.6) leads to

$$\begin{aligned}\mathcal{R}_D = \tau^2\mathcal{R}_d + \rho^{-1}\mathcal{R}_{D-d} - \frac{d}{2}\tau^4\partial_\mu\tau^{-2}\partial^\mu(\rho g_{mn})\rho^{-1}g^{mn} \\ - \frac{\tau^2}{4}\left(\rho^{-1}g^{mn}\partial(\rho g_{mn})\right)^2 - \nabla_\mu\left((d-1)\tau^4\partial^\mu\tau^{-2} + \tau^2\rho^{-1}g^{mn}\partial^\mu(\rho g_{mn})\right) \\ - \frac{(d+2)(d-1)}{4}\tau^6(\partial\tau^{-2})^2 + \frac{\tau^2}{4}\partial_\mu(\rho g_{mn})\partial^\mu(\rho^{-1}g^{mn}) .\end{aligned} \tag{3.10}$$

Inserting this expression into the D-dimensional action and performing partial integration results in

$$\begin{aligned}\int \mathrm{d}^d x\sqrt{-g_d}\,\tau^{-2}\mathcal{R}_D = \int \mathrm{d}^d x\sqrt{-g_d}\Bigg(\mathcal{R}_d + \tau^{-2}\rho^{-1}\mathcal{R}_{D-d} \\ - \frac{(d-1)(d-2)}{4}\tau^4(\partial\tau^{-2})^2 - \frac{d-2}{2}\tau^2\partial_\mu\tau^{-2}\partial^\mu(\rho g_{mn})\rho^{-1}g^{mn} \\ - \frac{1}{4}\left(\rho^{-1}g^{mn}\partial(\rho g_{mn})\right)^2 + \frac{1}{4}\partial_\mu(\rho g_{mn})\partial^\mu(\rho^{-1}g^{mn})\Bigg) ,\end{aligned} \tag{3.11}$$

with the prefactor τ^{-2} emerging from those in (3.1) and the process of dimensional reduction. For the sake of argument, we assume that the determinant of the metric tensor, $g_{D-d} = \det g_{mn}$, is independent of the external coordinates. It can be shown that $g^{mn}\partial_\mu g_{mn} = 0$, as $\partial_\mu \ln(\det M) = \mathrm{Tr}(M^{-1}\partial_\mu M)$ for an invertible matrix M. We then obtain

$$\begin{aligned}\int \mathrm{d}^d x\sqrt{-g_d}\,\tau^{-2}\mathcal{R}_D = \int \mathrm{d}^d x\sqrt{-g_d}\Bigg(\mathcal{R}_d + \tau^{-2}\rho^{-1}\mathcal{R}_{D-d} \\ - \frac{(d-1)(d-2)}{4}\tau^4(\partial\tau^{-2})^2 - \frac{(d-2)(D-d)}{2}\tau^2\rho^{-1}\partial_\mu\tau^{-2}\partial^\mu\rho \\ - \frac{(D-d)^2}{4}\rho^{-2}(\partial\rho)^2 - \frac{D-d}{4}\rho^{-2}(\partial\rho)^2 + \frac{1}{4}\partial_\mu(g_{mn})\partial^\mu(g^{mn})\Bigg) .\end{aligned} \tag{3.12}$$

This relation, together with the dilaton term in the equation (3.9) and simplifying the resulting expression, leads to the action of the d-dimensional effective theory,

$$\begin{aligned}S^{(d)} = \int \mathrm{d}^d x\sqrt{-g_d}\Bigg(\frac{M_p^2}{2}\mathcal{R}_d - \frac{M_p^2}{2}\Big((d-2)\tau^{-2}(\partial\tau)^2 \\ + \frac{D-d}{4}\rho^{-2}(\partial\rho)^2 - \frac{1}{4}\partial_\mu(g_{mn})\partial^\mu(g^{mn})\Big) - V\Bigg) ,\end{aligned} \tag{3.13}$$

with the scalar potential, as further specified in the following section. Before proceeding, we define another scalar field, σ, which is related to the internal geometry as described in [43, 44],

$$g_{mn}\mathrm{d}y^m\mathrm{d}y^n = \sigma^A\delta_{ab}e^{a_{||}}e^{b_{||}} + \sigma^B\delta_{cd}e^{c_\perp}e^{d_\perp} . \tag{3.14}$$

In this analysis, we use the orthonormal coframe, often referred to as the flat basis, $\{e^{a_{||}}\}, \{e^{a_\perp}\}$ of the compact manifold, such that the internal metric is given by $\mathrm{d}s^2_{D-d} = \delta_{ab}e^a e^b$ with one-forms $e^a = e^a{}_m(y)\mathrm{d}y^m$ in terms of the vielbeins $e^a{}_m(y)$ [34, 36]. The indices $a_{||}, b_{||}$ refer to the internal dimensions along a set of parallel sources, while $c_\perp, d_\perp$ denote those orthogonal to them. In anticipation of string compactifications on group manifolds, the spin connection can be described using parameters denoted by $f^a{}_{bc}$. Given the previous assumption, the exponents A, B are defined to ensure the determinant of the internal metric remains independent of the external coordinates, i.e.,

$$A = p_i + 1 - D\,, \qquad B = p_i + 1 - d\,. \tag{3.15}$$

The σ field refers to fluctuations of the internal volume wrapped by the O-planes/D-branes. Consequently, we define a σ_i for each set of parallel sources, indicating the dependence on the specific source configuration. This requires a slight generalization of the previously derived results to accommodate multiple sets of Op_i/Dp_i. For each set of parallel sources, which may vary in dimensionality p_i, we can define a fluctuation of the vielbeins according to [36],

$$e^{a_{||_i}}{}_m \to \sigma_i^{\frac{A_i}{2}} e^{a_{||_i}}{}_m\,, \qquad e^{a_{\perp_i}}{}_m \to \sigma_i^{\frac{B_i}{2}} e^{a_{\perp_i}}{}_m\,, \tag{3.16}$$

with $A_i = p_i + 1 - D$ and $B_i = p_i + 1 - d$, in analogy to (3.14) and (3.15). Given these fluctuations, each vielbein is scaled by powers of σ_i,

$$e^a{}_m \to \pi_a e^a{}_m\,, \qquad \text{with } \pi_a = \prod_i \sigma_i^{\frac{P_i(a)}{2}}\,, \tag{3.17}$$

where $P_i(a_{||_i}) = A_i$, $P_i(a_{\perp_i}) = B_i$, and the index a is not summed over. Note that these fields may not be linearly independent, as demonstrated by considering a source configuration in $D = 10, d = 4$ with two sets of O5-planes along internal dimensions 12, 34, and D7-branes along 1234[14]. In such cases, the independence of the internal volumes wrapped by the O-planes/D-branes is not guaranteed, resulting in a redundancy among the σ_i fields.

In terms of the field space metric defined in the action (3.2), this redundancy results in a vanishing determinant. To address this, a field redefinition is performed to eliminate redundant fields σ_x, which leads to vanishing metric coefficients along the directions $\partial_\mu \sigma_x$ and thus to a vanishing determinant. Before constructing the field space metric, it is necessary to identify a set of independent fields σ_m. This is done by the redefinition

$$\sigma_m \to \sigma_m \prod_x \sigma_x^{s_{xm}}\,, \qquad \sigma_x \to \sigma_x\,, \tag{3.18}$$

to isolate independent field components and remove σ_x from π_a, the constituents of the potential, if

$$P_x(a) + \sum_m s_{xm} P_m(a) = 0\,, \qquad \forall\ a, X\,. \tag{3.19}$$

After this transformation, we have a non-degenerate field space metric and a set of independent scalars. Although the field redefinition (3.18) is not the most general form, it has proven sufficient in several cases [35, 36].

[14] Unless otherwise specified, internal dimensions in source configuration examples are denoted by Arabic numerals starting with 1.

Finally, the kinetic term in the action (3.13) for a single σ field is given by

$$-\frac{1}{4}\partial_\mu(g_{mn})\partial^\mu(g^{mn}) = \frac{AB(A-B)}{4}\sigma^{-2}(\partial\sigma)^2 = \frac{1}{4}(D-p-1)(p+1-d)(D-d)\sigma^{-2}(\partial\sigma)^2\,, \tag{3.20}$$

as detailed in [45], leading to the transformation laws for canonically normalized fields,

$$\hat{\tau} = \sqrt{d-2}M_p \ln\tau\,, \qquad \hat{\rho} = \sqrt{\frac{D-d}{4}}M_p\ln\rho\,, \qquad \hat{\sigma} = \sqrt{\frac{-AB(B-A)}{4}}M_p\ln\sigma\,. \tag{3.21}$$

This results in a comprehensive understanding of the behavior of the scalar fields and their impact on the overall dynamics of the dimensionally reduced theory.

3.1.2 Scalar potential

In the following analysis, we focus on 10D type II SUGRA, detailed in Section 2.2. We extend the derivation of the universal potential $V(\rho,\tau)$ to arbitrary dimensions d, drawing on insights from [32,33]. This involves analyzing the contribution of each term in the 10D action to the scalar potential. As discussed earlier, the scaling of the Ricci scalar is given by (3.8), i.e.,

$$\mathcal{R}_{10} \to -\tau^{-2}\rho^{-1}\mathcal{R}_{10-d}\,, \tag{3.22}$$

where τ^{-2} emerges from the prefactor in the 10D action. This method is similarly applied to derive contributions to the potential from other terms in the action [49], including the fluxes,

$$|H_3|^2 \to -\tau^{-2}\rho^{-3}|H_3|^2\,, \qquad e^{2\phi}|F_q|^2 \to -\tau^{-d}\rho^{\frac{10-d-2q}{2}}g_s^2|F_q|^2\,, \tag{3.23}$$

and the sources,

$$e^{\phi}T_{10}^{(p_i)} \to -\tau^{-\frac{d+2}{2}}\rho^{\frac{2p_i-8-d}{4}}g_sT_{10}^{(p_i)}\,. \tag{3.24}$$

The powers of ρ, indicating fluctuations around the vacuum value of the field g_{mn}, arise either from the square of the fluxes (2.13) or from the property that $T_{10}^{(p_i)}$, as defined in (2.42), is inversely proportional to the transverse volume of the O-plane/D-brane. As outlined in Section 2.2 and Appendix A, the sources must satisfy $p_i+1\geq d$ in order to preserve maximal symmetry. For $d>q$, the fluxes are completely aligned along the internal directions, which requires $q\leq 10-d$. The contribution of spacetime-filling fluxes for $d\leq q$ will be discussed below.

Spacetime-filling fluxes, defined on-shell as Hodge duals (A.1) of the internal ones, introduce an additional d-dimensional boundary term in the on-shell action, ensuring gauge-independent boundary conditions, such as $\delta{*_4}F_4|_\infty = 0$. A detailed analysis for $d=4$ is given in [38]. The introduction of this new boundary term affects the scalar potential, effectively changing the expected contribution of spacetime-filling fluxes from $\varphi|F_{10-q}|^2$ to $-\varphi^{-1}|F_{10-q}|^2$. We summarize the contributions to the potential from spacetime-filling fluxes in the 10D action:

$$\begin{aligned} |H_3^d|^2 &\to -\tau^{2(d-1)}\rho^{d-3}|H_7|^2 \xrightarrow[\text{term}]{\text{boundary}} \tau^{2(1-d)}\rho^{3-d}|H_7|^2\,,\\ e^{2\phi}|F_q^d|^2 &\to -\tau^{d}\rho^{-\frac{10-d-2(10-q)}{2}}g_s^2|F_{10-q}|^2 \to \tau^{-d}\rho^{\frac{10-d-2(10-q)}{2}}g_s^2|F_{10-q}|^2\,, \end{aligned} \tag{3.25}$$

where we use the on-shell condition $|*_{10-d}F_{10-q}|^2 = |F_{10-q}|^2$, noting that F_q^{10} lacks a metric factor. Therefore, its powers in ρ, τ are determined solely by the dilaton prefactor and the square of the fluxes.

Having established the rules for contributions to the scalar potential, we define the potential in d dimensions as

$$V(\rho,\tau) = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10-d}y\sqrt{g_{10-d}}\, g_s^{-2}\Bigg(\tau^{-2}\left(-\rho^{-1}\mathcal{R}_{10-d} + \frac{1}{2}\rho^{-3}|H_3|^2\right) \\ + \frac{1}{2}\tau^{2(1-d)}\rho^{3-d}|H_7|^2 + \frac{g_s^2}{2}\tau^{-d}\sum_{q=0}^{10-d}\rho^{\frac{10-d-2q}{2}}|F_q|^2 - \tau^{-\frac{d+2}{2}}\sum_i \rho^{\frac{2p_i-8-d}{4}}\frac{g_s T_{10}^{(p_i)}}{p_i+1}\Bigg). \quad (3.26)$$

The dependence on the σ field, as specified in (3.14), has not yet been considered. Assuming a single set of sources with $T_{10}^{(p_i)} = T_{10}$, and therefore a single σ field, we extend the derivation of the σ dependence to arbitrary dimensions, following the approach in [38, 53]. For sources of dimensionality p_i, the internal flux is given by $F_q = \sum_{n=0}^{p_i-3} F_q^{(n)}$, where n denotes the number of legs along the source, and $F_q^{(0)} = F_q|_\perp$ represents the component orthogonal to the O-plane/D-brane. The square of the fluxes results in $|F_q|^2 = \sum_{n=0}^{p_i-3}|F_q^{(n)}|^2$ with

$$|F_q^{(n)}|^2 = \frac{1}{n!(q-n)!}F_{q\,a_{1||}\dots a_{n||}a_{n+1\perp}\dots a_{q\perp}}F_q^{\;a_{1||}\dots a_{n||}a_{n+1\perp}\dots a_{q\perp}}\,, \quad (3.27)$$

and similarly for the field strength H_3. This notation allows us to deduce fluctuations of σ around the background fluxes $H_3^{(n)}$ and $F_q^{(n)}$[15],

$$\begin{aligned} T_{10} &\to \sigma^{-\frac{1}{2}B(9-p_i)}T_{10}\,, \\ \left\{|H_3^{(n)}|^2, |F_q^{(n)}|^2\right\} &\to \sigma^{-An-B(q-n)} \times \left\{|H_3^{(n)}|^2, |F_q^{(n)}|^2\right\}, \\ |(*_{10-d}H_7)^{(n)}|^2 &\to -\sigma^{An+B(q-d-n)} \times |(*_{10-d}H_7)^{(n)}|^2\,. \end{aligned} \quad (3.30)$$

The latter also applies to $|(*_{10-d}F_{10-q})^{(n)}|^2$ and the coefficients A and B are defined in (3.15). Since $B = p_i + 1 - d$ specifies the number of internal dimensions wrapped by a set of sources, we confirm that

$$*_{10-d}F_{10-q}^{(n)} = (*_{10-d}F_{10-q})^{(\tilde{n})}\,, \quad (3.31)$$

[15]In cases involving multiple sets of parallel sources, the fluctuation of each contribution to the potential is expressed mathematically as

$$H_{3\,abc} \to (\pi_a\pi_b\pi_c)^{-1}H_{3\,abc}\,, \quad F_{q\,a_1\dots a_q} \to (\pi_{a_1}\dots\pi_{a_q})^{-1}F_{q\,a_1\dots a_q}\,, \quad f^a{}_{bc} \to \pi_a(\pi_b\pi_c)^{-1}f^a{}_{bc}\,, \quad (3.28)$$

where the latter transformation is especially relevant for group manifolds. The resulting expression for the internal Ricci scalar is derived using the established formula (3.34). The fluctuation of the source term $T_{10}^{(p_i)}$ reflects the fluctuation of $\mathrm{vol}_{||_i}$,

$$T_{10}^{(p_i)} \to T_{10}^{(p_i)}\prod_{a=a_{||_i}}\pi_a\,. \quad (3.29)$$

For an exhaustive discussion of sources of multiple dimensionalities p_i, and thus multiple fields, we refer to [36, 45].

with $\tilde{n} = p_i + 1 - d - n$ and

$$A\tilde{n} + B(q - d - \tilde{n}) = -An - B(10 - q - n)\,. \tag{3.32}$$

This framework allows us to rewrite the contribution of spacetime-filling fluxes in the potential and deduce

$$\begin{aligned} V(\rho,\tau,\sigma) = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10-d}y\sqrt{g_{10-d}}\, g_s^{-2}\Bigg(\tau^{-2}\Bigg(&-\rho^{-1}\mathcal{R}_{10-d}(\sigma) \\ &+ \frac{1}{2}\rho^{-3}\sum_n \sigma^{-An-B(3-n)}|H_3^{(n)}|^2\Bigg) + \frac{1}{2}\tau^{2-2d}\rho^{3-d}\sum_n \sigma^{-An-B(7-n)}|H_7^{(n)}|^2 \\ &+ \frac{g_s^2}{2}\tau^{-d}\sum_{q=0}^{10-d}\rho^{\frac{10-d-2q}{2}}\sum_n \sigma^{-An-B(q-n)}|F_q^{(n)}|^2 \\ &- \tau^{-\frac{d+2}{2}}\rho^{\frac{2p_i-8-d}{4}}\sigma^{\frac{1}{2}B(p_i-9)}\frac{g_s T_{10}}{p_i+1}\Bigg)\,. \end{aligned} \tag{3.33}$$

The dependence of the internal Ricci scalar $\mathcal{R}_{10-d}(\sigma)$ on the parameter σ is rather complex. While a comprehensive derivation is detailed in [53], our focus here is limited to compact group manifolds characterized by their structure constants $f^a{}_{bc}$, which are intrinsic to the underlying Lie algebra. This algebraic structure helps to identify the respective group manifold. However, it is important to note that these algebraic features alone do not fully define the global properties of the manifold, in particular its compactness. The expression for the Ricci scalar is given by [38, 53]

$$\mathcal{R}_{10-d} = \mathcal{R}_{||} + \mathcal{R}_{||}^{\perp} - \frac{1}{2}|f^{||}{}_{\perp\perp}|^2 - \delta^{cd} f^{b_\perp}{}_{a_{||}c_\perp} f^{a_{||}}{}_{b_\perp d_\perp}\,, \tag{3.34}$$

with the auxiliary variables defined as follows

$$\begin{aligned} 2\mathcal{R}_{||} &= -\delta^{cd} f^{a_{||}}{}_{b_{||}c_{||}} f^{b_{||}}{}_{a_{||}d_{||}} - \frac{1}{2}\delta_{ad}\delta^{be}\delta^{cg} f^{a_{||}}{}_{b_{||}c_{||}} f^{d_{||}}{}_{e_{||}g_{||}}\,, \\ 2\mathcal{R}_{||}^{\perp} &= -\delta^{cd} f^{b_\perp}{}_{a_\perp c_{||}} f^{a_\perp}{}_{b_\perp d_{||}} - \delta_{ah}\delta^{bg}\delta^{cd}\left(f^{h_\perp}{}_{g_\perp c_{||}} f^{a_\perp}{}_{b_\perp d_{||}} + f^{h_\perp}{}_{g_{||}c_{||}} f^{a_\perp}{}_{b_{||}d_{||}}\right)\,, \\ |f^{||}{}_{\perp\perp}|^2 &= \frac{1}{2}\delta^{ab}\delta^{cd}\delta_{ef} f^{e_{||}}{}_{a_\perp c_\perp} f^{f_{||}}{}_{b_\perp d_\perp}\,. \end{aligned} \tag{3.35}$$

The σ dependence can be derived explicitly from this decomposition,

$$\mathcal{R}_{10-d}(\sigma) = -\sigma^{-B}(\delta^{cd} f^{b_\perp}{}_{a_{||}c_\perp} f^{a_{||}}{}_{b_\perp d_\perp})^0 + \sigma^{-A}\left(\mathcal{R}_{||} + \mathcal{R}_{||}^{\perp}\right)^0 - \frac{1}{2}\sigma^{-2B+A}|f^{||0}{}_{\perp\perp}|^2\,. \tag{3.36}$$

The superscript ^{0} is used to denote background quantities. To avoid complicating the notation by cluttering the equations with unnecessary indices, we will not use this notation in the following discussion. Since all contributions to the potential for the scalar fields ρ, τ and σ are gathered in the equations (3.33) and (3.36), we introduce the simplified notation

$$\frac{\int \mathrm{d}^{10-d}y\sqrt{g_{10-d}}\,T_{10}}{\mathrm{vol}_{10-d}} \to T_{10}\,, \tag{3.37}$$

for each internal quantity in the potential. As a consequence, the prefactor in V can be replaced by $M_p^2/2$.

To summarize, our analysis in 10D type II SUGRA has highlighted the central role of two universal scalar fields in classical flux compactifications, our primary focus; the d-dimensional dilaton τ and the $(10-d)$-dimensional volume ρ. On the other hand, the field σ corresponds to the internal volume $\mathrm{vol}_{||}$ wrapped by a set of parallel O-planes/D-branes, or equivalently to $\mathrm{vol}_{\perp}$. With respect to the potential (3.33), our discussion is constrained to a single set of sources. For more complex configurations involving multiple intersecting sets of sources, we refer to the literature [34, 36, 45]. Using the simplified notation (3.37), we rewrite the potential (3.33),

$$
\begin{aligned}
2V(\hat{\rho},\hat{\tau},\hat{\sigma}) = &-\mathrm{e}^{\frac{-2}{\sqrt{d-2}}\hat{\tau}}\mathrm{e}^{\frac{-2}{\sqrt{10-d}}\hat{\rho}}\mathcal{R}_{10-d}(\hat{\sigma}) \\
&+\frac{1}{2}\mathrm{e}^{\frac{-2}{\sqrt{d-2}}\hat{\tau}}\mathrm{e}^{\frac{-12}{\sqrt{10-d}}\hat{\rho}}\sum_n \mathrm{e}^{(-An-B(3-n))\sqrt{\frac{-4}{AB(B-A)}}\hat{\sigma}}|H_3^{(n)}|^2\Big) \\
&+\frac{1}{2}\mathrm{e}^{\frac{2(1-d)}{\sqrt{d-2}}\hat{\tau}}\mathrm{e}^{\frac{2(3-d)}{\sqrt{10-d}}\hat{\rho}}\sum_n \mathrm{e}^{(-An-B(7-n))\sqrt{\frac{-4}{AB(B-A)}}\hat{\sigma}}|H_7^{(n)}|^2 \\
&+\frac{g_s^2}{2}\mathrm{e}^{\frac{-d}{\sqrt{d-2}}\hat{\tau}}\sum_{q=0}^{10-d}\mathrm{e}^{\frac{10-d-2q}{\sqrt{10-d}}\hat{\rho}}\sum_n \mathrm{e}^{(-An-B(q-n))\sqrt{\frac{-4}{AB(B-A)}}\hat{\sigma}}|F_q^{(n)}|^2 \\
&-\mathrm{e}^{-\frac{d+2}{2\sqrt{d-2}}\hat{\tau}}\mathrm{e}^{\frac{2p_i-8-d}{2\sqrt{10-d}}\hat{\rho}}\mathrm{e}^{\frac{1}{2}B(p_i-9)\sqrt{\frac{-4}{AB(B-A)}}\hat{\sigma}}\frac{g_s T_{10}}{p_i+1}
\end{aligned}
\tag{3.38}
$$

in Planckian units, with respect to the transformation laws (3.21) to canonically normalized fields. Moreover, if the compact space is a group manifold, the internal Ricci scalar (3.36) is given by

$$
\begin{aligned}
\mathcal{R}_{10-d}(\hat{\sigma}) = &-\mathrm{e}^{-\sqrt{\frac{-4B}{A(B-A)}}\hat{\sigma}}\left(\delta^{cd}f^{b_\perp}{}_{a_{||}c_\perp}f^{a_{||}}{}_{b_\perp d_\perp}\right)^0 \\
&+\mathrm{e}^{\sqrt{\frac{-4A}{B(B-A)}}\hat{\sigma}}\left(\mathcal{R}_{||}+\mathcal{R}_{||}^{\perp}\right)^0-\frac{1}{2}\mathrm{e}^{(-2B+A)\sqrt{\frac{-4}{AB(B-A)}}\hat{\sigma}}|f^{||0}{}_{\perp\perp}|^2 .
\end{aligned}
\tag{3.39}
$$

3.2 A new set of scalar fields (τ, r)

Having established the basic framework for deriving scalar potentials in flux compactifications of type II SUGRA, we now turn our attention to a new set of scalar fields. In addition to the d-dimensional dilaton τ, we introduce the radion r and extend the analysis of [38] to arbitrary dimensions d. Building on the methods of the previous section, we decompose the D-dimensional metric as follows,

$$
\mathrm{d}s_D^2 = \tau^{-2}(x)g_{\mu\nu}(x)\mathrm{d}x^\mu\mathrm{d}x^\nu + r^2(x)(e^1)^2 + \sum_{a=2}^{D-d}\delta_{ab}e^a e^b \,, \tag{3.40}
$$

with $e^a = e^a{}_m(y)\mathrm{d}y^m$. Going to the d-dimensional Einstein frame, we define the fluctuation around the background dilaton field according to

$$\mathrm{e}^{\delta\phi} = \tau^{\frac{2-d}{2}} r^{\frac{1}{2}} \, , \tag{3.41}$$

due to the expression for the Ricci scalar, $\mathcal{R}_D = \tau^2 \mathcal{R}_d + \mathcal{R}_{D-d}(r) + \ldots$, in D dimensions. This results in a prefactor of $-\tau^{-2}$ in the d-dimensional action, analogous to the equation (3.8).

3.2.1 Kinetic terms

We will now focus on the kinetic terms for these scalars. For simplicity and without loss of generality, we will work in a flat basis where $g_{\mu\nu} = \eta_{\mu\nu}$ and $e^a{}_m = \delta^a_m$. We analyze each term in the expression $\mathcal{R}_D + 4(\partial\phi)^2_D$ separately. The kinetic term of the D-dimensional dilaton is given by

$$4(\partial\phi)^2_D = (d-2)^2 \left(\partial\tau\right)^2 + 2(2-d)\tau^2 \partial_\mu \ln\tau \partial^\mu \ln r + \tau^2 \left(\partial \ln r\right)^2 \, , \tag{3.42}$$

while the explicit calculation of $\mathcal{R}_D$ results in

$$\begin{aligned} \mathcal{R}_D = \tau^2 \mathcal{R}_d + \mathcal{R}_{D-d}(r) + 2\tau^2 \partial_\mu \left((d-1)\partial^\mu \ln\tau - \partial^\mu \ln r\right) \\ - 2\tau^2 \left(\partial \ln r\right)^2 - (d-2)(d-1)\left(\partial\tau\right)^2 + 2(d-2)\tau^2 \partial_\mu \ln\tau \, \partial^\mu \ln r \, . \end{aligned} \tag{3.43}$$

By substituting these results into the equation (3.1) and performing partial integration, we obtain the d-dimensional action,

$$S^{(d)} = \int \mathrm{d}^d x \sqrt{-g_d} \left(\frac{M_p^2}{2} \mathcal{R}_d - \frac{M_p^2}{2} \left((d-2) \left(\partial \ln\tau\right)^2 + \left(\partial \ln r\right)^2 \right) - V \right) \, , \tag{3.44}$$

a crucial step in establishing the transformation laws for canonically normalized fields:

$$\hat{\tau} = \sqrt{d-2} M_p \ln\tau \, , \qquad \hat{r} = M_p \ln r \, . \tag{3.45}$$

3.2.2 Scalar potential

As we continue our analysis, we set $D = 10$ and introduce a new notation, $F_q^{(n)}$ and $H_3^{(n)}$, where $n = 0, 1$ denotes the number of legs along the internal dimension 1 parallel to the direction of the scalar field r. In this framework, the contributions to the scalar potential from the various terms in the 10D action are given by

$$\begin{aligned} |H_3|^2 &\to -\tau^{-2} \left(|H_3^{(0)}|^2 + r^{-2} |H_3^{(1)}|^2 \right) \, , \\ \mathrm{e}^{2\phi} |F_q|^2 &\to -\tau^{-d} g_s^2 \left(r |F_q^{(0)}|^2 + r^{-1} |F_q^{(1)}|^2 \right) \, , \end{aligned} \tag{3.46}$$

and for the sources,

$$\mathrm{e}^{\phi} T_{10}^{(p_i)} \to -\tau^{-\frac{d+2}{2}} \left(\delta_1^{||} r^{\frac{1}{2}} + \delta_1^{\perp} r^{-\frac{1}{2}} \right) g_s T_{10}^{(p_i)} \, . \tag{3.47}$$

For spacetime-filling fluxes, we derive the relations

$$\begin{aligned} \frac{1}{2}|H_3^d|^2 &\to \frac{1}{2}\tau^{2(1-d)}\left(|(*_{10-d}H_7)^{(0)}|^2 + r^2|(*_{10-d}H_7)^{(1)}|^2\right) , \\ e^{2\phi}\frac{1}{2}|F_q^d|^2 &\to \frac{1}{2}\tau^{-d}g_s^2\left(r^{-1}|(*_{10-d}F_{10-q})^{(0)}|^2 + r|(*_{10-d}F_{10-q})^{(1)}|^2\right) , \end{aligned} \tag{3.48}$$

using a similar reasoning as before. We can further simplify these relations using the equations

$$|(*_{10-d}H_7)^{(n)}|^2 = |H_7^{(|1-n|)}|^2 , \qquad |(*_{10-d}F_{10-q})^{(n)}|^2 = |F_{10-q}^{(|1-n|)}|^2 . \tag{3.49}$$

The resulting scalar potential in $D = 10$, including the scalar fields τ and r, is given by

$$\begin{aligned} V(\tau, r) = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10-d}y\sqrt{|g_{10-d}|}\, g_s^{-2}\Bigg(&\tau^{-2}\left(-\mathcal{R}_{10-d}(r) + \frac{1}{2}\left(|H_3^{(0)}|^2 + r^{-2}|H_3^{(1)}|^2\right)\right) \\ &+ \frac{1}{2}\tau^{2-2d}\left(|H_7^{(1)}|^2 + r^2|H_7^{(0)}|^2\right) + \tau^{-d}\frac{g_s^2}{2}\sum_{q=0}^{10-d}\left(r|F_q^{(0)}|^2 + r^{-1}|F_q^{(1)}|^2\right) \\ &- \tau^{-\frac{d+2}{2}}\sum_i\left(\delta_1^{||_i}r^{\frac{1}{2}} + \delta_1^{\perp_i}r^{-\frac{1}{2}}\right)\frac{g_s T_{10}^{(p_i)}}{p_i + 1}\Bigg) . \end{aligned} \tag{3.50}$$

To study the dependence of the internal Ricci scalar on r, we again assume that the internal space is a group manifold. In particular, the relevant expression in [38],

$$\mathcal{R}_{10-d}(r) = \mathcal{R}_{10-d}^0 + (r^{-2} - 1)\mathcal{R}_{11}^0 + \frac{1}{4}(2 - r^2 - r^{-2})\delta^{ik}\delta^{jl}({f^1}_{ij})^0({f^1}_{kl})^0 , \tag{3.51}$$

remains applicable and crucial to our analysis, since it is independent of the dimensionality of the external spacetime.

4 Constraints on (quasi-)de Sitter solutions in 10D supergravity

In this section, we explore the possibility of a d-dimensional dS solution in the classical regime of string theory, which provides a more simple and controlled framework compared to those involving (non-)perturbative elements. One question arises from the discrepancy between the familiar four dimensions of our universe and the ten dimensions proposed by string theory: Why prioritize four dimensions? In response, we study various constraints and possible exclusions of classical string backgrounds with a dS spacetime in $d \geq 3$. While demonstrating that a dS solution is excluded in SUGRA is sufficient to rule out its existence in a classical string context, the converse is not inherently true: it remains to be shown that any solution found in 10D SUGRA is consistent within the classical regime.

Extensive research has been devoted to classical dS solutions, aiming at both their identification and constraining them [52, 72, 74, 105], with pioneering efforts noted in [49, 106]. In light of these challenges, several no-go theorems have been formulated that impose restrictions on the internal manifold, fluxes or source content [38, 45, 53]. These theorems typically result from one of two approaches. The first method, which we adopt here, involves combining the classical 10D equations of motion and Bianchi identities to establish, under certain assumptions, an inequality of the form $\mathcal{R}_d \leq 0$ that effectively excludes classical de Sitter. An alternative method uses a d-dimensional effective theory with the respective (positive) scalar potential, which is further detailed in Section 5.

The structure of this section is as follows; in Section 4.1, we review the general compactification ansatz and outline the constraints relevant to our analysis. Section 4.2 is devoted to deriving no-go theorems against the existence of classical solutions with a d-dimensional dS spacetime, thus extending and refining previous theorems in $d = 4$. In Section 4.3, we apply these no-go theorems in various dimensions, with particular attention to the nature of the fluxes and sources, especially those preserving SUSY, as detailed in Section 4.3.2. Our results conclusively rule out classical dS solutions for $d \geq 7$, indicate limited possibilities for $d = 5, 6$ and suggest viable options for $d \leq 4$, while supporting conjectures from [35, 45]. Finally, Section 4.4 extends this discussion to quasi-dS solutions, providing a comprehensive exploration of the potential landscape in different dimensional settings in string theory.

4.1 Theoretical framework: conventions and 10D equations

We continue with the compactification ansatz previously established in A, imposing additional constraints detailed below, and focus on O-planes/D-branes in type II SUGRA. Although classical compactifications may include other sources such as NS5-branes, Kaluza-Klein monopoles, fundamental strings and anti-D-branes, we omit these from the current discussion for simplicity and to preserve parametric control [36]. It is important to note that the inclusion of O-planes and anti-D-branes introduces significant complexities, in particular the breaking of SUSY within effective theory. Furthermore, the roles of O-planes and D-branes are mutually coherent in the Bianchi identities and the equations of motion, which is crucial for the formulation of no-go theorems. However, this coherence is broken when anti-D-branes are included, as discussed in Footnote 9. We further restrict our analysis to smeared sources, assuming a constant background dilaton $g_s = e^{\phi}$ and the absence of a warp factor. The motivation for this approach has been extensively discussed before.

In the democratic formalism, O9-planes and D9-branes serve as electric sources for a C_{10} gauge potential, which, lacking a field strength in ten dimensions, does not support any propagating degrees of freedom. In order to prevent a singular behavior of the propagator, O9/D9 charge cancellation is a mandatory requirement [3], which is also confirmed by the Bianchi identity, leading to

$$T_{10}^{(9)} = 0\,. \tag{4.1}$$

Therefore, the mere presence of either O9 or D9 without the other is not viable within our framework. More details on this topic are provided in Section 4.3.

We then proceed to introduce several key equations from Appendix A, beginning with the dilaton equation of motion,

$$2\mathcal{R}_{10} + g_s \sum_i \frac{T_{10}^{(p_i)}}{p_i + 1} - |H_3^{10}|^2 = 0\,. \tag{4.2}$$

Using the trace-reversed Einstein equations described in (A.7), the 10D Einstein trace is given by

$$4\mathcal{R}_{10} + \frac{g_s}{2} T_{10} - |H_3^{10}|^2 - \frac{g_s^2}{2} \sum_{q=0}^{7} (5-q)|F_q|^2 = 0\,. \tag{4.3}$$

Similarly, the d-dimensional Einstein trace,

$$\mathcal{R}_d = \frac{d}{16} \left(g_s \sum_i \frac{7-p_i}{p_i+1} T_{10}^{(p_i)} - 2|H_3|^2 - 6|H_7|^2 + g_s^2 \sum_{q=0}^{7} (1-q)|F_q|^2 = 0 \right), \tag{4.4}$$

follows from the equation (A.11). These particular equations will be central to the arguments in the following sections. Furthermore, the implicit dependence of the non-vanishing source and flux content on the dimension of the external spacetime will be further explored.

4.2 Constraints on classical de Sitter in $d \geq 3$

A no-go theorem in theoretical physics establishes constraints that frame what is possible within a particular theoretical framework, often including inherent assumptions. In this context, no-go theorems serve as invaluable tools for formulating and testing scientific theories. More specifically, these theorems define a set of constraints that constrain classical string backgrounds with a dS spacetime, with notable insights provided in works such as [16, 32, 37, 85, 107].

In the following section, we aim to extend the work done in [38, 74, 105] and to formulate no-go theorems that exclude classical solutions with a d-dimensional dS spacetime.

While such theorems are typically derived from fundamental physical principles, our approach here relies mainly on algebraic manipulations of three crucial equations: (4.2), (4.3) and (4.4). By employing this method, we seek to elucidate the boundaries that limit the viability of classical dS solutions and explore possible ways to navigate around these restrictions.

No-go theorem by Maldacena and Nuñez

Given its relevance to this thesis, we seek to review the reasoning behind the Maldacena-Nuñez no-go theorem [10,11]. Here, we provide a concise outline of the argument. All 10D string theories, except for massive type IIA, obey the strong energy condition,

$$\mathcal{R}_{MN}\, u^M u^N \geq 0\,, \tag{4.5}$$

for any non-spacelike vector field u^M in ten dimensions, which characterizes the attractive nature of gravity [108]. In particular, this implies that the time component of the 10D Ricci tensor satisfies $\mathcal{R}_{00} \geq 0$.

To compactify a 10D gravitational theory with an Einstein-Hilbert term coupled to matter fields down to d dimensions, as outlined in equation (3.1), we use the metric ansatz presented in (2.15). The trace-reversed Einstein equations are then given by

$$\mathcal{R}_{MN} = T_{MN} - \frac{1}{8} g_{MN} T_P{}^P\,, \qquad \text{with } T_{MN} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{MN}}\,, \tag{4.6}$$

where T_{MN} represents the full stress-energy tensor of the theory, unlike (2.30) which refers only to sources. For a warped product metric, the Ricci tensor takes the form

$$\mathcal{R}_{\mu\nu} = \tilde{\mathcal{R}}_{\mu\nu} - \frac{g_{\mu\nu}}{d} e^{-dA} \nabla^2_{10-d} e^{dA}\,. \tag{4.7}$$

This equation relates the Ricci tensor of the warped metric to the one of the unwarped metric $\tilde{g}_{\mu\nu}$. The trace with $g^{\mu\nu}$ yields

$$\nabla^2_{10-d} e^{dA} = e^{(d-2)A}\tilde{\mathcal{R}}_d + e^{dA}\left(-T_\mu{}^\mu + \frac{d}{8} T_P{}^P\right) = e^{(d-2)A}\tilde{\mathcal{R}}_d + e^{dA} T\,, \tag{4.8}$$

where the trace T of the stress-energy tensor remains non-negative if the parent theory satisfies the strong energy condition. Assuming this holds for pure gravity, integrating (4.8) over the compact space results in

$$\left(\int_{\mathcal{M}_{10-d}} e^{(d-2)A}\right)\tilde{\mathcal{R}}_d = -\int_{\mathcal{M}_{10-d}} e^{dA}\, T\,, \tag{4.9}$$

which shows that a positive external scalar curvature is unfeasible.

How does this apply to supergravity? Because the graviton is governed by an Einstein-Hilbert term and obeys the strong energy condition, it interacts with various fields and localized sources. Maldacena and Nuñez showed that contributions from fluxes and scalar fields positively affect the overall trace of the stress-energy tensor [10,11]. Localized sources violating the strong energy condition require a negative tension T_i, leading to repulsive gravity. Hence, dS compactifications require the presence of Op^--planes. In summary, the Maldacena-Nuñez no-go theorem states that no classical string background with a dS spacetime can emerge from a non-singular wrapped compactification. We will obtain this constraint by carefully combining the equations of motion.

To simplify the analysis, we exploit the dilaton equation of motion (4.2) to eliminate the Ricci scalar $\mathcal{R}_{10}$ in the 10D Einstein trace (4.3),

$$g_s \sum_i \frac{p_i - 3}{p_i + 1} T_{10}^{(p_i)} + 2|H_3^{10}|^2 - g_s^2 \sum_{q=0}^{7} (5-q)|F_q|^2 = 0\,, \tag{4.10}$$

which then allows the elimination of the H_3 flux in (4.4), leading to

$$\mathcal{R}_d = \frac{d}{4}\left(g_s \sum_i \frac{T_{10}^{(p_i)}}{p_i+1} - g_s^2 \sum_{q=0}^{7} |F_q|^2 - 2|H_7|^2 \right)\,. \tag{4.11}$$

Given that the second and third term contribute negatively, a positive cosmological constant, and thus a dS spacetime, is only possible if the first term sufficiently compensates for these negative contributions. This leads to the Maldacena-Nuñez no-go theorem:

No-go theorem 1. There is no classical dS solution if $\forall\, p_i : T_{10}^{(p_i)} \leq 0$.

As noted above, the most straightforward way to satisfy this condition is to include negative tension sources, such as Op^--planes, whose contributions must exceed those of the Dp-branes[16]. As a consequence, the Maldacena-Nuñez no-go theorem can be generalized to arbitrary dimensions $d \geq 3$ and to sources of multiple dimensionalities.

Sources of single dimensionality

We continue our discussion with sources of single dimensionality p_i, which are either parallel or intersect in different sets. For this purpose, we simplify $\sum_i T_{10}^{(p_i)} = T_{10}$, while keeping in mind that $p_i \geq d-1$. We extend a no-go theorem, originally derived in four dimensions by [37] and in ten dimensions by [74], by first eliminating the source term in (4.2) using

[16]The number of O-planes/D-branes is constrained by the tadpole cancellation condition. Without fluxes, this is simplified to [109]

$$\sum \mu_i = \mu_{\mathrm{Dp}} \left(n_{\mathrm{Dp}} - \bar{n}_{\mathrm{Dp}} - 2^{p-5} \left(n_{\mathrm{Op}^-} - \bar{n}_{\mathrm{Op}^-} - n_{\mathrm{Op}^+} + \bar{n}_{\mathrm{Op}^+} \right) \right) = 0\,, \tag{4.12}$$

for a given set of parallel sources containing n_{Dp} Dp, $\bar{n}_{\mathrm{Dp}}$ anti-Dp, and multiple O-planes (n_{Op^-} Op^-, $\bar{n}_{\mathrm{Op}^-}$ anti-Op^-, n_{Op^+} Op^+, $\bar{n}_{\mathrm{Op}^+}$ anti-Op^+), while the total tension is given by

$$\begin{aligned} \sum T_i &= T_{\mathrm{Dp}} \left(n_{\mathrm{Dp}} + \bar{n}_{\mathrm{Dp}} - 2^{p-5} \left(n_{\mathrm{Op}^-} + \bar{n}_{\mathrm{Op}^-} - n_{\mathrm{Op}^+} - \bar{n}_{\mathrm{Op}^+} \right) \right) \\ &= 2T_{\mathrm{Dp}} \left(n_{\mathrm{Dp}} - 2^{p-5} \left(n_{\mathrm{Op}^-} - n_{\mathrm{Op}^+} \right) \right)\,. \end{aligned} \tag{4.13}$$

The requirement for negative tension is

$$\sum T_i < 0 \quad \Leftrightarrow \quad n_{\mathrm{Dp}} < 2^{p-5} \left(n_{\mathrm{Op}^-} - n_{\mathrm{Op}^+} \right)\,. \tag{4.14}$$

Our current analysis does not consider flux contributions in (4.12). Including these would modify (4.14) by adding terms proportional to n_{flux}.

the equation (4.3),

$$\left(-2\mathcal{R}_{10} + |H_3^{10}|^2\right)(p_i - 3) + 2|H_3^{10}|^2 - g_s^2 \sum_{q=0}^{7} (5-q)|F_q|^2 = 0\,. \tag{4.15}$$

With this result, we eliminate T_{10} from the equation (4.11),

$$\left(2 + \frac{4}{d}\right)\mathcal{R}_d = -2\mathcal{R}_{10-d} + |H_3|^2 - 3|H_7|^2 - g_s^2 \sum_{q=0}^{7} |F_q|^2\,. \tag{4.16}$$

By subtracting the equation (4.15) from (4.16), multiplied by $p_i - 3$, we obtain

$$\frac{4(p_i-3)}{d}\mathcal{R}_d = -2|H_3|^2 + 2(4-p_i)|H_7|^2 + g_s^2 \sum_{q=0}^{7} (8 - p_i - q)|F_q|^2\,. \tag{4.17}$$

This extends the approach in [74] to the more general dimension $d \geq 3$, allowing us to propose the following no-go theorem against classical string backgrounds with a dS spacetime of dimension $d \geq 3$:

No-go theorem 2. There is no classical dS solution for $p_i = 7, 8$ or 9 .

We also address another no-go theorem, extending the work in [74, 105] to arbitrary dimension $d \geq 3$. While we previously focused on sources of maximal dimensionality p_i, we now consider the minimal dimension $p_i = d-1$, where $\mathrm{vol}_{\perp_i} = \mathrm{vol}_{10-d}$. We then project the Bianchi identity of the sources (2.32)

$$\mathrm{d}F_{8-p_i} - H \wedge F_{6-p_i} = \varepsilon_{p_i} \frac{T_{10}}{p_i + 1} \mathrm{vol}_{\perp_i}\,, \qquad \text{with } \varepsilon_{p_i} = -(-1)^{\frac{(8-p_i)(7-p_i)}{2}}\,, \tag{4.18}$$

onto vol_{10-d}, which leads to the scalar equation

$$2g_s \frac{T_{10}}{p_i+1} = |H_3|^2 + g_s^2 |F_{6-p_i}|^2 - |*_{10-d}H + \varepsilon_{p_i} g_s F_{6-p_i}|^2 + 2\varepsilon_{p_i} g_s \left(\mathrm{d}F_{8-p_i}\right)\,, \tag{4.19}$$

with $\mathrm{d}F_{8-p_i} = \left(\mathrm{d}F_{8-p_i}\right)\mathrm{vol}_{10-d}$. Another important result is obtained by summing the equations (4.10) and (4.11) after multiplying the latter by $-4(p_i+1)/d$, which is given by

$$-\frac{4(p_i+1)}{d}\mathcal{R}_d = -4g_s \frac{T_{10}}{p_i+1} + 2|H_3|^2 + 2p_i|H_7|^2 + g_s^2 \sum_{q=0}^{7} (p_i + q - 4)|F_q|^2\,. \tag{4.20}$$

We use the equation (4.19) to extract

$$\begin{aligned}\mathcal{R}_d = -\frac{d}{2(p_i+1)}\Big(& -2g_s\varepsilon_{p_i}\left(\mathrm{d}F_{8-p_i}\right) + |*_{10-d}H + \varepsilon_{p_i} g_s F_{6-p_i}|^2 + p_i|H_7|^2 \\ &+ g_s^2\left(-|F_{2-p_i}|^2 + 2|F_{8-p_i}|^2 + 3|F_{10-p_i}|^2 + 4|F_{12-p_i}|^2 + 5|F_{14-p_i}|^2 + 6|F_{16-p_i}|^2\right)\Big)\,,\end{aligned} \tag{4.21}$$

where $2 \leq p_i \leq 9$. To develop this theorem further, we integrate over the internal space $\mathcal{M}_{10-d}$, noting that a (smeared) dS solution implies $\int_{\mathcal{M}_{10-d}} \mathrm{vol}_{10-d}\, \mathcal{R}_d = \mathcal{R}_d \int_{\mathcal{M}_{10-d}} \mathrm{vol}_{10-d}$.

Given our focus on $p_i = d-1$, we conclude that

$$\int_{\mathcal{M}_{10-d}} \mathrm{d}F_{8-p_i} = \int_{\mathcal{M}_{10-d}} \mathrm{d}F_{9-d} = \int_{\partial\mathcal{M}_{10-d}=0} F_{9-d} = 0\,, \tag{4.22}$$

which yields a negative integral of (4.21) for $d > 3$ or $p_i > 2$:

No-go theorem 3a. There is no classical dS solution for $p_i = d-1$ in $d \geq 4$.

Due to the subtleties in the equation (4.21), introduced by F_{p_i-2}, the extension of this no-go theorem to sources of a single dimensionality $p_i = 2$ leads to the following conclusion:

No-go theorem 3b. There is no classical dS solution for $p_i = 2$ in $d = 3$ with $F_0 = 0$.

Sources of multiple dimensionalities

In this short paragraph, we extend the analysis of the previous section to include sources of multiple dimensionalities, p_i and $\tilde{p}_j$. These results will be crucial in Section 4. We begin by generalizing the derivation of the equation (4.17) for a given p_i,

$$\frac{4(p_i-3)}{d}\mathcal{R}_d = g_s \sum_{\tilde{p}_j\neq p_i} \frac{p_i-\tilde{p}_j}{\tilde{p}_j+1} T_{10}^{(\tilde{p}_j)} - 2|H_3|^2 + 2(4-p_i)|H_7|^2 + g_s^2 \sum_{q=0}^{7}(8-p_i-q)|F_q|^2\,. \tag{4.23}$$

Similarly, we generalize the equation (4.20). By using the approach previously applied to sources of a single dimensionality, we add another source term,

$$\begin{aligned} -\frac{4(p_i+1)}{d}\mathcal{R}_d = -4g_s\frac{T_{10}^{(p_i)}}{p_i+1} + g_s \sum_{p'_j\neq p_i} \frac{\tilde{p}_j-p_i-4}{\tilde{p}_j+1} T_{10}^{(\tilde{p}_j)} \\ + 2|H_3|^2 + 2p_i|H_7|^2 + g_s^2\sum_{q=0}^{7}(p_i+q-4)|F_q|^2\,. \end{aligned} \tag{4.24}$$

Applying the Bianchi identity for $p_i = d-1$, including sources with $\tilde{p}_j > p_i$ requires a modification of the no-go theorem 3a to relax its constraints:

No-go theorem 4. There is no classical dS solution for $p_i = d-1$ in $d \geq 4$ and $\forall\, \tilde{p}_j$: $(\tilde{p}_j - p_i - 4)\, T_{10}^{(\tilde{p}_j)} \geq 0$.

This result is not entirely new, and is consistent with the discussion found in [105] for $d = 4$ in type IIB SUGRA.

Vanishing field strength

We now focus more closely on sources of single dimensionality p_i, in particular those that have not been addressed before. We extend algebraic no-go theorems derived in [74,110,111] for $d = 4$ to arbitrary dimension. For O6-planes/D6-branes, it is necessary to impose $F_0 \neq 0$ to ensure a positive scalar curvature and thus a classical dS solution. This result follows from the equation (4.17) for $p_i = 6$,

$$\frac{12}{d}\mathcal{R}_d = -2|H_3|^2 - 4|H_7|^2 + g_s^2\left(2F_0^2 - 2|F_4|^2 - 4|F_6|^2\right)\,. \tag{4.25}$$

Similarly, we require $F_1 \neq 0$ for O5-planes/D5-branes due to the equation (4.17) with $p_i = 5$, which leads to

$$\frac{8}{d}\mathcal{R}_d = -2|H_3|^2 - 2|H_7|^2 + g_s^2\left(2|F_1|^2 - 2|F_5|^2 - 4|F_7|^2\right) . \tag{4.26}$$

When we evaluate (4.17) for sources of dimensionality $p_i = 4$, we deduce

$$\frac{4}{d}\mathcal{R}_d = -2|H_3|^2 + g_s^2\left(4F_0^2 + 2|F_2|^2 - 2|F_6|^2\right) . \tag{4.27}$$

To circumvent the last no-go theorem and obtain a classical dS solution, it seems necessary to ensure that either $F_0 \neq 0$ or $F_2 \neq 0$. Revisiting the argument from [53], we note that for $p_i = 4$ the Bianchi identity, $\mathrm{d}F_0 = 0$, indicates a constant F_0 flux. However, due to the orientifold projection, F_0 must be odd over the O4-plane, resulting in $F_0 = 0$. Therefore, the presence of O4-planes/D4-branes requires $F_2 \neq 0$ to ensure that classical dS solutions exist. In summary, we can state the following no-go theorem for sources of single dimensionality p_i, provided that the condition $p_i \geq d-1$ is satisfied:

No-go theorem 5a. There is no classical dS solution for $p = 4, 5$ or 6 in $d \geq 3$ with $F_{6-p} = 0$.

While a similar no-go theorem cannot be found for O3-planes/D3-branes, the unique case of $p_i = 2$ for $d = 3$ yields an interesting result. Again, we consider the equation (4.17) for $p_i = 2$, which leads to

$$\frac{4}{d}\mathcal{R}_d = 2|H_3|^2 - 4|H_7|^2 - g_s^2\left(6F_0^2 + 4|F_2|^2 + 2|F_4|^2\right) . \tag{4.28}$$

Since all terms of the RR fluxes contribute negatively, we turn our attention to obtaining the necessary condition for the NSNS field strength:

No-go theorem 5b. There is no classical dS solution for $p_i = 2$ in $d = 3$ with $H = 0$.

This is a new and interesting result, the implications of which will be discussed in a moment. But first, let us take a look at another case.

Non-negative internal curvature

The no-go theorem presented in this section, which has been previously studied in [49] and derived in [37,74], is once again directed at sources of single dimensionality p_i. We proceed to combine the equations (4.17) and (4.16), where the latter is multiplied by 2, giving in the following relation,

$$\frac{d+p_i-1}{d}\mathcal{R}_d = -\mathcal{R}_{10-d} - \frac{p_i-1}{2}|H_7|^2 - \frac{g_s^2}{4}\sum_{q=0}^{7}(p_i+q-6)|F_q|^2 . \tag{4.29}$$

We take into account the possible effects of O-planes projecting out certain fluxes, a feature that is particularly apparent in the case of $p_i = 4$ and F_0, as discussed above. From this analysis we can conclude that

No-go theorem 6. There is no classical dS solution for $p_i \geq 4$, $d \geq 3$ with $\mathcal{R}_{10-d} \geq 0$.

This no-go theorem emphasizes the connection between the source configuration and the geometric and physical properties of the internal space, in particular the role of the internal Ricci curvature.

No-go theorem for heterotic strings

While we will not delve into an introduction to heterotic strings here, as there is already an extensive literature on the subject [5], it is crucial to clarify certain elements supporting the arguments for a no-go theorem against the existence of classical dS vacua in heterotic string theory at leading order in the α' expansion. The 10D effective action for heterotic strings at leading order in α' couples $\mathcal{N} = 1$ SUGRA to a 10D super-Yang-Mills theory [112]. Depending on the respective heterotic theory, the gauge group could be either $E_8 \times E_8$ or $SO(32)$. The bosonic part of the low-energy effective action at leading order in α' involves the bosonic fields of the 10D SUGRA multiplet – namely the metric g_{MN}, the dilaton ϕ and the NSNS two-form B_2. The kinetic terms for the fields in the Yang-Mills multiplet, including the gauge field A_M with field strength F_{MN}, appear at order α'

$$S^{(\alpha')} = -\frac{\alpha'}{8\kappa_{10}^2} \int_{\mathcal{M}_{10}} d^{10}x \sqrt{-g}\, e^{-2\phi}\, \mathrm{tr}_v |F_2|^2 \, . \tag{4.30}$$

Note that there are no RR fields in heterotic theories and the gauge trace tr_v is normalized to the vector representation of either $SO(32)$ or $SO(16)$, a subgroup of E_8 [102]. In the absence of excited gauge fields, at zeroth order in α', heterotic SUGRA effectively becomes a truncation of the NSNS sector of type II SUGRA. The d-dimensional Einstein trace (4.4) then yields

$$\mathcal{R}_d = -\frac{d}{8}\left(|H_3|^2 + 3|H_7|^2\right) , \tag{4.31}$$

which can be further simplified to

$$\mathcal{R}_d = -\frac{d}{2}|H_3|^2 \, , \tag{4.32}$$

using the equation (4.11). We conclude with the following no-go theorem:

No-go theorem 7. There is no classical dS solution in $d \geq 3$ in heterotic SUGRA at leading-order in the α' expansion .

Furthermore, for $d \geq 4$ and $H_7 = 0$, only Minkowski vacua are feasible in heterotic string theory at leading order in α'. On the other hand, for $d = 3$ and non-zero H and H_7, AdS solutions become possible, e.g. Freund-Rubin compactifications [113], as well as the pure NSNS configuration on $\mathrm{AdS}_3 \times S^3 \times T^4$ in [114].

4.3 Source configurations for classical de Sitter solutions

Building on the no-go theorems presented in the previous section, which challenge the existence of classical dS solutions, the following study attempts to constrain and potentially exclude such solutions in various dimensions. We begin by identifying the non-trivial source content and the relevant fluxes for each dimension, which are integral to the assumptions underlying these theorems.

In Section 4.3.1, we discuss the consequences of these no-go theorems for classical string backgrounds with a d-dimensional dS spacetime. Inspired by [49], which studies metastable dS solutions under SUSY-preserving source configurations, our analysis not only confirms some results for $d > 7$, but also introduces new insights related to the no-go theorem 3a. Throughout our discussion in Section 4.3.2, we draw parallels and identify differences with the results in [49]. We conclude this analysis in Section 4.4, where we extend the scope to quasi-de Sitter solutions, addressing one of the central topics of this section.

4.3.1 Existence constraints

We have summarized the respective source and flux content of type II supergravities in arbitrary dimensions in Tables 1 and 2. For the type IIA theory compactified to ten dimen-

Dimension	Theory	Fluxes	Source dimensionality
$d = 10$	IIA	F_0	$\emptyset$
	IIB	$\emptyset$	9
$d = 9$	IIA	F_0	8
	IIB	F_1	9
$d = 8$	IIA	F_0, F_2	8
	IIB	F_1	7, 9
$d = 7$	IIA	F_0, F_2, H_3	6, 8
	IIB	F_1, F_3, H_3	7, 9

Table 1: This table summarizes the source and flux content of type II supergravities in dimensions $d = 7$ to $d = 10$.

sions, hereafter referred to as type IIA$|_{10}$, the absence of sources rules out the existence of classical dS backgrounds, as dictated by the Maldacena-Nuñez no-go theorem 1.

Similarly, type IIB$|_{10}$ is subject to constraints due to the no-go theorem 2. This also applies to dS solutions in dimensions 9 and 8, although the constraints are limited to sources of single dimensionality in $d = 8$. In contrast, type IIB$|_8$ can be considered a priori as a potential avenue for solutions with sources of multiple dimensionalities, an option not covered in [49]. However, the combination of the equations (4.2), (4.3) and (4.11) leads to the expression

$$\mathcal{R}_8 = -g_s \frac{T_{10}^{(9)}}{10}\,, \tag{4.33}$$

which culminates in the no-go theorem 4.24 for $p_i = 7$, $\tilde{p}_j = 9$; note also (4.1). These observations suggest:

Constraint 1. There is no classical dS solution in $d = 8, 9$ or 10 .

We now turn to more intriguing cases starting with $d = 7$, focusing on sources of single dimensionality that invoke either the no-go theorem 2 or 3a, especially for $p_i = 6$. However, our main interest is in models involving sources of multiple dimensionalities. For type IIA$|_7$, the equation (4.23) yields

$$\mathcal{R}_7 = \frac{7}{6}\left(-g_s\frac{T_{10}^{(8)}}{9} - |H_3|^2 + g_s^2 F_0^2\right) = \frac{7}{10}\left(g_s\frac{T_{10}^{(6)}}{7} - |H_3|^2 - g_s^2|F_2|^2\right) , \tag{4.34}$$

which suggests that $T_{10}^{(6)} > 0$ for classical dS, while the theorem 4 implies $T_{10}^{(8)} > 0$, thus requiring $F_0 \neq 0$. Note that the presence of both the O6- and O8-planes is confirmed. The O8 involution (2.24) causes F_0 to be an odd function over its fixed points, while the O6 involution, with its transverse space spanning the entire $\mathcal{M}_{10-d}$, requires F_0 to be even. Despite these peculiar but compatible conditions, we conclude:

Constraint 2a. There is no classical dS solution in type IIA$|_7$ unless we have O6- and O8-planes along with $F_0 \neq 0$.

Further discussion in [35] draws parallels to similar constraints observed for O4- and O6-planes in $d = 4$, where classical dS solutions are viable, albeit with $F_0 = 0$.

For type IIB$|_7$ with sources of dimensionality $p_i = 7, 9$, our analysis becomes more complex; the equation (4.23) results in

$$\mathcal{R}_7 = -\frac{7}{8}\left(g_s\frac{T_{10}^{(9)}}{10} + |H_3|^2 + g_s^2|F_3|^2\right) = \frac{7}{12}\left(g_s\frac{T_{10}^{(7)}}{8} - |H_3|^2 - g_s^2\left(|F_1|^2 + 2|F_3|^2\right)\right) , \tag{4.35}$$

which leads to $T_{10}^{(7)} > 0$ and $T_{10}^{(9)} < 0$, indicating the presence of O7-planes and D9-branes. However, the F_1 Bianchi identity implies that $\mathrm{d}F_1 \sim T_{10}^{(7)}\mathrm{vol}_{\perp \mathrm{O}7}$, which requires $F_1 \neq 0$. This requirement excludes the presence of O9-planes, which would imply $F_1 = 0$, leaving only D9-branes as viable objects. This conclusion has been previously refuted around the equation (4.1):

Constraint 2b. There is no classical dS solution in type IIB$|_7$.

Dimension	Theory	Fluxes	Source dimensionality
$d = 6$	IIA	F_0, F_2, F_4, H_3	6, 8
	IIB	F_1, F_3, H_3	5, 7, 9
$d = 5$	IIA	F_0, F_2, F_4, H_3	4, 6, 8
	IIB	F_1, F_3, F_5, H_3	5, 7, 9
$d = 4$	IIA	F_0, F_2, F_4, F_6, H_3	4, 6, 8
	IIB	F_1, F_3, F_5, H_3	3, 5, 7, 9
$d = 3$	IIA	F_0, F_2, F_4, F_6, H_3, H_7	2, 4, 6, 8
	IIB	F_1, F_3, F_5, H_3, H_7	3, 5, 7, 9

Table 2: This table summarizes the source and flux content of type II supergravities in dimensions $d = 3$ to $d = 6$.

We then proceed to higher dimensional theories, starting with $d = 6$ in type IIA$|_6$, where classical dS solutions with sources of single dimensionality $p_i = 6$ are studied under the no-go theorem 2. Here, our focus shifts to configurations with multiple dimensionalities $p_i = 6, 8$, guided by the equation (4.23), (4.23),

$$\mathcal{R}_6 = \frac{3}{5}\left(g_s\frac{T_{10}^{(6)}}{7} - |H_3|^2 - g_s^2\left(|F_2|^2 + 2|F_4|^2\right)\right) = -g_s\frac{T_{10}^{(8)}}{9} - |H_3|^2 + g_s^2\left(F_0^2 - |F_4|^2\right) , \tag{4.36}$$

which provides the conditions $T_{10}^{(6)} > 0$, indicating the presence of O6-planes, and $g_s^2 F_0^2 - g_s T_{10}^{(8)}/9 > 0$. The second inequality, treated similarly to the previous case, employs the Bianchi identity $\mathrm{d}F_0 \sim T_{10}^{(8)}\mathrm{vol}_{\perp_{\mathrm{O8}}}$ to confirm $F_0 \neq 0$. The presence of O8-planes suggests a scenario similar to $d = 7$, but with the possibility of non-overlapping $p_i = 6, 8$ sources. Here, the localization of the O8-plane at a fixed point on the O6-plane results in F_0 being odd along the O6 direction due to the O8 involution. However, although $\sigma_{O_6}(F_0) = F_0$, the involution along the O6 is trivial and there are no complications from overlapping or non-overlapping sources with multiple dimensionalities $p_i = 6, 8$:

Constraint 3a. There is no classical dS solution in type IIA$|_6$ unless we have O6-planes and $F_0 \neq 0$.

The constraints applied in type IIB$|_6$, based on the no-go theorems 2 and 3a, exclude classical dS for single-dimensional sources, which leads us to consider configurations involving multiple dimensionalities. The equation (4.23),

$$\mathcal{R}_6 = \frac{1}{2}\left(g_s\frac{T_{10}^{(7)}}{8} + 2g_s\frac{T_{10}^{(5)}}{6} - |H_3|^2 - g_s^2\left(|F_1|^2 + 2|F_3|^2\right)\right) \tag{4.37}$$

$$= \frac{3}{4}\left(-g_s\frac{T_{10}^{(9)}}{10} + g_s\frac{T_{10}^{(5)}}{6} - |H_3|^2 - g_s^2|F_3|^2\right) \tag{4.38}$$

$$= \frac{3}{2}\left(-2g_s\frac{T_{10}^{(9)}}{10} - g_s\frac{T_{10}^{(7)}}{8} - |H_3|^2 + g_s^2|F_1|^2\right) , \tag{4.39}$$

reveals that $T_{10}^{(5)} > 0$ due to $T_{10}^{(9)} = 0$ and thus the presence of O5-planes. Furthermore, the constraint 4 and the equation (4.24) imply $T_{10}^{(7)} > 0$, thus indicating the presence of O7-planes. Analogous to type IIA$|_7$, this configuration requires $F_1 \neq 0$ due to the Bianchi identity, which effectively excludes $p_i = 9$ sources:

Constraint 3b. There is no classical dS solution in type IIB$|_6$ unless we have O5- and O7-planes along with $F_1 \neq 0$.

This comprehensive analysis sets the stage for further detailed studies of classical dS solutions in various dimensions, as well as for exploring the intriguing realm of $d = 5$ and below, where additional complexities and possibilities for dS solutions arise.

Turning to type IIA$|_5$, sources of single dimensionality $p_i = 4, 8$ are excluded by the theorems 3a and 2. However, for $p_i = 6$, the theorem of Maldacena and Nuñez 1 indicates the presence of O6-planes. This observation holds for configurations involving sources of multiple dimensionalities: for models with O4-planes/D4-branes, the no-go theorem 4 still requires O6-planes, independent of $p_i = 8$ sources. Even without $p_i = 4$ sources, the equation (4.24) with $p_i = 4$ leads to the same conclusion:

Constraint 4a. There is no classical dS solution in type IIA$|_5$ unless we have O6-planes.

The no-go theorems 1 and 2 also restrict single-dimensional sources to O5-planes in type IIB$|_5$. For multiple dimensionalities, we analyze the equation (4.23) for $p_i = 5, 7, 9$,

$$\mathcal{R}_5 = \frac{5}{12}\left(g_s\frac{T_{10}^{(7)}}{8} + 2g_s\frac{T_{10}^{(5)}}{6} - |H_3|^2 - g_s^2\left(|F_1|^2 + 2|F_3|^2 + 3|F_5|^2\right)\right) \tag{4.40}$$

$$= \frac{5}{8}\left(-g_s\frac{T_{10}^{(9)}}{10} + g_s\frac{T_{10}^{(5)}}{6} - |H_3|^2 - g_s^2(|F_3|^2 + 2|F_5|^2)\right) \tag{4.41}$$

$$= \frac{5}{4}\left(-2g_s\frac{T_{10}^{(9)}}{10} - g_s\frac{T_{10}^{(7)}}{8} - |H_3|^2 + g_s^2(|F_1|^2 - |F_5|^2)\right) , \tag{4.42}$$

where $T_{10}^{(9)} = 0$ due to (4.1). This analysis yields the conditions: $T_{10}^{(5)} > 0$, supporting the presence of O5-planes, and $g_s^2|F_1|^2 - g_s T_{10}^{(7)}/8 > 0$, interpreted as $F_1 \neq 0$ via the Bianchi identity. However, as mentioned above, this implies the absence of O9-planes/D9-branes. Therefore, classical dS solutions are only possible without $p_i = 9$ sources. We conclude:

Constraint 4b. There is no classical dS solution in type IIB$|_5$ unless we have O5-planes and $F_1 \neq 0$.

We now consider the phenomenologically relevant case of $d = 4$. Given the extensive research on this dimension, we refer to the vast literature on single-dimensional sources [45, 53], where only sources with $p_i = 4, 5$ or 6 are viable, with $p_i = 4$ subject to significant constraints [35]. On the other hand, configurations with sources of multiple dimensionalities offer additional possibilities for classical string backgrounds with a dS spacetime, as detailed in [35, 105].

Finally, we explore the intriguing case of $d = 3$. For a broad range of possibilities to obtain classical dS solutions, we turn to [115] in type IIA$|_3$, which includes anti-Dp-branes, and [116] in type IIB$|_3$. Although no solutions have been found in the latter setup, the introduction of a F_7 flux certainly enriches the discussion. It is important to note that in the previous section we have derived additional no-go theorems for $d = 3$, while some constraints are not applicable. In particular, in type IIA$|_3$ with $p_i = 2$, we address the no-go theorems 3b and 5b. Further discussion of this topic will continue in the next section.

4.3.2 Supersymmetric source configurations

SUSY-preserving string compactifications offer several compelling advantages that make them particularly attractive for theoretical and phenomenological studies [102, 117]. First, such configurations are inherently stable and free of tachyons, which simplifies their analysis. In the context of SUGRA, solutions satisfying first-order SUSY conditions also inherently satisfy the more complex second-order SUGRA equations of motion. Thus, the search for vacuum solutions in string theory often starts from configurations in which SUSY is partially preserved, facilitating the transition to broken SUSY rather than solving the full equations of motion outright. From a phenomenological perspective, supersymmetric backgrounds serve as a robust framework for constructing models in particle physics, especially when SUSY is broken at energy scales much lower than the fundamental string scale. Furthermore, even if SUSY does not manifest itself at the TeV scale, the study of SUSY-preserving models provides valuable insights into the dynamics of string compactifications. Therefore, the discussion of SUSY breaking remains a critical aspect of model building [35].

In our study of static brane and orientifold configurations, it is essential to assess their potential to preserve SUSY. Such a test of whether these configurations are mutually BPS is crucial for advancing our understanding of string compactifications and their practical applications in theoretical physics. Type II string theories each possess $\mathcal{N} = 2$ SUSY, or equivalently 32 supercharges [2, 3]. A single set of parallel branes halves these supersymmetries, while two orthogonal sets can potentially reduce them further, preserving only a quarter. This conservation requires that the sum of the Neumann-Dirichlet (ND) boundary conditions $\mathcal{N}_{ND}$ be a multiple of four. More specifically, the directions unique to a set must also be a multiple of four, otherwise SUSY is completely broken[17]. In our orthogonal coframe setup, SUSY checks are trivial due to the clear separation of d external and $p_i + 1 - d$ internal dimensions, which can be ND.

Let us demonstrate this with an example in $d = 4$; For configurations with sources of single dimensionality, two intersecting $p_i = 4$ sources give $\mathcal{N}_{ND} = 2$, breaking SUSY. For $p_i \geq 5$, preserving SUSY while ensuring that the total number of internal dimensions of two intersecting sets is less than six requires us to define $N = p_i - 5$ for each pair. Here, N represents the number of common internal dimensions that guarantees homogeneous overlap and the preservation of SUSY. This analysis extends to sources of multiple dimensionalities, applying the $\mathcal{N}_{ND}/4 \in \mathbb{Z}$ rule to determine those that preserve some SUSY, with each orthogonal pair preserving a quarter of the initial supersymmetries. We now explore the behavior of the parallel and orthogonal brane configurations described in Section 4.3.1.

We revisit the highest possible dimension, $d = 7$, for classical dS solutions in type IIA with O6-/O8-planes and a non-vanishing F_0 flux, as outlined in the constraint 2a. Due to the overlapping nature of the O6-/O8-planes, we find that $\mathcal{N}_{ND} = 2$, which excludes any SUSY-preserving source configuration in $d = 7$.

Constraint 5. There is no classical dS solution in a SUSY-preserving theory in $d = 7$.

In type IIA$|_6$, as detailed in the constraint 3a, the configurations include O6-planes with non-zero F_0 flux. When these planes intersect, they yield $\mathcal{N}_{ND} = 2$, breaking SUSY, which may change upon considering non-overlapping O8-planes, as detailed in Table 3 and below equation (4.36). The internal dimensions are given in Arabic numerals, starting with 1. Applying T-duality relations along any internal dimension leads to configurations involving O5- and O9-planes/D9-branes, or O7-planes/D7-branes, which, according to 3b, exclude the possibility of classical dS backgrounds. The likelihood of obtaining dS solutions with

[17] Including anti-D-branes in a configuration with O-planes also breaks SUSY, providing another argument for avoiding them.

Sources	Spacetime dimensions	1	2	3	4
O6, (D6)	x	x			
(O8, D8)	x		x	x	x

Table 3: Source configurations preserving SUSY for classical dS solutions in type IIA$|_6$ supergravity.

parallel O6-planes within a single set is also low, as indicated by [45] and further supported by [118]. However, the lack of a definitive (algebraic) no-go theorem means that we cannot categorically dismiss this configuration. It is also worth noting that [118] reports finding solutions involving KK5 monopoles and KKO5-planes in type IIA$|_6$ that preserve half of the supersymmetries. However, these solutions face challenges due to the potential non-compactness of the internal space.

In the context of type IIB$|_6$, the condition 3b suggests a source configuration that includes O5- and O7-planes. However, similar to observations in type IIA$|_7$, the spacetime-filling nature of the O5-planes rules out the preservation of SUSY.

Constraint 6. There is no classical dS solution within a SUSY-preserving theory in $d = 6$ unless from the source configurations in Table 3.

Finally, we explore type IIA$|_5$, emphasizing the necessity of O6-planes as outlined in the constraint 4a. The inclusion of O4-planes/D4-branes in SUSY-preserving configurations is ruled out due to the spacetime-filling nature of $p_i = 4$ sources. With respect to the equation (4.23) for $p_i = 6$,

$$\mathcal{R}_5 = \frac{5}{6}\left(g_s\frac{T_{10}^{(4)}}{5} - g_s\frac{T_{10}^{(8)}}{9} - |H_3|^2 + g_s^2\left(F_0^2 - |F_4|^2 - 2|F_6|^2\right)\right), \tag{4.43}$$

and with $T_{10}^{(4)} = 0$ we derive an additional condition, $g_s^2F_0^2 - g_sT_{10}^{(8)}/9 > 0$. Revisiting the discussion under (4.36), we can include O8-planes/D8-branes. However, the only configurations that preserve SUSY are those listed in Table 4, which require $\mathcal{N}_{ND} = 4$. In

Sources	Spacetime dimensions	1	2	3	4	5
O6, (D6)	x	x	x			
(O6, D6)	x			x	x	
O6, (D6)	x	x	x			
(O8, D8)	x		x	x	x	x

Table 4: Source configurations preserving SUSY for classical dS solutions in type IIA$|_5$ supergravity.

type IIB$|_5$, the relevant configuration outlined in the constraint 4b includes O5-planes and potential D5-branes, along with $p_i = 7$ sources. This setup mirrors considerations in type IIA$|_6$, where similar rules apply. For sources of single dimensionality $p_i = 5$, only one set of O5/D5 can preserve SUSY, while combinations of $p_i = 5, 7$ offer more diverse possibilities, summarized in Table 5. We draw the following conclusion:

Sources	Spacetime dimensions	1	2	3	4	5
O5, (D5)	x	x				
(O7, D7)	x		x	x	x	

Table 5: Source configurations preserving SUSY for classical dS solutions in type IIB$|_5$ supergravity.

Constraint 7. There is no classical dS solution within a SUSY-preserving theory in $d = 5$ unless from the source configurations in Tables 4 and 5.

Doubts remain for $d = 5$, similar to problems previously identified in type IIA$|_6$, since configurations with only parallel sources preclude classical dS solutions [45]. In addition, T-dualities applied to the source configurations listed in Tables 4 and 5 result in configurations that definitively exclude dS backgrounds.

4.3.3 Conclusion

In our analysis of flux compactifications to arbitrary dimension d, we initially reduced the number of sources and fields. In higher dimensions, this reduction is often consistent with the assumptions of the no-go theorems discussed in Section 4.2, allowing us to definitively rule out or severely constrain the possibility of d-dimensional dS solutions. Furthermore, when focusing on SUSY-preserving source configurations in Section 4.3.2, classical dS compactifications are unambiguously ruled out in $d = 7$ and in type IIB for $d = 6$. The few remaining viable source configurations that could support dS spacetimes are detailed in Table 3 for $d = 6$ and Tables 4 and 5 for $d = 5$. These results not only echo but also build on those of [49], and we intend to extend these results to quasi-dS solutions in Section 4.4.

Focusing our analysis on SUSY-preserving source configurations is a pragmatic approach that enhances the phenomenological relevance, as it potentially allows for a supersymmetric d-dimensional effective theory. For $d = 6$ and $d = 5$, the viable configurations typically involve at most two intersecting sets of sources, as shown in the tables above. This subtlety is dictated by the SUSY algebra which does not support only 4 supercharges in dimensions $d \geq 5$. Therefore, preserving SUSY in these dimensions usually allows no more than two intersecting sets of sources. This principle is critical in the context of Conjecture 4 in [35], which will be discussed in 4 dimensions.

Three conjectures concerning classical dS solutions have been proposed in [45], with our focus here on Conjecture 1. This conjecture posits the non-existence of classical dS solutions in configurations with only a single set of sources. Although it remains neither proven nor refuted, it is considered valid within the ansatz outlined in [45] and may have broader applicability. In our studies [35], we propose an extension of this conjecture:

Conjecture 4. There is no classical dS solution with two intersecting sets of sources.

Our confidence in this conjecture is supported by empirical data; none of the classical dS solutions classified in [35] have less than three sets of sources. Moreover, our dedicated search for dS solutions with only two sets of sources across various classes have been unsuccessful.

Further support for Conjecture 4 comes from T-duality analysis. Certain source configurations are T-dual to each other, and in particular, when restricted to exactly two source sets, either two O-planes or one O-plane with one D-brane, we identify two T-duality chains that include all configurations that allow exactly two sets, leaving no class isolated. The support for Conjecture 4 is based on the no-go theorems against classical dS solutions in these specific classes, as detailed in [105]. The T-duality relations suggest that similar constraints apply to related classes. However, a comprehensive T-duality relationship between these classes is not definitively established, since certain fields within one class may prevent a solution from being T-dual to the other, even though the sources themselves are potentially dualizable. For further details, see [35]. Despite these nuances, it remains a strong indication of a universal obstruction to dS solutions across all classes within these T-duality chains, each of which allows exactly two source sets.

An important outcome of Conjecture 4 is its implications for a 4D effective theory, where "effective" in this context refers to (consistent) truncations that preserve the contributions of sources without eliminating them. Such truncations typically lead to 4D gauged SUGRA, as noted in [45], which ensures that any solution in this 4D framework corresponds to one in the 10D theory. Indeed, if a classical dS solution involves three or more intersecting sets of sources, then, as discussed in Section 4.3.2, SUSY is broken by a factor of at least 8, and we can confidently conclude [35]:

Implication. A 4D effective theory derived from a classical string compactification, which includes a dS extremum, is inherently constrained to have $\mathcal{N} = 1$ SUSY.

However, a non-supersymmetric 4D theory, likely prone to instabilities, is also a theoretical possibility.

Nevertheless, many 4D gauged supergravities, defined by unique gaugings, often do not originate from classical compactifications. Identifying dS solutions in these theories, especially those having extended SUSY, $\mathcal{N} > 1$, poses significant challenges when strictly derived from classical compactifications. This difficulty is consistent with the Conjecture 4. Typically, finding (meta-)stable dS solutions in such supergravities involves elements such as non-compact gaugings, Fayet-Iliopoulos terms or non-geometric fluxes, whose higher-dimensional origins remain ambiguous or controversial, as discussed in [119–121].

The implications of Conjecture 4 are significant for phenomenology. Intersecting source configurations, which are frequently utilized in the construction of particle physics models, allow for chirality under the condition that $\mathcal{N} \leq 1$ in 4D [105,122,123]. Regarding classical dS solutions in higher dimensions, the supersymmetric source configurations discussed in Section 4.3.2 are typically restricted to at most two sets. If Conjectures 1 and 4 are validated, dS solutions would be precluded in dimensions $d \geq 5$, leaving $d = 4$ the highest viable dimension for such model.

4.4 Extension to quasi-de Sitter

The constraints on classical dS solutions discussed in Sections 4.3.1 and 4.3.2 naturally extend to quasi-dS. For this analysis, we transition from a 10D framework to a d-dimensional effective theory featuring a scalar potential, following dimensional reduction as detailed in Section 3. Here, a dS solution is defined by an extremum of the positive potential while quasi-dS is characterized by a small gradient $|\nabla V(\varphi)|$. This distinction will be elaborated upon shortly.

In our analysis, the constraints on the existence of classical dS solutions, as derived in previous sections, incorporate several critical components. These include a linear combination of the 10-dimensional equations of motion (4.2), (4.3) and (4.4), along with the Bianchi identities for the fluxes F_0, F_1 and F_{9-d}. Additionally, the rules of orientifold projection and the preservation of SUSY within the d-dimensional effective theory are fundamental.

Furthermore, we demonstrate that the dilaton equation of motion, given in (4.2), can be derived from a combination of the universal potential (3.26) and its derivatives

$$
\begin{aligned}
-\frac{2}{M_p^2}\left(\frac{2d}{d-2}V+\frac{2}{d-2}\tau\partial_\tau V\right) &= 2\tau^{-2}\rho^{-1}\mathcal{R}_{10-d} \\
&\quad -\tau^{-2}\rho^{-3}|H_3|^2+\tau^{2(1-d)}\rho^{3-d}|H_7|^2+\tau^{-\frac{d+2}{2}}g_s\sum_i\rho^{\frac{2p_i-8-d}{4}}\frac{T_{10}^{(p_i)}}{p_i+1}\,,
\end{aligned} \tag{4.44}
$$

evaluated on-shell where $\rho=\tau=1$ and using the expression (3.4). Employing a similar method, we deduce the 10-dimensional Einstein trace (4.3) from

$$
\begin{aligned}
-\frac{2}{M_p^2}\left(\frac{4d}{d-2}V+\frac{10-d}{2(d-2)}\tau\partial_\tau V+\rho\partial_\rho V\right) &= 4\tau^{-2}\rho^{-1}\mathcal{R}_{10-d}-\tau^{-2}\rho^{-3}|H_3|^2 \\
+\tau^{2(1-d)}\rho^{3-d}|H_7|^2+\frac{1}{2}\tau^{-\frac{d+2}{2}}g_s\sum_i\rho^{\frac{2p_i-8-d}{4}}T_{10}^{(p_i)} &-\frac{1}{2}\tau^{-d}g_s^2\sum_{q=0}^{10-d}\rho^{\frac{10-d-2q}{2}}(5-q)|F_q|^2\,,
\end{aligned} \tag{4.45}
$$

at the critical point. Concurrently, the combination

$$
\begin{aligned}
\frac{2}{M_p^2}\left(\frac{d}{d-2}V+\frac{d(10-d)}{16(d-2)}\tau\partial_\tau V+\frac{d}{8}\rho\partial_\rho V\right) &= \frac{d}{16}\Bigg(-2\tau^{-2}\rho^{-3}|H_3|^2-6\tau^{2(1-d)}\rho^{3-d}|H_7|^2 \\
+\tau^{-\frac{d+2}{2}}g_s\sum_i\rho^{\frac{2p_i-8-d}{4}}\frac{7-p_i}{p_i+1}T_{10}^{(p_i)} &+\tau^{-d}g_s^2\sum_{q=0}^{10-d}\rho^{\frac{10-d-2q}{2}}(1-q)|F_q|^2\Bigg)\,,
\end{aligned} \tag{4.46}
$$

yields the d-dimensional Einstein trace on-shell. The Bianchi identities remain applicable within the d-dimensional theory, albeit potentially modified by positive factors associated with the scalar fields. Consequently, the critical elements required for deriving no-go theorems in Section 4.2 continue to be relevant in the effective theory.

It is important to emphasize that the reasoning behind these constraints, leading to $\mathcal{R}_d\leq 0$ and thus excluding dS solutions, relies on the signs of individual terms in the equations rather than on their linear combinations. This indicates that the inclusion of scalar fields does not affect the results of these constraints. This stands in contrast to situations where constraints rely on combinations of terms with differing power scaling, as elaborated in [38] under the term "field-dependent" condition. In essence, we can replicate the derivation of the no-go theorems discussed earlier, which restrict the possibility of classical dS solutions, within a d-dimensional framework that involves a scalar potential and its derivatives. Rather than concluding with $\mathcal{R}_d \leq 0$ we derive a comparable on-shell statement that remains valid both at and away from the critical point (off-shell). In the coming section, we will further explore these constraints to either exclude or more tightly constrain d-dimensional quasi-dS solutions.

5 No-go theorems in lower-dimensional effective theories

In Sections 4.2 and 4.3.1, we established numerous no-go theorems against classical string backgrounds with a d-dimensional dS spacetime. Drawing on Conjecture 1 [45] and Conjecture 4 [35], we argue against the existence of such solutions in $d \geq 5$, suggesting that viable models, if any, would likely appear in $d \leq 4$. As noted above, no-go theorems can be derived in two ways: One approach involves combining 10-dimensional equations, and another uses a d-dimensional effective theory as derived in section 3. The latter approach leads, under some assumptions, to an inequality of the form

$$aV(\varphi) + \sum_i b_i \varphi^i \partial_{\varphi^i} V(\varphi) \leq 0 \,, \quad a > 0 \,, \; \exists \; b_i \neq 0 \,, \tag{5.1}$$

which effectively excludes dS critical points. In this section, we shall take the d-dimensional Planck mass to be equal to one. When evaluated on-shell, where $\varphi^i = 1$, this inequality can typically be matched with the 10-dimensional result, $\mathcal{R}_d \leq 0$. The benefit of employing a d-dimensional derivation is its capacity to exclude quasi-de Sitter solutions as discussed in greater detail in Sections 4.4 and 5.3. The equation (5.1) is typically rephrased in the form

$$|\nabla V(\hat{\varphi})| \geq c \; V(\hat{\varphi}) \,, \quad c \sim \mathcal{O}(1) \tag{5.2}$$

for canonically normalized fields $\hat{\varphi}^i$, thereby allowing the evaluation of a value of c. This rate is then compared to the bound proposed by the Trans-Planckian Censorship Conjecture (TCC) [58],

$$c_{\rm tcc} = \frac{2}{\sqrt{(d-1)(d-2)}} \,, \tag{5.3}$$

in the swampland program, as outlined in Section 5.2. Our results, summarized and discussed in Section 5.4, confirm that the TCC bound holds in all dimensions, with multiple cases of saturation already noted in $d = 4$. These checks support the validity of the TCC bound, although there is no fundamental principle that supergravity no-go theorems would reproduce it in $d \geq 3$. A more detailed discussion of this topic can be found in Section 5.4.1, which includes an intriguing result in $d = 3$, probably related to the unique properties of gravity in this dimension. After strengthening the TCC bound for dimensions $d \geq 4$, we proceed in Section 5.4.2 to contrast this bound with various proposals cited from the literature, in particular those found in [124]. In addition, our discussion will extend to the Distance Conjecture [125], focusing on a speculative d-dependent formulation for the lower bound on its rate λ. In Section 5.4.3, we explore an asymptotic upper bound on the gradient of the potential, a critical requirement for cosmic accelerated expansion.

5.1 Accelerated expansion in string cosmology

Following the notation in [126], we explore the d-dimensional uncompactified part of our universe. The Friedmann-Lemaître-Robertson-Walker (FLRW) metric,

$$ds_d^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 \left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega_{d-2} \right) \,, \tag{5.4}$$

describes an isotropic and homogeneous spacetime in spherical coordinates, including the scale factor $a(t)$, which depends on the proper cosmological time, and the parameter of spatial curvature k, which adopts values of $-1, 0, +1$, according to open, flat and closed universes. The effective action (3.2),

$$S^{(d)} = \int \mathrm{d}^d x \sqrt{-g_d} \left(\frac{1}{2} \mathcal{R}_d - \frac{1}{2} g^{\mu\nu} G_{ij} \partial_\mu \varphi^i \partial_\nu \varphi^j - V(\varphi) \right) , \tag{5.5}$$

repeated here for convenience, follows from consistent truncation of the 10-dimensional parent theory. It involves several minimally coupled scalar fields φ^i, a field space metric G_{ij} and an effective scalar potential $V(\varphi)$. The specific structure of the potential depends on the details of the string compactification, see Section 3. The scalar fields $\varphi^i = \varphi^i(t)$ are a priori functions of time alone. Such a theoretical framework can support solutions where the d-dimensional spacetime is maximally symmetric with a cosmological constant Λ_d. The action (5.5) allows us to derive the cosmological equations, which include the two Friedmann equations

$$\begin{gathered} \frac{(d-1)(d-2)}{2} \left(H^2 + \frac{k}{a^2} \right) - \frac{1}{2} G_{ij} \dot{\varphi}^i \dot{\varphi}^j - V(\varphi) = 0 \, , \\ (d-2) \left(\dot{H} - \frac{k}{a^2} \right) + G_{ij} \dot{\varphi}^i \dot{\varphi}^j = 0 \, , \end{gathered} \tag{5.6}$$

characterized by the Hubble parameter $H = \dot{a}/a$; the dots indicate derivatives by proper time. In addition, we consider the equations of motion for the scalar fields

$$\ddot{\varphi}^i + \Gamma^i{}_{jk} \dot{\varphi}^j \dot{\varphi}^k + (d-1) H \dot{\varphi}^i + \partial_{\varphi^i} V(\varphi) = 0 \, , \tag{5.7}$$

with indices lowered by G_{ij} and $\Gamma^i{}_{jk}$ representing the Christoffel symbols associated with the field space metric. To simplify these equations, we assume the energy-momentum tensor of a perfect fluid. Therefore, the energy density and pressure of the scalar fields are given by

$$\rho = \frac{1}{2} G_{ij} \dot{\varphi}^i \dot{\varphi}^j + V(\varphi) \, , \qquad p = \frac{1}{2} G_{ij} \dot{\varphi}^i \dot{\varphi}^j - V(\varphi) \, , \tag{5.8}$$

and the equation of state parameter $w = p/\rho$. In the following sections we will study positive scalar potentials, $V(\varphi) > 0$. In this model, a positive H is necessary to ensure an expanding universe, while $\ddot{a} > 0$ is required for acceleration. As a consequence, the second Friedmann equation imposes

$$w < -\frac{d-3}{d-1} \, , \tag{5.9}$$

leading to a constraint on the scalar potential, as identified in [124] and further elaborated on in Section 5.4.3. This inequality actually represents a violation of the strong energy condition (4.5) in d dimensions. For the sake of clarity, we first consider the dynamics under slow-roll conditions. This approximation relies on two key assumptions: First, the kinetic terms are negligible with respect to the potential, i.e.

$$V(\varphi) \gg \frac{1}{2} G_{ij} \dot{\varphi}^i \dot{\varphi}^j \, . \tag{5.10}$$

Second, the friction term in (5.7) dominates any term of second-order,

$$(d-1)H\dot{\varphi}^i \gg \ddot{\varphi}^i + \Gamma^i{}_{jk}\dot{\varphi}^j\dot{\varphi}^k\,. \tag{5.11}$$

These assumptions allow a simplification of the first Friedmann equation and the scalar field equation of motion,

$$\frac{(d-1)(d-2)}{2}H^2 - V(\varphi) = 0\,, \qquad (d-1)H\dot{\varphi}^i + \partial_{\varphi^i}V(\varphi) = 0\,, \tag{5.12}$$

which shows that the expansion of the universe can lead to near exponential growth, known as inflation. It is important to note, however, that inflation can also occur outside the slow-roll regime.

A solution to (5.6) with a d-dimensional de Sitter spacetime, corresponds to an extremum of the potential, $\nabla V(\varphi)|_0 = 0$, where the scalar fields lack kinetic energy. On the other hand, a quasi-de Sitter solution involves positive $V(\varphi)$ with small $|\nabla V(\varphi)|$, and (slowly) rolling fields. A natural question arises: Can a model consistent with these classical string backgrounds be derived directly from string theory?

5.2 Swampland criteria and de Sitter

Addressing the challenges of identifying viable solutions in string models is at the heart of the swampland program [56, 57], which strives to establish universal bounds on the scalar potentials within effective theories of quantum gravity. Although this task is inherently complex, it is possible to provide definitive answers in certain regions of field space. There is strong support for the notion that the potential $V(\varphi)$ exhibits a universal behavior near points of divergent geodesic distance in field space,

$$s(\varphi(t)) = \int_{t_0}^{t} \mathrm{d}t\,\sqrt{G_{ij}\dot{\varphi}^i\dot{\varphi}^j} \to \infty\,, \tag{5.13}$$

often referred to as the asymptotic limit. At these points, it has been observed that the potential satisfies

$$\gamma \equiv \left(\frac{|\nabla V(\varphi)|}{V(\varphi)}\right)_{s\to\infty} \geq c\,, \tag{5.14}$$

where c is a constant that can vary with d and we refer to γ as the (asymptotic) de Sitter coefficient [126]. Any gradient of $V(\varphi)$ satisfying (5.14) must neither vanish nor be small. Because of this, the inequality is in a sense an asymptotic formulation of the de Sitter Conjecture [30, 31], that postulates a fundamental obstacle to the existence of de Sitter solutions in regions where parametric control over corrections to the effective theory is maintained. This constraint highlights our interest in the classical regime, which is consistent with these asymptotics and where the bound in (5.14) is assumed to be valid. Therefore, this regime is suitable for testing the asymptotic de Sitter conjecture, a challenge we will face in the following sections. However, it is important to understand that the constraint (5.14) holds only within these asymptotic limits, and does not preclude de Sitter solutions elsewhere in field space where parametric control may be lost. Moreover, this conjecture is evident in various compactifications of string theory [38, 40, 55], where the potential consistently decreases exponentially at a rate γ as it approaches points of infinite geodesic distance.

We then discuss particular swampland conjectures and their refinements, which provide estimates for the bound on the de Sitter coefficient from various arguments:

- According to the Trans-Planckian Censorship Conjecture [58], any consistent effective theory of quantum gravity should prevent cosmological expansions that allow sub-Planckian fluctuations to expand beyond the Hubble scale, freeze out and classicalize. This conjecture sets a lower bound on the de Sitter coefficient, $\gamma \geq c_{\text{tcc}}$, as given in the equation (5.3).

- The Strong de Sitter Conjecture [124, 127] states that any effective theory that satisfies the asymptotic condition (5.14) with

$$c_{\text{strong}} = \frac{2}{\sqrt{d-2}}\,, \tag{5.15}$$

will uphold this constraint after dimensional reduction. This suggests that the de Sitter Conjecture is robust to dimensional reduction, provided that $c = c_{\text{strong}}$. Moreover, this condition implies that the strong energy condition, outlined in the equations (4.5) and (5.4.3), persists at late times in the asymptotic regions of field space, thus ruling out any possibility of asymptotic accelerated expansion. We will return to this topic in Section 5.4.3.

Given that $c_{\text{tcc}} < c_{\text{strong}}$, we will argue in Section 5.4.3 that the Trans-Planckian Censorship Conjecture still allows for a cosmic accelerated expansion. For the sake of completeness, we mention another swampland constraint in the context of a d-dimensional effective theory (3.2) coupled to Einstein gravity.

- The Distance Conjecture [125] proposes that the moduli space is inherently non-compact. More specifically, starting from any point $\varphi(t_0)$ in field space, it is always possible to find another point $\varphi(t)$ at an infinite geodesic distance $s(\varphi(t)) \to \infty$. This conjecture leads to a far-reaching consequence under the assumption that such asymptotic points exist in moduli space: Approaching a point at infinite geodesic distance in field space, an infinite tower of states, characterized by a mass scale m, becomes exponentially lighter,

$$m(\varphi(t)) \sim m(\varphi(t_0))\, \mathrm{e}^{-\lambda s(\varphi(t))}\,, \quad \text{as } s(\varphi(t)) \to \infty\,. \tag{5.16}$$

However, the conjecture does not specify the exponential rate $\lambda > 0$. We will return to this topic in Section 5.4.2.

Below we discuss whether the first bound, $\gamma \geq c_{\text{tcc}}$, is consistent with the no-go theorems derived earlier.

5.3 Scalar flux potentials and no-go theorems in supergravity

After setting up the framework of a d-dimensional effective theory in Section 3, we will revisit the no-go theorems derived from the 10D equations in Section 4 to constrain the existence of classical (quasi-)de Sitter solutions in an arbitrary dimension d. Our focus will be on sources of single dimensionality p_i. As discussed in Section 4.4, the d-dimensional representation of these no-go theorems, given by the inequality (5.1), replicates the constraints in the 10D theory, i.e. $\mathcal{R}_d \leq 0$, at the critical point of the potential.

Upon transforming to canonical fields, as detailed in equation (3.21) for (ρ, τ, σ) and in (3.45) for (τ, r), we derive the formal expression

$$aV(\hat{\varphi}) + \sum_i \hat{b}_i \partial_{\hat{\varphi}^i} V(\hat{\varphi}) \leq 0\,, \tag{5.17}$$

from the inequality (5.1) [45, 52]. The slope of the potential, $|\nabla V(\hat{\varphi})|/V(\hat{\varphi})$, in (5.14) is bounded by

$$c = \sqrt{\frac{a^2}{\sum_i \hat{b}_i^2}}\,. \tag{5.18}$$

As we will see in this section, this formalism is consistent with the asymptotic de Sitter Conjecture in equation (5.14), in particular with the bound (5.3) of the Trans-Planckian Censorship Conjecture [58]. Therefore, for any no-go theorem in arbitrary dimension d that relies on field-independent assumptions, we derive the respective value of c and compare it to the bound proposed by the TCC, as discussed in [38] for $d = 4$.

No-go theorem by Maldacena and Nuñez

According to the equations above, the scalar potential (3.26) and its derivatives can be combined to form

$$\begin{aligned} 2\,(2V + \tau\partial_\tau V) = \frac{d-2}{2}\Bigg(&\tau^{-\frac{d+2}{2}} \sum_i \rho^{\frac{2p_i-8-d}{4}} g_s \frac{T_{10}^{(p_i)}}{p_i+1} \\ &- 2\tau^{2-2d}\rho^{3-d}|H_7|^2 - \tau^{-d} g_s^2 \sum_q \rho^{\frac{10-d-2q}{2}} |F_q|^2\Bigg)\,. \end{aligned} \tag{5.19}$$

When evaluated on-shell, this leads to the statement (4.11) of the no-go theorem 1 by Maldacena and Nuñez, given by the inequality

$$2V + \sqrt{d-2}\;\partial_{\hat{\tau}} V \leq 0\,, \quad \text{if } \forall\, p_i:\; T_{10}^{(p_i)} \leq 0\,, \tag{5.20}$$

using canonical fields (3.21). The respective value of c is then obtained as $c^2 = 4/(d-2)$.

Sources of single dimensionality

The first constraint for sources of single dimensionality p_i results from the unique combination

$$2\left(\frac{2(p_i-3)}{d-2}V - \frac{d+4-2p_i}{2(d-2)}\tau\partial_\tau V + \rho\partial_\rho V\right) = -\tau^{-2}\rho^{-3}|H_3|^2$$
$$+ (4-p_i)\tau^{2-2d}\rho^{3-d}|H_7|^2 + \tau^{-d}\frac{1}{2}g_s^2\sum_q \rho^{\frac{10-d-2q}{2}}\left(8-p_i-q\right)|F_q|^2\,, \quad (5.21)$$

which, when evaluated on-shell, gives the equation (4.17) and eventually leads to the no-go theorem 2,

$$\frac{2(p_i-3)}{d-2}V - \frac{d+4-2p_i}{2\sqrt{d-2}}\partial_{\hat\tau}V + \sqrt{\frac{10-d}{4}}\partial_{\hat\rho}V \leq 0\,, \quad \text{if } p_i = 7, 8 \text{ or } 9\,. \tag{5.22}$$

The corresponding value of c is given by

$$c^2 = \frac{4(p_i-3)^2}{(d-2)((p_i-3)^2 + (p_i-5)(2-d))}\,. \tag{5.23}$$

Before we study this expression in more detail, let us first derive another constraint in form of no-go theorem 3a. To this end, we compute the following combination of the potential and its derivatives,

$$\begin{aligned} &2\left(\frac{2(p_i+1)}{d-2}V - \frac{d-4-2p_i}{2(d-2)}\tau\partial_\tau V + \rho\partial_\rho V\right) \\ &= -\left|\tau^{-1}\rho^{-\frac{3}{2}} *_{10-d} H_3 + \varepsilon_{p_i} g_s \tau^{-\frac{d}{2}}\rho^{\frac{2p_i-2-d}{4}}F_{6-p_i}\right|^2 + 2\tau^{-\frac{d+2}{2}}\rho^{\frac{2p_i-8-d}{4}} g_s\varepsilon_{p_i}(\mathrm{d}F_{8-p_i}) \\ &\quad - p_i\tau^{2-2d}\rho^{3-d}|H_7|^2 + \frac{1}{2}\tau^{-d}g_s^2\sum_{q\neq 6-p_i}\rho^{\frac{10-d-2q}{2}}|F_q|^2\left(4-p_i-q\right)\,. \end{aligned} \tag{5.24}$$

In this equation, we have employed the Bianchi identity (4.19), which has been multiplied by powers of the scalar fields and reorganized to conform to the off-shell formulation of the no-go theorem. The term containing $(\mathrm{d}F_{8-p_i})$ has a priori no definite sign. Similar to the 10-dimensional approach, we notice that the integration of $(\mathrm{d}F_{8-p_i})$ over the compact space, implicitly defined in (3.37), vanishes for $p_i = d-1$ according to the equation (4.22). Therefore, the linear combination (5.24) satisfies the inequality

$$\frac{2d}{d-2}V + \frac{d+2}{2\sqrt{d-2}}\partial_{\hat\tau}V + \sqrt{\frac{10-d}{4}}\partial_{\hat\rho}V \leq 0\,,$$
$$\text{if } d\geq 4 \;\;\&\;\; p_i = d-1, \text{ or } d=3 \;\;\&\;\; F_0 = 0\,, \quad (5.25)$$

leading to the no-go theorems 3a and 3b. The respective value of c is given by

$$c^2 = \frac{d^2}{(d-2)(d-1)}\,. \tag{5.26}$$

Vanishing field strength

Going back to the inequality (5.22), one confirms its validity for $p_i = 4, 5, 6$ with $F_{6-p_i} = 0$, thus successfully deriving the d-dimensional analog of the no-go theorem 5a. The denominator of the rate c in equation (5.23) has no roots in the range $3 \leq d \leq 10$, where the minimal dimensionality dictated by Lorentz invariance is $p_i = d - 1$. This guarantees the validity of the equation over the above range and allows us to study the dependence of the slope c on the dimensionality of the sources. We illustrate this relation in Figure 3, which shows that the plot of $c(p_i, d)$ has a minimum at $p_i = 3$ and a maximum at $p_i = 7$. It

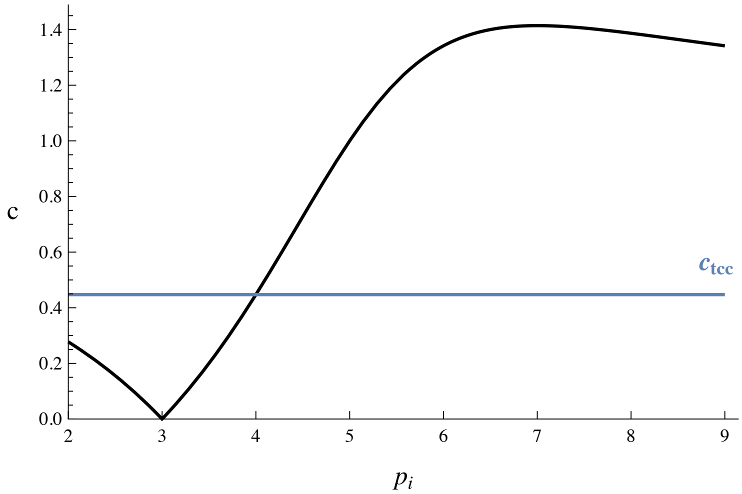


Figure 3: This figure shows the plot of $c(p_i, d = 6)$ in the equation (5.23) and the value of (5.3) for $d = 6$. The qualitative properties of this function remain consistent for dimensions $3 \leq d \leq 10$.

is important to remember that $p_i \geq d - 1$. As can be seen from equation (5.23) and the figure above, the minimal value of c in the range $4 \leq p_i \leq 9$ is reached for $p_i = 4$. This value exactly matches the rate (5.3) of the Trans-Planckian Censorship Conjecture.

In order to identify a further no-go theorem, we divide equation (5.21) by $(p_i - 3)$ and set $d = 3$. Given that $p_i \geq d - 1$, we may employ arguments similar to those previously discussed to derive the constraint 5b under the assumption $H_3 = 0$. The value of the rate in equation (5.23) is also applicable in this case, resulting in the following formula for $p_i = 2$,

$$c^2 = \frac{4}{(d-2)(3d-5)}\,, \tag{5.27}$$

in $d = 3$. It is remarkable that the value of c falls below the threshold of c_{tcc}, a fact already evident from Figure 3; Although the value for $p_i = 4$ satisfies the bound in equation 5.3, as discussed above, $c(p_i = 2, d = 6)$ remains below this threshold. This pattern is consistently observed across dimensions in the range $3 \leq d \leq 10$. We will resume this discussion later on.

Non-negative internal curvature

The no-go theorem 6 is given by the unique combination of the potential and its derivatives

$$\begin{aligned} 2\left(\frac{2(d+p_i-1)}{d-2}V - \frac{d-4-2p_i}{2(d-2)}\tau\partial_\tau V + \rho\partial_\rho V\right) &= -2\tau^{-2}\rho^{-1}\mathcal{R}_{10-d} \\ &+ (1-p_i)\tau^{2-2d}\rho^{3-d}|H_7|^2 + \frac{1}{2}\tau^{-d}g_s^2\sum_q \rho^{\frac{10-d-2q}{2}}\left(6-p_i-q\right)|F_q|^2\,, \end{aligned} \tag{5.28}$$

which is consistent with the on-shell equation (4.29) in the 10D case. Applying the transformation laws (3.21) to canonical fields, the above expression leads to the following inequality,

$$\frac{2(d+p_i-1)}{d-2}V - \frac{d-4-2p_i}{2\sqrt{d-2}}\partial_{\hat{\tau}}V + \sqrt{\frac{10-d}{4}}\partial_{\hat{\rho}}V \leq 0\,, \quad \text{if } \mathcal{R}_{10-d} \geq 0 \;\;\&\;\; p_i \geq 4\,, \tag{5.29}$$

which agrees with the corresponding no-go theorem. The c value in equation (5.18) is defined by

$$c^2 = \frac{4(d+p_i-1)^2}{(d-2)(-1+d-dp_i+4p_i+p_i^2)}\,. \tag{5.30}$$

Since the denominator has no roots in the desired range, the validity of the equation above is confirmed for $3 \leq d \leq 9$ and $p_i \geq d-1$.

No-go theorem for heterotic strings

As discussed earlier, the bosonic action of heterotic string theory at leading-order in α' is given by the NSNS sector in (2.18). Therefore, the potential (3.26) is reduced to the contribution from the internal Ricci scalar $\mathcal{R}_{10-d}$ and the H_3 flux term. The following combination of this potential and its derivatives,

$$2\left(V + \frac{4-d}{4}\tau\partial_\tau V + \frac{d-2}{2}\rho\partial_\rho V\right) = -\frac{d-2}{2}\tau^{-2}\rho^{-3}|H_3|^2\,, \tag{5.31}$$

equals the on-shell equation (4.32) in the 10D theory. Returning to canonical fields, the adjustment of the potential to heterotic strings allows us to reformulate the no-go theorem 7 in a d-dimensional framework,

$$V + \frac{(4-d)\sqrt{d-2}}{4}\partial_{\hat{\tau}}V + \frac{(d-2)\sqrt{10-d}}{4}\partial_{\hat{\rho}}V \leq 0\,. \tag{5.32}$$

In this context, the value of c is defined as

$$c^2 = \frac{4}{(d-2)(d-1)}\,. \tag{5.33}$$

At this point, we have successfully reproduced the no-go theorems excluding classical de Sitter solutions in arbitrary dimensions from their original 10-dimensional formulations in Section 4.2 within a d-dimensional framework. With these analytical tools at hand, we can now expand our studies to include two additional no-go theorems, originally presented in [38, 53] for $d = 4$.

No-go theorem for $\lambda \leq 0$

In this section we reproduce a no-go theorem, recently discussed in [38, 45, 53], within a d-dimensional effective theory. This constraint takes a more restricted approach than previous ones, especially targeting group manifolds for the internal space, which implies constant $f^a{}_{bc}$, and assuming constant fluxes. The relevant RR flux components entering the Bianchi identities are $F^{(0)}_{6-p_i}$ and $F^{(1)}_{8-p_i}$. Since the NSNS field strength is odd under the orientifold involution, its non-vanishing components are limited to $H_3^{(0)}$ and $H_3^{(2)}$. In the following combination of the potential (3.33) and its derivatives,

$$
\begin{aligned}
&2\bigg(\frac{4(B-A)}{(d-2)}V+\frac{(B-A)(d+2)}{2(d-2)}\tau\partial_\tau V-(A+B)\rho\partial_\rho V+2\sigma\partial_\sigma V\bigg)\\
&=(B-A)\Bigg(-2\tau^{-2}\rho^{-1}\left(\sigma^{-B}\delta^{cd}f^{b_\perp}{}_{a_\parallel c_\perp}f^{a_\parallel}{}_{b_\perp d_\perp}+\sigma^{A-2B}|f^{\parallel}{}_{\perp\perp}|^2\right)\\
&\quad-2\tau^{-2}\rho^{-3}\sigma^{-3B}|H_3^{(0)}|^2+(n-1-p_i)\tau^{2-2d}\rho^{3-d}\sum_n\sigma^{-An-B(7-n)}|H_7^{(n)}|^2\\
&\quad+4\tau^{-\frac{d+2}{2}}\rho^{\frac{2p_i-8-d}{4}}\sigma^{\frac{1}{2}B(p-9)}g_s\frac{T_{10}}{p_i+1}\\
&\quad+\frac{1}{2}\tau^{-d}g_s^2\sum_q\rho^{\frac{10-d-2q}{2}}\sum_n\sigma^{-An-B(q-n)}(2+2n-p_i-q)|F_q^{(n)}|^2\Bigg)\,,
\end{aligned}
\tag{5.34}
$$

we have replaced the internal curvature by the equation (3.36). Because of the similarities with the equation (5.24), we use the Bianchi identity (4.19), which has been multiplied by powers of the scalar fields and reorganized to conform to the off-shell formulation of the no-go theorem, where we replace $(\mathrm{d}F_{8-p_i})_\perp$ with

$$
\begin{aligned}
&2\tau^{-\frac{d+2}{2}}\rho^{\frac{2p_i-8-d}{4}}\sigma^{B\frac{p_i-9}{2}}\varepsilon_{p_i}g_s(\mathrm{d}F_{8-p_i})_\perp\\
&=-\sum_{a_\parallel}\left|\tau^{-1}\rho^{-\frac{1}{2}}\sigma^{\frac{A-2B}{2}}*_\perp(\mathrm{d}e^{a_\parallel})|_\perp-\tau^{-\frac{d}{2}}\rho^{\frac{2p-6-d}{4}}\sigma^{\frac{-A-B(7-p)}{2}}\varepsilon_{p_i}g_s\left(\iota_{\partial_{a_\parallel}}F_{8-p_i}^{(1)}\right)\right|^2\\
&\quad+\tau^{-d}\rho^{p_i-3-\frac{d}{2}}\sigma^{-A-B(7-p_i)}g_s^2|F_{8-p_i}^{(1)}|^2+\tau^{-2}\rho^{-1}\sigma^{A-2B}|f^{\parallel}{}_{\perp\perp}|^2\,,
\end{aligned}
\tag{5.35}
$$

in terms of the relevant RR flux components, $F_{8-p_i}^{(1)}$, defined above. Substituting the Bianchi identity into the equation (5.34) leads to

$$
\begin{aligned}
&\frac{2}{M_p^2}\bigg(\frac{4(B-A)}{(d-2)}V+\frac{(B-A)(d+2)}{2(d-2)}\tau\partial_\tau V-(A+B)\rho\partial_\rho V+2\sigma\partial_\sigma V\bigg)\\
&=(B-A)\Bigg(-2\left|\tau^{-1}\rho^{-\frac{3}{2}}\sigma^{\frac{-3}{2}B}*_\perp H_3^{(0)}+\varepsilon_{p_i}g_s\tau^{-\frac{d}{2}}\rho^{\frac{2p_i-2-d}{4}}\sigma^{B\frac{p_i-6}{2}}F_{6-p_i}^{(0)}\right|^2\\
&\quad-2\tau^{-2}\rho^{-1}\sigma^{-B}\delta^{cd}f^{b_\perp}{}_{a_\parallel c_\perp}f^{a_\parallel}{}_{b_\perp d_\perp}+(n-1-p_i)\tau^{2-2d}\rho^{3-d}\sum_n\sigma^{-An-B(7-n)}|H_7^{(n)}|^2\\
&\quad-2\sum_{a_\parallel}\left|\tau^{-1}\rho^{-\frac{1}{2}}\sigma^{\frac{A-2B}{2}}*_\perp(\mathrm{d}e^{a_\parallel})|_\perp-\tau^{-\frac{d}{2}}\rho^{\frac{2p_i-6-d}{4}}\sigma^{\frac{-A-B(7-p_i)}{2}}\varepsilon_{p_i}g_s\left(\iota_{\partial_{a_\parallel}}F_{8-p_i}^{(1)}\right)\right|^2\\
&\quad+\frac{1}{2}\tau^{-d}g_s^2\sum_{\substack{q\neq6-p_i\ \&\ n\neq0\\ q\neq8-p_i\ \&\ n\neq1}}\rho^{\frac{10-d-2q}{2}}\sigma^{-An-B(q-n)}(2+2n-p_i-q)|F_q^{(n)}|^2\Bigg)\,.
\end{aligned}
\tag{5.36}
$$

We will study each term of this analytical expression separately in terms of flux components. The H_7 field strength, which is only present in $d=3$, is proportional to vol_{10-d} and extends over each internal dimension wrapped by the sources. Therefore, the only non-vanishing component is found for $n=p_i+1-d$, where n is the number of internal dimensions parallel to the sources. The leading factor of the H_7 term is negative. Applying the same argument to the RR fluxes, the parameter n satisfies $2 + 2n - p_i - q \leq 0$, consistent with the more obvious conditions $n \leq q$ and $n \leq p_i + 1 - d$. Therefore, the only term whose sign is not fixed leads us to conclude that the existence of classical de Sitter solutions is excluded for

$$-\delta^{cd} f^{b_\perp}{}_{a_\parallel c_\perp} f^{a_\parallel}{}_{b_\perp d_\perp} \equiv \lambda \, |f^\parallel{}_{\perp\perp}|^2 \leq 0\,. \tag{5.37}$$

By extending the condition $|f^\parallel{}_{\perp\perp}|^2 \neq 0$, as derived in [74] for $d = 4$, to an arbitrary dimension d, the above inequality becomes equivalent to the condition $\lambda \leq 0$. Substituting this into the combination of the potential and the derivatives in the equation (5.36), we obtain the following no-go theorem,

$$\begin{aligned} &\frac{4(B-A)}{(d-2)} V + \frac{(B-A)(d+2)}{2\sqrt{d-2}} \partial_{\hat{\tau}} V - (A+B)\sqrt{\frac{10-d}{4}}\partial_{\hat{\rho}} V \\ &\qquad + \sqrt{-AB(B-A)}\partial_{\hat{\sigma}} V \leq 0\,, \quad \text{if } 3 \leq d \leq 9 \;\; \& \;\; \lambda \leq 0\,. \end{aligned} \tag{5.38}$$

The value of the rate in equation (5.18) results in

$$c^2 = \frac{4}{(d-2)(d-1)}\,. \tag{5.39}$$

Internal parallel Einstein equations

We introduce another set of scalar fields, (τ, r), which leads to the potential (3.50). With this setup we establish an additional no-go theorem. This constraint has been validated for $d = 4$ using the 10D equations of motion, as shown in [45], and in a 4-dimensional effective theory in [38]. We analyze the following linear combination of the potential and its derivatives,

$$\begin{aligned} 2\,(2V + \tau\partial_\tau V + (d-2) r \partial_r V) = -(d-2)\Bigg(&-\tau^{-2} r^2 \frac{1}{2}\delta^{ik}\delta^{jl} f^1{}_{ij} f^1{}_{kl} \\ &+ \tau^{-2} r^{-2} |H_3^{(1)}|^2 + \tau^{2-2d} |H_7^{(1)}|^2 + \tau^{-d} r^{-1} g_s^2 \sum_q |F_q^{(1)}|^2 \\ &- g_s \tau^{-\frac{d+2}{2}} r^{-\frac{1}{2}} \sum_i \frac{T_{10}^{(p_i)}}{p_i+1}\delta_1^{\perp_i} + \tau^{-2} r^{-2}\left(\frac{1}{2}\delta^{ik}\delta^{jl} f^1{}_{ij} f^1{}_{kl} - 2\mathcal{R}_{11}\right)\Bigg)\,, \end{aligned} \tag{5.40}$$

paying careful attention to the sign of each term in order to formulate a conclusion. We focus our analysis on sources of dimensionality $p_i \leq d$ and assume that all O-planes extend along the radion direction 1. This configuration ensures that $\delta_1^{\perp_i} = 0$, which leads to the elimination of the source term in (5.40). Note that sets containing only Dp_i are not relevant, since they satisfy $T_{10}^{(p_i)} < 0$. In addition, our assumption of a group manifold results in

$$\frac{1}{2}\delta^{ik}\delta^{jl} f^1{}_{ij} f^1{}_{kl} - 2\mathcal{R}_{11} \geq 0\,, \tag{5.41}$$

as detailed in [38]. In this case, only the first term may have the wrong sign. Using the transformation laws to canonical fields (3.45), we obtain the no-go theorem

$$2V + \sqrt{d-2}\,\partial_{\hat{\tau}} V + (d-2)\,\partial_{\hat{r}} V \leq 0\,, \quad \text{if } \forall\, i,j : \delta_1^{\perp i} = 0 \text{ and } {f^1}_{ij} = 0\,. \tag{5.42}$$

The corresponding value of c is

$$c^2 = \frac{4}{(d-2)(d-1)}\,. \tag{5.43}$$

5.4 Summary and discussion

In the previous section, we have successfully formulated no-go theorems that challenge the existence of classical string backgrounds with a d-dimensional (quasi-)de Sitter spacetime. These constraints arise from the dimensional reduction of 10D type II supergravity theories, of which the first five were supported by the 10D equations in Section 4.2. However, the assumptions we make do not depend on the scalar fields but rather on the 10D background. This approach extends the study recently presented in [38] for the 4-dimensional case and applies it more broadly. In Section 5.3, we define a unique c value for each no-go theorem, which corresponds to the parameter of the de Sitter Conjecture discussed in [30] and further explored in Section 5.2. In particular, we focus on comparing these values with the bound proposed by the Trans-Planckian Censorship Conjecture in [58].

In the following Section 5.4.1 we will discuss this comparison of rates in more detail, with special emphasis on the unique case of $d = 3$. In Section 5.4.2 we will extend our analysis to alternative values suggested in the literature and explore related conjectures. Our discussion concludes in Section 5.4.3, where we consider the implications for cosmological theories of accelerated expansion.

5.4.1 No-go theorems and the Trans-Planckian Censorship Conjecture

A summary of the c values derived for the no-go theorems discussed in this section is given in table 6. An important observation is that for dimensions from 4 to 10, all values of the rate (5.18) meet or exceed the lower TCC bound given by the equation (5.3). In particular, the cases where $c = c_{\text{tcc}}$ correspond exactly to those identified in $d = 4$ [38]. Note that there is no inherent reason for these results to coincide, since the analysis was performed purely within the framework of supergravity, which is supposedly unrelated to the quantum gravity and cosmological arguments that underlie the Trans-Planckian Censorship Conjecture. However, if the TCC holds, such a consistency underscores the swampland perspective, given that supergravity is the asymptotic, both perturbative and classical, limit of string theory. From this perspective, the c values derived in this work strongly verify the TCC hypothesis. Remarkably, the precise agreement of c_{tcc} in four dimensions, where $c = \sqrt{2/3}$, may be generalized to any formula across d dimensions, as investigated in Section 5.4.2. This alignment demonstrates a universal consistency with the Trans-Planckian Censorship Conjecture.

In addition, the Figure 4 shows how the values of c vary with the dimension d, relative

No-go theorem	Value of c, (5.18)
Maldacena-Nuñez	$\frac{2}{\sqrt{d-2}}$
$p_i = 7, 8$ or 9	$\frac{2(p_i-3)}{\sqrt{(d-2)((p_i-3)^2+(p_i-5)(2-d))}} \geq c_{\text{tcc}}$
$p_i = 4, 5$ or 6 with $F_{6-p_i} = 0$	
$p_i = 2$ with $H_3 = 0$ (in $d = 3$)	$\frac{2}{\sqrt{(d-2)(3d-5)}}(= 1) < c_{\text{tcc}}$
$p_i = d - 1$	$\frac{d}{\sqrt{(d-2)(d-1)}}$
$\mathcal{R}_{10-d} \geq 0$	$\frac{2(d+p_i-1)}{\sqrt{(d-2)(-1+d-dp_i+4p_i+p_i^2)}} > 1$
Heterotic strings at $(\alpha')^0$	$\frac{2}{\sqrt{(d-2)(d-1)}}$
$\lambda \leq 0$	
Internal parallel Einstein equations	

Table 6: This table lists the no-go theorems analyzed in this study, along with the d-dependent c values derived from (5.18).

to c_{tcc}. The case of $d = 3$ presents unique challenges. Our results indicate anomalies in this dimension, as outlined in the equation (5.27), where the no-go theorems suggest a violation of the TCC bound. However, the integrity of its algebraic derivation remains robust even in $d = 3$. On the other hand, the fundamental physical arguments based on preventing quantum fluctuations from becoming classical might not be applicable. Given the peculiar topological nature of gravity in $d = 3$, where gravitational fluctuations are inherently absent, it is plausible that the underlying reasoning of the TCC may not extend to this dimension. This extraordinary observation highlights a potential limitation in the application of the TCC to $d = 3$, as supergravity sensitively detects these subtleties by providing a counterexample to this bound.

The broader implications of swampland conjectures in $d = 3$ remain ambiguous. While generally applicable in dimensions 4 and above, and less so in dimensions 2 and below, $d = 3$ poses unique theoretical challenges, evident in the distinct properties of black holes and the absence of gravitational fluctuations, which complicate the application of standard swampland arguments. However, papers such as [128] have affirmed swampland conjectures in $d = 3$, while others like [129, 130] have applied these without contradiction. However, the peculiar no-go theorem and the associated value of c for $d = 3$, attributed to O2-planes/D2-branes exclusive to this dimension, suggest new dynamics that may conflict with established bounds such as the TCC. Further explorations in $d = 3$, such as those involving scale-separated AdS_3 solutions [131, 132], may reveal more instances that challenge swampland conjectures, especially those related to the TCC and the de Sitter Conjecture.

5.4.2 Exploring connections in the swampland

Testing the TCC bound (5.3), which establishes a minimum value of $c \geq c_{\text{tcc}}$, via SUGRA no-go theorems in various dimensions, offers substantial support for this proposal. However,

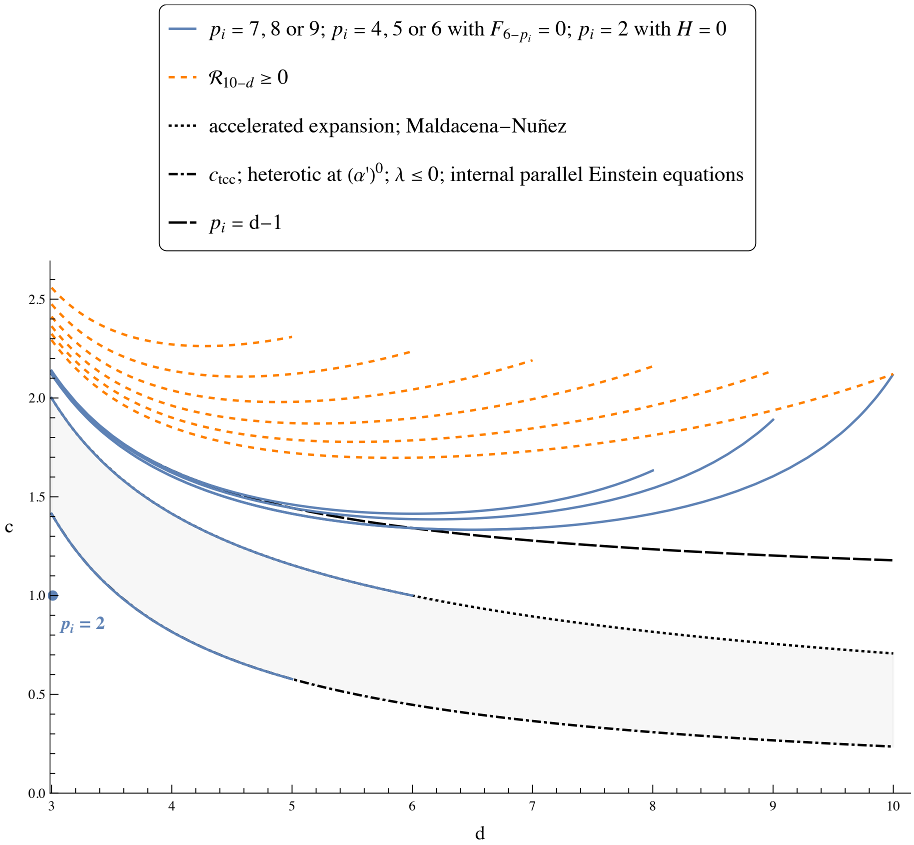


Figure 4: This figure plots the values of $c(d)$ for each no-go theorem listed in Table 6 as a function of the dimension d. The presentation is specified in the caption above, with colored lines representing p_i-dependent values corresponding to the dimensionality of the Op_i/Dp_i. Theoretical bounds are also included; the lower $c_{\rm tcc}$, the upper bound for accelerated expansion as mentioned in Section 5.4.3, and a shaded region where both bounds are satisfied. Note that the figure shows a violation of the TCC bound by a no-go theorem at $d = 3$.

the universal validity of this bound is not obvious. While saturation has been observed in $d = 4$ [38], one might have expected different expressions for arbitrary dimensions. For instance, investigations into the Distance Conjecture within the complex structure moduli space of Calabi-Yau manifolds have revealed potential alternatives, as suggested in [133, 134]. More specifically, these studies suggest that

$$c \geq \frac{4s}{\sqrt{10-d}}\,, \qquad s = \begin{cases} 1 & \text{for } \frac{10-d}{2} \text{ even} \\ \frac{1}{2} & \text{for } \frac{10-d}{2} \text{ odd} \end{cases}\,, \tag{5.44}$$

where $(10-d)/2$ denotes the complex dimension of the Calabi-Yau space. This formulation illustrates one among several proposals for different bounds on c, aligning with $c = \sqrt{2/3}$ in $d = 4$. It is noteworthy, however, that a coherent expression for c in arbitrary dimensions emerges consistently from the no-go theorems discussed in this thesis.

Recent studies, such as those in [124, 127] and further discussions in [60, 135], suggest a minimal c value (5.15) that surpasses the TCC bound. This value, proposed to remain invariant under dimensional reduction, is anticipated to hold in the asymptotics of field space and is applied considering the full gradient, $|\nabla V|$ – see also the discussion in Section 6.4. This approach necessitates a more stringent, thus higher, value of c. However, an in-depth analysis reveals that the derivation of the no-go theorems in Section 5.3 is effectively single field. More specifically, the relation

$$aV + \sum_i \hat{b}_i \partial_{\hat{\varphi}^i} V \leq 0 \quad \Leftrightarrow \quad cV + \partial_{\hat{t}} V \leq 0 \tag{5.45}$$

defines a single canonically normalized field $\hat{t}$ according to

$$\sum_i \hat{b}_i \partial_{\hat{\varphi}^i} = \sqrt{\sum_i \hat{b}_i^2}\, \partial_{\hat{t}}\,, \tag{5.46}$$

which successfully reproduces the Strong de Sitter Conjecture since $\partial_{\hat{t}} V \geq -|\nabla V|$. An alternative interpretation is that our approach focuses on the slope of the potential along a single field direction, while stabilizing other fields φ^i at their critical point values, i.e. $\varphi^i = 1$ or $\partial_{\varphi^i} V = 0$. In this case, $|\partial_{\hat{t}} V| = |\nabla V|$. This formulation has been rigorously validated for negative potentials in Section 6.5.

Moreover, a compelling relation between the Distance Conjecture [125] and the de Sitter Conjecture states that the mass scale of the tower of states is directly correlated with the scalar potential in the asymptotics of field space [31, 38, 58],

$$m \sim V^{\alpha}\,. \tag{5.47}$$

In string compactifications, scalar potentials are typically represented as sums of exponentials, with emphasis on the dominant term, $V \sim e^{-c\varphi}$, as one approaches the asymptotic limit[18]. Therefore, the relation (5.47) matches the decay rate of the Distance Conjecture, $m \sim e^{-\lambda\varphi}$ with $\lambda > 0$, with the c value of the de Sitter Conjecture such that $\lambda = \alpha c$, especially for their respective minimal values,

$$\lambda_{\text{min}} = \alpha\, c_{\text{min}}\,. \tag{5.48}$$

This reasoning fosters the assumption that $c_{\text{min}} = c_{\text{tcc}}$ with $\alpha = 1/2$, as validated in 4 dimensions [38]. Therefore, the robustness of the TCC bound in $d \geq 4$ motivates

$$\lambda_{\text{min}} = \frac{2\alpha}{\sqrt{(d-2)(d-1)}}\,, \tag{5.49}$$

with $\alpha = 1/2$ in $d = 4$, consistent with the potential range of α from $1/d$ to $1/2$ for (quasi-) de Sitter proposed in [124, 137, 138]. This leads to the prediction of a tower decaying at a rate $1/\sqrt{(d-2)(d-1)}$ in the asymptotics of field space.

In [135] an alternative interpretation is proposed by introducing a lower bound, $\lambda_{\text{min}} = 1/\sqrt{d-2}$, which represents the minimum rate of the lightest tower. This implies that although a tower can decay at a rate of $1/\sqrt{(d-2)(d-1)}$ in the asymptotics of field

[18] The asymptotics typically correspond to $|V| \to 0$ [136].

space, [135] argues that there will always be a lighter tower that decays at a faster rate, exceeding $1/\sqrt{d-2}$. This argument facilitates the validation of (5.48) with $\alpha = 1/2$, in the light of the bound (5.15) discussed earlier. We intend to revisit these complex relation between conjectures, associated decay rates and bounds in future work.

5.4.3 Criteria for asymptotic accelerated expansion

In cosmological terms, the constraints on $|\nabla V|/V$ strongly reduce, and may even preclude, the possibility of achieving a quasi-de Sitter spacetime or any form of accelerated expansion. Given a cosmological model in d dimensions, equipped with a FLRW metric and a single canonical scalar field[19] φ with a positive potential $V(\varphi)$, we obtain the upper bound

$$\frac{|V'|}{V} < \frac{2}{\sqrt{d-2}}\,, \tag{5.50}$$

an asymptotic condition for cosmic accelerated expansion [30, 60, 124]. From a physical perspective, the nature of this bound can be understood as follows: The constraint (5.9) requires that the kinetic energy is negligible. This condition would be violated if the slope of the potential were too steep, therefore justifying the upper bound on the gradient. To verify this claim, we re-evaluate the given bound and offer two separate derivations of (5.50).

A positive potential alone does not suffice for achieving cosmic accelerated expansion. The universe, if homogeneous, undergoes accelerated expansion provided that q, the deceleration parameter, is negative. Defined by

$$q \equiv -\frac{\ddot{a}a}{\dot{a}^2} = -1 - \frac{\dot{H}}{H^2} < 0 \quad \Leftrightarrow \quad \epsilon < 1\,, \tag{5.51}$$

with $\epsilon \equiv -\dot{H}/H^2$, this parameter is a dimensionless measure reflecting the acceleration rate of the expanding FLRW spacetime. The slow-roll approximation (5.10) and (5.11), which simplifies the equations of motion and guarantees that inflation will occur, then becomes

$$\epsilon \ll 1\,. \tag{5.52}$$

In addition, the following equation,

$$\epsilon = \frac{d-1}{2}\frac{\dot{\varphi}^2}{V} = \frac{d-2}{4}\frac{|V'|^2}{V^2}\,, \tag{5.53}$$

emerges from the simplified equations (5.12). Within the slow-roll approximation we expand w as

$$w = -1 + \frac{2\epsilon}{d-1} + \mathcal{O}(\epsilon^2) = -1 + \frac{d-2}{2(d-1)}\frac{|V'|^2}{V^2} + \mathcal{O}(\epsilon^2)\,. \tag{5.54}$$

At leading-order, the condition for accelerated expansion (5.9) eventually returns the bound (5.50). This result aligns with the established bounds for exponential potentials in $d = 4$, such as $|V'|/V \leq \sqrt{2}$, discussed extensively in [139–141].

[19] This implies a number of simplifications in Section 5.1: $G_{ij} = \delta_{ij}$, $\Gamma^i{}_{jk} = 0$, $\nabla V = V'$.

Following the work of [142, 143], which was originally restricted to 4 dimensions, we extend the derivation to arbitrary dimension $d \geq 3$ – see also the more recent work in [60]. By defining the variables

$$x = \frac{\dot{\varphi}}{\sqrt{6}H}\,, \qquad y = \frac{\sqrt{V(\varphi)}}{\sqrt{3}H}\,, \tag{5.55}$$

for $V > 0$, we analyze a system of equations that includes an energy density ρ_m and pressure p_m, in addition to the scalar field and potential. The derivatives of x and y with respect to $N = \ln a$ are given by

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}N} &= -\sqrt{\frac{3}{2}}\frac{V'}{V}y^2 - (d-1)x + \frac{x}{d-2}\left(6x^2 + \frac{\rho_m + p_m}{H^2}\right)\,, \\ \frac{\mathrm{d}y}{\mathrm{d}N} &= \sqrt{\frac{3}{2}}\frac{V'}{V}xy + \frac{y}{d-2}\left(6x^2 + \frac{\rho_m + p_m}{H^2}\right)\,, \end{aligned} \tag{5.56}$$

using the equation of motion of the scalar field and the second Friedmann equation. In addition, the contribution $\rho_m + p_m$ can be rewritten via the first Friedmann equation along with the parameter $w_m = p_m/\rho_m$. In this case, we choose to set $\rho_m = p_m = 0$ to match our initial assumptions. Identifying fixed points where $\mathrm{d}x/\mathrm{d}N = 0$ and $\mathrm{d}y/\mathrm{d}N = 0$ reveals the asymptotic behavior of the N-flow (time flow). In this context, the ratio V'/V should be regarded as an asymptotic expression that remains constant along the flow when considering an exponential potential. The resulting fixed points and the related equation of state parameters are given by

- for $V' \neq 0 : (x, y) = (0, 0)$, w undefined
- $(x, y) = \left(\pm\sqrt{\frac{(d-1)(d-2)}{6}}, 0\right)$, $w = 1$
- $(x, y) = \left(-\frac{(d-2)}{2\sqrt{6}}\frac{V'}{V}, \pm\frac{\sqrt{(d-2)}}{2\sqrt{6}}\sqrt{4(d-1) - (d-2)\left(\frac{V'}{V}\right)^2}\right)$, $w = -1 + \frac{d-2}{2(d-1)}\left(\frac{V'}{V}\right)^2$.

However, the last expression gives the only viable fixed point for $V > 0$ capable of achieving cosmic acceleration. This is confirmed by the condition (5.9), which in turn leads to the bound (5.50) on accelerated expansion.

These analyses reveal that each of the two derivations relies on assumptions, either the slow-roll approximation or an asymptotic limit to a fixed point. Whether these assumptions are equivalent remains to be seen. However, the last fixed point is consistent with (5.12). These assumptions open up potential ways to violate the bound (5.50) yet still obtain accelerated expansion; leaving the slow-roll regime or entering a transient phase of accelerated expansion. In addition, the constraint on accelerated expansion defined by provides a narrow operational range, while conforming to the TCC bound (5.3):

$$\frac{2}{\sqrt{(d-2)(d-1)}} \leq \frac{|V'|}{V} < \frac{2}{\sqrt{d-2}}\,, \tag{5.57}$$

illustrated in Figure 4. Moving beyond this upper bound using the strategies mentioned above could broaden the scope for viable models such as those based on quintessence.

6 Anti-Trans-Planckian Censorship and scalar potentials

The swampland program [56, 57] seeks to classify effective theories of quantum gravity. A key aspect of this research has been the formulation of criteria to characterize scalar potentials within these theories. In particular, some advances have proposed to limit the characterization of scalar potentials to the asymptotic limits of moduli space. A prominent example of this approach is the Trans-Planckian Censorship Conjecture [58] in Section 5.2, which provides conditions for $V > 0$ and $V' \leq 0$ for a single canonical field:

$$0 < V(\varphi) \leq e^{-c_{\rm tcc}|\varphi-\varphi_i|}\,, \quad \left\langle -\frac{V'}{V}\right\rangle_{\varphi\to\infty} \geq c_{\rm tcc}\,, \quad c_{\rm tcc} = \frac{2}{\sqrt{(d-1)(d-2)}}\,, \tag{6.1}$$

where $M_p = 1$. In contrast, this section shifts the focus to negative scalar potentials, which are capable of producing a contracting phase in the universe. Despite the formal similarities between positive and negative potentials, typically distinguished only by the signs and values of the coefficients, the functional relations with respect to the field variables remain generally consistent. Based on the literature [28], it has been postulated that negative potentials should conform to

$$\frac{|\nabla V|}{|V|} \geq c \quad \text{or} \quad \frac{\min\nabla\partial V}{|V|} \leq c'\,, \quad \text{with } c, c' \sim \mathcal{O}(1)\,, \tag{6.2}$$

in Planckian units, where $\min\nabla\partial V$ is the minimal eigenvalue of the mass matrix $M^i{}_j = G^{ik}\nabla_k\partial_j V$. In particular, at an AdS extremum of the potential, the first condition may be breached, necessitating compliance with the second. An alternative constraint noted in [144] prefers the maximal eigenvalue $\max\nabla\partial V$ to maintain consistency with the Breitenlohner-Freedmann bound (6.42). This is similar to the refined de Sitter conjecture [31],

$$\frac{|\nabla V|}{V} \geq c \quad \text{or} \quad \frac{\min\nabla\partial V}{V} \leq -c'\,, \quad \text{with } c, c' \sim \mathcal{O}(1)\,, \tag{6.3}$$

where the positive scalar potential of a theory coupled to gravity has to obey one of the two conditions. The Refined de Sitter Conjecture (6.3) is weakened by the implications of the TCC (6.1). In the second conjecture, the constraint on the first derivative of the potential is preserved, but applies only to the asymptotics of field space. The second condition in (6.3) is replaced by an upper bound on the lifetime, as detailed in [58].

On this basis, we introduce the Anti-Trans-Planckian Censorship Conjecture (ATCC), which extends the characterization to negative potentials. The same differences and relaxations apply to (6.2); the condition on $|\nabla V|/V$ becomes an asymptotic one and the second constraint is relaxed, in particular for AdS critical points. Before discussing the details of the ATCC, it is reasonable to revisit and refine the TCC. Both conjectures analyze solutions with a d-dimensional FLRW metric, considering cases of both expansion and contraction, with the refined TCC specifically addressing expanding universes [46]:

Refined Trans-Planckian Censorship Conjecture. In any effective theory of quantum gravity (5.5) with $V > 0$ admitting an expanding cosmology, modes shorter than the Planck length at t_i should not exceed the typical length scale of the universe at some later time t

without losing the validity of the effective theory.

In a mathematical formulation:

$$\frac{a(t_i)}{a(t)} \geq \frac{\sqrt{V(\varphi(t))}}{M_p^2}\,, \tag{6.4}$$

derived for the highly restrictive initial condition involving a Planckian wavelength and $M_p/\sqrt{V}$ representing the typical length scale of the universe, as a replacement for $1/H$ in the original TCC inequality [58].

The choice of $\sqrt{V}/M_p$ rather than H is justified by the limitations of H in providing a consistent measure of typical lengths outside (quasi-)de Sitter spacetimes. Furthermore, in settings close to (quasi-)de Sitter conditions, as explored in [58], this replacement does not lead to substantial differences and the characterization of the potential as described in equation (6.1) remains intact with this substitution.

This nuanced approach to the application of the TCC suggests that violations of (6.4) do not necessarily conflict with quantum gravity, but simply break the validity of the effective theory. This reformulation pragmatically claims that the usual energy scale (or energy cutoff) of the effective theory, given by V, should not exceed the Planck scale within its domain of validity. Therefore, we find that $1 \geq \sqrt{V}/M_p^2$, which is consistent with the equation (6.4) for an expanding universe. The ATCC thus introduces, through our refinement of the TCC, a solid physical argument underlying these proposals, based on the regime of validity of an effective theory of quantum gravity. The refinement of the TCC provides a theoretical foundation that could explain why the verification of the TCC bound, as discussed in Section 5, has been consistently successful.

Motivated by this conceptual and technical refinement, we now shift our focus to negative potentials and proceed to formulate the statement of the Anti-Trans-Planckian Censorship Conjecture.

6.1 The Anti-Trans-Planckian Censorship Conjecture (ATCC)

In this section, we first review the general cosmological framework outlined in Section 5.1. We then introduce the Anti-Trans-Planckian Censorship Conjecture in Section 6.1.2 and explore its multiple implications. In particular, we establish an explicit bound on the lifetime of a contracting universe and detail the asymptotic properties of negative scalar potentials in Section 6.1.3, based on an additional postulate. The discussion then concludes with an examination of the well-established AdS solution and two dynamical solutions in Section 6.1.4.

6.1.1 General framework

In this section we focus on effective theories of quantum gravity (5.5) with $3 \leq d \leq 10$. These models include scalar fields φ^i that are minimally coupled to gravity, leading to solutions with a d-dimensional maximally symmetric spacetime defined by a cosmological constant Λ_d. They are the critical points of the potential where $\nabla V = 0$ and $E_{\text{kin},\varphi^i} = 0$, satisfying the relation

$$\Lambda_d = \frac{d-2}{2d}\mathcal{R}_d = M_p^{-2} V|_0 \,. \tag{6.5}$$

In addition, we study more dynamical situations with scalars characterized by either rolling down or climbing up the potential. To treat these cases properly, we restrict our attention to metrics of the FLRW form (5.4). Finally, we consider a single, homogeneous scalar field φ. The dynamics is governed by the two Friedmann equations (5.6) and the equation of motion for the scalar (5.7), with the energy density and pressure given by the equation (5.8).

Most of the discussion in this section involves negative scalar potentials, $V < 0$, which have several notable implications. For one, the second Friedmann equation can be written as follows

$$(d-1)\frac{\ddot{a}}{a} = \frac{2}{d-2}V - \dot{\varphi}^2 \,, \tag{6.6}$$

which leads to the conclusion that $\dot{a} < 0$, indicating a decelerating universe. We will also study solutions with $\dot{\varphi} = 0$, allowing for negative energy densities. Under these constraints, the first Friedmann equation implies the choice of $k = -1$. While this value is in conflict with the observable universe, which favors $k = 0$, our intention is not to model a realistic cosmology. This choice also differs from the Trans-Planckian Censorship Conjecture, which assumes $k = 0$. To resolve this discrepancy, we will make a few minor adjustments to our arguments compared to [58]. In Section 6.1.4, we will study explicit solutions for this model, including the AdS spacetime.

6.1.2 Statement of ATCC

The Trans-Planckian Censorship Conjecture was introduced with a focus on an expanding universe and its implications for modes evolving from a sub-Planckian to a classical regime. Our current discussion shifts to a contracting spacetime, $\dot{a} < 0$, where we similarly study the evolution of modes to characterize negative scalar potentials. As recalled from Section 6.1.1, negative potentials inherently imply deceleration, $\ddot{a} < 0$, while several examples in Section 6.1.4 show a contracting phase under the condition $V < 0$, including the AdS solution and other dynamical models. This connection provides the basis for our discussion of contracting universes and leads us to formulate the ATCC [46]:

Anti-Trans-Planckian Censorship Conjecture. In any effective theory of quantum gravity (5.5) with $V < 0$ admitting a contracting cosmology, modes with a wavelength close to the typical length scale of the universe at t_i should not shrink to sub-Planckian levels at some later time $t > t_i$ without losing the validity of the effective theory.

This proposal refers to modes that are initially considered classical. Unlike de Sitter, where horizons facilitate the freezing and classicalization of modes, contracting universes with $V < 0$ lack an equivalent mechanism due to the absence of a horizon, as will be discussed in Section 6.1.4. Here, the concept of a classical regime is replaced by the regime of validity defined by the energy cutoff of the effective theory. This refined perspective on the ATCC also leads to the subtle revision of the TCC discussed around equation (6.4).

To emphasize, in contracting models with $V < 0$, traditional metrics such as the Hubble parameter H or horizon-based scales fail to provide a sensible typical length. Instead, the characteristic length scale is related to the cosmological constant (6.5), or more aptly, the inverse square root of the absolute value of the potential, $M_p/\sqrt{|V|}$. This adjustment also reflects the role of the potential as a typical energy scale in the effective theory. According to the ATCC, the initial scale $a(t_i)\lambda_0 \sim M_p/\sqrt{|V_i|}$, with $V_i = V(\varphi(t_i))$, should not shrink below the Planck length, $a(t)\lambda_0 \lesssim l_p = 1/M_p$, at any given time. In a mathematical formulation[20]:

$$\frac{a(t)}{a(t_i)} \geq \frac{\sqrt{|V_i|}}{M_p^2} \quad \Leftrightarrow \quad \int_{t_i}^{t} \mathrm{d}t'\, H(t') = \ln\frac{a(t)}{a(t_i)} \geq \ln\frac{\sqrt{|V_i|}}{M_p^2}\,, \qquad \forall\, t > t_i\,, \tag{6.7}$$

which is expected to apply in the regime of validity of the effective theory and becomes trivial for $M_p \to \infty$. This implies that the scale factor $a(t)$ cannot dwindle to zero, thus avoiding a potential Big Crunch and ensuring that the contraction stops at a Planckian threshold where the effective theory loses its meaning (see Section 6.1.4). We then turn to the broad implications of the Anti-Trans-Planckian Censorship Conjecture.

6.1.3 Immediate consequences

The ATCC provides a solid framework for the analysis of scalar potentials in the context of contracting cosmology. We first establish a bound on the lifetime that is consistent with the progression towards a final crunch in a finite period. In contrast to the simple characterization of positive potentials guided by the TCC, the ATCC requires a "second assumption". While this condition is inherently satisfied for $V > 0$, it introduces additional complexity for $V < 0$. We rigorously test this condition through detailed case studies of contracting universes, including the AdS solution and two dynamical models involving rolling scalar fields, in Section 6.1.4.

Upper bound on lifetime

Physically, it is reasonable to expect that a contracting and decelerating universe would have a finite lifetime. This expectation is supported by the behavior of the concave and positive scale factor $a(t)$, which starts from a finite initial value a_i and decreases to zero, or $a(t) < a_i$, within a finite period. By employing the ATCC, we can circumvent the complexities of deriving an explicit formula for this upper bound, using methods similar to those in [58].

In our model, in which the universe with $k = -1$ contains only one scalar field with corresponding potential $V(\varphi)$, the relation $\rho + p \geq 0$ coupled to the second Friedmann equation (5.6) implies that $\dot{H} < 0$, indicating a deceleration of the universe. Therefore,

[20] An earlier attempt to adapt the TCC to contracting universes by reversing time [58] suggested a different scaling relation (6.7), but did not alter the dynamics governed by $\ddot{a}$ or V. Given the importance of H in this attempt, the time-reversed TCC condition is inconsistent with our observations, leading us to rely on the latter.

$H_i > H_f$, where $H_i = H(t_i)$ and $H_f = H(t_f)$. This leads to the inequality

$$\int_{t_i}^{t_f} \mathrm{d}t' \, H(t') < H_i(t_f - t_i) \,, \tag{6.8}$$

which, combined with the ATCC bound (6.7), imposes the constraint

$$t_f - t_i < \frac{1}{|H_i|} \ln\left(\frac{M_p^2}{\sqrt{|V_i|}}\right) \tag{6.9}$$

on the lifetime of our contracting, decelerating universe. As understood by the ATCC, this bound dictates that beyond a certain period – defined by the inequality above – the universe enters a Planckian regime. It is at this point that our conventional low-energy theoretical framework ceases to be reliable. The refinement of the TCC above equation (6.4) allows for the same logic to be applied to the interpretation of the lifetime bound presented in [58]. However, this estimate is subject to ambiguity due to its reliance on the Hubble parameter $H(t)$. As detailed in Section 6.1.4, the value $|H_i|$ is typically very small, while the argument of the logarithmic function is large, suggesting a considerably large upper bound on the lifetime. Note that this estimation varies if t_i is chosen closer to the final crunch, resulting in a larger $|H_i|$ and consequently a tighter constraint on the lifetime.

Consequences of ATCC for scalar potentials

Starting from the Anti-Trans-Planckian Censorship Conjecture, we aim to derive universal properties of negative scalar potentials in any effective theory of quantum gravity. We assume, for the sake of argument, that

$$-\frac{k}{a^2}\frac{(d-1)(d-2)}{2} + \frac{V}{M_p^2} \geq 0 \,, \tag{6.10}$$

which becomes trivial as $M_p \to \infty$ for $k = 0, -1$. This condition, less intuitive than the inequality (6.7), is by definition satisfied for positive potentials with $k = 0$. However, we face some significant challenges and differences from the TCC, starting with (6.10), for negative potentials with $k = -1$, which is a necessary condition for AdS solutions and other contracting, decelerating universes. Although we have successfully verified this assumption for several cosmological solutions in Section 6.5, it is not intrinsically satisfied and will require further discussion to understand its motivation. For convenience, we will set the Planck mass to one in the rest of this section.

Under the previous assumption, we use the statement (6.7) of the Anti-Tans-Planckian Censorship Conjecture to characterize negative scalar potentials. Following the methods in [58], we assume a definite sign for the gradient V' and study, without loss of generality, a scalar field climbing up the potential, $V(\varphi) \geq V_i$. We choose the initial condition $\dot{\varphi} \geq 0$ and rewrite the inequality (6.10) in terms of the Friedmann equation for a contracting universe, $H \leq 0$:

$$H^2 \geq \frac{\dot{\varphi}^2}{(d-1)(d-2)} \quad \Leftrightarrow \quad H\mathrm{d}t \leq -\frac{\mathrm{d}\varphi}{\sqrt{(d-1)(d-2)}} \,. \tag{6.11}$$

We integrate this expression over a finite period and obtain

$$\int_{t_i}^{t} \mathrm{d}t'\, H \leq -\frac{\varphi - \varphi_i}{\sqrt{(d-1)(d-2)}} \,. \tag{6.12}$$

By applying the ATCC inequality (6.7) to this relation, we get

$$\ln\left(\sqrt{|V_i|}\right) \leq -\frac{|\varphi - \varphi_i|}{\sqrt{(d-1)(d-2)}} \,, \tag{6.13}$$

where we use the absolute value, $|\varphi - \varphi_i|$, to preserve the generality of the field direction. For $V' \geq 0$ and $V < 0$, this inequality imposes a lower bound on the potential,

$$V(\varphi) \gtrsim -\mathrm{e}^{-\frac{2|\varphi-\varphi_i|}{\sqrt{(d-1)(d-2)}}} \tag{6.14}$$

up to some constant of order one, which is illustrated in Figure 5 along with several exponential potentials in Planckian units. The potential has a lower bound that is a monotonically increasing exponential function. Before we move on, it is important to study

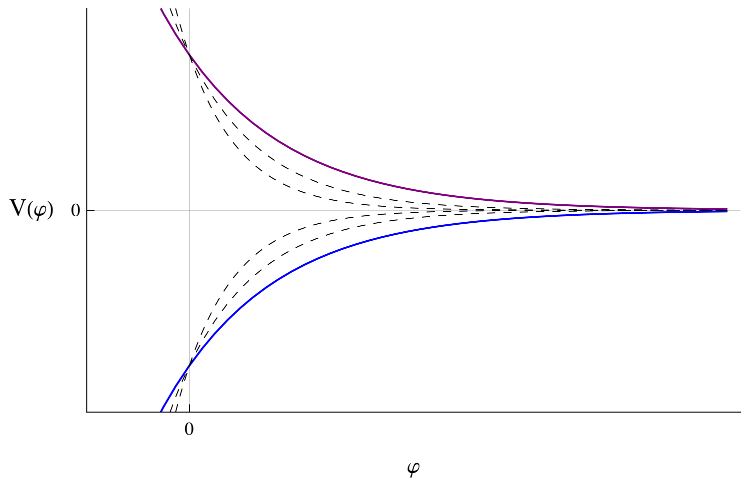


Figure 5: This figure illustrates the upper (TCC, purple) and lower (ATCC, blue) bounds on scalar potentials as defined by the equations (6.1) and (6.14). It also shows various exponential potentials in Planckian units, demonstrating compliance with these bounds for $\varphi \geq 0$.

some details of this exponential bound (6.14). For $\varphi = \varphi_i$ this simplifies to $V_i \gtrsim -1$, which is invoked implicitly around the equation (6.7) to differentiate $\sqrt{|V_i|}$ from the Planck scale. Since $V' \geq 0$, this bound remains valid over time for any field value $\varphi(t)$, provided that the effective theory, or equivalently the inequalities (6.7) and (6.10), retains its validity.

We then analyze the average value of the first derivative of the potential,

$$\left\langle -\frac{V'}{V} \right\rangle = \frac{1}{\Delta\varphi} \int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi}\, \frac{V'}{-V} = \frac{\ln(|V_i|)}{\Delta\varphi} - \frac{\ln(|V|)}{\Delta\varphi} \geq \frac{\ln(|V_i|)}{\Delta\varphi} + \frac{2}{\sqrt{(d-1)(d-2)}}\,, \quad (6.15)$$

to derive a constraint on the slope in the large field limit. This implies that $|\Delta\varphi| = |\varphi - \varphi_i| \neq 0$, which makes the inequality inapplicable at an AdS critical point. In the asymptotic limits of field space we get

$$\left\langle -\frac{V'}{V} \right\rangle_{\varphi\to\pm\infty} \geq \frac{2}{\sqrt{(d-1)(d-2)}}\,, \quad (6.16)$$

where we encounter two limits due to the possible directions in field space, corresponding to the sign of V', as the scalar field climbs up the potential. It should be noted that ascending the potential implies $V'\mathrm{d}\tilde{\varphi} \geq 0$, resulting in a positive average, which is consistent with the positive bound given in (6.16).

For exponential potentials, $V(\varphi) \sim V_0\, \mathrm{e}^{-c\varphi}$ with $V_0 < 0$, with the rate $c > 0$, typical in the large field limits of string compactifications ($\varphi \to \infty$), the asymptotic bound (6.16) imposes a constraint on the rate,

$$c \geq \frac{2}{\sqrt{(d-1)(d-2)}}\,. \quad (6.17)$$

This value corresponds to the rate of the Trans-Planckian Censorship Conjecture in equation (6.1). Our confidence in the validity of the bound (6.17) is strengthened by the fact that the same rate has been verified for positive potentials in the context of supergravity no-go theorems, as discussed in Section 5. However, it is important to emphasize that the inequality (6.16) does not hold for AdS solutions with $\dot{\varphi} = 0$. This loophole raises serious challenges for both the TCC and the ATCC. In particular, while the constraint on the slope does not completely exclude critical points corresponding to AdS solutions in the asymptotics of field space, it strongly restricts the permissible region for localizing such vacua.

As mentioned above, the ATCC weakens the stricter conditions previously set by (6.2) for the characterization of negative scalar potentials. The constraint on the first derivative of the potential evolves into an asymptotic requirement, while the second constraint is relaxed, particularly at AdS extrema, therefore shifting the focus away from the ongoing debate on scale separation for $V < 0$.

6.1.4 Examples from cosmology

In this section we validate the conditions, mathematical assumptions, and properties of negative potentials described above. We apply them to various solutions of the Friedmann equations (5.6) and the scalar field equation of motion (5.7). Our analysis focuses on a decelerating and contracting universe with $|V| < 1$, while working in Planckian units. The models considered include the AdS solution and two dynamical ones with variables $a(t)$ and $\varphi(t)$.

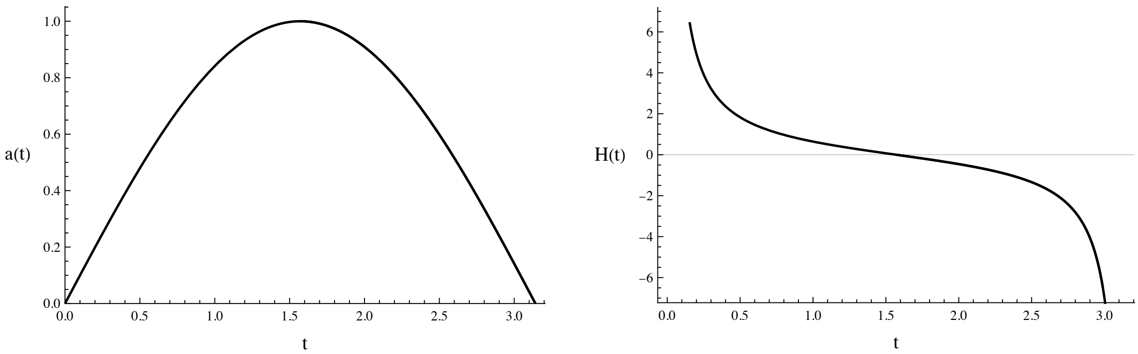


Figure 6: This figure shows the scale factor $a(t)$ and the Hubble parameter $H(t)$ for an anti-de Sitter solution with radius $l = 1$.

Anti-de Sitter spacetime

The anti-de Sitter spacetime, which is characterized by vanishing kinetic energy, negative potential and negative cosmological constant, $\Lambda_d = V|_0$, emerges as a solution to the Friedmann equations

$$\dot{a}^2 + \frac{a^2}{l^2} = 1\,, \qquad \ddot{a} + \frac{a}{l^2} = 0\,, \qquad l^2 = -\frac{(d-1)(d-2)}{2\Lambda_d}\,, \tag{6.18}$$

with the characteristic length scale l. This maximally symmetric spacetime, which may be regarded as a d-dimensional hyperboloid embedded in a $(d+1)$-dimensional flat spacetime,

$$\mathrm{d}s_d^2 = l^2 \left(-\mathrm{d}\psi^2 + \sin^2(\psi)\left(\mathrm{d}\chi^2 + \sinh^2(\chi)\mathrm{d}\Omega_{d-2}^2\right)\right)\,, \tag{6.19}$$

is equivalent to the FLRW metric for

$$k = -1\,, \qquad a(t) = l \sin\left(\frac{t}{l}\right)\,. \tag{6.20}$$

This configuration allows us to plot the functions $a(t)$ and $H(t)$ for a numerical AdS solution in Figure 6. In this example, the contracting phase starts at $t = \pi l/2$ and ends at the final crunch, $t_f = \pi l$. In contrast to a dS spacetime, the Hubble parameter is neither constant nor bounded, raising questions about the use of a horizon or $H(t)$ as a characteristic length. Indeed, both the particle horizon

$$h_p = a(t)\int_{t_i=0}^{t} \frac{\mathrm{d}t'}{a(t')} = l\sin\left(\frac{t}{l}\right) \times \ln\tan\left(\frac{t'}{2l}\right)\Bigg|_{t_i=0}^{t} \to\ \infty \tag{6.21}$$

and the event horizon

$$h_e = a(t)\int_{t}^{t_f=\pi l} \frac{\mathrm{d}t'}{a(t')} = l\sin\left(\frac{t}{l}\right) \times \ln\tan\left(\frac{t'}{2l}\right)\Bigg|_{t}^{t_f=\pi l} \to\ \infty \tag{6.22}$$

are unbounded. Instead, this confirms our proposal to use the scalar potential or the cosmological constant or the AdS radius l as the characteristic length in the development of the Anti-Trans-Planckian Censorship Conjecture. We then apply the arguments and conclusions of the previous sections to this analytical solution of the Friedmann equations and check their consistency. We start with the algebraic assumption in equation (6.10). For an AdS solution, this inequality becomes

$$l^2 \geq a^2 \quad \Leftrightarrow \quad \sin^2\left(\frac{t}{l}\right) \leq 1\,, \tag{6.23}$$

which applies universally. In the same way, the ATCC condition (6.7) is given by

$$\sin\left(\frac{t}{l}\right) \;\geq\; \frac{l_p}{l} \times \sqrt{\frac{(d-1)(d-2)}{2}}\; \sin\left(\frac{t_i}{l}\right) \equiv \frac{l_p}{l} \times \delta\,, \tag{6.24}$$

for small l_p/l. For the purposes of this argument, let us assume that the initial time is set near the maximum size of the AdS spacetime, ensuring we commence within the regime of validity of the effective theory, where $\sin(t_i/l) \lesssim 1$. Without loss of generality, we set $\delta = 1$ and define a time interval $\delta t = t_f - t$ that quantifies the distance from the end time. Taking the limit $\Delta t \to 0$, i.e. starting close to the Big Crunch, we obtain

$$\sin\left(\pi - \frac{\Delta t}{l}\right) = \sin\left(\frac{\Delta t}{l}\right) \sim \frac{\Delta t}{l} \geq \frac{l_p}{l}\,. \tag{6.25}$$

This threshold marks the entry into a Planckian regime near the final crunch, signaling the point at which our effective theory may no longer be reliable. Furthermore, when evaluating the lifetime bound (6.9), focusing on the contracting phase after $t_i > \pi l/2$ yields

$$t_f - t_i \;<\; l \left|\tan\left(\frac{t_i}{l}\right)\right| \ln\left(\frac{l}{l_p} \times \sqrt{\frac{2}{(d-1)(d-2)}}\right)\,, \tag{6.26}$$

where we use the relation (6.18) between the cosmological constant and characteristic AdS length scale. Evaluating the right-hand side poses a challenge due to its dependence on the initial time, which complicates the analysis. However, it is important to compare this to the total contraction time of AdS. In the case of an AdS solution, which aligns with the critical point of $V(\varphi)$ where $\dot{\varphi} = 0$, the bound on the potential (6.14) is specified as $\Lambda_d = V \geq -1$ in Planckian units. The conclusion follows from the premise that $-\Lambda_d$, the typical energy scale of effective theory, should remain below the Planck mass.

Although the condition (6.16) does not apply directly to anti-de Sitter, it is reassuring to note that this spacetime, considered one of the most well-established solutions within string theory, is consistent with the ATCC and its implications.

Dynamical solutions

We will now focus on two dynamical solutions derived numerically in $d = 4$ with $k = -1$ using the equations introduced earlier. Before entering exponential potentials, it is important to note that the second Friedmann equation is inherently satisfied by any solution of the first one and of the equation of motion (5.7) during both the contracting and the expanding phase, provided that $H(t) \neq 0$. Our discussion here focuses on the former case, characterized by $\dot{a}(t) < 0$. When exploring properties of potentials and initial conditions, it is important to clarify that our analysis is restricted to regions where $V < 0$, ignoring any positive part of the potential. This approach also allows for situations where the kinetic energy is negligible, requiring the choice of $\rho(t_i) < 0$. With respect to the field $\varphi(t)$, we consider two initial states – either climbing up or rolling down the potential gradient – with a preference for the latter. Note that these explicit initial conditions do not limit the generality of our approach. The logic applied and the results derived from the following numerical solutions remain valid for different initial configurations.

The finite lifetime of a contracting, decelerating universe has important consequences for our numerical analysis. The Big Crunch, characterized by $a(t_c) = 0$, inevitably arrives after a finite time, regardless of the initial conditions. However, our numerical solution will stop just before t_c due to divergences and singularities that arise as $a(t)$ approaches zero. With that in mind, we may proceed with a comprehensive study of each individual numerical solution.

Exponential potential. Solving the equations (5.6) and (5.7) requires us to specify the potential $V(\varphi)$. As discussed above, most potentials in string compactifications exhibit exponential behavior in the asymptotics of field space. Therefore, the potential

$$V(\varphi) = 0.04\,\mathrm{e}^{-1.74\varphi} - 0.05\,\mathrm{e}^{-0.87\varphi}\,, \tag{6.27}$$

as shown in Figure 7, is an obvious candidate for testing the Anti-Trans-Planckian Censorship Conjecture and its consequences. In order to obtain a numerical solution for the

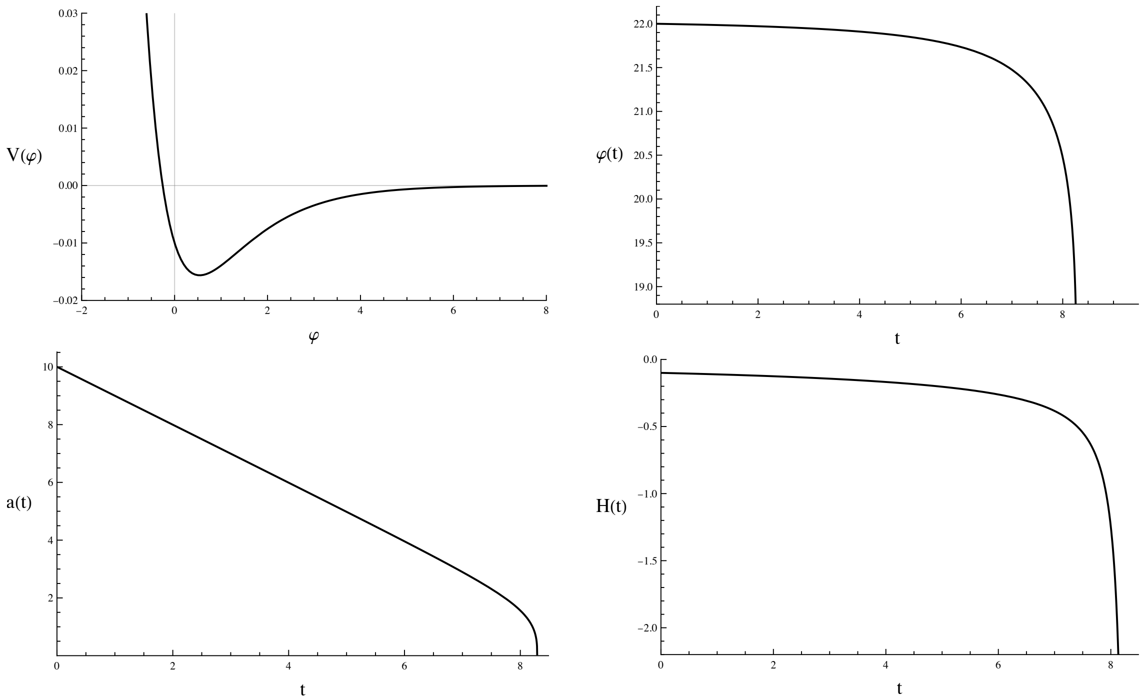


Figure 7: This figure shows the potential $V(\varphi)$, the scalar field $\varphi(t)$, the scale factor $a(t)$ and the Hubble parameter $H(t)$ for a dynamical solution with an exponential potential (6.27).

functions $\varphi(t)$, $a(t)$ and $H(t)$, as shown in Figure 7, we impose the initial conditions

$$\varphi(0) = 22\,, \qquad \dot{\varphi}(0) = -0.01\,, \qquad a(0) = 10 \tag{6.28}$$

at the time $t_i = 0$. Under these conditions, the field φ rolls down the potential, passes through the minimum and then rises slightly to the left of the critical point $V_{\text{min}} = -0.015625$ at $\varphi_{\text{min}} = 0.540234$. Passing through the minimum does not affect the behavior of the functions. Our numerical method breaks down at $t_f \approx 8.29096$ with

$$a(t_f) = 0.00035871\,, \qquad \varphi(t_f) = 0.0463959\,, \qquad V(\varphi(t_f)) = -0.0111242\,. \tag{6.29}$$

Moving on we will check the assumption (6.10) and the bound (6.14) within this dynamical model. In order to verify that the assumption holds over the entire period for which our numerical analysis is reliable, it is plotted in Figure 8. The statement of the ATCC,

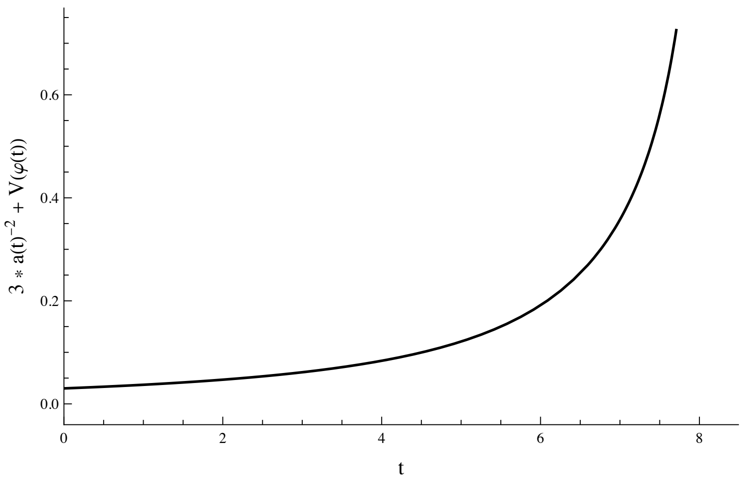


Figure 8: This figure shows a successful check of the assumption (6.10) for a dynamical solution with an exponential potential (6.27).

represented by the inequality (6.7), is also satisfied in the entire time interval $[t_i, t_f]$, which implies that

$$a(t) - \sqrt{|V_i|}a(t_i) \geq 0\,, \tag{6.30}$$

in the contracting spacetime, or $a(t_f) - \sqrt{|V_i|}a(t_i) = 2.02651 \times 10^{-4}$, where our numerical analysis fails. Because of the validity of the above conditions, we expect the bound (6.14) to hold for this dynamical solution as well. It remains interesting to see how. For the exponential potential, the asymptotic inequality (6.14) simplifies to a bound on the rate (6.17), which is consistent with the dominant term in (6.27) in the large field limit, i.e. $V(\varphi) \sim -0.05\; e^{-0.87\varphi}$.

KKLT-inspired potential. Inspired by the KKLT construction in [25], this paragraph studies the potential

$$V(\varphi) = 10^{12}\,\frac{\mathrm{e}^{-0.2\varphi}\left(-3\times 10^{-4}\,\mathrm{e}^{0.1\varphi} + 0.1\varphi + 3\right)}{6\varphi^2}\,, \tag{6.31}$$

which has a minimal value of $V_{\mathrm{min}} = -0.0199658$ at $\varphi_{\mathrm{min}} = 113.589$, as shown in Figure 9. Following the same method as before, we fix the initial conditions,

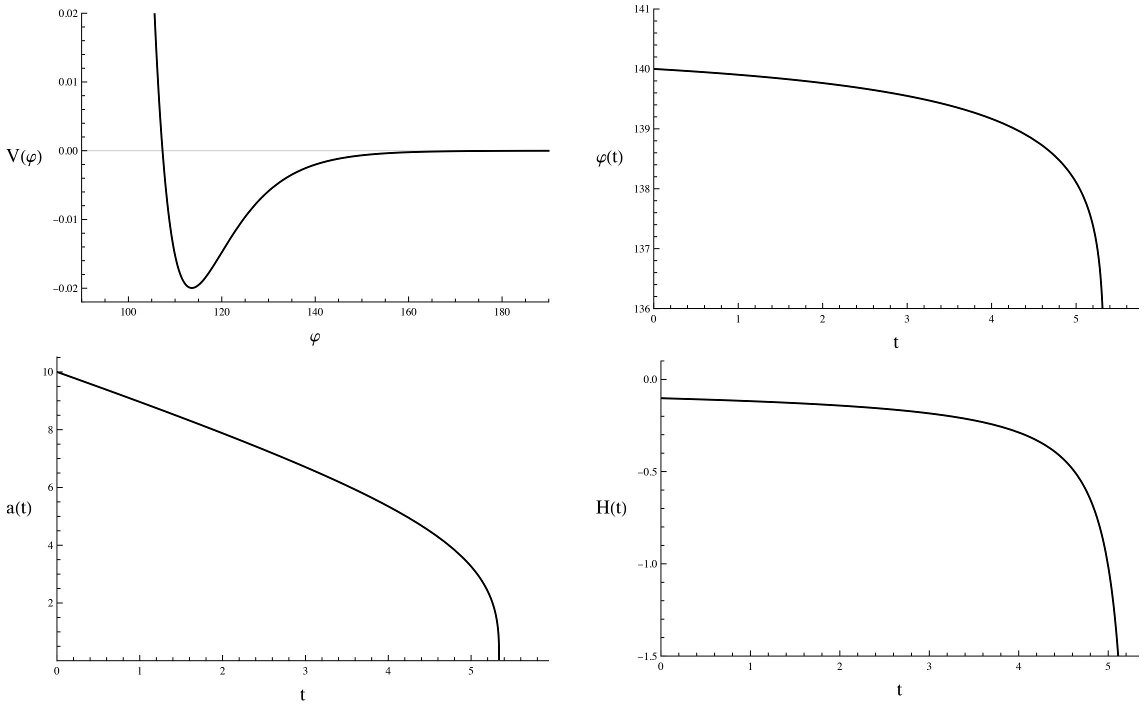


Figure 9: This figure shows the potential $V(\varphi)$, the scalar field $\varphi(t)$, the scale factor $a(t)$ and the Hubble parameter $H(t)$ for a dynamical solution with a KKLT-inspired potential (6.31).

$$\varphi(0) = 140\,, \qquad \dot{\varphi}(0) = -0.082\,, \qquad a(0) = 10\,, \tag{6.32}$$

and solve the equations of motion to obtain the numerical solutions for $\varphi(t)$, $a(t)$, and $H(t)$, as shown in Figure 9. Our numerical analysis stops at $t_f \approx 5.3379$ with

$$a(t_f) = 0.000627402\,, \qquad \varphi(t_f) = 117.175\,, \qquad V(\varphi(t_f)) = -0.0178131\,. \tag{6.33}$$

In contrast to the previous case, the field rolls down the potential without crossing its minimum. One might criticize our dynamical models for consistently showing the field rolling down the potential rather than climbing up, as was originally assumed when deriving the bounds in Section 6.1.3. However, as noted above, this behavior does not invalidate our results, and we can still explore the physics of negative potentials. By reversing the sign of the initial condition, $\dot{\varphi}(0) \to -\dot{\varphi}(0)$, we can also find solutions where the field climbs up the potential.

We then return to the discussion of (6.10) and study the consequences of the condition (6.7) for the potential. Verification of the assumption is straightforward by plotting the left-hand side over the entire period for which our numerical solution remains valid, as shown on the left in Figure 10. The numerical solution also verifies the statement of the ATCC, given by the inequality (6.7), up to the point where the universe approaches the Big

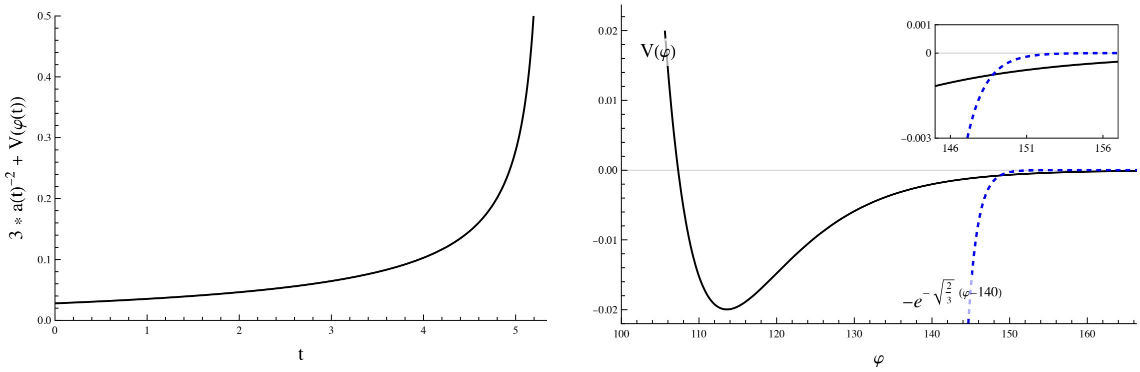


Figure 10: The left figure shows a successful check of the assumption (6.10) for a dynamical solution with a KKLT-inspired potential (6.31). The right figure shows a comparison between the potential and the exponential growth of the bound (6.14) with $\varphi_i = 140$. The intersection point of the two curves is at $\varphi = 148.79$.

Crunch, suggesting that the condition holds within the regime of validity of the effective theory.

Following the constraints previously validated up to the final crunch, we further discuss the bound (6.14) on the potential. Interestingly, this inequality is not satisfied by the KKLT-inspired potential, as shown in Figure 10. In particular, the potential crosses the bound set by (6.14) at $\varphi = 148.79$. A possible explanation for this violation of the constraint could be that our numerical solution only probes the region $\varphi < 148.79$ in field space where the relevant algebraic assumption and the bound (6.7) are satisfied. Therefore, the dynamical solution may not fully capture the anomalies due to its limited exploration of φ. The potential may be consistent within the effective theory of quantum gravity only for lower field values. The violation of (6.14) in regions of large field values is further proved by the asymptotic constraint (6.16), resulting in $\langle -V'/V \rangle_{\varphi\to\infty} = 0.1$ that is below the expected value of 0.82.

Furthermore, compared to the KKLT constructions analyzed in Table 7, this inconsistency becomes apparent at higher field values due to differences in V'' from the expected asymptotic behavior.

6.2 A new bound on V''

To extend the discussion of analytical bounds, we have derived an additional asymptotic bound on the second derivative of the scalar potential. This constraint is notable because it applies to both positive potentials, with $V' \leq 0$, and negative potentials, with $V' \geq 0$. Note that the chosen field direction does not limit the generality of our approach. For the sake of simplicity, we assume $\varphi - \varphi_i \geq 0$ in the asymptotic limit, $\varphi \to \infty$, which implies that the scalar field rolls down the potential for $V > 0$ and climbs up for $V < 0$. Our discussion begins with the equation

$$\int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi}\, \frac{V''}{V} = \int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi}\, \frac{V''V - V'V'}{V^2} + \int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi} \left(\frac{V'}{V}\right)^2$$

$$= \int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi} \left(\frac{V'}{V}\right)' + \int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi} \left|\frac{V'}{V}\right|^2 . \quad (6.34)$$

Using the Cauchy-Schwarz inequality,

$$\left(\int \mathrm{d}\tilde{\varphi}\, f(\tilde{\varphi})g(\tilde{\varphi})\right)^2 \leq \int \mathrm{d}\tilde{\varphi}\, f(\tilde{\varphi})^2 \int \mathrm{d}\tilde{\varphi}\, g(\tilde{\varphi})^2 , \quad (6.35)$$

with $f(\tilde{\varphi}) = |V'/V|$ and $g(\tilde{\varphi}) = 1$, we obtain the following inequality

$$\int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi} \frac{V''}{V} \geq \int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi} \left(\frac{V'}{V}\right)' + \frac{1}{\Delta\varphi}\left(\int_{\varphi_i}^{\varphi} \mathrm{d}\tilde{\varphi} \frac{|V'|}{|V|}\right)^2 = \frac{V'}{V}\bigg|_{\varphi_i}^{\varphi} + \Delta\varphi \left\langle \frac{|V'|}{|V|} \right\rangle^2 . \quad (6.36)$$

In summary, the resulting expression is given by

$$\left\langle \frac{V''}{V} \right\rangle \geq \frac{1}{\Delta\varphi} \frac{V'}{V}\bigg|_{\varphi_i}^{\varphi} + \left\langle \frac{|V'|}{|V|} \right\rangle^2 . \quad (6.37)$$

We propose a minor assumption that the ratio V'/V is locally bounded in the large field limit. This is obvious for potentials that exhibit asymptotic exponential behavior, which is common in string compactifications. In particular, we require that the first term on the right-hand side of the inequality (6.37) vanishes. If this condition is satisfied, we can apply the asymptotic (A)TCC bound (6.16) on the first derivative of the potential. Taking into account the signs of both V' and V, we eliminate the need for absolute values. For a single canonical field, this gives the local condition

$$\left\langle \frac{V''}{V} \right\rangle_{\varphi\to\infty} \geq \frac{4}{(d-1)(d-2)} , \quad (6.38)$$

which is the square of the exponential rate in the inequality (6.17). It is not surprising that when the potential is constrained by an exponential, as indicated in (6.14), there are natural constraints on both the first and second derivatives.

We will discuss the consequences of the asymptotic bound (6.38) for a single canonical scalar field. In this case, the mass of φ corresponds to the second derivative, $m^2 = V''$. We omit the averaging, which is justified for potentials with exponential behavior. However, one has to be careful when interpreting these results, since the bounds (6.16) and (6.38) are only valid in the asymptotics of field space and thus not directly at a critical point corresponding to an (anti-)de Sitter solution.

6.2.1 Consequences for $V > 0$

We establish a lower bound for positive potentials,

$$m^2 \geq \frac{4}{(d-1)(d-2)} V . \quad (6.39)$$

This inequality requires careful handling and cannot be applied directly to the extrema of scalar potentials. Therefore, it should be distinguished from the presence of tachyons in de Sitter solutions [45,52], or conjectures about their existence in the swampland program [31,145,146]. However, by continuity in field space one can extend the consequences of this bound to critical points, suggesting the existence of a state with $m^2 > 0$ in the vacuum solutions. This claim can be tested by analyzing the full mass spectrum in a dataset of de Sitter solutions [147]. Remarkably, each spectrum appears to contain a state that satisfies the bound (6.39).

Alternatively, for a tower of states with mass scale m in the asymptotic regions of field space, the condition (6.39) suggests the possibility of scale separation, consistent with the conjecture in [45]. The conjecture states that classical de Sitter backgrounds are likely to exhibit scale separation, in particular in the large field limit. According to our discussion, this scale separation would be local [54,148], but not parametrically controlled [40].

6.2.2 Consequences for $V < 0$

For negative potentials, there is an upper bound on the square of the mass,

$$m^2 \leq \frac{4}{(d-1)(d-2)} V \,. \tag{6.40}$$

This suggests the existence of a state with negative m^2. However, this result should be distinguished from those in the swampland program [27,28,144], which focus primarily on light states or scale separation.

6.3 Mass bound in AdS and a holographic perspective

As mentioned above, the inequality (6.40) is intended for use in asymptotic regions, away from any critical point of the potential. But due to continuity in the field space, this bound can have important consequences in such vacua. More precisely, the mass spectrum of any solution may undergo a continuous transition from regions where equation (6.40) is applicable to an extremum of the potential, which may cause slight deformations. The following section studies this extrapolation towards a critical point in more detail. Our analysis focuses on the possible effects for d-dimensional AdS solutions in $d \geq 4$, given the peculiarities of gravity in $d = 3$ and the violations of the TCC or ATCC outlined in Sections 5.3 and 6.5.

At the critical point of a negative potential, the d-dimensional AdS vacuum, hereafter simply referred to as AdS$_d$, is characterized by a cosmological constant $\Lambda_d = V|_0$, which is related to the length scale l as defined in the equation (6.18). For $d \geq 4$, the bound (6.40) is given by

$$m^2 l^2 \lesssim -2 \,. \tag{6.41}$$

In other words, there exists a scalar field with mass m that satisfies (6.41) in any d-dimensional AdS solution. The notation $\lesssim$ is deliberately used here to allow flexibility, given the complexity of modifications to the mass spectrum discussed earlier.

In flat or de Sitter spacetimes, scalar fields with negative m^2 typically imply instabilities due to an "upside-down" potential. In AdS$_d$, however, negative m^2 for a bulk field does not guarantee unstable solutions, provided that

$$m^2 l^2 \geq -\frac{(d-1)^2}{4}\,, \tag{6.42}$$

and that the field fluctuations follow a specific asymptotic behavior not discussed here. This is known as the Breitenlohner-Freedman (BF) bound [68,69]. For more details and a comprehensive overview, see [149,150]. The BF bound dictates that for $d \geq 4$, all perturbatively unstable AdS solutions adhere to the mass bound specified in (6.41). Therefore, we will limit our discussion to perturbatively stable solutions of quantum gravity and their respective mass spectra, which provide a structured overview that highlights compliance or violations of the bound (6.41). As part of this exercise, we will explore possible counterexamples and study whether and how significant changes in their mass spectra might occur as we navigate through field space. We aim to provide a better understanding of the challenges and complexities that arise in the discussion of both supersymmetric and non-supersymmetric solutions in string theory.

6.3.1 Supersymmetric AdS

In Table 7, we present perturbatively stable AdS solutions in dimensions $d \geq 4$, which are also supersymmetric. Most of these solutions satisfy our proposed bounds, except for certain well-known compactifications, which are discussed below:

- Supersymmetric AdS$_4$ in M-theory: The mass spectrum of AdS$_4 \times S^7$, with the $SO(8)$ symmetry group [151,163], is given by $\Delta = E_0$ and $m^2 l^2 = -2 + \text{Mass}^2/4$, using the equation (6.43) and the notation of [151]. This spectrum is consistent with the bound (6.41), since the lowest masses for scalar states $0^{\pm(1)}$ are $m^2 l^2 = -\frac{9}{4}$, corresponding to the BF bound, or $m^2 l^2 = -2$. In [152], four additional supersymmetric AdS$_4$ solutions have been studied in M-theory, in particular within maximal gauged supergravity, each obeying (6.41).
- AdS$_4 \times S^6$ in massive type IIA supergravity: The solutions differ in the residual symmetry group of the internal space and the preserved SUSY [153,154,164]. Mass spectra in [153] show that all seven supersymmetric solutions have a scalar field with $m^2 l^2 \leq -2$.
- KKLT, LVS, DGKT: These controversial AdS$_4$ solutions, along with their quantum gravity origins, have been discussed in [102,117], focusing on the control of perturbative and non-perturbative corrections, as well as the smearing of sources:
  - $*$ KKLT scenario [25]: Including non-perturbative corrections, both the moduli which define the shape of the complex structure of the Calabi-Yau manifold and the dilaton are stabilized by the tree-level potential. The mass spectrum for these scalar fields satisfies $m^2 l^2 \geq 0$ [25]. The condition $m^2 l^2 > 0$ also holds for the spectrum of the Kähler moduli [155].
  - $*$ LVS scenario [26]: This construction involves perturbative corrections and features light scalars with $m^2 l^2 \geq 0$ [156].

Dim.	Solution	Specification	$\mathcal{N}$	Spectrum	Lowest scalar mass (m^2l^2)
$d=4$	AdS$_4$, M-theory	$SO(8)$	8	[151, Tab. 4]	$-9/4$
		$SU(3)\times U(1)$	2	[152]	-2.222
		G_2	1		-2.242
		$U(1)\times U(1)$	1		-2.25
		$SO(3)$	1		-2.245
	AdS$_4\times S^6$, IIA	G_2	1	[153, App. B] [154, App. A]	-2.24158
		$SU(3)\times U(1)$	2		$-20/9$
		$SO(3)\times SO(3)$	3		$-9/4$
		$SU(3)$	1		$-20/9$
		$U(1)$	1		-2.23969
		$\emptyset$	1		-2.24943
		$U(1)$	1		-2.24908
	KKLT, IIB		1	[25, 155]	≥ 0
	LVS, IIB		1	[156, Sec. 2]	≥ 0
	DGKT, IIA		1	[70, 157]	> 0
	DGKT-like, IIA	Branch A1-S1	1	[158, Tab. 2]	-2
	S-fold, IIB	$U(1)\times U(1)$	1	[159]	-2
		$U(1)\times U(1)$	2		-2
		$SO(4)$	4		-2
$d=5$	AdS$_5\times S^5$, IIB	$SO(6)$	8	[160]	-4
		$SU(2)\times U(1)$	2	[161, Tab. D.4]	-4
$d=7$	AdS$_7\times S^3$, IIA		1	[162]	-8

Table 7: This table shows the properties of perturbatively stable supersymmetric anti-de Sitter solutions, including the symmetry group of the internal space, the preserved SUSY and the lowest m^2l^2 in the mass spectrum of each solution.

  * (Original) DGKT solution [70, 101]: Fluctuations of the metric and the dilaton satisfy $m^2l^2 > 0$, while the square of the masses of axionic fields depend on the sign of the fluxes. They remain positive in supersymmetric solutions [157].

  The mass spectra of all three solutions violate our fixed bound. If these solutions are indeed part of the landscape, the apparent violation of our bound could be explained by serious changes of the mass spectra as we approach the critical point in field space.

- Generalized DGKT solutions: Appears in three branches in $d = 4$; the supersymmetric branch has light scalars with $m^2l^2 \geq -2$, where the lowest mass is exactly -2 [158]. The non-supersymmetric branches are discussed separately.

- Uplift to type IIB string theory in the form of S-folds: For three supersymmetric AdS$_4$ solutions [159], uplifted to type IIB string theory as S-folds, corresponding to different symmetry groups, the lowest scalar mass satisfies $m^2l^2 = -2$. For more details see [165–167].

- Higher dimensional supersymmetric solutions:

  * AdS$_5 \times S^5$: The mass spectrum of solutions in $SO(6)$ gauged supergravity

[160, 168] and $SU(2) \times U(1)$ [169, 170] is consistent with the mass bound, see also [161].

∗ Supersymmetric AdS$_7$ solutions in type IIA string theory [162]: The dilaton field always satisfies bound (6.41).

6.3.2 Non-supersymmetric AdS

In Table 8, the majority of non-supersymmetric AdS solutions conform to the mass bound given in (6.41), especially when we consider possible changes in the mass spectrum that may occur during the motion through field space. For example, the lowest mass squared

Dim.	Solution	Specification	Non-pert. instability	Spectrum	Low. scalar mass (m^2l^2)
$d = 4$	AdS$_4$, M-theory	$SO(3) \times SO(3)$	[171]	[172]	$-12/7$
	AdS$_4 \times S^6$, IIA	G_2	[173]	[153, App. B] [154, App. A]	-1
		$SU(3)$			-1.58174
		$SO(3) \times U(1)$			-1.71379
		$SO(3)$			-1.71663
		$SU(3)$			-1.70679
		$SO(3) \times U(1)$			-1.70677
		$SO(3) \times SO(3)$			-1.96422
		$U(1)$			-2.18141
		$\emptyset$			-2.24727
	DGKT-like, IIA	Branch A1-S1	[174]	[158]	-2
		Branch A2-S1			0
$d = 7$	AdS$_7 \times S^3$, IIA	d = 2	[162]	[162]	-9
		d = 3			0

Table 8: This table shows the properties of perturbatively stable non-supersymmetric anti-de Sitter solutions, including the symmetry group of the internal space and the lowest m^2l^2 in the mass spectrum of each solution. References to potential non-perturbative instabilities are included where applicable.

is only slightly above -2. However, two exceptions do not have negative m^2 in their mass spectra. These solutions, among others, are non-perturbatively unstable according to the conjecture in [71] and are excluded from the landscape of viable quantum gravity solutions. The individual cases are discussed below:

- Non-supersymmetric AdS$_4$ in M-theory: Most are perturbatively unstable [152, 172], satisfying (6.41). An exception is the symmetry group $G = SO(3) \times SO(3)$ [175, 176], where the lowest scalar mass obeys $m^2l^2 = -12/7$ [171]. This solution, while showing a non-perturbative brane-jet instability [172], satisfies the bound if we allow slight modifications as we approach the AdS critical point.

- AdS$_4 \times S^6$ in massive type IIA supergravity: Many solutions contain a scalar field with $m^2l^2 \simeq -2$ [153], assuming some flexibility. Those characterized by the symmetry groups $G = G_2$ or $G = SU(3)$ have large lowest mass values. This problem has not been solved [177], even when higher order Kaluza-Klein modes are considered, as in [178]. This requires a more detailed analysis than is provided here to confirm that they do indeed respect the flexibility allowed by the mass bound. Although these solutions are not supersymmetric [71], they are perturbatively stable. However, the $G = G_2$ configuration in particular exhibits a non-perturbative instability. In contrast to the previously mentioned brane-jet instability [154], this instability manifests as a "bubble of nothing" [154, 173, 179], raising concerns about its validity.

- Generalized DGKT solutions: Two branches in $d = 4$ are non-supersymmetric; one has a scalar with $m^2 l^2 = -2$, while A2-S1 lacks any state with negative mass squared in the spectrum. The latter is a potential counterexample to the mass bound, although these non-supersymmetric solutions also exhibit non-perturbative instabilities.

- Higher dimensional non-supersymmetric solutions: In type IIA string theory, the dilaton field of non-supersymmetric AdS$_7$ solutions has a non-negative m^2 [162], yet these solutions suffer from non-perturbative instabilities related to NS5-brane bubbles. Furthermore, these configurations may contain several scalar fields associated with the representation of the $SU(2)$ R-symmetry or the presence of D8-branes. The masses of these fields may satisfy the bound (6.41).

These results raise the possibility that all (non-)supersymmetric counterexamples are eventually part of the landscape. If so, the modifications of the mass spectrum as we approach the critical point of the potential may be more significant than previously thought. Understanding this evolution, and why supersymmetric solutions have more consistent spectra while non-supersymmetric solutions do not, remains a promising direction for future study.

6.3.3 Holographic consequences

After applying the bound (6.41) to individual cases, our next task is to briefly explore its holographic interpretation within the dual conformal field theory (CFT) for dimensions $d \geq 4$. We start by revisiting the well-established relation between the masses of scalar fields and the conformal dimension Δ of an operator in the dual CFT,

$$\Delta(\Delta - (d-1)) = m^2 l^2 \,, \qquad \Delta_\pm = \frac{d-1}{2} \pm \frac{1}{2}\sqrt{(d-1)^2 + 4m^2 l^2} \,, \tag{6.43}$$

such that the presence of a state satisfying the inequality (6.41) results in

$$\Delta^0_- \leq \Delta \leq \Delta^0_+ \,, \qquad \Delta^0_\pm = \frac{d-1}{2} \pm \frac{1}{2}\sqrt{(d-1)^2 - 8} \,. \tag{6.44}$$

This relation is illustrated in Figure 11. In practice, the existence of a state whose mass satisfies the bound (6.41) indicates the presence of an operator in the dual CFT with the property (6.44). When considered together with the scalar unitarity bound[21], $\Delta \geq$

[21] In a quantum field theory, unitarity requires that all states within a given representation have a positive norm, which constrains the conformal dimensions of operators in the dual CFT.

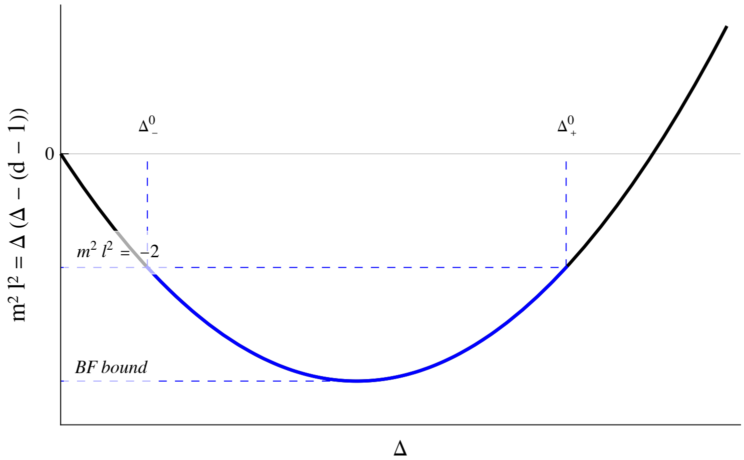


Figure 11: This figure plots m^2l^2 as a function of conformal dimension Δ. The curve follows a consistent pattern for all $d \geq 4$. The masses within the blue region are constrained by the lower Breitenlohner-Freedman bound, indicating perturbative stability, and by (6.41), along with the corresponding bounds on the conformal dimension as given by (6.44).

$(d-3)/2$, the mass bound for scalar fields in $d = 4$ guarantees the unitarity bound for the corresponding operators in the CFT. Furthermore, for $d = 4$, the bound in (6.44) leads to unique integer values $1 \leq \Delta \leq 2$. Reaching the mass bound, specifically $m^2l^2 = -2$, entails that the conformal dimensions are integers. This is a crucial property of scale-separated AdS solutions and their corresponding dual conformal field theories [157,180–183]. In these studies, the appearance of integer conformal dimensions is related to the unique nature of scale separation.

6.4 Multi-field models

In discussing the implications of the ATCC, we have simplified the problem by focusing only on a single, canonically normalized scalar field. This approach contrasts with the typical effective theories derived from string compactifications, which include multiple scalars. In the following, we aim to extend the results of the Sections 6.1 and 6.2 to a multi-field context. Our discussion will be brief, as a full treatment of this complex task is beyond the scope of this thesis. In the multi-field extension, we first focus on a 1D trajectory in field space, parametrized by an affine parameter $\hat{s}$ or a single canonical field direction. Here, as before, it is convenient to extend the arguments of [58], which focus on positive scalar potentials, to discuss negative potentials in a multi-field framework. Without loss of generality, we assume that $\nabla_{\hat{s}} V \geq 0$, where the gradient along the trajectory is given by

$$\nabla_{\hat{s}} V = \frac{\partial \varphi^i}{\partial \hat{s}} \partial_i V \,, \tag{6.45}$$

with $\partial_i = \partial / \partial \varphi^i$. We define the average in the multi-field extension as follows,

$$\left\langle \frac{\nabla_{\hat{s}} V}{V} \right\rangle = \frac{1}{\Delta \hat{s}} \int_{\hat{s}_i}^{\hat{s}} \mathrm{d}s \, \frac{\nabla_{\hat{s}} V}{V} \,, \tag{6.46}$$

where $\Delta\hat{s} = |\hat{s} - \hat{s}_i|$. This results in constraints on the negative potential,

$$V(\varphi(\hat{s})) \geq -\mathrm{e}^{-\frac{2|\hat{s}-\hat{s}_i|}{\sqrt{(d-1)(d-2)}}}, \tag{6.47}$$

and its first derivative,

$$\left\langle \frac{\nabla_{\hat{s}} V}{|V|} \right\rangle_{\hat{s}\to\pm\infty} \geq \frac{2}{\sqrt{(d-1)(d-2)}}. \tag{6.48}$$

Note that the exponential rate is consistent with single-field effective theories, as detailed in the equations (6.14) and (6.16). In the remainder of this section, we will address the challenges presented by this generalization to multiple fields.

This approach introduces an inherent ambiguity due to the flexibility in choosing different local paths. We opt to select a field direction in which the potential is represented as a negative, monotonically increasing exponential function. This choice is crucial for analyzing the asymptotic behavior of the potential, in particular within the ATCC framework, with respect to the exponential rate and the bound given above. Identifying such paths is challenging and significantly complicates the analysis. Beyond the multitude of possible paths, there remains the question of whether it is wise to limit ourselves to one particular field direction. This requires a mechanism that effectively stabilizes other fields at some finite values. This line of thought leads to the conclusion that the definition of the gradient given in (6.45) is not completely unambiguous.

Alternatively, defining

$$\nabla V(\varphi) = \sqrt{G_{ij}\partial^i V(\varphi)\partial^j V(\varphi)} \tag{6.49}$$

allows us to analyze derivatives across all fields in the multi-field space. This definition of the gradient is central to the Strong de Sitter Conjecture [124,127], which is characterized by an asymptotic bound with the rate (5.15),

$$\left(\frac{|\nabla V(\varphi)|}{V(\varphi)} \right)_{s\to\infty} \geq c_{\text{strong}}, \tag{6.50}$$

for $V > 0$ and any divergent geodesic distance s in field space. This is more stringent than the TCC bound (5.3), which aligns with $\nabla V \geq |\nabla_{\hat{s}} V|$. For negative potentials, this leads to a multi-field extension of the ATCC,

$$\left(\frac{\nabla V}{|V|} \right)_{s\to\infty} \geq \frac{2}{\sqrt{d-2}}. \tag{6.51}$$

As before, the discrepancy in the bounds is due to a different definition of the gradient. By stabilizing other fields, both definitions converge, possibly causing inconsistencies in the bounds (6.47) and (6.51). Recent work on Calabi-Yau compactifications in $d = 4$ [126] sheds light on this multi-field extension for $V > 0$ and the discrepancy between the different definitions of the gradient: Although the TCC has been extensively tested across many paths [38, 39], a direction among the scalar fields has been identified that violates the asymptotic behavior of the potential proposed by (5.14). Moreover, some scalar fields (e.g., Kähler fields) in this potential cannot be stabilized perturbatively, leaving ambiguity in defining the gradient.

The remaining challenge is to extend the asymptotic bound (6.38) on the second derivative of the potential to multiple fields. However, this requires a more comprehensive and careful analysis. From previous results, we can confidently state that for positive potentials

$$\left(\frac{\max \nabla \partial V}{V}\right)_{s\to\infty} \geq \frac{4}{(d-1)(d-2)}\,, \tag{6.52}$$

and for negative potentials

$$\left(\frac{\min \nabla \partial V}{|V|}\right)_{s\to\infty} \leq -\frac{4}{(d-1)(d-2)}\,. \tag{6.53}$$

Here, the mass matrix $M^i{}_j = G^{ik}\nabla_k \partial_j V$ is used. Note the difference to the bound in (6.2): While the inequality (6.53) aligns with the Refined de Sitter Conjecture for $V > 0$ [56], except for a constant of order one, the corresponding condition presented in (6.2) shows a clear distinction from our results when applied to negative potentials. We will revisit this distinction in the following section by exploring actual examples of string compactifications.

6.5 Examples from string theory

In Section 6.1.4, we have discussed the Anti-Trans-Planckian Censorship Conjecture and its implications for negative potentials in various solutions of the Friedmann equations and the scalar field equation of motion, independent of string theory. Building on these insights, we will now study scalar potentials typically found in string compactifications. We will test these potentials against the condition (6.14) and the asymptotic bounds given in the equations (6.16) and (6.48), the latter for multiple scalar fields. Before turning to specific examples, let us recall our expectations, which are based on fundamental concepts of string theory. Scalar potentials in the effective theory can show three distinct asymptotic behaviors: Divergence towards positive values, convergence to values infinitesimally close to zero ($\lim_{|\varphi|\to\infty} V \to 0^{\pm}$) or divergence towards negative values. The latter creates inconsistencies, while convergence to a finite constant is problematic as cosmological constants usually appear at critical points rather than a constant potential. Therefore, the ATCC restricts this to one viable case: a potential approaching 0^- in the asymptotic limit of field space.

Moving to canonical fields, potentials in string compactifications can typically be expressed as sums of exponential terms. For a single scalar field, the dominant contribution in the large field limit is $V \sim \mathrm{e}^{-c\varphi}$ with $c > 0$, thus simplifying the constraints (6.16) and (6.48), leading to the inequality (6.17) on the rate. However, with multiple scalar fields, the behavior of the potential does not consistently converge to 0^- for each field individually, but rather along a particular field direction, a linear combination of the fields. Our analysis will identify this direction while stabilizing the orthogonal directions at finite values. The gradient along this single field direction will be compared to the lower bound (6.16). As noted in Section 6.4, the inclusion of additional scalar fields increases the gradient $|\nabla V|$. Since we are testing lower bounds against single-field gradients, we find it appropriate to ignore the other orthogonal directions in our discussion.

6.5.1 Scalar potential for (ρ, τ, σ)

The semi-universal potential $V(\rho, \tau, \sigma)$, central to classical string compactifications across dimensions, is discussed in Section 3.1. Using canonically normalized fields, the potential is given in (3.38) and the internal Ricci scalar in (3.39). With this potential defined, we can check its compliance with the bound on the rate (6.17). We then study the asymptotic behavior of V along the axes of $\hat{\rho}$, $\hat{\tau}$ and $\hat{\sigma}$, as well as a yet unidentified direction $\hat{t}^1$. The asymptotic behavior in field space is defined by the limits $\hat{\tau}, \hat{\rho} \to \infty$ (large volume and small string coupling), while moving towards $-\infty$ violates the supergravity approximation in classical string theory [32, 33]. The σ field is unbounded in either direction, $\hat{\sigma} \to \pm\infty$.

Exponential rates

First we will test the bound in (6.17) along the directions $\hat{\rho}$, $\hat{\tau}$ and $\hat{\sigma}$. We then determine the sign of each term in the equations (3.38) and (3.39): Flux terms are always positive, while the source term and individual contributions to $\mathcal{R}_{10-d}(\hat{\sigma})$ can be negative. We proceed to analyze the exponential rates associated with these terms. The rate associated with $\hat{\tau}$ is $-(d+2)/(2\sqrt{d-2})$ for the source term and $-2/\sqrt{d-2}$ for the curvature term, both obeying the bound (6.17) for dimensions $d \geq 3$. However, the rate of $\hat{\rho}$ in the curvature term violates this bound in $d = 3$, which is attributed to the unique dynamics of gravity in this dimension. This result is consistent with the discussion of the Trans-Planckian Censorship Conjecture in Section 5.3 Since this issue has little relevance to our broader analysis, we omit a detailed discussion of $d = 3$ compactifications. For the source term, we note that $2p_i - 8 - d < 0$ is consistent with the desired asymptotic behavior, leading to a reformulation of the bound in (6.17),

$$\frac{(2p_i - 8 - d)^2}{4(10-d)} - \frac{4}{(d-1)(d-2)} \geq 0\,. \tag{6.54}$$

This condition is universally satisfied, with exceptions including the case of $d = 3$, $d = 7$ with $p_i = 7$ and $d = 4$ with $p_i = 5$. The latter two are resolved via the Bianchi identity (4.18) and do not strictly violate the ATCC bound. From the perspective of the effective theory this argument seems to stem from quantum gravity: In consistent string compactifications, the Bianchi identity requires that a non-vanishing source term is accompanied by additional fluxes. In the first case, involving sources of dimensionality $p_i = 7$, we find that $F_1 \neq 0$, magnetically sourced by the O7-planes. However, the RR flux provides a positive, monotonically increasing exponential contribution to the scalar potential, which dominates the source term in the large field limit. This situation remains irrelevant for the discussion of negative potentials. Turning to compactifications in $d = 4$, the F_3 flux is magnetically sourced by O5-planes/D5-branes. However, the Bianchi identity $dF_3 - H \wedge F_1 \propto T_{10}^{(5)}$ can also be satisfied by a combination of the F_1 and H flux, leading to a similar conclusion as above. Therefore, we assume $F_3 \neq 0$ in the absence of F_1. According to (3.38), the F_3 flux term lacks dependence on $\hat{\rho}$ and contributes to the potential as a positive constant relative to $\hat{\rho}$, leading to a positive potential in the large field limit.

With respect to the σ field, we obtain the condition $AB \neq 0$, or equivalently $d \leq p_i \leq 8$ in terms of the source dimensionality. The complexity of the curvature term requires a careful discussion of each contribution in the equation (3.39). Among these terms, only the first two can be negative, yet they exhibit opposing exponential behavior: as σ approaches infinity, one term increases while the other decreases. This dynamic suggests that if one term violates the ATCC bound, the other may counteract this violation by becoming dominant. Therefore, a thorough analysis of the signs and the intricate nature of the string compactification is required, which is beyond the scope of this thesis. Regarding the source term, a detailed evaluation shows that the reformulation of the bound (6.17) leads to

$$\frac{(9-p_i)(p_i+1-d)}{10-d} - \frac{4}{(d-1)(d-2)} \geq 0\,, \tag{6.55}$$

which is true for $d \leq p_i \leq 8$ and $d \geq 4$, with the exception of $d = 3$, which remains problematic. In conclusion, our study of the exponential rates explicitly present in the potential (3.38) for the fields $\hat{\rho}$, $\hat{\tau}$ and $\hat{\sigma}$ shows no significant violation of (6.17) for dimensions greater than three. Nonetheless, $d = 3$ remains a persistent challenge. It is important to remember that the potential is likely to exhibit the desired asymptotic behavior only along a certain direction in field space, a topic that we will explore in the following discussion.

Anti-de Sitter no-go theorems

Unlike de Sitter, AdS vacua are rarely constrained by no-go theorems, largely because these solutions are usually preferred by the equations of motion [184]. However, orientifold projections involving specific sets of O-planes can impose stringent constraints on the flux content, potentially excluding AdS solutions in certain source configurations. These models and their corresponding non-vanishing variables are extensively studied in [39], which uses an ansatz similar to the one in the first part of this thesis but with additional assumptions [45,74,105]. To summarize these simplifications, we use a smeared approximation and limit our focus to a 6D group manifold characterized by structure constants ${f^a}_{bc} = -{f^a}_{cb}$ of the underlying Lie algebra. This includes the simple example of a flat torus. Our analysis is further restricted to a basis where ${f^a}_{ab} = 0$, without summation [110]. In the smeared limit, the absence of a warp factor ensures that the dilaton remains constant, $e^\phi = g_s$, and so are the variables in the equations of motion:

$${f^a}_{bc}\,, \qquad H_{abc}\,, \qquad g_s F_{q\ a_1 \dots a_q}\,, \qquad g_s T_{10}^{(p_i)}\,. \tag{6.56}$$

For further details, refer to [39].

We begin with a source configuration consisting of three sets of O5-planes, outlined in Table 9, each intersecting the others within the internal space. When considering a single

O5-plane, only the variables

$$\text{O5}: \quad f^{a_{||}}{}_{b_\perp c_\perp},\, f^{a_\perp}{}_{b_\perp c_{||}}, \quad F_1^{(0)},\, F_3^{(1)},\, F_5^{(2)},\, H^{(0)},\, H^{(2)} \tag{6.57}$$

retain non-zero values. The numbers in brackets indicate the number of legs of the flux components along the sources. Therefore, the solution class for the source configuration in

Sources	Spacetime dimensions	1	2	3	4	5	6
O5, (D5)	x	x	x				
O5, (D5)	x			x	x		
O5, (D5)	x					x	x

Table 9: Source configuration in type IIB$|_4$ with 3 sets of O5-planes/D5-branes. Sources in brackets are not required a priori.

Table 9 is characterized by eight flux components and 24 structure constants:

$$\begin{aligned}
F_3: \quad & F_{3\,135},\quad F_{3\,136},\quad F_{3\,145},\quad F_{3\,146},\quad F_{3\,235},\quad F_{3\,236},\quad F_{3\,245},\quad F_{3\,246},\\
f^a{}_{bc}: \quad & f^3{}_{15},\quad f^3{}_{16},\quad f^3{}_{25},\quad f^3{}_{26},\quad f^4{}_{15},\quad f^4{}_{16},\quad f^4{}_{25},\quad f^4{}_{26},\quad f^1{}_{53},\\
& f^1{}_{63},\quad f^1{}_{54},\quad f^1{}_{64},\quad f^2{}_{53},\quad f^2{}_{63},\quad f^2{}_{54},\quad f^2{}_{64},\quad f^5{}_{13},\quad f^5{}_{23},\\
& f^5{}_{14},\quad f^5{}_{24},\quad f^6{}_{13},\quad f^6{}_{23},\quad f^6{}_{14},\quad f^6{}_{24}\,.
\end{aligned} \tag{6.58}$$

Since all internal directions are equivalent, this setup retains its generality. Previous work has shown that dS solutions are not feasible within this class [34]. Extending the analysis to AdS solutions, the equation (4.17) shows that

$$(p_i - 3)\mathcal{R}_4 = -2|H|^2 + g_s^2\left((7-p_i)|F_1|^2 + (5-p_i)|F_3|^2 + (3-p_i)|F_5|^2\right) \tag{6.59}$$

in $d = 4$ [105]. The configuration 9 projects out several fields, including $H = F_1 = F_5 = 0$ via (6.58), leading to

$$\mathcal{R}_4 = 0\,, \tag{6.60}$$

for $p_i = 5$. This illustrates a no-go theorem against classical AdS solutions due to excessive inclusion of orientifolds in different intersecting sets.

We then analyze a source configuration with one set of O4-planes/D4-branes along the internal direction 4 and two sets of O6-planes/D6-branes along directions 123 and 156, detailed in Table 10. The general nature of this class remains valid. An O4 projection

Sources	Spacetime dimensions	1	2	3	4	5	6
O4, (D4)	x				x		
O6, (D6)	x	x	x	x			
O6, (D6)	x	x				x	x

Table 10: Source configuration in type IIA$|_4$ with 1 set of O4-planes/D4-branes and 2 sets of O6-planes/D6-branes. Sources in brackets are not required a priori.

yields the following non-vanishing variables

$$\text{O4}: \quad f^{a_{||}}{}_{b_\perp c_\perp},\, f^{a_\perp}{}_{b_\perp c_{||}}, \quad F_2^{(0)},\, F_4^{(1)},\, H^{(0)}\,, \tag{6.61}$$

while an O6 projection is leading to

$$\text{O6}: \quad f^{a_{||}}{}_{b_\perp c_\perp},\, f^{a_\perp}{}_{b_\perp c_{||}},\, f^{a_{||}}{}_{b_{||} c_{||}}, \quad F_0^{(0)},\, F_2^{(1)},\, F_4^{(2)},\, F_6^{(3)},\, H^{(0)},\, H^{(2)}\,. \tag{6.62}$$

The non-zero variables for the source configuration in Table 10 include 12 flux components and 12 structure constants:

$$\begin{aligned}
F_2: &\quad F_{2\ 25}, \quad F_{2\ 26}, \quad F_{2\ 35}, \quad F_{2\ 36}, \\
F_4: &\quad F_{4\ 1245}, \quad F_{4\ 1246}, \quad F_{4\ 1345}, \quad F_{4\ 1346}, \\
H: &\quad H_{125}, \quad H_{126}, \quad H_{135}, \quad H_{136}, \\
f^a{}_{bc}: &\quad f^2{}_{45}, \quad f^2{}_{46}, \quad f^3{}_{45}, \quad f^3{}_{46}, \quad f^4{}_{52}, \quad f^4{}_{62}, \quad f^5{}_{42}, \quad f^6{}_{42}, \\
&\quad f^4{}_{53}, \quad f^4{}_{63}, \quad f^5{}_{43}, \quad f^6{}_{43}\,.
\end{aligned} \tag{6.63}$$

Note that this source configuration is T-dual to the previously discussed model in Table 9, which suggests a similar no-go theorem against AdS solutions. Combining the 4D trace (4.4) with the 6D (trace-reversed) Einstein equation (A.7), we deduce

$$\begin{aligned}
\mathcal{R}_{ab} = \frac{g_s^2}{2}\left(F_{2\ ac}F_{2\ b}{}^c + \frac{1}{3!}F_{4\ acde}F_{4\ b}{}^{cde}\right) + \frac{1}{4}H_{acd}H_b{}^{cd} \\
+ \frac{g_s}{2}\left(T_{ab} - \delta_{ab}\sum_i \frac{T_{10}^{(p_i)}}{p_i+1}\right) + \frac{\delta_{ab}}{4}\left(\mathcal{R}_4 + 2g_s^2|F_6|^2\right)\,.
\end{aligned} \tag{6.64}$$

The indices a, b denote the flat basis of the internal space. In this setup, where only sources of dimensionality $p_i = 6$ wrap the internal dimension 1, we get $T_{11} = T_{10}^{(6)}/7$, by implicitly summing over all sets of O6-planes/D6-branes parallel to 1. In addition, the equation (6.64) imposes further constraints on the Ricci tensor components, including $\mathcal{R}_{11} = 0$ due to $f^1{}_{ab} = f^a{}_{1b} = 0$, which are projected out by the O-planes [45]. For the source configuration in Table 10 and the flux components given in (6.63), where $F_0 = F_6 = 0$, the equation

$$\mathcal{R}_4 = g_s\frac{2T_{10}^{(4)}}{5} - 2|H|^2 - 2g_s^2|F_4|^2 \tag{6.65}$$

follows directly from (6.64). Note that F_2 is missing components along the direction 1. Comparing the equation (6.65) with the trace (4.23) for $p_i = 6$ yields $\mathcal{R}_4 = 0$, thus proving a no-go theorem against (anti-)de Sitter solutions and rendering Minkowski unique in both solution classes 9 and 10.

It would be instructive to extend these 10D no-go theorems to a 4D framework to compute the value of c in (5.18) and compare it to the rate (6.17) of the ATCC. This requires the tools from the Sections 3 and 5. As shown in equation (5.17), no-go theorems against classical (quasi-)anti-de Sitter solutions are formulated as

$$aV(\hat{\varphi}) + \sum_i b_i \partial_{\hat{\varphi}^i} V(\hat{\varphi}) \leq 0\,. \tag{6.66}$$

This formalism was introduced to compare no-go theorems with the underlying conjectures in the swampland program, such as the Trans-Planckian Censorship Conjecture. By rewriting the previous relation as

$$cV(\hat{t}^1) + \partial_{\hat{t}^1} V(\hat{t}^1) \leq 0 \tag{6.67}$$

in terms of the canonically normalized field direction $\hat{t}^1$, we have structured our comparative analysis. The scalar $\hat{t}^1$ is defined by

$$\sum_i b_i \partial_{\hat{\varphi}^i} = \sqrt{\sum_i b_i^2}\, \partial_{\hat{t}^1}\,, \tag{6.68}$$

whose exponential rate c in the potential is given in equation (5.18). We will apply this approach to the previously derived 10D no-go theorems against (quasi-)anti-de Sitter solutions in a 4D effective theory. We will then compare the exponential rate for a yet to be determined field direction with the bound (6.17) proposed by the ATCC.

For the source configuration 9, with three intersecting sets of O5-planes/D5-branes, we reformulate the 10D no-go theorem (6.60) in our 4D framework using the scalar potential $V(\hat{\rho}, \hat{\tau})$ defined in Section 3. This is leading to

$$2V + \frac{1}{\sqrt{2}} \partial_{\hat{\tau}} V + \sqrt{\frac{3}{2}} \partial_{\hat{\rho}} V = 0\,, \tag{6.69}$$

which vanishes for the configuration in question. The corresponding exponential rate $c = \sqrt{2}$ satisfies the ATCC bound (6.17) in $d = 4$. To determine the field direction t^1, we define the field space diffeomorphism for vectors $\partial_{\hat{\boldsymbol{\varphi}}}$ and 1-forms $d\hat{\boldsymbol{\varphi}}$ using the matrix [52]

$$P = \left(\frac{\partial \hat{\boldsymbol{\varphi}}}{\partial \hat{\boldsymbol{t}}} \right)\,, \qquad P^T \partial_{\hat{\boldsymbol{\varphi}}} = \partial_{\hat{\boldsymbol{t}}}\,, \qquad P^{-1} (d\hat{\boldsymbol{\varphi}})^T = (d\hat{\boldsymbol{t}})^T\,. \tag{6.70}$$

The first row of P^T comes from the equation (6.68), with its components $(P^T)_{ij}$ associated with the canonical fields $\hat{\varphi}^i$ and $\hat{t}^j$, which ensure that the matrix is orthonormal, $P^T = P^{-1}$. This property guarantees that the equation (6.68) is also valid for 1-forms and specifies the second row in P^T, leading to

$$\hat{t}^1 = \frac{b_1}{\sqrt{b_1^2 + b_2^2}} \hat{\varphi}^1 + \frac{b_2}{\sqrt{b_1^2 + b_2^2}} \hat{\varphi}^2\,, \qquad \hat{t}^2 = -\frac{b_2}{\sqrt{b_1^2 + b_2^2}} \hat{\varphi}^1 + \frac{b_1}{\sqrt{b_1^2 + b_2^2}} \hat{\varphi}^2\,, \tag{6.71}$$

as shown in [38]. For the source configuration 9, with $\hat{\varphi}^1 = \hat{\rho}$, $\hat{\varphi}^2 = \hat{\tau}$ and $\hat{t}^1 = \hat{t}$, $\hat{t}^2 = \hat{t}_\perp$, we obtain

$$\hat{t} = \frac{\sqrt{3}}{2} \hat{\rho} + \frac{1}{2} \hat{\tau}\,, \qquad \hat{t}_\perp = -\frac{1}{2} \hat{\rho} + \frac{\sqrt{3}}{2} \hat{\tau} \tag{6.72}$$

via the no-go theorem (6.69). The potential (3.38) expressed in these new fields is captured by the equation

$$2V(\hat{t},\hat{t}_\perp) = \mathrm{e}^{-\sqrt{2}\hat{t}}\left(-\mathrm{e}^{-\sqrt{\frac{2}{3}}\hat{t}_\perp}\mathcal{R}_6 - g_s\frac{T_{10}}{6}\mathrm{e}^{-\sqrt{\frac{8}{3}}\hat{t}_\perp} + \frac{1}{2}g_s^2|F_3|^2\mathrm{e}^{-\sqrt{6}\hat{t}_\perp}\right), \tag{6.73}$$

where the total prefactor reflects the exponential rate of $\hat{t}$, consistent with the bound (6.17) for $d = 4$. However, a complete comparison with the Anti-Trans-Planckian Censorship Conjecture requires evaluating the sign within the brackets, which depends on the details of the given string compactification.

Another no-go theorem in equation (6.65) applies to the solution class 10. We will reproduce this constraint in the 4D approach using the scalar potential

$$\begin{aligned}2V(\rho,\tau,\sigma_1,\sigma_2,\sigma_3) &= -\tau^{-2}\rho^{-1}\mathcal{R}_6(\sigma_1,\sigma_2,\sigma_3) + \frac{1}{2}\tau^{-2}\rho^{-3}\sigma_1^{-3}\sigma_2^3\sigma_3^3|H|^2\\ &+\frac{1}{2}\tau^{-4}\rho\sigma_1^{-2}|F_2|^2 + \frac{1}{2}\tau^{-4}\rho^{-1}\sigma_1^2|F_4|^2\\ &-\tau^{-3}\rho^{-1}\sigma_1^{\frac{-5}{2}}\sigma_2^{\frac{3}{2}}\sigma_3^{\frac{3}{2}}\frac{T_{10}^{(4)}}{5} - \tau^{-3}\sigma_1^{\frac{3}{2}}\sigma_2^{-\frac{9}{2}}\sigma_3^{-\frac{9}{2}}\left(\sigma_3^6\frac{T_{10}^{(6_2)}}{7} + \sigma_2^6\frac{T_{10}^{(6_3)}}{7}\right),\end{aligned} \tag{6.74}$$

where σ_1 is related to the set of O4-planes/D4-branes, while $\sigma_{i=2,3}$ are related to two intersecting sets of O6-planes/D6-branes with tension $T_{10}^{(6_{i=2,3})}$. For a group manifold, the Ricci scalar

$$\begin{aligned}\mathcal{R}_6(\sigma_1,\sigma_2,\sigma_3) &= R_1\sigma_1^5\sigma_2^{-9}\sigma_3^3 + R_2\sigma_1^{-7}\sigma_2^3\sigma_3^3 + R_3\sigma_1^5\sigma_2^3\sigma_3^{-9}\\ &+ R_4\,\sigma_1^{-1}\sigma_2^{-3}\sigma_3^3 + R_5\sigma_1^5\sigma_2^{-3}\sigma_3^{-3} + R_6\sigma_1^{-1}\sigma_2^3\sigma_3^{-3}\end{aligned} \tag{6.75}$$

is obtained in [36], using the following expressions

$$\begin{aligned}-2R_1 &= {f^2}_{45}{}^2 + {f^2}_{46}{}^2 + {f^3}_{45}{}^2 + {f^3}_{46}{}^2\,,\\ -2R_2 &= {f^4}_{25}{}^2 + {f^4}_{26}{}^2 + {f^4}_{35}{}^2 + {f^4}_{36}{}^2\,,\\ -2R_3 &= {f^5}_{24}{}^2 + {f^5}_{34}{}^2 + {f^6}_{24}{}^2 + {f^6}_{34}{}^2\,,\\ -R_4 &= {f^2}_{45}{f^4}_{25} + {f^2}_{46}{f^4}_{26} + {f^3}_{45}{f^4}_{35} + {f^3}_{46}{f^4}_{36}\,,\\ R_5 &= {f^2}_{45}{f^5}_{24} + {f^3}_{45}{f^5}_{34} + {f^2}_{46}{f^6}_{24} + {f^3}_{46}{f^6}_{34}\,,\\ -R_6 &= {f^4}_{25}{f^5}_{24} + {f^4}_{35}{f^5}_{34} + {f^4}_{26}{f^6}_{24} + {f^4}_{36}{f^6}_{34}\,.\end{aligned} \tag{6.76}$$

The 10D no-go theorem is given by the following linear combination of the potential (6.74) and its first derivatives,

$$V + \frac{1}{3}\rho\partial_\rho V + \frac{1}{4}\tau\partial_\tau V + \frac{1}{6}\sum_{i=1}^{3}\sigma_i\partial_{\sigma_i}V = 0\,, \tag{6.77}$$

which vanishes for the source configuration 10 with the variables in (6.63). To derive the corresponding value of c from the equation (6.67), we rewrite (6.77) in terms of canonically normalized fields. Although we know the transformation laws for ρ and τ, finding those for $\sigma_{i=1,2,3}$ requires diagonalizing the field space metric [36]

$$G_{ij} = \begin{pmatrix} \frac{3}{2}\rho^{-2} & 0 & 0 & 0 & 0 \\ 0 & 2\tau^{-2} & 0 & 0 & 0 \\ 0 & 0 & \frac{15}{2}\sigma_1^{-2} & -\frac{9}{2}(\sigma_1\sigma_2)^{-1} & -\frac{9}{2}(\sigma_1\sigma_3)^{-1} \\ 0 & 0 & -\frac{9}{2}(\sigma_1\sigma_2)^{-1} & \frac{27}{2}\sigma_2^{-2} & -\frac{9}{2}(\sigma_2\sigma_3)^{-1} \\ 0 & 0 & -\frac{9}{2}(\sigma_1\sigma_3)^{-1} & -\frac{9}{2}(\sigma_2\sigma_3)^{-1} & \frac{27}{2}\sigma_3^{-2} \end{pmatrix} . \tag{6.78}$$

More specifically, we solve the eigenvalue problem for the metric block of the scalars σ_i to determine the canonical $\hat{\sigma}_i$ fields satisfying $G_{ij}\partial\sigma_i\partial\sigma_j = \sum_i(\partial\hat{\sigma}_i)^2$. This leads to

$$\begin{gathered} \hat{\sigma}_1 = 3\ln\left(\frac{\sigma_2}{\sigma_3}\right) , \quad \hat{\sigma}_2 = \sqrt{\beta_2}\ln\left(\sigma_1^{\alpha_2}\sigma_2\sigma_3\right) , \quad \hat{\sigma}_3 = \sqrt{\beta_3}\ln\left(\sigma_1^{\alpha_3}\sigma_2\sigma_3\right) , \\ \alpha_2 = \frac{1}{6}(1-\sqrt{73}) , \ \alpha_3 = \frac{1}{6}(1+\sqrt{73}) , \ \beta_2 = \frac{9}{4}\left(1+\frac{7}{\sqrt{73}}\right) , \ \beta_3 = \frac{9}{4}\left(1-\frac{7}{\sqrt{73}}\right) . \end{gathered} \tag{6.79}$$

The transformation laws of the vectors $\partial_{\boldsymbol{\sigma}}, \partial_{\hat{\boldsymbol{\sigma}}}$ are obtained from the diffeomorphism matrix with components

$$\left(\frac{\partial\hat{\boldsymbol{\sigma}}}{\partial\boldsymbol{\sigma}}\right)_{ij} = \left(\frac{\partial\hat{\sigma}_i}{\partial\sigma_j}\right) = \begin{pmatrix} 0 & 3\sigma_2^{-1} & -2\sigma_3^{-1} \\ \alpha_2\sqrt{\beta_2}\sigma_1^{-1} & \sqrt{\beta_2}\sigma_2^{-1} & \sqrt{\beta_2}\sigma_3^{-1} \\ \alpha_3\sqrt{\beta_3}\sigma_1^{-1} & \sqrt{\beta_3}\sigma_2^{-1} & \sqrt{\beta_3}\sigma_3^{-1} \end{pmatrix} , \tag{6.80}$$

resulting in

$$\begin{pmatrix} \partial_{\sigma_1} \\ \partial_{\sigma_2} \\ \partial_{\sigma_3} \end{pmatrix} = \left(\frac{\partial\hat{\boldsymbol{\sigma}}}{\partial\boldsymbol{\sigma}}\right)^T \begin{pmatrix} \partial_{\hat{\sigma}_1} \\ \partial_{\hat{\sigma}_2} \\ \partial_{\hat{\sigma}_3} \end{pmatrix} = \begin{pmatrix} \sigma_1^{-1}(\alpha_2\sqrt{\beta_2}\partial_{\hat{\sigma}_2} + \alpha_3\sqrt{\beta_3}\partial_{\hat{\sigma}_3}) \\ \sigma_2^{-1}(3\partial_{\hat{\sigma}_1} + \sqrt{\beta_2}\partial_{\hat{\sigma}_2} + \sqrt{\beta_3}\partial_{\hat{\sigma}_3}) \\ \sigma_3^{-1}(-3\partial_{\hat{\sigma}_1} + \sqrt{\beta_2}\partial_{\hat{\sigma}_2} + \sqrt{\beta_3}\partial_{\hat{\sigma}_3}) \end{pmatrix} , \tag{6.81}$$

due to the equation (6.70). This transformation allows us to reformulate the combination (6.77) as

$$\begin{aligned} & V + \frac{1}{4}\tau\partial_\tau V + \frac{1}{3}\rho\partial_\rho V + \frac{1}{6}\sum_{I=1}^{3}\sigma_I\partial_{\sigma_I}V = 0 \\ & \qquad \Rightarrow \quad V + \frac{1}{2\sqrt{2}}\partial_{\hat{\tau}}V + \frac{1}{\sqrt{6}}\partial_{\hat{\rho}}V + \frac{2+\alpha_2}{6}\sqrt{\beta_2}\partial_{\hat{\sigma}_2}V + \frac{2+\alpha_3}{6}\sqrt{\beta_3}\partial_{\hat{\sigma}_3}V = 0 , \end{aligned} \tag{6.82}$$

in terms of canonical fields. The resulting value of c, derived from the equation (6.67) as

$$c = \sqrt{2} , \tag{6.83}$$

satisfies the rate (6.17) proposed by the ATCC in $d = 4$. This is expected since the no-go theorem is T-dual to the previous one in (6.69), which suggests the same rate c even if different equations are used in the 10D derivation. To clarify, this no-go theorem specifies a particular direction of a canonically normalized field, along with the associated exponential rate in the scalar potential given by (6.83). This definition is consistent with methods previously established.

6.5.2 DGKT-inspired potentials

In the last part of Section 6.5 we study a family of classical AdS solutions in a 4D effective theory, known as DGKT solutions [70, 101, 158]. Furthermore, due to the complexities previously outlined in $d = 3$, we also probe similar compactifications to a 3D AdS spacetime [131, 132].

The original DGKT solution

We begin our analysis with the simplest example of classical AdS_4 solutions, which are obtained by the compactification of type IIA SUGRA on $T^6/\mathbb{Z}_3^2$ to a 4D effective theory with reduced $\mathcal{N} = 1$ supersymmetry. This is the original DGKT solution [70, 97, 101]. The source content in this model is built entirely from smeared O6-planes/D6-branes. To stabilize the moduli, we require the presence of the NSNS field strength along with several RR fluxes of different form degrees,

$$H_3 = -p\beta_0\,, \qquad F_0 = m_0\,, \qquad F_2 = 0\,, \qquad F_4 = e_i\tilde{w}^i\,, \tag{6.84}$$

with parameters p, m_0, e_i. We define a basis β_0, representing the imaginary component of the holomorphic 3-form in a Calabi-Yau compactification, which is odd under the orientifold projection $\Omega_p(-1)^{F_L}\sigma_6$. This involves the target space involution

$$\sigma_6: \quad z_i \to -\bar{z}_i\,, \tag{6.85}$$

where $i = 1, 2, 3$. The z^i are the complex coordinates of the compact space, the three 2-tori. Moreover, we construct a basis $\{w_i\}$ of 2-cycles,

$$w_i = \left(\kappa\sqrt{3}\right)^{1/3} \mathrm{i}\,\mathrm{d}z^i \wedge \mathrm{d}\bar{z}^i\,, \qquad \kappa \equiv \int_{T^6/\mathbb{Z}_3^2} w^1 \wedge w^2 \wedge w^3\,, \tag{6.86}$$

which is odd under the transformation (6.85), where the intersection number κ is arbitrary. The dual basis $\{\tilde{w}_i\}$ of even 4-forms is given by

$$\tilde{w}_i = \left(\frac{3}{\kappa}\right)^{1/3} (\mathrm{i}\,\mathrm{d}z^j \wedge \mathrm{d}\bar{z}^j) \wedge (\mathrm{i}\,\mathrm{d}z^k \wedge \mathrm{d}\bar{z}^k)\,, \qquad \int_{T^6/\mathbb{Z}_3^2} w_i \wedge \tilde{w}^j = \delta_i^j\,, \tag{6.87}$$

where j, k are two indices, each taking the values 1, 2 or 3, different from i.

In the following discussion we briefly review and summarize several results of [70, 185] and the formalism in [157, 181]. The moduli space comprises two distinct components. The first part encompasses the vector multiplets, which include the Kähler moduli, while the second part consists of the hypermultiplets, incorporating the dilaton and the complex structure moduli. The metric on each component is determined by its respective Kähler potential, which will play an important role in defining the canonical fields. Due to the discrete symmetry of the internal space, the moduli of the effective theory are reduced to the γ_i fields and the scalars b_i, related to the NSNS 2-form potential

$$B_2 = \sum_{i=1}^{3} b_i w^i \,, \tag{6.88}$$

for each torus in $T^6 = T^2 \times T^2 \times T^2$. The former correspond to the sizes of each 2-torus and appear in the parameterization of the internal space metric,

$$\mathrm{d}s_6^2 = \sum_{i=1}^{3} \gamma_i \mathrm{d}z^i \mathrm{d}\bar{z}^i \,. \tag{6.89}$$

However, rather than using the three complex parameters γ_i, we consider the rescaled quantities

$$\upsilon_i = \frac{1}{2}\frac{1}{(\kappa\sqrt{3})^{1/3}}\gamma_i \,, \tag{6.90}$$

which are identified with the Kähler moduli of 4D supergravity. Therefore, the volume of the internal manifold results in

$$\mathrm{vol} \equiv \int_{T^6/\mathbb{Z}_3^2} \mathrm{d}^6 y \sqrt{g_6} = \kappa \upsilon_1 \upsilon_2 \upsilon_3 \,. \tag{6.91}$$

In addition, we consider the 4-dimensional dilaton D and its partner, the axion ξ, arising from the C_3 potential. The relation between these superfields and the 10D dilaton ϕ is given by

$$\mathrm{e}^D = \frac{\mathrm{e}^\phi}{\sqrt{\mathrm{vol}}} \,. \tag{6.92}$$

Note that there are no moduli associated with the RR 1-form C_1 and the complex structure moduli are projected out. Following the previous examples, our path is set. First we have to identify the kinetic terms of the scalars to derive the transformation laws of the canonical fields. Our strategy is based on the formalism of $\mathcal{N} = 1$ supergravity, as detailed in [185] and further supported by more recent work [157, 181]. Also have a look at the work in [99, 186, 187] for a better understanding of this section.

Before we continue, we briefly review the stability analysis in [70]. The scalar fields b_i, ξ are fixed by the AdS scale alone, independent of any field strength, while the few remaining moduli covering the spectrum of metric and dilaton fluctuations are stabilized by the fluxes. Again, it is important to remember that in AdS spacetime the presence of a tachyonic mode does not necessarily imply instability. Tachyonic fields with a negative m^2 beyond the Breitenlohner-Freedman bound (6.42) do not lead to unstable perturbations. We then derive the kinetic terms for the remaining moduli by introducing the superfields [157]

$$t_i = b_i + \mathrm{i}\upsilon_i \,, \quad S = \mathrm{e}^{-D} + \mathrm{i}\frac{\xi}{\sqrt{2}} \,, \quad \text{with } i = 1, 2, 3 \,. \tag{6.93}$$

The metric on the moduli space is defined with respect to the Kähler potential,

$$K = -\ln\left(\frac{4}{3} \times 6 \text{ vol}\right) - 4\ln\left(S + \bar{S}\right) \,, \tag{6.94}$$

whose derivatives define the Kähler metric $K_{I\bar{J}}$, according to

$$K_{S\bar{S}} = \frac{\partial^2 K}{\partial S \partial \bar{S}} = \mathrm{e}^{2D}\,, \qquad K_{t^i\bar{t}^i} = \frac{\partial^2 K}{\partial t^i \partial \bar{t}^i} = \frac{1}{4\upsilon_i^2}\,. \tag{6.95}$$

Therefore, the kinetic terms of the superfields Φ^I in equation (6.93) are given by

$$e^{-1}\mathcal{L}_{\text{kin}} = K_{I\bar{J}}\partial\Phi^I\partial\Phi^{\bar{J}} = (\partial D)^2 + \frac{1}{4}\sum_{i=1}^{3}\frac{1}{\upsilon_i^2}(\partial\upsilon_i)^2\,. \tag{6.96}$$

As mentioned above, the axion ξ and the b_i fields have no physical degrees of freedom. We also introduce the volume e of the external spacetime. Since the geometry of our problem is isotropic, we can reduce the degrees of freedom for the components of the rescaled metric (6.90) by fixing $\upsilon_i = \upsilon/|e_i|$. We are left with only one Kähler modulus υ. After introducing the scalar fields g and r [70],

$$\mathrm{e}^D = |p|\sqrt{\frac{|m_0|}{E}}g\,, \qquad \upsilon = \sqrt{\frac{E}{|m_0|}}r^2\,, \tag{6.97}$$

where we set $E = \upsilon^3/\mathrm{vol} = |e_1e_2e_3|/\kappa$, the Lagrangian of the kinetic terms can be rewritten in a simple form,

$$e^{-1}\mathcal{L}_{\text{kin}} = \frac{1}{2}\left(\frac{2}{g^2}(\partial g)^2 + \frac{6}{r^2}(\partial r)^2\right)\,. \tag{6.98}$$

From this equation, the transformation laws to canonical fields are easy to find,

$$r = \mathrm{e}^{\frac{1}{\sqrt{6}}\hat{r}}\,, \qquad g = \mathrm{e}^{\frac{1}{\sqrt{2}}\hat{g}}\,. \tag{6.99}$$

In the second part, we derive the scalar potential of the effective theory. The presence of a non-vanishing F_0 flux, characterized by the mass parameter m_0, requires the use of massive type IIA supergravity, with the action [70, 188]

$$S = S_{\text{bulk}} + S_{\text{loc}}^{(6)} = S_{\text{bulk}} + S_{\text{DBI}}^{(6)} + S_{\text{CS}}^{(6)}\,. \tag{6.100}$$

This equation includes the action of the bulk fields,

$$S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2}\int \mathrm{d}^{10}x\sqrt{-g}\left(\mathrm{e}^{-2\phi}\left(\mathcal{R} + 4(\partial\phi)^2 - \frac{1}{2}|H_3|^2\right) - \left(m_0^2 + |F_2|^2 + |F_4|^2\right)\right)\,, \tag{6.101}$$

in the string frame[22]. The Dirac-Born-Infeld and the Chern-Simons part of the O6-planes/D6-branes is given by

$$S_{\text{loc}}^{(6)} = -T_6\int_{\text{O6}}\mathrm{d}^7x\sqrt{-g_7}\,\mathrm{e}^{-\phi} + \sqrt{2}\,T_6\int_{\text{O6}}C_7\,. \tag{6.102}$$

[22]Note that we adhere to the conventions outlined in [185] for the RR fields, including the treatment of the Romans mass. In particular, there is a change in notation from the one used in the rest of this thesis, which follows Polchinski's conventions [2, 3]. This difference leads to the relation $C_q = C_q^{(\text{Polch.})}/\sqrt{2}$.

We can ignore the Chern-Simons term from this discussion since it does not affect our results. As shown in [70], O6-planes induce a tadpole for C_7 that can be eliminated by adding background fluxes. Therefore, we can safely ignore the second term in the equation (6.102). Having selected fluxes that are consistent with the tadpole cancellation condition, we proceed from the action (6.100) to systematically study the contribution of each term to the effective potential of the moduli fields. In order to define the potential, we assume that the scalars v_i, b_i, ϕ and ξ are independent of the internal coordinates. Because of the relation

$$\int_{T^6/\mathbb{Z}_3^2} \mathrm{d}^6 y \sqrt{g_6}\, \mathrm{e}^{-2\phi} = \mathrm{vol}\, \mathrm{e}^{-2\phi}\,, \tag{6.103}$$

the transition to the 4D Einstein frame implies the redefinition

$$g_{\mu\nu} = \frac{\mathrm{e}^{2\phi}}{\mathrm{vol}}\, \hat{g}_{\mu\nu}\,, \tag{6.104}$$

which leads to the following equation for the Ricci scalar

$$\sqrt{-g_4}\, \mathcal{R}_4 = \frac{\mathrm{e}^{2\phi}}{\mathrm{vol}} \sqrt{-\hat{g}_4}\, \hat{\mathcal{R}}_4\,. \tag{6.105}$$

Now that we have already introduced the kinetic term, we will focus on the contribution of the fluxes and sources to the effective potential. Substituting the background fluxes from (6.84) into the equation (6.101) we obtain

$$\begin{aligned} |F_4|^2\, \mathrm{vol}_{10} = F_4 \wedge *F_4 = e_i e_j (\tilde{w}^i \wedge *\tilde{w}^j) = \left(\sum_{i=1}^{3} e_i^2 v_i^2\right) (\mathrm{vol})^{-2}\,, \\ |H_3|^2\, \mathrm{vol}_{10} = H_3 \wedge *H_3 = p^2 (\beta_0 \wedge *\beta_0) = p^2 (\mathrm{vol})^{-1}\,. \end{aligned} \tag{6.106}$$

As a result, the contribution of the fluxes to the potential is given by

$$V = \underbrace{\frac{p^2}{4} \frac{\mathrm{e}^{2\phi}}{(\mathrm{vol})^2}}_{H\text{ flux}} + \underbrace{\frac{1}{2}\left(\sum_{i=1}^{3} e_i^2 v_i^2\right) \frac{\mathrm{e}^{4\phi}}{(\mathrm{vol})^3}}_{F_4\text{ flux}} + \underbrace{\frac{m_0^2}{2} \frac{\mathrm{e}^{4\phi}}{\mathrm{vol}}}_{\text{Romans}} + \ldots\,. \tag{6.107}$$

Since the spacetime-filling O6-planes wrap internal 3-cycles, the metric determinant can be decomposed as follows,

$$\sqrt{-g_7} = \sqrt{-g_4}\sqrt{g_3} = \frac{\mathrm{e}^{4\phi}}{(\mathrm{vol})^2} \sqrt{-\hat{g}_4}\sqrt{g_3}\,. \tag{6.108}$$

Substituting this expansion into the Dirac-Born-Infeld action (6.102),

$$S_{\mathrm{loc}}^{(6)} = -T_6 \int \mathrm{d}^4 x \sqrt{-\hat{g}_4} \left(\int \mathrm{d}^3 y \sqrt{g_3}\, \frac{\mathrm{e}^{3\phi}}{(\mathrm{vol})^2} \right)\,, \tag{6.109}$$

yields the contribution of the sources [189]

$$V_{\rm O6} = -2\kappa_{10}^2\, T_6 \frac{\mathrm{e}^{3\phi}}{(\mathrm{vol})^2} \int \mathrm{d}^3 y \sqrt{g_3}\,, \tag{6.110}$$

where the integral $\int \mathrm{d}^3 y \sqrt{g_3}$ scales with $(\mathrm{vol})^{1/2}$. Under the tadpole cancellation condition [70], the tension of the sources is given by $m_0 p = -2\kappa_{10}^2 \sqrt{2} T_6$, in terms of flux numbers (6.84). Therefore, the effective potential results in

$$V = \frac{p^2}{4} \frac{\mathrm{e}^{2\phi}}{(\mathrm{vol})^2} + \frac{1}{2}\left(\sum_{i=1}^{3} e_i^2 \upsilon_i^2\right) \frac{\mathrm{e}^{4\phi}}{(\mathrm{vol})^3} + \frac{m_0^2}{2} \frac{\mathrm{e}^{4\phi}}{\mathrm{vol}} - \sqrt{2}|m_0 p| \frac{\mathrm{e}^{3\phi}}{(\mathrm{vol})^{3/2}}\,. \tag{6.111}$$

This equation can be written as

$$V = \frac{p^2}{4}\frac{\mathrm{e}^{2D}}{\upsilon^3} E + \frac{3}{2}\frac{\mathrm{e}^{4D}}{\upsilon} E + \frac{m_0^2}{2}\frac{\mathrm{e}^{4D}}{E}\upsilon^3 - \sqrt{2}|m_0 p| \mathrm{e}^{3D}\,, \tag{6.112}$$

using the volume of the Calabi-Yau manifold (6.91) and the 4D dilaton (6.92), and is finally given by

$$\frac{1}{\lambda} V(g,r) = \frac{1}{4} g^2 r^{-6} + \frac{3}{2} g^4 r^{-2} + \frac{1}{2} g^4 r^6 - \sqrt{2} g^3\,, \tag{6.113}$$

due to the equation (6.97) and the definition of $\lambda = p^4 |m_0|^{5/2} E^{-3/2}$. Using the transformation laws for canonical fields (6.99), the scalar potential becomes

$$\frac{1}{\lambda} V(\hat{g},\hat{r}) = \underbrace{\frac{1}{4}\mathrm{e}^{\sqrt{2}\hat{g}}\mathrm{e}^{-\sqrt{6}\hat{r}}}_{H\ \text{flux}} + \underbrace{\frac{3}{2}\mathrm{e}^{2\sqrt{2}\hat{g}}\mathrm{e}^{-\sqrt{\frac{2}{3}}\hat{r}}}_{F_4\ \text{flux}} + \underbrace{\frac{1}{2}\mathrm{e}^{2\sqrt{2}\hat{g}}\mathrm{e}^{\sqrt{6}\hat{r}}}_{\text{Romans}} - \underbrace{\sqrt{2}\mathrm{e}^{\frac{3}{\sqrt{2}}\hat{g}}}_{\text{O6-planes}}\,. \tag{6.114}$$

Again, note that the conventions of 10D supergravity differ slightly from our approach, which may lead to small changes in the numerical coefficients of the potential. However, since our focus is on the exponential rates, these small differences do not significantly affect our analysis. In our discussion of the DGKT potential in (6.114), we observe no violation of the bound (6.17) proposed by the ATCC, a result that is not unexpected. The relation to the scalar fields ρ, τ can be clearly seen from the equations (3.38) and (6.113),

$$\rho \propto r^2\ (\upsilon)\,, \qquad \tau \propto g^{-1}\ (\mathrm{e}^{-D})\,. \tag{6.115}$$

As shown in Section 6.5.1, the exponential rates of these fields are consistent with the ATCC bound, thus confirming our intuition.

Similar to the previous sections, we study a specific direction in the field space, which shows the expected behavior of the potential in the asymptotics. From the on-shell equation

$$g\partial_g V + 2r\partial_r V = 0\,, \tag{6.116}$$

we derive the constraint

$$g r^6 = \frac{5}{4\sqrt{2}} \tag{6.117}$$

on the scalar fields. Because of the relation $\partial_\mu \hat{g} = -2\sqrt{3}\partial_\mu \hat{r}$, the kinetic term in the equation (6.98) becomes $(\partial\hat{g})^2 + (\partial\hat{r})^2 = 13(\partial\hat{r})^2 \equiv (\partial\hat{t})^2$, where we have introduced the new field $\hat{t}$ according to

$$\partial_\mu \hat{t} = \sqrt{13}\partial_\mu \hat{r} = -\frac{\sqrt{13}}{2\sqrt{3}}\partial_\mu \hat{g}\,. \tag{6.118}$$

By specifying $\partial_\mu \hat{t}_\perp = 0$, we remove any orthonormal direction $\hat{t}_\perp$. If we integrate the equation (6.118) and simplify the result by setting the integration constant to zero, we obtain

$$\hat{t} = \sqrt{13}\hat{r} = -\frac{\sqrt{13}}{2\sqrt{3}}\hat{g} + \sqrt{\frac{13}{6}}\ln\left(\frac{5}{4\sqrt{2}}\right)\,. \tag{6.119}$$

Finally, with this new field direction $\hat{t}$, we can write the potential as

$$\frac{1}{\lambda}V(\hat{t}) = \frac{1875}{2048}\mathrm{e}^{-\sqrt{\frac{26}{3}}\hat{t}} - \frac{975}{2048}\mathrm{e}^{-3\sqrt{\frac{6}{13}}\hat{t}}\,, \tag{6.120}$$

which has the expected behavior in the asymptotics of field space, as shown in Figure 12. The exponential rate of the term which dominates for $\hat{t} \to \infty$ satisfies the bound

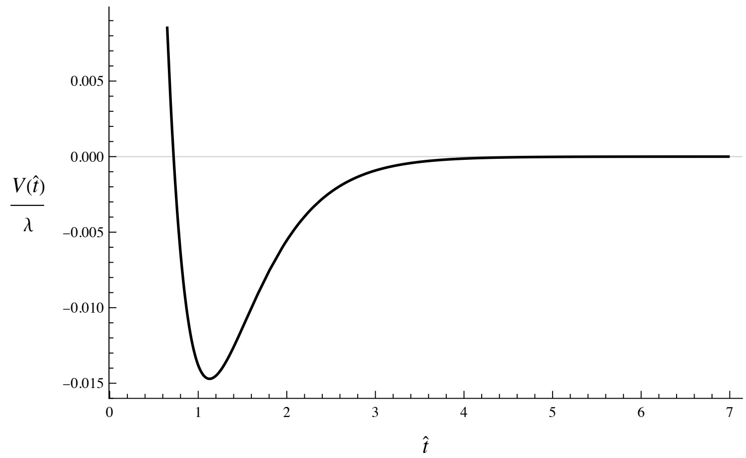


Figure 12: This figure shows the potential $V(\hat{t})$ as defined in (6.120), highlighting its expected behavior in the asymptotic regions of field space.

$$3\sqrt{\frac{6}{13}} > \sqrt{\frac{2}{3}} \tag{6.121}$$

proposed by the ATCC in $d = 4$. This finding is particularly noteworthy given the unique features of the proposed AdS_4 solutions, which include parametric control in the classical regime, complete moduli stabilization and scale separation.

DGKT-inspired AdS_3 solutions

After studying the DGKT construction in $d = 4$, we turn our attention to potentials derived from flux compactifications leading to DGKT-like AdS_3 vacua [131]. We continue our work in 10D type IIA supergravity, where the internal space is given by $T^7/\mathbb{Z}_2^3$ with G_2 holonomy, leading to AdS_3 vacua with minimal SUSY. The sources and fluxes of this model include smeared O2-/O6-planes and non-vanishing F_0, F_4 and H. These solutions retain some of the intriguing properties of the AdS_4 vacua summarized at the end of the previous section. We will avoid going into the details of the compactification. Instead, we will identify the independent moduli and their kinetic terms, which lead to the transformation laws for canonical fields. Finally, we will study the scalar potential, focusing in particular on its behavior along a specific direction in field space.

This model includes several scalar fields: The universal moduli x and y are given by

$$\frac{x}{\sqrt{7}} = -\frac{3}{8}\phi + \frac{\beta}{2}\upsilon\,, \qquad y = -\frac{1}{4}\phi - \frac{21\beta}{2}\upsilon\,. \tag{6.122}$$

in terms of the 10D dilaton ϕ and the internal volume υ, with $\alpha = \sqrt{7}/4$ and $\beta = -\alpha/7$. The latter appears in the decomposition of the 10D metric,

$$\mathrm{d}s_{10}^2 = \mathrm{e}^{2\alpha\upsilon}\mathrm{d}s_3^2 + \mathrm{e}^{2\beta\upsilon}\mathrm{d}\tilde{s}_7^2\,. \tag{6.123}$$

Furthermore, we introduce the scalar fields $s^{i=1,\dots,7}$, which correspond to the volumes of 3-cycles. Therefore, the volume of the internal manifold scales as

$$\mathrm{vol}_7 \sim \left(\prod_{i=1}^{7} s^i\right)^{\frac{1}{3}} \sim \left(\mathrm{e}^{\beta\upsilon}\right)^7\,, \tag{6.124}$$

which is consistent with the metric (6.123). Rather than keeping the scalars s^i, we define the unit volume fluctuations $\hat{s}^i$,

$$s^i = \tilde{s}^i\mathrm{e}^{3\beta\upsilon}\,, \qquad \prod_{i=1}^{7}\tilde{s}^i = 1\,, \tag{6.125}$$

which are moduli for the G_2 metric deformations. Note that only 6 moduli fields $\tilde{s}^{a=1,..,6}$ are linearly independent because of the second condition in the equation above. The Lagrangian for the kinetic terms of all 9 scalar fields is given by [131]

$$e^{-1}\mathcal{L}_{\mathrm{kin}} = -\frac{1}{4}(\partial x)^2 - \frac{1}{4}(\partial y)^2 - \frac{\delta_{ij}}{4\tilde{s}^i\tilde{s}^j}\partial_\mu\tilde{s}^i\partial^\mu\tilde{s}^j\,, \tag{6.126}$$

with the 3D volume e. With the equation (6.125) and the definition of the field space metric $\tilde{G}_{ab} = (\delta_{ab}+1)/(4\tilde{s}^a\partial\tilde{s}^b)$ [116] we conclude that

$$e^{-1}\mathcal{L}_{\mathrm{kin}} = -\frac{1}{4}(\partial x)^2 - \frac{1}{4}(\partial y)^2 - \tilde{G}_{ab}\partial_\mu\tilde{s}^a\partial^\mu\tilde{s}^b\,. \tag{6.127}$$

This results in the transformation laws to canonical fields,

$$\hat{x} = \frac{1}{\sqrt{2}}x\,, \qquad \hat{y} = \frac{1}{\sqrt{2}}y\,, \tag{6.128}$$

for the universal moduli x, y. Regarding the fields $\hat{s}^a$, we establish an orthonormal basis by addressing the eigenvalue problem defined by the field space metric $\tilde{G}_{ab}$. By additionally normalizing the eigenvectors by the square root of their respective eigenvalues, we derive the following expressions for the canonical fields $\hat{s}^a$,

$$
\begin{aligned}
\ln \tilde{s}^1 &= \frac{1}{\sqrt{21}}\hat{s}^1 - \hat{s}^2 - \frac{1}{\sqrt{3}}\hat{s}^3 - \frac{1}{\sqrt{6}}\hat{s}^4 - \frac{1}{\sqrt{10}}\hat{s}^5 - \frac{1}{\sqrt{15}}\hat{s}^6\,,\\
\ln \tilde{s}^2 &= \frac{1}{\sqrt{21}}\hat{s}^1 + \sqrt{\frac{5}{3}}\hat{s}^6\,,\\
\ln \tilde{s}^3 &= \frac{1}{\sqrt{21}}\hat{s}^1 + 2\sqrt{\frac{2}{5}}\hat{s}^5 - \frac{1}{\sqrt{15}}\hat{s}^6\,,\\
\ln \tilde{s}^4 &= \frac{1}{\sqrt{21}}\hat{s}^1 + \sqrt{\frac{3}{2}}\hat{s}^4 - \frac{1}{\sqrt{10}}\hat{s}^5 - \frac{1}{\sqrt{15}}\hat{s}^6\,,\\
\ln \tilde{s}^5 &= \frac{1}{\sqrt{21}}\hat{s}^1 + \frac{2}{\sqrt{3}}\hat{s}^3 - \frac{1}{\sqrt{6}}\hat{s}^4 - \frac{1}{\sqrt{10}}\hat{s}^5 - \frac{1}{\sqrt{15}}\hat{s}^6\,,\\
\ln \tilde{s}^6 &= \frac{1}{\sqrt{21}}\hat{s}^1 + \hat{s}^2 - \frac{1}{\sqrt{3}}\hat{s}^3 - \frac{1}{\sqrt{6}}\hat{s}^4 - \frac{1}{\sqrt{10}}\hat{s}^5 - \frac{1}{\sqrt{15}}\hat{s}^6\,,
\end{aligned}
\tag{6.129}
$$

where $2\tilde{G}_{ab}\partial_\mu \tilde{s}^a \partial^\mu \tilde{s}^b = \delta_{ab}\partial_\mu \hat{s}^a \partial^\mu \hat{s}^b$. The scalar potential is given by [131]

$$
V(\hat{x},\hat{y},\tilde{s}^a) = \underbrace{F(\tilde{s}^a)\mathrm{e}^{2\sqrt{2}\hat{y}-2\sqrt{\frac{2}{7}}\hat{x}}}_{F_4\ \text{flux}} + \underbrace{H(\tilde{s}^a)\mathrm{e}^{2\sqrt{2}\hat{y}+2\sqrt{\frac{2}{7}}\hat{x}}}_{H\ \text{flux}} + \underbrace{C\mathrm{e}^{\sqrt{2}\hat{y}-\sqrt{14}\hat{x}}}_{\text{Romans}} - \underbrace{T(\tilde{s}^a)\mathrm{e}^{\frac{3}{2}\sqrt{2}\hat{y}-\frac{5}{2}\sqrt{\frac{2}{7}}\hat{x}}}_{\text{O6-planes}}\,,
\tag{6.130}
$$

which introduces the Romans mass $C = m^2/16$ and the functions

$$
\begin{aligned}
F(\tilde{s}^a) &= \frac{f^2}{16}\left(\sum_{a=1}^{6}(\tilde{s}^a)^2 + 36\prod_{a=1}^{6}(\tilde{s}^a)^{-2}\right)\,,\\
H(\tilde{s}^a) &= \frac{h^2}{16}\left(\sum_{a=1}^{6}(\tilde{s}^a)^{-2} + \prod_{a=1}^{6}(\tilde{s}^a)^{2}\right)\,,\\
T(\hat{s}^a) &= \frac{hm}{8}\left(\sum_{a=1}^{6}(\tilde{s}^a)^{-1} + \prod_{a=1}^{6}\tilde{s}^a\right)\,.
\end{aligned}
\tag{6.131}
$$

In this case, $F_0 = m$ corresponds to the mass parameter, while f and h denote the flux quanta for F_4 and H_3, respectively, which have to be quantized accordingly. The term hm refers to the tension of the O6-planes due to the tadpole cancellation, or equivalently the Bianchi identity of the sources. We also find that $V_{\mathrm{O2}} = -V_{\mathrm{D2}}$ and the flux F_6 is trivial in the smeared limit. We compare the scalar potential $V(\hat{x},\hat{y},\tilde{s}^a)$ with (3.38) to obtain the equations

$$
\hat{\rho} = -\frac{1}{4\sqrt{2}}\left(\sqrt{7}\hat{y} + 5\hat{x}\right)\,, \qquad \hat{\tau} = -\frac{1}{4\sqrt{2}}\left(5\hat{y} - \sqrt{7}\hat{x}\right)
\tag{6.132}
$$

between the universal scalar fields in $d = 3$. The potential is shown in Figures 13 and 14, where we first have to fix the remaining flux numbers by the flux quantization conditions [131],

$$
h = (2\pi)^2 K\,, \qquad m = (2\pi)^{-1}M\,, \qquad f = (2\pi)^3 N\,, \qquad KM = 16\,,
\tag{6.133}
$$

with $N, K, M \in \mathbb{Z}$. The tadpole cancellation condition leads to the final relation. For illustration, we set $K = 16$, $M = N = 1$ where (6.130) reaches a critical point, suggesting the presence of an AdS vacuum. In our search for supersymmetric solutions, we have to ensure that the derivatives of the superpotential [131],

$$P = -\frac{f}{8}\mathrm{e}^{y-\frac{x}{\sqrt{7}}}\left(\sum_{a=1}^{6}\tilde{s}^a - 6\prod_{a=1}^{6}(\tilde{s}^a)^{-1}\right) + \frac{h}{8}\mathrm{e}^{y+\frac{x}{\sqrt{7}}}\left(\sum_{a=1}^{6}(\tilde{s}^a)^{-1} + \prod_{a=1}^{6}\tilde{s}^a\right) + \frac{m}{8}\mathrm{e}^{\frac{y}{2}-\frac{\sqrt{7}}{2}x}\,, \tag{6.134}$$

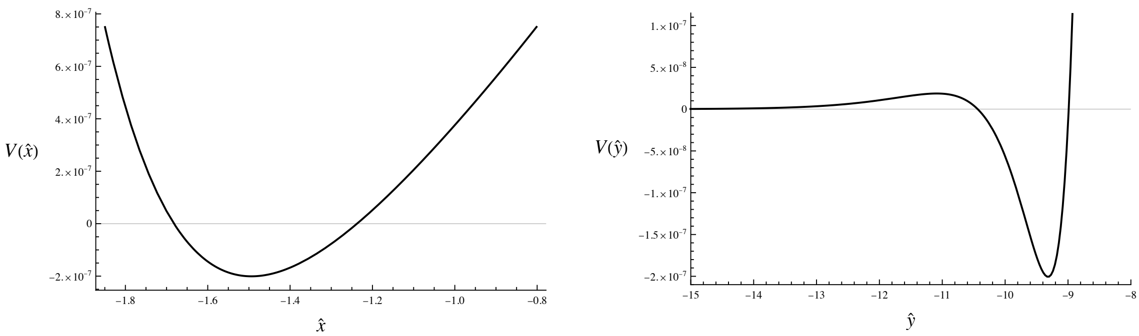
with respect to $\hat{x}$, $\hat{y}$ and $\tilde{s}^a$ vanish. In the framework of 3D massive supergravity, the scalar potential takes the from [131]

$$V(\varphi) = G^{ij}\partial_i P \partial_j P - 4P^2\,, \tag{6.135}$$

with the field space metric $G_{ij} = \mathrm{diag}(1/4, 1/4, \tilde{G}_{ab})$. The scalar potential (6.130) then results from our choice of P. The supersymmetric conditions, $\partial_i P = 0$, lead to a supersymmetric AdS vacua with

$$\frac{h}{f}e^{2\sqrt{\frac{2}{7}}\hat{x}_0} = 0.515696\,, \qquad \frac{m}{f}e^{-\frac{\sqrt{2}}{2}\hat{y}_0-\frac{5}{2}\sqrt{\frac{2}{7}}\hat{x}_0} = 3.43111\,, \qquad \tilde{s}_0^a = 1.32691\,. \tag{6.136}$$

We can see that $\hat{x}_0 = -1.49381$ and $\hat{y}_0 = -9.31713$. Unfortunately, the potential (6.130) does not show the expected behavior in the asymptotics of the field space along any field direction. A more detailed discussion of the exponential rates of the potential shows that the only negative term, the contribution of the O6-planes, is dominated by positive terms in the asymptotic limit of $\hat{x}$ and $\hat{y}$, as shown in Figure 13. Furthermore, the potential



(a) $V(\hat{x}, \hat{y}_0, \tilde{s}_0^a)$ (b) $V(\hat{x}_0, \hat{y}, \tilde{s}_0^a)$

Figure 13: This figure illustrates the potential defined in (6.130) as a function of a single canonical field direction, either $\hat{x}$ or $\hat{y}$, with all remaining fields held at their vacuum values.

exhibits the same asymptotic behavior for the scalar fields $\tilde{s}^a$, as shown in Figure 14. In conclusion, our analysis reveals no clear violation of the bound set by (6.17) in the field directions considered here. This result is unexpected since we are working in $d = 3$, which has been problematic before. However, using the same approach as for AdS$_4$, we explore a specific direction in the $\hat{x}\hat{y}$-plane of the field space.

We first revisit the detailed derivation of the potential (6.120) in the DGKT solution, starting with (6.114). Note that H_3, F_0 and the source term for the O6-planes exhibit identical field dependence with respect to $\hat{t}$, while the F_4 flux shows a different behavior.

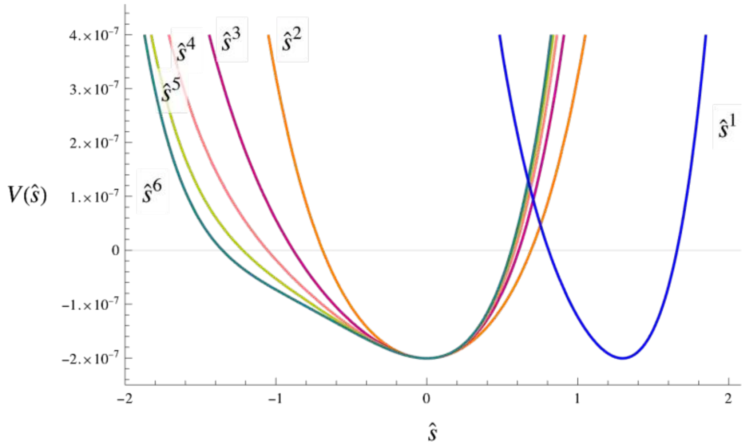


Figure 14: This figure illustrates the potential defined in (6.130) as a function of a single canonical field direction, $\hat{s}^a$, with all remaining fields held at their vacuum values. We refer to the relations specified in (6.129) to determine the dependence on $\hat{s}^a$.

With the same analytical criteria for the behavior of each term in the potential, we identify a specific field direction in $d = 3$ obeying this property. We start with the on-shell condition,

$$\partial_{\hat{x}} V + \frac{1}{\sqrt{7}} \partial_{\hat{y}} V = 0 \,, \tag{6.137}$$

for the potential (6.130), while stabilizing the other scalar fields, $\tilde{s}^a = \tilde{s}^a_0$, to focus on a single dimension in field space. This equation defines a new field direction by imposing the constraint

$$\hat{x} = -\frac{\sqrt{7}}{9}\hat{y} + \frac{\sqrt{14}}{9} \ln A \,, \tag{6.138}$$

characterized by a positive constant

$$A = \frac{-T(\tilde{s}^a_0) + \sqrt{96 C H(\tilde{s}^a_0) + T(\tilde{s}^a_0)^2}}{8 H(\tilde{s}^a_0)} \,, \tag{6.139}$$

which satisfies the quadratic equation $4A^2 H(\tilde{s}^a_0) + A T(\tilde{s}^a_0) - 6C = 0$. The potential along this direction is given by

$$V(\hat{y}) = F(\tilde{s}^a_0) A^{-\frac{4}{9}} e^{\frac{20\sqrt{2}}{9}\hat{y}} + A^{-\frac{14}{9}} \left(H(\tilde{s}^a_0) A^2 - T(\tilde{s}^a_0) A + C \right) e^{\frac{16\sqrt{2}}{9}\hat{y}} \,. \tag{6.140}$$

Similar to the potential (6.120), the second term dominates in the asymptotic regions of field space, which is defined by $\hat{y} \to -\infty$. This term is negative under the condition

$$H(\tilde{s}^a_0) A^2 - T(\tilde{s}^a_0) A + C \leq 0 \quad \Leftrightarrow \quad 1.38988 \leq 10^5 \times A \leq 13.7585 \,, \tag{6.141}$$

which allows us to test the rate (6.17) of the ATCC. According to our solution in the equation (6.136), we get $A = 3.78707 \cdot 10^{-5}$, which agrees with the bounds given in (6.141). Note that the notation in (6.140) can be misleading because the field direction defined in this way is not canonically normalized. This problem can be solved by implementing the redefinition

$$\hat{t} = \frac{2\sqrt{22}}{9}\hat{y}\,. \tag{6.142}$$

Therefore, the behavior of the potential in the asymptotics along $\hat{t}$ is dominated by a negative term, as shown in Figure 15. The exponential rate of this term satisfies the bound in (6.17), i.e.

$$\frac{16\sqrt{2}}{9} \times \frac{9}{2\sqrt{22}} > \sqrt{2}\,. \tag{6.143}$$

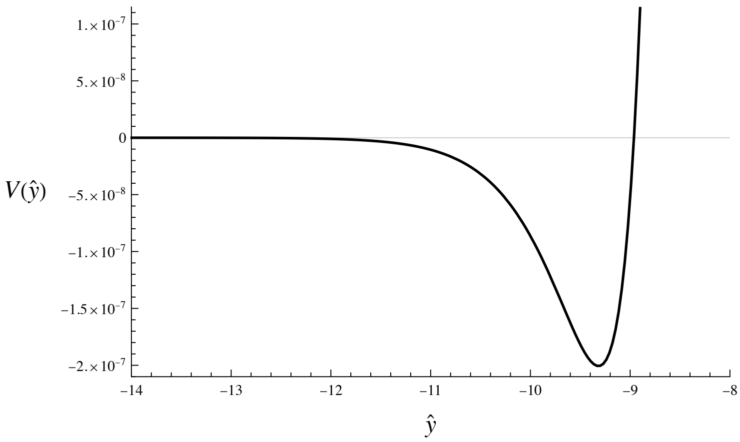


Figure 15: This figure shows the potential (6.140) along the field direction (6.138) with $A = 3.78707 \cdot 10^{-5}$.

7 Almost classical de Sitter

No-go theorems often preclude dS solutions in the classical regime. Typically, the few identified solutions either exhibit $\mathcal{O}(1)$ curvature and string coupling, or are perturbatively unstable. Notably, [72] claimed to have found reliable vacua within a simple "CDT1" model, where the equations of motion include the full nonlinear backreaction of the O8-planes. However, [73] pointed out a critical inconsistency: the localized sources in the CDT1 model appear in the 10D SUGRA equations in a manner that deviates from the conventional O-plane action at leading order in α', thus casting doubt on their authenticity as genuine O8-planes.

In an effort to address these concerns, [75] explored ambiguities in the SUGRA equations of motion that might allow source terms that violate the assumptions underlying the no-go theorem, as detailed in Section 2.3. Nevertheless, supported by forthcoming studies [95], we will argue that such ambiguities do not exist at the classical SUGRA level, suggesting that the O-planes should coincide with those postulated in the no-go theorem.

Furthermore, leading α' corrections to the O8/D8 actions could revise this conclusion. These corrections introduce additional couplings between the O8/D8 and the bulk fields, changing how these fields are sourced in the equations of motion. In the following sections, we will explore this notion by proposing a minimal extension of the classical dS scenario that incorporates leading α' corrections – the "almost classical" dS scenario – while assuming that higher-order corrections remain minimal. In addition, we will examine the "CDT2" model introduced by [75], which includes both O6- and O8-planes, to discuss whether it faces similar challenges to those identified in the CDT1 model.

7.1 Type II flux vacua with O8/D8

Motivated by the proposed existence of dS solutions within the CDT1 model [72], we investigate source configurations with O8$^{\pm}$-planes and D8-branes coupled with a non-vanishing F_0 flux. The internal space consists of $\mathcal{M}_5 \times S^1$, where $\mathcal{M}_5$ is a negatively curved Einstein space wrapped by two O8$^{\pm}$-planes, localized on a circle S^1 parameterized by the coordinate $z \in [0, 2\pi)$. This configuration is illustrated in Figure 16. Our analysis

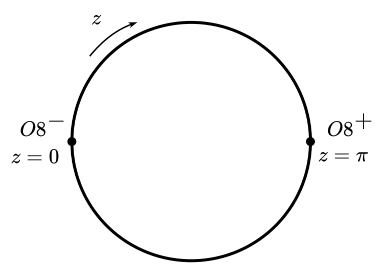


Figure 16: Schematic representation of the CDT1 model with sets of parallel O8$^{\pm}$-planes localized at $z = 0$ and $z = \pi$ along the internal circle S^1 [72].

will maintain a level of generality that refrains from specifying the nature of the sources – whether O-planes or D-branes – or their precise distribution. Moreover, we will not explicitly define the dimensions of the external and compact spaces, instead focusing solely on the mass parameter F_0 and excluding models with F_4 flux as detailed in [72].

The structure of this section is as follows: First, we will show that these models preclude classical dS solutions with $\mathcal{L}_{\alpha'^2,i} = 0$, since the equations of motion impose a vanishing cosmological constant when only leading-order terms are included in the effective action. This no-go theorem can be approached in two different ways. The first is to use the full 10D theory, including the fully backreacted 10D solution as presented in [73]. Alternatively, we can analyze the scalar potential in the d-dimensional effective theory derived from the dimensional reduction of type IIA SUGRA, as discussed in Section 3. In the latter approach, we assume that the smeared approximation is valid over most of the compact space, implying that backreaction corrections do not significantly alter the scalar potential. As explained in Section 2.4, the existence of such a smeared regime is crucial (though not always necessary) to avoid large singular holes and thereby ensure the reliability of classical SUGRA. This serves as an important consistency check for resolving classical equations of motion involving O-planes. Note that in the smeared limit both the warp factor and the dilaton are constants, simplifying the analytical expressions.

The classical no-go theorem, as discussed in [73, 75], suggests that the dS solutions explored in [72] may be feasible if non-standard source terms are included in the equations of motion. "Classicality" in this context refers to focusing only on the leading-order terms in the actions (2.18) and (2.21). Consequently, we will investigate whether this no-go theorem can be circumvented by integrating additional corrective terms into the action of the $\mathrm{O8}^{\pm}/\mathrm{D8}$.

Before we proceed, a final note: most of our previous analysis was done in the Einstein frame, which led us to omit the superscript E. In this section, however, the scenario is different. We will switch between frames as needed for each specific calculation, following the conventions established in the referenced papers [72,73,96]. For clarity, we will explicitly use superscript notation for quantities defined in the Einstein frame throughout this section.

7.1.1 Dimensional reduction and scalar potential

Before going into the derivation of the no-go theorem, we will first examine the scalar potential for our model in the smeared approximation, which also takes into account possible 4-derivative corrections. Our analysis will focus primarily on the two universal moduli ρ and τ, for which we derived the potential in Section 3. We will exclude additional scalar fields such as cycle volumes or axions from this study. In order to be consistent with the notation used in [96], we will slightly modify the definitions of these moduli. Specifically, we define $\tau = \mathrm{e}^{-\phi}$ to represent the dilaton modulus and set $g_{mn} = \rho^{\frac{2}{10-d}} \hat{g}_{mn}$, where $\rho = \int \mathrm{d}^{10-d}y \sqrt{g_{10-d}}$ ensures that $\hat{g}$ maintains unit volume. These adjustments do not significantly change the results of our analysis, although they deviate from our original definitions in Section 3.

In this discussion, we consider only the F_0 mass, ignoring other fluxes. The possible

4-derivative terms (2.26) are expressed as

$$\mathcal{L}_{\alpha'^2,i} = \left(c_{1i}(\mathrm{e}^{\phi}F_0)^4 + c_{2i}(\mathrm{e}^{\phi}F_0)^2\mathcal{R} + c_{3i}\mathcal{R}^2\right) , \tag{7.1}$$

where $\mathcal{R}^2$ includes any contraction of two Riemann tensors or their sum, involving either tangent or normal indices relative to the O8/D8. For the scalar potential corrections, only the internal components of the Riemann tensor are relevant; external components modify the Einstein-Hilbert term of the bulk action or contribute to curvature-squared terms and are considered irrelevant in the regime of small curvature. In addition, terms involving covariant derivatives are omitted since they scale similarly with the β field defined below and thus do not change our conclusions.

We continue with the action of type IIA supergravity as defined in the equations (2.18) and (2.21),

$$\begin{aligned} S \supset 2\pi \int \mathrm{d}^d x\sqrt{-g_d}\Bigg(&\tau^2\rho\mathcal{R}_d + \tau^2\rho^{\frac{8-d}{10-d}}\hat{\mathcal{R}}_{10-d} - \frac{1}{2}\rho F_0^2 \\ &- \tau\rho^{\frac{9-d}{10-d}}\sum_i \frac{T_i}{2\pi\hat{V}_i}\left(1 + c_{1i}\tau^{-4}F_0^4 + c_{2i}\tau^{-2}\rho^{-\frac{2}{10-d}}F_0^2\hat{\mathcal{R}}_{10-d} + c_{3i}\rho^{-\frac{4}{10-d}}\hat{\mathcal{R}}^2_{10-d}\right)\Bigg) . \end{aligned} \tag{7.2}$$

We perform the integration over the compact $(10-d)$-dimensional space using the simplified notation as defined in the equation (3.37) with $M_p^2/2 = 1$ and $2\kappa_{10}^2 = 1/2\pi$. By the definition

$$g_{\mu\nu} = \tau^{-\frac{4}{d-2}}\rho^{-\frac{2}{d-2}}g^E_{\mu\nu} , \tag{7.3}$$

we return to the Einstein frame, which in turn leads to the scalar potential

$$\begin{aligned} V = &-\tau^{-\frac{4}{d-2}}\rho^{-\frac{16}{(10-d)(d-2)}}\hat{\mathcal{R}}_{10-d} + \frac{1}{2}\tau^{-\frac{2d}{d-2}}\rho^{-\frac{2}{d-2}}F_0^2 \\ &+ \tau^{-\frac{d+2}{d-2}}\rho^{-\frac{18-d}{(10-d)(d-2)}}\sum_i\frac{T_i}{2\pi\hat{V}_i}\Big(1 + c_{1i}\tau^{-4}F_0^4 + c_{2i}\tau^{-2}\rho^{-\frac{2}{10-d}}F_0^2\hat{\mathcal{R}}_{10-d} \\ &+ c_{3i}\rho^{-\frac{4}{10-d}}\hat{\mathcal{R}}^2_{10-d}\Big) . \end{aligned} \tag{7.4}$$

The quantities labeled with hats are defined by the metric $\hat{g}_{mn}$, and the index i runs over all sets of O8$^{\pm}$-planes/D8-branes. The external Ricci scalar in the Einstein frame is represented by the on-shell potential, as detailed in equation (3.4). To understand the stability of dS solutions within these models and to study the properties of the critical points of the scalar potential, we introduce another field redefinition,

$$\rho = (\alpha\beta)^{10-d} , \qquad \tau = \beta . \tag{7.5}$$

This transformation leads to the following expression,

$$\begin{aligned} V = &\beta^{-\frac{20}{d-2}}\left(-\alpha^{-\frac{16}{d-2}}\hat{\mathcal{R}}_{10-d} + \frac{1}{2}\alpha^{-\frac{2(10-d)}{d-2}}F_0^2 + \alpha^{-\frac{18-d}{d-2}}T\right) \\ &+ \beta^{-\frac{4(d+3)}{d-2}}\alpha^{-\frac{18-d}{d-2}}\sum_i\frac{T_i}{2\pi\hat{V}_i}\left(c_{1i}F_0^4 + c_{2i}\alpha^{-2}F_0^2\hat{\mathcal{R}}_{10-d} + c_{3i}\alpha^{-4}\hat{\mathcal{R}}^2_{10-d}\right) , \end{aligned} \tag{7.6}$$

where we have introduced shorthand notation

$$T \equiv \sum_i \frac{T_i}{2\pi \hat{V}_i} \,. \tag{7.7}$$

Note that this formulation preserves the generality of the potential, especially with respect to the source content, without being specific to any particular model.

In the smeared approximation, the mass F_0 remains constant, but is odd under an O8 involution [90], leading to $F_0 = 0$. This is consistent with the results of [72], where F_0 changes sign over an O8-plane in the backreacted solution, effectively rendering its mean zero, thus identifying F_0 as a backreaction effect[23]. In any model with O8-planes, including the CDT1 model, the potential (7.6) is simplified to

$$V = \beta^{-\frac{20}{d-2}} \left(-\alpha^{-\frac{16}{d-2}} \hat{\mathcal{R}}_{10-d} + \alpha^{-\frac{18-d}{d-2}} T\right) + \beta^{-\frac{4(d+3)}{d-2}} \alpha^{-\frac{10+3d}{d-2}} \sum_i \frac{T_i}{2\pi \hat{V}_i} c_{3i} \hat{\mathcal{R}}^2_{10-d} \,, \tag{7.8}$$

except for source configurations containing only D8-branes, where F_0 does not necessarily vanish in the absence of the O8 involution. However, this setup often requires anti-D8-branes to cancel out the tadpole induced by D8-branes, a scenario typically fraught with instabilities due to open-string tachyons. Notably, in the CDT1 model [72], the leading source term in (7.8) effectively disappears because $T_{\text{O8}^+} + T_{\text{O8}^-} = 0$, and the circle volume V_i is the same for all O8$^\pm$-planes. If the coefficients c_{3i} are identical for all sources, the 4-derivative corrections are absent, resulting in a classical potential at least at the 4-derivative level. In addition, the tadpole cancellation implies a classical potential in other configurations, such as those involving only O8$^-$-planes/D8-branes. However, the appearance of anti-O8$^-$-planes or anti-D8-branes does not necessarily lead to a vanishing source term.

Our extensive discussion is intended to provide a thorough review of the theoretical framework and underlying motivations. In the following section, we will delve more deeply into the general potential (7.6), leaving aside the specific model discussed earlier. We will show that metastable dS solutions, whether classical or modified by 4-derivative corrections, are unattainable in models with O8$^\pm$/D8, regardless of the presence of a F_0 flux.

7.1.2 A classical no-go theorem

We begin our discussion by focusing on the classical theory, deliberately excluding α' corrections to the action and potential. In particular, we focus on configurations with $p_i = 8$ sources and F_0 flux, especially in the context of the CDT1 model. As noted above, we validate the same no-go theorem against classical dS vacua using two different methods. At first, we use the classical 10D equations of motion derived from (2.18) and (2.21) with $\mathcal{L}_{\alpha'^2,i} = 0$ and the presence of O8$^\pm$-planes/D8-branes along with the energy density F_0^2. This approach includes the complete backreaction of the sources. Our analysis studies a particular combination of the dilaton equation of motion and the 10-dimensional Einstein

[23] This observation agrees with the absence of a topological F_2 flux in compactifications with O6-planes [12, 15, 98]. It manifests itself as a backreaction effect.

equation, which leads to

$$\mathrm{e}^{-2A}\mathcal{R}_d = \frac{\mathrm{e}^{-dA+2\phi}}{\sqrt{g_{10-d}}}\partial_m\left(\sqrt{g_{10-d}}\mathrm{e}^{\frac{d-10}{5}\phi}\partial^m\mathrm{e}^{dA-\frac{d}{5}\phi}\right)\,, \tag{7.9}$$

in the string frame. Integrating this equation over the compact space after multiplying by $\mathrm{e}^{dA-2\phi}$ results in

$$\left(\int \mathrm{d}^{10-d}y\sqrt{g_{10-d}}\mathrm{e}^{(d-2)A-2\phi}\right)\mathcal{R}_d = 0\,. \tag{7.10}$$

At first glance, this equation suggests that no classical (anti-)de Sitter solutions are possible. However, a closer look reveals that the vacuum energy does not vanish completely; only the classical contribution vanishes, with string corrections still to be considered. Thus, equation (7.10) indicates that in models with O8$^{\pm}$-planes/D8-branes, such as CDT1, the sign of $\mathcal{R}_d$ or, when evaluated on-shell, V, is not determined by classical contributions alone. In a regime characterized by sufficiently large volume, or equivalently, small string coupling, the curvature over the entire 10D spacetime is small, as shown in the right plot of Figure 2. In this case, the sign of the vacuum energy is determined by next-to-leading-order terms in the α' expansion of the equations of motion [73]. These corrections include 8-derivative terms in the bulk and localized 4-derivative corrections to the source action. Due to their predominance, our analysis will henceforth focus mainly on these localized corrections.

In scenarios where g_s exceeds a critical threshold $g_{s,\mathrm{crit}}$, as shown in the left plot of Figure 2, the integration of the equation (7.9) over the internal space presents complexities [75]. These complications arise from the appearance of singular holes around O8$^-$-planes, rendering the α' expansion unreliable in this regime. To address this problem, we adapt our integration approach by restricting the integration up to the boundary of these singularities, introducing additional boundary terms $\mathcal{B}_i(\epsilon)$ into the equation (7.10). The diameter ϵ of these holes decreases as g_s approaches $g_{s,\mathrm{crit}}$ from above. Beyond this theoretical minimum, the α' expansion remains valid everywhere. For consistency, the boundary term $\mathcal{B}_i(0)$ must coincide with the correction terms in the α' expansion, specifically the 4-derivative terms and the higher-order corrections in the O8$^-$ action. These corrections represent the leading contributions to the equation (7.10) from the O8$^-$-planes. Even when slightly deviating from the limit of a vanishing hole diameter, where $g_s \gtrsim g_{s,\mathrm{crit}}$, it is expected that $\mathcal{B}_i(\epsilon) = \mathcal{B}_i(0) + \mathcal{O}(\epsilon)$ will approximately hold, suggesting that any corrections due to the holes are equivalent to localized higher-derivative corrections. Consequently, the sign of the vacuum energy, which remains undefined in classical calculations, can only be definitively resolved by analyzing the α' corrections. This conclusion holds in scenarios with sufficiently small singular regions where the perturbative approach of classical SUGRA, as discussed in this section and in [72, 73, 75], remains applicable.

As we revisit the d-dimensional effective theory and its scalar potential V in the following sections, it proves useful to repeat the no-go theorem discussed here in this dimensionally reduced framework, highlighting the similarities with our previous derivation. We proceed in the smeared limit, starting with the classical contributions to the scalar potential as given by the equation (7.6) with $c_{ai} = 0$,

$$V_{\text{class}} = \beta^{-\frac{20}{d-2}} \left(-\alpha^{-\frac{16}{d-2}} \hat{\mathcal{R}}_{10-d} + \frac{1}{2} \alpha^{-\frac{2(10-d)}{d-2}} F_0^2 + \alpha^{-\frac{18-d}{d-2}} T \right) . \tag{7.11}$$

This formulation deliberately omits any string corrections to (7.11), whether they arise in the bulk spacetime or from regions containing the O-planes and holes. When evaluated on-shell, we observe

$$\beta \partial_\beta V_{\text{class}} = -\frac{20}{d-2} V_{\text{class}} = 0 \quad \Rightarrow \quad V_{\text{class}}|_0 = 0 \, . \tag{7.12}$$

In other words, the classical vacuum energy vanishes on-shell, consistent with the result (7.10) derived from the 10D equations of motion. As before, we study the consequences of string corrections to V_{class},

$$V = V_{\text{class}} + \delta V_{\text{corr}} \, . \tag{7.13}$$

Note that the classical computation of the vacuum energy remains reliable only if all string corrections are indeed sub-leading. This requirement is mathematically expressed as

$$V_{\text{class}}|_0 \neq 0 \, , \qquad V_{\text{class}}|_0 \gg \delta V_{\text{corr}}|_0 \, , \tag{7.14}$$

for (anti-)de Sitter vacua, or in terms of the scalar field masses

$$(m^2)_{\text{class}} \neq 0 \, , \qquad (m^2)_{\text{class}} \gg \delta (m^2)_{\text{corr}} \, . \tag{7.15}$$

If the latter condition is not satisfied, the classical derivation alone cannot sufficiently confirm the stability of the solutions. Although the condition (7.14) is satisfied in certain cases, such as the classical AdS solutions in the DGKT-CFI class of type IIA SUGRA [70,101], the dS solutions of the O8/D8 models discussed here fail to meet this requirement. However, it is crucial to note that while the DGKT flux vacua pass these preliminary tests, as indicated by

$$V|_0 \approx V_{\text{class}}|_0 \, , \tag{7.16}$$

with only minor corrections [15], their non-perturbative stability remains unproven. Nevertheless, according to the no-go theorem outlined in (7.10) or (7.12), the leading contribution to the vacuum energy in the O8/D8 model is governed by string corrections, i.e.

$$V|_0 = \delta V_{\text{corr}}|_0 \, , \tag{7.17}$$

casting doubt on the existence of dS solutions at the classical level in these compactifications. Importantly, this scenario does not arise due to holes or regions of strong curvature around the orientifold where the classical SUGRA description fails due to non-perturbative short-distance physics. Instead, classical SUGRA might still be the dominant contributor to the vacuum energy, as observed in other models with different source configurations.

In the context of the CDT1 model [72], the existence of dS vacua has been proposed based on numerical solutions of the classical equations, assuming $\mathcal{R}_4 > 0$. An analytical solution supporting this claim appears in [92]. This corresponds to the implicit assumption that $\delta V_{\text{corr}}|_0 > 0$, according to the equation (7.17). However, [72] did not explicitly determine the sign of the string corrections to the potential, leaving open the possibility that $\delta V_{\text{corr}}|_0 \leq 0$. Thus, the classical calculations do not inherently favor (anti-)de Sitter or Minkowski solutions [73].

Although the CDT1 solutions satisfy the classical bulk equations, [73, 75] suggest a possible escape from the no-go constraints via non-standard boundary terms that are inconsistent with the classical actions of O8-planes/D8-branes. This prompts the inclusion of string corrections, especially at the 4-derivative level in the α' expansion of the O8/D8 action. By going beyond classical calculations and showing that these α' corrections contribute a positive $\delta V_{\text{corr}}|_0 > 0$ to the otherwise vanishing vacuum energy, we may uncover a feasible way around the no-go theorem (7.10), provided that higher-derivative corrections are adequately suppressed within the potential. While not strictly classical, these solutions are close to it, and we are willing to explore these prospects further.

7.1.3 Including α' corrections

Now that the context provides clear definitions of on-shell expressions, we will omit the notation $|_0$. As a first step, we revisit the classical potential described in (7.11) and examine the runaway behavior $V \sim \beta^{-\frac{4(d+3)}{d-2}}$. The only way to suppress this behavior is to satisfy the following condition at the critical point for α,

$$-\alpha^{-\frac{16}{d-2}}\hat{\mathcal{R}}_{10-d} + \frac{1}{2}\alpha^{-\frac{2(10-d)}{d-2}}F_0^2 + \alpha^{-\frac{18-d}{d-2}}T = 0\,, \tag{7.18}$$

Under this condition, β becomes a flat direction and $V = 0$ off-shell, consistent with the no-go theorem (7.12). This scenario mirrors the dynamics observed in the GKP vacua of type IIB supergravity [190], where the runaway behavior of the potential is similarly constrained by a condition analogous to (7.18), resulting in $V = 0$ and a flat direction.

Moving on to the general O8/D8 models with the scalar potential given by (7.6), including up to 4-derivative corrections, and considering the implications of (7.18), we see that β can now be stabilized. This implies that the runaway behavior can be suppressed, provided that the other scalar fields are stabilized so that the no-scale condition (7.18) is violated,

$$-\alpha^{-\frac{16}{d-2}}\hat{\mathcal{R}}_{10-d} + \frac{1}{2}\alpha^{-\frac{2(10-d)}{d-2}}F_0^2 + \alpha^{-\frac{18-d}{d-2}}T \equiv C \neq 0\,. \tag{7.19}$$

This adjustment is crucial to prevent the runaway behavior $V \sim \beta^{-\frac{4(d+3)}{d-2}}$. The stabilization of β also suggests that the 4-derivative terms are fine-tuned to the leading terms in (7.6) within this regime of small curvature and energy densities. However, we will not delve into a detailed analysis of this fine-tuning, nor will we specify the particular models in which it might be achievable, as we have a more pressing problem in the model under consideration. Satisfying the equation of motion $\partial_\beta V = 0$ leads to

$$\partial_\beta^2 V < 0\,, \tag{7.20}$$

for $V > 0$, as indicated by the on-shell equation

$$V = -\frac{(d-2)^2}{80(d+3)}\beta^2 \partial_\beta^2 V\,, \tag{7.21}$$

suggesting the presence of a tachyon according to Sylvester's criterion [34, 191, 192]. Consequently, if dS solutions exist within this framework, they are perturbatively unstable, making only anti-de Sitter or Minkowski vacua plausible outcomes. While we will not delve deeply into these vacua here, a brief comment is in order. In the regime of small curvature, the smeared CDT1 model, characterized by $F_0 = T = 0$, exhibits a runaway behavior unless $\mathcal{R}_{10-d} = 0$, leading to $V = 0$ off-shell. Therefore, at the 4-derivative level, only Minkowski solutions seem feasible, regardless of the stability analysis discussed earlier. For more complicated models with non-zero flux F_0 or source T, a quick analysis suggests that achieving a fine-tuned C, small string coupling, large volume or small backreaction is challenging, effectively ruling out self-consistent regimes. Furthermore, the possible presence of an open-string tachyon has not been addressed in this discussion.

In summary, our efforts to solve the problems identified in [73] by minimally extending the model in [72] have not been successful. Although theoretically feasible, departing from the simplicity of this model to explore configurations with corrections at higher orders introduces significant complexity. Specifically, we could consider incorporating α' corrections in the O8/D8 action beyond the 4-derivative level, or introducing higher-derivative corrections in the bulk action. Nevertheless, the need for further fine-tuning to balance various corrections at different levels further complicates the situation.

A potential limitation of our analysis is the reliance on the smeared approximation in the d-dimensional approach. Backreaction effects in the potential could significantly alter the results discussed here, possibly changing the vacuum energy and the mass of the scalar field β to circumvent (7.21) and challenge the no-go theorem (7.1.2) against dS solutions.However, the 10D derivation, as discussed in Section 7.1.2, does not make specific assumptions regarding the warp factor, the internal metric or the dilaton. Although backreaction effects are fully accounted for in this approach, they do not affect the classical vacuum energy. The backreaction mainly induces singular regions around the O8-planes, as shown in Figure 2, where string corrections are important. However, our analysis focuses on scenarios where classical SUGRA is reliable over most of the spacetime. This is particularly true in the limit of point-like singularities and small curvatures or energy densities. In this limit, the dominant contribution to the vacuum energy comes from the 4-derivative corrections in the actions of the O8-planes/D8-branes, as discussed earlier in the smeared approximation. Slight deviations to small but finite diameters of these singular holes are also expected to preserve this approximation. Consequently, the left-hand side of (7.21) remains largely unaffected by backreaction effects. However, the backreaction on the mass of the beta field remains ambiguous and require further investigation, which we leave to future studies [95].

In addition, we recognize that dS solutions may be viable in regimes where singular regions extend significantly over the 10D spacetime. In such cases, classical SUGRA models referred to in [72, 73, 75] become inappropriate. Addressing these scenarios would require a worldsheet analysis or other non-perturbative techniques. Given the comprehensive and detailed nature of such investigations, they are beyond the scope of the current work, which leads us to move on to the next model.

7.2 Type II flux vacua with O6/D6 and O8/D8

In this section, we focus on source configurations incorporating O8-planes/D8-branes and (anti-)O6-planes/D6-branes. Specifically, we critically evaluate the CDT2 model [75] to challenge the proposed existence of (almost) classical dS solutions. Employing methods akin to those discussed in the previous sections, we analyze both classical and α'-corrected O-planes/D-branes up to the 4-derivative order. Following the framework outlined in Section 7.1, we establish a no-go theorem against the existence of classical dS vacua within the CDT2 model. Our arguments are formulated using two distinct strategies: initially, we use the smeared approximation, under which the backreaction effects of the sources are considered negligible over the entire 10D spacetime. Then, we consider scenarios that account for the full backreaction from the orientifold planes, consistent with the methods employed in establishing the no-go theorem in Section 7.1.2 and detailed in [73]. We also demonstrate that the inclusion of α' corrections up to the 4-derivative order in the O-plane/D-brane actions does not alter this outcome.

Furthermore, we reveal that the numerical dS solutions within the CDT2 model circumvent the no-go theorem by including non-standard source terms that are incompatible with both classical and α'-corrected source actions. Due to the complexity of this approach, we refrain from exploring an extension of this model. Instead, our focus remains exclusively on the CDT2 construction. A detailed discussion of this extension is available in [47], where some constraints and assumptions of the CDT2 model are relaxed, particularly regarding the flux and source content as well as the geometry, to consider a more generalized model.

7.2.1 The CDT2 model

We commence with the metric ansatz of the CDT2 model as detailed in [75]. The 10D metric is given by

$$\mathrm{d}s^2_{10} = \mathrm{e}^{2A(z)} g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu + \mathrm{e}^{-2A(z)}\left(\mathrm{e}^{2\lambda_3(z)}\mathrm{d}s^2_{\kappa_3} + \mathrm{e}^{2\lambda_2(z)}\mathrm{d}s^2_{S^2} + R^2\mathrm{d}z^2\right) , \tag{7.22}$$

where κ_3 represents a 3D Einstein space with negative scalar curvature, $\mathcal{R}_{\kappa_3} < 0$, S^2 denotes a unit sphere and z parameterizes a 1D interval with $z \in [0, \pi]$. In this setting, R serves as a length scale and standard spherical coordinates (θ, ϕ) are used on S^2. The compactification space can conceptually be described as a $\kappa_3 \times S^2$ fibration over z. The metric (7.22) is crucial for calculating various curvatures and covariant derivatives. Note that the notation used here differs from the one in [75], where $\mathrm{e}^{2A(z)} = \mathrm{e}^{2W(z)}$ and $R^2 = \mathrm{e}^{2q_0}$.

Regarding the flux and source content, the configuration includes (anti-)O6$^-$-planes localized on S^2 and along z, as well as O8$^+$-planes transverse to z, all wrapping the 3D Einstein space κ_3. It is important to clarify that this description concerns the covering space. On the orientifold, both O6-planes are identified, and the O8-plane wraps $\kappa_3 \times \mathbb{RP}^2$. The orientifold projection is defined by $\Omega\sigma_8$ for O8-planes and $\Omega(-1)^{F_L}\sigma_6$ for O6-planes, as elaborated in [75]. As a reminder, Ω is the worldsheet parity operator, F_L is the left-moving fermion number and σ_i denotes the spacetime involution (2.24),

$$\sigma_8 : z \to \pi - z\,, \qquad \sigma_6 : (\theta, \phi) \to (\pi - \theta, \phi + \pi)\,. \tag{7.23}$$

The fixed loci of σ_6, at positions where $e^{3\lambda_2(z)} = 0$ and S^2 shrinks to a point, have codimension 3, corresponding to the (anti-)O6$^-$-planes. On the other hand, the fixed loci of σ_8, having codimension 1, define the O8$^+$-planes. The non-vanishing fluxes are given by

$$F_0(z)\,, \qquad F_2 = f_2(z)\,\mathrm{vol}_{S^2}\,, \qquad H_3 = h(z)\,\mathrm{d}z \wedge \mathrm{vol}_{S^2}\,. \tag{7.24}$$

Following the orientifold projection, which induces the transformations [90]

$$\begin{aligned} \Omega :& \quad F_0 \to -F_0\,,\ F_2 \to F_2\,,\ H_3 \to -H_3\,,\\ (-1)^{F_L} :& \quad F_0 \to -F_0\,,\ F_2 \to -F_2\,,\ H_3 \to H_3\,,\\ \sigma_6,\,\sigma_8 :& \quad \mathrm{d}z \to -\mathrm{d}z\,,\ \mathrm{vol}_{S^2} \to -\mathrm{vol}_{S^2}\,, \end{aligned} \tag{7.25}$$

the following conditions are necessary for the fluxes to comply with both orientifold projections,

$$F_0(z) = -F_0(\pi - z)\,, \qquad f_2(z) = f_2(\pi - z)\,, \qquad h(z) = h(\pi - z)\,. \tag{7.26}$$

7.2.2 Challenges in the smeared limit

It is appropriate to initiate our discussion on dS vacua in the smeared limit, characterized by negligible backreaction from the sources, thereby ensuring the validity of classical SUGRA everywhere. As elaborated in Section 2.4, the singular regions surrounding the O-planes are exceedingly small and expected to minimally impact the lower-dimensional effective theory, including the scalar potential. However, our analysis will demonstrate that the CDT2 model lacks non-trivial vacuum solutions in the smeared approximation, indicating that significant backreaction effects are inevitable over a large part of the 10D spacetime. This claim holds not only for classical source terms but also when considering 4-derivative corrections.

Turning to the details of the metric (7.22), the smeared limit implies a constant warp factor, $e^{2A(z)} = 1$, and a constant internal curvature. While consistent with the metric configuration in (7.22), these conditions do not impose a unique geometry. For instance, different compactifications may arise based on the specific choices of λ_2 and λ_3. Possible configurations include $\kappa_3 \times S^2 \times S^1$ for $e^{2\lambda_3(z)} = 1$ and $e^{2\lambda_2(z)} = \text{const.}$, or the product $S^4 \times S^2$ with constant curvature, for $e^{2\lambda_3(z)} = R^2 \sin^2(z)$ and $e^{2\lambda_2(z)} = \text{const.}$, where the 3D Einstein space is an S^3 and thus non-trivially fibered over z. The exact nature of the compact space in the smeared limit remains non-trivial and open to various geometric interpretations. A natural choice, however, is

$$\kappa_3 \times S^3\,, \quad \text{for } \mathrm{e}^{2\lambda_3(z)} = 1\,,\ \mathrm{e}^{2\lambda_2(z)} = R^2\sin^2(z)\,, \tag{7.27}$$

justified by the backreaction of the (anti-)O6 in (7.22), which causes the warp factor, dilaton and fluxes to vary with z while remaining independent of the angular coordinates of S^2. This variation is coherent only if S^2 reduces to a singular point at the locations of the (anti-)O6, that is, at the fixed points of σ_6 in the absence of backreaction. Consequently, (anti-)O6-planes would be incompatible with a constant, finite $\mathrm{e}^{2\lambda_2}$, necessitating that the S^2 is non-trivially fibered over z. This leads to the product space $\kappa_3 \times S^3$ with the metric

$$\mathrm{d}s^2_{10} = g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu + \mathrm{d}s^2_{\kappa_3} + R^2\left(\sin^2(z)\,\mathrm{d}s^2_{S^2} + \mathrm{d}z^2\right) \tag{7.28}$$

in the smeared limit, as shown in Figure 17. In the smeared approximation, the symmetry

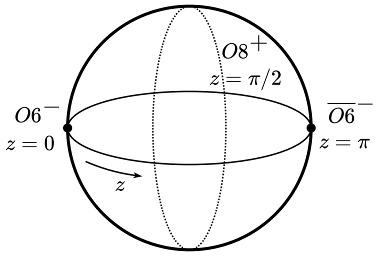


Figure 17: The internal space S^3 with an O6$^-$-plane, an anti-O6$^-$-plane and an O8$^+$-plane wrapping an S^2 [47].

of S^3 ensures that F_0, $|F_2|^2$ and $|H_3|^2$ remain constant. Following the flux ansatz specified in the equations (7.24) and (7.26), we find

$$F_0 = 0\,, \qquad F_2(z) = \tilde{f}_2\sin^2(z)\,\mathrm{vol}_{S^2}\,, \qquad H_3 = \tilde{h}\sin^2(z)\,\mathrm{d}z\wedge\mathrm{vol}_{S^2}\,, \tag{7.29}$$

determined by the constants $\tilde{f}_2$ and $\tilde{h}$. Furthermore, the smeared F_2 Bianchi identity, $\mathrm{d}F_2 = 0$, results from the cancellation of smeared contributions (zero modes) from O6$^-$ and anti-O6$^-$, leading to

$$F_0 = 0\,, \qquad F_2 = 0\,, \qquad H_3 = \tilde{h}\sin^2(z)\,\mathrm{d}z\wedge\mathrm{vol}_{S^2}\,. \tag{7.30}$$

Consequently, H_3 is topological on S^3, while F_0 and F_2 manifest as backreaction effects of the O-planes. Assuming a regime with negligible backreaction from the sources, the leading-order equations in the CDT2 model, as outlined in Section 2.2, are dominated by the zero modes of the O-plane sources. Higher modes are irrelevant, except in the vicinity of the O-planes. As defined in (2.42), it is practical to replace the delta-functions in the equations of motion with

$$\delta(\Sigma_{\mathrm{O6}^-}),\, \delta(\Sigma_{\overline{\mathrm{O6}}^-}) \to \frac{1}{V_{S^3}} = \frac{1}{2\pi^2R^3}\,, \qquad \delta(\Sigma_{\mathrm{O8}^+}) \to \frac{V_{S^2}}{V_{S^3}} = \frac{2}{\pi R}\,, \tag{7.31}$$

including the radius R of S^3 and $V_{S^2} = \int_{S^3} \mathrm{d}^3y \sqrt{g_{S^3}}\, \delta(\Sigma_{\mathrm{O8}^+})$.

Using the metric (7.28), the flux ansatz (7.29) and the leading behavior of the sources (7.31), the trace-reversed Einstein equations (2.29) simplify to

$$\mathcal{R}_{zz} = \frac{g_{zz}}{8}\left(3|H_3|^2 - \frac{14\mathrm{e}^{\phi}}{\pi^2 R^3} + \frac{144\mathrm{e}^{\phi}}{\pi R}\right), \quad \mathcal{R}_{ab} = \frac{g_{ab}}{8}\left(3|H_3|^2 - \frac{14\mathrm{e}^{\phi}}{\pi^2 R^3} + \frac{16\mathrm{e}^{\phi}}{\pi R}\right), \tag{7.32}$$

where a, b are the indices of S^2. As previously discussed, the S^2 must remain non-trivially fibered over z to form an S^3 in the smeared solution, leading to $\mathcal{R}_{ab} = 2g_{ab}/R^2$ and $\mathcal{R}_{zz} = 2g_{zz}/R^2$. However, this configuration is inconsistent with the equations (7.32) unless the terms related to the O8-plane are negligible. Given the necessity for a large R to ensure perturbative control of the α' expansion in the bulk, $1/R^3 \ll 1/R$ implies that the O6 terms must similarly vanish. Therefore, the smeared limit of vanishing backreaction of the O-planes in the CDT2 model requires

$$\delta(\Sigma_{\mathrm{O6}^-}),\, \delta(\Sigma_{\overline{\mathrm{O6}}^-}),\, \delta(\Sigma_{\mathrm{O8}^+}) \to 0\,, \tag{7.33}$$

in contrast to (7.31). In essence, the CDT2 model is incompatible with a smeared limit in which the predominant contributions from the O-planes are represented by their zero modes, as outlined in (7.31). The smeared approximation, defined by minimal backreaction effects from the sources, can only be achieved if all Fourier modes, including the zero modes, are negligible in the equations of motion.

An alternative perspective is provided by the F_0 Bianchi identity, which simplifies to $\mathrm{d}F_0 = 0$ following the application of (7.33). This outcome is consistent with our previous flux ansatz (7.29) in the smeared limit. However, replacing the localized O8-plane with (7.31) in the Bianchi identity yields $\mathrm{d}F_0 \sim \mathrm{d}z$, indicating that backreaction corrections to F_0 arise from the zero mode of the O8-plane. Such corrections should not appear, especially in the limit of negligible backreaction. In line with our analysis from the Einstein equations, we confirm that (7.33) is the appropriate approach when considering small backreaction effects.

Using the metric (7.28) and the flux ansatz (7.29), along with the prescription (7.33), we can analytically identify possible vacuum solutions. The equations of motion in Section 2.2 are given by

$$\begin{aligned} -\mathcal{R}_4 - \frac{1}{2}|H_3|^2 = 0\,, \qquad & 2\mathcal{R}_4 + 2\mathcal{R}_{\kappa_3} + 2\mathcal{R}_{S^3} - |H_3|^2 = 0\,, \\ -\mathcal{R}_{\kappa_3} - \frac{3}{8}|H_3|^2 = 0\,, \qquad & -\mathcal{R}_{S^3} + \frac{9}{8}|H_3|^2 = 0\,, \end{aligned} \tag{7.34}$$

which relate H_3 to the scalar curvatures of the internal and external spaces. This analysis confirms that the only viable solution is

$$\mathcal{R}_4 = \mathcal{R}_{\kappa_3} = \mathcal{R}_{S^3} = |H_3|^2 = 0\,. \tag{7.35}$$

Thus, in a regime where the O-plane backreaction is negligible, the CDT2 model fails to provide non-trivial vacuum solutions.

In the previous discussion we assumed that the classical source terms are dominant. However, our reasoning extends to scenarios involving four-derivative (or higher-order) corrections in the α' expansion of the O-plane action, especially in regimes of negligible curvature or field strengths where such corrections are next-to-leading-order relative to classical contributions. Given the vanishing effect of classical source terms in the equations (7.32), it follows that 4-derivative corrections would also be minimal, maintaining the integrity of our analytical framework. Some critics might contend that the α' expansion becomes unreliable near the O-planes, challenging our assumption that 4-derivative terms are irrelevant compared to classical sources. Nevertheless, our analysis is carried out in the smeared approximation, where point-like singularities are smoothed out, and only the uniform zero modes of the O-plane sources span the entire 10D spacetime. If the zero modes of the 4-derivative corrections were to surpass those of the classical sources, it would suggest a breakdown of the α' expansion over the entire 10D spacetime, not just near the O-planes. This would undermine our reliance on SUGRA for the CDT2 model. Therefore, we conclude that the inclusion of α'-corrected source terms does not yield reliable dS vacua within the CDT2 model in the smeared limit.

Despite these conclusions, it is crucial to acknowledge that this analysis does not completely rule out the possibility of (almost) classical dS configurations in the CDT2 model. Our assumptions, particularly regarding the behavior of the metric in the smeared approximation, have not been definitively proven. As discussed in Section 2.4, a regime where the smeared solution prevails is sufficient, though not always necessary, to avoid large singular regions where classical SUGRA becomes unreliable. In particular, objects with positive tension, such as the $O8^+$-plane, prevent the formation of holes and thus circumvent singularity issues. Hence, we might imagine a scenario where the $p_i = 6$ sources have negligible backreaction on most of the spacetime, avoiding large singularities, while the $O8^+$ backreaction remains significant without causing control issues. The arguments presented here do not entirely preclude the viability of these scenarios. To bridge this gap, the following section will introduce a another no-go theorem for the CDT2 model, addressing the complete nonlinear backreaction of the O-planes. This additional analysis will strengthen our previous conclusions, conclusively ruling out the existence of (almost) classical dS vacua in regions where SUGRA remains reliable throughout most of the 10D spacetime.

7.2.3 A classical no-go theorem

The following section presents an explicit no-go argument for the CDT2 model to address the shortcomings of the previous discussion. This argument does not rely on the smearing assumption and includes the full backreaction of the (anti-)$O6^-$-/$O8^+$-planes. Our approach mirrors the methods applied to the $O8^\pm$/D8 models in Section 7.1.2, following the strategy delineated by [73]. The idea is to manipulate the 10D equations in such a way that, upon integration over the compact space, they yield an expression for the external scalar curvature. Assuming that all singular regions where classical SUGRA fails remain small, we will employ the classical equations of motion, inclusive of classical source terms.

The first step is to form a combination of the 4D scalar curvature and $\mathcal{R}_{\kappa_3}/\mathcal{R}_{S^2}$, including a total derivative term. Using the equations (2.28) and (2.29), we deduce that

$$\begin{aligned}
\mathrm{e}^{-2A}\mathcal{R}_4 &= \frac{4}{3}\mathrm{e}^{2A-2\lambda_3}\mathcal{R}_{\kappa_3} + 4\frac{\mathrm{e}^{-4A+2\phi}}{\sqrt{g_6}}\partial_m\left(\mathrm{e}^{4A-2\phi}\sqrt{g_6}\,\partial^m\left(2A-\lambda_3\right)\right)\\
&= -\mathrm{e}^{2A-2\lambda_2} + \frac{\mathrm{e}^{-4A+2\phi}}{\sqrt{g_6}}\partial_m\left(\mathrm{e}^{4A-2\phi}\sqrt{g_6}\,\partial^m\left(3A+\lambda_2-\phi\right)\right)\,,
\end{aligned} \tag{7.36}$$

for any solution adhering to the ansatz presented in Section 7.2.1, particularly under the flux ansatz (7.24) and the assumption that all O-planes wrap κ_3. The metric (7.22) is used to evaluate scalar curvatures and covariant derivatives in these equations, with g_6 corresponding to the six internal components of (7.22).

When integrating the above equation over the internal manifold, we obtain

$$\begin{aligned}
\left(\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{2A-2\phi}\right)\mathcal{R}_4 &= \frac{4}{3}\left(\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{6A-2\lambda_3-2\phi}\right)\mathcal{R}_{\kappa_3}\\
&= -\int \mathrm{d}^6y\sqrt{g_6}\,\mathrm{e}^{6A-2\lambda_2-2\phi}\,,
\end{aligned} \tag{7.37}$$

where the compact space is assumed to be closed and smooth, as we aim to consider a compactification with a finite Kaluza-Klein scale. Consequently, the integral of the derivative in (7.36) vanishes and all volume factors in (7.37) are integrable. By additionally considering the relation between the (un)warped metrics using (7.22), $\sqrt{g_6} \sim \mathrm{e}^{-6A+2\lambda_2+3\lambda_3}$, results in the simplified integral

$$\left(\int \mathrm{d}z\,\mathrm{e}^{-4A+2\lambda_2+3\lambda_3-2\phi}\right)\mathcal{R}_4 = \frac{4}{3}\left(\int \mathrm{d}z\,\mathrm{e}^{2\lambda_2+\lambda_3-2\phi}\right)\mathcal{R}_{\kappa_3} = -\int \mathrm{d}z\,\mathrm{e}^{3\lambda_3-2\phi}\,. \tag{7.38}$$

Given that the dS solutions in [75] exhibit $\mathcal{R}_{\kappa_3} < 0$, they are inconsistent with the equation (7.38). In other words, the CDT2 model allows only classical AdS solutions. This raises the question whether other models where $\mathcal{R}_{\kappa_3} > 0$ could support dS solutions. We study these "CDT2-like" models in [47]. Furthermore, note that in the smeared approximation, the equation (7.38) simplifies to $\mathcal{R}_4 = 4\mathcal{R}_{\kappa_3}/3$, which coincides with the result obtained by combining the equations in (7.34).

It might be argued that integrating a classical equation over the entire internal space, including the singular regions associated with the (anti-)O6$^-$-planes where classical SUGRA is inapplicable, is problematic. This criticism was notably highlighted by [75] with respect to the equation (7.10), which relies on a similar formulation. To clarify our approach, consider integrating (7.36) up to the boundary of two singular holes with centers at $z = 0, \pi$ and diameter ϵ, that is, over a region where classical SUGRA remains reliable. In each equation of (7.38) appear two additional boundary terms associated with each (anti-)O6$^-$-plane and depending on the diameter of the holes we cut out. However, due to the orientifold projection (7.26), these terms are identical. Consequently, we obtain

$$\begin{aligned}
\left(\int_\epsilon^{\pi-\epsilon} \mathrm{d}z\,\mathrm{e}^{-4A+2\lambda_2+3\lambda_3-2\phi}\right)\mathcal{R}_4 &= \frac{4}{3}\left(\int_\epsilon^{\pi-\epsilon} \mathrm{d}z\,\mathrm{e}^{2\lambda_2+\lambda_3-2\phi}\right)\mathcal{R}_{\kappa_3} + \mathcal{B}^{(1)}(\epsilon)\\
&= -\int_\epsilon^{\pi-\epsilon} \mathrm{d}z\,\mathrm{e}^{3\lambda_3-2\phi} + \mathcal{B}^{(2)}(\epsilon)\,.
\end{aligned} \tag{7.39}$$

As mentioned above equation (2.46), taking the limit $g_s \to 0$ causes these singular regions and their boundaries to shrink to points, thus extending the scope of classical SUGRA over the entire spacetime. We will not go into the details of this limit, other than to require that it be one in which the backreaction from the O6-planes becomes small, thus ensuring that the singularities disappear. As discussed in Section 7.2.2, the backreaction of the O8$^+$ is not negligible in the CDT2 model without leading to trivial vacuum solutions; therefore, we allow it to remain finite, which means that the dilaton and other functions may vary considerably with z. Note that this does not contradict our discussion in Section 2.4, where the backreaction effects of the O8$^+$ dominate those of the O6-planes when the transverse volume R is sufficiently large.

In the limit where g_s approaches zero and the backreaction from the O6-planes is absent, the boundary terms become classically negligible, making these corrections also irrelevant for sufficiently small holes, where classical SUGRA retains its validity almost everywhere except in minuscule singular regions. Therefore, for sufficiently small ϵ and the analysis restricted to classical sources, the equation (7.38) holds and the singularities do not affect the classical no-go result. While O6-planes within these boundaries may introduce discontinuities that could prevent the boundary terms from vanishing as $g_s \to 0$, the particular combinations of the equations of motion ensure that no delta-function source terms appear at the classical level. Thus, the contribution of the O6-/O8-planes to (7.37) occurs at most at the 4-derivative level, which, as we will see in the following section, are generally insufficient to challenge the existing no-go theorem in the small-hole regime.

However, as g_s increases, causing the holes to expand and occupy a larger fraction of spacetime, thereby violating the Small-Hole Condition outlined in Section 2.4, string corrections to (7.38) become significant and we can no longer reliably predict dS vacua. Under such conditions, relying on classical SUGRA becomes unfeasible and using classical equations of motion to search for dS vacua, as done in [75], is no longer tenable. Instead, this scenario requires the use of non-perturbative techniques to accurately calculate the vacuum energy. Therefore, we conclude that (7.38) effectively excludes the existence of dS vacua in the CDT2 model in all regimes where classical SUGRA remains applicable.

7.2.4 Including α' corrections

We revisit the derivation of the no-go theorem (7.38), which we previously discussed, by including 4-derivative corrections (2.26) to the source action, while assuming that α' corrections in the bulk spacetime remain negligible. Our objective is to evaluate the impact of these modifications on the conclusion of the no-go theorem. The corrected equations of motion provide an analog of (7.36), specifically

$$\begin{aligned} \mathrm{e}^{-2A}\mathcal{R}_4 = &\frac{4}{3}\mathrm{e}^{2A-2\lambda_3}\mathcal{R}_{\kappa_3} + 4\frac{\mathrm{e}^{-4A+2\phi}}{\sqrt{g_6}}\partial_m\left(\mathrm{e}^{4A-2\phi}\sqrt{g_6}\,\partial^m\left(2A-\lambda_3\right)\right) \\ &-\sum_i \frac{T_i}{2\pi}\mathrm{e}^{2\phi}\delta(\Sigma_i)\left(\frac{4}{3}g^{xy}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta g^{xy}} - g^{\mu\nu}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta g^{\mu\nu}}\right), \end{aligned} \tag{7.40}$$

$$
\begin{aligned}
\mathrm{e}^{-2A}\mathcal{R}_4 =& -\mathrm{e}^{2A-2\lambda_2} + \frac{\mathrm{e}^{-4A+2\phi}}{\sqrt{g_6}}\partial_m\left(\mathrm{e}^{4A-2\phi}\sqrt{g_6}\,\partial^m\left(3A+\lambda_2-\phi\right)\right)\\
&-\sum_i \frac{T_i}{2\pi}\mathrm{e}^{2\phi}\delta(\Sigma_i)\left(\frac{1}{4}e^{-\phi}\mathcal{L}_{\alpha'^2,i} + \frac{1}{4}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta\phi} + \frac{1}{2}g^{xy}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta g^{xy}}\right.\\
&\left.+\frac{1}{2}g^{zz}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta g^{zz}} - \frac{1}{2}g^{\mu\nu}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta g^{\mu\nu}}\right)\,,
\end{aligned}
\tag{7.41}
$$

where x, y are the indices of the internal 3D Einstein space κ_3. In addition, we introduce another combination of the equations involving classical source terms,

$$
\begin{aligned}
\mathrm{e}^{-2A}\mathcal{R}_4 =& -\mathrm{e}^{2\phi}F_0^2 - \mathrm{e}^{4A-4\lambda_2+2\phi}f_2^2 + 4\frac{\mathrm{e}^{-4A+2\phi}}{\sqrt{g_6}}\partial_m\left(\mathrm{e}^{4A-2\phi}\sqrt{g_6}\,\partial^m A\right)\\
&-\sum_i\frac{T_i}{2\pi}e^{2\phi}\delta(\Sigma_i)\left(\mathrm{e}^{-\phi}+2\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i} + \frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta\phi} - g^{\mu\nu}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,i})}{\delta g^{\mu\nu}}\right)\,,
\end{aligned}
\tag{7.42}
$$

which will be important for the following discussion.

By integrating the above equation (7.40) over the internal manifold up to the boundary of the singular regions, we derive

$$
\begin{aligned}
&\left(\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{-4A+2\lambda_2+3\lambda_3-2\phi}\right)\mathcal{R}_4 = \frac{4}{3}\left(\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{2\lambda_2+\lambda_3-2\phi}\right)\mathcal{R}_{\kappa_3} + \mathcal{B}^{(1)}_{\mathrm{O6}^-}(\epsilon)\\
&\quad-\frac{T_{\mathrm{O8}^+}}{2\pi R}\mathrm{e}^{-A+2\lambda_2+3\lambda_3}\left(\frac{4}{3}g^{xy}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,\mathrm{O8}^+})}{\delta g^{xy}} - g^{\mu\nu}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,\mathrm{O8}^+})}{\delta g^{\mu\nu}}\right)\Bigg|_{z=\pi/2}\,,
\end{aligned}
\tag{7.43}
$$

including the boundary term $\mathcal{B}^{(1)}_{\mathrm{O6}^-}(\epsilon)$ associated with the (anti-)O6$^-$-planes. A similar expression arises for the other equations (7.41) and (7.42), where the boundary terms are defined by

$$
\begin{aligned}
\mathcal{B}^{(1)}_{\mathrm{O6}^-}(\epsilon) &= -\frac{8}{R^2}\mathrm{e}^{2\lambda_2+3\lambda_3-2\phi}(2A-\lambda_3)'\big|_{z=\epsilon}\,,\\
\mathcal{B}^{(2)}_{\mathrm{O6}^-}(\epsilon) &= -\frac{2}{R^2}\mathrm{e}^{2\lambda_2+3\lambda_3-2\phi}(3A+\lambda_2-\phi)'\big|_{z=\epsilon}\,,\\
\mathcal{B}^{(3)}_{\mathrm{O6}^-}(\epsilon) &= -\frac{8}{R^2}\mathrm{e}^{2\lambda_2+3\lambda_3-2\phi}A'\big|_{z=\epsilon}\,.
\end{aligned}
\tag{7.44}
$$

From the analysis of the equations (7.40) and (7.41) it is evident that there is a potential to circumvent the classical dS no-go theorem, provided that the terms associated with the (anti-)O6$^-$/O8$^+$ are sufficiently large to compensate the negative contributions within these equations. To explore this possibility, it is crucial to precisely define the boundary conditions near the O6-planes.

We use Gauss's law, or the divergence theorem of vector calculus, which establishes a mathematical relation between a volume integral and a surface integral over the boundary of that volume. Applying this theorem to the boundary terms (7.44) is challenging due to their complexity. By equating these terms with an integral over the enclosed volume, we effectively integrate over the problematic hole. However, for small singularities, where the string coupling approaches zero, the surface integral becomes identical to the contribution from a probe O6-plane, whose backreaction is negligible, closely matching string theory predictions [76–84]. Assuming these conditions retain their validity even at small, non-zero values of ϵ, representing the size of these singularities, the appropriate boundary integral is given by

$$\mathcal{B}^{(1)}_{\mathrm{O6}^-}(\epsilon) = -\frac{2T_{\mathrm{O6}^-}}{8\pi^2 R}\mathrm{e}^{A+3\lambda_3}\left(\frac{4}{3}g^{xy}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,\mathrm{O6}^-})}{\delta g^{xy}} - g^{\mu\nu}\frac{\delta(\mathrm{e}^{-\phi}\mathcal{L}_{\alpha'^2,\mathrm{O6}^-})}{\delta g^{\mu\nu}}\right)\bigg|_{z=\epsilon}, \tag{7.45}$$

and similarly for $\mathcal{B}^{(2)}_{\mathrm{O6}^-}$, $\mathcal{B}^{(3)}_{\mathrm{O6}^-}$, while ignoring corrections beyond the 4-derivative level. However, caution is advised when the singularities are not minimal, i.e., $\epsilon \sim \mathcal{O}(1)$, where the clear distinction between a weakly curved bulk and localized holes of O-planes becomes obscured.

Moreover, we observe that the boundary conditions set in the numerical dS solution [75] do not specify derivatives such as $(2A - \lambda_3)'$ near the singularities. This uncertainty allows for deviations of the boundary terms in (7.43) from the theoretical contributions expected from a standard O6-plane. This situation is similar to the unresolved permissive boundary conditions observed in the CDT1 model, where the sources are undefined in certain combinations of the equations of motion where the classical contributions from the O8-planes are canceled; see Section 2.3. Our analysis emphasizes the need for the derivatives to strictly conform to rigorous boundary conditions, which are determined by the higher-derivative terms described in (7.45). As a result, (7.43) leads to the following expression,

$$\begin{aligned}\left(\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{-4A+2\lambda_2+3\lambda_3-2\phi}\right)\mathcal{R}_4 &= \frac{4}{3}\left(\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{2\lambda_2+\lambda_3-2\phi}\right)\mathcal{R}_{\kappa_3}\\ &+\frac{2T_{\mathrm{O6}^-}}{8\pi^2 R}\mathrm{e}^{A+3\lambda_3-\phi}\mathcal{O}(E^2)_{\mathrm{O6}^-}\big|_{z=\epsilon} + \frac{T_{\mathrm{O8}^+}}{2\pi R}\mathrm{e}^{-A+2\lambda_2+3\lambda_3-\phi}\mathcal{O}(E^2)_{\mathrm{O8}^+}\big|_{z=\pi/2},\end{aligned} \tag{7.46}$$

in which the quadratic terms in the energy densities/curvature, $\mathcal{O}(E^2)_i$, arise from the α' corrections, $\mathcal{L}_{\alpha'^2,i}$.

We are now prepared to study whether 4-derivative corrections can circumvent the classical no-go theorem. According to equation (7.46), the only solution is AdS, unless the $\mathcal{O}(E^2)_i$ terms are sufficient to offset the integrated curvature. However, compensating a classical term with α' corrections while maintaining low energy densities and curvatures appears challenging. Nonetheless, warping effects might play a crucial role, potentially allowing a balance between terms of different orders in α'. This requires a more detailed discussion.

Starting from (7.42) and the equivalent expression (7.45) for $\mathcal{B}^{(3)}_{\mathrm{O6}^-}$, we obtain

$$\begin{aligned}\left(\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{-4A+2\lambda_2+3\lambda_3-2\phi}\right)\mathcal{R}_4 &= -\left(\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{-2A+2\lambda_2+\lambda_3}\right)F_0^2\\ &-\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{2A-2\lambda_2+\lambda_3}f_2^2 - \frac{2T_{\mathrm{O6}^-}}{8\pi^2 R}\mathrm{e}^{A+3\lambda_3-\phi}\left(1+\mathcal{O}(E^2)_{\mathrm{O6}^-}\right)\big|_{z=\epsilon}\end{aligned}$$

$$-\frac{T_{\mathrm{O8}^+}}{2\pi R}\mathrm{e}^{-A+2\lambda_2+3\lambda_3-\phi}\left(1+\mathcal{O}(E^2)_{\mathrm{O8}^+}\right)\big|_{z=\pi/2}\,, \tag{7.47}$$

which also includes classical source terms. For $E^2 \ll 1$, this equation suggests that

$$\frac{2|T_{\mathrm{O6}^-}|}{8\pi^2 R}\mathrm{e}^{A+3\lambda_3-\phi}\big|_{z=\epsilon} \gtrsim \frac{T_{\mathrm{O8}^+}}{2\pi R}\mathrm{e}^{-A+2\lambda_2+3\lambda_3-\phi}\big|_{z=\pi/2} \tag{7.48}$$

for de Sitter, which is why the α' corrections for the O8 can be ignored in following analysis. Using a similar result derived from (7.42) and the corresponding expression (7.45) for $\mathcal{B}^{(2)}_{\mathrm{O6}^-}$, we establish the de Sitter condition

$$\frac{2|T_{\mathrm{O6}^-}|}{8\pi^2 R}\mathrm{e}^{A+3\lambda_3-\phi}\big|_{z=\epsilon} \gtrsim \int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{3\lambda_3-2\phi}\,. \tag{7.49}$$

Given that supergravity is considered reliable throughout the integration interval, it is essential that both the string coupling and the energy densities remain weak across this domain, i.e.,

$$E \ll 1\,, \qquad \mathrm{e}^\phi \ll 1\,, \qquad \frac{\mathrm{e}^{2A}}{R^2}(\phi')^2 \ll 1\,. \tag{7.50}$$

The latter relation pertains to the energy density of the dilaton field. Additionally, the string-frame curvatures must also be small to ensure control; thus, terms entering curvature invariants such as the Ricci scalar must satisfy the following relation,

$$\frac{\mathrm{e}^{2A}}{R^2}(A')^2 \ll 1\,, \qquad \frac{\mathrm{e}^{2A}}{R^2}(\lambda_3')^2 \ll 1\,, \tag{7.51}$$

for all $z \in [\epsilon, \pi-\epsilon]$. To meaningfully discuss an effective supergravity solution, it is crucial that the relevant length scales are large in string units. This requirement is particularly important along the z interval, where (7.22) implies that

$$e^{-A}R \gg 1\,, \qquad \forall z \in [\epsilon, \pi-\epsilon]\,, \tag{7.52}$$

using the conditions (7.51). From these relations, it follows that $|(\mathrm{e}^{A+3\lambda_3-2\phi})'|/R \ll \mathrm{e}^{3\lambda_3-2\phi}$ within the interval where supergravity is reliable, leading to

$$\frac{1}{R}\left(\mathrm{e}^{A+3\lambda_3-2\phi}\big|_{z=\epsilon} - \mathrm{e}^{A+3\lambda_3-2\phi}\big|_{z_{\min}}\right) \ll \int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{3\lambda_3-2\phi}\,, \tag{7.53}$$

where $z_{\min} \in [\epsilon, \pi-\epsilon]$ corresponds to the global minimum of $\mathrm{e}^{A+3\lambda_3-2\phi}$. Furthermore, given that

$$\frac{1}{R}\mathrm{e}^{A+3\lambda_3-2\phi}\big|_{z_{\min}} \le \frac{1}{R(\pi-2\epsilon)}\int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{A+3\lambda_3-2\phi} \ll \int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{3\lambda_3-2\phi}\,, \tag{7.54}$$

we deduce

$$\frac{1}{R}\mathrm{e}^{A+3\lambda_3-2\phi}\big|_{z=\epsilon} \ll \int_\epsilon^{\pi-\epsilon}\mathrm{d}z\,\mathrm{e}^{3\lambda_3-2\phi}\,. \tag{7.55}$$

From equation (7.49), which posits that classical terms can be balanced with α' corrections, and equation (7.55), which provides a condition based on the reliability of supergravity within the specified interval, we derive that $\mathrm{e}^{\phi} E^2|_{z=\epsilon} \gg 1$. This result, however, stands in stark contradiction to the constraints outlined in equation (7.50).

In the CDT2 model, attempts to balance classical bulk terms with α' corrections arising from O6 holes do not yield a viable dS solution where string coupling, curvature and energy densities remain small throughout the bulk spacetime. The α' corrections are insufficient to compensate for the negative contributions from the classical terms. Furthermore, large warping effects are untenable, since they invariably lead to large curvature and energy densities within the bulk. Notably, this conclusion is reached independently of the specifics of the 4-derivative corrections.

Consequently, in scenarios where singularities are small and classical supergravity remains reliable over spacetime, (7.36) approximately holds. Even when considering 4-derivative or higher-order corrections to the O-plane source terms, the dS no-go theorem is not violated.

7.2.5 Non-standard sources

In the previous sections, we argued that the CDT2 model [75] is unlikely to support dS vacua within a regime where classical SUGRA is considered reliable. Nevertheless, [75] claims to have identified a numerical dS solution within this framework. This finding implies that at least one of the assumptions fundamental to our analysis may have been violated. In particular, we assumed that singular holes around the O-planes are sufficiently small to allow a meaningful description of classical SUGRA over most of the spacetime; the "Small-Hole Condition" discussed in Section 2.4. Furthermore, we have chosen the source terms in the equations of motion to reflect either classical or slightly α'-corrected contributions from the O-plane action, as detailed in Section 2.3. It is crucial to clarify that satisfying these criteria alone does not prove the non-perturbative existence of a SUGRA solution; rather, they serve as essential validity checks – necessary but not sufficient to ensure a reliable solution.

We want to study these assumptions within the CDT2 model to see if it is possible to circumvent the no-go theorem against (almost) classical dS solutions by considering compactifications with larger singular regions or source terms that differ significantly from the conventional O-plane action. However, these scenarios are generally regarded as unphysical, since they violate the self-consistency of SUGRA, or the physical validity of the sources becomes questionable.

The dS solution reported in [75] appears to involve non-standard sources. In particular, the behavior of the fields near the (anti-)O6$^-$ is unconventional. The warp factor $e^{-4A(z)}$ either remains finite or vanishes, while the exponential expression of the dilaton, e^{ϕ}, together with e^{λ_i}, decays at an unexpected rate as one approaches the O6-planes, leading to atypical irrational exponents that differ from classical SUGRA predictions. These anomalies suggest a break from conventional models, potentially undermining the credibility of this particular solution.

In [47, App. B], we have analytically replicated this behavior observed near the O6-plane by solving the equations locally using a method similar to that used in [193, App.

B] for a different compactification scenario. Our results are consistent with the behavior reported in [75]. We assume a power-law dependence for the fields near O6, as suggested in the latter reference. However, the possibility of boundary conditions that exhibit logarithmic scaling, and thus potentially more complex behavior, cannot be completely ruled out. For simplicity, we specify the source location at $z = 0$ to ensure consistency throughout our analysis without compromising generality. Under this assumption, both the dilaton field and the warp factors adhere to

$$\begin{gathered} \mathrm{e}^{-4A} \sim z^{-F+\frac{1}{3}\sqrt{-15F^2+48FM-60M^2-24F+24M+12}}\,, \qquad \mathrm{e}^{\phi} \sim z^{F}\,, \\ \mathrm{e}^{\lambda_2} \sim z^{M}\,, \qquad \mathrm{e}^{\lambda_3} \sim z^{\frac{1}{3}(2F-2M+1)}\,, \end{gathered} \tag{7.56}$$

characterized by parameters M and F, depending on f_2. In cases where $\lim_{z\to 0} f_2 \to 0$, we obtain

$$\max\left(0,\, M-\frac{1}{2}\right) < F \leq M - \frac{1}{2} + \frac{\sqrt{3-6M^2}}{2}\,, \qquad 0 < M < \frac{\sqrt{2}}{2}\,, \tag{7.57}$$

whereas for $\lim_{z\to 0} f_2$ remaining finite, we conclude

$$\begin{gathered} \max\left(0,\, \frac{8M-4}{5} + \frac{\sqrt{-9M^2-6M+9}}{10}\right) < F \leq M - \frac{1}{2} + \frac{\sqrt{3-6M^2}}{2}\,, \\ 0 < M < \frac{1+\sqrt{6}}{5}\,. \end{gathered} \tag{7.58}$$

The irrational exponents in (7.56) indicate deviations from conventional models. Typically, an O6-plane in flat space would yield

$$\text{for } z \to 0: \quad \mathrm{e}^{-4A} \sim z^{-1}\,, \qquad \mathrm{e}^{\phi} \sim z^{\frac{3}{4}}\,, \qquad \mathrm{e}^{\lambda_2} \sim z\,, \qquad \mathrm{e}^{\lambda_3} \sim z^{\frac{1}{2}}\,. \tag{7.59}$$

Identifying the specific delta-function sources required to generate the unconventional boundary conditions outlined in (7.56), and confirming that these sources are inconsistent with any known object in string theory, is crucial to further validate or refute the physical plausibility of these solutions under standard interpretations of string theory.

8 Summary and outlook

Our universe exhibits characteristics of a dS spacetime during certain epochs, notably the inflationary period and the ongoing accelerated expansion. For string theory to effectively model our observable universe, it must successfully integrate these de Sitter phases. This thesis has explored this requirement by studying dS solutions within the classical regime of string theory. In this section, we summarize our results and identify promising directions for future research.

In section 4, we used the 10D equations to derive no-go theorems that constrain the existence of classical dS solutions in spacetime dimensions $d \geq 3$. The following analysis of the field and source content in type II compactifications revealed a pronounced scarcity, especially in higher dimensions. This shortness often satisfies the assumptions underlying our no-go theorems, thus naturally excluding dS solutions in $d \geq 8$. For dimensions $d = 7, 6, 5$, viable configurations are severely constrained, especially when considering SUSY-preserving sources. Consequently, only a few source configurations remain viable in $d = 6, 5$, typically involving at most two intersecting source sets. This observation supports two conjectures in [35, 45] that exclude dS solutions in $d \geq 5$. Thus, $d = 4$ emerges as the highest feasible dimension for dS solutions, which represents an interesting area for further exploration.

This discussion opens up several avenues for future research. A focus on SUSY-preserving configurations could refine the search for classical dS solutions, particularly in higher dimensions where constraints on sources and fluxes are more stringent than in $d = 4$, a dimension known for its complexity. Future studies could aim to validate or refine the conjectures presented in [35,45]. In addition, extending this research to a stability analysis of classical dS solutions could shed light on whether the admissible solution classes might be perturbatively unstable [49,52]. Another compelling direction would be to explore the conjecture proposed in [194] that accelerated cosmological expansion is unattainable in the asymptotic regions of field space unless metastable dS vacua exist in higher dimensions.

In Section 5 we revisited established no-go theorems in the context of a d-dimensional effective theory derived from type II compactifications. For each theorem, we computed the value of c in the equation (5.2), which should be compared with the bound proposed by the Trans-Planckian Censorship Conjecture. Notably, for dimensions $4 \leq d \leq 10$, all derived values either meet or exceed the bound, with multiple instances of saturation. This agreement is particularly striking because our analysis was restricted to supergravity, independent of the cosmological principles integral to the TCC. Furthermore, in $d = 3$, our newly derived no-go theorem leads to a c-value that violates the TCC bound, underscoring the unique topological nature of gravity in this dimension. This discrepancy initiates a broader discussion about the applicability of swampland conjectures in $d = 3$, which remains an open question.

These studies set the stage for several interesting future research directions. One particularly promising avenue is to explore the unique properties of gravity in $d = 3$. Investigating other specific examples in $d = 3$ that challenge the swampland conjectures, in particular the TCC and the de Sitter conjecture, could prove enlightening. The type IIA scale-separated AdS solutions discussed in [131, 132] may provide relevant case studies. Another compelling path is the detailed analysis of the relation between the TCC bound and other proposals within the swampland [125]. Such an analysis could clarify how these different bounds overlap and differ, thus improving our understanding of their implications for cosmological models. This could be particularly important for the development of viable quintessence scenarios.

In our analysis of negative scalar potentials in Section 6, which are particularly relevant for AdS solutions, we found structural similarities with positive potentials. This observation suggests that analytical methods developed for positive scalar potentials could be effectively adapted to negative ones. Our models focus on contracting spacetimes, consistent with the Anti-Trans-Planckian Censorship Conjecture, which redefines our conventional understanding of contracting universes by emphasizing the need to preserve the validity of effective theory. We established a lower bound for negative potentials and explored its implications for their asymptotic behavior. In addition, we derived asymptotic conditions on the derivatives of the potential and studied their consequences and holographic implications.

Our recent discoveries, in particular the new condition on the second derivative of the potential (6.38) and the flexible mass bound (6.41) in an AdS solution, require further exploration of their broader implications. We observed that mass spectra are generally more stable in supersymmetric configurations and exhibit greater variability in non-supersymmetric scenarios. A detailed study of the evolution of the mass spectrum across the field space, especially towards AdS critical points, could provide profound insights. Given the prevalence of multi-field models in typical string compactifications, extending our analysis to multiple fields is crucial. A comprehensive study of these extensions, further characterizing scalar potentials in multi-field settings, and a broader application of the ATCC would be intriguing.

In the context of the CDT1/2 models [72, 75] discussed in Section 7, we have studied the feasibility of dS vacua in flux compactifications with O8/D8 and O6/D6. Our analysis shows that none of these models support classical metastable dS solutions. We sought to maintain control over the α' expansion and to obey the Small-Hole Condition [96], ensuring that any singularity near the O-planes that would perturb supergravity is small relative to the size of the compact space. This approach preserves the simplicity of the original models, while introducing "almost classical" scenarios that include α' corrections at the 4-derivative level to classical source terms, excluding higher-order corrections and those in the bulk. Our study of how these 4-derivative corrections affect the classical no-go results shows conclusively that dS vacua remain unattainable, unaffected by these modifications in both the original and generalized models. These results imply that the numerical dS solutions identified in [72, 75] likely originate from unphysical sources that are incompatible with the conventional O-planes in string theory.

While our study highlights the limitations of the almost classical dS scenario within these models, the implications of these findings warrant further investigation in other type II flux compactifications. Future studies should explore scenarios where classical conditions indicate either a Minkowski solution or a finely-tuned scenario where various classical terms nearly cancel out in the vacuum energy. The 4-derivative corrections should be able to raise the vacuum energy to a positive value without introducing instabilities. It is also imperative to derive the backreacted analogues in models where our no-go arguments were based on a smeared limit. Moreover, our results contribute to the emerging consensus that dS solutions are likely unattainable in perturbative regimes of string theory where the scalar potential is approximated by a few leading α' terms [40, 96, 103][24]. This consensus underscores the need for further investigation of the universal constraints that string theory imposes on cosmological models.

[24]See also [42] for possible counterexamples.

Appendices

Appendix A Standard formulation of type II string theory

In this appendix, we elucidate the compatibility of the democratic action [89, 90] with the standard formulations of (massive) type II SUGRA [3, 39, 74] previously used in this thesis. Specifically, we address the nuances associated with spacetime-filling fluxes. Given that the source content remains unchanged, it will not be further discussed in this section. To be consistent with the conventional formulations of type II SUGRA, it is necessary to break the democracy of the RR fluxes via the self-duality conditions defined in equation (2.19). The 10D NSNS flux is denoted by H_3^{10}, whereas the RR fluxes specified in equation (2.16) are given by F_q^{10} for $0 \leq q \leq 5$, therefore excluding field strengths for $q \geq 6$. In dimensions $d > 3$, the fluxes $F_{0,1,2}^{10} = F_{0,1,2}$ are purely internal, both in their components and coordinate dependencies, while H_3^{10} and $F_{3,4,5}^{10}$ may include d-dimensional components. The requirement of maximal symmetry mandates that only the spacetime-filling fluxes H_3^d and F_q^d are non-zero for $q \geq d$. We formulate this by expressing $H_3^{10} = H_3^d + H_3$ and $F_q^{10} = F_q^d + F_q$.

To elaborate further, by definition, F_q^d with $q \geq d$ is parallel to the external spacetime, that is vol_d, resulting in $q - d$ legs along the internal directions. To accommodate these fluxes, we introduce internal field strengths F_{10-q}, where $*_{10-d}F_{10-q}$ compensates for the additional degrees of freedom. More specifically, the internal fluxes F_{10-q} and H_7 are defined as

$$\begin{aligned} F_q^d &= (-1)^{\frac{q(q+1)}{2}} *_{10} F_{10-q} = (-1)^{\frac{q(q+1)}{2}} (-1)^{(10-q)d}\ \mathrm{vol}_d \wedge *_{10-d} F_{10-q}\,, \\ H_3^d &= *_{10} H_7 = (-1)^d\ \mathrm{vol}_d \wedge *_{10-d} H_7\,. \end{aligned} \tag{A.1}$$

This convention aligns with the self-duality relation for F_5^{10}, leading to

$$|F_q^{10}|^2 = |F_q|^2 - |F_{10-q}|^2\,, \qquad |H_3^{10}|^2 = |H_3|^2 - |H_7|^2\,. \tag{A.2}$$

In this framework, the bosonic part of the 10D bulk action in (2.17), which remains consistent with the equations of motion outlined in Section 2.2, is captured by

$$S_{\mathrm{bulk}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g} \left(\mathrm{e}^{-2\phi} \left(\mathcal{R} + 4(\partial\phi)^2 - \frac{1}{2}|H_3|^2 \right) - \frac{1}{2} \sum_{q=0}^{4} |F_q^{10}|^2 - \frac{1}{4} |F_5^{10}|^2 \right), \tag{A.3}$$

in the string frame. It is important to note that the democratic formalism, especially concerning the flux F_5^{10}, cannot be entirely disregarded. The above equation (A.3) includes a pseudo-action for F_5^{10}, necessitating the imposition of the self-duality relation,

$$F_5^{10} = - *_{10} F_5^{10} \ \Rightarrow\ |F_5^{10}|^2 = 0\,, \tag{A.4}$$

on-shell. In addition, we detail our compactification ansatz for the metric and the dilaton. The 10-dimensional metric (2.15), in the absence of any warp factor (smeared approximation), is assumed as

$$ds_{10}^2 = g_{\mu\nu}(x)dx^\mu dx^\nu + g_{mn}(x,y)dy^m dy^n \,. \tag{A.5}$$

At this point, without going into detail, we briefly revisit the dilaton equation of motion, as described in equation (2.27),

$$2\mathcal{R}_{10} + e^\phi \sum_i \frac{T_{10}^{(p_i)}}{p_i+1} - |H_3^{10}|^2 + 8\left(\nabla^2\phi - (\partial\phi)^2\right) = 0\,, \tag{A.6}$$

in the smeared limit, while the trace-reversed Einstein equation is given by

$$\begin{aligned} 2\mathcal{R}_{MN} = |H_3^{10}|^2_{MN} - \frac{g_{MN}}{4}|H_3^{10}|^2 + e^{2\phi}\sum_{q=0}^{4}\left(|F_q^{10}|^2_{MN} - \frac{q-1}{8}g_{MN}|F_q^{10}|^2\right) \\ + \frac{e^{2\phi}}{2}|F_5^{10}|^2_{MN} + e^\phi\left(T_{MN} - \frac{g_{MN}}{8}T_{10}\right) - 4\nabla_M\partial_N\phi - \frac{g_{MN}}{2}\left(\nabla^2\phi - 2(\partial\phi)^2\right) . \end{aligned} \tag{A.7}$$

The 10D Einstein trace is derived as follows,

$$4\mathcal{R}_{10} + \frac{e^\phi}{2}T_{10} - |H_3^{10}|^2 - \frac{e^{2\phi}}{2}\sum_{q=0}^{7}(5-q)|F_q|^2 + 18\nabla^2\phi - 20(\partial\phi)^2 = 0\,. \tag{A.8}$$

We redefine the source energy-momentum tensor T_{MN} using the orthonormal coframe discussed in Section 3 and apply the transformation $T_{AB} = e^M{}_A e^N{}_B T_{MN}$. The metric, as detailed in equation (A.5), facilitates the following decomposition,

$$\begin{aligned} T_{AB} = \delta_A^\alpha\delta_B^\beta\, T_{\alpha\beta} + \sum_i \delta_A^{a_{\parallel i}}\delta_B^{b_{\parallel i}}\, T^{(p_i)}_{a_{\parallel_i} b_{\parallel_i}} \\ \text{with}\quad T_{\alpha\beta} = \eta_{\alpha\beta}\sum_i \frac{T_{10}^{(p_i)}}{p_i+1} \quad\text{and}\quad T^{(p_i)}_{a_{\parallel_i} b_{\parallel_i}} = \delta_{a_{\parallel_i} b_{\parallel_i}}\frac{T_{10}^{(p_i)}}{p_i+1} \end{aligned} \tag{A.9}$$

and $T_{a_{\perp_i} b_{\perp_i}} = e^M{}_{A_{\perp_i}} e^N{}_{B_{\perp_i}} T_{MN} = 0$, where α, β correspond to the d-dimensional flat indices, while the indices $\{a_{||_i}, a_{\perp_i}\}$ pertain to the $(10-d)$-dimensional internal directions, either parallel or transverse to each source set i. From the equation (A.7) and

$$\frac{1}{(q-1)!}F^{10}_{q\ \mu P\ldots Q}F^{10\ P\ldots Q}_{q\ \nu} = -g_{\mu\nu}|F_{10-q}|^2\,, \tag{A.10}$$

we deduce the d-dimensional Einstein equation,

$$\begin{aligned} \mathcal{R}_{\mu\nu} = \frac{g_{\mu\nu}}{16}\Bigg(g_s\sum_i \frac{7-p_i}{p_i+1}\,T_{10}^{(p_i)} - 2|H_3|^2 - 6|H_7|^2 \\ + g_s^2\sum_{q=0}^{10-d}(1-q)|F_q|^2 - 4\nabla^2\phi + 8(\partial\phi)^2\Bigg) . \end{aligned} \tag{A.11}$$

Appendix B Résumé de la thèse

B.1 Vers une théorie unifiée

Dans le paysage de la physique théorique du XXe siècle, deux théories fondamentales se sont distinguées par leur vérification expérimentale rigoureuse et leurs profondes implications théoriques : la relativité générale et le modèle standard de la physique des particules. De façon remarquable, leurs prédictions n'ont, jusqu'à ce jour, jamais été mises en défaut par aucune expérience connue. La relativité générale a transformé notre compréhension de la gravité en tant que propriété géométrique de l'espace-temps, fournissant une explication sophistiquée aux phénomènes cosmologiques. Au niveau quantique, le modèle standard fournit un cadre théorique complet, qui repose sur la théorie des champs quantiques relativistes, et décrit l'interaction électromagnétique ainsi que les forces nucléaires faible et forte entre les particules élémentaires. Au prix de l'étendre pour y inclure des neutrinos massifs, ce modèle permet de reproduire le spectre des particules tel qu'on l'observe jusqu'aux échelles d'énergie atteignables actuellement – soit plusieurs milliers de gigaélectronvolts (GeV) – et se montre fiable pour expliquer les phénomènes physiques bien en dessous de l'échelle de Planck.

Malgré leurs remarquables succès, ces théories laissent d'importantes questions en suspens et sont confrontées à de sérieux défis lorsqu'elles sont étendues au-delà de leurs domaines de validité respectifs. La relativité générale et le modèle standard, en tant qu'entités distinctes, fournissent des descriptions incompatibles lorsque les effets quantiques et gravitationnels dominent, comme à proximité des singularités des trous noirs et dans les conditions de l'Univers primordial. Le modèle standard s'y révèle inadapté car il n'incorpore aucune description de l'interaction gravitationnelle, et n'est pas en mesure d'expliquer certaines observations cosmologiques, telles que les anomalies gravitationnelles que l'on attribue généralement à l'existence de matière noire, la densité d'énergie noire, ou encore l'expansion accélérée de l'Univers.

Bien que le modèle standard décrive efficacement trois des quatre interactions fondamentales à basse énergie, il souffre de lacunes théoriques qui compromettent son statut de théorie complète, au cœur desquelles on retrouve sa dépendance en une série de paramètres empiriques, tels que les constantes de couplage des particules élémentaires. Cela pose ce qu'il convient d'appeller un problème de « naturalité », car ces paramètres présentent d'importantes variations de magnitude qui ne sont pas prédites par la théorie, mais plutôt ajustées pour correspondre aux résultats expérimentaux. De telles incohérences ont conduit à l'émergence d'une idée selon laquelle il existerait une structure sous-jacente plus fondamentale que le modèle standard lui-même, peut-être régie par des propriétés de symétrie que le cadre actuel n'explique pas de manière satisfaisante.

Une voie de recherche qui dépasse le modèle standard et améliore notre compréhension des interactions fondamentales est celle des théories de grande unification (GUT, acronyme de l'anglais *grand unification theories*). Celles-ci proposent qu'au-delà d'une certaine échelle d'énergie, connue sous le nom d'« échelle GUT », les forces électromagnétique, faible et forte puissent converger en une force unique régie par un simple groupe de Lie. Le niveau d'énergie précis auquel l'unification pourrait se produire, en supposant qu'elle se réalise dans la nature, dépend des lois de la physique qui ont court à des échelles qui sont actuellement inexplorées, car hors de portée de nos capacités expérimentales. En supposant l'existence d'une vaste gamme d'énergies sans nouvelle physique (souvent appelée « désert ») et que la théorie unificatrice soit supersymétrique (voir ci-dessous), l'énergie d'unification est estimée à environ 10^{16} GeV[25]. Cette unification potentielle ferait du modèle standard une théorie effective, limite de basse énergie d'une théorie plus fondamentale, qui distinguerait les différentes forces observées dans la nature à mesure que le niveau d'énergie diminue.

Une « théorie du tout » visant à unifier les forces électrofaible et forte avec la gravité nécessite un niveau d'énergie nettement plus élevé, compte tenu de l'absence manifeste de forces gravitationnelles dans le modèle standard. Aux échelles d'énergie généralement explorées en physique des particules, la gravité reste négligeable, ses effets ne devenant comparables à ceux de la physique quantique qu'au voisinage de l'échelle de Planck, $M_p \approx 10^{19}$ GeV. Par conséquent, un modèle qui unifie toutes les interactions fondamentales, y compris la gravité, n'est pas considéré comme essentiel aux niveaux d'énergie couramment étudiés. Toutefois, ce point de vue change radicalement dans le contexte de densités d'énergie ou de courbures de l'espace-temps extrêmement élevées, comme celles que l'on pourrait rencontrer dans les trous noirs ou dans l'Univers primordial, où la théorie quantique interagit avec de puissants champs gravitationnels. Le principal défi que pose l'incorporation de la gravité dans un modèle unifié réside dans la nature (perturbativement) non-renormalisable de la relativité générale, gouvernée par l'action d'Einstein-Hilbert. Par conséquent, la quantification naïve d'un tel modèle devrait mener à des amplitudes divergentes, à moins d'une compensation inattendue des infinis. Cette limitation ne met pas seulement en évidence l'exclusion de la gravité du modèle standard en raison des différences d'échelles, mais également la nécessité de construire une nouvelle théorie complète qui traite la gravitation de manière quantique et de façon analogue aux autres interactions fondamentales – une théorie de la « gravité quantique » (GQ).

L'une des principales énigmes de la cosmologie moderne, connue sous le nom de « problème de la constante cosmologique », est l'écart important entre la valeur de la constante cosmologique prédite par la théorie et sa valeur observée, qui lui est inférieure de plusieurs ordres de grandeur. Des mesures précises, qui s'accordent avec le modèle standard de la cosmologie (ΛCDM), suggèrent que l'univers connaît actuellement une expansion accélérée. Sa géométrie peut donc être localement modélisée par un espace-temps de de Sitter (dS), solution maximalement symétrique des équations d'Einstein en présence d'une constante cosmologique strictement positive. L'expansion universelle peut ainsi être vue comme l'effet d'une telle constante, notée Λ, avec une échelle de masse correspondante, $M_\Lambda = \sqrt{\Lambda} \approx 10^{-12}$ GeV, plusieurs ordres de grandeur en-dessous de la masse de Planck. L'origine de cette échelle étonnamment petite et des propriétés de l'« énergie sombre » qu'on lui associe souvent, reste insaisissable et pose d'importants défis conceptuels. Ceux-

[25] L'accélérateur de particules le plus avancé, le grand collisionneur de hadrons (LHC, pour *large hadron collider*), atteint des énergies allant jusqu'à 10^5 GeV dans les collisions proton-proton. Cela place l'échelle hypothétique GUT à quelques ordres de grandeur seulement sous de l'échelle de Planck de 10^{19} GeV, et donc bien au-delà de la capacité opérationnelle de n'importe quel collisionneur actuel.

ci peuvent dépasser les cadres théoriques actuels, nécessitant éventuellement une théorie de la gravité quantique ou des dimensions spatiales supplémentaires.

Dans le cadre des discussions sur la structure sous-jacente du modèle standard, un autre problème théorique est celui du boson de Higgs, dont la découverte a été cruciale pour valider le mécanisme de brisure de la symétrie électrofaible. Avant sa confirmation expérimentale en 2012, la masse prédite du boson de Higgs nécessitait un ajustement à des valeurs extrêmement précises pour éviter d'importantes corrections quantiques susceptibles de déstabiliser l'échelle électrofaible, un problème connu sous le nom de « problème de la hiérarchie de Higgs ». Un tel niveau d'ajustement est considéré comme artificiel. La supersymétrie propose un cadre théorique qui pourrait résoudre ce problème en introduisant des superpartenaires pour chaque particule, équilibrant ainsi naturellement les corrections quantiques et stabilisant la masse du boson de Higgs.

La supersymétrie (SUSY) est une extension robuste des symétries du modèle standard de la physique des particules : elle se présente sous la forme d'une symétrie globale additionnelle qui étend le groupe de Poincaré en y ajoutant des générateurs fermioniques dans la super-algèbre de Lie correspondante, contournant ainsi les hypothèses du théorème *no-go* de Coleman-Mandula. Dans l'espace-temps à quatre dimensions, le nombre de générateurs de supersymétrie, noté $\mathcal{N}$, varie de un à quatre pour les champs de spin un, et jusqu'à huit pour les particules de spin deux telles que le graviton. Dans les modèles supersymétriques, les particules sont regroupées en multiplets contenant à la fois des fermions et des bosons, généralement en nombre égal et avec des masses identiques, ce qui garantit que chaque particule est appariée avec un superpartenaire dont le spin diffère d'un demi. Cependant, cette symétrie exacte n'est pas observée aux énergies typiques du modèle standard, ce qui suggère qu'elle doit être brisée à des échelles d'énergie plus élevées.

Malgré les complications introduites par la SUSY, notamment les multiples schémas de rupture et la prédiction de superpartenaires qui n'ont pas encore été observés, ces extensions présentent des avantages significatifs. La symétrie supplémentaire dans les théories de jauge supersymétriques impose des contraintes strictes, de sorte que les corrections quantiques sont généralement plus faciles à gérer en raison de la compensation entre les contributions bosoniques et fermioniques. Un autre atout de la SUSY est qu'elle permet de préciser certaines propriétés des théories GUT : les trois constantes de couplage du modèle standard convergent exactement vers un point unique lorsqu'elles sont évaluées à des échelles d'énergie plus élevées. Ces propriétés attrayantes ont fortement motivé l'adoption de la SUSY dans le modèle standard.

Considérer la SUSY comme une symétrie locale conduit directement à des théories de supergravité (SUGRA), plus contraignantes que la relativité générale conventionnelle. Notamment, sur un espace-temps à onze dimensions, la théorie est définie de manière unique par ses symétries. En dimension inférieure, différentes théories de supergravité peuvent être considérées, possiblement reliées entre elles. Bien que ces théories permettent d'étendre la SUSY dans un cadre qui inclut la gravité, leur renormalisation constitue à nouveau un sérieux défi. Elles offrent néanmoins une voie distinctive et prometteuse menant à une théorie potentielle de la GQ, en particulier dans les dimensions supérieures.

Il convient cependant de remarquer que la physique au-delà du modèle standard n'est pas intrinsèquement liée à la théorie de la GQ, en particulier si l'on considère les échelles concernées. Comme nous l'avons déjà stipulé, les effets de la GQ deviennent significatifs à l'échelle de Planck, alors que les expansions du modèle standard se concentrent généralement sur les phénomènes à l'échelle électrofaible[26]. Néanmoins, du point de vue de l'unification, ces deux domaines devraient finir par se recouper. De plus, être en mesure de développer une théorie de la GQ qui inclut également certains groupes de jauge offre la possibilité qu'à des énergies plus faibles, certaines extensions du modèle standard soient observables. La théorie en question est bien sûr la théorie des cordes [2–6], que nous introduisons dans le paragraphe suivant.

B.2 Théorie des cordes

La théorie des cordes se présente aujourd'hui comme un cadre formel pour une théorie unifiée de la gravité quantique aux profondes implications, dans laquelle les constituants élémentaires ne sont pas des particules ponctuelles, mais des « cordes » à une dimension, intégrées dans un espace-temps de dimension supérieure (espace-cible ou *target space*). La masse et la charge des particules sont définies par les oscillations de ces cordes excitées dans différents modes de vibration. Ces dernières fournissent également une solution aux divergences ultraviolettes (UV) rencontrées dans les théories de particules ponctuelles, qui se résolvent ici naturellement par la nature étendue des cordes. Par ailleurs, le spectre de la théorie des cordes contient intrinsèquement un tenseur symétrique de rang deux, le graviton, une particule quantique messagère de l'interaction gravitationnelle. Cette propriété implique qu'une fois quantifiée, cette théorie intègre naturellement la gravité parmi ses modes de masse nulle, tout en contournant les problèmes de renormalisabilité – les amplitudes quantiques des cordes étant finies.

Dans l'approche perturbative de la théorie des cordes, la supersymétrie est intégrée au modèle comme symétrie de l'espace-cible, qui comprend donc à la fois des degrés de liberté bosoniques et fermioniques. Cette symétrie est importante car elle garantit que toutes les théories des cordes cohérentes sont exemptes d'anomalies, mais elle requiert de travailler avec un espace-temps à dix dimensions. Parmi ces modèles, on trouve cinq théories des supercordes, dont les types IIA et IIB, qui sont caractérisés par la présence de $\mathcal{N} = 2$ générateurs de supersymétrie. Les théories de cordes hétérotiques de type I et II, quant à elles, ne font appel qu'à un seul générateur de supersymétrie, $\mathcal{N} = 1$ et se distinguent par leur groupe de jauge, qui sont respectivement $E_8 \times E_8$ et $SO(32)$.

Dans les régimes de basse énergie et de couplage faible[27], ces théories s'apparentent à des théories classiques de supergravité, bien que corrigées par des dérivées d'ordre supérieur et des effets quantiques perturbatifs et non-perturbatifs. Il faut pour cela que les modes massifs de la théorie de base soient suffisamment « lourds » pour pouvoir être né-

[26] Notons que le concept d'*species scale* peut modifier considérablement la relation entre ces échelles [1].

[27] Pour que l'approximation de la supergravité soit valable, la constante de couplage des cordes, g_s, doit être petite afin que la description théorique reste perturbative.

gligés, typiquement au-delà d'une échelle de masse fixée par la longueur des cordes, l_s. Il est remarquable qu'il s'agisse de la seule constante universelle de la théorie des cordes qui n'ait pas été fixée. Malgré l'absence d'une justification concluante, elle est communément associée à l'échelle de Planck. De plus, ces théories effectives, de même que la « théorie M » à onze dimensions[28], sont reliées entre elles par des dualités : elles sont donc considérées comme des représentations diverses d'une théorie unifiée soumise à des contraintes différentes.

Établir le lien avec l'Univers observable, en particulier la capacité de la théorie des cordes à reproduire les propriétés physiques connues dans sa limite de basse énergie, est l'un des principaux défis de cette approche. Alors que l'élaboration d'une théorie des supercordes cohérente requiert dix dimensions d'espace-temps, seules quatre de ces dimensions sont observées à l'échelle macroscopique. Les six dimensions restantes sont supposées compactes et repliées sur elles-mêmes. Elles forment donc une variété lisse et compacte, généralement appelée $\mathcal{M}_6$, et qui ne deviendrait observable qu'à des niveaux d'énergie supérieurs à ceux actuellement atteints par l'expérience. Par conséquent, $\mathcal{M}_6$ demeure indétectable, ce pourquoi nous l'appelons « espace interne ». L'espace-cible à dix dimensions est donc la variété obtenue par le produit $\mathcal{M}_4 \times \mathcal{M}_6$. En guise d'exemple, le produit de six cercles T^6 est une variété particulièrement simple où chaque paramètre géométrique, tel que le rayon moyen, introduit une échelle de compactification supplémentaire, M_c. Des configurations plus compliquées incluent les variétés de Calabi-Yau, qui sont favorisées afin de préserver la supersymétrie $\mathcal{N} = 1$ dans la théorie effective à quatre dimensions.

L'on obtient la limite de basse énergie de la théorie des cordes au moyen d'une réduction dimensionnelle afin d'obtenir une description effective à quatre dimensions. Une étape importante dans le développement de cette théorie effective est l'identification des modes « légers » (ou de masse nulle) de la théorie complète, qui se manifestent typiquement comme de petites fluctuations autour d'un champ de fond (on utilise souvent le vocable anglais *background*) stable ou d'un état de vide du potentiel. Seuls ces modes sont alors retenus dans la théorie, à condition que cette approche sélective maintienne la cohérence. Le résultat est une théorie effective robuste de basse énergie, la supergravité à quatre dimensions dans le cas présent. Il est essentiel de procéder avec prudence : il faut d'abord identifier les modes légers, puis déterminer la possibilité d'éliminer les interactions avec les modes massifs avant de tronquer le spectre. Pour illustrer le processus de réduction dimensionnelle à une théorie effective à quatre dimensions, il est instructif d'examiner un exemple simple dans lequel les modes de masse nulle jouent un rôle central.

Le concept de réduction dimensionnelle a vu le jour dans les années 1920 dans les travaux de Kaluza et Klein. Considérons une théorie des champs définie sur la variété $\mathcal{M}_4 \times S^1$, où S^1 représente un cercle de rayon R muni d'une coordonnée périodique que nous notons x^5. Dans ce cas, les champs peuvent être développés en série de Fourier dans la dimension compacte, chaque terme de la série $\sim \mathrm{e}^{-\mathrm{i}(n/R)x^5}$ représentant des moments quantifiés, n/R. Toute valeur du moment contribue à la masse m_n des modes correspon-

[28] Les objets fondamentaux de la « théorie M » sont des objets étendus de dimension supérieure à un, connus sous le nom de *M-branes*. La théorie effective de basse énergie est alors considérée comme la supergravité à onze dimensions.

dants dans la théorie effective à quatre dimensions, ${m_n}^2 \sim (n^2/R^2)$, en supposant que les champs sont sans masse dans la théorie originelle à cinq dimensions. L'action de cette dernière peut alors être intégrée sur la dimension compacte pour déduire une description effective en dimension inférieure. Cela revient à dire que les champs sont indépendants des coordonnées internes. Les modes de Kaluza-Klein plus élevés (correspondant à des valeurs plus élevées de n) contribuent de moins en moins à l'action à mesure que n grandit. En outre, un rayon plus petit réduit la contribution des modes avec $n \neq 0$. Ce processus est connu sous le nom de « réduction de Kaluza-Klein ».

Revenons à présent à la supergravité à quatre dimensions. Pour être cohérente avec (les extensions du) modèle standard, cette théorie effective inclut typiquement un seul générateur de supersymétrie ($\mathcal{N} = 1$), cette dernière devant être brisée à une échelle bien inférieure à l'échelle de compactification. Par conséquent, les compactifications de cordes sont essentielles pour aligner l'espace-temps théorique à dix dimensions avec le monde observable.

Cependant, ce processus pose plusieurs défis conceptuels, notamment en ce qui concerne la stabilisation des « champs de modules ». Ces scalaires sans masse, dénotés par φ^i dans l'action effective (bosonique) à quatre dimensions

$$S^{(4)} = \int \mathrm{d}^4x\sqrt{-g_4}\left(\frac{M_p^2}{2}\mathcal{R}_4 - \frac{1}{2}G_{ij}\partial_\mu\varphi^i\partial^\mu\varphi^j - V(\varphi)\right), \tag{B.1}$$

sont essentiels pour définir des propriétés telles que le volume de l'espace interne. Cependant, leur potentiel effectif, $V(\varphi)$, reste plat en raison de leur absence inhérente de masse. Par conséquent, leurs valeurs moyennes dans le vide et les paramètres physiques qu'ils contrôlent restent indéfinis, ce qui constitue un obstacle important au développement de modèles réalistes. L'existence de champs de modules contredit les observations empiriques, manifestant le besoin urgent de stratégies efficaces de stabilisation de ces modules à travers différentes compactifications de cordes.

Une approche prometteuse pour résoudre ce problème est celle des « compactifications de flux » [7–9], où des flux de *background* sont capables de générer un potentiel $V(\varphi)$ pour les champs de modules. Bien que cette méthode permette souvent de stabiliser avec succès de nombreux configurations de champs de modules, elle ne les traite pas tous de manière exhaustive, ce qui requiert des mesures supplémentaires telles que des corrections quantiques du potentiel effectif, qui s'avèrent difficiles à calculer avec précision. En outre, les potentiels générés par ces flux, ainsi que les effets des structures locales, peuvent involontairement déstabiliser certains modules, ce qui pourrait entraîner l'apparition de tachyons dans la théorie à quatre dimensions.

Ces complexités, ainsi que les subtilités des conditions de quantification des flux, rendent la construction explicite de modèles réalistes techniquement compliquée, en particulier dans les scénarios dépourvus de propriétés « protectrices » telles que la supersymétrie, qui pourraient empêcher la présence de tachyons. La recherche d'une stabilisation des modules qui soit totalement stable et réaliste est semée d'embûches, ce qui remet en perspective le défi permanent que représente la mise en correspondance des théories des cordes à dix dimensions avec dans les phénomènes observables dans notre Univers.

Les cordes hétérotiques sont importantes car elles comprennent des groupes de jauge non-abéliens, qui sont utiles pour construire des théories de grande unification et peuvent potentiellement correspondre aux interactions de jauge du modèle standard. Cependant, la réalisation de ce programme ambitieux reste incomplète. Entretemps, la théorie des cordes de type II, qui présente généralement des champs de jauge abéliens dans son spectre perturbatif, a gagné en importance à la suite des progrès réalisés dans le domaine de la physique non-perturbative, en particulier la correspondance AdS/CFT. La possibilité d'y introduire des groupes de jauge non-abéliens par le biais d'objets non-perturbatifs appelés « D-branes » a fait des théories de type II un cadre plus adaptable et prometteur pour modéliser efficacement le contenu en champs, les groupes de jauge et les couplages. Pour cette raison, cette thèse se concentrera principalement sur la supergravité de type II.

Parmi les objets décrits par la théorie des cordes de type II, initialement définie perturbativement, les D-branes sont des objets étendus indispensables, conçus comme des membranes solitoniques ou des hypersurfaces de différentes dimensions qui évoluent au cours du temps. Ces branes sont en mesure de supporter les extrémités des cordes ouvertes soumises à des conditions au bord de Dirichlet : elles peuvent ainsi être mises en mouvement grâce aux oscillations perturbatives des cordes qui leur sont attachées. En supergravité classique, la dynamique des D-branes est gouvernée par des champs scalaires et vectoriels provenant des modes de masse nulle des cordes ouvertes sur l'élément de volume sous-tendu par les branes. En particulier, les champs vectoriels peuvent conduire à des théories de jauge non-abéliennes, ce qui rend les D-branes essentielles pour l'exploration de phénomènes physiques complexes au-delà des limites de la théorie perturbative des cordes. Parallèlement, les champs scalaires agissent comme des modules qui déterminent la position de la brane dans l'espace-temps.

Ensuite, les plans orientifold (abrégés en O-plans ou *O-planes* en anglais) complètent les D-branes et les cordes fondamentales dans le bestiaire de la théorie des cordes. Les O-plans sont des *hypersurfaces* définies par des symétries spécifiques de l'espace-temps sous-jacent. Ils sont identifiés comme des points fixes au sein d'une variété, résultant de l'imposition d'une symétrie finie sur les champs, couplée à une inversion de l'orientation de la surface d'univers. Cette opération de symétrie implique non seulement des réflexions spatiales, mais également une inversion de l'orientation des cordes, ce qui affecte profondément les propriétés physiques et les types d'interactions autorisées dans les modèles de cordes.

Aussi bien les D-branes que les O-plans agissent comme des sources de charges Ramond-Ramond (RR), générant des champs électriques et magnétiques ; ils influencent également le champ gravitationnel en ajoutant de la tension, ce qui modifie la densité d'énergie globale du vide. Ensemble, ils facilitent les interactions de jauge non-abéliennes par le biais de champs vectoriels vivant sur les volumes d'Univers qu'ils définissent et modifient la construction théorique de l'Univers en altérant localement les propriétés de l'espace-temps. Ces propriétés uniques font des D-branes et des O-plans des composants indispensables dans le contexte plus large de la théorie des cordes. Ils interagissent de manière complexe avec d'autres objets étendus, tels que les « NS5-branes », les « monopôles KK » (ou « anti-D-branes », dénotés $\overline{\mathrm{D}}$-branes), qui sont tous des manifestations des symétries et dualités plus profondes, inhérentes à la théorie. Cependant, ces dernières ne sont pas pertinentes pour l'étude que nous développons ici.

Faisant suite à ce qui a été expliqué auparavant, les D-branes et les O-plans s'identifient à des sous-variétés de l'espace-temps à dix dimensions, ce qui joue un rôle central dans plusieurs aspects de la physique théorique. Leur localisation est essentielle pour assurer la cohérence du processus de compactification, en particulier dans les scénarios impliquant des flux de *background*. D'un point de vue phénoménologique, ces sources localisées présentent un certain nombre d'avantages : elles facilitent la brisure de supersymétrie tout en induisant une déformation significative de l'espace-temps[29], soulignant l'impact profond de ces structures sur le tissu de l'espace-temps. Dans le contexte plus général de la cosmologie des cordes, qui sera largement exploré dans cette thèse, l'existence de telles sources locales est essentielle pour développer une compréhension cohérente des propriétés fondamentales de l'Univers. Cependant, leur présence complique considérablement les équations du mouvement. Dès lors, obtenir une solution complète qui tienne compte de leurs effets représente une tâche bien ambitieuse, qui dépasse souvent ce que la supergravité peut actuellement offrir en termes de dérivations analytiques. Cette complexité est étudiée en plus grands détails dans cette thèse.

Une approche pratique des défis posés par les sources localisées consiste à les remplacer par une distribution de charge homogène qui « s'étale » aux dépens de la variété interne, une technique connue sous le nom d'approximation d'étalement (en anglais, *smeared approximation*) [12–15]. Dans cette méthode, les distributions *delta*, qui localisent les sources dans les équations de la supergravité, sont remplacées par des fonctions régulières. Le principal avantage de cette approche est la simplification significative des calculs, puisqu'elle permet de traiter les équations du mouvement de manière intégrée plutôt que différentielle. En outre, « l'étalement » facilite l'utilisation de méthodes bien établies telles que les troncatures cohérentes [16–18], alors que le traitement des sources localisées nécessiterait un cadre plus sophistiqué, tel que les compactifications déformées [19, 20], qui souffrent de quelques ambiguïtés dans leur traitement.

Cependant, cette simplification se fait au prix d'ignorer les effets de rétroaction de la source sur les champs internes et la géométrie de l'espace compact. Cette omission peut avoir des effets significatifs sur les observables du point de vue quadri-dimensionnel, telles que les valeurs moyennes des modules ou de la constante cosmologique. La question de savoir si ces effets de rétroaction peuvent être négligés ou non fait actuellement l'objet d'un débat, étant donné que les solutions obtenues dans l'approximation d'étalement ne satisfont pas aux équations de la théorie à dix dimensions, où la rétroaction complète est prise en compte. Par conséquent, la validité de ces solutions en tant que représentations exactes des scénarios localisés est toujours débattue, ce qui souligne le besoin impérieux de comprendre la rétroaction des sources localisées. Cette question revêt un intérêt d'autant plus grand que la majorité des solutions actuellement disponibles dans la littérature reposent sur cette

[29] Cette déformation est cruciale et ne peut généralement pas être réalisée par le truchement de la seule supergravité (se référer à la discussion du théorème *no-go* de Maldacena-Nuñez [10, 11] pour plus de détails).

approximation, et seul un nombre limité d'entre elles se concentrent sur des configurations entièrement localisées. Nous approfondirons ce sujet au cours de cette thèse.

B.3 Le « paysage » de la théorie des cordes

Le grand nombre de choix de compactifications possibles constitue un défi majeur pour dériver la physique de basse énergie à partir de la théorie des cordes. Il en résulte un ensemble d'environ 10^{500} vides compatibles[30], chacun possédant des propriétés physiques uniques [22]. Cet ensemble porte le nom métaphorique de « paysage » de la théorie des cordes. Il s'agit alors de comprendre comment fonctionne le processus de sélection d'une théorie effective à quatre dimensions qui corresponde à notre Univers observable au sein de ce vaste ensemble. En particulier, l'espace-temps quadri-dimensionnel choisi doit être phénoménologiquement viable.

Les espaces-temps à symétrie maximale sont importants dans ce contexte. Tout d'abord, l'espace-temps de Minkowski est la géométrie d'élection pour imiter les phénomènes du modèle standard. Les espaces-temps anti-de Sitter (AdS) sont particulièrement appropriés pour formuler des dualités holographiques [23, 24], certaines constructions impliquant l'espace-temps de de Sitter [25, 26] et des discussions sur la séparation des échelles [27, 28]. Cependant, la théorie des cordes doit également tenir compte de l'expansion accélérée de l'Univers, qui peut être obtenue soit en supposant qu'une constante cosmologique positive se manifeste aux échelles cosmologiques (ce qui écarte à la fois Minkowski et AdS), ou par l'évolution de champs scalaires dits « roulants » (le processus *slow roll* par exemple). Ce travail se concentre sur la première solution, en particulier sur les compactifications dans un espace-temps dS.

Malgré des recherches approfondies au cours des deux dernières décennies, la faisabilité des compactifications dS en théorie des cordes reste incertaine. Bien que plusieurs scénarios prometteurs aient été proposés (voir notamment [25, 26]), des modèles entièrement explicites de l'espace-temps dS n'ont pas encore été construits. En outre, plusieurs arguments théoriques semblent remettre en question la compatibilité de l'espace-temps dS avec la théorie des cordes [29–31].

Un écueil important réside dans le fait que les solutions dS sont intrinsèquement non-supersymétriques, ce qui les empêche de satisfaire toutes les équations du mouvement par rapport aux solutions supersymétriques. De plus, sans la SUSY, il n'y a pas de mécanisme intrinsèque pour assurer la stabilité des solutions, ce qui rend la stabilisation des modules problématique. En outre, les solutions dS sont exclues dans les compactifications classiques les plus simples [10, 11], et requièrent donc des corrections à l'approximation de la supergravité classique ou des sources localisées de tension négative.

En théorie des cordes, deux stratégies principales visant à obtenir des vides avec constante cosmologique positive sont mises en avant dans la littérature. La première consiste à inclure des termes correctifs impliquant des dérivées d'ordre supérieur et des effets non-perturbatifs dans l'approximation de la supergravité classique. Une autre approche notable est celle dite du « scénario dS classique » [32, 33], qui explore les compactifications en pré-

[30] Ce nombre peut en fait atteindre $10^{272\,000}$ ou plus [21].

sence de O-plans et de flux de *background* dans le régime classique de la théorie des cordes de type II [16,34–36]. La théorie effective en dimension inférieure est obtenue par réduction dimensionnelle de la supergravité de type II à dix dimensions incorporant des termes allant jusqu'au second ordre de dérivation, en incluant l'action des O-plans/D-branes localisés à l'ordre supérieur en la constante de couplage α'. Cette approche garantit que les corrections perturbatives et non-perturbatives des cordes restent minimales dans la théorie effective, bien qu'elles ne soient pas nécessairement présentes partout dans la théorie originelle à dix dimensions. Cela se produit si le volume considéré est suffisamment grand et le couplage α' suffisamment faible.

Le scénario dS classique offre plusieurs avantages pour la théorie des cordes de type IIA, notamment en fournissant des modèles explicites qui ne requièrent ni stabilisation des modules, ni calculs supplémentaires pour dériver la dépendance complète des modules en les corrections instantoniques, deux tâches habituellement complexes. Cependant, cette approche présente de sérieuses limitations. De nombreux modèles classiques se voient écartés par des théorèmes *no-go* qui excluent les solutions dS [32, 37–39]. Les quelques solutions identifiés sont souvent perturbativement instables, ou rencontrent des problèmes pour des configurations à courbure élevée ou couplage α' important.[31] Ces instabilités sont souvent associées à un tachyon [43, 44].

Malgré ces obstacles, le scénario dS classique n'a pas été explicitement exclu [45], ce qui laisse une certaine latitude pour de nouvelles recherches dans cette direction, ainsi que l'élaboration d'éventuels contre-exemples. Nous passons maintenant à la description du contenu de la thèse et de sa structure.

B.4 Plan et résumé

Cette thèse est basée sur les résultats de recherche originaux publiés dans [35,36,39,46,47].

Dans la section 2.2, nous clarifions les conventions utilisées et passons en revue quelques notions fondamentales concernant les compactifications de flux avec sources localisées en théorie des cordes de type II. Nous explorons ensuite, dans la section 2.3, les différentes conditions au bord utilisées dans cette thèse, avant de revisiter, dans la section 2.4, le raisonnement derrière l'utilisation de l'approximation d'étalement, une composante essentielle de notre approche analytique.

Enfin, dans la section 3, nous illustrons la procédure de réduction dimensionnelle en utilisant deux ensembles de champs scalaires. Notre objectif est d'établir les lois de transformation de ces champs dans la forme canonique et de dériver le potentiel correspondant d'une théorie effective à d dimensions, et régie par une action de la forme (B.1), pour chaque ensemble de champs scalaires.

[31] Voir aussi les arguments suggérant que les vides dS classiques en régime de courbure et couplage α' faibles ne sont pas possibles dans de nombreuses classes [40,41], ainsi que les contre-exemples possibles [42].

Contraintes sur les solutions (quasi-)de Sitter de la supergravité à dix dimensions

Comme nous l'avons rappelé, la théorie des cordes est traditionnellement formulée dans un espace-temps à dix dimensions, lequel diffère considérablement de l'Univers à quatre dimensions que nous observons. Cette remarque soulève des questions fondamentales sur la nature dimensionnelle de celui-ci ainsi que sur le paysage théorique de la théorie des cordes [48] : en particulier, pourquoi n'y a-t-il que quatre dimensions observables et non plus ? La théorie des cordes n'ayant pas de dimensionnalité privilégiée, nous en sommes réduits à étudier la viabilité de scénarios en dimensions supérieures dans ce contexte.

L'étude des solutions dS classiques en dimensions $d \geq 3$ n'est donc pas seulement une curiosité théorique, mais est bien motivée par plusieurs facteurs [49]. Actuellement, toutes les solutions explicites et robustes de la théorie des cordes présentent une constante cosmologique négative ou nulle, ce qui renforce l'affirmation largement répandue selon laquelle le « paysage » des vides est dépourvu de solutions dS métastables [30]. Pour remettre en cause cette dernière, il nous suffit de trouver un seul contre-exemple qui soit métastable, de Sitter et fiable, sans aucune contrainte sur sa dimension ou son échelle de brisure de SUSY.

Étudier l'espace des solutions en dimension supérieure offre la possibilité de simplifier les procédures de stabilisation des modules, flux et branes, et donc, potentiellement, la recherche de solutions dS. En outre, ces recherches pourraient fournir des modèles plus simples pour tester certaines propositions théoriques, telles que la correspondance dS/CFT [50], et même faciliter le développement de modèles de quintessence à quatre dimensions grâce à la réduction dimensionnelle [51]. En étendant notre recherche à des dimensions arbitraires, nous souhaitons être en mesure de répondre à des questions à la fois théoriques et phénoménologiques, approfondissant ainsi notre compréhension des espaces-temps dS en théorie des cordes et leurs implications pour notre Univers.

Dans la section 4, nous étudions la possibilité d'identifier des solutions (quasi-)dS classiques en dimensions arbitraires. Dans ce même but d'identifier et contraindre de telles configurations, plusieurs études se sont concentrées sur les solutions dS classiques en quatre dimensions. Pour une liste exhaustive de ces études, on pourra consulter [52]. Malgré ces efforts, l'entreprise n'a rencontré qu'un succès limité, ce qui a motivé le développement de nombreux théorèmes *no-go* qui imposent des contraintes sur les flux, les sources et les propriétés de la variété, nécessaires pour obtenir des configurations de supergravité consistantes [38, 45, 53].

Les travaux de recherche rassemblés dans cette thèse contribuent à cette ligne de recherche en dérivant des théorèmes *no-go* qui excluent effectivement les solutions dS en supergravité de type II. Alors que prouver qu'une solution dS ne se dérive pas de la supergravité est suffisant pour l'exclure de tout régime classique, le contraire n'est pas nécessairement vrai. En d'autres termes, dériver une solution dS de la supergravité à dix dimensions n'implique pas automatiquement qu'elle soit également solution de la théorie classique. Des arguments génériques allant dans ce sens ont été donnés dans [40, 45, 55] et l'on retrouve même des contre-exemples notoires à quatre dimensions [41, 54].

Notre discussion étend les théorèmes *no-go* bien établis pour $d = 4$ [38] à des dimensions d arbitraires telles que $3 \leq d \leq 10$, en introduisant de nouveaux éléments liés à cette extension. Cet effort s'appuie également sur les résultats fondamentaux de [49], où de telles extensions dimensionnelles ont été étudiées pour la première fois, et que nous reproduisons puis développons ici.

Dans la section 4.2, nous établissons à nouveau des théorèmes *no-go* en utilisant des équations valables en dix dimensions et qui, sous certaines hypothèses, conduisent à une inégalité $\mathcal{R}_d \leq 0$ pour le scalaire de courbure $\mathcal{R}_d$, écartant effectivement les solutions dS. Nous ne tiendrons pas compte des complications liées aux orientifolds étalés et à la rétroaction des sources localisées, et nous nous concentrerons sur les cas qui admettent des solutions dS classiques avec sources étalées. Des études plus élaborées d'approches non-classiques en dimensions supérieures pourraient également rencontrer des difficultés similaires : cela demeure ainsi un terrain fertile pour de futures investigations.

Dans la section 4.3, nous appliquons systématiquement ces théorèmes *no-go* en dimensions $d \geq 4$. Au fur et à mesure que la dimension augmente, les restrictions sur le flux et le contenu de la source deviennent plus strictes, satisfaisant souvent certaines hypothèses de ces théorèmes de façon automatique. Ceci est particulièrement manifeste dans la section 4.3.2, où nous nous concentrons sur les configurations qui préservent la supersymétrie.

Notre conclusion, résumée par la section 4.3.3, exclut explicitement l'existence de solutions dS classiques en dimensions $d \geq 7$, tandis que les possibilités en dimensions $d = 5$ et 6 restent limitées, voire parfois écartées par des conjectures théoriques supplémentaires, comme indiqué dans [35, 45]. Ces résultats suggèrent l'apparition d'un biais théorique vers $d \leq 4$, indiquant peut-être une préférence pour $d = 4$. Nous étendons ces observations aux solutions quasi-de Sitter dans la section 4.4.

Comportement asymptotique des potentiels de flux scalaires dans les théories effectives de dimension inférieure

La discussion que nous venons d'entretenir a révélé des défis importants dans la recherche de solutions (quasi-)dS classiques dans des dimensions arbitraires. Parallèlement, le programme « *swampland* » [56, 57] (vocable anglais pour « marécage », qui s'oppose donc au « paysage » décrit plus haut), vise à définir des critères permettant de déterminer si une théorie effective cohérente peut se dériver d'une limite de basse énergie de la théorie des cordes. Les théories qui ne répondent pas à ces critères sont considérées comme se situant dans le « *swampland* ». De ce point de vue, toutes les dimensions doivent être considérées sur un même pied d'égalité. Cela implique que la portée des conjectures du programme « *swampland* » s'étend aux compactifications de cordes sur des dimensions externes arbitraires, tant qu'il n'y a pas d'argument convaincant qui soit en accord avec le comportement quantique supposé de la gravité, qui dicterait une certaine préférence dimensionnelle.

Dans ce contexte, la « conjecture de Sitter » [30] propose un obstacle systématique aux solutions dS, qui prend la forme de l'inégalité suivante

$$M_p|\nabla V(\varphi)| \geq c\, V(\varphi)\,, \qquad c \sim \mathcal{O}(1)\,. \tag{B.2}$$

Bien qu'étudiée principalement en quatre dimensions [38], cette conjecture est en fait également valide pour toute dimension $d \geq 3$. Plus récemment, l'on s'est mis à penser que cette inégalité n'était valable que dans les limites asymptotiques de l'espace des modules : en témoigne la conjecture de censure transplanckienne (TCC) [58], qui introduit une borne inférieure sur la constante c,

$$c \geq c_0\,, \qquad c_0 = \frac{2}{\sqrt{(d-1)(d-2)}}\,. \tag{B.3}$$

Dans la section 5, nous testons rigoureusement cette borne à l'aide de théorèmes *no-go*. Si elle devait être validée dans notre Univers, cela nécessiterait de faire appel à des modèles cosmologiques plus compliqués [59, 60]. De plus, notre motivation pour étudier le régime classique de la théorie des cordes est basée sur le fait qu'il correspond à ces asymptotiques, où ces inégalités sont supposées valides, ce qui est idéal pour étudier la TCC.

Nous récapitulons les théorèmes *no-go* de la supergravité en utilisant une théorie effective à d dimensions avec $V(\varphi) > 0$, semblable à (B.1). Sous certaines hypothèses, nous dérivons une inégalité du type (B.2), ce qui révèle une valeur spécifique de c et exclut les points critiques de de Sitter. Cette dérivation s'accorde avec les inégalités obtenues en dix dimensions, et confirme que $|\nabla V(\varphi)|$ ne peut ni s'annuler, si prendre des valeurs trop faibles. Nous développons ce point dans la section 5.3, où nous expliquons comment de telles dérivations excluent également les solutions quasi-dS. Ces solutions sont caractérisées par un potentiel positif avec un gradient minimal et des champs qui « roulent » lentement (c'est-à-dire lorsque le potentiel $V(\varphi)$ domine sur les termes cinétiques). Alors que des analyses approfondies se sont précédemment concentrées sur le cas $d = 4$ [38], notre travail se fixe pour but de dériver une valeur de c dépendant de la dimension d, permettant ainsi une comparaison directe avec la borne inférieure prescrite par (B.3).

Nos résultats, résumés dans la section 5.4 et illustrés dans le tableau 6 et la figure 4, confirment la limite TCC pour les dimensions $d \geq 4$, avec de multiples cas de saturation, cohérents avec ceux observés dans le cas $d = 4$. Cette cohérence entre les dimensions sert de validation substantielle de la limite TCC et supporte la nature universelle des conjectures du « *swampland* ». Par ailleurs, dans la section 5.4.1, nous décrivons une anomalie intrigante en trois dimensions, où un théorème *no-go* nouvellement établi suggère une valeur de c en dessous du seuil fixé par la TCC, liée aux particularités de la gravité à trois dimensions.

Dans les sections 5.2 et 5.4.2, nous comparons la TCC avec d'autres propositions apparues dans la littérature, notamment la « *Swampland Distance Conjecture* ». Dans la section 5.4.3, nous discutons également une borne supérieure asymptotique sur $|\nabla V(\varphi)|$ nécessaire pour modéliser une accélération de l'expansion cosmologique. Lorsqu'elle est violée, cela suggère d'avoir recours à des scénarios cosmologiques alternatifs et ouvre d'autres pistes de recherche, en particulier si la borne TCC se maintient.

Nous nous intéressons ensuite aux potentiels scalaires négatifs. Les vides AdS entrent dans cette catégorie. Bien qu'ils puissent sembler moins pertinents pour les modèles cosmologiques[32], ils font par ailleurs l'objet d'études approfondies grâce aux progrès de l'holographie [23, 24, 67]. Les similitudes structurelles entre les potentiels scalaires négatifs et positifs dans la théorie des cordes, qui diffèrent principalement par des variations dans les valeurs et les signes des coefficients, suggèrent que les deux types pourraient bénéficier d'une approche unifiée.

Notre analyse utilise des modèles qui décrivent des espaces-temps caractérisés, soit par une expansion, soit par une contraction, en fonction du profil temporel du facteur d'échelle $a(t)$ présent dans la métrique. L'on peut trouver des discussions détaillées à ce sujet dans les sections 5.1 et 6.1.1. La notion de contraction des espaces-temps est particulièrement importante dans le contexte de la conjecture de censure anti-transplanckienne (ATCC) :

Conjecture de censure anti-transplanckienne. Dans toute théorie effective (de dimension inférieure) de la gravité quantique qui soit décrite par l'action (B.1) avec $V < 0$ admettant des solutions cosmologiques en contraction, les modes ayant une longueur d'onde proche de l'échelle de longueur typique de l'Univers à t_i ne devraient pas tomber sous l'échelle de Planck à un moment ultérieur $t > t_i$ sans compromettre la validité de la théorie effective.

Cette affirmation peut être exprimée analytiquement comme suit :

$$\frac{a(t)}{a(t_i)} \geq \frac{\sqrt{|V(\varphi(t_i))|}}{M_p^2}\,, \qquad \forall\, t > t_i\,. \tag{B.4}$$

L'ATCC modifie notre compréhension des Univers en contraction avec $V < 0$, auxquels manquent les mécanismes d'horizon utilisés dans le cas des espaces-temps dS, et tels que proposés dans la TCC originale [58]. Au lieu de cela, cette nouvelle conjecture insiste sur l'importance de maintenir la validité de la théorie effective en s'assurant que l'échelle d'énergie typique, c'est-à-dire le potentiel scalaire, reste inférieure à l'échelle de Planck. Nous élaborons à ce sujet dans la section 6.1.2, en soulignant que les solutions violant l'inégalité (B.4) sont considérées comme invalides. Ces considérations plus approfondies ne sont pas seulement spéculatives, mais basées sur les contraintes physiques qu'imposent les échelles fondamentales de la théorie.

Dans la section 6.1.3, nous dérivons une limite explicite sur la durée de vie de l'Univers, interprétée dans le contexte de la contraction spatio-temporelle. À partir de la contrainte (B.4), nous développons un formalisme pour caractériser les potentiels négatifs en dimensions $3 \leq d \leq 10$ en utilisant les unités de Planck et en nous concentrant sur un seul champ scalaire. Ce formalisme requiert une hypothèse sur V et $a(t)$ qui, bien que naturellement satisfaite pour $V > 0$, doit faire l'objet d'un examen minutieux pour $V < 0$. Ce faisant, nous dérivons une limite inférieure exponentielle, $V(\varphi) \geq -\mathrm{e}^{-c_0|\varphi-\varphi_i|}$, applicable à l'ensemble de l'espace des champs et qui conduit, par conséquent, à la condition suivante dans

[32] Voir cependant les modèles « *ekpyrosis* » et cosmologies « à rebonds » [61–66].

la limite asymptotique de l'espace des champs,

$$\left\langle -\frac{V'}{V} \right\rangle_{\varphi\to\pm\infty} \geq c_0\,, \tag{B.5}$$

reflétant celle de la TCC malgré des différences subtiles dans leurs dérivations. Cette condition n'exclut pas catégoriquement les points critiques AdS dans les limites asymptotiques, de la même manière que la TCC n'exclut pas explicitement les espaces dS. Au lieu de cela, elle impose des contraintes sur le comportement asymptotique du potentiel et fixe une borne inférieure aux taux exponentiels. Dans la section 6.1.4, nous vérifions la validité de ces contraintes ainsi que l'hypothèse précédente dans différents modèles cosmologiques, y compris les solutions AdS et les solutions dynamiques avec des champs « roulants ».

Dans la section 6.2, nous introduisons une nouvelle condition sur la dérivée seconde du potentiel,

$$\left\langle \frac{V''}{V} \right\rangle_{\varphi\to\infty} \geq \frac{4}{(d-1)(d-2)}\,. \tag{B.6}$$

Cette condition asymptotique permet d'affirmer que pour toute solution AdS en dimension d et de longueur typique l, il existe un champ scalaire de masse m satisfaisant à la condition suivante :

$$m^2 l^2 \lesssim -2\,. \tag{B.7}$$

Cette inégalité quelque peu flexible est satisfaite par les solutions perturbativement instables [68, 69].

Nous vérifions la contrainte (B.7) de manière exhaustive pour un ensemble de configurations perturbativement stables. La plupart des configurations supersymétriques s'y soumettent, malgré quelques exceptions, encore sujettes à débat [25, 26, 70]. En revanche, les modèles non supersymétriques requièrent souvent une plus grande flexibilité et peuvent présenter quelques déviations par rapport à la contrainte que nous avons dérivée. Cependant, nombre d'entre eux souffrent d'instabilités non-perturbatives [71]. Un résumé détaillé de ces exemples est donné dans les tableaux 7 et 8. Nous complétons cette analyse par la discussion des implications holographiques de cette contrainte pour une CFT duale dans la section 6.3.3.

Enfin, les sections 6.4 et 6.5 se penchent sur les modèles à champs multiples et appliquent l'ATCC à des compactifications de cordes spécifiques. Nous validons les bornes prescrites par l'ATCC pour un potentiel semi-universel, $V(\rho,\tau,\sigma)$, initialement dérivé dans la section 3, et dans le contexte des compactifications de flux conduisant aux solutions DGKT [70].

Scénario de Sitter « presque classique »

Récemment, les auteurs de [72] ont prétendu avoir identifié des vides dS classiques dans un modèle remarquablement simple, que nous noterons CDT1. Ce dernier utilisait la théorie des cordes de type IIA avec des O8-plans parallèles et la masse de Romans comme seul flux, simplifiant les équations du mouvement en équations différentielles ordinaires solubles, y compris la rétroaction non-linéaire des O-plans. Des analyses ultérieures ont cependant démontré que les sources localisées ne sont pas compatibles avec l'interprétation conventionnelle de la théorie des cordes des O8-plans à l'ordre dominant en α' [73]. Plus généralement, l'on a montré que l'action globale classique de type IIA, ainsi que l'action O8/D8 à l'ordre dominant dans l'expansion α', exclut les solutions dS classiques dans toutes les compactifications de flux sans sources de codimension supérieure [73, 74].

Dans un article ultérieur [75], une faille potentielle due à des ambiguïtés dans les équations de la supergravité a été suggérée, et des conditions aux bords « permissives », autorisant l'ajout potentiel de termes de source violant les hypothèses considérées dans [73], ont été proposées. Dans les sections 2.3 et 7.1.2, nous argumenterons que ces ambiguïtés ne sont pas évidentes au niveau de la supergravité classique, et que les sources doivent être conformes à celles spécifiées dans le théorème *no-go*.

Comme indiqué dans [73], cette conclusion pourrait être modifiée en considérant l'effet des corrections en α' dans l'action O-plan/D-brane, qui apparaissent notablement au quatrième ordre de dérivation [76–84]. Pour aller plus loin, nous introduisons dans la section 7.1.3 le scénario dS « presque » classique, une extension minimale du scénario dS classique qui incorpore des corrections de premier ordre en α' tout en négligeant les dérivées d'ordre supérieur. Bien que l'on puisse se préoccuper des régions où l'expansion en α' serait invalide, nous assertons que les termes classiques et leurs corrections jusqu'au quatrième ordre de dérivation dominent la contribution des O-plans à l'énergie du vide, les effets de la « région des trous » non-perturbative ne jouant un rôle que dans la physique à courte portée. Néanmoins, cette approche ne parvient toujours pas à produire des solutions dS métastables, confirmant ainsi la validité du théorème *no-go*.

Le modèle CDT2, qui comprend les deux O6-/O8-plans, est également abordé dans la section 7.2.1. Bien que plus complexe, il conserve la propriété du modèle CDT1 quant à la solvabilité des équations mais inclut également une rétroaction non-linéaire des O-plans. Cependant, comme dans sa précédente version, le modèle CDT2 n'échappe pas à un théorème classique *no-go* visant les solutions dS, et discuté dans la section 7.2.3, et ne produisant de ce fait que des solutions AdS. L'inclusion de termes de couplage impliquant quatre dérivées dans la section 7.2.4 ne résout pas ces problèmes, car leurs effets sont sous-dominants dans les régimes à faible courbure.

Cette section met en évidence les défis actuels pour obtenir des vides dS stables dans le cadre de la théorie des cordes, et illustre les limites des scénarios dS classiques et quasi-classiques. Bien qu'ils fournissent des informations précieuses, ces modèles n'offrent pas de voie viable vers des solutions dS stables, ce qui renforce la nécessité de poursuivre la recherche et l'exploration de stratégies alternatives dans ce domaine.

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