Dissertation

Rigid-foldable Quad Meshes with Control Polylines: Interactive Design and Motion Simulation

Ausgeführt zum Zwecke der Erlangung des akademischen Grades eines Doktors der technischen Wissenschaften unter der Leitung von

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Declaration of Authorship

I, Kiumars SHARIFMOGHADDAM, declare that this thesis titled, "Rigid-foldable Quad Meshes with Control Polylines: Interactive Design and Motion Simulation" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed:

Date:

Abstract

Generic discrete surfaces composed of quadrilateral plates connected by rotational joints in the combinatorics of a square grid are rigid, but there also exist special ones with 1-parametric flexibility. This dissertation focuses on two particular classes of so-called T-hedra (trapezoidal quad surfaces) and V-hedra (discrete Voss surfaces). T-hedra can be thought of as a generalization of discrete surfaces of revolution in such a way that the axis of rotation is not fixed at one point but rather sweeping a polyline path on the base plane. Moreover, the action does not need to be a pure rotation but can be combined with an axial dilatation. After applying these transformations to the breakpoints of a certain discrete profile curve, a flexible quad-surface with planar trapezoidal faces is obtained. Therefore, the design space of T-hedra also includes as subclasses discretized translational surfaces and moulding surfaces beside the already mentioned rotation surfaces. V-hedra are the discrete counterpart of Voss surfaces which carry conjugate nets of geodesics. In discrete case the opposite interior angles of a vertex star are equal. From a V-hedral vertex one can always generate an anti-V-hedral vertex with the same kinematics, in which the sum of corresponding opposite angles equal to π and therefore is a known case of valence four flat-foldable and developable origami vertex. The author developed Rhino/Grasshopper plugins, implemented with C-sharp, which make the design space of T-hedra, V-hedra and anti-V-hedra accessible for designers and engineers. The main components enable the user to design these quad surfaces interactively and visualize their deformation in real time based on a recursive parametrization of the quad-mesh vertices under the associated isometric deformation. Furthermore, this research investigates semi-discrete T-hedral surfaces and other topologies, such as tubular structures composed of T-hedra.

Acknowledgements

I am profoundly grateful for the collaborative effort that has led to the completion of this thesis. Many individuals have played pivotal roles in shaping both my research and personal development. Their contributions have been invaluable, and I extend my heartfelt appreciation to each of them.

I am deeply thankful to my supervisor, **Georg Nawratil**, for his continuous support and visionary insights. His belief in my abilities have been instrumental in realizing the potential of this work. Also, special thanks go to **Ivan Izmestiev**, whose insights have laid the theoretical groundwork for this research.

Tomohiro Tachi has been an exceptional mentor during my time in his lab, and I'm grateful for the opportunity to learn from him and his exceptionally talented lab members.

I also extend my thanks to **Simon Guest** for his valuable feedback as my thesis reviewer and project partner.

My gratitude extends to all my coauthors over the past four years, with particular appreciation for **Rupert Maleczek**, a wonderful person whom I enjoyed working with and for introducing me to the Structural Origami Gathering, where I started working with **Klara Mundilova**, **Tomohiro Tachi** and many others.

The stimulating environment within the research groups of Differential Geometry, Geometric Structures, and Applied Geometry and Geometric Computational Design (GCD) at TU Wien has been instrumental in my academic journey. I extend my sincere thanks to my colleagues, especially **Jonas Tervooren** for all the mathematical and friendly chats and **Doris Hotz**, **Martin Peternell** for their enriching conversations and support.

I am thankful to **Arvin Rasoulzadeh**, **Steffan Pillwein** and **Florian Rist**, for their valuable advice during challenging times, as well as to **Michael Wimmer** and **Milena Stavric** for providing opportunities within the ACD project and beyond my expectations. I am also grateful to the long list of coauthors within the ACD project, who have contributed significantly to my research journey. Special thanks to my fellow PhD students in other subprojects, **Khrystyna Vasylevska**, **Balint Kovacs**, and **Julian Jauk**, whom I enjoyed collaborating with.

Many individuals have significantly contributed to nurturing my love for mathematics and shaping my academic journey. My heartfelt appreciation goes to my math teachers in Iran, whose dedication inspired me to pursue mathematics as a passion. I owe special thanks to **Erfan Safar** and **Ali Ghassab** for introducing me to recreational mathematics, igniting a profound curiosity within me. **Amir Asghari**'s mentorship guided me through the intricacies of academia, while **Majid Alizadeh** and **Maryam Tahmasbi**, my professors at the University of Tehran and SBU played pivotal roles in my academic growth.

I am grateful for encounters that further fueled my passion, such as with **Ali Bahmani**, who introduced me to the scientific aspect of origami. **Tom Hull**'s influential work inspired me to explore origami within mathematics education, enriching my teaching practices. **Erik Demaine**'s Geometric Folding Algorithm course sparked my interest in computational geometry, shaping my academic trajectory and leading me to pursue the intersection of origami and geometry as focal areas of study.

To my family and friends in Iran, whose support and resilience continue to inspire me.

Lastly, to **Narges Lali**, my steadfast companion and greatest source of encouragement, I owe immeasurable gratitude for her unwavering belief in me over the past decade.

Contents

Dedicated to Narges, my beloved wife

Chapter 1

Introduction

1.1 Motivation

1.1.1 Advanced Computational Design

This dissertation stems from the author's engagement in the "Advanced Computational Design" project funded by the Austrian Science Fund (Grant No. SFB F77), which integrates various disciplines such as digital architecture, integrated building design, computer graphics, and more. The project aims to enhance design tools and processes through interdisciplinary collaboration.

In a broader context, the Architecture, Engineering, and Construction (AEC) industry significantly influences our built environment but lags behind in digital advancements. Conversely, the Information and Communication Technology (ICT) sector excels in digital innovation but struggles to tailor solutions to designers' needs.

The novelty of this project lies in its interdisciplinary approach, bridging architecture, computer science, mathematics, and engineering. By synthesizing diverse expertise and leveraging robust theoretical frameworks, the project goal is to bring more possibilities to the computational design tools. It seeks to provide timely feedback in early design stages and explore design spaces previously inaccessible. This dissertation delves into the contributions made within this innovative framework, aiming to advance both theoretical and practical applications in computational design.

1.1.2 Flexible Quad-Surfaces for Transformable Design

Transformable structures, increasingly prominent in engineering, architecture, and art, encompass a distinct class known as flexible quad-surfaces. These structures consist of quadrilateral plates interconnected by rotational joints, typically arranged in the square grid combinatorics. While a generic quad-surface is rigid, there are flexible classes exhibiting a single degree of freedom, enabling precise motion control. An exemplary application of such flexibility is evident in the Miura-ori pattern, notably utilized in solar panel configurations [8]. Despite its potential, industrial and artistic uses of flexible quad-surfaces has been severely limited due to the lack of both a comprehensive theoretical foundation and suitable design tools. A complete classification is presented by Izmestiev in [2]. Two important classes of such quad-meshes (T-hedra and V-hedra) were the subjects of study by one of the involving SFB subprojects (see Fig. 1.1). The author of this thesis was involved in theoretical studies, implementation of the results as plugins of a CAD software, digital design and fabrication of examples and facilitating the connections within subprojects.

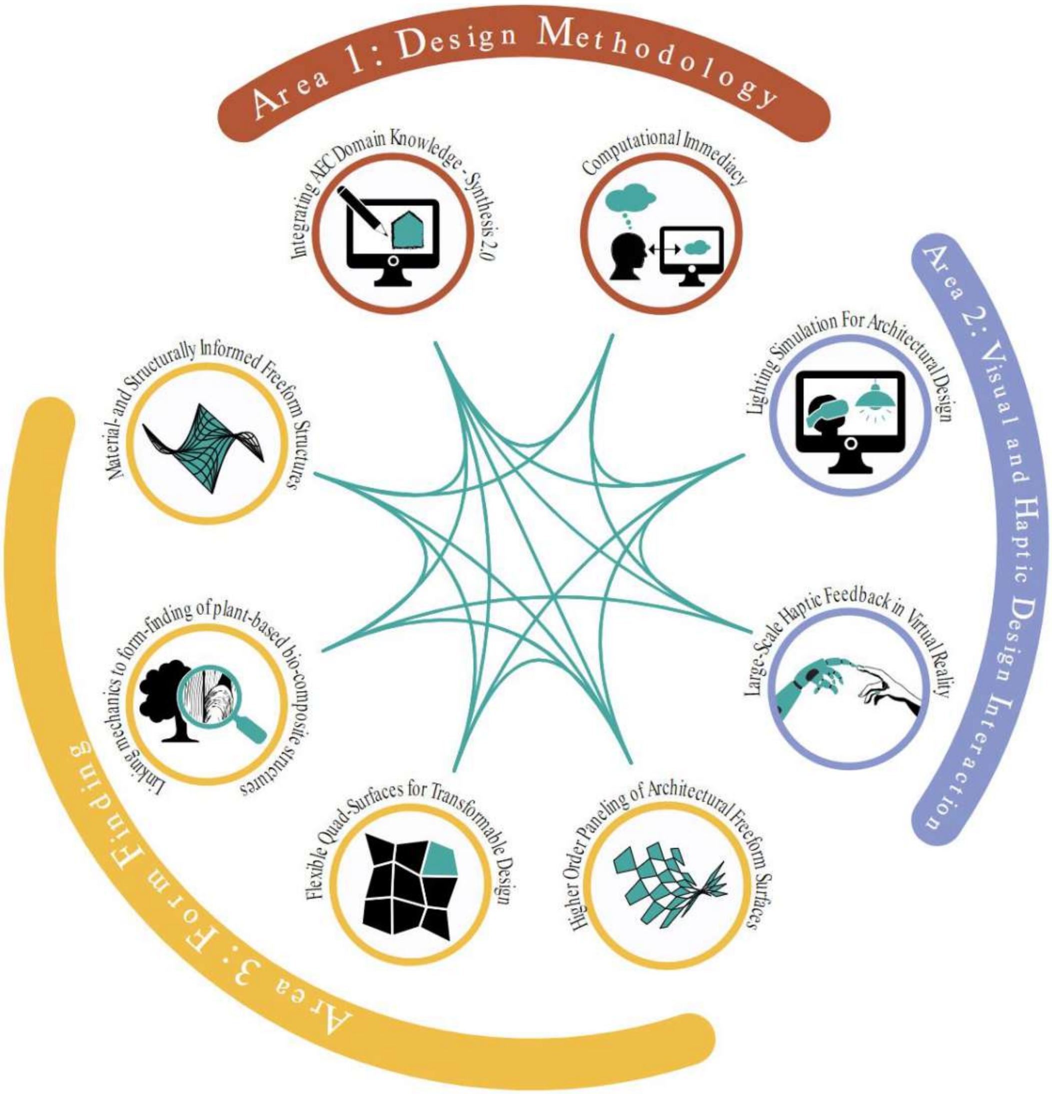

FIGURE 1.1: The subprojects of the SFB project "Advanced Computational design" in three research area.

1.2 Background

1.2.1 Isometric Deformation of Surfaces in Differential Geometry

In differential geometry, one studies the geometric properties of curves and surfaces and isometric deformation refers to a transformation of a surface in which intrinsic distances between points remain unchanged. In the context of surfaces in differential geometry, isometric deformation involves altering the shape of the surface without stretching or compressing it. This means that any distance measured on the original surface will be identical to the corresponding distance on the deformed surface. Isometric deformations provide insights into how surfaces maintain their intrinsic geometry even as they undergo changes, facilitating analysis in various fields such as physics, engineering, and computer graphics. The discrete counterpart of a surface is a quad mesh with the combinatorics of a square grid. More precisely a smooth surface in Euclidean space is usually defined as a map

$$\gamma : U \subset \mathbb{R}^2 \to \mathbb{R}^3, \quad (u, v) \mapsto \gamma(u, v),$$

while the discrete counterpart can be defined as

$$\gamma : U \subset \mathbb{Z}^2 \to \mathbb{R}^3, \quad (i, j) \mapsto \gamma(i, j),$$

with non-zero edge lengths. A smooth (or discrete quad-) surface $\gamma : U \to \mathbb{R}^3$ is flexible, if there exists a map

$$\gamma_t : (1 - \epsilon, 1 + \epsilon) \times U \to \mathbb{R}^3, \quad (t, p) \mapsto \gamma_t(p), \quad \epsilon \in \mathbb{R}^+,$$

that is continuous in t with:

1. γ_t is a smooth (or discrete quad-) surface for all t.
2. $\gamma_1 = \gamma$.
3. For $t_1, t_2 \in (-\epsilon, \epsilon)$, γ_{t_1} and γ_{t_2} are isometric but not congruent.

Two smooth surfaces $\gamma_{t_1}, \gamma_{t_2} : U \to \mathbb{R}^3$ are isometric if they have the same first fundamental form:

$$\begin{aligned} E &:= \langle \gamma'(u, v), \gamma'(u, v) \rangle, \\ F &:= \langle \gamma'(u, v), \dot{\gamma}(u, v) \rangle, \\ G &:= \langle \dot{\gamma}(u, v), \dot{\gamma}(u, v) \rangle. \end{aligned}$$

And two discrete quad surface are isometric if for all $(i, j) \in U$, the quads $Q_{ij}(t_1)$ and $Q_{ij}(t_2)$ are congruent.

1.2.2 Kinematics of Discrete Quad Surfaces

Isometric deformation of a discrete quad surface (mesh) preserves the rigidity of each quad and only changes the angle between neighboring quads. Therefore, from kinematics point of view, the quadrilateral faces, which in the studies of this dissertation are planar, can be replaced with rigid bodies and by implementing rotational joints on the edges, the movement of the mesh can be simulated. As it is mentioned in Section 1.1.2, not every such valence-four planar quad mesh is flexible with rigid motion. The necessary and sufficient condition for a planar quad surface to be flexible with this kinematic setting is that every (3×3) patch of quad face grid has a one degree of freedom motion [15], which is intrinsic as it is only dependent to the interior angles of the quads. Before Izmestiev, partial classifications of these patches were given by Stachel [19] and Nawratil [10, 11, 12] using a kinematic approach, dealing with spherical four-bar linkages which result from the spherical image of 2×2 patches of faces.

1.2.3 Rigid Origami

Conventional origami is a traditional art and craft governed by simple rules. Typically, it begins with a flat sheet of paper, allowing one to fold and shape it without the use of cuts or glue. While these rules may seem limiting at first, they not only foster creativity but also, due to their simplicity, efficiency, and reproducibility, make origami suitable for a wide range of design endeavors. Over the past few decades, origami has firmly entrenched itself within a diverse array of design and engineering disciplines. It has been explored and applied in the realm of engineering by mechanical, aerospace and structural engineers and many scientific and technology-related fields such as architecture, material science, computer science, robotics, medicine and biotechnology. On the other hand, due to its usual geometric aesthetics, it has become a design theme even if the design does not completely follow the origami rules and it is only origami-inspired. In scientific and computational origami communities, there is a growing interest in studying the kinematic aspects of movable origami-inspired geometries involving rigid bodies and rotational hinges. Although rigid origami often focuses on developable and/or flat-foldable vertices, there is currently a trend towards exploring non-developable and non-flat-foldable cases in this field. Examples of all the aforementioned cases, with conditions applied or relaxed, fall within the classes studied in this research. It is worth noting that the isometric deformation of a planar quad surface is equivalent to the rigid folding of a quad tessellation or corrugation origami pattern, such as Miura-ori, eggbox, or chickenwire.

1.2.4 Interactive Tools for Computer Aided Design

As the number and variety of applications of origami in design, architecture, and engineering increase, there is a growing demand for tools that provide access to the design space and enable motion simulation of such complex mechanisms. The majority of existing tools (see [22]) simulate the motion of a given origami crease pattern by enforcing local conditions and subsequently resorting to numerical methods to handle the extensive computations. Consequently, these tools are typically slow and imprecise when simulating large meshes. In contrast, the novel approach outlined in our published studies employs interactive tools (developed by the author) with analytic computations running in the background, resulting in greater accuracy and significantly faster performance.

1.3 Structure of the Thesis

This is a thesis by publication (cumulative). The following is the list of all accepted or published research essays of the author, during PhD studies, which based on the level of contribution the publications is divided into three groups:

1. Research essays published or accepted by peer reviewed journals/conference proceedings as the first author or in one case *, shared first authorship:

- Sharifmoghaddam, K., Nawratil, G., Rasoulzadeh, A. and Tervooren, J. (2021). Using flexible trapezoidal quad-surfaces for transformable design. Proceedings of the **IASS** Annual Symposium 2020/21 and the 7th International Conference on Spatial Structures (pp. 3236-3248).
- Sharifmoghaddam, K., Maleczek, R. and Nawratil, G. (2023). Generalizing rigid-foldable tubular structures of T-hedral type. **Mechanics Research Communications** 132, Paper No. 104151, 15 pp. DOI: 10.1016/j.mechrescom.2023.104151.
- Sharifmoghaddam, K., Mundilova, K., Nawratil, G. and Tachi, T. (2024) Woven Rigidly Foldable T-hedral Tubes Along Translational Surfaces. Proceedings of the 8th International Meeting on Origami in Science, Mathematics and Education-**8OSME** (accepted).
- Kilian, M., Nawratil, G., *Raffaelli, M., Rasoulzadeh, A. and *Sharifmoghaddam, K. (2024) Interactive design of discrete Voss nets and simulation of their rigid foldings. **Computer Aided Geometric Design** 10.1016/j.cagd.2024.102346.

2. Research essays in which the author had major contributions:

- Maleczek, R., Sharifmoghaddam, K., and Nawratil, G. (2022). Rapid prototyping for non-developable discrete and semi-discrete surfaces with an overconstrained mobility. Proceedings of the **IASS** Annual Symposium 2022 and the 13th Asian-Pacific Conference on Shell and Spatial Structures. (pp. 2302–2313).
- Maleczek, R., Sharifmoghaddam, K., Nawratil, G. and Preisinger, C. (2023) Bridging the gap–A study on foldable tubular bridges. Proceedings of **IASS** Annual Symposia 2023. (pp. 1676-1686)
- Vasylevska, K., Batik T., Brument, H., Sharifmoghaddam, K., Nawratil, G., Vonach, E., Mortezapoor, S. and Kaufmann, H. (2023) Action-Origami Inspired Haptic Devices for Virtual Reality. Extended abstract in **ACM SIGGRAPH** 2023 Emerging Technologies. DOI: 10.1145/3588037.3595393.

3. Research essays in which the author had minor contributions:

- Kovács, B. I., Sharifmoghaddam, K., Jauk, J., Erb, I., Stavric, M., Nawratil, G. and Ferschin P. (2022). Integrative Mixed Reality Sketching. In **Creative Construction e-Conference** 2022 (pp. 233-240). Budapest University of Technology and Economics. DOI: 10.3311/CCC2022-030.
- Ferschin, P., Suter, G., Palma, M., Erb, I., Hahn, D., Kovács, B., Nawratil, G. and Sharifmoghaddam, K. (2023). Transformable luminaire design: from digital sketch to fabrication through computation and simulation. Proceedings of the 41st Education and Research in Computer Aided Architectural Design in Europe **eCAADe** 2023, Volume 1, pp. 117–126. DOI: 10.52842/conf.ecaade.2023.1.117.
- Kovács, B. I., Erb, I., Sharifmoghaddam, K., Lipp, L., Wimmer, M. and Ferschin P. (2023). The Theatre Metaphor for Spatial Computing in Architectural Design. in **Creative Construction e-Conference** 2023, (pp. 674-683) Keszthely, Hungary. DOI: 10.3311/CCC2023-087.

- Vasylevska, K., Ghazanfari, M., Sharifmoghaddam, K., Mortezapoor, S., Vonach, E., Brument, H., Nawratil, G. and Kaufmann, H. (2024) Stiffness Simulation with Haptic Feedback Using Robotic Gripper and Paper Origami as End-Effector. Workshop on Novel Input Devices and Interaction Techniques – NIDIT at **IEEE VR**.

The following chapters (2,3,4,5) are dedicated to the first group in chronological order. A paragraph at the beginning of each chapter, briefly determines the contributions of the author of this thesis to the publication coming after. In the following Section 1.4, the contributions of the author to all three groups of papers is explained in more detail.

1.4 Contributions of the Thesis

The main contribution of this dissertation is an interactive design tool called "Scutes", which is a Rhino/Grasshopper plugin designed for creating, manipulating, and visualizing isometric deformation in discrete quad surfaces. To date, the plugin has effectively integrated two significant classes: T-hedra and V-hedra, which include anti-V-hedra and their hybrid counterparts, components to apply extra conditions of developability and flat-foldability and other pre- and post-processing components for special input preparations and output visualizations. Both T-hedra and V-hedra, along with their topological variations, can be uniquely determined by a few control polylines. Therefore, a forward design method is chosen for these tools.

1.4.1 T-hedra

In the first step of the project the co-authors in [17] studied and computed the explicit expressions of the construction and deformation of T-hedra. The geometry and kinematic behavior of T-hedra were first examined nearly a century ago by Graf and Sauer [13, 14].

The algorithms for constructing and deforming all different types of T-hedra were designed and implemented by the author of this thesis, along with other post-processing and feedback features, such as force transmission and flexion limit closeness. Additionally, the first author contributed to validating the formulas and generalizing them to cover special examples where the discrete version cannot be achieved through proper discretization of a smooth counterpart. The publication is included in Chapter 2.

1.4.2 T-hedral Tubes

After the first publication, further studies on T-hedra took place which led to a journal publication [16] reviewing the existing rigid foldable origami tubes and identifying the T-hedral tubes within those. In this publication, the semi-discrete version of T-hedra and T-hedral tubes has been investigated as well.

The first author made contributions to various aspects of the project, including theory development, literature review, model implementation, visualization, fabrication of physical models, and drafting of the paper. The publication is included in Chapter 3.

1.4.3 Topology Variations and T-hedral Complex Structures

In [16] sandwich structures and cellular materials containing T-hedral tubes with a variety of already existing aligned edge and face connections and zipper couplings on the generalized version of T-hedral tubes was studied. In [18] a novel variation of tubular assemblies called woven tubes was presented which offer the potential for stronger and more light weight structures owing to their woven topology.

The primary contribution of the first author included the conceptualization of weaving tubes, formulation of the geometric problem, investigation into developing the theory, visualization, fabrication of physical models, and drafting the paper. The publication is included in Chapter 4.

1.4.4 V-hedra

In the final year of studies, the author of this thesis incorporated a second class of discrete quad meshes into the plugin. The new components of the plugin are capable of generating and isometrically deforming a V-hedral or anti-V-hedral mesh, as well as hybrid versions. Although the construction and isometric deformation of this class have been investigated by others (e.g., [15]) and have even been utilized as a design tool in many research projects, such as [9], the authors of [3] present new cases of Voss and anti-Voss vertices and incorporate concave and flipped quads into the design tool.

The author of the dissertation at the hand, contributed to the algorithm design and implementation of the constructive algorithm, as well as theory development of novel vertex and face cases, visualization, digital design and physical models, and conducting the comparisons of the methods presented in the paper. The corresponding publication is included in Chapter 5.

1.4.5 Applications

Towards the application of the theoretical investigations and to showcase the usability of the interactive tools for designers, architects and engineers, the author of this thesis contributed to [6, 7] by implementing the bending energy calculations to the plugin, designing parametric examples and visualizations. Also towards the goals of the ACD project and to integrate the plugin with the tools developed by other subprojects, such as a virtual and augmented reality sketching app for early design in [4, 5] and lighting simulation in [1]. Finally, the entire process of parametric and digital design and fabrication of an origami prop based on zipper coupling of T-hedral tubes, carried out by the author of this thesis for a multi-disciplinary research on haptic feedback in virtual reality. The result was a robot end-effector giving programmable sense of stiffness as a haptic feedback, by adjusting the robot fingers. The prop was used in [20, 21] and won the best project award at ACM SIGGRAPH Conference Emerging technologies in August 2023.

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Chapter 2

Using Flexible Trapezoidal Quad-surfaces for Transformable Design

This chapter consists of the conference paper:

- Sharifmoghaddam, K., Nawratil, G., Rasoulzadeh, A. and Tervooren, J. (2021). Using flexible trapezoidal quad-surfaces for transformable design.

It is published in the proceedings of the **IASS** Annual Symposium 2020/21 and the 7th International Conference on Spatial Structures (pp. 3236-3248).

Contributions

The author of this thesis contributed to the validation of the theoretical results, developing the algorithms, implementation as a grasshopper plugin and designing the interface for visualization of the features.

Chapter 3

Generalizing Rigid-foldable Tubular Structures of T-hedral Type

This chapter consists of the journal article:

- Sharifmoghaddam, K., Maleczek, R. and Nawratil, G. (2023). Generalizing rigid-foldable tubular structures of T-hedral type.

It is published in the **Journal of Mechanics Research Communications** 132, Paper No. 104151, 15 pp. DOI: 10.1016/j.mechrescom.2023.104151.

Contributions

The author of this thesis contributed to theory development, literature review, model implementation, visualization, fabrication of physical models, and drafting of the paper.

Chapter 4

Woven Rigidly Foldable T-hedral Tubes Along Translational Surfaces

This chapter consists of the conference paper:

- Sharifmoghaddam, K., Mundilova, K., Nawratil, G. and Tachi, T. (2024). Woven Rigidly Foldable T-hedral Tubes Along Translational Surfaces.

It is accepted for the 8th International Meeting on Origami in Science, Mathematics and Education-**8OSME**.

Contributions

The author of this thesis contributed to the conceptualization of woven tubes, formulation of the geometric problem, investigation into developing the theory, visualization, fabrication of physical models, and drafting the paper.

Chapter 5

Interactive design of discrete Voss nets and simulation of their rigid foldings

This chapter consists of the journal article:

- Kilian, M., Nawratil, G., *Raffaelli, M., Rasoulzadeh, A. and *Sharifmoghaddam, K. (2024) Interactive design of discrete Voss nets and simulation of their rigid foldings.

It is published in the **Journal of Computer Aided Geometric Design** 111, Paper No. 102346, 24 pp. DOI: 10.1016/j.cagd.2024.102346.

Contributions

The author of this thesis contributed to the algorithm design and implementation of the constructive algorithm, as well as theory development of novel vertex and face cases, visualization, digital design and physical models, and conducting the comparisons of the methods presented in the paper. A more detailed contribution statement is given in page 24 of this article.