# A DISTRIBUTED JOINT INTEGRATED PROBABILISTIC DATA ASSOCIATION (JIPDA) FILTER WITH SOFT OBJECT ASSOCIATION

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## ABSTRACT

We propose a distributed multisensor joint integrated probabilistic data association (JIPDA) filter for multiobject tracking in decentralized sensor networks. Conventional Chernoff fusion of the posterior multiobject distributions of neighboring sensors presupposes a correct "hard" association of the objects tracked at the sensors. To avoid detrimental effects of an incorrect hard association, we develop a fusion method based on probabilistic ("soft") object association. Our numerical results demonstrate significant performance gains relative to the hard association approach.

***Index Terms—*** Distributed multiobject tracking, distributed multitarget tracking, data fusion, distributed JIPDA filter, probabilistic object association.

## 1. INTRODUCTION

The purpose of multiobject tracking is estimation of the time-dependent number and states of multiple objects based on noisy and cluttered sensor measurements [1–4]. In challenging scenarios, the use of multiple sensors is helpful or even necessary. In particular, decentralized sensor networks call for distributed multisensor algorithms where each sensor runs a local tracking filter and exchanges information with a set of neighboring sensors. Most distributed multiobject tracking methods fuse the posterior probability density functions (pdfs) of neighboring sensors [5–8].

The joint integrated probabilistic data association (JIPDA) filter [3, 9] is a powerful multiobject tracking method. In the case of distributed JIPDA filtering, the fusion of the posterior pdfs presupposes a correct association of the single-object state vectors at different sensors, whose correspondence is a priori unknown. To the best of our knowledge, only one method for distributed JIPDA filtering was proposed so far [10]. This method does not use hard association decisions; rather, it incorporates so-called fusion weights in an ad-hoc manner. These fusion weights can be interpreted as association probabilities. However, the complexity of the method scales exponentially with the number of object states.

Here, we propose a new probabilistic fusion method for distributed JIPDA filtering. Our method follows an approach that we previously introduced in [11] for soft association of labeled Bernoulli components in distributed labeled multi-Bernoulli filtering [12–14]. Soft (probabilistic) object association is performed by computing the posterior probability distribution of the possible object associations and incorporating it in the fused posterior probability distribution of the multiobject state. An efficient algorithm is obtained by approximating the association distribution by the product of its marginals. Numerical results demonstrate significant performance gains compared to JIPDA fusion using hard object association.

We will use the following basic notation. We write pdfs as $f(\cdot)$ and probability mass functions (pmfs) as $p(\cdot)$. Vectors are denoted by small boldface letters (e.g., $\boldsymbol{x}$). Randomness of scalar or vector variables is indicated by a sans serif font (e.g., $\mathsf{x}$ or $\mathbf{x}$). The superscript $^{\mathrm{T}}$ indicates transposition.

The paper is organized as follows. The single-sensor JIPDA filter is reviewed in Section 2. A basic distributed JIPDA filter relying on hard object association is presented in Section 3. In Section 4, we derive the proposed fusion method using soft object association. Finally, numerical results are presented in Section 5.

## 2. REVIEW OF THE JIPDA FILTER

We consider $I_k$ moving objects, where $k = 0, 1, \ldots$ is a discrete time index. The kinematic state $\mathbf{x}_{k,i}$ of object $i \in \mathcal{I}_k \triangleq \{1, \ldots, I_k\}$ at time $k$ usually comprises position, velocity, and possibly further variables. At any given time $k$, object $i \in \mathcal{I}_k$ may exist or not. In the JIPDA filter [3, 9], the multiobject state (i.e., the joint state of all objects) is described by the random vector $[\mathbf{x}_k^{\mathrm{T}}\ \mathbf{e}_k^{\mathrm{T}}]^{\mathrm{T}}$, where $\mathbf{x}_k \triangleq [\mathbf{x}_{k,1}^{\mathrm{T}} \cdots \mathbf{x}_{k,I_k}^{\mathrm{T}}]^{\mathrm{T}}$ is the joint kinematic state and $\mathbf{e}_k \triangleq [\mathsf{e}_{k,1} \cdots \mathsf{e}_{k,I_k}]^{\mathrm{T}}$ with $\mathsf{e}_{k,i} \in \{0,1\}$ is an existence indicator vector, i.e., $\mathsf{e}_{k,i} = 1$ if object $i$ exists at time $k$ and $\mathsf{e}_{k,i} = 0$ otherwise.

The single-sensor JIPDA filter estimates the multiobject state $[\mathbf{x}_k^{\mathrm{T}}\ \mathbf{e}_k^{\mathrm{T}}]^{\mathrm{T}}$ from measurements $\boldsymbol{z}_{1:k} \triangleq [\boldsymbol{z}_1^{\mathrm{T}} \cdots \boldsymbol{z}_k^{\mathrm{T}}]^{\mathrm{T}}$ with $\boldsymbol{z}_{k'} \triangleq [\boldsymbol{z}_{k',1}^{\mathrm{T}} \cdots \boldsymbol{z}_{k',M_{k'}}^{\mathrm{T}}]^{\mathrm{T}}$, where $M_{k'}$ is the number of measurements produced by the sensor at time $k' \in \{1, \ldots, k\}$. This estimation is based on the posterior pdf $f(\boldsymbol{x}_k, \boldsymbol{e}_k | \boldsymbol{z}_{1:k})$. The JIPDA filter approximates the states $[\mathbf{x}_{k,i}^{\mathrm{T}}\ \mathsf{e}_{k,i}]^{\mathrm{T}}$ of different objects $i$ as conditionally independently given $\boldsymbol{z}_{1:k}$, i.e.,

$$f(\boldsymbol{x}_k, \boldsymbol{e}_k | \boldsymbol{z}_{1:k}) = \prod_{i \in \mathcal{I}_k} f(\boldsymbol{x}_{k,i}, e_{k,i} | \boldsymbol{z}_{1:k}) = \prod_{i \in \mathcal{I}_k} f(\boldsymbol{x}_{k,i} | e_{k,i}, \boldsymbol{z}_{1:k})\, p(e_{k,i} | \boldsymbol{z}_{1:k}). \quad (1)$$

Here, in particular, $p(e_{k,i} = 1 | \boldsymbol{z}_{1:k})$ is the posterior probability that object $i$ exists, and $f(\boldsymbol{x}_{k,i} | e_{k,i} = 1, \boldsymbol{z}_{1:k})$ is modeled as a Gaussian pdf. On the other hand, $f(\boldsymbol{x}_{k,i} | e_{k,i} = 0, \boldsymbol{z}_{1:k})$ is obviously meaningless and therefore formally replaced by a "dummy pdf" $f_{\mathrm{D}}(\boldsymbol{x}_{k,i})$. The time-recursive calculation of $f(\boldsymbol{x}_k, \boldsymbol{e}_k | \boldsymbol{z}_{1:k})$ is described in [3, 9]. For later use, we note that $f(\boldsymbol{x}_k | \boldsymbol{z}_{1:k})$ can be obtained by marginalizing out $\mathbf{e}_k$ from $f(\boldsymbol{x}_k, \boldsymbol{e}_k | \boldsymbol{z}_{1:k})$, which yields

$$f(\boldsymbol{x}_k | \boldsymbol{z}_{1:k}) = \sum_{\boldsymbol{e}_k \in \{0,1\}^{I_k}} f(\boldsymbol{x}_k, \boldsymbol{e}_k | \boldsymbol{z}_{1:k}) = \prod_{i \in \mathcal{I}_k} \sum_{e_{k,i} \in \{0,1\}} f(\boldsymbol{x}_{k,i} | e_{k,i}, \boldsymbol{z}_{1:k})\, p(e_{k,i} | \boldsymbol{z}_{1:k})$$

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$$= \prod_{i\in\mathcal{I}_k} \big( f_{\mathrm{D}}(\boldsymbol{x}_{k,i})\, p(e_{k,i}\!=\!0|\boldsymbol{z}_{1:k}) + f(\boldsymbol{x}_{k,i}|e_{k,i}\!=\!1, \boldsymbol{z}_{1:k})\, p(e_{k,i}\!=\!1|\boldsymbol{z}_{1:k}) \big) . \quad (2)$$

## 3. DISTRIBUTED JIPDA FILTERING

We next consider distributed JIPDA filtering in a basic two-sensor scenario and propose a two-sensor fusion algorithm. This algorithm can be easily extended to distributed networkwide multisensor fusion by using it at each sensor in turn to perform pairwise fusion with each of the neighboring sensors in a recursive manner [11, 15].

Each sensor runs a local JIPDA filter. Suppressing the time index $k$ for simplicity, the multiobject states at sensors 1 and 2 at time $k$ are denoted as $[\mathbf{x}^{(1)\mathrm{T}}\, \mathbf{e}^{(1)\mathrm{T}}]^{\mathrm{T}}$ and $[\mathbf{x}^{(2)\mathrm{T}}\, \mathbf{e}^{(2)\mathrm{T}}]^{\mathrm{T}}$, respectively, with $\mathbf{x}^{(1)} = [\mathbf{x}_1^{(1)\mathrm{T}} \cdots \mathbf{x}_{I_1}^{(1)\mathrm{T}}]^{\mathrm{T}}$, $\mathbf{e}^{(1)} = [\mathrm{e}_1^{(1)} \cdots \mathrm{e}_{I_1}^{(1)}]^{\mathrm{T}}$, $\mathbf{x}^{(2)} = [\mathbf{x}_1^{(2)\mathrm{T}} \cdots \mathbf{x}_{I_2}^{(2)\mathrm{T}}]^{\mathrm{T}}$, and $\mathbf{e}^{(2)} = [\mathrm{e}_1^{(2)} \cdots \mathrm{e}_{I_2}^{(2)}]^{\mathrm{T}}$. We define $\mathcal{I}_1 \triangleq \{1,\dots,I_1\}$ and $\mathcal{I}_2 \triangleq \{1,\dots, I_2\}$, where $I_1$ and $I_2$ are the numbers of states at sensor 1 and 2, respectively. The measurement sequences $\boldsymbol{z}_{1:k}$ at sensor 1 and 2 are denoted as $\boldsymbol{z}^{(1)}$ and $\boldsymbol{z}^{(2)}$, respectively.

### 3.1. Chernoff Fusion of Posterior pdfs

According to (1), the posterior pdfs at the individual sensors are

$$f^{(s)}(\boldsymbol{x}^{(s)}, \boldsymbol{e}^{(s)}|\boldsymbol{z}^{(s)}) = \prod_{i\in\mathcal{I}_s} f^{(s)}(\boldsymbol{x}_i^{(s)}|e_i^{(s)}, \boldsymbol{z}^{(s)})\, p^{(s)}(e_i^{(s)}|\boldsymbol{z}^{(s)})\,, \quad (3)$$

for $s \in \{1,2\}$. In the proposed distributed JIPDA filter, $f^{(1)}(\boldsymbol{x}^{(1)}, \boldsymbol{e}^{(1)}|\boldsymbol{z}^{(1)})$ and $f^{(2)}(\boldsymbol{x}^{(2)}, \boldsymbol{e}^{(2)}|\boldsymbol{z}^{(2)})$ are combined into a fused posterior pdf $\tilde{f}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$. Here, $[\boldsymbol{x}^{\mathrm{T}}\, \boldsymbol{e}^{\mathrm{T}}]^{\mathrm{T}}$ is a "consensual" multiobject state and the tilde in $\tilde{f}$ indicates that the fused posterior pdf is different from the true posterior pdf $f(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$. Our fusion scheme is based on the Chernoff fusion rule [6]. In the idealistic case where $I_1 = I_2 = I$, $\boldsymbol{x}^{(1)} = \boldsymbol{x}^{(2)} = \boldsymbol{x}$, and $\boldsymbol{e}^{(1)} = \boldsymbol{e}^{(2)} = \boldsymbol{e}$, the Chernoff fusion rule is given by

$$\tilde{f}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = \frac{1}{C} \big(f^{(1)}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)})\big)^{\omega} \big(f^{(2)}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(2)})\big)^{1-\omega}, \quad (4)$$

with $C \triangleq \sum_{\boldsymbol{e}\in\{0,1\}^I} \int \big(f^{(1)}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)})\big)^{\omega} \big(f^{(2)}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(2)})\big)^{1-\omega} \mathrm{d}\boldsymbol{x}$ and some fixed $\omega \in [0,1]$. Next, we address the realistic case where this idealistic assumption is not satisfied.

### 3.2. Object Association

In a practical implementation of the distributed JIPDA filter, it is likely that $[\mathbf{x}_i^{(1)\mathrm{T}}\, \mathrm{e}_i^{(1)}]^{\mathrm{T}}$ tracked at sensor 1 and $[\mathbf{x}_i^{(2)\mathrm{T}}\, \mathrm{e}_i^{(2)}]^{\mathrm{T}}$ tracked at sensor 2 (for the same object index $i$) describe different objects because the order of the single-object states within the multiobject states $[\mathbf{x}^{(1)\mathrm{T}}\, \mathbf{e}^{(1)\mathrm{T}}]^{\mathrm{T}}$ and $[\mathbf{x}^{(2)\mathrm{T}}\, \mathbf{e}^{(2)\mathrm{T}}]^{\mathrm{T}}$ is usually different. Furthermore, there may be object states $[\mathbf{x}_i^{(s)\mathrm{T}}\, \mathrm{e}_i^{(s)}]^{\mathrm{T}}$ modeled by the local JIPDA filter at sensor $s \in \{1,2\}$ that do not correspond to an actually existing object, and, conversely, not every actually existing object may be taken into account by sensor $s$. As a consequence, it is likely that $I_1$ and $I_2$ are different, resulting in different dimensions of the multiobject state vectors $[\mathbf{x}^{(1)\mathrm{T}}\, \mathbf{e}^{(1)\mathrm{T}}]^{\mathrm{T}}$ and $[\mathbf{x}^{(2)\mathrm{T}}\, \mathbf{e}^{(2)\mathrm{T}}]^{\mathrm{T}}$. Therefore, application of the fusion rule in (4) presupposes a pertinent association of the states $[\mathbf{x}_i^{(1)\mathrm{T}}\, \mathrm{e}_i^{(1)}]^{\mathrm{T}}$ and $[\mathbf{x}_j^{(2)\mathrm{T}}\, \mathrm{e}_j^{(2)}]^{\mathrm{T}}$.

Without loss of generality, let us assume $I_2 \geq I_1$ and consider sensor 1 as a reference sensor. We will associate each state $[\mathbf{x}_i^{(1)\mathrm{T}}\, \mathrm{e}_i^{(1)}]^{\mathrm{T}}$, $i \in \mathcal{I}_1$ at sensor 1 with some state $[\mathbf{x}_j^{(2)\mathrm{T}}\, \mathrm{e}_j^{(2)}]^{\mathrm{T}} = [\mathbf{x}_{a_i}^{(2)\mathrm{T}}\, \mathrm{e}_{a_i}^{(2)}]^{\mathrm{T}}$ at sensor 2, where $j = a_i \in \mathcal{I}_2$ is a function of $i$. Let us combine the association variables $a_i \in \mathcal{I}_2$ for all $i \in \mathcal{I}_1$ into the *object association vector* $\boldsymbol{a} \triangleq [a_1 \cdots a_{I_1}]^{\mathrm{T}} \in \mathcal{I}_2^{I_1}$. The requirement that different states at sensor 1 are associated with different states at sensor 2 implies that all entries $a_i$ of $\boldsymbol{a}$ are different, which will be referred to as an *admissible* association. We also define the *association alphabet* $\mathcal{A}$ as the set of all admissible association vectors $\boldsymbol{a}$.

To account for the associations expressed by $\boldsymbol{a}$, we modify the Chernoff fusion rule in (4) according to

$$\tilde{f}_{\boldsymbol{a}}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = \frac{1}{C_{\boldsymbol{a}}} \big(f^{(1)}(\boldsymbol{x}^{(1)}\!=\!\boldsymbol{x}, \boldsymbol{e}^{(1)}\!=\!\boldsymbol{e}|\boldsymbol{z}^{(1)})\big)^{\omega} \times \big(f^{(2)}(\boldsymbol{x}_{\boldsymbol{a}}^{(2)}\!=\!\boldsymbol{x}, \boldsymbol{e}_{\boldsymbol{a}}^{(2)}\!=\!\boldsymbol{e}|\boldsymbol{z}^{(2)})\big)^{1-\omega}, \quad (5)$$

with $C_{\boldsymbol{a}} \triangleq \sum_{\boldsymbol{e}\in\{0,1\}^{I_1}} \int \big(f^{(1)}(\boldsymbol{x}^{(1)} = \boldsymbol{x}, \boldsymbol{e}^{(1)} = \boldsymbol{e}|\boldsymbol{z}^{(1)})\big)^{\omega} \times \big(f^{(2)}(\boldsymbol{x}_{\boldsymbol{a}}^{(2)} = \boldsymbol{x}, \boldsymbol{e}_{\boldsymbol{a}}^{(2)} = \boldsymbol{e}|\boldsymbol{z}^{(2)})\big)^{1-\omega} \mathrm{d}\boldsymbol{x}$. Here, $\boldsymbol{x} = [\boldsymbol{x}_1^{\mathrm{T}} \cdots \boldsymbol{x}_{I_1}^{\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{e} = [e_1 \cdots e_{I_1}]^{\mathrm{T}}$, $\boldsymbol{x}^{(1)} = [\boldsymbol{x}_1^{(1)\mathrm{T}} \cdots \boldsymbol{x}_{I_1}^{(1)\mathrm{T}}]^{\mathrm{T}}$, $\boldsymbol{e}^{(1)} = [e_1^{(1)} \cdots e_{I_1}^{(1)}]^{\mathrm{T}}$, $\boldsymbol{x}_{\boldsymbol{a}}^{(2)} = \big[\boldsymbol{x}_{a_1}^{(2)\mathrm{T}} \cdots \boldsymbol{x}_{a_{I_1}}^{(2)\mathrm{T}}\big]^{\mathrm{T}}$, and $\boldsymbol{e}_{\boldsymbol{a}}^{(2)} = \big[e_{a_1}^{(2)} \cdots e_{a_{I_1}}^{(2)}\big]^{\mathrm{T}}$. Inserting (3), it can be readily shown that (5) can be expressed as

$$\tilde{f}_{\boldsymbol{a}}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = \prod_{i\in\mathcal{I}_1} p_{i,a_i}(e_i)\, f_{i,a_i}(\boldsymbol{x}_i|e_i)\,, \quad (6)$$

where the fused existence pmfs $p_{i,a_i}(e_i)$ are given by

$$p_{i,a_i}(e_i) = \begin{cases} \frac{1}{C_{i,a_i}} \big(P_i^{(1)}(0)\big)^{\omega} \big(P_{a_i}^{(2)}(0)\big)^{1-\omega}, & e_i = 0\,, \\ \frac{C'_{i,a_i}}{C_{i,a_i}} \big(P_i^{(1)}(1)\big)^{\omega} \big(P_{a_i}^{(2)}(1)\big)^{1-\omega}, & e_i = 1\,, \end{cases} \quad (7)$$

with $P_i^{(s)}(e)$ being short for $p^{(s)}(e_i^{(s)}\!=\!e|\boldsymbol{z}^{(s)})$, and the fused spatial pdfs $f_{i,a_i}(\boldsymbol{x}_i|e_i)$ are given by

$$f_{i,a_i}(\boldsymbol{x}_i|e_i) = \begin{cases} f_{\mathrm{D}}(\boldsymbol{x}_i)\,, & e_i = 0\,, \\ \frac{1}{C'_{i,a_i}} \big(f^{(1)}(\boldsymbol{x}_i^{(1)}\!=\!\boldsymbol{x}_i|e_i^{(1)}\!=\!1, \boldsymbol{z}^{(1)})\big)^{\omega} \times \big(f^{(2)}(\boldsymbol{x}_{a_i}^{(2)}\!=\!\boldsymbol{x}_i|e_{a_i}^{(2)}\!=\!1, \boldsymbol{z}^{(2)})\big)^{1-\omega}, & e_i = 1\,. \end{cases} \quad (8)$$

Here, the constants in these expressions are

$$C_{i,a_i} \triangleq \big(P_i^{(1)}(0)\big)^{\omega} \big(P_{a_i}^{(2)}(0)\big)^{1-\omega} + \big(P_i^{(1)}(1)\big)^{\omega} \big(P_{a_i}^{(2)}(1)\big)^{1-\omega} C'_{i,a_i} \quad (9)$$

and

$$C'_{i,a_i} \triangleq \int \big(f^{(1)}(\boldsymbol{x}_i^{(1)}\!=\!\boldsymbol{x}_i|e_i^{(1)}\!=\!1, \boldsymbol{z}^{(1)})\big)^{\omega} \times \big(f^{(2)}(\boldsymbol{x}_{a_i}^{(2)}\!=\!\boldsymbol{x}_i|e_{a_i}^{(2)}\!=\!1, \boldsymbol{z}^{(2)})\big)^{1-\omega} \mathrm{d}\boldsymbol{x}_i\,. \quad (10)$$

We note that $p_{i,a_i}(0) + p_{i,a_i}(1) = 1$ and $C_{\boldsymbol{a}} = \prod_{i\in\mathcal{I}_1} C_{i,a_i}$. According to (6), calculating $\tilde{f}_{\boldsymbol{a}}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ amounts to calculating the single-object pmfs $p_{i,a_i}(e_i)$ and pdfs $f_{i,a_i}(\boldsymbol{x}_i|e_i)$.

## 4. FUSION WITH SOFT OBJECT ASSOCIATION

The correct association vector $\boldsymbol{a}$ is defined by the fact that, for each $i \in \mathcal{I}_1$, the states $[\mathbf{x}_i^{(1)\mathrm{T}}\, \mathrm{e}_i^{(1)}]^{\mathrm{T}}$ and $[\mathbf{x}_{a_i}^{(2)\mathrm{T}}\, \mathrm{e}_{a_i}^{(2)}]^{\mathrm{T}}$ describe the same object. In practice, the correct $\boldsymbol{a}$ is unknown at the sensors and must be estimated. However, a "hard" estimation can lead to an incorrect result, in which case the meaning of the expressions (7)–(10) and of the overall fusion result (6) is unclear and poor tracking performance of the distributed JIPDA filter will be obtained. Therefore, in the following, we propose to avoid a hard association and present a new "soft" (probabilistic) object association method. This approach is analogous in spirit to the probabilistic data association that is performed in many multiobject tracking methods to implicitly associate measurements with objects [1].

### 4.1. Basic Formulation

To develop the probabilistic object association method, we model $\boldsymbol{a}$ as a random vector $\mathsf{a}$. Furthermore, instead of the fused posterior pdf $\tilde{f}_{\boldsymbol{a}}(\boldsymbol{x}, \boldsymbol{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ in (5), we consider the joint fused posterior pdf of $\mathsf{x}$, $\mathsf{e}$, and $\mathsf{a}$, which is defined for $\boldsymbol{a} \in \mathcal{A}$ as

$$\tilde{f}(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{a}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) \triangleq \frac{1}{C'} \big(f^{(1)}(\boldsymbol{x}^{(1)} \!=\! \boldsymbol{x}, \boldsymbol{e}^{(1)} \!=\! \boldsymbol{e}|\boldsymbol{z}^{(1)})\big)^{\omega} \times \big(f^{(2)}(\boldsymbol{x}^{(2)}_{\boldsymbol{a}} \!=\! \boldsymbol{x}, \boldsymbol{e}^{(2)}_{\boldsymbol{a}} \!=\! \boldsymbol{e}|\boldsymbol{z}^{(2)})\big)^{1-\omega},$$

where $C' \triangleq \sum_{\boldsymbol{a} \in \mathcal{A}} C_{\boldsymbol{a}}$, and for $\boldsymbol{a} \in \mathcal{I}_2^{I_1} \setminus \mathcal{A}$ as $\tilde{f}(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{a}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) \triangleq 0$. Combining (5) and (6) gives $\big(f^{(1)}(\boldsymbol{x}^{(1)} \!=\! \boldsymbol{x}, \boldsymbol{e}^{(1)} \!=\! \boldsymbol{e}|\boldsymbol{z}^{(1)})\big)^{\omega} \times \big(f^{(2)}(\boldsymbol{x}^{(2)}_{\boldsymbol{a}} = \boldsymbol{x}, \boldsymbol{e}^{(2)}_{\boldsymbol{a}} = \boldsymbol{e}|\boldsymbol{z}^{(2)})\big)^{1-\omega} = C_{\boldsymbol{a}} \prod_{i \in \mathcal{I}_1} f_{i,a_i}(\boldsymbol{x}_i|e_i) \times p_{i,a_i}(e_i)$. By inserting $C_{\boldsymbol{a}} = \prod_{i \in \mathcal{I}_1} C_{i,a_i}$, we finally obtain

$$\tilde{f}(\boldsymbol{x}, \boldsymbol{e}, \boldsymbol{a}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = w_{\boldsymbol{e},\boldsymbol{a}} \prod_{i \in \mathcal{I}_1} f_{i,a_i}(\boldsymbol{x}_i|e_i), \tag{11}$$

with

$$w_{\boldsymbol{e},\boldsymbol{a}} = \frac{1}{C'} \prod_{i \in \mathcal{I}_1} C_{i,a_i}\, p_{i,a_i}(e_i), \tag{12}$$

where $C' = \sum_{\boldsymbol{a} \in \mathcal{A}} \prod_{i \in \mathcal{I}_1} C_{i,a_i}$.

### 4.2. Formulation Using the Augmented Association Vector

To enable an approximate calculation of $\tilde{f}(\boldsymbol{x}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$, we introduce the *augmented association vector* $\mathsf{a}' \triangleq [\mathsf{a}'_1 \cdots \mathsf{a}'_{I_1}]^{\mathrm{T}}$ with entries $\mathsf{a}'_i \in \mathcal{I}_{2,0} \triangleq \{0\} \cup \mathcal{I}_2$, where $\mathsf{a}'_i = 0$ additionally expresses the nonexistence of the $i$th object. An augmented association vector $\boldsymbol{a}'$ such that all nonzero entries $a'_i$ are different is said to be admissible, and the augmented association alphabet $\mathcal{A}'$ comprises all admissible $\boldsymbol{a}'$. Note that $\mathsf{a}'$ expresses the same information as the two vectors $\mathsf{e}$ and $\mathsf{a}$ together. For $\boldsymbol{a}' \in \mathcal{A}'$, we can reformulate Eq. (11) as

$$\tilde{f}(\boldsymbol{x}, \boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = w_{\boldsymbol{a}'} \prod_{i \in \mathcal{I}_1} f'_{i,a'_i}(\boldsymbol{x}_i), \tag{13}$$

where

$$f'_{i,a'_i}(\boldsymbol{x}_i) = \begin{cases} \frac{1}{C'_{i,a'_i}} \big(f^{(1)}(\boldsymbol{x}^{(1)}_i \!=\! \boldsymbol{x}_i|e^{(1)}_i \!=\! 1, \boldsymbol{z}^{(1)})\big)^{\omega} \\ \quad \times \big(f^{(2)}(\boldsymbol{x}^{(2)}_{a'_i} \!=\! \boldsymbol{x}_i|e^{(2)}_{a'_i} \!=\! 1, \boldsymbol{z}^{(2)})\big)^{1-\omega}, & a'_i \in \mathcal{I}_2, \\ f_{\mathrm{D}}(\boldsymbol{x}_i), & a'_i \!=\! 0, \end{cases} \tag{14}$$

with $C'_{i,a'_i}$ given by (10) with $a_i$ replaced by $a'_i$, and, consistently with (12),

$$w_{\boldsymbol{a}'} = \frac{1}{C''} \prod_{i \in \mathcal{I}_1} \beta_{i,a'_i},$$

with

$$\beta_{i,a'_i} \triangleq \begin{cases} \dfrac{\big(P^{(1)}_i(1)\big)^{\omega} \big(P^{(2)}_{a'_i}(1)\big)^{1-\omega} C'_{i,a'_i}}{\big(P^{(1)}_i(0)\big)^{\omega} \big(P^{(2)}_{a'_i}(0)\big)^{1-\omega}}, & a'_i \in \mathcal{I}_2, \\ 1, & a'_i = 0 \end{cases}$$

and

$$C'' \triangleq \sum_{\boldsymbol{a}' \in \mathcal{A}'} \prod_{i \in \mathcal{I}_1 : a'_i \in \mathcal{I}_2} \frac{C_{i,a'_i}}{\big(P^{(1)}_i(0)\big)^{\omega} \big(P^{(2)}_{a'_i}(0)\big)^{1-\omega}}.$$

For $\boldsymbol{a}' \in \mathcal{I}_{2,0}^{I_1} \setminus \mathcal{A}'$, we set $\tilde{f}(\boldsymbol{x}, \boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = 0$.

We now formally treat $\tilde{f}(\boldsymbol{x}, \boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ as the joint pdf of $\mathsf{x}$ and $\mathsf{a}'$ given $\mathsf{z}^{(1)} \!=\! \boldsymbol{z}^{(1)}$ and $\mathsf{z}^{(2)} \!=\! \boldsymbol{z}^{(2)}$, and thus obtain the factorization

$$\tilde{f}(\boldsymbol{x}, \boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = f(\boldsymbol{x}|\boldsymbol{a}', \boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})\, p(\boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}). \tag{15}$$

By comparison with (13), the two factors are seen to be

$$f(\boldsymbol{x}|\boldsymbol{a}', \boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = \prod_{i \in \mathcal{I}_1} f'_{i,a'_i}(\boldsymbol{x}_i) \tag{16}$$

and

$$p(\boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = \begin{cases} w_{\boldsymbol{a}'} = \frac{1}{C''} \prod_{i \in \mathcal{I}_1} \beta_{i,a'_i}, & \boldsymbol{a}' \in \mathcal{A}', \\ 0, & \text{otherwise}, \end{cases}$$

consistently with $\tilde{f}(\boldsymbol{x}, \boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = 0$ for $\boldsymbol{a}' \in \mathcal{I}_{2,0}^{I_1} \setminus \mathcal{A}'$.

### 4.3. Approximate Calculation of $\tilde{f}(\boldsymbol{x}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$

We now define the unconditional marginal fused posterior pdf $\tilde{f}(\boldsymbol{x}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ by marginalizing out the association vector $\mathsf{a}'$ from $\tilde{f}(\boldsymbol{x}, \boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$, yielding

$$\begin{aligned} \tilde{f}(\boldsymbol{x}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) &= \sum_{\boldsymbol{a}' \in \mathcal{I}_{2,0}^{I_1}} \tilde{f}(\boldsymbol{x}, \boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) \\ &= \sum_{\boldsymbol{a}' \in \mathcal{I}_{2,0}^{I_1}} p(\boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) \prod_{i \in \mathcal{I}_1} f'_{i,a'_i}(\boldsymbol{x}_i), \end{aligned} \tag{17}$$

where (15) and (16) were used. Finally, to obtain the proposed probabilistic object association scheme for the JIPDA filter, we approximate $p(\boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ by the product of its marginals, i.e.,

$$p(\boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) \approx \prod_{i \in \mathcal{I}_1} p(a'_i), \quad \boldsymbol{a}' \in \mathcal{I}_{2,0}^{I_1}, \tag{18}$$

where

$$p(a'_i) = \sum_{\boldsymbol{a}'^{\sim i} \in \mathcal{I}_{2,0}^{I_1 - 1}} p(\boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}). \tag{19}$$

Here, $\boldsymbol{a}'^{\sim i}$ denotes the vector $\boldsymbol{a}'$ with the $i$th entry, $a'_i$, removed, and $p(a'_i)$ is short for $p(a'_i|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$. Next, we insert (18) into (17) and use $\sum_{\boldsymbol{a}' \in \mathcal{I}_{2,0}^{I_1}} = \sum_{a'_1 \in \mathcal{I}_{2,0}} \cdots \sum_{a'_{I_1} \in \mathcal{I}_{2,0}}$, to obtain

$$\begin{aligned} \tilde{f}(\boldsymbol{x}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) &\approx \sum_{\boldsymbol{a}' \in \mathcal{I}_{2,0}^{I_1}} \prod_{i \in \mathcal{I}_1} p(a'_i)\, f'_{i,a'_i}(\boldsymbol{x}_i) \\ &= \prod_{i \in \mathcal{I}_1} \sum_{a'_i \in \mathcal{I}_{2,0}} p(a'_i)\, f'_{i,a'_i}(\boldsymbol{x}_i) \\ &= \prod_{i \in \mathcal{I}_1} \bigg(p(a'_i \!=\! 0)\, f_{\mathrm{D}}(\boldsymbol{x}_i) + \sum_{a'_i \in \mathcal{I}_2} p(a'_i)\, f'_{i,a'_i}(\boldsymbol{x}_i)\bigg). \end{aligned}$$

This expression can be written in the form of (2), i.e.,

$$\begin{aligned} \tilde{f}(\boldsymbol{x}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = \prod_{i \in \mathcal{I}_1} \big(& f_{\mathrm{D}}(\boldsymbol{x}_i)\, p_i(e_i \!=\! 0) \\ &+ f_i(\boldsymbol{x}_i|e_i \!=\! 1)\, p_i(e_i \!=\! 1)\big), \end{aligned} \tag{20}$$

where the fused existence pmfs $p_i(e_i)$ are given by

$$p_i(e_i \!=\! 0) = p(a'_i \!=\! 0), \tag{21}$$

$$p_i(e_i \!=\! 1) = 1 - p_i(e_i \!=\! 0) = \sum_{a'_i \in \mathcal{I}_2} p(a'_i) \tag{22}$$

and the fused spatial pdfs $f_i(\boldsymbol{x}_i|e_i=1)$ are given by

$$f_i(\boldsymbol{x}_i|e_i=1) = \frac{1}{p_i(e_i=1)} \sum_{a'_i \in \mathcal{I}_2} p(a'_i)\, f'_{i,a'_i}(\boldsymbol{x}_i)\,, \tag{23}$$

both for $i \in \mathcal{I}_1$.

Our ultimate goal is to obtain the joint fused posterior pdf $\tilde{f}(\boldsymbol{x}, \mathbf{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$. In (20), the marginal fused posterior pdf $\tilde{f}(\boldsymbol{x}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ is expressed in the same form as the marginal posterior pdf $f(\boldsymbol{x}_k|\boldsymbol{z}_{1:k})$ in (2), which was obtained by marginalizing out $\mathbf{e}$ from the joint posterior pdf $f(\boldsymbol{x}_k, \boldsymbol{e}_k|\boldsymbol{z}_{1:k})$ in (1). This provides a motivation for writing $\tilde{f}(\boldsymbol{x}, \mathbf{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ in the form of (1), i.e.,

$$\tilde{f}(\boldsymbol{x}, \mathbf{e}|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)}) = \prod_{i \in \mathcal{I}_1} f_i(\boldsymbol{x}_i|e_i)\, p_i(e_i)\,, \tag{24}$$

where $f_i(\boldsymbol{x}_i|e_i=0) = f_{\mathrm{D}}(\boldsymbol{x}_i)$, $f_i(\boldsymbol{x}_i|e_i=1)$ is given by (23), and $p_i(e_i)$ is given by (21) and (22). Indeed, one can easily verify that (20) is obtained by marginalizing out $\mathbf{e}$ from (24).

The proposed JIPDA fusion scheme with probabilistic object association is constituted by expressions (24) and (23) together with the expressions of $f'_{i,a'_i}(\boldsymbol{x}_i)$ in (14) and the expressions of $p_i(e_i)$ in (21) and (22). According to (22), each fused existence probability $p_i(e_i=1)$ is the sum of the $I_2$ marginal association probabilities $p(a'_i)$, $a'_i \in \mathcal{I}_2$ in (19). Furthermore, according to (23), each fused state pdf $f_i(\boldsymbol{x}_i|e_i=1)$ is a weighted sum of $I_2$ pdfs $f'_{i,a'_i}(\boldsymbol{x}_i)$, $a'_i \in \mathcal{I}_2$, which are obtained by the Chernoff-type pdf fusion in (14) while the weights are given by $p(a'_i)/p_i(e_i=1)$ with $p_i(e_i=1) = \sum_{a'_i \in \mathcal{I}_2} p(a'_i)$. A highly accurate approximation to the marginal association probabilities $p(a'_i)$ can be calculated by means of an efficient belief propagation-based algorithm.

Our JIPDA fusion scheme is valid for posterior pdfs of general form, even though in the JIPDA filter the posterior pdfs are Gaussian. The restriction to Gaussian posterior pdfs leads to a low-complexity version of our fusion scheme. This version, as well as the belief propagation-based algorithm for approximate marginalization mentioned above, will be described in a future publication.

## 5. NUMERICAL RESULTS

We consider a two-dimensional (2-D) simulation scenario with a region of interest (ROI) given by $[-150, 150] \times [-150, 150]$ and three sensors located at $\boldsymbol{p}^{(1)} = [-50 \;\; 0]^{\mathrm{T}}$, $\boldsymbol{p}^{(2)} = [0 \;\; 0]^{\mathrm{T}}$, and $\boldsymbol{p}^{(3)} = [50 \;\; 0]^{\mathrm{T}}$. We simulated ten objects during 200 time steps. This choice allows us to demonstrate the strengths of our algorithm while not requiring long simulation times. In each simulation run, the objects appear at various time steps before time step 40 and at randomly chosen positions in the area $[-50, 50] \times [-50, 50]$, and they disappear at various time steps after time step 150. The object states $\mathbf{x}_k = [\mathsf{x}_{k,1} \; \mathsf{x}_{k,2} \; \dot{\mathsf{x}}_{k,1} \; \dot{\mathsf{x}}_{k,2}]^{\mathrm{T}}$ comprise the 2-D position $(\mathsf{x}_{k,1}, \mathsf{x}_{k,2})$ and velocity $(\dot{\mathsf{x}}_{k,1}, \dot{\mathsf{x}}_{k,2})$ and evolve according to the nearly constant velocity motion model [16, Sec. 6.3.2]. An object is detected by sensor $s \in \{1, 2, 3\}$ with a probability $p_{\mathrm{D}}^{(s)}(\boldsymbol{x}_k) = p_{\mathrm{D}}(\boldsymbol{x}_k)$ of 0.9 or 0.7; the resulting measurement is $\mathbf{z}_k^{(s)} = [\mathsf{x}_{k,1} \; \mathsf{x}_{k,2}]^{\mathrm{T}} + \mathbf{v}_k^{(s)}$, where the measurement noise $\mathbf{v}_k^{(s)} = [\mathsf{v}_{k,1}^{(s)} \; \mathsf{v}_{k,2}^{(s)}]^{\mathrm{T}}$ is independent and identically distributed zero-mean Gaussian with standard deviations $\sigma_{v_1} = \sigma_{v_2} = 2$. The clutter measurements are uniformly distributed on the ROI with mean number $\mu_{\mathrm{C}}$ equal to 5 or 25.

We compare the distributed JIPDA filter using our proposed fusion method with soft object association (abbreviated as S-JIPDA) with a distributed JIPDA filter using Chernoff fusion with hard object association (H-JIPDA) and with a centralized multisensor JIDPA filter based on the iterated-corrector approach [1, 3] (C-JIPDA). We were not able to simulate the method proposed in [10] because the exponential dependency of its complexity on the number of object states causes it to be numerically infeasible in our simulation scenario. S-JIPDA and H-JIPDA employ the following fusion schedule: first, sensors 1 and 3 send their posterior pdfs to sensor 2, which recursively fuses them with its own posterior pdf; then, sensor 2 sends the fused posterior pdf to sensors 1 and 3, which fuse it with their own posterior pdfs. The fusion parameter $\omega$ is set to 0.5. For the three filters, Fig. 1 shows the mean optimal subpattern assignment (MOSPA) error [17] with cutoff parameter $c = 20$ and order $p = 2$, averaged over the three sensors and 1000 simulation runs. It can be seen that the MOSPA error of S-JIPDA is much smaller than that of H-JIPDA and also much closer to that of C-JIPDA. Furthermore, in the more challenging scenario considered in Fig. 1(b), the MOSPA error of S-JIPDA and C-JIPDA is only slightly larger than in Fig. 1(a). We can conclude from these results that the proposed JIPDA fusion method using soft object association significantly outperforms the method using hard object association.

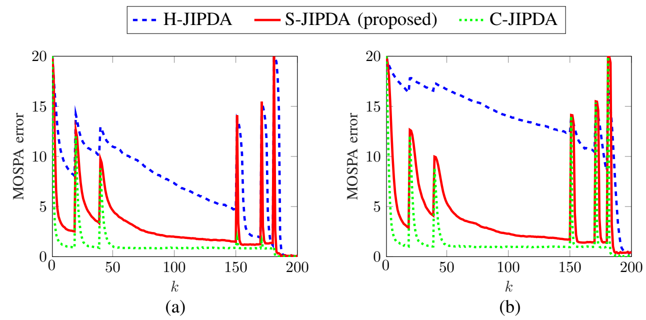


**Fig. 1**: MOSPA error versus time for (a) $p_{\mathrm{D}}(\boldsymbol{x}_k) = 0.9$ and $\mu_{\mathrm{C}} = 5$ and (b) $p_{\mathrm{D}}(\boldsymbol{x}_k) = 0.7$ and $\mu_{\mathrm{C}} = 25$.

As mentioned in Section 4.3, S-JIPDA can be efficiently implemented using a belief propagation-based marginalization of the object association distribution $p(\boldsymbol{a}'|\boldsymbol{z}^{(1)}, \boldsymbol{z}^{(2)})$ (see (19)). The overall complexity of the resulting S-JIPDA implementation scales only linearly in the number of object states. By contrast, the complexity of the method in [10] scales exponentially. Finally, H-JIPDA involves a two-dimensional assignment problem, whose approximate solution typically scales quadratically; nevertheless, in many scenarios, the total runtime of H-JIPDA can be smaller than that of S-JIPDA. A detailed comparison of the runtimes of the various filters will be presented in a future publication.

## 6. CONCLUSION

In distributed JIPDA filters employing Chernoff fusion of the local posterior multiobject distributions, incorrect results of "hard" object association may lead to a performance loss. To avoid this loss, we developed a fusion method that is based on probabilistic ("soft") object association. Our derivation of this method involved the reformulation of the fused multiobject distribution in terms of an object association distribution and the approximation of the latter by the product of its marginals. The proposed soft-association fusion method is applicable to general nonlinear and non-Gaussian system models. Numerical results demonstrated a significant performance advantage over conventional fusion using hard object association. We note that a more detailed presentation and additional results, including an efficient belief propagation algorithm for approximately marginalizing the object association distribution and a low-complexity version of our fusion method for Gaussian posterior pdfs, will be presented in a future publication.

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