The Filtering Effect on Simulated Signals under Consideration of Entropy Methods

Alexander Edthofer[1*], Andreas Körner[1]

[1]Institute of Analysis and Scientific Computing, TU Wien, Wiedner Hauptstraße 8-10, 1040 Vienna, Austria;
[*]*alexander.edthofer@tuwien.ac.at*

Abstract. In this work, we investigate the impact of lowpass filters on two entropy methods, Permutation Entropy (PE) and Entropy of Difference (EoD), using simulated noise signals. Colored noise, specifically white, pink and brown noise, was generated and filtered with passband frequencies of 30 Hz, 40 Hz and 50 Hz, alongside unfiltered signals. The PE and EoD values were computed to analyze the effects of filtering. The results indicate that both entropy measures decrease with lower passband frequencies. PE can effectively distinguish between pink and brown noise with and without a lowpass filter, while EoD shows similar differentiation only with filtering. These findings highlight the sensitivity of entropy measures to lowpass filtering, with implications for their application in EEG analysis.

Introduction

Simulated signals such as noise are often used for modeling and comparison of situations or occurrences of real-world phenomena. Pink noise is very common in biosystems, as these are stochastic, self-organizing and their equilibrium is at the lowest energy level possible [1]. In the context of biomedicine, noise can be thought of as an idealized or abstracted signal. The brain activity of awake humans resembles pink noise when measured by an electroencephalogram (EEG) [2]. During unconsciousness, higher frequencies are not as present compared to the awake stage. Therefore, brain activity in this stage can be better compared to brown noise [3].

In the field of EEG analysis, usually there are bandpass filters applied on the measured signals [4]. The permutation entropy (PE) [5], first introduced in 2002, is a commonly used parameter in research in this field of application [6, 7], the entropy of difference (EoD) [8] is new and not yet established in EEG analysis, but seems promising. In this work, we investigate the impact of lowpass filters on these two entropy methods under the use of raw simulated signals using colored noise.

The effect of linear filters on white noise using the PE has already been shown in [9]. However, no other colored noise was considered. In the case of EoD, no such research has been conducted.

The computations were performed on a laptop with 16 GB RAM, an AMD Ryzen 5 5500U processor with operating system Microsoft Windows 10 Pro using MATLAB version R2023b.

1 Methods

1.1 Noise Signals

Stochastic processes can be used to generate noise signals. The most common ones for model analysis are white, pink and brown noise [10], which also coincide with the simulated signals for the application of EEG analysis. Their power spectral densities (PSD) in general are given by $S(f) = \frac{L(f)}{|f|^\alpha}$ with L being a positive, slowly varying or even constant function. White noise has a uniform distribution of the frequencies, i.e. $S_w(f) = L(f)$, which means $\alpha = 0$. A mathematical description is derived by the time-derivative of a Brownian motion process [11]. Pink noise is defined with $\alpha = 1$ as $S_p(f) = \frac{L(f)}{|f|}$, i.e. higher frequencies appear with lower amplitude. Brown noise has an even stronger decrease than pink as $\alpha = 2$, which results in $S_b(f) = \frac{L(f)}{|f|^2}$. The simulated signals in this work are generated using the MATLAB function `dsp.ColoredNoise`.

The noise signals are compared with two entropy methods. There are different lowpass-filters applied to the signals, which are compared as well. For this, the MATLAB function `lowpass` is used.

1.2 Permutation Entropy

The PE was first introduced in [5]. A given times series $(x_t) = (x_1, ..., x_N)$ is divided in tuples of length m, which is called the order. For each tuple, an ordinal pattern is determined, for length m there are $m!$ possible combinations. The PE is defined as

$$PE = -\frac{1}{\log(m!)} \sum_{i=1}^{m!} p_i \log p_i, \tag{1}$$

where p_i defines the probability of occurrence of pattern i and the base of the logarithm is two. The coefficient $-\frac{1}{\log(m!)}$ represents a normalization factor, such that $PE \in [0,1]$. If two values in one tuple are equal, i.e. $x_i = x_j$, we choose the rule that $x_i < x_j$ for $i < j$. A detailed mathematical description is given in [7].

1.3 Entropy of Difference

An alteration of the PE is the EoD, defined by Pasquale Nardone in [8]. Again, a time series (x_t) is divided in the same amount of tuples of length m. For this method, one considers the neighboring values within a tuple and compares them. The encoding is given only by $+$ and $-$, depending if there is an increase or decrease between the values. If two neighboring values would be equal, we again apply the rule that $x_i < x_j$ for $i < j$. For tuples of length m, there are 2^{m-1} possible patterns that can be achieved. The EoD is then defined as

$$EoD = -\frac{1}{m-1} \sum_{i=1}^{2^{m-1}} p_i \log p_i, \tag{2}$$

where p_i again defines the probability of pattern i. The base of the logarithm is two, such that the coefficient $-\frac{1}{m-1}$ is also a normalization factor, i.e. $EoD \in [0,1]$.

2 Results

For our study, colored noise with $5 \cdot 10^6$ sample points was created. Firstly, the simulated signal is lowpass filtered by a passband frequency of either 30 Hz, 40 Hz or 50 Hz. Considering also unfiltered signals, this makes a total of four different scenarios. These passband frequencies were chosen as these are also used in the application of EEG analysis [4]. Secondly, the sample was rearranged to 1665 vectors each containing 3000 values. Comparing such a vector to a recorded signal with a sampling frequency of 200 Hz, this would correspond to 15 s. Next, the simulated signals were decoded in the respective patterns of the PE and EoD and afterwards the corresponding entropy value were calculated. The results of the two different entropies are given in the case of orders $m = 3$ and $m = 7$. For the PE, the most common orders are between 3 and 7 [5]. A graphical representation of the results for the 1665 samples is given in Figure 1 for $m = 3$ using boxplots, which were created using the MATLAB function `boxplotgroup`. The y-axis refers to the entropy value, which are defined in equations (1) and (2), shown between 0.5 and 1 as these are the minimal and maximal value that appear in our study. The four different filter scenarios are indicated on the x-axis.

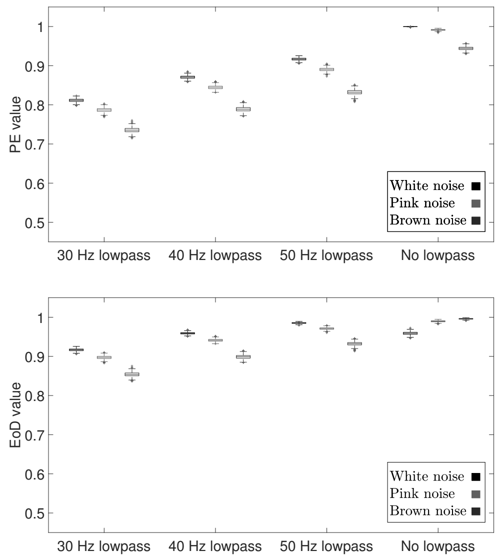

Figure 1: Entropy values of order $m = 3$ and different filters for white, pink and brown noise.

For order 3, a decrease of the PE value from white to pink to brown noise is observable, independent of the applied filter. EoD does not show the same behavior, as for no lowpass filter, there is an increase of the values from white to brown noise. In general, a decrease in the lower passband frequencies is observable for PE and EoD with a more prominent decrease in the case of PE. This is reasonable because, when lowpass filtering is done, fewer patterns occur. The PE can distinguish between pink and brown noise for all four scenarios as none of the respective boxplots overlap. The EoD manages this task only for the three scenarios in which lowpass filtering was applied. For the PE with $m = 3$ and no filtering, the results coincide with the ones of [10].

The results for order $m = 7$ are shown in Figure 2, created with the same function and settings as before. The PE for order 7 shows a similar course to $m = 3$ although the decrease for a lower passband frequency is even stronger. However, it still separates pink and brown noise well, as the boxplots again are not overlapping. The EoD for order $m = 7$ cannot distinguish between pink and brown noise even for the filtering scenarios. The values for white noise are in all cases lower than those for pink noise.

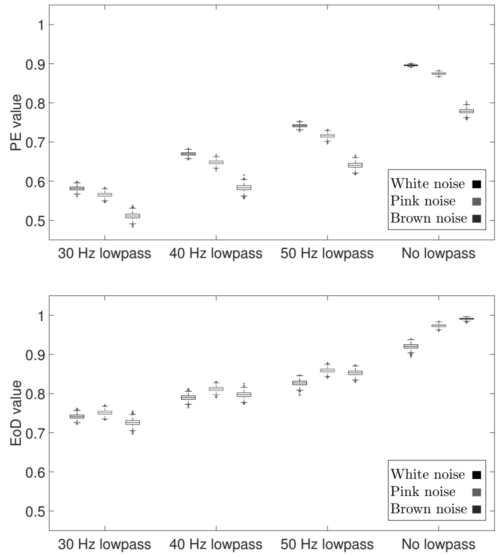

Figure 2: Entropy values of order $m = 7$ and different filters for white, pink and brown noise.

3 Discussion

In this work, we tested different lowpass filters on colored noise and investigated their impact on PE and EoD. Passband frequencies of 30 Hz, 40 Hz and 50 Hz were compared with no lowpass filtering, as these three are most commonly used in the field of EEG analysis [4]. The results show that the EoD behaves differently, if no filter is set. For any other of the three scenarios, EoD behaves similar to the PE for order $m = 3$. For the order $m = 7$, the results differ, as with noise does not achieve the highest EoD value, but pink noise.

One can see as well that the lower the passband frequency was set for the lowpass filter, the lower the entropy values get. For the PE and white noise, this was already indicated in [9]. We also showed the effect on other types of noise as well as a similar impact on the EoD. However, the decrease in value in the latter case is not as strong as for the PE.

We also considered a highpass filter, usually there is a passband frequency of 0.5 Hz, but this did not show any effect on the values of the two entropy methods.

The quality of EoD in comparison to PE, especially in the application of EEG analysis, will be examined in more detail in future studies.

Acknowledgement

The authors would like to thank their research partner the AG Neuromonitoring of Dr. Matthias Kreuzer from the Technical University of Munich for their continued guidance in the field of EEG analysis and, additionally, Clara Horvath for proofreading.

References

[1] Szendro P, Vincze G, Szasz A. Pink-noise behaviour of biosystems. *European Biophysics Journal*. 2001; 30:227–231.

[2] Buzsaki G. *Rhythms of the Brain*. Oxford university press. 2006.

[3] Colombo MA, Napolitani M, Boly M, Gosseries O, Casarotto S, Rosanova M, Brichant JF, Boveroux P, Rex S, Laureys S, et al. The spectral exponent of the resting EEG indexes the presence of consciousness during unresponsiveness induced by propofol, xenon, and ketamine. *NeuroImage*. 2019;189:631–644.

[4] Sanei S, Chambers JA. *EEG signal processing*. John Wiley & Sons. 2013.

[5] Bandt C, Pompe B. Permutation entropy: a natural complexity measure for time series. *Physical review letters*. 2002;88(17):174102.

[6] Berger S, Schneider G, Kochs EF, Jordan D. Permutation entropy: too complex a measure for EEG time series? *Entropy*. 2017;19(12):692.

[7] Edthofer A, Feldhammer I, Fenzl T, Körner A, Kreuzer M. Permutation Entropy as a Conceptual Model to Analyse Brain Activity in Sleep. *arXiv preprint arXiv:231207129*. 2023;pp. arXiv–2312.

[8] Nardone P, Sonnino G. Entropy of Difference: A New Tool for Measuring Complexity. *Axioms*. 2024; 13(2):130.

[9] Dávalos A, Jabloun M, Ravier P, Buttelli O. The impact of linear filter preprocessing in the interpretation of permutation entropy. *Entropy*. 2021;23(7):787.

[10] Chen Z, Ma X, Fu J, Li Y. Ensemble Improved Permutation Entropy: A New Approach for Time Series Analysis. *Entropy*. 2023;25(8):1175.

[11] Kasdin NJ. Discrete simulation of colored noise and stochastic processes and 1/f/sup/spl alpha//power law noise generation. *Proceedings of the IEEE*. 1995; 83(5):802–827.