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ORIGINAL ARTICLE

An analytical approach for the performance calculation of an RCS/RS with several picking stations

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Abstract

Robotic Compact Storage and Retrieval Systems (RCS/RS) offer numerous advantages, including high performance, scalability, and availability, which are essential for modern logistics and warehousing. However, information on the potential performance of RCS/RS is limited, primarily due to the diverse range of configurations available. This paper aims to address this gap by developing an analytical approach to predict the throughput of an RCS/RS with multiple robots serving several picking stations. The approach considers various parameters such as grid size, stack height, number of robots, and filling degree, alongside kinematic data. The cycle time for each robot is calculated assuming a uniform distribution of container stacks. Subsequently, a queueing system with limited capacity is constructed using performance data from a single robot. The analytical approach is validated using a discrete event simulation model of an RCS/RS. Following the validation, an extensive parameter variation and application example are conducted to demonstrate the versatility of the approach. This method offers a straightforward and efficient set of formulas for determining RCS/RS throughput, easily solvable using standard table or algebra programs.

Keywords Automated warehouses · RCS/RS · Cycle time model · Queueing theory · Grid-based storage system

1 Introduction

The ongoing supply chain issues continue to heavily impact logistics today. Scalability stands out as a key feature for addressing long delivery times, extended downtimes, and short-term fluctuations. Warehouses serve as a bridge to manage time and maintain the readiness of goods, albeit with the costs always remaining in focus. Robotic compact storage and retrieval systems (RCS/RS), as depicted in Fig. 1, emerge as a scalable warehousing solution. These systems facilitate high throughputs, rapid order fulfilment, enhanced redundancy through multiple operating robots, and scalability across throughput, storage capacity, energy demand, and costs. The installation of RCS/RS is still on the rise. In block-arranged warehouses, goods are stored in standardised plastic containers stacked onto each other. They are operated by robots from above, functioning on the goods-to-person principle. The system is adaptable to nearly any hall size, ranging from small-scale to large, with its overall performance contingent upon demand.

Nowadays, the layout and design process is typically undertaken by material handling providers who utilise simulation models to gather insights into potential throughput based on specific warehouse characteristics outlined by the customer. The primary input parameters include required storage capacity, anticipated throughput, article distribution, and demand structure. Currently, there is a lack of universally valid information from suppliers, industrial or commercial standards, or market research regarding the performance of such systems. Moreover, scientific investigations addressing the performance of RCS/R systems are scarce. Few, if any, offer readily accessible tools for analytically predicting throughput based on input parameter settings. Especially, considering several picking stations, there are no straightforward approaches that are solvable with standard calculation tools.

Therefore, this paper aims to develop an analytical approach to calculate the performance of an RCS/RS with

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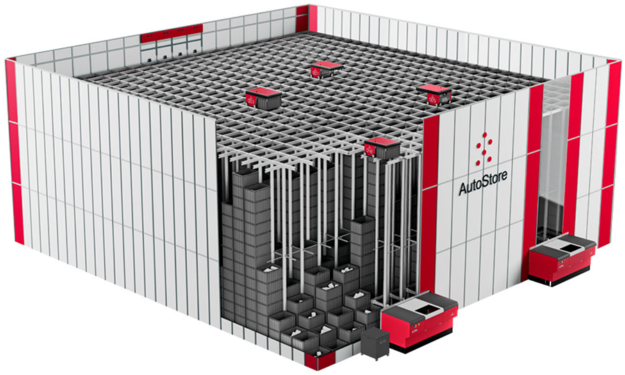


Fig. 1 Example of an RCS/RS. Source: AutoStore [1]

several robots operating on the grid serving a variable number of picking stations. The analytical formulas are validated by comparing their results with those of a numerical discrete event simulation (DES). The simulation, representing the state-of-the-art for performance prediction of automated storage systems, was modelled in *SIMIO*.

The approach presents a straightforward and efficient method for determining throughput. This is a novel contribution, not least due to its ability to be efficiently solved using standard algebra or spreadsheet software. Notable parameters that can be adjusted include stack height, the number of stacks along the horizontal axes, and the filling degree. Furthermore, the optimal number of robots for a given parameter setting can be evaluated.

Based on the abbreviations listed in Table 1, Sect. 2 provides an overview of the scientific literature concerning RCS/RS and queueing approaches for multiple service stations. Section 3 offers a detailed description of the system under investigation and the underlying assumptions. Section 4 outlines the analytical approach as per the specifications in Sect. 3. To validate the analytical approach, Sect. 5 conducts a numerical study, extensive parameter variation, and provides an application example. The parameters used are sourced from a European RCS/RS supplier. Finally, Sect. 6 offers a summary and outlines potential options for future research to conclude the paper.

Table 1 Abbreviations

AS/RS	Automatic storage and retrieval system
AVS/RS	Automatic vehicle storage and retrieval system
CTM	Cycle time model
DCC	Dual command cycle
DES	Discrete event simulation
I/O point	In- and output point
I/O shaft	In- and output shaft
ORCS/RS	Overhead robotic compact storage and retrieval systems
RCS/RS	Robotic compact storage and retrieval system
SBS/RS	Shuttle-based storage and retrieval system
SCC	Single command cycle

2 Literature review

An RCS/RS is defined as a three-dimensional automatic storage and retrieval system using an orthogonal grid as a railway network for the robots responsible for handling the containers. In general, automated warehouses have been discussed scientifically many times over. RCS/RS, as one subtype representing a static storage system under the goods-to-person principle with a block layout, have only been the subject of a few scientific investigations. Nonetheless, there are some papers dealing with RCS/RS.

Beckschäfer et al. [2], Galka and Scherbath [3], Chen et al. [4], Trost et al. [5], and Kartnig et al. [6] all developed a DES to gain insights into the performance of an RCS/RS. Yener and Yazgan [7] proposed a simulation-based optimisation method and investigated the performance of an RCS/RS with a focus on return relocation strategies. The main conclusion drawn is that a return relocation should be conducted whenever there is an opportunity, representing the state-of-the-art practice. Such storage systems, for example, *AutoStore*, perform return relocations in the reverse sequence of the relocation immediately after retrieval.

Ko et al. [8] adopted a different approach to investigate RCS/RS and proposed a roll-out heuristic algorithm to determine the optimal order sequencing within an RCS/RS. Hameed et al. [9] developed a numerical performance calculation approach using an optimal path algorithm for robot routing and compared the impact of a collision avoidance system within the robots. For a specific testing scenario, the total throughput decreased by around 10% when obstacles were considered compared to when they were neglected.

Galka and Scherbath [3] noted that obtaining an analytical approach to determine throughput is complicated due to the large number of parameters and the high complexity of the system.

All the aforementioned papers share the commonality of either developing numerical simulations such as DES or heuristics to investigate throughput or processes. None of them provided analytical approaches to calculate or approximate the performance of an RCS/RS. However, two scientific investigations have presented analytical approaches on the performance of RCS/RS. One is by Trost and Eder [10], and the other is by Zou et al. [11].

Zou et al. [11] proposed a performance approach for RCS/RS using a semi-open-queueing network. Mutual hindrances of the robots and, thus, congestion were not further considered since the number of robots was small concerning the grid. The approach is neither easy nor quickly solvable with standard algebra programs, which raises questions about its practicality, not least because the solution has to be obtained numerically.

Trost and Eder [10] developed a CTM to determine the throughput of an RCS/RS with one operating robot serving one picking station. This approach offers a straightforward and quickly solvable method to determine the performance of a single robot operating in an RCS/RS. The presented formulas are based on CTM for AS/RS and SBS/RS, and they have been significantly expanded and adapted for RCS/RS. The approach was tested and validated through a three-stage process to ensure the model accurately captures the robot's movement along the grid, potential relocations, and the overall throughput for a single robot.

Trost and Eder's approach [10] was further developed with an M|M|1|K single queue with limited capacity to consider the interaction of the robots serving one picking station [12]. Although the assumption of Poisson-distributed arrival and service times was deemed a good compromise, the authors asserted that an M|G|1|K model with generally distributed arrival times would yield higher approximation quality. A broad range of parameter variations was examined, demonstrating that the maximum error rate remains below 10%, even when applying the M|M|1|K model.

In investigating the steady state of a system, queueing theory is often applied in material handling and warehouse logistics to consider various interacting processes [13]. It offers numerous performance evaluation models, such as single queueing models utilising Markov chains, open, semi-open, or closed queueing networks. An open-queueing network allows input and output from outside the system boundary, whereas in a closed-queueing network, neither input nor output is possible. Mixed networks with different classes of users permit, for example, new input from outside into the system for one user class and departure from the system for another user class [14]. Table 2 provides some queueing terms:

Table 2 Abbreviations of queueing systems

CQN	Closed queueing network
OQN	Open queueing network
SQ	Single queue
SQLC	Single queue with limited capacity
SOQN	Semi-open-queueing network
A	Arrival process
B	Service process
C	Number of parallel stations
K	Capacity of the system
M	Markovian
D	Deterministic
Ek	Erlang
G	General
GI	General independent and identically distributed
FCFS	First Come First Served
LIFO	Last In First Out

Considering more than one picking station and several operating robots poses a particular challenge in material handling and warehousing research. Due to the limited number of scientific investigations on RCS/RS in this context, the research has been extended to other storage systems such as AVS/RS and SBS/RS. Papers dealing with SBS/RS mainly investigated the performance of one aisle, or in cases with more than one aisle, there were lifts for every aisle, as seen in the work of Eder [15]. Cross-aisle SBS/RS (aisle-free) with several lifts have hardly been studied. While Heragu et al. [16] developed an OQN with general distributed arrival and service rate to predict the performance of an AVS/RS, Ekren and Akpunar [17] measured the performance of SBS/RS. Their approach has to be solved with the Witt algorithm.

Several other papers discuss storage systems and deploy queueing models, but none consider more than one lift or picking station. A systematic literature survey examined by Amjath et al. [27] confirms this. Only a few approaches deal with queueing models with more than one server in the context of storage systems, warehouses, and distribution centres. Their review was expanded to include material handling systems in the manufacturing environment, mining and harvesting, and container terminals.

Manufacturing systems are often modelled with CQN. Nazzal [19], Raman [20], Tu et al. [23], and Mohammadi et al. [25] all assumed general distributed arrival and service processes for performance evaluation in manufacturing systems. In contrast, Govind et al. [18], Bedell and Smith [21], Smith and Kerbache [22], Smith and Barnes [24], and Zhang et al. [26] applied M|G|m|K models with Poisson-distributed arrivals. While most of those papers investigating manufacturing systems calculate the crucial Work-In-Progress key performance indicator, Smith and Kerbache [22] and Smith and Barnes [24] conducted analyses of the networks.

A Markovian distribution was also assumed by Trost and Eder [12]. The authors mentioned that a generally distributed arrival time would yield more accurate results since the robot's arrival on the grid within an RCS/RS depends on various factors such as the grid size, the stack height, the robot's velocity, acceleration/deceleration rate, and more.

Building upon Trost and Eder [12], the straightforward and rapid method of throughput approximation should be extended to multiple picking stations. Transitioning from Markovian arrival processes to a general distribution dependent on the variation coefficient could enhance the approximation quality. This would lead to a multiple server problem of type G|M|m|K. Such multi-server performance models with finite waiting space are valuable tools for various applications including manufacturing, telecommunications, transportation and logistics, material handling, and facility modelling [28].

As found in the literature (e.g. Smith [29]), an exact solution for these systems is only possible for exceptional cases such as exponential arrival rate, i.e. M|M|m|K, a single server, i.e. M|G|1|K, or infinite queue capacity resulting in the M|G|m model. Kimura [30] proposed a transform-free approximation for the M|G|m|K queue with finite waiting spaces. He considered $m \geq 1$ parallel servers and $K = m + r$ waiting spaces with $r \geq 0$ under the FCFS rule. The approach is exact for the special cases mentioned earlier. Smith [29] derived blocking probability models for $m = 1$ up to $m = 10$ with one equation for each m. Subsequently, Smith [28] developed a two-moment approach to obtain closed-form solutions for performance evaluation.

Since there is no readily solvable closed-form expression for performance evaluation using an M|G|m|K model, this paper builds upon the approach from Trost and Eder [12] and expands it to multiple picking stations, leading to an M|M|m|K model. The results from this paper's model are also compared with the approximation equations from Smith [29], assuming a general distribution for the arrival process.

The open-queueing system with limited capacity to approximate the possible throughput and the optimal number of robots operating on an RCS/RS grid under consideration of several picking stations is a seminal contribution to the scientific research of automated warehouses.

The model provides a straightforward and rapid calculation method to determine the performance of an RCS/RS with adequate accuracy for a variable number of robots operating on the grid, assuming a homogeneous article demand structure along the three axes and Markovian distributions for both arrival and service processes. This paper's novelty lies in the possibility of approximating the expected performance for every RCS/R system and any input parameters, regardless of size or kinematic data.

Table 3 provides an overview of existing literature regarding material handling and manufacturing systems.

3 System description

RCS/RS systems combine numerous advantages, including high storage density and minimal space requirements, enhanced reliability due to high redundancy, exceptional performance potential, and low energy consumption. These favourable attributes are further complemented by the system's simple and modular design, facilitating easy scalability and flexible expandability. Additionally, an RCS/RS operates

Table 3 Literature overview

Author	Year	System	Method	Model	Validation with DES
Heragu et al. [16]	2011	AVS/RS	OQN	GI|G|m	✓
Govind et al. [18]	2011	Manufacturing	CQN	M|G|m	✓
Nazzal [19]	2011	Manufacturing	CQN	G|G|m	
Raman [20]	2011	Manufacturing	CQN	G|G|m	
Bedell and Smith [21]	2012	Manufacturing	CQN	M|G|m|K	✓
Smith and Kerbache [22]	2012	Manufacturing	CQN	M|G|m|K	✓
Tu et al. [23]	2013	Manufacturing	CQN	GI|G|m	✓
Smith and Barnes [24]	2014	Manufacturing	CQN	M|G|m|K	✓
Beckschäfer et al. [2]	2017	RCS/RS	DES		
Zou et al. [11]	2018	RCS/RS	SOQN	M|G|m	✓
Mohammadi et al. [25]	2020	Manufacturing	CQN	GI|G|m	
Zhang et al. [26]	2020	Manufacturing	CQN	M|G|m|K	✓
Galka and Scherbath [3]	2021	RCS/RS	DES		
Ekren and Akpunar [17]	2021	SBS/RS	OQN	G|G|m	
Chen et al. [4]	2022	RCS/RS	DES		
Trost et al. [5]	2023	RCS/RS	DES		
Eder [15]	2023	SBS/RS	SQLC	M|G|1|K	✓
Kartnig et al. [6]	2023	RCS/RS	DES		
Trost and Eder [10]	2024	RCS/RS	CTM		✓
Trost and Eder [12]	2024	RCS/RS	SQLC	M|M|1|K	✓
This paper	2024	RCS/RS	SQLC	M|M|m|K	✓

under the goods-to-person principle, with robots functioning autonomously on the grid [6]. Figure 2 provides a visual representation of an RCS/RS system, illustrating three picking stations and multiple operating robots.

RCS/R systems are designed with simplicity in mind and consist of four fundamental components [5]:

- Grid: The aluminium or steel grid forms the foundation of the storage system, serving as an orthogonal railway for the robots.
- Robots: These are responsible for transporting containers, which hold the goods, to and from the stacks to the picking stations.
- Containers: Standardised plastic containers, typically measuring 600 by 400 mms (LxW) with variable heights ranging from 200 to 425 mms.
- Picking station with I/O shaft: Serving as an input/output point, the picking station is connected to the grid level via the I/O shaft.

When a new article needs to be stored in the warehouse, it is placed into a container at the picking station and then picked up by a robot. The robot lifts the container through the I/O shaft and transports it to its designated stack. The I/O shafts are typically located at one of the grid's wider edges. If a new order arrives, the required container is requested accordingly.

The robot waiting next to the stack where the order is stored is assigned to retrieve the required container. If the container is not readily accessible, relocations may be necessary. Once the robot gains access to the ordered container, it is lowered through the I/O shaft to the picking station. Some systems, particularly those with an inhomogeneous article demand structure, also return relocate previously relocated containers to their original stack. However, this paper, inspired by the findings of Trost and Eder [10], focuses on storage systems with a homogeneous access structure, omitting return relocations as it could potentially decrease throughput further.

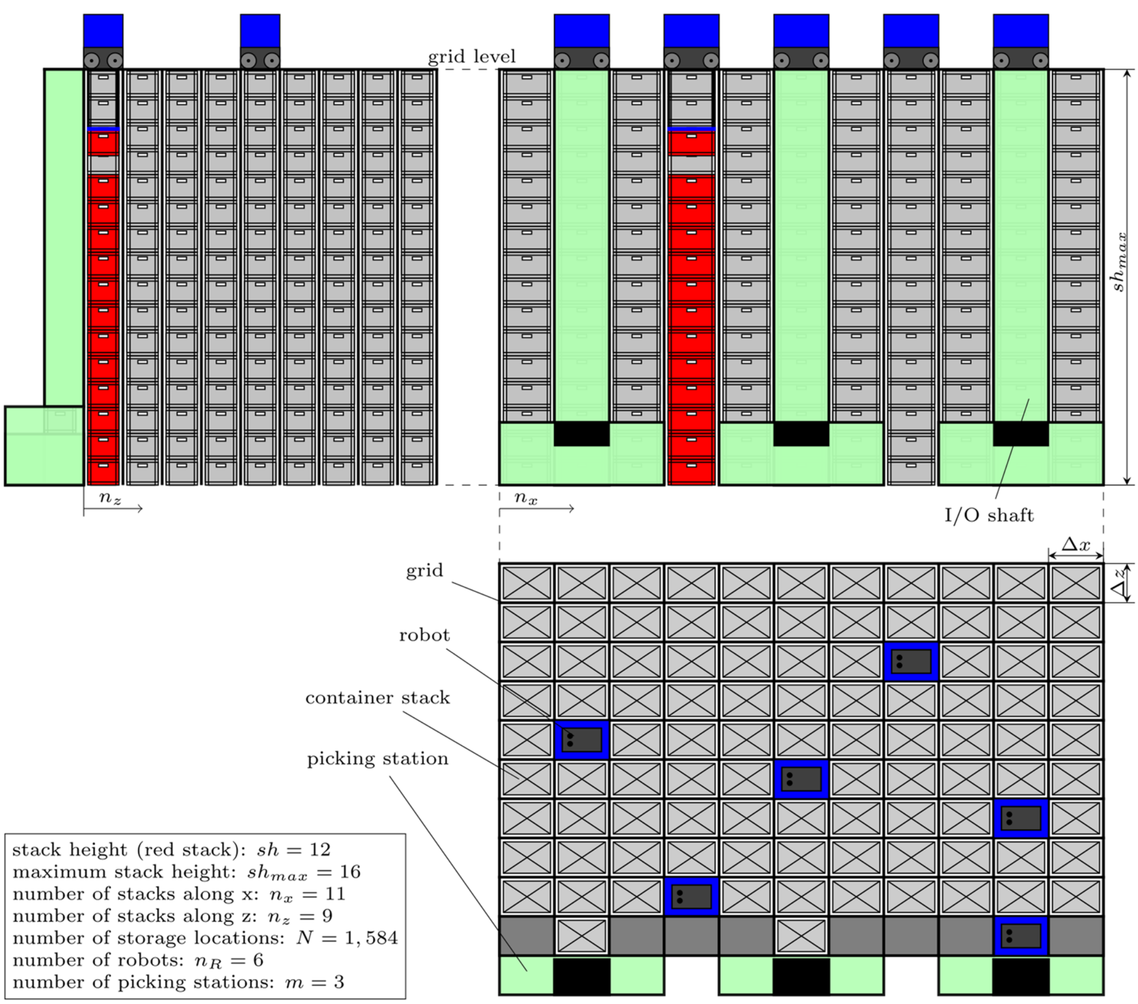


Fig. 2 Robotic compact storage and retrieval system with several robots on the grid

This study will primarily focus on determining the optimal number of robots and picking stations. The key assumptions of this research are outlined below:

- The robots work in a DCC under the FCFS rule.
- The systems dwell point is in front of the picking stations.
- Every picking station can be used for storage and retrieval.
- Every picking station consists of one I/O shaft.
- The picking stations are located along one of the grid's wide edges.
- There are always totes waiting at the dwell point in the pre-warehouse zone.
- The robots pick up a new container to be stored after dropping off a required container.
- The robot's velocity is constant. If not, a realistic velocity rate has to be calculated.
- The robots routing along the grid are without collisions.
- The containers are ordered evenly distributed based on a homogeneous access distribution.
- The order to relocate containers is evenly distributed among all stored containers.
- The container to be relocated is relocated to the nearest available storage location.
- The filling degree is limited to a specific value to ensure that relocations can be done.

4 Analytical approach

This section outlines the analytical approach aimed at calculating the performance of an RCS/R system with multiple robots serving several picking stations at one edge of the grid. To determine the possible throughput of an RCS/RS, the cycle time of one robot must be calculated first. For this purpose, some of the formulas introduced by Trost and Eder [10] were slightly adapted and expanded.

In the analytical approach presented, collisions, congestion, or deadlock situations of the robots on the grid are excluded, as the routing logic of real systems and the simulation process ensure this. Furthermore, the primary objective of the analytical approach is to serve as a performance approximation tool to provide insights into the maximum throughput. This is achieved by constructing a multi-queue model with limited capacity. The process is illustrated in Fig. 3, where K represents the maximum number of robots waiting in front of the I/O shafts plus the one positioned on each of the I/O shafts ($K = n_R + m$).

The following performance calculation for RCS/RS can be split up into the following steps below:

- Cycle time of one robot (arrival rate):
  - Ride time from the I/O shaft to the stack (storage)
  - Ride time from the stack to another stack (DCC)

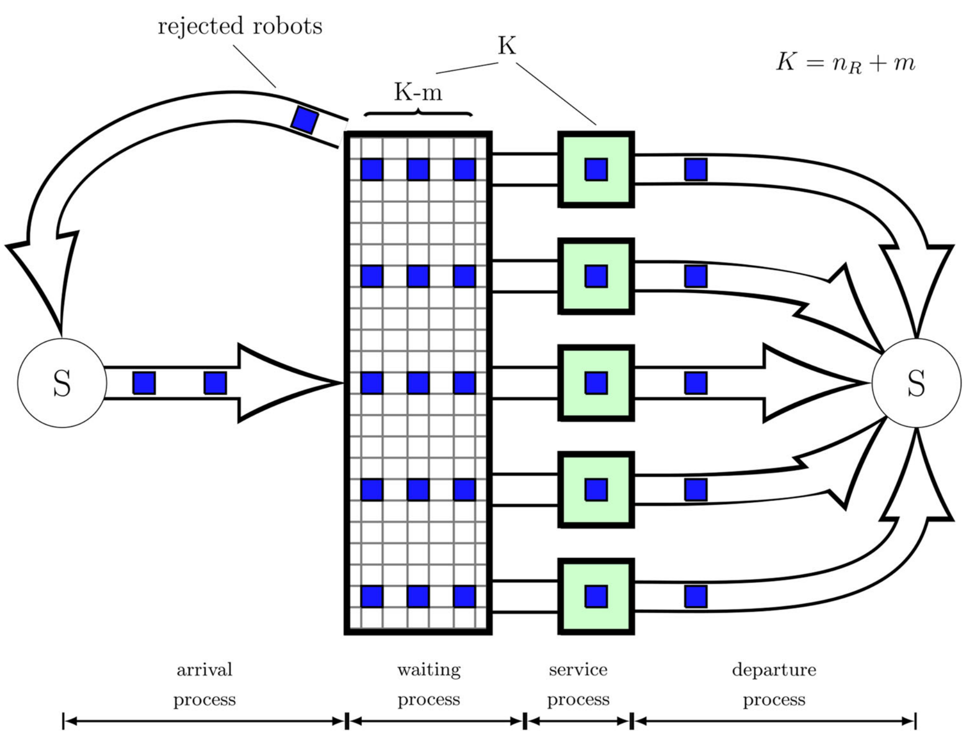


Fig. 3 Multi queue model with limited capacity

  - Ride time from the stack to the I/O shaft (retrieval)
  - Transfer time of the container up from/down onto the stack
  - Probability of relocations
  - Ride time in the relocation cycle
- Time on the I/O shaft (service rate)
- Throughput determined with an M|M|m|K queuing model

The notation used in this approach can be found in Table 4.

Table 4 Notation

Δx	Distance between two stacks along the x-axis
Δz	Distance between two stacks along the z-axis
ϑ	Throughput of the RCS/RS
λ	Arrival rate
μ	Service rate
ρ	Utilisation rate of the picking station
A	Space demand
a_R	Robot's acceleration rate for horizontal ride
a_T	Robot's acceleration rate for container transfer
f	Filling degree
h_C	Height of a storage container
K	Capacity of the queueing system
k_0	Position of the I/O shaft along the x-axis
k_m	Factor considering more than one I/O shaft
m	Number of picking stations with I/O shaft
N	Number of storage locations
n_R	Number of robots operating on the grid
n_{R_opt}	Optimal number of robots
n_x	Number of grid elements along the x-axis
n_z	Number of grid elements along the z-axis
p_0	Probability for an empty system
p_k	Blocking probability
sh	Storage height of a container stack
t_A	Arrival time
t_{CX}	Container exchange time at the picking station
t_L	Time to lock and unlock a container
t_{R_rel}	Robot's ride time in a relocation cycle
$t_{R_{dual}}$	Additional ride time of a robot in a DCC
$t_{R_{stor}}$	Robot's ride time in an SCC for storaging
$t_{R_{retr}}$	Robot's ride time in an SCC for retrieval
t_S	Service time at the I/O shaft
t_T	Container transfer time
t_{WX}	Wheel exchange time of a robot
V	Volume demand
v_R	Robot's velocity rate for horizontal ride
v_T	Robot's velocity rate for container transfer
w_{rel}	Probability of a relocation cycle

4.1 Cycle time calculation

The cycle time for a single robot operating within a DCC without return relocations can be determined using Eq. 1, as described by Trost and Eder [10]:

$$t_A = t_{R_{stor}} + t_{R_{dual}} + t_{R_{retr}} + 2 \cdot t_T + w_{rel} \cdot (t_{R\_rel} + 2 \cdot t_T) + t_S \tag{1}$$

Following the methodology of Trost and Eder [10], it is necessary to differentiate between ride time and transfer time functions, considering whether the maximum speed is reached and the trapezoidal drive mode is employed, or if short distances prompt the use of the triangular drive. Equation 2 illustrates the two cases considered by the $\mathbf{t}(j)$ function for the robot's horizontal ride:

$$\mathbf{t}(j) = \begin{cases} 2 \cdot \sqrt{\frac{j}{a_R}} & \text{for } j < \frac{v_R^2}{a_R} \\ \frac{j}{v_R} + \frac{v_R}{a_R} & \text{for } j \geq \frac{v_R^2}{a_R} \end{cases} \tag{2}$$

The variable j represents the distance travelled by the robot, and the function $\mathbf{t}(j)$ returns the corresponding time.

Furthermore, accounting for an acceleration/deceleration rate during the lifting and lowering of the container necessitates another distinction. Equation 3 describes the $\mathbf{g}(y)$ function for the transfer time:

$$\mathbf{g}(y) = \begin{cases} 2 \cdot \sqrt{\frac{y}{a_T}} & \text{for } y < \frac{v_T^2}{a_T} \\ \frac{y}{v_T} + \frac{v_T}{a_T} & \text{for } y \geq \frac{v_T^2}{a_T} \end{cases} \tag{3}$$

Depending on the height y, the function $\mathbf{t}(j)$ evaluates the time required for the lifting and lowering.

4.1.1 Mean ride time

Based on Trost and Eder [10], the ride time of one robot from the picking station to a grid element, or vice versa, can be calculated using Eq. 4. The time function (Eq. 2) determines the corresponding time required for the ride based on the distance. If the direction must change once, meaning the robot also travels along the x-axis, an additional time component must be considered. Therefore, for $|k - k_0| > 0$, the last term evaluates to $\mathbf{sign}\,(|k - k_0|) = 1$.

$$t_{R_{stor}} = \frac{1}{n_x} \cdot \frac{1}{n_z} \cdot \sum_{k=1}^{n_x} \sum_{l=1}^{n_z} \mathbf{t}((|k - k_0|) \cdot \Delta x) + \mathbf{t}(l \cdot \Delta z) + t_{WX} \cdot \mathbf{sign}\,(|k - k_0|) \tag{4}$$

Considering the variable number of I/O shafts with picking stations along the grid, it's essential to account for the influence of their locations. This is addressed by Eq. 5.

$$t_{R_{retr}} = \frac{m}{n_x} \cdot \frac{1}{n_z} \cdot \sum_{k=1}^{\frac{n_x}{m}} \sum_{l=1}^{n_z} \mathbf{t}((|k - k_m|) \cdot \Delta x) + \mathbf{t}(l \cdot \Delta z) + t_{WX} \cdot \mathbf{sign}\,(|k - k_m|) \tag{5}$$

The number of stacks along the x-axis is divided into m sub-zones, denoted by the factor k_m. Apart from this, the equation operates similarly to the one presented in Eq. 4.

In addition to Eqs. 4 and 5, for systems operating in DCC, it's necessary to consider the robot's ride time from the storage stack to the retrieval stack. Equation 6 illustrates the impact of this additional ride in a DCC.

$$t_{R_{dual}} = \frac{1}{n_x^2} \cdot \frac{1}{n_z^2} \cdot \sum_{k=1}^{n_x} \sum_{l=1}^{n_x} \sum_{m=1}^{n_z} \sum_{n=1}^{n_z} \mathbf{t}((|m - n|) \cdot \Delta z) + \mathbf{t}((|k - l|) \cdot \Delta x) + t_{WX} \cdot \mathbf{sign}\,((|k - l|) \cdot (|m - n|)) \tag{6}$$

Equation 6 encompasses three distinct scenarios. In cases where there is no change of direction, indicated by $|k-l| = 0$ or $|m - n| = 0$, the last term evaluates to zero. This occurs when the robot travels solely along the x- or z-axis. However, the last term becomes non-zero when a change of direction is required, transitioning either from the x- to z-axis or vice versa.

4.1.2 Mean time for container transfer

Based on Trost and Eder [10], the mean time for the transfer of the containers can be calculated. Utilising a binomial coefficient allowed for the derivation of a single equation applicable to various stack heights (n). This equation underwent validation for theoretical storage heights of $sh = 100$. Moreover, constant velocity was assumed. This approach also considers an acceleration/deceleration rate of the lifting and lowering device, which is accounted for by the $\mathbf{g}(y)$ function within Eq. 7.

$$t_T = t_L + \sum_{n=1}^{sh} \sum_{i=0}^{n-1} \frac{1}{sh + 4 \cdot i} \cdot \binom{sh-1}{i} \cdot f^{sh-1-i} \cdot (1 - f)^i \cdot 2 \cdot \mathbf{g}(h_C \cdot n) \tag{7}$$

In this context, t_L represents the time needed to open and close the locking claws, while f denotes the filling degree of the storage system. Additionally, h_C represents the height of the container and is multiplied by the indices n from the first sum to compute the time necessary for lifting and lowering.

4.1.3 Relocation cycle

If the stack height is greater than 1 ($sh > 1$), it's possible that relocations may be necessary to retrieve the required container. The probability of a relocation cycle, denoted as w_{rel}, is calculated identically to Trost and Eder [10]. The total relocation time comprises the sum of the container transfer time (t_T) and the robot's ride time for the relocations.

Their assumption that the stacks, where the relocation containers are transported to, are along one of the four outgoing paths enables high performance since no directional changes are necessary. The typical high stack heights in RCS/RS also ensure high filling degrees for accessible relocation stacks.

Instead of an equally distributed access structure over the stack height resulting in a stochastic relocation probability, any other relocation probability can be applied using the factor w_{rel}. For example, an 80/20 access distribution over a stack height of $sh = 10$ results in a relocation probability of $w_{rel} = 1.5$ relocations for one retrieval. Conversely, assuming a 50/50 access structure as mentioned above, the probability of a relocation calculates to approximately $w_{rel} \sim 5$.

4.2 Service time calculation

Equation 8 represents the service time of a robot on the I/O shaft.

$$t_S = t_{CX} + 2 \cdot t_L + 2 \cdot \mathbf{g}(h_C \cdot n) \cdot sh \tag{8}$$

Therein, t_{CX} denotes the container exchange time within the picking station, while t_L describes the unlocking of the retrieval container and the locking of the storage container. The third term represents the time for lifting and lowering the old and the new containers through the I/O shaft. The $\mathbf{g}(y)$ function again considers the acceleration/deceleration of the lifting and lowering device according to Eq. 3.

4.3 Multi queue model M|M|m|K

The open-queueing model is based on the interaction of two processes within the storage system:

- The robot's arrival rate at the I/O shafts.
- The service time on the I/O shafts.

Based on the arrival and service processes, the Kendall notation A|B|C|K describes the characteristics of the queueing system. According to Baum [14], A stands for the arrival process, B for the service process, C for the number of parallel service stations, and K for the queue length. This approach assumes an M|M|m|K model with Markov characteristics for the arrival and service processes (Poisson distributed).

A more precise solution would be, as mentioned in Sect. 2, implementing generally distributed arrival times of the robots by applying an M|G|m|K model. Section 5.1 presents a comparison of the simulation results with those from the analytical approach.

The assumption of exponentially distributed arrival and service rates is necessary, as stated by Trost and Eder [10]. This is due to the impossibility of analytically evaluating the coefficient of variation, given the extensive range of variation parameters. Furthermore, as discussed in Sect. 2, exact analytical solutions for multi-server M|G|m|K problems with m parallel service stations are only feasible for special cases. The M|M|m|K model represents one such special case. While approaches exist for a finite number of service stations, a precise formula as a closed expression is not available. However, the M|M|m|K model presented in this section offers an easy and quick analytical tool with sufficient accuracy to approximate the throughput of an RCS/RS for any given parameter setting. The developed equations can be solved using standard algebra and table calculation tools, which is also a novelty.

The arrival rate of the robots at the I/O shafts is determined by the reciprocal value of the expectation of one robot's cycle time multiplied by the number of robots operating on the grid, as depicted by Eq. 9:

$$\lambda = n_R \cdot \frac{1}{t_A} \tag{9}$$

Similarly, the service rate, which denotes the potential number of pickings per time unit, is calculated by multiplying the service rate per picking station by the number of operating picking stations with I/O shafts (Eq. 10):

$$\mu = m \cdot \frac{1}{t_S} \tag{10}$$

The utilisation rate of the I/O shaft is defined as the ratio of the arrival rate to the service rate and given by Eq. 11:

$$\rho = \frac{\lambda}{\mu} = n_R \cdot \frac{t_S}{t_A} \cdot \frac{1}{m} \tag{11}$$

The probability of emptiness p_0 for the M|M|m|K model is calculated by Eq. 12 as follows: [14]

$$p_0 = \left[\sum_{i=0}^{m-1} \frac{\rho^i}{i!} + \frac{m^m}{m!} \cdot \sum_{j=m+1}^{n_R+m} \frac{\rho^j}{m^j} \right] \tag{12}$$

The throughput, denoted by ϑ, can be calculated using following expression (Eq. 13):

$$\vartheta = \mu \cdot (1 - p_0) \tag{13}$$

The equation above illustrates that the throughput is determined by multiplying the service rate μ multiplied with the term $(1 - p_0)$.

5 Numerical study

Section 5 aims to conduct a numerical study for both the validation of the analytical approach and the demonstration of its scope. The section is divided into three parts: Firstly, the analytical approach is validated using a numerical discrete event simulation (Sect. 5.1). Secondly, a parameter variation is performed to demonstrate the sensitivity of the parameters and provide insights into the expected performance of such storage systems (Sect. 5.2). Subsequently, an application example is presented in Sect. 5.3 to illustrate the practical application of this approach. The parameters utilised for the numerical study are detailed in Table 5:

Table 5 Parameters for the RCS/RS

Parameter	Value
Number of stacks along the x-axis	$n_x \in \{10, 15, ..., 50\}$
Number of stacks along the z-axis	$n_z \in \{10, 15, ..., 50\}$
Storage height of a container stack	$sh \in \{1...25\}$
Filling degree	$f \in \{70\%, ..., 98\%\}$
Distance between two grid elements along the x-axis	$\Delta x = 0.7m$
Distance between two grid elements along the z-axis	$\Delta z = 0.5m$
Container height	$h_C = 330mm$
Robot horizontal velocity rate	$v_R = 3.1\frac{m}{s}$
Robot lifting and lowering velocity rate	$v_T = 1.6\frac{m}{s}$
Robot's horizontal acceleration/deceleration rate	$a_R = 0.8\frac{m}{s^2}$
Acceleration/deceleration rate of the lifting/lowering device	$a_T = 2.5\frac{m}{s^2}$
Robot time to lock/unlock the container	$t_L = 1s$
Robot wheel change time	$t_{WX} = 1s$

The results will be compared with 30 independent simulation model scenarios to validate the analytical approach. Therefore, a simulation model in the DES simulation software SIMIO (version 15) was modelled. The rebuilt RCS/R system for the simulation includes the four essential components. The system's necessary logic was implemented in the background. The simulation time varies depending on the complexity of the scenario, ranging from a few minutes to several hours.

The containers were evenly distributed over all stacks and storage heights. All the parameters from Table 5, such as the number of stacks along both horizontal axes, the stack height, the filling degree, etc., can be varied in the simulation. If a robot queue arises in front of one of the I/O shafts, the robots are assigned to wait on storage stacks that are not assigned next to the I/O shaft. Whenever the robot is assigned to a new order and leaves the I/O shaft, the next robot can drive to the I/O shaft.

5.1 Validation

Following the discussion in Sect. 4.3, the accuracy of the analytical approach shall be determined. This entails validating the M|M|m|K model. The primary focus of the validation is the number of picking stations with I/O shafts m and the number of robots n_R. The analytical approach, which includes the cycle time and the queuing model, has been validated in other studies, such as [10] and [12] or [31], across a wide range of parameters, including grid size, stack height, and filling degree. The main requirements for the approximation of the performance of a whole system are the arrival rate λ as given by Eq. 9 and the service rate μ (Eq. 10). Hence, variations in the aforementioned parameters will affect only those two previously validated interim results. This investigation extends the analysis from a single picking station with an I/O shaft to multiple picking stations, enabling an approximation of the performance of an entire RCS/R system. The performance ϑ represents the throughput of the entire RCS/R system. Consequently, an increasing number of robots results in higher throughput.

Figure 4 provides a comparative analysis between simulation outcomes and those derived from the analytical approach. The graph illustrates the system's throughput across different configurations of I/O shaft placements along a single edge of the 50 by 50 grid. For this evaluation, a practical stack height of $sh = 16$ and a 90% total filling degree was selected.

As depicted in Fig. 4, the analytical approach consistently underestimates the simulation results, providing a safety margin. The maximum estimation error is less than 10%, occurring at approximately 20 robots and $m = 2$ I/O shafts. Notably, as the number of shafts increases, the estimation error diminishes. Table 8 presents the corresponding data from Fig. 4 alongside the estimation errors.

Although the M|M|m|K model, as discussed in Sect. 4, may not be the optimal choice, it offers a satisfactory level of accuracy for estimating RCS/RS throughput. Despite its slight estimation error, the model's advantage lies in its provision of a closed expression, enabling rapid and straightforward approximation using standard calculation software.

Figure 5 compares the analytical approach using an M|D|m|K model with Dirac-distributed service time, an M|G|m|K model, and the M|M|m|K model against results obtained from numerical simulation. For this comparison,

Fig. 4 Throughput depending on the number of robots for a different number of I/O shafts

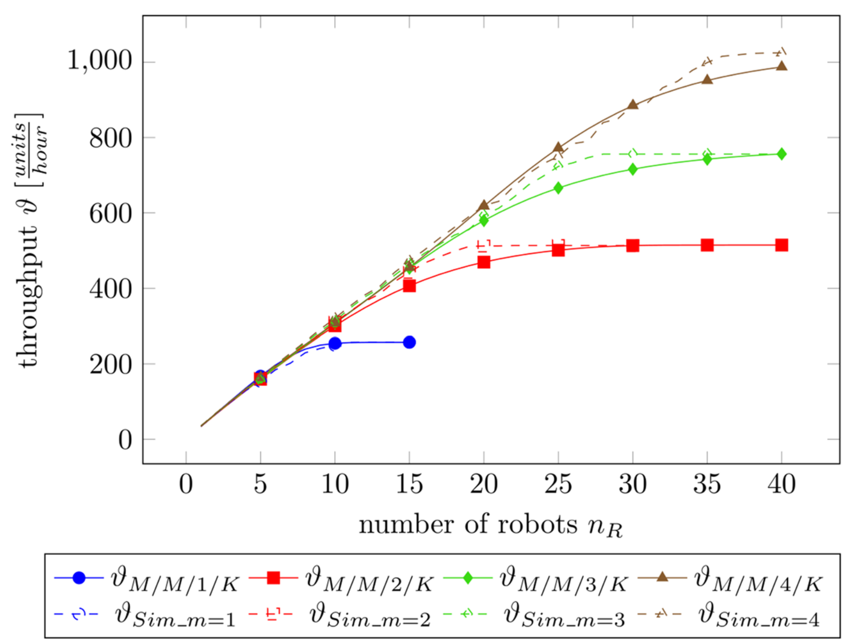
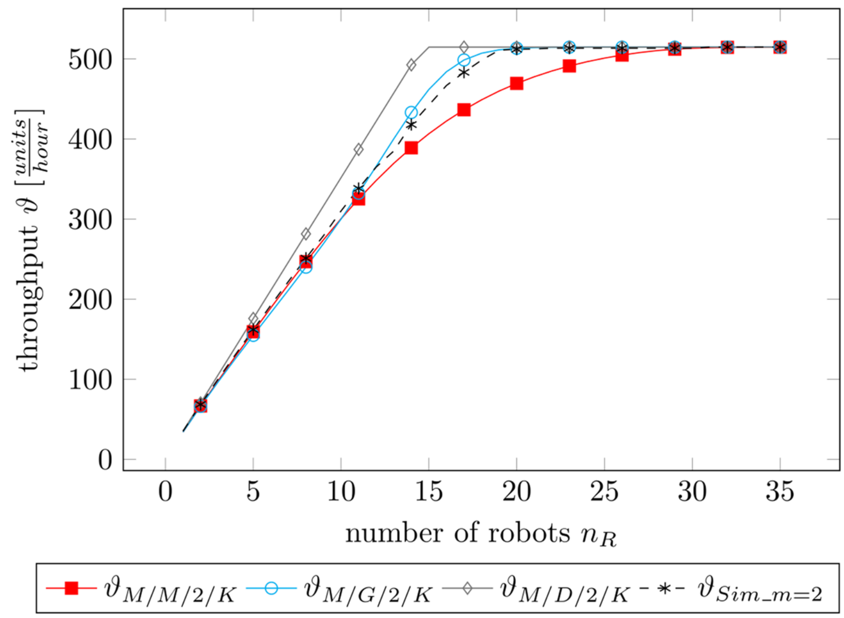


Fig. 5 Throughput depending on the number of robots testing different queueing models



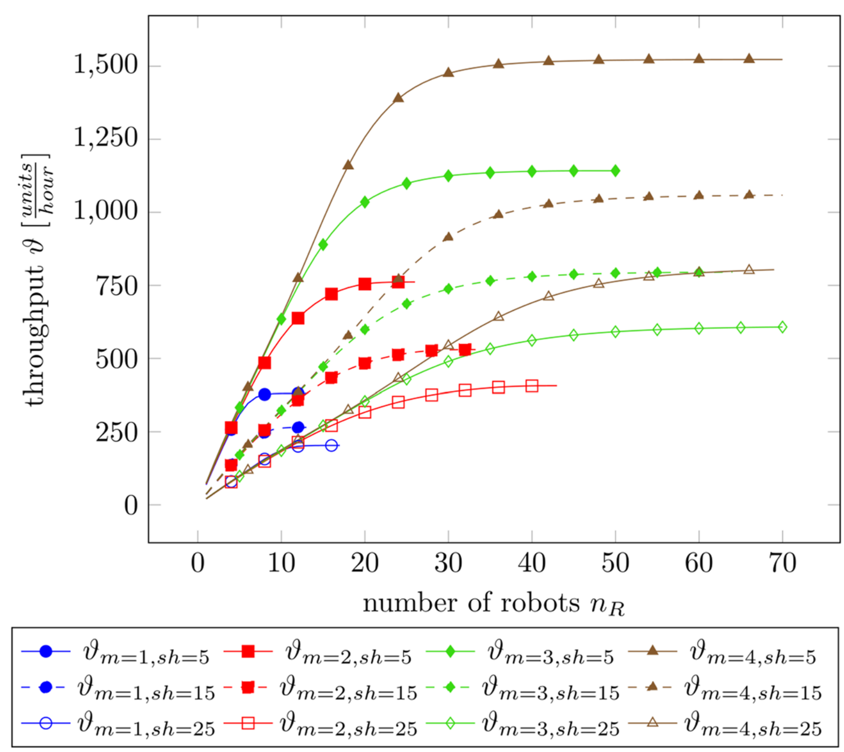
the number of stations was set to $m = 2$, while maintaining all other parameters consistent with those used in Fig. 4. Table 9 presents the data corresponding to Fig. 5.

As depicted in Fig. 5, the M|G|m|K model demonstrates the highest level of accuracy, as anticipated. Conversely, the M|D|m|K model tends to overestimate performance, while the M|M|m|K model consistently underestimates it. Notably, the estimation error is more than halved when using the M|M|m|K model. Moreover, larger values of m result in significantly reduced estimation errors.

5.2 Parameter variation

After validating the analytical approach, the next step involves varying parameters. All subsequent results are derived from the analytical approach. As outlined in the literature review (Sect. 2) and the system description (Sect. 3), the

Fig. 6 Throughput depending on the number of robots for different stack heights and a different number of I/O shafts (50x50, $f = 90\%$)



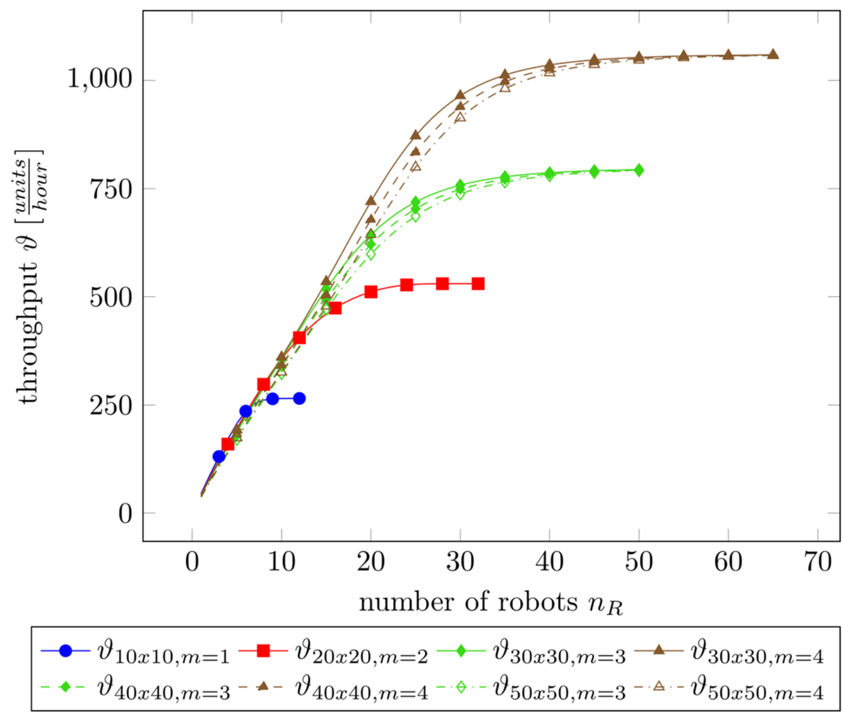


Fig. 7 Throughput depending on the number of robots for different grid sizes and a different number of I/O shafts for ($f = 90\%, sh = 15$)

stack height is a key parameter of an RCS/RS. Figure 6 illustrates the throughput of an RCS/RS in relation to the number of robots, considering different numbers of I/O shafts and three distinct stack heights. The grid size was 50 by 50, accommodating a maximum of 2,500 stacks, with a filling degree of 90%.

As depicted in Fig. 6, the smallest stack height exhibits the steepest increase in throughput, allowing for the highest

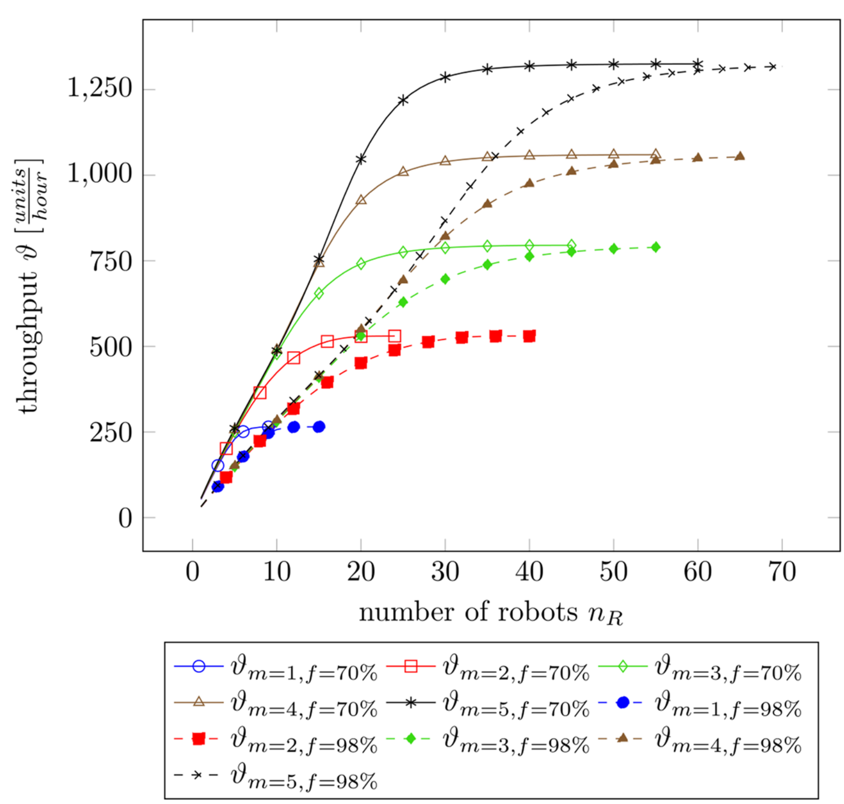


Fig. 8 Throughput depending on the number of robots for two different filling degrees and a different number of I/O shafts (50x50, $sh = 15$)

Table 6 Requirements for the application example

Parameter	Value
Storage capacity	$N = 30,000$
Stack height	$sh \in \{5...25\}$
Stacks along x	$n_x \in \{10, 15, ..., 50\}$
Stacks along z	$n_z \in \{10, 15, ..., 50\}$
Filling degree	$f = 90\%$
Picking stations	$m \in \{1...5\}$

achievable throughput. The optimal number of robots can be approximated by a straight line. The curves terminate when the absolute difference in throughput between the current and previous values becomes smaller than 10%.

Another critical parameter of an RCS/RS is the grid size. Previous investigations have demonstrated that the grid size's impact on throughput is insignificant when more than one robot is operational. Figure 7 illustrates the throughput variation with the number of robots for different grid sizes and various numbers of I/O shafts, assuming a stack height of $sh = 15$.

As anticipated, the grid size has a limited impact on the performance of an RCS/RS. Smaller grid sizes result in steeper curves and higher picks per hour, but all curves converge toward the same limit. However, when there's more than one I/O shaft, the grid size's importance increases because it restricts the number of robots and I/O shafts that can be accommodated. For instance, a 10 by 10 grid cannot efficiently operate with four picking stations with I/O shafts and 80 robots to reach maximum performance. Simulations have revealed that the ratio of the number of stacks to the number of robots should exceed five. For a 10 by 10 grid (i.e., 100 stacks), a maximum of 20 robots should be operating, resulting in one or two picking stations with I/O shafts.

Another crucial parameter of any storage system is the filling degree. Figure 8 illustrates the throughput of an RCS/RS based on the number of robots for two different filling degrees, with a stack height set to $sh = 15$ (50 by 50 grid).

Figure 8 illustrates that as the filling degree increases, the number of required robots also increases, and the throughput curves become flatter. This can be attributed to the increased number of necessary relocations needed to retrieve a container.

5.3 Application example

This example aims to demonstrate an application of the analytical approach while showcasing its ease, speed, and informational value. To achieve this, a scenario is defined based on specific practical requirements. The objective is to determine the optimal storage system considering various parameters such as performance, space, or volume demand. Table 6 outlines the requirements for the storage system:

The storage capacity aims to accommodate approximately 30,000 containers with a maximum deviation of $\pm$ 1%. The stack height can vary realistically from 5 to 25, and the filling degree is set at 90%. The number of stacks along both directions can be adjusted between 10 and 50 in increments of 5.

Table 7 and Fig. 9 illustrate the outcomes of the application example.

Figure 10 illustrates seven scenarios as examples for $m = 2$ picking stations, aiming to clarify the applicability and feasibility and highlight the advantages of the analytical approach.

The smallest system, with a storage capacity of 30,000 containers, is a 40 by 30 grid with a stack height of $sh = 25$. This system occupies 476 m^2 of space but offers the lowest performance and requires the highest number of robots. Conversely, a 50 by 50 grid with a stack height of $sh = 12$ demands 945 m^2 of space. Despite the larger footprint, its performance is nearly one-third higher with five operating picking stations, requiring only 53 robots compared to 70 for the smallest scenario. An intermediate option between space efficiency and performance could be scenario four: a 45 by 35 grid with a stack height of $sh = 19$ containers. Operating with three picking stations would necessitate 47 robots

Table 7 Results of the application example

	Layout							ϑ_{max} [1/h]					n_R [1]				
no	n_x	n_z	sh	N	A [m^2]	V [m^3]	$m =$	1	2	3	4	5	1	2	3	4	5
1	40	30	25	30,000	476	4879		203	407	610	813	1017	14	37	55	64	70
2	50	25	24	30,000	508	5013		208	416	625	833	1041	12	34	52	61	68
3	50	30	20	30,000	595	5117		230	460	691	921	1151	11	31	48	56	62
4	45	35	19	29,925	614	5080		236	473	709	946	1182	10	30	47	55	61
5	50	35	17	29,750	683	5194		250	500	750	1000	1250	10	29	45	53	58
6	50	40	15	30,000	770	5352		265	530	795	1061	1326	10	27	44	51	56
7	50	50	12	30,000	945	5632		292	583	875	1167	1459	9	26	42	49	53

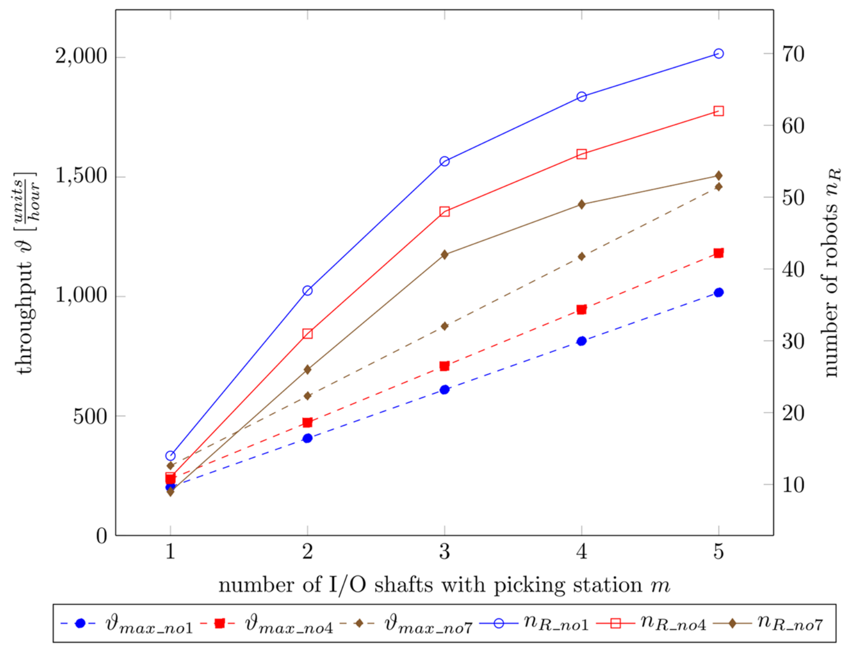


Fig. 9 Throughput and required number of robots for three selected application example scenarios

to retrieve over 700 containers per hour, resulting in a space requirement of 614 m^2, approximately one-third smaller than that of the largest system.

6 Conclusion

There are hardly any statements on the performance of RCS/R systems, neither from the sales side nor from science. Most scholarly investigations have focused on specific system configurations with default settings.

This paper aimed to address this gap by presenting a reasonably accurate analytical approach for assessing RCS/RS performance. Building upon the cycle time model (CTM) proposed by [10], this study has expanded it with queueing theory principles, specifically a multi-queue model with limited capacity (M|M|m|K). This allowed for the prediction of robot behaviour at the I/O shafts and subsequent performance calculation. A notable innovation was the incorporation of multiple robots serving several picking stations with I/O shafts. By leveraging the arrival rate of robots and the service rate of stations, a closed-form expression for

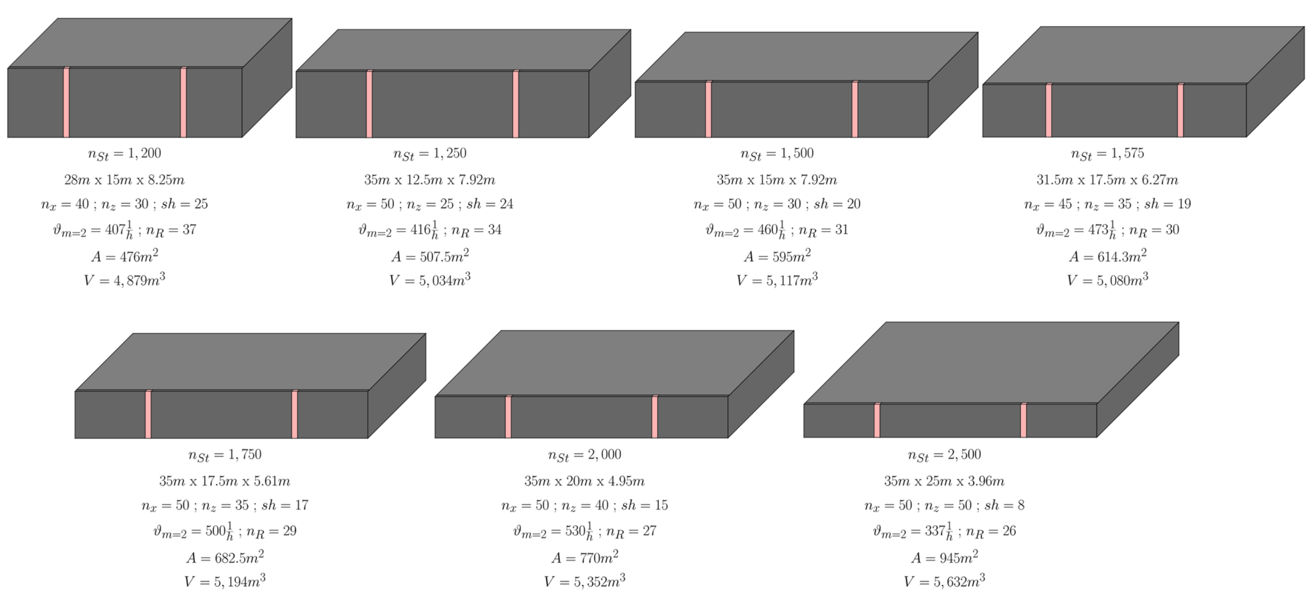


Fig. 10 Different scenarios of the application example

performance determination was derived and validated against Discrete Event Simulation (DES) results.

The analytical approximation presented here serves as a valuable, expedient tool for both determining throughput for a given system size and evaluating optimal system configurations under different parameter settings. Traditionally, such assessments have relied on time- and computationally-intensive simulations.

Looking ahead, future research could explore the impact of placing picking stations with I/O shafts along multiple grid edges, potentially yielding significant insights. Additionally, investigating robot routing on the grid and potential congestion issues could prove worthwhile. Furthermore, implementing a class-based article distribution, such as the ABC classification, has the potential to enhance system performance by minimising relocations.

Appendix

Table 8 Validation of the analytical approach

		Throughput [$^1/_h$] (m=1)			Throughput [$^1/_h$] (m=2)			Throughput [$^1/_h$] (m=3)			Throughput [$^1/_h$] (m=4)		
		ϑ_{Sim}	ϑ_{MM1K}	Error	ϑ_{Sim}	ϑ_{MM2K}	Error	ϑ_{Sim}	ϑ_{MM3K}	Error	ϑ_{Sim}	ϑ_{MM4K}	Error
n_R	1	34.9	34.1	2.27%	35.2	34.1	3.05%	35.4	34.7	2.03%	35.4	35.0	1.25%
	2	67.7	68.3	0.91%	68.6	66.7	2.83%	68.9	67.9	1.50%	69.2	68.7	0.68%
	3	98.9	102.3	3.43%	100.4	98.2	2.22%	101.5	99.8	1.72%	102.0	101.4	0.61%
	4	129.1	135.6	5.07%	131.3	129.0	1.75%	133.4	130.6	2.11%	134.0	133.0	0.76%
	5	154.1	167.6	8.79%	162.0	159.3	1.65%	155.2	160.6	3.50%	165.8	163.6	1.31%
	6	184.4	197.0	6.81%	192.5	189.2	1.74%	195.3	190.2	2.63%	197.4	193.5	2.00%
	7	201.9	221.5	9.73%	222.1	218.4	1.68%	225.8	219.4	2.84%	228.4	222.6	2.53%
	8	229.6	239.3	4.24%	251.2	246.8	1.74%	256.3	248.5	3.03%	259.3	251.3	3.08%
	9	241.0	249.8	3.67%	279.3	274.3	1.80%	285.6	277.7	2.75%	291.0	279.7	3.88%
	10	249.6	254.7	2.06%	310.2	300.5	3.14%	313.1	307.1	1.93%	320.5	308.0	3.89%
	11	256.9	256.6	0.13%	338.3	325.2	3.88%	347.1	336.5	3.05%	352.5	336.5	4.54%
	12	257.4	257.1	0.10%	365.0	348.2	4.59%	377.0	366.0	2.91%	382.2	365.3	4.42%
	13	257.4	257.3	0.05%	385.0	369.6	4.01%	399.7	395.5	1.06%	404.9	394.6	2.53%
	14	257.5	257.3	0.07%	418.0	389.0	6.94%	434.5	424.6	2.28%	442.0	424.6	3.93%
	15	257.5	257.3	0.07%	442.4	406.6	8.09%	464.6	453.2	2.45%	472.9	455.4	3.71%
	16				466.8	422.4	9.51%	492.5	481.0	2.33%	501.4	486.9	2.90%
	17				483.3	436.5	9.69%	516.0	507.8	1.59%	532.3	519.0	2.49%
	18				498.5	448.9	9.95%	540.7	533.3	1.37%	561.0	551.8	1.64%
	19				510.9	459.9	9.99%	555.9	557.3	0.25%	590.3	584.9	0.92%
	20				512.1	469.4	8.34%	593.6	579.7	2.34%	619.7	618.0	0.28%
	21				512.4	477.7	6.76%	612.5	600.4	1.97%	630.8	650.8	3.17%
	22				513.4	484.9	5.54%	641.4	619.4	3.43%	670.1	683.0	1.92%
	23				513.4	491.2	4.33%	674.5	636.7	5.61%	704.6	714.1	1.35%
	24				513.4	496.5	3.29%	697.9	652.2	6.54%	730.9	744.0	1.79%
	25				513.4	501.0	2.41%	723.3	666.2	7.89%	749.9	772.2	2.97%
	26				513.4	504.8	1.67%	733.1	678.7	7.42%	784.0	798.6	1.87%
	27				513.4	507.9	1.06%	746.7	689.8	7.62%	790.1	823.1	4.18%
	28				513.4	510.4	0.59%	756.1	699.7	7.46%	839.6	845.6	0.72%
	29				513.4	512.1	0.24%	756.1	708.4	6.31%	850.2	866.1	1.87%
	30				513.4	513.3	0.02%	756.1	716.0	5.30%	886.0	884.6	0.15%
	31				513.4	514.0	0.12%	756.1	722.8	4.41%	892.9	901.3	0.94%

Table 8 continued

	Throughput [1/h] (m=1)			Throughput [1/h] (m=2)			Throughput [1/h] (m=3)			Throughput [1/h] (m=4)		
	ϑ_{Sim}	ϑ_{MM1K}	Error	ϑ_{Sim}	ϑ_{MM2K}	Error	ϑ_{Sim}	ϑ_{MM3K}	Error	ϑ_{Sim}	ϑ_{MM4K}	Error
32				513.4	514.4	0.19%	756.1	728.7	3.62%	932.1	916.1	1.71%
33				513.4	514.5	0.22%	756.1	734.0	2.93%	949.2	929.3	2.09%
34				513.4	514.6	0.24%	756.1	738.5	2.32%	978.5	941.1	3.83%
35				513.4	514.6	0.24%	756.1	742.6	1.79%	999.0	951.4	4.77%
36				513.4	514.6	0.24%	756.1	746.1	1.32%	1015.6	960.5	5.42%
37				513.4	514.7	0.24%	756.1	749.2	0.91%	1019.2	968.5	4.97%
38				513.4	514.7	0.24%	756.1	752.0	0.55%	1023.3	975.6	4.66%
39				513.4	514.7	0.24%	756.1	754.4	0.23%	1024.3	981.8	4.15%
40				513.4	514.7	0.24%	756.1	756.5	0.05%	1024.6	987.3	3.64%

Table 9 Comparison of different analytical models

		DES [1/h] (m=2)	Throughput [1/h] (m=2)		Throughput [1/h] (m=2)		Throughput [1/h] (m=2)	
		ϑ_{Sim}	ϑ_{MM2K}	Error	ϑ_{MD2K}	Error	ϑ_{MG2K}	Error
n_R	1	35.2	34.1	3.05%	35.2	0.05%	34.0	3.28%
	2	68.6	66.7	2.83%	70.4	2.57%	66.1	3.61%
	3	100.4	98.2	2.22%	105.5	5.13%	96.7	3.72%
	4	131.3	129.0	1.75%	140.7	7.18%	126.1	3.99%
	5	162.0	159.3	1.65%	175.9	8.59%	154.7	4.51%
	6	192.5	189.2	1.74%	211.1	9.66%	182.9	4.98%
	7	222.1	218.4	1.68%	246.3	10.89%	211.1	4.95%
	8	251.2	246.8	1.74%	281.5	12.05%	239.7	4.59%
	9	279.3	274.3	1.80%	316.6	13.37%	269.0	3.68%
	10	310.2	300.5	3.14%	351.8	13.42%	299.5	3.46%
	11	338.3	325.2	3.88%	387.0	14.40%	331.2	2.10%
	12	365.0	348.2	4.59%	422.2	15.67%	364.0	0.27%
	13	385.0	369.6	4.01%	457.4	18.80%	397.0	3.12%
	14	418.0	389.0	6.94%	492.6	17.84%	428.4	2.49%
	15	442.4	406.6	8.09%	514.7	16.33%	455.8	3.03%
	16	466.8	422.4	9.51%	514.7	10.25%	477.2	2.23%
	17	483.3	436.5	9.69%	514.7	6.49%	492.0	1.80%
	18	498.5	448.9	9.95%	514.7	3.24%	501.0	0.51%
	19	510.9	459.9	9.99%	514.7	0.73%	506.0	0.96%
	20	512.1	469.4	8.34%	514.7	0.50%	508.5	0.70%
	21	512.4	477.7	6.76%	514.7	0.44%	509.7	0.52%
	22	513.4	484.9	5.54%	514.7	0.24%	510.2	0.61%
	23	513.4	491.2	4.33%	514.7	0.24%	510.5	0.57%
	24	513.4	496.5	3.29%	514.7	0.24%	510.6	0.55%
	25	513.4	501.0	2.41%	514.7	0.24%	510.6	0.54%
	26	513.4	504.8	1.67%	514.7	0.24%	510.6	0.54%
	27	513.4	507.9	1.06%	514.7	0.24%	510.6	0.54%

Table 9 continued

	DES [1/h] (m=2)	Throughput [1/h] (m=2)		Throughput [1/h] (m=2)		Throughput [1/h] (m=2)	
	ϑ_{Sim}	ϑ_{MM2K}	Error	ϑ_{MD2K}	Error	ϑ_{MG2K}	Error
28	513.4	510.4	0.59%	514.7	0.24%	510.6	0.54%
29	513.4	512.1	0.24%	514.7	0.24%	510.6	0.54%
30	513.4	513.3	0.02%	514.7	0.24%	510.6	0.54%
31	513.4	514.0	0.12%	514.7	0.24%	510.6	0.54%
32	513.4	514.4	0.19%	514.7	0.24%	510.6	0.54%
33	513.4	514.5	0.22%	514.7	0.24%	510.6	0.54%
34	513.4	514.6	0.24%	514.7	0.24%	510.6	0.54%
35	513.4	514.6	0.24%	514.7	0.24%	510.6	0.54%

Author contribution All authors contributed to the study's conception and design. Material preparation, data collection and analysis were performed by Philipp Trost and Michael Eder. Philipp Trost wrote the first draft of the manuscript, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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Declarations

Conflict of interest The authors declare no competing interests.

References

1. AutoStore. Last visited: 04 Jul 2024. Online. Available from: https://de.autostoresystem.com
2. Beckschaefer M, Malberg S, Tierney K, Weskamp C (2017) Simulating Storage Policies for an Automated Grid-Based Warehouse System. In: Bektaş T, Coniglio S, Martinez-Sykora A, Voß S (eds) Computational Logistics. vol. Computational Logistics: 8th International Conference, ICCL 2017. Southampton, UK,: Springer International Publishing, pp 468–482. Available from: https://link.springer.com/chapter/10.1007/978-3-319-68496-3_31#citeas
3. Galka S, Scherbarth C (2021) Simulationsbasierte Untersuchung der Grenzproduktivität von Robotern in einem AutoStore-Lagersystem. In: Franke J, Schuderer P (eds) Simulation in Produktion und Logistik 2021: Erlangen, 15.-17. September 2021. Cuvillier Verlag. vol. Proceedings of the ASIM Fachtagung Simulation in Produktion und Logistik of 2021. ASIM. Goettingen: Cuvillier Verlag, pp 197–206
4. Chen X, Yang P, Shao Z (2022) Simulation-based time-efficient and energy-efficient performance analysis of an overhead robotic compact storage and retrieval system. Simul Modell Pract Theory 119:102560. https://doi.org/10.1016/j.simpat.2022.102560
5. Trost P, Karting G, Eder M (2023) Simulation study of RCS/R-systems with several robots serving one picking station. FME Trans 51(2):201–210. https://doi.org/10.5937/fme2302201t
6. Kartnig G, Trost P, Eder M (2023) RCS/RS under throughput investigation. In: Proceedings of the 16th international material handling research colloquium (IMHRC) and the 11th international scientific symposium on logistics (ISSL). Dresden, Saxony, Germany, pp 1–8
7. Yener F, Yazgan HR (2023) Simulation of re-arrangement and healing in robotic compact bin-storage system. Int J Simul Modell 22(1):100–109. https://doi.org/10.2507/ijsimm22-1-635
8. Ko D, Han J (2022) A rollout heuristic algorithm for order sequencing in robotic compact storage and retrieval systems. Expert Syst Appl 203:117396. https://doi.org/10.1016/j.eswa.2022.117396
9. Hameed H, Rashid A, Amry KA (2020) Automatic storage and retrieval system using the optimal path algorithm. 3D SCEEER Conference sceeer(3d):125–133. https://doi.org/10.37917/ijeee.sceeer.3rd.18
10. Trost P, Eder M (2024) An analytical performance approach for RCS/RS with one robot serving multiple stack heights under a one-path relocation strategy. Sci Rep 14(1). https://doi.org/10.1038/s41598-024-53884-6
11. Zou B, Koster RD, Xu X (2018) Operating policies in robotic compact storage and retrieval systems. Transp Sci 52(4):788–811. https://doi.org/10.1287/trsc.2017.0786
12. Trost P, Eder M (2024) A performance calculation approach for a robotic compact storage and retrieval system (RCS/RS) serving one picking station. Prod Manuf Res. https://doi.org/10.1080/21693277.2024.2336056
13. Arnold D, Furmans K (2019) Materialfluss in Logistiksystemen. Springer, Berlin Heidelberg
14. Baum D (2013) Grundlagen der Warteschlangentheorie. Springer, Berlin Heidelberg
15. Eder M (2023) An analytical approach of multiple-aisle shuttle-based storage and retrieval systems. Int J Adv Manuf Technol 127(3–4):1585–1596. https://doi.org/10.1007/s00170-023-11485-z

16. Heragu SS, Cai X, Krishnamurthy A, Malmborg CJ (2011) Analytical models for analysis of automated warehouse material handling systems. Int J Prod Res 49(22):6833–6861. https://doi.org/10.1080/00207543.2010.518994
17. Ekren BY, Akpunar A (2021) An open queuing network-based tool for performance estimations in a shuttle-based storage and retrieval system. Appl Math Modell 89:1678–1695. https://doi.org/10.1016/j.apm.2020.07.055
18. Govind N, Roeder TM, Schruben LW (2011) A simulation-based closed queueing network approximation of semiconductor automated material handling systems. IEEE Trans Semicond Manuf 24(1):5–13. https://doi.org/10.1109/tsm.2010.2089659
19. Nazzal D (2011) A closed queueing network approach to analyzing multi-vehicle material handling systems. IIE Trans 43(10):721–738. https://doi.org/10.1080/0740817x.2011.566907
20. Raman D (2011) Integrated optimisation of facilities layout and material handling system. In: 2011 IEEE international conference on industrial engineering and engineering management, pp 758–762
21. Bedell P, Smith JM (2012) Topological arrangements of M/G/c/K, M/G/c/c queues in transportation and material handling systems. Comput Oper Res 39(11):2800–2819. https://doi.org/10.1016/j.cor.2012.02.009
22. Smith JM, Kerbache L (2012) State-dependent models of material handling systems in closed queueing networks. Int J Prod Res 50(2):461–484. https://doi.org/10.1080/00207543.2010.535041
23. Tu YM, Lu CW, Lee AHI (2013) AMHS capacity determination model for wafer fabrication based on production performance optimization. Int J Prod Res 51(18):5520–5535. https://doi.org/10.1080/00207543.2013.784416
24. Smith JM, Barnes R (2014) Optimal server allocation in closed finite queueing networks. Flexible Serv Manuf J 27(1):58–85. https://doi.org/10.1007/s10696-014-9202-2
25. Mohammadi M, Dauzère-pérès S, Yugma C, Karimi-Mamaghan M (2020) A queue-based aggregation approach for performance evaluation of a production system with an AMHS. Comput Oper Res 115:104838. https://doi.org/10.1016/j.cor.2019.104838
26. Zhang HY, Xi SH, Chen QX, Smith JM, Mao N, Li X (2020) Performance analysis of a flexible flow shop with random and state-dependent batch transport. Int J Prod Res 59(4):982–1002. https://doi.org/10.1080/00207543.2020.1712488
27. Amjath M, Kerbache L, Elomri A, Smith JM (2023) Queueing network models for the analysis and optimisation of material handling systems: a systematic literature review. Flexible Serv Manuf J. https://doi.org/10.1007/s10696-023-09505-x
28. Smith JM (2008) M/G/c/K performance models in manufacturing and service systems. Asia-Pacific J Oper Res 25(04):531–561. https://doi.org/10.1142/s0217595908001857
29. Smith JM (2003) M/G/c/K blocking probability models and system performance. Perform Eval. 52(4):237–267. https://doi.org/10.1016/s0166-5316(02)00190-6
30. Kimura T (1996) A transform-free approximation for the finite capacity M/G/s queue. Oper Res 44(6):984–988. https://doi.org/10.1287/opre.44.6.984
31. Eder M (2020) An approach for performance evaluation of SBS/RS with shuttle vehicles serving multiple tiers of multiple-deep storage rack. Int J Adv Manuf Technol 110(11–12):3241–3256. https://doi.org/10.1007/s00170-020-06033-y