ORIGINAL ARTICLE



Analytical examination of shuttle-based storage and retrieval systems with multiple-capacity lifts

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Received: 2 February 2024 / Accepted: 21 June 2024 $\ensuremath{\mathbb{C}}$ The Author(s) 2024

Abstract

The unbroken trend towards e-commerce and the ever-increasing demands on storage technology present the providers of storage systems with ever greater challenges. One way to meet these requirements is to significantly enhance warehouses' handling systems. Multiple capacity lifts in tier-captive single-aisle shuttle-based storage and retrieval systems (SBS/RS) are a method to raise the throughput performance of such systems. Therefore, a straightforward calculation approach is presented based on a cycle time model combined with a time-continuous single open Markovian queue with limited capacity. A comparison of the invented approach to a discrete-event simulation (DES) is made to validate it. Finally, a numerical example of the influences of the different handling capacities and their usage in the design process of SBS/RS is depicted. The result is that with double the handling capacity, the throughput increases by about 50%. Another result is sorting the stops within one handling cycle, which leads to a significantly higher throughput.

Keywords SBS/RS · Tier-captive autonomous vehicle · Queueing system · Discrete spatial value

1 Introduction

There is an unbroken trend towards e-commerce and ongoing efforts on the supply chain. This subsequently leads to increasing demands on storage systems. One requirement is the detailed performance of the storage system and the reachable throughput. Here, Carlo [1] and Lerher [2, 3] argue that the vertical conveyor respective to the lifts determines the reachable throughput. That is not the whole truth because, as Eder [4] wrote in 2019, the interactions of the lifts with the shuttles and the shuttles themselves also influence the throughput that can be achieved. However, this statement shows the importance of the performance of the lifts and its influence on the achievable throughput of tier-captive singleaisle shuttle-based storage and retrieval systems (SBS/RS). It should be noted here that many existing warehouses are limited by their geometric dimensions and the transporter specifications by the lift performance. On the one hand, the performance of the lifts can be increased via thekinematic data of the lifts, but this is not infinitely possible and sensible. On the other hand, the number of lifts or their capacity can be increased. This study deals with precisely this: an increase in the capacity of the lifts and the influence on the reachable throughput of a storage aisle. Further aspects discussed in this publication are the influence of sorting the stops within a processing cycle over the height and, secondly, the impact of the removals within a batch sequence of the order list. These two points become more important and relevant, especially with higher lift capacities.

The main aim of this study is to present a straightforward, accurate analytical calculation approach that determines the throughput of a tier-captive single-aisle shuttle-based storage and retrieval system with multiple-capacity lifts. The depicted approach is validated through a comparison with a discrete-event simulation (DES) to show the common accuracy requirements. A European material handling provider gives the data used.

This paper is organized as follows. Section 2 presents existing literature discussing tier-captive shuttle-based storage and retrieval systems. The following Section 3 describes the concerned storage system in details and lists the underlying assumptions. The analytical approach is depicted in Section 4. Section 5 presents different examples to see the approximation quality and the influence of the various param-

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eters on the throughput of a single aisle. The last Section 6 assumes this paper and gives an outlook on further research.

2 Literature

The following section gives an overview of how the performance of SBS/RS can be determined. SBS/RS with advanced lift designs are not the only ones mentioned here. There are two ways to assess the performance of tier-captive shuttlebased storage and retrieval systems.

The first way is to simulate a SBS/RS with a discrete-event simulation (DES). Many publications discuss SBS/RS with this kind of approach (e.g., Ekren et al. [5–8], Marchet et al. [9], Lerher et al. [10–15], Kriehn et al. [16], Ha et al. [17], Eroglu et al. [18], Ning et al. [19], and Jerman et al. [20]). Only two papers deal with advanced lift configuration in this approach. Here, Ning et al. [19] presents a simulation study with multiple-lifts. Jerman et al. [20] instead advance the lifts due to their moveability. This means that the lifts are no longer situated fixedly. Nevertheless, these approaches will not be treated in more detail because of their different aims.

Analytical approaches are the second possibility in discussing tier-captive shuttle-based storage and retrieval systems. Here, diversification has to be made.

Cycle time models of the transportation systems in SBS/RS are the first analytical approach. These approaches discuss the subsystems, lifts, and shuttles and do not mention the interactions between these two subsystems. Many papers deal with such cycle time models such as, e.g., Sari et al. [21], Lerher [2, 3, 22, 23], Lerher et al. [24, 25], Borovinšek [26], Ekren et al. [27], and Manzini et al. [28]. Papers with this kind of approach are not discussed in more detail here.

Queueing networks to discuss tier-captive shuttle-based storage and retrieval systems are the second analytical approach. These queueing networks take the interactions into account. The primary outcome of such models is the time to store or retrieve a tote to and out of the storage. Besides this, it is not possible to determine the reachable throughput of a storage system. Heragu et al. [29], Marchet et al. [30], Wang et al. [31], Ekren et al. [32], Epp et al. [33], and Tappia et al. [34] deal with such analytical approach. They are not directly comparable because of the different aims of queueing networks.

Single queueing models with limited capacity are the third kind of approach mentioned here. This approach considers the lifts and shuttles' interactions, and they can determine the reachable throughput of tier-captive shuttle-based storage and retrieval systems. Eder published a bunch of papers that deal with such an approach [4, 35–39]. He discusses different configurations of the SBS/RS. Here, various con-

The International Journal of Advanced Manufacturing Technology

Except for Ning et al. [19], who presents a simulation study with multiple-lifts, and Jerman et al. [20], who discusses moveable lift, no other publication threat SBS/RS with advanced lift configurations. The importance of the lift configuration can also be proven by the fact that for a long time, the lifts were considered the bottleneck of the systems [1–3]. This was only refuted by the publication by Eder [4], which considers the interactions of the subsystems (lifts and shuttles). This paper discusses lifts with different handling capacities because there is no existing literature on advanced lift configurations. A single queue with limited capacity is used to mention all relevant processes of the tiercaptive shuttle-based storage and retrieval system. In order, the approach of Eder [4] is advanced.

3 System description

A tier-captive shuttle-based storage and retrieval system with higher lift capacities is presented in Fig. 1. Such a system has a rack where totes can be stored on each side of the aisle. In the discussed system, the rack can hold a single deep. This is followed by the buffer spaces, which serve as a link between the lifts and shuttles. Here, one side is for the storage process, and one is for the retrieval process. In our case, the buffer capacity (three totes) is the same as the lift capacity, which causes the lifts to be able to load and unload by one stop. The next part is the lifts. Again, one is for the storage and retrieval process. In the shown SBS/RS, every lift has a capacity of three. The last part of the system is the I/O point, which is positioned at the bottom of the SBS/RS. This point is the link to the pre-storage area, which this paper does not discuss further.

The main assumptions are based on a SBS/RS produced by a European material handling provider. These assumptions are similar to Eder [4, 35–39].

- There is one lift for the input and one for the output, which serves the transactions under a first come, first served rule in a single command cycle.
- The shuttles serve the transactions under a first come, first served single and dual command cycle rule.
- The I/O point is the dwell point of the input lift.



- The point of service completion is the dwell point of the output lift and the shuttles.
- The lifts and shuttles' accelerate/decelerate rates are averaged over time.
- The lifts and shuttles' velocities are averaged over time and exhibit the same behavior as the real.
- The time needed to transfer a tote to and from the lift is assumed.
- The waiting queue before the I/O point is always filled. This assumption is necessary to eliminate the influence of the pre-storage area.
- The totes are stored evenly distributed over the entire storage rack.
- The order of totes is evenly distributed among all totes in the rack.

Another assumption that has to be explained is the sorting over the height of the orders. This means the orders are so sorted within one handling cycle that the lifts do not have to change their direction during the handling cycle. In other words, the lift load/unload is while lifting or lowering down. A change of direction is not possible after starting load-/unloading and before the end of load-/unloading.

The second unique assumption that can be made is that the tote must not be retrieved in the ordered order but rather in a batch of, e.g., three following orders. For example, if the requested order is 1, 2, and 3, the retrieved totes can have the following order, e.g., 3, 1, and 2. This kind of batch sequencing is interesting for the higher handling capacities of the lift because when two orders within the batch are located in the same tier, the lift can take over both totes by one stop.

4 Analytical approach

The analytical approach discussed here is based on Eder [4]. This approach uses an open queueing model with limited capacity (M|G|1|K). The calculation process of this approach is as follows:

- The inter-arrival time to a single tier
- The shuttle service time
- The open queueing model M|G|1|K

Several adoptions must be made to adopt this approach to lifts with higher handling capacity, especially the calculation approach of the inter-arrival times to a tier, which must be advanced. Table 1 depicts all notations used in the following. The equations taken directly from Eder [4] are also listed to enable the entire approach's comprehensibility. Parts 4.2 and 4.3 were taken directly from Eder [4] and presented to provide a complete explanation of the calculation approach. The reason for using this approach is that it allows for a closed-form solution that describes the interactions between lifts and shuttles with high accuracy. Furthermore, this approach can also be solved with simple calculation programs. The novelty

Δx	Horizontal distance between storage slots
Δy	Vertical distance between tiers
θ	Throughput of a storage section
ϑ_{tier}	Throughput of a single tier with multiple-aisles
ρ	Utilization rate of the shuttle
<i>a_{lift}</i>	Acceleration/deceleration rate of the lifts
a _{shuttle}	Acceleration/deceleration rate of the shuttles
Κ	Capacity of the queueing system in a tier
$l_{I/O}$	Vertical distance between the I/O point and the first tier
n _{batch}	Number of orders in which an order has to be retrieved
n _{buf}	Number of buffer slots per tier
n _{cap}	Minimum of the lift capacity and the buffer capacity
n _{command}	Number of command within a handling cycle
n _{Lcap}	Capacity of the lift
n _{slot}	Number of slots on each side of an aisle per tier
n _{tier}	Number of tiers in the respective storage section
p_0	Probability of emptiness of a queueing system in a tier
p_k	Blocking probability of a queueing system in a tier
t_A	Inter-arrival time to a tier
t_{t_L}	Transfer time of a tote to and from the lift
t_{t_S}	Transfer time of a tote to and from the shuttle
t _{liftsc}	Time of a lift needed for a single command cycle
$t_{lift_{2C}}$	Time of a lift needed for a dual command cycle
t _{lift_{MC}}	Time of a lift needed for a M command cycle
$t_{R_{L_{SC}}}$	Ride time of a lift at single command cycle
$t_{R_{L_{DC}}}$	Ride time of a lift at dual command cycle
$t_{R_{L_{MC}}}$	Ride time of a lift at M command cycle
t _{Rs sc}	Ride time of a shuttle at single command cycle
$t_{R_{S},DC}$	Ride time of a shuttle at dual command cycle
t_S	Service time of a shuttle in a tier
$t_{S_{SC}}$	Shuttle service time at single command cycle
$t_{S_{DC}}$	Shuttle service time at dual command cycle
t _{shuttlesc}	Time of a shuttle for a single command cycle
t _{shuttleDC}	Time of of a shuttle for a dual command cycle
S	Coefficient of variation of the shuttle cycle times
v_{lift}	Velocity of the lifts
$v_{shuttle}$	Velocity of the shuttles

 Table 1
 Notation of the tier-captive single-aisle tier-captive shuttlebased storage and retrieval system

4.1.1 Cycle time of the lift with single command cycle

$$t_{lift_{SC}} = 2 \cdot t_{R_{L_{SC}}} + t_{t_L} \tag{1}$$

The mean ride time for a single command cycle is as follows:

$$t_{R_{L_{SC}}} = \frac{1}{n_{tier}} \sum_{k=1}^{n_{tier}} \mathbf{t}(|l_{I/O} + (k-1) \cdot \Delta y|)$$
(2)

To consider the different speed profiles of the lift, the function $\mathbf{t}(l)$ has to be divided into two ranges. One range for distances less than $l < \frac{v^2}{a}$:

$$\mathbf{t}(l) = 2\sqrt{\frac{l}{a_{lift}}}\tag{3}$$

The other range for larger distances:

$$\mathbf{t}(l) = \frac{l}{v_{lift}} + \frac{v_{lift}}{a_{lift}}$$
(4)

4.1.2 Cycle time for a dual command cycle

$$t_{lift_{2C}} = 2 \cdot t_{R_{L_{SC}}} + t_{R_{L_{DC}}} + t_{t_L} \tag{5}$$

Time for the ride between the two drop-off/pick-up positions

$$t_{R_{L_{DC}}} = \frac{1}{n_{tier}^2} \sum_{k=1}^{n_{tier}} \sum_{l=1}^{n_{tier}} \mathbf{t}(|(k \cdot \Delta y - l \cdot \Delta y)|)$$
(6)

4.1.3 Cycle time for a multiple command cycle

The index M stands for the number of commands within one handling cycle

$$t_{lift_{MC}} = 2 \cdot t_{R_{L_{SC}}} + \sum_{m=2}^{n_{command}} t_{R_{L_{MC}m}} + t_{t_L}$$
(7)

Time for the ride between the drop-off/pick-up positions at multiple command cycles with sorted positions

$$t_{R_{L}_{MC}m} =$$

$$\frac{1}{n_{tier}^{m}} \sum_{k=1}^{n_{tier}} \sum_{l=1}^{n_{tier}} \mathbf{t}(|(k \cdot \Delta y - l \cdot \Delta y)|) \\ \cdot (-1)^{m-1} \cdot \frac{((|k-l|) - n_{tier})^{m-1}}{n_{tier} - (|k-l|)}$$
(8)

Time for the ride between the drop-off/pick-up positions at multiple command cycles without sorted positions

$$t_{R_{L_{MC}}} = \frac{1}{n_{tier}^2} \sum_{k=1}^{n_{tier}} \sum_{l=1}^{n_{tier}} \mathbf{t}(|(k \cdot \Delta y - l \cdot \Delta y)|)$$
(9)

of this calculation method is explained in the following section and relates to including the different handling capacities of the lifts.

4.1 Inter-arrival time

The first point of the analytical approach is the inter-arrival time determined by the lifts. The main part is the cycle time of the lifts with the different command strategies.

4.1.4 Inter-arrival time for a fully ordered order sequence

The mean inter-arrival time to a tier is the cycle time of the lifts with several commands, the same as the lift's capacity, times the number of tiers divided by the lift's capacity.

This considers that one lift serves all tiers

$$t_A = t_{lift_{MC}} \cdot \frac{n_{tier}}{n_{L_{cap}}} \tag{10}$$

4.1.5 Inter-arrival time for an order retrieved within a batch order sequence

The mean inter-arrival time to a tier is the cycle time of the lift with different command sizes dependent on the lift's capacity and the capacity of the puffer spaces in the tiers.

$$t_{A} = \frac{n_{tier}}{n_{L_{cap}}}$$

$$\cdot \left\{ \sum_{M=2}^{n_{cap}} \left[t_{lift_{MC}} \cdot \frac{1}{n_{tier}^{n_{cap}-M}} \right]^{n_{batch}-1-n_{cap}+M} \left(1 - \frac{1}{n_{tier}} \right)^{n_{batch}-1-n_{cap}+M} \cdot \left(\frac{n_{batch}-1}{n_{cap}-M} \right) \right] + t_{lift_{1C}} \cdot \sum_{M=n_{cap}-1}^{n_{batch}-1} \left[\frac{1}{n_{tier}^{M}} \left(1 - \frac{1}{n_{tier}} \right)^{n_{batch}-1-M} \cdot \left(\frac{n_{batch}-1}{M} \right) \right] \right\}$$

$$(11)$$

The parameter n_{cap} is depending on the load capacity of the lift $n_{L_{cap}}$ and the capacity of the buffer slots in the tiers n_{buffer} .

$$n_{cap} = \begin{cases} n_{L_{cap}}, & n_{L_{cap}} < n_{buf} \\ n_{buf}, & n_{L_{cap}} \ge n_{buf} \end{cases}$$
(12)

4.2 Service time for a single deep rack

The second part of this calculation approach is the service time of the shuttles. This part contains the times for riding from A to B and transferring the totes to and from the shuttle. For a single command cycle, the following equation is made [4]:

$$t_{shuttle_{SC}} = 2 \cdot t_{R_{S-SC}} + t_{t_S} \tag{13}$$

In the meantime, the ride is due to the following [4]:

$$t_{R_{S_SC}} = \frac{1}{n_{slot}} \sum_{k=1}^{n_{slot}} \mathbf{t}(k \cdot \Delta x)$$
(14)

To consider the different equations depending on the distances, Eqs. 3 and 4 have to be used [4].

The service time of the shuttles is as follows [4]:

$$t_{S_{SC}} = 2 \cdot t_{shuttle_{SC}} \tag{15}$$

For the dual command cycle, the following equation is made.

$$t_{shuttle_{DC}} = 2 \cdot t_{R_{S_SC}} + t_{R_{S_DC}} + 2 \cdot t_{t_S} \tag{16}$$

This equation is the cycle time for a single command cycle extended by time for the ride between the slot where a tote is transferred from the shuttle and the slot where the next tote shall be retrieved.

The mean time for this is due to [4]

$$t_{R_{S_DC}} = \frac{1}{n_{slot}^2} \sum_{k=1}^{n_{slot}} \sum_{l=1}^{n_{slot}} \mathbf{t}(|(k \cdot \Delta x - l \cdot \Delta x)|)$$
(17)

The mean service time of a dual command cycle is as follows:

$$t_{S_{DC}} = t_{shuttle_{DC}} \tag{18}$$

4.3 Open queueing model M|G|1|K

The interactions between the lift and the shuttles and the influence of the buffer slots are taken into account via a timecontinuous open queueing model with limited capacity. Here, the throughput of a single tier is determined with an M|G|1|K queueing model and, further on, assumed to be the throughput of the whole aisle. The calculation approach can be taken directly from Eder [4]. Due to the equations of the interarrival time and its fitness to a single tier, this can be made.

The determination of the throughput of a single tier is due to the following [40]:

$$\vartheta_{tier} = \frac{1}{t_A} \cdot (1 - p_k) = \frac{1}{t_S} \cdot (1 - p_0)$$
 (19)

The blocking probability p_k of a queueing system is as follows [41]:

$$p_{k} = \frac{\rho_{2} + \sqrt{\rho \cdot e^{-s^{2}} \cdot s^{2}} - \sqrt{\rho \cdot e^{-s^{2}} + 2K}}{\rho_{2} + \sqrt{\rho \cdot e^{-s^{2}} \cdot s^{2}} - \sqrt{\rho \cdot e^{-s^{2}}} \cdot (\rho - 1)}{\rho_{2} \cdot \frac{\sqrt{\rho \cdot e^{-s^{2}} \cdot s^{2}} - \sqrt{\rho \cdot e^{-s^{2}} + K + 1}}{\rho_{2} \cdot \sqrt{\rho \cdot e^{-s^{2}} \cdot s^{2}} - \sqrt{\rho \cdot e^{-s^{2}} + K + 1}} - 1}$$
(20)

This equation describes the status of a filled system; no tote can enter the system.

The utilization rate of the shuttles is [4]

$$\rho = \frac{t_S}{t_A} \tag{21}$$

K is the capacity of the queueing system. It is the sum of the number of buffer slots and the handling capacity of the shuttles [4].

$$K = n_{buf} + 1 \tag{22}$$

s is the coefficient of variation of the service time distribution. This coefficient has to be gained by a simulation model or via an equation from Eder [4].

The probability of emptiness of the queueing system describes the status that the server has to wait because there is no tote in the queuing system [41]:

$$p_{0} = \frac{\rho - 1}{2 \cdot \frac{\sqrt{\rho \cdot e^{-s^{2}} \cdot s^{2}} - \sqrt{\rho \cdot e^{-s^{2}} + K + 1}}{\rho^{2} \cdot \frac{\sqrt{\rho \cdot e^{-s^{2}} \cdot s^{2}} - \sqrt{\rho \cdot e^{-s^{2}}} - 1}}$$
(23)

The throughput of a storage section is equal to the throughput of one tier multiplied by the number of tiers [4]

$$\vartheta = \vartheta_{tier} \cdot n_{tier} \tag{24}$$

5 Numerical study

This part aims to depict the accuracy of the invented approach and, as a second aim, the influence of the different lift capacities on the throughput and the rack design for the highest throughput. These investigations are essential to understand the entire system. Here, simulation models were developed,

 Table 2
 Tested parameter configurations of the tier-captive single-aisle

 SBS/RS with different capacities of the lift

Parameter	Value
Number of tiers	$n_{tier} \in \{4-50\}$
Number of slots per tier on each side of the aisle	$n_{slot} = 200$
Number of totes that can be transported at once on the lift	$n_{L_{cap}} = \in \{1, 2, 3\}$
Number of buffers per tier on each side of the aisle	$n_{buf} = 3$
Distance between two storage slots	$\Delta x = 0, 5m$
Distance between two tiers	$\Delta y = 0, 4m$
Vertical distance between the first tier and the I/O point	$l_{I/O} = 0m$
Lift velocity	$v_{lift} = 5\frac{m}{s}$
Lift acceleration/deceleration rate	$a_{lift} = 7 \frac{m}{s^2}$
Time to transfer a tote to and from the lift	$t_{t_L} = 1.4s$
Shuttle velocity	$v_{shuttle} = 2\frac{m}{s}$
Shuttle acceleration/deceleration rate	$a_{shuttle} = 2\frac{m}{s^2}$
Time needed for transferring a tote to and from the shuttle out of the buffer/rack	$t_{t_S} = 8.4s$

and the results were compared to the analytical approaches. A European material handling provider gave the input parameter for the comparison, and the numerical example is depicted in Table 2.

5.1 Approximation quality

A comparison is made to a discrete-event simulation model (DES) to depict the approximation quality. Here, primarily, the lifts were modeled to gain results. Therefore, different lift handling capacities were chosen. Table 3 shows this comparison's results. The results are also visualized in Figs. 4 and 5.

Here, three different scenarios were chosen to depict the accuracy. The first is with the parameter that the lift only performs a triangular ride (see Eq. 3 and Fig. 2). This velocity scenario has the behavior that the maximum velocity can not be reached during the length of the ride. In this scenario, it can be seen that with a higher number of handling capacities, there is an error in low digits. This estimation error gets even less when combining the ride time with the handling times. In the depicted example, it is -5% at the most. The second scenario is a system where only trapezoid rides are performed (see Eq. 4 and Fig. 3). This ride configuration assumes that the acceleration is high compared to the maximum reachable velocity of the lift so that every length of the lift ride reaches the maximum velocity, and the velocity profile looks like a trapezoid. In this scenario, the approximation is exact, meaning no approximation error exists. The third scenario is with real data from a European material handling provider. Depending on the length of the ride, this velocity scenario has a combination of the two mentioned speed profiles above. This means at low length, there is a triangle velocity profile, and at longer rides, there is a trapezoid speed profile. The lack of approximation is in the middle of the two scenarios above. It is -2%. This leads to the assumption that the analytical approach has a high approximation quality (Figs. 4 and 5).

5.2 Numerical example

Influence of the handling capacity

The first example in this paper tries to depict the influence of the different handling capacities on the reachable throughput.

Here, four different capacities were chosen to give an overview. All these four scenarios have in common that the totes are sorted over height, and the order batch size is 10. In Table 4 and in Fig. 6, the results are shown. There, it can be seen that with a higher capacity, the throughput increases. Also, it can be seen that with low storage racks, the throughput is the same because only the shuttles determine the throughput. On the other hand, the throughput is mainly determined by the lifts at a higher storage rack, so

	Triangle ride	e of the lift		Mixed ride o	f the lift		Trapezoid rio	le of the lift	
v_{lift}	$10\frac{m}{s}$			$5\frac{m}{s}$			$5\frac{m}{s}$		
a_{lift}	$1\frac{m}{s^2}$			$7 \frac{m}{s^2}$			$1000 \frac{m}{s^2}$		
Ntier	100			100			100		
Δy	0.4 m			0.4 m			0.4 m		
tun-/load	1.4 m			1.4 m			1.4 m		
Ride time of the lift									
	Simulation	Analytical approach	Estimation error	Simulation	Analytical approach	Estimation error	Simulation	Analytical approach	Estimation error
Single command cycle	17.00 s	16.99 s	0%	9.50 s	9.49 s	0%	8.08 s	8.08 s	0%
Dual command cycle	23.72 s	23.73 s	0%	12.86 s	12.85 s	0%	$10.77 \mathrm{~s}$	10.75 s	0%
Tribe command cycle	28.29 s	27.58 s	-3%	14.85 s	14.64 s	-1%	12.08 s	12.08 s	0%
Fourfold command cycle	31.92 s	30.14 s	-6%	16.29 s	15.77 s	-3%	12.90 s	12.88 s	0%
Cycle time of the lift									
	Simulation	Analytical approach	Estimation error	Simulation	Analytical approach	Estimation error	Simulation	Analytical approach	Estimation error
Single command cycle	$19.80\mathrm{s}$	19.79 s	0%	12.30 s	12.29 s	0%	10.88 s	10.88 s	0%
Dual command cycle	27.92 s	27.93 s	0%	17.06 s	17.05 s	0%	$14.97 \mathrm{s}$	14.95 s	0%
Tribe command cycle	33.89 s	33.18 s	-2%	20.45 s	$20.24\mathrm{s}$	-1%	17.68 s	17.68 s	0%
Fourfold command cycle	$38.92\mathrm{s}$	37.14 s	-5%	$23.29\mathrm{s}$	22.77 s	-2%	19.90 s	19.88 s	0%0

 Table 3
 Approximation quality of the ride time and the cycle time approach at multiple command cycles



Fig. 2 Velocity over time in triangle form

the difference between the four scenarios is also given. The highest throughput for all scenarios is at the middle height of the rack. With increasing handling capacity, the height for the highest throughput increases.

Influence of the order strategy and the sorting of the orders over the height

The second example shows the aim of delivering the influence of the strategy to order the orders over the height of the rack or not. Here, the two parameters, sorting the totes over the height and the ordered order batch sequence, are varied. The dimensions of the storage rack were chosen to achieve the highest throughput with the same storage capacity of



Fig. 3 Velocity over time in trapezoid form



Fig. 4 Approximation quality of the ride time approach at multiple command cycles



Fig. 5 Approximation quality of the cycle time approach at multiple command cycles

Table 4Throughput over rackheight with different capacitiesof the lift and sorted handlingpoint and an order batch size of10

Rack height	Lift capacity = 1	Lift capacity = 2	Lift capacity = 3	Lift capacity = 4
1.2m	$166\frac{totes}{L}$	$166\frac{totes}{L}$	$166\frac{totes}{L}$	$166\frac{totes}{L}$
2.0 m	$248 \frac{totes}{h}$	$249 \frac{totes}{h}$	$249\frac{totes}{h}$	$249 \frac{totes}{h}$
2.8 m	$318\frac{totes}{h}$	$332\frac{totes}{h}$	$332\frac{totes}{h}$	$332 \frac{totes}{h}$
3.6m	$369\frac{totes}{h}$	$414 \frac{totes}{h}$	$415\frac{totes}{h}$	$415\frac{totes}{b}$
4.4 m	$405 \frac{totes}{h}$	$492\frac{totes}{h}$	$498 \frac{totes}{h}$	$498 \frac{totes}{h}$
5.2 m	$429\frac{totes}{h}$	$559 \frac{totes}{h}$	$579\frac{totes}{h}$	$581 \frac{totes}{h}$
6.0 m	$445\frac{totes}{h}$	$612 \frac{totes}{h}$	$656\frac{totes}{h}$	$663 \frac{totes}{h}$
6.8 m	$455\frac{totes}{h}$	$650\frac{totes}{h}$	$722\frac{totes}{h}$	$743 \frac{totes}{h}$
7.6 m	$460\frac{totes}{h}$	$675 \frac{totes}{h}$	$774\frac{totes}{h}$	$815 \frac{totes}{h}$
8.4 m	$463\frac{totes}{h}$	$690\frac{totes}{h}$	$810\frac{totes}{h}$	$874\frac{totes}{h}$
9.2 m	$462\frac{totes}{h}$	$697 \frac{totes}{h}$	$833\frac{totes}{h}$	$918 \frac{totes}{h}$
10.0 m	$460\frac{totes}{h}$	$699 \frac{totes}{h}$	$844\frac{totes}{h}$	$946\frac{totes}{h}$
10.8 m	$457 \frac{totes}{h}$	$697 \frac{totes}{h}$	$848 \frac{totes}{h}$	$960\frac{totes}{h}$
11.6 m	$453\frac{totes}{h}$	$691 \frac{totes}{h}$	$845 \frac{totes}{h}$	$965 \frac{totes}{h}$
12.4 m	$448 \frac{totes}{h}$	$684 \frac{totes}{h}$	$839\frac{totes}{h}$	$962\frac{totes}{h}$
13.2 m	$443\frac{totes}{h}$	$676 \frac{totes}{h}$	$830\frac{totes}{h}$	$955\frac{totes}{h}$
14.0 m	$438 \frac{totes}{h}$	$666\frac{totes}{h}$	$820\frac{totes}{h}$	$945 \frac{totes}{h}$
14.8 m	$432\frac{totes}{h}$	$657 \frac{totes}{h}$	$809\frac{totes}{h}$	$934\frac{totes}{h}$
15.6 m	$426\frac{totes}{h}$	$647 \frac{totes}{h}$	$797 \frac{totes}{h}$	$921\frac{totes}{h}$
16.4 m	$420\frac{totes}{h}$	$636\frac{totes}{h}$	$785 \frac{totes}{h}$	$908 \frac{totes}{h}$
17.2 m	$414\frac{totes}{h}$	$626 \frac{totes}{h}$	$773 \frac{totes}{h}$	$895\frac{totes}{h}$
18.0 m	$408 \frac{totes}{h}$	$616\frac{totes}{h}$	$761 \frac{totes}{h}$	$882\frac{totes}{h}$
18.8 m	$402\frac{totes}{h}$	$606\frac{totes}{h}$	$749\frac{totes}{h}$	$869\frac{totes}{h}$
19.6 m	$396\frac{totes}{h}$	$596\frac{totes}{h}$	$737 \frac{totes}{h}$	$856\frac{totes}{h}$
20.4 m	$390\frac{totes}{h}$	$587 \frac{totes}{h}$	$726\frac{totes}{h}$	$844\frac{totes}{h}$

20,000 totes per aisle. Table 5 lists this example's results. Figure 7 depicts this comparison. It can be seen that all scenarios have the same throughput with a lift capacity of 1. At



Fig. 6 Throughput over rack height with different capacities of the lift and sorted handling point and an order batch size of 10

a capacity of two totes, the influence of the ordered order batch sequence influences the reachable performance. Here, the fully ordered order sequence achieves the lowest throughput, and a batch sequence of 10 in which the requested order has to be retrieved achieves the highest throughput. With further ingression of the handling capacity, sorting the orders over the height substantially influences the throughput more than the batch sequencing ordered order strategy.

Influence of the lifts handling capacities on the rack dimensions

The third example of the different handling capacities of the lifts discusses the influence on the rack dimensions. In Table 5, the results of this example are depicted, and Fig. 8 shows these results. It can be seen that with an increase in the handling capacity of the lift, the throughput of the lift increases, and so on, as well as the performance of the aisle. Furthermore, the height of the rack increases for higher throughput. The example depicted here has a storage capacity of 20,000 totes per aisle.

Capacity of the lift	Fully sequence chaotic	Fully sequence sorted	Batch sequence 5 chaotic	Batch sequence 10 chaotic	Batch sequence 5 sorted	Batch sequence 10 sorted
1	$13.6 \text{ m} \times 147 \text{ m}$ $427 \frac{totes}{h}$	$13.6 \text{ m} \times 147 \text{ m}$ $427 \frac{totes}{h}$	$13.6 \text{ m} \times 147 \text{ m}$ $427 \frac{totes}{h}$	$13.6 \text{ m} \times 147 \text{ m}$ $427 \frac{totes}{h}$	$13.6 \text{ m} \times 147 \text{ m}$ $427 \frac{totes}{h}$	$13.6 \text{ m} \times 147 \text{ m}$ $427 \frac{totes}{h}$
2	$14 \text{ m} \times 143 \text{ m}$ $629 \frac{totes}{h}$	$14 \text{ m} \times 143 \text{ m}$ $629 \frac{totes}{h}$	$14 \text{ m} \times 143 \text{ m}$ $638 \frac{totes}{h}$	$14 \text{ m} \times 143 \text{ m}$ $638 \frac{totes}{h}$	$14 \text{ m} \times 143 \text{ m}$ $648 \frac{totes}{h}$	$14 \text{ m} \times 143 \text{ m}$ $648 \frac{totes}{h}$
3	$14 \text{ m} \times 143 \text{ m}$ $748 \frac{totes}{h}$	$14.4 \text{ m} \times 139 \text{ m}$ $782 \frac{totes}{h}$	$14.4 \text{ m} \times 139 \text{ m}$ $756 \frac{totes}{h}$	$14.4 \text{ m} \times 139 \text{ m}$ $767 \frac{totes}{h}$	$14.8 \text{ m} \times 135 \text{ m}$ $788 \frac{totes}{h}$	$14.8 \text{ m} \times 135 \text{ m}$ $795 \frac{totes}{h}$
4	$14.4 \text{ m} \times 139 \text{ m}$ $826 \frac{totes}{h}$	$\frac{15.2 \text{ m} \times 132 \text{ m}}{906 \frac{totes}{h}}$	$14.4 \text{ m} \times 139 \text{ m}$ $833 \frac{totes}{h}$	$14.4 \text{ m} \times 139 \text{ m}$ $843 \frac{totes}{h}$	$15.2 \text{ m} \times 132 \text{ m}$ $910 \frac{totes}{h}$	$\frac{15.2 \text{ m} \times 132 \text{ m}}{914 \frac{totes}{h}}$

Table 5 Storage dimensions and throughput with different capacity of the lift and different handling strategies with a storage capacity of 20,000

6 Conclusion

The increasing demands on the supply chain and the logistics sector lead to an industry-driven further development of more performance and, simultaneously, lower use of resources. One way to increase the performance of tier-captive shuttlebased storage and retrieval systems is to expand the vertical conveyors respective to the lifts. Here, in particular, we aim to improve their handling capacity. No study in the literature can be found that discusses such systems using an analytical approach.

This paper uses an analytical approach to describe tiercaptive shuttle-based storage and retrieval systems with different lift handling capacities. The approach is based on the three-step calculation approach from Eder [4].

• Determination of the inter-arrival times to a tier



Fig.7 Storage dimensions and throughput with different capacity of the lift and different handling strategies with a storage capacity of 20,000

- Determination of the service times of the tiers
- Determination of the throughput of an aisle through a queueing model with limited capacity

The main novelty of this paper relates to the determination of the inter-arrival time. The loading capacities according to the different configurations have been considered here. Furthermore, a processing strategy for the retrieval process has also been developed and compared with the full-order sequence. This processing strategy sorts the stops within a lift cycle one after the other so that no unnecessary direction changes are made. The second retrieval strategy examined is that if the lift has a higher capacity, it is relevant if it can be loaded from one level in one go. This leads to a reduction in the number of stops within a handling cycle. This was solved using a so-called batch sequence order. This strategy states that an order will be delivered within the next, e.g., five orders must be retrieved, and the complete ordering sequence does not have to be adhered to. These two options for changing the processing sequence lead to significantly higher throughputs with the same technical effort.

The increase of the throughput performance is not in the same relation as the increase of the lift capacity. For example, doubling the lift capacity increases performance by around 50%, and a further doubling leads to roughly a doubling of primary performance. This relation is significant when designing new-tier-captive shuttle-based storage and retrieval systems.

Its speed and ease of use characterize the presented approach. It enables the rapid determination of the performance of machines under consideration of given boundary conditions.

In the industrial environment, it is beneficial as it provides engineers and designers with an efficient tool for quick design evaluation and optimization. In summary, the chosen method is a valuable tool for engineers and designers in mechanical **Fig. 8** Storage dimensions and throughput with different capacity of the lift and different handling strategies with a storage capacity of 20,000



engineering due to its speed, simplicity, and flexibility. It is important to note that the approximation errors of this analytical approach can be neglected due to the aim of generating results in no time. In contrast, a simulation study can deliver a higher approximation quality, but determining the throughput of different storage systems and comparing them is very timeconsuming. Further studies can extend the SBS/RS towards a higher number of lifts with different handling capacities and their influence on the reachable throughput of such a system. Another interesting case for studies based on the developed approach in this paper is the implementation of shuttles with higher handling capacities and, therefore, a higher number of commands per handling cycle.

Author contribution All authors contributed to the study's conception and design. Material preparation, data collection, and analysis were performed by Michael Eder. Michael Eder wrote the first draft of the manuscript, and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

Funding Open access funding is provided by TU Wien (TUW). The TU Wien University Library supported this work through its Open Access Funding Program.

Availability of data and material Not applicable

Code availability Not applicable

Declarations

Ethics approval Not applicable

Consent to participate Not applicable

Consent for publication Not applicable

Competing interests The author declares no competing interests.

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