A Harmonic Balance Method for an Eddy Current Problem with Circuit Coupling

Der Wissenschaftsfonds.

Karl Hollaus, Valentin Hanser, Michael Leumüller and Markus Schöbinger

Introduction

A technique to solve a nonlinear eddy current problem (ECP) in the steady state coupled with a voltage-driven circuit using the harmonic balance method (HBM) with the fixed-point method (FPM) is presented. The HBM is preferred to avoid long simulation times of a time stepping method (TSM) [1] to reach the steady state. Exploiting the FPM the entire nonlinear system of the finite element method (FEM) can be split into small ones according to the considered harmonics [2]. The harmonics of the unknown circuit currents are also included in the small systems. A slice model of a synchronous machine is studied to demonstrate the feasibility.

Numerical Techniques

A 3D nonlinear ECP connected to a three-phase power supply via resistors R_s is solved, where the magnetic vector potential (MVP) $\boldsymbol{A}$ and the currents of the three phases i^U, i^V and i^W are unknown.

A) Eddy Current Problem:

The partial differential equation with $\boldsymbol{A}$ reads as

$$\operatorname{curl} \nu(\boldsymbol{A}) \operatorname{curl}(\boldsymbol{A}) + \sigma \frac{\partial}{\partial_t} \boldsymbol{A} = \boldsymbol{J}_0.$$

B) Field Circuit Coupling:

The equations for coupling the ECP with the voltage-driven circuits are

$$i^X R_s + N_T N_S \frac{\partial}{\partial_t} \int_\Omega \boldsymbol{h}_{BS}^X \operatorname{curl}(\boldsymbol{A})\, d\Omega = u^X, \qquad (1)$$

where $X \in \{U, V, W\}$.

C) Weak Form:

The field circuit coupling (1) is integrated over the time and multiplied by -1 to obtain a symmetric FEM system:

Find $\boldsymbol{A}(\boldsymbol{x}, t) \in V$, $i^U(t)$, $i^V(t)$ and $i^W(t) \in C(0, T)$, so that

$$N_S \int_\Omega \nu_{FP} \operatorname{curl}(\boldsymbol{A}) \operatorname{curl}(\boldsymbol{v})\, d\Omega - N_S \int_\Omega \sigma \frac{\partial}{\partial t} \boldsymbol{A} \boldsymbol{v}\, d\Omega$$
$$-N_T N_S \int_\Omega (i^U \boldsymbol{h}_{BS}^U + i^V \boldsymbol{h}_{BS}^V + i^W \boldsymbol{h}_{BS}^W) \operatorname{curl}(\boldsymbol{v})\, d\Omega =$$
$$N_S \int_\Omega (\nu_{FP} - \nu(|\boldsymbol{B}|)) \operatorname{curl}(\boldsymbol{A}) \operatorname{curl}(\boldsymbol{v})\, d\Omega,$$
$$-N_T N_S \int_\Omega \boldsymbol{h}_{BS}^X \operatorname{curl}(\boldsymbol{A})\, d\Omega\, q^X - R_s \int_0^t i^X\, dt\, q^X = -\int_0^t u^X\, dt\, q^X$$

for all $\boldsymbol{v}(\boldsymbol{x}, t) \in V$ and $q^X(t) \in C(0, T)$ with suitable initial and boundary conditions holds, where $V = C(H(\operatorname{curl}, \Omega), (0, T))$, T is the simulated time, ν the magnetic reluctivity, $\boldsymbol{h}_{BS}^X$ the Biot-Savart field of the unit current of 1A, u^X a supply voltage, and N_S and N_T are the number of sheets in the iron core and turns in the windings, respectively.

D) Harmonic Balance Method:

The HBM is based on a truncated Fourier series

$$u(\boldsymbol{x}, t) = u_0(\boldsymbol{x}) + \sum_{\substack{l=1,\\ k=2l-1}}^{N} u_k^c(\boldsymbol{x}) \cos(k\omega t) + u_k^s(\boldsymbol{x}) \sin(k\omega t)$$

for the MVP $\boldsymbol{A}(\boldsymbol{x}, t)$ and the currents $i^X(t)$, where ω is the angular frequency and $\boldsymbol{x}$ the field point. Due to the FPM with a suitably chosen fixed-point reluctivity ν_{FP} the nonlinearity can be moved to the right hand side yielding the k^{th}-harmonic block

$$\begin{pmatrix} S_k & k\omega M_k - \Lambda_k^X \\ -\Lambda_k^X & diag(R_s) \end{pmatrix} \begin{pmatrix} \boldsymbol{a}_k \\ \boldsymbol{i}_k^X \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}_k(\nu(|\boldsymbol{B}|)) \\ \delta_{k1} \boldsymbol{u}_1^X \end{pmatrix} \qquad (2)$$

with $\boldsymbol{u}_1^X = (u_1^U, u_1^V, u_1^W)^T$, $\boldsymbol{i}_k^X = (i_k^U, i_k^V, i_k^W)^T$ and the Kronecker delta δ_{ij}. The entire nonlinear system with N harmonic blocks (2) is iteratively solved block by block with ascending harmonic order until it converges.

Numerical Example

A slice model of a synchronous machine [1] serves as a numerical example. Selected parameters are the fixed-point reluctivity ν_{FP}, the number of considered harmonics of $N = 3$, the FEM order of one, the conductivity of iron $\sigma = 2.08 \cdot 10^6$ S/m in S, $R_s = 1\Omega$, $N_T = 20$, $N_S = 200$ and the frequency of $f = 50$Hz.

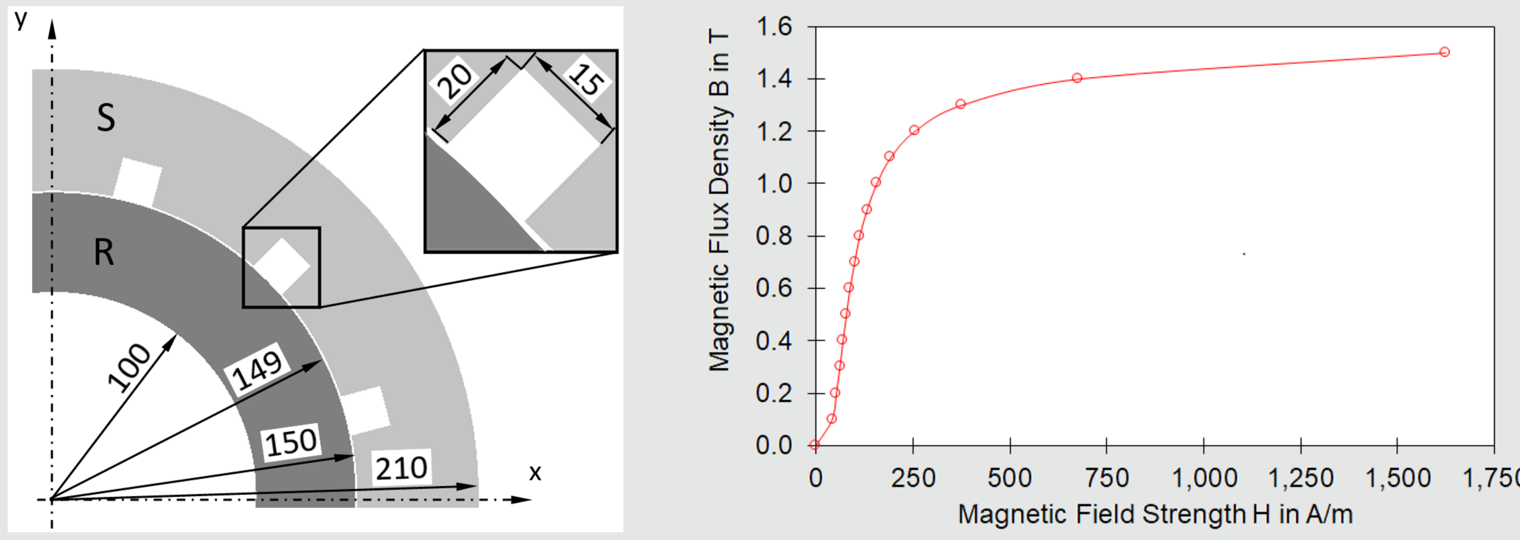

Fig. 1: One fourth of the slice model (left) with stator S and rotor R, dimensions are in mm. BH-curve (right).

Simulation Results

The comparison of the circuit currents i^X in Fig. 2 shows a good agreement for the steady state. Simulation results are summarized in Tabs. I and II.

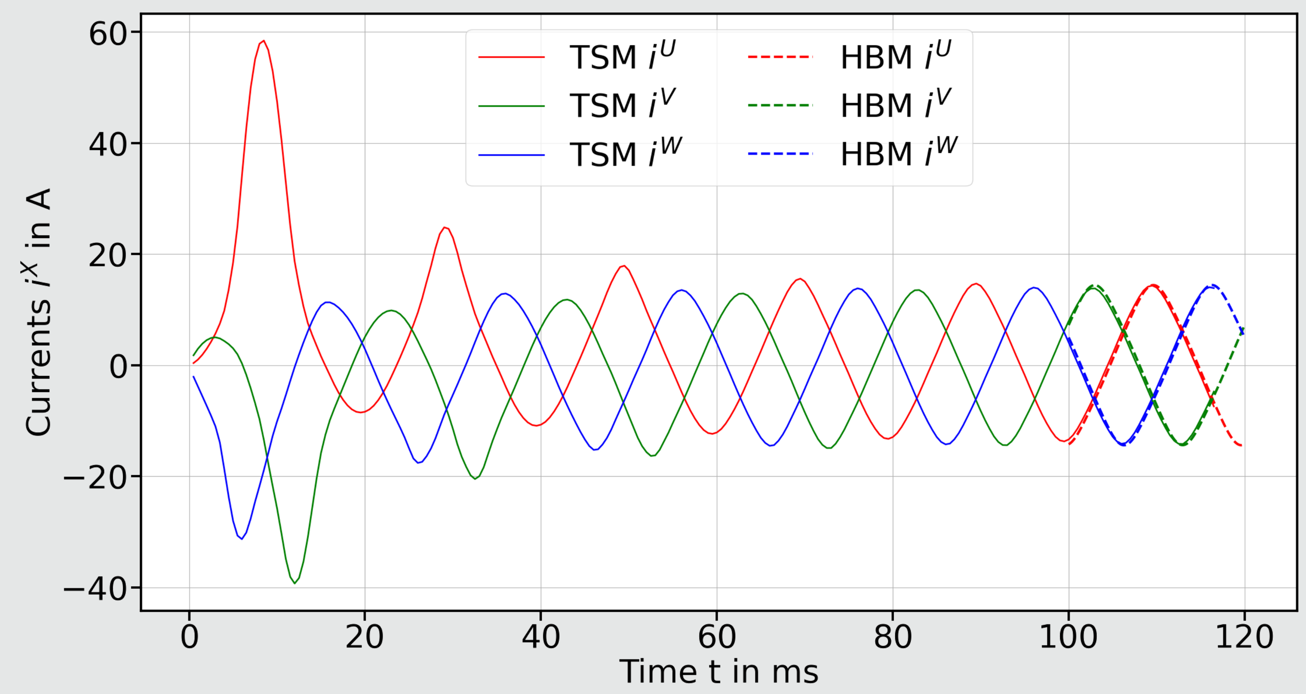

Fig. 2: Currents i^X of TSM and HBM with the peak value of the supply voltages of $\hat{u}_1^X$=150V.

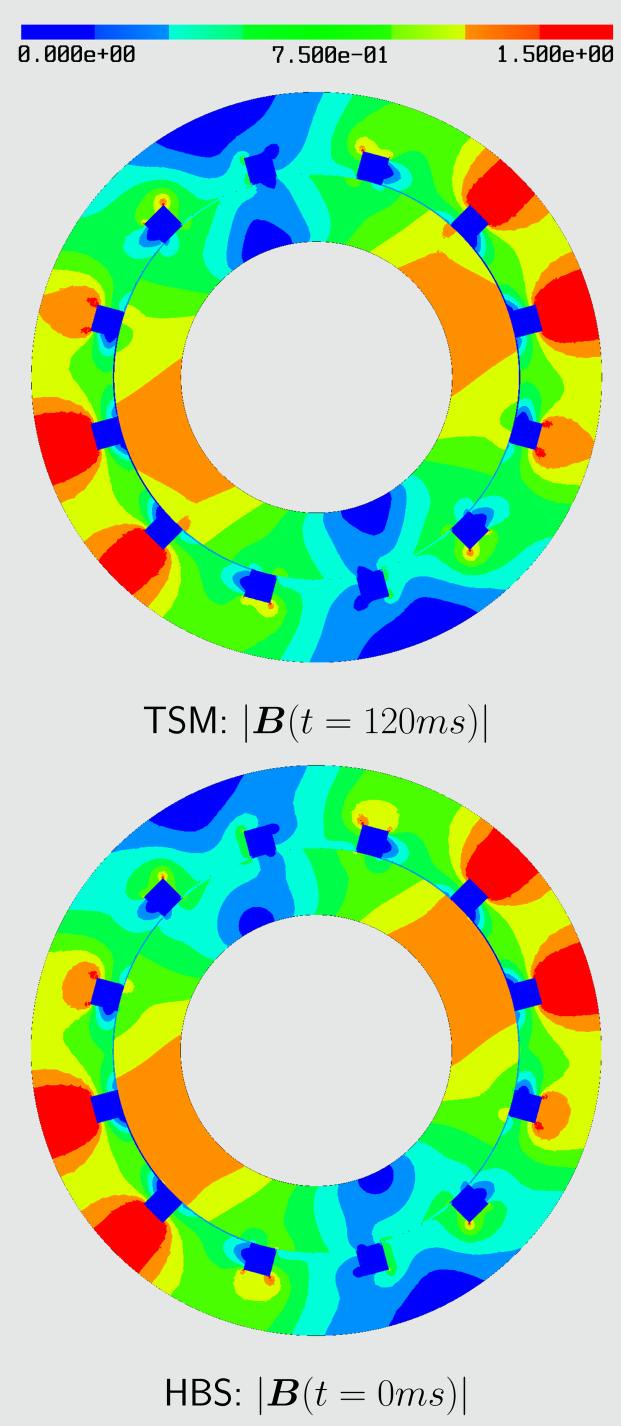

Fig. 3: Magnetic flux density $|\boldsymbol{B}|$ distributions.

TABLE I
Comparison of TSM with HBM

	P_{Loss}	DOFs	NLI	CT
Method	W	-	-	s
TSM	21.07	264,671	4,371	32,011
HBM	21.27	1,588,026	22	1,816

TABLE II
Convergence of HBM

ν_{FP}	P_{Loss}	CC	PM	α	NLI
m/H	W	-	-	-	-
5,000	13.91	n	n	1.0	37
3,000	14.04	n	n	1.0	32
5,000	21.23	y	n	1.0	51
4,750	21.23	y	n	1.0	49
2,000	21.24	y	y	1.0	27
1,300	21.28	y	y	1.0	25
800	21.27	y	y	0.5	31
800	21.27	y	y*)	0.5	22

P Losses, NLI no. nonlinear iterations, CT computation time, CC circuit coupling, PM Preis' method [3], relaxation method: $u = \alpha u + (1 - \alpha) u_0$, n no, y yes, *) double material updates

References

[1] K. Hollaus, V. Hanser, and M. Schöbinger, "Effective Material and Static Magnetic Field for the 2D/1D-Problem of Laminated Electrical Machines," *IEEE Trans. Magn.*, 2024, accepted for publication.

[2] S. Ausserhofer, O. Bíró, and K. Preis, "An Efficient Harmonic Balance Method for Nonlinear Eddy-Current Problems," *IEEE Trans. Magn.*, vol. 43, no. 4, pp. 1229–1232, 2007.

[3] O. Bíró and K. Preis, "Finite element calculation of time-periodic 3d eddy currents in nonlinear media," in *Advanced Computational Electromagnetics*, T. Homna, Ed. Budapest, Hungary: Elsevier, July 1995, pp. 62–74.

Supported by the Austrian Science Fund (FWF), Project P 31926

karl.hollaus@tuwien.ac.at