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Modeling Judicial Discretion with Nuanced Permissions

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Abstract. Judicial discretion is a central question in both the theory and the practice of law, but it received very little explicit attention from AI&Law yet. What is more, it is often considered as the limitation of what can be formalized in law, which might have serious implications for the future of computational law. In this paper, we introduce a deontic logic extended with nuanced permissions pursuing to grasp the characteristics, normative framework of and reasoning process in the discretionary decision-making of the judge. We illustrate the modeling capacity of the Discretionary Judicial Decision Logic (DJDL) by formalizing examples from an area of law where discretion plays an openly crucial role: family law, more precisely child custody cases.

Keywords. discretion, nuanced permissions, logical modeling, deontic logic

1. Judicial Decision-Making, Discretion in Child Custody, and Our Approach

When describing the judge's role in the law, discretion is traditionally considered among the limitations of norm-based derivation [1,2,3,4,5], which is otherwise taken as a central operating mode of judicial decision-making and reasoning in the continental legal tradition. Does being a limitation for norm-based derivation mean being the limitation of logical modeling as well? While this question has not been in the forefront of legal KR, recent legislation put it on the table. The notion of Automated Decision Making (ADM) was made known by the GDPR, which, however, left it undefined. The last two years have witnessed various regulatory initiatives all around Europe to delineate ADM and mark off its application's boundaries. A comparative analysis [6] and volume of reports [7] highlight the trend of allowing the application of ADM in public law only if the used AI is based on rule- and knowledge-based systems. However, the cases normally involving discretionary power are excluded even from symbolic AI-based ADM. This suggests that legal knowledge representation (KR) does not—or maybe even cannot—provide tools for grasping and representing what happens when exercising discretionary powers, without which no automation is imaginable. This paper is a first step in challenging this impression by investigating some relevant aspects of discretion and how they relate to those phenomena that in computational law we traditionally consider graspable through formal languages, specifically in deontic logic.

A conference paper's space does not allow for going into the centuries-long discussion in legal theory of discretion's characteristics, thus we only highlight some fundamental points. One crucial point is that a discretionary decision is not made *contrary to* the law, it is exactly the law that imposes it. Another crucial aspect is that, according to the common understanding, discretion suggests some kind of *freedom*: it seems at first sight—given the lack of norms from which the decision would be derivable—that it is up to the judge, to her own assessment. But, of course, this does not mean arbitrariness: decisions made with discretionary power do show reasoning patterns and *can be wrong*. We approach this twoness (freedom still not arbitrary) with a formalism pursuing to grasp the basic characteristics of this strange kind of normative space and its boundaries.

In family law, the role of discretion is very obviously present and openly crucial. In a child custody trial, there is only one principle to serve: the best interest of the child. Given the variety of features influencing what is in the best interest of a child in a given family (divorce) situation, the Hungarian[1] statutory law—both the Civil Code and the Family Act—dedicatedly leaves it to the judge to deliberate these features pointing out only three aspects of what the 'best interest' consists of: *ensuring the child's physical, mental, and moral development in the most favorable way*. It was the Supreme Court's Directive nr 17[2] that provided some further details pointing out that the judge must carefully examine whether the parents' personalities, lifestyles, and moral qualities—witnessed in their actions—make them suitable for raising the child. The Directive also prescribed to examine the housing and financial situation of the parties, as to which parent's environment is more secure for the maintenance, care, and health care of the child. A crucial aspect is the opinion of the child herself about which parent she would like to live with, which the judge has to take into account. What this judicial obligation exactly means though depends on some future aspects, like the age and the judgmental capacity of the child (in terms of maturity but also being or not under the influence, effect of trauma, etc.).[3]

Directive 17 also provides an important framing regarding how *free* the judge is in her deliberation, which practically designates the content and boundaries of discretion. It starts with demanding *being comprehensive* in the deliberation: the court must decide by exploring and considering together all the circumstances affecting the child's life. But then adds that the requirement of taking all the circumstances into account does not contradict the fact—and so such an assessment—that some circumstances affecting the child's life may be particularly important in some specific cases. However, the Directive also points out that giving over-excessive weight to a particular circumstance and ignoring other aspects goes against properly enforcing the child's best interest when deciding custody. It is crucial to see what this set of provisions declares: after requiring comprehensivity, it affirms discretion in the assessment, particularly in assigning the importance, weight of each factor. In our formal approach, we will highlight these aspects, and in the examples show boundaries this set of provisions imposes on the freedom of discretion: it does not mean arbitrariness, randomness, or personal taste. This means that a decision made using discretionary power still can be subject to review in appeal: it can be wrong.

In this paper, we use an extended deontic logic to describe the choice a judge can (or cannot make); hence, the permission modality will be our central notion. The basic per-

[1] To stay consistent, we have looked into one national legal system but it could have been any one of them, we suppose that the main setup and considerations are similar (at least in the European countries).

[2] https://kuria-birosag.hu/hu/iranyelvek/17-szamu-iranyelv Strictly speaking this Directive is not applicable anymore, but judges tend to act still in its spirit hence its content still prevails in the Hungarian judicature.

[3] The variance of the meaning of this judicial obligation and its representation using bipolar formal argumentation has been explored in [8]

mission indicates that in a custody case, the judge can choose Parent A and can choose Parent B for custody.[4] That is, we will use this (or any other) modality only with this specific scope. In some special situations, a judge cannot choose a parent, that is, there is a prohibition to make a decision to give custody to him. This prohibition can be statutorily established; for instance, a parent who is in prison is, by definition, unfit to have custody. Even more rarely, the prohibition can come from a previous court decision: the judge can exclude a parent, who commits a crime against his child, from custody and also for future children of his. If there is no prohibition, the judge can choose either parent to give the custody to. How the deliberation works will be described by three types of permission: *better-not permissions*, *rather-so permissions*, and *neutral permissions*. This variety allows us to express how taking into account different aspects in a case makes the freedom (permission) to choose one or another parent nuanced—leading up finally to a decision. Better-not permissions are based on properties that a parent has that are not great, that is, are against the parent's fit due to showing in a direction where the child's development could be suboptimal, but it is still permitted to give the child to this parent. For example, living in a sketchy neighborhood or working a lot. The rather-so permissions are based on properties that are permitted to have and even count in favor of the parent. For example, being supportive about the child's connection with the other parent or having a high income. Lastly, neutral permissions are established by properties that should not be evaluated, neither positively, nor negatively. For example, the European Convention on Human Rights excludes ethnicity from the aspects to be evaluated. We refer to these as *nuanced permissions*. Next to assessing characteristics as positive or negative, when describing judicial evaluation, we find explicit references to weights; hence the permissions will carry a weight in the formalism to be comparable and to establish the final decision.

The paper is organized as follows. Sec. 2 discusses examples of real-life child custody cases. Sec. 3 introduces our formal approach. Sec. 3.1 recalls *SDL*, our base logic, and then Sec. 3.2 extends SDL to include the nuanced permissions as well as weighted formulas. Sec. 4 puts our framework into practice by formalizing the cases of Sec. 2.

2. Child Custody Cases

We introduce two—naturally, significantly simplified—examples from real-life custody cases from Hungarian case law and describe the reasoning patterns of different judges.

Example 1 (Case 1). *There are two children. Parent A's personality is balanced and reliable, and her parenting style is warm and lenient. Parent B has a psychiatric condition, his obsessive-compulsive disorder is known, and treated with drugs, therefore, his personality is balanced, but he still can respond unpredictably in stressful situations. Despite the differences, both parents have been found suitable for raising children in the average range, however, the judge found that raising two children alone would mean an elevated level of stress resulting in threatening the stability of Parent B. The smaller child is only 3 years old, at which age children need primarily their mother. And since the judge found that the connection between the siblings is strong, separating them would be traumatic, she decided to give custody of both of them to Parent A.*

[4] For simplifying, we do not consider the possibility of (equally) shared custody cases, however, where more than two agents can get the custody can be handled after generalizing the formalism.

The following case shows two turns of reasoning as the sentence of the first instance court was appealed. We chose this to point out how a decision maker who is "free" can still be wrong, that is, to approach the boundaries of the "freedom" in discretion.

Example 2 (Case 2)**.** *In this case, both parents can provide good conditions objectively (income, neighborhood, etc) as well as both being capable of raising children appropriately (both in the average range). Where differences can be found are the following: Parent B does not have a good relationship with the child (actually hit him once in a conflict). And Parent A, when the deterioration of the marriage seemed to be final, decided to move first to Austria, then to Germany with the child without the consent of Parent B. The judge at the court of the first instance found that moving abroad was not only illegal but also exceptionally estranging the child from Parent B. While the child declared his wish to live with Parent A, the judge found that this is due to this conscious estrangement hence not to be taken seriously and thus granted full custody to Parent B.*

Parent A appealed the ruling of the first instance. The appeal court reevaluated the situation and found that the first judge had given unjustifiedly great weight to the move and the implied estrangement not evaluating the properly the opinion of the child himself who is old enough to have a reliable opinion immune to significant manipulation, having an established point of view in which the bad behavior of Parent B rather played a role. Hence, the appeal judge granted full custody to Parent A overwriting the first judgment.

3. Formal Framework

Here, we present the formal framework for modeling the discretionary judicial decision-making process. First, in Section 3.1, we recall Standard Deontic Logic (*SDL*) [9], the best-known deontic logic. Then, in Section 3.2, we extend this logic to model key aspects of the judicial decision-making process, including the properties held/conditions provided by the parents, the classification of these properties and conditions as positive or negative, and the weight assigned to each, resulting in the logic *DJDL*.

3.1. Base Logic: Standard Deontic Logic (SDL)

The formulas of *SDL* are built using the following grammar (*Atom* is the set of atomic propositions): $\phi ::= p \in Atom \mid \neg\phi \mid (\phi \vee \phi) \mid \mathscr{P}\phi$. Here, $\neg$ is the classical negation, $\vee$ the disjunction, and the connectives $\wedge, \rightarrow, \leftrightarrow$ as well as the constants $\top$ and $\bot$ are classically defined as usual. We take the permissions operator $\mathscr{P}\phi$ to mean "it is permitted (for the judge) to make a decision that results in ϕ being true"; for example, it is permitted to give the child to a parent that has a good income. Prohibition to make a decision that results in ϕ is defined as the absence of permission $\mathscr{F}\phi := \neg\mathscr{P}\phi$. The obligation to make a decision that results in ϕ being true is $\mathscr{O}\phi := \neg\mathscr{P}\neg\phi$. The logic *SDL* is axiomatized by the modal logic *KD*, and modeled using Kripke semantics with a serial relation.

Unlike the common practice in modern descriptions of *SDL*, where obligation is treated as the undefined modality, we introduced permission as the undefined one. This approach is of course not novel; the "father of deontic logic", von Wright himself introduced his first system this way in 1951 [9]. This approach fits the centrality of permission in our proposal. Defining prohibition as the lack of permission is usually an unwanted consequence of such an approach though converging to the closed-world assumption.

This approach in our case can be defended as one highlighting that, in the reason-giving where the judge explains how she concluded a decision, she has to list and explain all her considerations (resulting in the form of various nuanced permissions introduced below in 3.2). That is, in this interpretation, these permissions are indeed strong permissions (even if they are not explicitly imposed in the statutes as we usually take when talking about strong permissions). The reader still not liking having the permission as the undefined modality can take obligation being so, this will not affect the results described below.

3.2. Extension: Discretionary Judicial Decision Logic DJDL

In order to express the properties of the family and of the different parents, e.g. their mental state, financial situation, etc., we extend the logic *SDL* with a global modality, $\Box\phi$, meaning that it is legally established that ϕ is true. We use the atoms A and B to say "the child goes to Parent A resp. Parent B" and, for simplicity, we take the properties to be atomic formulas p_1, p_2, p_3, ... Then the formula $\Box(A \to p)$ expresses: "it is legally established that if a child goes to Parent A, a child goes to a place where p is true".[5] The modality $\Box$ is, as usual, axiomatized by the modal logic S5. Its dual, $\Diamond\phi$, means "it is legally possible that ϕ is true" (it is not settled in the eye of the law that $\neg\phi$).

Within the permission, we distinguish between better-not permission (where certain judicial decisions are discouraged), rather-so permissions (where the decisions are encouraged), and neutral permissions. To define these different permissions, we extend the logic with ceteris-paribus preferences, which isolate the effect of a single property. This allows the court to fairly compare parents based on specific characteristics, like stability, without being influenced by other factors like income or education. Ceteris paribus preferences in logic were first introduced in [11] and later incorporated into modal logic in [12]. From [12], we use the modality $\boxdot^{\Gamma}\phi$, where Γ is a set of *SDL*-formulas. This operator is read as "in all better worlds agreeing on the truth of the formulas in Γ, ϕ is true". Its dual, $\diamond^{\Gamma}\phi$, means "there is a better world that agrees on Γ, where ϕ is true". This preference order is reflexive and transitive, and, therefore, $\boxdot^{\Gamma}$ is axiomatized by S4.

Additionally, to express the importance of each property, we introduce formulas of the form $\langle p, n\rangle$. Here, p is an atom, and n is a natural number between 1 and 100 representing the weight of p. We recognize that other approaches, such as using real numbers, rational numbers, or larger ranges, are also possible, and worth exploring in future work, as well as the use of compound formulas over atomic propositions. Moreover, we introduce formulas $p \succeq q$ to express an ordering of the weights of the properties. Using such an expression, we can indicate constraints on the judge's evaluation established, for instance, by case law as a fixed hierarchy and prevent a judge from making arbitrary decisions based on subjective weights. However, within the constraints, assigning the weights accounts for one parameter of judicial discretion itself. We restrict this comparison to atoms, as these express the properties of the parents. The grammar of the language $\mathscr{L}_{DJDL}$ is the following:

$$\phi ::= p \in Atom \mid (p \succeq p) \mid (\phi \vee \phi) \mid \neg\phi \mid \mathscr{P}\phi \mid \Box\phi \mid \boxdot^{\Gamma}\phi \mid \{\langle p, n\rangle : 1 \leq n \leq 100\}$$

when $\Gamma = \emptyset$, we write $\boxdot\phi$, and we write $p \succ q := p \succeq q \wedge \neg(q \succeq p)$.

[5] Interpreting the $\Box$ modality as *legally established* was introduced to computational law in [10]. It is to be understood as proven at the trial, settled by the judge, statutorily established, etc. showing a settlement in the eye of the law.

Definition 3 (DJDL)**.** *The logic DJDL extends the logic SDL with the S5 axioms for $\Box$, the S4-axioms for $\boxdot^\Gamma$, and the following:*

I.	$\langle p,n\rangle \wedge \langle q,m\rangle \rightarrow p \succeq q$ *for* $n \geq m$	***VI.***	$\Box\phi \rightarrow \boxdot\phi$
II.	$\langle p,n\rangle \wedge \langle q,m\rangle \rightarrow p \succ q$ *for* $n > m$	***VII.***	$\boxdot^\Delta\phi \rightarrow \boxdot^\Gamma\phi$ *where* $\Delta \subseteq \Gamma$
III.	$p \succeq q \wedge q \succeq r \rightarrow p \succeq r$	***VIII.***	$\gamma \rightarrow \boxdot^\Gamma\gamma$
IV.	$p \succeq p$	***IX.***	$\neg\gamma \rightarrow \boxdot^\Gamma\neg\gamma$
V.	$p \succeq q \vee q \succeq p$		

The inference rules are modus ponens, $\mathcal{O}$-, $\Box$- and $\boxdot^\Gamma$-necessitation.

Axioms I. and II. say that the ordering of two atoms is derived from the weight. As the weight is a natural number, the ordering $\succeq$ has the same properties as the ordering of natural numbers. Therefore, axioms III.- V. express that $\succeq$ is a transitive, reflexive and total order. Axioms VI.- IX. are imported from [12]. Axiom VI. describes the interaction between $\Box$ and $\boxdot^\Gamma$: when ϕ is true in all worlds, it is true in all better worlds. Additionally, if ϕ is true in all better worlds that agree on the formulas in Δ and $\Delta \subseteq \Gamma$, ϕ is true in all better worlds that agree on Γ (Ax. VII.). Axioms VIII. resp. IX. say that if $\gamma \in \Gamma$ and γ resp. $\neg\gamma$ is true, then in all better worlds that agree on the formulas in Γ (and thus also γ), γ resp. $\neg\gamma$ is true. *DJDL* is sound and complete w.r.t to the following semantics (see proof in the online appendix).

Definition 4 (Semantics)**.** *A DJDL-model $F = \langle W,R,P,f,V\rangle$ is a tuple where $W \neq \emptyset$ is a set of worlds $w,v,u,\ldots$, $R \subseteq W \times W$ a serial relation, $P \subseteq W \times W$ a transitive and reflexive relation between worlds, $f : Atom \rightarrow \{1,\ldots,100\}$ a function, and V a valuation function $V : Atom \rightarrow P(W)$.*

Definition 5. *Let $M = \langle W,R,P,f,V\rangle$ be a DJDL-model. The satisfaction at any $w \in W$:*

$M,w \models p$	*iff*	$w \in V(p)$ *for* $p \in Atom$
$M,w \models \neg\phi$	*iff*	$M,w \not\models \phi$
$M,w \models \phi \vee \psi$	*iff*	$M,w \models \phi$ *or* $M,w \models \psi$
$M,w \models \mathscr{P}\phi$	*iff*	$\exists u \in W$ *such that* wRu *and* $M,u \models \phi$
$M,w \models \Box\phi$	*iff*	$\forall u \in W\ M,u \models \phi$
$M,w \models \boxdot^\Gamma\phi$	*iff*	$\forall u \in W$ *if* $w \equiv^M_\Gamma u$ *and* wPu, *then* $M,u \models \phi$
$M,w \models \langle p,n\rangle$	*iff*	$f(p) = n$ *for* $p \in Atom$
$M,w \models p \succeq q$	*iff*	$f(p) \geq f(q)$ *for* $p,q \in Atom$

Where $w \equiv^M_\Gamma v$ is true iff for all $\gamma \in \Gamma$ $M,w \models \gamma$ iff $M,v \models \gamma$, and $\geq$ is a transitive, reflexive, total ordering on the natural numbers. We say that a formula ϕ is valid, written $\models \phi$, if $M,w \models \phi$ for all worlds w of any DJDL-model M.

3.2.1. Nuanced permissions

To define the three permissions, we need to express the preference of a formula ϕ over its negation. We denote this as $Pref(\phi)$, and define it below, using the ceteris paribus modality $\boxdot^\Gamma$. Intuitively, when comparing two formulas ϕ and its negation our objective is to ensure agreement on the truth of all formulas, except for ϕ. This reflects our concern to refrain from comparing any two arbitrary worlds; rather, we view it as a decision-making process: a choice has to be made between doing ϕ or $\neg\phi$, and therefore, we focus on comparing worlds where this is the only different aspect.

Definition 6. *For ϕ is p or $\neg p$, $g(\phi)$ is the set Atom without p, i.e. $g(\phi) = Atom \setminus \{p\}$. Pref is defined as follows:* $Pref(\phi) := \Box(\phi \to \boxminus^{g(\phi)}\phi) \wedge \Diamond(\neg\phi \wedge \diamondminus^{g(\phi)}\phi)$

The intuition behind $Pref(\phi)$ is as follows. Take ϕ to be p and consider two scenarios, one where p is true and a better one agreeing on all atoms except the truth of p. If the second scenario is considered "better," then p cannot be false in it, as this suggests that $\neg p$ is preferred over p. Moreover, the second conjunct says that there is a world where $\neg p$ is true, and a better one where p is true. This formula ensures that if p is preferred over $\neg p$, $\neg p$ is not preferred over p. We use $Pref$ to define the three permissions, and the combination of these two conjuncts guarantees a clear differentiation between them, and that better-not, rather-so, and neutral permissions are all distinct.

Definition 7 (Nuanced Permissions)**.** *These are:*

- *The rather-so permission:* $\mathscr{P}^+(p) := \mathscr{P}p \wedge Pref(p)$
- *The better-not permission:* $\mathscr{P}^-(p) := \mathscr{P}p \wedge Pref(\neg p)$
- *The neutral permission:* $\mathscr{P}^0(p) := \mathscr{P}p \wedge \neg\mathscr{P}^+(p) \wedge \neg\mathscr{P}^-(p)$

A formula p is rather-so permitted, when p, everything else being equal, is preferred over $\neg p$ and thus $Pref(p)$ is true. Similarly for better-not permissions. For a neutral permission, there is no preference between p and $\neg p$. Additionally, not every formula p that is preferred over its negation (or vice versa) is a permitted action, as we see in the semantics in Def. 5. Therefore, we add the requirement that $\mathscr{P}p$ is true.

3.2.2. Final Decision

Thus, the judge has collected the facts about the family, the law, and the nuanced permissions for each property with weights assigned to them. We refer to the set of formulas containing all this information as Δ. We use Δ, and all that can be derived from Δ, to make the final decision. The derivation of a formula ϕ from a set of *DJDL*-formulas Δ', written as $\Delta' \vdash \phi$, is defined as usual. The judge extracts a final decision from the properties and the summation of their weights. For a property p such that $\langle p,n\rangle$ and $\Box(A \to p)$ are true, Parent A gets $+n$ points when $\mathscr{P}^+(p)$ is true, and $-n$ points if $\mathscr{P}^-(p)$ is true. We take $Weight = \{\langle p,n\rangle : \langle p,n\rangle \in \Delta\}$ to be the set of weights that the judge has assigned to each property. The points are calculated as follows: $points(par) = \sum_{\langle p,n\rangle \in S^+(par)} n - \sum_{\langle p,m\rangle \in S^-(par)} m$ where $S^*(par) = \{\langle p,n\rangle \in Weight : \Delta \vdash \mathscr{P}^*(p) \wedge \Box(par \to p)\}$ for $* \in \{+,-\}$ and $par \in \{A,B\}$. Then, $points(A) > points(B)$ implies that Parent A gets custody, and vice versa. The judge only does this calculation if it is not prohibited for the judge to give custody to any of the parents.

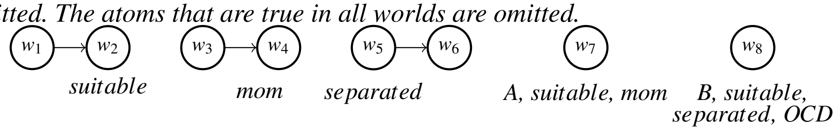
4. Framework into Practice

We formalize the reasoning of judges in the cases of Section 2. We do this by translating the case descriptions into formulas and providing a model such that the formulas are true in it. Additionally, by doing this, we show the consistency of our framework.

Example 8 (Case 1)**.** *We start by describing the formulas in Δ. Parent A is the mother and is suitable to raise a child, which we formalize as $\Box(A \to mom)$ and $\Box(A \to suitable)$.*

Parent B is also suitable to raise a child, has a treated obsessive-compulsive disorder, and can only take care of one child. This means that if a child goes to Parent B, the children are separated. We formalize these as $\Box(B \rightarrow suitable)$, $\Box(B \rightarrow OCD)$ *and* $\Box(B \rightarrow separated)$. *It is most important that the parent that the child goes to is a suitable parent. Then, whether the children go to the mom or whether the children are separated is less important than whether the parents are suitable. Additionally, because the OCD is treated, this property is not being held against Parent B.*

As there are two different children, we consider what is best for each child individually. The first child, $child1$, *is very young, and therefore, it is preferred that the child stays with the mother. Formally, we express this as* $\Box(child1 \rightarrow young)$ *and* $\Box(young \rightarrow \mathscr{P}^+(mom))$. *Additionally, we have the nuanced permissions* $\mathscr{P}^+(suitable)$, $\mathscr{P}^-(separated)$ *and* $\mathscr{P}^0(OCD)$. *A possible assignment is as follows:* $\langle suitable, 100\rangle$, $\langle mom, 10\rangle$, $\langle separated, 10\rangle$. *We model* Δ *in* M_1, *where* $M_1 = \langle W,R, P,f,V\rangle$ *such that:* $W = \{w_i : 1 \leq i \leq 8\}$, wRw_2, wRw_4, wRw_5 *for all* $w \in W$, *and* $f(suitable) = 100$, $f(mom) = 10$, *and* $f(separated) = 10$. *Note that while the function* f *is defined for all atoms* $p \in Atom$, *we only care about the properties that are rather-so and better-not permitted and do not explicitly state the weight of the others. The truth of the atoms in each world:* $V(suitable) = \{w_2, w_7, w_8\}$, $V(mom) = \{w_4, w_7\}$, $V(child1) = V(young) = W$, $V(separated) = \{w_5, w_8\}$, $V(OCD) = \{w_8\}$, $V(A) = \{w_7\}$ *and* $V(B) = \{w_8\}$. *The arrows of the relation* P *are depicted in the picture below, where the reflexive arrows are omitted. The atoms that are true in all worlds are omitted.*



From this assignment, we obtain $points(A) = f(suitable) + f(mom) = 110$ *and* $points(B) = f(suitable) - f(separated) = 90$, *and Parent A gets full custody. Additionally, we see that if Parent A had not been suitable, the child would have gone to Parent B, and if Parent B had been able to take care of both children, custody would still go to Parent A (the child's age being the bigger factor).*

For the second child, $child2$, *the valuation function changes in the following atoms:* $V(child2)$=W, $V(young) = \emptyset$, $V(mom) = \{w_7\}$, $V(child1) = \{w_4, w_7\}$. *Now, we additionally have* $\Box(A \rightarrow child1)$ *(meaning that if the child goes to Parent A, the child reunites with child1), and* $\mathscr{P}^+(child1)$ *(instead of* $\mathscr{P}^+(mom)$*). Therefore,* $points(A) > points(B)$ *and custody of child 2 goes to Parent A, as well.*

By formalizing Example 2, we show how our framework behaves when the boundaries of judicial discretion's freedom are reached making the judge's reasoning wrong.

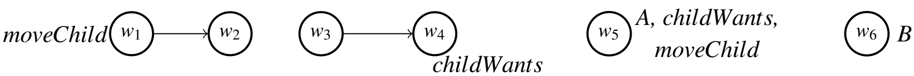
Example 9 (Case 2)**.** *Again, we start by describing the formulas in* Δ. *The child wants to be with Parent A, formalized as* $\Box(A \rightarrow childWants)$, *and Parent A moved the child illegally away from Parent B,* $\Box(A \rightarrow moveChild)$, *where* $\mathscr{P}^-(moveChild)$. *Both parents are suitable to raise the child. If a child is manipulated by a parent then the opinion of the child is not taken into account* $\Box(manipulated \rightarrow \mathscr{P}^0(childWants))$. *However, when a child's opinion is well founded (solid, well-reasoned, hence reliable), their preference needs to be taken into account more than all other properties, formalized as* $\Box(wellFounded \rightarrow \mathscr{P}^+(childWants) \land childWants \succeq p)$.

According to the first judge, the child has been manipulated by Parent A, resulting in $\mathscr{P}^0(childWants)$ *to be true. We model this in* M_2, *where* $M_2 = \langle W,R,P,f,V\rangle$, *such that*

$W = \{w_i : 1 \leq i \leq 4\}$, wRw_3 for all $w \in W$, $V(manipulated) = W$, $V(oldEnough) = \emptyset$, and the truth valuation of the remaining atoms is depicted in the figure below, as well as preference relation P, for which the reflexive arrows are omitted.

As there is only one better-not permission in M_2, $\mathscr{P}^-(moveChild)$, we conclude that $points(B) > points(A)$, and Parent B gets custody over the child.

However, the appeal judge reviewed the case and concluded the first judge did not take into account that Parent B had hit the child and the child was old enough to make a well-founded opinion, represented as $\Box(oldEnough \wedge badBehavior \rightarrow wellFounded)$, which resulted in $\mathscr{P}^+(childWants)$ to be true. The reasoning of the appeal court is shown in the following model $M_3 = \langle W, R, P, f, V\rangle$, such that $W = \{w_i : 1 \leq i \leq 6\}$,$wRw_1$, wRw_4 and wRw_5 for all $w \in W$. We take f such that $f(childWants) = 100$ and $f(moveChild) = 60$. Then, $V(manipulated) = \emptyset$, $V(oldEnough) = V(badBehavior) = V(wellFounded) = W$, and the truth valuation of the remaining atoms is depicted in the figure below, as well as preference relation P, for which the reflexive arrows are omitted.



We see that $M_3 \models \mathscr{P}^+(childWants)$ and thus $M_3 \not\models \mathscr{P}^0(childWants)$. Additionally, we see that $M_3 \models childWants \succeq p$ for all $p \in Atom$, as the assignment of childWants is the maximal amount of points. It follows that $points(A) = 100 - 60 = 40$, which is higher than $points(B) = 0$, and custody goes to Parent A.

5. Discussion, Related Work, and Conclusion

Discretion has received very little *explicit* attention in AI&Law so far, the notable exceptions coming from common law. From the AI side, a set of papers around the millennium addressed the issue of discretion directly in the context of prediction in Australian (family) case law using neural networks for knowledge discovery from database [13,14,15,16]. From the legal side of AI&Law, there is reflection to a greater extent, for instance, in [17] we can find an (informal, legal theoretical) analysis of some decision-supporting systems used in criminal sentencing in Commonwealth countries and their relation to the concept of similarity, case-based reasoning, and judicial discretion, but, indirectly, several studies in [18] address discretion as well. To our knowledge, no one tried to provide a modeling of discretion with (deontic) logic before.

Moving to the formalism side, the combination of preferences and deontic logic has been a recurring theme in the literature. Preference semantics have frequently been used to define dyadic deontic logic operators, as seen in works like [19], [20] and [21]. In these studies, deontic operators are defined using a preference relation between worlds, rather than the typical Kripke relation. The work in [21] even incorporates ceteris paribus preferences, defining "best" worlds based on this preference relation. However, our approach differs by attaching preferences exclusively to the permissions operator.

The three types of permission we define have appeared in various forms within philosophy, logic, and legal reasoning. For instance, better-not permissions and supererogatory actions, akin to our rather-so permissions, have been discussed for over two millennia by Sanskrit philosophers of the Mīmāṃsā school [22], and suboptimal permissions (actions that are minimally acceptable but sufficient) and supererogatory actions have been semantically explored by [23,24]. Furthermore, bilateral permissions, or liberties—similar to our neutral permissions—feature in the legal reasoning of [25] and [26], further described in [27]. Moreover, Meinong classified normative modalities into four categories: meritorious, required, excusable, and inexcusable, described in [28], originally from [29]. Also assigning weights to formulas has been done in, e.g., [30] or [31].

This paper assigns weights to the nuanced permissions—better-not, rather-so, and neutral—introduced in [32]. Here we present a sound and complete formalization, which we use to describe the discretionary reasoning process of the judge in a highly discretionary area, custody cases, highlighting the characteristics of "freedom" and still being restricted in how she makes the decision. This—to our knowledge—is the first attempt to use deontic logic explicitly to capture discretionary reasoning.

Of course, this model has various limitations paving several paths to future work. Formal-wise, for instance, the actual decision after summing up the weights, strictly speaking, "happens" outside of the logic. This on the one hand, does not contradict the idea of discretion falling outside of norm-based derivation, on the other hand though, we are going to investigate how to bring it inside the logic, and, optimally bridge it to a conclusion spelling out an *obligation* to give the custody to a given parent showing the binding nature of the judge's own discretionary reasoning leading to her decision. Also, as we extend *SDL*, our logic carries on all the commonly-known problems of *SDL*. Therefore, it is worth exploring which other logic to extend. For example, we used monadic operators while both the weight of a property as well as the type of permission that is assigned to it can be different in a different context (as we saw in Example 9, where the child's ability to form well-founded opinions caused a different type of permission). Dyadic operators are specifically suitable to model these and, therefore, worth investigating for future work. Conceptuality-wise, there are other notions handled in deontic logic that can play an important role: the judge operates in her discretion exercising her special *power* in the normative position sense [33,34], hence using that formal theory in capturing discretion is obviously worth investigating. Last but not least, about the formalism, deontic logic is of course not the only formalism that legal KR can offer: formal argumentation has several aspects suggesting its suitability for modeling discretionary decision-making, thus this is also among the first directions our present research is going to take.

This paper has a scope dedicated to a paradigmatic area of discretionary reasoning in single-case decision making. While we believe that many of the here presented formal insights can be generalized to cover other areas of law with explicit discretion (given, for instance, the binary-ness of a judicial decision both in civil and criminal law), first steps in future work will be taken investigating those other areas as well to identify further characteristsics. It is important to note that we by no means suggest that the here proposed formalism, even through developing applications, makes discretionary decision-making automatizable. We intentionally chose the area of law where, according to our current knowledge, automation is the least desirable or even imaginable: we aimed to show that, even there, discretion does not mean arbitrariness or randomness, and does not fall completely out of the scope what can be captured by using the tools of deontic logic and legal KR in general. But, instead of robots deciding child custody cases, what we do claim is that it is worth investigating where exactly the limitations of (deontic logic for) legal KR are and enable computational law to conquer new territories.

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