Neutron Interferometry for Understanding Quantum Mechanics

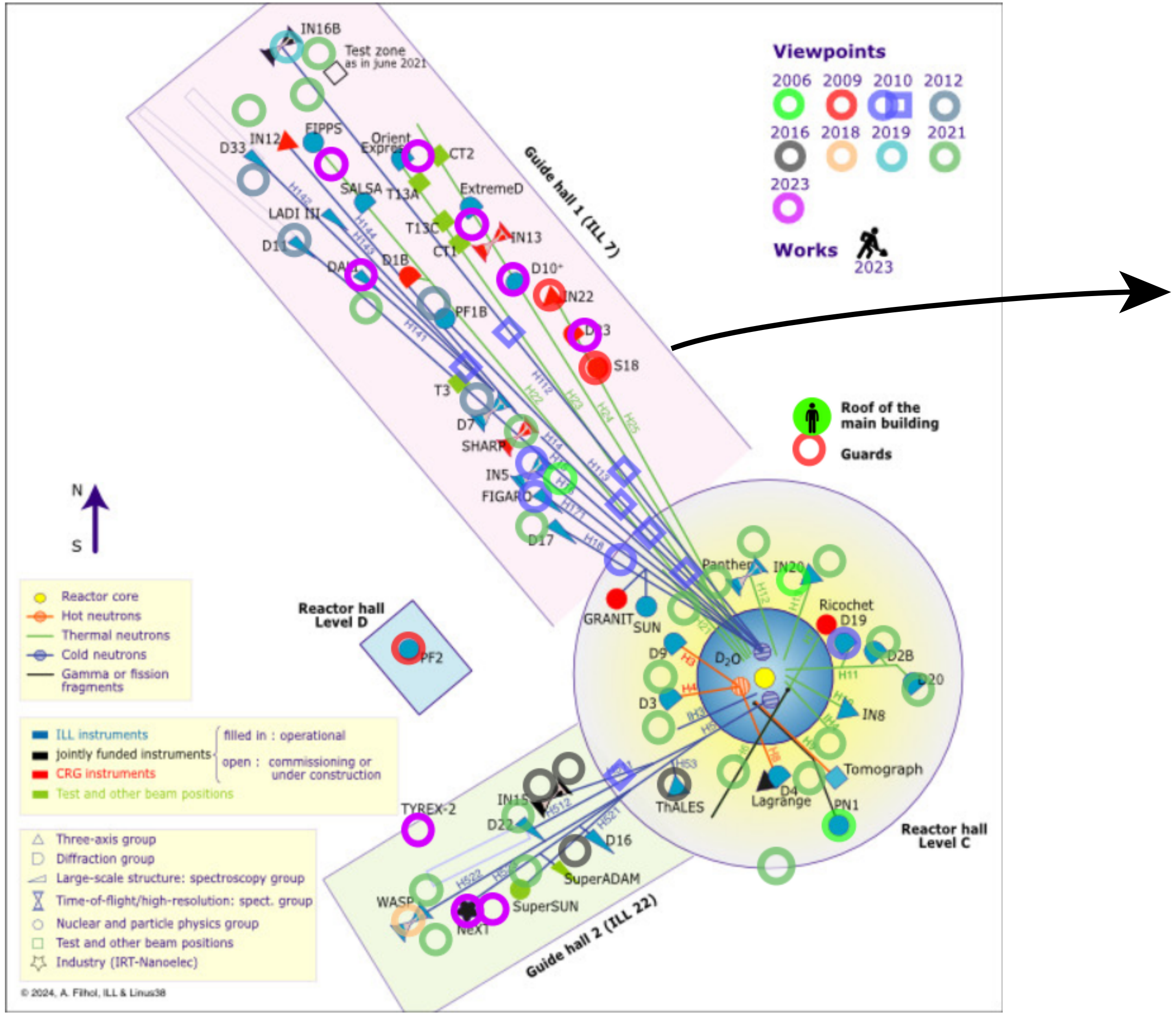
Hartmut Lemmel

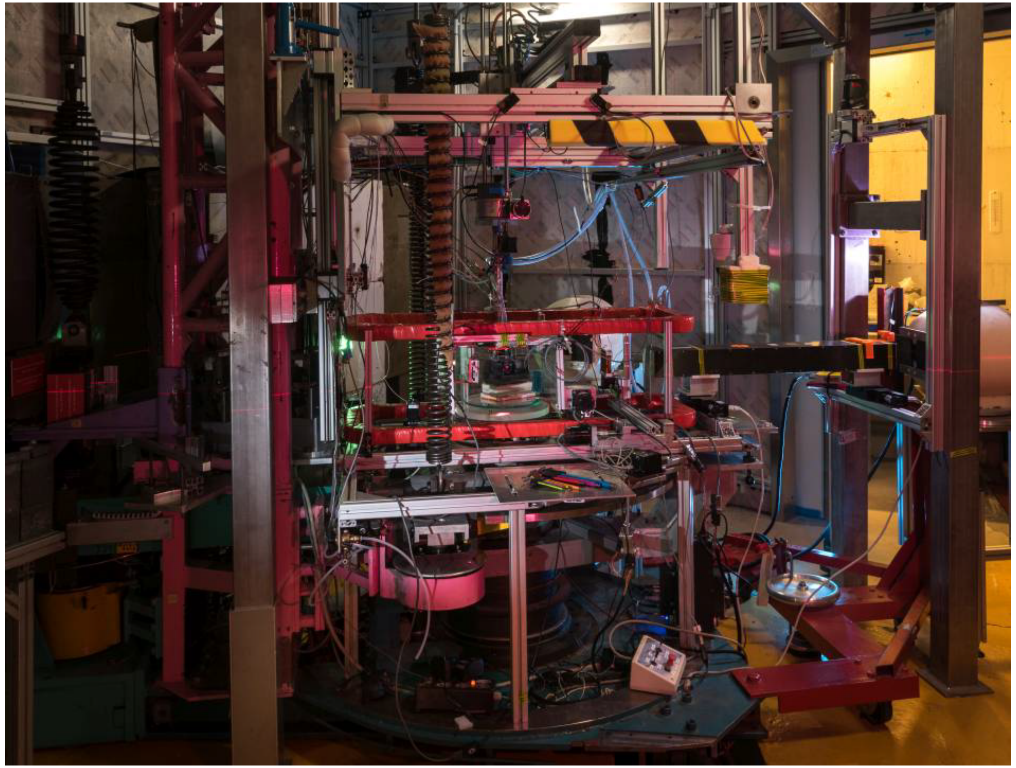
Atominstitut, TU Wien, Austria
Institut Laue-Langevin, Grenoble, France
hartmut.lemmel@tuwien.ac.at

Introduction
- Perfect crystal interferometers

Applications
- Historic experiments
- Quantum mechanics

Neutron Interferometry Setup S18 @ ILL

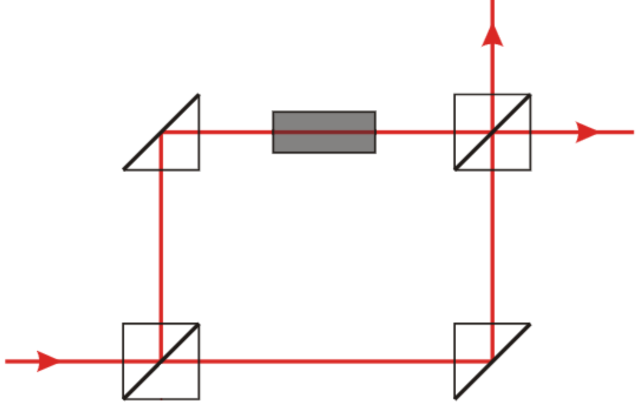




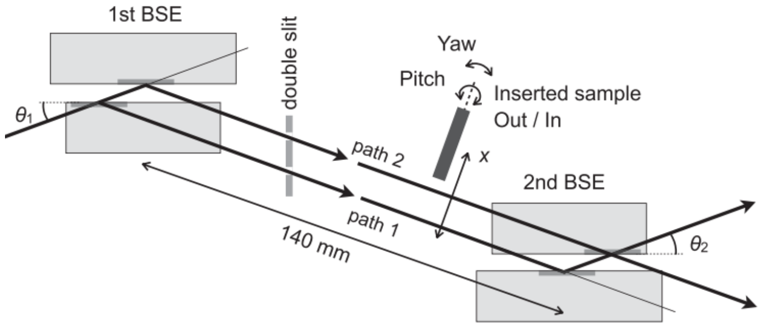
S18: CRG instrument, TU Wien, Atominstitut

Neutron beam splitters

Mach-Zehnder type interferometer

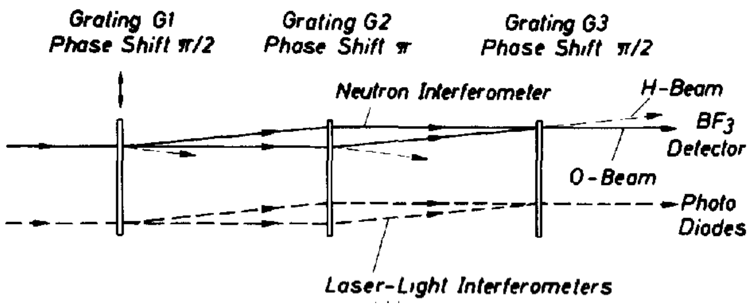


Semitransparent mirror



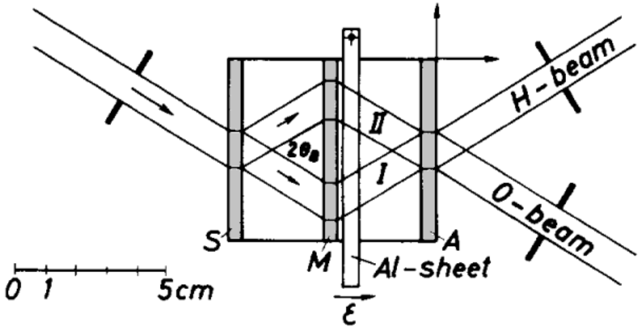
Fujiie, PRL 132, 023402 (2024)

Diffraction grating



Gruber et al, Phys. Lett. A 140, 363 (1989)

Bragg diffraction on crystals



Rauch, Treimer & Bonse, Phys. Lett. 47A, 369 (1974)
Rauch & Werner, Oxford Univ. Press 2014

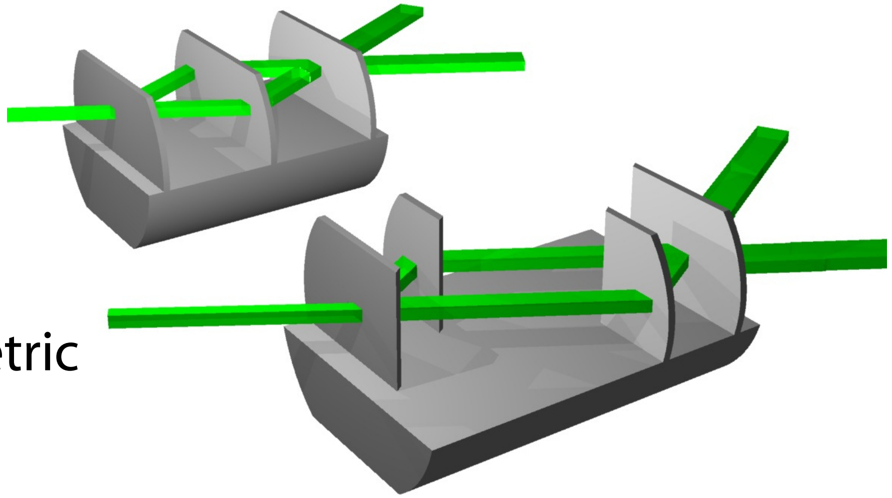
sub-mm beam separation

10 cm beam separation
Setups: ILL, NIST

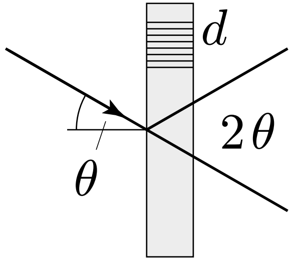
Perfect crystal neutron interferometers

symmetric

skew-symmetric



Bragg's law: $\lambda = 2\,d\,\sin\theta$

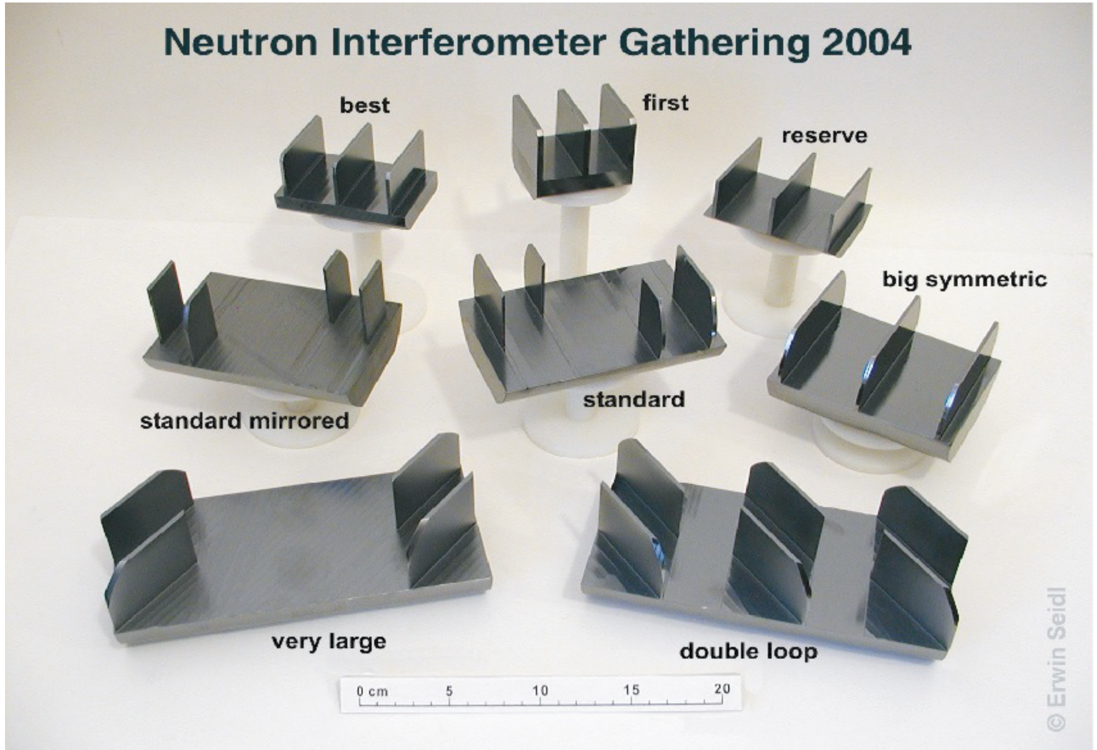


silicon 220:

$$d = 1.9\,\text{Å} = 0.19\,\text{nm}$$

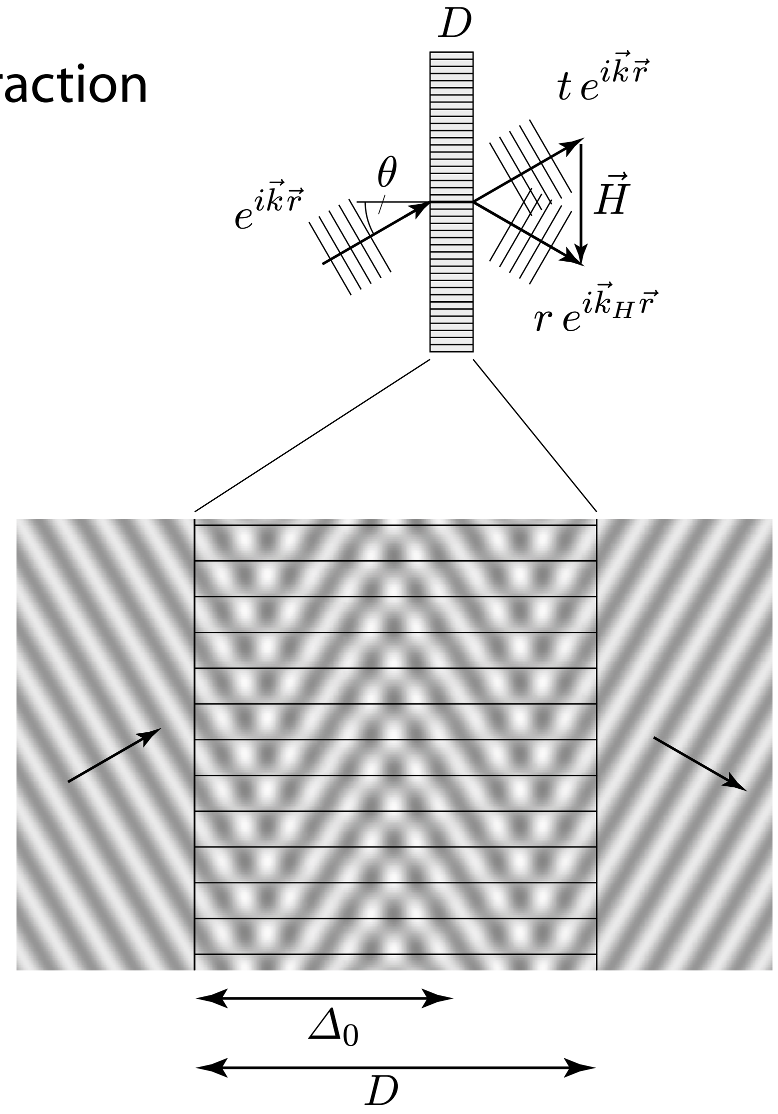
$$\theta = 30°: \quad \lambda = 1.9\,\text{Å}$$

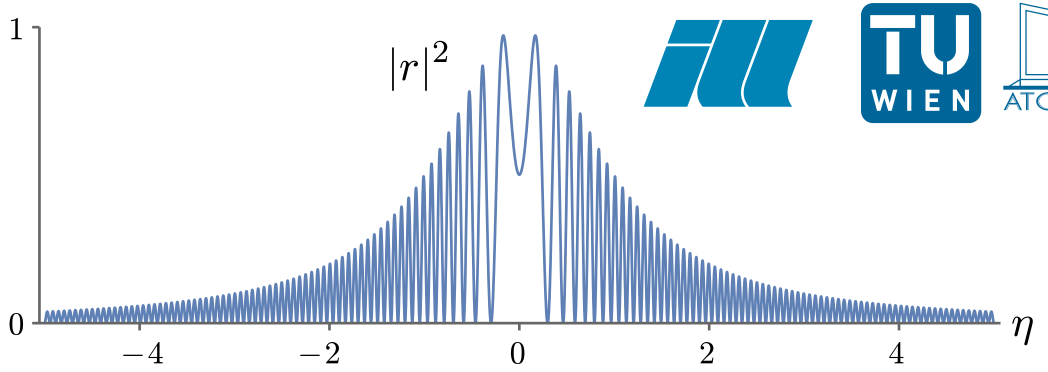
$$\theta = 45°: \quad \lambda = 2.7\,\text{Å}$$



Single beam splitter

Dynamical diffraction





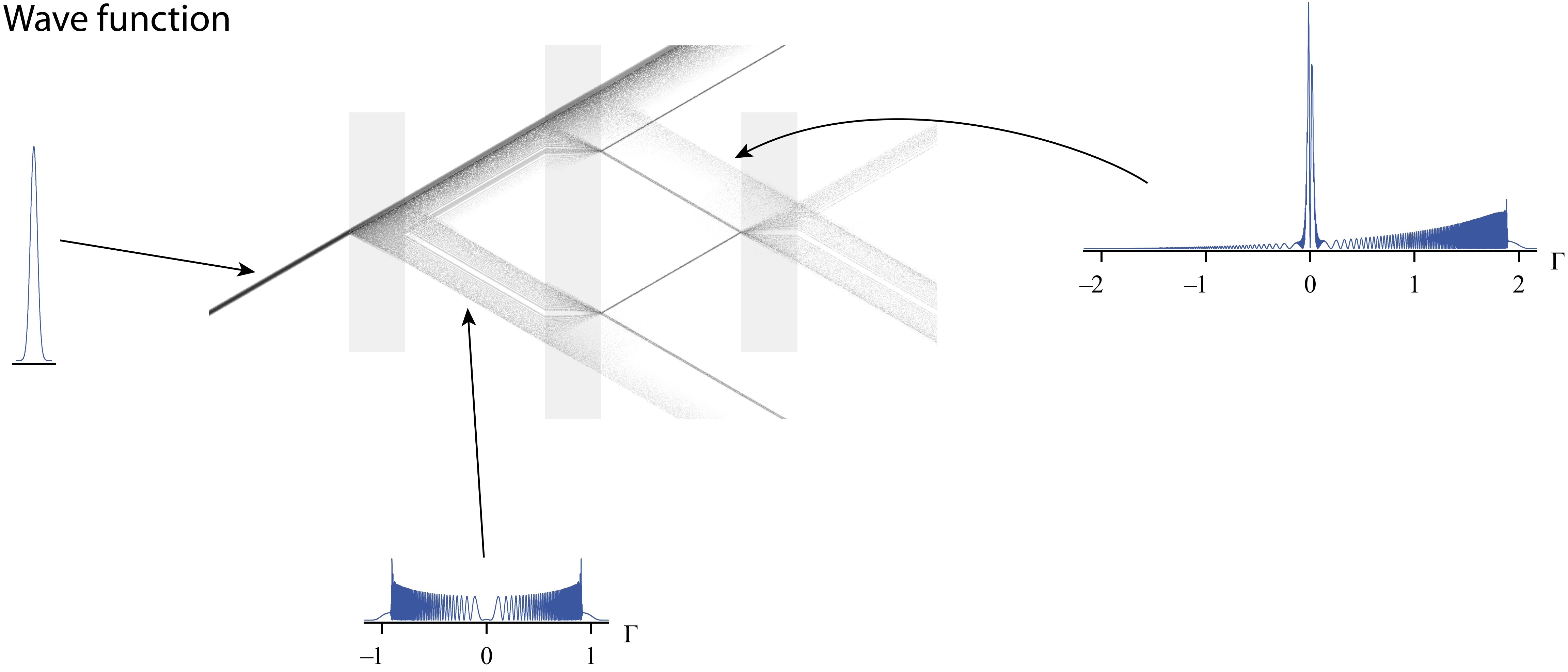
$$t = e^{i(-A-A\eta)}\left\{\cos\left(A\sqrt{1+\eta^2}\right) + \frac{i\,\eta}{\sqrt{1+\eta^2}}\sin\left(A\sqrt{1+\eta^2}\right)\right\}$$

$$r = e^{i(-A+A\,\eta+k\,z_0\,\eta\,v_0/\cos\theta_B)}\frac{-i}{\sqrt{1+\eta^2}}\sin\left(A\sqrt{1+\eta^2}\right)$$

$$\eta = -\delta\theta\,\sin(2\theta_B)/v_0$$

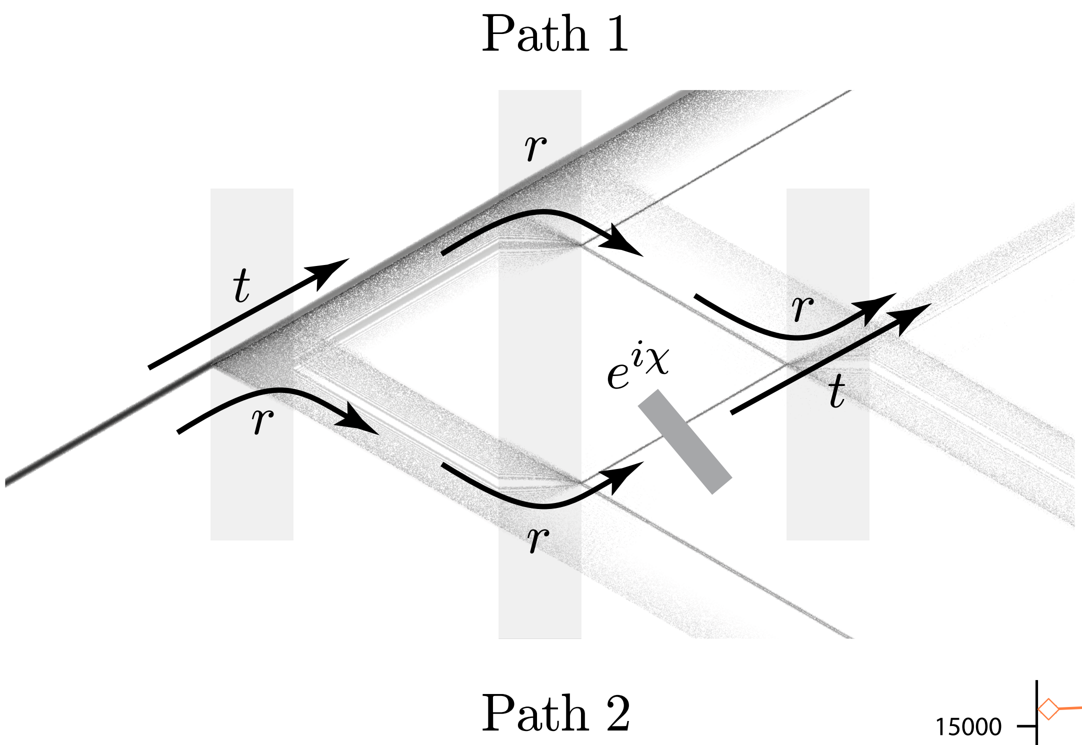
$$A = \frac{\pi D}{\Delta_0} = \frac{D\,k\,v_0}{2\cos\theta_B} \qquad v_0 = \frac{V_0}{E}$$

Wave function



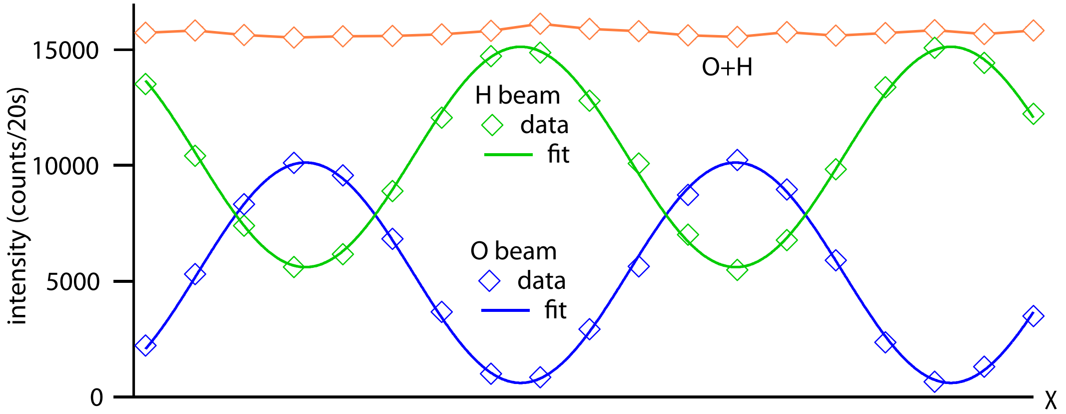
Whole interferometer

In practice



$$\psi_O = t\,r\,r + r\,r\,t\,e^{i\chi}$$
$$= t\,r\,r(1+e^{i\chi})$$
$$I_O = |\psi_O|^2 \propto (1 + V\cos\chi)$$

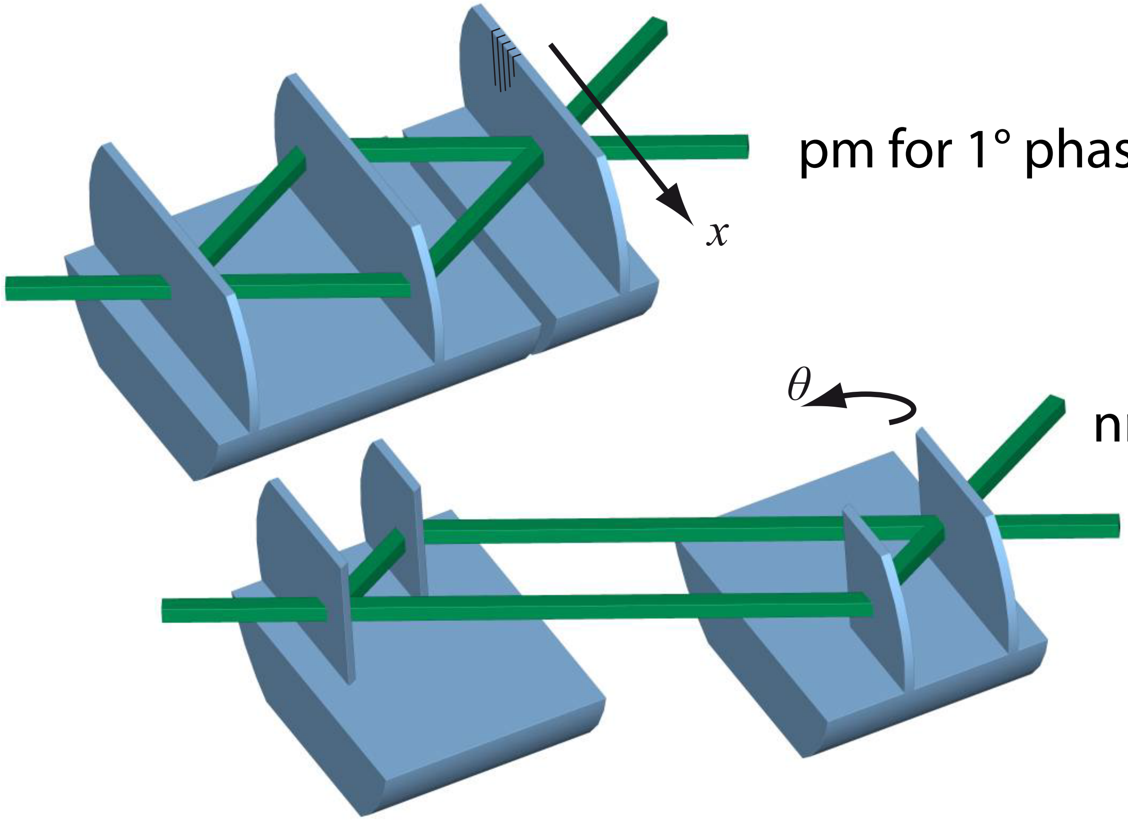
$$\psi_H = t\,r\,t + r\,r\,r\,e^{i\chi}$$



Stability

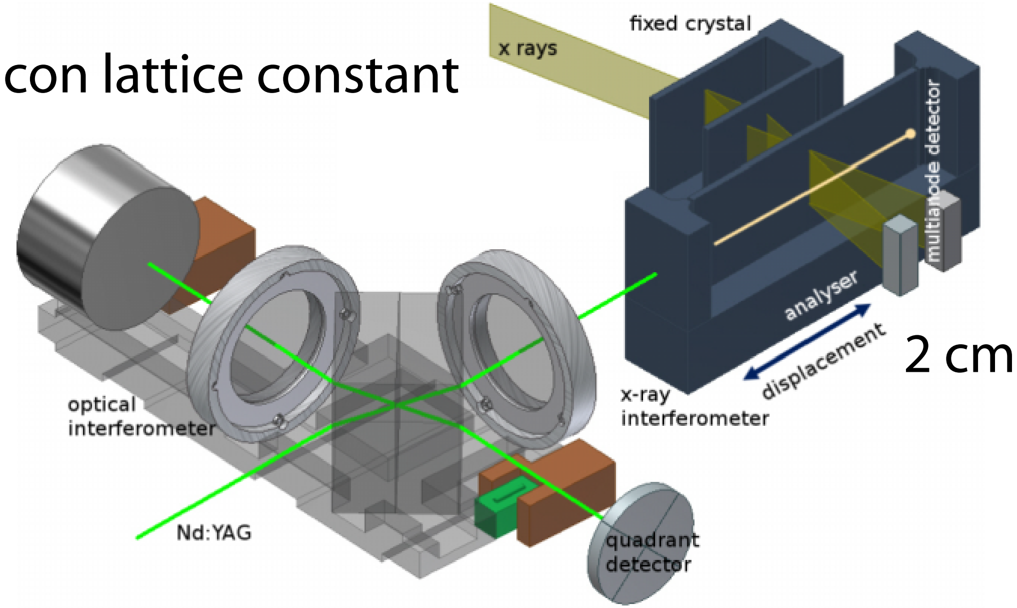
Phase and contrast influenced by

- lamella geometries
- lamella positions
- lamella angles
- vibrations
- temperature gradient
- temperature drift



Split-crystal interferometers

- Measurement of Silicon lattice constant



INRIM Torino (with x rays)
for Avogadro project

Massa et al., J. Phys. Chem. Ref. Data 44, 031208 (2015)

2 cm movement $\Rightarrow$ 10^8 finges

d220 = 192 014 711.98(34)

- Split-crystal neutron interferometer @ S18

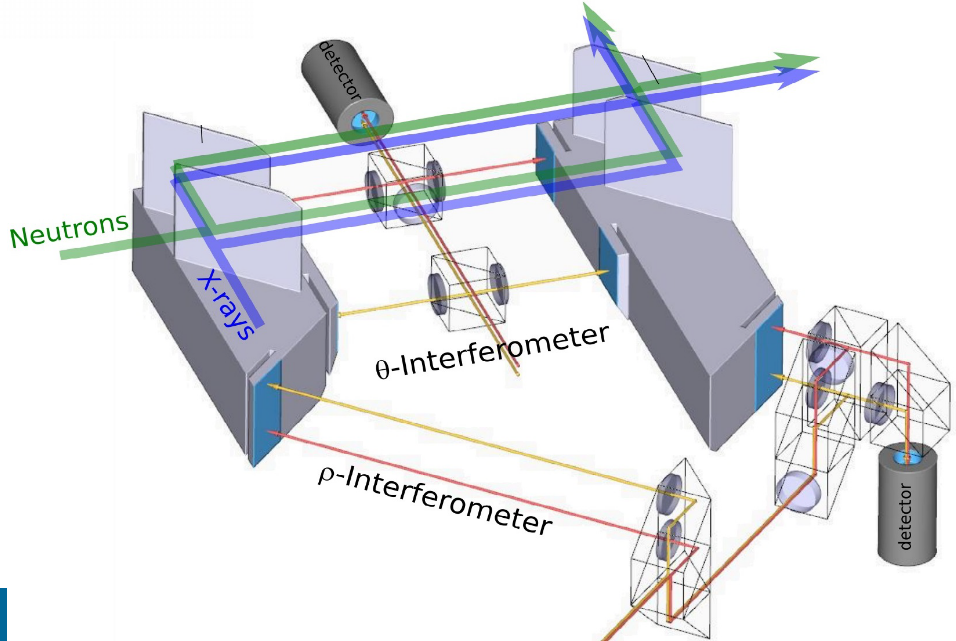
  larger interferometer loop

  more sample space

  neqstPi collaboration TU Wien, INRIM, ILL

  proof of principle

  Lemmel et al., J. Appl. Cryst. 55 (2022) 870–875

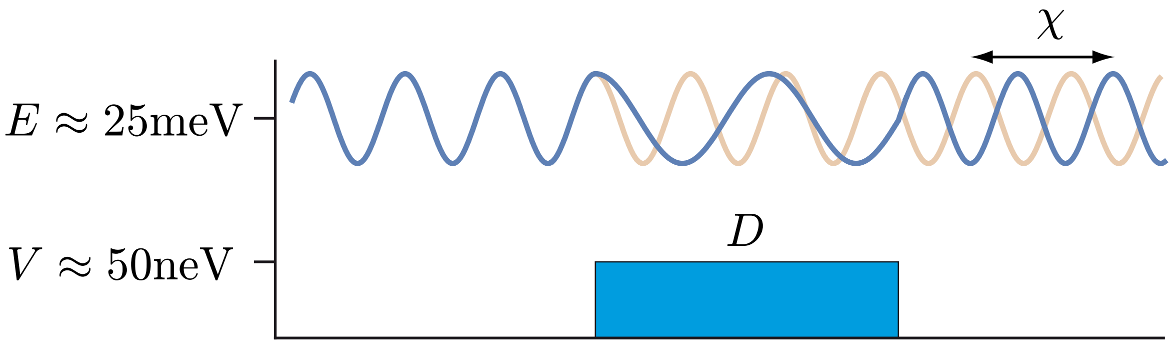


Applications

Scattering length measurement

Coherent scattering length b_c

- Describes interaction of n with matter
- Depends on nuclid
- Cannot be calculated from scratch (except lightest nuclids)
- Neutron interferometry



$$V = \frac{2\pi\hbar^2 N\, b_c}{m} \qquad \chi = -\lambda N\, b_c D$$

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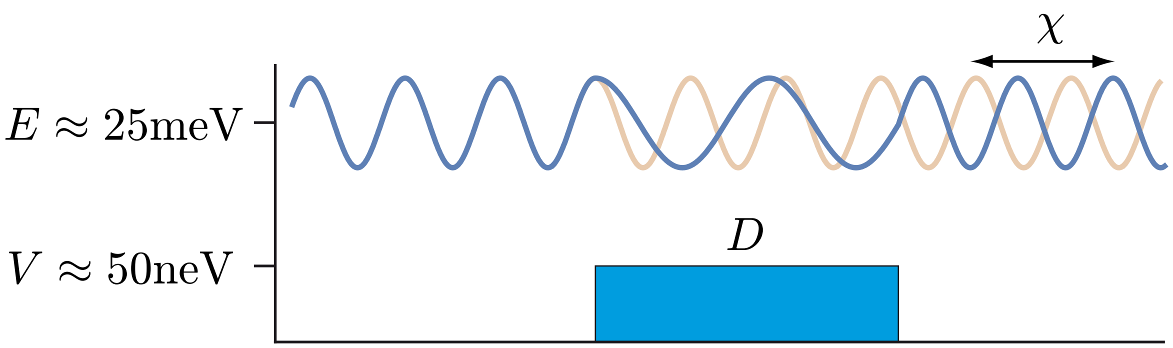

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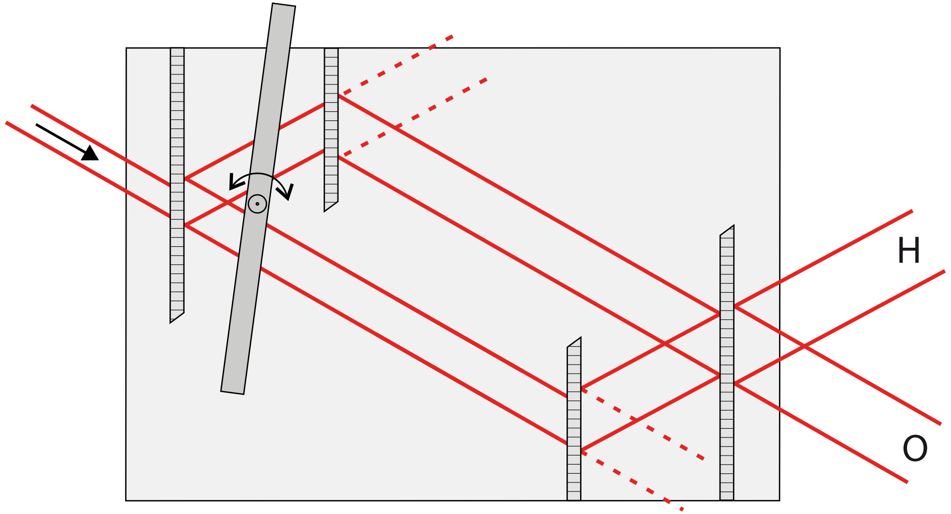

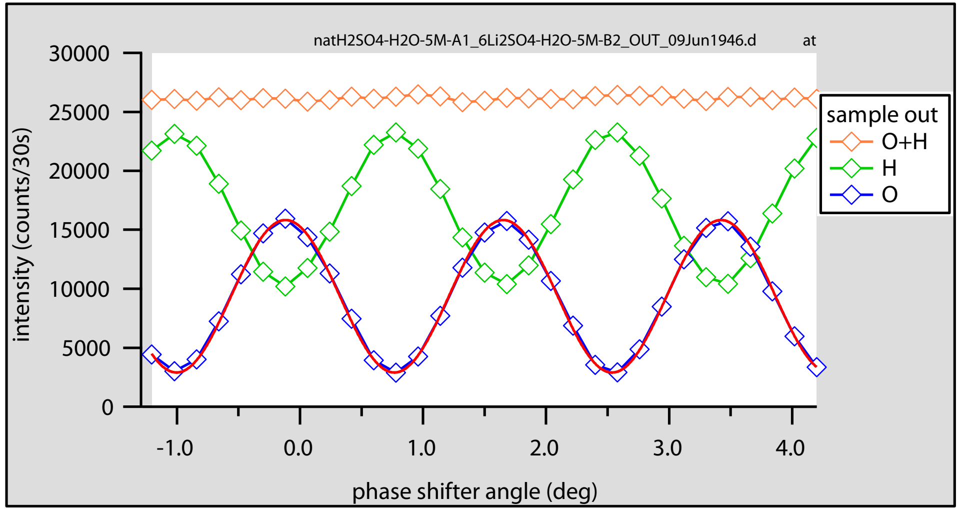

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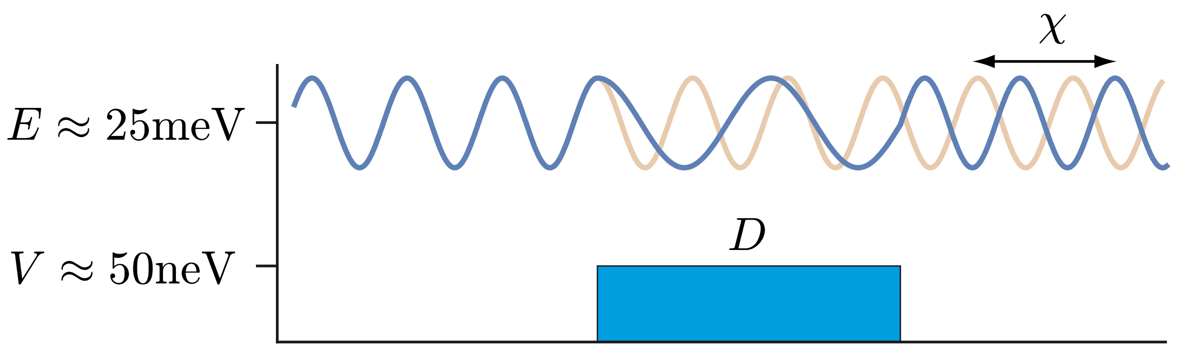

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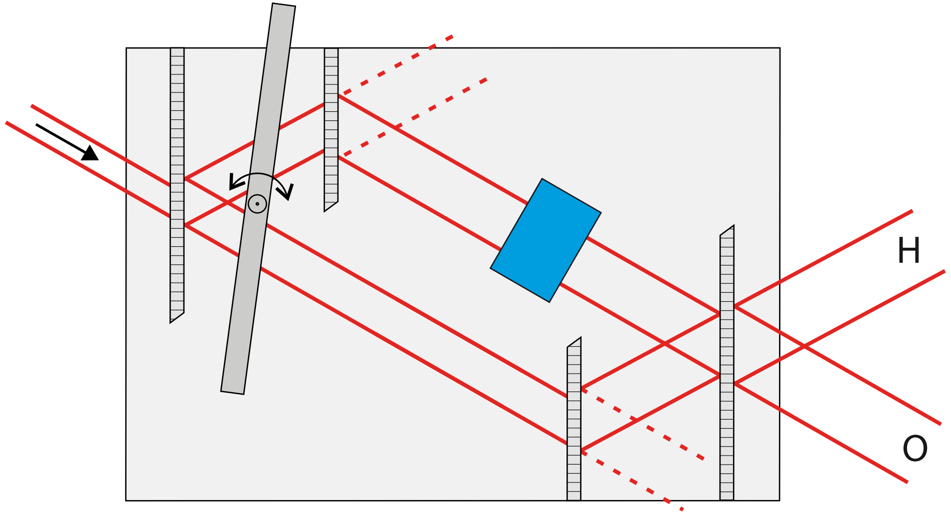
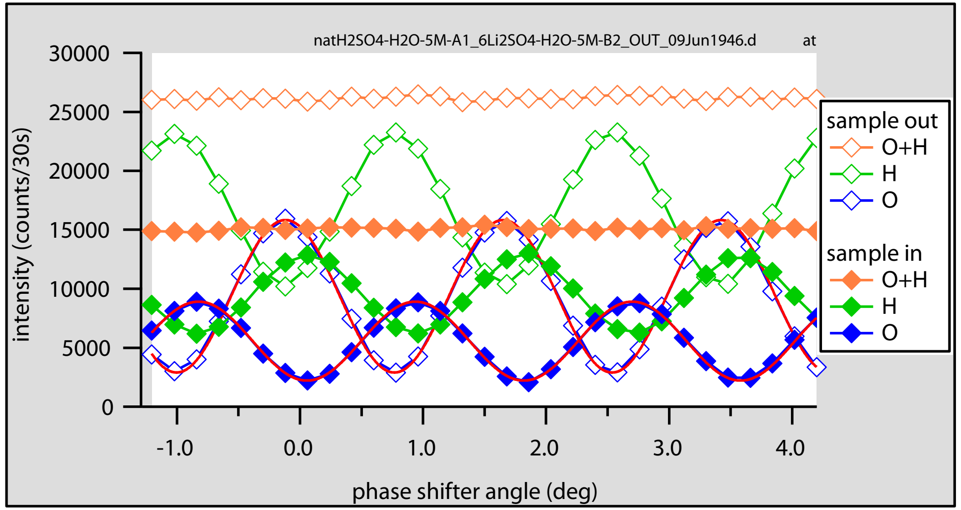
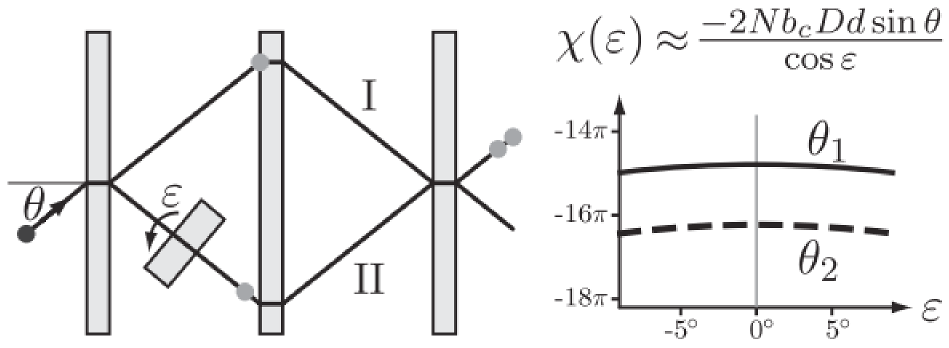




Scattering length measurement -- Accuracy

$\chi = -\lambda N b_c D$

- sample perp. to beam



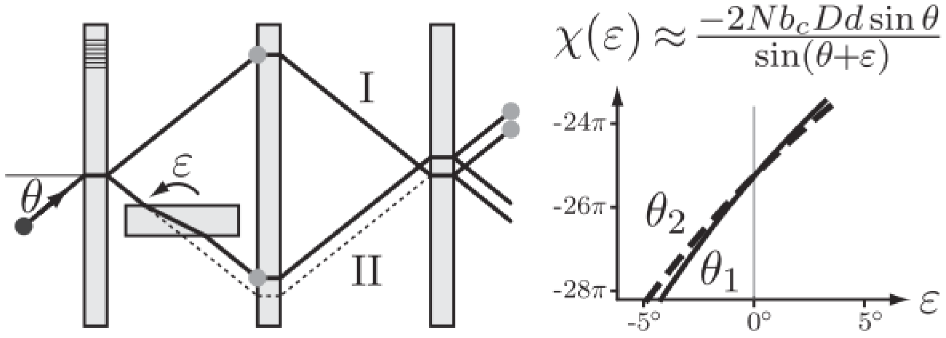
gaseous samples, 3H: accuracy = 5e-3

Rauch et al., Phys. Let. B 165 (1985) 39-42

liquid samples, 17O, 18O: accuracy = 1e-3

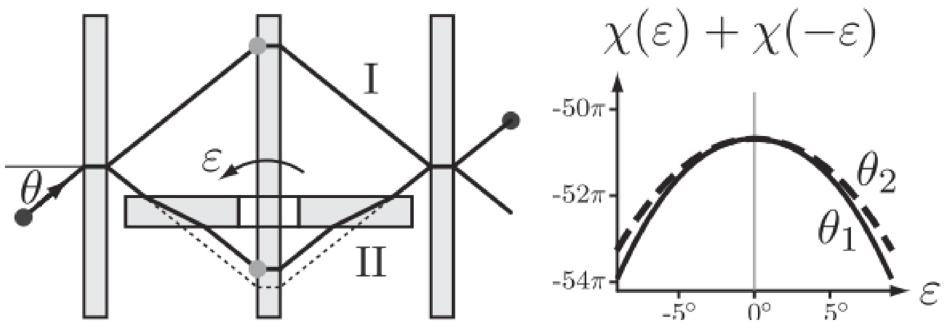
Fischer et al, J. Phys.: Condens. Matter 24 (2012) 505105

- sample parallel to crystal planes



→

solid sample, Si: accuracy = 1e-5



Lemmel et al, PRA 82 (2010) 033626; Lemmel, J. Opt. 16 (2014) 105704
Abbas et al., AIP Conf. Proc. 1349 (2011) 501–502

COW experiment

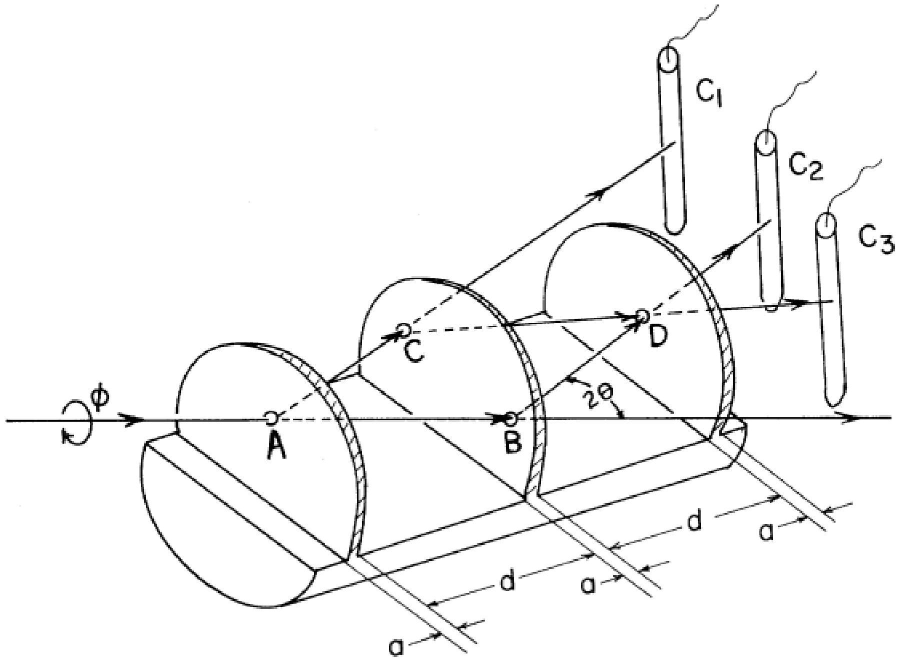
- Phase shift due to gravity

VOLUME 34, NUMBER 23 PHYSICAL REVIEW LETTERS 9 JUNE 1975

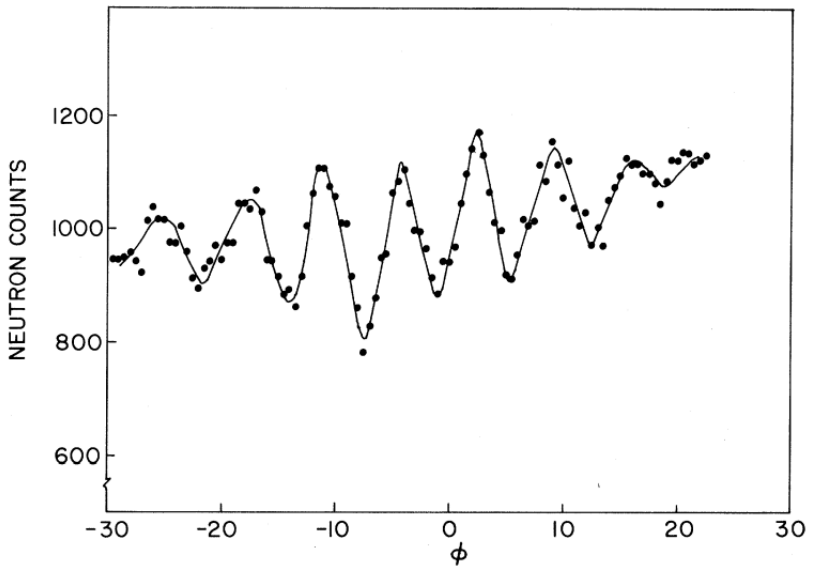
Observation of Gravitationally Induced Quantum Interference*

R. Colella and A. W. Overhauser
Department of Physics, Purdue University, West Lafayette, Indiana 47907

S. A. Werner
Scientific Research Staff, Ford Motor Company, Dearborn, Michigan 48121



$$\chi = -\frac{m^2 g\, A \sin\phi}{\hbar^2 k}$$



4π Symmetry

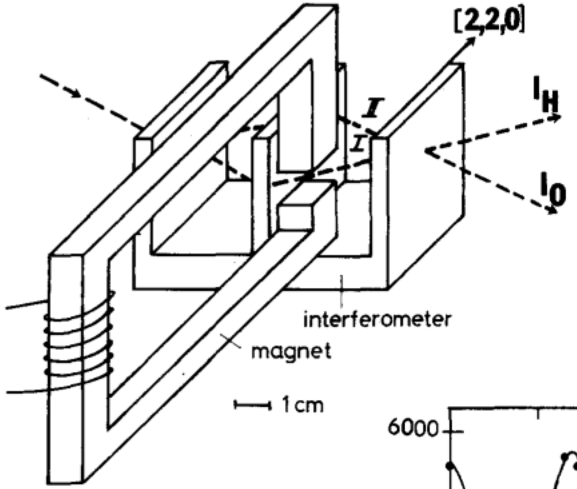
Volume 54A, number 6 PHYSICS LETTERS 20 October 1975

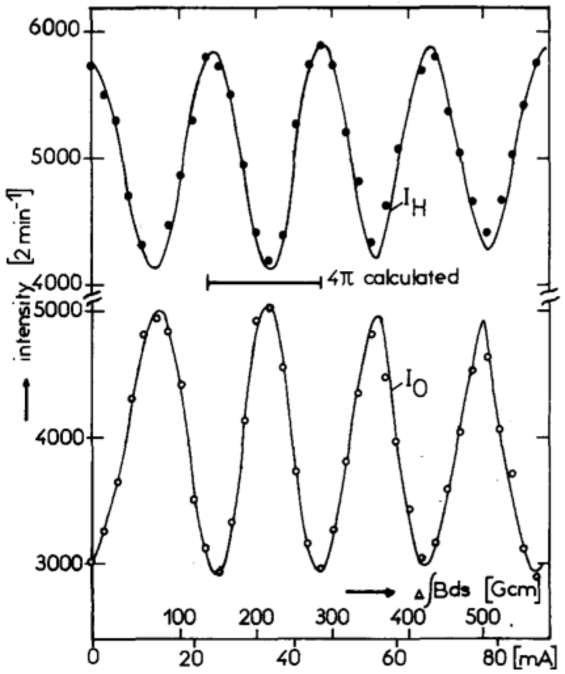
VERIFICATION OF COHERENT SPINOR ROTATION OF FERMIONS ☆

H. RAUCH, A. ZEILINGER, G. BADUREK, A. WILFING
Atominstitut der Oesterreichischen Hochschulen, A-1020 Wien, Austria

W. BAUSPIESS
Institut für Physik, Universität, D-46 Dortmund 50, Germany
and Institut Laue-Langevin, F-38042 Grenoble, France

U. BONSE
Institut für Physik, Universität, D-46 Dortmund 50, Germany

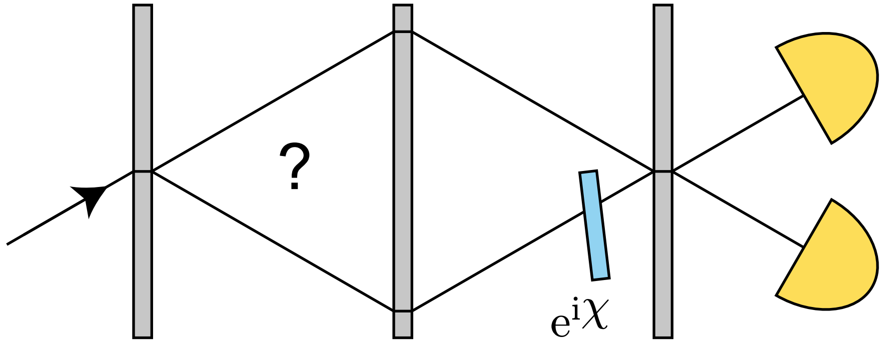




spin rotation by 2π = phase factor of −1

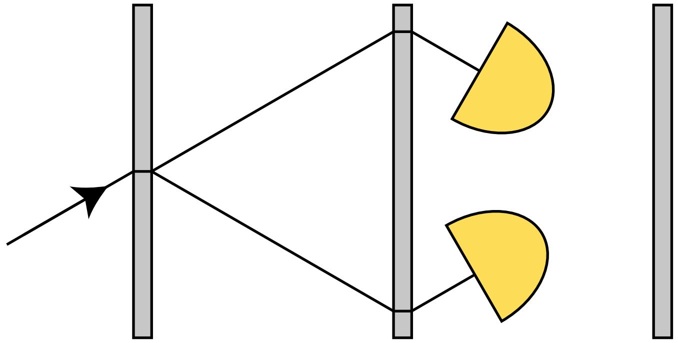
Which-way experiment

- Two-path interferometer
- Single-particle interference
  (double slit experiment)



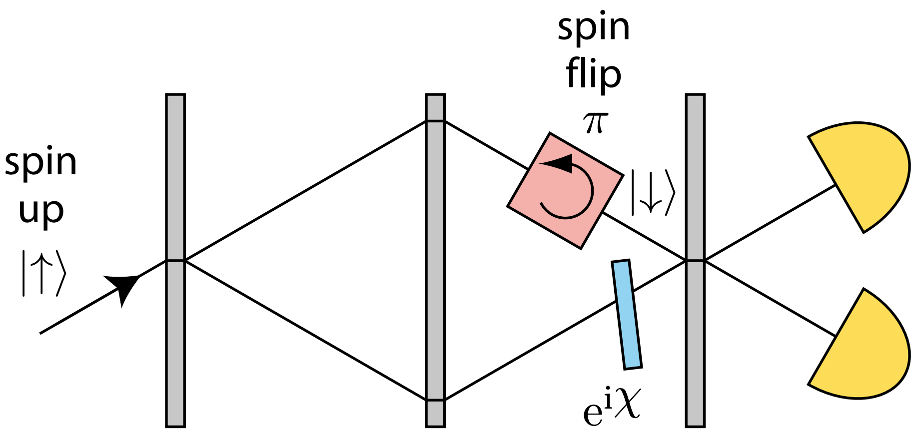
Which-way experiment

Detection = absorption



Which-way experiment

Path marking by spin flip



$$\psi_1 = |\uparrow\rangle$$
$$\psi_2 = |\downarrow\rangle$$

$$\psi_O = \psi_1 + e^{i\chi}\,\psi_2$$

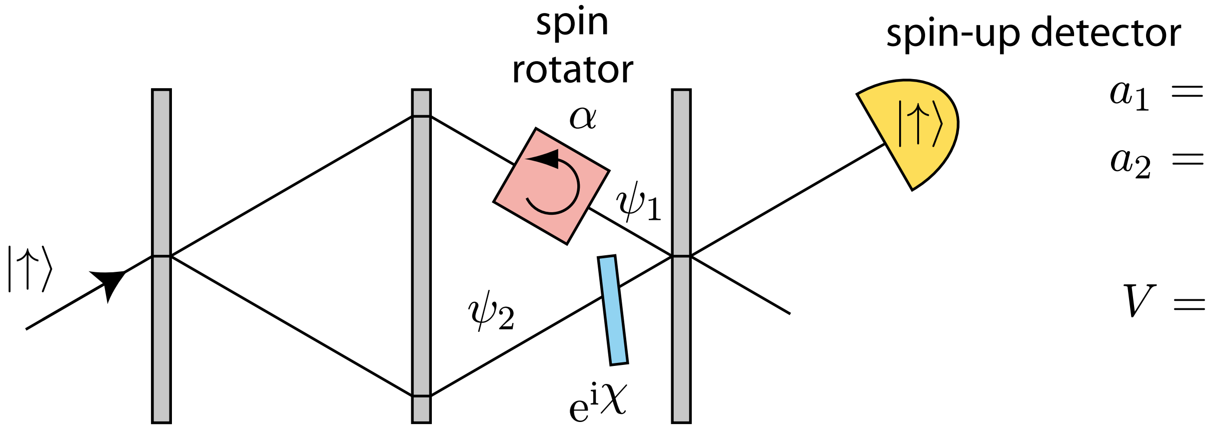
$$I_O \propto 1 + \underbrace{|\langle\uparrow|\downarrow\rangle|^2}_{V=0} \cos\chi$$

Which-way experiment

Path marking by spin rotation

$$\alpha = -2\mu B t/\hbar$$

$$\psi_1 = \cos\tfrac{\alpha}{2}\,|\!\uparrow\rangle + \mathrm{i}\,\sin\tfrac{\alpha}{2}\,|\!\downarrow\rangle$$
$$\psi_2 = |\!\uparrow\rangle$$



$$a_1 = \cos\tfrac{\alpha}{2}$$
$$a_2 = 1$$

$$V = \frac{2a_1 a_2}{a_1^2 + a_2^2}$$

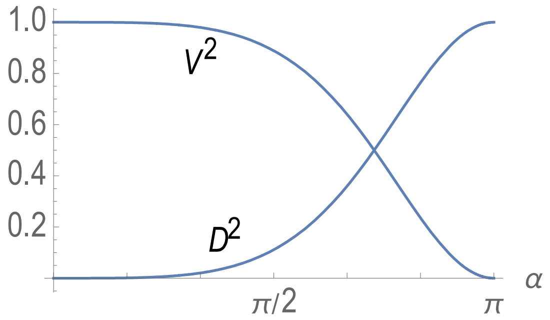
visibility

$$D = \frac{|a_1^2 - a_2^2|}{a_1^2 + a_2^2}$$

distinguishability

Greenberger-Englert relation $\quad V^2 + D^2 \leq 1$

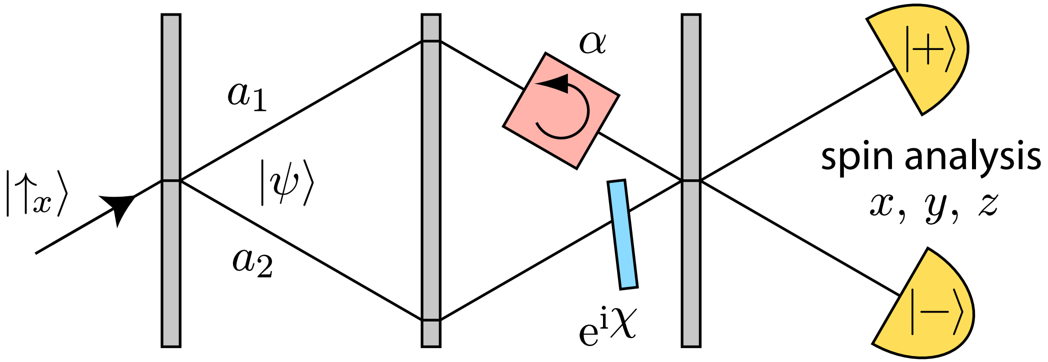
Greenberger and Yasin, Phys. Lett. A 128 391 (1988)
Englert, PRL 77, 2154 (1996)



„weak measurement“
gives
„weak value“

Which-way by „weak measurement“ Aharonov et al, PRL 60, (1988) 1351

Observable (presence in path 1) is weakly coupled to a meter (spin) $\Rightarrow$ Path-1 presence = Re of weak value $\omega_{1\pm} = \frac{\langle \pm | \hat{\Pi}_1 | \psi \rangle}{\langle \pm | \psi \rangle} \overset{(\chi = 0)}{=} \frac{a_1}{a_1 \pm a_2}$



for small α, $\chi = 0$

$\langle \hat{\sigma}_{x\pm} \rangle \approx 1$

$\langle \hat{\sigma}_{y\pm} \rangle \approx -\alpha \, \mathrm{Re}\, \omega_{1\pm}$

$\langle \hat{\sigma}_{z\pm} \rangle = 0$

Weak values

Recall: observable b

operator $\hat{B}$ in QM

eigenvalues and -states

$$\hat{B}|b_n\rangle = \lambda_n |b_n\rangle$$

state

$$|\psi\rangle = \sum_n a_n |b_n\rangle$$

$$a_n = \langle b_n|\psi\rangle$$

expectation value

$$\langle \hat{B} \rangle = \frac{\langle\psi|\hat{B}|\psi\rangle}{\langle\psi|\psi\rangle} = \sum_n p_n \lambda_n \quad \text{(average)}$$

$$p_n = |a_n|^2$$

Apply to which-way question:

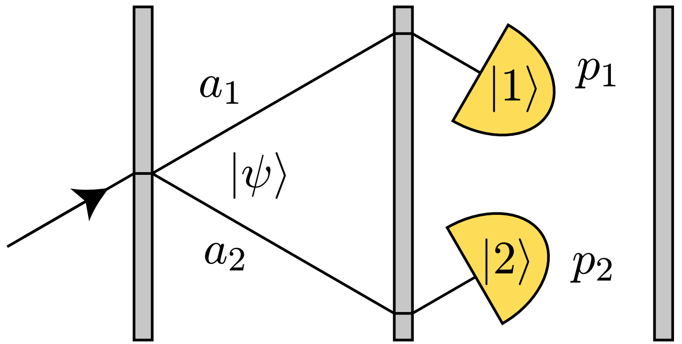
path projection $\hat{\Pi}_1$

$$\hat{\Pi}_1|1\rangle = 1 \cdot |1\rangle$$
$$\hat{\Pi}_1|2\rangle = 0 \cdot |2\rangle$$

$$|\psi\rangle = a_1|1\rangle + a_2|2\rangle$$
$$p_{1,2} = |a_{1,2}|^2$$

$$\langle \hat{\Pi}_1 \rangle = \frac{\langle\psi|\hat{\Pi}_1|\psi\rangle}{\langle\psi|\psi\rangle} = p_1 \cdot 1 + p_2 \cdot 0$$

average presence in path 1



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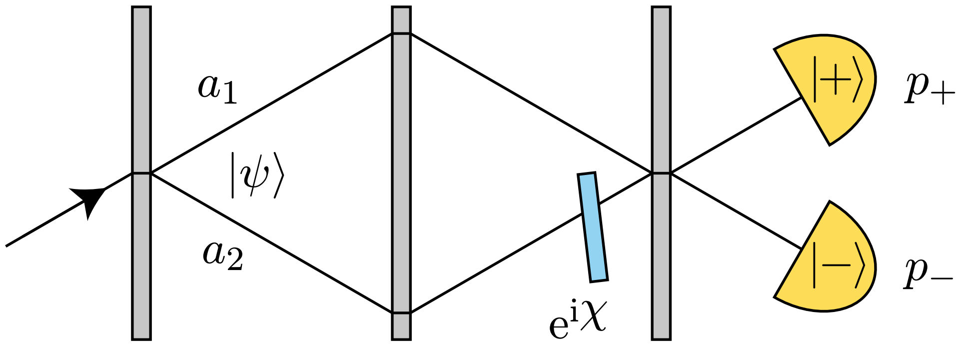
average presence in path 1



measure in the exit beams:

$$|+\rangle = \frac{1}{\sqrt{2}}\Big(|1\rangle + e^{i\chi}|2\rangle\Big)$$

$$|-\rangle = \frac{1}{\sqrt{2}}\Big(|1\rangle - e^{i\chi}|2\rangle\Big)$$

$$\langle\hat{\Pi}_1\rangle = \langle\psi|\overbrace{\sum_\pm |\pm\rangle\langle\pm|}^{\hat{1}}\hat{\Pi}_1|\psi\rangle$$

$$= \sum_\pm \underbrace{\Big|\langle\psi|\pm\rangle\Big|^2}_{p_\pm} \underbrace{\frac{\langle\pm|\hat{\Pi}_1|\psi\rangle}{\langle\pm|\psi\rangle}}_{\omega_{1\pm}} \quad \text{„weak value“}$$

$$= p_+\cdot\omega_{1+} + p_-\cdot\omega_{1-} = p_1$$

weak values $\omega_{1\pm}$ = path-1-presence of a neutron detected in $|\pm\rangle$ state assigned in retrospect (post selection)

Weak values

Example:

$$\omega_{1\pm} = \frac{\langle \pm|\hat{\Pi}_1|\psi\rangle}{\langle \pm|\psi\rangle} \overset{(\chi=0)}{=} \frac{a_1}{a_1 \pm a_2}$$

$$\omega_{2\pm} = 1 - \omega_{1\pm}$$

		$(\chi = 0)$		
balanced $a_1 = a_2$	$p_1 = 1/2$	$p_+ = 1$	$\omega_{1+} = 1/2$	$\omega_{2+} = 1/2$
	$p_2 = 1/2$	$p_- = 0$	$\omega_{1-} = \infty$	$\omega_{2-} = \infty$
unbalanced $a_1/a_2 = 2$	$p_1 = 4/5$	$p_+ = 9/10$	$\omega_{1+} = 2/3$	$\omega_{2+} = 1/3$
	$p_2 = 1/5$	$p_- = 1/10$	$\omega_{1-} = 2$	$\omega_{2-} = -1$

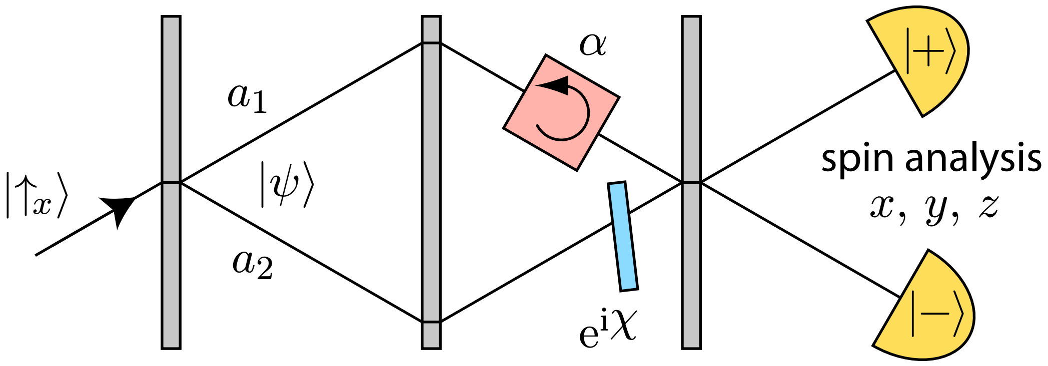
outside eigenvalue spectrum

necessary for correct expectation values:

$$\langle \hat{\Pi}_1 \rangle = p_1 = p_+\omega_{1+} + p_-\omega_{1-}$$

$$\langle \hat{\Pi}_2 \rangle = p_2 = p_+\omega_{2+} + p_-\omega_{2-}$$

path presence = interaction strength



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$$\langle \hat{\sigma}_{x\pm} \rangle \approx 1$$

$$\langle \hat{\sigma}_{y\pm} \rangle \approx -\alpha\,\omega_{1\pm}$$

$$\langle \hat{\sigma}_{z\pm} \rangle = 0$$

weak measurement

variance at maximum ($\approx |\uparrow_x\rangle$ measured in y direction)

weak value = ensemble property

Weak values

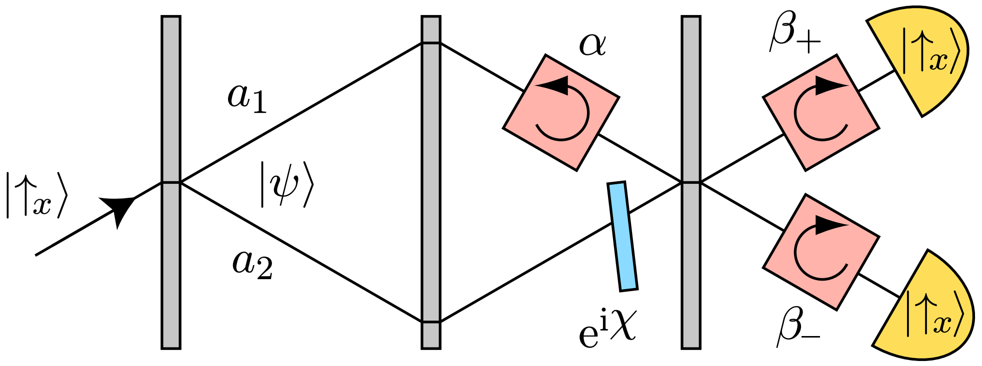
Example:

		$(\chi = 0)$		
balanced $a_1 = a_2$	$p_1 = 1/2$	$p_+ = 1$	$\omega_{1+} = 1/2$	$\omega_{2+} = 1/2$
	$p_2 = 1/2$	$p_- = 0$	$\omega_{1-} = \infty$	$\omega_{2-} = \infty$
unbalanced $a_1/a_2 = 2$	$p_1 = 4/5$	$p_+ = 9/10$	$\omega_{1+} = 2/3$	$\omega_{2+} = 1/3$
	$p_2 = 1/5$	$p_- = 1/10$	$\omega_{1-} = 2$	$\omega_{2-} = -1$

outside eigenvalue spectrum

$$\omega_{1\pm} = \frac{\langle \pm | \hat{\Pi}_1 | \psi \rangle}{\langle \pm | \psi \rangle} \overset{(\chi=0)}{=} \frac{a_1}{a_1 \pm a_2}$$

$$\omega_{2\pm} = 1 - \omega_{1\pm}$$



for small α, $\chi = 0$ and $\beta_\pm = \alpha\, \omega_{1\pm}$

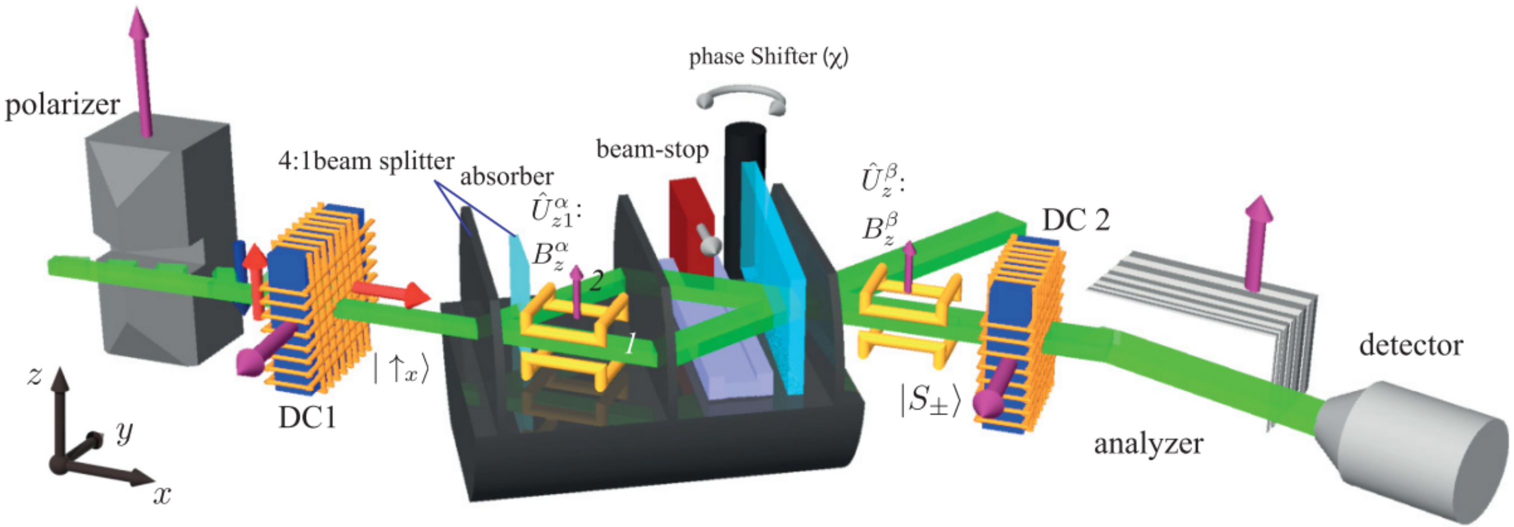
$$\langle \hat{\sigma}_{x\pm} \rangle = 1$$
$$\langle \hat{\sigma}_{y\pm} \rangle = 0$$
$$\langle \hat{\sigma}_{z\pm} \rangle = 0$$

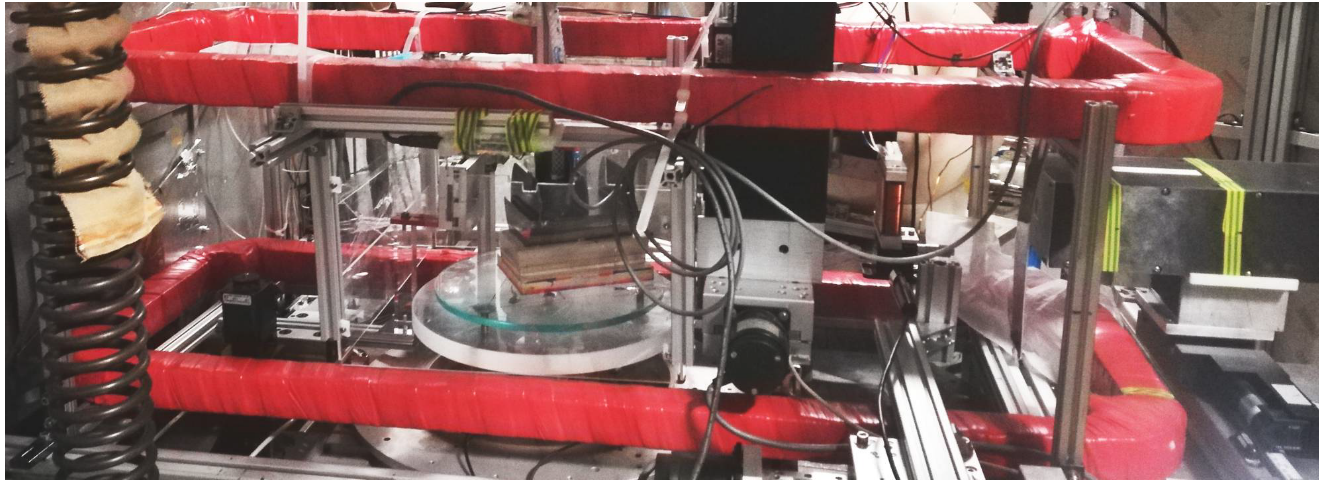
variance vanishes
average = individual property

feedback compensation scheme
H. F. Hofmann, Phys. Rev. Research 3 (2021) L012011

experiment @ S18, ILL
H. Lemmel et al., Phys. Rev. Research 4 (2022) 023075

Feedback compensation – Setup



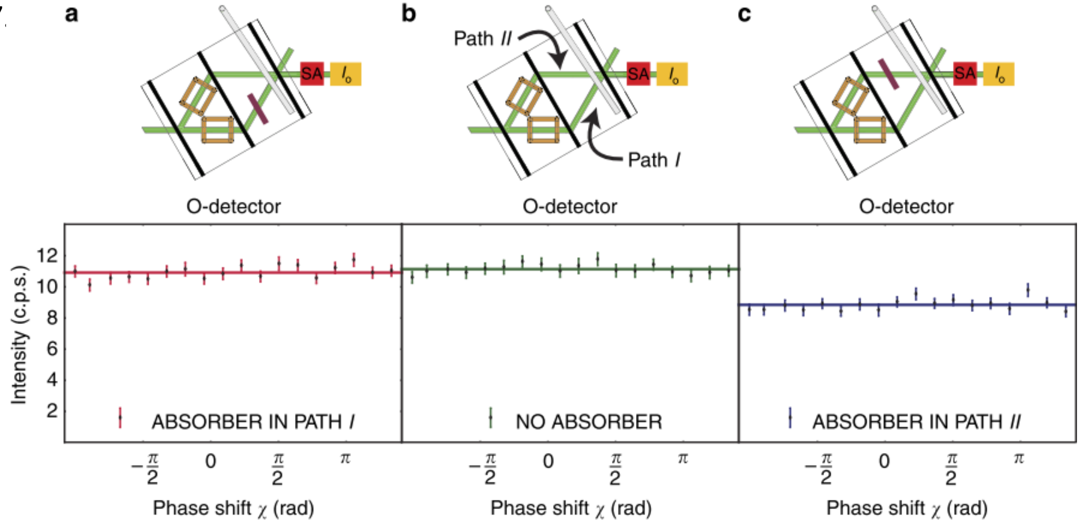


H. Lemmel et al., Phys. Rev. Research 4 (2022) 023075, @S18, ILL

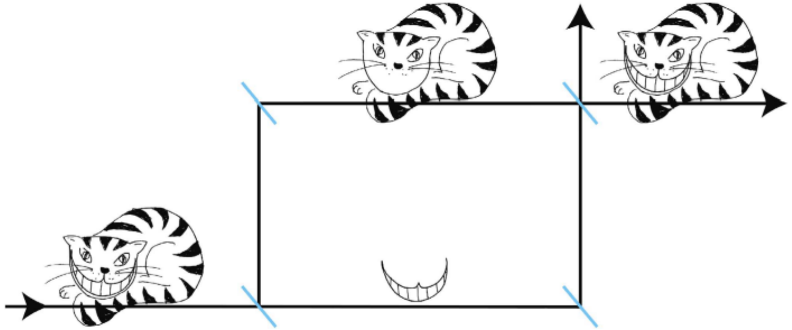
Quantum Cheshire Cat

Aharonov (2005)
Quantum Paradoxes, ch. 17

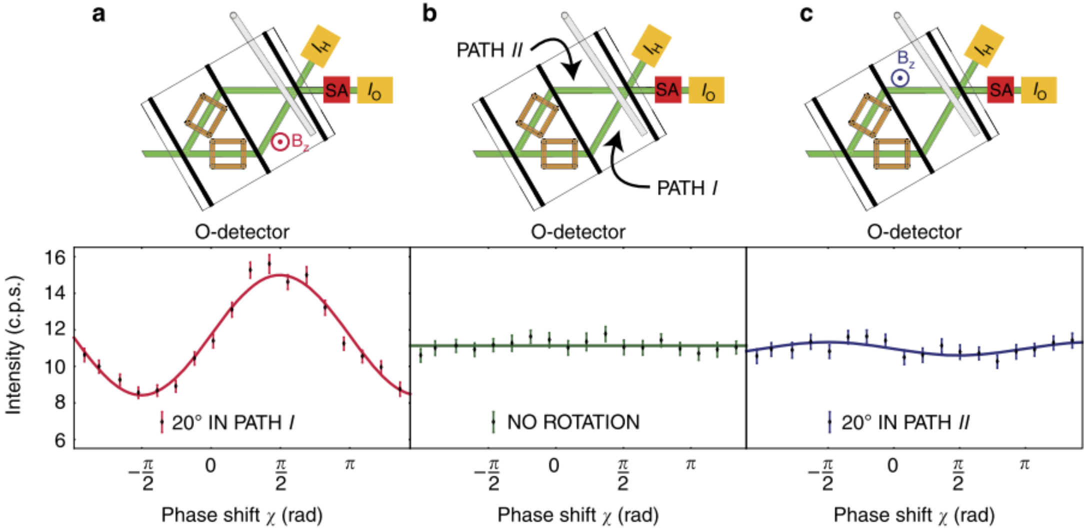
Absorber
has effect
only in path 2



→ neutron in path 2



Magnetic field
has effect
only in path 1



→ spin in path 1

experiment @ S18, ILL

Denkmayr et al., nature comm. (2014) 5492

Thank you for your attention.