

On Stewartson's collision problem on a sphere

This talk is about recent progress made on a long-standing fundamental problem in boundary layer theory and beyond. We consider the flow induced by a rigid sphere that spins around its diameter in an otherwise quiescent Newtonian fluid of uniform properties and filling an unbounded domain. Here we distinguish (roughly) between the unsteady flow due to start-up from rest, where the angular velocity increases to finally assume a constant value, and the, potentially attained, steady one. The associated Reynolds number (Re), as the only parameter at play, shall take on arbitrarily large values. We tackle this classical problem by solving the full Navier-Stokes equations using a finite-volume technique, which allows for reliable numerical predictions for values of Re exceeding 300,000, as well as by asymptotic analysis.

For comparison with previous results, three different starting conditions initiate the numerical computations: impulsively (classical situation), staggered impulsively, ramp-type,. The analytical description of the start-up phase in the first case is revisited and extended to finite- Re effects. One thereby finds that the flow meets both the axial and the equatorial symmetry automatically.

An intriguing unsettled question concerns the equatorial collision and the associated ejection of a radial jet. Stewartson (1958) proposed a flow structure under the assumption of stationarity that shed light on the severe challenges arising when it comes to its self-consistent completion. The associated controversy about his and Smith & Duck's (1977) conflicting structure, mostly regarding the presence and structure of the toroidal eddies, has gained attraction ever since. In contrast to claims made very recently, we demonstrate theoretically why the latter, resorting to free viscous-inviscid interaction, is to be favoured. However, we furthermore demonstrate that perfect stationarity cannot exist, which is also substantiated by the numerical results and the well-known equatorial finite-time break-up that terminates the solution of the problem governing the longitudinal boundary layer. First steps concerning its regularisation seem to resolve that discrepancy. In a nutshell, the present findings suggest a jet moving constantly into the fluid at rest while it produces vortices travelling back towards the equator. We believe that they finally provoke laminar-turbulent transition once the symmetries are relaxed. Finally, we give a brief account of how these results are modified if the sphere is axially squeezed or stretched. This is a topic of ongoing activities.

This research is carried out jointly by Bernhard Scheichl (TU Wien) and Christian Klettner and Frank T. Smith (both UCL).