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Optimization-based Insights on Renewable Integration in Fossil-Fuel-Powered Microgrids under Market Power Conditions: A Lebanese Case Study

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Preamble

This master thesis forms the basis for a peer-reviewed conference paper which, at the time of this thesis completion, is under review with the International Conference on the European Energy Market (Contribution ID 694; submitted February 10th 2025).
The submitted conference paper is entitled:

> *Renewable Integration in Fossil-Fuel-Powered Microgrids under Market Power Conditions*

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Abstract

Microgrids provide a reliable supply alternative to unstable public electricity grids in developing countries such as Lebanon. However, privately owned microgrids often prioritize fossil fuel generation over the integration of renewable energy, as they aim to maximize profit by increasing the utilization of their existing diesel generators. This study employs a mixed integer linear programming model to optimize the operation of a microgrid under market power conditions imposed by its owner. It also examines the interactions between microgrid owners, household consumers, and household prosumers with rooftop solar PV. Results indicate a transition from diesel generators to solar PV, yet generation remains largely controlled by microgrid owners, limiting prosumer contributions. Although the unutilized rooftop PV generation capacity decreases, the reduction remains modest. Furthermore, profit-driven disconnections of unprofitable prosumers increase as penalties for unmet demand become more severe. The findings highlight the need for policy interventions in developing countries to further improve rooftop photovoltaic utilization and ensure equal access to electricity for all microgrid customers.

Kurzfassung

Microgrids bieten eine zuverlässige Versorgungsalternative zu instabilen öffentlichen Stromnetzen in Entwicklungsländern wie dem Libanon. In vielen Fällen priorisieren diese Microgrids (häufig in privatem Besitz) jedoch fossile Stromproduktion gegenüber der Integration erneuerbarer Energien. Diese Arbeit entwickelt und analysiert ein gemischt-ganzzahliges lineares Optimierungsmodell zur Optimierung des Betriebs eines Microgrids, welches unter Marktmacht von seinem Besitzer betrieben wird. Dabei werden die Wechselwirkungen zwischen Microgrid-Betreibern und angeschlossenen Haushalten untersucht. Die Ergebnisse zeigen einen Übergang von Dieselgeneratoren zu Photovoltaikanlagen in der Stromversorgung, wobei die Produktion aus erneuerbaren Energien weitgehend in der Hand der Microgrid-Betreiber bleibt. Obwohl die ungenutzte Photovoltaikkapazität von Prosumern durch Einspeisung in das Microgrid verringert wird, bleibt dieser Beitrag relativ gering. Zudem steigt die Tendenz zur gewinnorientierten Netztrennung von unrentablen Prosumern, insbesondere wenn Strafgebühren für nicht gedeckte Stromnachfragen zunehmen. Die Ergebnisse unterstreichen die Notwendigkeit politischer Maßnahmen in Entwicklungsländern, um die Nutzung von Photovoltaikanlagen durch Microgrid-Kunden zu fördern und allen Microgrid-Nutzern einen gleichberechtigten Zugang zu Elektrizität zu gewährleisten.

Contents

Abbreviations

MCG	Microgrid
DG	Diesel Generator
PV	Photovoltaic
UD	Unmet Demand
MILP	Mixed Integer Linear Programming
NPV	Net Present Value
LCOE	Levelized Costs of Electricity
DKN	Deir Kanoun al Naher
LBP	Lebanese Pound
CAPEX	Capital Expenditures
OPEX	Operational Expenditures
SoC	State of Charge
kWp	Kilo-Watt Peak

1. Introduction and Background

1.1. Case of Lebanon

Many governments in developing countries fail to provide reliable electricity supply to their citizens [1]. Lebanon is one such example, where the public grid has experienced a near-total collapse due to political instability and economic crises, originating well in advance of the recent war [2]. As a result, citizens are forced to search for supply alternatives in order to maintain access to electricity during periods of public grid failure. On the one hand, small, parallel local grids developed, centered around one or more private diesel generators serving a single neighborhood or village [3]. These privately owned local grids can be classified as diesel-based microgrids. Their owners exercise full control over energy distribution and customers' access to the grid, with minimal regulatory oversight. On the other hand, citizens have increasingly pursued self-sufficiency by installing off-grid rooftop photovoltaic systems and battery storage [4]. However, their self-production conflicts with the interests of the microgrid owners, who seek to increase the utilization of their diesel generators to maximize profit. As a consequence, there is a risk of electricity-producing consumers (prosumers) being disconnected from the microgrid. The integration of renewable energy into diesel-based microgrids –whether through renewable energy systems installed by the microgrid or through integrating the rooftop solar photovoltaic (PV) output of prosumers– remains uncertain in Lebanon's energy system.

The core objective of this research is to investigate the integration of renewable energy into fossil-fuel-powered microgrids in Lebanon, with a particular focus on the control exerted by profit-driven microgrid owners and the influence of various customer types on microgrid operations. The research is guided by two key questions:

1. To what extent do profit-maximizing microgrid owners integrate renewable energy into their fossil fuel-powered microgrids where they exercise market power?
2. What are the dominant mechanisms for integrating renewable energy into these microgrids, considering the trade-offs between microgrid-owned generation systems and granting prosumers access to the infrastructure to consume their surplus generation?

The key novelty of this work is accounting for the strategic behavior of microgrid owners, particularly their ability to intentionally withhold supply or disconnect grid customers whose demand profiles do not align with the microgrid owner's profit-maximizing strategy. While existing literature primarily focuses on cost minimization while ensuring demand coverage in a microgrid, this study explicitly prioritizes profit maximization as the central objective of microgrid operation. A mixed-integer linear programming (MILP) model is developed to optimize the generation mix and operational strategy of a microgrid, while considering a diversified technology portfolio that includes both fossil and renewable energy sources, along with storage capabilities.

This introduction concludes with a brief review of the literature on microgrids. It is followed by an outline of the methodology presented in Chapter 2, which details the mathematical framework of the optimization model and the precomputations of the input parameters. Chapter 3 introduces the case study from Lebanon, providing a comprehensive numerical elaboration of the input parameters. The results of the analysis are presented in Chapter 4, followed by a discussion of their implications in Chapter 5. Finally, Chapter 6 concludes the study and outlines possible future research.

1.2. Background Microgrids

1.2.1. Applications

Microgrids are widely recognized as a viable solution for enhancing electrification in various countries. In nations with conditions similar to Lebanon, such as Nigeria and Yemen [5, 6], microgrids have primarily emerged as a response to the failure of the public grid. Other developing regions additionally deploy microgrids to electrify remote areas lacking grid access [7]. The ongoing Russian invasion of Ukraine has further intensified research on microgrids as a strategy to enhance the resilience of energy supply against attacks on critical infrastructure. Doronina et al. emphasize the positive impact of microgrid deployment for supporting islanding and black start capabilities within the Ukrainian power grid [8].

1.2.2. Renewable Energy Integration

Distributed renewable energy resources are well-established as a beneficial component of microgrids, enhancing accessibility and reducing reliance on fossil fuels [9]. In addition, they contribute to the reduction of operational expenditures (OPEX) and overall lifecycle costs in microgrids [10]. Solar PV proves particularly cost-effective, benefiting from declining panel prices and high energy yield potential in regions with rich solar radiation [11]. Case studies from various developing regions confirm these benefits while also highlighting challenges associated with purely solar PV-powered microgrids, including reliability and financing constraints [12]. The growth of renewable microgrids is facilitated in developing countries and conflict-prone regions through regulatory frameworks that prioritize decentralized and sustainable energy systems [13]. Furthermore, an analysis of solar PV microgrids in Iraq further underscores the potential of well-regulated microgrids in delivering reliable and sustainable energy [14].

1.2.3. Technologies and Grid Modes

Microgrids employ renewable technologies such as solar PV, wind power, and biomass alongside conventional fossil-based generation [15]. Among these, solar PV is regarded as the most cost-effective electricity generation option, particularly when compared to wind power and diesel-based systems [16]. To achieve high penetration of renewable energy and improve grid resilience, microgrids often incorporate storage technologies, including batteries [17] and thermal storage systems [18], in combination with advanced load management strategies. Microgrids can operate in island mode, where they are disconnected from the main grid and function independently. When connected to the main public grid, power can be supplied externally without activating internal generation technologies. This integration enhances the microgrid's ability to balance supply and demand, optimizing energy utilization and providing economic benefits [19].

1.2.4. Market Power and Prosumer Relations

The conflict of interest between microgrid owners and prosumers, along with the extensive control exercised by owners over the grid, remains an underexplored research area. While numerous studies propose microgrid optimization models that incorporate various technologies, such as a multi-objective optimization model [20], none of them fully account for the control exerted by microgrid owners—a critical factor in developing countries with minimal regulatory oversight. Prosumer interactions within microgrid systems have been analyzed through a prosumer-centric approach, providing valuable insights into their behavior in such systems [21]. Additionally, research has explored barriers to private sector investments and their impact on microgrid operations [22].

2. Methodology

The microgrid concept incorporated in the modeling framework (illustrated in Figure 2.1) includes the grid itself, connections to consumer and prosumer households, and deployable assets from the technology portfolio: diesel generators, photovoltaic systems, and battery storage units. All these elements fall under the control of the microgrid owner, enabling the MILP optimization model to regulate the deployment of generation assets, grid operation, and grid access.

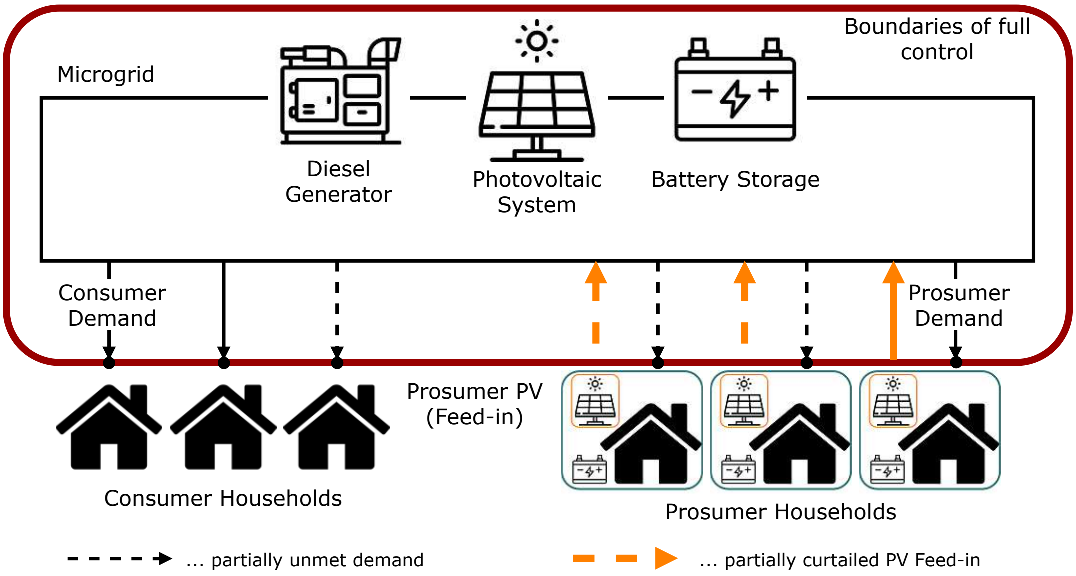

Figure 2.1.: Main concept of the studied microgrid

The model determines the optimal strategy for meeting microgrid demand while maximizing profit. This includes investment in assets from the generation portfolio and their cost-optimal utilization, purchase of surplus production from prosumers, or modification of the customer base within the grid. In addition to supplying the energy demand of customers, there is an option to leave certain demand unsupplied while incurring a penalty.

The following subsections offer an in-depth description of the model's essential functions.

A comprehensive listing of all decision variables, input parameters, and employed sets is provided in the appendix of this work.

2.1. Objective Function

The structure of the microgrid is configured in Equation in 2.1 by maximizing the net present value (NPV) of the income and costs related to its operation. Revenues R_y stem from supplying electricity from a type of generation $disp_{g,y,d,h}$ or through the battery $b^{out}_{y,d,h}$ to grid customers at a set tariff ρ. The yearly expenses include capital costs C^C_y for new generation capacity $a_{g,y}$, along with fixed and variable operational costs C^{OF}_y and C^{OV}_y for installed generation capacity $c_{g,y}$. Variable operational costs also include fuel expenses for diesel generators and excess PV production fed-in from prosumers, which is compensated through a feed-in tariff. A comprehensive explanation of the modeling approach for both expenses is presented in Subsections 2.2.4 and 2.2.3. Costs sustained from not supplying C^{UD}_y originate by choosing not to satisfy customer demand $ud_{y,d,h}$, thereby accepting a penalty ϵ per kWh.

The optimization model employs representative days as opposed to using the entire time series. Each representative day modeled is scaled by the number of calendar days ω_d it represents. In the following framework, index y represents the corresponding year, while d denotes the representative day and h the respective hour.

$$\max_{\mathcal{S}} \sum_{y} \left[\left(R_y - \left(C^C_y + C^{OF}_y + C^{OV}_y + C^{UD}_y\right)\right) \frac{1}{(1+\gamma)^y} \right] \tag{2.1}$$

With:

$$R_y = \rho \cdot \sum_{d\in\mathcal{D}} \omega_d \sum_{h\in\mathcal{H}} \Big(\sum_{g\in\mathcal{G}g} disp_{g,y,d,h} \Big) + b^{out}_{y,d,h} - b^{in}_{y,d,h}$$

$$C^C_y = \sum_{g\in\mathcal{G}} a_{g,y} \cdot \lambda^C_g$$

$$C^{OF}_y = \sum_{g\in\mathcal{G}} c_{g,y} \cdot \lambda^{OF}_g$$

$$C^{OV}_y = \sum_{d\in\mathcal{D}} \omega_d \sum_{h\in\mathcal{H}} \Bigg(\Big(\sum_{g\in\mathcal{G}g} disp_{g,y,d,h} \cdot \lambda^{OV}_g \Big) + \Big(\big(b^{in}_{y,d,h} + b^{out}_{y,d,h}\big) \cdot \lambda^{OV}_{g=B} \Big) +$$
$$\Big(disp_{g=DG,y,d,h} \cdot \sum_{k\in\mathcal{K}} \big(bin^{DG}_{k,y,d,h} \cdot \alpha_k\big) \cdot \pi \Big) + \Big(\sum_{i\in\mathcal{I}} fi_{i,y,d,h} \cdot \tau \Big) \Bigg)$$

$$C_y^{UD} = \sum_{d \in \mathcal{D}} \omega_d \sum_{h \in \mathcal{H}} ud_{y,d,h} \cdot \epsilon$$

$$s \geq 0 \quad \forall s \in \mathcal{S}$$

2.2. Constraints

The model operates under a set of constraints which can be classified into the following categories:

2.2.1. Generation-Demand Balance

Equation 2.2 ensures the balance between generation and consumption within the microgrid. Grid consumption includes the demand from customers and the charging activity of batteries $b^{in}_{y,d,h}$. Given that grid connections fall within the microgrid's control (refer to Figure 2.1), the microgrid determines the number of connected households $n^{house}_{i,y}$. Consequently, customer demand equates to the cumulative demand of all connected households. The dispatch by multiple generation technologies $disp_{g,y,d,h}$ along with the battery output $b^{out}_{y,d,h}$ forms the supply side of the microgrid. Alternatively, the model may opt not to fulfill customer demand with $ud_{y,d,h}$.

$$\Big(\sum_{i \in \mathcal{I}} n^{house}_{i,y} \cdot \mu_{i,d,h}\Big) + b^{in}_{y,d,h} = \Big(\sum_{g \in \mathcal{G}g} disp_{g,y,d,h}\Big) + b^{out}_{y,d,h} \quad + ud_{y,d,h} \quad \forall y, d, h \tag{2.2}$$

$$q_{dem} = \quad q_{sup} \quad + q_{nsup} \tag{2.3}$$

In other words, the microgrids demand q_{dem} in 2.3 is comprised of two components: the energy supplied to customers q_{sup} and the energy that remains unsupplied q_{nsup}. Every component generates costs that are taken into account in the objective function presented in 2.1. Meeting certain electricity demand can lead to costs that exceed its resulting revenue. In such cases, opting not to supply q_{nsup} and incurring a penalty for the amount of energy may be more beneficial.

2.2.2. Generation Technologies

The microgrid has the option to dispatch electricity from the following generation technologies: Diesel generators (DG), self-installed photovoltaics (PV), and surplus feed-in from customers (FI). Equations 2.4 - 2.6 define the upper limit on available electricity generation for each technology. While the diesel generators' output is just limited by their installed capacity $c_{g=DG,y}$ (2.4), PV generation is also affected by its capacity factor

$\phi_{d,h}$, which considers the fluctuation in generation across daily and seasonal periods (2.5). Total customer surplus feed-in is calculated by summing the consumed feed-in $fi_{i,y,d,h}$ from all household types.

$$disp_{g=DG,y,d,h} \leq c_{g=DG,y} \qquad \forall y, d, h \tag{2.4}$$

$$disp_{g=PV,y,d,h} \leq c_{g=PV,y} \cdot \phi_{d,h} \qquad \forall y, d, h \tag{2.5}$$

$$disp_{g=FI,y,d,h} = \sum_{i \in \mathcal{I}} fi_{i,y,d,h} \qquad \forall y, d, h \tag{2.6}$$

Equation 2.7 assigns a specific constraint to the diesel generators' annual added capacity $a_{g=DG,y}$. The size requirement dictates that any new DG capacity must align with feasible DG increments α_j^C, thereby preventing small-scale capacity expansions. The microgrid's available area for installing PV is restricted in 2.8 to a maximum value of χ^{avL}.

$$a_{g=DG,y} = \sum_{j \in \mathcal{J}} c_{j,y}^{DGSteps} \cdot \alpha_j^C \qquad \forall y \tag{2.7}$$

$$c_{g=PV,y} \cdot \chi^{useL} \leq \chi^{avL} \qquad \forall y \tag{2.8}$$

2.2.3. Surplus PV Feed-in from Prosumers

The amount of surplus PV fed into the grid is dictated by the microgrid. Equation 2.9 limits the feed-in per household type $fi_{i,y,d,h}$ to the available surplus generation per household $\sigma_{i,d,h}$ multiplied by the number of connected households. The allowed feed-in $fi_{i,y,d,h}$ is optimized for demand coverage at low costs, resulting in prosumer households' excess generation partly not being utilized.

$$fi_{i,y,d,h} \leq n_{i,y}^{house} \cdot \sigma_{i,d,h} \qquad \forall i, y, d, h \tag{2.9}$$

The model can choose excess PV production from prosumers to be beneficial for reducing supply costs. Equation 2.10 prevents the microgrid from connecting prosumers purely for the utilization of their surplus production without addressing their demand. It requires that the percentage of annual energy supplied to households must exceed the percentage of annual surplus generation $fi_{i,y,d,h}$ utilized by the microgrid. Both variables are normalized by the annual amount of prosumer demand $\mu_{i,d,h}$ and surplus generation $\sigma_{i,d,h}$ connected to the microgrid.

$$1 - \frac{\sum_{d \in \mathcal{D}} \omega_d \sum_{h \in \mathcal{H}} ud_{y,d,h}}{n_{i,y}^{house} \cdot \sum_{d \in \mathcal{D}} \omega_d \sum_{h \in \mathcal{H}} \mu_{i,d,h}} \geq \frac{\sum_{d \in \mathcal{D}} \omega_d \sum_{h \in \mathcal{H}} (\sum_{i \in \mathcal{I}} fi_{i,y,d,h})}{n_{i,y}^{house} \cdot \sum_{d \in \mathcal{D}} \omega_d \sum_{h \in \mathcal{H}} \sigma_{i,d,h}} \quad \forall i, y \tag{2.10}$$

2.2.4. Fuel Consumption Curve of Diesel Generators

The diesel generators' increased fuel consumption at lower utilization levels is modeled using a two-step consumption curve:

In the objective function (Equation 2.1), fuel consumption is considered under the operational costs C_y^{OV}. When the DG's utilization surpasses or falls short of 25% of the total installed DG capacity, the corresponding fuel consumption value in α_k is applied to calculate the associated costs. This is achieved by introducing the binary variables $bin_{k,y,d,h}^{DG}$. These variables are multiplied by the set of fuel consumption coefficients α_k, in order to choose the correct consumption for each time step in the analyzed period. Equations 2.11-2.13 ensure the correct setting of the binary variables.

$$\sum_{k \in \mathcal{K}} bin_{k,y,d,h}^{DG} = 1 \quad \forall y, d, h \tag{2.11}$$

$$disp_{g=DG,y,d,h} \leq c_{g=DG,y} \cdot 0.25 + M \cdot (1 - bin_{k=0,y,d,h}^{DG}) \quad \forall y, d, h \tag{2.12}$$

$$disp_{g=DG,y,d,h} \geq c_{g=DG,y} \cdot 0.25 - M \cdot (1 - bin_{k=1,y,d,h}^{DG}) \quad \forall y, d, h \tag{2.13}$$

Additional constraints regarding the retirement of power generation systems and the operation of the microgrid's battery systems are comprehensively listed in the appendix.

2.3. Input Parameters

2.3.1. Prosumer Household Model

Prosumer households are equipped with a photovoltaic system and battery storage, which is why they require a reduced demand from the microgrid compared to consumer households. Additionally, they offer self-generated amounts of power to feed into the grid.

The prosumer grid demand $\mu_{i=PH,d,h}$ and surplus energy $\sigma_{i=PH,d,h}$ are calculated in the following model and used as input parameters in the main optimization model.

Symbol	Name	Unit
$p_{d,h}^{PV}$	PV Generation of Household at day d and hour h	kWh
$q_{d,h}^{resH}$	Residual Prosumer Demand at day d and hour h	kWh
$soc_{d,h}^{PH}$	State of Charge of Prosumer Battery at day d and hour h	kWh

Table 2.1.: Variables: prosumer household model

The main parameters characterizing the electricity system within a prosumer household are detailed in Table 2.1. The residual demand from prosumers, as presented in Equation 2.15, is calculated by taking the actual electricity demand of the household and subtracting the electricity production obtained from their PV systems. This demand is positive when the generation is less than the household's consumption and negative if the household produces more electricity than it consumes.

$$p_{d,h}^{PV} = \theta_{i=PH}^{PV} \cdot \phi_{d,h} \qquad \forall d,h \qquad (2.14)$$

$$q_{d,h}^{resH} = \mu_{i=CH,d,h} - p_{d,h}^{PV} \qquad \forall d,h \qquad (2.15)$$

In Equation 2.16 the battery's state of charge is increased or reduced by the residual demand, unless that value exceeds the battery's minimal or maximal storage limits. Surplus generation intended for microgrid feed-in is determined by the actual surplus generation minus any available battery storage capacity, as presented in Equation 2.17. It is evident that this parameter is set to zero in the case of more available storage capacity than surplus generation or a positive residual demand. The energy demand drawn from the microgrid is determined in Equation 2.18 by subtracting the energy stored within the battery from the residual demand. If the stored energy meets the entire residual demand or in cases where the residual demand is negative, there will be no demand from the microgrid.

$$soc_{d,h}^{PH} = \begin{cases} \min\left(soc_{d,h-1}^{PH} - q_{d,h}^{resH}, \theta_{i=PH}^{Bat}\right) & \text{for } q_{d,h}^{resH} < 0, \\ \max\left(\theta_{i=PH}^{Bat} \cdot \beta_{msoc}, soc_{d,h-1}^{PH} - q_{d,h}^{resH}\right) & \text{for } q_{d,h}^{resH} \geq 0. \end{cases} \tag{2.16}$$

$$\sigma_{i=PH,d,h} = \begin{cases} \max\left(0, -q_{d,h}^{resH} - (\theta_{i=PH}^{Bat} - soc_{d,h}^{PH})\right) & \text{for } q_{d,h}^{resH} < 0, \\ 0 & \text{for } q_{d,h}^{resH} \geq 0. \end{cases} \tag{2.17}$$

$$\mu_{i=PH,d,h} = \begin{cases} 0 & \text{for } q_{d,h}^{resH} < 0, \\ q_{d,h}^{resH} - \min\left(soc_{d,h}^{PH} - \theta_{i=PH}^{Bat} \cdot \beta_{msoc}, q_{d,h}^{resH}\right) & \text{for } q_{d,h}^{resH} \geq 0. \end{cases} \tag{2.18}$$

The resulting values of $\mu_{i=PH,d,h}$ and $\sigma_{i=PH,d,h}$ are utilized in the main optimization model detailed in Sections 2.1 and 2.2. The impact of prosumer-owned assets on the household's microgrid consumption is illustrated in the following chapter in Figure 3.2.

2.3.2. Price Elastic Microgrid Demand

Considering the microgrid's monopolistic market position and minimal regulatory oversight in developing countries, a microgrid owner may opt for electricity pricing as an additional decision variable to optimize. On the other hand, customers generally show a price-elastic consumption behavior and adjust their demand in response to the height of their expenses. The topic of demand elasticity was a major point of discussion while evaluating the case study.

This theoretical framework introduces a method for incorporating electricity price as a decision variable and integrating price-elastic demand into the microgrid model. Although it was not applied deriving the majority of the results –only as a scenario in Section 4.4– it presents a foundation for future research.

Symbol	Name	Type	Unit
E	Elasticity of Demand	Parameter	
p	Electricity Price in MCG	Variable	\$/kWh
q_d	Elastic MCG Demand of entire representative Period d	Variable	kWh
p^{hist}	Historic Electricity Price Value	Parameter	\$/kWh
q_d^{hist}	Historic MCG Demand of entire representative Period d	Parameter	kWh
$\mu_{i,d,h}^{el}$	Elastic MCG Demand of household of type i at day d and hour h	Parameter	kWh

Table 2.2.: Variables and parameters: demand elasticity

The elasticity of demand is a dimensionless parameter that quantifies the percentage variation in demand in response to a percentage variation in price. Demand and price variations are calculated in relation to a historical point in time where these indicators were determined. We assume the elasticity parameter to be constant and therefore the demand response to a price change to be linear. The price elastic demand of a representative period $q_d(p)$ is derived in Equations 2.19 and 2.20.

$$E = \frac{\Delta q(\%)}{\Delta p(\%)} = \frac{\frac{q_d^{hist} - q_d}{q_d^{hist}}}{\frac{p^{hist} - p}{p^{hist}}} \tag{2.19}$$

$$\Leftrightarrow \quad q_d(p) = p \cdot E \frac{q_d^{hist}}{p^{hist}} + q_d^{hist} \cdot (1 - E) \tag{2.20}$$

The model must determine the optimal electricity price to maximize profit under elastic customer demand, which leads to non-linearity. The linear price curve in 2.21 (derived from 2.20) is discretized to values in ρ_i^{steps} in order to linearize the problem.

$$p(q_d) = q_d \cdot \frac{p^{hist}}{q_d^{hist} E} - p^{hist} \cdot (\frac{1}{E} - 1) \tag{2.21}$$

For the incorporation of price elastic demand, two areas in the main optimization model have to be adapted: The electricity price found in the generated Revenues R_y in the objective function 2.1 and the customer demand in the balance constraint 2.2.

The adjustable electricity price ρ^{el} is calculated in Equation 2.22. This is accomplished using the binary variable bin_i^{price}, which is multiplied by discrete points from the previously determined price curve.

$$\rho^{el} = \sum_i bin_i^{price} \cdot \rho_i^{steps} \tag{2.22}$$

The correct setting of the binary variables is ensured by the same method applied in Section 2.2.4. To achieve this, the decision variables $disp_{g,y,d,h}$, $ud_{y,d,h}$, $b_{y,d,h}^{out}$, and $b_{y,d,h}^{in}$ are aggregated on an annual basis. Consequently, despite the method utilizing significantly fewer binary variables than in Section 2.2.4, the problem remains computationally intensive.

The integration of price-elastic customer demand into the balance constraint is achieved by multiplying the hourly customer demand by the ratio of price-elastic demand to historical demand.

$$\mu_{i,d,h}^{el} = \mu_{i,d,h} \cdot \frac{q_d(p)}{q_d^{hist}} \tag{2.23}$$

3. Empirical Scale

The framework presented in Chapter 2 is broadly applicable to island microgrids that incorporate the technologies outlined in the main concept (Figure 2.1). However, the broad-reaching control of the microgrid owner makes it more applicable for deployment in developing countries with limited regulatory oversight. This chapter outlines the applied empirical scale of the modeling framework to generate the results in Chapter 4. Special focus is placed on Lebanon, where a case study from there provides data on grid customers.

The following tariffs and cost parameters do not factor in annual inflation rates. The extended time horizon was chosen to account for the retirement of existing technologies and their replacement. Nevertheless, the sum of annual revenues and expenses is discounted by an interest rate of 11% for the NPV calculation.

3.1. Case Study Lebanon

In this study, the optimized microgrid is scaled after an already existing microgrid located in Deir Kanoun al Naher (DKN), a town in southern Lebanon. DKN, home to approximately 15 000 residents, initiated in 2012 a diesel generator load-sharing system for supply during public grid outages. In 2022, the municipality initiated the integration of solar PV into its energy mix. Currently, eight diesel generators supply a continuous 1800 kW of electricity, alongside 490 kWp of deployed solar PV capacity. The customer base includes 1350 grid users, consisting of approximately 1150 residential units and 200 industrial units. 550 residential units have installed off-grid rooftop solar PV systems with a combined power of 1500 kWp.

Data loggers were deployed within DKN's microgrid to collect model input data on generation and consumption of various customer types. However, the 2024 war in Lebanon has resulted in the evacuation of Deir Kanoun al Naher, like the majority of South Lebanese villages. As a result, the initial plan for data collection was hindered, since loggers were no longer tracking the electric load and supply of the town. Consequently, the customer base of the modeled microgrid was reduced to only 1150 households whose demand patterns were obtained from literature.

Type	Number	Avg. PV Capacity
Residential Consumer	600	0 kWp
Residential Prosumer	550	2.7 kWp
Total	1150	1500 kWp

Table 3.1.: Case study customer base

3.1.1. Demand Profiles

As previously stated, the model simulates representative days of microgrid operations on an hourly resolution throughout a year. A span of 15 years is modeled to analyze the phase-out of current generation assets and the investment into new infrastructure for the microgrid. Year 0 represents the existing assets in the grid at the start of the optimization. The model aggregates each season into a representative period. Due to similarities in average temperature and demand patterns in the Middle East during spring and autumn [23], they are combined into one representative day. Therefore, the framework models three distinct representative days capturing the core characteristics of all four seasons.

$$\text{Years} \quad \mathcal{Y} = [0:15] \tag{3.1}$$
$$\text{Days} \quad \mathcal{D} = [0:2] \tag{3.2}$$
$$\text{Hours} \quad \mathcal{H} = [0:23] \tag{3.3}$$

Acquiring seasonally categorized residential demand data from the Middle East proved challenging. Rafiq et al. classified residential demand data from Dubai into cooling and non-cooling households, further subdividing them into four household categories for summer and winter [23] . While economic disparities between Dubai and Lebanon are evident, this classification framework provides a basis to develop a demand profile adapted to Lebanese consumption patterns.

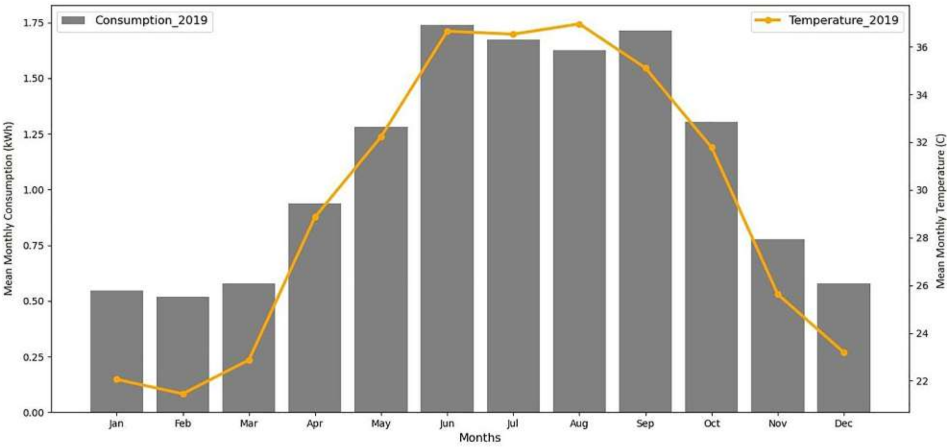

Figure 3.1.: Mean monthly electricity consumption in sampled households from literature [23]

Figure 3.1 illustrates the mean monthly consumption of the sampled dataset from [23]. This visualization allows for the identification of high-demand summer months (June–September), low-demand winter months (December–March), and transitional spring/autumn months (April–May and October–November). Consequently, a representative summer day is weighted at 122, a spring/autumn day at 122, and a winter day at 121, corresponding to the number of days within each respective period.

The households in the case study are divided into 600 cooling and 550 non-cooling units. Table 3.2 presents their further classification into clusters from the case study [23]. The two clusters with the most unrealistic demand patterns for Lebanon were excluded.

Type	Cluster 1	Cluster 2	Cluster 3	Cluster 4
Cooling	-	100	300	200
non Cooling	250	200	100	-

Table 3.2.: Allocation of households in demand clusters from literature

Figure 3.2 illustrates the derived seasonal demand profiles. The left side represents the demand profile of a consumer household that exclusively relies on the microgrid for its electricity needs. The right profiles reflect seasonal grid consumption of a prosumer household, which only consumes from the microgrid during periods when its photovoltaic system and battery storage cannot provide sufficient power.

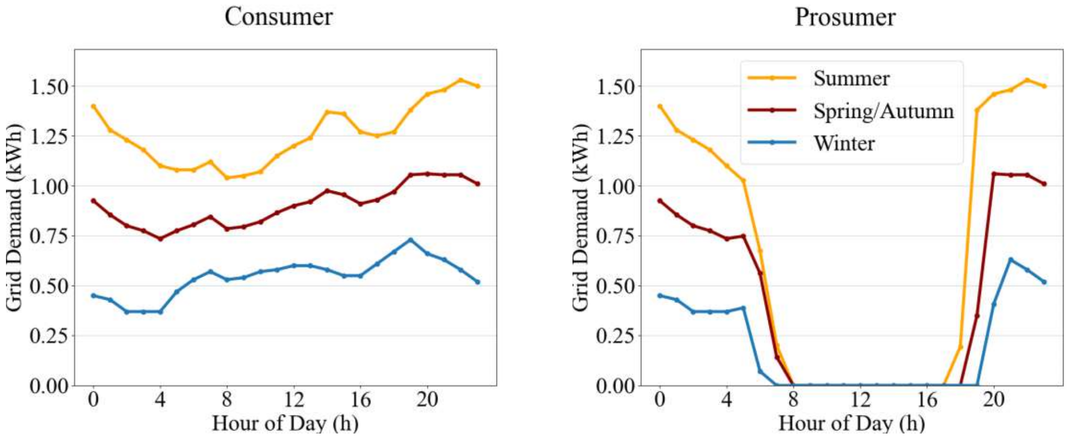

Figure 3.2.: Seasonal demand profiles for consumer- and prosumer households

3.1.2. Pricing and Tariffs

In the framework, the electricity price for grid customers is set at 0.39 $/kWh, based on the tariff from August 2024 in the case study's system. Accordingly, the fuel cost for diesel, also taken from August 2024, is 1 454 000 LBP per 20 liters [24], equivalent to 0.81 $ per liter.

Prosumers receive a feed-in tariff for each kilowatt-hour of surplus production fed into the grid. This tariff is set at the levelized cost of electricity (LCOE) of PV generation, which is 0.103 $/kWh (Equation 3.4).

$$\text{LCOE}_{PV} = \frac{\lambda^{C}_{g=PV} \cdot AF + \lambda^{OF}_{g=PV}}{\phi_y \cdot 8760} + \lambda^{OV}_{g=PV} \tag{3.4}$$

$$AF = \frac{(1+\gamma)^{\nu_{g=PV}} \cdot \gamma}{(1+\gamma)^{\nu_{g=PV}} - 1} \tag{3.5}$$

The penalty for unsupplied customer demand is set at 0.1 $/kWh. Given the model's high sensitivity to this parameter, a sensitivity analysis is presented in section 4.2 of the results.

3.2. Technology Portfolio Configuration

3.2.1. Investment and Operation Costs

The costs for installing and operating assets from the technology portfolio are listed in Table 3.4. Data for PV systems and battery storage units are sourced from the global and national energy systems techno-economic database [25]. The costs and lifetime associated with diesel generators are documented in [26] and [20].

	CAPEX $/kW	Fix- OPEX $/kW-yr	Var- OPEX $/kWh	Lifetime yr
Diesel Generator	800	20	0.06	10
Photovoltaic System	1000	20	0.02	20
Battery Storage	350	60	0.00	5

Table 3.3.: Technology costs for microgrid-owned assets

3.2.2. Diesel Generator

The efficiency of diesel generators significantly varies based on the specific model and lifespan of the unit. In their study on hybrid energy microgrids, Premadasa et al. use four parameters to characterize fuel consumption [20]. However, substantial decreases in consumption were observed merely from 25% to 50% utilization range, leading to the choice to differentiate solely between below and above 25% utilization levels.

Utilization Rate	≤25%	>25%
Consumption per kWh_{el}	0.35L	0.25L

Table 3.4.: Fuel consumption at different utilization levels of diesel generators

The capacity of diesel generators can only be expanded in increments of 50 kilowatts, preventing small-scale expansions and better reflecting the typical sizes of diesel generators installed in microgrids.

3.2.3. Photovoltaic System

PV production on an hourly basis is obtained from [27] which uses the MERRA-2 database. The hourly values are aggregated into representative clusters corresponding to specific time intervals, as elaborated in Subsection 3.1.1.

Additionally, photovoltaic systems require 8 m^2/kWp of land, as reported by [28]. This metric is applied to limit the amount of PV installation by the microgrid.

3.2.4. Battery Storage

The characteristics of the battery storage are shown in Table 3.5. Power ratings for both the charging and discharging phases are identical, with the same efficiency for both processes. The battery's capacity is configured to be six times larger than its installed power. Additionally, there is a minimal capacity requirement of 500 kWh to document only significant battery system installations.

Efficiency	Minimum SoC	Ratio Capacity/Power	Minimum Capacity
85%	20%	6 kWh/kW	500 kWh

Table 3.5.: Parameters for battery operation

4. Results

This chapter presents the findings of the analyzed Lebanese case study and initially discusses their implications. The discussion chapter later provides a more in-depth analysis of specific topics. The results focus on the integration of renewable energy and prosumer households into the microgrid. Specifically, the analysis examines the number of households connected to the network and the extent of unmet customer demand. Representative days from the modeled dataset illustrate the hourly utilization of the generation portfolio and unfulfilled demand during all four seasons. This study also evaluates the impact of prosumer connections to the microgrid. Additionally, a sensitivity analysis explores the deployment of non-PV technologies from the portfolio under varying input parameter combinations.

The parameter values for the main optimization appear in this chapter as *Base Scenario*.

4.1. Microgrid Operation over Representative Days

Photovoltaic electricity generation completely covers the microgrid's demand during its operational hours in Figure 4.1. The majority of PV-generated electricity within the grid is sourced from self-installed systems as opposed to supply from prosumer households. The diesel generators primarily supply the demand during periods lacking photovoltaic production. It predominantly operates at its full capacity of 950 kW when activated, except during the winter when demand is generally lower. Additionally, the diesel generators aim to maintain a minimum utilization rate of 25% to reduce fuel consumption. Notably, the grid does not install a battery storage system.

Table 4.1 illustrates the deployment of the technology portfolio in the microgrid over the planning horizon of 15 years. IC represents the initial capacity in the microgrid before the start of the optimization process. Existing diesel generators and photovoltaic systems have a remaining lifetime of 5 years, after which their initial capacities are retired from the grid. Even after the retirement of the initial capacity, the microgrid rebuilds nearly all of the retired DG capacity, reaching 950 kW. Solar-PV is immediately increased from 430 kWh to 1000 kWh in Year 1, and subsequently reduced to 950 kWh after the retirement of the initial capacity.

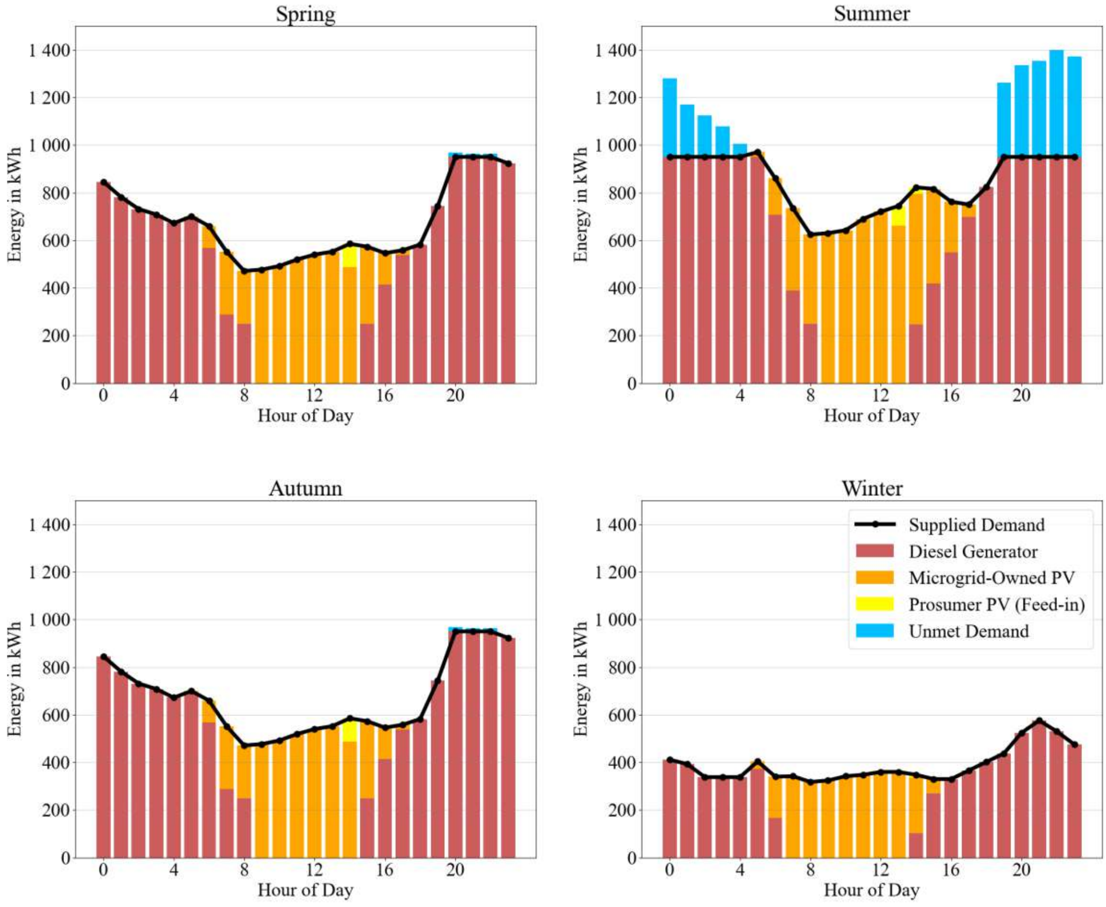

Figure 4.1.: Demand coverage in the microgrid on representative days in spring, summer, autumn and winter

Increased household cooling drives significantly higher nighttime demand in summer compared to other seasons. The model determines that nighttime demand peaks in summer are more cost-effective to leave unmet, despite incurring a penalty of 0.1 $/kWh. This is highlighted in Figure 4.1 by the blue bars. In the intermediate seasons spring and autumn, the system leaves only minimal amounts of demand in the evening unmet. During winter, the installed generation capacities are sufficient to cover the entire load.

Year	IC	1	2	3	4	5	6	7
Diesel Generators (kW)	1000	1000	1000	1000	1000	1000	950	950
Self-Installed PV (kWp)	430	1000	1000	1000	1000	1000	950	950
Battery Storage (kWh)	0	0	0	0	0	0	0	0
Year	8	9	10	11	12	13	14	15
Diesel Generators (kW)	950	950	950	950	950	950	950	950
Self-Installed PV (kWp)	950	950	950	950	950	950	950	950
Battery Storage (kWh)	0	0	0	0	0	0	0	0

Table 4.1.: Deployment of microgrid-owned technologies over the 15-year planing horizon

The extent of unmet demand is driven by the penalty parameter. Below, we present different sensitivities regarding its impact and examine the conditions under which customers are disconnected from the microgrid.

4.2. Connected and Disconnected Customers

A sensitivity analysis of the penalty for unmet demand, presented in Figure 4.2, illustrates the range of connected and disconnected customers in the microgrid. The figure is divided into two parts: (top) displays the number of connected consumer and prosumer households, while (bottom) illustrates the total energy demand not delivered to the different customer types in the microgrid. The following notable observations emerge from the analysis:

The number of consumers within the microgrid (represented by the blue line in the upper figure) remains unaffected by variations in the penalty for unmet demand. Regardless of the penalty parameter, it remains cost-optimal for the microgrid owner to fully meet the demand of all consumers. In contrast, the number of prosumers shows a significant dependence on the penalty parameter. When the penalty is low (e.g., 0.05 $/kWh), the number of connected prosumer households is maximized at 550. As the penalty increases, this number decreases, reaching 130 at a penalty of 0.35 $/kWh. Increasing the penalty beyond this threshold does not lead to further reductions in the number of prosumers.

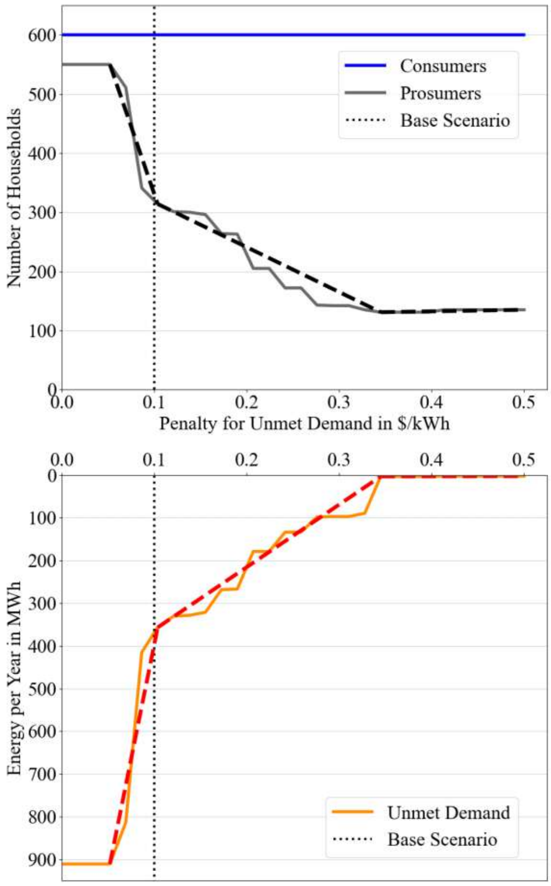

Figure 4.2.: (Top) Number of connected consumer and prosumer households, (Bottom) Total annual unmet energy demand across different customers as a function of the unmet demand penalty

Additionally, the unmet demand of connected customers is also highly sensitive to the penalty parameter. With a penalty of 0.05 $/kWh (or lower), the unmet demand is the highest at 900 MWh/year. As the penalty increases, this value decreases progressively, reaching 350 MWh/year at a penalty of 0.1 $/kWh and reducing to zero (0 MWh/year) at a penalty of 0.35 $/kWh. It is important to note that this unmet demand refers only to customers that are connected, excluding those who are completely disconnected.

The disconnection of up to 420 prosumer households results in a significant loss of grid demand and excess PV generation. We quantify these metrics for an individual prosumer household in the following subsection.

4.3. Impact of Prosumers

Although prosumer households maintain a reduced demand due to their own energy generation, they still offer economic opportunities for the microgrid. As demonstrated in Table 4.2, the microgrid supplies annually 2540 kWh to a prosumer household, while it chooses to leave 1150 kWh unsupplied. The grid only consumes 100 kWh of surplus PV production. It leaves 430 kWh annually unutilized, since most PV generation in the grid originates from microgrid-owned systems.

	Demand	Unmet Demand	Unutilized PV Generation
disconnected	7880 kWh	3690 kWh	530 kWh
connected		1150 kWh	430 kWh

Table 4.2.: Annual demand coverage and PV utilization of a single prosumer household without- and with microgrid connection

Figure 4.3 illustrates the unmet demand per prosumer household during the summer, presented at an hourly resolution. For comparison, it also includes a prosumer household's microgrid demand during the same season.

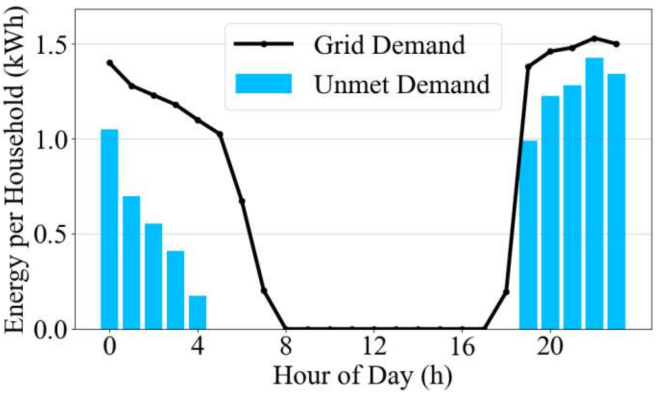

Figure 4.3.: Prosumer grid demand and unmet demand per connected prosumer household (unmet demand only distributed over prosumers) in summer

The evaluation of prosumers' energy demand and PV generation reveals a significant amount of unused surplus PV energy, along with unmet demand during periods of inactive photovoltaic generation. Storing this surplus energy for use during inactive periods presents a trade-off with the diesel generators, which supply electricity during these times in the base scenario. The next subsection outlines the conditions for installing battery storage or expanding the capacity of diesel generators.

4.4. Scenarios for Electricity Supply During Inactive PV-Production

In the base scenario configuration, the optimization model does not opt for the installation of battery storage within the microgrid nor the expansion of diesel generators beyond their initially installed capacity. Tables 4.4 and 4.5 present the expansion of diesel generator capacity and battery storage installation across different combinations of input parameters. The nine analyzed scenarios include low and high variations for the five selected parameters (Table 4.3), except for the feed-in tariff and unmet demand penalty, where the high scenario is trivial or already explored. Additionally, this analysis includes a scenario that incorporates price-elastic customer demand and treats the electricity price as a decision variable (detailed in Subsection 2.3.2).

Parameter	Variation	Value
Feed-in Tariff	set to 0	0 $/kWh
Unmet Demand Penalty	set to 0	0 $/kWh
Diesel Price low	× 0.5	0.405 $/L
Diesel Price high	× 1.5	1.215 $/L
Electricity Price low	× 0.5	0.195 $/kWh
Electricity Price high	× 1.5	0.585 $/kWh
Num. Prosumers low	Share: 25%	288
Num. Prosumers high	Share: 75%	862

Table 4.3.: Parameter variation in scenarios

The model expands capacity of diesel generators in scenarios with high electricity prices or low diesel prices. A low number of prosumers also frequently drives capacity expansion. Conversely, a high number of prosumers, high diesel prices, or price-elastic microgrid demand reduce the incentive for expanding the capacity of diesel generators.

Base Scenario: **No DG Expansion**		Feed-in Tariff 0 $/kWh	UD Penalty 0 $/kWh	Diesel Price low	Diesel Price high	Electricity Price low	Electricity Price high	Number Prosumers low	Number Prosumers high	Price Elastic Demand
Feed-in Tariff:	0 $/kWh									
UD Penalty:	0 $/kWh									
Diesel Price	low	X		X						
	high									
Electricity Price	low									
	high	X	X	X	X		X			
Number Prosumers	low	X		X			X	X		
	high			X			X			
Price Elastic Demand				X						

Table 4.4.: Diesel generators capacity expansion in different scenarios
X... Expansion of DG capacity beyond its initial installed value
[Grey]... Infeasible or repetitive scenario

To include only significant battery system installations, this analysis documents only systems that have a capacity of at least 500 kWh. Battery systems are installed in scenarios with high electricity prices or a large number of prosumers. Likewise, high diesel prices often lead to battery system installations across most scenarios due to increased operational costs for the competing flexible generation technology. Unlike DG capacity expansion, the grid installs battery systems in all scenarios with price-elastic customer demand. On the other hand, scenarios with no penalty for unmet demand or a low number of prosumers reduce the appeal of installing a battery system.

Base Scenario: **No Battery Installation**		Feed-in Tariff 0 $/kWh	UD Penalty 0 $/kWh	Diesel Price low	Diesel Price high	Electricity Price low	Electricity Price high	Number Prosumers low	Number Prosumers high	Price Elastic Demand
Feed-in Tariff:	0 $/kWh									
UD Penalty:	0 $/kWh									
Diesel Price	low									
	high	X			X					
Electricity Price	low									
	high	X	X	X	X		X			
Number Prosumers	low						X			
	high	X	X	X	X		X		X	
Price Elastic Demand		X	X	X	X			X	X	X

Table 4.5.: Battery storage installation in different scenarios
X... Installation of a microgrid-owned battery system
[Grey]... Infeasible or repetitive scenario

The absence of a penalty for unmet demand removes the incentive to install new capacity from either technology, as unfilled demand does not carry economic consequence.

Low electricity prices seem to have the greatest impact against both expansion of the diesel generators and installation of a battery system. However, it is important to emphasize that these electricity prices fall below the generation costs of all technologies, and as a result, no generation technology is implemented in the microgrid.

4.4.1. System Operation with a Battery Storage

Figure 4.4 illustrates demand coverage for an exemplary battery storage deployment under high electricity prices. Green and purple bars indicate battery charging and discharging. In summer, the battery significantly reduces unmet demand by storing PV generation for nighttime use. However, the battery serves only a complementary role, while the diesel generators remain the primary source of power during periods without PV generation.

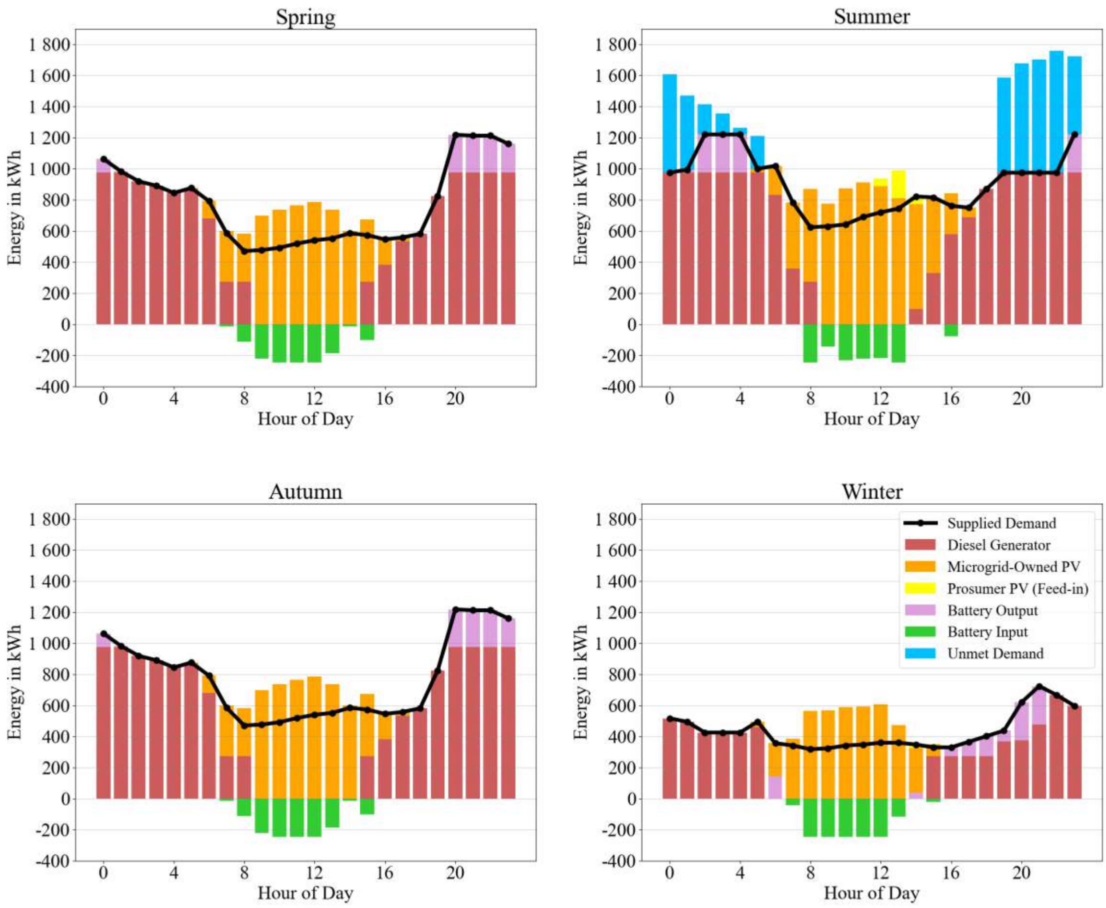

Figure 4.4.: Scenario **Electricity Price High** year 4: Demand coverage in the microgrid on representative days in spring, summer, autumn and winter

The annual deployment of technologies in Table 4.6 provides valuable insights. It is important to consider the retirement of initial capacities, including diesel generators and microgrid-owned PV systems, after five years, as well as the five-year lifetime of battery storage. During the initial period, when these capacities remain in the grid, high levels of PV and battery storage are deployed. After retiring the initial photovoltaic capacity, the capacity of diesel generators increases to 1550 kW, while the battery storage capacity decreases to 750 kWh, and only 1000 kWp of PV is used.

Year	IC	1	2	3	4	5	6	7
Diesel Generators (kW)	1000	1100	1100	1100	1100	1100	1550	1550
Self-Installed PV (kWp)	430	1250	1250	1250	1250	1250	1000	1000
Battery Storage (kWh)	0	1500	1500	1500	1500	1500	750	750
Year	8	9	10	11	12	13	14	15
Diesel Generators (kW)	1550	1550	1550	1450	1450	1450	1450	1450
Self-Installed PV (kWp)	1000	1000	1000	1000	1000	1000	1000	1000
Battery Storage (kWh)	750	750	750	600	600	600	600	600

Table 4.6.: Scenario **Electricity Price High**: Deployment of microgrid-owned technologies over the 15-year planing horizon

In this high electricity price scenario, all 550 prosumer households are connected to the microgrid, and a battery system is deployed. This aligns with previous findings, where a high number of prosumers led to battery storage installation. Due to its flexibility and ability to operate profitably even at low utilization, battery storage effectively shifts renewable generation to nighttime. This capability makes it well-suited to complement the fluctuating demand profiles of prosumers.

4.4.2. Price Elastic Microgrid Demand

In the price-elastic demand scenario, the optimization process adjusts electricity prices to maximize profit. As the electricity price rises, customer demand decreases. Table 4.7 compares the fixed price in the base scenario with the optimized price using the elasticity framework from Subsection 2.3.2. The optimized electricity price increases nearly by a factor of six, while annual microgrid demand drops by over 2000 MWh, from 6760 MWh to 4550 MWh. The properties of the elasticity framework –such as high revenues and reduced overall grid demand– demonstrate benefits for battery system installation instead of expanding capacity of diesel generators.

	Base Scenario	Elasticity Scenario
Electricity Price	0.39 $/kWh	2.2 $/kWh
Annual MCG Demand	6760 MWh	4550 MWh

Table 4.7.: Scenario **Price Elastic Demand**: Electricity price and microgrid demand

Even under the assumption of no regulatory constraints and full market power of the microgrid owner, the electricity price of 2.2 $/kWh significantly exceeds what grid customers would be willing to pay. The primary reason for the model's inaccurate pricing is the assumption of constant demand elasticity, which results in a linear demand curve as a function of the price. The elasticity parameter derived from the case study only provides insights into customer demand reactions at the price levels where it was determined. At significantly higher price levels, like the model's optimal electricity price, customer demand likely responds with a different elasticity. While demand at the case study's price level is highly inelastic, with an elasticity of -0.1, the parameter is expected to be significantly higher at elevated prices. A more elastic customer response to price increases would compel the model to set lower electricity prices to maintain grid demand at an optimal level for profit maximization.

However, the results illustrate the fundamental behavior of an elastic demand response to the electricity price, consistent with the applied elasticity framework. Additionally, the framework is utilized in the scenario analysis for evaluating battery storage deployment and capacity expansion of diesel generators. In this context, its properties —such as high revenues and reduced overall grid demand— demonstrate benefits for battery system installation instead of expanding capacity of diesel generators.

5. Discussion

This chapter discusses key findings from the results, including the sources of renewable electricity in the grid and the allocation of unmet demand. It also elaborates on the exogenous setting of tariffs and prices.

5.1. Main Source of Renewable Electricity Generation

The results of this study show that photovoltaic generation within the microgrid originates primarily from microgrid-owned systems rather than surplus feed-in from prosumers (as illustrated in Figure 4.1). One contributing factor influencing this trend is the feed-in tariff, which is based on the levelized cost of electricity for PV. This tariff structure does not provide a clear advantage for surplus PV generation by prosumers over microgrid-owned PV systems. However, even when the feed-in tariff is set below the LCOE for PV, the full potential of prosumer surplus generation remains underutilized. The main reason is the limited availability of hours with surplus generation from prosumers. Prosumer feed-in is too sporadic to meet demand on a constant level, leading the microgrid to install its own PV system to ensure a stable and sufficient energy supply through renewables. Once installed, these systems continue to be utilized even when prosumer feed-in is available at a lower tariff than the LCOE of microgrid-owned generation.

5.2. Unmet Demand of Customer Types

The model does not account for the distribution of unmet demand $ud_{y,d,h}$ among different customer types. Instead, it considers unmet demand at the microgrid level, meaning that any shortage in supply affects consumer and prosumer households equally. From the microgrid owner's perspective, prosumer households are less favorable to supply due to their limited hours of grid demand, which occur only when solar PV generation is inactive. During these periods, additional generation capacity from diesel generators or battery storage is required to supply all customers. However, this additional capacity is not optimally utilized, as prosumers primarily rely on their own generation and storage. Consequently, this results in electricity demand peaks that impose costs exceeding the resulting revenue. In this case, it is more beneficial for the microgrid to not fulfill supply and incur a penalty for the unmet energy demand. Prosumers contribute to this issue by their fluctuating reliance on the grid. Therefore, if feasible, the microgrid is expected to prioritize supplying its connected consumer households over its prosumer households. Figure 4.3 further supports this assumption by illustrating the simultaneous occurrence of unmet demand per connected prosumer household and the grid demand of prosumers. Notably, the hourly unmet demand per prosumer household never exceeds it's grid demand. Therefore, for the current microgrid configuration, it can be assumed that the demand of consumer households is fully met, while prosumer households experience the entirety of the unmet demand in the microgrid.

5.3. Feed-in Tariff and Unmet Demand Penalty Values

The Lebanese case study offers valuable insights into tariff structures, particularly for electricity pricing within the microgrid and the cost of diesel fuel. However, other tariffs, such as for fed-in electricity from prosumers or the penalty for unmet demand, could not be incorporated due to the absence of relevant regulatory frameworks or implementation mechanisms in the case study's environment. The determination of these two values was guided by the following considerations:

The feed-in tariff that prosumers receive for their surplus photovoltaic generation fed into the grid typically ranges between 0 $/kWh and the LCOE of PV. Prosumers who are concerned about the possibility of grid disconnection may be willing to accept lower compensation for their surplus electricity or even provide it for free in exchange for continued grid access. However, prosumer households have already made capital investments in their energy systems and demonstrated aspirations for self-sufficiency. As a result, they may be unwilling to donate their entire surplus to the microgrid while simultaneously paying up to four times their LCOE for electricity drawn from the microgrid. Therefore the feed-in tariff that prosumers receive for surplus PV generation consumed by the microgrid is set at the LCOE of PV (0.103 $/kWh). This rate reflects the actual cost of energy generation incurred by prosumers and is only a quarter of the electricity price within the microgrid.

Modeling frameworks often incorporate unmet demand as a measure to maintain model feasibility, typically assigning a high penalty to ensure that the majority of demand is met. In this study, unmet demand is utilized as an additional optimization tool for the microgrid owner, given that regulations do not mandate full demand fulfillment and alternatives to the microgrid are limited. The sensitivity analysis of the unmet demand penalty in Section 4.2 revealed a steep decline in annual unmet demand within the grid up to a penalty of 0.1 $/kWh, while more than half of the prosumers remained connected. Consequently, the penalty for unmet demand is set at 0.1 $/kWh to keep unmet demand at a moderate extent while maintaining a high amount of prosumers in the microgrid.

6. Conclusion

This thesis has presented a Lebanese microgrid model operating under market power to explore the opportunities and challenges of integrating renewable energy into the supply mix. Customer demand in this analysis was based on literature-derived profiles. These profiles reflect generalized consumption patterns, rather than the specific consumer behavior of Lebanese citizens. The analysis modeled three distinct representative days that reflect the core characteristics of all four seasons. This approach provided a broad reflection of seasonal variations while keeping the computational efforts of the model constrained. An increased number of representative days could capture day-to-day fluctuations in energy consumption for example the variations between weekdays and weekends.

The model identified a clear optimal strategy to meet microgrid demand using a technology portfolio that included both fossil-based and renewable generation, while maximizing profit. Furthermore, the analysis investigated the relationship between prosumers with self-owned solar PV generation and the profit-maximizing strategy of the microgrid owner, offering valuable and unique insights into microgrid access and demand fulfillment. The following four key conclusions summarize the main insights and implications of the study:

- The profit-maximizing strategy of the microgrid owner leads to an increase in solar PV generation, facilitating a transition from fossil-fuel-based to renewable-based energy supply. Photovoltaic production is the main source of electricity in the microgrid during PV's operating hours. This transition yields clear economic benefits for the microgrid owner, even when accounting for efficiency variations associated with utilization of diesel generators.
- Results indicate that the majority of solar PV generation originates from microgrid-owned capacity rather than prosumer-owned systems, due to the limited hours of surplus generation from prosumers. Only a small fraction of the generation from prosumers' solar PV installations is integrated into the microgrid's supply mix.
- During non-operational hours of PV, the primary source of electricity within the microgrid are the diesel generators. Factors such as rising electricity prices, an increasing number of prosumers in the customer base, or higher operational costs of the diesel generators incentivize the installation of battery storage systems to enable the distribution of renewable electricity production throughout the day.

- The analysis also reveals many disconnected prosumers from the grid due to an unprofitable demand profile from the microgrid owner's perspective. As a result, a substantial portion of prosumer-owned solar PV capacity remains underutilized.

In conclusion, despite the absence of a regulatory authority, microgrids that incorporate solar PV systems –including the surplus generation fed-in from connected prosumers– demonstrate clear improvements over the existing diesel-based microgrids for all parties. However, additional policy interventions are necessary to further reduce the unutilized PV generation potential on prosumer household rooftops and ensure equal access to electricity for all microgrid customers.

7. Future Work

7.1. Incorporate Public Grid Connection

Future research should extend the current framework by incorporating a microgrid connection to the public grid, as the existing model considers only an island microgrid. Although the public grid may be unreliable and inconsistent, its availability could significantly impact microgrid profitability and should be analyzed. A stochastic modeling approach would allow for the integration of grid variability and unreliability, providing a more comprehensive assessment. Given that microgrids in Lebanon have emerged as a response to public grid instability, it is essential for the modeling framework to account for this critical system parameter.

7.2. Consider Unmet Demand and Surplus Feed-in

The modeling of unmet energy demand for grid customers relies on several assumptions, including the absence of customer response to unsupplied demand. In reality, unmet demand could incentivize customers to invest in their own electricity generation, potentially becoming prosumers with less favorable demand profiles or even achieving complete self-sufficiency. To better reflect these dynamics, future work should incorporate a customer response to unmet demand into the framework, potentially by dynamically adjusting the penalty for unmet demand. In addition, future research should improve the modeling of surplus feed-in from consumers. The current feed-in tariff is set at an optimistically high value, which may not accurately reflect real-world conditions. Moreover, a precise framework is needed to determine the extent to which prosumers should be supplied when contributing surplus energy to the grid. Currently, this is regulated by Equation 2.10, which further restricts surplus feed-in due to the high level of unmet demand within the grid.

7.3. Use Empirical Demand Data

To enhance the accuracy of the microgrid demand, future research should incorporate empirical data obtained from logging devices or customized surveys to better reflect real-world consumption patterns. Furthermore, the number of representative days modeled should be increased to capture seasonal and weekly variations in energy consumption patterns.

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Figure 2.1 has been designed using resources from Flaticon.com.

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Statutory Declaration

I declare that I have authored this thesis independently, that I have not used other than the declared sources/resources, and that I have explicitly marked all material which has been quoted either literally or by content from the used sources.

Wien, ______________________ ______________________

Date Signature

Appendix A.

Model Notation

Symbol	Variable name	Unit
$a_{g,y}$	Added capacity of technology g by DGC during year y	kW
$c_{g,y}$	Capacity of technology g installed by DGC during year y	kW
$r_{g,y}$	Retired capacity of technology g by DGC during year y	kW
$disp_{g,y,d,h}$	Dispatched power from generator g at year y, day d and hour h	kW
$fi_{i,y,d,h}$	Fed-in Energy from prosumer i at year y, day d and hour h	kWh
$ud_{y,d,h}$	Unmet Demand of prosumers at year y, day d and hour h	kWh
$soc_{y,d,h}$	Battery State of charge at year y, day d and hour h	kWh
$b^{in}_{y,d,h}$	Battery input at year y, day d and hour h	kW
$b^{out}_{y,d,h}$	Battery output at year y, day d and hour h	kW
$n^{house}_{i,y}$	Number of households of type i during year y	
$bin^{DG}_{k,y,d,h}$	Binary variable for the k^{th} constraint on the Diesel Generator heat rate curve at year y, day d and hour h	
bin^{B}_{y}	Binary variable for the installation of a Battery System at year y	
$c^{DGSteps}_{j,y}$	Integer variable for the installed j^{th} quantity step of DG Capacity at year y	

Table A.1.: Decision Variables

Symbol	Parameter name	Unit
ρ	Electricity Price in Microgrid	USD/kWh
ϵ	Penatly for Unmet Prosumer Demand	USD/kWh
τ	Feed- in Tarif for Prosumer Surplus PV	USD/kWh
κ_g	Initial installed capacity of technology g in MC	kW
λ_g^C	Unit capital cost of technology g	USD/kW
λ_g^{OF}	Unit fixed operation cost of technology g	USD/kW/y
λ_g^{OV}	Unit variable operation cost of technology g	USD/kWh
ν_g	Lifetime of technology g	years
ν_g^0	Remaining lifetime of installed technology g at year 0	years
π	Price of diesel	USD/L
α_k^{HR}	Heat rate of the diesel generator on the k^{th} portion of the heat rate curve	L/kWh
α_j^C	k^{th} Possible DG Capacity additions	kW
θ_i^{PV}	Average PV capacity of households of type i	kW
θ_i^{Bat}	Average Battery capacity of households of type i	kWh
Ω_i	Total number of households of type i available to the microgrid	
$\mu_{i,d,h}$	Electricity Grid Demand of household of type i at day d and hour h	kW
$\sigma_{i,d,h}$	Surplus Energy of household of type i, at day d and hour h	kWh
$\phi_{d,h}$	Capacity factor of PV at day d and hour h	
χ^{useL}	Landuse of PV	m^2/kW
χ^{avL}	Available Land for PV	m^2
β^{msoc}	Battery Minimum state of charge	
β^{rcp}	Ratio Battery Capacity / Battery Power	
β^{eff}	Battery Charging and discharging efficiency	
β^{min}	Minimum Battery Capacity for Installation	kWh
ω_d	Number of representative day d in a year	
γ	Interest rate	
Υ	Planning horizon	

Table A.2.: Model Parameters

Symbol	Set name
$\mathcal{D}$	Representative day within a year
$\mathcal{G}$	All generation technologies (Diesel generator DG, photovoltaic cells PV, Prosumer PV Feed-in FI, MC Batteries B)
$\mathcal{G}g$	Non-storage technologies (DG, PV, FI)
$\mathcal{G}o$	Owned technologies (DG, PV, B)
$\mathcal{H}$	Hours in a representative period
$\mathcal{I}$	Household types (Consumer Household CH, Prosumer Household PH)
$\mathcal{K}$	Values of the DG Heatrate - Curve
$\mathcal{C}$	Values for DG Capacity addition
$\mathcal{S}$	All decision variables
$\mathcal{Y}$	Years

Table A.3.: Derived sets

Appendix B.

Mathematical Formulation

Capacity Constraints

$$c_{g,y} = c_{g,y-1} + a_{g,y} - r_{g,y} \qquad \forall g, y \qquad \text{(B.1)}$$

$$c_{g,y=0} = \kappa_g \qquad \forall g \qquad \text{(B.2)}$$

$$b^{in}_{y,d,h} \leq c_{g=B,y} \qquad \forall y, d, h \qquad \text{(B.3)}$$

$$b^{out}_{y,d,h} \leq c_{g=B,y} \qquad \forall y, d, h \qquad \text{(B.4)}$$

$$n^{house}_{i,y} \leq \Omega_i \qquad \forall \qquad i, y \qquad \text{(B.5)}$$

Battery Constraints

$$soc_{y,d,h} = soc_{y,d,h-1} + \beta^{eff} \cdot b^{in}_{y,d,h} - \frac{b^{out}_{y,d,h}}{\beta^{eff}} \qquad \forall y, d, h \qquad \text{(B.6)}$$

$$soc_{y,d,h=0} = soc_{y,d,h=23} + \beta^{eff} \cdot b^{in}_{y,d,h=0} - \frac{b^{out}_{y,d,h=0}}{\beta^{eff}} \qquad \forall y, d \qquad \text{(B.7)}$$

$$soc_{y,d,h} \geq c_{g=B,y} \cdot \beta^{rcp} \cdot \beta^{msoc} \qquad \forall y, d, h \qquad \text{(B.8)}$$

$$soc_{y,d,h} \leq c_{g=B,y} \cdot \beta^{rcp} \qquad \forall y, d, h \qquad \text{(B.9)}$$

$$a_{g=B,y} \leq bin^B_y \cdot M \qquad \forall y \qquad \text{(B.10)}$$

$$a_{g=B,y} \geq bin^B_y \cdot \frac{\beta^{min}}{\beta^{rcp}} \qquad \forall y \qquad \text{(B.11)}$$

Retirement Constraints

$$r_{g,y=\nu^0_g} = \kappa_g \qquad \forall g \in \mathcal{G}o \qquad \text{(B.12)}$$

$$r_{g,y} = 0 \qquad \forall g \in \mathcal{G}o, y \in [0, \nu^0_g - 1] \qquad \text{(B.13)}$$

$$r_{g,y} = 0 \qquad \forall g \in \mathcal{G}o, y \in [\nu 0_g + 1, \nu_g - 1] \qquad \text{(B.14)}$$

$$r_{g,y} = a_{g,y-\nu_g} \qquad \forall g \in \mathcal{G}o, y \in [\nu_g - 1, \Upsilon] \qquad \text{(B.15)}$$