TECHNISCHE UNIVERSITÄT WIEN

architektur + raumplanung

DIPLOMARBEIT
(Diploma Thesis)

Minimalflächen in der Architektur als mobile, wandelbare, ressourcenschonende Bauten

Mimimal surfaces in architecture as mobile, transformable and resource-saving structures

ausgeführt zum Zwecke der Erlangung des akademischen Grades
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Abstract

The aim of the work is to realize an architecture that is ecologically and socially sustainable, that addresses the improvement of the urban climate and resource-efficient building, and that can change places. This is achieved through the study of a construction method that is very rare. The edifices built so far are spatially unique and structurally demanding. Designs whose basic geometry corresponds to that of minimal surfaces are under investigation.

We begin with example projects that were built with the same intention for the future. Consequently, we consider the concepts from two scientific fields that explain the form with the smallest possible area. We focus on the geometric derivation in order to unfold its exceptional suitability for building structures.

After studying four types, we decide on a module that will serve as the building block for the pavilion construction in full scale. In this part we devote ourselves to the architectural design and structural performance of the selected minimal surface. We then focus on the architectural design and performance of the selected minimal area. The modules will have the same geometry and size and can be assembled in various ways.

Finally, we examine different typologies. As more and more becomes possible, we aim to provide a broader perspective on what could still be spatially realizable.

The last two pages are dedicated to the summary of the work. The results and insights are substantiated and discussed. This leads to new thoughts and ideas around the topics of economy and elegance, environment and future, modularity and resources in construction.

Kurzfassung

Das Ziel der Arbeit ist, eine Architektur ökologisch und sozial nachhaltig zu realisieren, die für die Themen Verbesserung des urbanen Klimas und ressourcengerechtes Bauen steht und wandeln kann. Dies geschieht mithilfe der Studie einer Bauweise, die sehr selten ist. Es sind Strukturen, deren Basisgeometrie einer Minimalfläche entsprechen, die nach wie vor zu der Forschung gehören. Die bisher realisierten Bauten sind räumlich einzigartig und konstruktiv anspruchsvoll.

Wir beginnen mit Beispielprojekte, die mit der gleichen Intention für die Zukunft gebaut wurden. Folglich ziehen wir die Begriffe zweier Wissenschaftsgebiete in Betracht, die die Form mit kleinstmöglichem Inhalt erklären. Wir konzentrieren uns auf die geometrische Herleitung, um die hervorragenden Eignungen der baulichen Strukturen zu entfalten.

Nach unserer Studie von vier Arten, entscheiden wir uns für ein Modul, das der Baustein für den Pavillonbau im echten Maßstab sein wird. Danach widmen wir uns der architektonischen Gestaltung und Performance der ausgewählten Minimalfläche. Die Module werden die gleiche Geometrie und Größe aufweisen und können in unterschiedliche Art und Weise zusammengebaut werden.

Zum Schluss schauen wir uns unterschiedliche Typologien an. Da immer mehr und mehr möglich ist und um eine umfangreichere Vorstellung geben zu können, was räumlich noch realisierbar wäre.

Die zwei letzten Seiten widmen sich der Zusammenfassung der Arbeit. Die Ergebnisse und Erkenntnisse werden daraus belegt und diskutiert. Es kommt zu neuen Gedanken und Ideen rund um die Themen Wirtschaftlichkeit und Eleganz, Umwelt und Zukunft, Modularität und Ressourcen beim Bauen.

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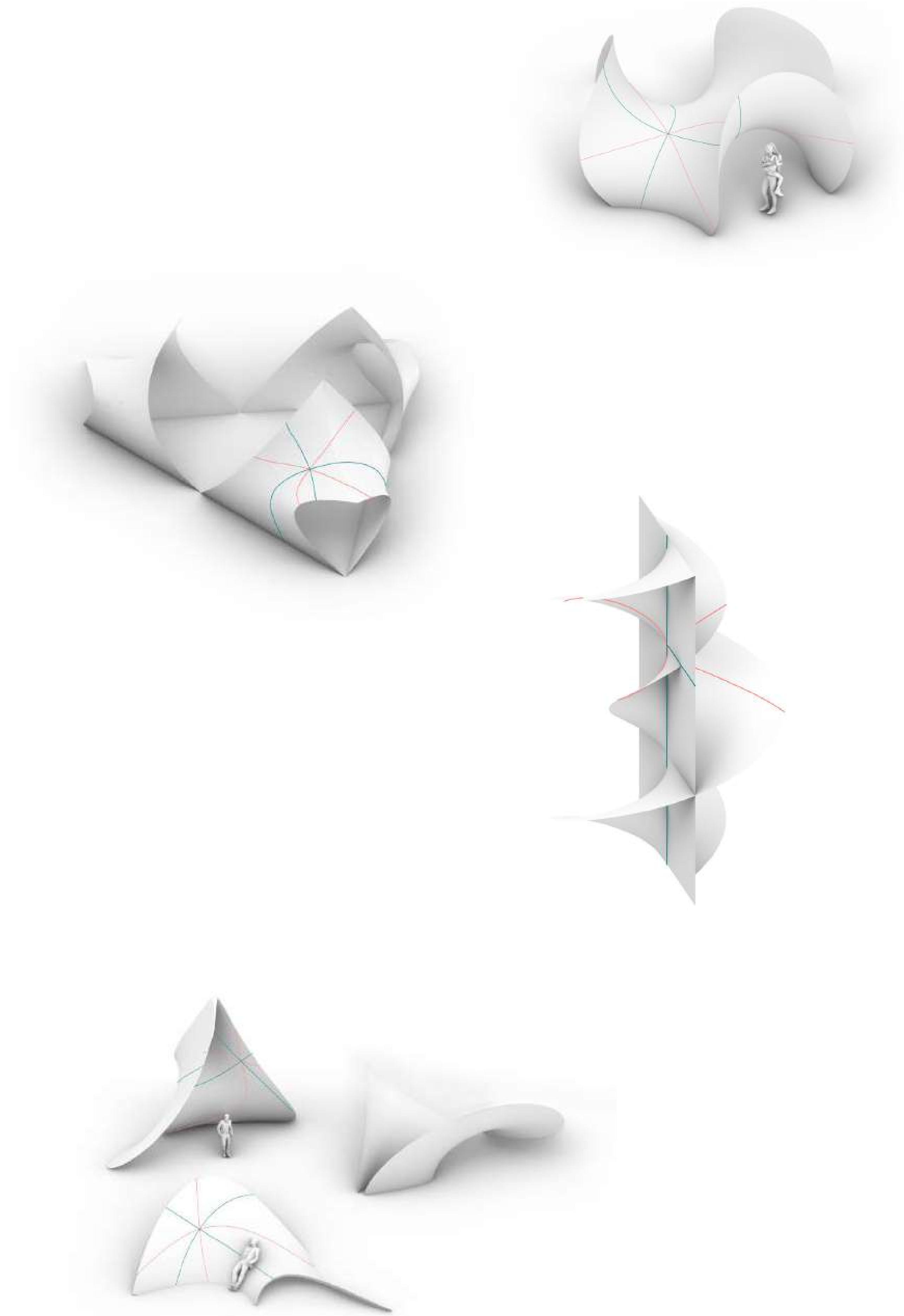

Fig.4 Minimal surfaces, examples of main curvature lines and asymptotic curves at selected points

1. References

This section shows examples of innovative projects in structural, material, spacious and aesthetical point of view, projects "doing more with less".

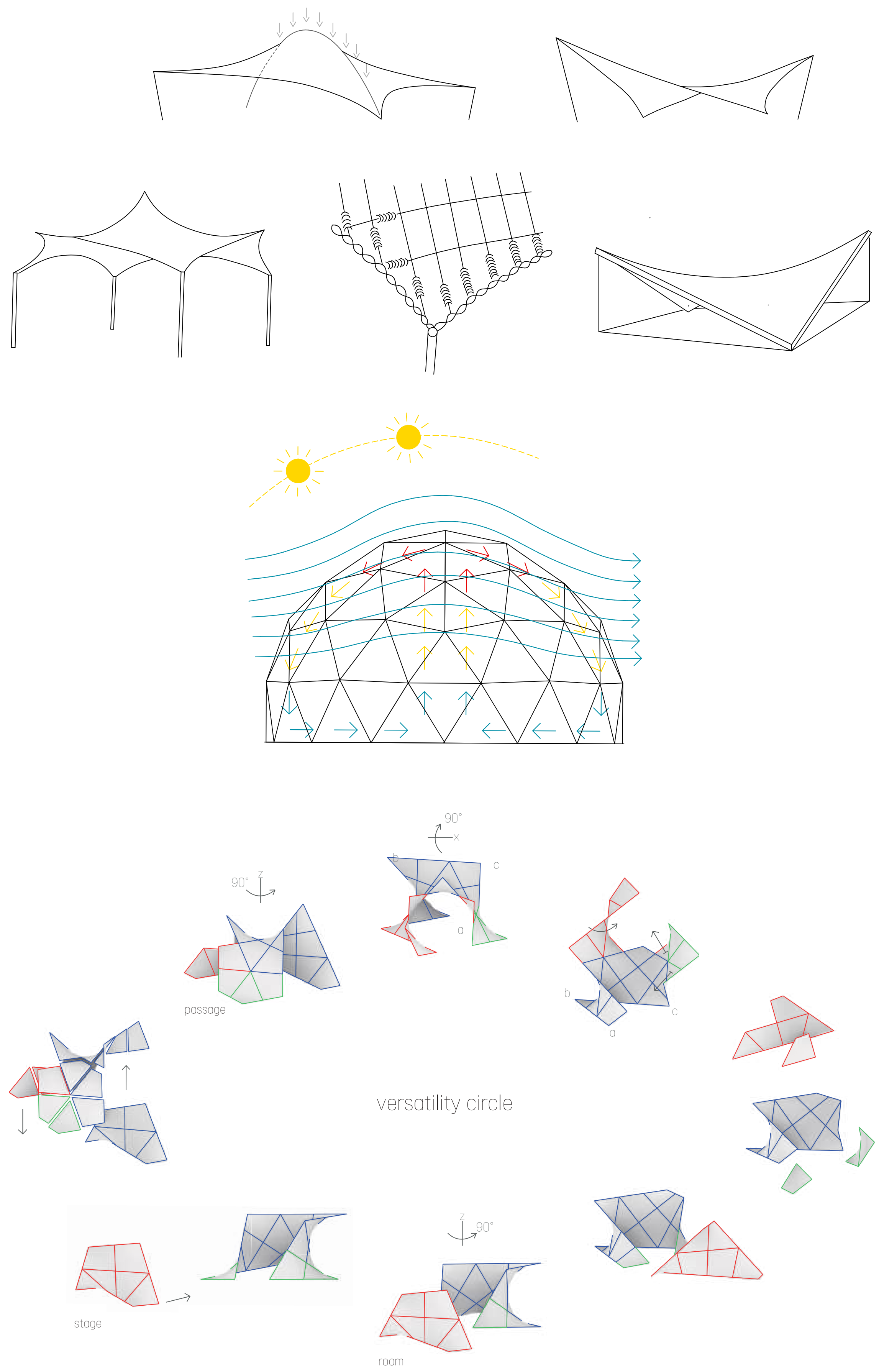

Fig.5 Diagrams of the reference projects

1.1 Frei Otto

Together with his team Frei Otto was doing experiments with forming bubbles for decades. It has been shown that self-generating and self-optimising structures, such as the soap films (fig.6) can be very efficient when designing with the minimal possible material. The way he realised such a natural edifices was to sometimes do microscope experiments, simulations, calculations, research throughout different disciplines and building of models.[1] In other words, to observe and analyse nature in order to repeat its principles.

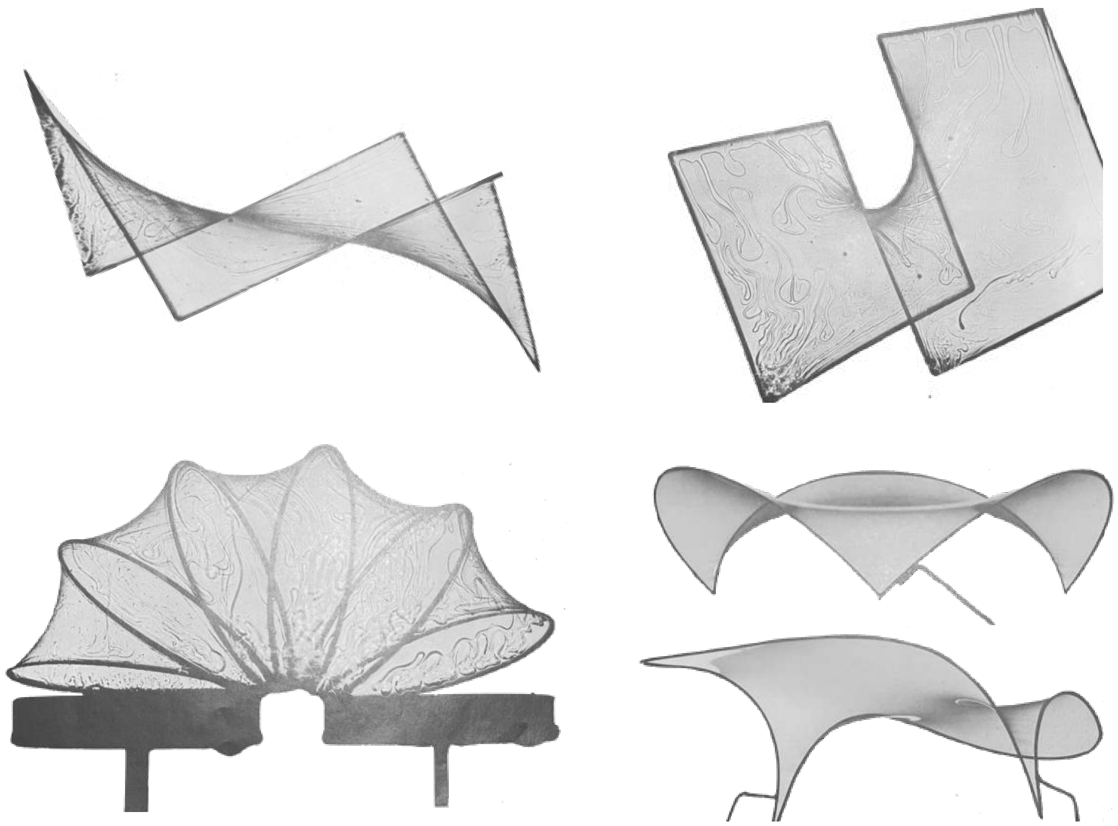

Fig.6 Experiments with soap films at IL Stuttgart (reference: *Bach, et.al.*,Seifenblasen Forming Bubles)[2]

The work of the pioneer of lightweight construction gave access and understanding of previously hardly explored areas of construction. Three example projects follow, where the structural behaviour of minimal surfaces were only able to be comprehended only by building physical models. These projects are analyzed by Martin Schuster in his coursework: "Designgeschichte: Frei Otto". Before delving into this, it is important to note that, strictly speaking, classic membrane structures do not adhere to an ideal minimum geometry because they have two primary load-bearing directions (warp and weft).

'Der Musikpavillon' (fig.7) was build 1955 for "Bundesgartenschau" in Kassel and it is his first experimental project with the geometry of minimal surfaces. 1mm thick cotton fabric, 18m in length. Extremely durable even in the strongest gusts of wind. Furthermore, the stretched membrane had very good acoustic properties.

'Der Eingangsbogen' (fig.8) or the Entrance arches spanned 34m wide and covered an area of 698m^2. The construction of 19cm thick steel pipe was stabilized by a glass silk fabric that was divided in two parts on both vault sides. Its ends were stretched over two 3m high cable trestles at a distance of 12m on both sides of the arch. This weightless structure had to be cut and processed with the greatest precision because any kind of wrinkles cannot be removed by tensioning due to the fabric`s lack of elasticity.

Initially the Dance pavilion (fig.9) was planned to remain only for one season. Six 10m long construction masts support a 1000 m2 cotton sailcloth membrane. The 12 segments are arranged around a central ring and mirror each other in pairs so that their outer borders undulate. The 28 m2 central opening is held fast with a cable tension ring that is suspended from the masts with ridge cables. Tow cables secure the placement of the ring and prestress the membrane. [3]

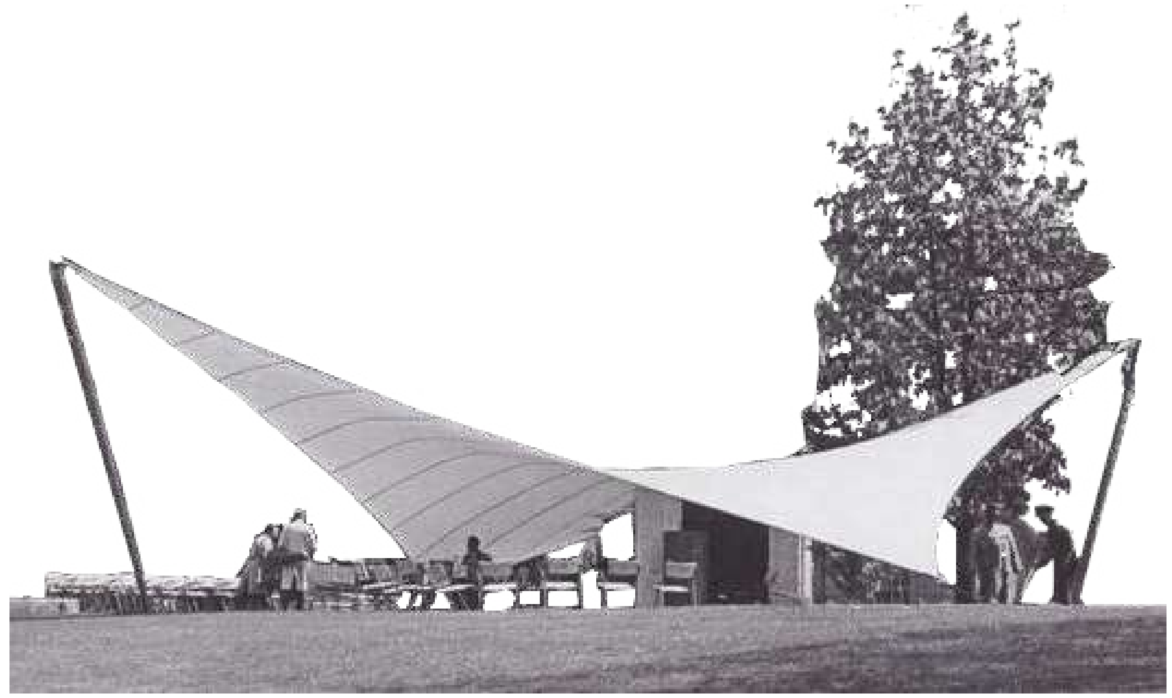

Fig.7 The musicpavilion (reference: *Meissner, et al.*, Frei Otto- forschen, bauen, konstruieren,15)[4]

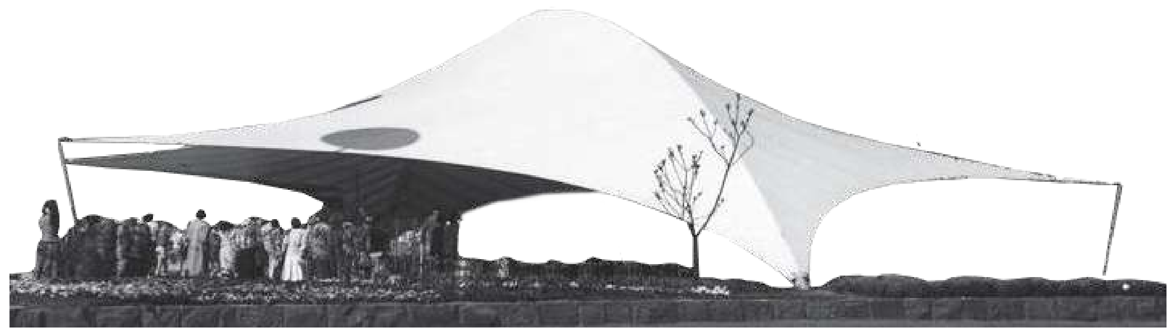

Fig.8 The entrance arches (reference: *Meissner, et al.*, Frei Otto- forschen, bauen, konstruieren, 41)[5]

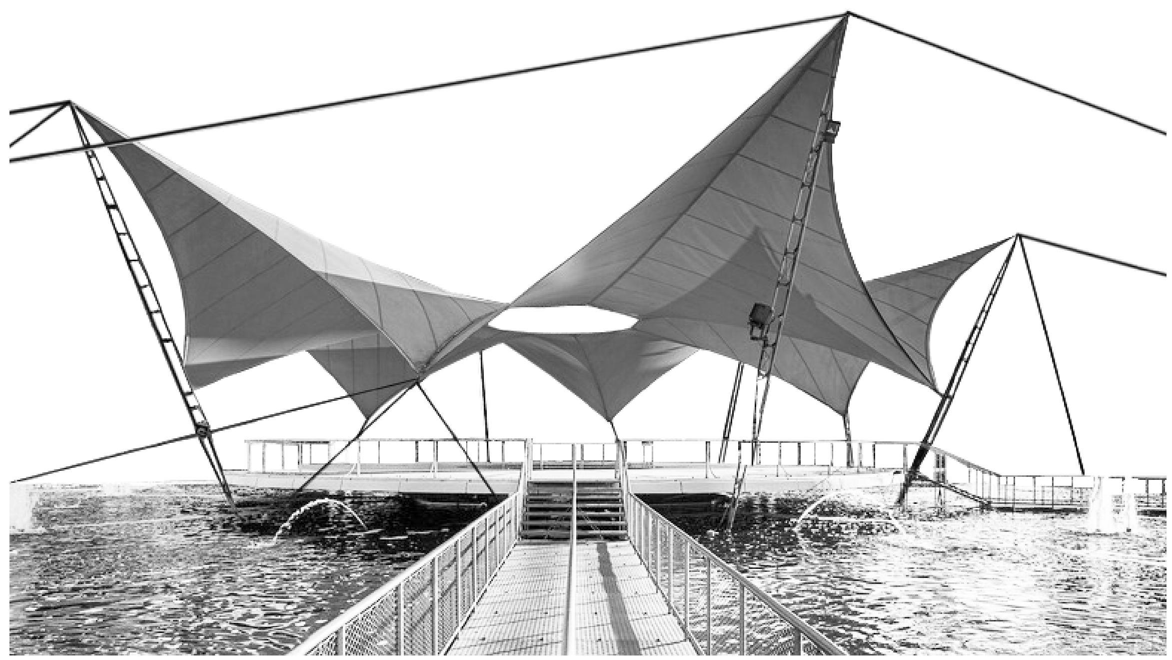

Fig.9 The dance pavilion (reference: *Meissner, et al.*, Frei Otto- forschen, bauen, konstruieren, 41)[6]

1.2 Richard Buckminster Fuller

Richard Buckminster Fuller worked on problems such as global transportation, communications and power transmission. As such he needed a two-dimensional representation of the Earth. Sphere or spherical surfaces are not easy to develop, but he found a way to create a map with very little distortions. He used a kind of polyhedral map projection of the continents onto a developed icosahedron. This is one example of his obsession with the world map and the globe. Fuller`s most famous design has also something in common with the Platonic solids and the Earth`s form.[7] The convex polyhedrons named by the ancient philosopher inherit congruent regular polygons as faces and at each vertex the same number of faces meet. We will look at more of their properties in chapter three: "Case studies" and five: "Excursus:...". It is important during the research phase that their natural beauty and intrinsic logic serve as an optimal scaffolding for the innovative project.

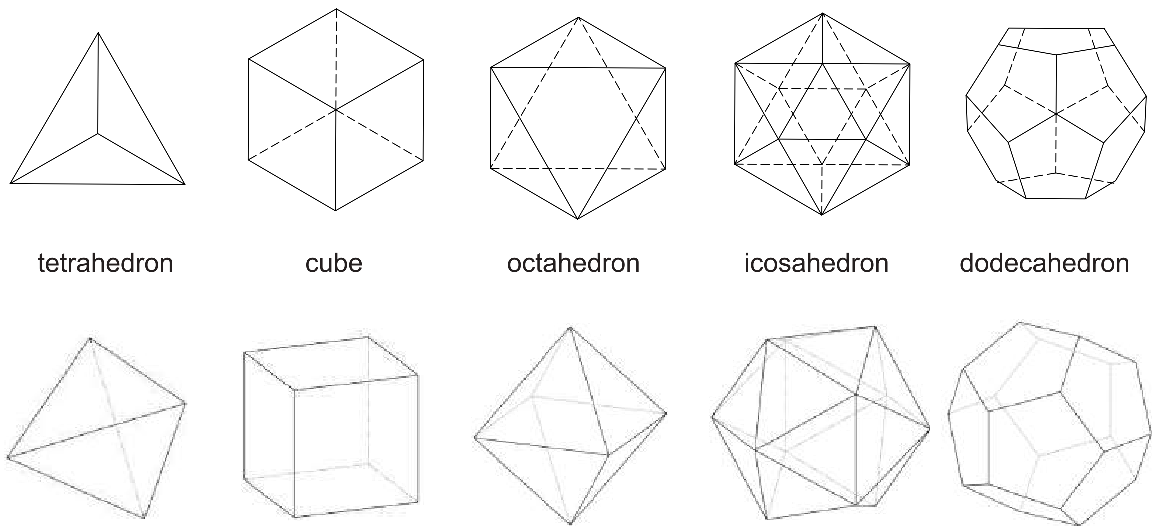

Fig.10 The platonic solids; side and perspective viuw

Buckminster Fuller extrapolated the geometry from the solids by truncating one solid and dividing its sides. In the example of the icosahedron by dividing each side of each triangle by two, a 2-frequency dome is created (fig.11). By dividing it by three a 3-frequency dome and so on. By counting the number of struts between the centers of each pentagon the frequency of any dome can be determined. The higher the frequency, the more spherical the dome is.[8]

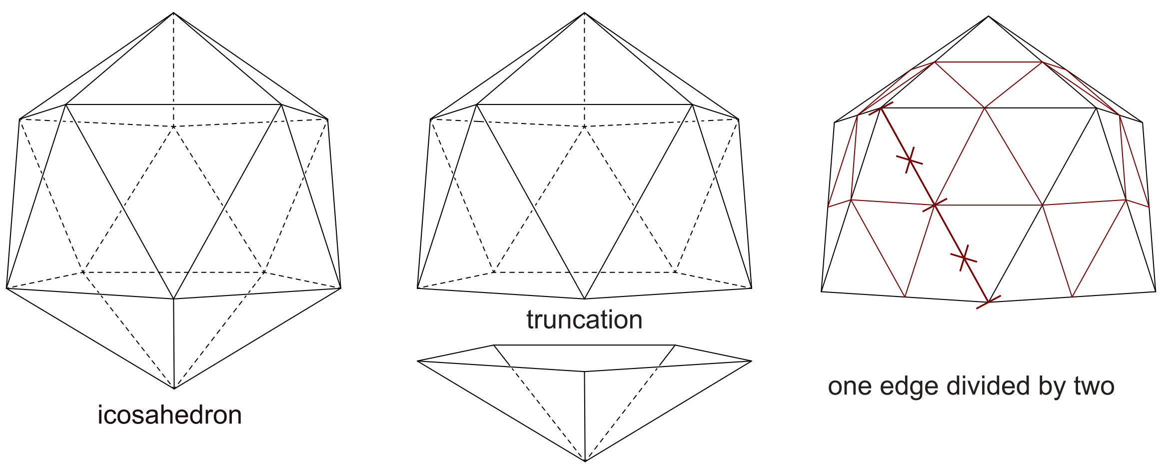

Fig.11 Truncating of an icosahedron and dividing each edge by two (redrawn from reference: https://www.youtube.com/watch?v=zpfql-Be5rA&t=91s)[9]

Paul Robinson is researching and providing information about building of geodesic domes:

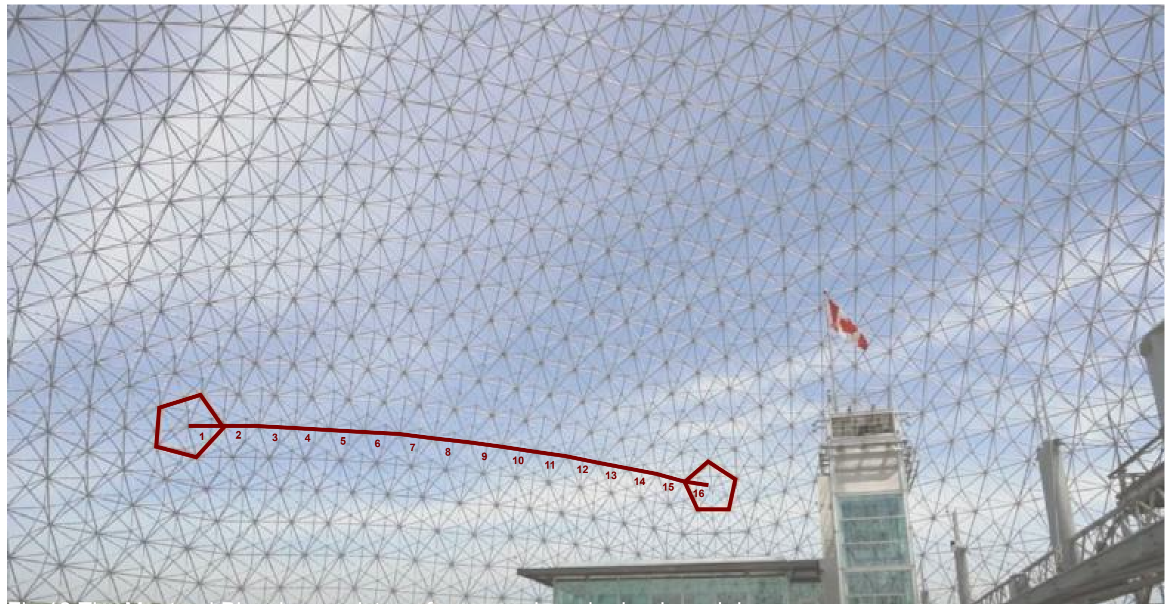

Fig.12 The Montreal Biosphere - sixteen frequency icosahedronbased dome

Fig.13 The Eden Project- nine frequency icosahedronbased dome

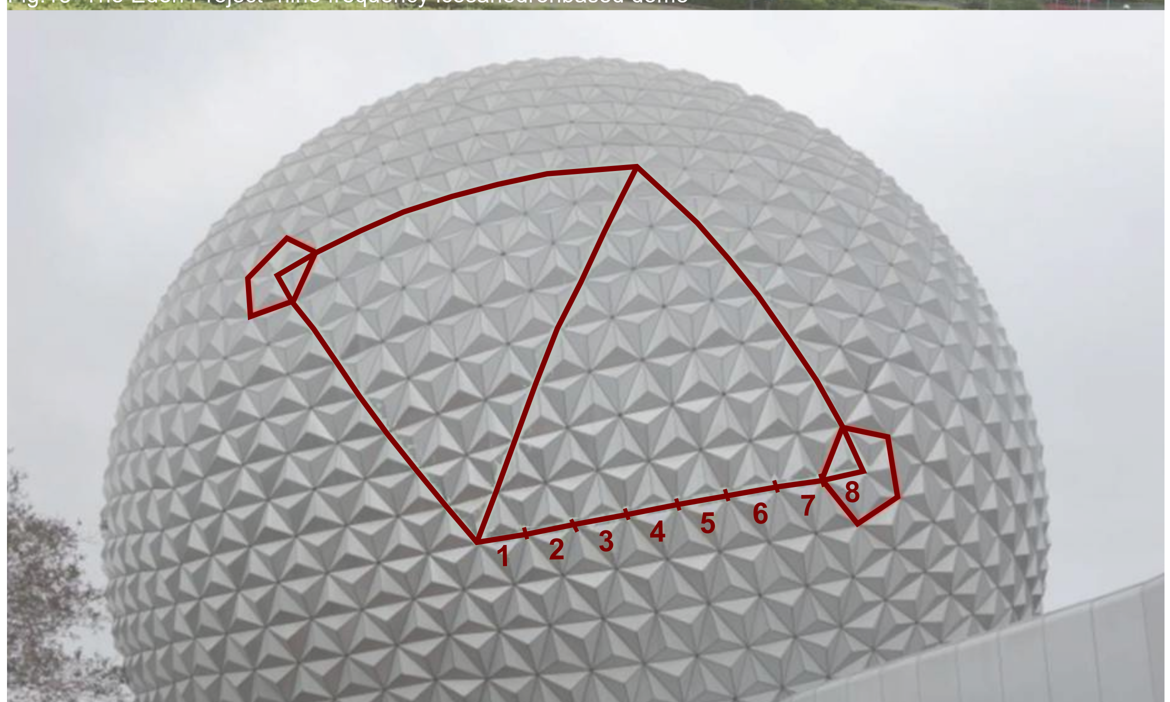

Fig.14 The Epcot Theme park- pentakis dodecahedron (page reference: https://www.youtube.com/watch?v=Ub86TsNBuC8&t=220s)[10]

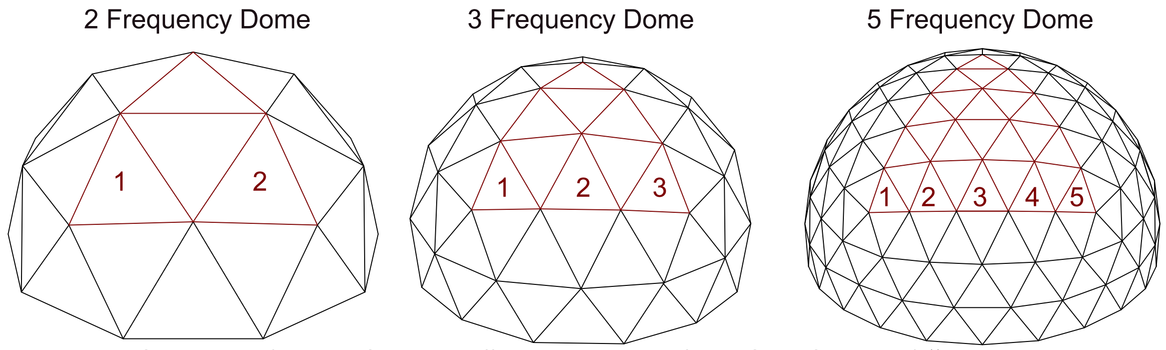

Fig.15 Dome frequencies (redrawn from: https://www.youtube.com/watch?v=zpfql-Be5rA) [11]

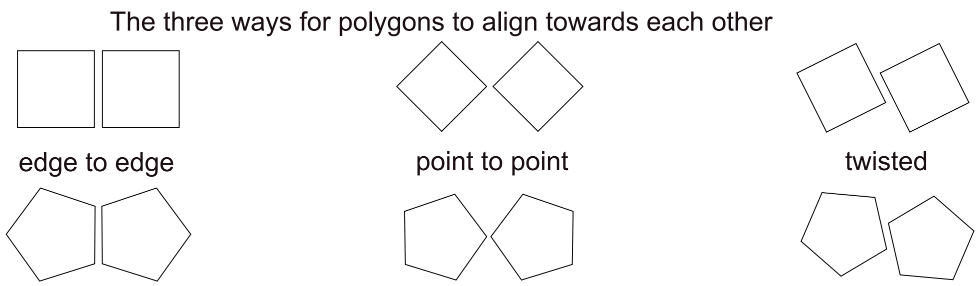

Fig.16 Possible polygon joints (reference: https://www.youtube.com/watch?v=Ub86TsNBuC8&t=220s) [12]

Such a dome encloses the most volume for a given surface area and the circle-footprint - the most area for a given perimeter. They are also very structurally efficient. Due to their spherical shape and the pattern that constitutes them, the geodesic domes inherit self-support with remarkable resistance. Triangles are rigid shapes regardless of the connections at the vertices.[13] The efficient distribution of stress across the interlocked struts creates a structure that is lightweight, yet provides free open space inside. Fuller`s design covers vast spaces and spans great distances using minimal materials.

As everything in the world the domes also had imperfections. They were not suitable for crowded urban spaces because of echoes. Odours or fires spread evenly inside the hemispheres. Because of the many joints and connections of the elements, leaks were a problem.[14] After years of designing many geodesic domes for industry and the military, in 1966 he began something new. Together with a specialist in fiberglass and the architect Norman Foster he designed the Fly`s Eye Dome (fig 17). 11,5m x 15.2m x 15.2m lightweight fiberglass construction with circular openings. Thanks to the pre-cast pieces, or in other words modules, the problems described improved significantly.[15]

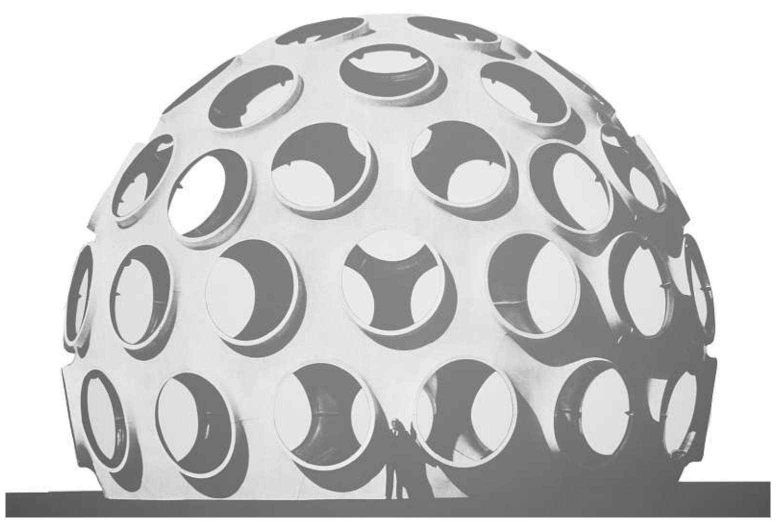

Fig.17 Buckminster Fuller and a Fly's Eye Dome (reference: https://www.fullerdome.org/blog/tag/fly+eye+dome)[16]

1.3 Versatile spaces continuity

The work of Frei Otto and Richard Buckminster Fuller is innovative. They invented new architecture with a minimal manner. Of course they didn't start from scratch and relied on some established building principles. Their constructed edifices are outstanding, because of the facts explained in the previous two chapters. Some may call into question the functionality of the architecture. For various reasons it is not habitual for us to live, use, or experience curved spaces. Possibly because it is just not common for us to imagine how to do that? Since summer term 2021 students and tutors of the design studio 'versatile spaces' (Research Unit of Building Construction and Design 2 and Research Unit Structural Design and Timber Engineering at TU Wien) are developing the intentions described in the previous pages. With the addition of more flexibility in the architecture plus the reusing of materials. Bellow you can see a very brief explanation about the geometrical development of the project sequence because I dedicated my time on this part. Double curved structures were created by using initially straight elements that were interwoven together into a pattern (fig.18 left). The stripes are following the asymptotic curves, describing the minimal surface. To make the construction method faster, to minimize the material consumption even more and to create further variable and easily changeable design, the pattern was divided into modules. (fig.18 right, fig.19, fig.20)

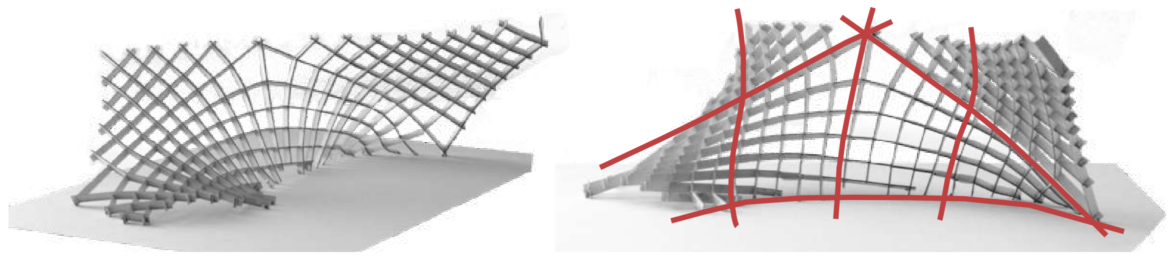

Fig.18 Left: asymptotic gridshell, right: the gridshell divided into modules (ressource: Versatile spaces SS2022)

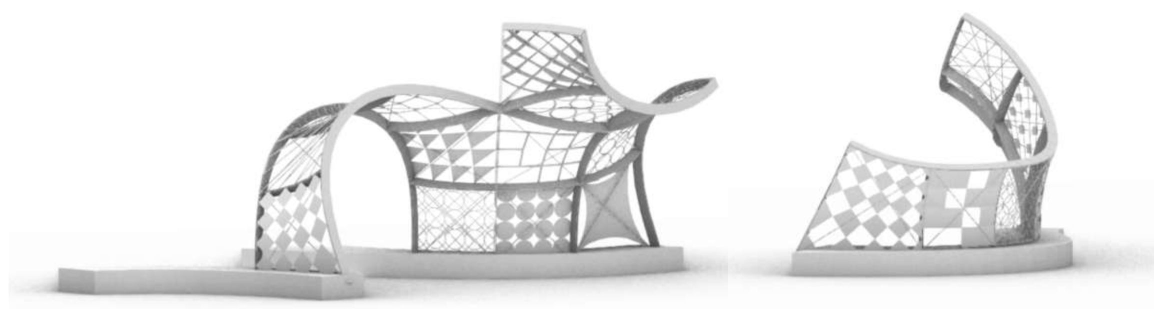

Fig.19 Variant A with modules (ressource: Versatile spaces WS2023)

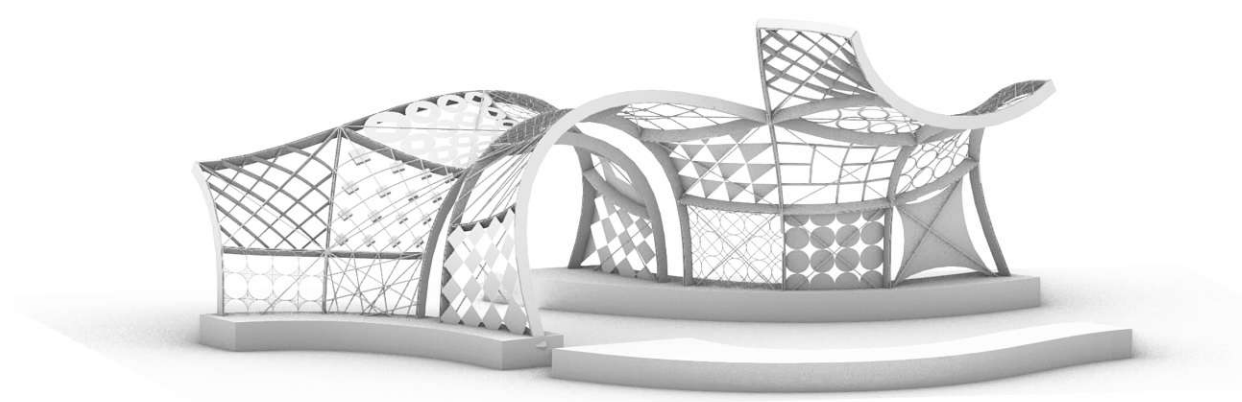

Fig.20 Variant B with modules (ressource: Versatile spaces WS2023)

1.4 Ve.sh pavilion

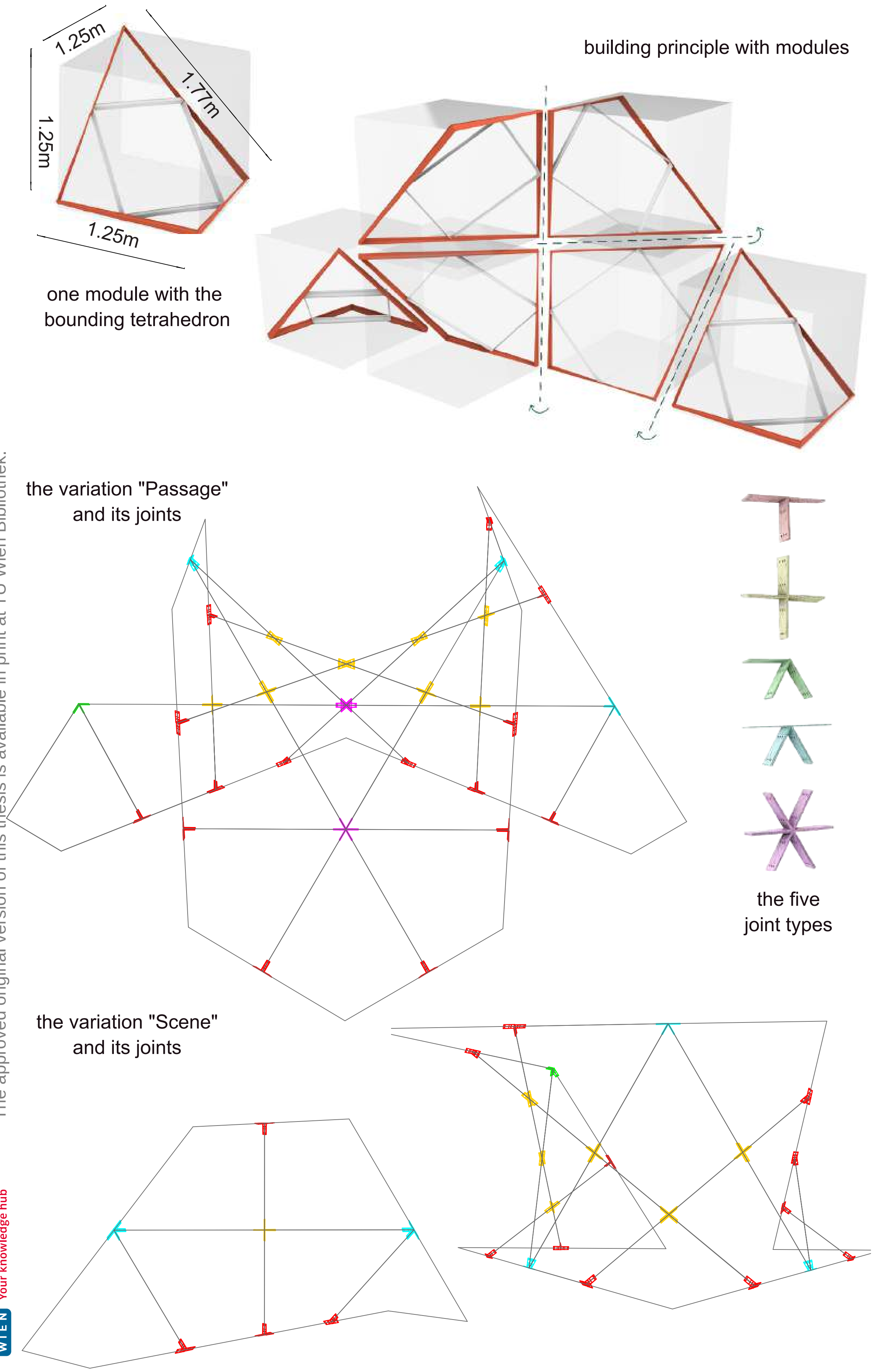

Fig.21 Up: way to build geometries via the modules; bottom: the connections of the variants

Fig.22 The opening of ve.sh in front of the TU Wien (ressource: Versatile spaces SS2023)

In summer term 2023 the design studio team built ve.sh with modules. Addressing current discourses and areas of tension, the project is the first modular architecture built with reused materials that follows the geometry of minimal surfaces. With 22 building units and between 70.00 kg - 1400.00 kg CO_2-emissions saved, the pavilion embodied its programme.[17] During nine days lunch dialogues were held, where the role of architecture in creating sustainable future was the main topic. Thanks to the concept it was very straightforward to reassemble the geometry. It took about 3 hours of work for 5 students to deconstruct the pavilion completely (unscrewing 900 screws, planting its flowers on Karlsplatz, relocating 262 kg of foundation stones and modules to a distant city district). It is important to note that the rebuilding of one variation into another was not working smoothly, five joint types and tolerances were needed (fig.21). The desire to examine new variations, building units and details remained left in the air. A proposal of enhancing the ve.sh pavilion is studied during the work on the following diploma project.

Fig.23 View of Ve.sh in front of the Otto Wagner Pavillon Karlsplatz (ressource: Versatile spaces SS2023)

1.5 Summary of the references

The reference projects showed examples of how to build experimental, minimalistic and beautiful. Obviously, on our round planet, curved architecture is structurally and functionally outstanding. When it comes to aesthetics, they look more natural to the eye. What is completely straight in nature? Following the motives of the projects, we now start to investigate and work with minimal surfaces in architecture.

1.-1.5 Chapter references:

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[3] *Schuster*, Seminararbeit Designgeschichte: Frei Otto, 2.Projekte Frei Ottos und des Instituts für Leichte Flächentragwerke,http://www.aspekt1.net/ms/fo_ref/projekte.html (10.2024)

[4] *Meissner, et al.*, Frei Otto- forschen, bauen, konstruieren , Edition Detail [1] (2015) 15 (128)

[5] *Meissner, et al.*, Frei Otto- forschen, bauen, konstruieren , Edition Detail [1] (2015) 41 (128)

[6] *Meissner, et al.*, Frei Otto- forschen, bauen, konstruieren , Edition Detail [1] (2015) 43 (128)

[7] Ananthasuresh, Buckminster Fuller and his Fabulous Designs, Resonance (2015) 9 (25)

[8] *Pacific Domes*, Geodesic Dome Frequencies Explained, https://www.youtube.com/watch?v=zpfql-Be5rA&t=91s (12.2024)

[9] *Pacific Domes*, Geodesic Dome Frequencies Explained, https://www.youtube.com/watch?v=zpfql-Be5rA&t=91s (12.2024)

[10] *Robinson*, How to work out the geometry of any geodesic dome, https://www.youtube.com/watch?v=Ub86TsNBuC8&t=220s (12.2024)

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[13] *Laila*, *et.al.*, The Constructive Advantages of Buckminster Fuller's Geodesic Domes and Their Relationship to the Built Environment Ergonomics, Advantages in Ergonomics in Design, Springer (2018) 358 (588)

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[15] *Crystal Bridges Museum of American Art,* Fly's Eye Dome | Research Guide (2023) 1 (2)

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Versatile spaces SS2022: Integratives Entwerfen trespassing grounds - creating versatile spaces
Lecturers: Dr.Ing. DI Sandra Häuplik-Meusburger, Univ. Prof. DI Peter Bauer , DI Marilies Frei, DI Lukas Zeilbauer; Students: Mikhail Danilenko, Clarissa Fabri, Ekaterina Mihaylova, Martina Zalevska

Versatile spaces WS2023: Entwerfen Constructing Versatile Space(s)
Lecturers: Dr.Ing. DI Sandra Häuplik-Meusburger, Univ. Prof. DI Peter Bauer , DI Marilies Frei, DI Lukas Zeilbauer; Students: Almas Azzahra, Sara Borjanovic, Valentin Burtscher, Till Caspary, Eralba Jonuzi, Marcos Luis Aleman , Ekaterina Mihaylova, Milomir Vincent Milenkovic, Uros Miletic, Edna Nineska, Johannes Pelz, Tura Bou Rissech, Peter Schandl, Susanna Cara Schmadalla

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2 Minimal surfaces

To find a better, quicker, faster, easier way and to find an optimum, is something people long for. Every day we are trying to find the shortest distance to a goal, or to finish a task in the least amount of time. Minimal surfaces inherit the smallest possible area for a given boundary. No other geometry exists with smaller surface area. Therefore the amount of material and weight are reduced to a minimum. Because of their physical and geometric properties, they can often be found in the universe (fig.24).

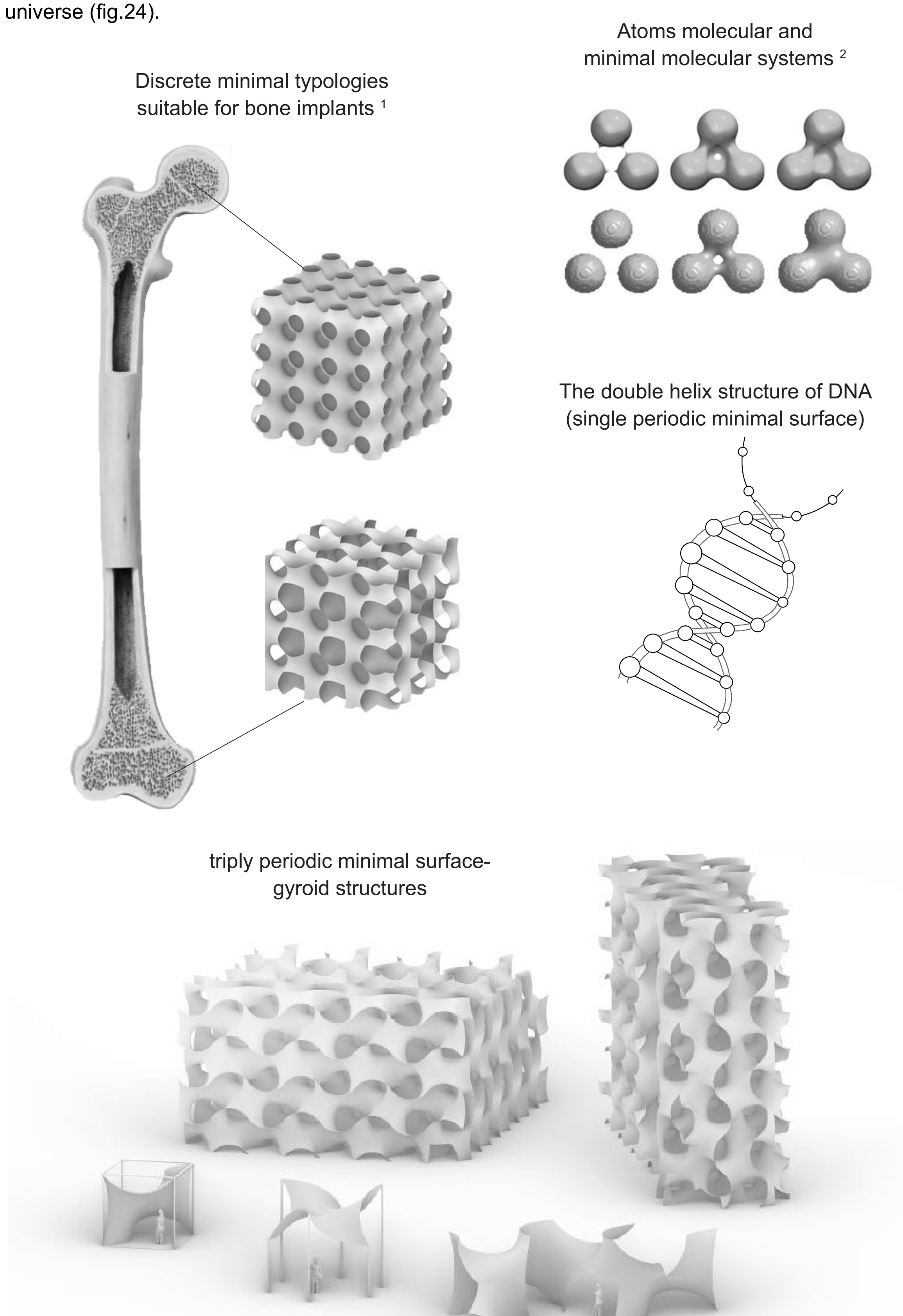

Fig.24 minimal surfaces as bone implants, in a molecular system, DNA, triply periodic minimal surface (bone reference: *Myaing*, Skeletal system)[3]

2.1 In science

"In 1760 the versatile mathematician Joseph-Louis de Lagrange (1736-1813),..., recognized that the problem of area minimizing is a characteristic model problem for many practical phenomena and that a detailed study of its properties would lead to far-reaching insight into many other so called variational problems. He opened up the mathematical investigation of minimal surfaces".[4]
One easy and very popular method to visualize them is to create a soap film between boundaries of wire (fig.6). They have, by their nature, zero mean curvature, i.e. the maximum and the minimum values of the curvature at this point are zero. Nowadays, they are a primary element in the physical simulation of compound polymers, black holes, protein folding or general relativity. "In engineering mechanics, for instance, the torsional stress of a bar or tube may be determined using a soap bubble above a similar plan in combination with the membrane analogy described by Prandtl. In electrical engineering equipotential surfaces are of importance and have the form of minimal surfaces." [5]
The complex geometries also have diverse range of applications in nanotechnology, molecular engineering, medicine, design and architecture. Light roof construction, form-finding models and pavillons are built following the geometry of minimal surfaces.

2.2 In architecture

According to the work of Vitruvius (Roman architect and engineer known for his multi-volume work De architectura) there are three principles that good architecture should have- strength, utility and beauty. How do minimal surfaces respond to this thriad?
When it comes to strength, we firstly see double curvature of the same magnitude. So, they inherit balanced surface tension that is stabilising the construction at each point. As geometric entities they can be described with curves, circles, lines, ect. Beams following the asymptotic curves of the geometry consist of initially straight boards. In order to follow the geometry they need to be bent and torsioned to form the right network. This kind of reshaping of the beams can have stiffening effect for the structure. E.g. when normal forces are applied to it, or materials with non-linear elastic behaviour are in use. The beams meet orthogonally, joints of 90-degree angle can be advantageous. Taking into account all the geometric and static factors can reduce realization costs.
Light roof construction, form-finding models, headwalls, sidewalls and pavillons are build following the geometry of minimal surfaces. No other geometry exists with smaller surface area, therefore the amount of material and weight are theoretically reduced to a minimum. Some kinds can be self-organizing, e.g. the building principle of ve.sh, what makes the building principle simultanious. When building with modules, different variations for diverse functions are possible.

As we have seen so far in this diploma thesis, curved architecture inherits natural aesthetics. Curves soften the view and in my opinion they reduce psychological stress. Interestingly, static arches distribute loads by transferring weight along a curve, a load path that helps reduce tensile stress[6]

There are not many built examples of edifices with the smallest possible area. Therefore, the spaces and atmospheres that they create inherit new qualities. If we go into nature, we can see spirals, twists, waves, optimum shapes and we feel good there.

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[5] *Bach, et.al.* , Seifenblasen Forming Bubles, IL, Stuttgart ,Karl Krämer Verlag (1988) 7,11 (400)

[6] *Kaarwan*, Structural Dynamics of Arches: A Comprehensive Guide, https://www.kaarwan.com/blog/architecture/structural-dynamics-of-arches-comprehensive-guide?id=567

Chemistry and mathematics explain minimal surfaces in different languages. If we understand the basics we can build them fast and properly. There are forces and theorems that generate the surface and the interaction between its components is exceptional.

2.3 Chemical reaction

"A thin sheet of water packed between two layers of detergent solution is what a soap film is. Detergent molecules are amphipathic, meaning they have hydrophilic (attracted to water) heads and hydrophobic (avoiding water) tails. The last tend to crowd to the surface of the soap film and stick out away from the layer of water. As a result, H2O molecules separate from each other (fig.25). The increased distance between the water molecules causes a decrease in surface tension, enabling soap films to form" [1]. They take a spherical shape, because the volume with the smallest surface area is a sphere.

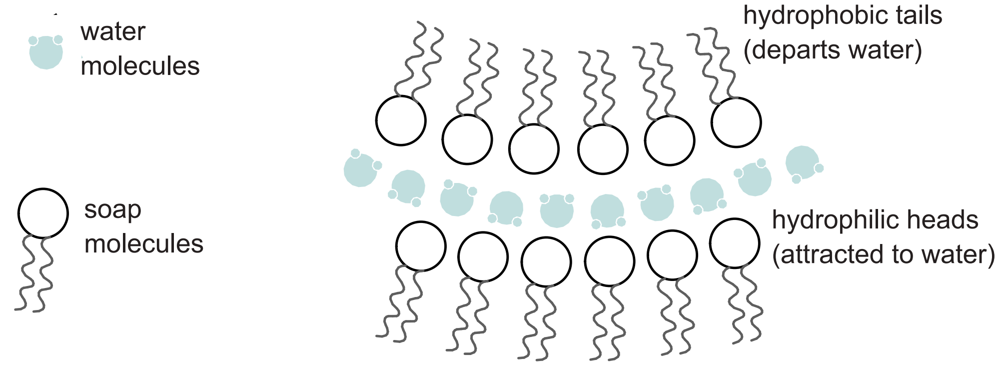

Fig.25 Chemical structure of the soap film

“When a soap film is formed between wire edges, they are bent towards each other by the action of the surface tension.... the bending curvature is determined by the relative strength of the surface tension to the tension of the wire.” [2] Because of these properties, soap films are excellent toy, but also models used for flow and energetic analysis.

The strength of the bubbles depends on the solution proportion. A recipe for “super bubbles” investigated by Fred Juergens, Dept. of Chemistry, University of Wisconsin-Madison, calls for a proportion even without water, namely 4:2:1 = glycerine : liquid : syrup.[3] When there is a boundary curve, the pressure on the concave surface will always be greater than the one on the convex surface due to the surface tension.[4]

Following the example that nature gives us, we can conclude, that every minimal area can be designed by fastening a isotropic fabric onto a suitable surface edge. Under the influence of the edge forces, due to fastening it tightly, a homogeneous skin in equilibrium is possible. The relative strength of the surface tension to the tension of the wire determinates the bending curvature.[5]

2.4 Mathematical explanation

The matematical explanation of minimal surfaces is not so tangible and easy visualisable as the chemical one, but it gives us the knowledge how to build structures in human scale with the smallest amount of material. Somethimes we use more algebra in order to be faster in programming minimal surfaces. A good visual understanding, of the geometric entities explained in the next pages, is needed in order to be able to build them correctly and so to profit from their advantages. All this because of the distinctive curvature behaviour of minimal surfaces.

1 point

A point is an idealization of an exact position in R3 , without size. One-dimensional curves, two-dimensional surfaces, and higher-dimensional objects consist out of points. A point can also be determined by the intersection of two curves or three surfaces, called a vertex or corner.[6]

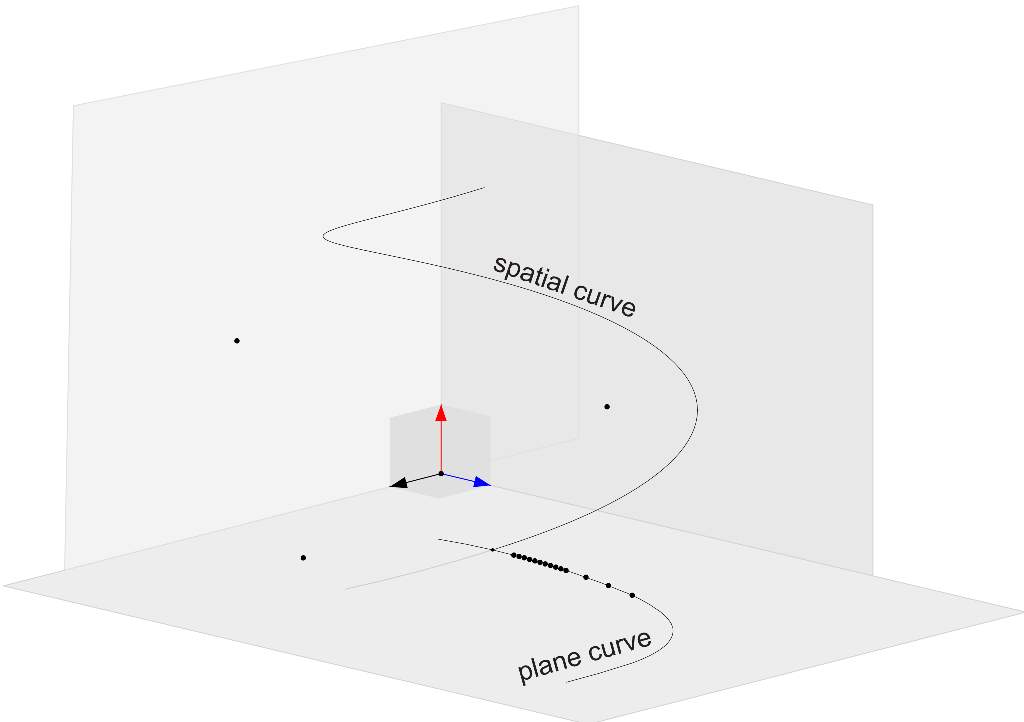

Fig.26 Plane and spatial curve

2 curves

A curve can be explained as a connected series of points. They all can lay on a plane or be positioned in R3, in other words curves can be planar or spacial. To know their properties will prepare us to understand the theory regarding surfaces. *Namely, curves can span and describe surfaces.* The degree of deviation from being straight can be measured via curvature. For calculating it we study the theory of osculating plane and osculating circle.

3 osculating plane and circle

In the book "Architectural geometry" from Bentley Institute Press the authors explain the construction of osculating plane and osculating circle in the following way: let $\mathbf{P_c}$ be a discrete spacial curve and **c** a spatial curve. We will refine $\mathbf{P_c}$ into **c** by fixing $\mathbf{C_2}$ at its place and redrawing the other vertices of $\mathbf{P_c}$. ***The consecutive vertices*** C_1, C_2, C_3 ***of*** P_c ***define a plane*** o_2***. There is a circle*** k_2 ***(lying in*** o_2***) passing through*** C_1, C_2 ***and*** C_3. The bisecting planes $\mathbf{B_{12}}$ of $\mathbf{C_1C_2}$ and $\mathbf{B_{23}}$ of $\mathbf{C_2C_3}$ intersect each other at the axis $\mathbf{a_2}$ (fig.27). We refine $\mathbf{P_c}$ by intersecting $\mathbf{B_{12}}$ and $\mathbf{B_{23}}$ with curve **c** at points $\mathbf{C_1}$\` and $\mathbf{C_3}$\`. Points $\mathbf{C_1}$\` and $\mathbf{C_3}$\`are the vertices of the refined discrete spacial curve $\mathbf{P_c}$\`. The same is done for each of the vertices. At the last step the refined $\mathbf{P_c}$ coincide with the spatial curve **c**. (fig.29)

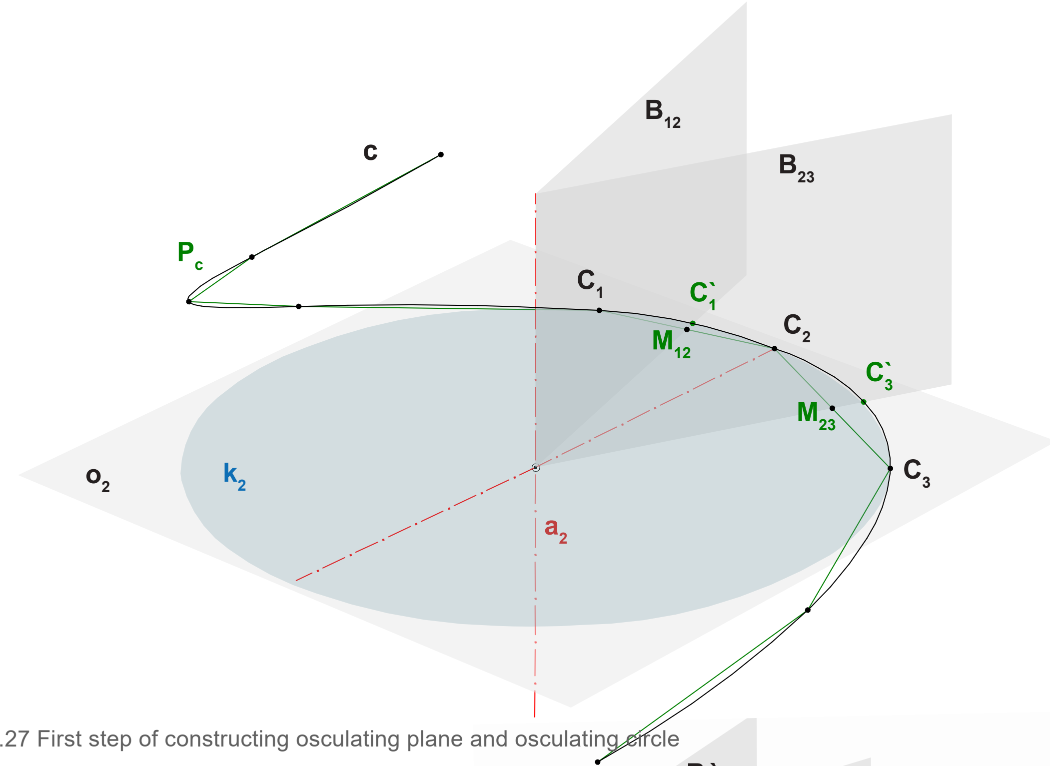

Fig.27 First step of constructing osculating plane and osculating circle

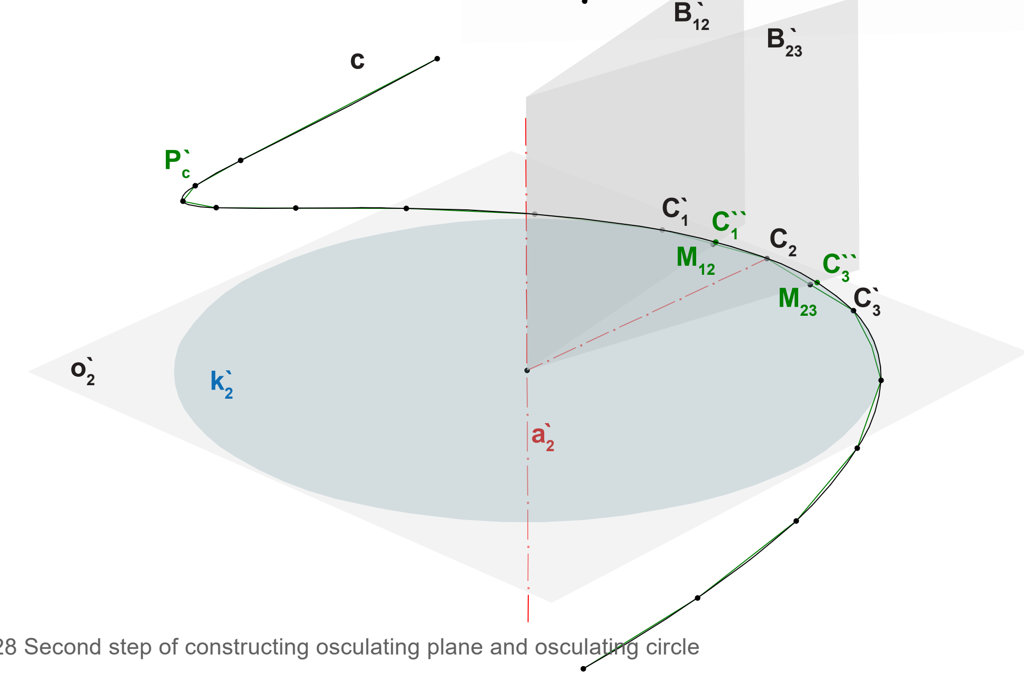

Fig.28 Second step of constructing osculating plane and osculating circle

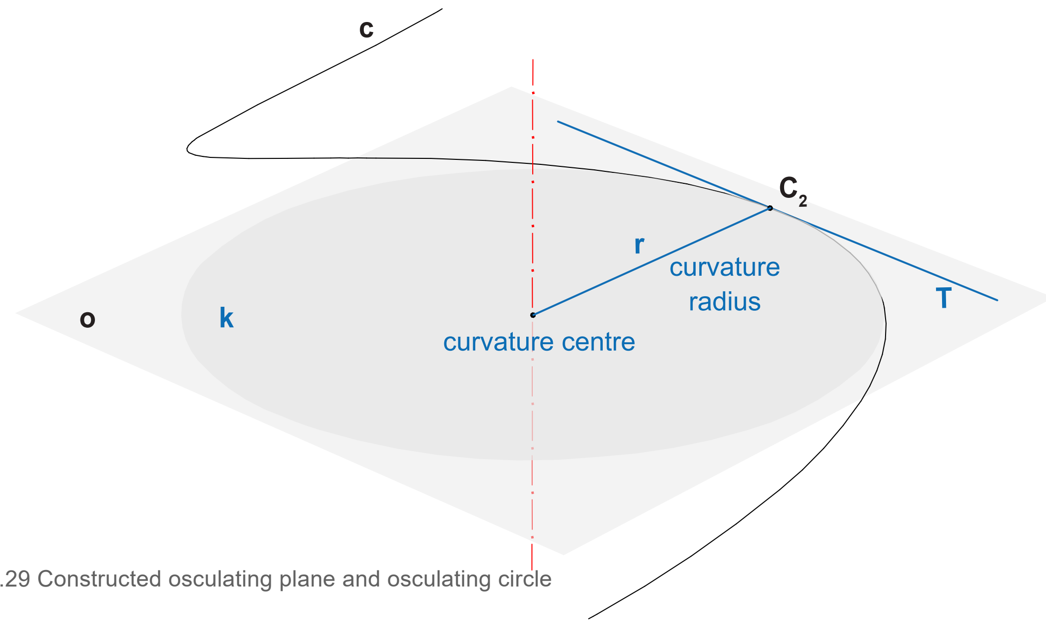

Fig.29 Constructed osculating plane and osculating circle

As a result the circle **k** passes through curve **c** at point $\mathbf{C_2}$ and so it becomes the osculating circle of **c.** If the (curvature) radius of the osculating circle is **r**, the curvature **k** of the curve **c** at the point $\mathbf{C_2}$ is defined as the reciprocal value of the radius.

$$\mathbf{k} = \frac{1}{\mathbf{r}}$$

There are different ways to prove the equality - via trigonometry, via refining a discrete curve, or via the parametric representation of the curve plus calculation with its derivatives. ***It is important that the curvature measures the local directional change of the tangent. Further control and design of the curve's behaviour is possible when knowing the value k.*** [7]

4 Frenet-Serret frame

When working with space curves, we can describe the points that they inherit not only by tangent line and curvature. *In three dimensional space three vectors describe a position on a curve at a selected point.* Let's consider the normal plane **n** that intersects the tangent line **T** at curve point **P** at right angle. The normal plane **n** and the osculating plane **o** cross each other along the principle normal **N** of the curve **c**. A straight line normal to **o** at point **P** is parallel to the axis **a** and intersects **N** and **T** at right angle. We call it the binormal **B** of the curve **c** at point **P**. [8] The tangent **T**, the principle normal **N** and the binormal **B** define the Frenet frame of the curve at point **P**. ***The unit vector T is pointing in the direction of motion, it tells us the direction in which the geometry is going. The normal unit vector n is the derivative of T and captures the way in which the tangent vector is itself changing. The aforementioned two vectors - T and N - define a plane. The binormal vector B is normal to that plane, because it is the cross product of T and N. The plane is constantly changing along the curve. This movement in captured via B.[9] The Frenet frame is constantly rotating along the curve, depicting positions (points) on the geometry.*** Not only **T** and **N** define a plane. **T** and **B**, **N** and **B** as well (fig.30).

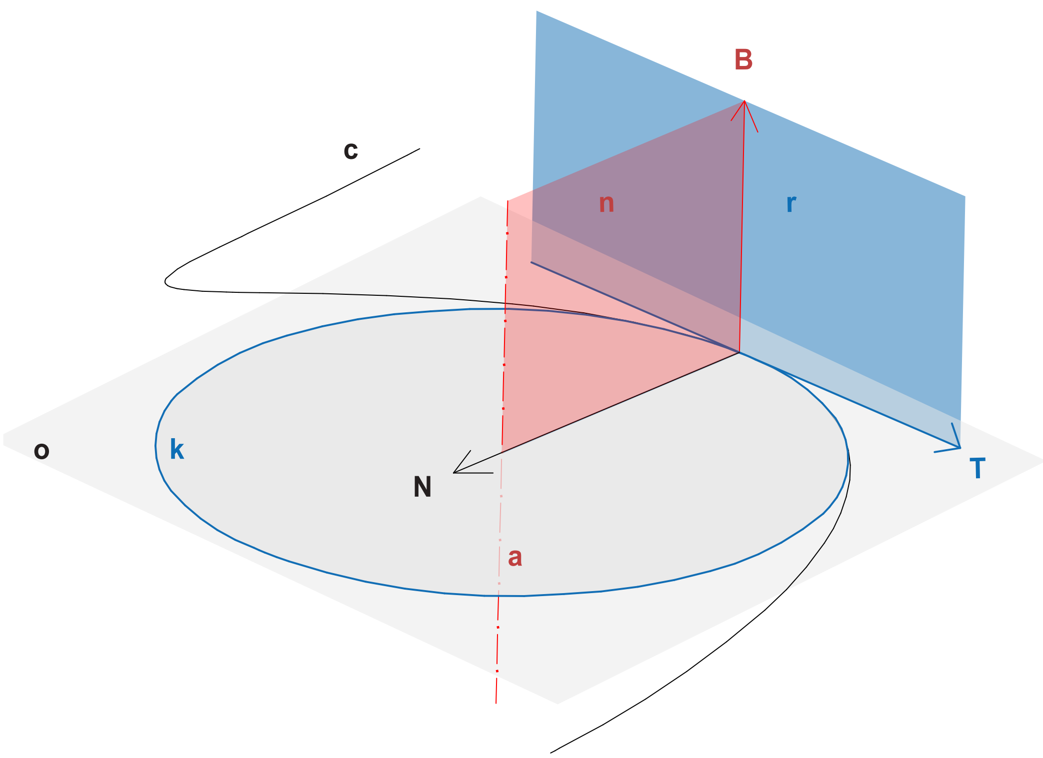

Fig.30 Frenet-Serret frame

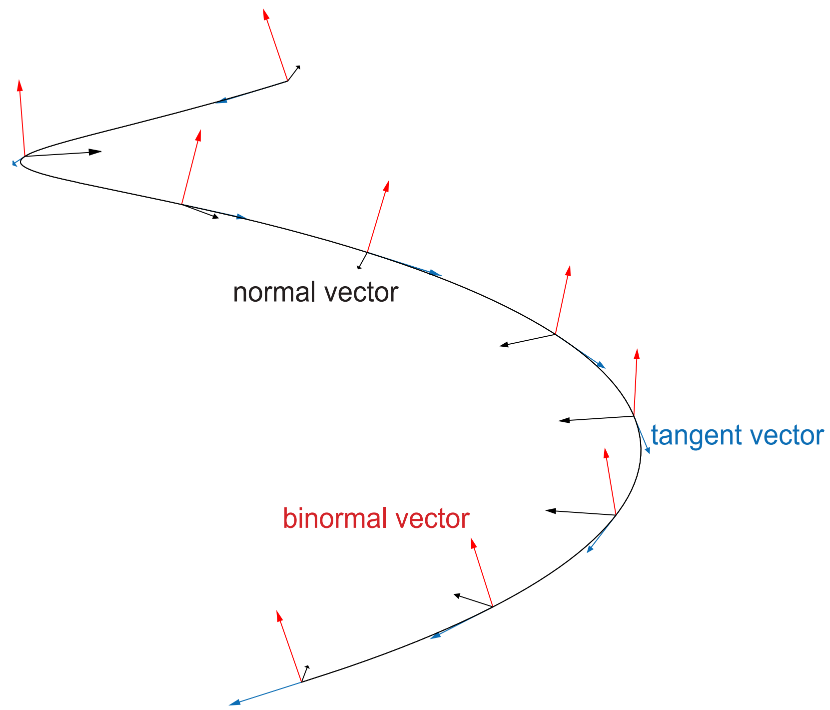

Fig.31 Frenet-Serret frames on selected points

5 Osculating curves

The authors from Bentley Institute Press explain further in the book "Architectural geometry" the theory about osculating curves and osculating parabola. *"There are many curves that touch the given curve **c** at a chosen point **P** and have the same curvature **k** and osculating circle there... the osculating circle itself is one example, but there are infinitely many osculating curves."* The Taylor`s theorem proves that if two curves **c** and **d** meet at point **P** they have the same tangent at point **P**. This information help in understanding the curvature of surfaces, where again infinitely many curves intersect each other at one point.

6 Osculating parabola

If a parabola $\mathbf{y=(1/2)X^2}$ is given, than the osculating circle at its vertex **(0,0)** has radius of **1** (fig.30). Meaning the curvature value at **(0,0)** is also **1** $\mathbf{(k = 1/r = 1/1)}$. Following the equation we can say that the curvature value of a parabola $\mathbf{y=(k/2)X^2}$ at its origin is k. Interestigly **k** is the second derivative of the function $\mathbf{g(X)=(k/2)X^2}$. (first derivative $\mathbf{=2(k/2)X = kX}$, second derivative = $\mathbf{X^{-1}kX=k}$) . For this case the second derivative gives us the value of the curvature at the origin point.[10] Knowing the aforementioned fact we can start the discusion of surface curvature.

7 Osculating paraboloid

We select point **P** of surface **s** and make **P** be the origin of a coordinate system **(X,Y)** , such as **(X,Y)** is a tangent plane of **s** at point **O** (fig.31). The surface $\mathbf{P(\alpha)}$ obtained by the straight line **l** and the parabola **q** is an osculating paraboloid of **s** at **P** with an equation form of: $\mathbf{Z = aX^2 + bXY + cY^2}$ All paraboloid surfaces have two symetry planes. We choose the **XY-plane** so that **XY** and **XZ** are the symetry planes of $\mathbf{P(\alpha)}$. This removes **bxy** from the sum above**.** We denote $\mathbf{a = k_1/2}$, $\mathbf{c= k_2/2}$ and now surface **P** has the simple equation $\mathbf{Z = (k_1 / 2)\ X^2 + (k_2 / 2)\ Y^2}$ These curvatures $\mathbf{k_1}$and $\mathbf{k_2}$ are called principal curvatures of **P** and **s** at point **P**. The **X-** and the **Y-** axes are called principle directions at **P**.[11] (Fig.32)

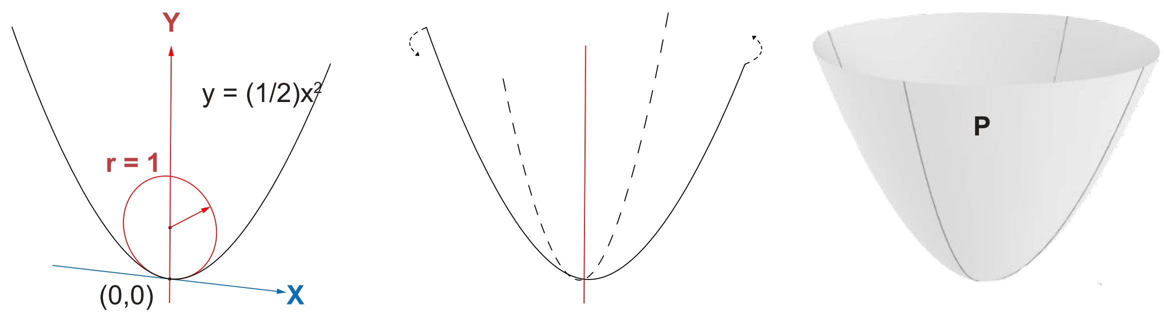

Fig.32 From parabola to paraboloid

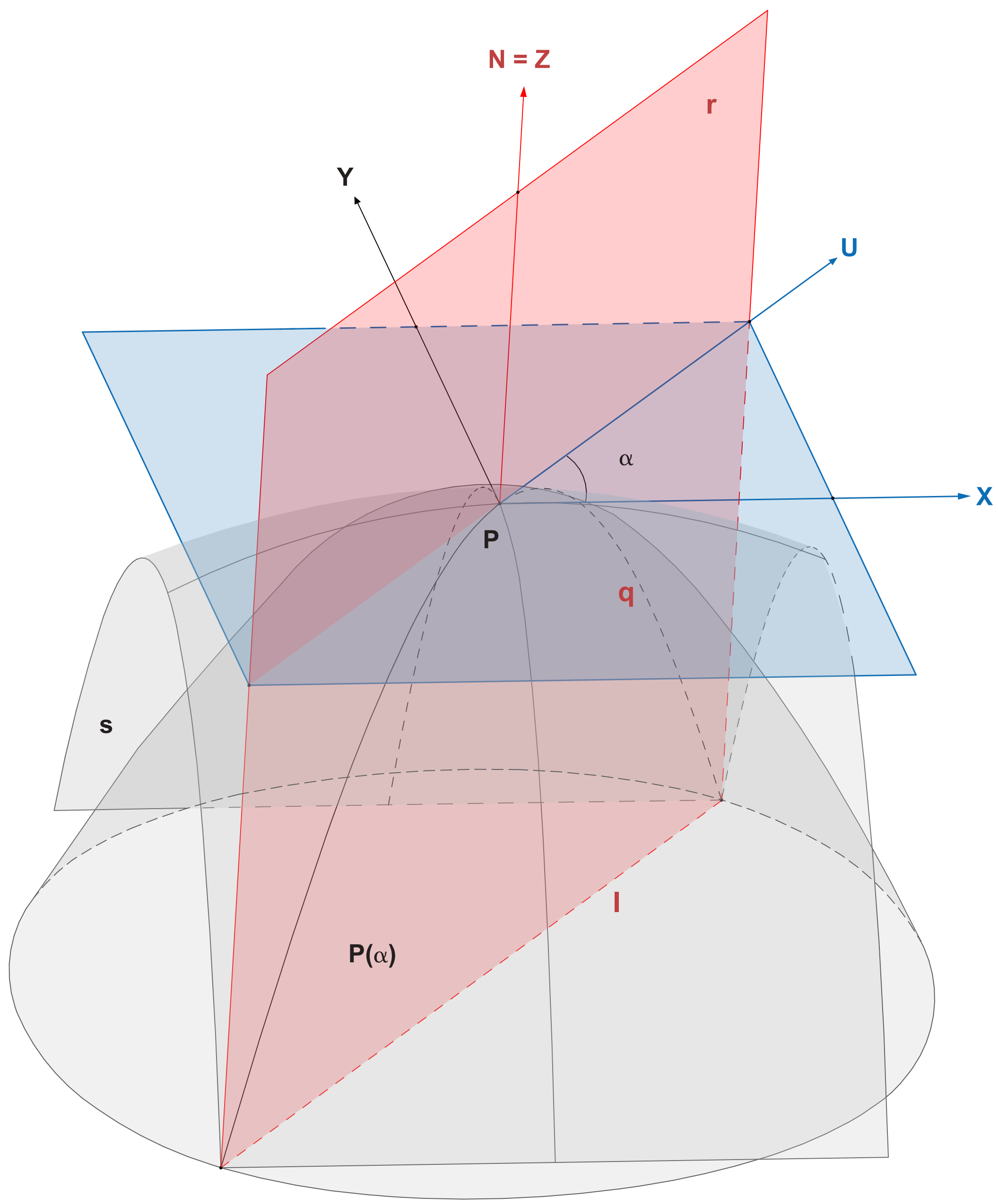

Fig.33 Finding normal curvature

8 Normal surface curvature

To investigate along which directions does the surface bend the least or the most we study normal surface curvature.[12] The normal surface curvature k_n measures how much a surface is curving. The value can be positive, negative, or zero. Let us have a look on fig.33 again. surface **s** is given. Plane **r** intersects **s** through the normal vector of point **P**. We make **P** the orirgin of a coordinate system, where the **Z-axis** is the normal vector of point **P** at surface **s**. We may use the osculating paraboloid instead of **s**.The plane **r** can be defined by its angle α against the **X-axis**. At fig.33 we see the trigonometric relation **X=u.cos**α and **Y=u.sin**α· We can insert this into the previous equation from point 7 (osculating parabola).

$Z = (k_1/2)X^2 + (k_2/2)Y^2$ and get for the parabola **p**(α): **$Z = 1/2\,[\,k_1(\cos\alpha)^2 + k_2(\sin\alpha)^2\,]\,u^2$**. It curvature at the origin is the normal curvature $k_n(\alpha)$: **$k_n(\alpha) = k_1(\cos\alpha)^2 + k_2(\sin\alpha)^2$**."Hence, knowing the principle curvatures k_1 and k_2 we can compute the normal curvature $k_n(\alpha)$ to any given direction angle (α)." [13]

Knowing the theory explained so far, we can plan networks on surfaces, following the principle curvature which implemented correctly, can have many advantages when building. More about this topic can be found in the next chapter: "Special curves on surfaces". Furthermore, from the principal curvatures k_1 and k_2 we can derive two important quantities — the Gaussian curvature and the Mean curvature for the curvature analysis of surfaces.

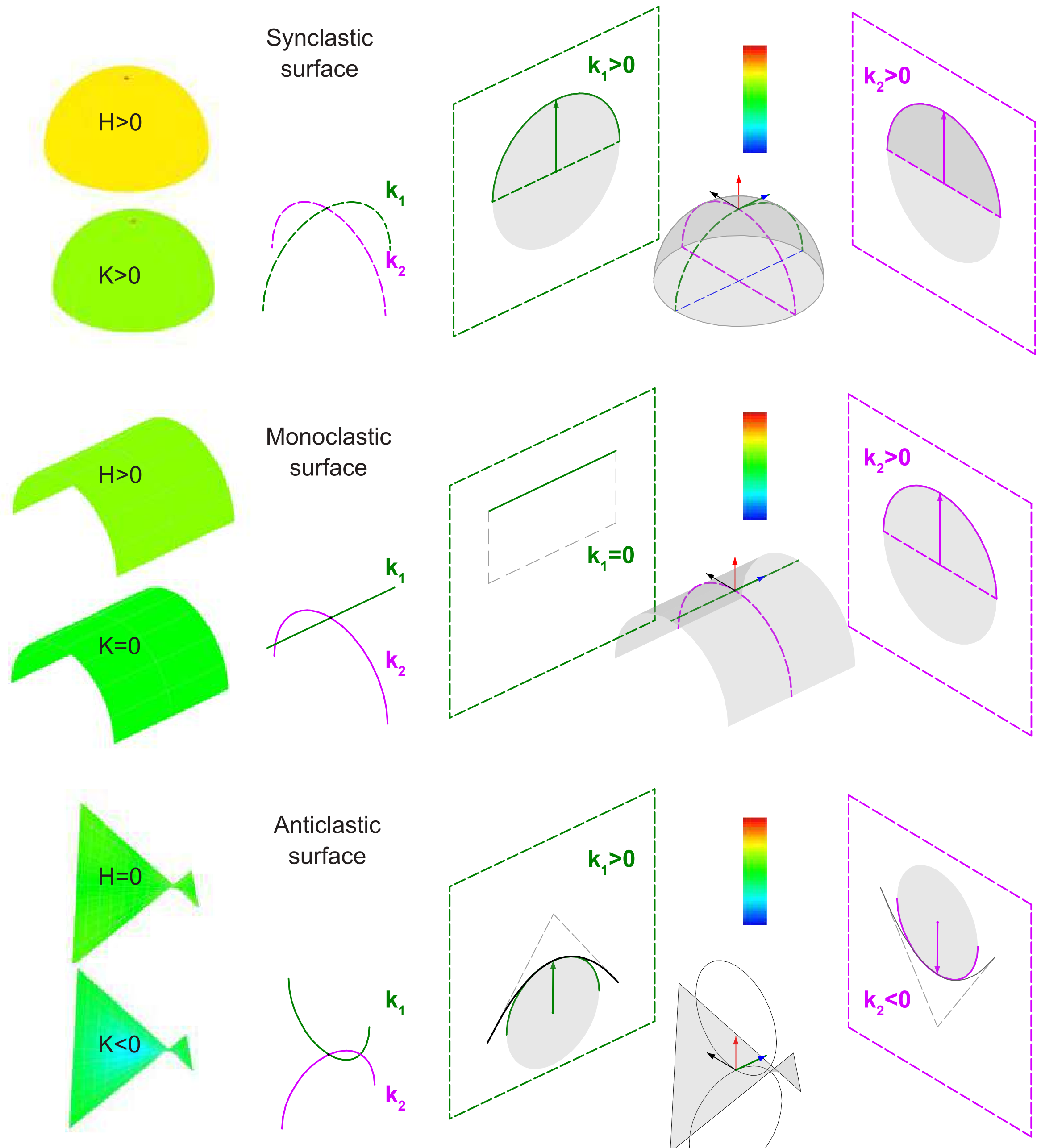

Fig.34 Analysis of Curvatures on Synclastic, Monoclastic, and Anticlastic Surfaces[14]

9 Gaussian curvature

The Gaussian curvature calculates the geometric mean of the principal curvatures: H = $\mathbf{k_1 \times k_2}$. It "is a property intrinsic to a given surface. The curvature encodes information about how the surface "unfolds", responds to stresses and shears, and how a particle moves across the surface." [15] It is good for identifying inflections and for locating saddle surfaces (blue/purple colours). [16] (FIg.34)

10 Mean curvature

The Mean curvature calculates the arithmetic mean of the principal curvatures: $\mathbf{H=(k_1 + k_2)/2}$. If $\mathbf{H = 0}$ we have a minimal surface, that " has vanishing Mean curvature in each of its points. It is equivalent to $\mathbf{k_1 = - k_2}$ .Unless we have a flat point ($\mathbf{k_1 = k_2 = 0}$), the two principle curvatures have different signs. Therefore a generic surface point of a minimal surface must be a hyperbolic point (saddle-like)." [17] (FIg.34)

11 Umbilic points

The authors from Bentley Institute Press explain that if $k_1 = k_2$ we have a surface point called an umbilic. Directions of principle curvature are not uniquely determined here. Network of curvature lines has a singularity at that point. The osculating paraboloid at an umbilic is either a paraboloid of revolution or a plane, resulting in the same curvature behavior as that of a sphere or a plane. Consequently, the network of principal curvature lines also exhibits a singularity at this point.[18]

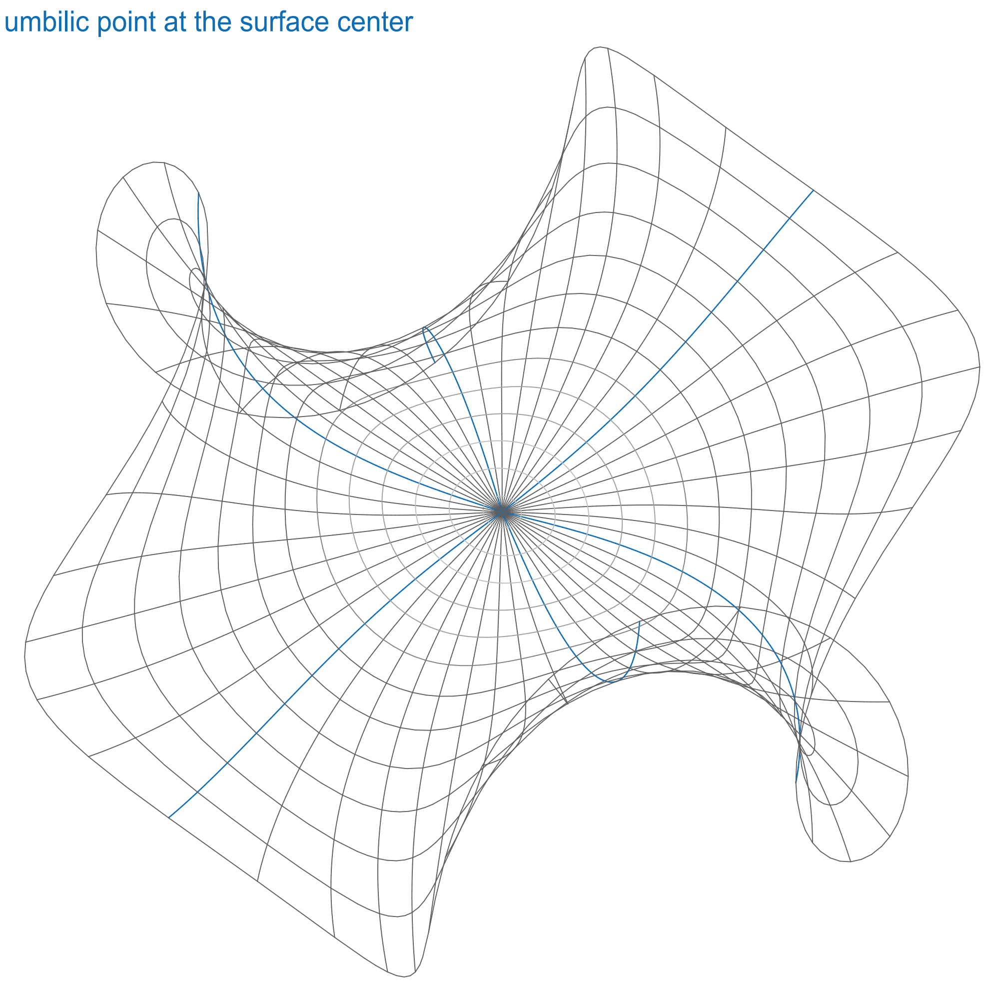

Fig.35 Umbilic point

12 Minimal surfaces

On page 648 from the book "Architectural geometry" are listed the most fundamental properties of minimal surfaces: "(1) A minimal surface has vanishing mean curvature in each of its points....
(2)The two principle curvatures have different signs. Therefore, a generic surface point **P o**f a minimal surface must be a hyperbolic point... (3)We see that in each point of a minimal surface the asymptotic directions are orthogonal. In other words, the bisecting lines of the always orthogonal principal directions are the asymptotic directions. The asymptotic curve network and the network of principal curvature lines can form the basis for realisation of minimal surfaces as frameworks of rigid straight rods with flexible connections. Applying appropriate forces at the boundary vertices, such frameworks may be brought into static equilibrium." [19]

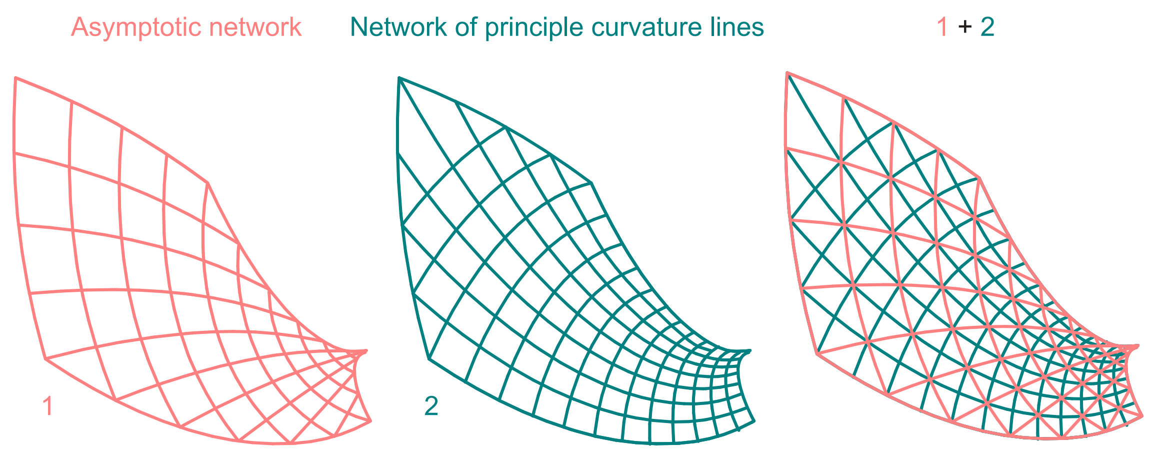

Fig.36 Networks on a minimal surface

2.5 Summary of the definitions

The mathematical explanation requires an understanding of some geometric entities. A brief explanation based on the chapters 2.2-2.4 can be described as follows:
A point represents an exact position in three-dimensional space, and a curve can be thought of as a smoothly connected series of points.

If two two-dimensional geometric objects (such as lines, circles, or polygons) intersect, they generally do so at a finite number of points. For example, two distinct non-parallel lines in a plane intersect at exactly one point, while two circles can intersect at zero, one, or two points. More generally, infinitely many geometric objects can pass through the same position in space, each with different orientations or curvatures.

Geometric objects can also osculate at a point, meaning they touch smoothly without crossing. In osculation, two shapes share not only a common point but also the same tangent direction and curvature at that location. A key example is the osculating circle, which approximates a curve at a given point by matching its curvature. The construction of an osculating circle naturally leads to the Frenet-Serret frame, which consists of three vectors: the tangent (T), normal (N), and binormal (B). These vectors provide a complete local description of how the curve evolves in space.

The concept of osculating geometries extends to surfaces, where curvature plays a fundamental role in understanding their behavior. This is especially useful for planning and designing networks on surfaces. A three-dimensional surface can be characterized using different types of curvature, such as normal curvature, Gaussian curvature, and mean curvature, each providing valuable geometric insights.

2.3-2.5 Chapter references:

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[3] *Katz, et.al.,*The Chemistry (and a little physics) of Soap Bubbles (2020) 8 (15)

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[5] *Sane, et.al.*, Surface tension of flowing soap films (2017) 9 (14)

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[8] *Pottmann, et.al.,* Architectural geometry (2007) 229 ff (724)

[9] *Bazett, Torsion:* How curves twist in space, and the TNB or Frenet Frame, https://www.youtube.com/watch?v=VIqA8U9ozIA&t=3s (01.2025) 00:00 to 03:30 minutes

[10] *Pottmann, et.al.,* Architectural geometry (2007) 487 ff (724)

[11] *Pottmann, et.al.,* Architectural geometry (2007) 489 ff (724)

[12] *Crane,* A Quick and Dirty Introduction to the Curvature of Surfaces , http://wordpress.discretization.de/geometryprocessingandapplicationsws19/a-quick-and-dirty-introduction-to-the-curvature-of-surfaces/(01.2025)

[13] *Pottmann, et.al.,* Architectural geometry (2007) 489 ff (724)

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[16] *Autodesk 2014*, Theory builder, Surface Curvature Evaluation Shader, https://www.aliasworkbench.com/theoryBuilders/TB7_evaluate3.htm, (02.2025)

[17] *Pottmann, et.al.,* Architectural geometry (2007) 648 ff (724)

[18] *Pottmann, et.al.,* Architectural geometry (2007) 501 ff (724)

[19] *Pottmann, et.al.,* Architectural geometry (2007) 648 (724)

2.6 Special curves on surfaces

An infinitely number of surface-curves exist. Three kinds with special properties are exceptional when designing special models. They show great potential to be build from a developable strips and are explained as follows:

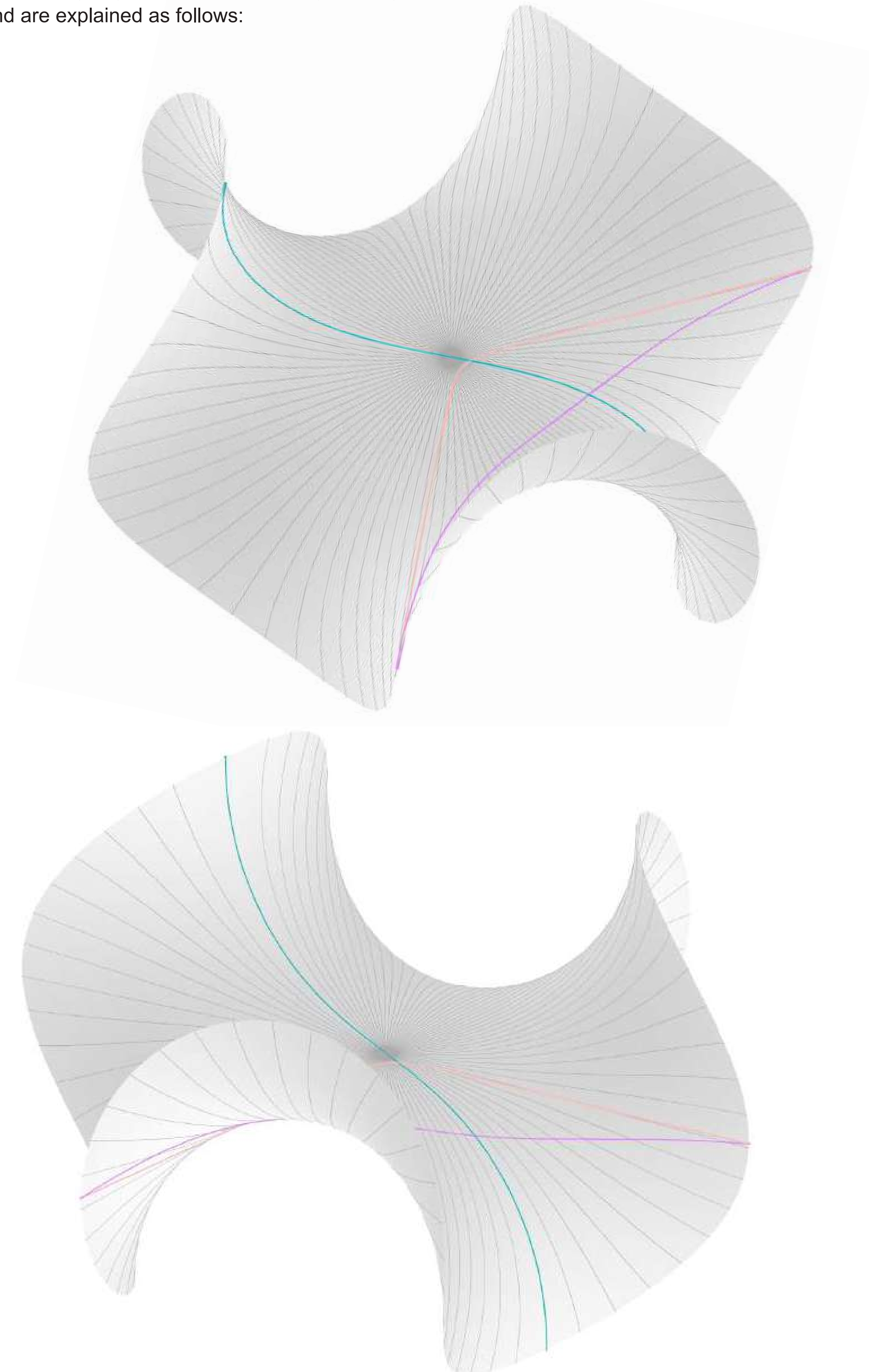


Fig.37 Curves on minimal surface- geodesic (purple), principal curvature line (blue), assymptotic curve(coral)

1 Geodesic curves

Explanation

A geodesic curve stands for the shortest path between two points on a surface. The term comes from the science concerned with measurements of the earth's surface- geodesy.[1]

Construction and facts

By laying a straight strip of paper over a smooth surface a geodesic curve is obtained. Hence, they can be described as fair curves on surfaces. The mathematical formulation of geodesic cruves involves calculus of variations (field of mathematical analysis to find maximal and minimal of functionals). There the geodesic is the curve that minimizes the distance integral between two points. Every surface`s line or curve can be parametrized so that it is geodetic.They 'move' with constant speed on the underlying geometry and its geodetic curvature is zero $\mathbf{k_g}$ everywhere.[2] For other curves, $\mathbf{k_g}$ measures how far the curve is from being a geodesic and stands for "tangential curvature of a curve ". It quantifies the deviation from straightness." Zero for straight, positive for right turn and negative for left turn in the tangential plane." [3] Girders following geodetic curves can bend vertically and twists, but they cannot be displaced laterally. [B]

Fig.38 Fig.36 Behavoiur of profile following geodesic curve [B]

Practical use

The connection to the general notion of distance has found extensive applications in science, engineering, art, architecture and structural design (e.g., domes). For instinctive segmentation into patches in use come geodetics. They are commonly used in buildings as beam layouts, support structures, panelization techniques, cladding systems, segmentation and shape analysis. [4] Another practical application of geodesic principles is, for example, the fact that airplanes use curved routes to minimize travel distance and time.[5] The idea of combining minimal surfaces and geodesic boundaries may appear attractive- the approximation of minimal surfaces from geodesics, for applications in order to minimize material consumption.

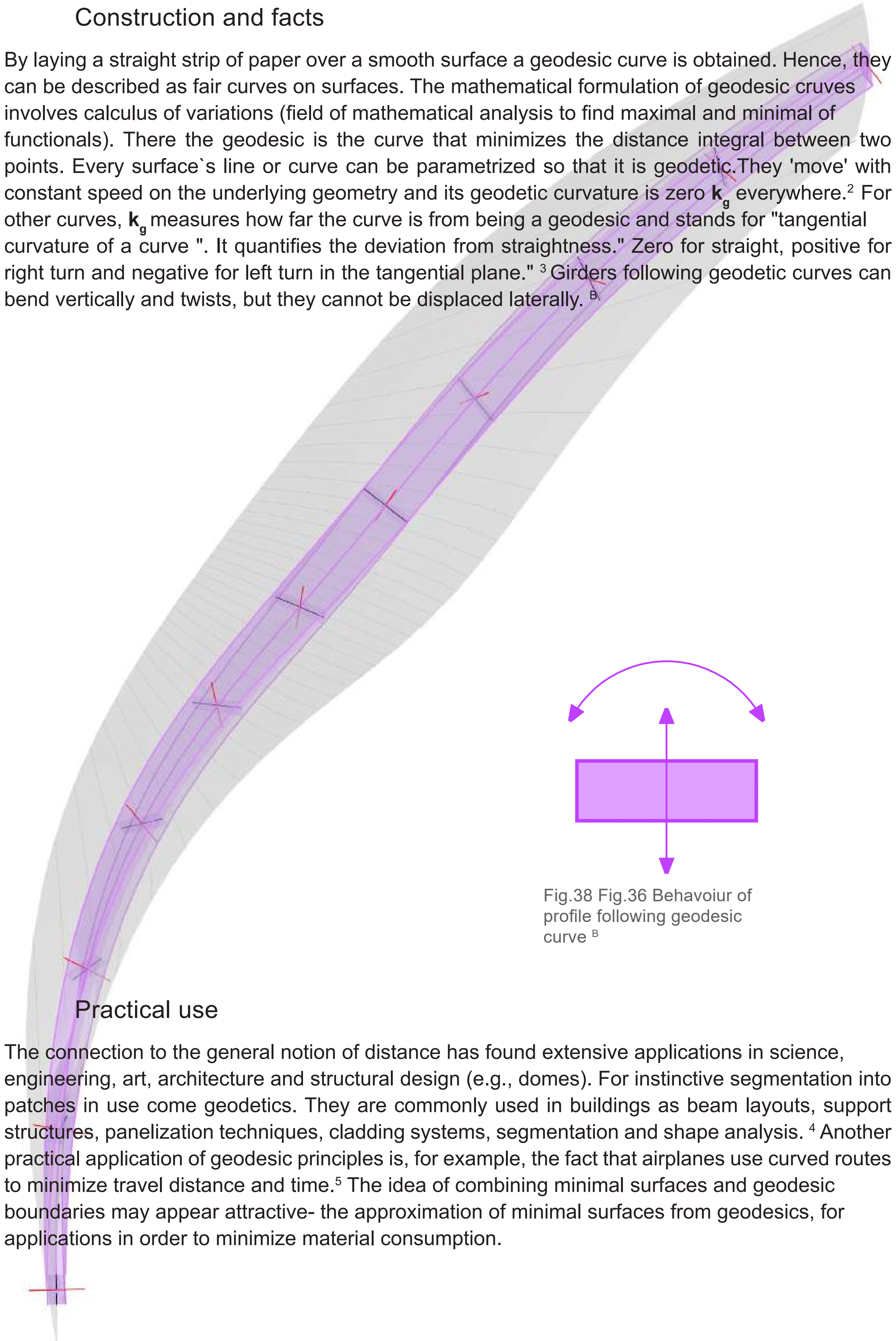


Fig.39 Geodetic girder with Darboux frames (background)

1 points on the curve

2 Darboux frame at the points

3 oriented profile sections

4 girder on the surface

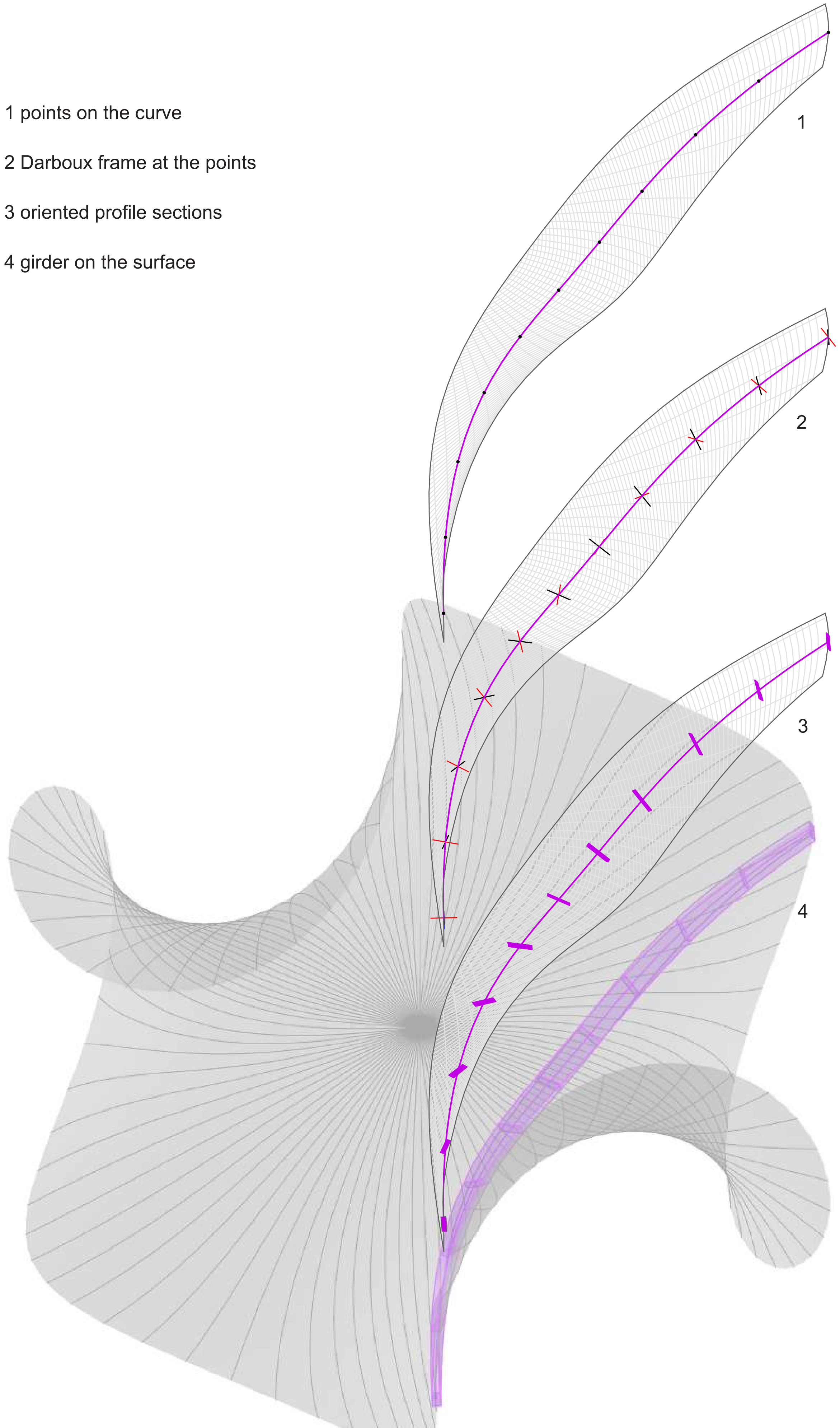


Fig.40 Constructing steps of a geodetic girder on an Enneper surface

2 Principle curvature lines
Explanation

A principal curvature curve is a curve on a surface that follows the direction of maximum or minimum curvature at each point. Their tangents are always in the direction of principle curvature. That is the reason, why we do not see the T axes of the Frenet frames at the presented surface. For each non-umbilical point there are two principal directions. They are orthogonal to each other. When having surface of revolution every line of curvature is a geodesic.

Construction and facts

The principle curvatures k_1,k_2 can be retrieved at any point on the surface via calculation consisting the coefficients of the first and second fundamental form of the surface. Girders following principle curvature lines can distort along their main directions, but they cannot twist. Perfect profile sections would be pipe-profiles. [B]

Practical use

Orthogonal nets following the principal curvature lines are advantageous both for manufacturing reasons and structural efficiency. For example, planar cladding panels and structure connections can be prefabricated. " Principal meshes in equilibrium under vertical loads are discrete representations of membrane surfaces where principal stress and principal curvature directions agree. There they follow these principal directions." The most efficient method of bearing loads in a framework is through axial forces. In such a way the beam cross section is used to the highest capacity and it offers the highest stiffness. For this reason, principal stress and curvature directions should coincide. [6]

Fig.41 Behaviour of profile following principle curvature line [B]

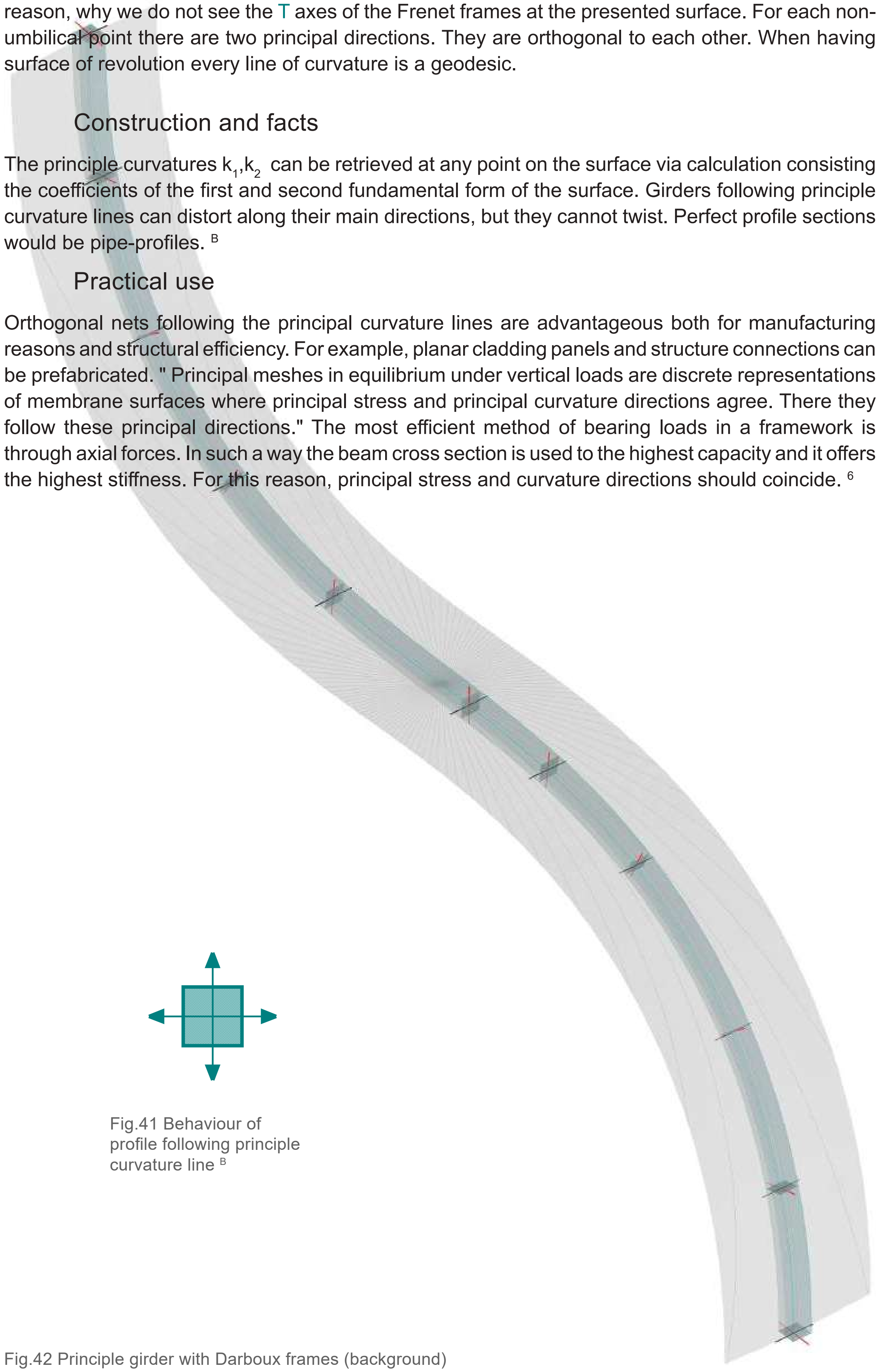


Fig.42 Principle girder with Darboux frames (background)

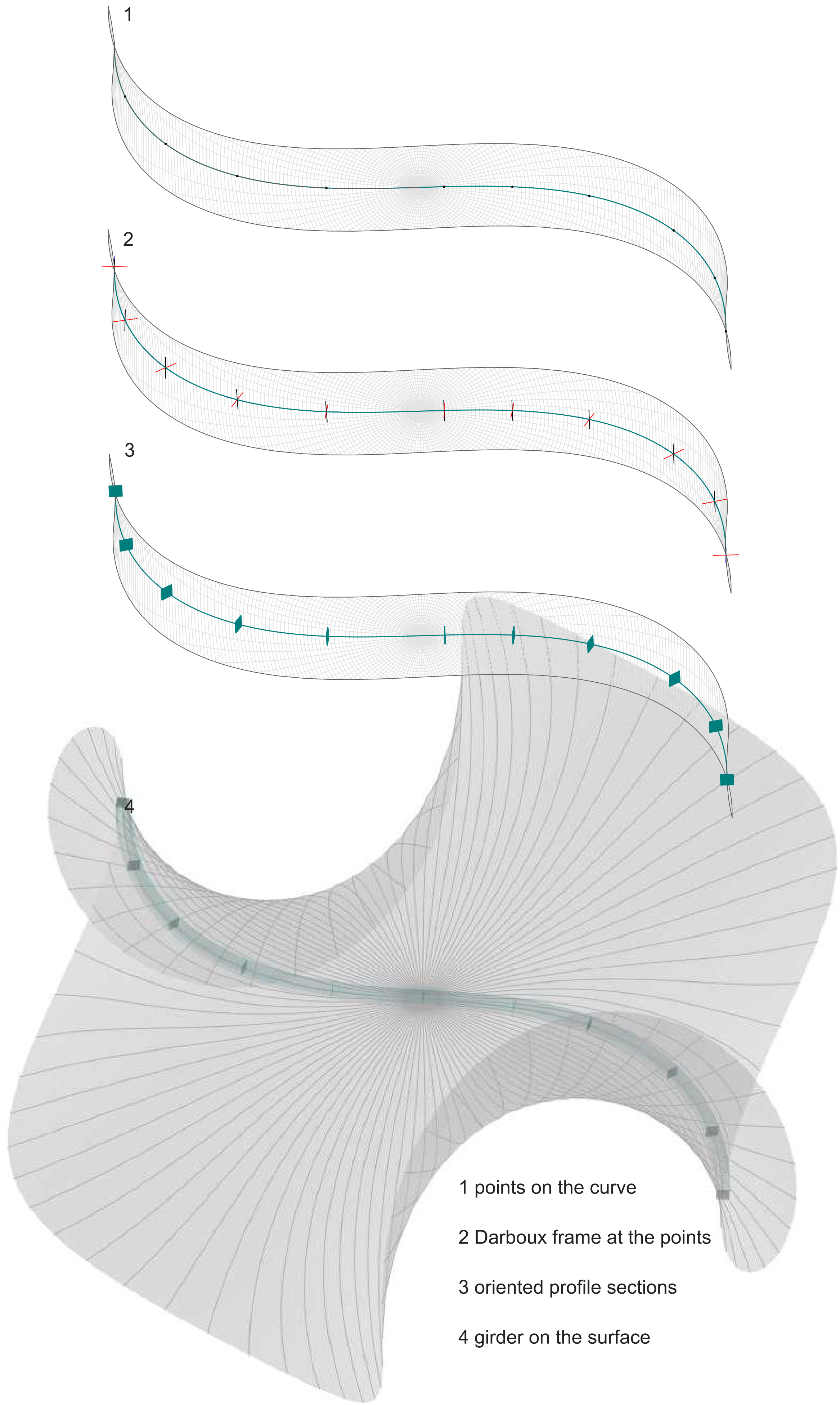


Fig.43 Constructing steps of a principle girder on an Enneper surface

3 Asymptotic curves

Explanation

An asymptotic curve is a curve on a surface where the normal curvature is zero at every point. In general, in mathematics an asymptote is a straight line that approaches a given curve, but does not meet it. Similarly, a asymptotic curves on surfaces are tangents to the corresponding asymptotic direction of the underlined surfaces.

Construction and facts

As for the principle curvatures lines, asymptotic curves can be retrieved at any point on the surface via calculation consisting the coefficients of the first and second fundamental form of the surface. Another way to design such a network is via the Euler's formula: " $k_n(\alpha) = k_1(\cos\alpha)2 + k_2(\sin\alpha)2$ can be used to compute the angles a between the principle direction and the asymptotic direction. We have to solve $k_n(\alpha) = 0$. " [7] Meaning that if we have k_1 and k_2 we can derive the asymptotic directions at surface points. Girders following asymptotic curves can distort laterally, they can be twisted in the weak axis, but cannot bend in the strong axis.[B]

Practical use

One fascinating construction method is that we can assemble slender, flat girders on the ground and afterwards deform them into an asymptotic doubly curved network. It offers simplification of fabrication and construction. This kind of reshaping of the beams is having stiffening effect on the structure, when normal forces are applied to it, or materials with non-linear elastic behaviour are in use. The girders meet orthogonally and joints of 90-degree angle can be advantageous.

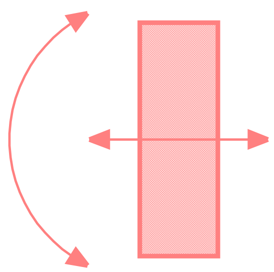


Fig.44 Behavoiur of profile following asymptotic curve [B]

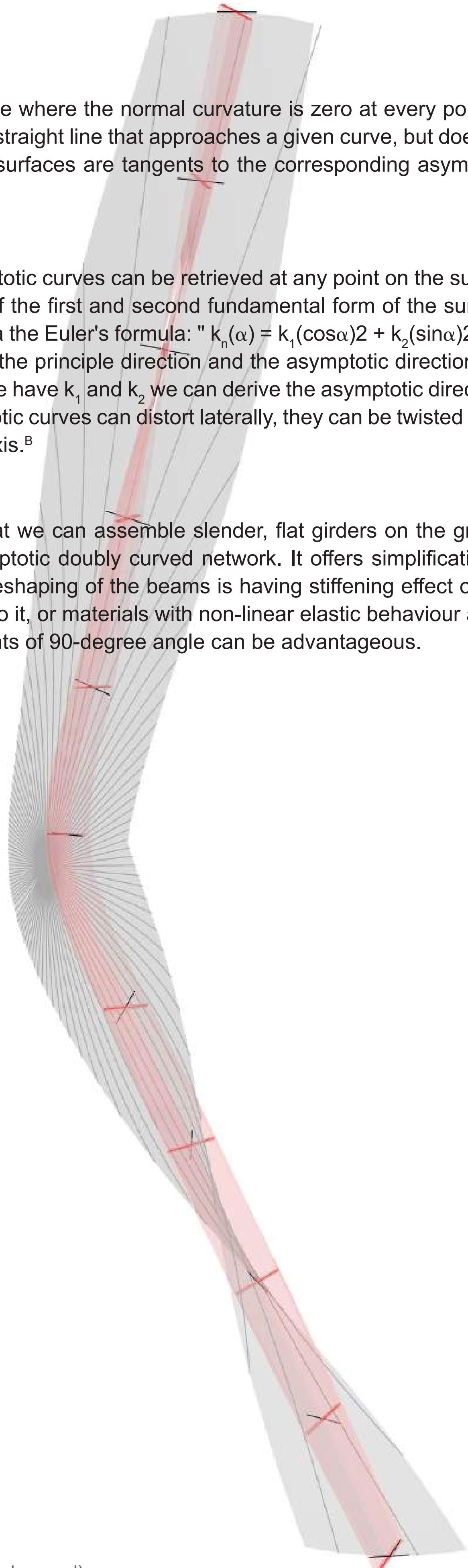


Fig.45 Asymptotic girder with Darboux frames (Background)

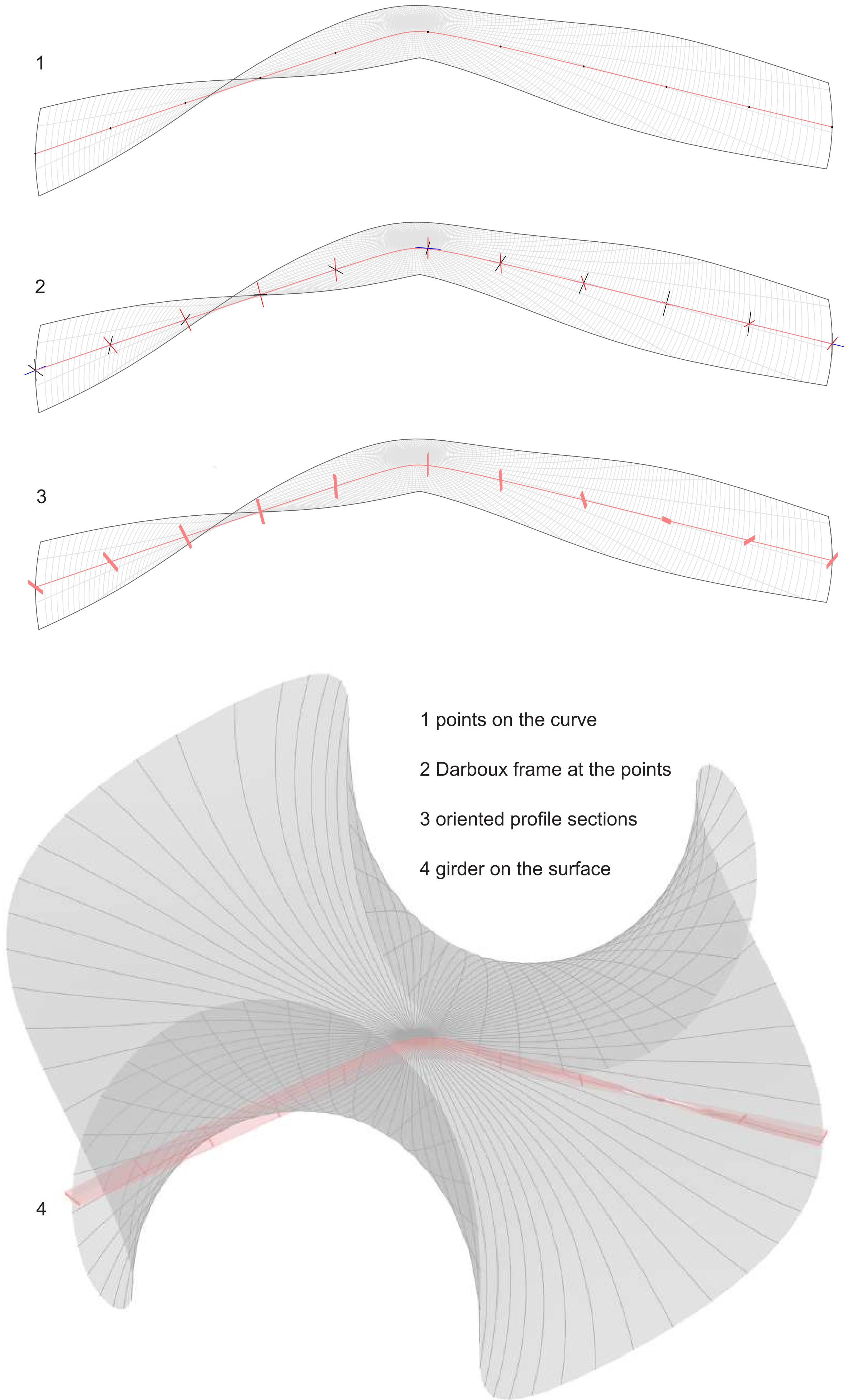


Fig.46 Constructing steps of an assymptotic girder on an Enneper surface

2.7 Summary of the special curves

Constructing flat strips into a spatial geometry is a smooth movement that can stop when the initial parts are accurately interwoven together. By working with only three examples of the three curve types we clearly see their behviour. They can meet at one point and share one binormal vector (asymptotic curve and principal curvature line), but they follow different directions. They can start and end at the same point (asymptotic curve and geodetic line), but travel across the surface distinctively. This is due to the construction principles of each of them. Sometimes they can coincide. Surfaces of revolution, such as the Enneper surface below, have the property that every line of curvature is a geodesic one. Depending on the profiles we design on the three types of curves, we can influence the equilibrium state in which a grid made out of the curves will naturally stay.[8] The elastic deformation of the initial flat materials enables the creation of doubly-curved structures.

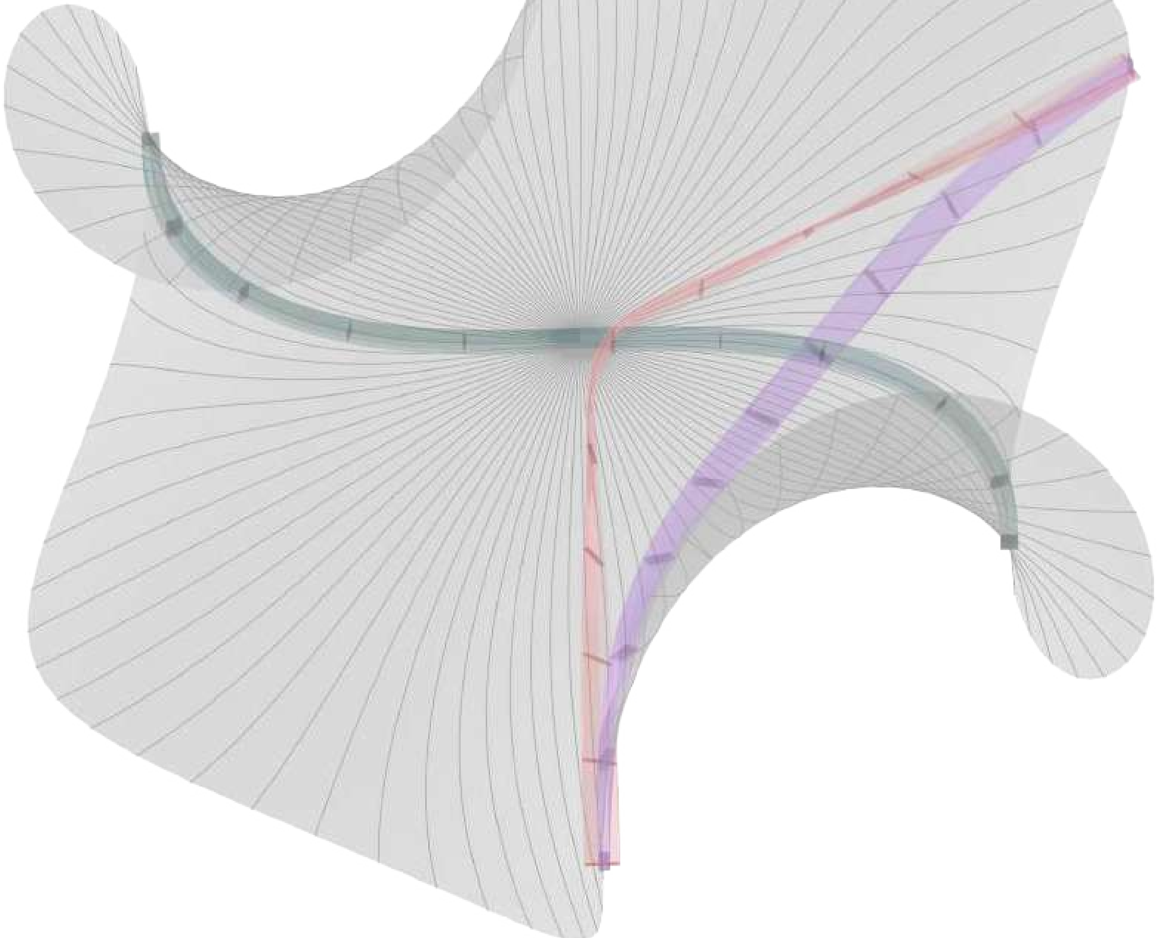


Fig.47 Girders oriented on minimal surface following the geodesic curve (purple), principal curvature line (blue), asymptotic curve (coral)

2.6-2.7 Chapter references:

[B] *Bauer*, Introductory lecture, Studio Trespassing grounds - creating versatile spaces (2022) TU Wien

[1,2,4] *Jia*, Geodesics (2024) 1 ff (9)

[3] *Narasimham*, https://math.stackexchange.com/questions/3948561/the-meaning-of-geodesic-curvature-for-a-geodesic-curve (01 2025)

[5] *GISGeography*, Why Are Great Circles the Shortest Flight Path?, https://math.univ-lyon1.fr/~alachal/diaporamas/diaporama_cartographie3/Great_Circles.htm (02.2025)

[6] *Pellis, et.al.*, Aligning principal stress and curvature directions (2018) 1,2,6,7 (15)

[7] *Pottmann, et.al.*, Architectural geometry (2007) 491 (724)

[8] *Computer Graphics at TU Wien*, Eike Schling (University of Hong Kong) - Geometry Design Structure, https://www.youtube.com/watch?v=U-TbUt74d9k (01.2025) 28:25 to 29:50 minutes

3. Casestudies

In order to present general properties in context, the following chapter examines four different cases of geometry: the Möbius strip (3.1), the Enneper surface (3.2), the Batwing surface (3.3), and a minimal surface formed by tetrahedron edges (3.4). For each of these surface geometries, the history, characteristics, and aspects of modularity are considered. In addition, a possible building definition is described.

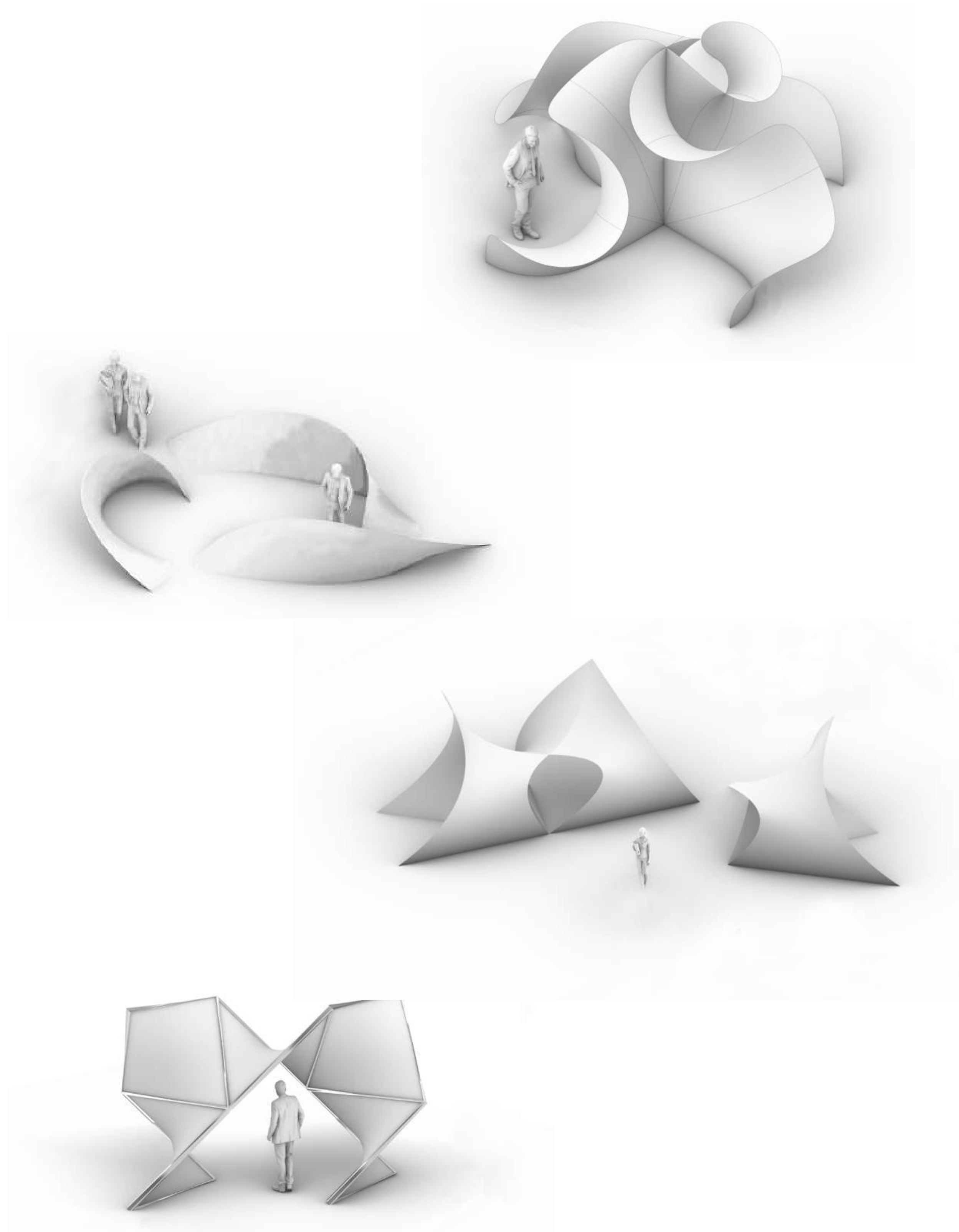


Fig.48 The four casestudies

3.1 Möbius strip

History

The first source of a Möbius strip , band or loop is dated to 200–250 CE. Since then it has inspired artists, graphic designers, architects, writers, ect. A mosaic portrays Aion, from a Roman villa in Sentinumi, holding a Möbius strip (the work is located at the Glyptothek Museum in Munich). A lot of art pieces from ancient Rome present a coiled ribbon with different number of twists. August Ferdinant Möbius made the rediscovery of it in 1858. He published it first and nowadays the geometry is named after him.[1]

Characteristics

A Möbius strip, as shown in Figure 58, is a very special kind of loop that has only one surface and one edge. If we start moving around the surface, by the time we get back to where we started, we have been flipped over. The geometry has only one closed boundary curve (fig.49). The saying 'there are two sides to everything' is irrelevant for this example. There is an 'inside' and 'outside' of the surface. The following observation may be also interesting: Every flat surface (set of points lying on a plane) is a minimal surface. A piece of paper lying on a table is also a minimal surface. If we take a strip, twist it half, one, or more times, then join the ends together we create a Mörbius strip. So, this even twist-movement spans a minimal surface in three dimensional space. F. López-one-ended Klein Bottle, the Kusner's spheres with planar ends, the López-Martín slab surface and the Henneberg surface are examples of a non- orientable surfaces.[2]

Modularity

A possibility is to divide the strip into segments of same geometry (triangles, rectangles ect.) and then assemble them following the twist of the surface (fig.51,fig.52). The Euler characteristic of the Möbius strip is zero. For any subdivision of the strip the numbers V of vertices, E for edges, and F for faces satisfy: $V - E + F = 0$ [3]

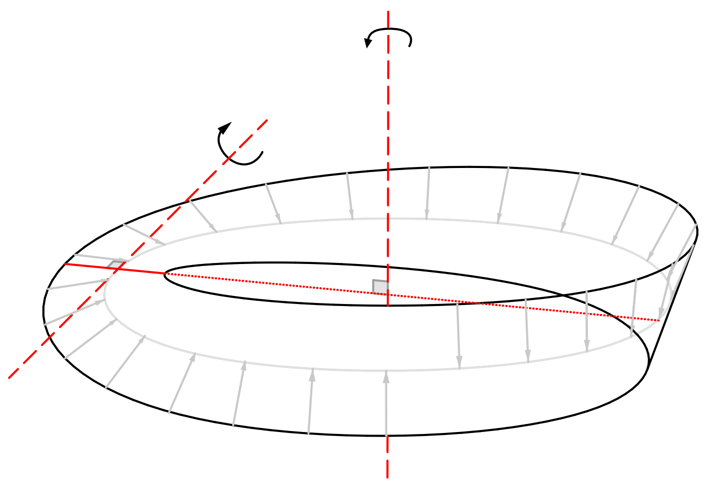


Fig.49 Non-orientable surface

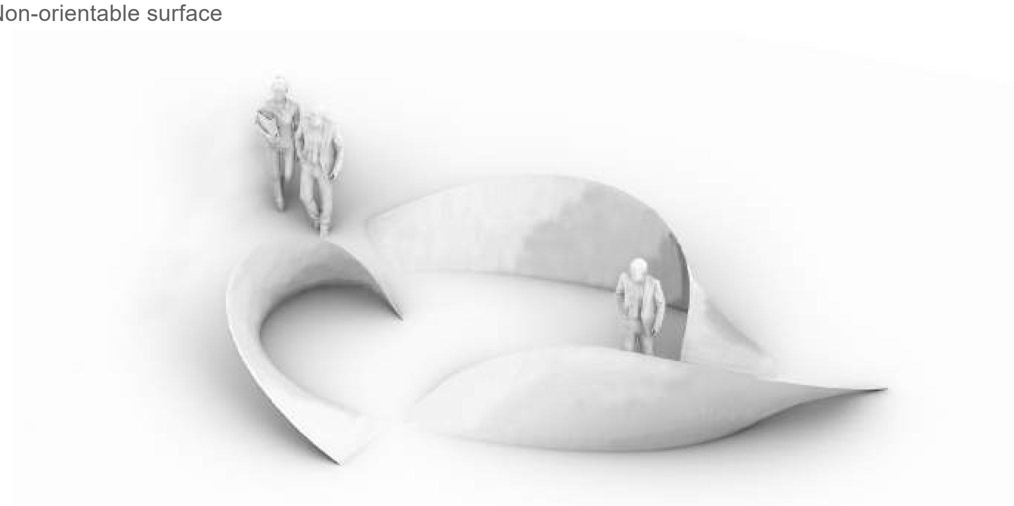


Fig.50 Perspective of the Möbius strip with 4*Pi turns

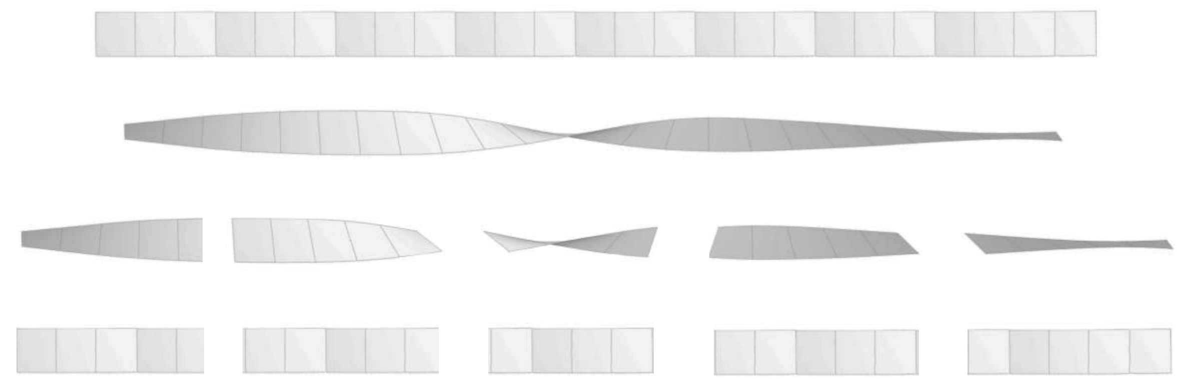


Fig.51 Divisions of the surface

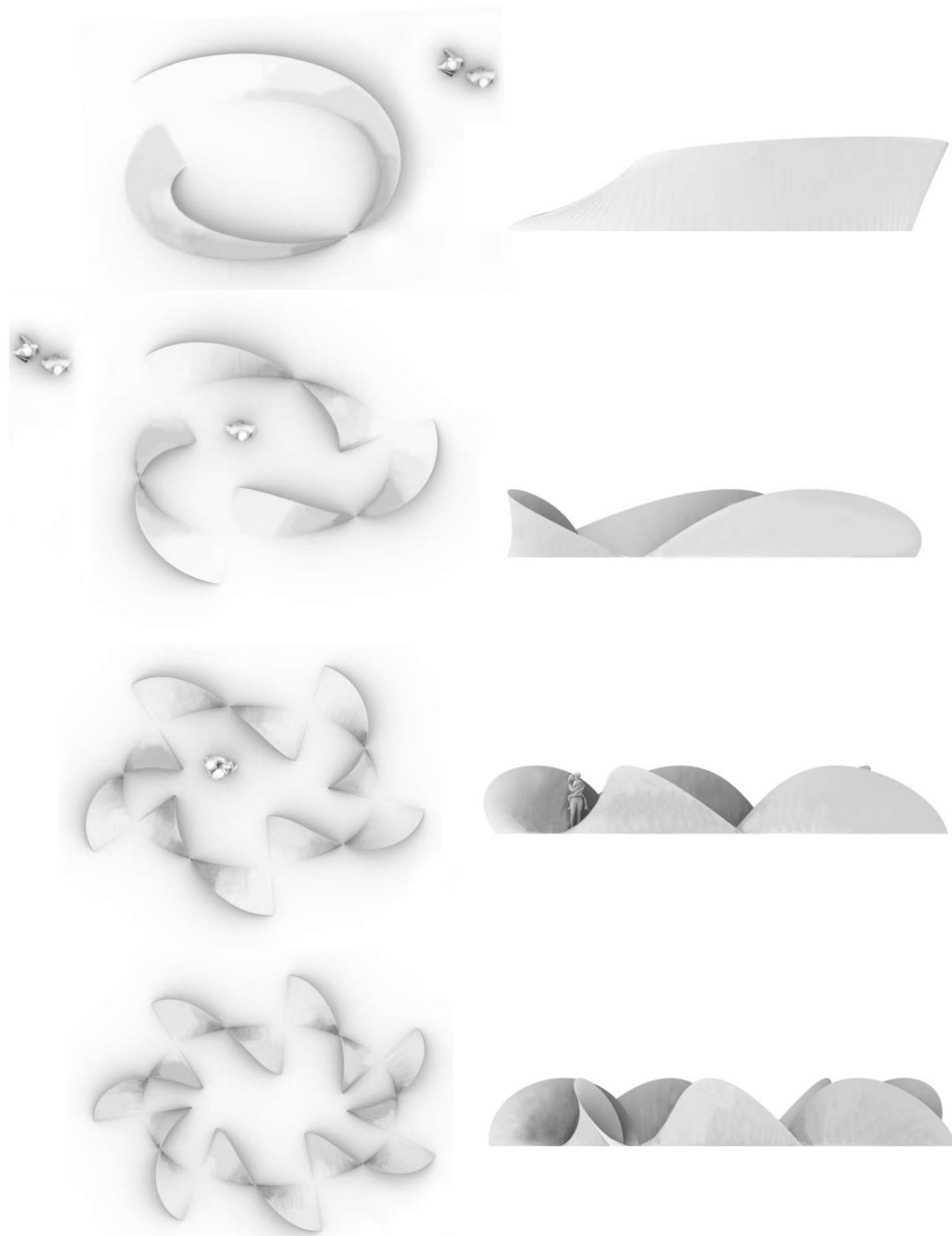
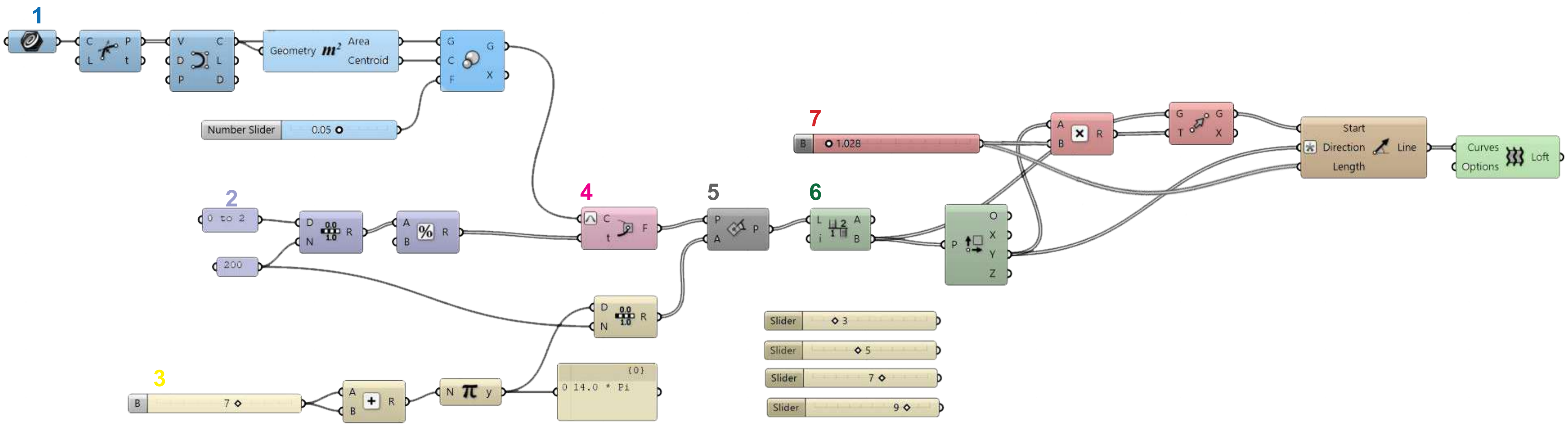
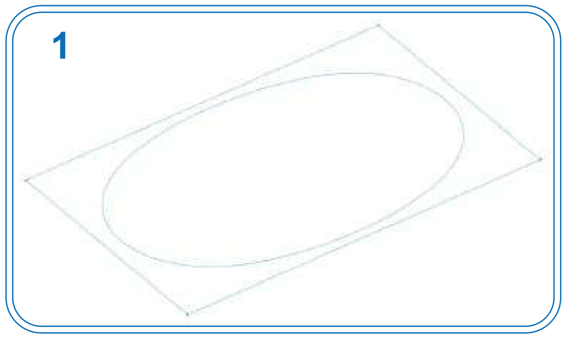
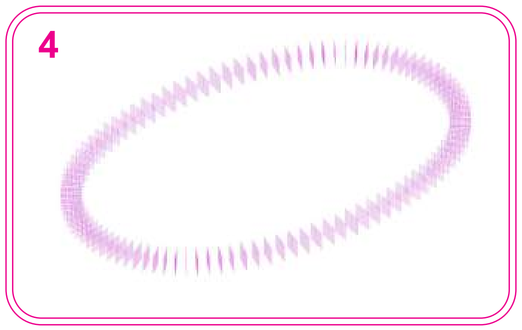


Fig.52 Top and side views of variations

Grasshopper script

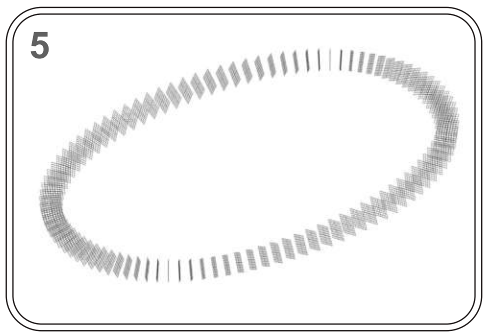






2

Creating a range of numbers with a domain from 0 to 2 and 200 steps. Modulus is dividing the numbers with 1 and returns only the remainder. It depends how many frames are modelled from this step. The components output 201 numbers between zero and two. 201 is also the amount of planes to be constructed at '4'.



3

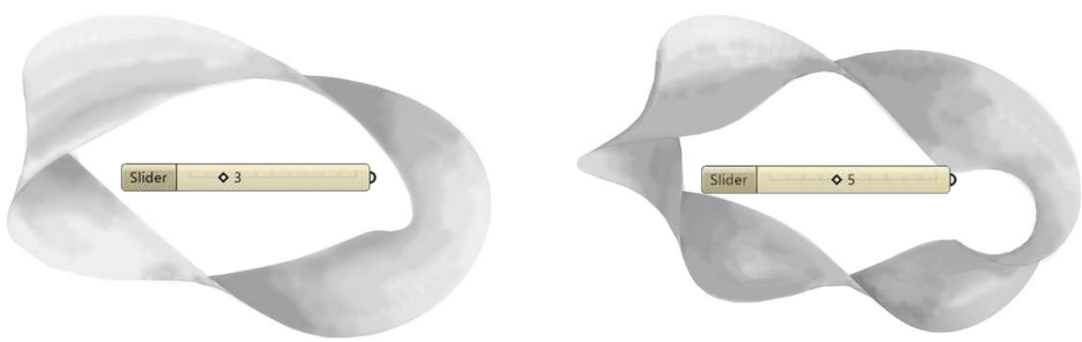
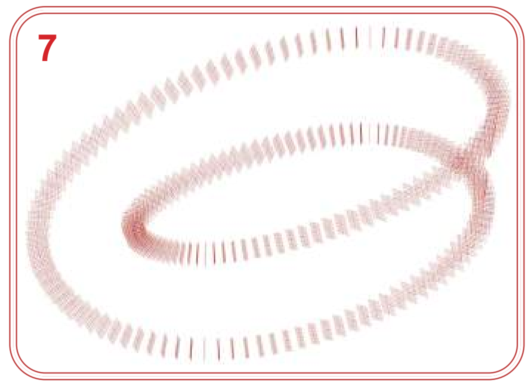
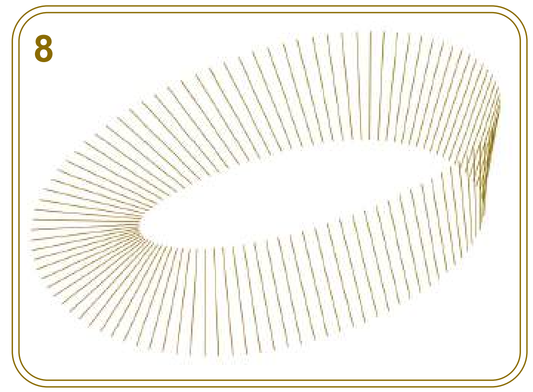


Fig.53 Building definition of the Möbius script in Grasshopper with explanations





6

Splitting the list of planes into two. 'A' contains the first plane, 'B' the other 200, that are all deconstructed into their components. The y-vector of every plane is used at step number 8 as a direction for creating a line .The 'direction' input holds '-' as expression. The 'length' input holds 'x2' as expression.

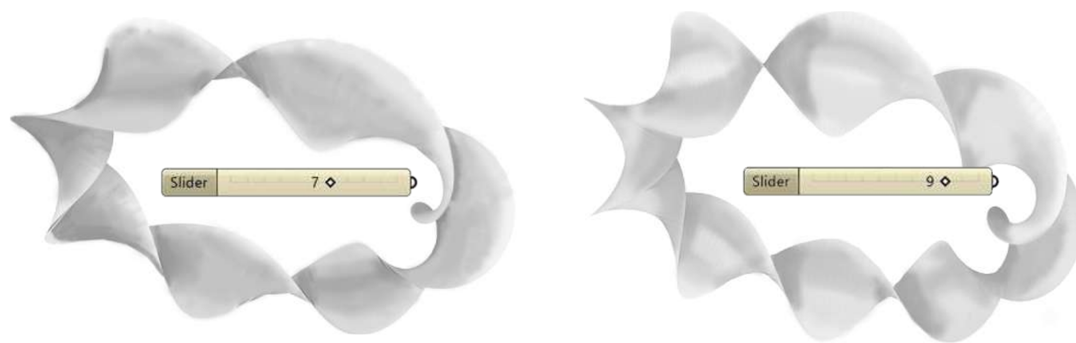


Fig.54 Building definition of the Möbius script in Grasshopper with explanations

Grasshopper legend

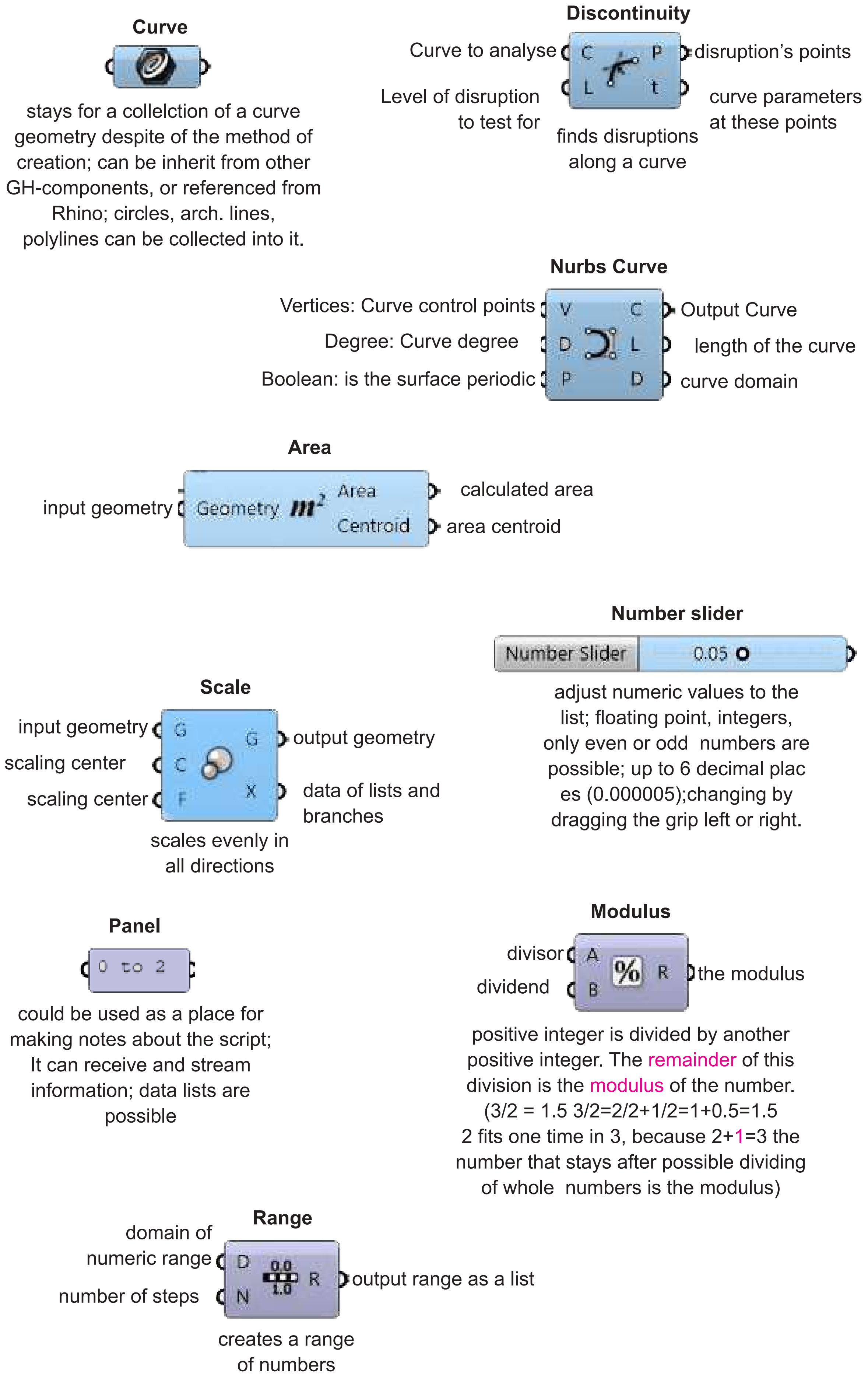


Fig.55 Grasshopper legend of the components- first part

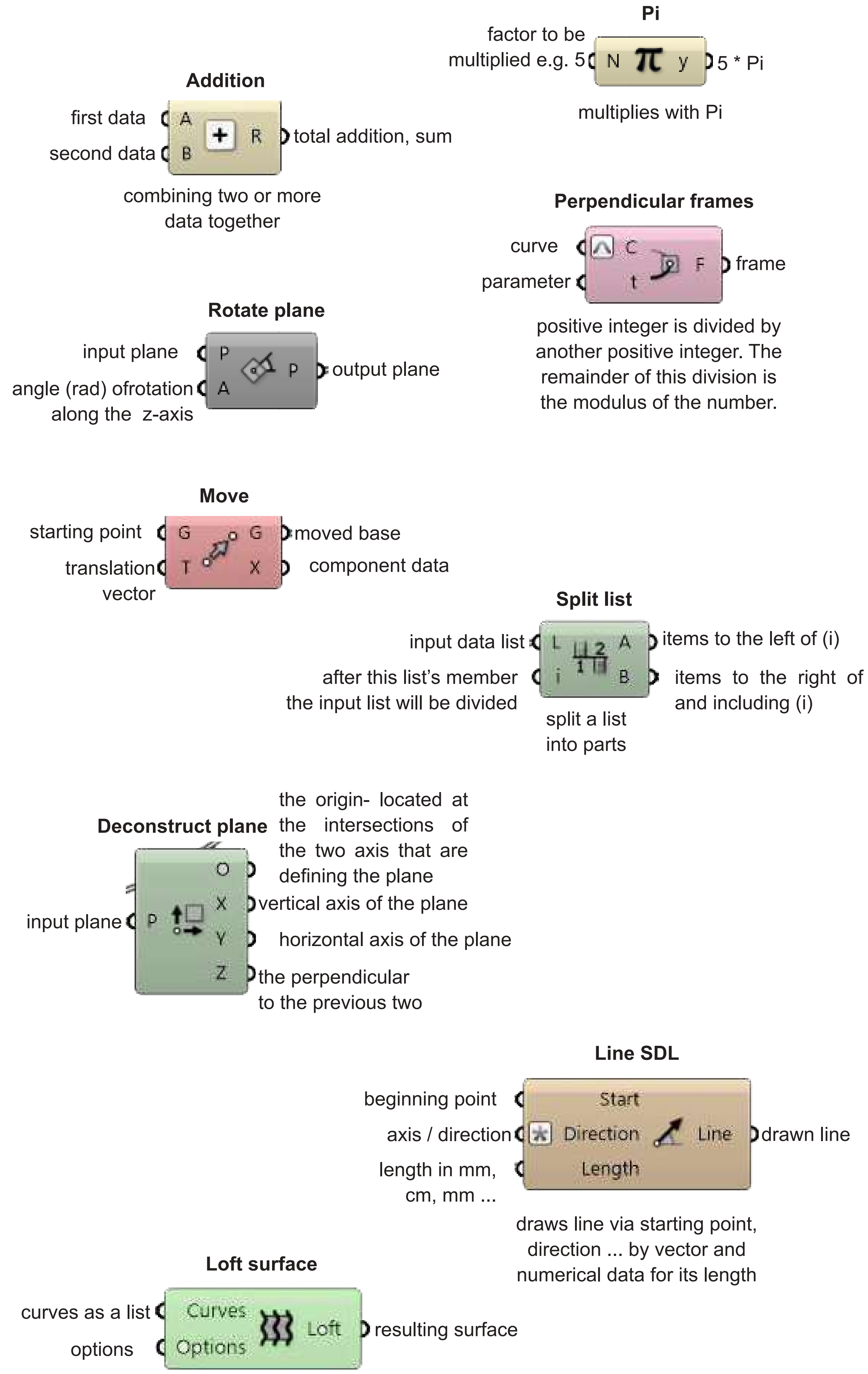


Fig.56 Grasshopper legend of the components-second part

3.2 Enneper surface

History

It is named by the mathematician Alfred Enneper who discovered it in 1863 and introduced it in 1864 in connection with the minimal surface theory.

Characteristics

The Enneper surface is extraordinary among minimal surfaces because of its symmetry and possibility for self-intersection. The geometry can extend infinitely in all directions, but it is usually visualized within a finite boundary for having a better overview (fig.57). It doesn’t have a distinct “inside” or “outside” like a sphere does. It can resemble a saddle or a series of undulating waves.[4] "It can be geometrically defined as the envelope of the mediatrix planes of two points located on two homofocal parabolas (i.e. parabolas the planes of which are perpendicular and such that the vertex of one passes by the focus of the other one;...)".[5] The classic Enneper surface is of order 2. It has four waves, two by two in opposite directions (fig.57).

Modularity

Modularity depends on how many elements are repeated around a circle. Some module`s geometries makethe building process of real structures easier, so we divide the surface along the symmetry axis.

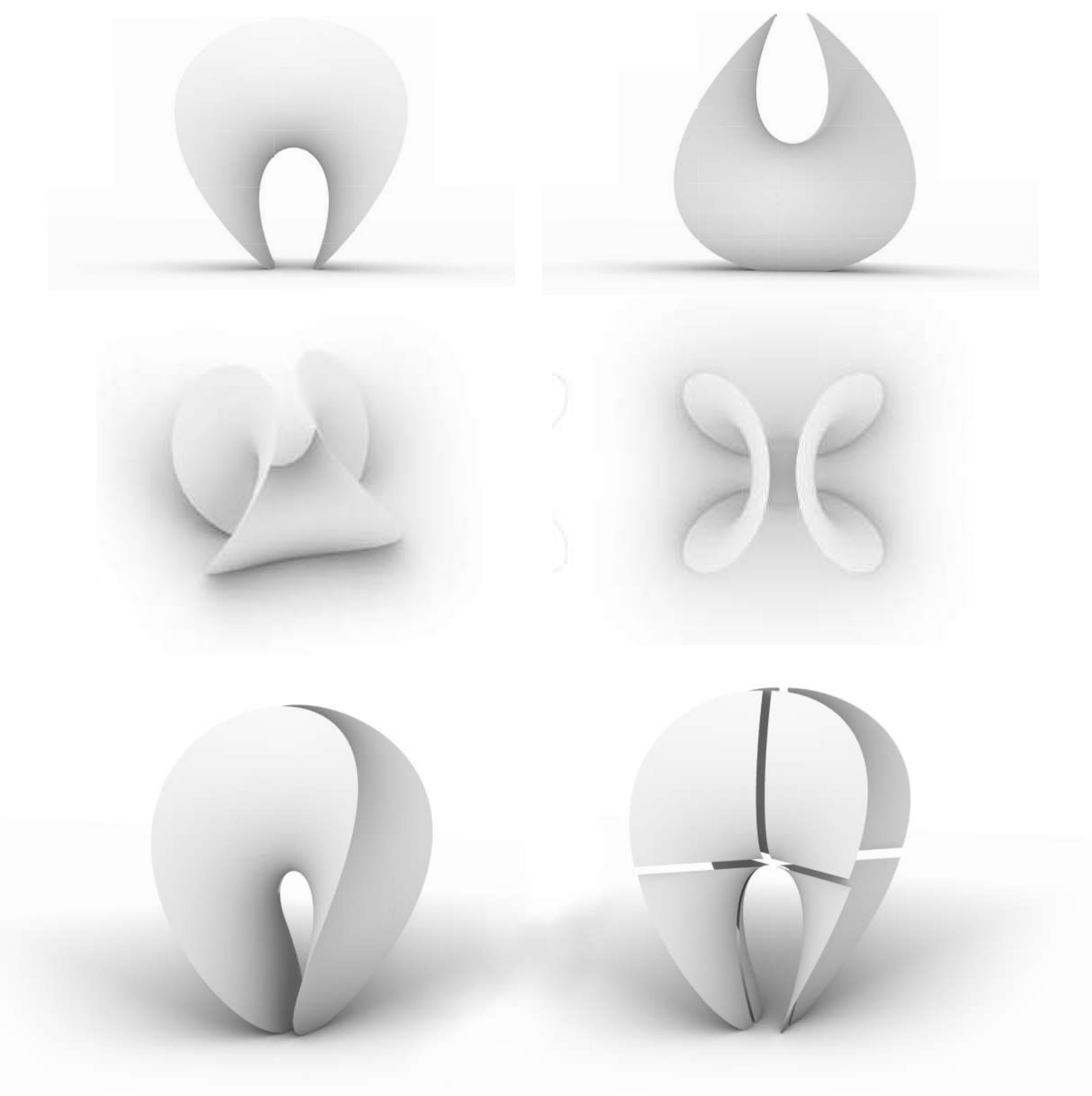


Fig.57 Up: side views; middle: perspective and top view; bottom: perspective and division of the Enneper surface

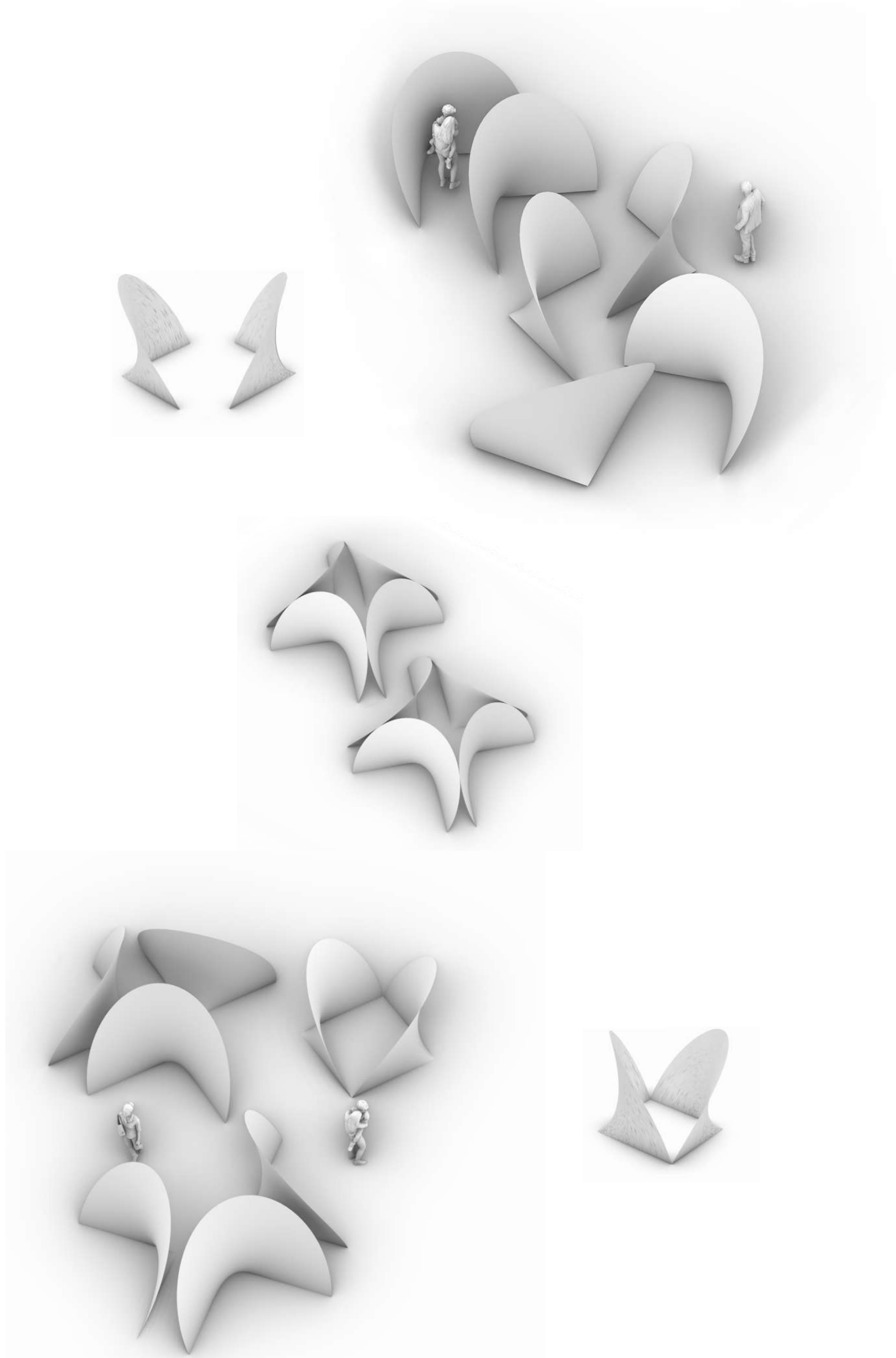


Fig.58 Perspectives of variations

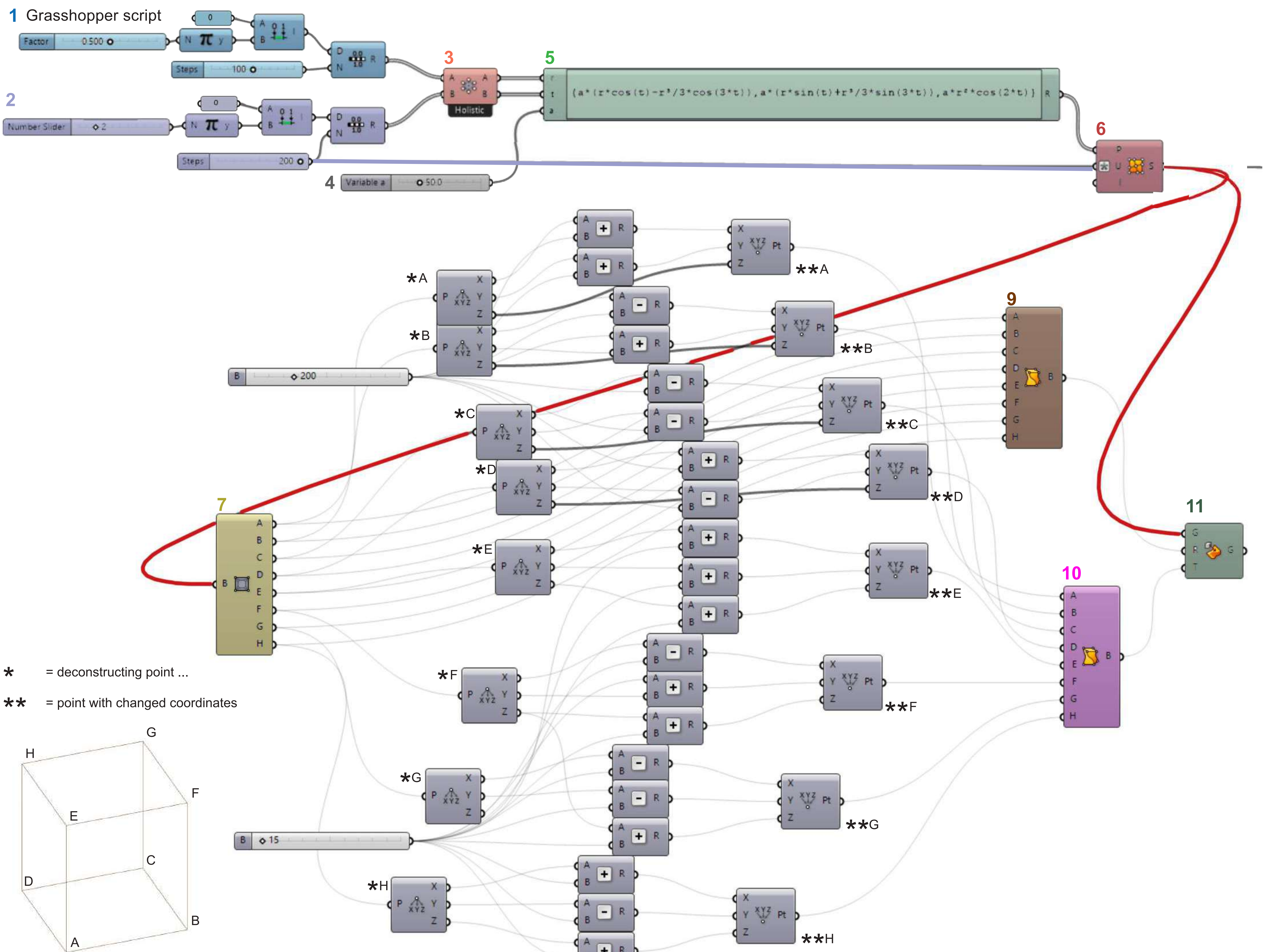
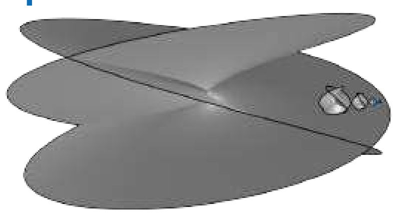
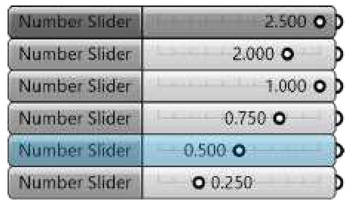


Fig.59 Building definition of the Ennepers script in Grasshopper

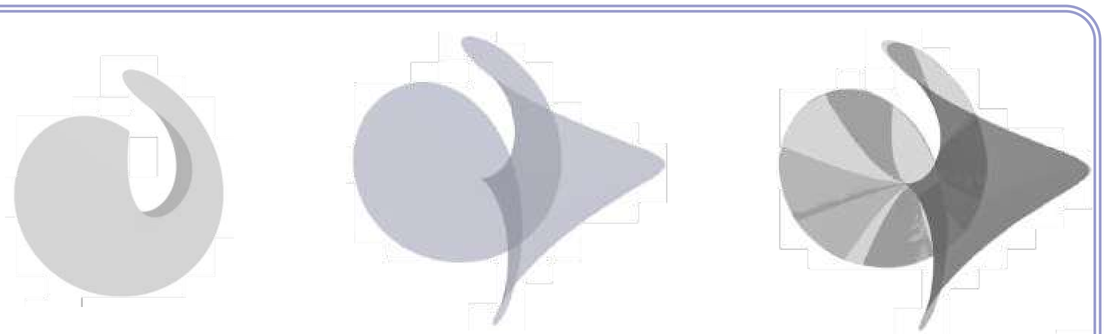
1



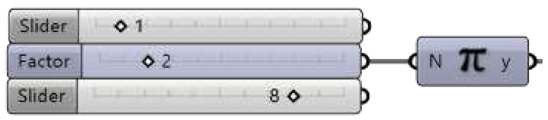
Numeric domain between '0' and '0.5*Pi' is created. The size of the surface of step ...depends on this step and whether it will be self-intersecting or not.

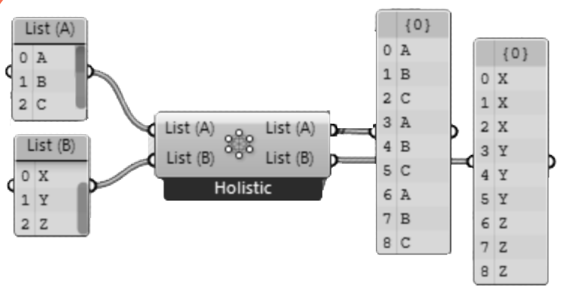


2



The second numeric domain is also influencing the distribution of points in space. '1' multiplied by two outputs at step ... a half Enneper surface. '2' outputs a whole surface. Even numbers more than two create layers of the geometry. Both domains stay for the revolution of the surface.





3

The holistic cross reference component makes all possible connections between the input lists. Mathematically this is similar to multiplication.

4 can change the scale of the model independently (it is not connected with other functions)

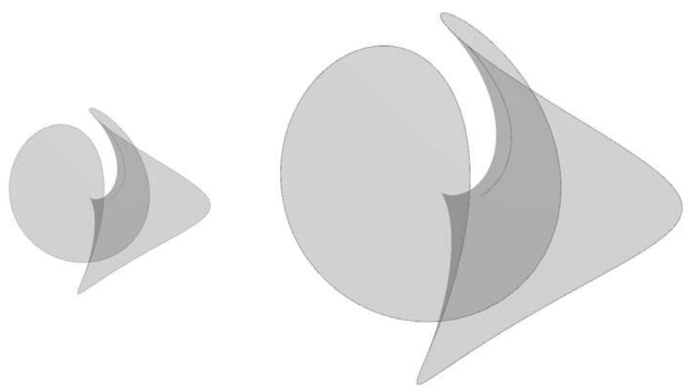
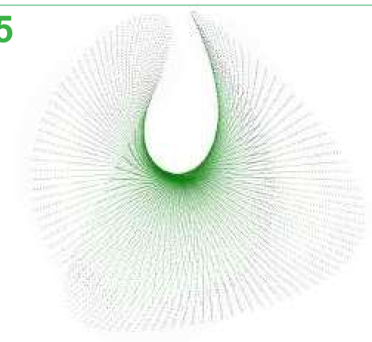


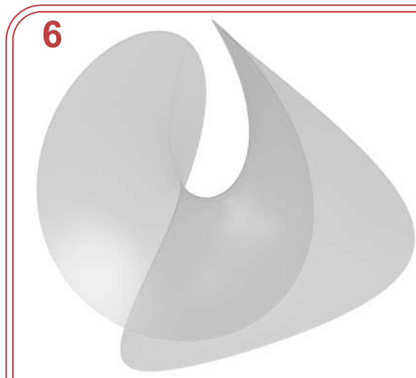
Fig.60 Explanations of the building definition

5



The mathematic formula is arranging points in R3 space, that are creating a grid of circles and curves. A nurbs surface is computed following the network.

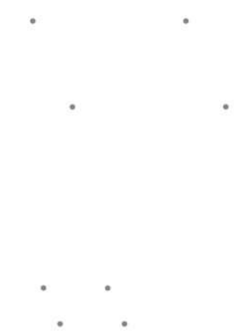
6



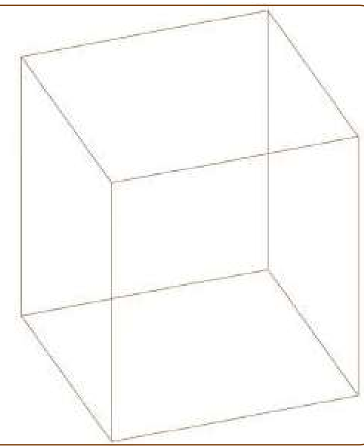
A suface from points is createdThe expression symbol at the 'U' - input stays for 'x+1' Otherwise the script is not working.

7

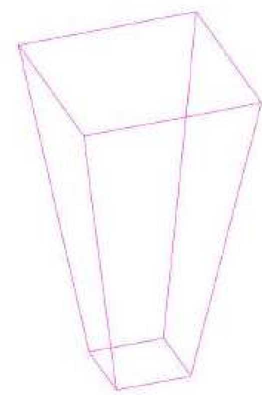
8



9



10



11

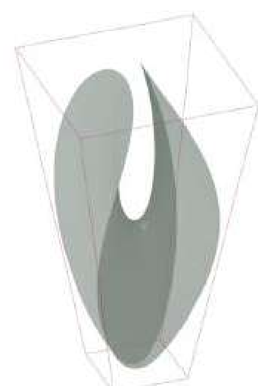


Fig.61 Explanations of the building definition

Grasshopper legend

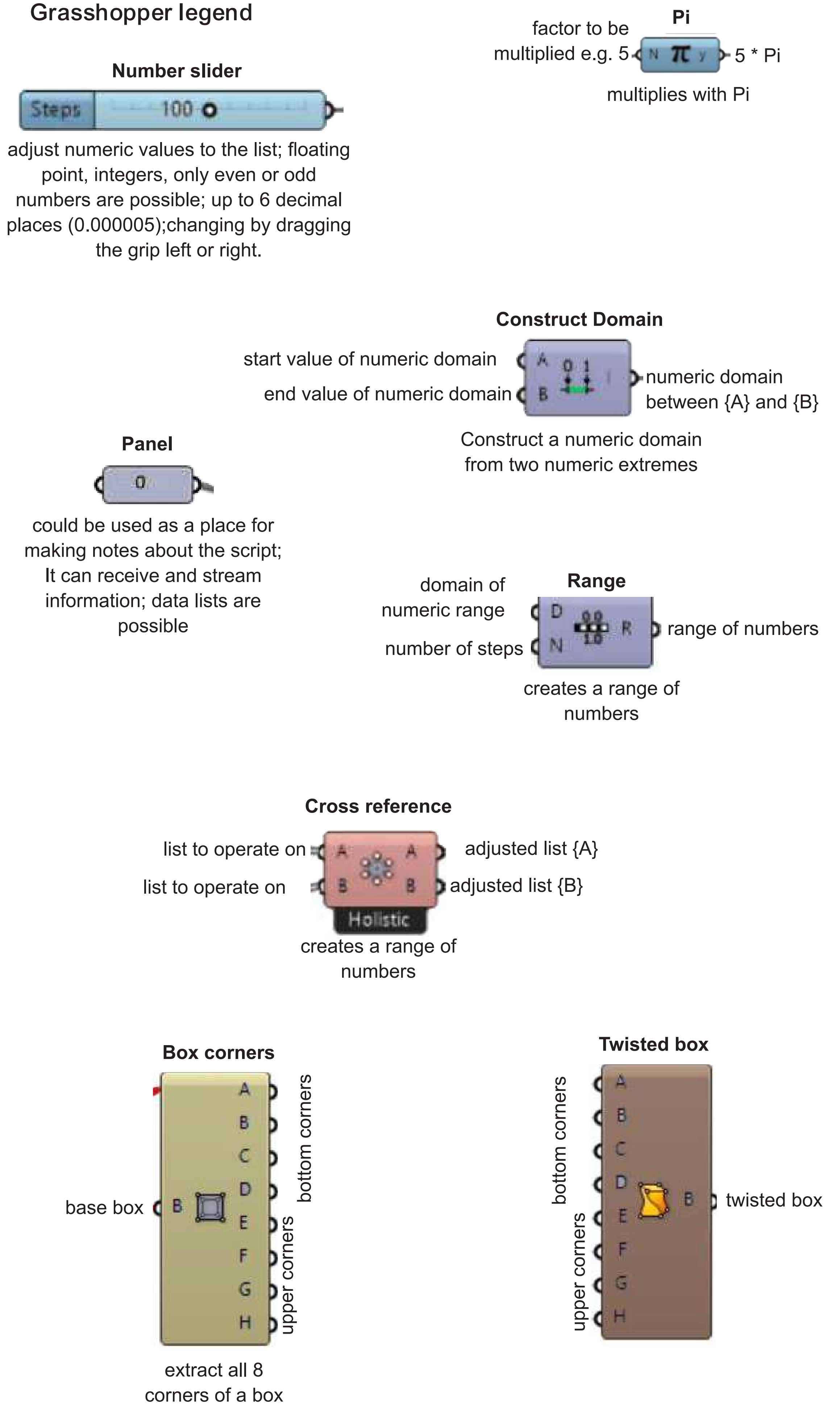


Fig.62 Grasshopper legend of the components

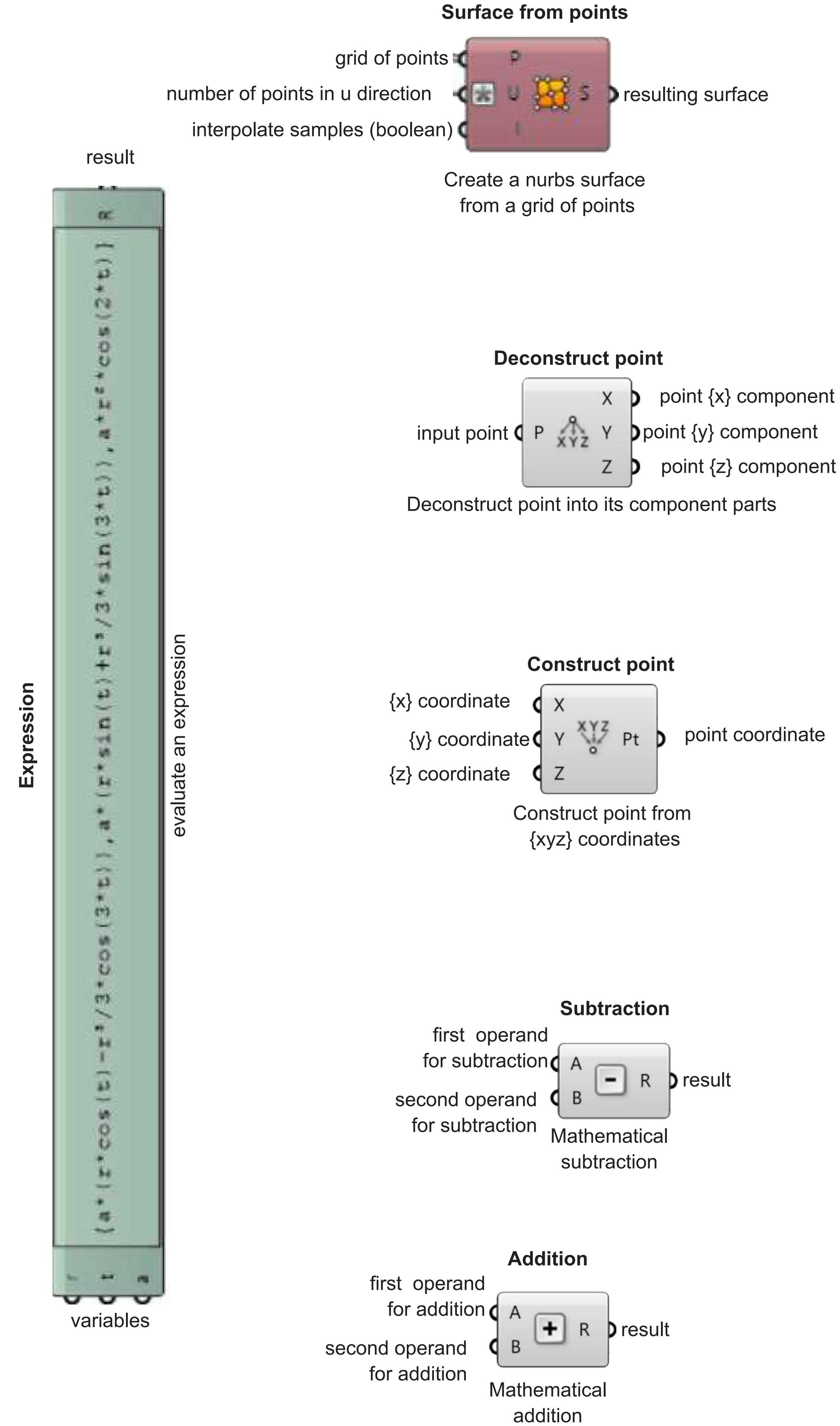
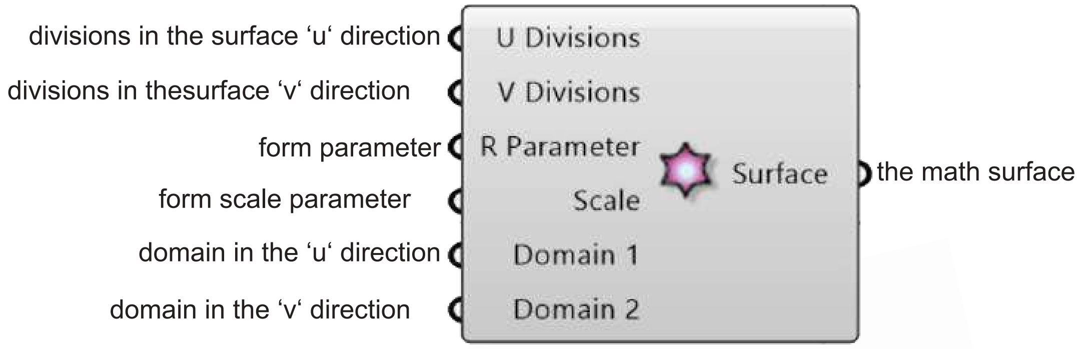


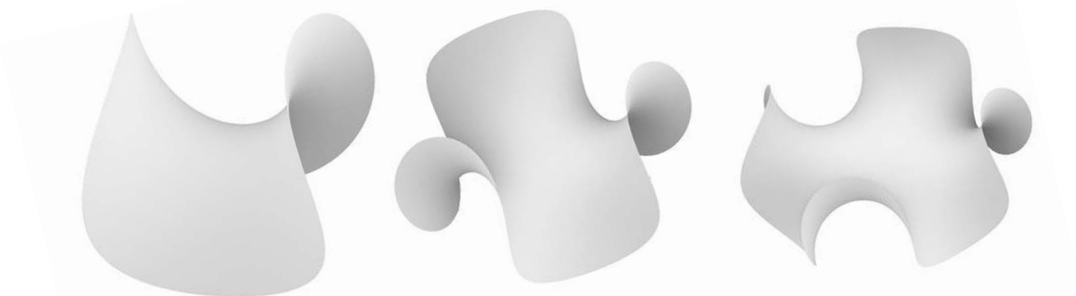
Fig.63 Grasshopper legend of the components

One very simple way nowadays of visual programming the surface is given from "LunchBox", a Grasshopper plug-in, that travels over mathematical shapes, paneling systems and structures.

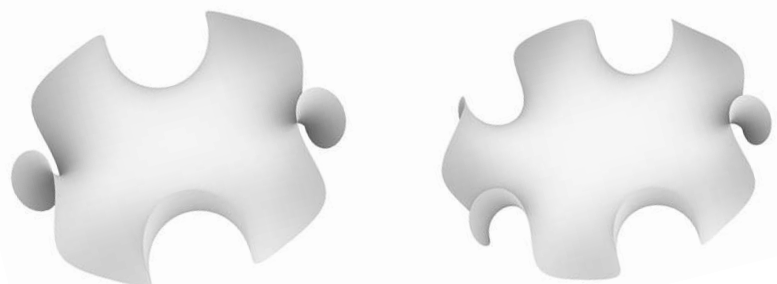
Enneper surface



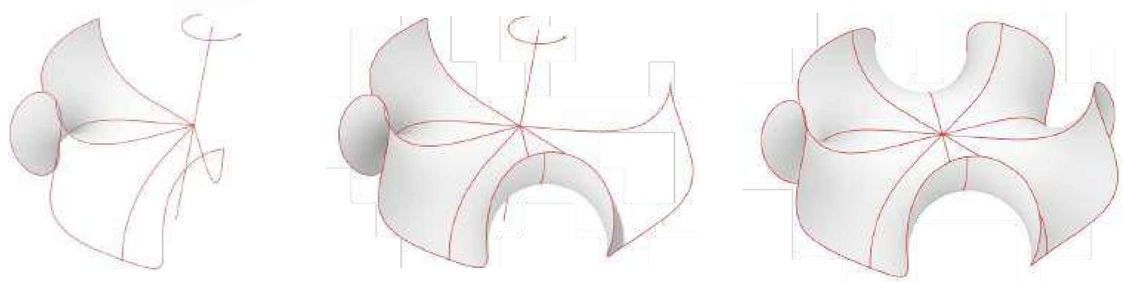
create a parameter Enneper surface



R parameter = 2 R parameter = 3 R parameter = 4

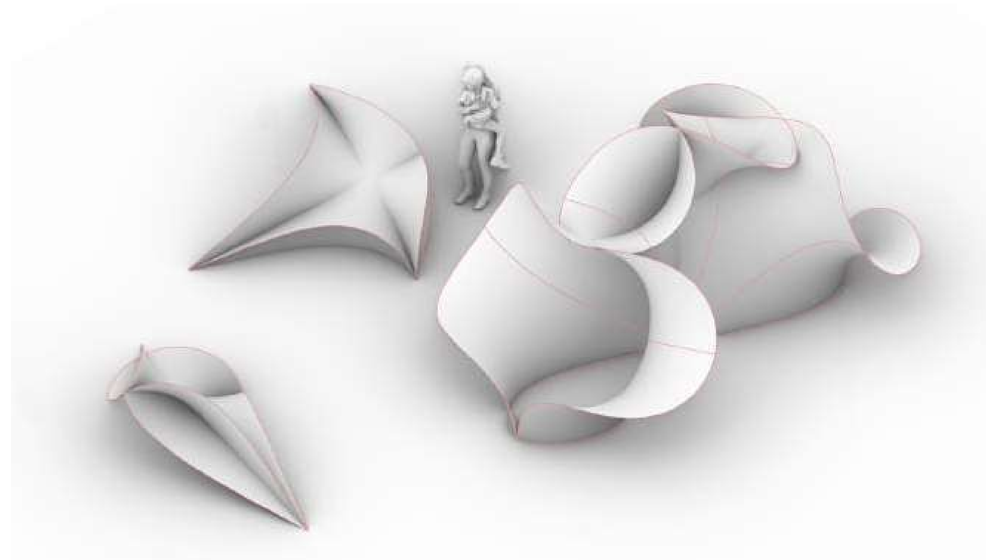


R parameter = 5 R parameter = 6



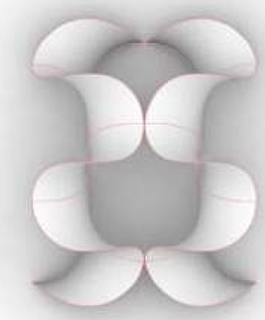
rotating the module of surface with ‘R parameter 5’ four times in 36°

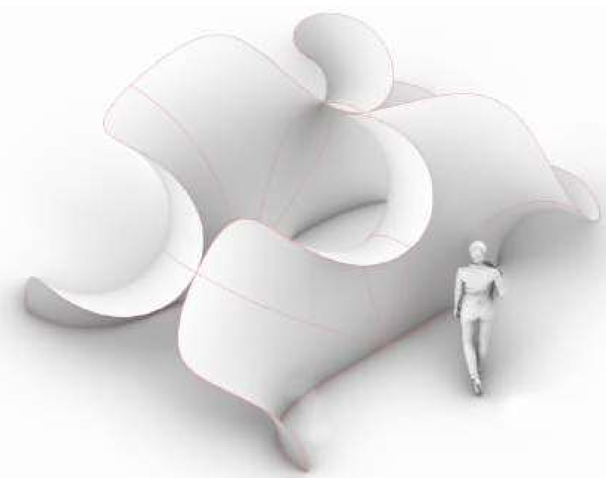
Fig.64 Enneper plugin in Grasshopper; middle: surfaces of different parameters; modules

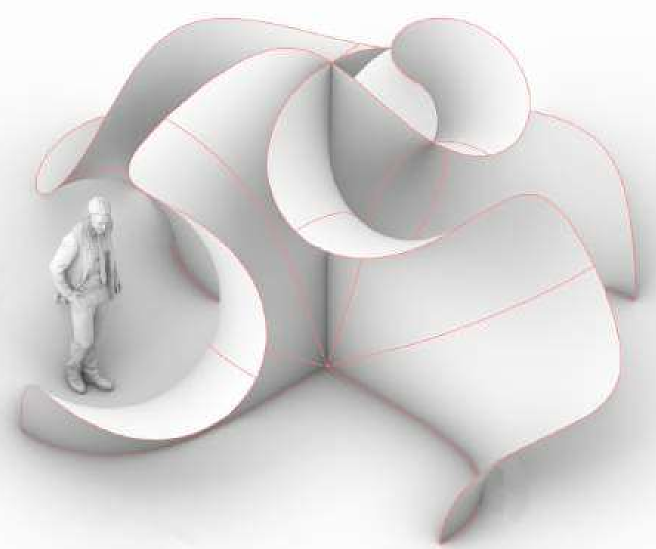


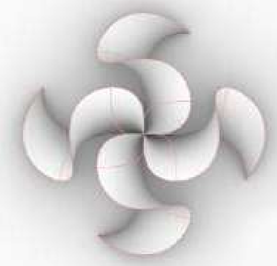
variation ‘cross’

variation ‘in contact’









variation ‘rose’

Fig.65 Perspectives of variations, top views

3.3 Batwing

History

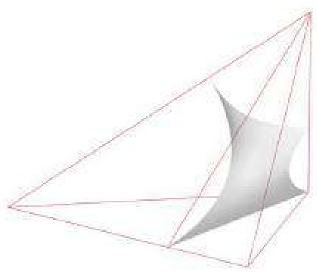
It was introduced by Alfred Enneper in 1864 in connection with minimal surface theory. The Batwing Surface is more complex than the Enneper surface and does not self-intersect, leading to shape that resembles a stretched, curved "wings"(fig.65).

Characteristics

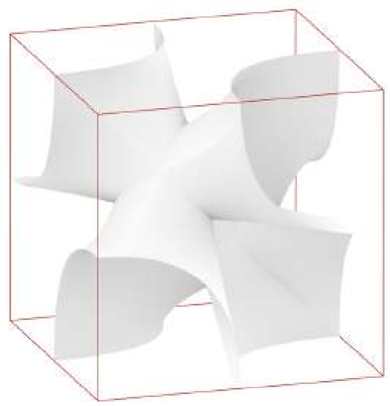
It is a triply periodic minimal surface, in the sense of repeating themselves in three dimensions
It has a crystalline structure. Two fundamental regions, in other words two modules, placed next to each other look like “batwings” and fit in a tetrahedron. Twelve surfaces can be arranged in a cube, or slightly flattened octahedron (fig.60). Brakke’s Pseudo-Batwing Surface, Schoen’s Batwing-41 Surface (higher genus version of the surface), Schoen’s Batwing-57 Surface. They are higher genus version of the surface, respectively with genus 41 and 57.[6] It is a cubic minimal surface, straight lines or holes are connecting elements.

Modularity

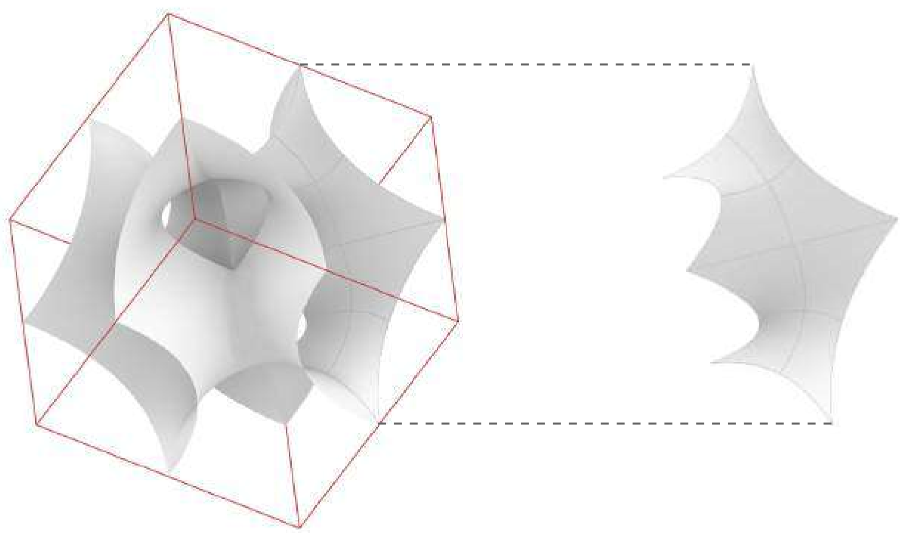
Being triply periodic means that it is appearing at intervals in three dimensions and a finite module can be bounded by a solid. Diving the intervals from each other could be a division in itself. There are many possibilities: splitting up one by one, two by two, three by three and so on (fig.69).



module and 1/8 unit cell

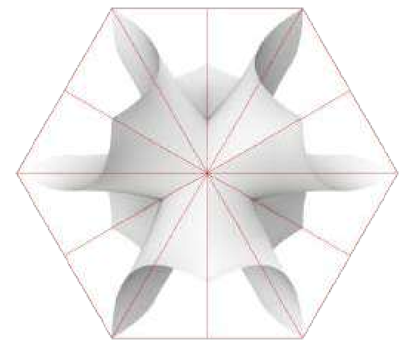


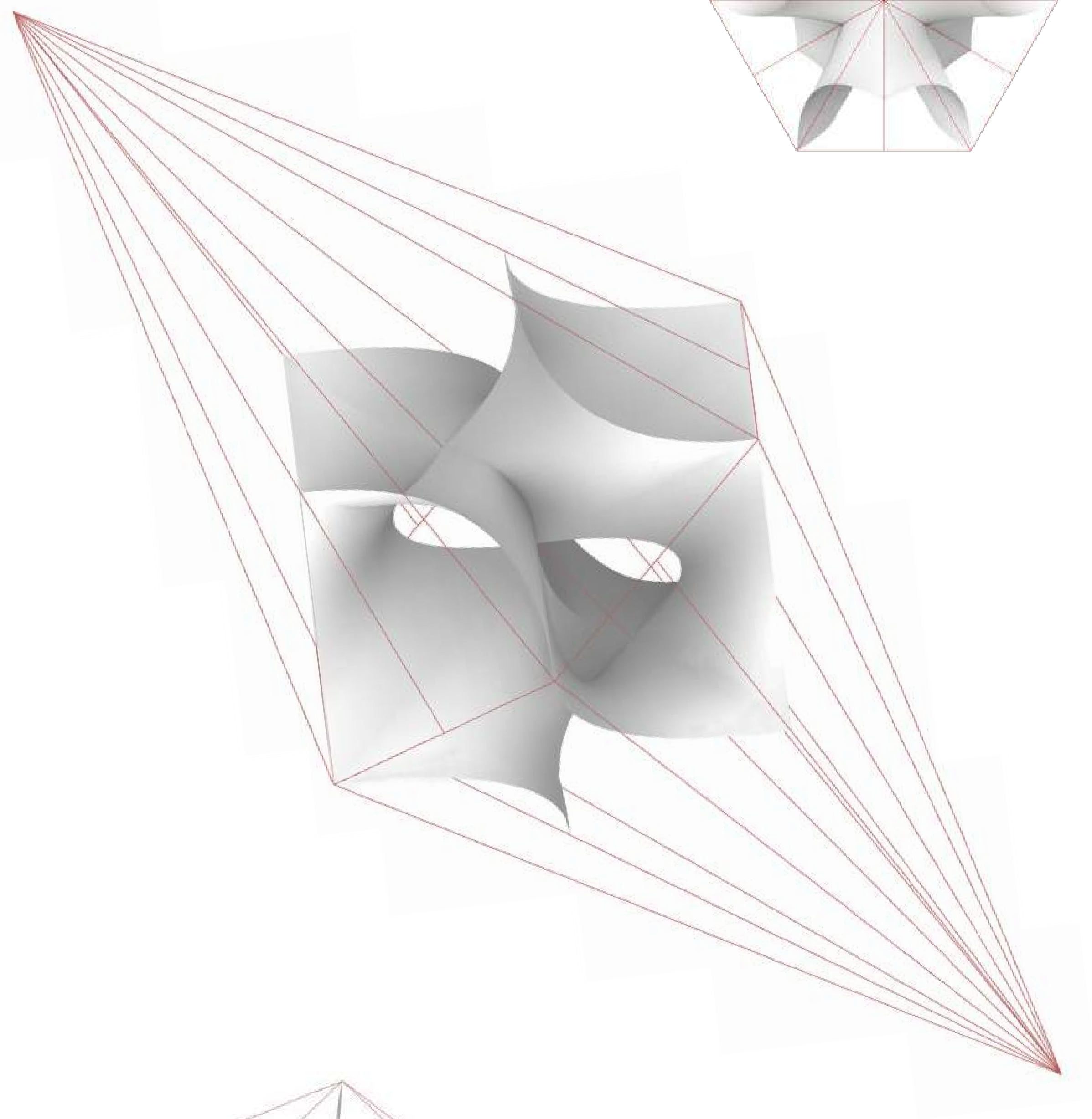
fundamental region and a full unit cell



The reason for the name: two fundamental regions looking like batwings.

Fig.66 One modulein a unit cell; Batwing surface in a cube; Name Origin





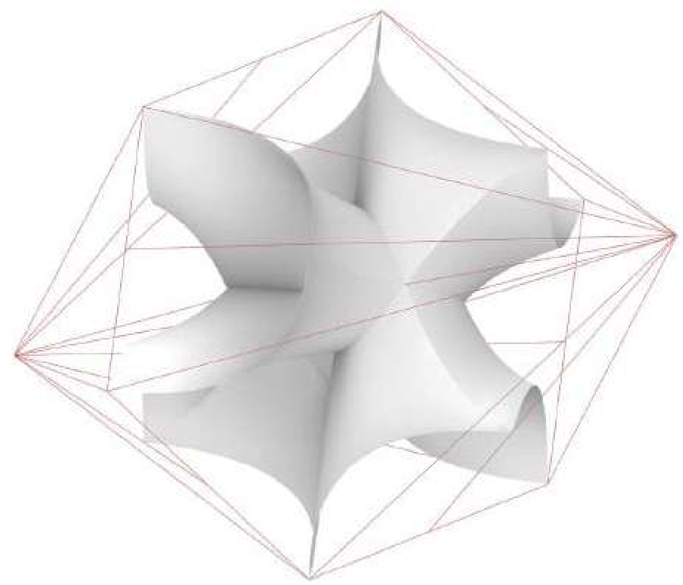


Fig.67 Batwing surface inscribed in two pyramids

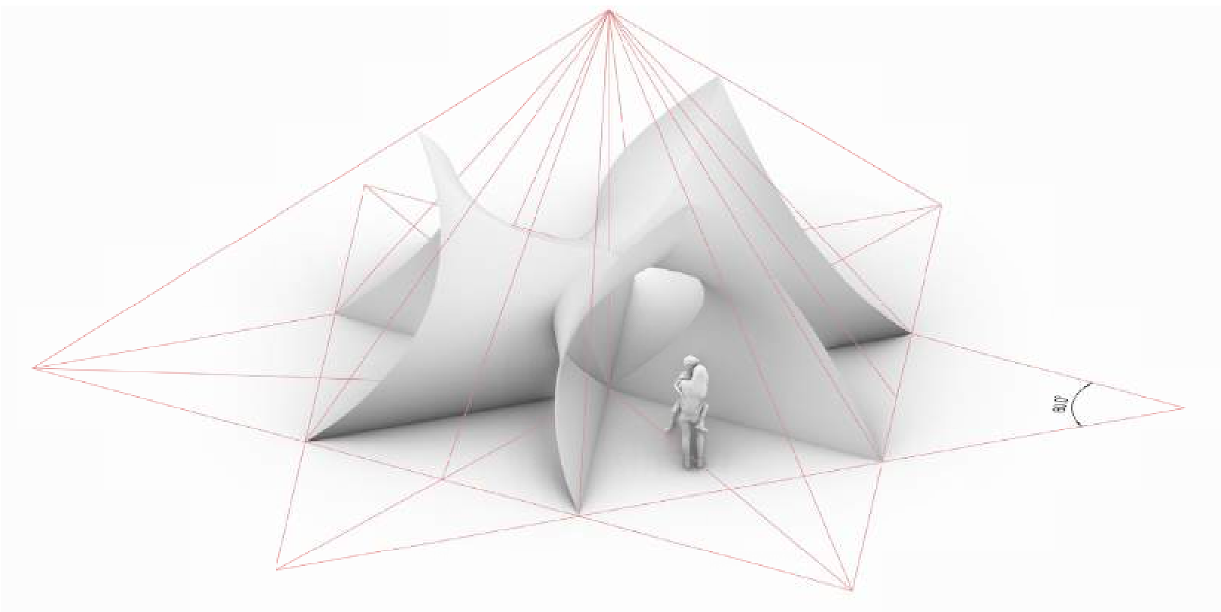


Fig.68 Relations of the surface out of six modules

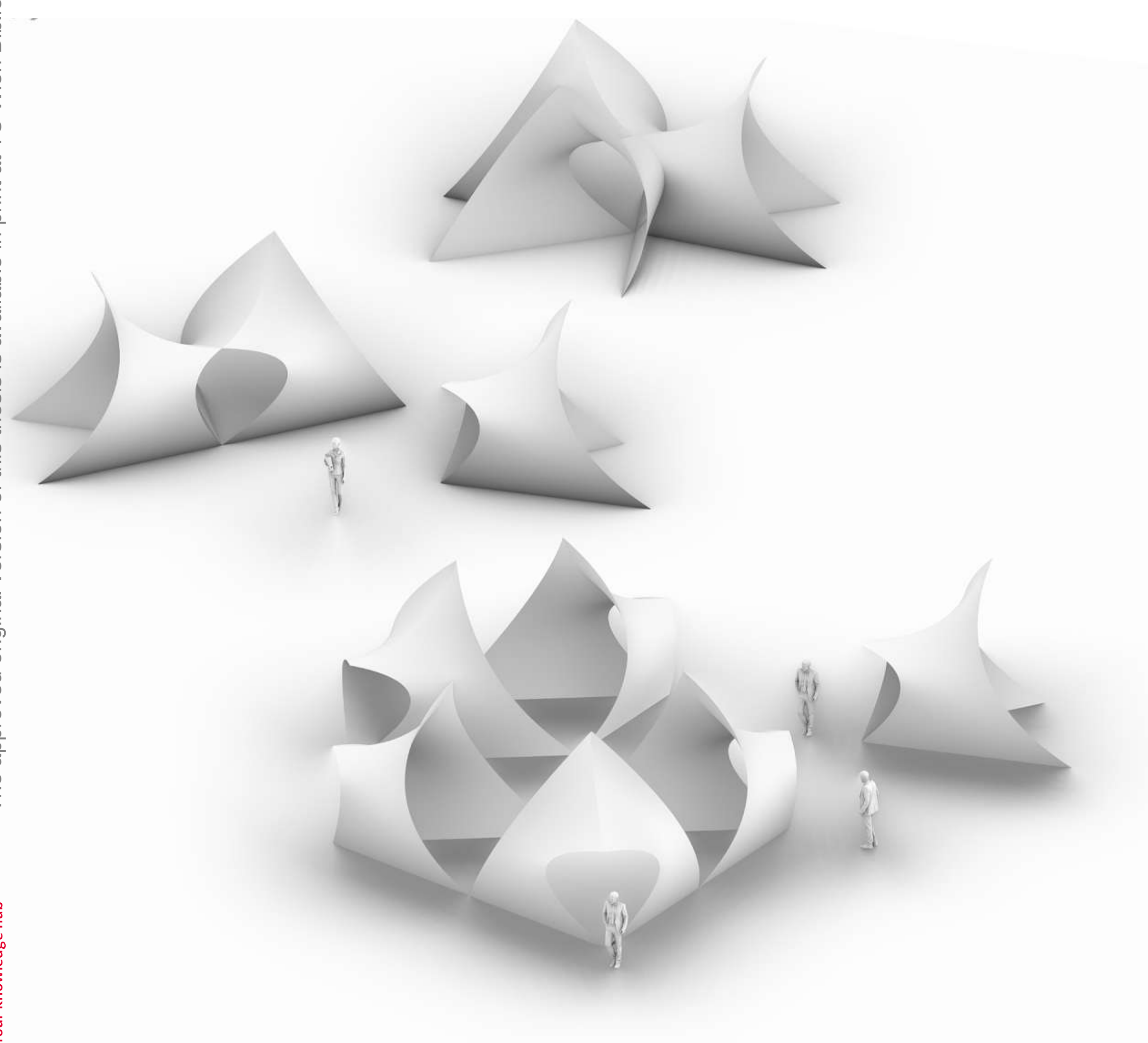
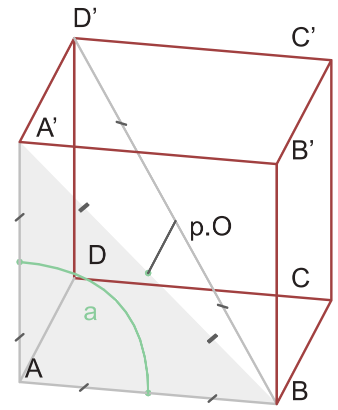
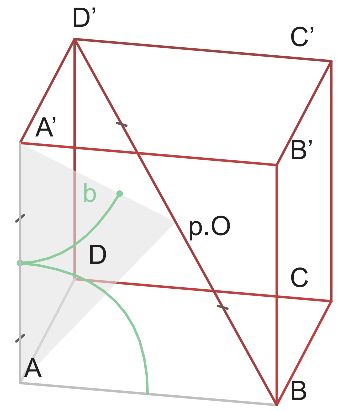


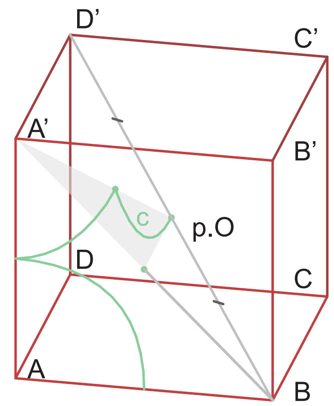
Fig.69 Perspective of modules



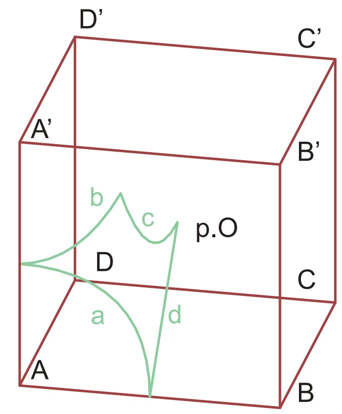
Arc ‘a’ of a quarter circle between two middle points of the neighborhood edges



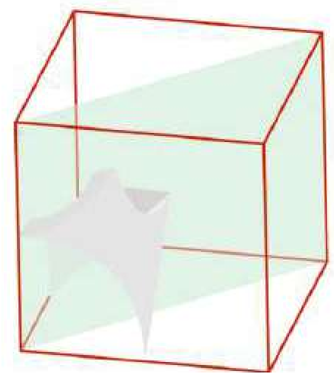
‘b’ curve of a quarter circle between two middle points of triangle AOA’



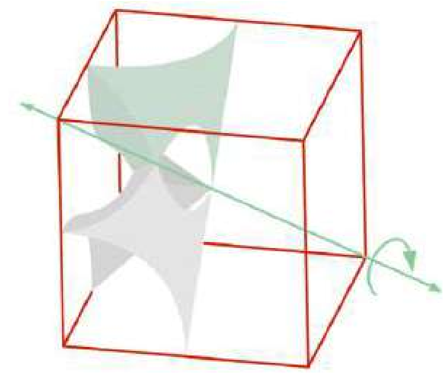
Bezier curve ‘c’ with control points middle of A’O, centre of ABA’B’ and p.O



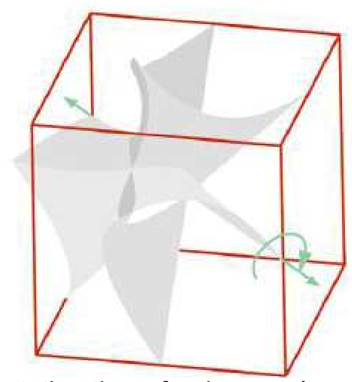
meridian ‘d’ to side AB and the polygon with edges ‘a’,’b’,’c’,’d’



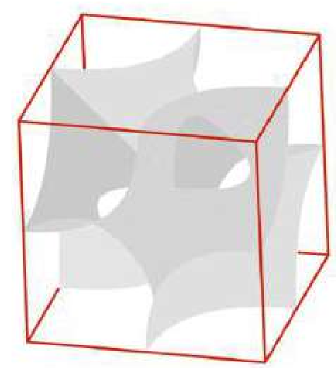
mirrorring the surface



rotating the first two surfaces in 120°



rotating three fundamental regions / six surfaces in 180°



twelve Batwing surfaces in a cube

Fig.70 Evolution of the Schoen’s Batwing surface drawn in Rhinoceros

3.4 Minimal surface from tetrahedron

History

All Platonic solids were well known to the ancient Greeks, described by Plato in his works ca. 350 BC. Predating him, the neolithic people of Scotland developed the five solids a thousand years earlier. Archaeological artifacts in a form of a stone models are kept in the Ashmolean Museum in Oxford (Atiyah and Sutcliffe 2003).[7] Minimal surfaces with a bounding hexahedron (cube) geomerty were investigated for first time while doing the reference project ve.sh (fig.21).

Characteristics

A tetrahedron is a regular triangular pyramid whose base is also a triangle. The only Platonic solid where all four vertices are equidistant from each other. It can be folded into three dimensional geometry by using a two dimensional geometric net (fig.71). All the interior angles of a tetrahedron are 60° each which results in a minimal surface with 60° corner angles as well. [8] Its edges are four neighbouring edges of the tetrahedron, so two others remain. The area of the minimal surface multiplied by 2.7 gives the area of the tetrahedron.

Modularity

Similar to ve.sh by rotating the bounding tetrahedron along one edge the geometry grows. The tetrahedron is the dual of the cube. When needed one can profit from both bounding boxes. While rotating the diagonal of the box, or the edge of the tetrahedron, the bounding geometries create sculptures.

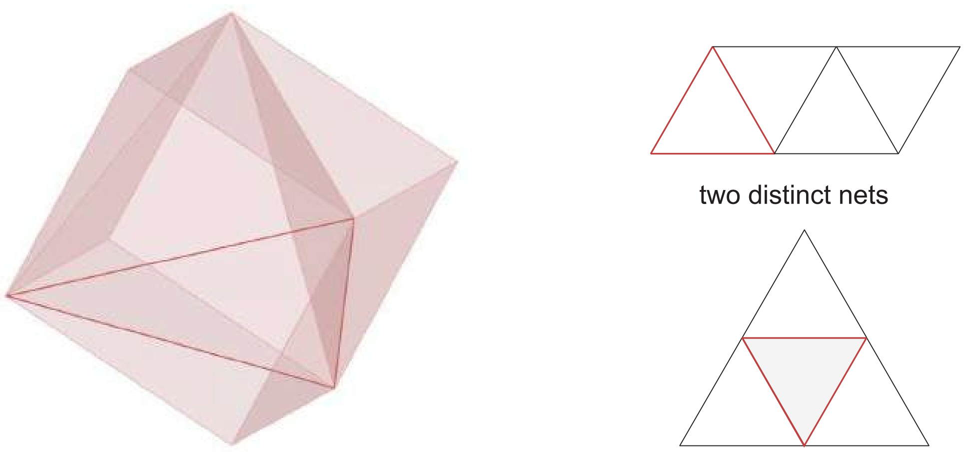


Fig.71 Left: tetrahedron in a hexahedron; right: the planar geometric nets

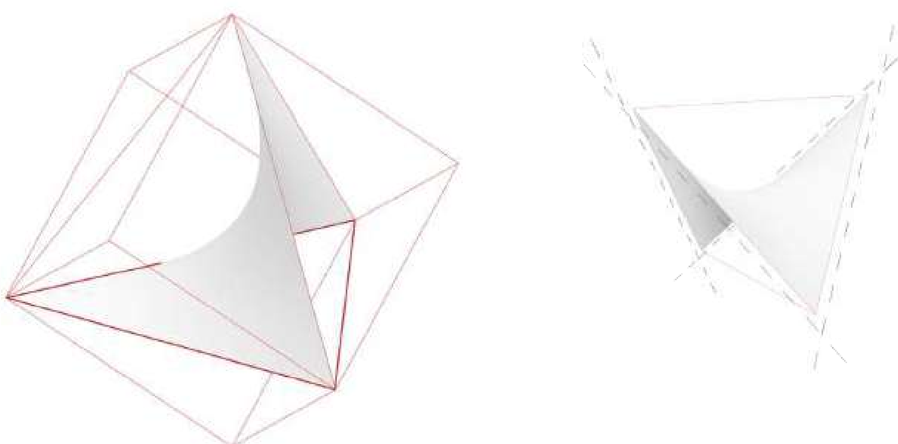


Fig.72 Left: minimal surface from tetrahedron edges; right: edges as a rotational axes

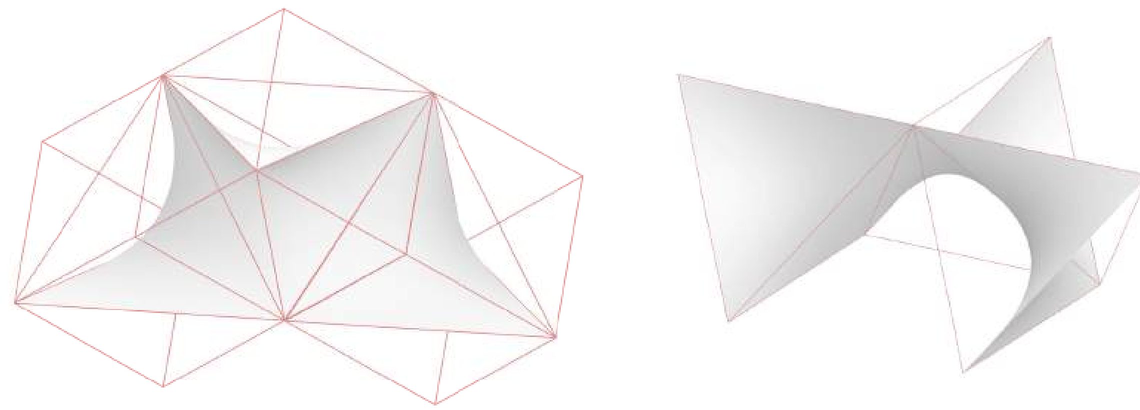


Fig.73 Two variants made with three modules

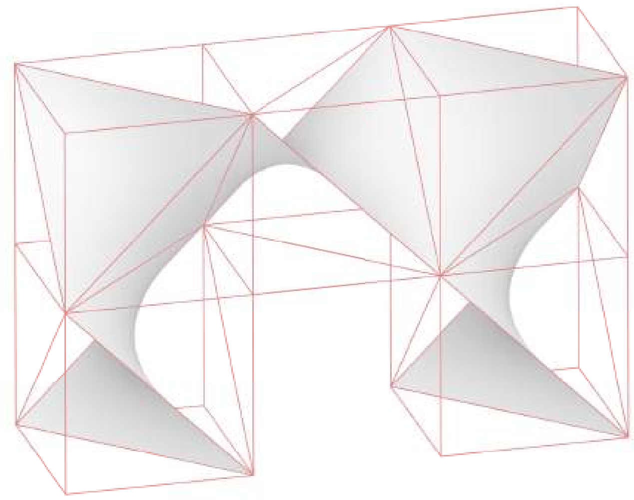
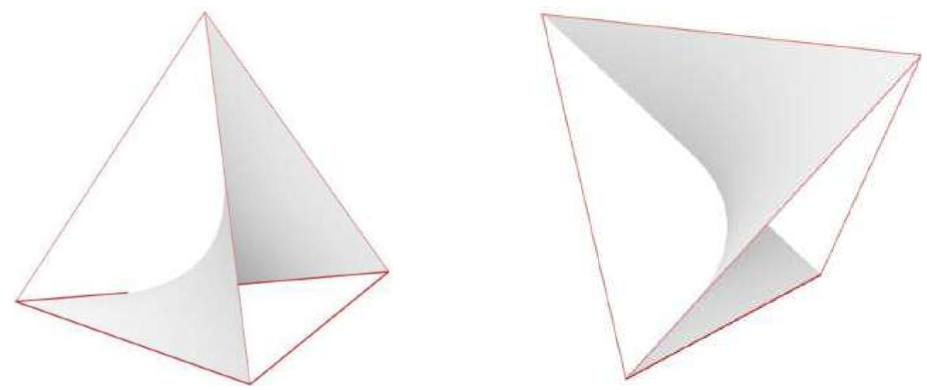


Fig.74 Variant made with five modules



There are four ways for one module to meet the ground: with 2 edges, 1 edge, 2 vertices, 1 vertex

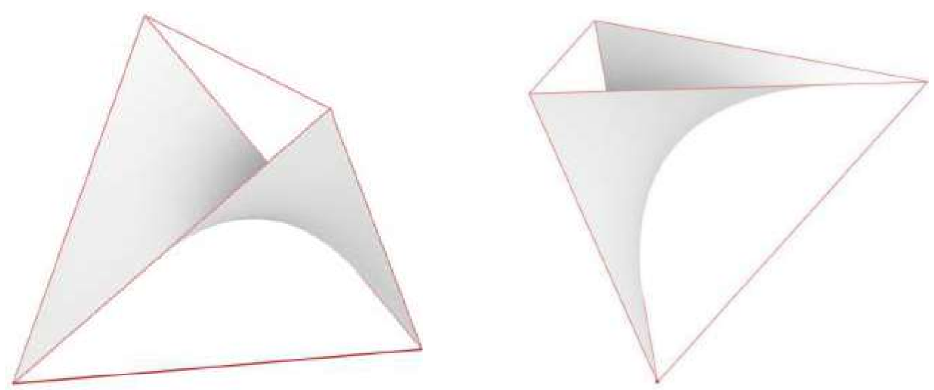


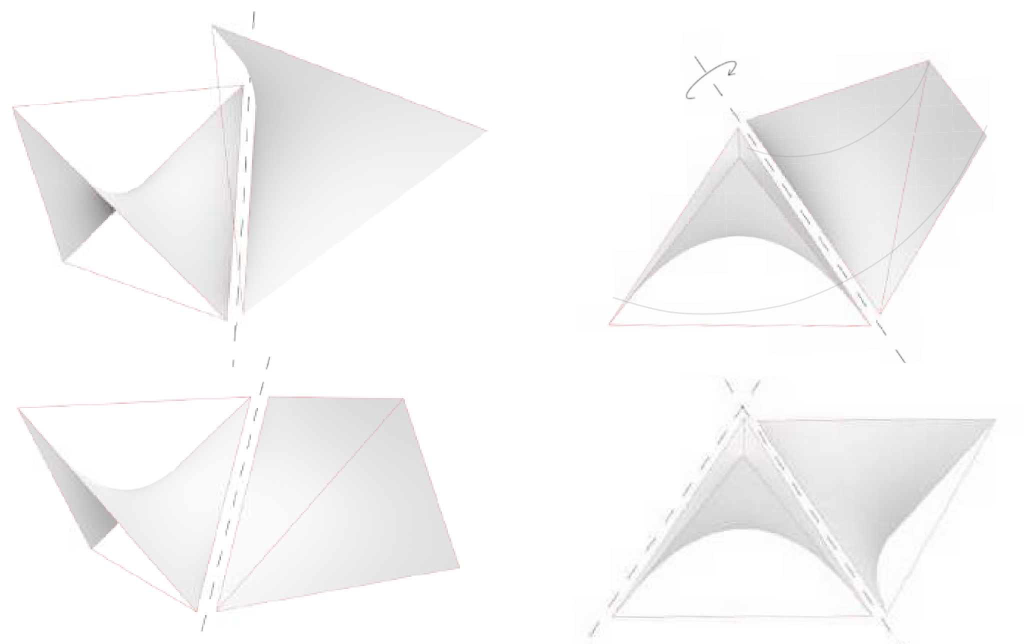
Fig.75 The four ways for one module to meet the ground

1. initial patches

start with two opposite edges

start with two adjacent edges

2. rotation of the first edge in 180°



3. rotation of the second edge in 180°

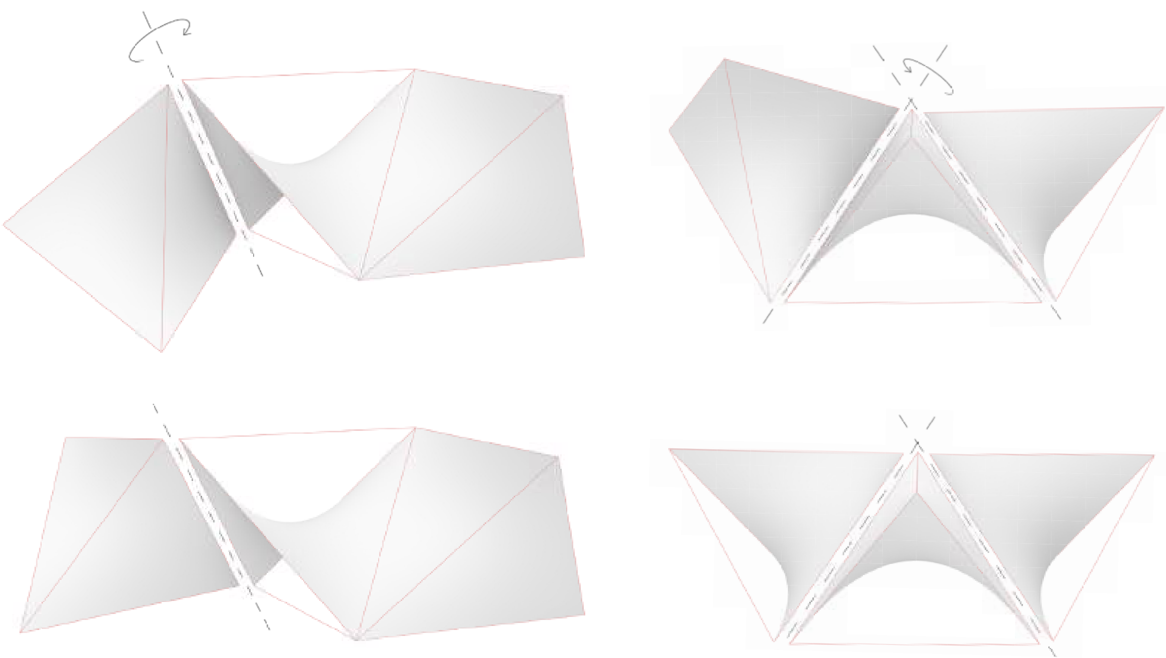
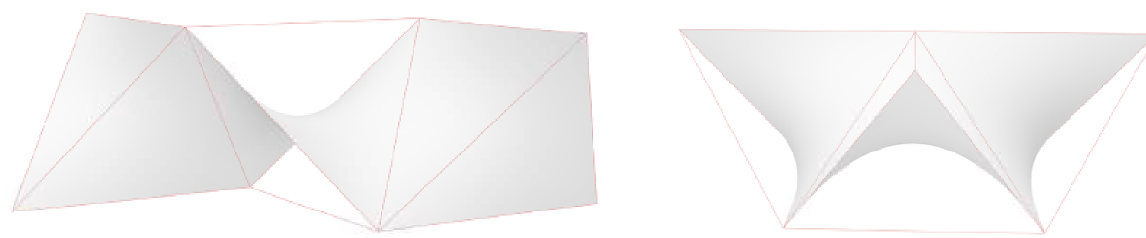
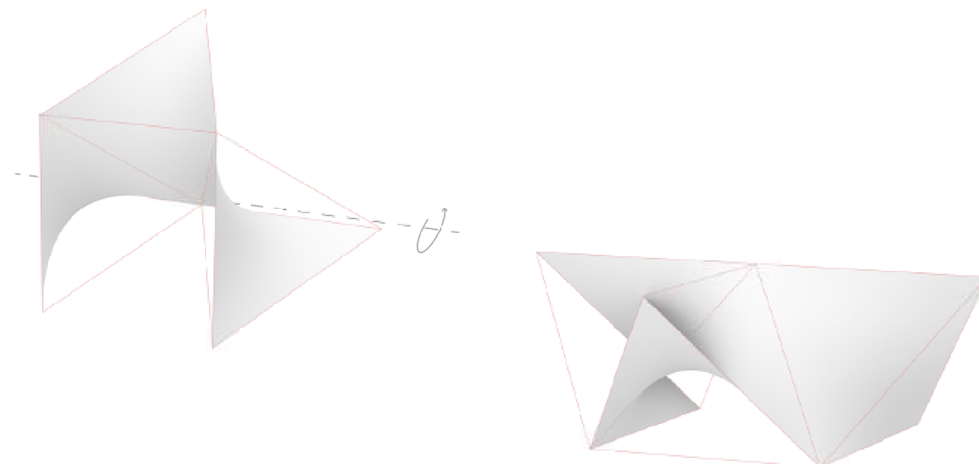


Fig.76 Building steps of the variants "lace" and "elan"

4. end result



5. rotation of the whole model in 90°



6. moving the middle part on the ground

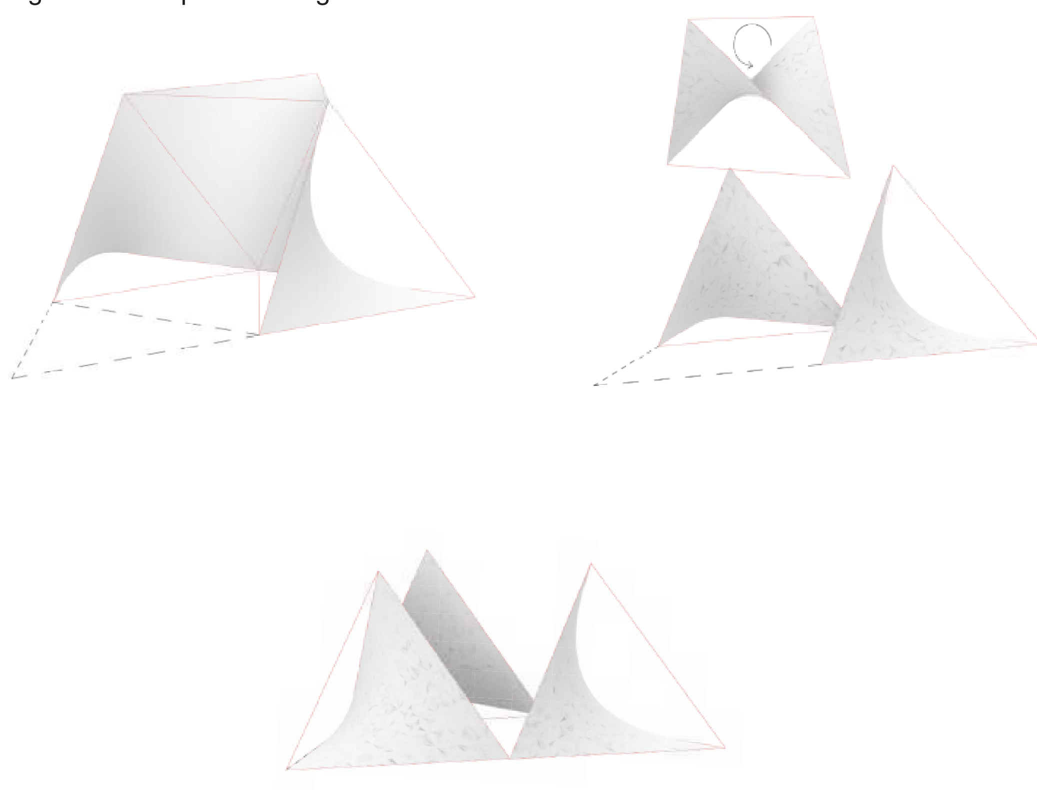
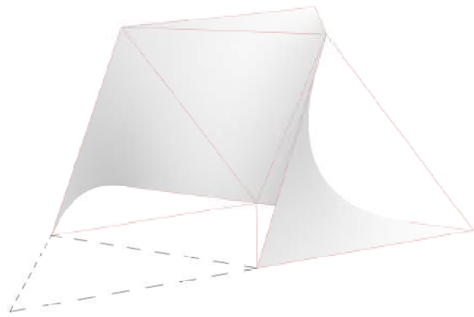
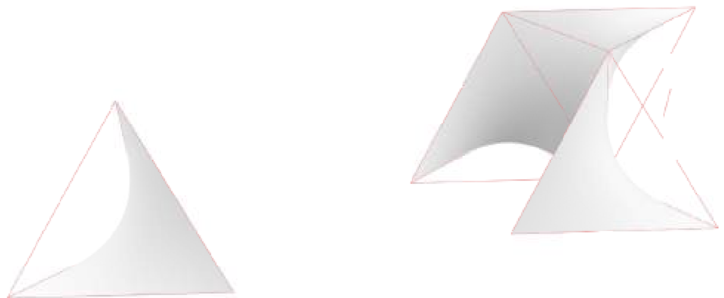


Fig.77 Transforming steps from "elan" into, "shelter" and "lilium"

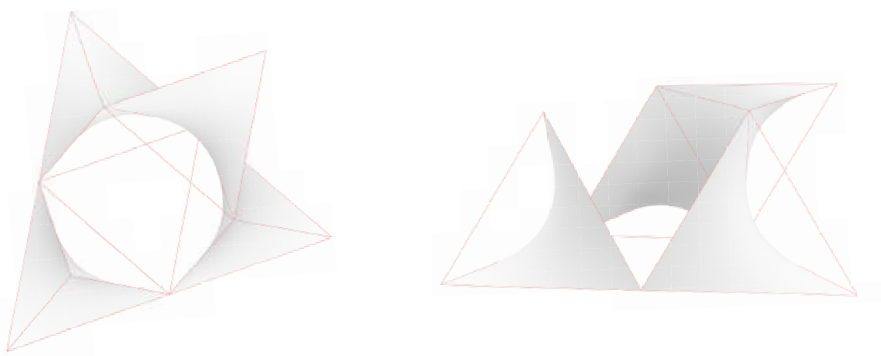
7. starting with the variation "shelter"



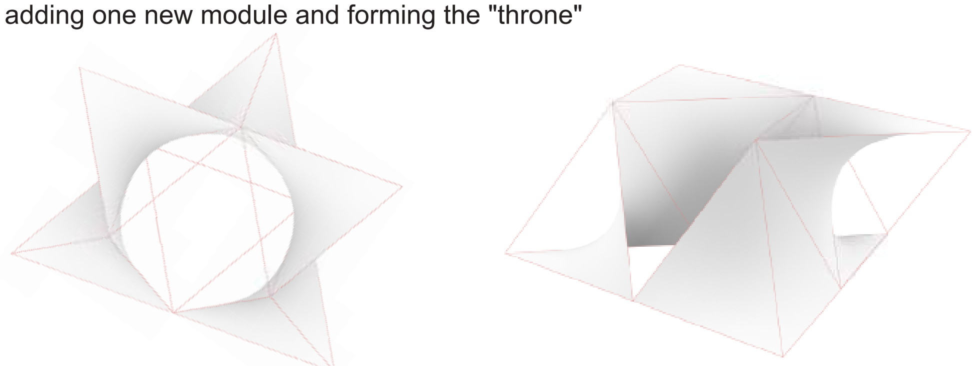
8. adding one new module to get the variation "approaching"



9. moving closer and forming the "silhouette"



10. adding one new module and forming the "throne"



11. adding one new module and forming the "flower"

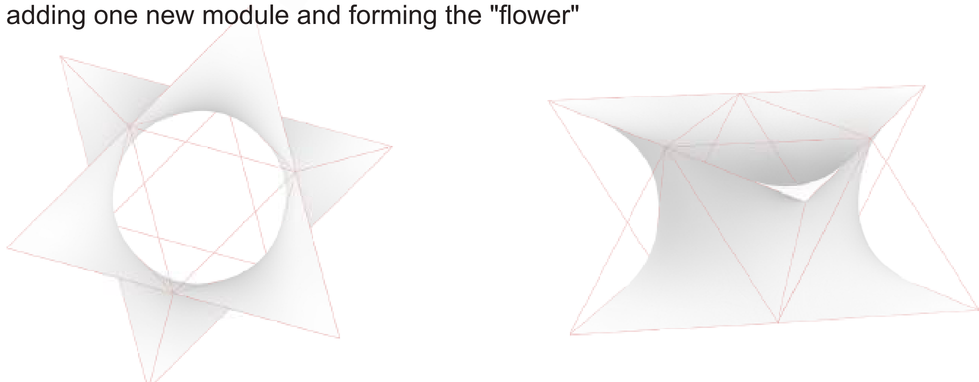
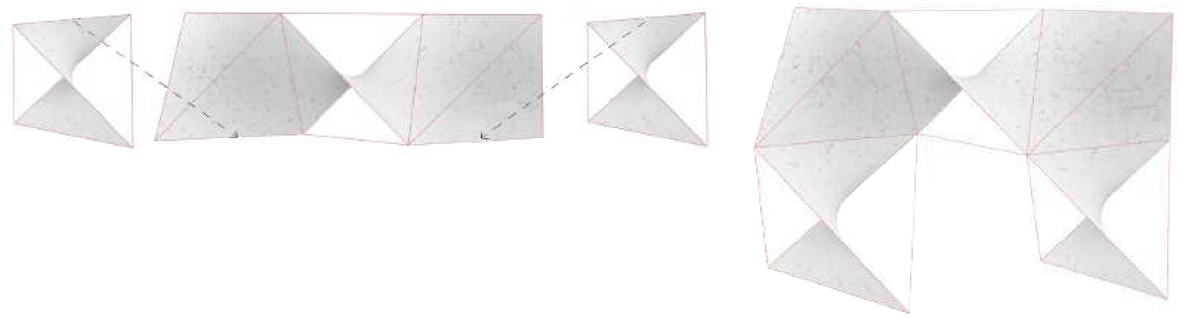
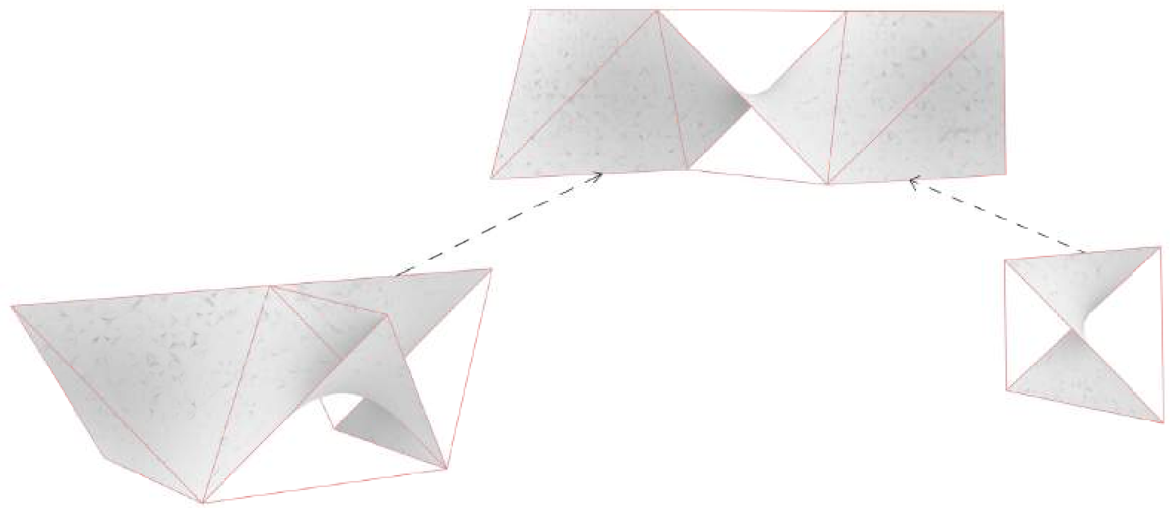


Fig.78 Transforming steps from "shelter" into "approaching" , "silhouette" , "throne" and "flower"

12. transforming the variation "lace" plus two modules into the variation "wings"



13. transforming into the "bird"



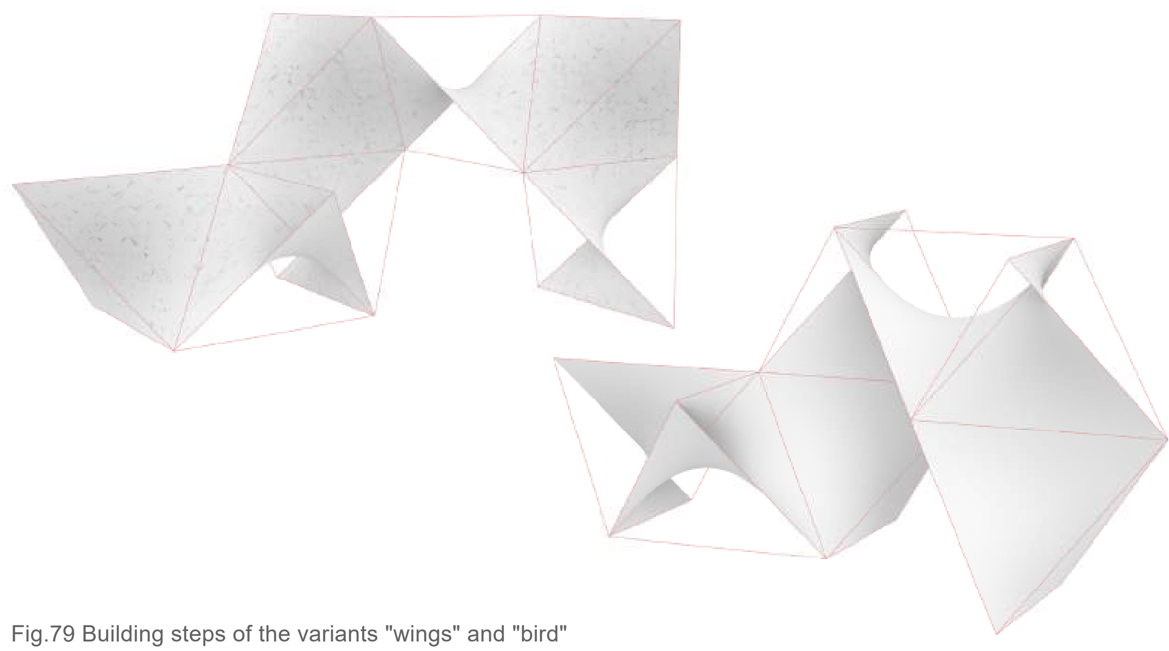


Fig.79 Building steps of the variants "wings" and "bird"

Grasshopper script

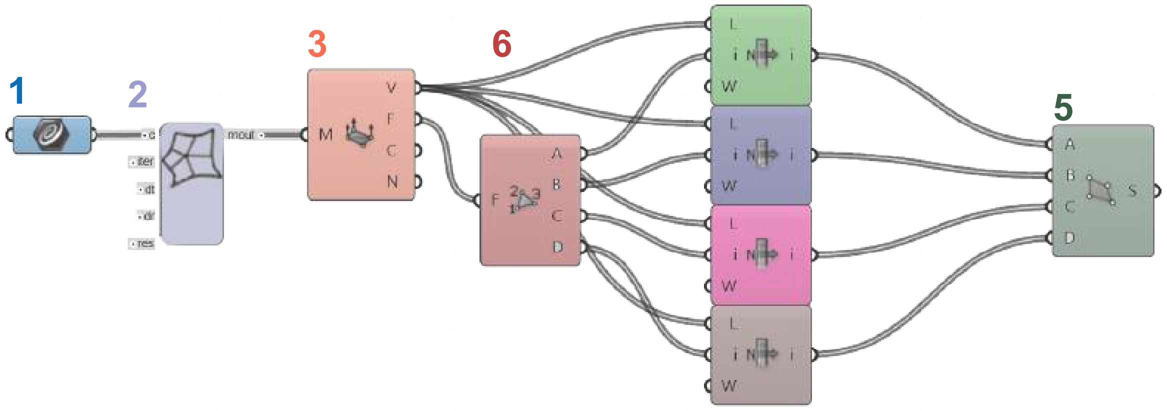


Fig.80 Building definition of a minimal surface from edges in Grasshopper

Four neighborhood edges of the tetrahedron are selected (black) and two other remain. We span a minimal surface from the edges using the Grasshopper plugin 'milpede'. The output is a mesh- a collection of vertices and polygons that define the shape of the object. Next step is to deconstruct the mesh into its component parts. The vertices are sorted in four lists. They are the four inputs of the component "4 point surface" (fig.80,fig.81).

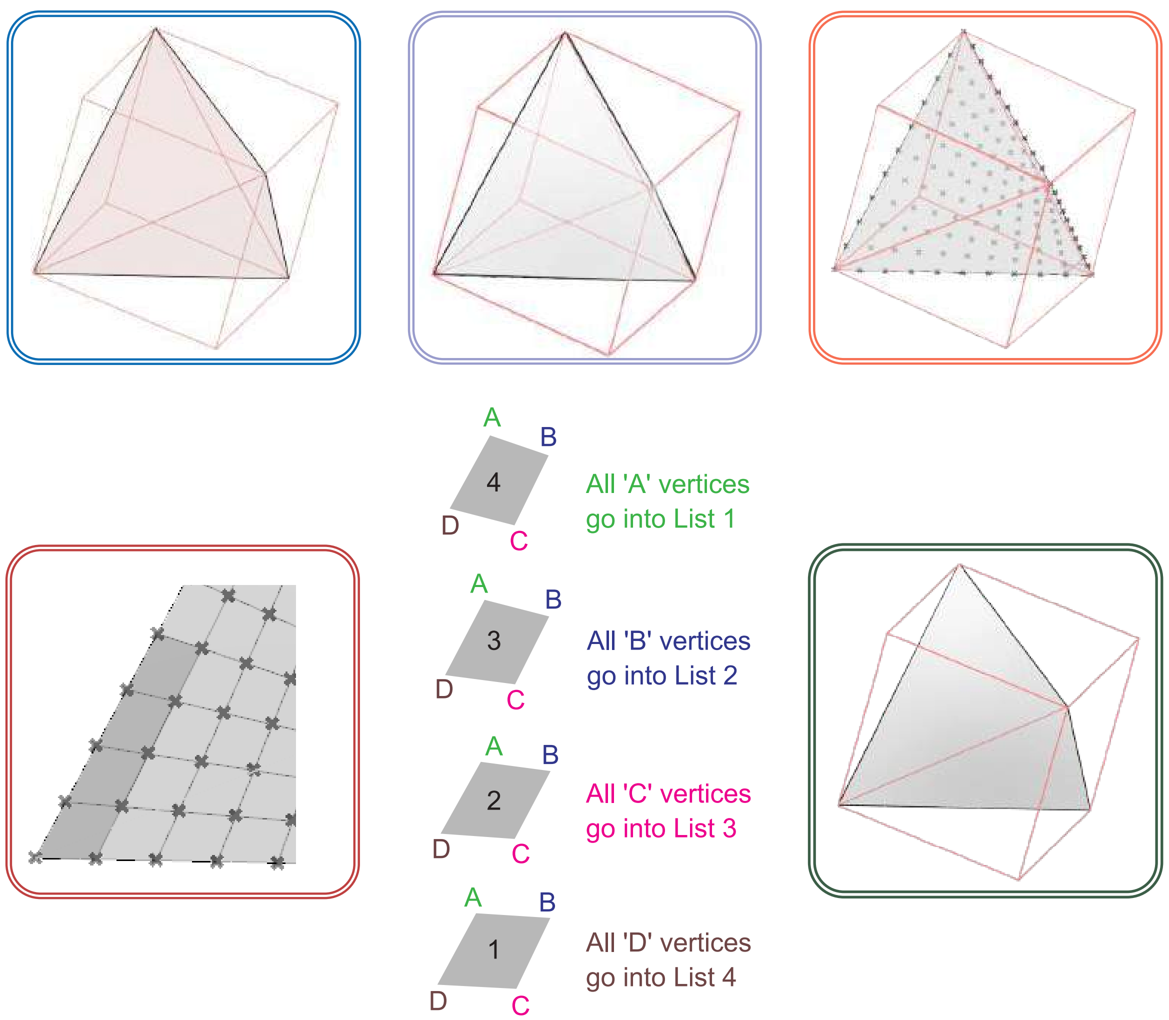


Fig.81 Explanational graphics for building of a minimal surface from edges in Grasshopper

Grasshopper legend

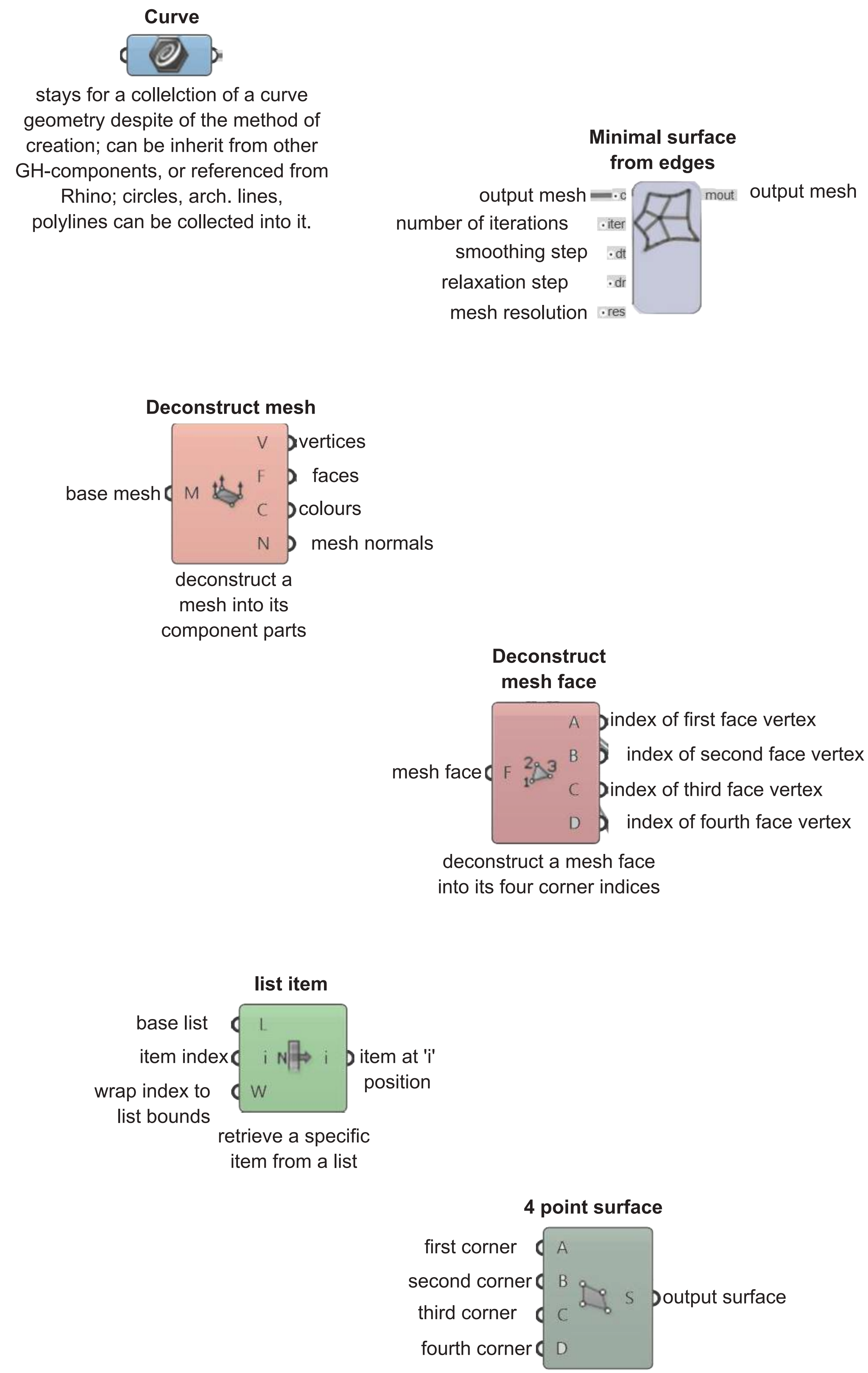


Fig.82 Grasshopper legend

3.5 Summary and Outlook

After exploring various spatial transformations with distinct building logics, the choice is clear—case number four presents a minimal surface that stands robustly upright. Unlike other configurations, this surface does not need to be divided into parts; instead, it remains embedded as a whole.

Its design is easy to replicate due to consistent lengths, angles, and inherited straight lines. The surface outlines align with four of the six tetrahedral edges, allowing for modular rearrangement. By rotating the initial module 180°, new variations can be created. The attachment system functions along the edges, where two modules meet, enabling flexible connections.

Despite this promising solution, curiosity remains about minimal surfaces in architecture and how the other three case studies behave in space. The following pages explore these concepts further, with visualized ideas and proposed functions inspired by the models

3.-3.6 chapter references:
[1] *Ristov S.*, The endless ribbon, https://takeinmind.com/the-endless-ribbon/ (08.2024)

[2] *WordPress,* Non-orientable, https://minimalsurfaces.blog/home/repository/non-orientable/ (11.2024)

[3] *Wikipedia,* Möbius strip, https://en.wikipedia.org/wiki/M%C3%B6bius_strip (01.2025)

A script reference for generating a Möbius strip (pages 46-47):
Piker, Generating a Möbius strip with different parameters, https://discourse.mcneel.com/t/generating-a-mobius-strip-with-different-parameters/138875/2 (11.2024)

[4] *BioArch Studio*, Enneper surface, https://bioarch.studio/enneper-surface-1 (12.2024)

[5] *Ferreol,* Enneper surface, https://mathcurve.com/surfaces.gb/enneper/enneper.shtml (12.2024)

A script reference for generating a Enneper surface (pages 52-53) :
Lopez Coronel, Enneper Modify, https://www.grasshopper3d.com/forum/topics/enneper-surface?id=2985220%3ATopic%3A1785824&page=2#comments (11.2024)

[6] *Brakke,* Triply Periodic Minimal Surfaces - Batwing Family, http://kenbrakke.com/evolver/examples/periodic/batwing.html (05.2024)

A drawing steps for a Batwing surface in Rhinoceros (page 63) :
Koshgarian, MINIMAL SURFACE_BATWING`S evolution, https://robertkoshgarian.smugmug.com/UCLAMasters/UCLA-WINTER-Qtr/Batwing/i-zzH4pPj/A (05.2024)

[8] *Weisstein*, Platonic Solid, From MathWorld--A Wolfram Web Resource https://mathworld.wolfram.com/PlatonicSolid.html (01.2025)

[7] *Cuemath*,Tetrahedron https://www.cuemath.com/geometry/tetrahedron/ (01.2025)

People for the renderings (pages 73-85):
3D HAMSTER, https://3dhamster.com/ (09.2024)

People for the grayscale graphics:
renderpeople,posed people, https://renderpeople.com/free-3d-people/?srsltid=AfmBOoqOVQ33fW3zPtdN7pdWyOH6dNyh6KaLLoxh90a-fFpDXDgpqMGG%20people (05.2024)

Fig.83 Visualisation of Mörbius strip, Enneper and the Batwing surface

3.6 Example designs

1 Möbius recess

The Möbius strip as an attraction to a restful place in nature. It is built out of aluminium plates. The first perspective is showing the version with separated from each other five modules arranged around a lake. The alumimium sheets are crossing each other. The geometry curvature is inviting visitors to lean on the minimal surface and to contemplate the surroundings. At the second perspective the stripes are rebuild so they can assemble the whole Möbius strip. This time the geometry twists, which can be compared with waves, surrounding the water. Guests can swim around the geometry, following its shape. Why not even climb on the structure and jump into the water?

Fig.84 Möbius recess as a water attraction place

Fig.85 Möbius recess as a water attraction place in nature

2 Enneper flower

A flower surrounded by flowers. Waves are transmitting energy. Here the surface waves that look like rose petals are transferring the energy of nature. In form of a verdures- vertically, horizontally - on a human eye level. The vertical gardens serve as screen plants, to look at or take from. It is not taking any space on the ground at all. Some parts could be covered and used as a screen- for an outdoor cinema or for a colourful light show on the surface. The second variation is a more enclosed one, also having one small, sheltered part.

Fig.86 Enneper flower around more flowers

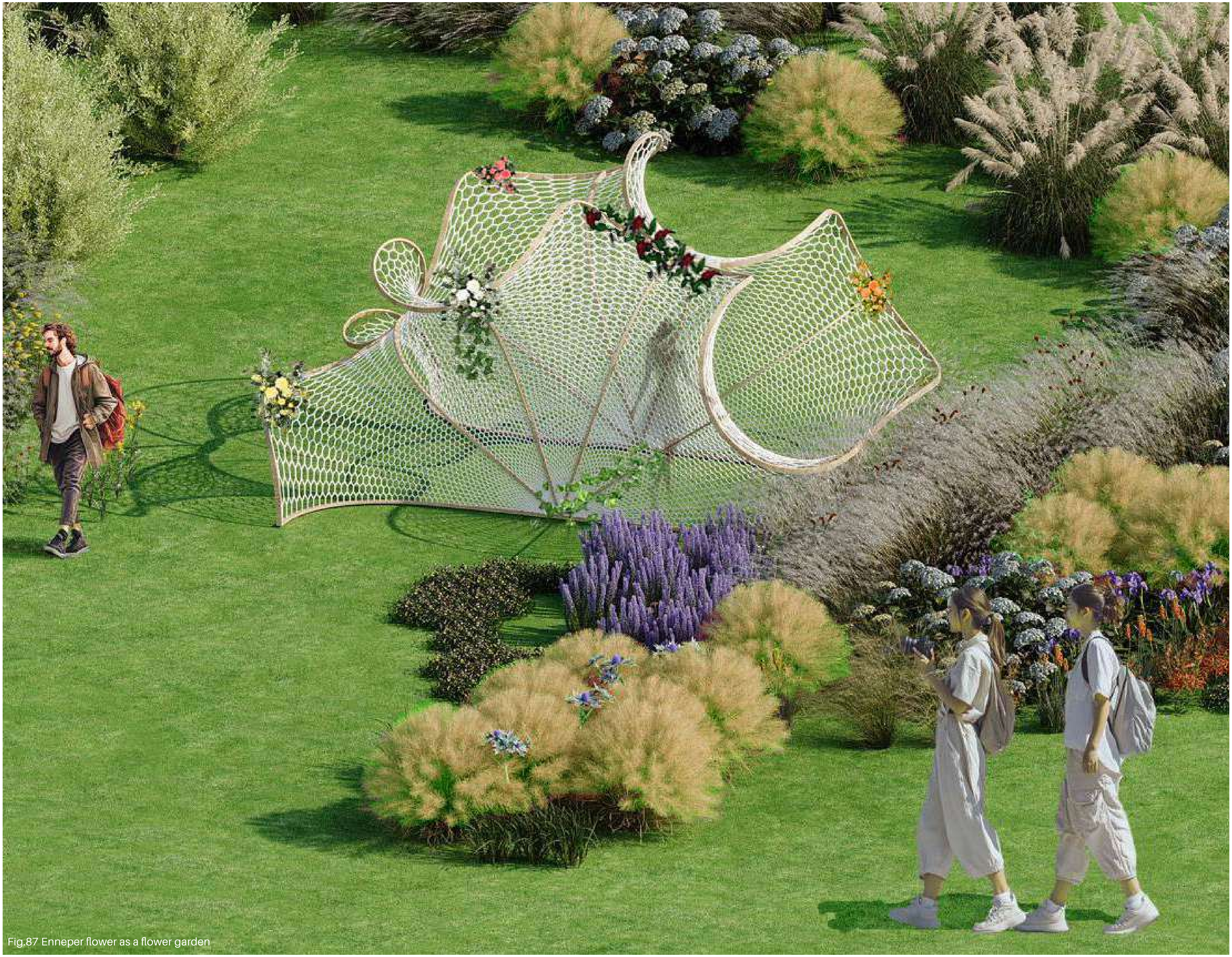


Fig.87 Enneper flower as a flower garden

3 Batwings play

A rendering depicting children experiencing the doubly curved surface through playing. An interesting and challenging geometry for kids to climb, slide and hop around. Possible materials are wood or fabrics. There are six modules of the first variation, followed by 12 at the second. Of course, swings, hanging ropes or membranes could be fastened to the facility. Minimal surface as a space, creating a harmonious, aesthetical and durable area for curious minds

Fig.88 Batwings play as a children playground (six modules)

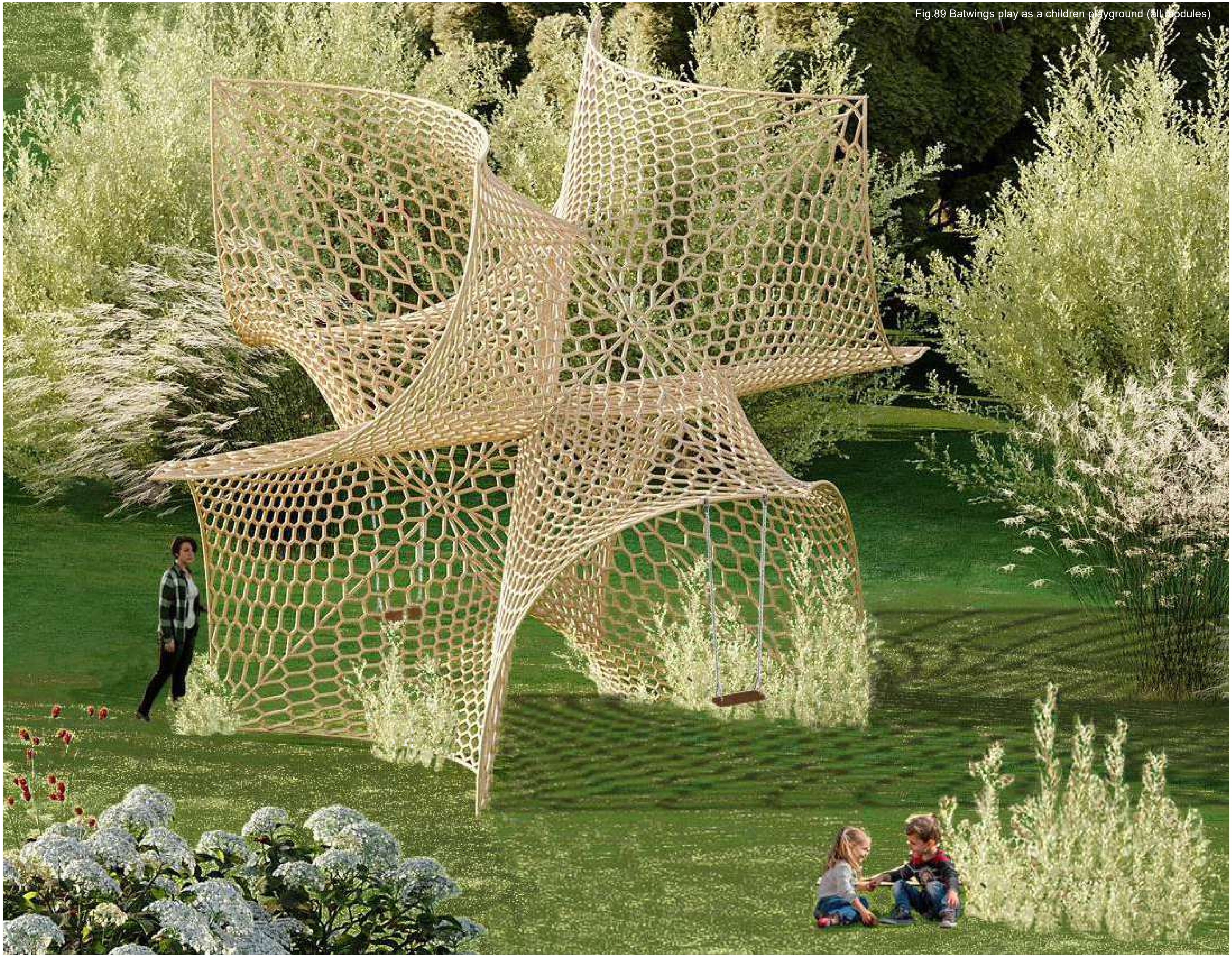


Fig.89 Batwings play as a children playground (all modules)

4. Building process

The following pages are presenting a chronologically ordered list of the events that happened, after the decision of which module to build was made. The main rules followed to solve the probems were to create clean, simple, beguiling details, that are allowing the straightforward modularity of the structure. We start with the equipment used while building:

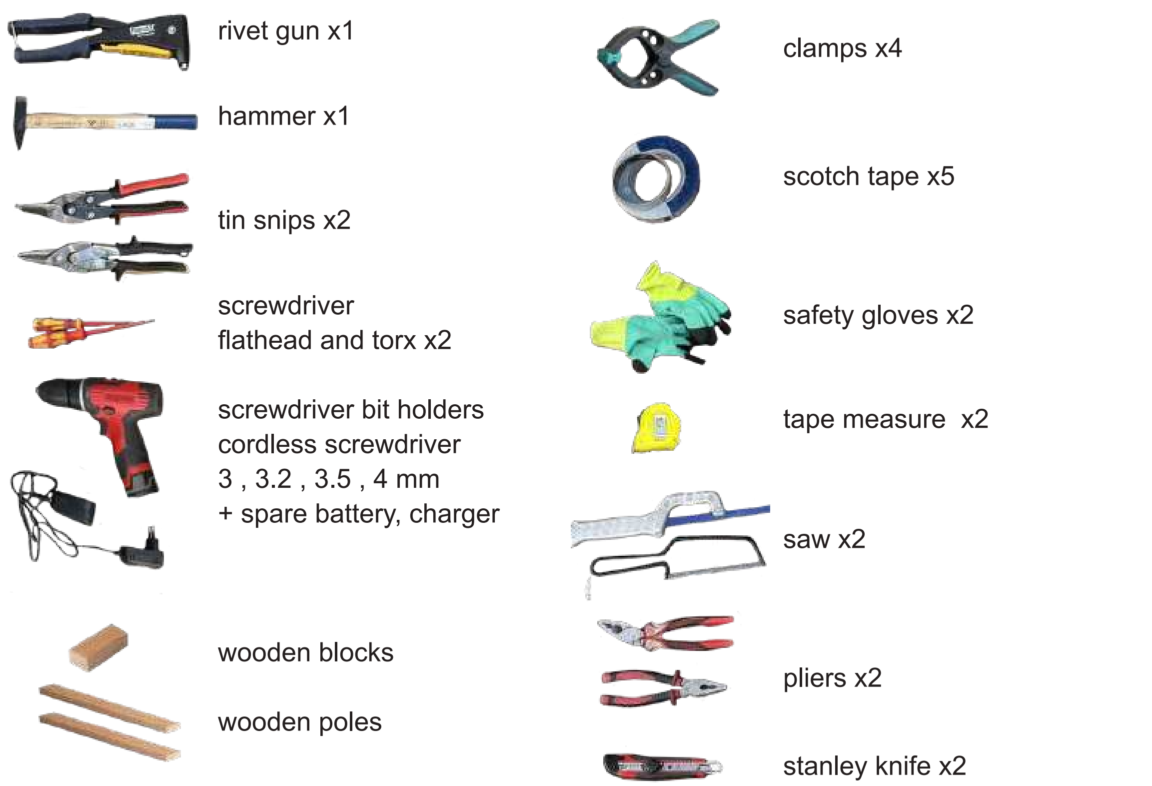


Fig.90 Tools used for building

4.1 Logistic

Fig.91 Tools used for building The ‘raw’ material, four packages each with 22 blinds

1 Picking up

The material was picked up from *Materialnomaden (arvestMAP Genossenschaft zur Vermittlung von ReUse-Bauteilen eG; 2017)*. There was no requirement for the tilting-mechanism, so it was removed, partly because of its weight. The length of the blinds (1.1m) allows to encase one set of twenty two pieces in two plastic bags of a 60-litre volume (fig.91 right). Four such sets fit effortlessly in a normal car boot. Conveying by carrying them, using a trolley and taking the bus or the suburban railway is also possible.

Tools: flathead screwdriver

Fig.92 Taking off the tilting mechanism, unriveting the brackets

2 Cleaning

Following the project concept for conserving the natural resources as much as possible, blinds were firstly cleaned up under the rain or by hand. Subsequently a high pressure water machine did the final touch of washing the raw material, reclaimed froma building scheduled for demolition. The final preparation step was to get rid of the brackets and the rubber strip.

Tools: gloves, drill, maker knife

Fig.93 Cleaning with rain and afterwards with high-pressure water

4.2 Module frames

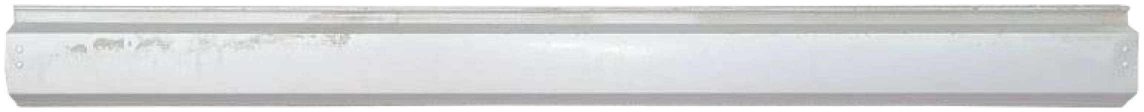


Fig.94 One aluminium blind - frontside

After computing the minimal surface, it is time to orient the material along the module edges. The aluminium sunblinds used for the building of ve.sh come into use. With length of 1100 mm and thickness of 0.8 mm they are easy to work with. Most importantly reused building materials are used. The aluminum blind slats from the demolition of the former OMV building in Vienna were repurposed. By reusing these slats, a sustainable design solution is created, and a resource-efficient construction approach is being investigated.

Fig.95 One aluminium blind - backside

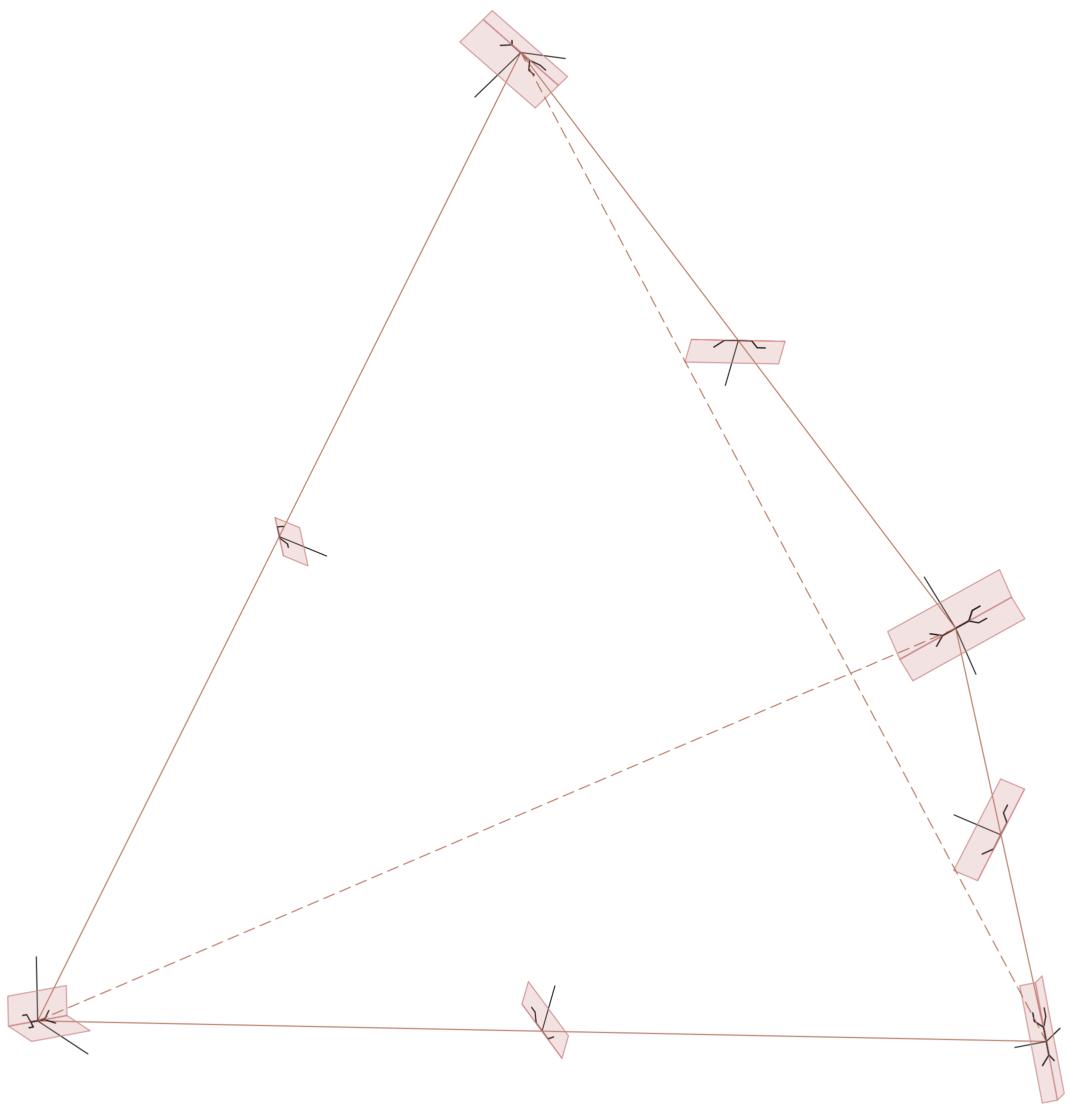


Fig.96 Orientation of the blind profile at selected points

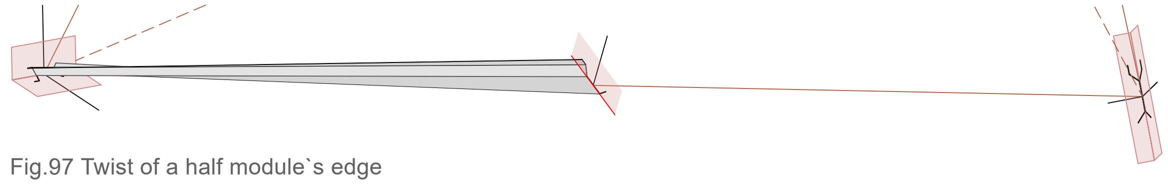


Fig.97 Twist of a half module`s edge

To orient the non-symmetrical blind profile we compute the binormal vector and the normal vector at each vertex and at the middle of the edges. Afterwards we orient the profile, so that the binormal vector at the mentioned points coincide with the z-axis of the profile. We do the same alignment with the normal vectors.

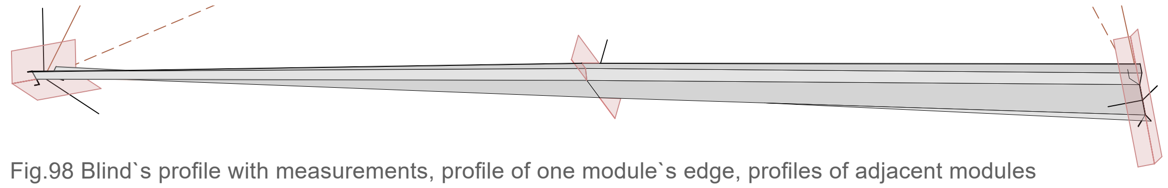


Fig.98 Blind`s profile with measurements, profile of one module`s edge, profiles of adjacent modules

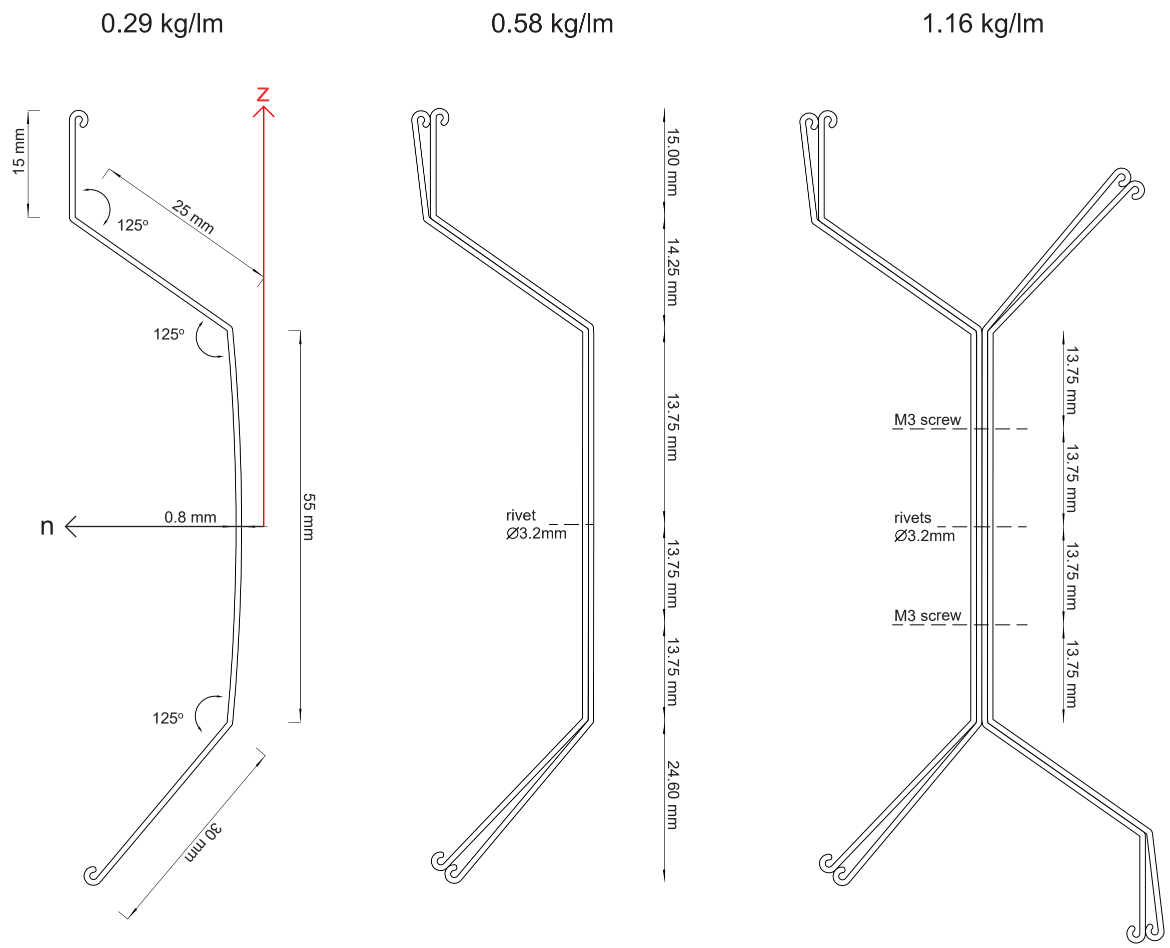


Fig.99 Blind`s profile with measurements, profile of one module`s edge, profiles of adjacent modules

8 inner blinds:
4 folded at the corners +
4 straight in the middle

For building the correct twisting of the edge the corner dimensions (53cm and 56cm) should be built exact !

5.07 kg/frame
16 blinds of
1.1m length

8 outer straight blinds

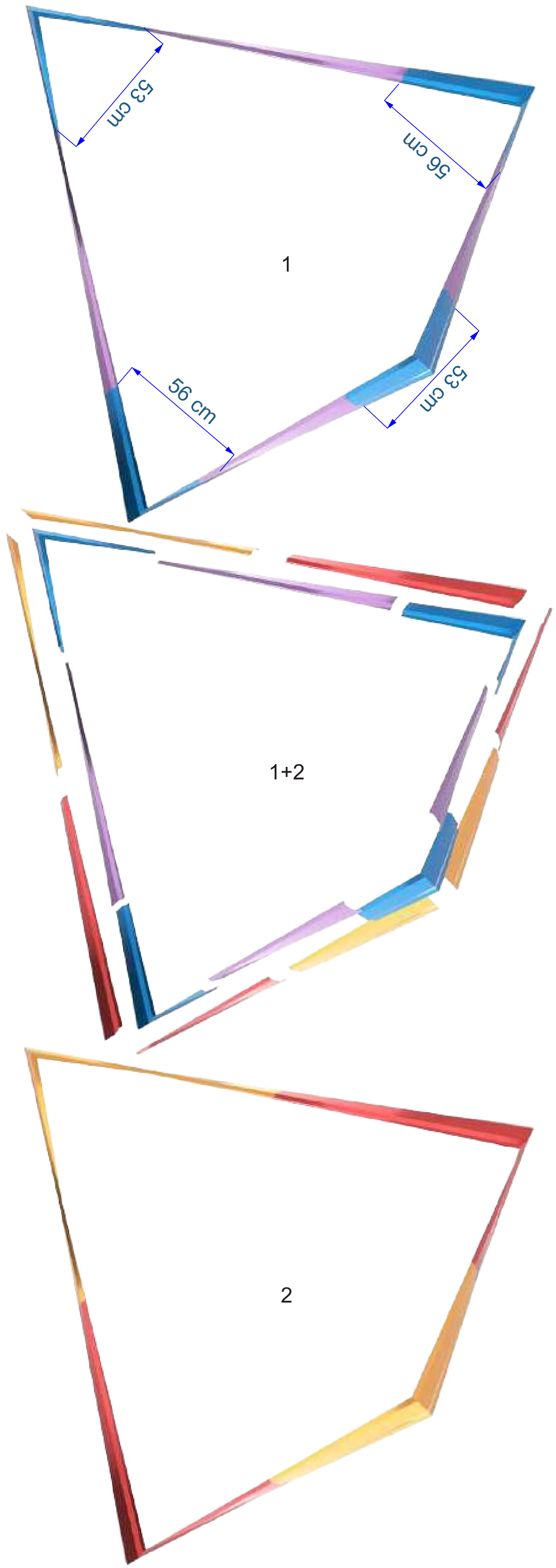


Fig.100 Up: inner eight blinds, bottom: outer eight blinds, middle: all blinds in two layers

We build one module frame in the following way: the size of one module is chosen in relation to the blind size and to the inner pavilion's height of the roofed variation 'Bird', namely 2.35m. The same thickness and strength of each edge, along with minimal possible blind corrections were preferred. These criteria resulted in an edge length of 2.20 m - the length of two blinds. Two layers of material, four folded in the middle blinds and twenty unchanged blinds are the constitutive elements of one frame. The inner pieces overlap the outer ones until the middle of each element and vice versa (see fig.100). Four 1.10m long blinds were bended in the middle in 60° degree in order to build the corners. That means that one angle side is 55 cm long.

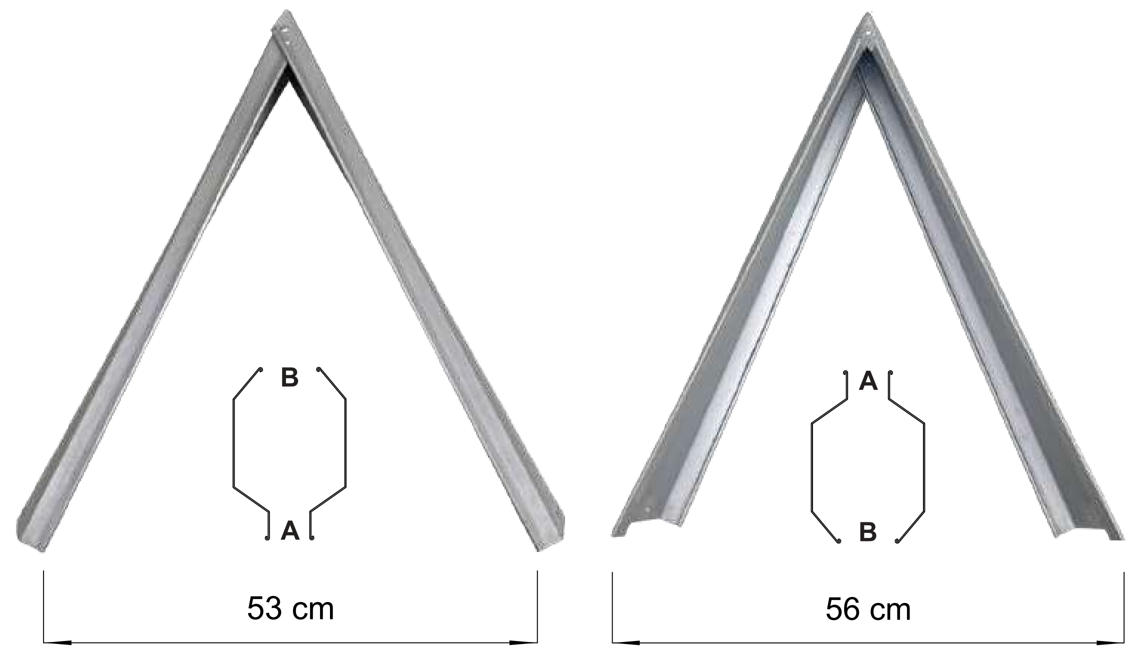


Fig.101 Down and upper part of a module`s corner

According to the Rhino drawing, the distance between the upper profile`s end points (A) of the folded blind and its bottom ones (B) is respectively 53cm and 56cm. This applies only for two opposite corners of a module. The other two were built the other way around - the distance between the end points (B) of the folded blind and the end points (A) is respectively 53cm and 56cm. In this way the frame`s twisting is correct. The span is not the same (3cm difference), because of the non-symmetric beam profile. (see fig.101)
The other whole-size blinds are riveted to the folded one in the middle. The aluminium piece automatically follows the twist without any resistance at all. The middle inside blind is the one that connects the outer whole-size blinds together. It takes place between the two angles.(see fig.100 middle)

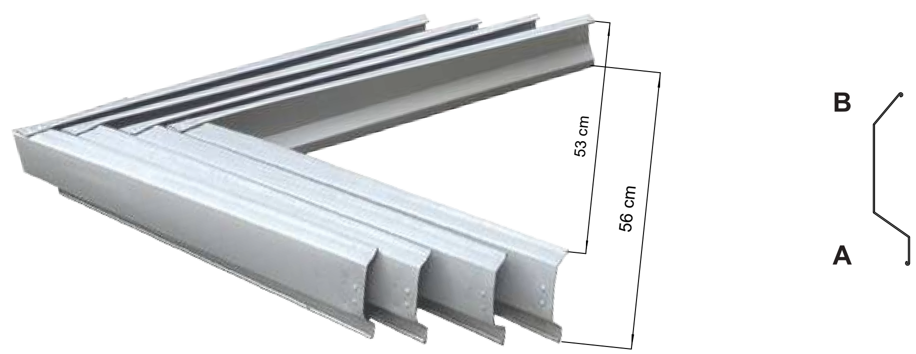


Fig.102 Four corners twisted in the same way, prepared for two modules; B-A orientation of the angles

1.the four corners with the bounding tetrahedron

2.the outer eight blinds

3.the middle four blinds

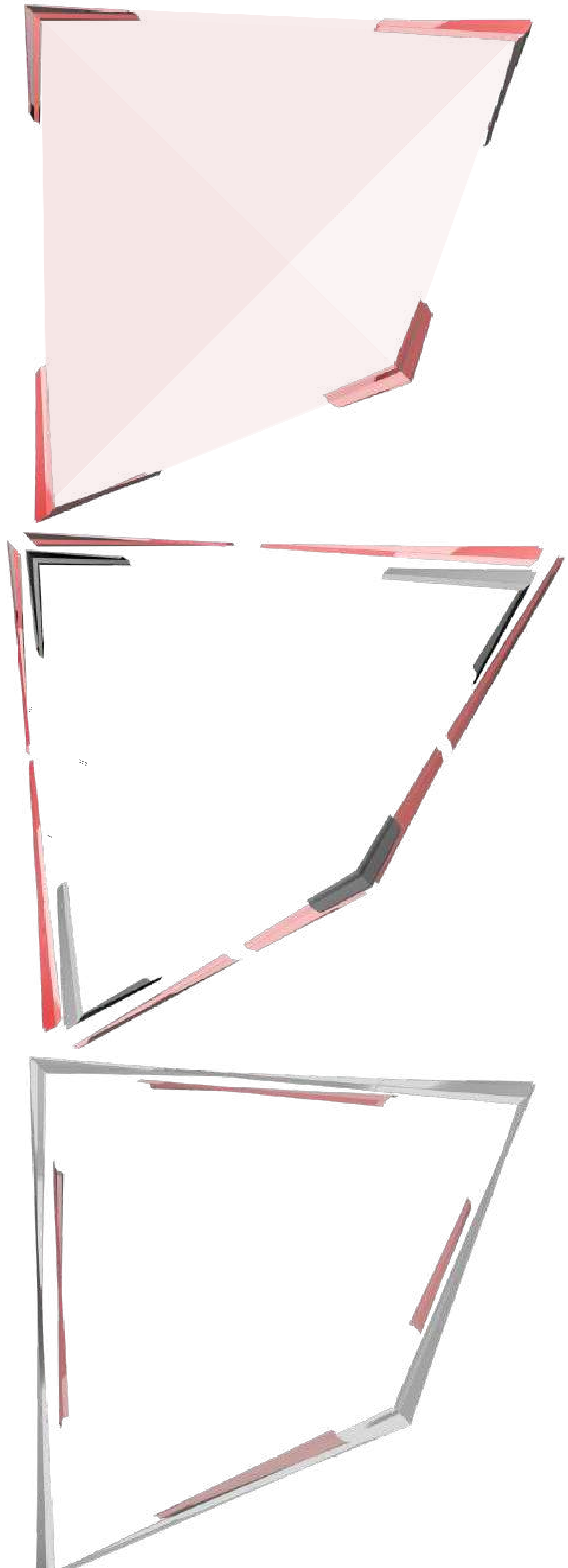


Fig.103 The sequence of attaching the blinds together

As we can see one module is not stable on its own and two different bracing systems were designed.

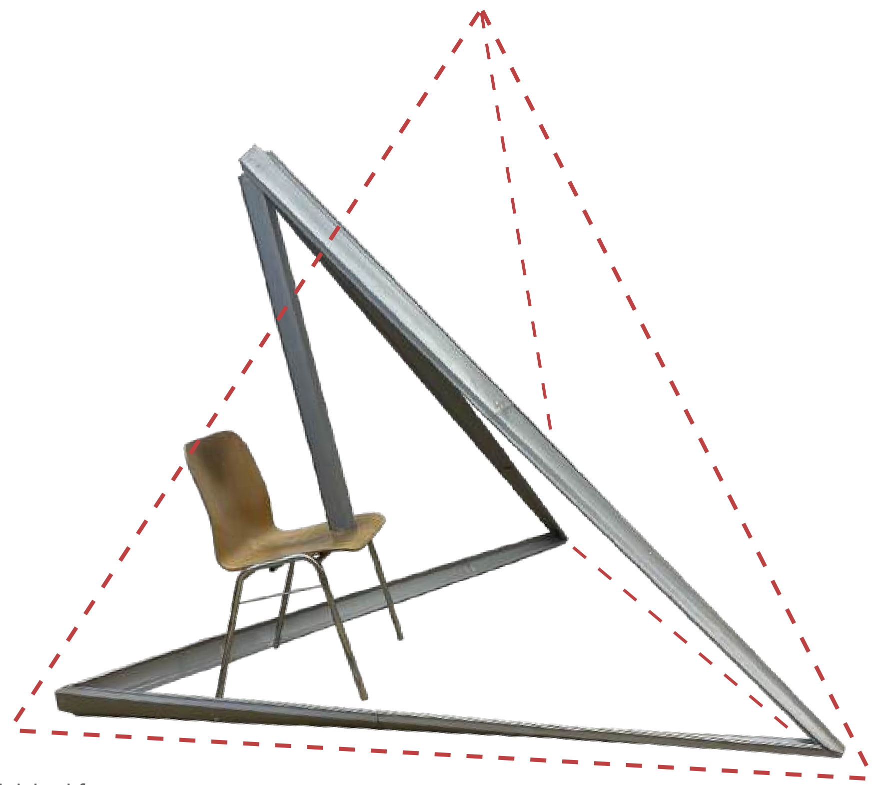


Fig.104 A finished frame

Materials for one module: 16 aluminium blinds, rivets: 3x4 corners DM4mm + 4x8 outer blinds DM3.2mm +4x4 middle inner blinds DM3.2mm = 52 rivets; Tools: rivet gun, hammer before riveting the angles, tin snips, screwdriver, wooden blocks, wooden poles for the tetrahedrons - scaffolding, clamps, tape measure, pliers

Fig.105 A finished frame with supporting rods forming a tetrahedron.

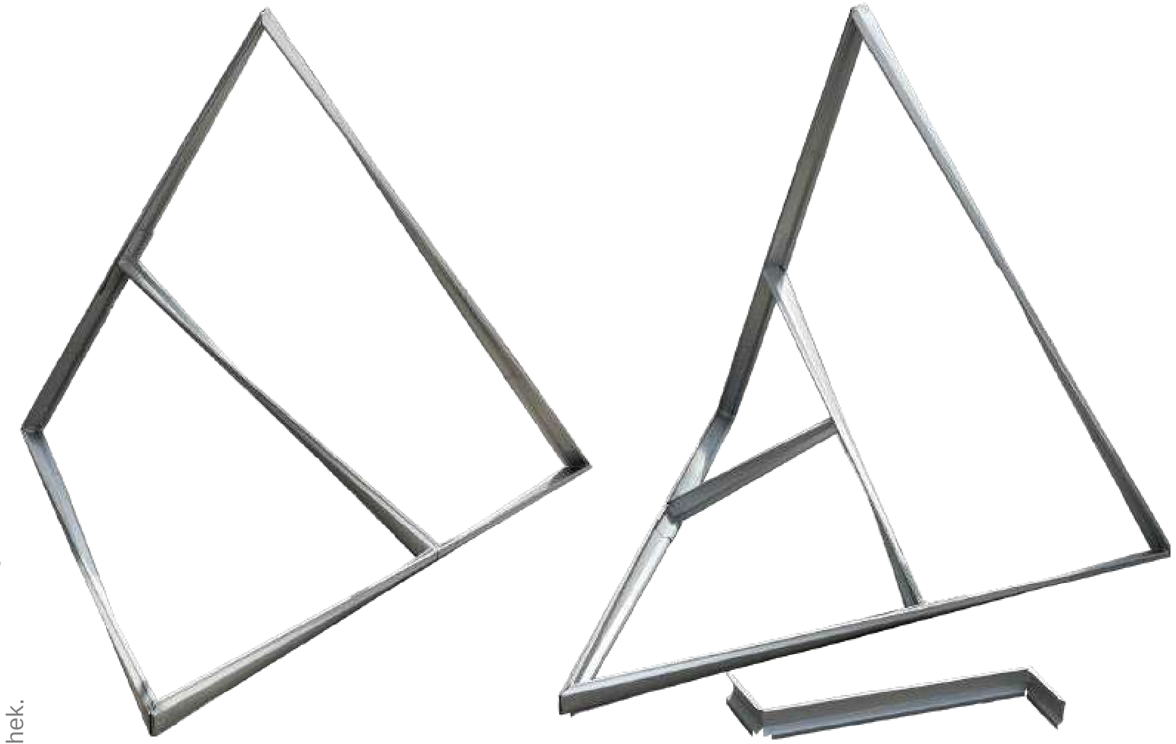


Fig.106 Left: half a bracing; right: three quarters

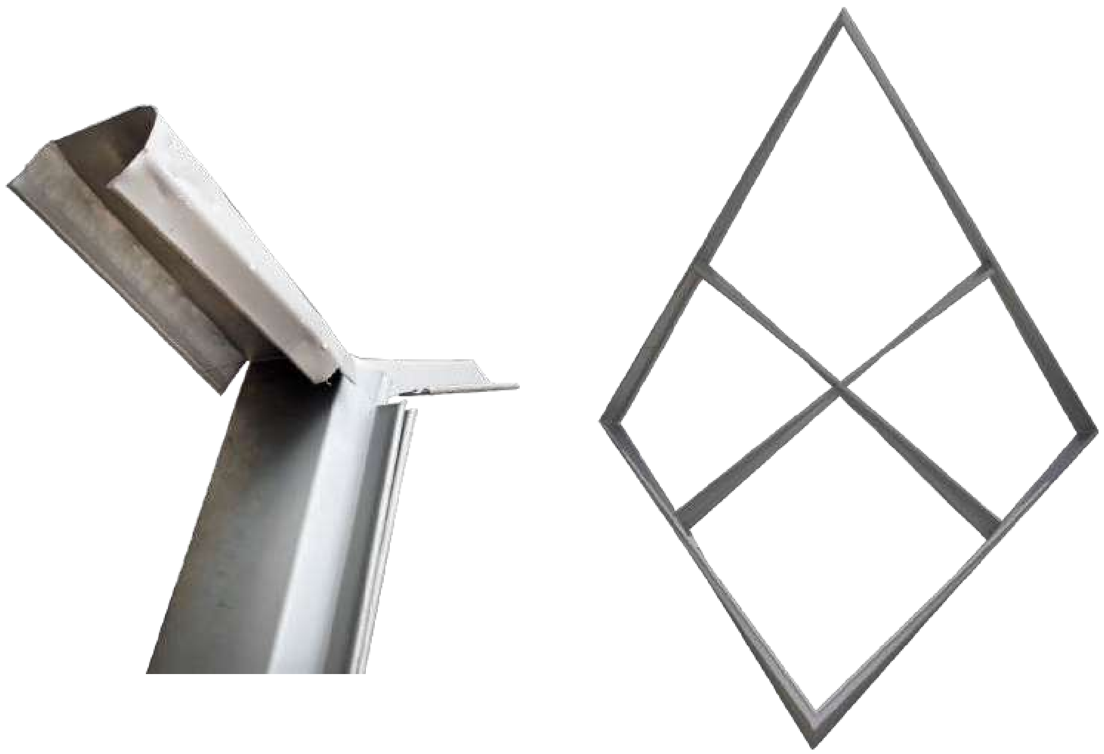


Fig.107 Reshaped end of a bracing element; right: built experiment.

4.3 Two experimental models

The one that connects the middle points of each edge is following the asymptotic curves that are describing the surface. The blinds are oriented upright on the underlying surface. This bracing type was more difficult to be built because of the blind's asymmetric profile. (see fig.107) To carry it through a door with a width of 90cm was manageable if there was at least 2.5 m^2 free space before and after the opening so that one can rotate the module while passing it through.

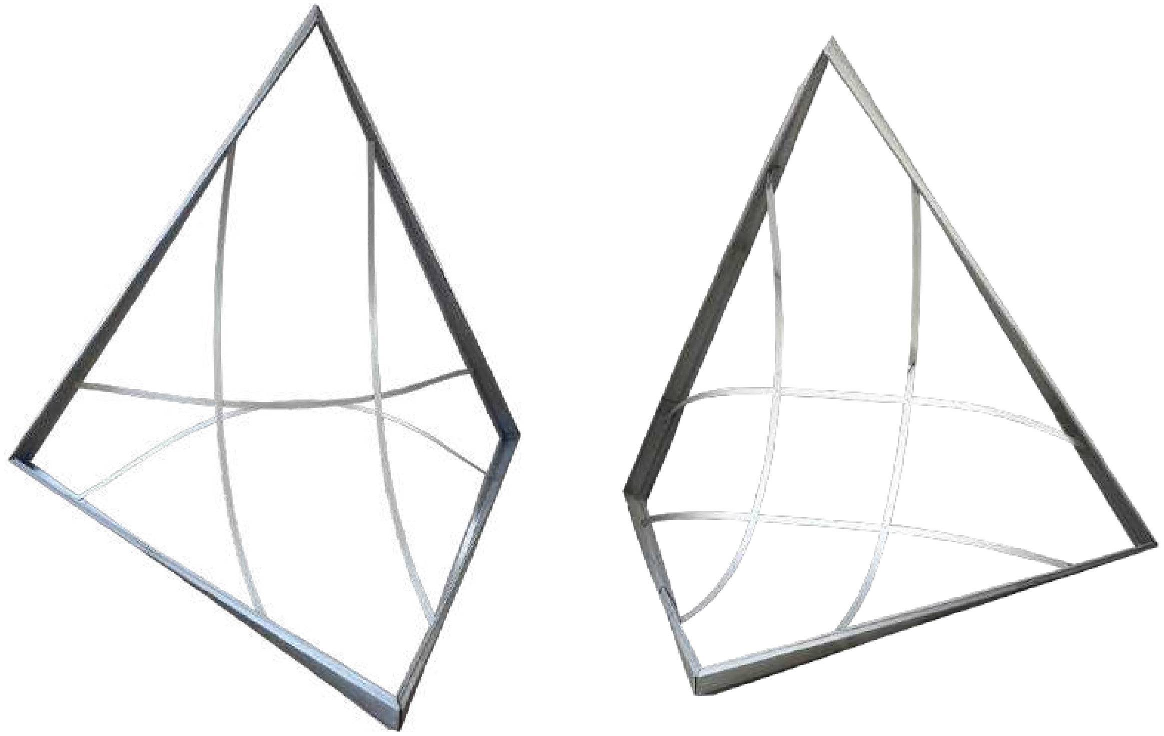


Fig.108 Inner stripes riveted for the frame. Attaching them together was not possible; right: bracing pattern successfully fixed to the frame

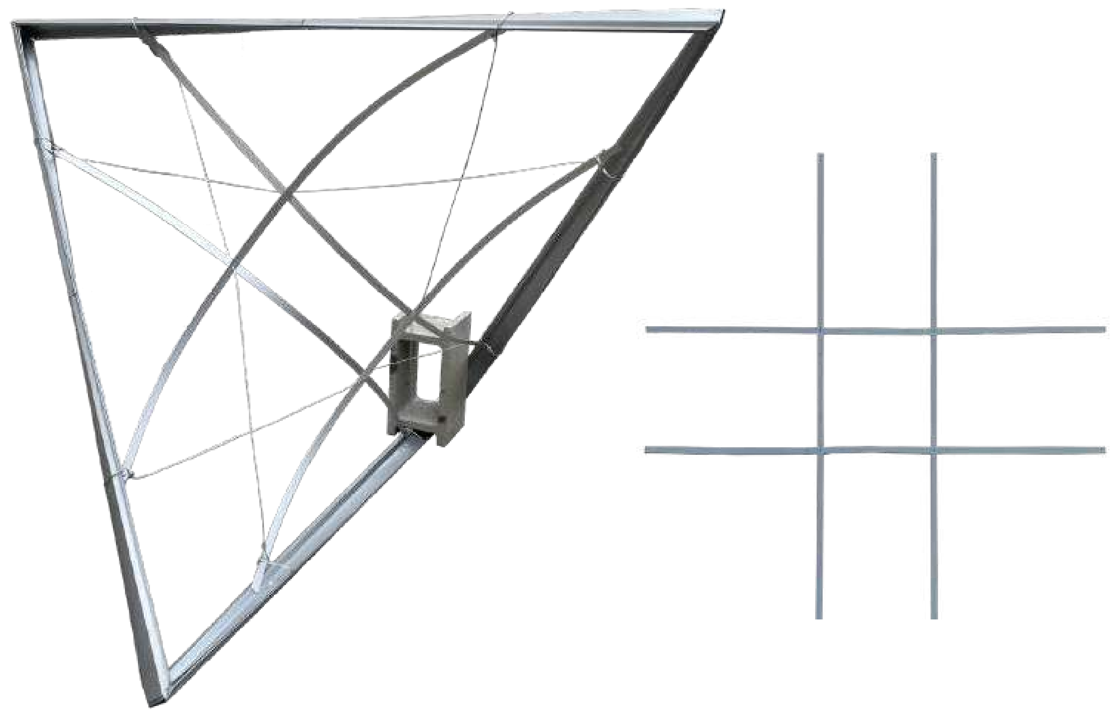


Fig.109 Left: First tryout with a steel cable and a footing; right: the aluminium stripes screwed together on the ground

While doing the second bracing-system experiment, the search was for something more elegant and easier. The pattern was chosen because of the need to connect 2x2 edges as well as the stripe`s length. First the inner pattern was screwed together on the ground then its ends are connected to two points of the surface edges (see fig.109). With 2 500 mm length, 25 mm width and 2 mm thickness, the aluminium belts were not steadily holding the sunblinds. The module itself was not standing in the right position, meaning it was not possible to fit it in a regular tetrahedron. If the inner squares are triangulated with tension-elements (e.g. membrane) it would do so.

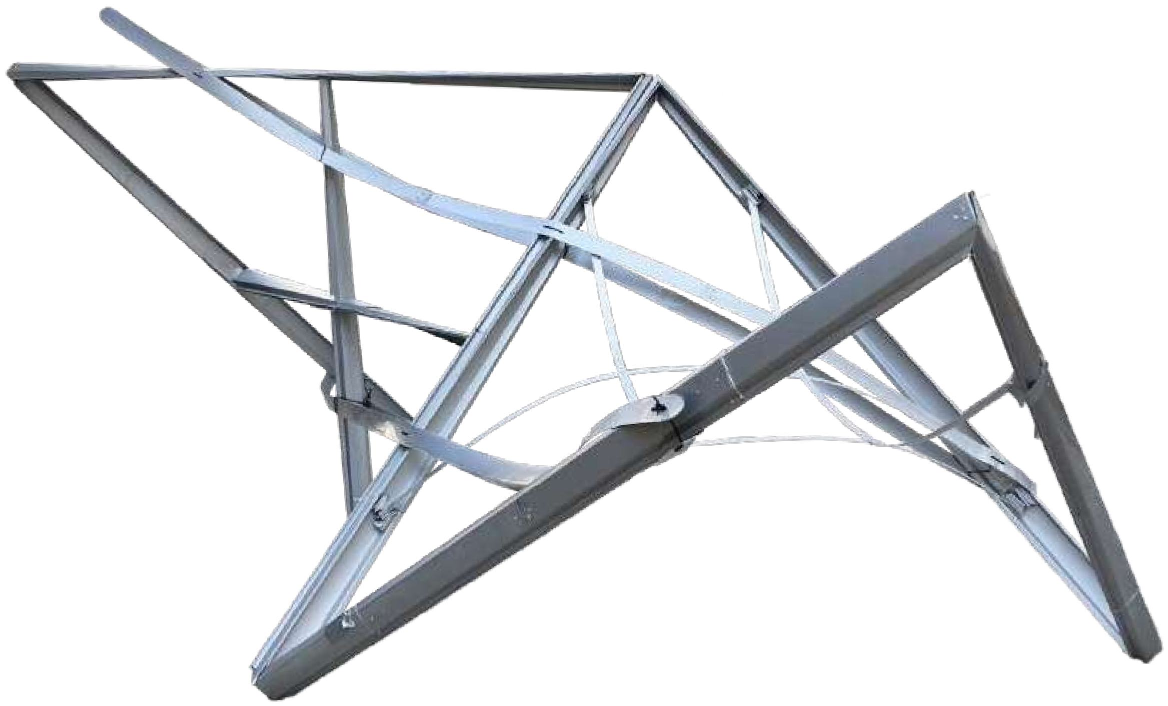


Fig.110 The two experimental models connected through aluminium stripes together

The connection between the modules can be seen as a continuation of both experiments of bracing geometries. A structure which proceeds through all modules, that is removable, but arrangeable, that can be stiff in one direction, but flexible in others was to be designed and dimensioned. The support at the middle edge points plus two more looked promising. The profile design of the second built experiment was simple, clear and beautifully following the minimal surface typology. At Fig. 110 aluminium stripes flow from one patch to another, connect the modules to each other and stabilizing the structure. The aluminium stripes are just tied to the modules. They are the long middle part of the blind's profile - cut out from the folded parts. 1.1m long piece plus 1.1m long piece overlap for 5 cm and lengthen together via two screws M3

.

Tools: rivet gun, hammer, tin snips, drill, screwdriver flathead and torx, wooden blocks, clamps, scotch tape, safety gloves, tape measure, pliers

The connection and the exact dimensions of everything in the module was first discussed with the tutors, then proved in Karamba. The Finite element models were done similarly to the programmed structural models of ve.sh in SS2023. The three tasks that were done in Karamba were: 1. Examination of other materials as possible connections; 2. Positions at the module`s edges; 3.Bracing layout.

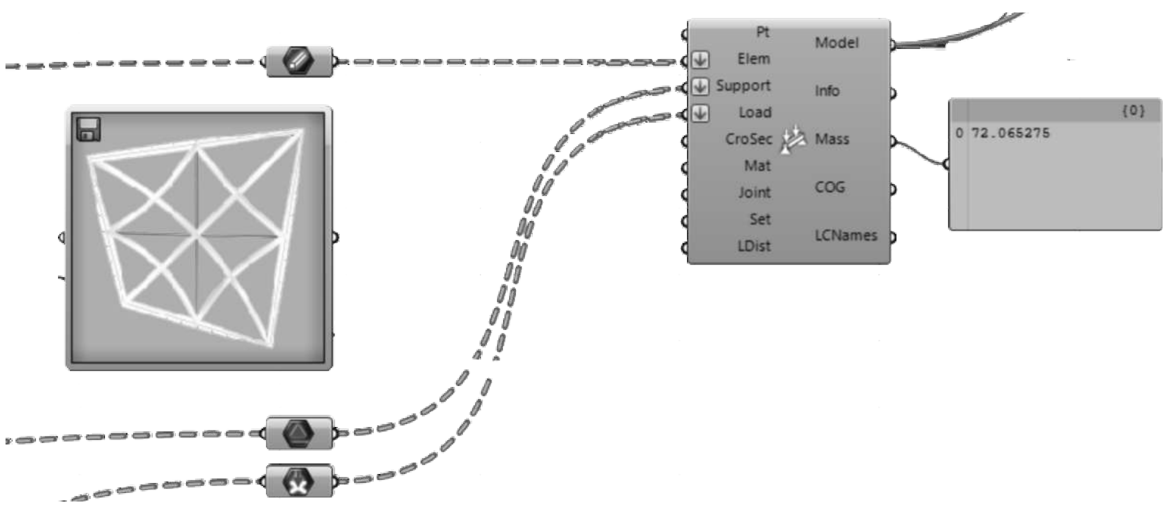


Fig.111 Selected bracing system and mass of the model in Karamba

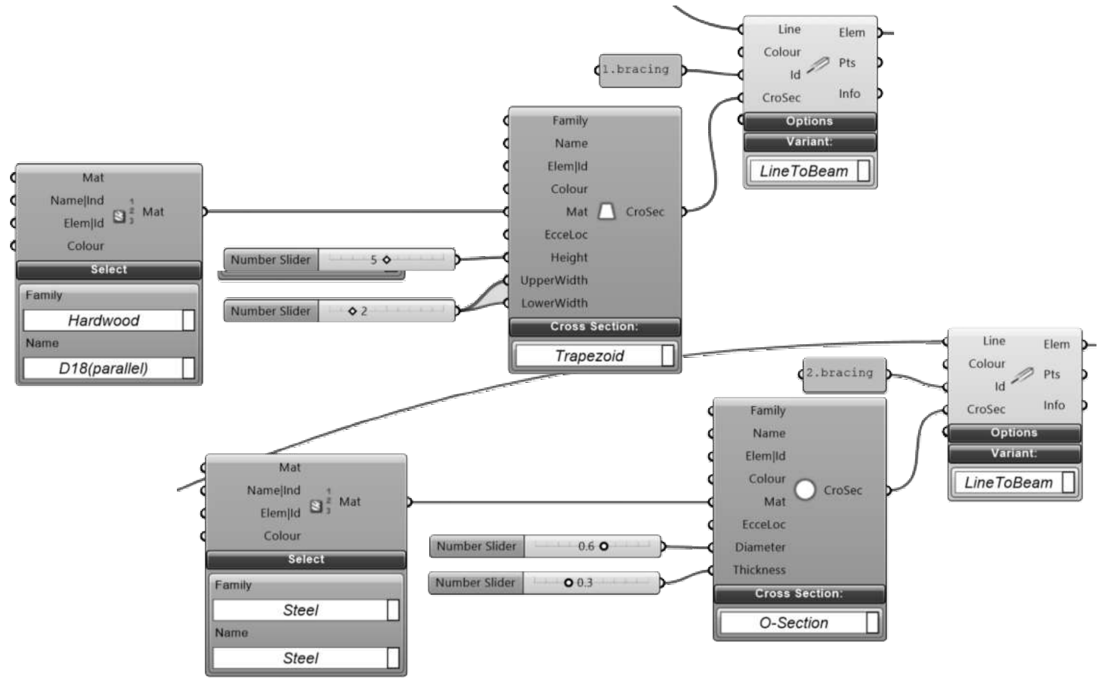


Fig.113 Computing the bracing in Karamba

4.4 New seven frames

To build the new seven frames took fourteen hours of work for two people. The most time consuming step was to build the angles. Marking where to connect the blinds and attaching them together followed;

Tools: rivet gun, hammer before riveting the angles,tin snips, screwdriver, wooden blocks, clamps, scotch tape, safety gloves, tape measure, pliers

Fig.112 All nine frames

4.5 Foundation

The form of the inner angle`s space is taken. Thus, the foundation design follows the natural form of the pavilion and fits slightly in one module. At the corners of one module we had singularity points. The side walls of the footing are again made out of bent in the middle blinds. So, at the ground, the angles have three layers of aluminium. The upper and bottom triangular sides are from four peaces of flattened blinds, cut out in triangular and trapezoid shapes. The aluminium profile is flattened very easily using pliers. They are placed tightly next to each other and riveted to the side foundation`s walls. Inside of the shape could be placed weights such as gravel, stones and bricks ect. The pavilion`s base follows the form of minimal surfaces, staying true to the materiality and idea of the project. They can change positions- by unscrewing and moving them from one module to another when reassembling the pavilion`s variations.

Improvement ideas: Using screws instead of rivets, levels of the triangular face`s position providing the places for fixing the elements together with long holes.

Materials: around four aluminium blinds, rivets DM3.2mm; for the filling: crushed stones, gravel; Tools: rivet gun, tin snips, hammer, drill, wooden blocks, scotch tape, tape measure, pliers;

Fig.114 Foundation positioned in a module ant its inner room

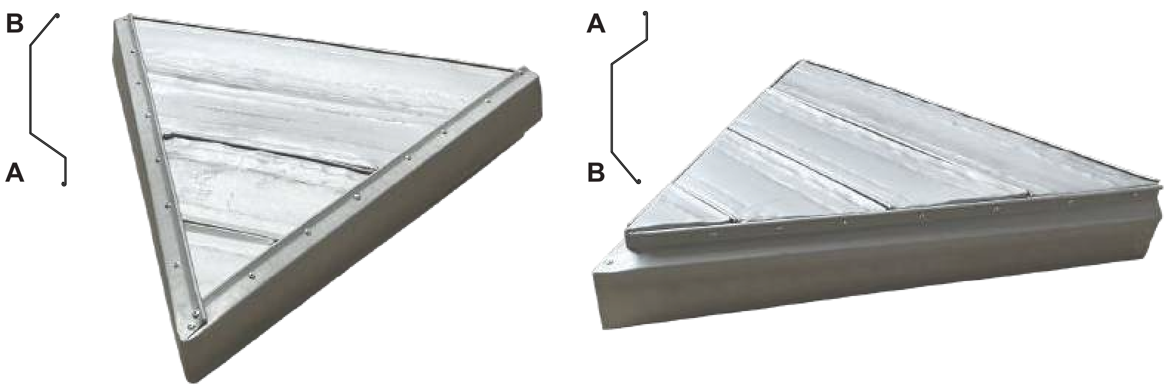


Fig.115 Both sides of a foundation

4.6 Membrane

A membrane is designed to cover and protect from the weather. Modules can be covered with cotton fabric, shielding the space from direct sunlight. The first tryout was done with rectangular textiles fastened between the inner and outer layers of blinds at the module edges. Of course, there was a gap between the actual surface and the improvised membrane at the center area of the geometry. The tailor from "Mode Atelier Mass- und Änderungsschneiderei" at Berggasse 27, 1090 Wien explained how to create a fabric piece that follows the desired curved typology. The whole surface has to be divided in segments (fig.116). Their exact measurements plus 2 cm offset at every piece`s border have to be pinned to the fabric and cut out with scissors. Last step is to stitch them together using a sewing machine. Firstly, a model in 1:10 was made.

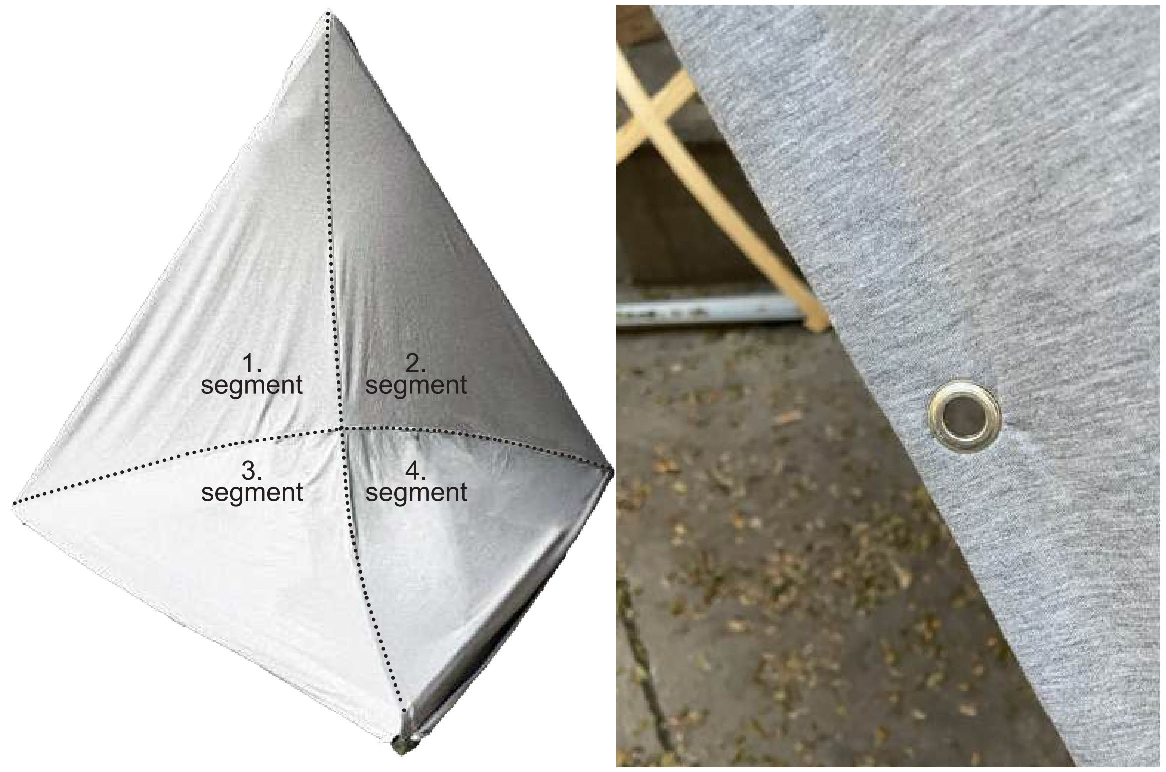


Fig.116 The membrane on the second bracing experiment

Improvement ideas: Perhaps next time when designing a membrane, I would read the chemical definition of minimal surfaces and rethink how to use the formula from page 24 in order to improve the design.

Materials: fabric triangular pieces: 2.20 m hypotenuse, 0.78 height, 1.39 m cathetus, thread, eyelets DM10x19mm, rope; Tools: sewing kit, hammer, setter, anvil, scissors; Improvement idea: a membrane, that not only serves as a weather protection, but also interacts with the supporting structure, could be approached. The minimal surfaces can be materialized with thin membranes carrying only tension. They can span large distances while requiring little material.

Fig.117 The membrane and the 'twist' variation of the pavilion.

4.7 Building the bracing

The modules 3x3x1 were assembled together and prepared for the building of the biggest variation ‘bird‘ with seven modules. The steel cables length span is 1.55m (single module on the ground), two times 3.10m (three modules on the ground.....) and 4.65m (for the three upper modules in a line). Scotch Aluminium Tape fixed the edges of individual modules so they can lay (friction)-tight next to each other without fixing them entirely. One module is not stable by itself because of its dimensions (2.20m x 2.20m x 2.20m x 2.20m). Supporting objects such as chairs or tables came into use. Consequently, the steel cables were not stretched. Around 30% of the wood stripes were connected to the modules. The result was promising, the modules started to straighten themselves up.

Tools: rivet gun, tin snips, hammer, drill, flathead and torque screwdriver, wooden blocks, scotch tape, tape measure, pliers

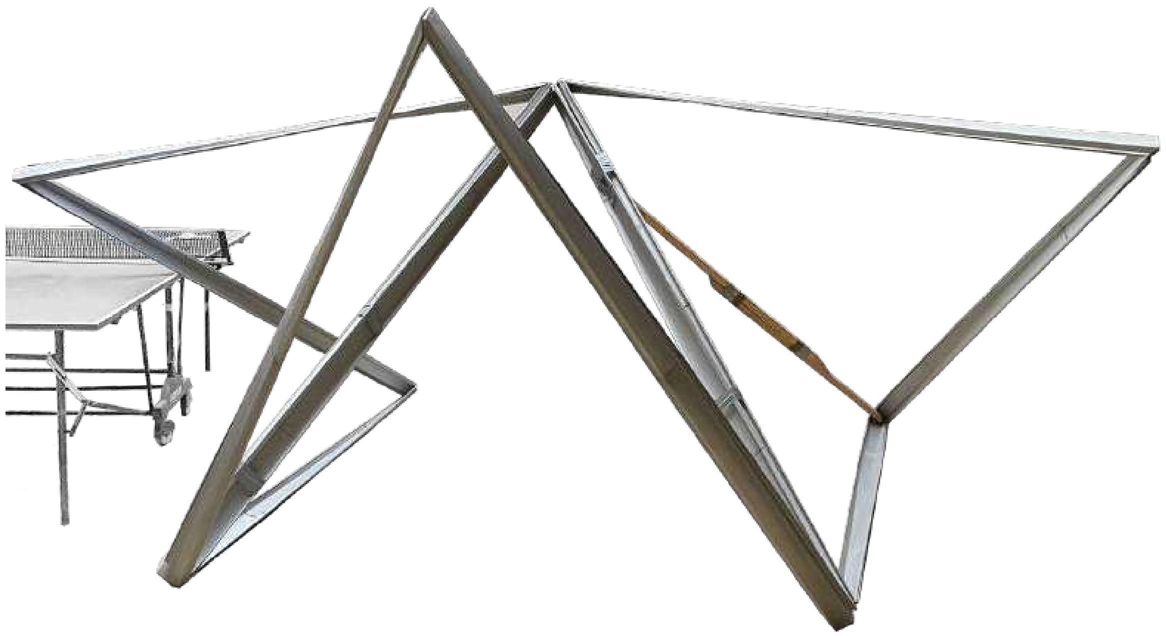


Fig.118 Three modules connected together only with scotch tape (leaning on a chairs)

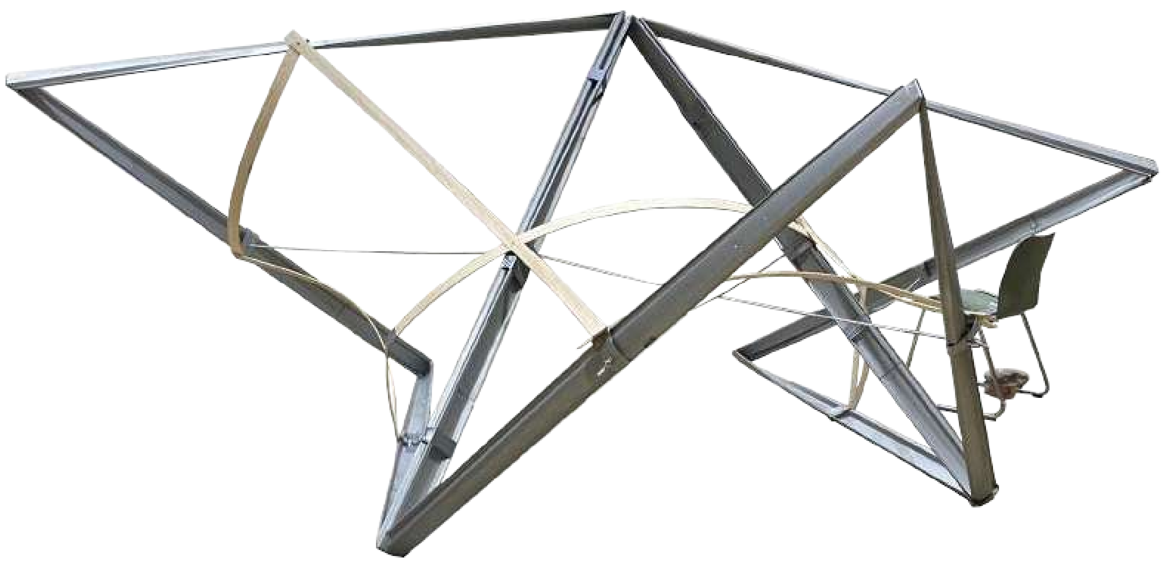


Fig.119 Three modules partially connected together via wood stripes

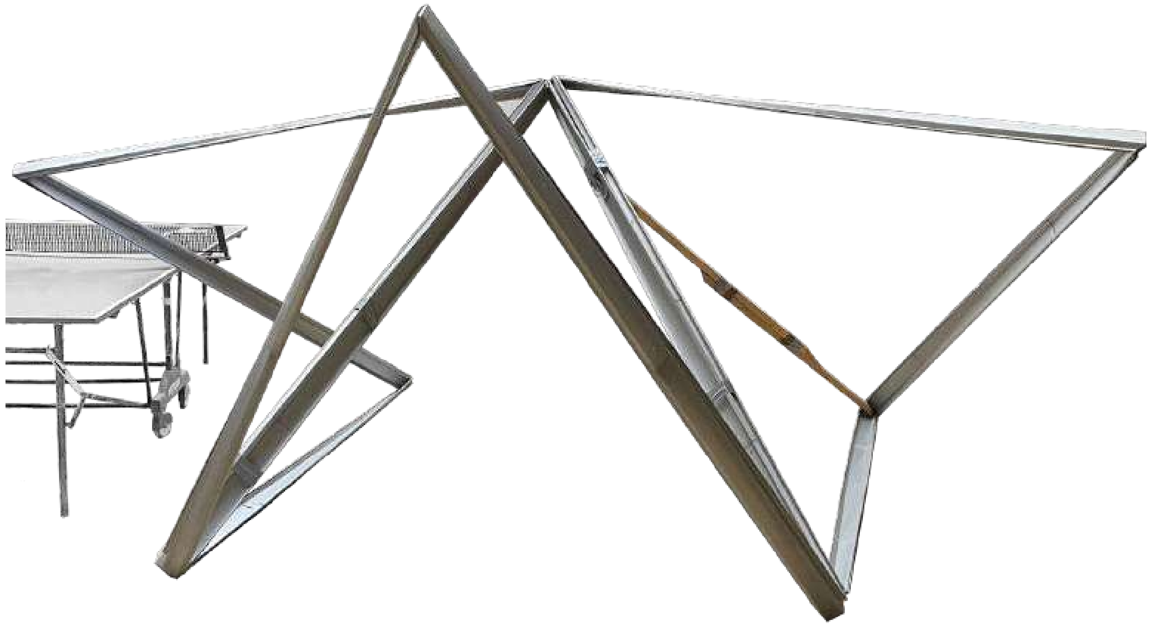
Fig.120 Left: transportation of the three partially connected bottom modules (variation 'bird'); right: transportation of the three separated modules

1 Transportation

After partially building the bracing, a transportation of the models followed. Two people moved the three single modules (the two working modules of the bracing system and the other one left alone) stacked up on each other. The other six, connected together via the steel cables, were split into two groups, each consisting of three modules and were transported by three people when the car traffic on the streets was very light. It was possible because the three openings (entrance door in Paniglgasse 12, entrance of the corridor to TFVA, entrance door of TFVA), through which assembled together modules had to go, have a width of 1.25m , 2.10m , 2.45 m. The space before & after the openings have the width dimensions of: 3.00 & 3.00m , 3.40 & 2.20m , 3.40 & 3.40m . The other key point that made passing through the openings possible was the modules were partially built so it was feasible to pull or bend them into the required direction.

Tools: rope to tie up the single modules together.

Fig.121 Left: transportation of the three partially connected bottom modules (variation 'bird'); right: transportation of the three upper modules



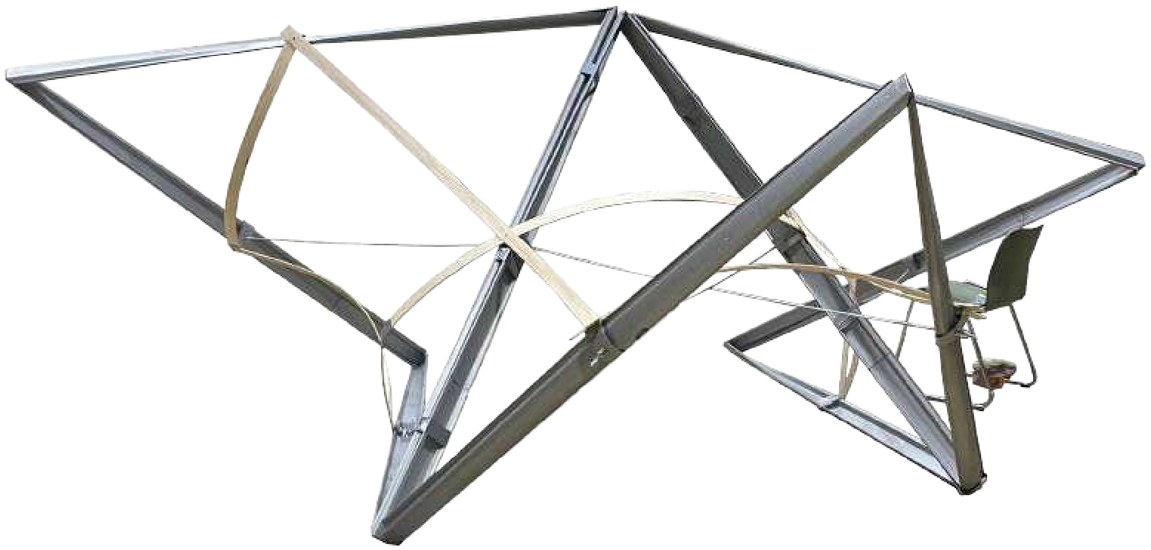


Fig.122 Three modules tied up together via steel and wood

2 Angles - preparation for attaching the bracing

Fig.123 Process of building the angles: bending the flattened blind, plates, riveting of the angles

Smaller pieces of flattened blinds are used as connecting element between the wood diagonal bracing and the frames. There is no need to cut through the blind to get the wanted triangular shape. Bending them up to 360 degrees a couple of times until it breaks along the axis of bending is enough. Then the parts are riveted to the angles of the asymptotic frame. The idea was born from a trial on the construction site in order to save time and work. Not to change the module frame or the bracing stripe in any way was also a requirement. The rectangular piece of wood was placed between the angle arms symmetrically. In such a manner the bottom face of the timber band and an edge of the aluminium profile flange lay in one plane. Logically, a plate was created and used as a connecting element.

Materials for one module: 1 aluminium blinds, 4 rivets DM4mm; Tools: rivet gun, hammer, tin snips, screwdriver, pliers Improvement idea: Instead of rivets, screws and nuts could be used, so the detail can be reconstructed with no need of removing elements.

Fig.124 Angles in a completed modules

3 Steel, wood and aluminium - attaching the bracing

Fig.125 Crosspoint of steel and wood

Following the results of the karamba model, wood and steel cables are used to stabilize the modules completely. After shifting the models to courtyard number 3 of the TU Wien, the whole bracing was firmly fixed to the aluminium module edges. At first the angles were riveted to the frames, then the diagonal wood stripes were screwed into them. The 6 cm width "belts", wrapped around the twisted blinds, to fix the timber elements passing through the middle of the edges. After that all the crossing points of the wood stripes were screwed together. It was a delight to the eye to see how the steel cables were slowly tensing up while installing the other part of the bracing. The 'crumpled' aluminium module edges reshaped into the planned geometry by adding the strengthening elements.

Materials for one module: 8.82m wood length (diagonals:1.79m x2 + middle lines: 1.16m x4 + fixing area:0.6cm);(Specification: wooden strip (pine / spruce)240x0,5x4,6cm, spruce/pine, solid, untreated, versatile, easy editing, screwing/gluing/nailing as a mounting type, can be painted over, weight (net) 264 g, not quite straight) 1.55mx2 steel cable: Corrosion resistant, galvanised steel cables with 6mm dia. are ideal for simple assemblies. It can easily be used to connect, hold and press wire, steel ropes and aluminium blinds. It has many uses and it is reclaimable. It has a 6×7 wire strand construction, which makes it very robust while also being flexible. (Specification: weight (net):135g, load capacity: up to 420kg, high life expectancy, high abrasion resistance, UV-resistant), M3 screws/ nuts/bolts x 21 pieces 4,aluminium band, 4 rivets for the bands, steel clamps 2 or 4Tools: rivet gun, hammer, tin snips,drill, flathead and torx screwdriver, tape measure , saw; Improvement idea: Connecting the steel with the wood at the point where they meet.

Fig.126 Connection between two modules

This solution emerged from several tasks such as: the need of fixing the bracing using one technique on a non-symmetrical profile; the connection is not fully fastened (movable along the edge) until every part is connected to the model; it stays true to the materiality and the concept of reusing; it can be fixed at a single edge but also on two neighbouring modules. For this reason, stripes of the widest blinds area were produced. They were twined round an edge. After screwing the wood on them, the so called `connection`s belts` are fastened with rivets so that they cannot move. (fig.126) At this step screws would again allow the reusing of the strips. They cover the location where the steel cable intersects the aluminium.

Materials (one joint): one M4/screw+nut+bolt, wood, aluminium bands,2 rivets DM3mm Tools: rivet gun, hammer, tin snips, drill, flathead and torque screwdriver, pliers Improvement: Perforated metal bands with min. width 6cm are used, so there is not only no need to hollow before screwing and but also a variety of possible places to fasten the connection.

4.8 Relocating

There are a few steps that even one person can take in order to prepare the modules for transportation in such a way that they take as little space as possible. Marking the elements belonging together can accelerate the rebuilding process when the last built variation is to be built again. The stripes connecting the modules have to be disjoined from the structure. The wood that is used for strengthening of single modules could be unscrewed only sideways. A folding of two opposite module's angles into each other is possible and single modules can pass through a standardized door. For the same reason, four rivets (two by two from opposite sides) are removed. Using only screws, instead of any rivets, would make the whole structure entirely rebuildable. All modules fit in a box truck. If there is time to stack them closely next to each other, three people can help loading them on the transportation vehicle, and a van will be sufficient. An important thing to consider is the traffic volume at the starting and the end points. Depending on this, with a smaller or bigger car, in less or in more time, a careful process of the conveyance could be followed.

Tools: screwdriver , scotch tape, flathead and torque screwdriver

Fig.127 The detached connection stripes, one disassembled module, unriveting at one angle’ side

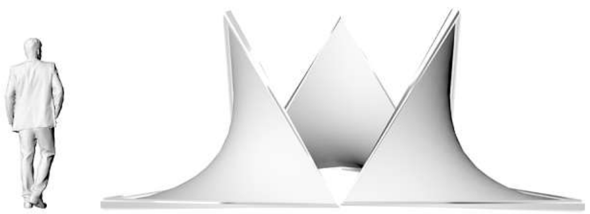
Fig.128 Disassembled wood bracing

Fig.129 minimalistic dismantleling of every module

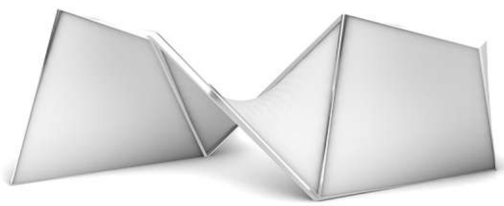
Fig.130 The modules in their storage place

4.9 Summary of the built models

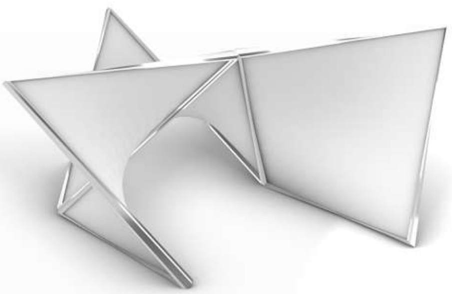
The following pages show a collection of the built variations. Proposed names are given according to the associations that the models evoke. They are all very different from each other which demonstrate the possibilities that the concept is giving. Firstly we recall to our minds their computational visualisations, afterwards we will see the photos of the real models.



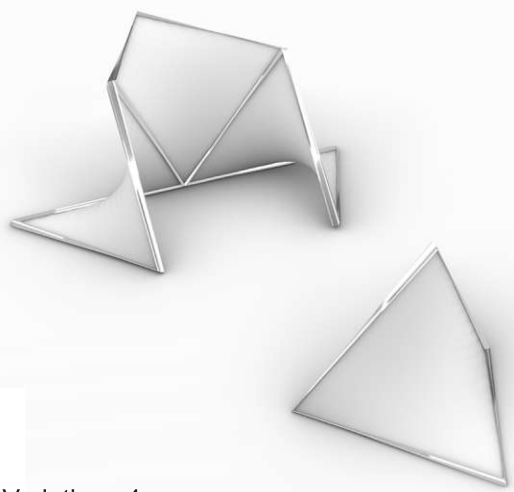
Variations 1
Lilium
3 modules



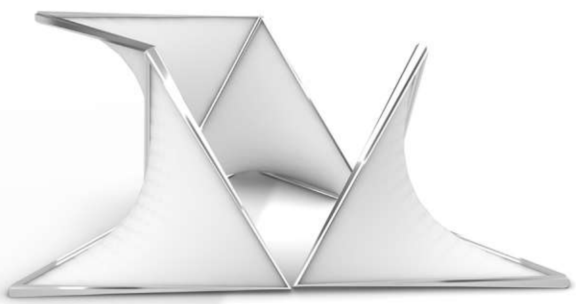
Variations 2
Lace
3 modules



Variations 3
Elan
3 modules



Variations 4
Approching
4 modules



Variations 5
Silhouette
4 modules

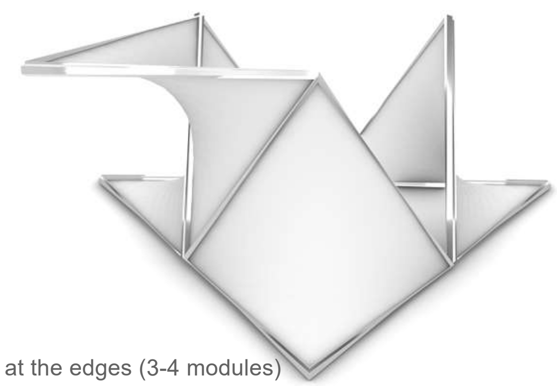
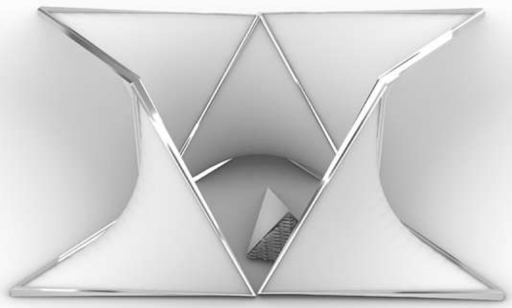
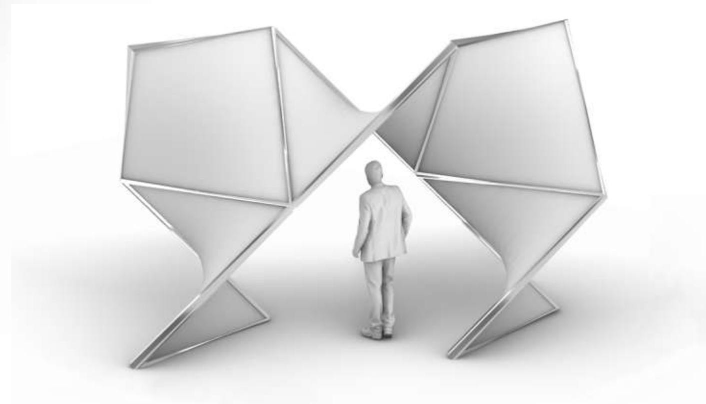


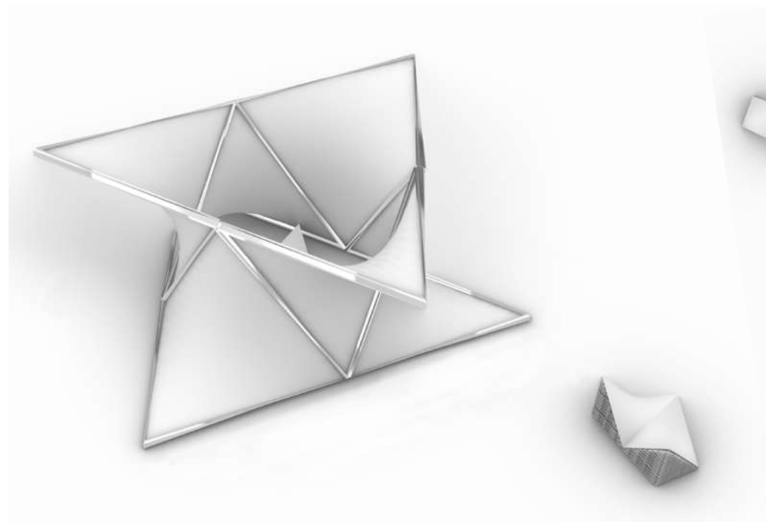
Fig.131 The variations with the computed blind`s geometry at the edges (3-4 modules)

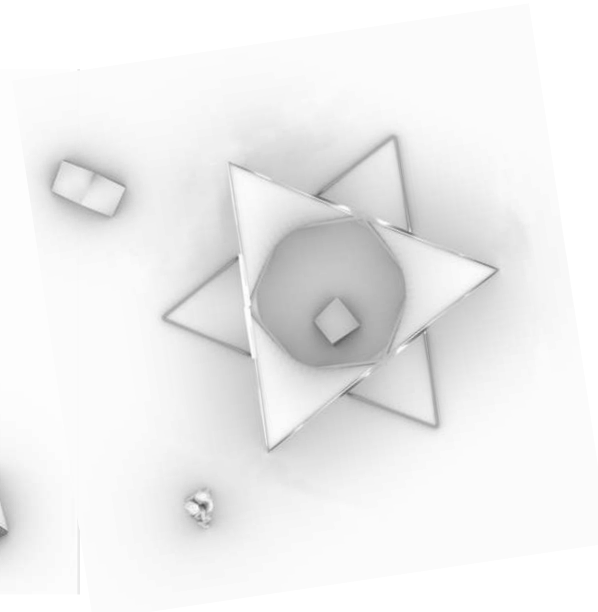


Variations 6
Throne
5 modules



Variations 7
Wings
5 modules





Variations 8
Flower
6 modules

Variations 9
Bird
7 modules

Fig.132 The variations with the computed blind`s geometry at the edges (5-7 modules)

Lilium

Fig.133 "Lilium" with 3 modules

Shelter

Fig.134 "Shelter" with 3 modules

Lace

Fig.135 "Lace" with 3 modules

Elan

Fig.136 "Elan" with 3 modules

Silhouette

Fig.137 "Silhouette" with 4 modules

Approaching

Fig.138 "Approaching" with 4 modules

Throne

Fig.139 "Throne" with 5 modules

Wings

Fig.140 "Wings" with 5 modules

Flower

Fig.141 'Flower' with 6 modules

Bird

Fig.142 "Bird" with 7 modules

Bird

Fig.143 "Bird" left front viuw

Bird

Fig.144 "Bird" right back viuw

Habitual reasons led to the building of the variation “Bird“ as the biggest and representational one: time, place and wishes. For the first two points two experimental and seven final modules were feasible. The intention of the real-scale-modeling part was the construction of an arch-like structure.

Fig.145 Looking at the sky from the "Bird"

5. Excursus: Typologies of minimal surfaces

The final part presents an excursus and digresses on different typologies. As more and more becomes possible, the aim is to provide a broader perspective on what can still be spatially realized. Minimal surfaces can be highly diverse—they can be spanned at boundaries, and in some cases, they can also grow, be endless, or remain limited.

We can observe the types of curves that often define a minimal surface, such as circles, straight lines, and waves with the same amplitude. A catenoid (with circular ends) can be transformed into a helicoid (with spiral ends), demonstrating that such spatial transformations are also possible.

The influence of typology helps us understand how the structural behavior of a design will perform—whether singularities restrict movement or allow for greater openness. In the following, we will examine some typologies from the perspectives of stability, boundary curves, and discretization. The goal is to gain insights and knowledge about what is possible.

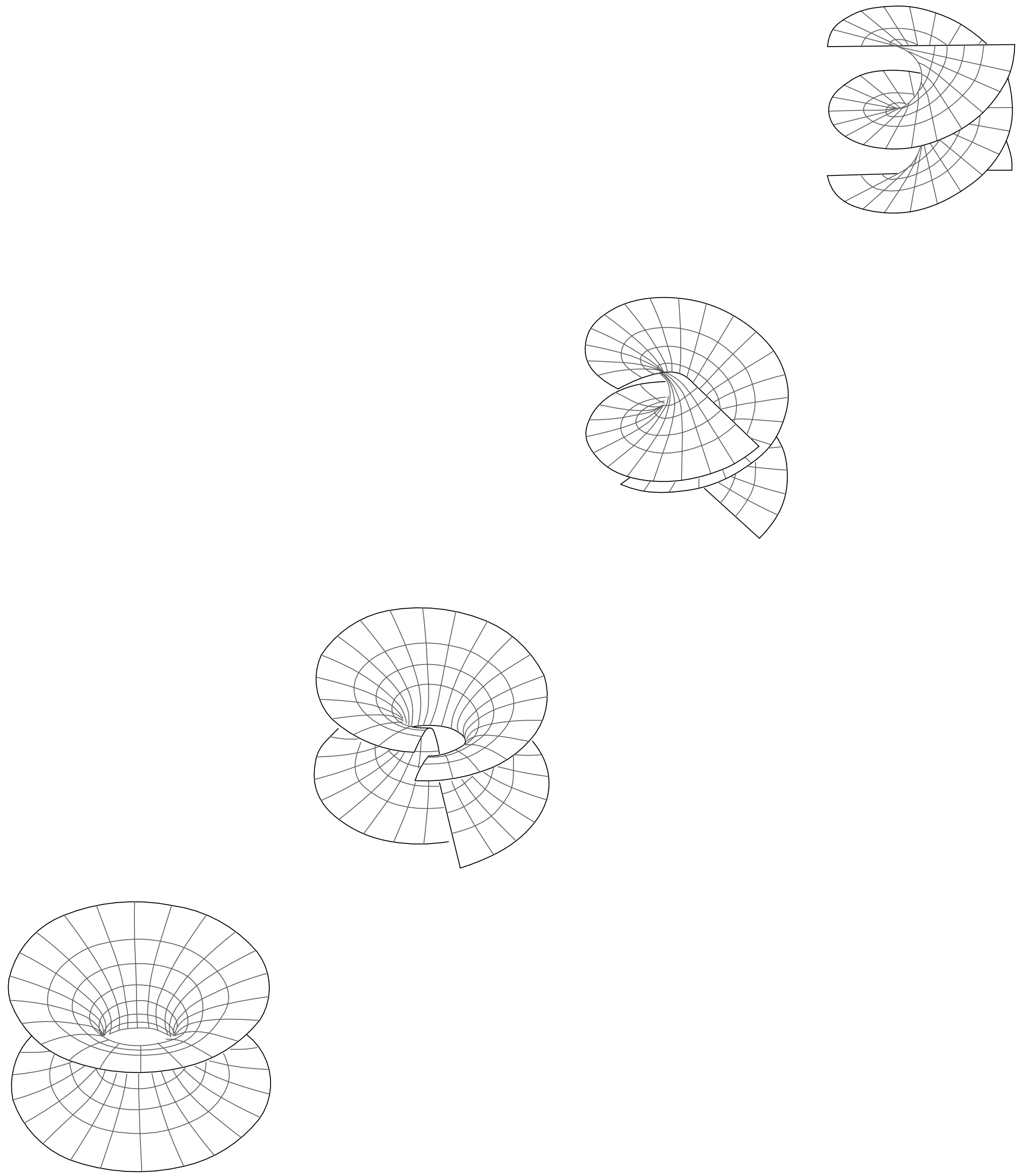


Fig.146 Bending of an Catenoid into a part of helicois without changing the lengths of curves (reference: *Pottmann, et.al.*,Architectural geometry)[1]

5.1 In terms of stability

Little attention has been paid to the mathematical study of unstable minimal surfaces. The transition from a stable to an unstable one may be very important in solving technical problems, e.g. bifurcation problems. By definition, the average of the two principal curvatures of a minimal surface is zero. $H = (k_1+k_2) / 2 = 0 \Rightarrow k_1 = - k_2$. "In classical Differential Geometry k_1 & k_2 are the maximum and minimum of the of the curvature at any given point on a surface. They measure how the surface bends by different amounts in different directions at that point." [2] The mean curvature of a minimal surface with the smallest possible area disappears at every point. The statement $H = 0$ is a necessary one, but not the only one sufficient condition for a surface to be minimal. (All comditions are listed in chapter 2.4.12) Basically, all surfaces that meet the conditions $H = 0$ and $\iint dA = min$ are stationary stable minimal surfaces. There are no directions in which the geometry can decrease its area. Such surfaces that meet the curvature condition, but do not satisfy the minimum property, are called unstable stationary minimal surfaces.[3] More tangible explanation is to create a soap film 'S' with a boundary 'b'. By reshaping 'b', 'S' also will change. Not every continuous change of form of a contour leads to a continual change the soap film itself. The reason for the irregular modifications is that at the time of deformation the surface can become unstable. The topological type and the connectivity will change. [4]

5.2 In terms of form

The information in the following chapter, along with the five definitions below, is primarily taken from a blog on WordPress about minimal surfaces.[5] Reference links are cited for each typology using superscripts.

There are plenty of minimal surfaces, described by a mathematical equation. According to their similarities and differences in appearance they can be divided into typologies. The next pages describe their classifications based on appearance and shows examples of main curvature lines and asymptotic curves at selected points. On this page we explore the definitions of some terms: 1. A surface is embedded when it can be placed in space without self-intersection. 2. The opposite is true when it is not-embeded. 3. Genus. in plural genera, stays for a number of "holes" of a surface, for example, a sphere has genus of zero. 4. Winding index is an integer representing the total number of times that a curve travels counterclockwise around a point. 5. Flux is the process of flowing in or flowing out of direction.

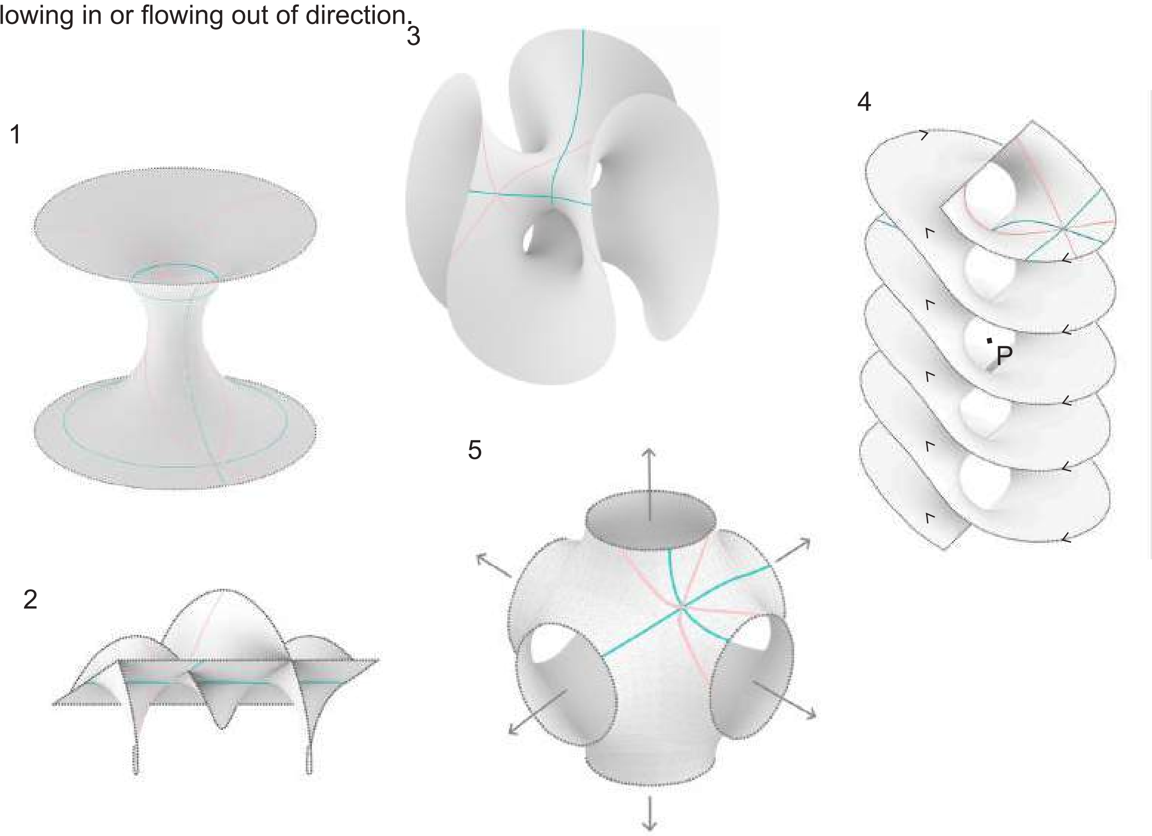


Fig.147 Graphical representation of the words (1-5): embedded, non-embedded,genus, winding index, flux

1 Spheres

They are conformally equivalent to a sphere. Conformally is a surface that can be mapped onto another surface so that all angles between intersecting curves remain unchanged. They can have an infinitely amount of punctures: Enneper ends look like a circle with sides sent in opposite directions, so a wave results. The surface holds the name of his inventor- Alfred Enneper and it is made by revolving a generatrix one full time around an axis of rotation. All its curvature lines are planar, and it can also be described parametrically. Sections of the geometry made by its symmetry planes are geodesics and principal curvature lines.[6] Of course, a combination of ends are possible- straight lines plus Enneper ends, etc. They often have winding indexes starting by number two. Catenoid comes from the Latin word 'catena' with the meaning of chain. Because it is arising by rotating a catenary curve about an axis- a surface of revolution. It can grow exponentially in radius while linearly in height, can be stable nonstable and it can be bent into piece of a helicoid.[7]

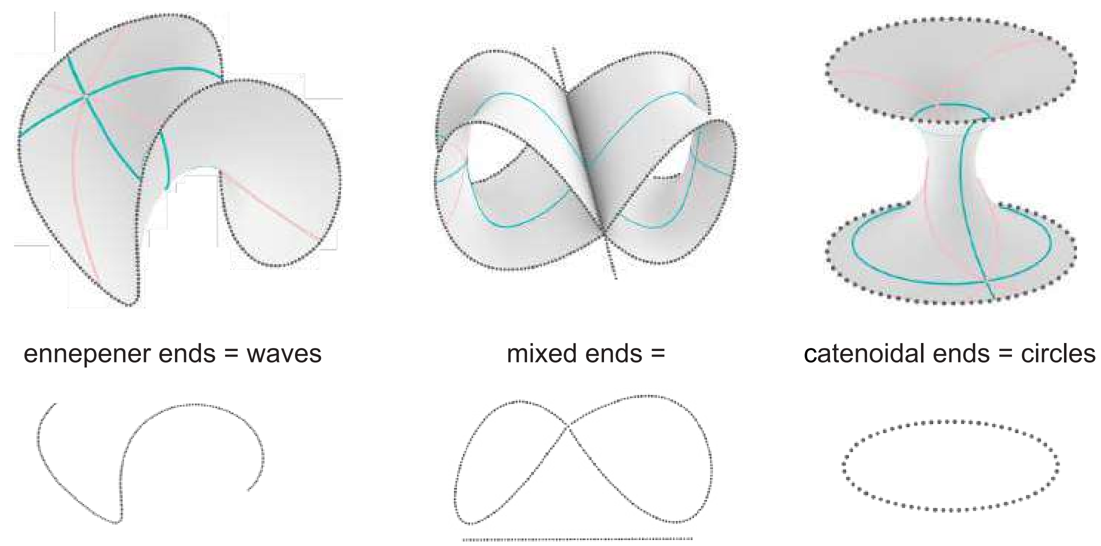


Fig.148 Surfaces of the typology "sphere"

2 Non-orientable

A continuous choice of orientation cannot be assigned. Topologically interesting but not embedded. Examples are the Klein Bottle, the Kusner's Spheres with Planar Ends and the Henneberg Surface. The Björling surfaces are also non-orientable. "In differential geometry, the Björling problem is the problem of finding a minimal surface passing through a given curve with prescribed tangent planes." [8] Shown is an example of a trefoil Knot without twists. The faster the spinning normal, the more the twists are. Its core curve is the edge of a Möbius strip with 3 half-twists. Other examples of Björling surfaces are: Cycloid (Catalan's Surface), Deltoid, Closed Cycloids, Quatrefoil Knot, Basic Torus Knots, Tweaked Torus Knots, Logarithmic Helix (Breiner-Kleene Surface) and spirals such as Archimedean Spiral, Logarithmic Spiral and Clothoid. They are all named by their core curves.[9]

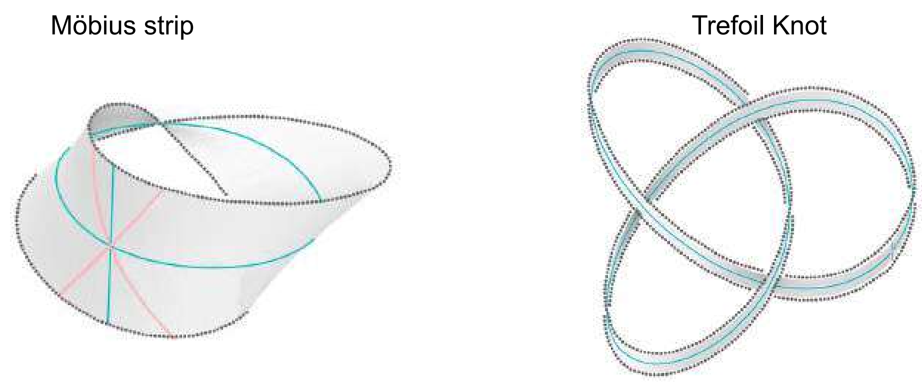


Fig.149 Surfaces of the typology 'non-orientable'

3 Symmetrizations

Typical for the typology is the increased surface complexity by expanding its symmetry.[10] On the left we see k-Noids surface. Symmetry is generated by reflections at 'n' vertical planes and rotations about horizontal lines. The number of genera is eaqual to 'n'-1. On the right we see another dislodged example of a symmetrized Chen-Gackstatter. It has two genera and similarly to Enneper surface there is only one edge curve. [11]

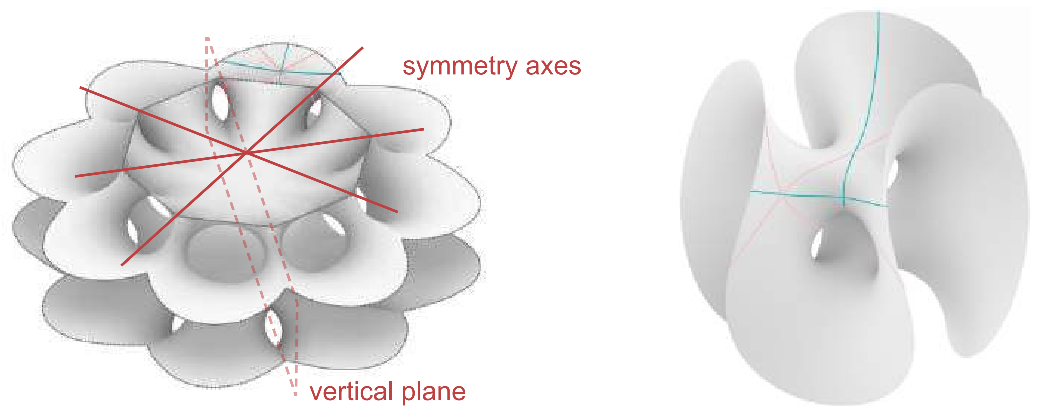


Fig.150 Left: k-noid surface; right: symmetrized Chen-Gackstatter

4 Singly Periodic

The singly periodic minimal surfaces can increase in three-dimensional world under transition in only one direction. They can have many different ends, e.g. planar, annular, helicoidal, periodic enneper ends, mixed ends.[12] For example, the Riemann's Singly Periodic Surface grows vertically and has only circular horizontal cross sections. Catenoid like tunnels serve as a connecting element.[13] The Scherk's Singly Periodic Surface was first described in 1835 by Heinrich Ferdinand Scherk and it looks similar to two intersecting planes. Again Catenoid like tunnels serve as a connecting element. The Helicoid was discovered by Jean Baptiste Meusnier in 1776; together with the plane it is the only one ruled minimal surface. All horizontal lines, that are apropos straight, are symmetry lines; the mean curvature along them is automatically zero. It can be rebuilt into catenoid.[15] The last example is of a mixed ends Half-Twisted Scherk Surface with self-intersections happening along straight lines.

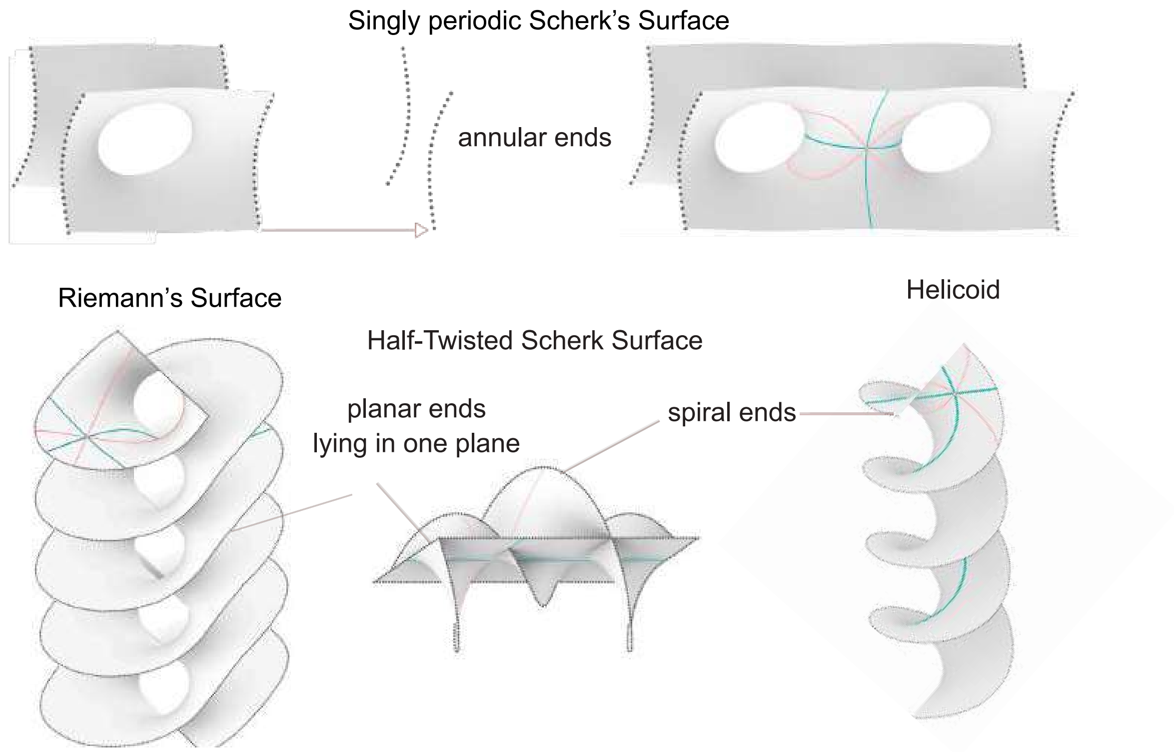


Fig.151 Singly periodic surfaces

5 Doubly Periodic

They grow in two linearly independent directions and different ends are possible. They can be embedded if they have annular ends (also called Scherk ends). The first example shows the simplest doubly periodic surfaces with only catenoidal ends and one planar end.[16] The second example - Scherk's Doubly Periodic Surface- is found by Heinrich Ferdinand Scherk. It has very simple mathematical equation and genus of 0. [17]

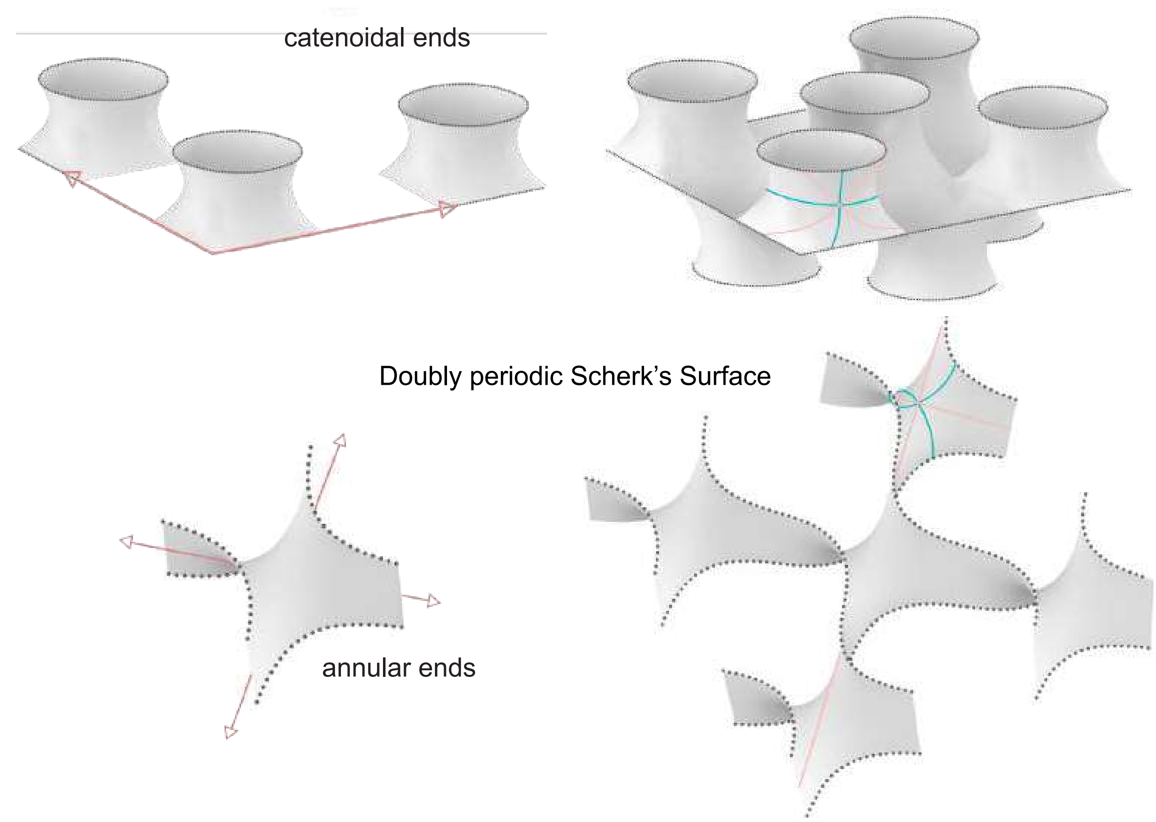


Fig.152 Doubly periodic surfaces

6 Triply Periodic

The examples of this kind of minimal surfaces can grow in three independent directions by translation. Their unit cell characteristics and relative density significantly affect the structure's stiffness. Logically they can be described in solids such as cubes, prisms, polyhedrons, etc. 3-13. The example below represents the Schwarz P surface. Hermann Amandus found it in 1867 . It is generated using symmetry axes and it can be inscribed in a unit cell or in a regular octahedron.[18]

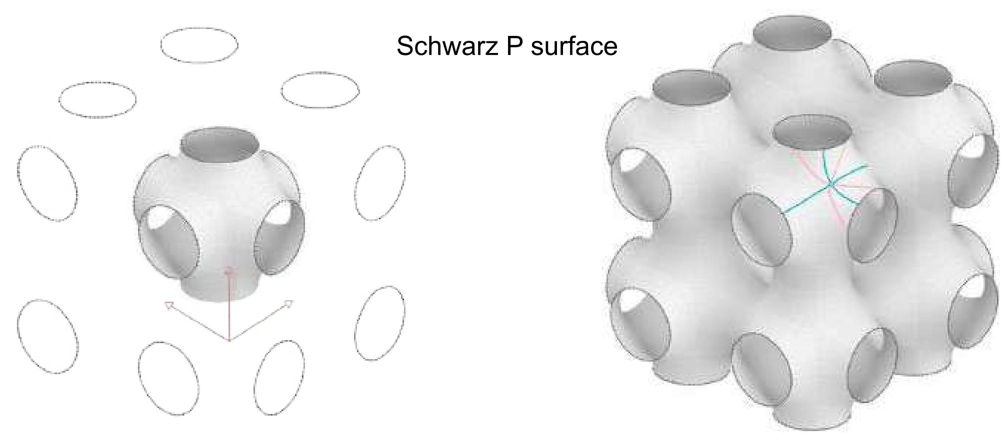


Fig.153 Triply periodic surface

7 Tori

Tori is the plural form of tourus. As well created by rotation, this time has an intersection in form of genera. The Costa surface on the next page (bottom two rows) has one catenoidal and one planar end and it is obviously an embedded one. Tori has two catenoidal and one planar end. It is similar to catenoid, but with an intersection in the middle. [19] First described by Celso José da Cost in 1982. Other examples of Tori surfaces are Chen-Gackstatter Surface Costa-Hoffman-Karcher Tori, Toroidal k-Noids, Costa-Enneper, 4-Ended Tori, Torus with one Catenoid and one Enneper end, Torus with Two Enneper Ends, The Genus One Helicoid [20]

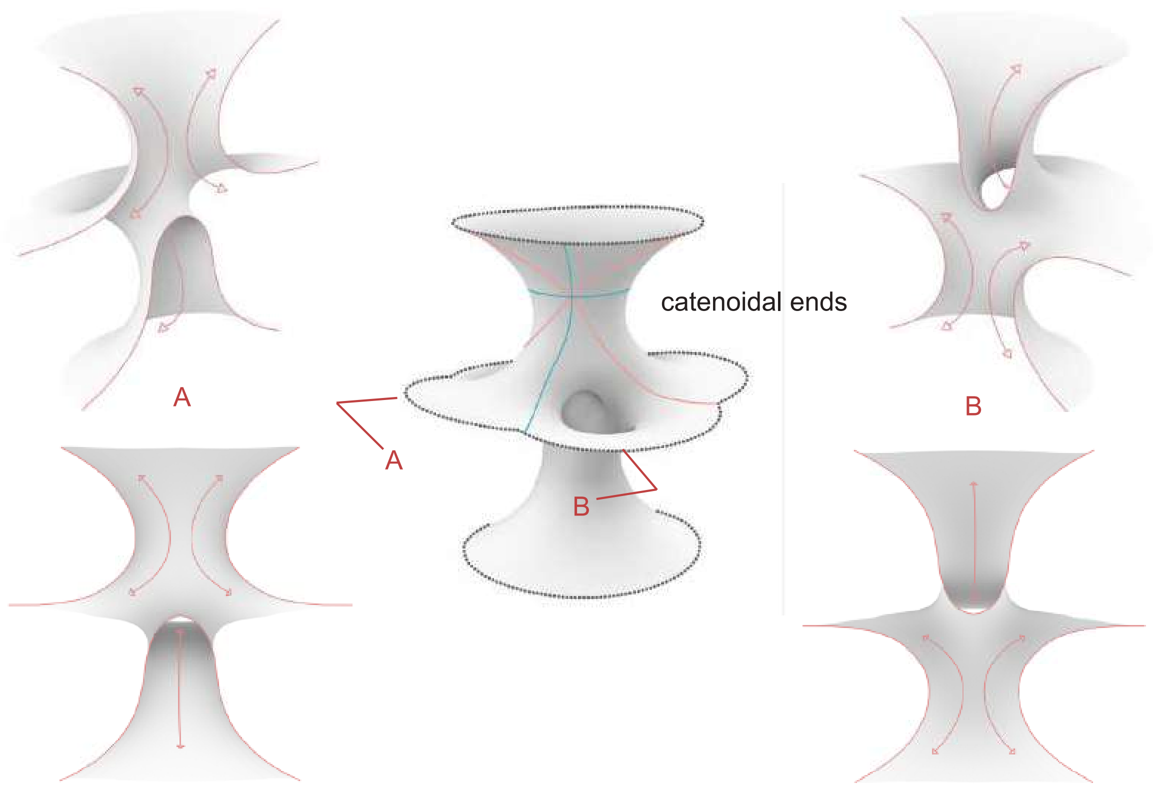


Fig.154 Costa surface

8 Higher genus

They are similar to the building principle of the previous two types (rotational surfaces). As their name suggests - higher genus - they have higher number of genera. The example surface is called Costa-Wohlgemuth surface, that is having two catenoidal and two planar ends and it is vertically symmetrical. [21]

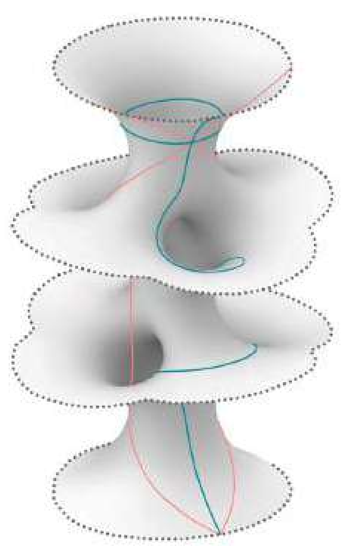


Fig.155 Costa-Wohlgemuth surface

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[1] *Pottmann, et.al.,* Architectural geometry (2007) 653 (724)

[2] *Verzea, et.al.,* Minimal surfaces, MATH 580: Partial Dierential Equations (2012) 2 (11)

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[4] *Fomenko, et.al.,* Translations of mathematical monographs,American Mathematical Society [93] (2005) 96 (155)

[5] *WordPress*, Repository, https://minimalsurfaces.blog/home/repository/ (08.2024)

[6] *WordPress*, Enneper Surface, https://minimalsurfaces.blog/home/repository/spheres/enneper-surface/ (08.2024)

[7] *Wikipedia*, Catenoid , https://en.wikipedia.org/wiki/Catenoid (08.2024)

[8] *Wikipedia*, Björling problem, https://en.wikipedia.org/wiki/Bj%C3%B6rling_problem (08.2024)

[9] *WordPress*, Björling Surfaces, https://minimalsurfaces.blog/home/repository/bjorling-surfaces/ (08.2024)

[10] *WordPress*, Symmetrizations, https://minimalsurfaces.blog/home/repository/symmetrizations/ (08.2024)

[11] *WordPress*, Symmetrized Chen-Gackstatter, https://minimalsurfaces.blog/home/repository/symmetrizations/symmetrized-chen-gackstatter/ (08.2024)

[12] *WordPress*, Singly Periodic, https://minimalsurfaces.blog/home/repository/singly-periodic/ (08.2025)

[13] *WordPress*, https://minimalsurfaces.blog/home/repository/singly-periodic/riemanns-singly-periodic-surface/ (08.2025)

[14] *WordPress*, Scherk's Singly Periodic Surface, https://minimalsurfaces.blog/home/repository/singly-periodic/scherks-singly-periodic-surface/ (08.2025)

[15] *WordPress*, Helicoid, https://minimalsurfaces.blog/home/repository/singly-periodic/the-helicoid/ (08.2025)

[16] *WordPress*, Plane with Catenoids,https://minimalsurfaces.blog/home/repository/doubly-periodic/plane-with-catenoids/ (08.2025)

[17] *WordPress*, Scherk's Doubly Periodic Surface https://minimalsurfaces.blog/home/repository/doubly-periodic/scherks-doubly-periodic-surface/ (08.2025)

[18] *WordPress*, Schwarz P-Surface, https://minimalsurfaces.blog/home/repository/triply-periodic/schwarz-p-surface/ (08.2025)

[19] *WordPress*, The Costa Surface, https://minimalsurfaces.blog/home/repository/tori/the-costa-surface/ (08.2025)

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Symmetrizations: *Piker,* Chen-Gackstatter, https://discourse.mcneel.com/t/minimal-surface-strips/144823/3 (11.2024)

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Tori: *Piker,* Discrete Costa , https://discourse.mcneel.com/t/isothermic-costa-surface/88957/24 (03.2024)

4.3 In terms of discretization

We have considered so far, a lot of information about smooth surfaces. There are two more types of discretization. One is consisting of straight lines and planar surfaces. The second class is composed of one smooth and one discrete parameter.

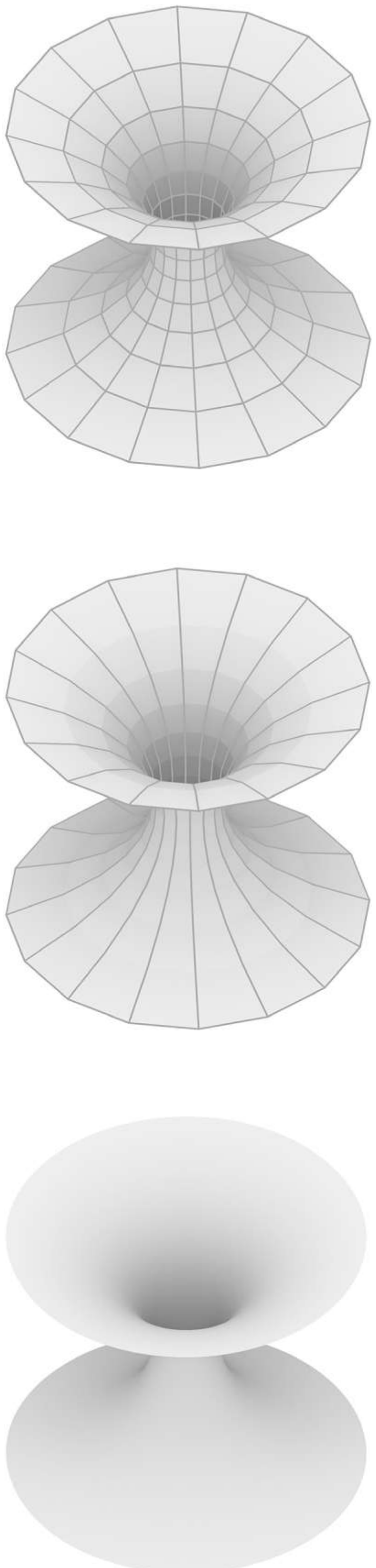

Fig.156 Discrete, semi-discrete and smooth Catenoid

1 discrete

A single definition of discrete minimal surfaces does not exist, because every surface can be discretized in different ways. On the one side, each of the numerous properties of the smooth geometry can be discretized, on the other, the resulting subdivision theory is very rich. Extraordinary would be a discretization that transfers more than one of the minimal surface characteristics to the discrete setting. Possible representations are triangle meshes, PQ - isothermic meshes, s-isothemic minimal surfaces with prescribed combinatorics and conical meshes.[1]

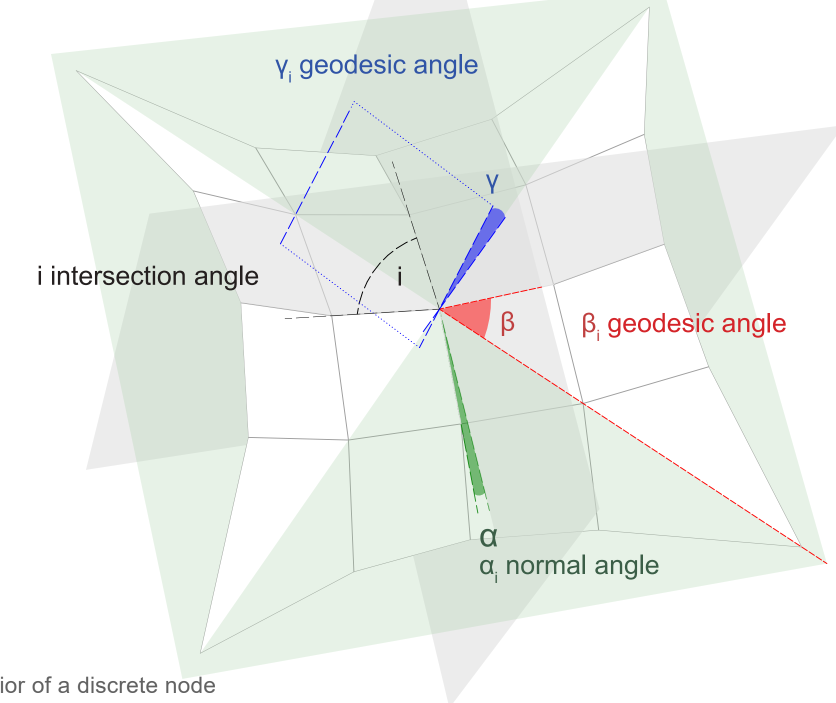

Fig.157 Behavior of a discrete node

The curves shaping the minimal surface become straight lines and its complexity is taken up by the nodes. We can define three node angles which are related to the three curvatures of a respective smooth segmentation. They are measured in relation to the node axis and its corresponding tangent plane. Elke Schling and Rainer Barthel and explains in the paper "Repetitive Structures" the relation inteh following way:

The normal angle **α** is connected with the normal curvature of the discrete surface and quantifies the deviation of each edge from the tangent plane at the node.

The geodesic angle **β** is related to the geodesic curvature of the surface and stands for the degree between an edge and a traversal node inside the tangent plane.

The torsion angle **γ** is associated with the geodesic torsion of the surface. It measures the deviation of subsequent node axes within the normal plane of their connecting (straight) edge.

The angle **i** measures the intersection between the edges.

All four angles explain the behavior of a discrete node.

Discrete minimal geometries are useful in computational graphics, mesh generation, architectural geometry. Fabricating from sheet material and straight lines allows fast and precise work. A mesh vertex where the central plane of emanating beams all pass through the axis of the node is an optimal solution when building networks of minimal possible area.[2]

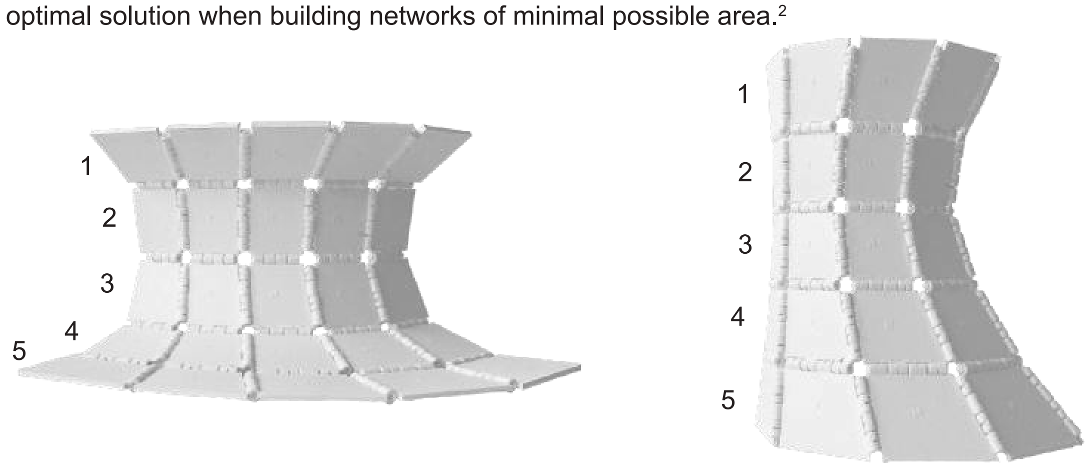

Fig.158 Discrete transformable surface; rows numbered (reference: *Maleczek, et.al.,* 11) [3]

2 semi-discrete

The semi-discrete minimal surfaces in Euclidean space are consisting of one smooth parameter that belongs to the set of real numbers (ϵ R , any number that can be found in the real world) and one discrete parameter belonging to the set of integers {Z = -3, -2, -1, 0, 1, 2, 3}. The aforementioned surface could be built with strips glued together from individual developable bands. In such a model one direction is discretized into individual "steps", the other is continuous and smooth. Their grid is consisting of lines along one axis and curves along the other. If a semi-discrete surface is isothermic, one advantage will be circularity. As a consequence there exists a parallel surface s which is inscribed to the unit sphere, which means that more relations between the entities creating the geometry exist.[4]

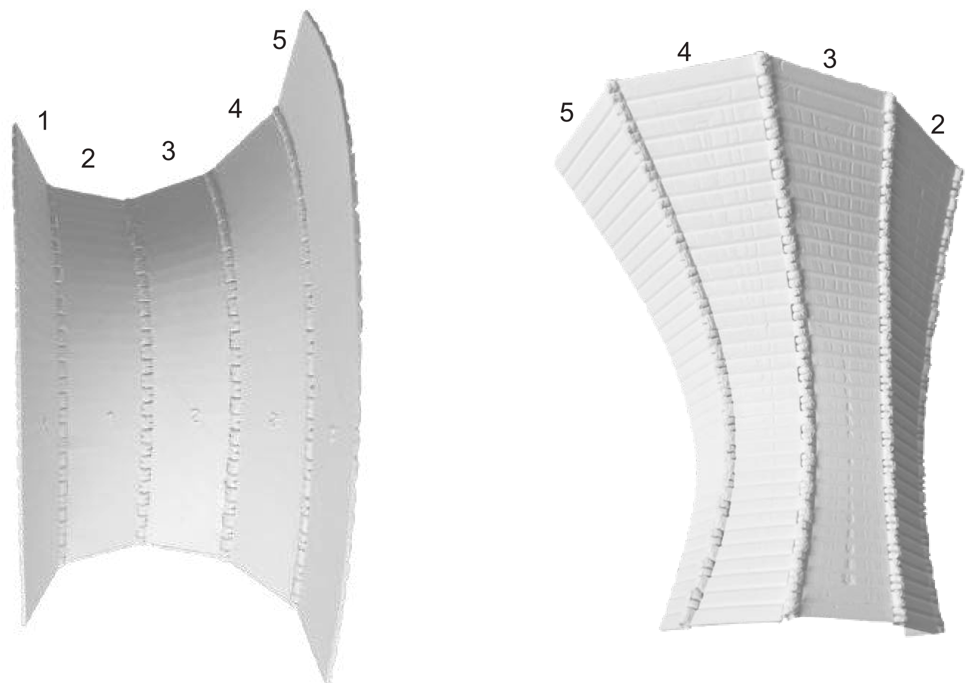

Fig.159 Semi-discrete transformable surface; column numbered (reference: *Maleczek, et.al.,* 11) [5]

In Figures 151 and 152, we see a discrete model and a semi-discrete model, respectively. The hinges of the first model can be rotated, allowing the height of the model to be significantly adjusted. The hinges in one direction are fully 3D-printed, yet remain rotatable. In the other direction, the hinges are 3D-printed separately and later assembled. As mentioned, all hinges can rotate. To model the discrete parameter of the semi-discrete model, five flat pieces are also 3D-printed. To reproduce the smooth parameter of the geometry, separately 3D-printed hinges are once again used.

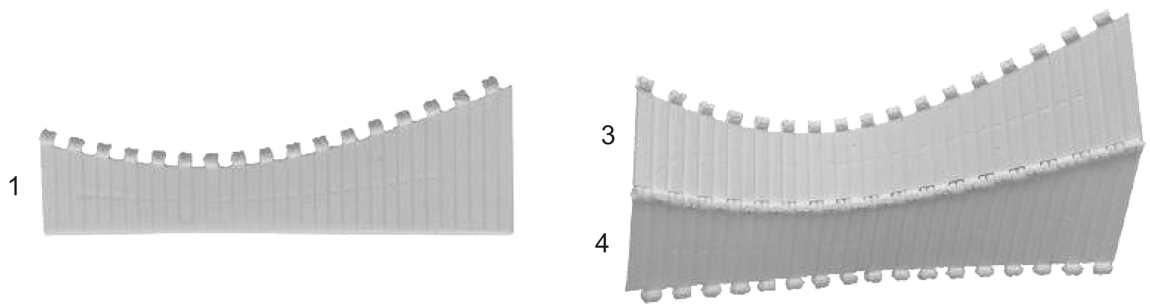

Fig.160 Pieces "1","2" and "3" of the Semi-discrete transformable surface (reference: *Maleczek, et.al.,* 11) [6]

Structural origami, technical folding, transformable design are keywords for the industrial application of both discrete and semi-discrete surfaces. Tailored sheets are connected together via hinges in order to make the geometry deployable. Interesting is the fact that "the energy stored in a surface **S** bent from an inextensible plate of area **A**, can be expressed in terms of the mean curvature **H** and Gaussian curvature **K**. Both curvatures encode information about how the surfaces reveal in space, what responds to stresses, shears and how particles move across the geometry.[7]

4.4 Spanning the Platonic Solids

Here we draw a close of the typological summary of minimal surfaces and create something new. Similarly to the work of Buckminster Fuller, the platonic solids will serve as scaffolding for a new geometric formation. While doing ve.sh in summer term 2023, a minimal surface was spanned from six adjacent hexahedron edges. The surface was divided into 6 pieces - modules (see page 18). In this chapter we will consider what else is possible with the regular polyhedrons. They have some similarities with minimal surfaces, namely they inherit internal logic, look strong and beautiful.

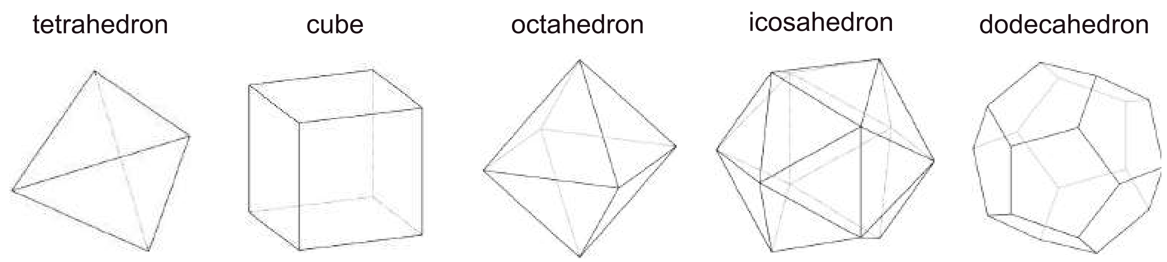

Fig.161 The platonic solids- perspective viuw

The Platonic solids are named after the ancient philosopher of the Classical period Plato, because he wrote about them in the dialogue Timaeus c. 360 B.C. Through all the millenniums they are examples of timeless symmetry and elegance. The alluring simplicity together with the intrinsic reasoning makes the shapes easy to understand. The tetrahedron, hexahedron, octahedron, dodecahedron and Icosahedron have unique properties such as:

-their symmetry: they have one face type, same edge length, equal dihedral edges, congruent vertex pyramids
-they have three related spheres with the same origin point, namely the circumsphere (all vertices), the middlesphere (edge-midpoints) and the insphere (face-midpoints)
-the face center points of each Platonic solid are the vertices of another solid. The tetrahedron is self-dual. The cube and the octahedron are duals to each other, the icosahedron and the dodecahedron as well.
-the Euler formula is true(for all polyhedra without holes). The number of vertices minus the number of edges plus the number of faces is always equal to 2: v(vertices) - e(edges)+ f(faces) = 2 For example, a dodecahedron has v=20, e=30, and f=12 => 20 - 30 + 12 = 2
-all the vertices are surrounded by the same number of faces. [8]

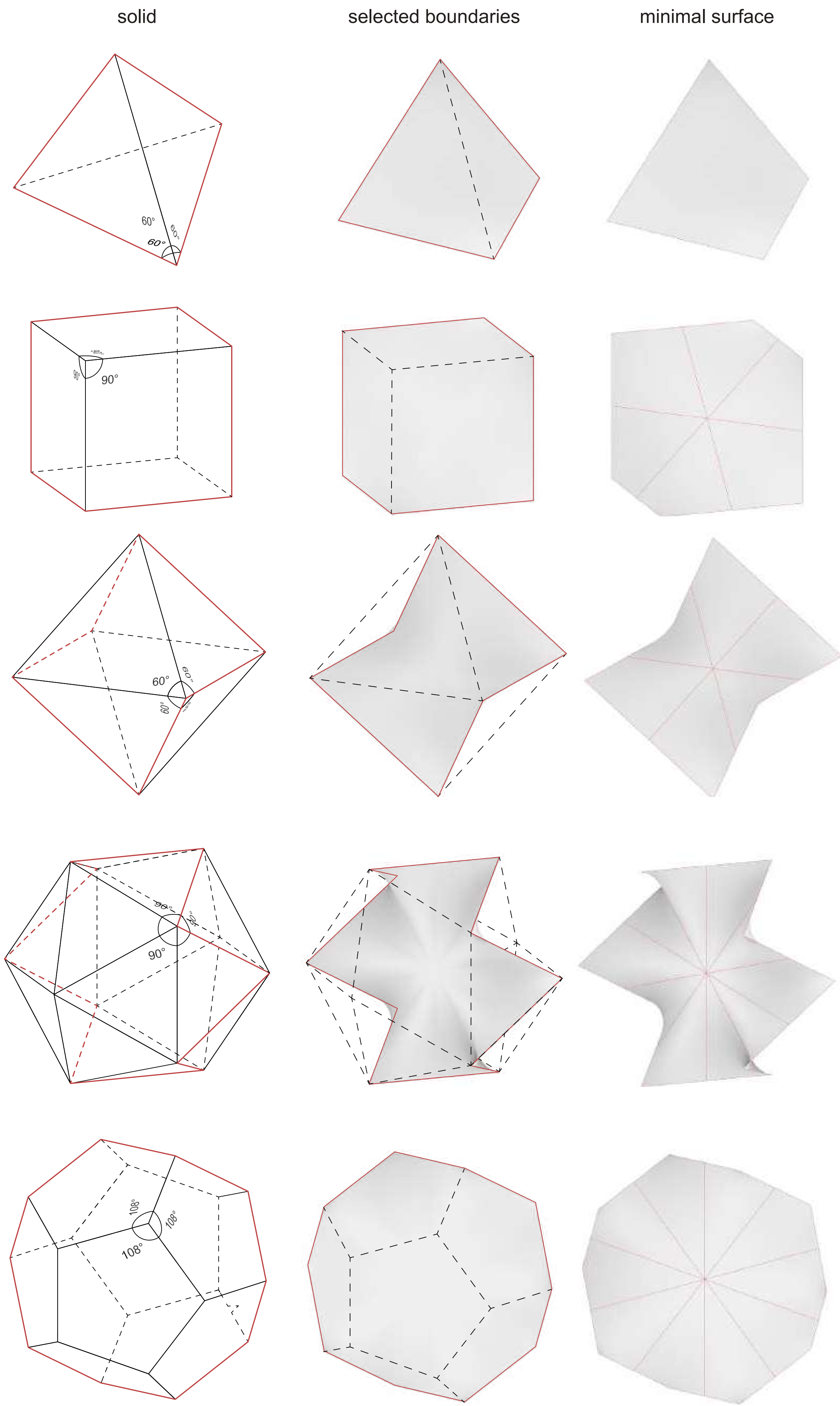

Fig.162 The platonic solids and minimal surfaces from selected boundaries.

1 Tetrahedron

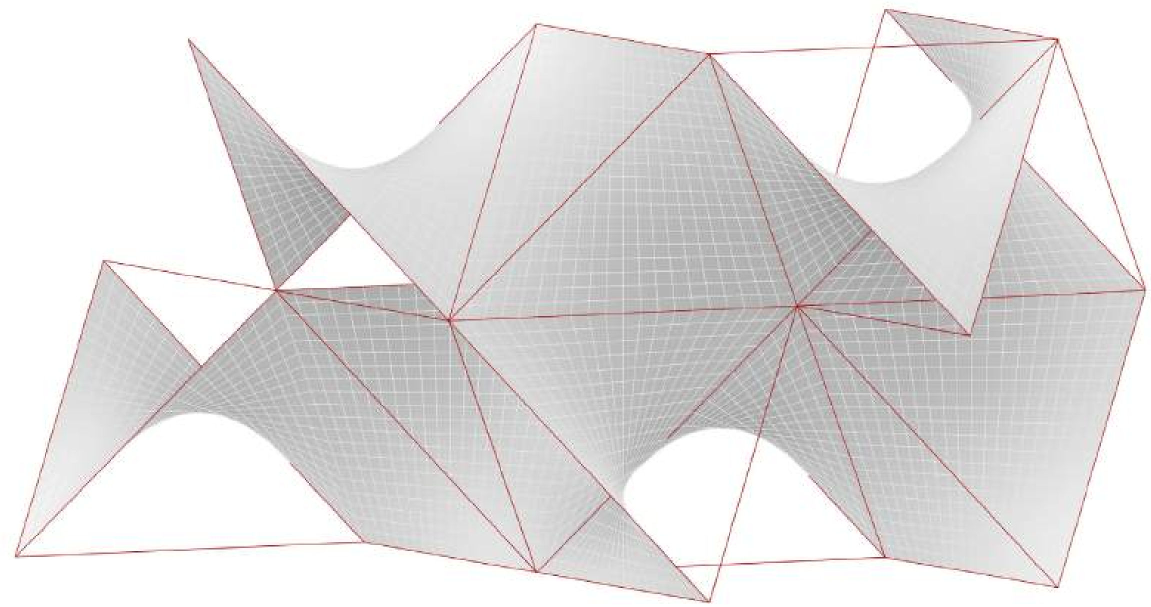

Fig.163 The platonic solids and minimal surfaces from selected boundaries.

2 Hexahedron

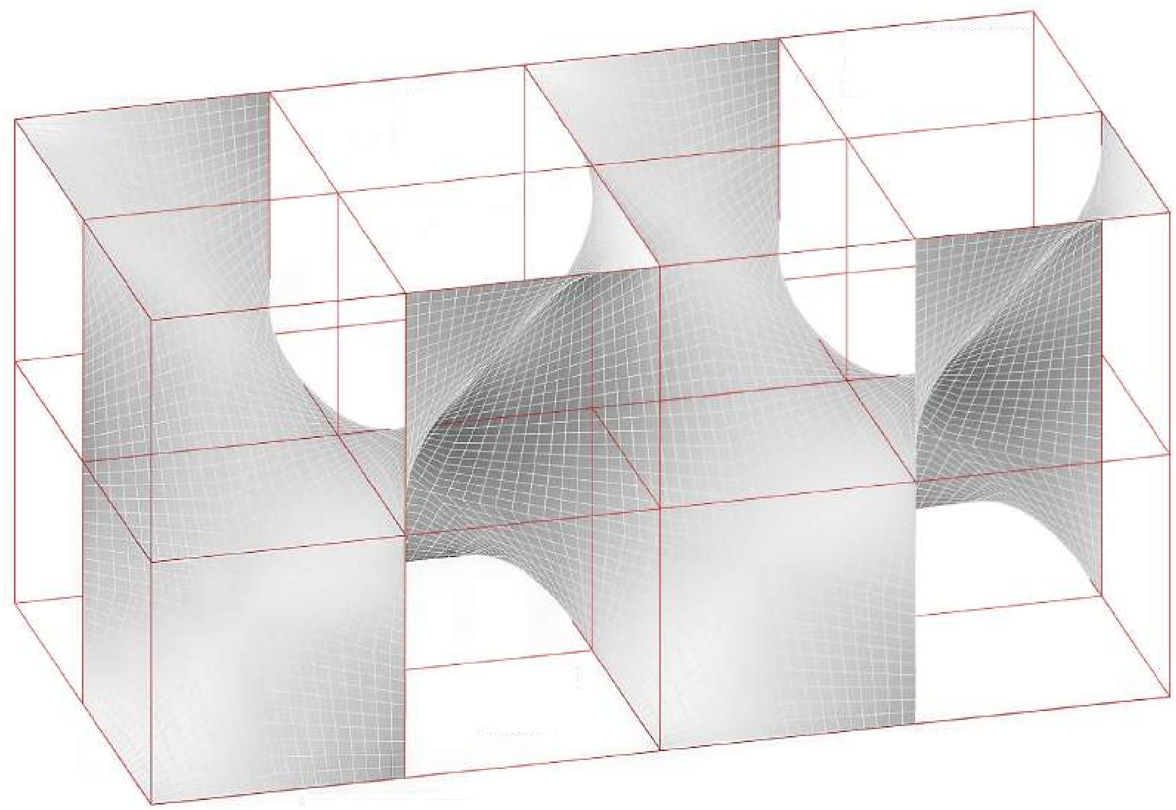

Fig.164 The platonic solids and minimal surfaces from selected boundaries.

3 Octahedron

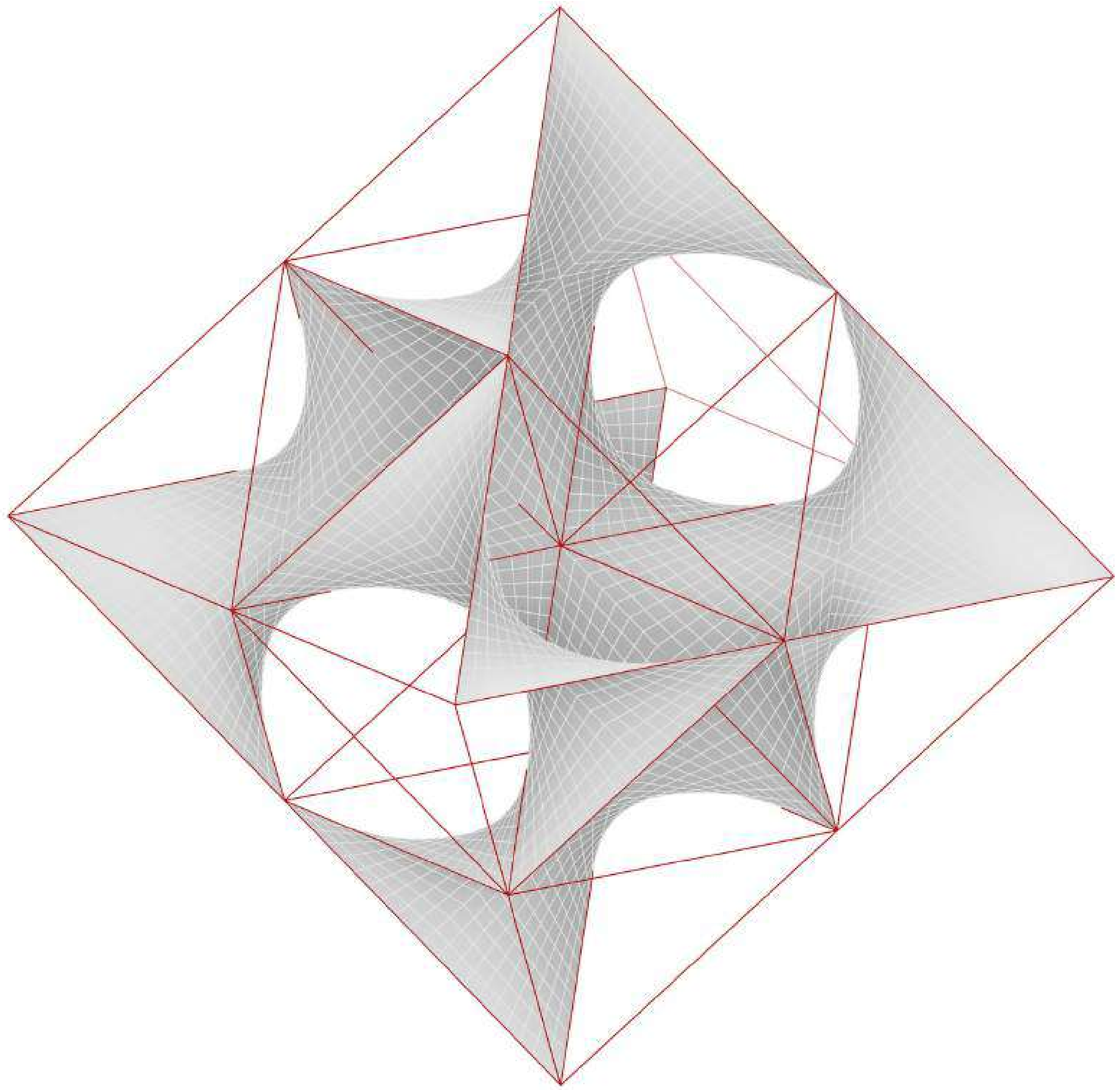

Fig.165 The platonic solids and minimal surfaces from selected boundaries.

4 Icosahedron

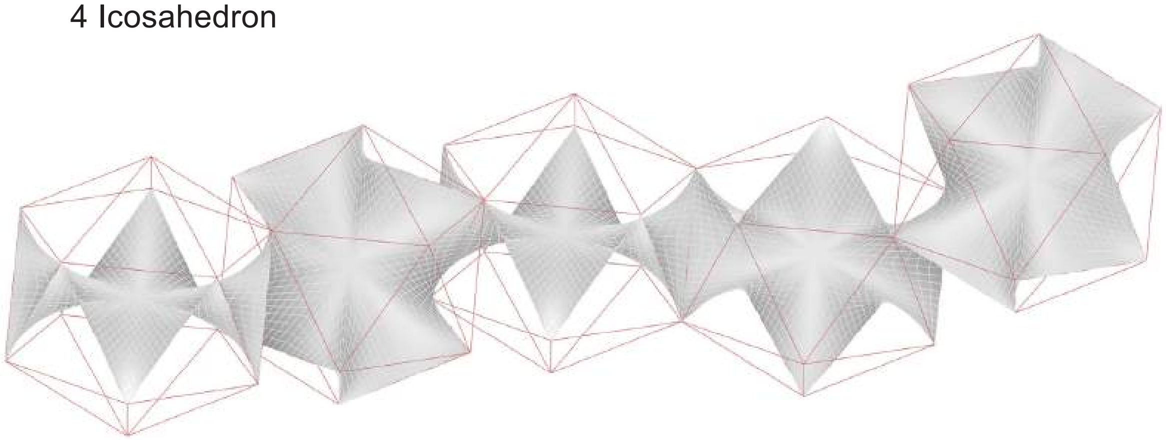

Fig.166 The platonic solids and minimal surfaces from selected boundaries.

5 Dodecahedron

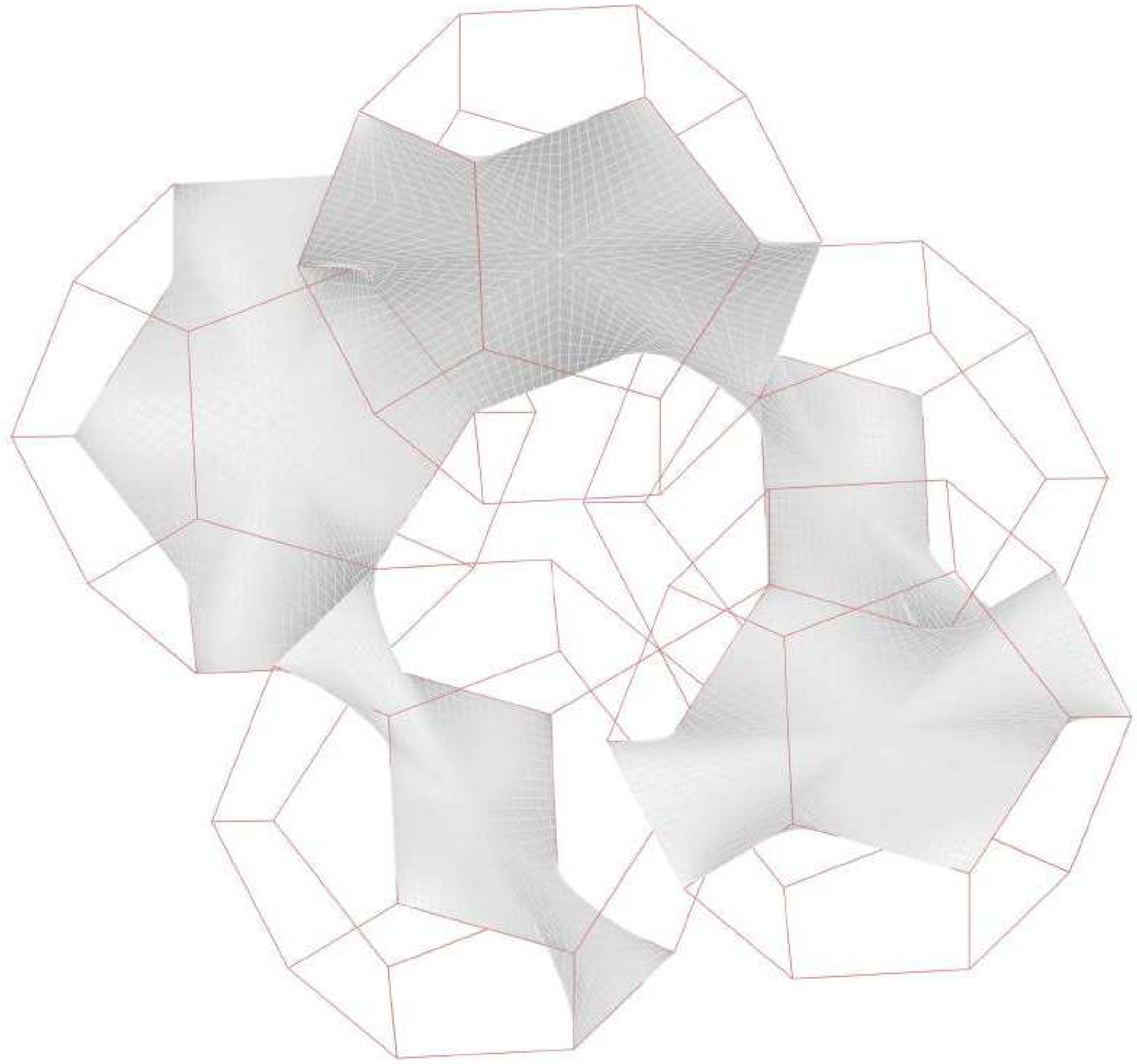

Fig.167 The platonic solids and minimal surfaces from selected boundaries.

We see that the minimal surfaces created with an icosahedron- and a dodecahedron- boundaries are flatter. To choose other edges may be advantageous. It is quite interesting that minimal surfaces formed from octahedron can also form ab octahedron-structure with their bounding geometries. The ve.sh was modelled with the help of a hexahedron. The geometry formed upon the tetrahedron is symmetrical and all four vertices are equidistant from each other. The sides meet at each vertex with the same length and angle to create equal forces.

4.3-4.4 Capter references::

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[8] *Pottmann, et.al.,* Architectural geometry (2007) 85 ff (724)

5 Summary and final conclusion

The intention of this work was to provide research, analysis and experimentation regarding the topic of minimal surfaces and their potential use in the field of mobile, transformable and resource-saving architecture.

The first part of this thesis reviews realised innovative projects that are reshaping our imagination of architecture in structural, functional and aesthetics point of view. The ve.sh project is also an explicit example of reusing resources when building. All the examples give us a perception of what is intended in the diploma thesis.

Next a broader definition of minimal surfaces is taken into consideration by breaking down the geometry into its smallest units. The interconnection between the geometries describing it, can create optimum structure networks when building architecture.

The mentioned properties influence the aesthetics, use and structural performance of the structure. They also have an effect on eminent curves on these surfaces. Depending on the profiles we design on the three types of curves, we can influence the equilibrium state and composition of the edifice. The elastic deformation of the initial flat materials enables the creation of double curved surfaces. This kind of material transformation during construction may have structural advantages if we choose the correct materials.

We take a look on four kinds of minimal surfaces: one non-orientable, one non-embedded, one triply periodic and one created with boundaries of tetrahedron are researched. Finally, we chose one to build because of the flexibility provided by the surface modules assembled together. Firstly, two modules, stabilized in different ways, were constructed. After discussions with the tutors, computational and real-life tryouts, the designs for the bracing and for the foundation were chosen. The exact connections between the wood slats, the steel cables and the aluminium blinds were born on the construction site. Rationales for this design decision were made by observing the form of all building materials, remembering the theory about osculating geometries and the intention to smoothly follow the minimal surface as much as possible. The foundation, the bracing and the connections are within in the width of a built module. The intention was to make the details look pleasant and harmonious with the whole structure. Another important rule in realizing the modules was to adhere to the virtual models. By working precisely and building the parts with the exact dimensions, the modules fit very well together.

The hinged connection between the frames and the bracing allowed for the smooth attachment of the modules. The variations stand stable in the courtyard of the university, but I cannot guarantee the same result at a windy riverbank. We can see from the photos that all modules were assembled together without foundations. Nevertheless, even the biggest variation stands in the planned design, supported by couple of stones. For me, the module represents a semidiscrete minimal surface, where the steel cables and the aluminium blinds, parallel to each other, represent the discrete parameter. The wood slats stand for the smooth parameter.

The final part of the diploma project is a brief excursus on the typologies of minimal surfaces. A variety of these surfaces are illustrated and analysed. They can be categorized based on their boundaries, the way they weave through three-dimensional space, their genera, frequency, crosssections, construction methods, and more. We observe a classification system that can be further expanded when one begins to model minimal surfaces independently—this type of geometry can be adapted to any boundary. Next, the Platonic solids are used as a scaffolding for generating new minimal surfaces. Space frame structures can be easily constructed using polyhedra that have been established over millennia. This is achieved by duplicating the initial solid and rotating it according to its inherent logic. As possibilities continue to expand, the goal of this excursus is to offer a broader perspective on what could still be spatially realized.

For the entire diploma project more than 50 kg recycled aluminium was used (modules, angles, foundations, connections, etc.). That means we saved between 100 to 200kg of CO_2 emissions. For the future I would also work on the steel cable connection. Cable claps came into use due to time constraints. They were hidden within the aluminium 'bends' for aesthetic reasons. The cables that cross each other in the middle of one module may also be connected with each other and with the wood.(see fig. 125) It would be interesting to investigate new designs with the other Platonic solids as bounding geometry.

The presented work was possible due to the knowledge acquired during the studio sequence 'Versatile spaces'. In addition, the skills accumulated during this time accelerated the building process. It was a blissful experience to build our knowledge regarding minimal surfaces in architecture gradually. A similar comparison for this experience would be the process of stabilizing the modules. The crumbled, vapid aluminium frames with an edge length of 2.2m were reshaped into the planned geometry by adding the bracing.

7 List of abbreviations

fig.	figure
R^3	three-dimensional space
kg	kilograms
m^2	square meter
m	meter/s
cm	centimetre/s
mm	millimetre/s
K	Gaussian curvature
H	Mean curvature
r	radius
k	curvature
k_1	principle curvature in the first direction
k_2	principle curvature in the second direction
MAX	maximum
MIN	minimum
SS	summer semester
WS	winter semester
$\in R$	is an element of the set of real numbers
$\in Z$	is an element of the set of integers
3D	three dimentional
$\iint$	double integral
dA	differential with respect to area

8 List of figures