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Oberseminar im Institut für Angewandte Analysis

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Im Rahmen des Oberseminars findet am Montag, den **13. Januar 2025** folgender Vortrag statt:

Artur Stephan

On time-splitting methods for gradient flows with two dissipation mechanisms

A gradient system $(X, \mathcal{E}, \mathcal{R})$ consists of a state space X (a separable, reflexive Banach space), an energy functional $\mathcal{E} : \mathrm{dom}(\mathcal{E}) \ \subset X \to \mathbb{R} \cup \{\infty\}$ and a dissipation potential $\mathcal{R} : X \to [0, \infty[$, which is convex, lower semicontinuous and satisfies $\mathcal{R}(0) = 0$. The associated gradient-flow equation is then given by

$$0 \in \partial\mathcal{R}(u'(t)) + \mathrm{D}\mathcal{E}(u(t)) \quad \text{or equivalently} \quad u'(t) \in \partial\mathcal{R}^*(-\mathrm{D}\mathcal{E}(u(t))).$$

In my talk we are interested in the case where the dual dissipation potential $\mathcal{R}^*$ is given by the sum $\mathcal{R}^* = \mathcal{R}_1^* + \mathcal{R}_2^*$ for two dissipation potentials $\mathcal{R}_j : X_j \to [0, \infty[$, $X_j \subset X$. This splitting provides also a decomposition of the right-hand side of the combined gradient-flow equation $u'(t) \in \partial(\mathcal{R}_1^* + \mathcal{R}_2^*)(-\mathrm{D}\mathcal{E}(u(t))) = \partial\mathcal{R}_1^*(-\mathrm{D}\mathcal{E}(u(t))) + \partial\mathcal{R}_2^*(-\mathrm{D}\mathcal{E}(u(t)))$, and enables to construct solutions via a split-step method. For this let $\tau = T/N$ define a uniform partition of the interval $[0, T]$. Starting from an initial datum $u_0 \in \mathrm{dom}(\mathcal{E})$, we define a piecewise constant time-discrete solution $U_\tau : [0, T] \to \mathrm{dom}(\mathcal{E}) \subset X_1$ by setting $U_\tau(0) = u_0$ and by performing the *Alternating Minimizing Movement* of the form

$$\begin{aligned}
&U_\tau(t) = U_k^1 \text{ for } t \in](k-1)\tau, (k-1/2)\tau], \quad U_\tau(t) = U_k^2 \text{ for } t \in](k-1/2)\tau, k\tau] \\
&\text{where } U_k^1 \in \mathrm{Argmin}_{U \in X_1} \{ \tfrac{\tau}{2} \mathcal{R}_1(\tfrac{2}{\tau}(U - U_{k-1}^2)) + \mathcal{E}((k-1/2)\tau, U) \} \\
&\text{and } U_k^1 \in \mathrm{Argmin}_{U \in X_2} \{ \tfrac{\tau}{2} \mathcal{R}_2(\tfrac{2}{\tau}(U - U_k^1)) + \mathcal{E}(k\tau, U) \}.
\end{aligned}$$

In my talk I will show how the curves U_τ indeed converge to the solution of the combined gradient-flow equation (involving $\mathcal{R}^* = \mathcal{R}_1^* + \mathcal{R}_2^*$) in the limit $N \to \infty$. The analysis relies on methods from the calculus of variations, and the usage of the energy-dissipation principle for gradient flows.
This is joint work with Alexander Mielke (Berlin) and Riccarda Rossi (Brescia).

Der Vortrag findet in **Raum E20, Helmholtzstr. 18** statt.
Beginn: 16 Uhr (c.t.). Alle Interessierten sind herzlich eingeladen.

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