Entanglement of massive, charged particles via a lossy conductive wire

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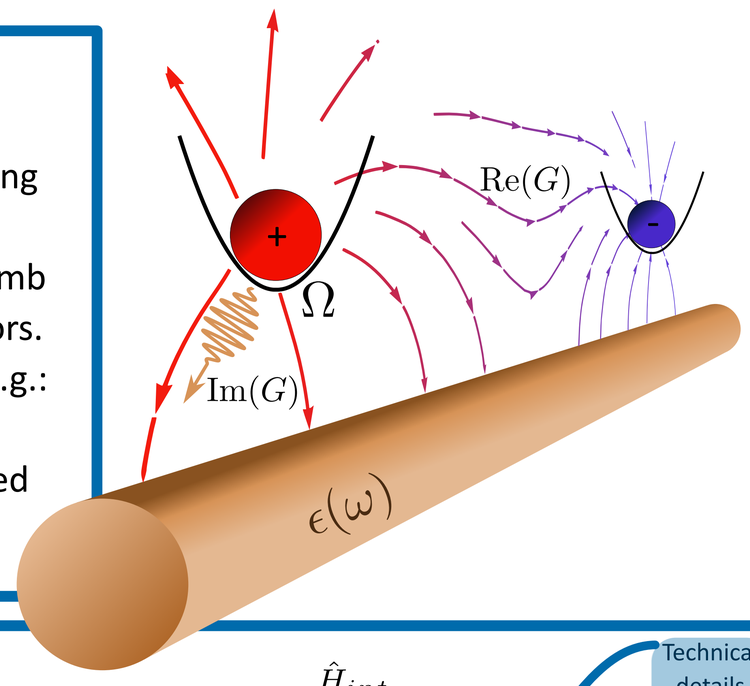
Motivation & Introduction

Gravity and the Coulomb force are both **central forces**. Therefore, generating **entanglement in mesoscopic bodies** with the latter is a first crucial step towards testing the quantumness of the former [1, 2]. In free space, Coulomb coupling decays as r^{-3} making this challenging especially with bulk resonators. Here we propose to **use a wire to enhance its range and tunability** (with e.g.: variable resistor elements). Although our results are general, we use parameters for levitated optomechanics where the preparation of entangled states would be a especially coveted milestone.



Parameter		Value
Conductivity	σ	59.6 MS/m
Wire radius	R	6 μm
Wire distance	d	0.9 μm
Particle charges	q	100 e
Particle Masses	m	2×10^{-18} kg
Trapping frequency	Ω	$2\pi \times 100$ kHz
Temperature	T	1.0 μK

Theory

Quasielectrostatic Hamiltonian: $\hat{H} = \overbrace{\hat{H}^{(1)}_{h.osc.} + \hat{H}^{(2)}_{h.osc.} + \hat{H}^{(12)}_{Coulomb}}^{\hat{H}_S} + \hat{H}_{wire} + \overbrace{\sum_i q_i \hat{\phi}_{wire}(\hat{r}_i)}^{\hat{H}_{int}}$

Green's function solves the electrostatic boundary condition problem with wire permittivity $\epsilon(\omega) = \epsilon_0 + i\frac{\sigma}{\omega}$

$\nabla \cdot \epsilon_0 \epsilon(\mathbf{r},\omega) \nabla g(\mathbf{r},\mathbf{r}',\omega) = -\delta(\mathbf{r}-\mathbf{r}')$ and the **Green's tensor** $\mathbf{G}(\mathbf{r},\mathbf{r}',\omega) \approx -\frac{1}{\mu_0\omega^2}\frac{\partial}{\partial \mathbf{r}} \otimes \frac{\partial}{\partial \mathbf{r}'} g(\mathbf{r},\mathbf{r}',\omega)$

Technical details

Macroscopic quantum electrodynamics (MQED):

Build lossy quantum field using Green's tensor on quantized fluctuating sources. [3]

$\hat{\phi}(\mathbf{r},\omega) = \int d^3\mathbf{r}' (\nabla' g(\mathbf{r},\mathbf{r}',\omega)) \hat{\mathbf{P}}_n(\mathbf{r}',\omega)$

Tracing out the wire, leaves us with the following **master equation:**

$$\frac{d\hat{\rho}}{dt} = -\frac{i}{\hbar}\left[\hat{H}_S + \sum_{ij}\delta\hat{H}_{ij}, \hat{\rho}\right] + \underbrace{\sum_{i,j\in\{1,2\}}\left(-\frac{\Gamma_{ij}}{2}\left[\hat{x}^i,\left[\hat{x}^j,\hat{\rho}\right]\right] + i\mathcal{N}_{ij}\left(\mathcal{D}_{\hat{x}^i\hat{x}^j}\hat{\rho} - \mathcal{D}_{\hat{x}^j\hat{x}^i}\hat{\rho}\right)\right)}_{\text{Decoherence due to wire}}$$

modified Hamiltonian; Lindblad dissipators

$$+\sum_i\left(\bar{n}\gamma\mathcal{D}_{\hat{b}_i^\dagger\hat{b}_i}\hat{\rho} + (\bar{n}+1)\gamma\mathcal{D}_{\hat{b}_i\hat{b}_i^\dagger}\hat{\rho}\right)$$ ← thermal damping due to other sources (i.e.: intrinsic damping of the oscillators)

Master equation rates

- $\Gamma_{ij} = 2\frac{q_i q_j x_{0,i} x_{0,j}}{\hbar}\bar{n}\,\mu_0\Omega_j^2\,\underbrace{\mathrm{Im}(G(\mathbf{R}_0^i,\mathbf{R}_0^j,\Omega_j)}_{\text{losses}} = \bar{n}\Gamma_{ij}^0$
- $\mathcal{N}_{ij} = \frac{q_i q_j x_{0,i} x_{0,j}}{2\hbar}\mu_0\Omega_j^2\,\underbrace{\mathrm{Re}(G(\mathbf{R}_0^i,\mathbf{R}_0^j,\Omega_j))}_{\text{image charge response}}$
- $\delta\hat{H}_{ij} = \underbrace{\frac{\hbar}{4}\Gamma_{ij}^0\frac{1}{2}\left(\hat{x}^i\hat{p}^j + \hat{p}^j\hat{x}^i\right)}_{\text{squeezing term}} - \underbrace{\hbar\mathcal{N}_{ij}\,\hat{x}^i\hat{x}^j}_{\text{frequency shift and image-charge interaction}}$

Entanglement

Quadratic master equation implies **Gaussian** evolution.

For Gaussian states, the **Simon-Duan criterion** gives a **necessary and sufficient** condition for entanglement [4].

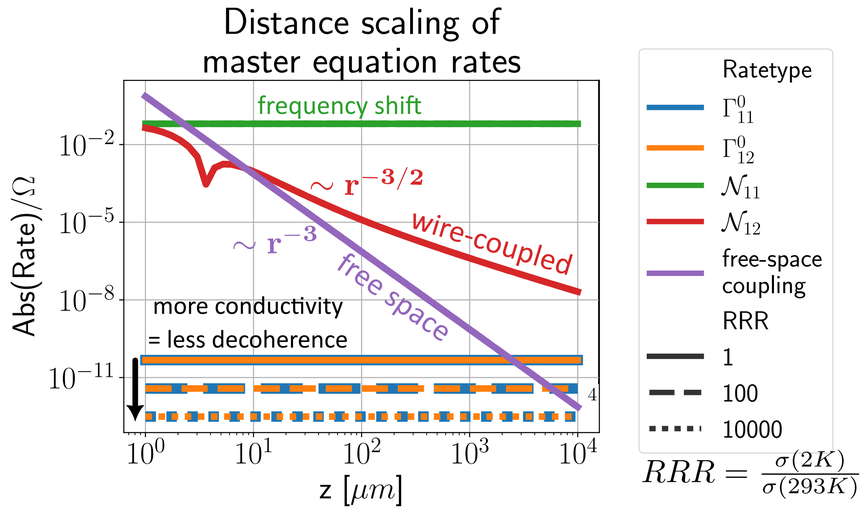
$C_{jk} = \frac{1}{2}\langle(S_j - \langle S_j\rangle))(S_k - \langle S_k\rangle)) + (S_k - \langle S_k\rangle))(S_j - \langle S_j\rangle))\rangle,$

where $\vec{S} = (\hat{x}_1, \hat{p}_1, \hat{x}_2, \hat{p}_2)$

Entanglement $\Leftrightarrow$ $\nu < 1/2$,
where ν is the symplectic eigenvalue of C_{jk}.

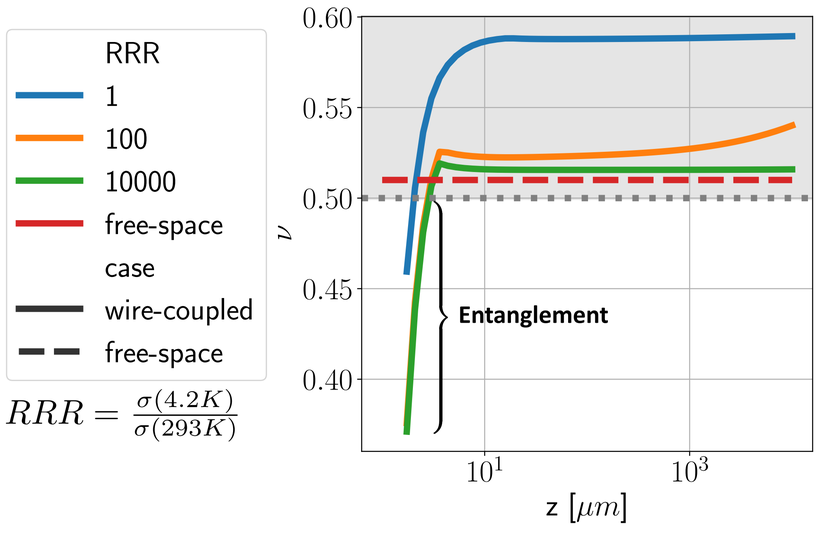
Variable	Description
$i = 1,2$	Particle index
$\boldsymbol{r}, \boldsymbol{r}'$	Position vectors
$\hat{x}_i, \hat{p}_i$	Position/ momentum quadratures
$\hat{b}_i, \hat{b}_i^\dagger$	Corresponding Ladder operators
$\bar{n}$	Mean BE-occupation number
γ	Thermal damping rate
$x_{0,i}$	zero-point motion
q_i	charge
R_0^i	Equillibrium position (with wire)

Distance scaling of master equation rates

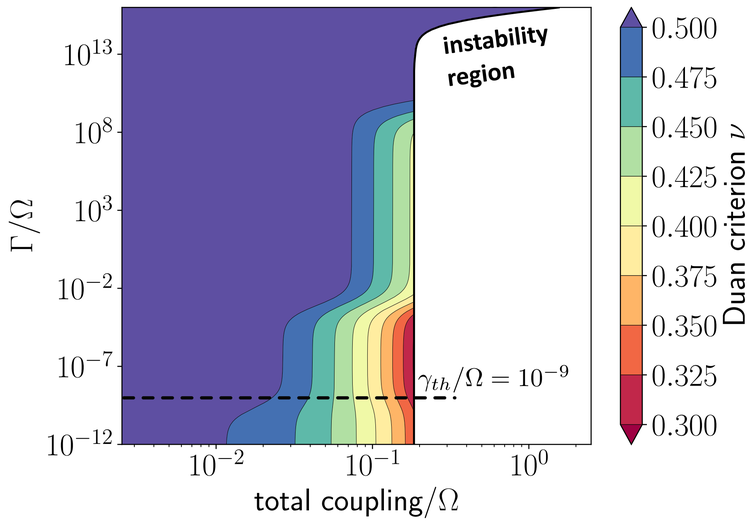


Results

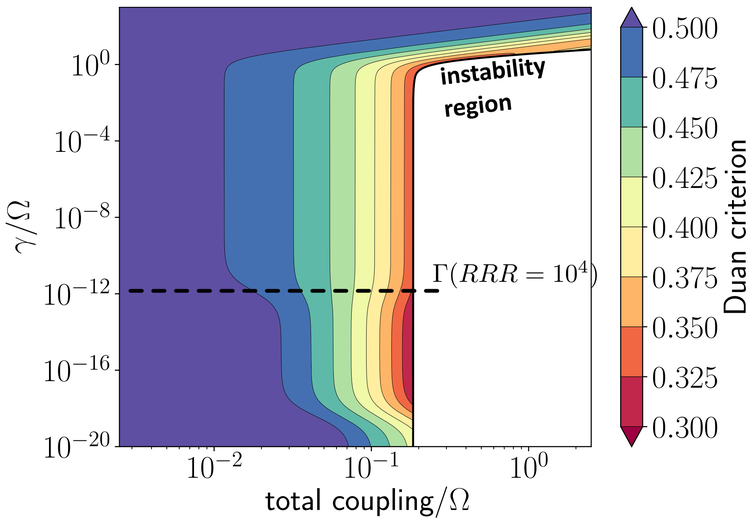
Steady state entanglement over distance (calculated using Green's function)



Steady state entanglement for different decoherence and coupling rates



Steady state entanglement for different thermal damping and coupling rates



Conclusion & Outlook

The **decoherence rate** Γ due to the wire can be made **smaller than the wire mediated coupling rate** $\mathcal{N}$, and when it is also smaller than the thermal damping the **effect on entanglement is negligible**.

The wire helps with entanglement and increases the range of the interaction.

In the future, circuitry could be refined for entanglement over longer range.

References

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Contact & Funding

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