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A Rolling Investment Approach for the Stepwise Implementation of District Heating Grids

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Table of Contents

Abstract

The expansion of district heating (DH) grids represents significant capital investments and requires strategic planning to optimize return on investment over the depreciation time. A critical challenge in DH grid expansion is determining the most economically viable sequence for gradually connecting new subareas – smaller areas in a certain predefined area, for which a plan of district heating grid expansion exists – while considering real-world constraints, such as workforce availability and annual investment limitations.

The primary research question addressed in this thesis is: *"What is the most economical sequence for expanding district heating networks within a predefined area, given constraints on annual pipeline construction lengths?"* This work aims to develop a methodology that optimizes the phased expansion of district heating infrastructure, ensuring profitability while balancing financial and construction constraints.

To achieve this goal, an optimization model based on Mixed-Integer Linear Programming (MILP) was developed to simulate and determine the optimal sequence of DH grid expansion. The model takes into account annual cash flow and workforce availability based on which net present value (NPV) and levelized cost of heat (LCOH) are calculated and evaluated. The model was implemented using Python with the CVXPY optimization framework and Gurobi solver. An exemplary case study was conducted on a part of the district heating grid in the Romanian city of Brașov, where different pipeline expansion rates were tested to analyse their economic impact.

The results demonstrate that there exists a lower limit for annual pipeline expansion below which the investment is not profitable within the examined timeframe. The NPV analysis indicates that slower expansion leads to delayed profitability, while higher annual expansion rates yield greater long-term financial benefits but require a larger upfront investment. The LCOH results confirm that increasing the expansion rate reduces heating costs due to economies of scale.

This research provides a novel contribution by addressing the gap in DH planning literature related to sequential investment under practical constraints. The findings highlight the importance of strategic scheduling of grid expansion to ensure financial viability, offering valuable insights for policymakers, urban planners, and investors in district heating infrastructure.

Abbreviations List

Abbreviation	Meaning
DH	District Heating
NPV	Net Present Value
LCOH / LCODH	Levelized Cost of Heat / Levelized Cost of District Heating
MILP	Mixed-Integer Linear Programming
CVXPY	Convex Optimization Modelling Tool in Python
LP	Linear Programming
DP	Dynamic Programming
GIS	Geographic Information System
CHP	Combined Heat and Power
CF	Cash Flow
EUR	Euro (currency)
MWh	Megawatt-hour
mL	Maximum Pipeline Length built per Year
DN	(fr. Diamètre Nominal) Nominal Diameter
FGW	Fachverband der Gas- und Wärmeversorgungsunternehmungen (en. Austrian Association of Gas and District Heating Supply Companies)

1. Introduction

District heating (DH) is seen as a critical investment in reaching goals of decarbonising the heating sector to achieve the C02 neutrality goals of EU until 2050 [1]. There have been significant scientific endeavours contributing to the topic of planning the extension of DH grid and its profitability in different areas [1]. None of them filled the scientific gap of planning the building process gradually, as it should be, over the course of several years.

This thesis investigates the most profitable way of sequentially connecting the DH grid in an area where the plans for DH infrastructure already exist [2], accounting for staged investment decisions over multiple time steps.

While district heating expansion is a well-researched topic, previous studies have primarily focused on large-scale system design without incorporating the rolling investment constraints that characterize real-world infrastructure projects [11]. Several studies analyse the potential for district heating grid expansion across Europe, highlighting the economic and policy factors that drive its adoption. Kranzl and Fallahnejad et. al. [13] assess the district heating network potential in the EU-27, emphasizing that the feasibility of expansion depends on factors like heat demand density, energy efficiency measures, and infrastructure costs. These findings underline the importance of developing optimized expansion strategies that align with marked conditions and available resources.

A large body of research has explored optimization models for DH network planning, primarily through Mixed-Integer programming approaches. For instance, large-scale district heating network optimization models focus on minimizing investment and operational costs while determining the optimal layout of new infrastructure [2]. However, these models often assume a single-stage investment process, whereas real-world DH network expansion occurs incrementally due to budgetary, workforce and technical constraints. Other studies introduce nonlinear approaches to account for thermal and hydraulic constraints in DH network expansion. These models incorporate heat loss calculations, pressure drop effects, and network flow dynamics, improving physical accuracy but significantly increasing computational complexity [14]. While such approaches provide detailed system behaviour insights, they are less suited for long-term investment planning under financial and construction constraints.

Another important distinction in the literature is the focus on capacity expansion vs. network expansion. Some studies optimize energy centre operations, determining the best configuration of heat production assets such as CHP plants (Combined Heat and Power), boilers, and storage systems. While capacity planning is crucial for operational efficiency, it does not address the sequential expansion of physical DH pipelines under manpower and financial constraints [16].

A more relevant set of studies explores phased network expansion strategies, such as stochastic optimization of DH network investments [11]. These works integrate uncertainty into decision-making, allowing for flexible investment phasing based on future demand predictions and energy price fluctuations. However, these approaches introduce higher computational demands and are less structured for deterministic long-term planning.

This study contributes to the field by developing a Mixed-Integer Linear Programming (MILP) model for rolling investment planning of DH networks. Unlike previous static optimization models, this approach accounts for manpower limitations, staged investment strategies, and construction pacing, ensuring that the expansion process aligns with real-world financial and logistical constraints. By integrating constraints related to annual pipeline construction limits and investment phasing, the model provides a practical framework for maximizing cash flow and therefore calculating Net Present Value (NPV) while ensuring the feasibility of sequential network expansion.

The case study presented in this work focuses on the city of Brasov, Romania, one of six pilot cities analysed within the EU Horizon 2020 project progRESsHEAT. The project aimed to support long-term decarbonization strategies for heating and cooling systems in European municipalities. In Brasov, a detailed local energy strategy was developed based on local energy demand, infrastructure, and stakeholder input [32].

The Python Code made for in scope of this work is reachable at following link: https://github.com/Marin1919/MarinMatic_MasterThesis_FinalCode.

1.1 Motivation

District heating (DH) is expected to play a key role in decarbonizing the building sector in European cities. According to the DECARB21 study [24], Vienna aims to achieve climate neutrality by 2040, with district heating projected to cover approximately 56% of the city's total low-temperature heat demand. To reach this target, a coordinated transformation is required—combining investments in renewable heat production technologies (e.g., deep geothermal, large-scale heat pumps), the phasing out of fossil fuels, the refurbishment of the building stock, and the expansion of the DH network. The latter is a major infrastructure challenge: approximately

€750 million is allocated for expanding the DH grid alone, as part of a broader €2.5 billion investment into Vienna's DH system.

Although Vienna already has approximately 1,200 km of district heating grid, making it one of the most developed cities in Europe in this field, further expansion is explicitly planned. As outlined in the Phasing Out Gas Heating and Cooling Vienna 2040 strategy, this includes both densification of the existing network and targeted extensions to ensure DH coverage in all relevant urban areas by 2030 and 2040 [25].

Motivation for this thesis is based on the research gap in existing research on district heating (DH) network expansion. Specifically, prior studies, such as the work of Fallahnejad et. al. [1], have largely focused on assessing the economic feasibility and expansion potential of DH networks but have not considered the practical constraints associated with incremental expanding of district heating grid over multiple years. In reality, district heating grids require a phased approach to expansion due to construction logistics, financial limitations and most importantly – workforce constraints.

Current DH expansion models focus more on a static investment strategy and finding the best ways to expand the district heating grid, but do not consider the last part of investment planning, which is scheduling the building process over a course of investment period and examining it from the economical point of view. Expansion of DH grid is driven by various dynamic factors, including limited availability of qualified workforce, seasonal restrictions on construction, and constraints on annual investment capacity, all summarized in the "Maximal length per timestep" constraint explained in the Chapter 3 "Methodology". The ability to optimize the sequence of grid expansion while accounting for these constraints has not been sufficiently examined in the literature. This thesis addresses the described research gap by developing an optimization model for rolling investment approach to expand the DH grid that integrates workforce availability constraints into decision making process. The proposed model explicitly considers the maximum feasible pipeline construction length per year, based on the available workforce and construction capacity. This ensures more practical and implementable strategy for DH infrastructure development and ensures planning that aligns with real-world constraints. By incorporating these elements, this work contributes to improving the planning methodologies for DH grid expansion, allowing the optimization of investment schedules in an economically viable way. The findings of this research are particularly relevant in contexts where labour shortages or financial constraints necessitate a gradual, stepwise approach to infrastructure expansion.

1.2 Research question and objective

The research question of this work is:

“What is the most economical sequence for expanding district heating networks within a predefined area of examination, given constraints on annual pipeline construction lengths?”

As discussed in Chapter 1.1 (Motivation), the annual limit on construction length reflects a combination of workforce availability, investment capacity, and seasonal restrictions. If any of these factors are reduced, the amount of pipeline that can be installed each year decreases accordingly. On the other hand, the term “most economical” in this context refers to optimal phasing of expansion that maximizes the sum over all cash flows each year and calculates Net Present Value to determine if a given amount of pipeline makes profits over the investment periods. As shown in Chapter 4. Results, it will also be determined, under which approximate amount of pipeline built per year the whole investment should appear as unprofitable over the period of 30 years. In other words, this work seeks to identify the optimal strategy that balances profitability, network coverage, and resource constraints over time. Predefined area of examination refers to the area where the plan is defined on how the DH grid should be constructed, meaning it is not expected to expand over the borders of this defined area, or examine other potential areas based on population density, as do other works. This area is characterized by an existing heat demand, predefined potential network routes, costs and types of pipelines as well as digging costs, generation and distribution costs.

Based on the research question outlined above, the primary aim of this study is to design and implement methodologies for exploring and analysing the most economical way of district grid expansion sequential construction process and to investigate the potential district heating grid investment for a case study of a district in the city of Brasov, Romania.

This aim is reached by developing an optimization model that finds the most profitable sequence of connecting the smaller areas of the whole researched area to the source of district heating, in such manner that the earnings from smaller areas - in further text also called “subareas” – are maximized. The baseline of this model is to connect first the most lucrative subareas, taking in consideration that these are able to make most profit over the course of investment period. In the end, there are different investment strategies compared, based on different maximal pipeline length available each year for all of which the Net Present Value (NPV) and Levelized Cost of Heat (LCOH) is calculated and examined.

1.3 Structure of this work

This thesis is structured to present a comprehensive overview of the development, implementation, and application of an optimization model for District Heating grid expansion. Each chapter builds upon the previous one, presenting the problem, methodology, and results in a logical progression.

Chapter 1: Introduction outlines the motivation behind this research and defines the key research question: determining the most economical sequence for district heating grid expansion under pipeline construction constraints.

Chapter 2: State of the Art reviews existing methodologies and tools for planning district heating networks, identifying gaps that this work seeks to address, particularly in the incremental development of DH grids.

Chapter 3: Methodology describes the technical approach to solving the problem, including key assumptions, simplifications, and the optimization model itself. It also covers the tools used, such as Gurobi and CVXPY, and details the input and output data preparation.

Chapter 4: Results present the outputs of the model, including a detailed analysis of net present value (NPV) trends for different construction scenarios, providing insights into the economic impact of different annual pipeline construction limits.

Chapter 5: Conclusion summarizes the findings, discusses the broader implications of the work, and suggests directions for future research in district heating grid planning.

2. State of the art

The following chapter reviews current methodologies, technologies, and models employed in the economic and technical assessment of DH systems. However, the existing studies often overlooked the scientific gap of planning and analysing the investment process of DH grid in the investment period. The motivation for planning and expanding DH grids in Europe are the EU's 2050 climate neutrality goals, where decarbonisation of the heating sector plays a crucial role [1]. Gaballo et al. [9] evaluated the role of DH in achieving a carbon-neutral energy system by 2050, demonstrating the fact that district heating share of total heat demand could increase from 8% to 40% through coordinated system expansion and sector coupling, focusing particularly on leveraging waste heat from electrolysers.

2.1 Optimization Approaches for DH expansion planning

Various optimization models, such as the one developed by Dorfner and Hamacher [2], focus on determining the cost-optimal structure and size of district heating (DH) grids for a certain area. These models employ Mixed-Integer Linear Programming (MILP) approaches for optimization of the pipe routes and dimensions based on heat demand and cost parameters given in this area. The DHMIN model, a notable open-source tool, builds on these methods, enabling detailed techno-economic analyses for planning of district heating grids. DHMIN optimization model can also be useful for planning potential DH grid expansions.

The effective width concept, introduced by Persson and Werner [3], estimates DH grid costs based on demographic data. It provides a simplified method to assess investment costs in areas without existing DH infrastructure by considering the ratio between land area and trench length. This method is particularly effective for pre-feasibility studies but has limitations in handling detailed network configurations. Persson and Werner [17] further developed this approach into an analytical model to estimate DH distribution capital costs using spatial and demographic indicators like population density and effective width. Their study shows that DH remains cost-effective in dense urban areas but becomes economically unviable in low-density zones. These insights are particularly relevant for a rolling investment approach, where distribution costs significantly influence long-term profitability.

Fallahnejad et al. [4] compared the effective width approach to detailed optimization models (e.g., DHMIN). While both approaches deliver similar patterns in areas with varying heat densities and plot ratios, optimization models like DHMIN offer more precise results by incorporating supply-side data and detailed spatial configurations, whereas DHMIN has significantly more CPU demand compared to the effective width approach. Being more precise, DHMIN approach to modelling DH grids also requires more extensive set on input data, that may not always be available, especially for rural areas. This highlights the trade-offs between simplicity and precision in different modelling approaches.

Fallahnejad et al. [18] proposed a GIS-based method to assess DH expansion potential by applying cost ceilings for both distribution and transmission lines. Their model distinguishes between infrastructure types and uses optimization to identify cost-effective network extensions based on heat density and market share assumptions. While insightful for constrained planning, it depends heavily on externally defined cost limits and spatial inputs. In contrast, the rolling investment model in this thesis does not impose cost ceilings but evaluates investment outcomes across pre-defined routing scenarios, offering flexibility in long-term cost analysis.

A recent study by Lambert and Spliethoff [12] introduces a nonlinear optimization method for district heating systems, incorporating graph preprocessing to reduce computational effort. This method optimizes the network's topology, pipe sizing, and operational parameters, achieving fast convergence using an interior point algorithm. Applied to a small fictional town, the method reduced the network from 72 to 69 pipes and completed the optimization in 19.37 seconds. The results demonstrate the efficiency of the method in minimizing investment and operational costs, suggesting its potential for larger district applications and improved discrete pipe sizing. Again it misses to fill the gap – as do all other previous mentioned works – of scheduling the investment and construction works based on real life constraints.Chicherin et al. [21] developed a GIS-based MILP model for district heating network planning that minimizes infrastructure costs while accounting for spatial and thermal constraints. Their approach integrates pipe sizing, routing, and investment optimization across different network components, using a scenario-based decision-support system. While the model effectively optimizes static network layout under real-world conditions, it does not address sequential pipeline construction or workforce-related investment constraints. The model presented in this thesis complements this approach by introducing time-phased investment planning that aligns network expansion with yearly implementation limits and real-world construction feasibility.

2.2 Static vs. sequential investment approaches

Existing studies often model static heat demand scenarios, assuming that investment occurs in a single-stage process. However, to provide a more realistic understanding of DH grid feasibility, works such as Fallahnejad et al. [1] have introduced dynamic considerations, such as evolving market shares and energy-saving measures over the investment horizon. On the other hand, these approaches did not consider the planning of successive investment in the building of the DH grid, based on human resources constraints, which is thoroughly researched in this work.

Some models attempt to optimize both District heating network expansion and heat production capacity, but they do not take into account investment constraints related to workforce and budget limitations. For example, Model Predictive Control (MPC) frameworks, such as those presented in dynamic energy system models, apply optimization to long-term investment planning of energy centres, optimizing heat production and storage capacities [16]. Although models as MPC allow for adaptive decision making, they need much more computing efforts and are not designed for network expansion problems involving sequential pipeline construction over the investment period.

In contrast, this thesis employs a MILP-based rolling investment model, ensuring that network expansion decisions align with pipeline construction constraints, such as manpower and investment management, in form of financial resources, while maintaining computational feasibility. By focusing on stepwise network expansion, this study addresses a gap in the literature – how to optimize the phased deployment of DH infrastructure under real-world investment limitations.

Sequential investment approach gives important insight into the upfront investment amount, which is valuable to make a decision on building the infrastructure in different areas. Büchele et al. [19] examined the modernization of the DH system in Brașov, Romania and found that only integrated policy packages—combining long-term planning support, infrastructure investment subsidies, and fossil fuel disincentives—can effectively restore and expand DH in aging networks. They stress that for such policies to succeed, detailed knowledge of network investment costs is essential. The model developed in this thesis contributes to this need by providing transparent, scenario-based assessments of investment costs over time, helping decision-makers evaluate which policy mixes are economically justifiable for DH development.

While strategic planning approaches such as Büchele et al. [20] provide valuable high-level guidance for defining DH-suitable zones and estimating supply mixes, they lack the resolution required for detailed investment or infrastructure rollout planning. The authors themselves note that their simplified GIS-based analysis cannot reflect real connection costs at the building level or guide step-by-step network expansion. Building on this, the model presented in this thesis operates at the level of construction phases, enabling the estimation of upfront investment needs over time. By incorporating workforce constraints and cost accumulation per scenario, it bridges the gap between strategic heat planning and practical investment decision-making.

2.3 Different optimization goals and constraints

Another prominent approach is presented by Roland and Schmidt [14], who developed a Mixed Integer Non-linear Programming (MINLP) model for district heating network expansion. Their model focuses on detailed hydraulic and thermal behaviour through full non-linear physical equations, focusing on static expansion of three-shaped networks. In this way it also decides the future of expansion based on high physical accuracy, especially in modelling pressure losses, thermal losses, and energy flows, however it does not address rolling investment planning over multiple periods or consider manpower constraints.

Further relevant contribution to district heating network expansion modelling is provided by Delangle et. al. [15], who developed a MILP based optimization framework for a marginal expansion of an existing DH network. Their model optimizes both the network spatial growth and the design of the associated energy center, focusing on either maximizing cost savings or minimizing greenhouse gas (GHG) emissions. Their model is running on an investment period of 12 years, offering detailed insights into the interactions between expansion strategies and technology operations under different policy and market conditions. Still, as other previously mentioned models, it misses to fill the gap on construction constraints and manpower constraints, which are taken into consideration in the model built in terms of this work.

While some of the models also predict and take in consideration the changes in heat energy demand over the years, the model developed in this work does not take any changes in consideration but rather focuses solely on investment construction. Opposed to the optimization model developed by Delangle et. al. [15] this work focuses on optimization maximizing cash flow (CF) for each investment step and not maximizing net present value (NPV), which is only calculated after the CF optimization to evaluate which of scenarios would be feasible.

Case studies, such as the DH network optimization in Munich, Germany [2] and Brasov, Romania [4] have demonstrated the applicability of both effective width and detailed optimization modelling method, more precisely DHMIN. These studies underline the importance of customizing model parameters to the specific geographical and economic conditions of the area under analysis. Both approaches give a base to the following optimization method, developed for this research, which observes the incremental connection of district heating subareas to the district heating grid. The base for this work is the optimal plan for DH grid provided as output from one of above-mentioned methods.

3. Methodology

Three important considerations are singled out in this chapter to provide comprehensive overview of the methodology used to develop the model and address the challenges associated with rolling investment in district heating grids. These are firstly assumptions, simplifications and limitations, secondly the model itself and last the input data preparation.

As each engineering endeavour must strike a balance between efficiency and precision, so was also the case in this work of developing an optimization model to plan an investment for district heating grid expansion over the years. There are surely more than many ways to approach the same problem, where the goal should be not to over-engineer it. More precisely, the engineering challenge lies in tailoring the model to provide actionable insights without overcomplicating or overloading the analysis.

This chapter will go on to establish the foundation for the analysis by defining the key assumptions and simplifications that define the scope of the work as mentioned earlier, not to overcomplicate the task for its real purpose. Furthermore, it is going to explain and present the Mixed-Integer linear programming model (MILP), which is designed to optimize the grid expansion and define the sequence of connection for all the subareas in the observed district heating region. In the subchapter 3.3 the input data preparation is explained together with *cvxpy* library used for optimization in the python code prepared in scope of this work.

Finally, the method is applied to an area of the city of Brasov in Romania, where a detailed sensitivity analysis is made based on four scenarios of different annual pipe-building capacities. Another goal of the analysis was to try to find an approximate border of unprofitability for this investment. More precisely, the amount of pipes built annually, for which the investment is unprofitable over the course of investment period has been defined based on the sensitivity analysis in the case study in Chapter 3.4.

3.1 Key assumptions, simplifications and limitations

Defining key assumptions and simplifications narrows the scope of the problem solving making the model more precise in optimizing the solution for an exact set of potential problems. This chapter outlines the key assumptions and simplifications concerned in creating the model in such way that it works as precisely as possible, not using incredible amounts of CPU capacity.

3.1.1 Maximal length constraint

This model is created with a goal of making sensitivity analysis on input information from a part of the DH grid in Brasov, Romania. The sensitivity analysis is conducted on the main parameter, which in this case is the most amount of DH pipes that is possible to build in each step of a time, in this model assumed as a year. In the model definition this parameter is called “maximal length per period”.

Because the model is created using MILP to deal with sequential problem, the model gets a discrete nature, meaning the decision variables are set and changed at the end of each sequence and are prepared for the next one. This may lead to some problems considering accuracy, because no two subareas that are connected in a row cannot be connected in one time step. This is because, as already mentioned, the decision variables are set at the end of each time step, which means that the second subarea in this row would “see” the first one connected only at the end of the step, although the connection of both may fit into the limits of “maximal length per period” in one step. This problem is tackled with the “resolution factor” explained in the next chapter 3.1.2.

3.1.2 Discrete time steps and resolution factor

The accuracy of model results is restricted by the discrete time steps, which is usual for linear programming [5]. A subarea is considered active and is seen as a consumer in the next step after the whole edge that connects it has been built. An edge is in the simplified form a pipe that connects two subareas or a subarea directly to the source of energy. Energy can be provided to the subarea either directly, if it is connected directly to the source of energy, or through a connection to another active subarea. The model takes as input further information about edges, that present pipelines and nodes that present subareas. Input data is explained more thoroughly in the chapter 3.3.

The resolution factor is a parameter defined in this work that makes it possible to define the resolution of the model. It allows the real step to be broken into more smaller steps, each having its own maximal length per period that equals the initial maximal length per period, or step divided by the resolution factor. In this manner the connected demand per step is also divided by the resolution factor, whereas the number of initial steps is multiplied by the resolution factor. This method allows the model to connect more than one subarea in a row, when it is possible – setting the decision variables not once in each initial step, but resolution factor times more.

3.1.3 Residual length

Maximal length per period is set as the maximal amount of pipe that is possible to build in each time step.

In this model subareas are assumed as complete units, which one reached with the pipelines become active and consume energy, therefore create income. The internal distribution grid within each subarea is not considered in this work.

As it is a sequential model solved by MILP method, it could be assumed as more non-sequential MILP models for each iteration or time step, tied together with input information that each iteration sets as input for the next one. The most important in this case would be residual length per period, which equals the maximal length per period subtracted by the amount that was in fact built in a certain period or time step.

This means that there is no possibility for an edge or pipeline to be partly built, but there is residual length as a parameter that regulates the equation. It also means that in some steps in the model it is possible that there is more than maximal length per period being built, but in that case, there needs to be an illusory deficit in the amount of pipeline built in the previous step. This is the way model interprets the maximal length per period. It is also a obvious example of the borders between Dynamic Programming and Mixed-Integer Linear Programming. In the case of MILP, it is not possible to predict what would be built in the next step and build a part of that edge. Residual length is in this manner the most significant intermediary between two steps, enabling this MILP model to solve a sequential problem.

3.1.4 Limitations

While the model effectively addresses the sequential optimization of district heating grid expansion, it has certain limitations. The use of MILP restricts the granularity of decisions within each time step, and the resolution factor increases computational complexity.

Furthermore, the model does not account for dynamic changes in heat demand or variations in construction costs over time. These limitations, while manageable, should be considered when interpreting results or applying the model to other case studies.

3.2 Model overview

The problem that this work tries to solve lies on the edge of Mixed-integer Linear Programming (MILP) and Dynamic Programming (DP) solutions. Both are mathematical approaches to solving an optimization problem, where MILP is more suitable for problems with linear relationships and constraints that area global and interconnected and DP is used for problems with a sequential, stage-wise nature and optimal substructure.

Based on this fact, the problem solved in this work could possibly be more efficiently solved with DP method, nevertheless it is modelled as a MILP. This choice is based on the specific requirements of planning a rolling investment, where the model needs to account for global constraints across multiple time steps. The structure of the MILP model, particularly its sequential, multi-period formulation, draws conceptual inspiration from classical examples presented by Williams [23], especially those involving block-angular structures used in investment and production planning. While the problem lies at the intersection of MILP and DP approaches—being complex enough to challenge DP but manageable with MILP—MILP was chosen due to its ability to handle interconnected constraints and decision variables effectively. Although MILP can be computationally intensive, its robustness makes it well-suited for this scenario, where the model is intended to be run infrequently for strategic planning.

This work uses a MILP method for solving a sequential, stage-wise nature problem of connecting a set of subareas as part of a larger area to the heat source in such manner that the profits are maximized in each observed time step with regard of constraints such as manpower.

It basically uses MILP method to solve a sequence of smaller non-sequential problems, setting the initial values and input parameters for the next iteration.

In this chapter all components of the optimization model are described, mathematical formulas for objective function and constraints are depicted so as the exact idea of modelling sequential network development problem in form MILP. As already mentioned, in each time step a static problem of nearest possible connections is solved, regarding the simplifications mentioned above.

Furthermore, the concept of the model is illustrated using the simplest possible network as an example, providing a clear and straightforward explanation of its underlying idea.

3.2.1 Parameters

Parameters used in this optimization model are following:

- ***τ***: Number of intervals, or time steps
- ***N***: Set of nodes in the network, representing actors in the DH grid such as consumers and producers – energy sources and subareas
- ***E***: Set of edges in the network, representing connection pipes between consumers and producers of energy
- ***l***[j]: Length of a certain edge j in meters
- ***L***[i, j]: The matrix represents the lengths of edges (in meters) between each pair of nodes i and j in the given network. For node pairs that do not form an edge, the corresponding entry in the matrix L is set to 0. However, these zero entries are less critical, as the X matrix ensures the model only considers existing edges, as discussed in the following sections.
- R_Q: Heat related revenue – revenue matrix, dimensions N × τ, where the first column defines revenue for all nodes and repeats itself t-times. Elementwise multiplication with decision variable Y gives revenue for all operational nodes in the step t
- p_Q: heat price in EUR/MWh
- C_{pipe}: Excavation and pipe installation costs. Defined for nodes, as reach costs for each node, the costs for building an edge reaching a certain node. For initial source nodes this costs equal zero. C_{pipe} is a matrix dimensions τ × N × E, that corresponds by size with matrices X and ΔX
- L_{max}: Maximum construction capacity per time interval in meters
- $L_{res}[t]$: Residual length, unused in the past timestep. Described in the previous chapter. For the first timestep it is set initially as zero.

3.2.2 Decision variables

Following is described the decision variables for this optimization model:

- $X[t,i,j]$: Binary variable, equals 1 if edge j at node i exists at the beginning of interval t, and 0 otherwise
- $\Delta X[t,i,j]$: Binary variable, equals 1 if edge j at node i is constructed during time interval t, and 0 otherwise

- $Y[i,t]$: Binary variable, equals 1 if node i is operational at the beginning of time interval t, and 0 otherwise
- $\Delta Y[i,t]$: Binary variable, equals 1if node i is made operational during time interval t, and 0 otherwise
- $L_{res}[t]$: Continuous variable representing the unused pipe length at time interval t, described in chapter 3.1.3.

Before delving deeper into the detailed description of the model, I would like to briefly elaborate on the conceptual foundation of the decision variables introduced above and their relationship to network development. These variables are designed to capture the dynamic processes governing the evolution of the network allowing us to systematically represent key decisions, such as when and where to activate nodes and edges, as well as the operational status of the network over time.

To briefly describe what and how X and Y work, I am going to outline a simple example of a network with 4 nodes and 3 edges with a random number of timesteps, as timesteps are not crucial for the following description.

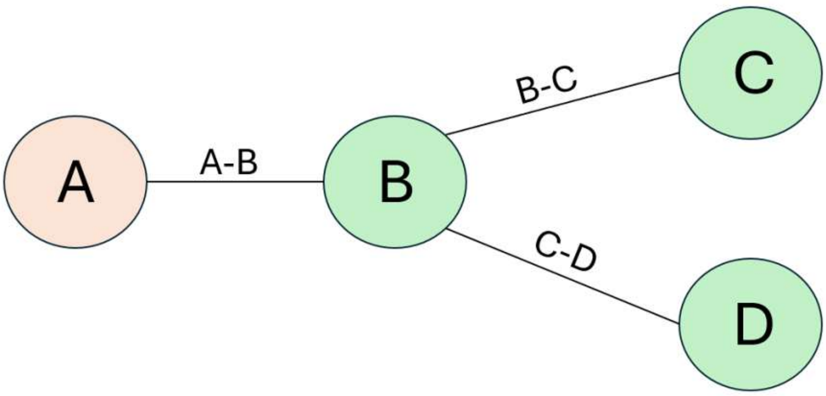

Figure 1 Example of a simple network of 4 nodes and 3 edges

Figure 1 depicts a simple network of 4 nodes and 3 edges, each having its own unique name. Node A is coloured differently than other nodes to highlight it as an initial source node in the time step t = 0. For a network like this one in Figure 1, the X matrix for $t = 0$ is going to look as follows, where edges and node names are temporary set in the first row/column to better understand how it works:

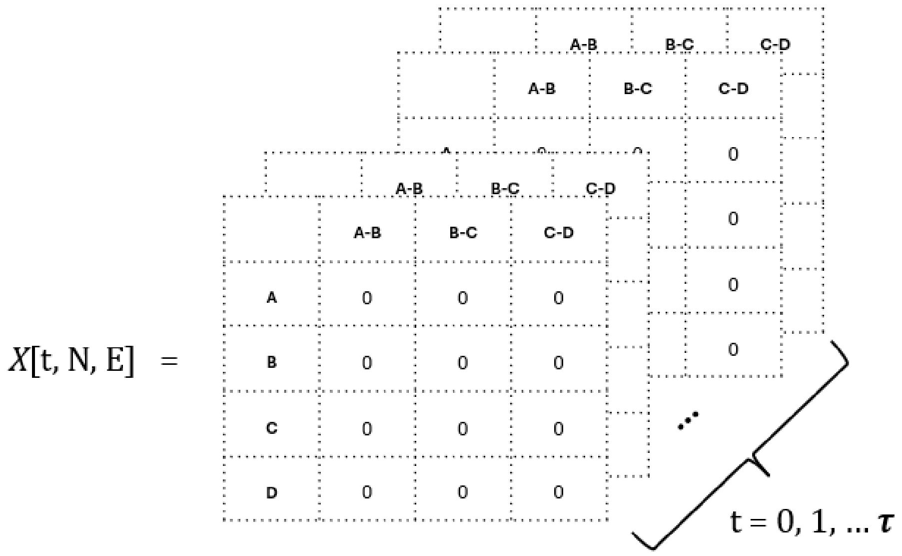

Figure 2 X-Matrix for simplified network depicted in Figure 1

Each layer of the matrix X depicted in Figure 2 represents the state of connections for a specific time step in the optimization process. Row A is supposed consistently to remain zero for all edges and time steps because node A does not have any incoming edges; it only has outgoing edges connecting to other nodes, as node A is initially defined as a source node.

The matrix $\Delta X[t, i, j]$ operates in a similar manner to the X matrix. However, each layer in ΔX exclusively captures changes that occur in the X matrix between consecutive time steps. In essence, it highlights the newly established connections or edges activated at each step, effectively tracking the incremental development of the network.

In addition, the matrices $Y[N, t]$ and $\Delta Y[N, t]$ are also introduced. These are one-dimensional matrices, with rows representing nodes and columns representing time steps, indicating which nodes are operational at each time step. Notably, the $Y[N, t]$ matrix should have a value of 1 in every time step for row A, as node A is a source node and must remain operational throughout all time steps.

$Y[\mathrm{N}, \mathrm{t}] =$

	t=0	t=1	...	...	t=τ
A	1	1	1	1	1
B	0	1	1	1	1
C	0	0	0	1	1
D	0	0	1	1	1

Figure 3 Y Matrix for a simple network depicted in Figure 1

These two matrix frameworks form the backbone of the optimization model developed in this work. In this form they allow MILP method to be used for a sequential optimization problem. Building upon them, the constraints of the mode, as well as the objective function, will be outlined in the following section.

3.2.3 Objective function

The objective of this model is to maximize profits at each step, with the steps constrained most importantly by the amount of pipeline that can be constructed. To calculate the profits, the objective function considers all operational nodes, determining the energy demand at each time step and the resulting revenue. This revenue is offset by expenses such as generation and distribution costs.

Additionally, pipeline-related costs, including excavation and pipe installation costs, are accounted for at each step. Due to the Mixed-Integer Linear Programming (MILP) formulation of this sequential problem, excavation and pipe installation costs are tied to the residual length variable in the model and are incurred when the decision is made to connect a new node. Consequently, the decision variable X does not appear explicitly in the objective function. However, X is closely linked to Y, as reflected in the constraints, that are presented in the following part of this chapter, that integrate these two critical decision variables, which are central to the model.

In the detailed calculation of costs provided by the model, excavation and pipe installation costs are distributed across all relevant time steps, extrapolated from the steps where these activities actually occur. This distribution accounts for the constraint that limits the maximum amount of pipeline that can be constructed in each time step, ensuring a realistic allocation of these costs over time.

This issue also arises from the limitations of the MILP approach to solving a sequential optimization model. However, it should not lead to significant errors in the calculation.

Taking all this into consideration, following is presented the objective function:

$$max \sum_{t=1}^{\tau} \sum_{i=1}^{N} (R_Q[i,t] \cdot Y[i,t] \; - \; C_{grid}[i,j,t] \cdot \Delta X[i,j,t] - (C_{gen}[i,t] + C_{dist}[i,t]) \cdot Y[i,t]) \tag{1}$$

Where $R_Q[i,t]$ is defined as:

$$R_Q[i,t] = Q_{heat}[i,t] \cdot p_Q[i] \tag{2}$$

- $Q_{heat}[i,t]$: heat demand for nodes in timesteps, also as a matrix with identical columns
- $c_{gen}[i,t]$: heat generation costs in $[EUR/MWh]$
- $c_{dist}[i,t]$: heat distribution costs in $[EUR/MWh]$
- $C_{gen}[i,t]$: heat generation costs in $[EUR]$
- $C_{dist}[i,t]$: heat distribution costs in $[EUR]$
- $R_Q[i,t]$: Heat related revenue – revenue [EUR]

And $C_{grid}[i,t]$ as:

$$C_{grid}[i,t] = c_{exc}[i,t] + c_{pipe}[i,t] \tag{3}$$

- $C_{grid}[i,t]$: costs of pipe installation and excavation costs as a matrix with identical columns in EUR.

- $c_{exc}[i,t]$: excavation costs as a matrix with identical columns, for nodes. Presenting the costs of leading-edge to each node in EUR.
- $c_{pipe}[i,t]$: pipe installation costs also in the matrix form with identical columns, where rows represent nodes and the costs represent the costs of reaching each node, more precisely the cost of the pipe for an edge that supplies a certain node in EUR.

3.2.4 Constraints

In the following subchapter, constraints of the model are described, with mathematical formulas that describe them.

This model is structured around six different groups of constraints: boundary constraints, static graph constraints, persistence constraints, constraints that define variables ΔX and ΔY, neighbour node constraints and length capacity constraints.

The greatest challenge in creating an optimization model is not to over-engineer the set of constraints, more specifically to define necessary and sufficient number of constraints to make model functional.

3.2.4.1 Boundary constraints:

$$Y[i,0] = 1 \quad \forall i \in N_{source} \tag{4}$$

$$Y[i,0] = 0 \quad \forall i \in N \setminus N_{source} \tag{5}$$

- N_{source}: set of nodes that are marked as source nodes in the input excel file, discussed more precisely in the chapter "Input data preparation".

These two formulas making boundary constraints are nothing more than setting the stage of the graph of nodes and edges, clarifying which nodes are active in the first time-step or rather reading which ones are set to be source nodes from the input excel file.

The third one in this set of constraints sets all edges decision variables X to be zero, meaning none of the edges exists in the beginning of the optimization.

$$X[0, i, j] = 0$$
$$\forall i \ \in N, \forall j \ \in E \quad (6)$$
$$s.t. \quad \{ t = 0$$

3.2.4.2 Static graph constraints:

The second group of constraints called “static graph constraints” makes an image of possible edges connecting each node with every other in the node set and sets X as zeros for all edges from this imagined larger set if a certain edge does not exist in the input excel file.

$$X[t, i, j] = 0$$
$$\forall t \quad (7.1)$$
$$\forall E_{i,j} \ \notin E$$

$$\Delta X[t, i, j] = 0$$
$$\forall t \quad (7.2)$$
$$\forall E_{i,j} \ \notin E$$

- $E_{i,j}$: an edge between the nodes i *and* j equivalent to $E_{j,i}$

3.2.4.3 Persistence constraints:

Persistence constraints define the consequent development of nodes and edges throughout the timesteps.

For example, if a node has become operational at a certain moment, it is supposed to stay operational for the rest of the iterations:

$$Y[i, t] \geq Y[i, t - 1]$$
$$\forall i, t \quad (8)$$

Same principle stands for the edges, if an edge was marked as built at a certain timestep, it remains such for the rest of the iterations:

$$X[t,i,j] \geq X[t-1,i,j] \quad \forall i,j,t \tag{9}$$

In this manner the model is constrained to be able to achieve consequent building of the edges and connecting nodes.

3.2.4.4 Constraints that define variables ΔX and ΔY

When defining ΔX and ΔY it may seem that they in essence are not real decision variables, but only some kind of derivates of decision variables. Non the less, they also play a considerable role in objective function and practically decide a lot about the costs.

Constraints defining ΔX and ΔY may look a lot like real discrete derivations of discrete functions X and Y.

$$\Delta Y[i,t] = Y[i,t+1] - Y[i,t] \quad \forall i,t \tag{10}$$

$$\Delta X[t,i,j] = X[t+1,i,j] - X[t,i,j] \quad \forall i,j,t \tag{11}$$

3.2.4.5 Neighbour constraints

Neighbour constraints limit the activation of a node if the reaching edge or a neighbour edge was not activated first.

$$\Delta Y[i,t] \leq \sum_{j=1}^{E} \Delta X[t,i,j] \quad \forall i \in N, \forall t \in \{0, \dots, \tau - 1\} \tag{12}$$

In the other form the constraints also do not allow an edge to be constructed if there is not at least one node active, from the two the edge is supposed to connect.

$$\Delta X[t, i_a, j] \leq Y[i_a, t] + Y[i_b, t] \quad \forall E_{ab} \in E \quad \forall j \in E \tag{13}$$

$$\Delta X[t, i_b, j] \leq Y[i_a, t] + Y[i_b, t] \quad \forall E_{ab} \in E \quad \forall j \in E \tag{14}$$

3.2.4.6 Length capacity constraints:

Viewed from point of analysis and research question for this work, following are maybe the most important of all constraints, limiting construction to maximal length per timestep.

$$\sum_{j \in E} \Delta X[0, i, j] \cdot L[i, j] \leq L_{max} \quad \forall i \in N \tag{15}$$

Above mentioned constraint limits the construction in the first step for $t = 0$ to be below the maximal possible length of pipeline which is to be built in each timestep.

In this equation L_{max} is not indexed with timestep, as it is assumed that maximal length per period is constant over all iterations. If necessary, it would also be easily possible to adapt the model to give different maximal length per period parameters for each timestep.

Once again, the model does not assume partial construction of edges before they are fully built but saves the unused length from the past step in a helping variable called residual length per period, which is indexed for each timestep, and connects a node, when residual length added to maximal length overreaches some of the lengths of edges that should be constructed. This is since the model in form of MILP is solving sequential optimization problem.

$$\sum_{j \in E} \Delta X[t,i,j] \cdot L[i,j] \leq L_{max} + L_{res}[t-1] \quad (16)$$

$$\forall i \in N, \forall t \in \{1, \dots, \tau\}$$

3.2.5 Implementation tools: Gurobi and CVXPY

In this chapter I am going to write about the solver that is used in this model and the modelling library CVXPY.

Gurobi optimizer is one of the leading optimization solvers in the industry, offering state-of-the-art tools for solving linear programming (LP), mixed-integer linear programming (MILP), and other advanced optimization problems [7].

It was chosen as a perfect optimizer for this project because of its performance and ease of integration with python programming language, through which the model was programmed. Gurobi exists also for other common programming languages.

CVXPY, a Python-embedded modelling language for convex optimization problems, was selected as the primary framework for modelling and solving the optimization problem in the project firstly due to its flexibility and because it is easy to use [8]. Constraints and mathematical formulas are written with CVXPY exactly the same way as the are in the initial mathematical form. Other solutions like Pyomo, also a modelling language for optimization problems, are robust but involve more verbose syntax, which can complicate the development of highly dynamic models.

3.3 Input and Output of the model

3.3.1 Input Data preparation

In this chapter, I describe the preparation of input data for the optimization model, detailing how the network characteristics and parameters are structured to support the analysis. Preparing input data is a crucial step, as it ensures the model's flexibility and applicability to a wide range of district heating networks.

By creating a standardized and well-structured dataset, the methodology allows for consistent representation of different networks, enabling comparative analyses and scalability to various case studies. This approach ensures that the model remains robust and adaptable, regardless of the specific characteristics of the grid under consideration.

As for each network, the basis that describes it are its nodes and edges. Both nodes and edges are attributed with different kinds of parameters that are going to be described further on in this chapter.

Preparation of input data involves extracting, formatting, and validating data to reflect the physical and operational characteristics of the district heating network. In this input data excel file, there are minimum inputs from the physical layer of information required as well as a couple of rules to be followed in order to make the optimization model operational. In the following chapter, where a case study is conducted, the input data was filtered from a GIS file, which describes the network of a city of Brasov in Romania.

Following are going to be presented the three sheets of input excel file, which describe the important things about the district heating network and be backbone of interpreting the network to the model. The first sheet is called "Node Data" presenting the information and attributes for nodes. The second sheet describes the coincidence of the nodes described in the first sheet and is called "Edge Data". The third and last sheet is called "Parameters" and allows the input of most important parameters for the optimization model, such as "maximal length" and "factor".

Table 1 Example of node input data

Node Name	Heat Demand [MWh]	Source	Reach Costs [kEUR]	Distribution Costs [EUR/MWh]	Distribution Costs [kEUR]
A	0,00	True	0,00	0,00	0,00
B	6 398,28	False	6 161,04	17,80	113,86
C	0,00	False	91,45	0,00	0,00
D	10 221,00	False	796,06	20,04	204,82
E	5 940,97	False	784,97	21,30	126,55
F	7 885,12	False	438,81	21,21	167,20
...	...	...	...	...	...

Before explaining the above Table 1, it is important to understand that there are three significant types of nodes: source node, sink node and junction nodes. Source node produces energy, that is the most important in the first timestep, to know which nodes are initially set as source nodes, all others are either sink nodes or junction nodes. Sink nodes consume energy and pass the not used energy to other nodes if they are connected to them, acting as sources for them from the optimization model point of view. There are also junction nodes, which neither consume nor produce energy, but act as a crossing point for network to develop as it is supposed to.

The content of Table 1, which is used as an example of the definition of nodes in a network is:

- Node Name: defines a unique node name in the grid, each of which should have its own name which does not appear more than once in the list.
- Heat Demand [MWh]: heat demand for each of the nodes expressed in MWh. For nodes that are initially a source and for junction nodes this parameter is set to be 0,0001, which is insignificant number opposed to other amounts of heat demand for sink nodes. This ensures that the solver does not prematurely stop if it fails to identify potential profits in the immediate neighbouring nodes, allowing it to consider connections beyond the closest nodes in the further steps, taking into consideration a number close to zero, that does not affect the calculation of the given input data.
- Source: this Boolean parameter is set to be “True” if a node is set to be an initial source, or rather a source and an active node in the first step. It is also possible to set more than one node as an initial source, as well as also to start simulation from a grid that is already partly built, setting “True” for all nodes that are active or connected at the time.
- Reach Costs [EUR]: the costs of reaching a certain node, more specifically the costs of building a node that supplies and connects a node to the grid. The fact was already mentioned in the chapter “Key assumptions, simplifications and limitations” that each node is only to be supplied from one edge.
- Distribution Costs [EUR/MWh]: the costs of distribution of energy inside of a subarea, from the model point of view – “inside of a node”.
- Distribution Costs [EUR]: distribution costs in EUR meaning annual distribution costs for each subarea, represented in network model as a node.

On the second sheet in the input excel file there is information about the correlation of the above-described nodes, more precisely information about the edges of the network.

Table 2 Example of the edge input data

Edge Name	Start Node	End Node	Edge Length [m]	Pipe Costs [EUR/m]	Excavation Costs [EUR/m]	End Node Reach Costs [kEUR]
EA	A	B	2132,58	2343	546	6 161,04
EB	B	C	31,65	2343	546	91,45
EC	B	D	613,77	957	340	796,06
ED	C	E	508,07	1161	384	784,97
EE	E	F	407,44	775	302	438,81
...	...	...	...	...	...	...

The content of the Table 2, used as an example for describing the input file has following meaning:

- Edge Name: The unique name of each edge, not allowing for two edges to have the same name.
- Start Node and End Node: To prevent the creation of duplicate edges resulting from two possible directions between nodes, a unified fictitious direction is assigned to each edge by defining a designated start node and end node.
- Edge Length [m]: The length of each edge in meters
- Pipe Costs [EUR/m]: The costs of pipes for each edge vary in the case study scenario, primarily depending on the specific area of the map they traverse. The pipeline extending from the initial source node to the first neighbouring node has the largest diameter, as it is designed to supply all subsequent nodes. Consequently, it is also the most expensive pipeline in the network.
- Excavation Costs [EUR/m]: The excavation costs along the pipeline route, measured in EUR per meter, vary depending on the type of pipeline used for different edges.
- End Node Reach Costs [EUR]: As already mentioned in the node sheet, there are again the same Reach Costs calculated and ready as input, also linked to the "Node Data" sheet.

On the third and the last sheet key input parameters for this optimization model are listed.

Table 3 Example of key parameters input data

Parameter Name	Quantity	Unit
inv_period	30	y
max_length	300	m
heat_price	89	EUR/MWh
factor_mL	3	
gen_cost	43	EUR/MWh

- Investment Period: The total time horizon for the investment, set to 30 years in this example.
- Maximal Length (max_length): The maximum allowable length of pipeline construction per year, set at 300 meters for this example.
- Heat Price (heat_price): The revenue granted per megawatt-hour (MWh) of heat delivered, valued at EUR 89 in the Table 3.
- Resolution Factor (factor_mL): A scaling parameter allowing more accurate results, explained in the chapter "Key assumptions, simplifications and limitations".
- Generation Costs (gen_cost): The cost of generating heat per megawatt-hour (MWh), specified in the table as EUR 43.

The important thing in setting these parameters, presented in the Table 3, is to set Maximal Length and Resolution Factor in a manner that they practically define an unseen parameter, which is the Maximal Length in the scaled model. So, for example, if we set the stage as shown in the Table 3Table 3, for Maximal Length to be 300 meters and Resolution Factor to amount 3, the model is going to work in steps of 100 meters, more specifically the resolution of one step is going to be 100 meters.

Just like that, the model is going to create for itself three times more timesteps. Meaning, if there is Investment period set to be 30 years, model is not going to assume that each step lasts a year, but rather that each step lasts 4 months, due to scaling with Resolution Factor, resulting with 90 steps at the end, meaning 90 iterations of the model.

In this manner we achieve the resolution of the step seen from model point of to be 4 months with maximal possible length built in a step as 100 meters. Again, all of this is a solution to using MILP in optimization problem of iterative nature. Doing the optimization in this way, the model refreshes its state three times per year, and not once each year.

Nevertheless, all nodes connected in a year start being active only at the end of the year, does not matter if they were reached at the beginning or the end of the year. So from the analytical point of view, resolution of the model is in time steps of a one year and Maximal Length of 300 meters.

The user of this model should be aware that with larger Resolution Factor, the process becomes slower, and the optimization becomes more CPU intense. But also, there should be made thoughts about the relationship between Maximal Length and Resolution Factor, to make the scaled Maximal Length small enough for the case study example. It should somehow be equal to the median of the edge lengths of similar edge types. More on this topic follows in the case study chapter.

Usually if there is one main edge leading from the initial source to the rest of the network, its length is not interesting for setting up *Resolution Factor/Maximal Length* relationship. It only starts making a difference in the part of the network where the branching becomes more frequent. We tend to take the lengths of the edges in this area and calculate their median setting it to be the scaled Maximal Length with relationship:

$$scaled\ Maximal\ Length = \frac{Maximal\ Length}{Resolution\ Factor} \quad (17)$$

3.3.2 Model Output Data

In this chapter the outputs of the model are discussed. The output of the model is not only important because of validation of model's functionality but also provides critical insights for decision making in this case.

Table 4 Output data example

Time Step	Built Edges	Connected Nodes
0		A
1	EA	A
2		A, B
3	EB	A, B
4		A, B, C
...	...	...

The Table 4 describes an output data from the model, giving exact information of the sequence of connection of nodes and built edges over the iteration period.

Given this kind of output, the exact costs and further investment analysis takes place in a non-automatic manner possibly in an Excel data sheet. Some of these results and investment analysis are going to be discussed in the chapter "Case Study".

Model output data is further going to be evaluated by two main metrics, Net Present Value (NPV) and Levelized Cost of Heat (LCOH) which are calculated as described in the Equation 18 and Equation 19.

Net Present Value (NPV) is a financial metric used to evaluate profitability of an investment by calculating the present value of all future cash flows [6].

$$NPV = \sum_t \frac{CF_t}{(1+r)^t} - C_0 \tag{18}$$

where

- CF_t presents cash flow for each period of the investment period
- t presents the investment period
- r presents the discount rate
- C_0 presents the initial investment cost

Levelized Cost of Heat (LCOH) is the average cost per unit of heat produced over the period of investment. It helps compare different heating technologies among other [6].

It is calculated by Equation 19.:

$$LCOH = \frac{\sum_t \frac{C_t + O_t}{(1+r)^t}}{\sum_t \frac{Q_t}{(1+r)^t}} \tag{19}$$

where

- C_t presents capital costs in a certain year
- O_t presents operational costs in a certain year
- Q_t presents supplied heat in MWh in a certain year
- r presents the discount rate

3.4 Case study

A case study serves as a practical demonstration of above-described optimization model in a real-world example. This case study was conducted on a plan of DH grid expansion in one district of Brasov in Romania.

The input data used in this thesis - including heat demand, subarea layout, annual energy needs, and grid expansion scenarios - are derived directly from the technical deliverables of the progRESsHEAT project, particularly the local heating strategy for Brasov and associated model output [30], [31]. The project combined local data collection, spatial energy planning, and stakeholder consultations to define scenarios for district heating revitalization, which serve as the basis for the MILP-based rolling investment model developed in this work.

Input of a map was given in form of a GIS file, depicting the main subareas of a larger area and all of the nodes and edges describing connection points of subareas and the pipelines between them. Number of nodes in the GIS file was larger than needed for input file for optimization model described in the previous chapter, so some of them were not considered when creating the input file. But if one node was eliminated, attention was paid that the edges leading to and from it were of the same pipe type, where their lengths were summed up to one edge.

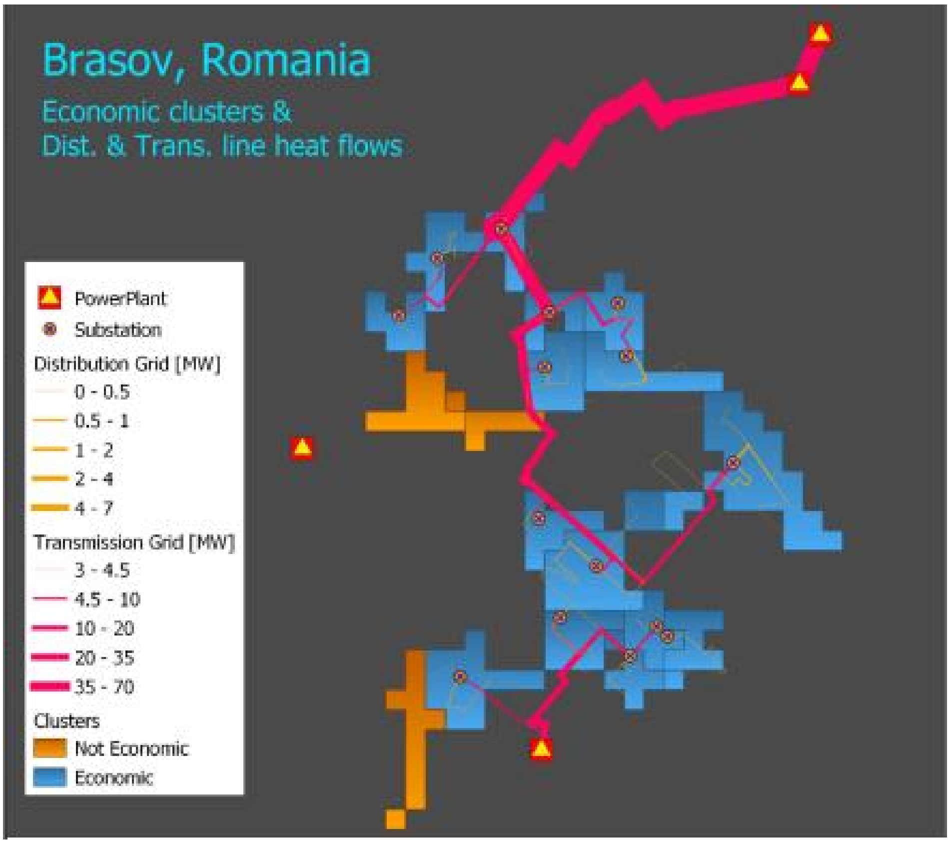

Figure 4 Brasov network expansion plan [10]

In the Figure 4 [10] a district in Brasov, Romania is shown, consisting of two separate DH networks, with two separate sources. In this chapter the optimization model is applied only to above DH network plan, more precisely on the larger of the two seen in the Figure 4.

In the legend in Figure 4, we see the "Power Plant" and "Substation", which equals to "initial source nodes" and "nodes" in the description of the model. There are also no "Substations" in

“Not Economic” subareas seen in the Figure 4, as these are defined not to be profitable in the analysis and planning of the network. There are also different types of pipelines described in the legend of the Figure 4, which reflect to larger pipeline costs also considered in the input file of the optimization model.

This work focuses only on the sequence of connection of “Economic” subareas in the plan.

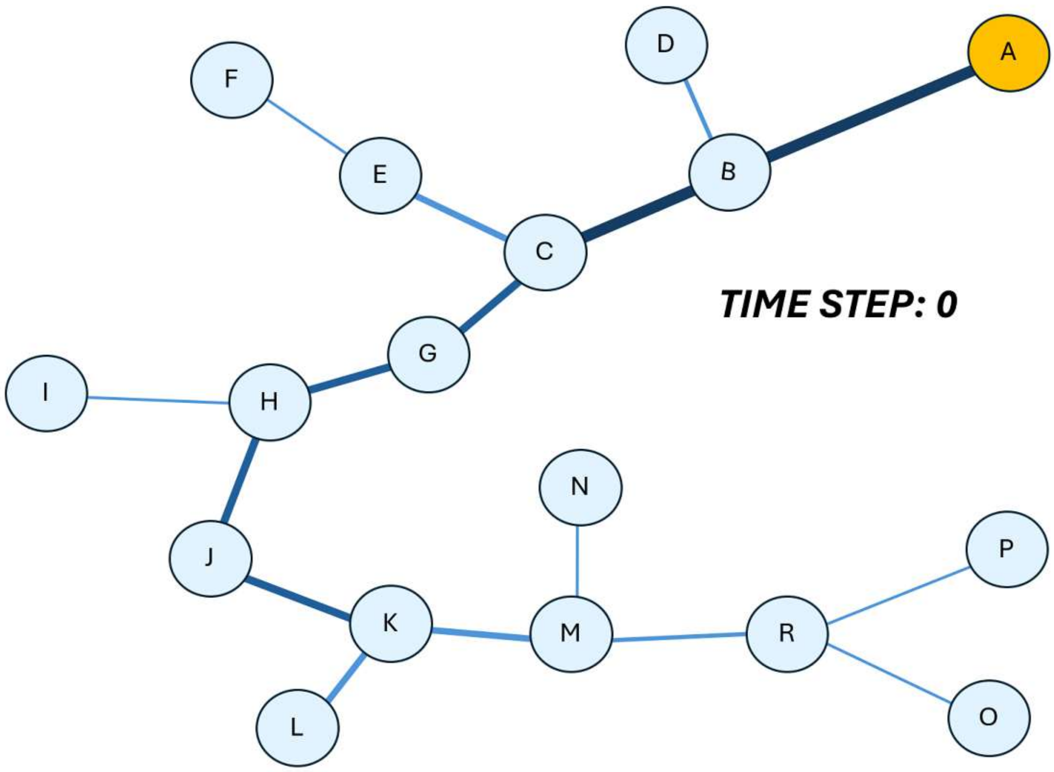

Figure 5 Brasov grid interpreted from the model point of view

Once when the model takes input data as they are described in the previous chapter, it creates a simplified grid as shown in the Figure 5. The blue nodes are “Substations” in the above shown Figure 4, where the only yellow one in Figure 5 presents initial source node or “Power Plant” in Figure 4.

Following tables Table 5, Table 6 and Table 7 show the exact input data needed to interpret the upper DH heating grid in presented in the Figure 4.

Table 5 Heat price, source costs and depreciation rate input data

Heat source yearly fix costs	162 500	EUR
Heat source variable costs	43	EUR/MWh
Heat price	89	EUR/MWh
Investment period	30	years
Depreciation rate	5	%

Table 5 shows the input heat source costs and heat sales price for the case study of part of the Brasov Grid in Rumania and Table 6 and Table 7 describe the particular network dimensions and relations in the case of Brasov grid.

Table 6 Node input data for case study of Brasov district

Node Name	Heat Demand [MWh]	Source	Reach Costs [kEUR]	Distribution Costs [EUR/MWh]	Distribution Costs [kEUR]
A	0,00	True	0,00	0,00	0,00
B	6 398,28	False	6161,04	17,80	113,86
C	0,00	False	91,45	0,00	0,00
D	10 221,00	False	796,06	20,04	204,82
E	5 940,97	False	784,97	21,30	126,55
F	7 885,12	False	438,81	21,20	167,20
G	7 998,56	False	751,40	24,45	195,53
H	0,00	False	331,48	0,00	0,00
I	6 442,10	False	580,43	17,98	115,81
J	6 487,23	False	1807,00	22,44	145,56
K	0,00	False	861,86	0,00	0,00
L	14 965,06	False	80,50	24,26	363,00
M	6 349,76	False	902,15	23,17	147,16
N	6 422,47	False	294,65	19,46	124,97
R	0,00	False	20,51	0,00	0,00
O	6 489,05	False	331,94	19,91	129,20
P	8 632,13	False	589,33	21,36	184,36

Reach Costs [kEUR] described in Table 6 are the same as End Node Reach Costs [kEUR] described in Table 7 which present the costs of reaching a certain node from the only node leading to it. "Source" in Table 6 is a "True/False" variable that tells which nodes are sources in the initial time step. Described like this Table 6 and Table 7 make a clear input and representation of a certain area on which the optimization model could be ran.

Table 7 Edge input data for case study of Brasov district

Edge Name	Start Node	End Node	Edge Length [m]	Pipe type (diameter)	Pipe Costs [EUR/m]	Excavation Costs [EUR/m]	End Node Reach Costs [kEUR]
EA	A	B	2 132,58	DN350	2 343	546	6161,04
EB	B	C	31,65	DN350	2 343	546	91,45
EC	B	D	613,77	DN150	957	340	796,06
ED	C	E	508,07	DN200	1 161	384	784,97
EE	E	F	407,44	DN125	775	302	438,81
EF	C	G	361,42	DN250	1 649	430	751,40
EG	G	H	159,44	DN250	1 649	430	331,48
EH	H	I	538,93	DN125	775	302	580,43
EI	H	J	869,17	DN250	1 649	430	1807,00
EJ	J	K	414,56	DN250	1 649	430	861,86
EK	K	L	52,11	DN200	1 161	384	80,50
EL	K	M	583,91	DN200	1 161	384	902,15
EM	M	N	273,59	DN125	775	302	294,65
EN	M	R	15,81	DN150	957	340	20,51
EO	R	O	308,21	DN125	775	302	331,94
EP	R	P	547,20	DN125	775	302	589,33

One of the important factors in the Table 7 is the "Pipeline Type" column, defining different types of pipes which lead to oscillations in excavation and pipeline cost graph presented in Figure 8Figure 8. DN stands for "nominal diameter" and the number following defines the diameter of the pipe in millimetres.

Furthermore, the cost calculations are done based on different results for each of four cases. Net present values (NPV) and Levelized costs of heat (LCOH)

4. Results

This chapter presents the results of the optimization model applied to the district heating network expansion. The key findings include the profitability of different expansion sequences, the impact of workforce constraints, and the sensitivity of the results to investment limits. The results are examined in two subchapters, considering the initial value of the heat price in the first subchapter and the 10% fluctuations in heat price in the second subchapter.

4.1 Results for initial heat price

The results of the optimization model are analysed through the net present value (NPV), a critical metric for assessing the profitability of district heating (DH) grid investments and Levelized Cost of Heat (LCOH), expressing the average cost of producing one unit of heat in five different scenarios This section presents the NPV and LCOH trends for five scenarios, each defined by a different maximum allowable annual pipeline length: 300m, 400m, 500m, 600m, and 700m.

By examining these cases, the aim is to highlight the relationship between construction capacity and the economic viability of the investment. The NPV graph offers a visual representation of how the profitability evolves as the annual construction limit increases, capturing the trade-off between higher upfront investments and long-term returns. Furthermore, these results provide a deeper understanding of the optimal balance between pipeline construction pace and financial outcomes.Through this analysis, the results not only validate the optimization model but also offer practical insights for strategic planning of DH grid expansion under real-world constraints.

Before going on to explain the main results, NPV and LCOH, I am going to describe the results that led to calculating NPV and LCOH. One of the most important outputs of the model is the connection sequence of subareas, that is most visible through the Figure 6"Annual connected heat demand in the researched area". Figure 6 shows that as expected the case of 700 meters a year reaches the full demand of the researched area the fastest, having almost no periods in which nothing is built, except in years 0-2 and 5-6, whereas the case of 300 meters a year has many periods of building, yet no connections made. This means that most of the routes between the subareas are shorter than 700 meters.

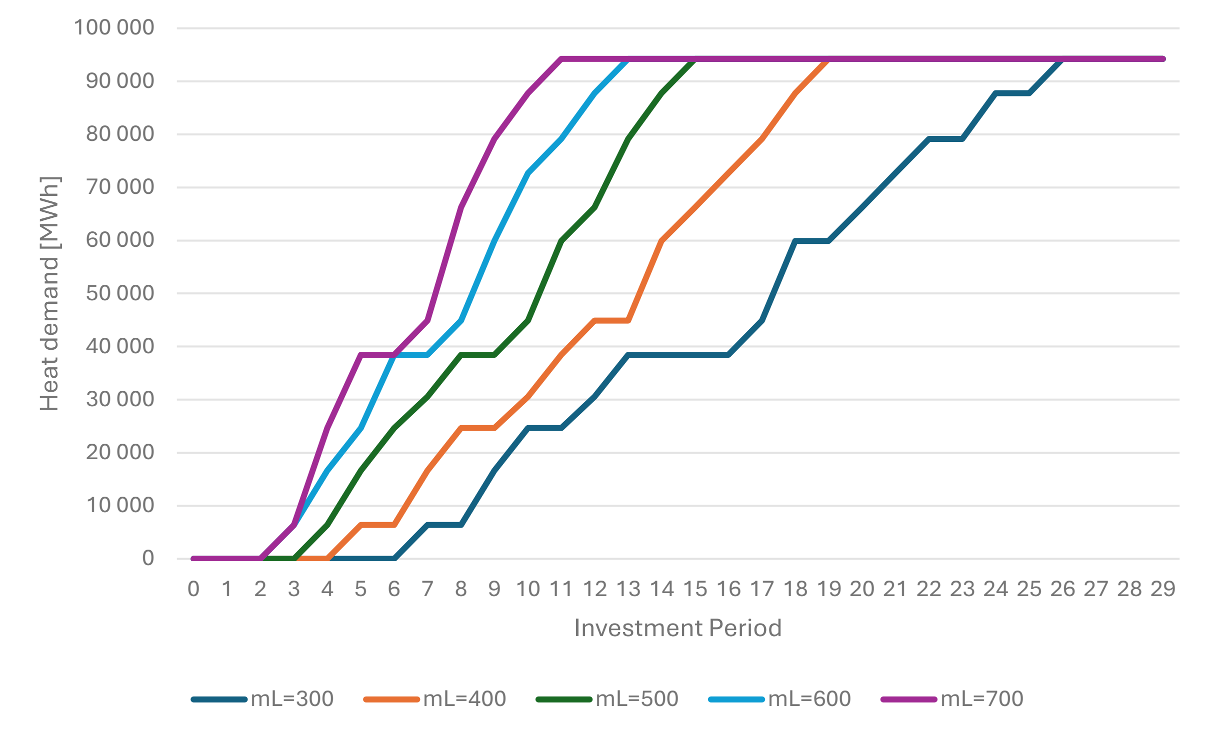

Figure 6 Annual heat demand in the researched area

Figure 7 shows the connection sequence of the grid for maximal yearly length of 300 meters in 0, 8, 14, 21, 23 and 26th time step. These steps were chosen on purpose to show how the connection varies from building the core pipeline starting from the source node "A" to "B" and then connecting peripheral nodes, which pipelines are cheaper to build. Table 7 "End Node Reach Costs" and "Pipe type" back up this statement with the costs of reaching a certain node. The clear difference in pipeline type is also visible in the upper part of Figure 4 which shows the same part of the grid as in the Figure 5, just in the different format. The red line starting from the upper right part of the Figure 4 and going down to the first blue rectangle depicts the most expensive connection shown as "A" – "B" in the Figure 5.

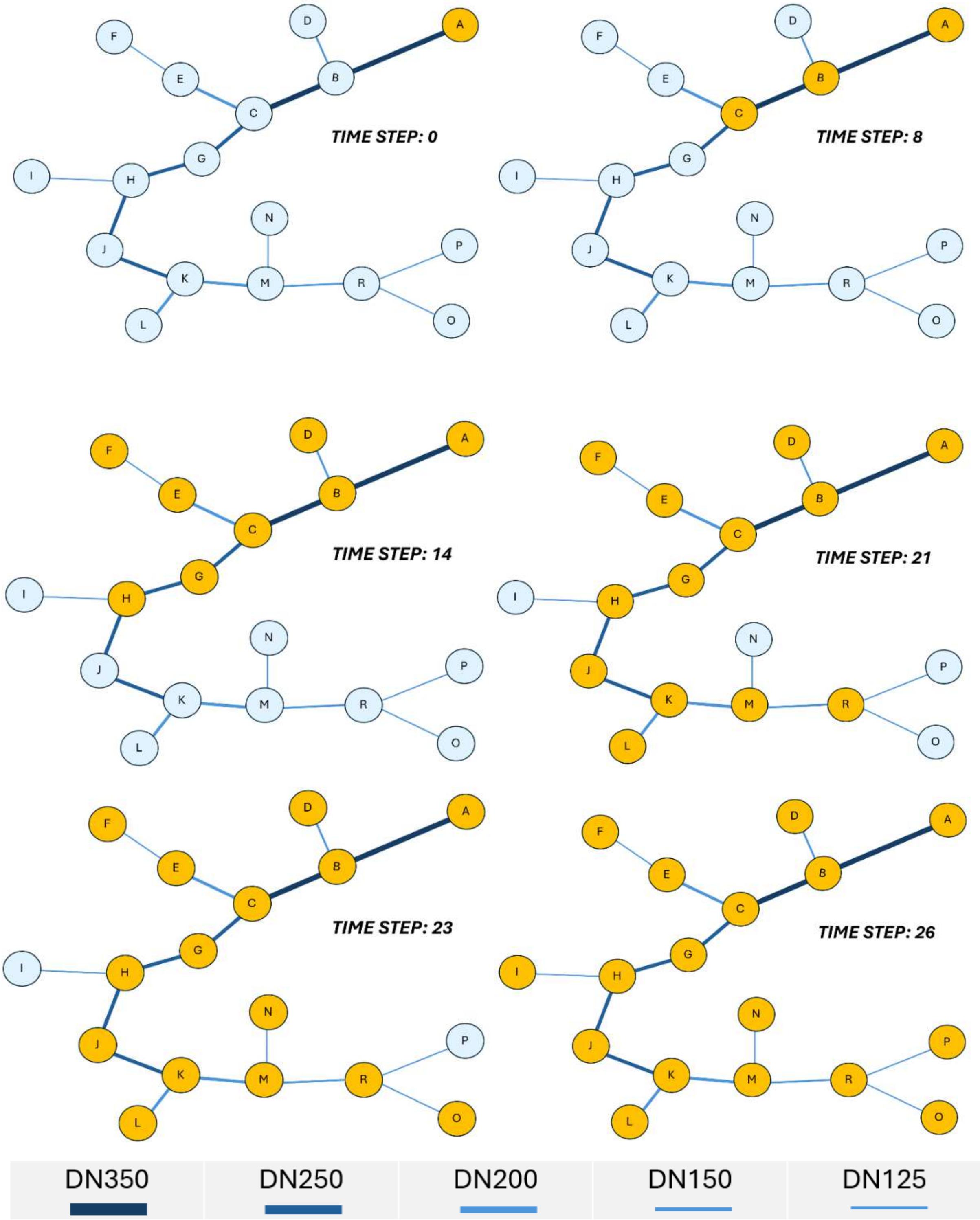

Figure 7 Network development for time steps 0, 4, 6, 9,11 and 13 for scenario of connecting 300 m per year

Following Figure 8 illustrates "Excavation and pipe costs" over the investment period. In contrast to steadily increasing annual heat demand, the costs of excavation and pipelines do not follow a consistent upward trend due to varied types of pipelines being installed. As depicted in Table 7, row "Pipe Costs [EUR/m]" and row "Excavation Costs [EUR/m]" different pipeline routes in the researched area have a distinct cost structure, reflecting variations in pipeline diameters and installation conditions. For example, primary pipelines that connect main heat sources to larger subareas tend to be more expensive, both in terms of material cost and excavation, as they require larger diameters to accommodate higher flow rates. Conversely, pipelines extending to smaller subareas with less demand can be made from more cost-efficient materials and smaller diameters, reducing installation costs.

The cost fluctuations observed in Figure 8 are thus largely influenced by these variations in pipeline specifications shown as different connection lines in Figure 7and then used to clarify these connections in Figure 7. The initial years of the investment period often involve constructing larger-diameter pipes for the core of the grid, leading to higher costs. As the grid expands into secondary areas, smaller pipes can be used, and the costs per meter of pipe installation generally decrease.

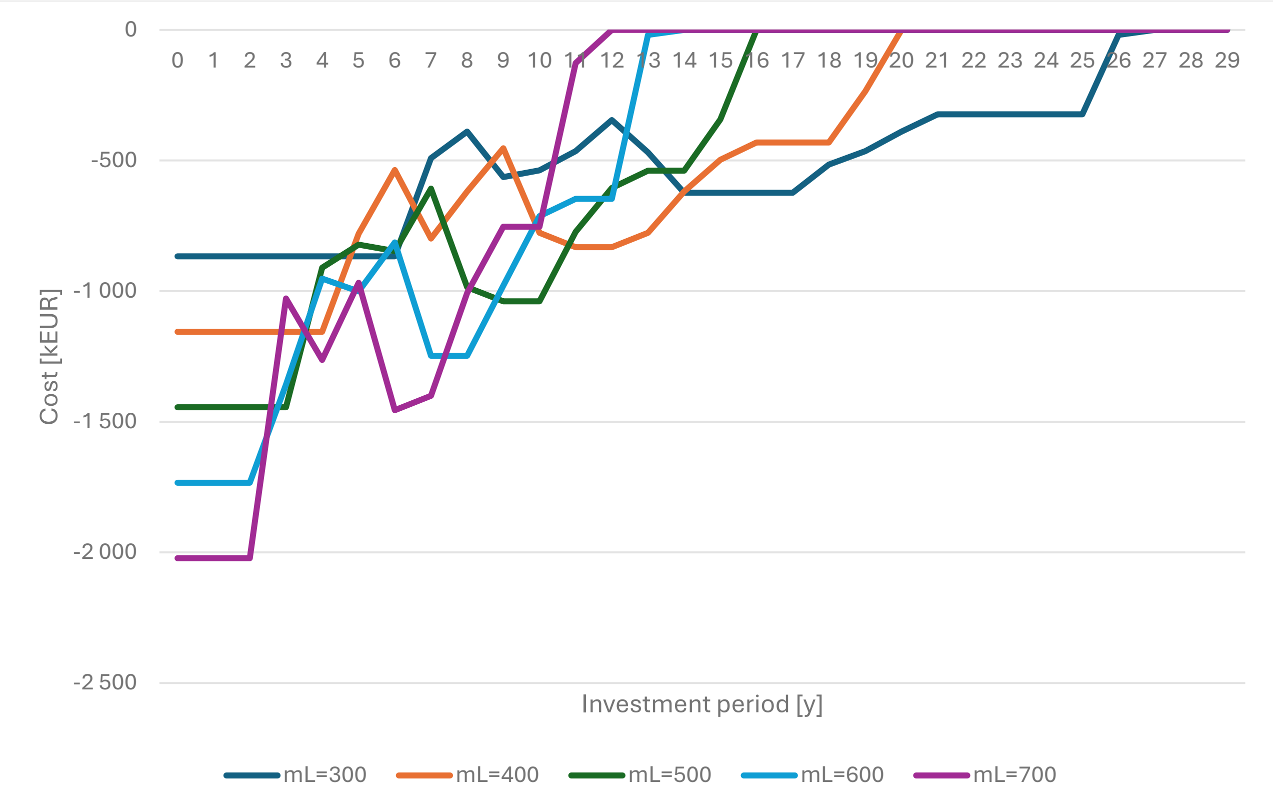

Figure 8 Excavation and pipe costs

Figure 9 shows the pipeline installation rate, or rather the cumulative length of pipeline installed in meters over the investment period for all five different scenarios, defined by the maximum pipeline lengths per year. This graph underscores the relationship between expansion speed and the timeline for achieving full network coverage. The quicker the installation rate, the faster the network grows, allowing it to meet heat demand of the whole research area sooner.

In the case study of Brasov there is a total of 7.817,9 meters of pipeline to be installed, which is installed within 12 years in case of 700 meters a year.

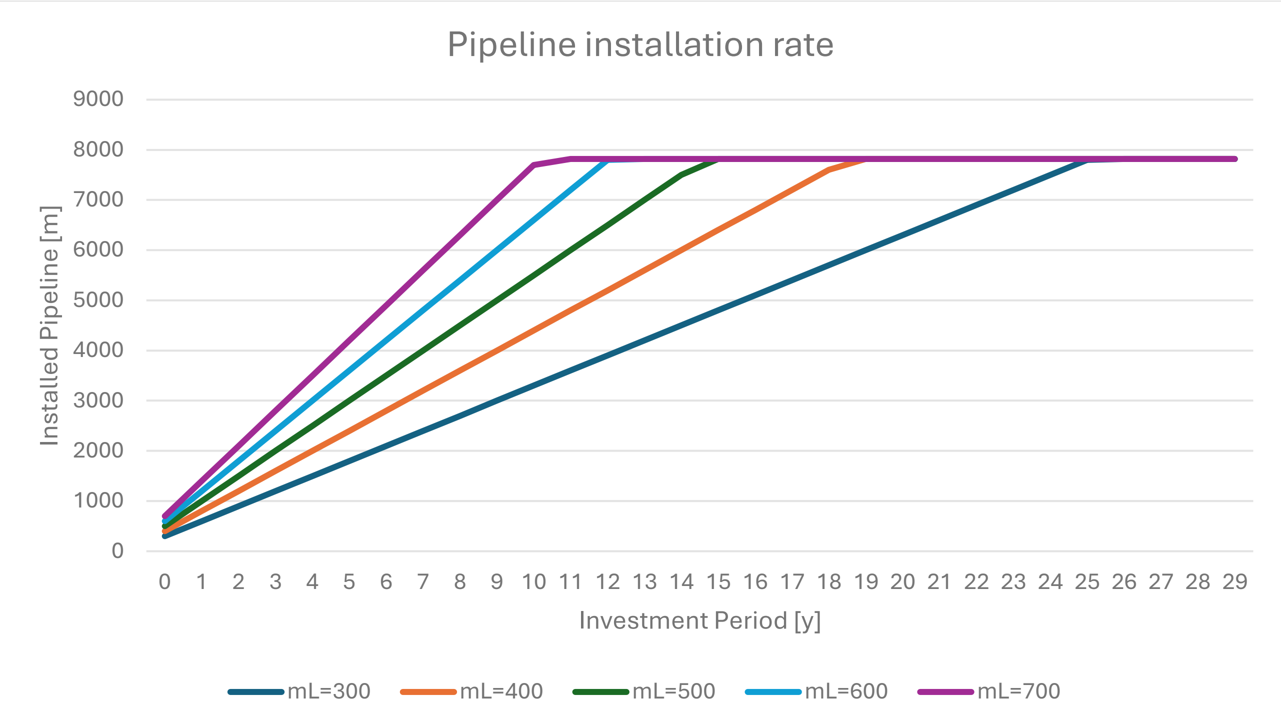

Figure 9 Pipeline installation rate

Figure 10 shows the "Annual Cash Flow", more precisely the annual financial inflows and outflows in EUR over the investment period for different construction rates per year. The main factors of influence for this graph are the excavation and pipeline costs presented in the Figure 8 and the Figure 6 presenting the annual heat demand, as all other profits and costs such as generating costs, distribution costs and profits from sold unit of heat are linearly depending on the annual heat demand graph. This is why we could see almost the same fluctuations in Cash Flow as in the Excavation and Pipeline costs presented in Figure 8.

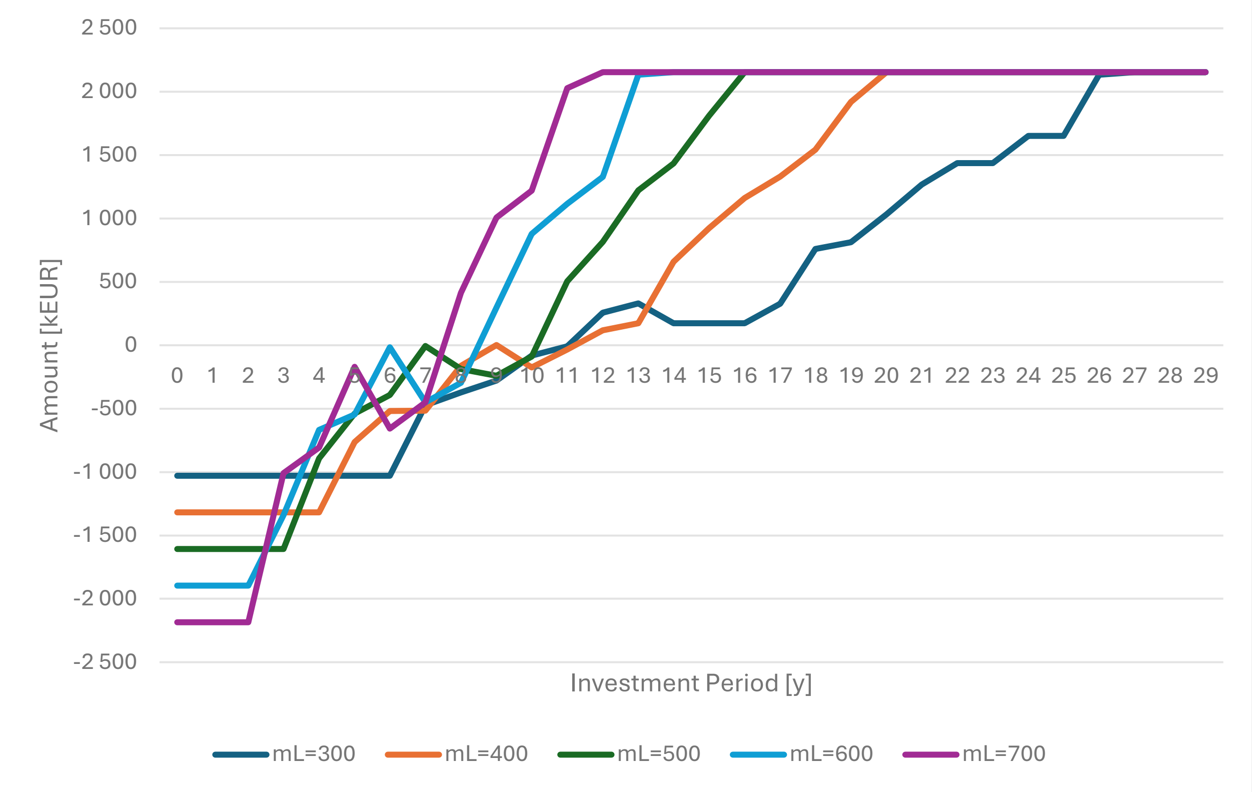

Figure 10 Annual Cash Flow

Figure 11 shows five different construction scenarios ranging from 300 meters to 700 meters per year, differing in 100 meters for each scenario from 300 to 700 meters. The worst-case scenario is building 300 meters annually, with the investment not getting profitable in the researched period at all, resulting with NPV of – 672 800 EUR in the final year of investment. Therefore, constructing less than 300 meters yearly is not feasible for Brasov district in Romania. Conversely, building 700 meters a year leads to faster returns but requires significantly more capital investment.

This finding indicates that areas identified as suitable for district heating grid expansion must be carefully scheduled and planned. It is essential that the expansion does not consider lengths shorter than 300 meters, aligning with the initial concept of this work.

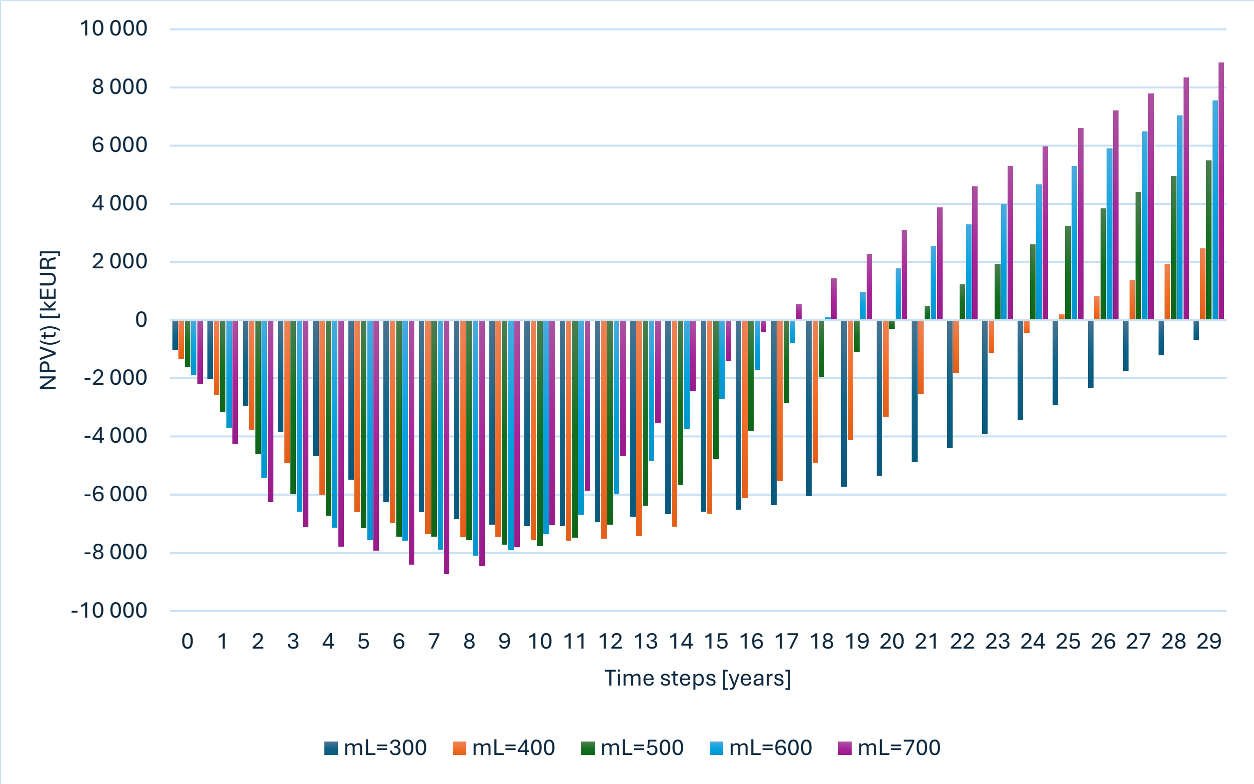

Figure 11 NPV function for different scenarios over investment period

The following Figure 12 presents the Levelized Cost of Heat (LCOH) for varying annual maximal lengths built, providing insight into the cost efficiency of the district heating system. The LCOH, which reflects the average cost per unit of heat supplied, shows a clear trend: higher expansion rates result in a reduction of LCOH, indicating the benefits of economies of scale. Faster expansion reduces per-unit costs over time, highlighting the financial advantages of investing in larger construction rates. This emphasizes the need for careful planning to balance the pace of expansion with long-term cost-effectiveness.

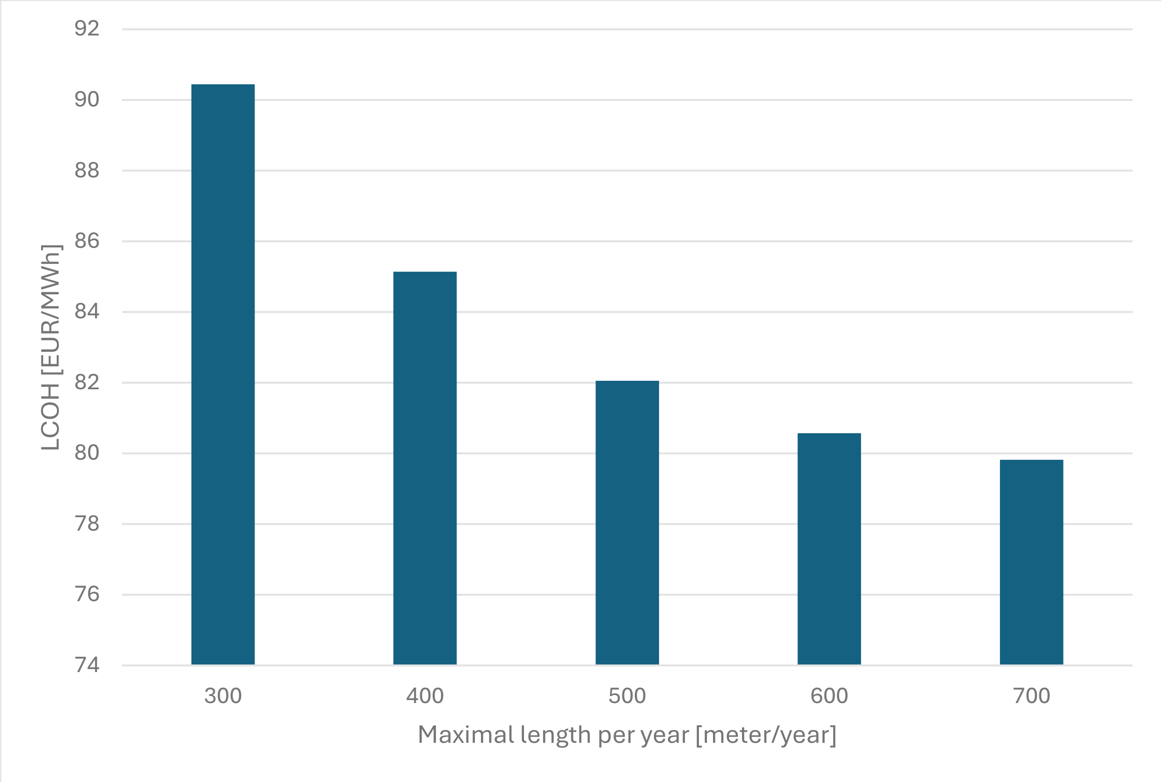

Figure 12 LCOH for different maximal lengths per year scenarios

4.2 Heat price fluctuations in ± 10% from the initial value

In this section, I analyse the impact of 10% fluctuations in heat price on the Net Present Value (NPV) of the district heating system. To assess the sensitivity of the investment to changes in the market price of heat, two scenarios are considered: one where the heat price increases by 10% and another where it decreases by 10%. The results will highlight how these price fluctuations affect the overall profitability of the system. The starting price of heat is 89 EUR/MWh as it was stated and analysed in the previous chapters. For the first scenario I simulate the decrease in price for 10% resulting with the heat price of 80,1 EUR/MWh. Figure 13 shows new results of NPV for all five scenarios in 3 cases of heat price. As seen in the Figure 13 low pipeline expansion rates as 300 and 400 meters a year are not viable by year 20, regardless the heat price oscillations. Maximal yearly built length of 500 meters could be considered a tipping point of this research, because it is profitable under favourable pricing. These results support the thesis argument that slower investment may lower upfront costs but leads to underutilization and long-term unprofitability especially when fixed infrastructure costs are significant.

Figure 13 also demonstrates the importance of pricing scenarios in determining DH expansion feasibility, underlining the need for sensitivity analysis in investment planning.

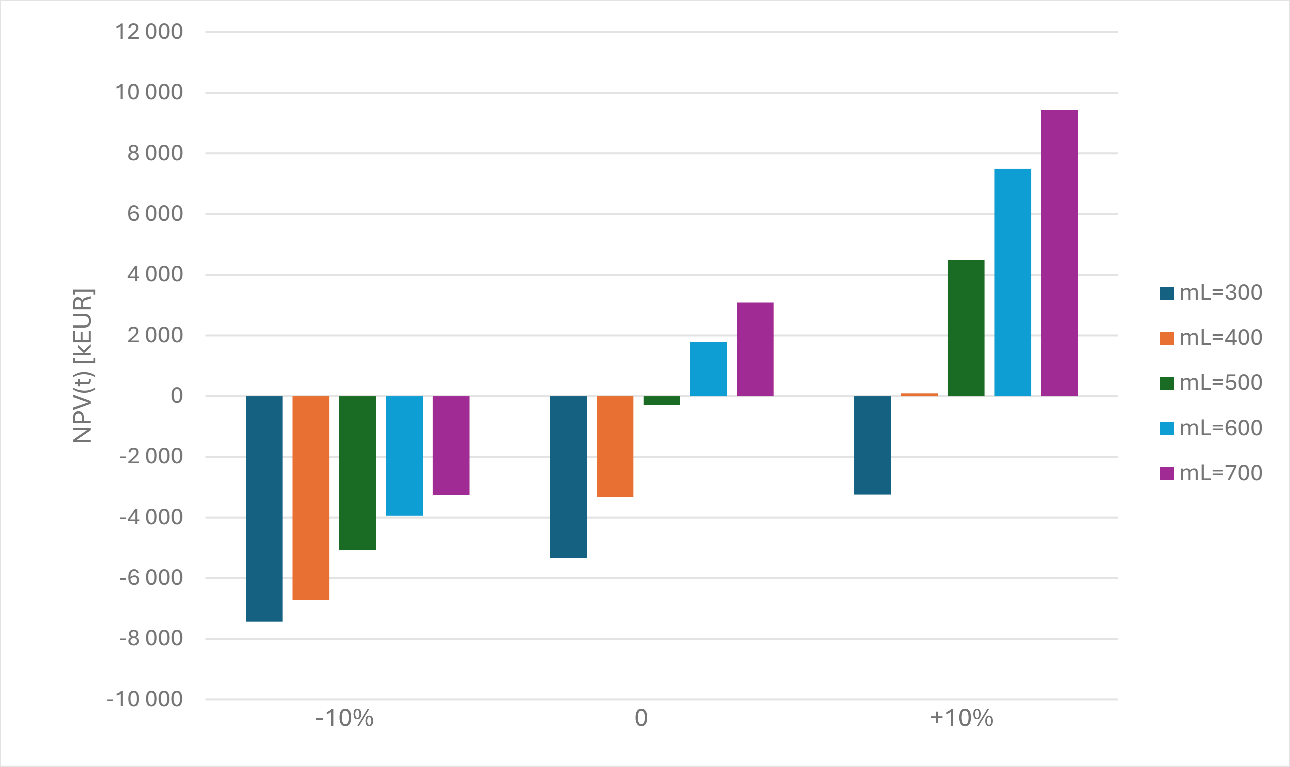

Figure 13 NPV for year 20 with district heat price variations of ±10%

5. Discussion

5.1 Implications for DH planning

The results confirm that there is a minimal annual expansion threshold, below which the investment is unprofitable. In this study, the exact amount of expansion rate has not been explicitly found, as this was not the goal of the study but in the process of evaluating the results one could agree that this threshold should not be far under the lowest examined scenario of 300 meters a year. This finding emphasizes the importance of strategic planning of district heating network expansion, to avoid unprofitable scenarios.

The comparison of different scenarios highlights the trade-off between short-term financial feasibility and long-term cost efficiency. While the scenario of expanding the grid with highest rate each year leads to greater long-term profits, it also requires substantial upfront investment, which may sometimes be scarce. As district heating grids are expanded sequentially due to workforce availability and financial limitations, investors should consider an optimal expansion rate that maximizes cash flow while considering realistic resource constraints.

Moreover, the speed of DH network rollout in practice depends on several local factors, including the timing of customer contracts, availability of generation capacity, internal planning resources, and permit approvals. Infrastructure coordination with other utilities, material supply issues, and shortages in skilled workers can further delay implementation. Construction conditions such as soil type and pipe diameter also heavily influence buildout speed—expansion in dense inner cities tends to be slower than in greenfield zones. These real-world factors must be considered when applying or interpreting model results outside the case study area.

5.2 Limitations and future research

The rolling investment MILP framework presented in this work provides a robust tool for long-term strategic planning of district heating (DH) expansion. By integrating investment decisions with the timing of expansions, the model captures the dynamics and economic trade-offs of gradual infrastructure build-out. It focuses on the sequencing of pipeline construction under realistic constraints such as limited workforce capacity and annual investment budgets. The model results provide insights in the minimum construction limits and investment capital needed to plan an economically viable DH investment. While this work provides a robust framework for sequential DH expansion planning, there are some limitations that should be addressed in future research.

For example, fixed heat demand assumption – this work does not assume any changes in demand on heat in the area of expansion, due to demographic shifts, energy policy changes and energy technology improvements. Furthermore, this work maximizes the cash flow in each investment year, whereas it could be maximizing the NPV, which is mathematically more challenging and is out of the scope of this work.

The model makes output based on the fixed area of research and does not consider any other constraints such as cost limitations, investment limitations or CO2 emissions. It rather strongly focuses on the workforce capacities and gives insights in the needed upfront investment for different project scenarios.

Further enhancements of the model could include Dynamic Programming method instead of MILP, which is naturally better suited for sequential decision-making. This way some non-linearity and uncertainty in heat demand could be included in the model, which could occur as a result of heat saving and housing renovation policies.

In addition to the previously discussed extensions, future development of the presented MILP model could also include multi-objective optimization. While the current model focuses solely on economic profitability by maximizing cash flow, it could be expanded to include environmental objectives through the implementation of weighted objective functions such as those developed in [22] and [26]. This would allow simultaneous evaluation of both economic and environmental criteria, such as CO2 emissions. Moreover, early-stage planning tools such as *Heat4Future* [27] could serve as a useful complement to detailed optimization models, especially for preliminary scenario screening under limited data availability. These tools may help identify viable strategic directions before applying infrastructure-level, resource-constrained planning models like the one presented in this thesis.

6. Conclusion

This thesis set out to answer the research question: *What is the most economical sequence for expanding district heating networks within a predefined area, given constraints on annual pipeline construction lengths?*

The findings contribute to the research gap concerning the sequential or incremental expansion of district heating grids under real-world constraints, specifically workforce availability and investment capacity. To address this, a single-objective MILP optimization model was developed, offering a novel approach for expanding the DH grid along a predefined route over the investment period in the most economically efficient way. Notably, while slower expansion scenarios such as 300 m/year may minimize upfront yearly investment, they fail to recover fixed infrastructure costs over the investment period and result in negative cash flow, making them economically unviable in the long term.

Although the case study was conducted on the upper part of a district in Brasov, Romania, the model would work the same for any other city or area that could be described with the mentioned input data. It would also be possible to run the model as an extension regarding more than one source nodes or "active" connected nodes.

This study has analysed district heating grid expansion strategies by evaluating the Net Present Value (NPV) and the Levelized Cost of Heat (LCOH) for five different pipeline expansion scenarios of 300, 400, 500, 600 and 700 meters constructed per year.

To enable broader applicability, the "maximum length per year" parameter can be interpreted as a relative share of the total grid length per year, rather than an absolute distance. For instance, in Brasov, the full grid length of 7818 meters in observed area corresponds to buildout times of 26, 16, and 12 years under annual expansion limits of 300, 500, and 700 meters respectively. This framing allows planners in other cities to scale results appropriately, such as interpreting 300 meters/year as ~4% of total length. In Austria, current district heating utilities plan to expand their 6100 km network for further 1300 km by an average of 123 km/year between 2024–2033, which equals roughly 2% of the already existing pipeline length annually [28], and approximately 9.5% of the total planned expansion volume.. This benchmarking supports the idea of using relative expansion rates (e.g., %/year) as a planning metric. This corresponds well with the assumption of building 700m a year in case study of Brasov, which amounts approximately 9% of the whole assumed building length of 7818 m.

As already mentioned, this "Maximal length pro period" incorporates both the workforce and the investment capacity constraints. The results of this work provide crucial insights into the financial viability of different investment strategies.

6.1 NPV Analysis

The results shown in Chapter 4 (Figure 11) indicate that the profitability of the DH investment depends significantly on the annual expansion rate, or rather "Maximal length per period" as called in the model. The first scenario of the lowest pipeline expansion rate of 300 meters per year barely becomes profitable towards the end of the investment period in year 26 of 30.

Conversely, higher expansion rates of 600 or 700 meters per year accelerate profitability and generate higher NPV values but also require a larger upfront capital investment.

6.2 LCOH Analysis

As the annual maximal built length increases, the Levelized Cost of Heat decreases. This trend could also be interpreted as the effect of economies of scale – with larger annual investments leading to lower long-term heat production costs. This is presented in Figure 12. Examination of LCOH or in this case LCODH (Levelized costs of district heating) helps the researcher or energy utility to decide which source of energy it is going to choose for a given area.

As shown in the IRENA REmap analysis [29], renewable-based district heating systems are expected to become increasingly cost-competitive by 2030, particularly in countries like Germany, where geothermal and solar thermal options show LCOHs in the range of 67–100 EUR/MWh, regarding the conversion rate of 0.93 USD/EUR, approaching the costs of conventional systems when externalities are considered.
This reinforces the relevance of long-term strategic models, such as the rolling investment approach proposed in this work, which allows city-level planners to evaluate not only economic feasibility under investment constraints, but also the transition pathway toward low-carbon heat supply in line with policy goals.

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