The Stückelberg Path to Pure de Sitter Supergravity

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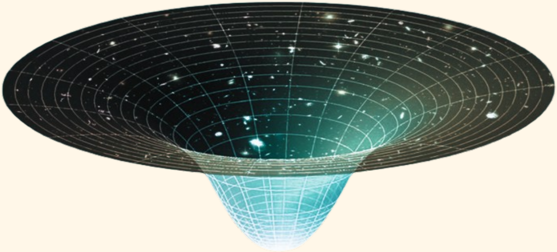
Introduction



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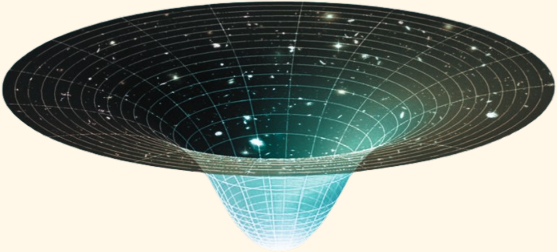

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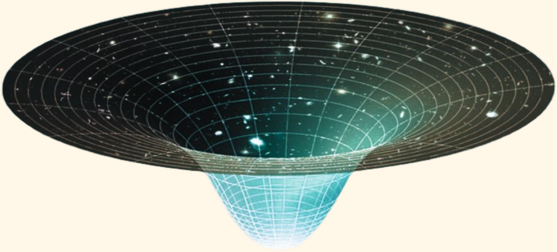

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- Also, according to inflation theory, the early universe underwent an exponential expansion
- Accelerated expansion of universe $\Rightarrow$ de-Sitter spacetime
- But linearly realised SUSY w/o scalar fields does not allow positive cosmological constant

No-go Results for Linear Superalgebra in dS_4

Linearly realised $\mathcal{N}$=1 SUSY does not allow for dS_4 solutions.

No Majorana Killing Spinor

- *Majorana Killing spinors*, needed for linear realisation of $\mathcal{N}$=1 SUSY in curved Lorentzian spacetimes, *do not exist in* dS_4.

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No Unitary Representation

- *Linear* dS_4 *super-algebra* in which the $\{Q, \bar{Q}\}$ anti-commutator closes on the generators of the $SO(4,1)$ dS isometry group, *does not have unitary representations.*

No-go Results for Linear Superalgebra in dS_4

In dS/AdS algebras translations have a non-zero commutator:

$$[P_\mu, P_\nu] = s \frac{1}{4L^2} M_{\mu\nu} \quad \text{where} \quad s = \begin{cases} -1 & \text{for dS} \\ +1 & \text{for AdS} \end{cases}$$

The Lie algebras are $SO(4,1)$ for dS and $SO(3,2)$ for AdS.

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Linear SUSY algebra:

$$[P_\mu, Q_\alpha] = \frac{1}{4L}(\gamma_\mu Q)_\alpha \qquad [M_{\mu\nu}, Q_\alpha] = -(\gamma_{\mu\nu})_\alpha{}^\beta Q_\beta$$

$$\{Q_\alpha, Q_\beta\} = -\frac{1}{2}(\gamma^\mu)_{\alpha\beta}P_\mu - \frac{1}{8L}(\gamma^{\mu\nu})_{\alpha\beta}M_{\mu\nu} \qquad [P_\mu, P_\nu] = 0$$

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Jacobi Identity Forbids Linear Superalgebra in dS_4

On embedding the dS/AdS algebra in the above linear SUSY algebra, the Jacobi identity

$$[P_\mu, P_\nu, Q] = 0$$

fixes $s = 1$. Therefore, linear $\mathcal{N}$=1 super-dS algebra does not exist in $4D$.

Move to non-linear SUSY

- Pure dS SUGRA was constructed by realising $\mathcal{N}$=1 SUSY non-linearly
- Different methods to realise SUSY non-linearly:
    - Nilpotent superfields
    - Goldstino brane action
    - Stückelberging unimodular supergravity

Motivation

- Different methods for realising $\mathcal{N}$=1 SUSY non-linearly
- Do different constructions give the same action?
- How to compare the actions?

Brief Historical Review

- In 2015 Bergshoeff, Freedman, Kallosh & Proeyen presented dS SUGRA for the first time using superconformal methods.
- Later in 2015 Bandos, Martucci, Sorokin & Tonin presented dS SUGRA by coupling a goldstino 3-brane to minimal supergravity.
- A lot of work has been done on dS SUGRA in recent years [Antoniadis, Dudas, Farakos, Ferrara, Hasegawa, Kehagias, Kuzenko, Porrati, Sagnotti, Scalisi, Wrase, Yamada, ...'15-'21]
- Cosmological and inflationary models in dS SUGRA [Andriot, Antoniadis, Dudas, Ferrara, Sagnotti, Buchmuller, Heurtier, Wieck, Ferrara, Kallosh, Linde, Thaler, Zavala, Zwirner, ...'15-'21]
- Brane models [Angelantonj, Antoniadis, Dudas, Mourad, Parameswaran, Pradisi, Riccioni, Sagnotti, Uranga, Vercnocke, Zavala, ...'99-'21]

Outline

Nilpotent Superfield Construction

In superconformal model we use 3 multiplets:

1) chiral compensating multiplet $\{X^0, \chi^0, F^0\}$,
2) nilpotent chiral multiplet $S = \{X^1, \mathcal{G}, F^1\}$,
3) Lagrange multiplier multiplet $\{\Lambda, \chi^\Lambda, F^\Lambda\}$.

The Lagrangian is [E Bergshoeff, D Freedman, R Kallosh & A Proeyen '15]

$$\mathcal{L} = [\tfrac{1}{2}\eta_{IJ} X^I \bar{F}^J]_F + [\mathcal{W}(X^I)]_F + [\Lambda (X^1)^2]_F$$

where $I, J = 0, 1$; $\eta_{IJ} = \text{diag}(-1, 1)$ and the superpotential $\mathcal{W}$ is

$$\mathcal{W} = a\,(X^0)^3 + b\,(X^0)^2 X^1$$

where a and b are arbitrary constants.

Nilpotent Superfield

$$S = X^1 + \sqrt{2}\,\theta\,\mathcal{G} + \theta^2 F^1$$

Nilpotency constraint: $S^2 = 0$

$$\Rightarrow (X^1)^2 + 2\sqrt{2}\,\theta\,\mathcal{G}X^1 + \theta^2\,(2\,F^1X^1 - \mathcal{G}^2) = 0$$

$$\Rightarrow X^1 = \frac{\mathcal{G}^2}{2F^1}$$

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This eliminates the fundamental scalar partner of the goldstino $\mathcal{G}$ and hence SUSY is realised non-linearly.

It gives solutions with the cosmological constant

$$\Lambda = |b|^2 - |a|^2$$

Goldstino Brane Action in Supergravity

$$S_B = S_{SG} + S_{VA}$$ [I Bandos, L Martucci, D Sorokin, M Tonin '15]

$$= \underbrace{-\frac{3}{2\kappa^2}\int d^8z \,\mathrm{Ber}\, E}_{\text{Standard pure SUGRA}} \; - \; \underbrace{\frac{2\, m^{(B)}}{\kappa^2}\left(\int d^6\zeta_L \mathcal{E} + \text{h.c.}\right)}_{\text{AdS cosmological constant term}}$$

$$\underbrace{-f^2 \int d^4\xi \,\det \boldsymbol{E}(z(\xi))}_{\text{Goldstino brane coupled to SUGRA}}$$

ξ^i are the 3-brane worldvolume coordinates with $i = 0, 1, 2, 3$. Coupling to supergravity is given via the embedding in the bulk superspace as

$$\xi^i \to z^M(\xi) = (x^\mu(\xi), \theta^\alpha(\xi), \bar{\theta}^{\dot{\beta}}(\xi))$$

Goldstino Brane Action in Supergravity

The solutions to the equations of motion of the auxilliary fields show that they belong to nilpotent superfields.

S_B perturbed up to the 3rd order in fluctuations, is:

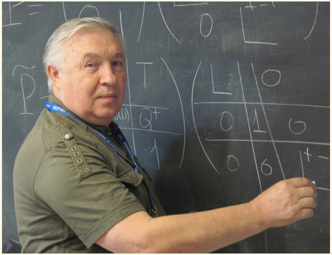
$$S = \frac{1}{2\kappa^2}\int d^4x\,\Big[\big\{\sqrt{-g}\,R - \varepsilon^{\mu\nu\rho\lambda}\left(\psi_\mu\sigma_\nu\mathcal{D}_\rho\bar{\psi}_\lambda + h.c.\right)\big\}^{(3)} - \left\{2\sqrt{-g}\left(\boldsymbol{\Lambda}_2 - \frac{m^2}{3}\right)\right\}^{(3)}$$
$$+\big\{\tfrac{2}{3}m\psi_\mu\sigma^{\mu\nu}\psi_\nu + 2i\boldsymbol{\Lambda}_2\breve{\mathcal{G}}\sigma^\mu\bar{\psi}_\mu - 2i\boldsymbol{\Lambda}_2\breve{\mathcal{G}}\sigma^\mu\boldsymbol{D}_\mu\bar{\breve{\mathcal{G}}} + \tfrac{4}{3}m\boldsymbol{\Lambda}_2\breve{\mathcal{G}}^2$$
$$+\ \tfrac{1}{3}hm\psi_\mu\sigma^{\mu\nu}\psi_\nu + \tfrac{2}{3}m\psi_\mu h_\rho{}^{[\mu}\sigma^{\nu]\rho}\psi_\nu + \boldsymbol{\Lambda}_2(ih\breve{\mathcal{G}}\sigma^\mu\bar{\psi}_\mu - i\breve{\mathcal{G}}h^\mu{}_\nu\sigma^\nu\bar{\psi}_\mu$$
$$- ih\breve{\mathcal{G}}\sigma^\mu\boldsymbol{D}_\mu\bar{\breve{\mathcal{G}}} + i\breve{\mathcal{G}}h^\mu{}_\nu\sigma^\nu\boldsymbol{D}_\mu\bar{\breve{\mathcal{G}}} + 2i\breve{\mathcal{G}}\sigma^\mu\bar{\omega}^{(1)}_\mu\bar{\breve{\mathcal{G}}} + \tfrac{2}{3}hm\breve{\mathcal{G}}^2) + h.c.\big\}\Big]$$

$\breve{\mathcal{G}}$ is the goldstino.

Volkov-Akulov Lagrangian

In 1972 **Dmitrij Vasilievich Volkov** and **Vladimir P. Akulov** developed the **Volkov-Akulov Lagrangian** formalism. It realizes supersymmetry entirely with a fermion.

Volkov



Akulov

Spontaneous breaking of SUSY produces a Goldstone field. Volkov-Akulov formalism enables the construction of the Lagrangian of the Goldstone field. SUSY is realised non-linearly with just one fermion and no boson.

Constructing Volkov-Akulov Action

$\mathcal{N} = 1$ in $D = 4$

SUSY transformations:

$$\delta x^a = i\left(\epsilon\,\sigma^a\,\bar\theta - \bar\epsilon\,\sigma^a\,\theta\right), \quad \delta\theta^\alpha = \epsilon^\alpha$$

Replace the Grassman coordinate θ with the field $\chi(x)$.

$$\theta^\alpha \to \kappa\,\chi^\alpha(x)$$

$$\delta x^a = i\,\kappa^2\left(\epsilon\,\sigma^a\,\bar\chi - \chi\,\sigma^a\,\bar\epsilon\right), \quad \delta\chi^\alpha = \epsilon^\alpha + i\,\kappa^2\left(\epsilon\,\sigma^a\,\bar\chi - \chi\,\sigma^a\,\bar\epsilon\right)\partial_a\chi^\alpha$$

It is easy to check that the above transformation realises SUSY algebra.

$$\left(\delta_\epsilon\delta_\eta - \delta_\eta\delta_\epsilon\right)\chi = 2i\kappa^2\left(\epsilon\,\sigma^a\,\bar\eta - \eta\,\sigma^a\,\bar\epsilon\right)\partial_a\chi^\alpha$$

$$\{Q_\alpha\,,\bar Q_{\dot\beta}\} = 2\,\sigma_{\alpha\dot\beta}{}^a\,P_a$$

Q_α $(\alpha = 1, 2)$ are Weyl spinor generators
P_a are translation generators.

Constructing Volkov-Akulov Action

Find a SUSY-invariant Cartan one-form:

$$g = e^{i\,x^a P_a}\, e^{i\,\theta Q}\, e^{-i\,\bar\theta \bar Q}$$

$$\Omega = -ig^{-1}dg = E^a P_a + E^\alpha Q_\alpha + E_{\dot\alpha} Q^{\dot\alpha}$$

$$E^a = dx^a + i\kappa^2 \left(\chi\,\sigma^a\, d\bar\chi - d\chi\,\sigma^a\,\bar\chi\right)$$

Construct Volkov-Akulov Action

In D dimensions:

$$S = \frac{1}{\kappa^2 D!}\int \varepsilon_{a_1,\ldots a_D}\, E^{a_1}\wedge E^{a_2}\ldots\wedge E^{a_D} = -\frac{1}{\kappa^2}\int d^D x\ \det E^a_m$$

Unimodular Gravity

Why Unimodular Gravity?

Einstein-Hilbert action: $S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\,(R - 2\,\Lambda)$

Λ is a constant to begin with, so can have only one value throughout.

Unimodular gravity action:

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} R - 2\,\Lambda(x)\,(\sqrt{-g} - \epsilon_0)$$

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Λ has emerged as a constant of integration from within the theory itself.

Unimodular gravity has the advantage of allowing at once for both positive and negative Λ.

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where $\Lambda(x)$ is a Lagrange multiplier field imposing the unimodularity condition:

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ϵ_0 is a constant, traditionally set to unity. But it breaks diffeomorphism invariance because of the term

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How to restore the broken diffeomorphism invariance?

Symmetry Breaking

Free Maxwell Lagrangian:

$$\mathcal{L}_M = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \qquad \text{where } F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

It is gauge invariant under the transformation $A_\mu \to A_\mu + \partial_\mu \alpha$.

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$$\mathcal{L}_M = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu$$
$$\xrightarrow{A_\mu \to A_\mu + \partial_\mu \alpha} -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}(A_\mu - \partial_\mu \alpha)(A^\mu - \partial^\mu \alpha)$$

Restoration of Broken Symmetry via the Stückelberg Procedure

Promote the parameter α to a field f.

$$\tilde{\mathcal{L}}_M = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}(A_\mu - \partial_\mu f)(A^\mu - \partial^\mu f)$$

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$\tilde{\mathcal{L}}_M$ is invariant under

$$\delta A_\mu = \partial_\mu \alpha\,, \quad \delta f = -\alpha\,.$$

So now we have a gauge-invariant Lagrangian $\tilde{\mathcal{L}}_M$ with a mass term.

Perturbative Approach

Unimodular Gravity and the Stückelberg Procedure

Under active diffeomorphism tranformation, Λ transforms as

$$\Lambda \to \Lambda' = \Lambda - \xi^\mu \partial_\mu \Lambda + \frac{1}{2}\xi^\nu \partial_\nu \left(\xi^\mu \partial_\mu \Lambda\right) + \ldots$$

We promote the diffeo parameter ξ^μ to the field ϕ^μ. Then the action becomes

$$S_\phi = \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{-g}R - 2\Lambda\sqrt{-g} + 2\,\Lambda\,\epsilon_0 \left[1 + \partial_\mu \phi^\mu + \frac{1}{2}\phi^\nu \partial_\nu \partial_\mu \phi^\mu + \frac{1}{2}\left(\partial_\mu \phi^\mu\right)\left(\partial_\nu \phi^\nu\right) + \ldots\right]\right]$$

It is invariant up to the relevant order, when ϕ^μ transforms as

$$\delta\phi^\mu = -\,\xi^\mu - \frac{1}{2}\xi^\nu \partial_\nu \phi^\mu + \frac{1}{2}\phi^\nu \partial_\nu \xi^\mu + \ldots$$

Unimodular Gravity and the Stückelberg Procedure

We can also do passive diffeomorphism transformation.

$$x^\mu \to \hat{x}^\mu(x) \xrightarrow{\text{Stückelberg}} s^\mu(x)$$

Then the Stückelberged action is

$$S_s = \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{-g}\mathcal{R} - 2\Lambda \left(\sqrt{-g} - \left| \mathrm{Det}\left(\frac{\partial s}{\partial x} \right) \right| \epsilon_0 \right) \right] ,$$

Using $s^\mu = x^\mu + \phi^\mu + \ldots$ the two Stückelberged actions are identical order by order in ϕ^μ.

Stückelberged Unimodular Supergravity

Perturbative Approach

$\mathcal{N} = 1$ Supergravity Action

Now we supersymmetrise the theory and see what solutions we get.

$\mathcal{N} = 1$ supergravity action

$$S = -\frac{6}{8\pi G_N} \int d^4x\, d^2\Theta\, \mathcal{E}\mathcal{R} + h.c.$$

Volume element factor $\mathcal{E}$ is

$$\begin{aligned} \mathcal{E} &= \mathcal{F}_0 + \sqrt{2}\,\Theta\mathcal{F}_1 + \Theta\Theta\mathcal{F}_2, \qquad \text{with} \\ \mathcal{F}_0 &= \tfrac{1}{2}e\,, \\ \mathcal{F}_1 &= \tfrac{i\sqrt{2}}{4}e\sigma^\mu\bar{\psi}_\mu\,, \\ \mathcal{F}_2 &= -\tfrac{1}{2}eM^* - \tfrac{1}{8}e\bar{\psi}_\mu\left(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu\right)\bar{\psi}_\nu\,. \end{aligned}$$

$\mathcal{N}$=1 Supergravity Action

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$$S = -\frac{6}{8\pi G_N} \int d^4x \, d^2\Theta \, \mathcal{E}\mathcal{R} + h.c.$$

Superfield $\mathcal{R}$ is

$$\begin{aligned}
\mathcal{R} &= -\frac{1}{6}(\mathcal{R}_0 \; + \; \Theta\mathcal{R}_1 \; + \; \Theta\Theta\mathcal{R}_2), \qquad \text{with} \\
\mathcal{R}_0 &= M \, , \\
\mathcal{R}_1 &= \sigma^\mu \bar{\sigma}^\nu \psi_{\mu\nu} - i\sigma^\mu \bar{\psi}_\mu M + i\psi_\mu b^\mu \, , \\
\mathcal{R}_2 &= \frac{1}{2}R + i\bar{\psi}^\mu \bar{\sigma}^\nu \psi_{\mu\nu} + \frac{2}{3}MM^* + \frac{1}{3}b_\mu b^\mu - ie_a^\mu \mathcal{D}_\mu b^a + \frac{1}{2}\bar{\psi}\bar{\psi}M \\
&\quad - \frac{1}{2}\psi_\mu \sigma^\mu \bar{\psi}_\nu b^\nu + \frac{1}{8}\varepsilon^{\mu\nu\rho\sigma}(\bar{\psi}_\mu \bar{\sigma}_\nu \psi_{\rho\sigma} + \psi_\mu \sigma_\nu \bar{\psi}_{\rho\sigma}) \, .
\end{aligned}$$

Supergravity multiplet: $\varphi_{sg} = (e_\mu^a, \psi_\mu^\alpha, b_\mu, M)$.

$\mathcal{N} = 1$ Unimodular Supergravity Action

$\mathcal{N} = 1$ unimodular supergravity action [S. Nagy, A. Padilla, I. Zavala '19]

$$S = -\frac{6}{8\pi G_N} \int d^4x\, d^2\Theta \left[\mathcal{E}\,\mathcal{R} + \frac{1}{6}\Lambda\left(\mathcal{E} - \mathcal{E}_0\right)\right] + h.c.$$

where

$$\mathcal{E}_0 = \epsilon_0 - \frac{1}{2} m\Theta^2 \,.$$

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The Lagrange multiplier field Λ is

$$\Lambda = \Lambda_0 + \sqrt{2} \, \Theta \Lambda_1 + \Lambda_2 \Theta^2$$

$$\Lambda_0 \big|_\infty = K_0 \, , \qquad \Lambda_1 \big|_\infty = 0 \, , \qquad \Lambda_2 \big|_\infty = K_2 \, .$$

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Varying over Λ, we get

$$\mathcal{E} = \mathcal{E}_0 \, .$$

Super-Stückelberg Procedure

Stückelberg trick is performed up to the 2^{nd} order in Stückelberg fields, so the diffeo and SUSY transformations of the superfield components are derived up to the 2^{nd} order in ξ^μ and ϵ.

Then we promote the diffeo and SUSY transformation parameters to fields:

$$\xi^\mu \to \phi^\mu \quad \text{and} \quad \epsilon \to \zeta$$

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Symmetry breaking part: $\frac{1}{16\pi G_N}\int d^4x\,(2\Lambda_2\epsilon_0 - m\Lambda_0 + h.c.)$

The action can then be constructed perturbatively as:

$$\begin{aligned}16\pi G_N\mathcal{L} =&\sqrt{-g}\Big[R - \tfrac{2}{3}M^* M + \tfrac{2}{3}b^\mu b_\mu + \varepsilon^{\mu\nu\rho\sigma}\Big(\bar{\psi}_\mu\bar{\sigma}_\nu\tilde{\mathcal{D}}_\rho\psi_\sigma - \psi_\mu\sigma_\nu\tilde{\mathcal{D}}_\rho\bar{\psi}_\sigma\Big)\Big]\\ &+ \tfrac{1}{2}\sqrt{-g}\Big[-2\Lambda_2 + \sqrt{2}i\Lambda_1\sigma^\mu\bar{\psi}_\mu + 2\Lambda_0\left(\bar{\psi}_\mu\bar{\sigma}^{\mu\nu}\bar{\psi}_\nu + M^*\right) + h.c.\Big]\\ &+ 2\left[\Lambda_2 + \delta^{(\epsilon\to\zeta,\,\xi^\mu\to\phi^\mu)}\Lambda_2\right]\epsilon_0 - m\left[\Lambda_0 + \delta^{(\epsilon\to\zeta,\,\xi^\mu\to\phi^\mu)}\Lambda_0\right] + h.c.\end{aligned}$$

de Sitter Solutions

$$
\begin{aligned}
G_{\mu\nu} + g_{\mu\nu}\left[\tfrac{1}{3}MM^* + \tfrac{2}{3}b^\rho b_\rho + \mathrm{Re}(\Lambda_2 - \Lambda_0 M^*)\right] & \\
+b_\mu b_\nu &= 0\,, \\
-\tfrac{2}{3}M + \Lambda_0 &= 0\,, \\
b_\mu &= 0\,, \\
-\partial_\mu\Lambda_2 + m\partial_\mu\Lambda_0 - \tfrac{1}{2}\partial_\nu\phi^\nu\partial_\mu\Lambda_2 + \tfrac{1}{2}\partial_\mu\left(\phi^\nu\partial_\nu\Lambda_2\right) & \\
+\tfrac{m}{2}\partial_\nu\phi^\nu\partial_\mu\Lambda_0 - \tfrac{m}{2}\partial_\mu\left(\phi^\nu\partial_\nu\Lambda_0\right) + h.c. &= 0\,, \\
\sqrt{-g}M^* - m - m\partial_\mu\phi^\mu - \tfrac{m}{2}\partial_\mu\left[\phi^\mu\partial_\nu\phi^\nu\right] &= 0\,, \\
-\sqrt{-g} + 1 + \partial_\mu\phi^\mu + \tfrac{1}{2}\partial_\mu\left(\phi^\mu\partial_\nu\phi^\nu\right) &= 0\,.
\end{aligned}
$$

$$
\begin{aligned}
\langle g_{\mu\nu}\rangle &= \bar{g}_{\mu\nu}, & &\text{with} \quad \sqrt{-\bar{g}} = 1\,, \\
\langle M\rangle &= m\,, \\
\langle\Lambda_0\rangle &= \frac{2}{3}m = K_0\,, \\
\langle\Lambda_2\rangle &= \boldsymbol{\Lambda}_2 = K_2, & &\text{with} \quad \mathrm{Im}(\Lambda_2) = 0\,.
\end{aligned}
$$

$$
c.c. = \boldsymbol{\Lambda}_2 - \frac{1}{3}m^2 \;=\; K_2 - \frac{3}{4}K_0^2
$$

Stückelberged Unimodular Supergravity Action

We finally arrive at the following action: [S. Bansal, S. Nagy, A. Padilla, I. Zavala '20]

$$
\begin{aligned}
S = \frac{1}{2\kappa^2}\int d^4x\,\Big[& \{\sqrt{-g}\,R - \varepsilon^{\mu\nu\rho\lambda}(\psi_\mu\sigma_\nu\mathcal{D}_\rho\bar{\psi}_\lambda + h.c.)\}^{(3)} - \left\{2\sqrt{-g}\left(\boldsymbol{\Lambda}_2 - \frac{m^2}{3}\right)\right\}^{(3)} \\
& + \{\tfrac{2}{3}m\psi_\mu\sigma^{\mu\nu}\psi_\nu + 2i\boldsymbol{\Lambda}_2\breve{\mathcal{G}}\sigma^\mu\bar{\psi}_\mu - 2i\boldsymbol{\Lambda}_2\breve{\mathcal{G}}\sigma^\mu\boldsymbol{D}_\mu\bar{\breve{\mathcal{G}}} + \tfrac{4}{3}m\boldsymbol{\Lambda}_2\breve{\mathcal{G}}^2 \\
& + \tfrac{1}{3}hm\psi_\mu\sigma^{\mu\nu}\psi_\nu + \tfrac{2}{3}m\psi_\mu h_\rho{}^{[\mu}\sigma^{\nu]\rho}\psi_\nu + \boldsymbol{\Lambda}_2(ih\breve{\mathcal{G}}\sigma^\mu\bar{\psi}_\mu - i\breve{\mathcal{G}}h^\mu{}_\nu\sigma^\nu\bar{\psi}_\mu \\
& - ih\breve{\mathcal{G}}\sigma^\mu\boldsymbol{D}_\mu\bar{\breve{\mathcal{G}}} + i\breve{\mathcal{G}}h^\mu{}_\nu\sigma^\nu\boldsymbol{D}_\mu\bar{\breve{\mathcal{G}}} + 2i\breve{\mathcal{G}}\sigma^\mu\bar{\omega}^{(1)}_\mu\bar{\breve{\mathcal{G}}} + \tfrac{2}{3}hm\breve{\mathcal{G}}^2) + h.c.\}\Big]\,.
\end{aligned}
$$

Same as the Goldstino brane action! [I Bandos, L Martucci, D Sorokin, M Tonin '15]

Complete Action

Stückelberged Unimodular Gravity Action

to All Orders

Unimodular Gravity

Unimodular Gravity Action:

$$S = \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{-g} R - 2\Lambda(x) \left(\sqrt{-g} - \epsilon_0 \right) \right]$$

where $\Lambda(x)$ is a Lagrange multiplier field imposing the unimodularity condition:

$$\sqrt{-g} = \epsilon_0 .$$

Under a finite diffeomorphism $\Lambda(x)$ transforms as

$$\Lambda(x) \to \Lambda'(x) = e^{\xi^\mu(x)\partial_\mu} \Lambda(x) ,$$

where $\xi^\mu(x)$, a 4-vector, is the diffeomorphism parameter.

Diffeomorphism Transformation of the Action

$$S = \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{-g}\, R(x) - 2\sqrt{-g}\, \Lambda(x) + 2\, \epsilon_0\, \Lambda(x)\right]$$

$$\Big\downarrow \text{Diffeomorphism}$$

$$S = \frac{1}{16\pi G_N} \int d^4x \left[\sqrt{-g}\, R(x) - 2\sqrt{-g}\, \Lambda(x) + 2\, \epsilon_0\, e^{\xi^\nu(x)\partial_\nu} \Lambda(x)\right]$$

How to restore the broken diffeomorphism invariance?

Applying Stückelberg Procedure to Unimodular Gravity Action

Coming back to our unimodular gravity action, we apply the Stückelberg procedure by promoting the diffeomorphism parameter $\xi^\mu(x)$ to Stückelberg fields $\phi^\mu(x)$:

$$e^{\xi^\mu(x)\boldsymbol{\partial}_\mu}\Lambda(x) \quad \rightarrow \quad e^{\phi^\mu(x)\partial_\mu}\Lambda(x)$$

where unlike $\xi^\mu(x)$ which is a 4-vector, $\phi^\mu(x)$ is a set of four fields $-\phi^0(x)$, $\phi^1(x)$, $\phi^2(x)$ and $\phi^3(x)$.

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where unlike $\xi^\mu(x)$ which is a 4-vector, $\phi^\mu(x)$ is a set of four fields $-\ \phi^0(x)$, $\phi^1(x)$, $\phi^2(x)$ and $\phi^3(x)$.

On applying the Stückelberg procedure on the transformed unimodular gravity action, we get,

$$S = \frac{1}{16\pi G_N}\int d^4x \left[\sqrt{-g}R - 2\sqrt{-g}\Lambda(x) + 2\,\epsilon_0\, e^{\phi^\nu(x)\partial_\nu}\Lambda(x)\right].$$

Diffeomorphism Transformation of the Stückelberg Fields

Diffeomorphism invariance of the Stückelberged action requires that

$$e^{-\phi'^{\nu}(x)\partial_\nu}\Lambda'(x) = e^{-\phi^{\nu}(x)\partial_\nu}\Lambda(x).$$

Using the fact that $\Lambda(x) = e^{\xi^{\nu}(x)\partial_\nu}\Lambda'(x)$, we immediately infer that

$$e^{-\phi'^{\nu}(x)\partial_\nu} = e^{-\phi^{\nu}(x)\partial_\nu} e^{\xi^{\nu}(x)\partial_\nu} \,.$$

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Diffeomorphism invariance of the Stückelberged action requires that

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$$e^{-\phi'^{\nu}(x)\,\partial_\nu} = e^{-\phi^{\nu}(x)\partial_\nu}e^{\xi^{\nu}(x)\partial_\nu}\,.$$

Using the integral form of the Baker-Campbell-Hausdorff formula, and working to linear order in ξ, we get,

$$\phi'^{\nu}(x)\partial_\nu = \phi^{\nu}(x)\partial_\nu - \frac{\mathrm{ad}_{(\phi^{\nu}(x)\partial_\nu)}}{1 - e^{\mathrm{ad}_{(\phi^{\nu}(x)\partial_\nu)}}}\xi^{\nu}(x)\partial_\nu$$

where $\mathrm{ad}_X(Y) = [X, Y]$

Diffeomorphism Transformation of the Stückelberg Fields

Diffeomorphism invariance of the Stückelberged action requires that

$$(e^{-\phi^\nu(x)\,\partial_\nu})'\Lambda'(x) = e^{-\phi^\nu(x)\partial_\nu}\Lambda(x).$$

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$$\begin{aligned}\phi'^\nu(x)\partial_\nu &= \phi^\nu(x)\partial_\nu - \frac{\mathrm{ad}_{(\phi^\nu(x)\partial_\nu)}}{1-e^{\mathrm{ad}_{(\phi^\nu(x)\partial_\nu)}}}\xi^\nu(x)\partial_\nu \\ &= \phi^\nu(x)\partial_\nu + \sum_{k=0}^{\infty}\frac{B_k^+(-1)^k\mathrm{ad}^k_{(\phi^\nu(x)\partial_\nu)}}{k!}\xi^\nu(x)\partial_\nu\,,\end{aligned}$$

where $\mathrm{ad}_X(Y) = [X, Y]$ and B_k^+ are the Bernoulli numbers,

$$B_0^+ = 1,\;\; B_1^+ = \frac{1}{2},\;\; B_2^+ = \frac{1}{6},\;\; B_3^+ = 0,\;\; B_4^+ = -\frac{1}{30},\;\; \ldots\quad .$$

Stückelberged Unimodular Supergravity Action to All Orders

$\mathcal{N}$=1 Supergravity Action

$\mathcal{N}$=1 supergravity action

$$S = -\tfrac{6}{8\pi G_N} \int d^4x\, d^2\Theta\, \mathcal{E}\mathcal{R} + h.c.$$

Infinitesimal (diffeomorphism + SUSY) = Infinitesimal SUGRA:

$$\delta\mathcal{E} = -\partial_M \left[(-1)^M\, \Xi^M \mathcal{E} \right] , \quad \text{where} \quad (-1)^M = \begin{cases} 1, & M = \mu\,, \\ -1, & M = \alpha\,. \end{cases}$$

$\mathcal{N}$=1 Supergravity Action

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The superfield Ξ^M, expressed in terms of a novel matrix $O_{\bar{N}}{}^M$ introduced by us:

$$\Xi^M \equiv \boldsymbol{\xi}^{\bar{N}} O_{\bar{N}}{}^M \, , \quad \text{where} \quad \boldsymbol{\xi}^{\bar{N}} \equiv \begin{pmatrix} \xi^\mu \\ \epsilon^\alpha \\ \bar{\epsilon}^{\dot{\alpha}} \end{pmatrix} .$$

ξ^μ: diffeomorphism parameter , $\epsilon^\alpha, \bar{\epsilon}^{\dot{\alpha}}$: local SUSY parameters

$\mathcal{N}$=1 Supergravity Action

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The superfield Ξ^M, expressed in terms of a novel matrix ${O_{\bar{N}}}^M$ introduced by us:

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ξ^μ: diffeomorphism parameter , $\epsilon^\alpha, \bar{\epsilon}^{\dot{\alpha}}$: local SUSY parameters

Components of matrix ${O_{\bar{N}}}^M$:

$${O_\nu}^M = \begin{cases} \delta^\mu_\nu\,, & M{=}\mu \\ 0\,, & M{=}\alpha \end{cases} \quad , \quad {O_\beta}^M = \begin{cases} 0\,, & M{=}\mu \\ \delta^\alpha_\beta + \frac{1}{3}\Theta^2 M^*\delta^\alpha_\beta\,, & M{=}\alpha \end{cases}$$

and

$${O_{\dot{\beta}}}^M = \begin{cases} 2\,i\,\Theta^\beta\sigma^\mu_{\beta\dot{\beta}} + \Theta^2(\bar{\psi}_\nu\bar{\sigma}^\mu\sigma^\nu)_{\dot{\beta}}\,, & M{=}\mu \\ i\,\Theta^\beta\psi^\alpha_\mu\sigma^\mu_{\beta\dot{\beta}} - i\,\Theta^2{\omega_\mu}^{\alpha\beta}\sigma^\mu_{\beta\dot{\beta}} - \frac{1}{2}\Theta^2\psi^\alpha_\nu(\bar{\psi}_\mu\bar{\sigma}^\nu\sigma^\mu)_{\dot{\beta}} + \frac{1}{6}\Theta^2 b_\mu\varepsilon^{\alpha\gamma}\sigma^\mu_{\gamma\dot{\beta}}\,, & M{=}\alpha\,. \end{cases}$$

$\mathcal{N}$=1 Unimodular Supergravity Action

$\mathcal{N}$=1 unimodular supergravity action [S. Nagy, A. Padilla, I. Zavala '19]

$$S = -\frac{6}{8\pi G_N}\int d^4x\, d^2\Theta\left[\mathcal{E}\,\mathcal{R} + \frac{1}{6}\Lambda\left(\mathcal{E} - \mathcal{E}_0\right)\right] + h.c.$$

Varying over Λ, we get

$$\mathcal{E} = \mathcal{E}_0\,.$$

$\mathcal{N}$=1 Unimodular Supergravity Action

Unimodularity condition: $\mathcal{E} = \mathcal{E}_0$

In components it reads:

$$\begin{aligned} \tfrac{1}{2}e =& \epsilon_0\,, \\ \tfrac{i\sqrt{2}}{4}e\sigma^\mu\bar{\psi}_\mu =& 0\,, \\ -\tfrac{1}{2}e\,M^* - \tfrac{1}{8}e\bar{\psi}_\mu\left(\bar{\sigma}^\mu\sigma^\nu - \bar{\sigma}^\nu\sigma^\mu\right)\bar{\psi}_\nu =& \tfrac{i}{2}m\,. \end{aligned}$$

The unimodular supergravity action is invariant under a restricted set of SUSY and diffeo transformations,

$$\delta\mathcal{E} = 0 \quad \text{where} \quad \delta\mathcal{E} = -\partial_M\left[(-1)^M\,\Xi^M\mathcal{E}\right].$$

This invariance preserves the conditions listed above.

Supergravity Transformation of Supergravity Multiplet

Supergravity multiplet: $\varphi_{sg} = (e_\mu^a, \psi_\mu^\alpha, b_\mu, M)$.

Infinitesimal supergravity transformation of φ_{sg}: $\boldsymbol{\delta}_{\boldsymbol{\xi}} \varphi_{sg} = \boldsymbol{\delta}_\xi \varphi_{sg} + \boldsymbol{\delta}_{(\epsilon,\bar{\epsilon})} \varphi_{sg}$

$$\begin{aligned}
\boldsymbol{\delta}_\xi e_\mu^a &= -\xi^\nu \partial_\nu e_\mu^a - (\partial_\mu \xi^\nu) e_\nu^a \,, \\
\boldsymbol{\delta}_\xi \psi_\mu^\alpha &= -\xi^\nu \partial_\nu \psi_\mu^\alpha - (\partial_\mu \xi^\nu) \psi_\nu^\alpha \,, \\
\boldsymbol{\delta}_\xi b_\mu &= -\xi^\nu \partial_\nu b_\mu - (\partial_\mu \xi^\nu) b_\mu \,, \\
\boldsymbol{\delta}_\xi M &= -\xi^\mu \partial_\mu M \,,
\end{aligned}$$

$$\begin{aligned}
\boldsymbol{\delta}_{(\epsilon,\bar{\epsilon})} e_\mu^a &= i \left(\psi_\mu \sigma^a \bar{\epsilon} - \epsilon \sigma^a \bar{\psi}_\mu \right) \,, \\
\boldsymbol{\delta}_{(\epsilon,\bar{\epsilon})} \psi_\mu^\alpha &= -2 \, \mathscr{D}_\mu \epsilon^\alpha + \tfrac{i}{3} M (\varepsilon \sigma_\mu \bar{\epsilon})^\alpha + i b_\mu \epsilon^\alpha + \tfrac{i}{3} b^\nu \left(\epsilon \, \sigma_\nu \bar{\sigma}_\mu \right)^\alpha \,, \\
\boldsymbol{\delta}_{(\epsilon,\bar{\epsilon})} M &= -\epsilon \left(\sigma^\mu \bar{\sigma}^\nu \psi_{\mu\nu} + i b^\mu \psi_\mu - i \sigma^\mu \bar{\psi}_\mu M \right) \,, \\
\boldsymbol{\delta}_{(\epsilon,\bar{\epsilon})} b_{\alpha\dot{\alpha}} &= \epsilon^\delta \left[\tfrac{3}{4} \bar{\psi}_\alpha{}^{\dot{\gamma}}{}_{\delta\dot{\gamma}\dot{\alpha}} + \tfrac{1}{4} \varepsilon_{\delta\alpha} \bar{\psi}^{\gamma\dot{\gamma}}{}_{\gamma\dot{\alpha}\dot{\gamma}} - \tfrac{i}{2} M^* \psi_{\alpha\dot{\alpha}\delta} + \tfrac{i}{4} \left(\bar{\psi}_{\alpha\dot{\rho}}{}^{\dot{\rho}} b_{\delta\dot{\alpha}} + \bar{\psi}_{\delta\dot{\rho}}{}^{\dot{\rho}} b_{\alpha\dot{\alpha}} - \bar{\psi}_\delta{}^{\dot{\rho}}{}_{\dot{\alpha}} b_{\alpha\dot{\rho}} \right) \right] \\
&\quad - \bar{\epsilon}^{\dot{\delta}} \left[\tfrac{3}{4} \psi^\gamma{}_{\dot{\delta}\gamma\dot{\alpha}\alpha} + \tfrac{1}{4} \varepsilon_{\dot{\delta}\dot{\alpha}} \psi_\alpha{}^{\dot{\gamma}\gamma}{}_{\dot{\gamma}\gamma} + \tfrac{i}{2} M \bar{\psi}_{\alpha\dot{\alpha}\dot{\delta}} - \tfrac{i}{4} \left(\psi_{\rho\dot{\alpha}}{}^\rho b_{\alpha\dot{\delta}} + \psi_{\rho\dot{\delta}}{}^\rho b_{\alpha\dot{\alpha}} - \psi^\rho{}_{\dot{\delta}\alpha} b_{\rho\dot{\alpha}} \right) \right].
\end{aligned}$$

Supergravity Transformation of Chiral Superfield

SUGRA transformation of chiral superfield

$$\text{Infinitesimal:} \quad \Lambda(Z) \to \tilde{\Lambda}(Z) = \Lambda(Z) + \boldsymbol{\delta_\xi}\Lambda(Z) = \Lambda(Z) - \Xi^M \partial_M \Lambda(Z)\,,$$

with $Z^M = (x^\mu, \Theta^\alpha)$.

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The infinitesimal passive transformation corresponding to the above active transformation, i.e.

$$Z^M \to \tilde{Z}^M = Z^M - \boldsymbol{\delta_\xi} Z^M = Z^M + \Xi^M,$$

is contractible.

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$$Z^M \to Z'^M = e^{-\boldsymbol{\delta_\xi}} Z^M.$$

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is contractible. Therefore its finite version can be obtained via exponentiation:

$$Z^M \to Z'^M = e^{-\delta_\xi} Z^M.$$

This finite passive transformation induces the following finite active transformation.

$$\text{Finite:} \quad \Lambda(Z) \to \Lambda'(Z) = e^{\delta_\xi} \Lambda(Z)\,.$$

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Supergravity Transformation of Chiral Superfield

SUGRA transformation of chiral superfield

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$$\text{Finite:}\quad \Lambda(Z) \to \Lambda'(Z) = e^{\boldsymbol{\delta_\xi}}\Lambda(Z)$$

with $Z^M = (x^\mu, \Theta^\alpha)$.

The composition of two finite transformations follows the Baker-Campbell-Hausdorff formula:

$$e^{\boldsymbol{\delta_{\xi_1}}} e^{\boldsymbol{\delta_{\xi_2}}} = e^{\boldsymbol{\delta_{\xi_1}}+\boldsymbol{\delta_{\xi_2}}+\frac{1}{2}[\boldsymbol{\delta_{\xi_1}},\boldsymbol{\delta_{\xi_2}}]+\frac{1}{12}[\boldsymbol{\delta_{\xi_1}}[\boldsymbol{\delta_{\xi_1}},\boldsymbol{\delta_{\xi_2}}]]-\frac{1}{12}[\boldsymbol{\delta_{\xi_2}}[\boldsymbol{\delta_{\xi_1}},\boldsymbol{\delta_{\xi_2}}]]+\ldots}$$

where the ... denote higher order commutators.

Supergravity Transformation of Chiral Superfield

Finite SUGRA transformation: $\Lambda'(Z) = e^{\delta_\xi}\Lambda(Z)$

$$[\delta_{\xi_1}, \delta_{\xi_2}] = \delta_{\xi_3(\xi_1,\xi_2)} = -\Xi_3^M \partial_M.$$

Supergravity Transformation of Chiral Superfield

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$$[\delta_{\xi_1}, \delta_{\xi_2}] = \delta_{\xi_3(\xi_1,\xi_2)} = -\Xi_3^M \partial_M .$$

$$\Xi_3^M = [\Xi_1, \Xi_2]_{S\mathcal{L}} + \delta_{\xi_1} \Xi_2^M - \delta_{\xi_2} \Xi_1^M \,,$$

$[\,,]_{S\mathcal{L}}$: superspace generalisation of the standard Lie bracket for vector fields.

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$[\,,]_{S\mathcal{L}}$: superspace generalisation of the standard Lie bracket for vector fields.

The additional terms are a consequence of the fact that, unlike in General Relativity, our transformation parameters now depend on supergravity fields.

The above commutator brackets are just the supergravity version of deformed brackets for field dependent parameters, which have appeared in [S. Nagy, J. Peraza & G. Pizzolo '24, M. Campiglia & J. Peraza '21, G. Barnich and C. Troessaert '10, etc.]

SUGRA Transformation of the Action

$$S = -\tfrac{6}{8\pi G_N} \int d^6 Z \left(\mathcal{E}\mathcal{R} + \frac{1}{6}\Lambda\,\mathcal{E} - \frac{1}{6}\,\mathcal{E}_0\,\Lambda \right) + h.c.$$

$$\Big\downarrow \text{SUGRA transformation}$$

$$S = -\tfrac{6}{8\pi G_N} \int d^6 Z \left(\mathcal{E}\mathcal{R} + \frac{1}{6}\Lambda\,\mathcal{E} - \frac{1}{6}\,\mathcal{E}_0\,e^{\delta_\xi}\Lambda \right) + h.c.$$

How to restore the broken supergravity invariance?

Restoring Broken Supergravity Invariance

Super-Stückelberg procedure comes to the rescue!

Applying the Stückleberg Procedure

Promote the SUGRA transformation parameters to Stückelberg fields:

Ernst Stückelberg

$$\boldsymbol{\xi}^{\bar{N}} = \begin{pmatrix} \xi^{\mu} \\ \epsilon^{\alpha} \\ \bar{\epsilon}^{\dot{\alpha}} \end{pmatrix} \quad \longrightarrow \quad \boldsymbol{\phi}^{\bar{N}} = \begin{pmatrix} \phi^{\mu} \\ \zeta^{\alpha} \\ \bar{\zeta}^{\dot{\alpha}} \end{pmatrix}$$

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$$\Xi^M \equiv \boldsymbol{\xi}^{\bar{N}} O_{\bar{N}}{}^M \longrightarrow \Phi^M \equiv O^M{}_{\bar{N}}\, \boldsymbol{\phi}^{\bar{N}}$$

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$$\phi^M = \Phi^M\big|_{\Theta=0} = O^M{}_{\bar{N}}\big|_{\Theta=0}\, \boldsymbol{\phi}^{\bar{N}} = \begin{pmatrix} \phi^\mu \\ \zeta^\alpha \end{pmatrix}$$

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$$\phi^{M} = \Phi^{M}\big|_{\Theta=0} = O^{M}{}_{\bar{N}}\big|_{\Theta=0}\, \boldsymbol{\phi}^{\bar{N}} = \begin{pmatrix} \phi^{\mu} \\ \zeta^{\alpha} \end{pmatrix}$$

In the end, we arrive at the final form of the Stückelberged action:

$$S = -\tfrac{6}{8\pi G_N} \int d^6 Z \left(\mathcal{E}\mathcal{R} + \frac{1}{6}\,\mathcal{E}\,\Lambda - \frac{1}{6}\,\mathcal{E}_0 e^{\delta_\Phi} \Lambda \right) + h.c.$$

SUGRA Transformation of the Stückelberg Fields?

New fields – New SUGRA transformations!

What are the SUGRA transformations of the Stückelberg fields?

$$\delta_{\xi}\phi^{M} = ?$$

Deriving $\delta_\xi \phi^M$

Supergravity invariance of the Stückelberged action requires that

$$e^{\delta'_\phi} \Lambda'(Z) = e^{\delta_\phi} \Lambda(Z).$$

Since $\Lambda'(Z) = e^{\boldsymbol{\delta_\xi}} \Lambda(Z)$, we get,

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Using the integral Baker–Campbell–Hausdorff formula, i.e., given $e^Z = e^X e^Y$,

$$Z = X + \left(\int_0^1 \mathcal{B}(e^{\mathrm{ad}_X} e^{t\mathrm{ad}_Y})\, dt \right) Y\,, \quad \text{where} \quad \mathcal{B}(x) = \frac{x \log(x)}{x-1},$$

we get,

$$\delta'_\phi = \delta_\phi - \left[\int_0^1 \mathcal{B}\left(e^{\mathrm{ad}_{\delta_\phi}} e^{-t\,\mathrm{ad}_{\delta_\xi}} \right) dt \right] \delta_\xi\,.$$

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Linearizing in ξ, we get,

$$\delta'_\phi = \delta_\phi - \mathcal{B}\left(e^{\mathrm{ad}_{\delta_\phi}} \right) \delta_\xi.$$

Deriving $\delta_\xi \phi^M$

We have $\delta'_\phi = \delta_\phi - \mathcal{B}\left(e^{\mathrm{ad}_{\delta_\phi}}\right)\delta_{\boldsymbol{\xi}}$. Using the fact that

$$\mathcal{B}\left(e^y\right) = \frac{y}{1-e^{-y}} = \sum_{k=0}^{\infty} \frac{B_k^+ y^k}{k!} ,$$

where B_k^+ are the Bernoulli numbers

$$B_0^+ = 1,\ \ B_1^+ = \frac{1}{2},\ \ B_2^+ = \frac{1}{6},\ \ B_3^+ = 0,\ \ B_4^+ = -\frac{1}{30},\ \ \ldots,$$

we arrive at the following expression:

$$\delta'_\phi = \delta_\phi - \sum_{k=0}^{\infty} \frac{B_k^+}{k!}\, \mathrm{ad}^k_{\delta_\phi} \delta_{\boldsymbol{\xi}} ,$$

where

$$\mathrm{ad}^k_{\delta_\phi} \delta_{\boldsymbol{\xi}} = [\delta_\phi, \ldots [\delta_\phi, [\delta_\phi, \delta_{\boldsymbol{\xi}} \underbrace{]] \ldots]}_{k \text{ times}} .$$

Deriving $\boldsymbol{\delta}_\xi \phi^M$

Derived expression: $\delta'_{\boldsymbol{\phi}} = \delta_{\boldsymbol{\phi}} - \sum_{k=0}^{\infty} \frac{B_k^+}{k!} \, \mathrm{ad}^k_{\delta_{\boldsymbol{\phi}}} \delta_{\boldsymbol{\xi}}$.

We know,

$$\begin{aligned} \delta_{\boldsymbol{\phi}} \xrightarrow{\text{SUGRA}} \delta'_{\boldsymbol{\phi}} &= \delta_{\boldsymbol{\phi}} + \boldsymbol{\delta}_{\boldsymbol{\xi}}(\delta_{\boldsymbol{\phi}}) \\ &= \delta_{\boldsymbol{\phi}} - \mathrm{ad}_{\delta_{\boldsymbol{\phi}}} \delta_{\boldsymbol{\xi}} - O^M{}_{\bar{N}} \, \boldsymbol{\delta}_{\boldsymbol{\xi}} \phi^{\bar{N}} \, \partial_M . \end{aligned}$$

Deriving $\delta_\xi \phi^M$

Derived expression: $\delta'_\phi = \delta_\phi - \sum_{k=0}^{\infty} \frac{B_k^+}{k!} \, \mathrm{ad}_{\delta_\phi}^k \delta_{\boldsymbol{\xi}}$.

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$$\delta_\phi \xrightarrow{\text{SUGRA}} \delta'_\phi = \delta_\phi + \boldsymbol{\delta_\xi}(\delta_\phi)$$
$$= \delta_\phi - \mathrm{ad}_{\delta_\phi} \delta_{\boldsymbol{\xi}} - O^M{}_{\bar{N}} \, \boldsymbol{\delta_\xi} \phi^{\bar{N}} \, \partial_M .$$

$\Rightarrow \boldsymbol{\delta_\xi} \phi^M$:

$$\boldsymbol{\delta_\xi} \phi^M \partial_M = (O^M{}_{\bar{N}} \, \delta_{\boldsymbol{\xi}} \phi^{\bar{N}} \, \partial_M)|_{\Theta=0} = \sum_{k=0}^{\infty} \frac{B_k^-}{k!} \, \mathrm{ad}_{\delta_\phi}^k \delta_{\boldsymbol{\xi}} \Big|_{\Theta=0}$$

where B_k^- differ from B_k^+ only for $k = 1$: $B_k^- = -1/2$ and $B_k^+ = 1/2$.

SUGRA Transformation of the Stückelberg Fields

$$\delta_\xi \phi^M = \sum_{k=0}^{\infty} \delta_\xi^{(k)} \phi^M = -\sum_{k=0}^{\infty} \frac{B_k^-}{k!} \Xi^{(k)M} \Big|_{\Theta=0}$$

where

$$\Xi^{(k)M} = \Xi^{(k-1)N} \partial_N \Phi^M - \Phi^N \partial_N \Xi^{(k-1)M} - \frac{\partial \Xi^{(k-1)M}}{\partial \varphi_{sg}} \delta_\xi \varphi_{sg} \, .$$

SUGRA Transformation of the Stückelberg Fields

$$\boldsymbol{\delta}_{\boldsymbol{\xi}}\phi^M = \sum_{k=0}^{\infty}\boldsymbol{\delta}_{\boldsymbol{\xi}}^{(k)}\phi^M = -\sum_{k=0}^{\infty}\frac{B_k^-}{k!}\Xi^{(k)M}\Big|_{\Theta=0}$$

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Using the recursive formula we get explicit expressions for the supergravity transformations of the Stückelberg fields, to all orders. For e.g.,

$$k=0: \quad \begin{pmatrix}\boldsymbol{\delta}^{(0)}\phi^\mu \\ \boldsymbol{\delta}^{(0)}\zeta^\alpha\end{pmatrix} = \begin{pmatrix}-\xi^\mu \\ -\epsilon^\alpha\end{pmatrix}$$

and

$$k=1: \quad \begin{pmatrix}\boldsymbol{\delta}^{(1)}\phi^\nu \\ \boldsymbol{\delta}^{(1)}\zeta^\alpha\end{pmatrix} = \begin{pmatrix}\frac{1}{2}\left(\phi^\mu\partial_\mu\xi^\nu - \xi^\mu\partial_\mu\phi^\nu\right) + i\left(\zeta\sigma^\mu\bar{\epsilon} - \epsilon\sigma^\mu\bar{\zeta}\right) \\ \frac{1}{2}\left(\phi^\mu\partial_\mu\epsilon^\alpha - \xi^\mu\partial_\mu\zeta^\alpha\right) + \frac{i}{2}\left(\epsilon\sigma^\mu\bar{\zeta} - \zeta\sigma^\mu\bar{\epsilon}\right)\psi_\mu^\alpha\end{pmatrix}.$$

Summary

To summarise, we have the full expression for invariant $\mathcal{N} = 1$ SUGRA action:

$$S = -\tfrac{6}{8\pi G_N} \int d^6 Z \left(\mathcal{E}\mathcal{R} + \frac{1}{6}\,\mathcal{E}\,\Lambda - \frac{1}{6}\,\mathcal{E}_0 e^{\delta_\phi}\Lambda \right) + h.c.$$

It admits a maximally symmetric solution, with the cosmological constant given by

$$\Lambda_{\text{eff}} = \boldsymbol{\Lambda}_2 - \frac{1}{3} m^2 \, .$$

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We have taken a significant step forward by constructing the super-Stückelberg action to all orders, providing the complete $\mathcal{N}=1$ supergravity action.

This represents a substantial advance in the development of a complete and consistent framework for de Sitter vacua in supergravity.

Absence of Pathological Terms

Possible extra term [Volkov, Soroka '73, '74] that didn't show up:

$$\sqrt{\tilde{e}}\,\tilde{g}^{\mu\nu}\tilde{\psi}_\mu\tilde{\psi}_\nu\,,$$

where the invariant vielbein and gravitino combinations are

$$\begin{aligned}\tilde{e}^a_\mu &= e^a_\mu + \tilde{\mathcal{D}}_\mu X^a + 2i\theta\sigma^a\bar{\psi}_\mu - 2i\psi_\mu\sigma^a\bar{\theta} + i\theta\sigma^a\tilde{\mathcal{D}}_\mu\bar{\theta} - i\tilde{\mathcal{D}}_\mu\theta\sigma^a\bar{\theta}\,,\\ \tilde{\psi}_\mu &= \psi_\mu + \tilde{\mathcal{D}}_\mu\theta\,.\end{aligned}$$

Ghost field?

Discussion

- If the full pure de Sitter actions obtained via different methods match up to all orders, it indicates the existence of an underlying comprehensive theory at higher energies, such as a string-theory model.
- The existence of such a string-theory model was corroborated in [I Bandos, M Heller, S Kuzenko, L Martucci, D Sorokin '16] where special cases of the pure dS SUGRA coupled to matter, were shown to match certain parts of the 4D effective action for an anti-D3-brane coming from the flux compactifications of 10D type IIB SUGRA.
- Find the $10D$ supergravity and string theory counterparts of this $4D$ pure dS supergravity, in search of dS vacua.
- It would also be worthwhile to use the pure dS SUGRA action for constructing phenomenological actions in inflationary cosmology.

Thank you!