Simulating multilayered inelastic pavements by a dynamic ALE formulation

A. Anantheswar, I. Wollny & M. Kaliske
Institute for Structural Analysis, Technische Universität Dresden, Dresden, Saxony, Germany

ABSTRACT: Pavements are typically long structures, and simulating their transient response when subjected to moving vehicle loads, using conventional techniques can be quite arduous. These techniques would necessitate the discretization of the complete pavement that is along the path of the moving load. Thus, depending on the velocity of the vehicle, considerably large meshes would need to be employed and solved. In recent developments, a dynamic ALE formulation has been put forth to improve efficiency by only considering the relevant region of the pavement around the load. The work at hand further highlights the capabilities of the dynamic ALE formulation by applying the dynamic ALE formulation to multilayered inelastic pavements.

1 INTRODUCTION

Pavements are part of the critical infrastructure of a country, and fulfill the important task of providing connectivity as a medium for transport of people and goods. The suitable design of pavements is therefore crucial. In order to facilitate the design of such structures, techniques like the Finite Element Method (FEM) in a Lagrangian setting are typically used. When simulating the response of the pavement subjected to a moving load, if these traditional techniques are used, the associated domain of the pavement that needs to be discretized is quite large, and therefore computationally inefficient. Additionally, restrictions are imposed on the mesh discretization when the vehicle load is accelerating or decelerating. Further, a moving load formulation is needed to simulate the movement of the vehicle on the pavement. The speed of the vehicle also imposes restrictions on the time steps of the simulation. The recent development proposed by (Anantheswar et al. 2024a) utilizes the Arbitrary Lagrangian Eulerian (ALE) simulation strategy to improve computational efficiency.

Traditionally, ALE simulation techniques are used in the field of fluid mechanics (Benson 1989, Venkatasubban 1995, Souli et al. 2000, Codina et al. 2009, Basting et al. 2017). Another typical use-case of ALE strategy is as a mesh adaptation technique, to improve the mesh quality when extreme distortions are encountered (Liu et al. 1986, Rodríguez-Ferran et al. 1998, Bayoumi & Gadala 2004, Donea et al. 2004, Nazem et al. 2009, Berger & Kaliske 2022). The pioneering work of (Nackenhorst 2004) described the use of the ALE methodology to improve efficiency in analyses of rolling tire structures. Following this, (Wollny & Kaliske 2013, Wollny et al. 2016) implemented the ALE formulation for pavements considering constant velocity load movement. Recent developments by (Anantheswar et al. 2024a) extend the ALE formulation for pavements to the dynamic case, and further to inelastic materials (Anantheswar et al. 2024b). This contribution highlights applying the dynamic ALE formulation to multilayered inelastic pavements.

2 THE MOVING ALE REFERENCE FRAME

The central theme involved in adopting the dynamic ALE formulation for pavements is a change in the reference frame of the observer. Instead of a stationary reference frame as in the conventional Lagrangian formulation, the reference frame moves with the vehicle load in the ALE formulation. An observer in this moving reference frame would perceive the load as stationary, while the material of the pavement would appear to flow beneath the load. The main advantage offered by this perspective is that only a relevant portion of the pavement in the vicinity of the moving vehicle load would need to be considered in the analyses. This is shown to significantly improve computational efficiency (Anantheswar et al. 2024a). A kinematic description of the various configurations involved when adopting the ALE formulation for pavements is shown in Figure 1.

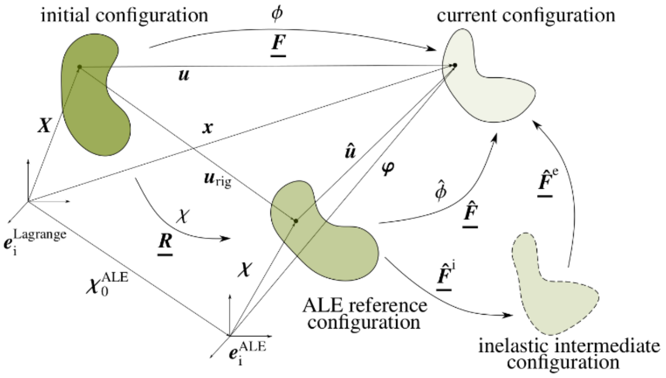

Figure 1. Kinematics of the moving ALE reference frame.

It should be noted that in Figure 1, for pavements, there is no rigid body displacements (translations or rotations) and so $\boldsymbol{u}_{\text{rig}} = \boldsymbol{0}$. This implies that the initial configuration and the ALE reference configuration are the same. But, over time, the ALE reference frame $\boldsymbol{e}_{\text{i}}^{\text{ALE}}$ travels with the same velocity as the vehicle load. Thus, a new portion of the pavement material in the vicinity of the load would be considered at each time step. The velocity with which the material of the pavement appears to flow through the mesh (in a Finite Element framework) is termed 'guiding velocity' (Nackenhorst 2004), given by

$$\boldsymbol{w} = \left.\frac{\partial \boldsymbol{\chi}}{\partial t}\right|_{\boldsymbol{X}} = \left.\frac{\partial\left(\boldsymbol{X} + \boldsymbol{u}_{\text{rig}} - \boldsymbol{\chi}_0^{\text{ALE}}\right)}{\partial t}\right|_{\boldsymbol{X}} = -\left.\frac{\partial \boldsymbol{\chi}_0^{\text{ALE}}}{\partial t}\right|_{\boldsymbol{X}} \quad , \tag{1}$$

where the positions and displacements of a material point in various configurations depicted in Figure 1, are related by

$$\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{u} = \boldsymbol{X} + \boldsymbol{u}_{\text{rig}} + \hat{\boldsymbol{u}} = \boldsymbol{\chi}_0^{\text{ALE}} + \boldsymbol{\chi} + \hat{\boldsymbol{u}} \quad . \tag{2}$$

Further, adhering to the balance of linear momentum, the weak formulation in the ALE reference configuration can be expressed as

$$\int_{\chi(\mathcal{B})} \hat{\rho}\dot{\boldsymbol{v}} \cdot \boldsymbol{\eta} d\hat{V} + \int_{\chi(\mathcal{B})} \hat{\boldsymbol{P}} : \text{Grad}\boldsymbol{\eta} d\hat{V} = \int_{\chi(\mathcal{B})} \hat{\rho}\boldsymbol{b} \cdot \boldsymbol{\eta} d\hat{V} + \int_{\partial\chi(\mathcal{B})} \hat{\boldsymbol{T}} \cdot \boldsymbol{\eta} d\hat{A} \quad , \tag{3}$$

where $\hat{\rho}$ is the density, $\dot{\boldsymbol{v}}$ denotes the acceleration, $\hat{\boldsymbol{P}}$ refers to the second Piola-Kirchhoff stress, $\boldsymbol{b}$ refers to body forces, $\hat{\boldsymbol{T}}$ denotes surface traction, $\boldsymbol{\eta}$ refers to an arbitrary test function, and $d\hat{V}$ and $d\hat{A}$ are infinitesimal volume and area elements, respectively, on domain $\chi(\mathcal{B})$ in the ALE reference configuration. In Equation 3, the terms on the left-hand side describe the internal forces developed in response to the externally applied forces, which are on the right-hand side. Of particular interest is the first term in Equation 3, which refers to the inertial forces. This term depends on the acceleration, which is defined as the material time derivative of the velocity field. One characteristic of the ALE approach is that whenever a material time derivative of a quantity f is encountered, advection effects need to be considered. For pavements, where the advection velocity is known, this is described as

$$\dot{f} = \left.\frac{\partial f}{\partial t}\right|_{\boldsymbol{X}} = \left.\frac{\partial f}{\partial t}\right|_{\boldsymbol{\chi}} + \text{Grad}f \cdot \boldsymbol{w} \quad . \tag{4}$$

Thus, according to Equation 4, the velocity and acceleration fields need to be advected through the mesh. This results in additional terms that need to be considered during the linearization and implementation into a finite element framework (Anantheswar et al. 2024a). Additionally, since the displacement field at any given time step depends on the displacement field at the previous time step, a suitable update of the displacement field also needs to be carried out.

Moreover, when inelastic material models are used, evolution of inelastic effects (like viscosity, plasticity etc.) are typically expressed as rate equations of certain internal variables $\boldsymbol{\alpha}$. Therefore, advection of such internal variables also needs to be accounted for, using

$$\dot{\boldsymbol{\alpha}} = \left.\frac{\partial \boldsymbol{\alpha}}{\partial t}\right|_{\boldsymbol{X}} = \left.\frac{\partial \boldsymbol{\alpha}}{\partial t}\right|_{\boldsymbol{\chi}} + \text{Grad}\boldsymbol{\alpha} \cdot \boldsymbol{w} \quad . \tag{5}$$

The advection procedure for internal variables is as per the Gauss point sub-mesh interpolation technique (DGPA) from the work of (Anantheswar et al. 2024b). This technique utilizes an operator split to first solve for internal variables in the Lagrangian phase. Then, interpolation and update of the internal variables in a sub-mesh of Gauss points takes place during the Eulerian phase. This procedure is performed after every iteration in the global Newton-Raphson solution scheme.

When multiple inelastic materials are used (as in the case of pavements, see Figure 2), advection of the internal variables of each layer should be treated separately. For the DGPA scheme, this means that each material layer gets its own sub-mesh of Gauss points, where the interpolation and update procedure is carried out. This ensures that the advection procedure does not transfer internal variables of one material to another. The extension of the DGPA scheme to allow for multiple materials is the novelty of the work at hand. Figure 2 illustrates the use of separate Gauss point sub-meshes for advection of internal variables in each layer of the pavement, for a simple mesh in two dimensions.

3 NUMERICAL STUDY

In this study, a four layered asphalt pavement with similar structure as in Figure 2 is analyzed. The non-linear viscoelastic material described in (Anantheswar et al. 2024b) with one viscous branch is used to model each of the layers.

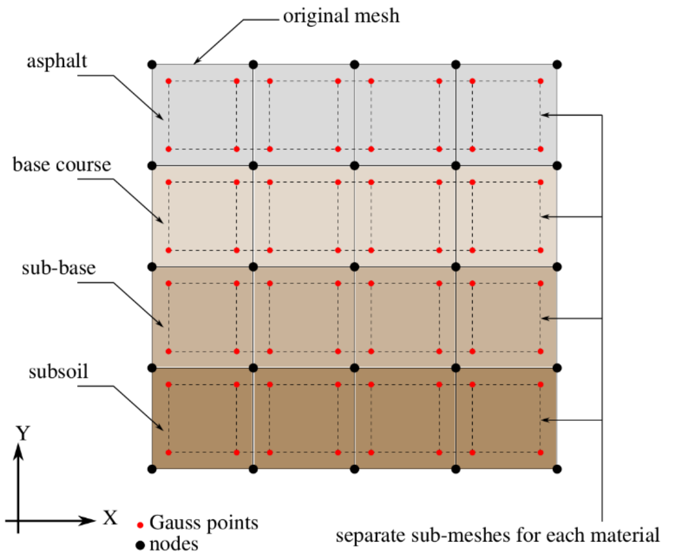

Figure 2. Separate Gauss point sub-meshes for each material, to accurately perform advection of internal variables.

The material parameters of the layers are listed in Table 1. The mesh and loading are depicted in Figure 3. The geometry of the specimen is a cuboid 8 m x 8 m x 3 m along x-, y- and z-directions, respectively. The boundary conditions are such that all surfaces except the top surface are restrained from translation in the direction normal to the surface. In the analysis, the transient response of the pavement when subjected to a moving truck tire load is simulated. The load is initially applied as a ramp over a period of 1 s. Then, it is maintained at this constant amplitude for the rest of the simulation. A guiding velocity is applied such that the load appears to accelerate from 0 m/s to 16.667 m/s (60 km/h) starting at 2 s, over a period of 2 s. Then, this velocity is maintained until 10 s. The guiding velocity is, then, ramped down such that the load appears to decelerate to 0 m/s over a period of 1.2 s. It is then maintained at 0 m/s for the rest of the simulation, ending at total time of 12.5 s. The Newmark time integration scheme (Newmark 1959) with a time step of 0.1 s and linear eight node brick type finite elements are used in the simulation.

Table 1. Material properties used in the simulation.

Layer	Elastic branch			Viscous branch	
	$\hat{\rho}$ kg/m^3	κ* MPa	μ* MPa	μ_v* MPa	η_v* Ns/m^2
Asphalt	2.3E+3	822.50	200.00	179.62	1.0E+8
Base course	2.2E+3	175.83	40.00	31.15	1.0E+8
Sub-base	2.0E+3	105.50	24.00	18.69	1.0E+8
Subsoil	1.9E+3	60.83	14.00	14.08	1.0E+8

* κ: Bulk modulus, μ: Shear modulus, μ_v: Shear modulus of viscous branch, η_v: Viscous parameter.

The thickness of the subsoil layer is 2 m, the sub-base course is 0.4 m, and both the base course and the asphalt layer on top have thicknesses of 0.3 m each.

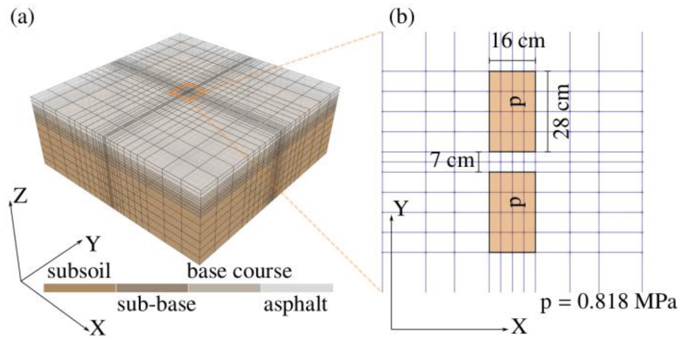

Figure 3. (a) Perspective view of the mesh and (b) top view (close-up) of the loaded area in the centre of the mesh.

The mesh used has 10192 finite elements, and the simulation took approximately 4.12 hours to run on a desktop computer with an Intel Core i5 10400 processor and 32 GB of RAM. The obtained results in terms of the displacements at the centre of load between the two tires are shown in Figure 4. Contours of the strain component e_{zz} are depicted in Figure 5, at various time points in the simulation.

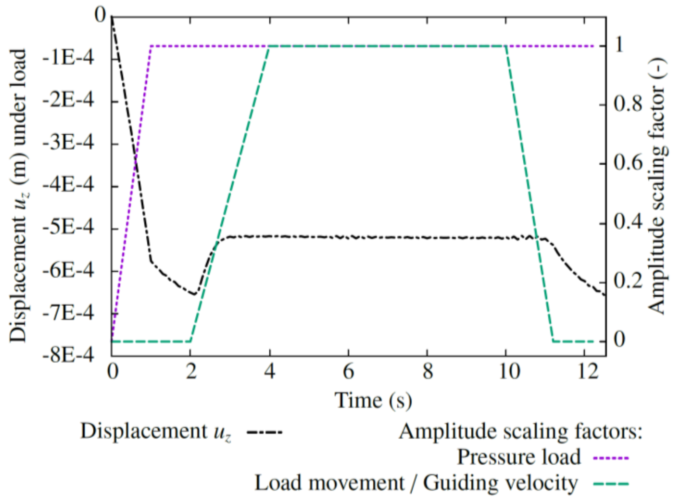

Figure 4. Displacement component u_z of the central node under the load plotted against time.

The results shown in Figures 4, 5 clearly demonstrate the capability of the dynamic ALE framework to simulate the transient response of inelastic multilayered pavement structures in a computationally efficient manner. It is worth mentioning, that if a corresponding simulation was run using conventional techniques, it would necessitate the discretization of a domain of length 154.67 m. With the ALE approach, this length is reduced to just 8 m, making the simulation possible on a desktop computer, even without parallelization. Further, the conventional techniques would necessitate the implementation of a cumbersome moving load formulation. This imposes further restrictions on the time step size and discretization used in the conventional simulation, as a sufficiently fine mesh would be required to ensure that the load is applied on the nodes in synchronicity with its movement as well. The need to discretize a large domain, along with restrictions imposed on time step and fineness of the mesh, are overcome using the dynamic ALE formulation. This formulation offers a substantial improvement to computational performance, when simulating the transient response of inelastic multilayered pavement structures.

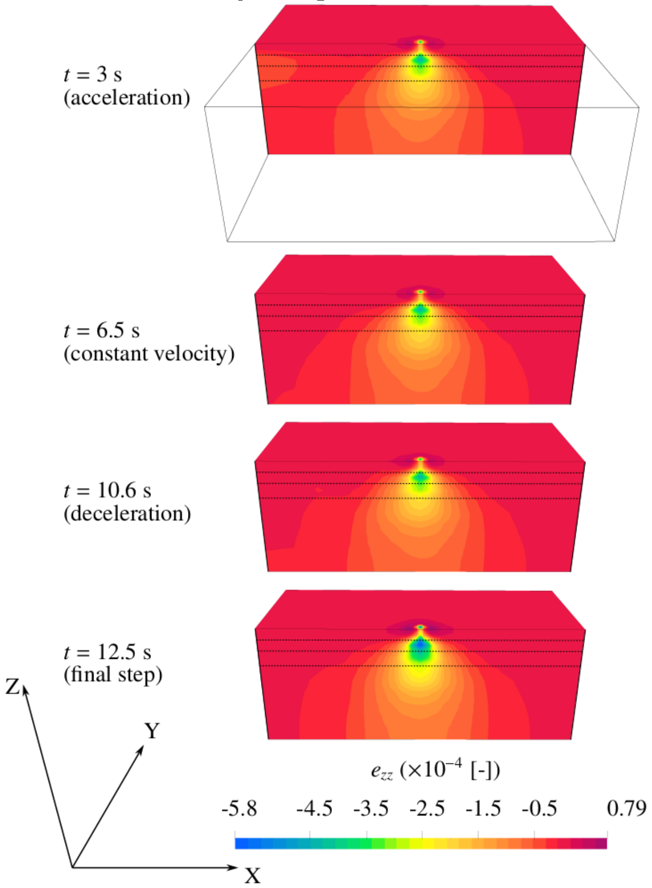

Figure 5. Contour plots of strain component e_{zz} at various stages in the simulation.

Future research towards improving the performance of the ALE formulation could be in the direction of utilizing model order reduction techniques or through parallelization. This is relevant in application cases such as in digital twins, which require fast and efficient calculations. These tools would indubitably lead to improved and informed decisions by engineers and policy makers alike.

4 ACKNOWLEDGEMENTS

This contribution has been developed under the research project No. 453596084 (SFB/TRR 339, Subproject A01), which has been granted by the German Research Foundation (Deutsche Forschungsgemeinschaft). This financial support is gratefully acknowledged.

5 REFERENCES

Anantheswar, A., Wollny, I., & Kaliske, M. 2024a. A dynamic ALE formulation for structures under moving loads. *Computational Mechanics*, 73:139–157.

Anantheswar, A., Wollny, I., & Kaliske, M. 2024b. Treatment of Inelastic Material Models Within a Dynamic ALE Formulation for Structures Subjected to Moving Loads. *International Journal for Numerical Methods in Engineering*, e7599.

Basting, S., Quaini, A., Čanić, S., & Glowinski, R. 2017. Extended ALE method for fluid-structure interaction problems with large structural displacements. *Journal of Computational Physics*, 331:312–336.

Bayoumi, H. & Gadala, M. 2004. A complete finite element treatment for the fully coupled implicit ALE formulation. *Computational Mechanics*, 33:435–452.

Benson, D. 1989. An efficient, accurate, simple ALE method for nonlinear finite element programs. *Computer Methods in Applied Mechanics and Engineering*, 72:305–350.

Berger, T. & Kaliske, M. 2022. An arbitrary Lagrangian Eulerian formulation for tire production simulation. *Finite Elements in Analysis and Design*, 204:103742.

Codina, R., Houzeaux, G., Coppola-Owen, H., & Baiges, J. 2009. The fixed-mesh ALE approach for the numerical approximation of flows in moving domains. *Journal of Computational Physics*, 228:1591–1611.

Donea, J., Huerta, A., Ponthot, J., & Rodríguez-Ferran, A. 2004. Arbitrary Lagrangian-Eulerian Methods, Chapter 14. Wiley Online Library, Encyclopedia of Computational Mechanics.

Liu, W., Belytschko, T., & Chang, H. 1986. An arbitrary Lagrangian-Eulerian finite element method for path-dependent materials. *Computer Methods in Applied Mechanics and Engineering*, 58:227–245.

Nackenhorst, U. 2004. The ALE-formulation of bodies in rolling contact: Theoretical foundations and finite element approach. *Computer Methods in Applied Mechanics and Engineering*, 193:4299–4322.

Nazem, M., Carter, J., & Airey, D. 2009. Arbitrary Lagrangian–Eulerian method for dynamic analysis of geotechnical problems. *Computers and Geotechnics*, 36:549–557.

Newmark, N. 1959. A method of computation for structural dynamics. *Journal of the Engineering Mechanics Division*, 85:67–94.

Rodríguez-Ferran, A., Casadei, F., & Huerta, A. 1998. ALE stress update for transient and quasistatic processes. *International Journal for Numerical Methods in Engineering*, 43:241–262.

Souli, M., Ouahsine, A., & Lewin, L. 2000. ALE formulation for fluid–structure interaction problems. *Computer Methods in Applied Mechanics and Engineering*, 190:659–675.

Venkatasubban, C. 1995. A new finite element formulation for ALE (arbitrary Lagrangian Eulerian) compressible fluid mechanics. *International Journal of Engineering Science*, 33:1743–1762.

Wollny, I. & Kaliske, M. 2013. Numerical simulation of pavement structures with inelastic material behaviour under rolling tyres based on an arbitrary Lagrangian Eulerian (ALE) formulation. *Road Materials and Pavement Design*, 14:71–89.

Wollny, I., Hartung, F., & Kaliske, M. 2016. Numerical modeling of inelastic structures at loading of steady state rolling. *Computational Mechanics*, 57:867–886.