Towards real-time structural simulations for the digital twin of the road

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ABSTRACT: In the context of digital twins, fast simulations are essential to obtain meaningful information about a system in near real-time. To achieve this goal in the context of a digital twin of a road system, a combination of the Arbitrary Lagrangian Eulerian (ALE) method and proper orthogonal decomposition (POD)-based model order reduction is proposed. In a numerical example, the combination of the two methods is shown to reduce the computation time of a simulation significantly, introducing only a small approximation error. Furthermore, it is demonstrated that the framework can predict unseen structural behavior without a vast amount of data.

1 INTRODUCTION

Fast, efficient simulations are crucial in technologies such as digital shadows or twins. These allow realistic predictions that are also accurate, therefore enabling governing authorities to take quick, meaningful and impactful decisions based on sound and structured logic. Particularly for pavement structures subject to moving wheel loads, the Arbitrary Lagrangian Eulerian methodology has been proven to be far more efficient than conventional simulation techniques (Wollny et al. 2016, Anantheswar et al. 2024). However, this methodology is still incapable of real-time simulations, and a further speedup is necessary. In this work, the application of Model Order Reduction (MOR) techniques to ALE simulations of the pavement structure is explored, to reduce the computational effort even further.

2 ALE FORMULATION

The concept that is at the core of ALE simulations of pavements is the adoption of a moving reference frame, see Figure 1.

This moving reference frame conveniently shows the same velocity as the applied wheel load. Thus, to an observer in this moving reference frame, the load would appear stationary, and the material of the pavement would appear to flow under the load. The main advantage of this ALE technique is that only the relevant region of the pavement in the immediate vicinity of the load needs to be discretized and simulated. This is in contrast to conventional simulation techniques, where the entire structure in the path of the wheel load would need to be discretized and analyzed. For a detailed description of the implementation of the ALE formulation, the reader may refer to the work of (Anantheswar et al. 2024).

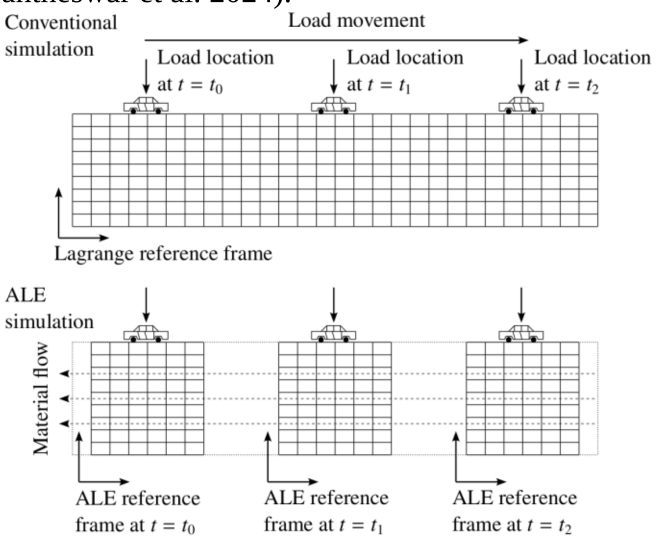

Figure 1. Moving reference frame in ALE simulations.

One interesting aspect of the ALE formulation is that it does not change the overall structure of the global system of equations in a nonlinear finite element framework. This means that the system can still be assembled into the well-known system of equations that has to be solved in every iteration of the solution scheme

$$\underline{\boldsymbol{M}}\,\Delta\ddot{\boldsymbol{u}} + \underline{\boldsymbol{D}}\,\Delta\dot{\boldsymbol{u}} + \underline{\boldsymbol{K}}_T\,\Delta\boldsymbol{u} = \boldsymbol{g}, \tag{1}$$

where $\underline{\boldsymbol{M}}$, $\underline{\boldsymbol{D}}$ and $\underline{\boldsymbol{K}}_T$ refer to the global mass, damping and tangential stiffness matrices, respectively, and $\Delta\ddot{\boldsymbol{u}}$, $\Delta\dot{\boldsymbol{u}}$, $\Delta\boldsymbol{u}$ and $\boldsymbol{g}$ denote the assembled incremental nodal vectors of acceleration, velocity, displacement and the residual vector, respectively. The Newmark-beta method is used for the time-integration, which results in the system of equations

$$\underline{\boldsymbol{K}}_{T,dyn}\,\Delta\boldsymbol{u} = \boldsymbol{g}, \tag{2}$$

with the $(n \times n)$-dimensional dynamic tangential stiffness matrix $\underline{\boldsymbol{K}}_{T,dyn}$. With the global system of equations in this standard structure, it is possible to apply MOR techniques in a relatively simple and straightforward manner.

3 POD-BASED MOR

In this contribution, the proper orthogonal decomposition (POD) method is applied to the problem at hand according to (Radermacher & Reese 2013) and (Kehls et al. 2023). The POD is a projection-based technique for reduced order modeling. In the following, the POD will be shortly explained for the present case. For a more detailed description of POD and projection-based MOR in general, the reader is kindly referred to the work of (Benner et al. 2015) and (Schilders 2008). For the problem described in Equation (2), it is assumed that an $(n \times m)$-dimensional projection matrix $\underline{\boldsymbol{\Phi}}$ can be found, such that the Galerkin projection of Equation (2) leads to the reduced system of equations

$$\underline{\boldsymbol{\Phi}}^T\underline{\boldsymbol{K}}_{T,dyn}\underline{\boldsymbol{\Phi}}\,\Delta\boldsymbol{u}_{red} = \underline{\boldsymbol{\Phi}}^T\boldsymbol{g}. \tag{3}$$

After the solution has been computed in the reduced subspace, the (m)-dimensional reduced solution vector $\Delta\boldsymbol{u}_{red}$ can be projected to the (n)-dimensional full solution space by the relation $\Delta\boldsymbol{U} = \underline{\boldsymbol{\Phi}}\,\Delta\boldsymbol{u}_{red}$. If $m << n$, solving the system of equations in the reduced subspace is much faster than solving it in the full solution space, leading to a significant reduction in computation time.

To construct the projection matrix $\underline{\boldsymbol{\Phi}}$, solution vectors $\boldsymbol{u}_i$ are sampled in precomputations and collected in the so-called snapshot matrix $\underline{\boldsymbol{D}} = [\boldsymbol{u}_1\ \boldsymbol{u}_2 \ldots \boldsymbol{u}_l]$. The sampled solution vectors can be e.g. time-series solutions from a precomputation but also solutions from simulations with varying material parameters or in the case of ALE varying material flow velocities. A singular value decomposition is then applied to the snapshot matrix such that $\underline{\boldsymbol{D}} = \underline{\boldsymbol{V}}\,\underline{\boldsymbol{\Sigma}}\,\underline{\boldsymbol{W}}^T$. The matrices $\underline{\boldsymbol{V}}$ and $\underline{\boldsymbol{W}}^T$ contain the left and right singular vectors and the matrix $\underline{\boldsymbol{\Sigma}}$ contains the decreasing singular values on its diagonal. Each singular value Σ_{ii} corresponds to a singular vector $\boldsymbol{v}_i$ and indicates its importance for the reconstruction of the snapshot matrix $\underline{\boldsymbol{D}}$. The number of POD modes $\boldsymbol{v}_i$ that leads to a good approximation of sampled snapshots can, therefore, be derived from the decay of the singular values. At last, the projection matrix is truncated at the specified index m such that the projection matrix is defined as $\underline{\boldsymbol{\Phi}} = [\boldsymbol{v}_1\ \boldsymbol{v}_2 \ldots \boldsymbol{v}_m]$.

4 NUMERICAL EXAMPLE

To test the proposed methodology, both methods presented above are implemented into the finite element research software *FEAP* (Taylor 2014) and a numerical example is computed and analyzed. The example is based on (Anantheswar et al. 2024) and uses the same material described therein. The boundary value problem is depicted in Figure 2. The structure is fixed on all sides and on the bottom and the distributed load p is applied in the center of the upper surface. The ALE material guiding velocity is denoted by w. The structure is simulated for t = 12.5 s with time increments of Δt = 0.1 s. At the beginning of the simulation, the load is increased linearly until the desired value P = 200 MPa is reached at t = 1 s and held constant thereafter. The guiding velocity starts increasing linearly after t = 2 s and reaches its maximum value at t = 4 s. It is then constant until t = 10 s, after which it linearly decreases until the material stops 'flowing' through the mesh at t = 11.2 s. Due to the time discretization, one precomputation yields 125 solution states that are used to construct the snapshot matrix and, therefore, the projection matrix. For the first investigation, only one precomputation is conducted where the material guiding velocity is chosen as w = 25 m/s. The influence of the number of POD modes m is then investigated by running the simulation with the same guiding velocity but an increasing number of modes.

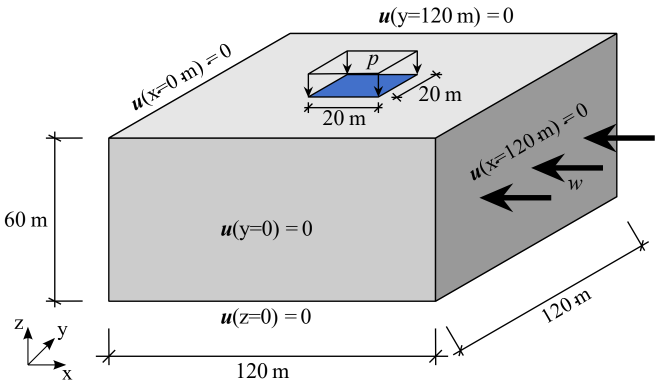

Figure 2. Geometry and boundary conditions of the example.

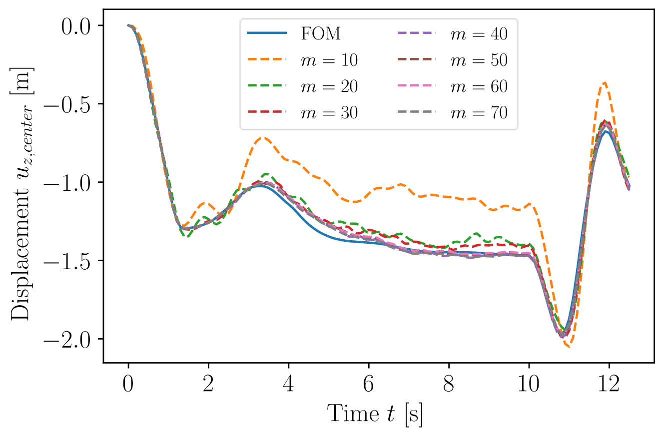

Figure 3. Comparison of the displacement $u_{z,center}$ over time t for reduced order models with an increasing number of modes m.

In Figure 3, the results of the investigation are shown by plotting the displacement in z-direction in the center of the upper surface of the structure over the time t. It can be seen that with m = 10 POD modes, the displacements are very far from the reference solution of the full order model (FOM). Increasing the number of modes, the curves get closer to the reference solution. Using m = 40 or more modes in the ROMs, the curves show good agreement with the reference. To illustrate this relation, the average error as well as the simulation time ratio of the ROMs over the whole simulation are shown in Figure 4. The error is defined as

$$\epsilon = \frac{1}{n_t}\sum_{i=1}^{n_t} \frac{u^i_{z,center,prec} - u^i_{z,center,POD}}{u^i_{z,center,prec}} \qquad (3)$$

where $u_{z,center,prec}$ and $u_{z,center,POD}$ describe the displacement in the center of the upper surface of the full order precomputation and the POD-reduced simulation, respectively. n_t describes the number of total timesteps. The simulation time ratio is calculated as

$$\tau = \frac{T_{POD}}{T_{FOM}}, \qquad (3)$$

where T denotes the CPU time a FOM or a POD-reduced simulation took. It can be seen that with m = 40 and more POD modes the error is about 1 % and approximately 80 % of simulation time is saved.

Until now, the ROM only reconstructed the results that are contained in the snapshots. To investigate the predictive qualities of the proposed approach, the same snapshot matrix as before is used, but the guiding velocity is changed to w = 20 m/s and w = 30 m/s, respectively. Figure 5 shows the results of the reduced order simulation (m = 60) as well as the results of a FOM with the same guiding velocities to test the accuracy of the ROMs. It can be seen that even though the ROMs are used to predict unseen cases, the qualitative behavior of the structure is captured nicely. Especially the minima and maxima approximated by the ROMs are well aligned with the corresponding reference result. Looking at the overall agreement of the displacement field in z-direction in

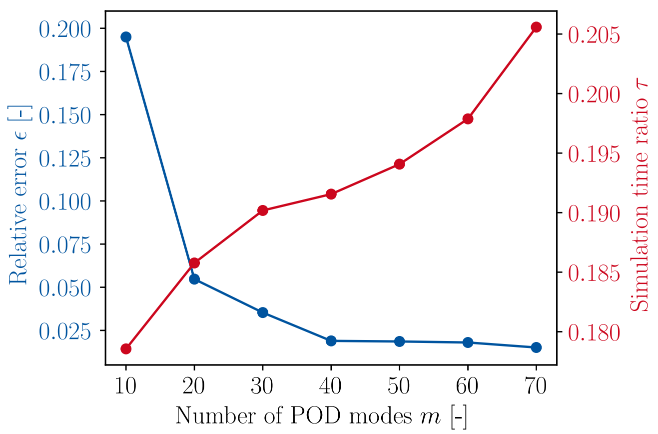

Figure 4. Relative error and simulation time ratio of the reduced order models.

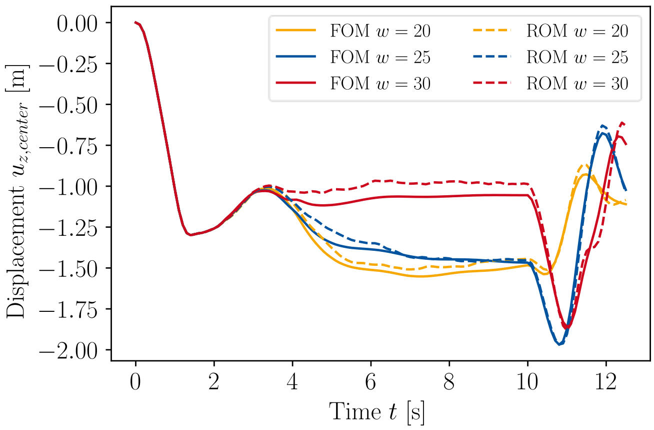

Figure 5. Reduced order simulation for different guiding velocities. All ROMs are created only with the snapshots from the full order ALE simulation with guiding velocity w = 25 m/s and m = 60 POD modes are used.

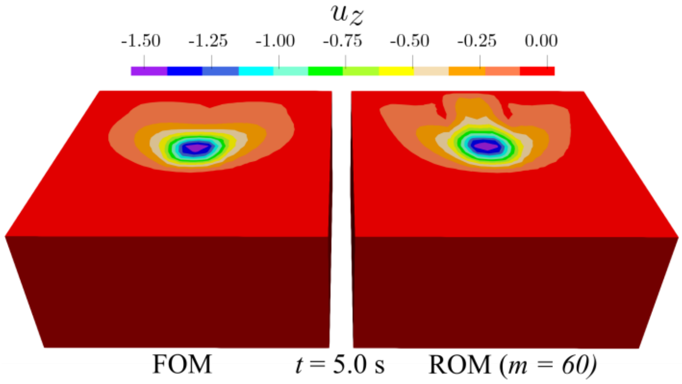

Figure 6. Comparison of the displacement field u_z of the full order model and reduced order model with a guiding velocity of w = 20 m/s and m = 60 POD modes at time t = 5.0 s.

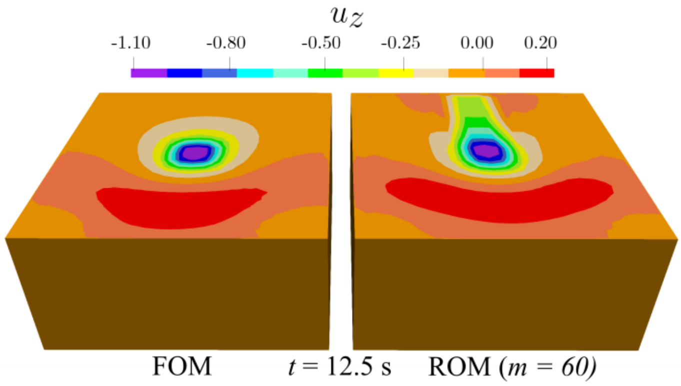

Figure 7. Comparison of the displacement field u_z of the full order model and reduced order model with a guiding velocity of w = 20 m/s and m = 60 POD modes at time t = 12.5 s.

Figures 6 and 7, it is seen that the displacements in the center of the structure are approximated with high accuracy, while the displacements behind the region, where the load is applied, show some inaccuracies.

Lastly, it is investigated whether the accuracy of the ROM can be increased by using more snapshots. Therefore, the snapshots of the full order simulations with guiding velocities w = 20 m/s and w = 30 m/s are used to construct the snapshot matrix and a reduced order simulation with guiding velocity w = 25 m/s is carried out to check the accuracy. The results of this study are shown in Figure 8, and it is seen that the ROM based on the two simulations has better agreement with the reference solution than the ROM based on the single simulation that it is reconstructing. It should be noted that both ROMs used the same number of POD modes ($m = 60$).

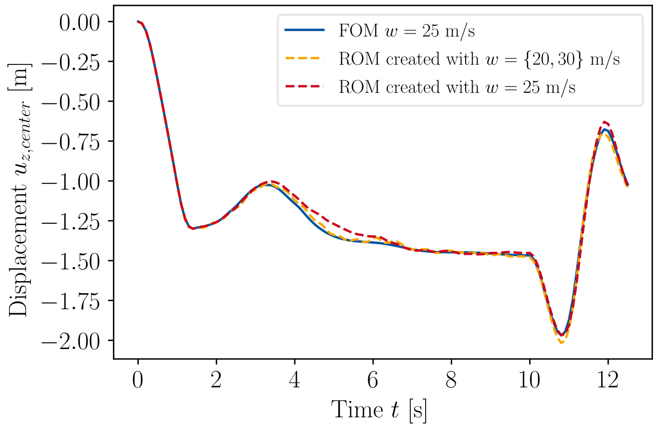

Figure 8. Comparison of two ROMs created from different snapshot matrices for a guiding velocity of $w = 25$ m/s. One ROM is created by taking the snapshots from the full order simulation with $w = 25$ m/s, whereas the other one is created by taking the snapshots from full order simulations with $w = 20$ m/s and $w = 30$ m/s.

5 CONCLUSION

In this work, an approach to accelerate simulations of road structures is presented. To this end, POD is applied to a problem which has been defined in the ALE framework. It was investigated whether this methodology can reduce the simulation time, while maintaining high accuracy. The results of a numerical example show that the structural response of the full order simulation can be approximated by the ROM with minimal error of about 1 % whilst saving about 80 % of simulation time. It is also shown that the ROM can be used to predict unseen behavior although this shows slightly higher errors. Lastly it is shown that the snapshot creation process plays an important role in creating a performant ROM. In future works it will be investigated whether the approach can be developed further to obtain higher accuracy. For example, a more sophisticated snapshot sampling approach could already improve the ROM significantly. It might also be worthwhile to use clustering approaches on a structural level to approximate the structural behavior in certain regions better. Another important aspect is the extension to hyper-reduction, where the number of element evaluations is reduced, saving more computation time. Here, one major challenge will be the hyper-reduction of simulations including inelastic materials, as the inelastic evolution of the material is commonly modeled by internal history variables. When the material is ‘flowing’ through the mesh, so do the internal variables corresponding to an integration point. If only some elements are then evaluated, the material history is lost or incorrect when it ‘flows’ though an element that is not evaluated.

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